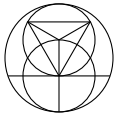
Two Theories, One World



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Gravity, the Quantum and the Mistake Behind the Search for One Theory

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The Argument in Brief

The Question

Physics rests on two theories, and each has passed every test set for it within its domain. General relativity is the theory of gravity: it says that gravity is the shape of space and time. Quantum theory is the theory of matter and light: it says that what a measurement finds comes in separate, countable steps. For more than ninety years physicists have tried to make the two theories one, and the method they have chosen is the method that worked for every other force: to quantize gravity, so that the geometry of space and time becomes a quantum system in its own right.

This book asks a question that comes before that search. Is the arena of space and time the kind of thing that can be quantized at all? The answer it gives is that the arena is the context of every measurement, and that a context cannot take the character of the content found in it.

The Answer in a Page

Every measurement is a comparison: the act of finding whether two things are the same or different, in some respect. Every comparison finds some content in some context. The content is what the comparison finds; the context is the respect in which the two things compared are two, such as two places or two moments.

A comparison, perfected at its foundation, can have only one of two characters. It can be continuously asymmetric: smooth, and different from place to place. Or it can be discontinuously symmetric: exactly repeating, and coming in countable steps. The four characters, continuity, discontinuity, symmetry and asymmetry, can be paired in ten ways. These two hold together, and the

theorem names two of the others as forbidden: continuity with symmetry, and asymmetry with discontinuity. This is the Incommensuration Theorem of *An Immanent Metaphysics*, and it rests on nothing but the structure of comparison itself.

The two theories of physics fall on the two sides of the theorem. I identify quantum theory as the description of the content of physics, whose findings are discontinuously symmetric, and general relativity as the description of its context, the arena, which is continuously asymmetric. Gravity carries four marks that any arena must carry: it couples to everything; nothing shields it; the mass it pulls on is the mass that resists being moved; and it has no charge of its own that comes in steps. So gravity is the shape of the arena, and not one more force acting within it.

Every theory has two levels, its mathematics and its findings, and the two always have opposite characters. To quantize a field is to change, for both levels of its theory together, which character each level holds exact. Light can be quantized, because light is content, found within a context that the change leaves in place. The arena cannot be quantized. A quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. A direction here is a direction of space or the direction of time. The change would also remove the setting in which every description is made. To quantize the arena is therefore to apply a concept where the concept does not belong. That is a category error, and it is the error behind the search for one theory.

The claim is exact. The arena has no least length, area or duration, and nothing is a quantum of the arena itself, at any scale. Gravitation is not quantized. Small waves of the arena can be described by quanta, and the word "graviton" names at most a quantum of that description, never a quantum of the arena. The phonon, a quantum of the description of the vibrations of a crystal, is the nearest sense by which the word can be made intelligible. A crystal is a held structure, whose atoms the bonds between them hold in place; the arena is not a held structure, so even that sense does not make the graviton a quantum of the arena. A detector that registers a gravitational wave in single clicks would not count against the account.

What remains when the error is removed is definite. Each alternative of a quantum mass carries its own smooth geometry. Two masses related only through gravity should become entangled, with a phase of about a third of a radian for the design proposed in 2017. Later designs use other masses, separations and times, and the same formula gives their phases. The Planck scale is where the two descriptions meet, and the arena has no least step there. The measurement problem is what a smooth mathematics leaves over when its findings come in steps, and each interpretation of quantum theory is a choice about which kind of time to set aside. Each program of quantum gravity found something real, and each met the difficulty that its first step predicts. The correspondence between gravity in a region and a quantum theory on its boundary translates between two descriptions without merging them. A black hole hides, while it stands, the information that falls into it, and destroys none.

The two theories are two descriptions of one world, each capturing what the other leaves out. They are joined side by side, from the one act of comparison, and the Incommensuration Theorem forbids fusing them into a single description.

How the Book Is Set Out

The book has seven parts, and each builds on the one before.

Part I, "The Search", sets out the two theories, the history of the attempts to make them one, and what a category error is, with the whole argument in seven sentences.

Part II, "The Ground", starts from nothing. It shows that every measurement is a comparison with six parts, sets out the Incommensuration Theorem with its ten candidate pairings and two survivors, and quotes the three axioms.

Part III, "The Diagnosis", draws the line between content and context, shows why the arena bends and why gravity carries the four marks, checks the two levels of a theory against six theories, and states the category error in full.

Part IV, "What Is Left When the Error Is Removed", gives one geometry for each alternative of the matter, the experiment that will test it, what the word "graviton" can name, and the meaning of the Planck scale.

Part V, "The Measurement Problem", finds three kinds of time in physics, shows why a measurement has one outcome, and reads the interpretations of quantum theory, and the theorems of Bell and his successors, in their light.

Part VI, "The Search Revisited", puts four questions to ten programs of quantum gravity, credits each with what it found, and sets out the unity the theorem permits: two descriptions joined, and never merged.

Part VII, "Black Holes, Objections and Tests", follows the information that falls into a black hole, answers the strongest objections, and gives each prediction with its evidence and the experiments that will decide it.

The book makes three kinds of claim, and it keeps them apart. Claims about the structure of comparison hold wherever comparison occurs, and they have no physical premise. Claims of established physics are taken as experiment has confirmed them. The steps that join the two are identifications, and I make each one in the first person, beside the observation that would test it.

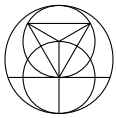
Each chapter opens with a question a reader brings, answers it, shows why the answer holds, and closes with its key idea. At the back of the book, "The Numbers" gives every number the book prints, with its source or its calculation; the notes tie each step of the ground to *An Immanent Metaphysics* by aphorism, and "The Aphorisms Cited" prints each of those aphorisms in full; and a glossary holds every term the book defines.

The Technical Form

This book gives the whole argument in prose, and nothing in it waits on another book. Its companion, *The Unquantized Arena; A Structural Diagnosis of Quantum Gravity and the Measurement Problem*, sets out the technical form of every step: the ground as numbered premises, the argument as propositions, the equations, the checks against six theories, the literature, and every objec-

tion a competent physicist raises, at full strength. Where a reader may want that form, the notes name its chapter beside a plain statement of what is there.

The ground that Part II sets out is given at length in three books: *Comparison; The Necessary Basis*, on comparison and its parts; *Incommensuration; The Theorem and the Three Axioms*, which proves the theorem step by step and derives the three axioms from it; and *Measurement; The Steps from Comparison to Physics*, which takes the steps from comparison to physics. Every step this book takes is made in it, and it stands complete without them; where a reader wants the longer reasons, the notes name the chapter of each beside a plain statement of what is there.



PART I

THE SEARCH



Chapter 1

Two Theories That Never Fail

Two Theories

Physics rests on two theories, and each has passed every test that has been set for it within its domain. General relativity is the theory of gravity. Quantum theory is the theory of matter and light. Between them they account for a range of measurements that runs from the colors of atoms to the ripples sent out when two black holes merge.

The two theories are also strikingly different. Relativity describes a world that is smooth, in which a small step in space or time brings only a small change. Quantum theory describes a world in which what is found comes in

separate, countable steps. For about a century physicists have expected that one of the two must give way, or that both must give way to a single deeper theory.

This book gives a different answer. The two theories are two descriptions of one world, and each captures what the other leaves out. Relativity describes the setting in which every measurement is made. Quantum theory describes what a measurement finds in that setting. To make them one by giving the setting the character of what is found in it is to apply a concept where the concept does not belong. Philosophers call that a category error, and Chapter 3 sets out exactly what the term means.

Two words need a fixed meaning before we begin. Physicists speak of an observer, and of what an observation finds. In this book an observer, or comparer, is anything that makes a comparison: a person, an instrument, or an atom that absorbs a particle of light. A finding is what one act of comparison yields: one outcome selected from among its possibilities. Neither word requires a mind, and neither ever means one here. Chapter 4 shows why every measurement is a comparison.

The Theory of the Arena

General relativity says that gravity is the shape of space and time. Einstein completed the theory in 1915. Before him, gravity was a force that one mass exerted on another across empty space. In his theory there is no such force. Matter and energy shape the geometry of space and time around them, and a body left to itself follows the straightest path that the geometry allows. The Earth circles the Sun because the geometry near the Sun bends the Earth's path into an orbit.

I call space and time together the arena. The arena is the setting in which everything that is measured is measured: every length is a length in it, and every duration is a duration in it. Relativity is the theory of the arena's shape, and that shape is what we feel as gravity.

Two features of the arena matter for everything that follows. The arena is smooth: in relativity, a small step in position or in time brings a small change in the geometry, never a jump. And the arena differs from place to place: the geometry near the Sun is not the geometry near the Earth, and in general no two regions of it need be exactly alike.

The confirmations began with the theory itself. In 1915 Einstein computed that the orbit of Mercury should turn slowly around the Sun by 43 seconds of arc per century more than Newton's gravity and the pull of the other planets allow, and astronomers had already measured that excess. In 1919 Dyson, Eddington and Davidson photographed stars beside the Sun during a total eclipse, and found their light bent by the Sun's gravity by about the amount the theory predicts. In 1960 Pound and Rebka sent light down a tower at Harvard and measured the small shift in its frequency that relativity predicts for light falling through a height.

You use relativity whenever your phone finds where you are. The satellites of the Global Positioning System carry atomic clocks. In the weaker gravity of their orbits those clocks run faster than clocks on the ground, and their speed makes them run a little slower. The net gain is about 38 millionths of a second each day. Light travels more than eleven kilometers in that time, so the system corrects every clock for relativity; without the correction, its positions would drift by more than eleven kilometers in a single day.

The theory has since been tested where gravity is strongest. On 14 September 2015 the two detectors of LIGO, the American observatory for gravitational waves, in Louisiana and in Washington State, registered a ripple of the arena itself. It came from two black holes, of about 36 and 29 times the mass of the Sun, merging about 1.3 billion light years away. The ripple stretched and squeezed each detector by about one part in a thousand billion billion, and its shape matched the shape relativity computes for such a merger. In merging, the pair turned about three Suns' worth of mass into the energy of the waves. In 2019 the Event Horizon Telescope published an image of the dark shadow at the center of the galaxy M87. It is cast by a black hole of about six and a half billion times the mass of the Sun, and it is the shadow relativity predicts.

The Theory of What Is Found

Quantum theory says that what a measurement finds comes in separate steps. Light of a given color is absorbed and emitted in definite packets of energy, and each packet is called a quantum. An atom can hold only certain energies, so the light it gives out comes in sharp lines of color rather than a smooth rainbow. Every electron has exactly the same charge and exactly the same mass as every other electron. Exact values that repeat without variation, and steps that can be counted: that is the quantum character.

Before a measurement, quantum theory describes a system by a smooth mathematical wave, which spreads, overlaps itself and sets the odds of each possible outcome. At a measurement, one outcome is found. Send electrons one at a time toward a screen through two narrow slits. Each electron arrives at one place, as a single dot. The pattern that many dots build up is the pattern of a wave passing through both slits. Until it is found, the electron is in what physicists call a superposition: a combination of alternatives, here the alternatives of passing through one slit or the other.

Quantum theory makes some of the most precise predictions in science. An electron is a tiny magnet. In 2023 Fan, Myers, Sukra and Gabrielse measured the strength of that magnet to about a tenth of a part in a trillion. Quantum electrodynamics, the quantum theory of electrons and light, computes the same quantity, and the computed value agrees with the measured one to about one part in a trillion. Every laser, every transistor and every chemical bond is an application of quantum theory.

One Experiment, Both Theories

The two theories work together every day, without strain. In 2010 Chou, Hume, Rosenband and Wineland compared two clocks, each ticking with the light absorbed by a single aluminum ion, and raised one of them by 33 centimeters. The raised clock ran faster by about four parts in a hundred million billion, the amount relativity gives for that change of height.

Look at what each theory did in that experiment. Quantum theory supplied the ticking: the ion absorbs light only at one exact frequency, the same for every such ion. Relativity supplied the difference between the two rates: the arena is shaped a little differently 33 centimeters higher. What was found was quantum. Where it was found was the arena. That division of labor is the subject of this book.

Where the Two Seem to Collide

The difficulty appears only where both theories must speak about the same thing. Three places are usually named.

The first is a mass in a superposition. Quantum theory allows a small mass to be in a combination of two places. In relativity, the arena is shaped by where the mass is. So where, for such a mass, is its gravity? Chapter 12 answers that question, and Chapter 13 describes the experiment now being built to test the answer.

The second is a black hole. Relativity describes its horizon, the surface from inside which nothing escapes. In 1974 Stephen Hawking showed that quantum fields near the horizon make every black hole glow faintly, so that a black hole slowly gives up its mass. That raises the question of what becomes of the information carried by what fell in. Chapter 25 answers it.

The third is a scale far below anything yet measured. Every object has a gravitational size, the size at which its own gravity would become strong, and a quantum size, the size over which quantum theory spreads it. For an object of about twenty-two millionths of a gram, the two sizes are equal. There both theories claim the same object. Chapter 15 shows what happens at that scale.

A reader who knows astronomy will think here of three further questions: the cosmological constant, dark matter and dark energy. They are questions of cosmology, the science of the universe as a whole, and they lie outside this book. Its argument needs none of them, and it makes no claim about any of them.

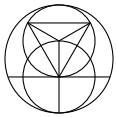
The usual response has been to extend one theory over the other. Every other force of nature was brought into quantum theory, so gravity, it seemed, must be brought in as well. The program of quantum gravity is the search for a single theory in which the arena itself is quantized, that is, given the quantum character of the matter and light within it. Chapter 2 tells the history of that search and credits what it has achieved.

The Answer This Book Gives

Every measurement finds something, and it finds it in a setting that makes one finding distinct from another: here rather than there, now rather than then. I call what is found the content of a measurement, and the setting in which it is found its context; Chapter 4 makes both words exact, and Chapter 7 applies them to physics. Chapter 3 states the answer of this book whole, in seven sentences, and the chapters after it establish each sentence in turn. Among the consequences is a prediction for an experiment being built: two small masses related only through gravity should become entangled, their alternatives linked so that neither can be described alone.

> **Key idea.** General relativity and quantum theory are the two best-confirmed theories in science, from clocks in orbit and the ripples of merging black holes to the magnetism of a single electron. Relativity describes the arena, the setting of every measurement; quantum theory describes what a measurement finds there. This book argues that they are two descriptions of one world, each capturing what the other leaves out, and that the search for one theory in which the arena is quantized is a category error.

Why, then, have physicists spent more than ninety years trying to make the two theories one?



Chapter 2

Ninety Years of Trying

Why have physicists tried to make relativity and quantum theory one? Because one method brought every other force of nature into quantum theory, and gravity looked like the next case for the same method. This chapter tells the history of that attempt, names the programs it produced and what each achieved, and identifies the one assumption that all of them share.

The Method That Worked

Quantum theory began as a theory of particles: electrons, atoms, packets of light. From the late 1920s it became a theory of fields as well. A field is a quantity spread through space and time, with a value at every place and moment, as the electric and magnetic field is. To quantize a field is to give it the quantum character: its waves are then found in countable packets, and its exact symmetries fix which packets can be found. The packets of the electromagnetic field are photons.

The method had to overcome one obstacle. Its first calculations gave infinite answers for quantities that are plainly finite, such as the mass and charge of the electron. In the late 1940s Tomonaga, Schwinger, Feynman and Dyson showed how to absorb those infinities into a few quantities that are measured once and then held fixed. A theory whose infinities can be absorbed in that way, into finitely many measured quantities, is called renormalizable. The result, quantum electrodynamics, gives the electron's magnetism to the precision described in Chapter 1.

The same method then took in the two other forces of particle physics. In the 1960s and 1970s the weak force, which drives some radioactive decays, and the strong force, which binds the nucleus, were written as quantized fields of the same kind, and both theories are renormalizable. Together with quantum electrodynamics they make up what physicists call the Standard Model, and every one of its predicted particles has been found. Three forces, one method, every test passed. Gravity was counted as the fourth force, and the method seemed ready for it.

The First Attempts

The first technical attempt came quickly. In 1930 Léon Rosenfeld applied the new method for quantizing fields to the small waves of Einstein's geometry, treated as ripples on a fixed, flat background. In 1936 Matvei Bronstein carried the same calculation further, and he saw its limit clearly. Gravity acts on its own energy, so its equations are not those of small ripples alone. A full treatment would have to quantize the geometry itself, with no fixed background beneath it, and Bronstein suggested that this might mean giving up the geometry Einstein used.

The quantum of that description of the small waves was given a name, the graviton. In the description of small waves, the graviton is a packet with no mass, as the photon is, and with twice the photon's spin. Spin is the built-in rotation that a particle carries. That description has stood ever since, and this book keeps it only as a description: the graviton names at most a quantum of a description, never a quantum of the arena and never a particle that carries gravity, as Chapter 14 explains. The question that occupied the next nine decades was a different one: what a full quantum theory of the geometry itself would be.

Two Roads

Two roads were tried, and both were traveled with great skill.

The first road treats gravity as small waves on a fixed background and quantizes the waves, as the other forces were quantized. Through the 1960s Feynman, DeWitt and others worked out its rules. In 1974 ’t Hooft and Veltman computed its first corrections for gravity with matter, and found infinities that cannot be absorbed in the usual way. In 1986 Goroff and Sagnotti computed the corrections at the next order for gravity alone, and found the same. Gravity, quantized by the method that worked for the other forces, is not renormalizable.

The consequence is easy to state. Each further order of calculation needs a new quantity measured by experiment before it can predict anything, so the full theory would need infinitely many such quantities. At low energies the extra quantities hardly matter, and predictions can still be made. At very high energies they all matter together, and the direct method stops predicting.

That road led to a real achievement all the same. In 1994 John Donoghue showed how to treat the description of small waves of gravity by quanta as an effective theory: a description that holds at energies far below a stated limit, whose quantum corrections can be computed there. In that form, as he wrote, gravity is a well-behaved quantum field theory at ordinary energies. The corrections it predicts are far too small to measure, but they can be computed, and they are finite.

The second road writes relativity as a space whose shape changes in time, and quantizes the shape itself. In 1967 Bryce DeWitt published the equation this road leads to, now called the Wheeler–DeWitt equation. In it the time in which a quantum state usually evolves has disappeared. Physicists call that the problem of time. The road clarified, more sharply than anything before it, what a quantum state of the whole geometry would have to be.

The Programs

From these two roads grew the programs of quantum gravity that are pursued today. Each is a body of work by many researchers, and each has achieved something real.

String theory replaces point-like particles by tiny vibrating strings. In 1974 Scherk and Schwarz found that one vibration of a string behaves as a graviton, and proposed that strings describe gravity. In 1984 Green and Schwarz showed that a class of string theories is free of a defect, called an anomaly, that would make them inconsistent, and string theory became a candidate theory of all the forces. In 1996 Strominger and Vafa counted the internal states of a class of black holes in string theory and recovered the entropy that Bekenstein and Hawking had assigned to them from the area of their horizons. In 1997 Juan Maldacena proposed, in a paper published in 1998, that a theory with gravity, in a space of a particular shape, is equivalent to a quantum theory without gravity on that space's boundary, and a large body of later work supports the proposal. Chapter 23 returns to that correspondence.

Loop quantum gravity follows the second road. In 1986 Abhay Ashtekar rewrote Einstein's theory in new variables that made a quantization with no fixed background possible. In 1995 Rovelli and Smolin computed, within that quantization, quantities representing area and volume, and found that the theory gives them only separate, discrete values. The program's achievement is a quantization that never places the geometry on a background.

Asymptotic safety asks whether the infinities of the first road might be tamed after all. In 1979 Steven Weinberg proposed that gravity might approach a fixed point at high energies, where its strength stops growing and the theory stays predictive. In 1998 Martin Reuter gave the program its working tools, and calculations since then have found evidence for such a fixed point in simplified versions of the theory.

Other programs make the arena discrete from the start. In 1987 Bombelli, Lee, Meyer and Sorkin proposed that space and time are a causal set: a vast collection of separate events, ordered only by which can influence which. In 2004 Ambjørn, Jurkiewicz and Loll built geometries from small identical pieces, summed over all the ways of fitting them together while keeping the order of cause and effect, and found an extended four-dimensional world emerging from the sum. Still others are pursued: the noncommutative geometry of Alain Connes, the twistor theory of Roger Penrose, and newer work

that derives the interactions of gravitons directly from their symmetries. Chapter 22 returns to ten programs, each with its achievement first and then the difficulty its starting point leads one to expect.

The Assumption They Share

The programs differ in almost everything: in their mathematics, in their pictures of space and time, in what they take as given. They share one assumption. Each takes gravity to be quantized as the other forces were, so that the geometry of the arena becomes a quantum system in its own right, with quantum states and quantum steps of its own. I call this the shared assumption.

Physicists give three reasons for it, and each deserves a fair statement.

The first is uniformity. Every other force yielded to quantization, and a world with one set of rules seems to call for one kind of theory. Electromagnetism shows why the argument persuades. The classical theory of light explains a great deal, but some effects, such as an excited atom giving out light into empty space with no light falling on it, are accounted for in full only by the quantized field. Physicists expected gravity to hold effects of the same kind.

The second is consistency. In 1957, at a conference on gravitation at Chapel Hill in North Carolina, Richard Feynman imagined a small ball, a centimeter across, put into a superposition of two places by a quantum event, and asked what its gravitational field would then be. If quantum theory holds for the ball, he argued, the field too must be in a superposition, so gravity must be quantized; and he urged that an experiment be designed in which a gravitational link shows quantum interference. The experiment of Chapter 13 is an experiment of that kind. In 1977 Eppley and Hannah argued that a gravitational field that is not quantized, acting on matter that is, would either fail to conserve momentum, or let a measurement beat the uncertainty principle, or send signals faster than light. They concluded that gravity must be quantized. Chapter 12 shows that the account of this book, in which each alternative of the matter carries its own geometry, sends no signal faster than light and meets the other two horns as well.

The third is the meeting of scales. Planck's constant sets the size of every quantum step: the energy of one packet of light is Planck's constant multiplied by the light's frequency. Newton's constant of gravity, Planck's constant and the speed of light combine into a single length, the Planck length. It is about 1.6×10^{-35} meters: a decimal point, thirty-four zeros, and then the digits 16. The standard view expects the quantum character of gravity to show itself there, and inside black holes, where relativity's geometry becomes extremely curved. This book reads the Planck length differently, as the place where the two descriptions meet, where an object's gravitational size equals its quantum size. Chapter 15 sets that reading out, and on it the arena has no least step there or anywhere else.

The assumption has had careful dissenters. In 1995 Ted Jacobson derived Einstein's equation from the thermodynamics of horizons, and suggested that the equation describes the arena as an equation of state describes a gas, so that it may not be the right thing to quantize. In 1996 Roger Penrose proposed that gravity limits how long a mass can stay in a superposition of two places. In 2023 Jonathan Oppenheim published a theory in which a classical arena acts on quantum matter consistently, at the price of a random element in the arena's motion. Each of them takes the classical character of the arena seriously. For the experiments now being built, Penrose's account and Oppenheim's predict that two masses related only through gravity will not become entangled; the account of this book predicts that they will.

What the History Shows

More than ninety years of work have produced a great deal. They have given physics the effective theory that describes small waves by quanta and a counting of black-hole states. They have given it a correspondence between gravity in a region and a quantum theory on its boundary, quantizations with no fixed background, and mathematics that has enriched other fields. Each of these is an achievement, and each is credited in this book where it arises.

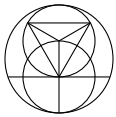
No program has yet produced a tested prediction of a quantum step of the arena itself, and no experiment has found one. I read that history as what the diagnosis of this book leads one to expect. The diagnosis does not rest on the history. It rests on the structure of comparison, which Part II sets out, and on the two theories as they stand. The history is the case the diagnosis explains.

The difficulties the programs meet also fall into a pattern. The direct method meets infinities it cannot absorb. The canonical road loses the time in which a quantum state evolves. The programs that build the arena from discrete pieces must then recover a smooth arena from them. I notice that each difficulty appears at the step where a program asks the arena itself to carry a quantum character. Chapter 21 states four questions that bring that pattern into the open, and Chapter 22 applies them to each program in turn.

The shared assumption is natural. It extended the most successful method in physics to the one force that had not yet received it. This book argues that the assumption asks a question of the wrong kind of thing: it asks the setting of every measurement to take the character of what is measured in it.

> **Key idea.** Quantizing fields brought three forces of nature into one confirmed theory, and from 1930 physicists set out to quantize gravity in the same way. The direct method meets infinities it cannot absorb; the programs that followed, from string theory and loop quantum gravity to asymptotic safety and causal sets, have each achieved something real. All of them share one assumption: that the arena itself is to be quantized as the other forces were.

What does it mean to ask a question of the wrong kind of thing?



Chapter 3

What a Category Error Is

What does it mean to ask a question of the wrong kind of thing? It means applying a concept where the concept does not belong. A category is a kind of thing, together with the concepts that apply to things of that kind. A category error is a question that applies a concept where the concept does not belong. This chapter shows what such an error looks like in ordinary life, then states the whole argument of this book in seven sentences.

The Visitor Who Asked for the University

The philosopher Gilbert Ryle gave the best-known example in 1949, in the first chapter of his book *The Concept of Mind*. A visitor to Oxford is shown the colleges, the libraries, the laboratories and the offices. At the end of the tour the visitor asks: "But where is the University?"

The visitor has made no mistake of fact. Every building is where the guide said it was. The mistake is one of kind. The University is the way the colleges, libraries and laboratories are organized and work together; it is not one more building beside them. To look for it as a building is to apply the concept of a building to something of another kind. The question has good grammar and a reasonable motive, and it has no answer of the kind it asks for.

Ryle used the example to diagnose a confusion about the mind. The form of the mistake is general, and it can be made in any subject, including physics.

Ordinary Examples

Category errors are easy to recognize once the categories are in view.

What color is the number seven? Numbers have order, size and factors, and color is not among the concepts that apply to them. How much does the offside rule weigh? A rule of football can be stated, kept or broken, and it has no weight. Which note of a song is its melody? The melody is the order of the notes in time, so no single note can be it.

Physics has examples of its own. What is the temperature of a single molecule? A molecule has a speed and an energy. Temperature describes how energy is shared among very many molecules, so a single molecule has no temperature in the ordinary sense of the word. The question is not hard; it applies to one molecule a concept that belongs to many.

The history of physics holds a larger case. In the nineteenth century, light was known to be a wave, and every wave then known was the motion of some material medium, as sound is a motion of the air. So physicists asked what light was a motion of, and they named the medium the ether. They built ingenious models of it, and in 1887 Michelson and Morley set out to measure the Earth's motion through it. They found none. In 1905 Einstein showed that the electromagnetic field needs no medium at all: it is a thing of its own kind, not the motion of a material. The question about the ether was not answered. It was dissolved, and Maxwell's equations for light, the real finding behind it, stood unchanged.

Four features mark every case. The question sounds sensible. The effort to answer it can be sincere, and it can be very clever. Every answer of the kind asked for fails, and each failure teaches something, but no single failure tells the inquirer to stop. And when the categories are seen, the question is dissolved rather than answered, and nothing real is lost: the colleges still stand, the molecule still has its energy, and light still obeys Maxwell's equations.

The people who ask such questions are often the most capable in their field. The ether was sought by the best physicists of their century, with the best instruments of their time. A category error comes naturally to capable

people, because it extends a concept that has worked very well into the one place where it does not apply. That is the shape of the error this book describes.

What Is Found and Where It Is Found

This book is about two categories that every measurement contains: what the measurement finds, and the setting in which it finds it. Chapter 1 called the first the content of a measurement and the second its context.

A piano shows the difference. Its notes come in steps: one key, then the next, with nothing between them. The time in which the notes are played does not come in keys. A pianist can hold a note for any length of time, and can begin the next note at any moment. To ask which key the time is would apply to the setting of the music a concept that belongs to the notes.

Physics has the same two categories. Its content is what measurements find: particles, charges, energies, the light an atom gives out. Its context is the arena of space and time, in which every one of those findings is made. Quantum theory describes the content, and its findings come in steps. Relativity describes the context, and its arena is smooth. To ask that the arena come in the steps that the content comes in is to ask of the setting a question that belongs to what is found in it. That is the category error this book diagnoses.

The piano is an illustration, and the argument does not rest on it. The argument rests on what every comparison contains and what it allows, which Part II sets out from the beginning. Here is the whole of it, in advance.

The Argument in Seven Sentences

1. Every measurement is a comparison, and a comparison can be perfected at its foundation in only two ways: as smooth and different from place to place, or as exactly repeating and coming in countable steps.

2. The other two ways of joining these characters, smooth with exactly repeating, and stepwise with nothing repeating, cannot be perfected in any comparison.
3. General relativity has the first character, and quantum theory has the second.
4. What a measurement finds is its content, and the setting that makes two findings two is its context; matter and light are the content of physics, and the arena of space and time, whose shape is gravity, is its context.
5. To quantize gravity as the other forces were quantized is to ask the arena either to keep its smoothness and gain the exact sameness of a quantum field, or to keep its unevenness and gain a quantum field's steps, with both perfected in one description.
6. So a quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both.
7. The search for a quantized arena therefore applies to the context of physics a treatment that belongs to its content, and that is a category error.

A direction, in the sixth sentence, is a direction of space or the direction of time. Each sentence is a step of the book. The first two are the Incommensuration Theorem, a result about comparison itself, which Chapter 5 sets out in full. The third is shown in Chapter 5 and checked against six theories in Chapter 10. The fourth is the work of Chapters 7 to 9, which show why gravity is the arena and not a force within it. The last three are drawn together in Chapter 11, which shows what quantizing does and why the arena cannot take it.

The claim is exact. The arena has no least step: no smallest length, area or duration of its own. Nothing is a quantum of the arena itself, at any scale. Small waves of the arena can still be described by quanta; those quanta are quanta of the description, never quanta of the arena, and Chapter 14 shows why.

What Remains When the Error Is Removed

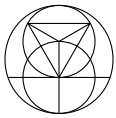
When the question of the wrong kind is set aside, the two theories stand exactly as experiment has confirmed them. Nothing in either has to be given up.

Gravity still acts on quantum matter, and quantum matter still shapes the arena. Chapter 12 shows that when a mass stands in a superposition, each of its alternatives carries its own smooth geometry. Chapter 13 describes the experiment, now being built, in which two small masses related only through gravity should become entangled. Chapter 24 shows the kind of unity that remains: two descriptions joined side by side from one source, each keeping its own character, and never one description that merges both.

What remains, in short, is a relationship, not a gap. The search was looking for one more building, when what it needed was the way the buildings stand together.

> **Key idea.** A category error applies a concept where the concept does not belong, as Ryle's visitor did in asking where the University is. Every measurement contains two categories: what it finds, its content, and the setting it finds it in, its context. Quantum theory describes the content and relativity describes the context, and to ask the arena to come in the steps of its content is a category error. The whole argument fits in seven sentences, and the rest of the book establishes each one.

To see why comparison allows only two characters at its foundation, we need the ground: what every measurement has in common.



PART II

THE GROUND



Chapter 4

Everything Is Found by Comparison

What do all measurements have in common? Every measurement is a comparison. This chapter says what a comparison is, shows that comparison, interaction and relation are one act, and names the six parts that every comparison contains, with what each part is in physics.

The Simplest Act

A comparison is the act of finding whether two things are the same or different, in some respect. It is the simplest act there is, and every other act of knowing is built from it. To know anything at all is to have found that something is the same as something else, or different from it.

Instruments compare as surely as minds do. A pan balance compares: it sets two loads against each other, and it tips, or it stays level. A spirit level compares: its bubble sits between the marks when a surface matches the horizontal, and drifts when it does not. A piano tuner compares two strings by ear, listening for the slow beating that two nearly equal notes make together, and the beating stops when the two notes are the same. A smoke alarm compares the air with a threshold, and it sounds or it stays silent. In each case two things are brought together, and a sameness or a difference is found.

Whatever makes a comparison is a comparer. A comparer may be a person, an instrument, or any physical system that registers one thing against another. That is why Chapter 1 could say that an observer is anything that makes a comparison. The words observer and comparer name a position inside the world, and not a mind, and a finding is an act made from such a position.

Science Is Made of Comparisons

Science proceeds by observation, hypothesis and test, and each of the three is a comparison. An observation compares a quantity with a standard: a length with a measuring rod, a duration with the ticking of a clock, the weight of a sample with a standard mass. A hypothesis is compared with what has been observed. A test compares a prediction with an outcome, and an experiment is designed so that the two can be set side by side.

Comparison is therefore the material science is made of, not one tool among others. Take the comparison out of an observation and nothing has been observed. Take it out of a test and nothing has been tested. Whatever every comparison must involve, every measurement must involve, and so must every law that measurements establish.

A law of physics is itself a finding of comparison. It says what a measurement keeps finding the same when its circumstances change. The laboratory may be in a deep mine or on a mountain top, the experiment may be done this year or a century from now, and the apparatus may face east or north.

That is the route by which this book passes from comparison to physics: through measurement, which is comparison, and through law, which is what comparison finds to stay the same.

One Act, Three Names

Comparison, interaction and relation are one act, named three ways. *An Immanent Metaphysics* states this as its Root Tautology: comparison is isomorphic with interaction and relation. Two things are isomorphic when they have the same form, so that each can be read as the other. A tautology is a statement true by the meaning of its terms, and this one is the root because everything in the ground grows from it.

Watch the pan balance again. As a comparison, it finds one load heavier than the other. As an interaction, something happens: each load presses on its pan, and the beam tips. As a relation, the tilt of the beam holds the two loads in a standing connection, the one heavier, the other lighter. There is one event here, described three ways: as a finding, as a happening, and as a connection.

The Root Tautology cannot be denied without being used. To deny it is to compare the claim with something and find the two different, and that denial is itself an act of comparison, with an interaction and a relation inside it. The premise is in use in every attempt to reject it.

It also binds the world, and not only what we know of the world. Physics reaches the world only through interactions: light striking a detector, a particle crossing a chamber, a planet tugging on a spacecraft. Every interaction is a comparison. An atom struck by light shows it plainly. The atom takes in the light only when the light's energy matches the gap between two of its own levels, and lets it pass otherwise. Its absorbing is a finding of same or different, made by the atom, with no mind anywhere. So whatever comparison requires, every finding of physics carries, and a limit on comparison is a limit on what physics can find.

Six Parts of Every Comparison

Every comparison contains six parts, and they come in three pairs.

The first pair is the verdict. A comparison finds the two things the same, or it finds them different. Sameness and difference are the first two parts.

The second pair is what the verdict is on, and in what respect. The verdict is on something, and what the verdict is on is the content of the comparison. The two things are two in some respect, and that respect is the context of the comparison. A tide gauge in a harbor compares the height of the water at noon with its height at midnight. The height of the water is the content. Noon and midnight are the context. The gauge can find the height the same or different; it cannot find noon and midnight to be one time.

The content and the context cannot be one respect. If the respect judged were the respect that makes the two things two, the verdict would be fixed before the comparison was made, and a verdict fixed in advance is no verdict.

The third pair is the two sides of the act. The subjective is the side from which the comparison is made, and the objective is the side about which it is made. For the tide gauge, the subjective is the gauge itself: its float, its mark on the harbor wall, its clock. The objective is the water it registers.

These six are the intrinsics of comparison: sameness, difference, content, context, the subjective and the objective. *An Immanent Metaphysics* names them, and they are what comparison is, not additions to it. They are necessary and sufficient for any comparison whatever. The count is easy to check. A comparison yields a verdict, same or different: two parts. The verdict is on something, in some respect that is not the thing judged: two more. The comparison is made, by a comparing, of something compared: two more. Anything else a comparison might involve, such as a unit, a scale or a number, is built from these six and comes after them.

Each pair is inseparable. Sameness and difference are inseparable: wherever one appears, the other is implicit. Content and context are inseparable: where there is content, there is context, and where there is context, there is content. The subjective and the objective are inseparable: there is no comparing without something compared, and nothing is compared without a comparing. Each pair is also distinct, and no pair can stand in for another.

The Six in Physics

Because every measurement is a comparison, each of the six has a plain meaning in physics.

Part	In any comparison	In a physical measurement
Sameness and difference	the verdict	a reading that matches its standard or does not; a count registered or not registered
Content	what the verdict is on	the quantity measured: a mass, a charge, an energy, a position
Context	the respect in which the two things are two	the places, times, orientations or settings across which the quantity is compared
The subjective	the side from which the comparison is made	the instrument: its position, its state of motion, its setting
The objective	the side about which it is made	the system registered: an electron, a planet, a beam of light

The table states an application, not a likeness. A measurement in physics is a comparison, so its parts are the six parts of comparison, under their names in physics. A measurement of a planet's position has a content, the position, and a context, the moments at which the position is compared. It has a subjective side, the telescope and its frame, and an objective side, the planet. Its verdict is the match or mismatch between the position found and the position predicted.

The two clocks of Chapter 1 make the same point. The content of that comparison was the rate of ticking. Its context was the two heights, 33 centimeters apart. Its subjective side was the arrangement that set the clocks against each other, the optical fiber that carried one clock's ticking to the other, and its objective side was the two ions whose light set the rates. Its verdict was "different": the higher clock ran faster.

A law, in these terms, is a sameness of content across a difference of context. The conservation of energy finds the content, the total energy, the same across a difference in the context, the moment of measurement. Chapter 5 gives that kind of sameness its name, symmetry, and shows what it can and cannot be joined with.

The second pair matters most for this book. Every measurement finds some content in some context. Chapter 1 used those two words in a plain sense, for what is found and for the setting in which it is found, and here they have their exact sense, as two of the six parts of every comparison. Chapter 7 shows what they pick out in physics as a whole.

Identifications, in the First Person

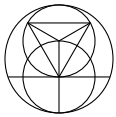
This book makes three kinds of claim, and it keeps them apart.

Claims about the structure of comparison hold wherever comparison occurs. They have no physical premise, and they rest on the ground set out in this Part. Claims of established physics, such as the precision of relativity and quantum theory, are taken as experiment has confirmed them. The third kind links the two. An identification is a stated step from the structure of comparison to a physical quantity, made in the first person, with the observation that would test it.

I make each identification in the first person, and I say so: “I identify”, “I take”, “I reason that”. The reader can then see exactly where the argument rests on a reading of the world, and which observation would bear on that reading. Chapter 5 makes the first identification of the book: the two characters that comparison allows, with the two theories of physics.

Key idea. Every measurement is a comparison: the act of finding whether two things are the same or different, in some respect. Comparison, interaction and relation are one act, so a limit on comparison is a limit on what physics can find. Every comparison has six parts in three inseparable pairs: the verdict (sameness and difference), what it is on and in what respect (content and context), and the sides from which and about which it is made (the subjective and the objective).

What happens when sameness and difference are applied to content and context? Four characters appear, and Chapter 5 shows that only two pairings of them can be perfected.



Chapter 5

Symmetry and Continuity Cannot Both Be Perfected

Why can physics not have everything at once? Because comparison allows only two characters at its foundation. Sameness and difference, applied to content and context, give four characters. Of the ten ways of pairing them, exactly two hold together, and two pairings in particular are forbidden. This result is the Incommensuration Theorem, and this chapter sets it out in full, with the reason it is settled.

Four Characters

A comparison gives two verdicts that matter here: one on the content, and one on the context. Each verdict is sameness or difference. Hold the stance of the comparison fixed, the same comparer in the same setting, and the two verdicts combine in four ways. Each combination is a character that a comparison can find, and each has a name.

Continuity is a sameness of content where there is a sameness of context. Read at the finest grain, it says that a small enough change of context brings only a small change of content. Nothing is sudden.

Discontinuity is a difference of content where there is a sameness of context. The content changes though the context has not. Something is sudden. The word covers three familiar cases, and each is a different content in one unchanged setting: a jump between two separate levels with nothing between them, a click that comes whole when a detector registers, and two identical set-ups that give different results.

Symmetry is a sameness of content where there is a difference of context. The content stays the same though the context has changed. Something is invariant.

Asymmetry is a difference of content where there is a difference of context. The content changes with the context. Nothing is invariant.

The four fit in a square, with the verdict on the content down the side and the verdict on the context across the top.

	Same context	**Different context**
Same content	continuity	symmetry
Different content	discontinuity	asymmetry

Each of the four is called a compound, because each is made of two verdicts. You meet all four every day. A dimmer is continuous: a small turn of the knob makes a small change in the light. A light switch is discontinuous: at one point in its throw, the light goes from off to on. A meter stick is symmetric: it measures the same length in every country and every year. A coastline is asymmetric: no stretch of it repeats another.

Physics uses all four words. In this book each names a character of what a comparison finds; physicists also call their equations symmetric and continuous, a second usage that Chapter 10, "How a Theory Sees", keeps apart from the first. Continuity is the character of smooth space and time, and of the calculus written on them. Discontinuity is the character of the quantum, the packet that is either found or not. Symmetry is the character of the conservation laws: in 1918 Emmy Noether showed that a law that holds the same at every moment conserves energy, and that a law that holds the same at every place conserves momentum. Asymmetry is the character of every record: a broken cup, a footprint, a written page, none of which repeats what came before it.

Ten Ways to Pair Them

Now ask which of the four can be joined, as the character of a single comparison perfected at its foundation. Four compounds can be paired in ten ways: each with itself, which makes four pairings, and each with each of the others, which makes six. The six cross-pairings are continuity with asymmetry, symmetry with discontinuity, continuity with symmetry, discontinuity with asymmetry, continuity with discontinuity, and symmetry with asymmetry.

A pairing of two compounds carries four verdicts: two on the content, one from each compound, and two on the context, one from each compound. That makes the question easy to settle, because there are only ten cases, and each can be examined in a few lines.

Three Tests

Three tests decide every case.

The first test: a compound paired with itself says nothing new. Continuity with continuity is continuity again. This removes the four self-pairings.

The second test: a compound paired with its direct opposite contradicts itself. Continuity finds the content the same within the same context; discontinuity finds it different within the same context. Together they say both of one case, which is a contradiction. Symmetry with asymmetry fails in the same way, across a change of context. This removes two more.

The third test rests on nothing but the inseparability of sameness and difference, which Chapter 4 found in every comparison. A pairing gives verdicts in two respects, content and context, and the inseparability must hold in each. So among the two verdicts on the content, a sameness must appear with a difference, and among the two verdicts on the context, a sameness must appear with a difference. A pairing in which the content is found only the same, or only different, has lost the inseparability of sameness and difference in the very respect in which its verdict is given.

Apply the third test to continuity with symmetry. Continuity finds the content the same, and symmetry finds the content the same. Sameness of content appears twice, and difference of content not at all. The pairing fails. Take

the tide gauge of Chapter 4. Continuity says: the same height, at the same moment. Symmetry says: the same height, at different moments. Put the two together, perfected, and the height is "same" twice and never "different". That is what the third test forbids.

Apply it to discontinuity with asymmetry: difference of content twice, and sameness of content not at all. That pairing fails too. These two are removed by the third test alone. The third test also fails all six pairings that the first two tests removed, so for pairings, inseparability alone, applied in each respect, reaches the same result as all three tests together.

The ten pairings and their verdicts:

Pairing	Verdicts on content	Verdicts on context	Result
continuity with asymmetry	same, different	same, different	holds
symmetry with discontinuity	same, different	different, same	holds
continuity with symmetry	same, same	same, different	forbidden
discontinuity with asymmetry	different, different	same, different	forbidden
continuity with discontinuity	same, different	same, same	fails
symmetry with asymmetry	same, different	different, different	fails
each compound with itself (four pairings)	one verdict repeated	one verdict repeated	fails

Eight pairings fail. Six of them fail as plain repetitions or contradictions. The two marked forbidden pass the first two tests and fail only the third, and the theorem names them.

The Two That Survive

Of the ten pairings, two hold.

Continuity with asymmetry is smooth, connected and without jumps, and different from place to place, with nothing exactly repeated. Symmetry with discontinuity is exactly invariant and exactly repeating, and it comes in separate, countable steps. I call a comparison of the first kind continuously asymmetric, and a comparison of the second kind discontinuously symmetric.

Both survivors pass the third test, and the check takes a moment. Continuity with asymmetry gives the content "same" and "different", and the context "same" and "different". Symmetry with discontinuity gives the content "same" and "different", and the context "different" and "same". In each, sameness appears with difference in both respects.

You can picture each one. Walk along a shore: step by step the coastline changes without a jump, and no stretch of it repeats another. That is continuity with asymmetry. Climb a ladder: every rung is the same as the last, and there is nothing to stand on between the rungs. That is symmetry with discontinuity.

The two survivors have one form. In each, the second compound is the exact complement of the first: wherever one finds sameness, the other finds difference, in content and in context alike. Every other pairing repeats a verdict somewhere, and the third test removes it. The two survivors are exhaustive and exclusive: any comparison perfected at its foundation has one of the two characters, and no third.

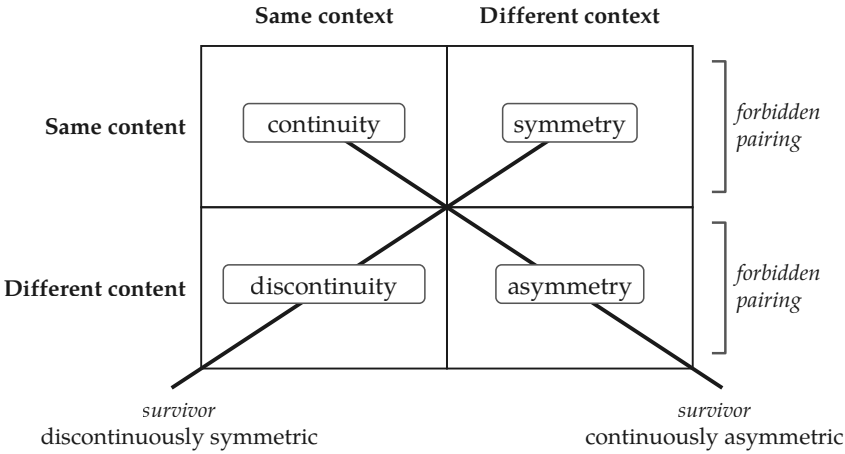


Figure 5.1. *The square of the four compounds. The two forbidden pairings lie along its rows, where both compounds give the same verdict on the content; the two survivors lie on its diagonals, where each compound is the exact complement of the other.*

The Theorem

The Incommensuration Theorem. Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric.

The theorem names two forbidden pairings, and this book refers to them often. The first is continuity with symmetry: smooth and exactly invariant, both perfected together. The second is asymmetry with discontinuity: stepwise and with nothing invariant, both perfected together.

The word that carries the weight is *perfected*. To be perfected is to hold exactly, at the foundation of a description. The theorem does not say that symmetry and continuity never appear together. Real systems show both, approximately, all the time: a smooth wave in a pond spreads out almost evenly in every direction. The theorem says that the two cannot both be exact, at the foundation of one and the same comparison.

Why the Theorem Is Settled

The Incommensuration Theorem is settled, and the reason is plain. The word covers three things, and each holds in its own way. The count of ten cases is settled as a count is settled. The three tests are argued from the inseparability of sameness and difference. The match of the two surviving characters to the two theories of physics is an identification, made at the end of this chapter, and observation tests it. The rest of this section concerns the first two. The theorem has no physical premise. It uses the meanings of sameness, difference, content and context, and the inseparability of sameness and difference, and nothing else. No step appeals to any fact about the physical world, so no fact about the physical world can unsettle it.

It is also finite. There are ten cases and three tests, and every case can be checked in a few lines by anyone who has read this far. The check comes out the same each time it is made. A result of that kind is settled in the way a count is settled: by the count itself, with no need to trust whoever made it.

And it governs every challenge made to it. An objection to the theorem compares the theorem with something and finds them different. The objection is itself a comparison, with the same six parts and the same inseparability, so the same ten pairings and the same three tests apply to it. Since every measurement is a comparison, the theorem applies to every measurement that physics has made or will make.

The appendix "The Ten Cases" lists the ten pairings as pairs of verdicts, each with the test that removes or keeps it, so that a reader can check every case on one page. The full enumeration, with its table of pairings, is given in *An Immanent Metaphysics*, and its aphorisms are printed at the back of this book, in "The Aphorisms Cited". A reader who wants the same steps stated as numbered premises, with the table, will find them in *The Unquantized Arena*, Chapter 1, "Comparison, the Intrinsics and the Incommensuration Theorem".

A Limit You Can Hear

Music offers a limit with the same shape, and only the shape carries over.

A pure fifth, the interval from C up to G, is a ratio of frequencies of three to two. Climb twelve pure fifths from a low C and you should arrive, seven octaves higher, at C again. You do not quite arrive. Twelve fifths multiply the frequency by a little under 129.75, while seven octaves multiply it by exactly 128. The overshoot is about a quarter of a semitone, and musicians call it the Pythagorean comma. No tuning keeps every fifth pure and also closes the circle back onto its starting note. The modern piano spreads the shortfall evenly, narrowing every fifth by about a fiftieth of a semitone, so that the circle closes and no fifth is quite pure.

Two demands, each easy to meet alone, cannot both be met exactly together. That is the form of the Incommensuration Theorem. The theorem itself rests on comparison, not on music, and it holds wherever any comparison is made.

The Name

Two quantities are incommensurable when no unit, however small, measures both of them a whole number of times. They share no common measure. The Incommensuration Theorem names a gap of that kind between two charac-

ters: symmetry and continuity have no common perfection. The gap is a feature of comparison itself, and no improvement in instruments or in mathematics can close it.

Two Characters, Two Theories

Physics has two great theories, and they fall on the two sides of the theorem.

I identify general relativity with the continuously asymmetric character. Its arena is smooth: a small step in position or time brings only a small change in the geometry. And its arena differs from place to place: in general no two regions of it are exactly alike. The observations that bear on this identification are the tests of Chapter 1, from Mercury's orbit to the ripples of merging black holes, each of which found the geometry smooth and shaped by what it holds.

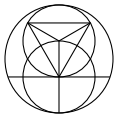
I identify quantum theory with the discontinuously symmetric character. What it finds comes in steps: one outcome, one packet, one sharp line of color. And what it finds is exactly invariant: every electron has the same charge and the same mass as every other. The observations that bear on this identification are the countable findings of every quantum experiment, and the exactness of the charges and masses they find.

The identification explains why the two theories have resisted being merged. A single theory in which the arena itself came in the steps of quantum theory would have to find the arena both smooth and exactly repeating, or both stepwise and nowhere invariant. Such a quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. Part III makes that argument in full.

The theorem permits the continuous symmetries of which physics is full. A sphere looks the same however far it is turned, and the angle of turning varies smoothly. It permits them because a physical theory has two levels, its mathematics and its findings, and the theorem concerns what a comparison finds. Chapter 6 establishes the two levels and shows where each character sits.

Key idea. Sameness and difference, applied to content and context, give four compounds: continuity, discontinuity, symmetry and asymmetry. Of their ten pairings, exactly two hold: continuously asymmetric, the character of relativity's arena, and discontinuously symmetric, the character of what quantum theory finds. Continuity with symmetry, and asymmetry with discontinuity, are forbidden. This is the Incommensuration Theorem; it rests on nothing but the inseparability of sameness and difference, and so it holds for every comparison.

What roles do the parts of a comparison play, and in what order is a finding made? Chapter 6 answers, and in doing so shows why every theory has two levels.



Chapter 6

Three Roles and the Order of Finding

What does a comparison need in order to happen? It needs a stance to be made from, an act that makes it, and a result that can be laid out and kept. Those are three roles, and the three pairs of intrinsics fill them. This chapter names the roles, sets out the order in which a finding is made, and quotes the three axioms that state how the roles stand to one another. From the axioms it draws two distinctions that the rest of the book relies on: the difference between what exists and what is real, and the two levels of every physical theory.

Three Roles

The three pairs of intrinsics work together, each pair playing a role with respect to the others, and the same three roles appear wherever comparison occurs. *An Immanent Metaphysics* calls the roles the modalities, and names them the omniscient (structure), the immanent (the act) and the transcendent (the stance with what is possible). The names are technical terms, and they carry no theological meaning in this book.

The omniscient is the role of what is laid out as structure: what can be held in view as a whole, like a finished drawing or a ledger. The immanent is the role of the act as it happens: the comparing itself, here and now, as it occurs. The transcendent is the role of the stance and of the range of what is possible: the position from which an act proceeds, and everything that could occur, beyond any one instance of it.

Which pair fills which role depends on what is in focus. The main focus of this book is the act of measurement: comparison as it is performed, by an instrument or any other comparer, here and now. In that focus, sameness and difference are immanent, because the finding is the act. Content and context are omniscient, because they are the finding laid out as structure. The subjective and the objective are transcendent, because they are the stance from which the measurement is made and the referent it is about.

In physics I identify the omniscient role with structure and with the conservation laws, the immanent role with interaction and force, and the transcendent role with possibility: the range of configurations a system could take, and the conditions within which events occur. The observation that bears on this identification is that every physical theory has all three: laws that hold across circumstances, interactions that happen, and a space of possible states within which they happen.

Which Concept Takes Which Role

The binding of the three roles is not the same in every argument, and the reason is plain. Which concept takes which role depends on the focus of the argument being made: what the comparison is taken to be about. Different arguments are not all versions of one argument, so no concept need take the same role in every focus. Three concepts form a triple, a set that fills the three roles, when there is one identified focus in which all three axioms, quoted below, hold of them. In another focus an argument may assert less. It may assert only the third axiom, so that the three concepts are only distinct, inseparable and non-interchangeable. Or it may assert the third and the second and leave open which concept is the act: the three then keep their order around the cycle described below, and any of the three bindings that keep that order may stand.

The foci this book uses, and what stands in each role in each:

Focus	**Structure (omniscient)**	**The act (immanent)**	**The stance with what is possible (transcendent)**
Measurement (this chapter)	content and context	sameness and difference	the subjective and the objective
Physics at large (this chapter)	conservation laws and structure	interaction and force	possibility, and the conditions within which events occur
A theory in use (this chapter)	its mathematics	each finding	the prepared setting, with the outcomes it allows
The two laws (this chapter and Chapter 8)	content, held to symmetry	the comparing	context, held to continuity
The six measures (Chapter 7)	pattern and space	force and time	probability and possibility
Time (Chapter 16)	geometric time	the time of findings	evolution time
Counting the information of a region (Chapter 25)	the count made cell by cell, from a vantage that holds the whole region	the count made from a place, by what crosses its boundary	not used

All three axioms hold in each focus of the table but the last, so each binding in those foci is fixed. In the last focus the argument asserts only the first and the third axioms, and it uses only two of the roles. A concept can stand in one role in one focus and in another role in another, and the two bindings then belong to two arguments, each made in its own focus.

The Order of Finding

In every comparison, the three roles engage in a definite order. A game of chess shows the order plainly.

The position on the board offers a range of legal moves. That range is the transcendent: what could happen next, from this stance. A player makes one move. That is the immanent: the act, done. The move is written into the score, and the board now shows a new position. That is the omniscient: the act laid out as structure, kept. And the new position offers a new range of moves, from which the next move is made.

Every comparison runs the same way. First there is a stance: a comparer, and something to be compared. From that stance a finding is made: something is the same, or something is different. The finding, once made, is laid out as structure: what was compared, and in what respect. And what is laid out reshapes the stance from which the next comparison is made.

The order is one of precedence, not of clock time. The cycle can be entered at any point, and it goes on, comparison after comparison. Its direction cannot be reversed. A later move in chess can bring back an earlier position, and the score still records every move that led there. In the large world of space and time, I identify that one-way order with the one direction of time. The observation that bears on it is that every record ever found is a record of the past.

The Three Axioms

An axiom here is a statement that holds of every comparison whatever. The three axioms of the Immanent Metaphysics were not chosen. Each can be found by looking at what a comparison contains, and they are easiest to meet in reverse order.

Chapter 4 found that the three pairs are distinct, that no comparison lacks any of them, and that no pair can stand in for another. Stated of the three roles, that is the third axiom. The roles are distinct: each is itself and not another. They are inseparable: none occurs without the other two. And they are non-interchangeable: none can do the work of another.

The second axiom is the order of finding just described. The stance precedes the act, the act precedes the laid-out finding, and the laid-out finding precedes the next stance. In the names of the roles: the transcendent precedes the immanent, the immanent precedes the omniscient, and the omniscient precedes the transcendent.

The first axiom places the act first. The four compounds of Chapter 5 are all made from comparison: each is sameness or difference applied to content and context. Comparison is not one of the four. It is the act from which all four are made, and so it is prior to each of them. In the focus of the two laws, where content is held to symmetry and context to continuity, two of the compounds carry the characters of the other two roles. Symmetry, sameness across a change of context, is what can be repeated, shared and laid out: the omniscient character. Continuity, sameness without a break, is what is unbroken and encompassing: the transcendent character. The Incommensuration Theorem showed that these two cannot both be perfected.

Two things related in that way are called conjugate. Two things are conjugate when each can be perfected only at the expense of the other, and neither is prior to the other, as symmetry and continuity are. So the immanent is more fundamental than the other two roles, and the omniscient and the transcendent are conjugate.

Two more words are needed before the exact statements. A class is a kind, standing ready as a whole; an instance is one particular case of a kind, here and now. The game of chess is a class, and the game being played on this board is an instance of it.

The three axioms, in the wording of *An Immanent Metaphysics*:

Axiom I. The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another.

Axiom II. A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent.

Axiom III. The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable.

In the first axiom, "and/or" means of either one and of both together. In the third, the bracket means that the statement holds both of the classes and of the instances.

In plain words, the first axiom says that the act comes first, and that structure and stance are conjugate. The second says that the stance comes before the finding, the finding comes before the record, and the record shapes the next stance. The third says that the three roles are distinct, inseparable, and that none can do another's work. The three are three faces of one structural fact about comparison, and so about every measurement. They are not statements about any particular physical system.

The axioms involve no circle. The roles were named from the pairs, and the axioms are stated in terms of the roles, but a circle needs an order: something assumed first and derived from later. The pairs, their roles and the pattern of their relations are present together in every comparison, with no one of them first. Naming them makes explicit what every comparison already contains.

What Exists and What Is Real

The axioms give two words exact and different meanings.

To exist is to stand as structure: timeless, the same in every instance, able to be held in view as a whole, like the pattern of a law, or the state of a system at one instant. To say that something is real is to say that it is given in an actual act of comparison: here, now, with the future still open. A law exists; a measurement is real.

Music shows the difference. The score of a sonata exists: it is the same in every copy, and it can be studied whole, note by note, at any hour. A performance of the sonata is real: it happens once, in one hall, on one evening, and while it is under way the next bar has not yet been played. The score is the omniscient role; the performance is the immanent. Neither can replace the other.

The two go with two kinds of reasoning. Mathematics reasons about structure, and within its structure a deduction is certain. Reasoning about the world must also ask whether its premises hold in the world, and the passage from a structure to the world always carries some risk. That is why this book states each such passage as an identification, in the first person, and names the observation that would test it.

The distinction also places the comparer. A deduction fixes its subject from outside: its terms do not change, and its domain is closed. The physical world is not reached that way. It is the whole of what comparison can reach from within it, and every measurement of it is made from inside, by a comparer that is part of it.

History, on this account, is structure. The past is known only through the marks it leaves in the present. A tree's rings are such marks: counted now, they let us reconstruct a sequence of wet years and dry years, one consistent history, and that history exists. The counting of the rings, here and now, is real.

The Two Levels of a Theory

Every physical theory has two levels. The first is its mathematics, which physicists call its formalism. The second is its findings: the outcomes of the actual measurements by which it is tested. Each finding is one such outcome, and the outcomes the theory says a measurement can have are its range of find-

ings; when this book speaks of the character of a theory's findings, it means the character of that range. The mathematics of a theory exists, and its findings are real.

The reason is the distinction just drawn. The mathematics of a theory is structure: its equations are the same for everyone who uses them, they can be held in view as a whole, and they say nothing about which instant is now. A finding is an act: a click in a detector, a reading on a dial, a shift in a clock's rate, each made here and now, by one comparer, once. A prediction joins the two levels by a further comparison, of a number computed in the mathematics with a number found in a measurement.

The third axiom then says how the two levels stand. They are distinct: a computed value is not a measured one. They are inseparable: the mathematics of a theory with no findings is pure mathematics, not physics, and findings with no mathematics are a list, not a theory. And they are non-interchangeable: no amount of calculation makes a finding, and no finding is a calculation.

Both theories of Chapter 1 show the two levels plainly. In quantum theory the mathematics is a smooth wave of possibilities, which is computed and never itself seen. Every finding of the theory is one outcome: one dot on the screen, one click, one sharp line of color. In general relativity the mathematics is written in coordinates, labels for places and times that can be changed freely without changing anything that is measured. That freedom to relabel is a symmetry of the mathematics. The findings of the theory are the geometry itself: the paths of light, the rates of clocks, the orbits of planets, which are smooth and differ from place to place.

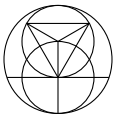
The two levels answer the question left at the end of Chapter 5. The Incommensuration Theorem concerns what a comparison finds, so it applies at the level of a theory's findings. A continuous symmetry, such as the symmetry of a sphere turned through any angle, has its two characters at the two levels of one theory. In a quantum theory of matter, the continuity of turning lies in the mathematics, where a smooth turning acts on the wave. The symmetry lies in what is found: the amount of rotation a particle carries, its angular momentum, comes in exact, equal steps. In general relativity it is the other way round: the symmetry lies in the mathematics, as the freedom to relabel,

and the continuity lies in the geometry that is found. Either way the two characters sit at different levels, and the theorem allows exactly that. What it forbids is both characters perfected together in what a single theory finds.

Later chapters rest on these two levels. Chapter 10 shows that in every established theory the two levels carry the two conjugate characters, one in the mathematics and the other in the findings, and checks that against six theories. Chapter 11 shows that to quantize a field is to exchange the characters of both levels together, changing which character each level holds exact, and that the arena cannot take that mode exchange. The same ground, stated as numbered premises with the axioms and the two levels, is set out in *The Unquantized Arena*, Chapter 2, "The Three Axioms and the Two Levels of a Theory".

> **Key idea.** Every comparison has three roles: the omniscient (what is laid out as structure), the immanent (the act as it happens), and the transcendent (the stance and the range of what is possible). The three axioms state how the roles stand: the act comes first, structure and stance are conjugate, a finding is made in a one-way order, and the three roles are distinct, inseparable and non-interchangeable. What exists is structure, and what is real is given in an act; so every physical theory has two levels, its mathematics, which exists, and its findings, which are real.

With the ground in place, Part III asks what the difference is between a thing and the place where it is found.



PART III

THE DIAGNOSIS



Chapter 7

What Is Measured, and Where

What is the difference between a thing and the place where it is found? The thing is the content of a comparison, and the place is its context. Every measurement has both, and physics divides along the line between them. This chapter draws that line, states the two laws that govern each side of it, and shows why one side comes in steps and the other is smooth.

Two Sides of Every Finding

Part II set out the ground, and this chapter builds on three of its words. Every measurement is a comparison. Whatever makes it is a comparer, which may be any instrument or physical system, and never needs to be a mind. What one comparison yields is a finding.

Two of the six intrinsics of comparison concern us here. Chapter 4 called what the verdict is on the content, and the respect in which the two things compared are two the context, and it showed that the two can never be one respect.

Take two photographs of one street, taken a year apart. The street is the content: the comparison can find it changed or unchanged. The two dates are the context: they are what makes the two photographs two. The comparison can find a new shop on the corner; it cannot find the two dates to be one date.

Physics is full of the same division. The charge of a particle, the count on a detector, the reading of a balance: these are contents. The place and the time at which each is found, and the range of outcomes that could have been found there, are their contexts. This is an application of the intrinsics, and not a likeness drawn between two subjects: every measurement is a comparison, and every comparison has a content and a context.

Content and context are assigned within each comparison. A thermometer dipped into a bath finds the temperature of the bath as its content. A chemist who then times one reaction in that bath, and the same reaction in a bath kept warmer, uses the two temperatures as the context of the timings: they are what makes the two timings two. The roles belong to the act of comparing, and each act gives them out afresh. One setting keeps its role in every comparison, the comparisons that measure it included: the arena of space and time, as Chapter 9 shows.

The Two Laws

Two laws govern the two sides. *An Immanent Metaphysics* states them in one line each.

> **The two laws.** Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context.

Recall the four compounds of Part II, each defined with the stance of the comparing held the same. Symmetry is a sameness of content across a difference of context: something is invariant. Continuity is a sameness of content within a sameness of context: nothing is sudden. So the first law says that what is found must stay the same when the place or the time of the finding changes. The second says that the settings in which things are found join one another without a break.

The first law is why physics can have results at all. The charge of an electron measured in a laboratory in Chile is the charge of an electron measured in Finland, this year or a century from now. A content that changed with every change of setting would be a new fact at each measurement, and no two comparers could share it.

The second law is why the settings form one world. The kitchen where you stand joins the hallway beside it, and this moment joins the next. A context with a gap in it would leave two settings that no comparison could connect, and the findings made in them could never be set side by side.

Being joined is not yet being smooth. A row of stepping stones is joined from one end to the other, and it still comes in steps. The reason the setting has no steps is the inseparability of sameness and difference, which holds in the context as in every part of a comparison. So every difference of context holds a sameness within it: two places that differ also share something, and a finer comparison can be made between them. A smallest step would be a difference of context with no sameness inside it, and the inseparability of sameness and difference allows none. So the settings of the world have no smallest step: they are smooth, and not only joined.

An Immanent Metaphysics adds a characterization of each side. Content is local and interexclusionary: two contents exclude one another from the same place, and content is understood in terms of symmetry. Context is non-local and intercompositional: contexts compose with one another, and context is understood in terms of continuity.

You meet both properties at any table. Two coins cannot lie on the same spot of it; one pushes the other aside. The length of the table and its width, though, compose into its surface, and a morning and an afternoon compose into one day. Things crowd each other out; settings combine.

One more principle binds the two sides together. Chapter 4 found content and context inseparable, and *An Immanent Metaphysics* states it in these words: "Where there is content, there must also be context. Where there is context there must also be content." This book calls it the inseparability principle. Every finding has both, and neither side can be removed while the other is kept.

Why Content Comes in Steps

The first law has a consequence that physics met long before it had a reason for it. Content is governed by symmetry, so what a comparison finds as content is an invariant: something that stays the same across a change of context.

Invariants sort things into kinds. An electron is an electron in every laboratory and every galaxy; that sameness is what the word names. Belonging to a kind is all or nothing. A particle carries a given label or it does not, and there is no particle that is two thirds of an electron. So a domain governed by symmetry is a domain of classes, and a classification has gaps between its classes.

The Incommensuration Theorem says the same thing at its root. That theorem, the center of Part II, says that symmetry and continuity cannot both be perfected in one comparison, and neither can asymmetry and discontinuity, where perfected means held exactly, at the foundation of a description. The theorem's two surviving pairings join symmetry with discontinuity, and continuity with asymmetry. Perfected symmetry therefore travels with discontinuity. I reason that content, perfected at its foundation, is discontinuously symmetric: exact labels, repeated without variation, coming in countable steps.

Physics shows this everywhere it looks at content closely. Electric charge comes in whole multiples of one third of the electron's charge. Spin comes in whole and half units. Hydrogen in a distant galaxy emits the same pattern of sharp lines as hydrogen in a lamp, shifted in color by its long journey and not reshaped. The discreteness of the quantum is the character that the first law, through the theorem, gives to content.

One distinction keeps this exact. What a body carries as its own comes in steps: its kind, its charge, its spin, the mass at rest of its kind. What a body has by its place in the arena varies smoothly: where it is, and how fast it moves. The first belongs to the content; the second is the content's relation to its context.

Why Context Comes Smooth

The second law has the matching consequence. Context is governed by continuity: within one unbroken stretch of context, nothing jumps. A continuous range does not sort into classes. Between any two places there is a third, and between any two moments there is another.

The theorem pairs perfected continuity with asymmetry. I reason that context, perfected at its foundation, is continuously asymmetric: smooth, with no gaps, and different from place to place, with no two places exactly alike.

Space and time together form the setting of every physical event. Chapter 1 called them the arena: the setting in which everything that is measured is measured. Every measurement of the arena so far finds it smooth. It also finds it uneven. A pendulum swings at a slightly different rate on a mountain than at sea level, and a satellite's clock runs at a different rate from the same clock on the ground. The arena is smooth, and it is different from place to place. Chapter 8 shows why it has that second property, and Chapter 9 shows that its unevenness is what we call gravity.

Six Measures in a Grid

The two sides have finer structure. The step to them is an identification, made in the first person as Chapter 4 set out, with the observation that would test it. I identify six measures of every physical measurement, in two rows of three.

The contents are pattern, force and probability. Pattern is what is laid out: mass, structure, the inertia of a body. Force is the directed flow of change. Probability is how the odds are spread among the alternatives that a finding could select.

The contexts are space, time and possibility. Space is where pattern is laid out. Time is the ordering of before and after, in which force acts. Possibility is the range of what a comparison could find. Each content is measured in its own context: pattern in space, force in time, probability in possibility.

The three columns are the three roles of every comparison, from Part II. The omniscient role is what is laid out as structure, and it holds pattern and space. The immanent role is the act as it happens, and it holds force and time. The transcendent role is the stance and the range of what is possible, and it holds probability and possibility.

The six measures, by row and by role:

	Omniscient (laid out)	**Immanent (happening)**	**Transcendent (possible)**
Content	pattern	force	probability
Context	space	time	possibility

The grid is an identification, and every new kind of measurement tests it: each must find its place in one of the six cells. The argument of this book uses only the two rows.

The Laws Belong to the Rows

A column names a role; a row names a law. All three contents are governed by symmetry, and all three contexts are governed by continuity. Whether a measure comes in steps is decided by its row.

Look down any column and you see one measure in steps above one that is smooth. Pattern comes in kinds, and it is laid out in a smooth space. Force, in particle physics, is exchanged in countable quanta, and it acts in smooth

time. Probability is spread over outcomes that are counted one by one, while the possibility in which it is laid out is smooth: the proportions in which a quantum state holds its alternatives can take any value. The quantum is discrete because of its row.

This gives the line on which physics divides. The matter of the world, and the three forces of particle physics (the electromagnetic, the strong and the weak), are content. They are governed by the law of symmetry, so their perfected description comes in steps, and they can be quantized. The arena is context. It is governed by the law of continuity, so its perfected description is smooth, and it cannot be quantized in the same sense. Chapter 11 says exactly what that sense is.

You can see the division in any room. The chair, the light from the window, the air: each is made of particles of a few kinds, every particle of a kind the same as every other. The room in which they sit, and the afternoon in which you sit among them, come in no such pieces. They are one smooth setting, and no two parts of it are exactly the same.

Two Theories, Two Sides

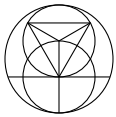
The division gives each of the two great theories its place. Quantum theory describes content in the character the first law gives it. Its findings are discrete: a detector clicks or it does not, an atom is in one level or another, and every electron carries exactly the charge of every other. General relativity describes the arena in the character the second law gives it. Its findings are smooth: a length, a duration, a curvature, each varying without a jump from one place to the next, and none repeated exactly elsewhere.

I identify the two theories with the two sides of the division in that way. Quantum theory is the theory of what is found; general relativity is the theory of where it is found. The identification is tested every time either theory is tested, because it says what kind of finding each will deliver: quantum theory, findings in exact steps; general relativity, findings that vary smoothly. Each theory has passed every test it has been put to, in its own kind of finding.

This is why the two theories look so different. They are the accounts of the two sides of every comparison, and every comparison needs both sides, so neither theory can take the other's place. You have used both in the last minute: the light reaching your eye arrived in countable quanta, and it crossed a smooth stretch of space and time to get there.

> **Key idea.** Every measurement has a content, what is found, and a context, where and in what respect it is found. The law of symmetry governs content and the law of continuity governs context. So perfected content comes in exact, countable steps, and perfected context is smooth and different from place to place. The six measures fill a grid of two rows and three roles, and the law that decides whether a measure comes in steps is the law of its row.

The arena is the context of everything found; the next question is why it has a shape, and why its shape follows the matter in it.



Chapter 8

Why the Arena Bends

Why does matter shape space and time? Because two demands of every comparison are held in full, and the Incommensuration Theorem allows no single character to meet both. The comparing must connect without a break, and the content it finds must stay the same for every comparer. What neither of them can carry is carried by the context, and the context of physics is the arena. General relativity, as an established theory, says how the arena carries it. This chapter gives the reason beneath that. The diagnosis of this book, completed in Chapter 11, stands on its own ground without this chapter; what this chapter adds is the reason the arena takes a shape at all.

A Relation in Need of a Reason

Since Albert Einstein wrote down his field equations in 1915, physics has known how matter and the arena are related. At each place, the equations tie one part of the curvature of space and time to the energy and momentum of what is there. The arena carries the rest of its curvature on into the empty space around. The shaped arena in turn guides what moves in it: a falling body follows the straightest path the arena allows.

The relation has passed every test put to it. Light passing the Sun is bent. A clock lower down runs slower than a clock higher up. Two black holes spiralling together shake the arena, and the waves of that shaking cross more than a billion light years to reach detectors on Earth, as the LIGO and Virgo collaborations first reported in 2016.

The equations say how much the arena bends, and in what form. They do not say why the setting of things should take any notice of the things in it. A setting that took no notice is easy to imagine. Isaac Newton's space was one: a fixed stage, the same everywhere, unmoved by the actors on it. The question of this chapter is why the world's stage is not that kind of stage.

The Comparing Connects

Part II found three roles in every comparison. The immanent role is the act as it happens. The omniscient role is what is laid out as structure: the finding, once made. The transcendent role is the stance, and the range of what is possible, from which the act proceeds.

This chapter takes one focus throughout, the focus of the two laws that Chapter 6 named: the comparison as a whole, with its comparing, its content and its context. All three axioms hold in that focus, so the binding of its roles is fixed. In that focus the comparing is the act, in the immanent role; the content, laid out once it is found, is structure, in the omniscient role; and the context, within which the act proceeds, is the stance with what is possible, in the transcendent role. That is the binding Chapter 6 set out for this focus.

Begin with the act. To compare is to hold two things in one act. A connection with a break in it connects nothing, since the two sides of the break are not held together. So the act of comparing brings continuity: it connects what it compares without a break.

Think of a pan balance. Its beam joins the two pans, and the verdict it gives, heavier on this side or that, travels along the beam. Saw the beam in two and the balance weighs nothing; each pan simply drops. The beam is the balance's act of comparing, and it works only while it is one unbroken piece.

Every comparison demands that its comparing connect without a break, since an act that did not connect would find nothing at all. But the comparing perfects no character of its own. It is the act of joining, not something laid out, and the continuity it brings is perfected in the context, where it connects.

The Content Holds Together

Now the content. The second axiom sets an order in every comparison: the stance comes first, then the act, then the finding laid out as structure, and the laid-out finding shapes the stance from which the next comparison is made. So the content comes after the act that finds it. It is what remains, laid out, for every later comparison to use.

Laid out in that way, the content is held to symmetry. It must be the same for every comparer who finds it, and consistent from one entry in the record to the next, so that the findings of many comparers make one history. This is the law of symmetry of Chapter 7, seen from the side of the record.

This demand is held in full. A content that was not consistent would be no single history. Two comparers could each find something, and never find that they had found the same thing. Two witnesses to one event must be able to reconcile their accounts, or there was no one event for them to witness.

Neither Can Carry Both

So every comparison makes two demands that admit no exception: that the comparing connect without a break, and that the content be the same for every comparer. The Incommensuration Theorem says that symmetry and continuity cannot both be perfected in one comparison. Perfected symmetry goes with discontinuity, and perfected continuity goes with asymmetry.

The first axiom states the same relation between the roles. In plain words, it says that the act comes first, and that what is laid out and the stance from which acts proceed are conjugate: each can be perfected only at the expense of the other, as symmetry and continuity are. The third axiom says that the three roles are distinct, inseparable, and that none can do another's work.

Put these together. The theorem is about a comparison, so it holds of the comparison as a whole, and not only of each of its parts taken alone. It allows a symmetric content beside a continuous context, because the two characters then belong to two different things, the content and the setting in which it sits, as it allows the mathematics of a theory and what the theory finds to carry the two conjugate characters (Chapter 10). But wherever continuity is

perfected in a comparison, asymmetry must travel with that continuity somewhere in the comparison, since continuity with asymmetry is the only surviving pairing that contains continuity. The continuity that the comparing brings is perfected in the comparison, though not in the comparing itself, so the comparison must carry an asymmetry somewhere. Asymmetry, by its definition, is a difference of content laid out across a difference of context. It cannot sit in the content, which keeps its sameness. It cannot sit in the comparing either, since the comparing is the act of joining, and not something laid out from place to place. And by the third axiom, neither the content nor the comparing can hand its own work to the other, or lay it down.

The Context Carries the Rest

One place remains: the context. I reason that the asymmetry which the comparing and the symmetric content cannot carry is carried by the context. In the focus of the two laws, where all three axioms hold, the character of continuity belongs to the context, in the transcendent role, as Chapter 6 found. The comparing connects without a break, but it perfects no character of its own: the continuity it brings is perfected in the context, where the comparing connects. There that continuity travels with the asymmetry that neither the content nor the comparing can carry. And the context gives up the symmetry that the content keeps. So the context is continuously asymmetric: smooth, and different from place to place and from moment to moment.

This is the character Chapter 7 found for context from the law of continuity. Here it appears again, from a different direction, and with something added: a reason for the asymmetry to follow the content. The arena is where the consistency of the content is paid for in asymmetry. Where there is content, the arena differs from one place to the next. The arena bends.

The established theory agrees at the one place where the payment is not needed. In general relativity the flat arena of special relativity, the same at every place and time, is the idealized case of an arena with no content anywhere and no waves crossing it. Put content into it, and its uniformity goes.

The bending then reaches beyond the content into the empty space around it: the space around the Sun holds no matter and is curved by the Sun, and a gravitational wave curves the empty space it crosses.

The same reasoning says why the bending is smooth and never stepped. The continuity that the comparing brings is perfected in the context, since the context is where the comparing connects. A bend that came in steps would put a break into the setting, and at the break the comparing would have nothing to join. So the arena may differ from place to place without limit, but it differs gradually, with no jump anywhere.

Notice what kind of explanation this is. It does not say how much the arena bends, or by what equation. It says why there must be a bending, and why the bending must be smooth, and why it must go with what is found there. Those three facts come from the structure of comparison alone.

Measured or Not

The arena bends wherever there is content, whether or not anyone measures it. The reason given here does not wait for a laboratory. By the Root Tautology, comparison, interaction and relation are one act under three names, so the comparisons in question are all the interactions by which content is what it is. A star in a galaxy that no telescope has seen still holds together; its particles still interact, and its content is still one consistent thing. The arena around it bends accordingly.

So the bending is a property of the world, and not of our looking at it. General relativity says the same in its own terms: the part of the curvature that the content at a place produces there goes with the energy and momentum at that place, and neither waits on a measurement.

What the Established Theory Adds

General relativity says how. Its equations tie one part of the curvature of the arena at each place to the energy and momentum of the content at that place, and they say how the rest of the curvature spreads through the arena. I take the form of that relation from the established theory, as the established physics it is.

The established theory adds one more thing that fits the reason given here. A change in the shape of the arena spreads through the arena, as a wave that travels at the speed of light. The waves have been detected. In 2017 a merger of two neutron stars sent gravitational waves and light toward us together, and after a journey of many millions of years they arrived within two seconds of each other. That fixes the two speeds as equal to within a few parts in a million billion. The arena carries its asymmetry smoothly, from place to place, at a finite speed.

One number enters from outside the reasoning: Newton's constant, which sets how strongly a given content bends the arena. It is measured, by weighing the pull between masses in the laboratory, and nothing in this book derives it. The ground of comparison fixes what role the arena plays, and not how large any quantity in it is: its axioms are structural, and they carry no units. Sizes come from measurement, as they do in every other part of physics.

So the two kinds of account divide the work cleanly. The ground of comparison says why the arena has a shape that follows its content. The established theory says what that shape is. Neither needs the other to do its own part.

You Live in a Bent Arena

You can feel the arena's asymmetry in your hand. Hold a pen at arm's length and let go. It falls because the arena at the height of your hand differs from the arena at the height of the floor, and the pen follows the straightest path that the difference allows.

Clocks show the same difference more delicately. In 2010 Chin-Wen Chou and colleagues at the National Institute of Standards and Technology raised one of two optical clocks by about a third of a meter and measured it running faster than the lower one, by the amount general relativity gives. The satellites of the navigation system in your phone carry clocks whose rates are set to allow for the same effect, or the positions they give would soon be far wrong.

In each case the arena behaves as the reason says it must. It is smooth: the clock rate changes by no step as the height changes. It is asymmetric: no two heights above the Earth are exactly alike. And the asymmetry is there because the Earth is there, a great body of content whose consistency the arena around it pays for.

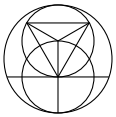
What This Gives the Diagnosis

The arena's character is the work that the context does in every comparison, in any world where comparisons are made: it carries the asymmetry that lets the content stay consistent while the comparing stays connected.

That gives a first view of the error this book diagnoses. To quantize the arena is to give it the character of content: exact steps, each the same as the next. A quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. An arena of that character would also have given up the very asymmetry it exists to carry. The asymmetry would then have to sit in the content, or in the comparing, and the reasoning of this chapter leaves room for it in neither. Chapter 11 makes that argument in full and on its own ground: from the two levels of a theory, and from what a context is. This chapter shows the same fact from the side of the work the context does.

Key idea. Every comparison connects without a break, and its content must be the same for every comparer, so the content is symmetric. In the focus of the two laws, where all three axioms hold, the comparing, the content and the context fill the three roles, and the Incommensuration Theorem, being about a comparison, holds of the comparison as a whole. The comparing perfects no character of its own: the continuity it brings is perfected in the context, and there it must travel with an asymmetry, which the third axiom forbids the content or the comparing to take on. So the context is continuously asymmetric: it is smooth and differs from place to place, following the content in it. General relativity, as established, says what shape the arena takes; the structure of comparison says why it must take one.

If the shape of the arena is what we call gravity, any arena should carry marks by which it can be recognized; the next chapter derives four, and checks gravity against them.



Chapter 9

Four Marks of the Arena

How do we know that gravity is the arena, and not one more force acting within it? Because any arena must carry four marks, and gravity carries all four. An arena couples to everything; nothing can shield it; the charge by which it holds a body and the inertia of that body are one quantity; and it has no charge of its own that comes in steps. No force of particle physics carries any of the four. This chapter derives each mark for any arena first, before gravity is named, and then checks gravity against it.

		the three forces of particle physics		
	Gravity	**Electromagnetism**	**Strong force**	**Weak force**
Couples to everything	●	○	○	○
Nothing shields it	●	○	○	○
One mass	●	○	○	○
No charge in steps	●	○	○	○

● carries the mark ○ does not carry it

Figure 9.1. *The four marks of the arena, set beside gravity and the three forces of particle physics. Gravity carries all four, and none of the three forces carries any of them.*

Any Arena, Before Gravity

Chapter 7 gave the materials. Content and context are inseparable: where there is content, there must also be context, and the reverse. Content is governed by the law of symmetry; context by the law of continuity. Content is local, and two contents exclude one another from the same place; context is non-local, and contexts compose with one another.

Now consider any phenomenon that plays the role of context for the content of some domain. Call it that domain's arena. The question is what such a thing must be like, whatever domain it belongs to, and whatever name it is later given.

The first answer is that it must exist. The matter of our world, and the three forces of particle physics, are content: they are governed by symmetry, they come in steps, and their charges are exact labels. By the inseparability principle, content does not occur without context. So our world must contain a phenomenon that plays the role of context for all of that content. The ground does not name it. It says only that there must be one, and it says what it must be like.

The First Mark: It Couples to Everything

Every content is found in a context; there is no content without one. So an arena takes part in every content of its domain, without exception. Nothing in the domain can be neutral to it, because to be neutral to the context would be to be found nowhere.

The forces of particle physics are selective. Electromagnetism acts only on electric charge, so a neutron, a neutrino and a ray of light pass through an electric field undeflected. The strong force acts only on color charge, so an electron does not feel it at all. Each of these forces has neutral objects, and most of the world is neutral to at least one of them.

Gravity has no neutral objects. It acts on every form of matter and on light: starlight passing the Sun is bent, and light climbing out of a gravitational field loses energy. It acts on the energy of motion and the energy of binding, as well as on the matter at rest. It acts on antimatter, which is the hardest case to

test. In 2023 the ALPHA-g experiment at CERN, the European laboratory for particle physics near Geneva, released atoms of antihydrogen in a tall vertical trap and watched them fall. They fell downwards, at a rate consistent with the rate for ordinary matter, as Anderson and colleagues reported.

The Second Mark: Nothing Shields It

To shield a phenomenon from a region is to make a region from which the phenomenon is absent. A region of space, though, is itself a structure of context: it is a stretch of the arena. So to shield the arena from a region would be to make a region with no context in it, and a region with no context is no region at all. I reason that an arena cannot be shielded, since a shield for it would have to remove the very thing that makes the shielded place a place.

The established physics of shielding gives a picture of the same fact. A metal box keeps electric fields out because the metal holds charges of both signs, free to move. A field from outside pushes the positive charges one way and the negative charges the other, and they settle where the field they make cancels the outside field within the box. That is why your phone can lose its signal in a lift.

Look at what the box needs. It needs an opposite to the source: charges of the other sign. And it needs parts that the field moves apart: positive charges going one way, negative the other. Gravity offers neither. It couples to energy, and energy has one sign: every body carries positive energy, matter and antimatter alike. ALPHA-g's falling antihydrogen is the check: antimatter is no opposite source. And every body falls alike in a given field, so no arrangement of bodies is pulled into the opposed pattern that cancellation needs. The MICROSCOPE satellite, described below, is the check of that.

The tides show the same fact from another side. The Moon stretches the oceans and the solid Earth because the arena differs from place to place across a large body: the near side is drawn toward the Moon a little more strongly than the far side. Every part is still drawn toward the source, and no part is pushed away from it. A tide can stretch a body; it can never set up the opposed charges that would cancel a field.

So no arrangement of matter covers a source of gravity. This physical picture leans on the third mark, below, which is why I give it as a picture that agrees with the structural argument, and not as a separate derivation. The structural argument says why no shield can exist; the picture shows, in the workings of a box, why none of the means a shield would need are available. There is no gravitational Faraday cage, and the two accounts say why.

The Third Mark: One Mass

A body that feels a force of the content kind has two separate properties. One is its charge, which classifies it: a label it carries as content. The other is its inertia, which is how strongly it resists a change in its motion; motion is a change of the body's place in the arena, so inertia is a property of the content's relation to its context, not a label the content carries. A label of content and a relation of content to context are distinct, by the distinctness of content and context found in Chapter 4. Nothing in the ground ties the charge to the inertia, and the ratio of the one to the other is free to differ from one body to the next.

Physics shows exactly this. The electron and the proton carry charges of equal size and opposite sign, yet the proton has about 1836 times the inertia of the electron. In one electric field the two respond very differently, because their charge and their inertia are independent quantities.

For an arena the case is different. What couples a body to the arena is how much the body takes part in the arena. What resists a change of the body's place in the arena is, again, how much the body takes part in the arena. I identify the two as one quantity, the body's participation in its context. So, for an arena, a body's charge and its inertia are equal, whatever the body, and every body falls alike.

This is the equivalence principle. In its weak form it says that gravitational mass and inertial mass are equal. In the form Einstein made the foundation of general relativity, it says that, over a small enough region, falling freely in gravity cannot be told apart from floating free of it. That stronger form follows too. A change of motion is a change of the body's context, and gravity is the context itself, so over a small region the two are changes of one kind, and

nothing tells them apart. An electric field can always be told apart from a change of motion, since a charged body and a neutral body respond to it differently.

The check is among the most precise in all of science. The MICROSCOPE satellite, whose final results Pierre Touboul and colleagues reported in 2022, carried test masses of titanium and platinum in orbit between 2016 and 2018. The two fell alike to within a few parts in a million billion. General relativity takes the equivalence principle as its starting point. The ground of comparison gives it a reason: for an arena, there is only one mass to measure.

The Fourth Mark: No Charge in Steps

Content is governed by symmetry, and Chapter 7 showed that symmetry gives exact labels in steps. So the charges of the content forces come in steps. Electric charge comes in whole multiples of one third of the electron's charge. Color charge comes as one of three kinds. Each label is exact, the same for every particle of a kind, and countable.

An arena is governed by continuity, and continuity gives no such labels. So whatever couples a body to an arena is no stepped label. It varies continuously, taking any value in its range.

Gravity couples to energy and momentum, and they vary continuously. The masses of the kinds of particle at rest form a list. Gravity, though, couples to the whole energy of a body: its matter at rest, its motion, its heat, the energy that binds its parts. A hot brick is heavier than the same brick cold, by an amount far too small to weigh but real. A nucleus of helium is lighter than the four particles that make it, because of the energy that binds them. The energy of motion can take any value, and no label sets the total in steps. The energy that binds a nucleus does come in steps, and those steps belong to the nucleus, which is content, and not to gravity. Gravity has a strength, set by Newton's constant, but what it couples to is no stepped label of the kind electric charge is.

The Four Together

Each mark, the reason any arena carries it, and the way gravity shows it:

Mark	Why any arena has it	How gravity shows it
It couples to everything	No content is found without context	No neutral objects; antimatter falls (ALPHA-g, 2023)
Nothing shields it	A region with no context is no region	No gravitational Faraday cage; energy has one sign
One mass	Charge and inertia both measure participation in the context	Titanium and platinum fall alike (MICROSCOPE, 2022)
No charge in steps	Continuity gives no stepped labels	It couples to energy, which varies continuously

The four marks are four consequences of one fact: the phenomenon in question is the context, and not a content within it. That is why they come together, and why no force of particle physics has even one of them.

The Identification

The ground says that our world must contain an arena, and that the arena must carry four marks. Physics has exactly one phenomenon that carries them. I identify gravity as the arena of physics, the context of every physical measurement, by that match.

A physicist will press one objection here, and it deserves its full strength. A gravitational-wave detector compares the arena at one moment with the arena at another, and its verdict is on the shape of the arena. So the detector seems to take the geometry as the content of its comparison, and the geometry would then be content after all.

What the detector finds is a change in the phase of its light, the point in its wave that the light has reached when it returns, and the light is content. It finds that change against the arena as its context, as every comparison does. A small wave of the geometry about a smooth background can be described as

if it were content, and a description of that kind can carry quanta: they are quanta of the description only, never quanta of the arena. The arena as the background of every comparison stays context, and the detector's own light is laid out in it.

The one empirical input to the identification is this match itself: that gravity is the thing that fills the place the ground marks out. The ground derived what any arena must be like, without naming gravity, and gravity turned out to be like that.

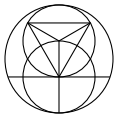
General relativity already treats gravity in this way. It describes gravity as the geometry of space and time, and each of the four marks is built into that description: a geometry is shared by everything in it, has no outside from which to screen it, moves every body along the same paths, and is measured by lengths and durations rather than by a stepped label. What the ground adds is the reason the setting of things must carry these marks, and the recognition that gravity carries them because it is that setting.

The identification makes predictions, and the experiments named above are its present tests. It predicts that no shield for gravity will ever be found; that no two materials will ever be found to fall differently, at any precision; and that nothing will ever be found that gravity leaves untouched. Each of these is an observation that physics goes on making, more sharply each decade.

So gravity is the shape of the arena, and not a force that acts within it. Newton described it as a force, and at the scale of planets and falling apples that description works very well. Its marks, though, are the marks of a setting. An astronaut on the space station floats because the astronaut and the station fall together, along the shape of the arena, and nothing pushes either of them.

> **Key idea.** Any arena, derived from the ground before gravity is named, must couple to everything, admit no shield, have its charge and its inertia as one quantity, and carry no charge in steps. Gravity carries all four, and no force of particle physics carries any of them. Antimatter falls down, titanium and platinum fall alike, and no arrangement of matter covers a source of gravity. I identify gravity as the arena by that match: the context of physics, and not a content within it.

If the two great theories are the theories of the two sides, a question follows: why does each of them have smooth mathematics and discrete findings, or the reverse?



Chapter 10

How a Theory Sees

Why does every theory of physics have smooth mathematics and discrete findings, or the reverse? Because, as I identify it, a theory is itself a comparison, of its mathematics with its findings, and a character compared with itself finds nothing new. So the mathematics of a theory is always in the character conjugate to the character of its findings. This chapter states that principle, shows how it settles a puzzle in both great theories, and checks it against six theories of physics.

A Puzzle in Both Theories

Each of the two theories seems to contain both characters together. In quantum theory, the wave that describes a system changes smoothly, with no jumps, from moment to moment; yet what a measurement finds comes in steps. In general relativity, the geometry is smooth; yet the equations rest on a perfect symmetry, since they keep their form however the points of space and time are labeled.

That looks like trouble. The Incommensuration Theorem says that symmetry and continuity cannot both be perfected in one comparison, and here each theory appears to perfect both. Part II answered the question in brief, and this chapter gives the answer in full: each character lives at a different level of the theory.

Two Levels of a Theory

Part II drew the distinction. A physical theory has two levels. One is its mathematics: the apparatus in which it is written, with its space of possible states, its equations of change, and the symmetries that shape those equations. The other is its findings: the outcomes of actual comparisons, each one of the outcomes the theory says a comparison can have. Those possible outcomes together are the theory's range of findings, and the character of a theory's findings is the character of that range.

Every mathematics of physics uses both smooth quantities and symmetries. What sets its character is which of the two it holds exact against its findings.

The two levels can have different characters, and the Incommensuration Theorem concerns what is found. It forbids symmetry and continuity perfected together in the findings of one theory. It permits a theory whose findings are of one character and whose mathematics is of the other.

Why the Levels Must Differ

A theory holds its mathematics against its findings, and judges where the two agree. I identify a theory, for that reason, as a comparison of its mathematics with its findings, and the Incommensuration Theorem applies to it. Its first test removes every pairing of a character with itself, because such a pairing says nothing new: it is the character applied to itself.

Apply that to a theory. A mathematics in the same character as its findings would be that character applied to itself, and it could discover nothing about the findings that it did not already hold. I take each level of a theory, held exact at its foundation, to be perfected in one of the two characters the theorem allows: continuously asymmetric, or discontinuously symmetric. The first test forbids the mathematics to repeat the character of its findings, and only one character is then left, so the mathematics carries the other one, the character conjugate to its findings. Symmetric mathematics probes continuous findings.

Continuous mathematics probes symmetric findings. The six theories at the end of this chapter, each read as physicists already write it, show this arrangement.

A clock checked against itself is always right. It agrees with itself to the last tick, and the agreement tells you nothing about the time. To learn whether it keeps time, you must hold it against something of another kind: the turning of the Earth, the swing of a pendulum, the vibration of an atom. A theory is in the same position. Its mathematics learns about its findings only by being unlike them.

The cross-modal principle. The mathematics of a theory and its findings are always in conjugate characters. The reach of the mathematics into the findings is dense, and never exhaustive.

Dense means that the mathematics can come as close as you like to any finding. Never exhaustive means that it never holds the findings entirely within itself: something of the findings always lies beyond what the mathematics contains. A polygon with more and more sides comes as close as you like to a circle, and at no number of sides is it a circle. The last part of this chapter shows what that something is, for each theory.

You meet the principle in a weather forecast. The forecast is smooth: a seventy per cent chance of rain, falling gradually to forty by evening. What you find, when you step outside, comes in steps: it rains on you, or it does not. The forecast reaches as close to the day as the forecasters' skill allows, and the day itself is still something the forecast does not contain.

You meet it in the reverse direction in a street map. The map is printed on a grid of identical squares, each the same as the next. The streets it describes wind and branch, and no stretch of them repeats another. The grid's exact sameness lets the map locate every bend of streets that never repeat.

The Two Theories Through the Principle

Quantum theory finds in steps. Its findings are discrete: an atom is found in one level or another, a detector clicks or it does not. They carry exact labels: every electron has exactly the charge of every other. Its findings are discontinuously symmetric. Its mathematics is continuous: a state can hold its alternatives in any proportions, and it changes smoothly in time between findings.

An electron's spin shows both levels plainly. Measured along any one direction, the spin is found up or down, and never anything between. The state that gives the odds of those two findings can be any blend of the two, and in a magnetic field the blend turns smoothly, like the hand of a clock. The findings come in two exact steps; the mathematics that predicts them flows.

General relativity finds smoothly. Its findings are lengths, durations and curvatures, each varying without a jump from place to place, and in general no two places alike. Its findings are continuously asymmetric. Its mathematics is symmetric: its equations keep the same form under any relabelling of the points of space and time, and the same form in every small, freely falling laboratory.

Relabelling is easy to picture. The lines of latitude and longitude on a globe could be drawn in many other ways, and the coastlines and mountains would stay exactly as they are. General relativity's equations hold in every such choice of lines. That perfect indifference to labels is a symmetry of the mathematics. The coastlines themselves, the curved and uneven geometry, are what the theory finds.

So the puzzle dissolves. Quantum theory's continuity lives in its mathematics, and its symmetry in its findings. General relativity's symmetry lives in its mathematics, and its continuity in its findings. Neither theory perfects both characters in what it finds. The continuous symmetries that physics prizes are real, and they belong to the mathematics of the theories that use them.

Chapter 7 gave the other half of the reasoning. The law of symmetry and the theorem make perfected content discontinuously symmetric; the law of continuity and the theorem make perfected context continuously asymmetric.

Add the cross-modal principle, and each theory's mathematics follows from its findings. A theory of content finds in steps and calculates smoothly. A theory of context finds smoothly and calculates by symmetry. Since content and context are the only two sides, there are only these two arrangements.

Each arrangement then has one plain check. Quantum theory, as the theory of content, should find in steps: it does, in every discrete spectrum ever measured. General relativity, as the theory of context, should find smoothly: it does, in every measured length and duration. The two theories have exactly the arrangements the reasoning gives them.

Continuous Symmetries in Their Place

The principle also shows how the two levels work together. Emmy Noether showed in 1918 that each continuous symmetry of a theory's equations brings a conserved quantity. Equations that keep their form from moment to moment conserve energy; equations that keep their form from place to place conserve momentum.

Read that through the two levels. The continuous symmetry belongs to the mathematics: it is a property of the equations. What it delivers to the findings is a quantity that comes out exactly the same before and after every change, an exact label of the kind the law of symmetry gives to content. The smooth group acts in the mathematics, and the findings receive an exact invariant. Each level keeps its own character, and the link between them makes the theory useful.

Six Theories

The principle is a claim about every theory of physics, so it can be checked beyond the two great ones. Here are six.

Six theories, the character of what each finds, and the character of its mathematics:

Theory	What it finds	Its mathematics
Quantum mechanics	Discrete levels, exact labels, outcomes one by one: in steps, symmetric	Smooth states and smooth change: continuous
General relativity	Lengths, durations, curvature, varying smoothly: continuous, asymmetric	Same form under any relabelling of points: symmetric
Classical mechanics	Positions and speeds varying smoothly along paths: continuous, asymmetric	Fixed rules for trading one set of variables for another: symmetric
Classical electromagnetism	Electric and magnetic fields, waves of every strength: continuous, asymmetric	Gauge symmetry and the symmetry of special relativity: symmetric
Quantum electrodynamics	Photons counted one by one, charge exact: in steps, symmetric	Smooth states, sums over every path: continuous
Thermodynamics	Temperature, pressure, volume, varying smoothly and one way in time: continuous, asymmetric	Same form when a system is doubled; exact exchange of one description for another: symmetric

In all six, the two levels are in conjugate characters. The check involves no fitting: in each case both characters are read off the theory as physicists already write it, and the character given to the mathematics is the one it holds exact against the findings.

Two rows of the table deserve a second look. Classical mechanics and classical electromagnetism describe content: bodies and the electromagnetic field. Yet they find in the character of context, smoothly. The principle has no quar-

rel with that, since their mathematics is conjugate to their findings, as it must be. The question is why content can be described in two ways at all, and Chapter 11 answers it.

What Each Theory Leaves Over

The reach of a theory's mathematics into its findings is dense, and never exhaustive. So each theory leaves something over, at the place where its mathematics meets the limit of its own character.

For quantum theory, what is left over is the finding itself. The smooth mathematics gives the odds of each discrete outcome with extraordinary precision. In 2023 Xing Fan, Gerald Gabrielse and colleagues measured the magnetism of a single electron, and the quantum theory of light and charge agrees with that measurement to about one part in a trillion. Yet the mathematics gives odds, and the finding selects one outcome from among them. The selection is not written in the smooth mathematics. That is the measurement problem, and Part V takes it up as this residual.

For general relativity, what is left over is the singularity. Roger Penrose showed in 1965 that, inside a black hole, the equations of the established theory, followed far enough, give paths that cannot be continued, and in the cases worked out in full, a curvature that grows without limit along them. There the smooth description reaches the edge of its domain. The symmetric mathematics carries the smooth geometry with great power, and it cannot carry a geometry whose differences from place to place grow without bound.

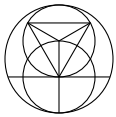
Each residual has the character of the theory's own mathematics meeting its limit. Quantum theory's continuous mathematics meets a discrete selection it does not contain. General relativity's symmetric mathematics meets an asymmetry it cannot follow to the end. The two great difficulties of the two theories are one pattern, seen from its two sides.

Every theory that holds conjugate levels against each other leaves a residual, because the reach of its mathematics is dense and never exhaustive. So each residual marks the edge of a theory's reach, at the place the principle predicts. The two residuals are often read as signs that each theory waits for a

deeper one to close a gap. On the principle, they are what seeing by conjugate levels always leaves over, and each sits exactly where the principle says it must. Parts V and VII return to each residual in turn.

> **Key idea.** I identify a theory as a comparison of its mathematics with its findings. Each level, held exact, takes one of the two characters the theorem allows, and a character compared with itself finds nothing new, so the two levels of every theory are in conjugate characters: this is the cross-modal principle. Quantum theory finds in steps and calculates smoothly; general relativity finds smoothly and calculates by symmetry. Six theories of physics show the same conjugate arrangement. Each theory leaves a residual where its mathematics meets its own limit: the finding, for quantum theory; the singularity, for general relativity.

If quantizing a theory changes its character, the next question is what exactly the change is, and why the arena cannot undergo it.



Chapter 11

What Quantizing Does, and Why the Arena Cannot Take It

Why can light be quantized, and space not? Because quantizing is a mode exchange of both levels of a theory together: its findings change from smooth to stepped, and its mathematics changes from holding its symmetries exact to holding its continuity exact. A description of content can take that mode exchange, because content is found within a context that the change leaves alone. The arena is that context. A quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. A mode exchange of the arena would also remove the setting in which every description is made. This chapter shows quantizing at work on light, then shows why the arena admits no mode exchange, and ends with the category error stated in full.

Quantizing, Shown on Light

Classical electromagnetism, the theory of light and charge completed in the nineteenth century, finds smoothly. Its electric and magnetic fields can have any strength, and its waves any size, however small. Its mathematics is symmetric: its equations keep their form under a change of the field's description called a gauge change, and under the changes of viewpoint of special relativity. Its findings are continuous and asymmetric; its mathematics is symmetric.

Quantum electrodynamics describes the same light and the same charges, and finds in steps. Light arrives in photons, counted one by one. Charge is exact, the same on every electron. Its mathematics is continuous: states that blend their alternatives in any proportions, and a sum taken smoothly over every path a photon or an electron could follow. Its findings are discontinuously symmetric; its mathematics is continuous.

What quantizing is. Quantizing a theory exchanges the characters of both its levels together. Its findings change from continuously asymmetric to discontinuously symmetric, and its mathematics changes from holding its symmetries exact to holding its continuity exact. The two levels stay conjugate; what changes is which level holds which character.

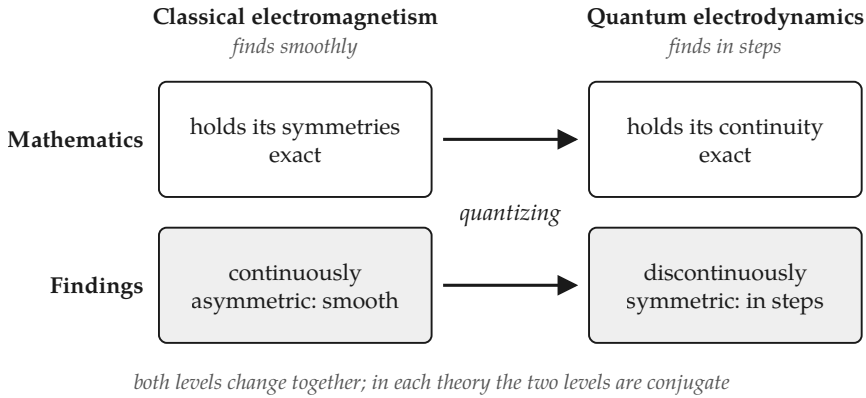


Figure 11.1. *Quantizing exchanges the characters of both levels of a theory together, shown here for the theory of light. Classical electromagnetism finds smoothly and holds its symmetries exact; quantum electrodynamics finds in steps and holds its continuity exact.*

A physicist will object that the classical theory and the quantum theory keep the same symmetries and the same calculus. That is so, and the mode exchange lies elsewhere. What changes is which of the two the mathematics

holds exact against the findings. In the classical theory, exact symmetries of the equations place findings that vary smoothly. In the quantum theory, a continuous space of states gives the odds of findings that come in exact steps.

You can watch light show its steps in a photograph. In bright light an image forms smoothly, and the classical picture of light describes it well. Turn the light down far enough, and the same image builds up as a scatter of separate dots, each the arrival of one photon, until the dots crowd together into the smooth picture. In the classical picture, the equations of the field hold their symmetries exact, and the brightness at each point of the image varies smoothly. In the quantum picture, a smoothly varying state gives the odds of a dot at each point, and each dot is one whole photon. The light was always content, and dim light shows its steps.

Why Classical Theories Work, and Where They Stop

Chapter 10 left a question open: classical mechanics and classical electromagnetism describe content, and yet they find smoothly, in the character of context. How can content be described in two ways?

Because it is content. A body or a field is something found within its context; it does not make up the context. So its description can pass by the mode exchange from one arrangement of the two levels to the other, and the context in which it is described stays as it was. Quantizing changes how content is seen, and leaves untouched the setting in which it is seen.

The classical description is valid. It captures content in the conjugate character, with smooth findings and symmetric mathematics, and for most of the world that is all anyone needs: bridges, orbits and radio are designed with it. The quantum description is the one in content's own character. So the classical description reaches its limits exactly where content's own steps begin to matter.

Two of those limits shaped modern physics. The classical theory of heat radiation predicted that a warm oven should hold an unlimited amount of energy at short wavelengths, an excess that physicists later called the ultraviolet catastrophe. Planck's law of 1900, which has no such excess, rests on energy exchanged in steps: Max Planck let the energy that passes between the radi-

ation and the walls of the oven come in steps, and he published the law in full in 1901. The classical picture of the atom gave a second limit: an electron circling a nucleus should radiate its energy away and spiral inwards, in about a hundred-billionth of a second. Niels Bohr in 1913 gave the atom separate levels, and the atom became stable.

In both cases the limit was removed by giving content its own character. The steps were already there; the classical description had been seeing them through the conjugate character.

The Check Against Content and Context

Five theories, the side each describes, and whether its findings have the character that side gives them:

Theory	Side it describes	Character of its findings	Agreement
Quantum mechanics	Content	In steps, symmetric: content's own character	Agrees
Quantum electrodynamics	Content	In steps, symmetric: content's own character	Agrees
Classical mechanics	Content	Smooth: the conjugate character, valid for bodies much larger than atoms	Agrees, as a classical description of content
Classical electromagnetism	Content	Smooth: the conjugate character, valid until light is dim or energetic enough to show its photons	Agrees, as a classical description of content
General relativity	Context	Smooth, asymmetric: context's own character	Agrees

The pattern in the table is the pattern of physics. Classical mechanics was made into quantum mechanics. Classical electromagnetism was made into quantum electrodynamics. Each mode exchange took a description of content

into content's own character, and each removed the limits of the description it replaced. The arena has a smooth description, general relativity, and that description is already in the character of what it describes. Ninety years of work on quantizing the arena, told in Chapter 2, have produced a great deal of physics. The arena's tested description is still the smooth one.

Why the Arena Admits No Mode Exchange

There are two reasons, and each is enough.

The first comes from the theorem. The arena is context, and its findings are continuously asymmetric: smooth, and different from place to place. To quantize it is to give its description findings in exact steps, with exact labels: the character of content. The one description would then have to carry both. If it kept its smoothness and gained content's exact sameness, it would carry continuity with symmetry. If it kept its unevenness and gained content's steps, it would carry asymmetry with discontinuity. Those are the two pairings that the Incommensuration Theorem forbids. A quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both.

The second comes from what a context is. Every description of content is laid out somewhere: in space, in time, in a range of possibilities. The context is that somewhere. When a description of content takes the mode exchange, its context stays in place to receive the new description. When the context itself is to take it, nothing stays in place. The change would have to be made in the very setting it was changing, and the continuity of the arena is the condition under which any description, of either arrangement, exists. The context of a comparison is where all describing happens, so it cannot be one more thing described in a new character. Such a mode exchange is incoherent, which is a different thing from being hard.

Chapter 8 adds a third view of the same fact. The arena carries the asymmetry that keeps the content consistent while the comparing stays connected. An arena made into content would have given up that asymmetry, and the asymmetry would have nowhere left to go.

Exactly What Is Excluded

The account predicts a great deal about gravity and the quantum, and Part IV sets out what. When matter is in two places at once, each alternative carries its own smooth geometry. Two small masses, each in two places and related only by gravity, should become entangled, and an experiment to test that is being built. The small waves of the arena can be described by quanta, and those quanta are quanta of the description. A detector that registers a gravitational wave in single clicks would not count against the account, and it would not show gravitation quantized, because a classical wave acting on a quantum detector gives the same clicks.

Physicists already work with such a description. Applied to the small waves of the arena about a smooth background, the standard method of quantizing gives a working account at the energies of every experiment so far, and it yields quantum corrections to Newton's law far too small to measure. That account is admitted here as a description. Its quanta are quanta of the waves' description, and none of them is a step of the arena itself or a quantum of the arena (the effective theory and its corrections are set out in *The Unquantized Arena*, Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale").

What is excluded is exact, and it has two parts. The arena has no least step: no smallest length, area or duration. And nothing is a quantum of the arena itself, at any scale.

At the scale physicists call the Planck scale, an object's gravitational size equals its quantum size, so neither description dominates and both are relevant. At every length far larger than the Planck length, the description of small waves by quanta and this account predict the same. The category error lies in the program of carrying a quantized arena through that scale and beyond, as the one description of everything. Chapter 15 sets out where the two descriptions meet.

The Chain in Full

Here is the whole argument of Part II and Part III, in nine steps.

1. The Incommensuration Theorem leaves two pairings that can be perfected at the foundation of a comparison: continuity with asymmetry, and symmetry with discontinuity. Continuity with symmetry, and asymmetry with discontinuity, are forbidden.
2. The law of symmetry governs content, and the law of continuity governs context.
3. So perfected content is discontinuously symmetric, coming in exact steps, and perfected context is continuously asymmetric, smooth and different from place to place.
4. The mathematics of a theory is always in the character conjugate to its findings: the cross-modal principle.
5. Content can be described in either arrangement of the two levels, because it is found within a context. Quantizing is the mode exchange of both levels together that brings a description of content into content's own character.
6. Gravity is the arena: it carries the four marks that any context must carry.
7. To quantize gravity is to give the description of the arena the character of content by a mode exchange.
8. A quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. The mode exchange would also remove the setting in which every description is made.
9. So the program of quantizing the arena asks a question of the wrong kind about it.

Each step rests on the ground of comparison or on established physics, and each identification in it is made in the first person, with the observations that test it named where they arise.

The Objection From the Theory of Light

The strongest objection to the chain comes from quantum electrodynamics, and it runs as follows. Quantum electrodynamics has a quantum foundation, and it recovers the smooth fields of the classical theory only as a limit, when photons are many. A quantum theory of gravity asks the same of the arena: a quantum foundation, with smooth geometry recovered as a limit. So, the objection concludes, an argument that condemns the second condemns the first, and quantum electrodynamics is among the best-tested theories in all of science.

The chain condemns quantizing the arena, and it never condemns quantizing a field of content on the arena. The electromagnetic field is content: it is found within the arena, at places and at times. Content's perfected character is discontinuous symmetry, so for light the quantum theory is the foundational form and the classical theory is its limit, and that is the order quantum electrodynamics has. The arena is the context of every finding, the findings of quantum electrodynamics included, and its perfected character is continuous asymmetry. A program that makes the arena quantum at its foundation, and recovers smooth geometry only as a limit, puts the character of content at the foundation of the arena, where the character of context is the one perfected. That inversion is the category error. Quantum electrodynamics makes no such inversion: its photons are counted at places and times that the arena supplies, and the arena keeps its own character throughout.

The Category Error, Stated in Full

Quantum theory and general relativity are the theories of the two sides of every comparison. Quantum theory describes what is found. General relativity describes where it is found. What is found comes in exact steps, because the law of symmetry governs it. Where it is found is smooth and different from place to place, because the law of continuity governs it.

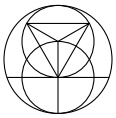
Quantizing is the mode exchange that gives a description of content its own character. It worked for matter, and it worked for light, because matter and light are content. To quantize the arena is to ask the setting of every finding to take on the character of the things found in it. That asks of the place where things are found what can be asked only of the things found there.

A category error is a question that applies a concept where the concept does not belong. The quantum is a concept that belongs to content. Applied to the arena, it asks the context to be what only content can be, and the Incommensuration Theorem forbids the description that would result. The search for a quantum theory of space and time, carried through the Planck scale as the one description of everything, rests on that category error.

So the two theories are two descriptions of one world, each perfect in its own character, each capturing what the other leaves out. They are to be joined side by side, and the Incommensuration Theorem forbids fusing them into one.

> **Key idea.** Quantizing exchanges the characters of both levels of a theory together: its findings from smooth to stepped, and its mathematics from holding its symmetries exact to holding its continuity exact. Content can take that mode exchange, because it is found within a context the change leaves in place, and the classical theories of matter are valid descriptions of content in the conjugate character, with the limits that quantizing removed. The arena is the context itself. A quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. A mode exchange of it would also remove the setting of every description. What is excluded is a least step of the arena and a quantum of the arena at any scale; a quantum field of content on the arena, as in quantum electrodynamics, is untouched. To quantize the arena is to apply the concept of the quantum where it does not belong: a category error.

With the error removed, what is left? Part IV begins with the simplest case: a mass in two places at once, and the question of where its gravity is.



PART IV

WHAT IS LEFT WHEN THE ERROR IS REMOVED



Chapter 12

One Geometry for Each Alternative

If matter can be in two places at once, where is its gravity? In both places, one geometry for each. Each alternative of the matter carries its own smooth geometry, shaped by that alternative's own energy. This chapter gives the reason for that answer, tells the experiment that ruled out the obvious rival, and shows why the answer sends no signal faster than light.

Two Places, Two Shapes

Quantum theory often describes a body as being in two places at once. The phrase is a shorthand. What the theory describes is two alternatives: in one the body is here, and in the other it is there. The theory holds both altern-

atives together, with a definite relation between them, until a finding selects one. Chapter 1 gave such a combination of alternatives its name, a superposition.

Part III identified the arena of space and time as the context of physics, and gravity as the shape of that arena. I will call the shape of the arena in a region its geometry. A body with energy curves the geometry around it; that is what general relativity, as an established theory, describes. So a body held in two alternatives raises a plain question. Which geometry goes with it: the one shaped by the body here, the one shaped by the body there, or some blend of the two?

The answer I give is the first two, each in its own alternative. In the alternative where the body is here, the geometry is the one a body here would shape. In the alternative where the body is there, the geometry is the one a body there would shape. Neither alternative carries the other's geometry, and no alternative carries a blend.

The Reason From the Ground

The reason comes from the inseparability of content and context. Where there is content, there is context, and where there is context, there is content. That principle holds of every comparison, so it holds of every alternative a comparison could find.

Each alternative of the matter is a possible content: it is one of the things a finding could find, and, found, it is content. So each alternative has a context. Contexts join into one only where what they hold is found together. The alternatives of the matter exclude each other: the body is never found both here and there. So no context is the context of both alternatives, and each alternative has a context of its own. The geometry is the context of the matter. So each alternative has its own geometry, shaped through Einstein's equations by the energy of that alternative alone.

A shared geometry would break the principle in a way that is easy to see. Take one geometry serving both alternatives, shaped by the body here and the body there together. Then the alternative in which the body is here would carry, over there, curvature of the kind that only a mass at that place pro-

duces, where in that alternative there is no mass to produce it. That curvature would be context with no content of its own, which is exactly what inseparability rules out.

Curvature in empty space is another kind, and general relativity is full of it. The space around the Sun holds no matter and is curved, and a passing gravitational wave curves the empty space it crosses. In each case the arena carries and spreads curvature that content elsewhere produced, and each alternative of the matter carries that kind of curvature around its own body.

Gravity can also be its own source. A strong wave of gravity carries energy, and enough of it gathered in one place could form a black hole with no matter in it at all. I reason that such curvature belongs to the context of the content that produced it, and carries that content's alternatives. Where no content is in superposition, there is one geometry. A black hole formed from waves has the alternatives of whatever made the waves.

An architect drawing two possible sites for one house makes the same point. Each drawing shows the house and the shadow the house casts at noon. If the two drawings are laid over each other and averaged, the result shows half a shadow on each site, cast by half a house on each site. No house that could be built casts that shadow. Each drawing is right about its own shadow, and the average is right about nothing.

This answer quantizes nothing. Within each alternative the geometry is smooth, continuous and different from place to place, exactly as general relativity describes it, with no least step. The separateness of the alternatives belongs to the matter, whose findings come in discrete steps, as content does. The geometry follows the matter's alternatives; it does not acquire steps of its own.

When the matter is sharply one alternative, as a star, a planet or a black hole of definite mass is, there is one alternative and one geometry, and that geometry is the familiar one. Every calculation that astronomers and physicists make with Einstein's equations for stars, planets, galaxies and black holes stands as it is.

The Average Geometry

There is a simpler-looking way to join quantum matter to smooth geometry, and it was proposed early, in the 1960s. Christian Møller in 1962 and Léon Rosenfeld in 1963 kept a single geometry for all the alternatives together, shaped by the average of the alternatives' energies, each weighted by its probability. Their proposal was a careful and natural first step, and it gives the right answer whenever the matter is sharply one alternative.

For a body in a superposition, though, the averaged geometry is the blended drawing. A body that is here with probability one half and there with probability one half would shape the geometry of half a body here and half a body there. Two things count against that geometry: an experiment, and the speed of light.

The Experiment of Page and Geilker

In 1981 Don Page and C. D. Geilker put the averaged geometry to a direct test, with a kind of instrument more than a century and a half old. Henry Cavendish had used a torsion balance in 1798 to weigh the Earth. A light bar hangs level from a thin wire, with a small lead ball at each end. Large lead balls placed near the small ones pull them sideways, and the bar turns a little until the twist of the wire balances the pull. The turn of the bar shows the gravity of the large balls.

Page and Geilker let a quantum event decide where the large balls went. Before each run, a Geiger counter registered the decays of a radioactive source, and its reading chose between two positions. In one position the large balls turned the bar one way; in the other position they turned it the other way. A radioactive decay is a quantum event: until a finding registers it, quantum theory describes both outcomes together.

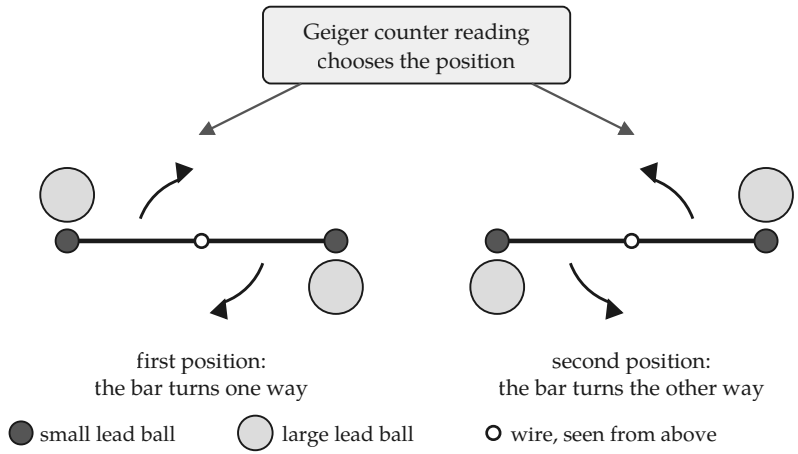


Figure 12.1. *The torsion balance of Page and Geilker, seen from above. The reading of a Geiger counter put the large lead balls in one of two positions, and the two positions turn the bar in opposite directions.*

Now follow the averaged geometry, in the form in which the quantum description is never reduced to one outcome. The large balls are then in both positions, each with weight one half. Their averaged pull on the bar is half a pull one way and half a pull the other way, which is no pull at all. On that account the bar should not turn.

The bar turned on every run. Sometimes it turned one way and sometimes the other, about equally often, and each time it turned the way the actual position of the large balls required. The gravity followed the outcome, not the average.

Page and Geilker titled their paper "Indirect Evidence for Quantum Gravity". What the experiment shows directly is that the field follows the outcome of the quantum event. A geometry for each alternative gives exactly that result. In each alternative the large balls are in one definite position and shape

the geometry of that position, and on each run the reading of the Geiger counter selects which alternative is found. The result needs no geometry with steps; it needs only that each alternative of the matter has its own geometry.

No Signal Faster Than Light

A defender of the averaged geometry has one move left. Let the weights of the alternatives change when a finding is made, so that after the Geiger counter registers, the average becomes the actual position. That version escapes the experiment and meets the second difficulty, which is the speed of light.

Put a body here into a superposition of two places, entangled with a partner particle in a laboratory on the Moon, which light takes a little more than a second to reach. The experimenter on the Moon can choose between two questions to put to the partner. One question leaves the body here, on the averaged account, sharply in one place or the other. The other question leaves it in an even combination of both. Under an averaged geometry whose weights change with the distant finding, the gravity here would then differ with the question asked. After the first question it would point toward one place or the other; after the second it would point toward the midpoint.

A sensitive balance here could then read which question was asked on the Moon, before any light from the Moon could bring the news. That is a signal faster than light. Relativity excludes it, and no experiment has ever found one. Mohammad Bahrami and colleagues set out this consequence in 2014: a single geometry shaped by the quantum state, together with the change of weights at a finding, sends signals faster than light.

So the averaged geometry fails either way. If its weights never change, it fails the experiment of Page and Geilker. If its weights change when a distant finding is made, it sends signals faster than light.

A geometry for each alternative meets neither difficulty. No geometry is shaped by the weights, so no geometry has a change of weights to follow. Each alternative already carries its geometry before any finding is made. A finding selects one outcome from among its possibilities: something possible becomes manifest for the comparer that finds. The finding changes no geo-

metry, here or on the Moon. What a distant finding changes for a comparer here is its record, and the record changes only when a signal carrying the finding arrives. That signal travels no faster than light, so it arrives at the earliest on the light cone of the finding, the set of places and times that light from the finding can reach. Each alternative carries its own geometry, and so its own light cones, and a finding is made on the light cone of the alternative it selects. Within one geometry the light cone is the same for every comparer, however each one moves. Where the geometries of the alternatives differ, the order of two events can differ from one alternative to the other, and Magdalena Zych, Fabio Costa, Igor Pikovski and Časlav Brukner showed in 2019 how a mass held in two places can leave that order itself in superposition. I follow an event from one alternative to the next by what happens in it, and to which bodies, and not by its position on a map.

Until that news arrives, everything a comparer here can find follows the ordinary rules of quantum theory, and those rules give the same odds here whichever question was asked on the Moon. No comparison registers a single "now" spread across space, on which a selection would be made for every comparer together, and this account needs none.

The argument of Eppley and Hannah, met in Chapter 2, has two other horns, and this account meets them too. Both horns use a gravitational wave of very short wavelength as a probe of a quantum body, and both assume one classical field for all of the body's alternatives. On this account each alternative of the body carries its own geometry, so the wave is scattered in each alternative separately, and the probe is drawn into the body's alternatives as any quantum probe is. Reading the probe is then an ordinary finding, and it keeps the uncertainty principle and the conservation of momentum, as every quantum measurement does. James Mattingly showed in 2006 that an apparatus able to make their measurement would be so massive that it would lie inside its own horizon, as a black hole.

What Is Left

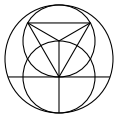
Remove the category error and this is what remains. The matter is described by quantum theory, with its alternatives and their findings. Each alternative of the matter carries its own smooth geometry, shaped by its own energy through Einstein's equations. The definite relation between the alternatives is kept. That is the whole configuration.

It is not an approximation to a quantum theory of geometry, because on this account there is no quantum theory of geometry for it to approximate. It is the meeting of the two theories as each one stands: quantum theory for the content, general relativity for the context, and inseparability joining each content to its context.

One feature of this configuration does more than remove a difficulty. The relation between the alternatives is kept, and each alternative carries a geometry of its own. So two bodies, each in two places and related only by gravity, should end up joined in the way quantum theory calls entanglement. That is a prediction, and an experiment to test it is being built.

> **Key idea.** Each alternative of the matter carries its own smooth geometry, shaped by that alternative's own energy, because each alternative is a content and every content has its context. The averaged geometry of Møller and Rosenfeld fails the experiment of Page and Geilker (1981), in which gravity followed the outcome of a quantum event, not the average over outcomes. A version whose weights change at a finding would send signals faster than light. A geometry for each alternative quantizes nothing, changes at no finding, and sends no signal faster than light.

If gravity follows each alternative separately, two masses related only by gravity should become entangled, and the next chapter describes the experiment that will show whether they do.



Chapter 13

The Experiment Being Built

Can a geometry for each alternative be tested? Yes, and the test has been designed. Take two tiny diamonds, put each one in two places, and hold them close enough that gravity is the only thing relating them. The account of the last chapter predicts that the two diamonds become entangled, with a phase of about a third of a radian for the design proposed in 2017. A single shared geometry predicts no entanglement at all.

Two Diamonds, Each in Two Places

The question is older than the design. It is the question Feynman asked at Chapel Hill in 1957, met in Chapter 2: if a ball is in two places, what is its gravitational field? He reasoned that there must be an amplitude for the field, and so that the field must be quantized. A geometry for each alternative gives his amplitude another reading: one smooth field for each place of the ball, with the relation between the two places kept.

In 2017 two groups published the idea side by side. Sougato Bose and nine colleagues proposed the experiment in detail, and Chiara Marletto and Vlatko Vedral gave the argument for what its result would mean. The leading candidate in the design of Bose and colleagues is a speck of diamond with a mass of about a hundredth of a billionth of a gram, about two thousandths of a millimeter across, far thinner than a hair.

Each speck carries one flaw in its crystal: a nitrogen atom sitting beside a gap where a carbon atom should be. That flaw carries a spin, the built-in rotation that Chapter 2 defined, and its spin makes the flaw a tiny magnet. The

spin can point up, point down, or be held in both alternatives together. A magnetic field that changes strength across space pushes the diamond one way if its spin points up, and the other way if its spin points down. With the spin held in both alternatives, the diamond is carried along both paths, and ends up in two places about a quarter of a millimeter apart.

Two such diamonds are set side by side, their two pairs of places in one line, with their centers a little less than half a millimeter apart. They are held so for about two and a half seconds. Then each diamond's two paths are brought back together, and each spin is read. One run gives two readings. Many runs give the pattern in the readings, and the pattern shows whether the diamonds were entangled.

The hard part is to keep everything except gravity away. A single stray molecule of air striking a diamond, or a single unit of heat radiation absorbed by it, would register which place the diamond was in, and its two alternatives would no longer recombine. So the diamonds must be held in a near-perfect vacuum and kept very cold. Electric forces between the two diamonds also join them, and at shorter distances they would far outweigh gravity. Thomas van de Kamp and colleagues proposed in 2020 to put a thin conducting plate between the two diamonds to block those forces, so that the diamonds could be smaller. The plate blocks electric forces. Nothing blocks gravity, which is one of the four marks of the arena found in Chapter 9, "Four Marks of the Arena".

How Gravity Writes a Phase

To see what the experiment measures, one more idea is needed. Every alternative in quantum theory carries a phase. A phase is a turning quantity, like the hand of a clock, and the rate at which an alternative's hand turns is set by the energy of that alternative. When alternatives are brought back together, the angles between their hands decide the odds of what is found.

Two diamonds, each in two places, make four joint alternatives. Call each diamond's place nearer the other diamond its near place, and the other its far place. In one joint alternative both diamonds are in their near places, about a fifth of a millimeter apart. In another both are in their far places, about seven tenths of a millimeter apart. In the two remaining alternatives one diamond is near and the other far, and the two are a little less than half a millimeter apart.

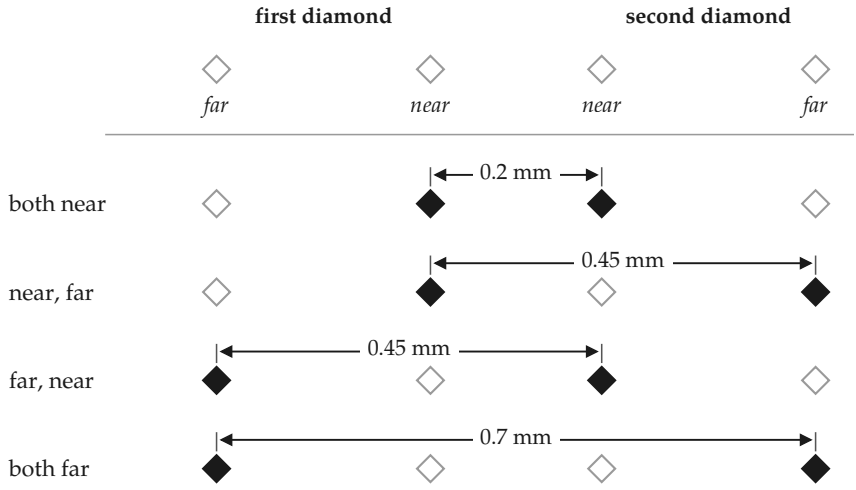


Figure 13.1. *Two diamonds, each in two places, make four joint alternatives at three distances. The diamonds are about a fifth of a millimeter apart when both are near, about seven tenths of a millimeter apart when both are far, and a little less than half a millimeter apart in each mixed alternative.*

Gravity between the two diamonds has an energy, and the energy depends on their distance. The closer the diamonds, the larger the energy of their attraction. So in each joint alternative the clock hand turns at its own rate, and over two and a half seconds the four hands drift out of step with one another.

The drift is tiny, and still large enough to find. A quantum phase counts energy multiplied by time, in units of Planck's constant, the constant that sets the size of every quantum step. The energy of attraction between two such diamonds is far too small to feel, but Planck's constant is smaller still on the scale of everyday things, and two and a half seconds is a long time for a body to stay in two places. The design balances three demands: diamonds heavy

enough for their gravity to turn the phase, light enough to be put in two places at all, and far enough apart that the electric forces between them stay below their gravity.

Nothing in the calculation goes beyond Newton's law of gravity. In each joint alternative the two diamonds are at one definite distance, the geometry of that alternative is the one their masses shape at that distance, and for slow bodies that geometry gives Newton's energy of attraction. The phase of each alternative is that energy, times the time, in units of Planck's constant.

Part of the drift of the four hands tells nothing about the two diamonds together, because it could be produced, or undone, by adjusting each diamond on its own. One combination of the four drifts cannot be produced that way. Add the drift of the alternative with both diamonds near to the drift of the alternative with both far, and subtract the drifts of the two mixed alternatives. If that combination is not zero, the two spins are entangled.

Two systems are entangled when their alternatives are joined so that neither has a state of its own. The findings on the two are then correlated more tightly than any exchange of ordinary signals between them could arrange. For the design proposed in 2017 by Bose and colleagues, the combination comes to about a third of a radian: about eighteen degrees, or a twentieth of a full turn of the clock hand. Later designs use other masses, separations and times, and the same formula gives their phases. That is the prediction of this account for the experiment (the calculation is set out in *The Unquantized Arena*, Chapter 8, "The Entanglement Test").

Why One Geometry Cannot Entangle

The argument of Marletto and Vedral turns on one principle of quantum theory. Two systems that interact only through a go-between in one definite state cannot become entangled through it. Such a go-between can carry ordinary signals, and ordinary signals can make the two systems agree; they cannot make the two entangled. Two people in separate rooms who pass written notes can coordinate their answers to any question, and still no exchange of notes can give their answers the tighter correlation of entangled systems.

The averaged geometry of Chapter 12 is a go-between of that kind. It is one geometry, the same for all four joint alternatives. The pull between the diamonds is then the same in every alternative, the four clock hands turn alike, and the combination that signals entanglement stays at zero. Any account in which the geometry is a go-between in one definite state, the same for all the alternatives, predicts no entanglement. Jonathan Oppenheim's theory of 2023, in which geometry stays classical while matter is quantum, is one such account.

A geometry for each alternative is a go-between of the other kind. Each joint alternative carries its own geometry, shaped by the diamonds where they are in that alternative. So each joint alternative has its own energy of attraction and its own turning rate, and the relations among the alternatives are kept. The combination grows, to about a third of a radian for the design proposed in 2017, and the diamonds become entangled.

A third family of proposals predicts no entanglement for another reason. Roger Penrose argued in 1996 that gravity itself brings a superposition of a mass in two places to an end, sooner the more the two places differ in their gravity. For diamonds of this mass held this far apart, his estimate ends each superposition within a small fraction of a second, long before two and a half seconds have passed and the entanglement has had time to build.

What the Result Separates

So the experiment separates the accounts into two groups. In the first group are the accounts with a geometry for each alternative of the matter, and they predict entanglement. In the second group are the accounts with one geometry for all the alternatives, and the accounts in which gravity ends superpositions; they predict none. A clean result, either way, removes one group.

The prediction of this account is entanglement, with a phase of about a third of a radian for the design proposed in 2017.

What the Result Leaves Together

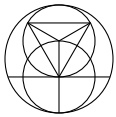
The experiment does not separate this account from linearized quantum gravity. Linearized quantum gravity describes the small waves of gravity on a smooth background by quanta, the description Chapter 14 takes up. For two slow masses this far apart, it uses the same energy of attraction in each joint alternative, and so it predicts the same phase. At every length far larger than the Planck length, the two accounts make the same predictions. They differ in what lies at the Planck scale and beyond, which is the subject of Chapter 15.

Marletto and Vedral conclude that if gravity entangles two masses, gravity cannot be a classical go-between in one definite state. I agree with that conclusion as they state it. On this account the geometry takes a different form in each alternative of the matter, and the relations among the alternatives are kept. Bose and colleagues titled their proposal a witness for quantum gravity. On this account, what the entanglement would witness is a geometry for each alternative. It would not show a step of the arena itself, or a quantum of the arena at any scale. The entanglement is written by the matter's alternatives, each with its own smooth geometry.

The proposals of 2017 have not yet been carried out. Work on their separate parts is under way: holding diamonds in vacuum, controlling their spins, and splitting and rejoining their paths. When the whole experiment runs, it will decide between the two groups of accounts.

> **Key idea.** Two diamonds, each in two places and related only by gravity, give four joint alternatives at three different distances. With a geometry for each alternative, each joint alternative turns its phase at its own rate, and the diamonds become entangled, with a phase of about a third of a radian for the design proposed in 2017 by Bose and colleagues. One shared geometry, or gravity that ends superpositions, predicts no entanglement. Linearized quantum gravity predicts the same entanglement as this account, so the experiment does not separate the two.

Linearized quantum gravity speaks of gravitons, and the next chapter asks what the word "graviton" can name on this account, and what a detector's clicks would mean.



Chapter 14

Waves Without Particles of Space

Are there gravitons? Gravitation is not quantized. Physicists can describe small waves of geometry by quanta, and the word "graviton" names at most a quantum of that description. It never names a piece of space and time, a quantum of the arena, or a particle that carries gravity. A detector that registers a gravitational wave in single clicks would not count against this account, and it would not show that gravitation is quantized.

The Objection as a Reader Puts It

The objection is a strong one, and it deserves its full form. On 14 September 2015 the two LIGO detectors described in Chapter 1 registered a wave of gravity from two black holes merging far away, and the LIGO and Virgo collaborations reported it in 2016. The objection starts from that wave. It says that every other kind of wave in physics has turned out to come in quanta: light comes in photons, and sound in a crystal comes in phonons. It concludes that waves of gravity must come in gravitons, and that if gravitons exist, gravity is quantized like everything else.

The objection holds one true thing: small waves of geometry can be described by quanta, and the description works within its range. Its conclusion does not follow. Light is a field within the arena, and sound in a crystal is a vibration of a crystal's atoms; the arena is neither. To see why the conclusion fails, it helps to look closely at what a phonon is, because the phonon is the objection's own example.

What a Phonon Is

A crystal is a lattice of atoms held in place by the bonds between them. Strike it, and a wave of motion runs through the lattice: each atom swings a little about its place and passes the swing to its neighbors. That wave is sound.

Quantum theory says that each way the lattice can vibrate takes up energy only in equal steps. One step of energy in one way of vibrating is called a phonon. Phonons are counted and used every day in physics. They carry heat through a crystal, and they account for how little heat a very cold solid can hold.

Yet nobody concludes from phonons that a crystal is made of phonons. The crystal is made of atoms. The phonon is a step in the energy of one way the atoms can move together; it is a quantum of the description of the motion. Nor does anyone conclude that the spacing between the atoms comes in phonon-sized steps. The phonon counts the energy of a vibration, and says nothing about a least distance in the lattice.

A crystal is a held structure: a structure whose parts are held in place by bonds, as the bonds of the crystal hold its atoms in the lattice. A phonon is a quantum of the description of how a held structure vibrates.

The Small Waves of Geometry

A gravitational wave is a ripple in the geometry itself. As it passes, it stretches distances in one direction and squeezes them in the other, and then the reverse. The wave LIGO registered in 2015 changed the length of each four-kilometer arm of the detector by far less than the width of a proton. That is a very small wave on a very smooth background.

Physics has a careful description of such small waves. John Donoghue showed in 1994 how to treat general relativity as an effective theory, the kind of description met in Chapter 2: one that holds within a stated range, here gentle curvature and small waves, and that states its own range. Within that range, physicists apply quantum theory to the description of the waves in the usual way. In that description each way the geometry can ripple takes up en-

ergy only in equal steps, and one step is the graviton that Chapter 2 named: a quantum of the description. The same description gives tiny quantum corrections to Newton's law of gravity. At a distance of one meter, the corrections are about one part in a number seventy digits long.

The phonon is the nearest sense by which the word "graviton" can be made intelligible: a phonon is a quantum of the description of how a held structure vibrates. The arena is not a held structure. It has no parts that bonds hold in place, as the atoms of a crystal are held, and its shape is set by the matter and energy in it. So even the phonon's sense does not make the graviton a quantum of the arena. A graviton names at most a quantum of the description of a wave. It is not a step in the extent of the geometry: the lengths, areas and durations of the arena keep no least value.

The Incommensuration Theorem allows this description, as a description. What it forbids, as Chapter 11 found, is a quantized arena: the arena given steps of its own, as content has. A description that counts the energy of a small ripple in steps, on a background that stays smooth, gives the arena no steps: the steps belong to the description, and the ripple is a ripple of an arena that is not quantized.

Where the Description of Small Waves Ends

The description of small waves rests on one condition: the waves are small compared with the smooth geometry they ride on. That condition holds everywhere physics has looked, by an enormous margin. The range of the description ends at the Planck curvature: the curvature at which the quantum corrections become as large as the effects they correct. There a ripple would be as large as the geometry it rides on, and the difference between a wave and the background that carries it disappears. At that point the description of small waves reaches the edge of its domain.

So the word "graviton" has a use within that range, as a name for a quantum of the wave's description, and no use outside it. This account takes no step from the description to the arena: it never treats the graviton as a

quantum of gravitation, as a building block of space and time, or the description of small waves as the start of an exact quantum theory of the whole arena. That step would be the category error of Chapter 11, applied to waves.

Two Questions Kept Apart

The objection runs two questions together, and the answer keeps them apart. The first question is whether small waves of geometry can be described by quanta. The second is whether gravitation, the arena itself, comes in steps. The answer to the first is yes, within the range of the description of small waves, and the steps belong to the description. The answer to the second is no, at every scale.

Light shows why the two questions differ. Quantizing light was the right step, as Chapter 11 found, because the electromagnetic field is a field within the arena: a content, whose findings come in countable steps. Quantizing light gives the field steps; it gives the space the light crosses none. A gravitational wave is different in kind. It is not a field within the arena; it is a ripple of the arena. Counting the energy of the ripple in steps is a way of describing the ripple, and the arena that ripples stays smooth.

A sound wave in a crystal and a gravitational wave differ in kind. The crystal is a held structure, built of atoms that bonds hold in place, and the phonon is a quantum of the description of how those atoms move together. On this account the arena is built of no smaller pieces. It has no parts of the kind the atoms are to the crystal, because it is the context in which every part is found. So the phonon makes the word “graviton” intelligible, and it makes nothing of the arena a quantum.

The smooth description also accounts for everything LIGO and Virgo have measured. In the description by quanta, a wave strong enough to register in those detectors is counted as so many gravitons that the count plays no part in what the detectors see. In the same way, a beam of ordinary light carries so many photons that the steps of its energy play no part in its brightness.

Has Anyone Detected a Graviton?

No single graviton has ever been detected. Freeman Dyson asked whether one could be detected even in principle, and suspected that none could. In a lecture of 2012, published in 2013, he showed that a detector of the LIGO kind cannot register a single graviton. Such a detector measures the distance between two mirrors. To make the quantum noise small enough to see one graviton, the mirrors would have to be so heavy that they would fall together into a black hole. He examined two other kinds of detector and did not claim that either could do it.

Tony Rothman and Stephen Boughn had worked through Dyson's question in 2006. They looked at a detector that absorbs a single graviton, as an atom absorbs a single photon. They found that a detector with the mass of Jupiter, in close orbit around a neutron star, might register at most about one graviton in a century. Without shielding, neutrinos passing through it would cause a billion trillion trillion events or more for each graviton event. Their conclusion was that Dyson's suspicion is very likely right.

In 2024 Germain Tobar, Sreenath Manikandan, Thomas Beitel and Igor Pikovski proposed a different route. A heavy resonator, cooled to its lowest state of vibration, is watched for single jumps of energy at the moment LIGO registers a passing wave. In their description by quanta, a wave from merging neutron stars carries so many gravitons that a resonator of this kind would absorb one with high probability. Its authors argue that this signature could be observed in a laboratory experiment.

Daniel Carney, Valerie Domcke and Nicholas Rodd asked, also in 2024, what such a click would show. Their answer is that a click from a single absorbed quantum does not by itself show that gravity is quantized. A classical wave acting on a quantum detector gives the same clicks. Light shows the same thing: the photoelectric effect, in which light knocks single electrons out of a metal, is explained equally well by a classical wave of light acting on the quantized atoms of the metal.

The Ruling on Detection

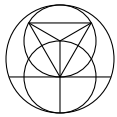
The account's position on gravitons can now be stated exactly.

Gravitons and their detection. Gravitation is not quantized. Small waves of geometry can be described by quanta, and a graviton is at most a quantum of that description, never a quantum of the arena. A detector that registers a gravitational wave in single clicks would not count against this account. It would not show gravitation quantized, because a classical wave acting on a quantum detector gives the same clicks: what the detector registers is a step of its own, a step of content, against the arena as its context. What the account excludes is a step of the arena itself, and a quantum of the arena at any scale. No detection of a small wave can find either.

So the place to test this account is not the graviton detector. The tests are the entanglement experiment of Chapter 13, "The Experiment Being Built", and the smoothness of space and time, which Chapter 15 takes up.

Key idea. Gravitation is not quantized. A graviton is at most the quantum of a description of a small wave of geometry, never a quantum of the arena. The phonon, a quantum of the description of how a held structure vibrates, gives the word its nearest sense, and the arena is not a held structure. A graviton is a step in the description of one way of rippling, not a step in the extent of space and time. The description holds while the waves are small compared with the geometry they ride on, and reaches the edge of its domain at the Planck curvature. No graviton has been detected, and a single detection is extraordinarily hard (Dyson 2013; Rothman and Boughn 2006). A detection would not count against this account and would not show gravitation quantized, because a classical wave acting on a quantum detector gives the same clicks.

The description of small waves ends where curvature reaches the Planck scale, and the next chapter asks what the Planck scale is.



Chapter 15

Where the Two Descriptions Meet

What happens at the smallest scales? The two descriptions of the world meet there. The Planck scale is where an object's gravitational size equals its quantum size; there neither description dominates, and both are relevant. The Planck length is not a least length; space and time have none. The role of the Planck scale is fixed by the ground of this book, and its size by the measured value of Newton's constant.

Two Sizes for Every Mass

Every mass has two lengths that belong to it, one from each theory.

The first is its gravitational size. It measures how strongly the mass curves the geometry around it: squeezed inside about twice this length, the mass would be a black hole. The gravitational size grows with the mass. For the Earth it is about four and a half millimeters, and for the Sun about one and a half kilometers.

The second is its quantum size. It is the length within which quantum theory will not let the mass be pinned down without spending enough energy to make new particles of its kind. The quantum size shrinks as the mass grows. For a proton it is about an eighth of the proton's own width. For the Earth it is so small that writing it as a fraction of a meter takes sixty-seven zeros after the decimal point.

Compare the two sizes and each familiar thing falls on one side. For a proton, the quantum size is larger than the gravitational size by a factor thirty-nine digits long. Quantum theory describes the proton, and the proton's own

gravity plays no part at any length where the proton can be found. For the Earth, the gravitational size is the larger, by a factor sixty-five digits long. General relativity describes the Earth's gravity, and the Earth's quantum spread plays no part in it.

Since one size grows with the mass and the other shrinks, there is one mass at which they are equal. That mass is the Planck mass, about twenty-two millionths of a gram. It is not small by everyday standards: a grain of quartz sand a quarter of a millimeter across has about that mass. At the Planck mass, both sizes come to the same length, the Planck length, about a hundred billion billion times smaller than a proton. Max Planck first wrote down these quantities in 1899, as units of length, mass and time built from the constants of nature alone.

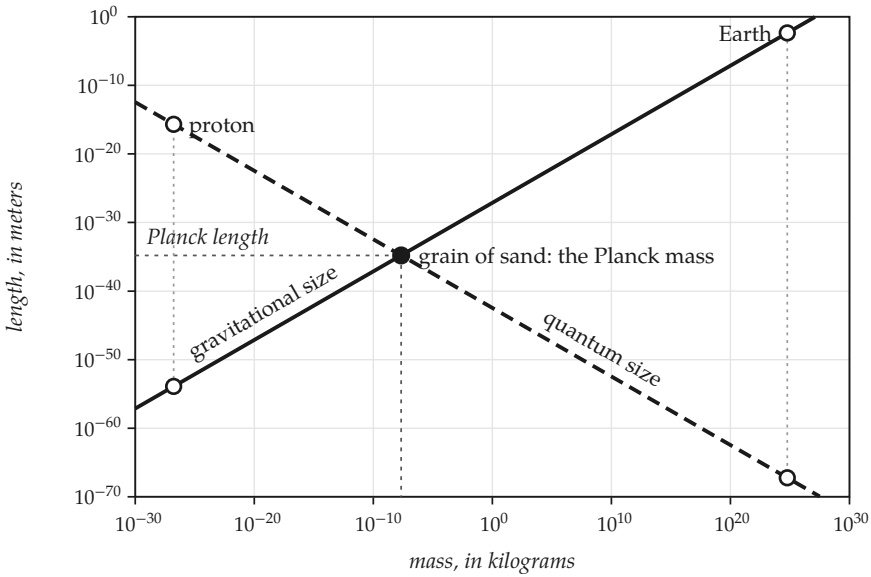


Figure 15.1. *The gravitational size of an object rises with its mass and its quantum size falls, on axes marked in powers of ten; the two cross at the Planck mass, where both equal the Planck length. The proton lies where the quantum size is far the larger, the Earth lies where the gravitational size is far the larger, and a grain of sand a quarter of a millimeter across has about the Planck mass.*

The grain of sand is far larger than either of its two sizes, so for the grain neither size is anything a measurement would notice. The two sizes meet only for a mass of the Planck mass held within a region of the Planck length. That is the Planck scale: a meeting of two lengths that, for every familiar thing, are far apart.

What the Meeting Means

Part II found that comparison, perfected at its foundation, takes one of two characters. It is either continuously asymmetric or discontinuously symmetric. I identify general relativity with the first character and quantum theory with the second. Each description is perfected in its own character, and each is the right description wherever its character dominates.

The two sizes of a mass say which character dominates. Where the quantum size is much the larger, the discrete findings of quantum theory govern, and the object's own gravity is a faint background. Where the gravitational size is much the larger, the smooth geometry governs, and the object's quantum spread is too small to matter. At the Planck scale itself, where the two sizes are equal, neither description dominates, and both are relevant.

The Incommensuration Theorem says what follows. A single description that carried both characters there, perfected together in what it finds, would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. So the Planck scale is not the doorway to a third theory that replaces the other two. It is the place where two descriptions are both needed and stay two. What is new at the Planck scale is not a new kind of physics waiting to be found; it is the meeting of two descriptions, each complete in its own character.

This also says how far the smooth description holds. The quantum corrections to Einstein's equations, the ones described in Chapter 14, "Waves Without Particles of Space", grow with the curvature of the geometry. Far below the Planck curvature they are negligible: for the attraction of two masses a meter apart, they are about one part in a number seventy digits long. They

become as large as the effects they correct only at the Planck curvature itself. So the smooth description holds wherever the curvature is below the Planck curvature, which is everywhere it has been tested, by a vast margin.

Where the curvature would reach the Planck value, as general relativity says it does at a black hole's singularity, the smooth description reaches the edge of its domain. That edge belongs to the description: every description has a domain, and this is where the domain of the smooth description ends. I claim no third description beyond that edge, and no new geometry there. The structure of comparison still holds there, because every finding, at any scale, is a comparison.

Not a Least Length

The Planck length bounds how sharply a comparison made from inside the world can place a body, and the arena itself keeps no least length. The Planck length is often described as the shortest length there can be. The standard view, as many physicists state it, adds that space and time dissolve there into a foam of quantum fluctuations. The argument behind that view is worth stating. To find where something is to within a very short distance, a probe of very short wavelength is needed, and a short wavelength means a high energy. To probe a region as small as the Planck length, the probe would carry so much energy into so small a region that it would make a black hole larger than the region probed.

I read that argument for what it shows. It concerns how sharply a comparison made from inside the world can place something. It is a bound on findings, not a step of space. The Planck length is the bound of every comparison made from a place inside the world, and the arena in which those comparisons are made has no least length, area or duration.

A light microscope shows the difference. An ordinary light microscope cannot resolve details much finer than about half the wavelength of the light it uses. Nobody concludes that a cell is built of tiles that size. The limit belongs to the way the cell is looked at, and the cell is as fine as it is. In the same way, the Planck length limits how finely a comparison can place a body, and the arena keeps no least step.

The ground gives the reason. A least step would make the arena discrete, the character of content, while what is found of the arena stays continuous and different from place to place, the character of context. One description of the arena would then carry both characters together, and so it would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. That is the quantized arena of Chapter 11.

Observation agrees. In 2009 the Fermi space telescope caught a burst of gamma rays, cataloged as GRB 090510 (the letters for gamma-ray burst, the digits for its date), whose light had traveled for billions of years. An electronvolt is the energy an electron gains in crossing one volt. A photon carrying about 31 billion electronvolts arrived within about a second of the low-energy light of the burst. If space had a grain at the Planck length that made the speed of light depend on its energy, in the simplest way such a grain would, that photon would have arrived noticeably late. It did not, and the Fermi team concluded that any such grain must be smaller than the Planck length (the evidence is set out in Chapter 27, "What the World Will Show").

Its Role and Its Size

The ground of this book fixes the role of the Planck scale: it is where the two descriptions meet. The ground also fixes gravity's role, by the four marks found in Chapter 9: gravity couples to everything, nothing shields it, inertial and gravitational mass are one, and it has no charge of its own that comes in steps. The ground does not fix the size of the Planck scale in meters or kilograms, and it does not need to.

The reason is that the axioms are structural, not dimensional. They concern relations among comparisons: which character goes with which, which pairings survive, what comes before what. An absolute magnitude is a relation of a different kind, between the world and a unit of measurement brought to it. A description made only of relations can fix every proportion of the world and still fit a world scaled up or down as a whole. One datum read off the world is needed to fix the scale.

That datum is Newton's constant, the number that sets the strength of gravity. It is measured, with torsion balances that descend from the one Cavendish built, and with other methods. Its value is known to about two parts in a hundred thousand, less precisely than any other constant of nature in common use. The axioms are about comparison as such, not about any particular domain of comparison. The numerical value of Newton's constant selects the domain: it says at what scale the structure of comparison is realized in this world. From that value, with the speed of light and Planck's constant, the Planck length and the Planck mass follow.

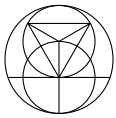
A map makes the same division. A surveyor can draw every proportion of a valley correctly, every bend of the river and every angle between the roads, and the map still needs one line in its legend that says how many meters one centimeter stands for. That line is read off the ground, not off the drawing. The axioms draw the proportions; the measured value of Newton's constant writes the legend.

What the Account Claims Here

At every length far larger than the Planck length, this account and linearized quantum gravity make the same predictions, as Chapter 13 found for the entanglement experiment. They differ about the Planck scale and what lies past it. Linearized quantum gravity, extended into a complete quantum theory of gravity, would carry one quantized description across the Planck scale. This account holds that two descriptions meet there and stay two, and that the arena has no least step at any scale. The claim that quantizing the arena is a category error is a claim about that meeting.

Key idea. Every mass has a gravitational size, which grows with the mass, and a quantum size, which shrinks with it. The two are equal at the Planck mass, about twenty-two millionths of a gram, where both equal the Planck length. At the Planck scale neither description dominates and both are relevant, and the Incommensuration Theorem forbids one description that perfects both. The Planck length bounds how finely a comparison made from inside the world can place a body; the arena itself has no least length, area or duration. The role of the Planck scale is fixed by the ground; its size comes from the measured value of Newton's constant, because the axioms are structural, not dimensional.

Part IV has shown what is left when the error is removed; Part V turns to the measurement problem, beginning with the question of why physics has trouble with time.



PART V

THE MEASUREMENT PROBLEM



Chapter 16

Three Kinds of Time

One Word for Three Things

Why does physics have trouble with time? Because it uses one word for three different things, and each of its two great theories holds only two of them.

Picture a laboratory bench with a lump of radioactive material on it, a counter beside the lump, and a clock on the wall. Each of the three goes with a kind of time, and the three kinds are not the same.

The clock on the wall measures the first kind. Put an identical clock in the orbit of a navigation satellite and it runs faster than the one on the bench, by about 38 millionths of a second a day once both its speed and its height are counted, as Chapter 1 described for the satellites of the Global Positioning System. Those satellite clocks are adjusted before launch to allow for exactly

this; Neil Ashby set out the whole accounting in 2003. This is the time of the arena. It belongs to the shape of space and time, and it differs from place to place.

The counter measures the second kind. Each click is a finding: an atom has decayed, the counter has registered it, and a record now exists that did not exist a moment before. Before the click and after it are different, and the difference cannot be undone.

Between the clicks, quantum theory describes a third kind. The chance that a given atom has decayed grows smoothly from moment to moment, and the wave that carries those chances changes without a jump. That smooth change has a time of its own: the variable of the equation of Erwin Schrödinger, which says how the wave changes.

Physicists use the single word "time" for all three. I find that they are three kinds of time, that they stand to one another exactly as the three roles of a comparison do, and that the difference between them is the source of the measurement problem. This chapter sets out the three kinds and shows which two each theory holds.

The Time of Geometry

The first kind I call geometric time. Geometric time is time as a direction within the arena: one of the four directions of space and time that general relativity describes. It is the time of the paths that bodies take through the arena, of the intervals that clocks measure along those paths, and of light cones, the sets of places and times that light from each event can reach (Chapter 12).

Geometric time is continuous. Between any two moments on a clock's path there are others, and the geometry has no jumps. Geometric time is also asymmetric, in the sense of Chapter 5: what the geometry shows differs from place to place and from moment to moment, so that no stretch of it exactly repeats another. The clock in orbit and the clock on the bench run at different rates because their paths through the arena differ, and the arena itself differs along those paths.

The equations of general relativity read the same with the direction of time reversed. That symmetry belongs to the law, the mathematics in which the theory is written, and not to the geometry the theory finds. Chapter 10, "How a Theory Sees", showed that every theory has these two levels, its mathematics and its findings, and that their characters are opposite. In general relativity the law carries the symmetry, and the geometry found carries continuity with asymmetry. So geometric time, as found, is continuous and asymmetric.

Chapter 6 found that which concept takes which role depends on the focus of the argument. This chapter's focus is time, and in it all three axioms hold. In the terms of the three roles of Chapter 6, geometric time is time laid out as structure: the whole four-dimensional pattern of paths and intervals, held in view as one map. I identify geometric time with the role of structure, the omniscient. The time of the grid in Chapter 7, the time in which force acts, is the order of happening. Geometric time is that order laid out as one map, which is why it stands in the role of structure.

The Time of Findings

The second kind I call the time of findings. Physicists call it event time. A finding is an act of comparison that leaves a record: the counter clicks, a grain of photographic film darkens, a molecule in a detector changes its state. As everywhere in this book, a finding is made by any instrument or interaction, and it needs no mind. The time of findings is the order of before and after that findings make.

The time of findings has the character of content. A finding is discrete: one click, a count of one, one outcome from a definite list of possible outcomes. Its record is symmetric: once made, it reads the same for every comparer that later consults it. So the time of findings is discontinuous and symmetric, the character that Chapter 5 found in everything quantum theory finds.

The time of findings also has a direction: the past is fixed, and the future is open. That direction does not come from anything inside a single finding. It comes from the order in which the roles of a comparison engage. Chapter 6 set out that order as the second axiom: first the range of what is possible, then

the act, then the record, which shapes the range for the next act. The order is one of precedence, not of clock time, and it cannot be reversed. I identify that one-way order with the direction of time.

Findings do not happen inside a time that waits for them; the order of before and after is made by the findings themselves. I identify the time of findings with the role of the act, the immanent.

The Time of Evolution

The third kind I call evolution time. Evolution time is the variable of Schrödinger's equation: the time along which the quantum state changes smoothly between findings. The quantum state is quantum theory's description of a system, the wave of the theory. It describes a range of possible findings and gives each of them its odds.

Evolution time is continuous. The state changes without a jump, and in the mathematics its change can be run backward as well as forward, because the equation keeps the total of the odds fixed and loses nothing. Evolution time belongs to the mathematics of quantum theory, the level that Chapter 10 found to be smooth, and not to its findings, which come in steps.

What evolution time ranges over is not space. The possibilities that a state describes form a space of their own, with one independent direction for each of a set of outcomes that exclude one another, and that space is not a region of the arena. Evolution time is the time of possibility: of what could be found, and with what odds. I identify evolution time with the role of the stance with what is possible, the transcendent.

Distinct, Inseparable, Not Interchangeable

Three kinds of time, then, for three roles. *An Immanent Metaphysics* states how the three roles stand to one another in its third axiom (aphorism 1.3-1):

Axiom III. The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable.

The three kinds of time meet each of the three conditions, and each condition can be checked in physics.

They are distinct, because they are different kinds of mathematical object. Geometric time is a coordinate: a label that a smooth map of the arena gives to its points. A finding is a record: something that exists after the act and did not exist before it. Evolution time is a parameter: the variable of a smooth family of changes of the quantum state. A label is not a record, a record is not a parameter, and a parameter of possibility is not a direction of space.

They are inseparable, because each needs the other two. A finding needs a state whose possibilities it selects among, and a place and moment in the arena where it is made. A geometry is known only through the findings that mark it, as the clock's ticks mark its path, and through the changes that fill the stretches between them. The changing state needs findings that put its odds to use, and clocks of the arena that index its change.

Take any one away, and the other two lose their use. A state that is never found predicts nothing that anyone can check. A geometry that no finding marks is a map that nothing reads. Findings with no arena have no before, no after, and no distance between them.

They are not interchangeable, because none can do another's work. A coordinate selects no outcome: labeling a moment does not decide what the counter will show at it. A finding cannot serve as the variable of smooth change, because a record once made is not unmade by running an equation backward. And the space of possibilities cannot serve as a direction of the arena, because nothing travels through it from one place to another.

Each Theory Holds Two

Each of the two great theories holds two of the three kinds of time and takes the third as given from outside.

General relativity holds geometric time as what it finds. It holds the time of findings as events: it places each finding at a point of the arena, with a definite position among the light cones. It has no evolution time, because it contains no range of possibilities and no odds. It takes each finding as given and says where the finding lies.

Quantum theory holds evolution time in its mathematics and the time of findings in its outcomes, joined by the rule that gives the odds of each finding (Chapter 17). It does not hold geometric time. The clock of quantum theory is a setting supplied from outside, like the clock on the laboratory wall, and the theory describes the state's change along it without describing the clock.
The pattern fits in a short table.

	Geometric time (structure)	**Time of findings (the act)**	**Evolution time (possibility)**
General relativity	held: what the theory finds	held: findings placed as events	absent: no range of possibilities
Quantum theory	absent: a clock supplied from outside	held: outcomes, with their odds	held: the smooth change of the state

The two theories share the time of findings, and each describes the finding in its own way: general relativity says where a finding lies, and quantum theory says with what odds each outcome is found. The time that each theory leaves out is the time that the other supplies. Quantum theory leaves out the time of structure that general relativity finds; general relativity leaves out the time of possibility that quantum theory carries.

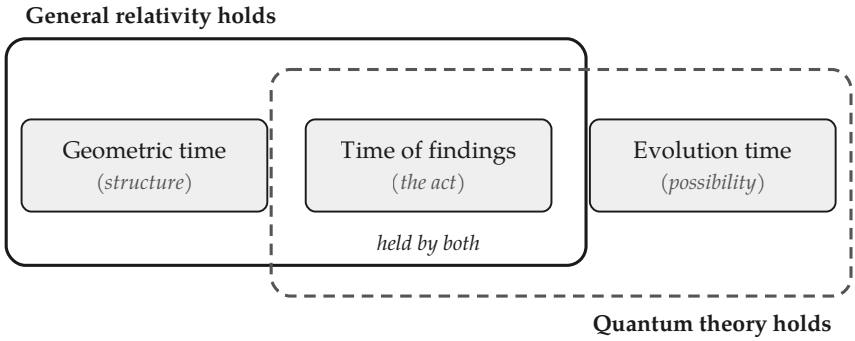


Figure 16.1. *The three kinds of time and the two theories. General relativity holds geometric time and the time of findings, quantum theory holds the time of findings and evolution time, and the time of findings is held by both.*

Physics has already seen what happens when the arena is put inside a quantum state. The time disappears. The canonical quantization of gravity, set out by Bryce DeWitt in 1967, gives an equation for a quantum state of the geometry that contains no time at all. It is the Wheeler–DeWitt equation of Chapter 2, and the puzzle it poses is the problem of time named there. On this account the puzzle is what the category error looks like from the side of time. The variable of evolution was a time of the arena, and the arena has been turned into the thing that was to evolve.

Why No Single Time Holds All Three

No single description can hold all three kinds of time as its own, because one time that was all three would carry both of the pairings that the Incommensuration Theorem forbids.

Consider what such a time would have to be. It would have to be perfectly continuous, as geometric time and evolution time are, and perfectly symmetric, as the record of a finding is. That is continuity with symmetry, the first forbidden pairing. It would also have to be perfectly asymmetric, as geometric time is, and perfectly discrete, as findings are. That is asymmetry with discontinuity, the second forbidden pairing.

I call such a time a unified time: one time in which all three kinds are perfected together, in one description. A description that asked one time to be both smooth geometry and discrete record would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. A unified time asks all four of one time, so it would carry both at its foundation, and so no description has one.

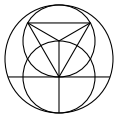
It follows that any description holding two kinds of time must treat the third as external or as primitive. External means supplied from outside as a setting, and not described. Primitive means laid down as a rule, and not derived from anything else in the description. Quantum theory treats geometric time as external and the step from odds to a finding as primitive. General relativity treats evolution as external: it contains no possibilities to evolve.

This is why physics has trouble with time. Each theory is exact in its own domain, and each kind of time is right where it is used. The trouble begins when one time is asked to do the work of all three. A single theory of gravity and the quantum would need one, and Chapter 11 found the same requirement from the side of the arena: a quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. The three kinds of time show the same thing from the side of time.

For the reader who wants the three kinds of time as mathematical objects, with their irreducibility set out step by step, the treatment is in *The Unquantized Arena*, Chapter 10, "Three Times".

Key idea. Physics uses one word for three kinds of time: geometric time, which is continuous and asymmetric; the time of findings, which is discrete and symmetric and carries the direction of time; and evolution time, the smooth variable of quantum theory's possibilities. They correspond to the three roles of a comparison, and like those roles they are distinct, inseparable and not interchangeable. General relativity holds geometric time and findings; quantum theory holds findings and evolution. One time that held all three would carry both forbidden pairings, so no description has one.

If the wave of quantum theory changes smoothly and its findings come in steps, how does a measurement come to have one outcome?



Chapter 17

Why a Measurement Has an Outcome

One Dot at a Time

In 1989 Akira Tonomura and his colleagues sent electrons through a double slit one at a time, so that each electron made its passage on its own. Behind the slits stood a screen that recorded each arrival.

Every electron arrived as a single dot, at a single place. No electron ever arrived as a smear across the screen, and no electron ever arrived in two places. At first the dots looked scattered at random. As thousands of them accumulated, they built up a pattern of bright and dark stripes, the interference pattern of a wave that had passed through both slits together.

Quantum theory predicts the stripes exactly. Its description of each electron is a wave that passes through both slits, spreads over the whole screen, and is strong where the stripes are bright and weak where they are dark. The wave gives the odds of a dot landing at each place. What the screen records, each time, is one dot.

That is the measurement problem, in one experiment. The theory's description is smooth and spread out; each finding is single and sharp. How does a smooth wave give one definite result?

The Problem in Plain Words

The difficulty is sharper than it first looks. Schrödinger's equation, which says how the wave changes, is smooth, and it is linear. Linear means this: if the equation carries one starting wave to one result, and another starting wave to another result, then it carries the sum of the two starting waves to the sum of the two results.

Now apply the equation to the electron and the screen together. A wave that would put the dot here, added to a wave that would put the dot there, is carried by the equation to a sum: the screen with a dot here, plus the screen with a dot there. The equation never produces one dot. It produces every dot the odds allow, each with its weight. Yet every run of the experiment shows exactly one.

So three statements cannot all be kept together: that the wave is a complete description of the electron and the screen; that the wave always changes by Schrödinger's equation; and that each run has one definite outcome. Erwin Schrödinger put the same difficulty in 1935 in the form of his cat, whose life depends on one atom and whose description, by the equation alone, becomes a sum of a living cat and a dead one.

The usual question is a question about a process: what happens, inside the wave or the apparatus, to turn the spread wave into one dot? The interpretations of quantum theory, which Chapter 19 sets out one by one, are different answers to that question, or different reasons for thinking it needs no answer. This chapter gives the answer that the ground of this book gives.

Two Levels, and What Is Left Over

The measurement problem is what a smooth mathematics leaves over when the findings it describes come in steps.

Chapter 10, "How a Theory Sees", showed that every physical theory has two levels, its mathematics and its findings, and that the two always have opposite characters. In quantum theory the mathematics is smooth: the wave, the space of possibilities, the continuous change of evolution time. The findings are discrete: one dot, one click, one outcome from a definite list.

A mathematics of one character can describe findings of the other character as closely as anyone likes, but it cannot become them. Something is always left over. Chapter 10 called what is left over the residual of the theory: the point where its mathematics, having come as close as it can, meets the findings it describes and does not contain them.

I identify the measurement problem as the residual of quantum theory. Its smooth wave comes as close to the discrete finding as a smooth description can, and the selection of one outcome is not a step in the wave's smooth change. General relativity has a residual of the opposite kind: the singularity, where its smooth description reaches the edge of its domain. The two residuals are a matched pair, because the two theories are.

The Finding Comes First

The wave does not produce the finding. The finding is the act, and the wave describes the possibilities the act selects among.

The first axiom of *An Immanent Metaphysics* (aphorism 1.3-1) states the order: "The immanent is more fundamental than the omniscient and/or the transcendent." In plain words, the immanent (the act) comes before both the omniscient (structure) and the transcendent (the stance with what is possible). The finding is an act of comparison; the wave is quantum theory's description of possibility. So the finding is not something the wave makes. The wave describes the range from which each finding is made, and the odds of each outcome in that range.

A finding is one outcome selected from among its possibilities. When the electron reaches the screen, the screen compares, and something that was possible becomes manifest for that comparer: a dot, here, and a record that now reads the same for every later comparison that reaches it. The finding is made by the screen, as an interaction, and it needs no mind.

Now look again at the usual question. It asks for a process, running in some one time, that carries a smooth wave into a discrete dot. A process of that kind would make one time serve both as evolution time, which is smooth, and as the time of findings, which comes in steps with a record that is the same for all. That is the unified time of Chapter 16, and a unified time

needs the pairings the Incommensuration Theorem forbids. Within quantum theory as it stands, the question asks for a process that the theory's equation does not contain. A process of that kind can be written only by changing the theory, as the collapse models of Chapter 19 do, and that chapter sets out what each such change sets aside and what it costs. The finding needs no such process, because the finding is prior to the description of it.

What a Finding Changes, and What It Does Not

A finding changes the record of the comparer that makes it. It changes no geometry.

Chapter 12, "One Geometry for Each Alternative", showed that when matter is in several alternatives together, each alternative carries its own classical geometry, shaped by that alternative's own energy. A finding selects among alternatives that already carry their geometries. So nothing in the arena changes when a finding is made, here or anywhere else.

A finding made far away reaches a comparer here only as news, carried by some signal, and the news arrives on the comparer's light cone at the earliest. This is established physics, and the account needs nothing further. No comparison registers a surface of simultaneity, a slice of the arena on which every finding would be settled for every comparer together, and the account needs none. When two comparers later bring their records together, at a meeting that lies in the future of both findings, their records of each finding agree.

This is where the measurement problem meets gravity. Because each alternative carries its own geometry, there is nothing in the arena for a finding to reshape. The single averaged geometry that Chapter 12 set aside gives all the alternatives one shape, set by their weighted average, and in it a finding would have to reshape the arena wherever the alternatives reached. The experiment of Don Page and C. D. Geilker in 1981 found the field following the actual outcome, and not the average, and the account given here agrees with it.

A Feature of Time, Not a Gap in Knowledge

The measurement problem is a feature of the structure of time. It is not a gap in what physicists know, waiting for a cleverer mechanism to fill it.

The usual question presupposes one time, inside which a mechanism would run and at the end of which the wave would have become a dot. There is no such time. There are three kinds of time, and Chapter 16 showed that they are distinct, inseparable and not interchangeable. The smooth change of the wave runs in evolution time. The dot belongs to the time of findings. The screen on which the dot appears is placed in geometric time. Each is exact in its own role, and none of them can be turned into another.

So the measurement problem is resolved without a mechanism for the passage from wave to dot. The resolution shows that the demand for such a mechanism rests on the premise of a single time within which the mechanism would operate. That premise is the unified time, and the Incommensuration Theorem rules it out. What remains is what every experiment shows: a smooth description of possibilities, with odds that hold for the many, and a finding, each time, that is one.

The precision of quantum theory shows how close the smooth side comes. Its predictions agree with measurement more closely than almost any others in science. That closeness is what the two levels lead one to expect: a mathematics of one character can approach findings of the other without limit, and still not contain them.

The Rule That Gives the Odds

What joins the smooth wave to the discrete findings is a rule: the odds of each outcome are the square of the size of the wave for that outcome. Max Born stated it in 1926, and it is called the Born rule, or the probability rule.

The rule is the bridge between quantum theory's two levels. On one side is the wave, in the smooth mathematics; on the other side are the findings, in steps. The rule gives the odds of each finding. It does not give the finding, and nothing in the mathematics does. That is the residual again, seen from the side of the rule.

The rule cannot be obtained from Schrödinger's equation alone, because the equation stays on one side of the bridge. It is not arbitrary either. In 1957 Andrew Gleason took the space of possibilities in the form quantum theory gives it, and asked how odds could be assigned to findings so that the odds of an outcome do not depend on which other outcomes are measured alongside it. He showed that there is only one way, and that the one way is Born's rule. Gleason's result needs at least three outcomes to choose among; in 2003 Paul Busch extended it to every case, two outcomes included, by allowing the more general measurements that quantum theory uses.

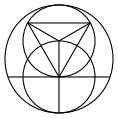
I notice that the condition in Gleason's result is a symmetry. It asks that the same outcome carry the same odds in every measurement it belongs to: the same content, across different contexts. A bridge to findings that are discrete and symmetric must itself carry their symmetry. So the rule that turns the continuous description into the odds of a finding is fixed once its symmetry is granted. This is an identification about why the rule has the form it has. It does not derive the space of possibilities; it takes that space as quantum theory gives it.

Recognize the rule in the experiment you began with. Each electron gave one dot, and nothing in the wave said where the next would fall. The many dots together followed the square of the wave, stripe by stripe, as closely as the count allowed. The rule speaks about the many; the finding is each one.

For the reader who wants the problem in its standard technical forms, and the results of Gleason and Busch stated exactly, the treatment is in *The Unquantized Arena*, Chapter 11, "The Measurement Problem as a Cross-Modal Residual".

> **Key idea.** The measurement problem is what quantum theory's smooth mathematics leaves over when its findings come in steps: the residual of the theory. The wave does not produce the finding; the finding is the act, one outcome selected from among its possibilities, and the wave describes those possibilities and their odds. A finding changes the comparer's record and no geometry, and news of it travels no faster than light. Born's rule is the bridge that gives the odds, and its form is fixed by the symmetry that the bridge must carry.

If a smooth theory cannot contain the finding, how close can it come?



Chapter 18

How Close a Smooth Theory Comes

Fringes That Fade

In 2003 Klaus Hornberger, Markus Arndt, Anton Zeilinger and their colleagues in Vienna sent molecules of carbon seventy through an interferometer. Each molecule is a closed cage of seventy carbon atoms. An interferometer is an apparatus that splits a beam into paths and brings the paths back together. With a good vacuum, the molecules built up interference fringes, as Tonomura's electrons did, the mark of a wave that had traveled more than one path.

Then the experimenters let a little gas into the chamber. As the pressure rose, the fringes faded. Each stray gas molecule that struck a carbon cage carried away a trace of which path the cage had taken. The more traces there were in the surrounding gas, the weaker the fringes became, and the fading followed the calculation quantitatively.

Nothing in that experiment went outside Schrödinger's equation. The equation, applied to the cages and the gas together, predicts the fading, step by step. This is decoherence, and it is the closest a smooth theory comes to the finding.

What Decoherence Explains

Decoherence explains why the world of instruments and records looks classical, and it does so inside quantum theory, with no added rule.

Dieter Zeh pointed the way in 1970, and Wojciech Zurek, Erich Joos and Zeh developed it in the years after. The idea is simple. No real system is alone. A pointer, a dust grain or a molecule is struck all the time by light and by gas, and each collision copies a little information about the system into its surroundings. When the surroundings carry a record of which alternative the system is in, the alternatives stop interfering with one another in anything that can be observed on the system alone.

Decoherence explains three things. It explains why interference fades for large objects, and how fast: Joos and Zeh calculated in 1985 that for a dust grain in air the fading is all but immediate. It explains why detectors show sharp positions and not blends of positions: the surroundings pick out the alternatives that survive repeated copying, which Zurek called pointer states. And it explains why many comparers can read the same result. In Zurek's account of 2009, which he called quantum Darwinism, the surroundings hold many copies of the same record, so that each comparer can consult a copy of its own, and every copy says the same thing.

I notice that the last of the three is the symmetry of the record that Chapter 16 found in the time of findings: a record reads the same for every comparer that consults it. Decoherence shows how a smooth description comes to carry that symmetry in the surroundings of every finding. It is a large achievement, and it is why the boundary between the quantum and the classical is no longer a mystery of scale.

Decoherence also explains why the experiment of Chapter 13 is so hard to build. There, two small masses are each held in two places, and each alternative of the masses carries its own geometry, so that gravity between the masses can entangle them. A single stray molecule or a single photon of heat that records where a mass is would make the alternatives stop interfering, and the entanglement could not be seen. So the masses must be kept in a vacuum colder and emptier than almost any on Earth, for the few seconds the entanglement needs to build. The same smooth process that makes the everyday world look classical is what the experimenters must hold off.

Where It Stops

Decoherence brings the smooth description to the edge of the finding, and it stops there.

After decoherence, the description of the whole, system and surroundings together, is still a sum of alternatives, each with its own record copied into the gas and the light. The alternatives no longer interfere, so a description of the system alone looks exactly like a list of ordinary odds: this outcome with this chance, that outcome with that chance. But a list of odds is not an outcome. Nothing in the smooth description selects one entry from the list.

Return to Schrödinger's cat. Decoherence says a great deal about it. The cat is warm, it breathes, and it is struck every instant by light and air, so its living and dead alternatives stop interfering long before anyone opens the box. Each alternative then carries its own record, written into the air, the light and the walls of the box. What decoherence does not say is which of the two records the opened box will show. That is left to the finding.

The point can be put exactly. A list of odds for the system alone arises in two different situations. It arises when one outcome has occurred and the comparer does not yet know which. It arises equally when the whole is still a sum of alternatives, each with its record, and no outcome has been selected. The description of the system alone reads the same in both situations, so it cannot tell them apart. Decoherence delivers the second situation. The first situation needs a finding, and decoherence does not supply one.

Maximilian Schlosshauer, in a careful review of 2005, set out what decoherence does and does not settle, and found that its bearing on the problem of outcomes had remained a matter of controversy. On the ground of this book the answer is definite. Decoherence gets as close as a continuous description can get to the discrete finding. It does not cross into it.

That is what the two levels lead one to expect. Chapter 17 identified the measurement problem as the residual of quantum theory: the point where its smooth mathematics, having come as close as it can, meets findings of the opposite character and does not contain them. Decoherence is the process by

which the mathematics comes that close. The finding itself is the act: one outcome selected from among its possibilities, manifest for the comparer that makes it.

Decoherence therefore marks the exact place where every interpretation of quantum theory does its work. Up to that place, all of them agree, because all of them use the same smooth equation and the same fading of interference. At that place, each adds something, or declines to add anything, and what each adds is a choice about the three kinds of time. Chapter 19 takes the choices one by one.

An Open Future Is Real

Which outcome a finding selects is not fixed in advance. The randomness of quantum findings is the openness of the future, and not a gap in knowledge.

The second axiom of *An Immanent Metaphysics* (aphorism 1.3-1) states the order: "A class of the transcendent will precede an instance of the immanent." In plain words, the transcendent, the stance with what is possible, comes first, as a kind, and the immanent, the act, comes after it, as one particular case. A class stands ready as a whole. It does not single out which of its instances will occur. So the range of possibilities that precedes a finding does not contain the choice of the finding's outcome.

The Incommensuration Theorem gives the same result from another side. Chapter 5 defined a discontinuity as a difference of content where the context is the same. What is found of content is discontinuous and symmetric, with both characters perfected together. So, among the findings of content, the same context does not fix the same content: two preparations made exactly alike do not fix one outcome. No description of the context, however complete, fixes every finding. A description that did fix every finding would make every sameness of context a sameness of content, which is continuity, held together with the symmetry of content: continuity with symmetry, the first forbidden pairing. The outcome of each finding is open: nothing a comparison can register determines it.

You have met this openness in the radioactive lump of Chapter 16. Its atoms are identical, and quantum theory gives each the same odds of decaying in the next second. Half of them will be gone after a fixed time, the half-life, and that time can be measured to great precision. Which atom goes next is fixed by nothing: no atom carries a clock inside it that the others lack. The law is exact about the many, and the finding is open for each one.

Bohm's account of quantum theory, which Chapter 19 sets out, is deterministic in its mathematics, and it gives the same findings as standard quantum theory. Its determinism belongs to its model of the world, and no finding can establish it, because the hidden positions it uses are distributed exactly as the odds of Born's rule require. So Bohm's account holds its determinism in its mathematics, where no finding reaches. In what is found, the outcome of each finding stays open.

Causation Established, Determinism Never

Science establishes causation every day. It never establishes determinism.

Causation is a relation with a direction between findings: this finding, and then that one, where the first makes a difference to the second. You press a switch and the lamp lights. A beam of protons strikes a target and the detectors downstream begin to count. Each of these is a pair of findings in order, and the order runs one way, the way of the order of the three roles that Chapter 16 identified with the direction of time. Causation is found in every experiment, as a finding that follows a preparation.

Determinism is a different claim. It says that a complete specification of everything at one moment fixes everything at every other moment. No observation ever delivers a complete specification of everything. Every finding is one comparison, made from one place inside the world, of one content in one context. So determinism is never observed, while causation is observed every day.

The difference is one of kind, and better instruments do not remove it. A complete specification of everything would have to include the instrument that made it, and the act of making it, and the record it left. A comparer is always part of the world it compares, so its findings are always of something

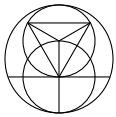
other than the whole. A claim of determinism therefore goes beyond every finding that could support it, while a claim of causation is checked by the next finding.

This is why the form of quantum law is right as it stands. A law that gives the odds of findings, with a direction from one finding to the next, is causal without being deterministic. That is the correct form for a law made of findings, and quantum theory has had that form from Born onward.

For the reader who wants decoherence in its technical form, with the literature of Zeh, Zurek, and Joos and Zeh, the treatment is in *The Unquantized Arena*, Chapter 11, "The Measurement Problem as a Cross-Modal Residual". The case for causation without determinism is set out in its Chapter 13, "Bell, Kochen and Specker, and PBR as Instances".

> **Key idea.** Decoherence explains, inside quantum theory, why interference fades for large objects, why detectors show sharp results, and why many comparers find the same record. It brings the smooth description to the edge of the finding and stops there: a list of odds is not an outcome. The outcome is not fixed in advance, because a class of possibilities does not single out its instance, and because the same context does not fix the same content among the findings of content, so that no description of the context fixes every finding. Science establishes causation, a relation with a direction between findings; it never establishes determinism.

If the finding cannot be derived from the wave, what are the interpretations of quantum theory doing?



Chapter 19

The Interpretations as Choices About Time

Six Answers to One Question

Which interpretation of quantum theory is right? Each of the major interpretations is a choice about the three kinds of time, and each meets its own difficulty exactly where the time it sets aside returns.

An interpretation of quantum theory is an account of what the theory describes, added to the mathematics and the probability rule that every interpretation shares. All of them predict the same findings in every experiment yet performed, except the collapse models, which predict small departures that experiments are now testing. So the choice among them is a choice about what the theory says the world is, and in particular about where the finding comes from.

Chapter 16 found three kinds of time, and in the focus of time it gave each one of the three roles: geometric time, the time of structure; the time of findings, the time of the act; and evolution time, the time of possibility. Each interpretation takes one or two of these as basic, and then either drops the others or absorbs them into the ones it keeps. In every case its achievements come from what it keeps, and its characteristic difficulty comes from what it sets aside. This chapter takes six interpretations in turn, each with its achievement first.

Copenhagen and the Cut

The Copenhagen interpretation, shaped by Niels Bohr and Werner Heisenberg in the 1920s, takes the finding as basic. The wave is a tool for calculating the odds of findings; the findings themselves are recorded by classical instruments, in plain language, in a classical arena.

Its achievement is that it matches what physicists do. Every experiment ends in a record that can be read and reported, and Copenhagen puts that record at the center. By the first axiom of *An Immanent Metaphysics*, which places the act before both structure and possibility, Copenhagen is the interpretation closest to the ground of this book. It does not try to derive the finding from the wave.

Copenhagen draws a line, the Heisenberg cut, between the system described by the wave and the instrument that records the finding. In 1932 John von Neumann showed that the cut can be moved: the instrument can be described by the wave too, and the line placed further along, at a second instrument or a third, without changing any prediction. I identify the Heisenberg cut as the boundary between the two levels of quantum theory: on one side the smooth description of possibility, on the other the discrete record of the act. Its mobility shows that it is not a place in the laboratory. It is a boundary between two characters, and it can be drawn wherever a smooth description meets a finding.

What Copenhagen sets aside is evolution time as a description of the world. Between findings, the wave is only a tool. The difficulty that follows is the one its critics have always pressed: Copenhagen gives no account of what happens between findings, and no rule for where the cut belongs. Possibility is left with no standing in the world.

Many Worlds

The many-worlds interpretation, proposed by Hugh Everett in 1957 and developed by Bryce DeWitt, David Deutsch and David Wallace, takes evolution time as basic and makes everything else follow from it. The wave always

changes by Schrödinger's equation, with no exception. Every outcome occurs, each in its own branch, and a comparer in each branch records the outcome of that branch.

Its achievement is economy. It needs no extra rule, no cut and no special role for instruments. It applies the same equation to an atom and to the whole universe, and with decoherence (Chapter 18) it explains why each branch looks classical from inside. It takes the mathematics of quantum theory with complete seriousness.

What it sets aside is the time of findings. The finding is absorbed into evolution: a finding becomes a branching of the wave, and no outcome is selected. Its two characteristic difficulties are exactly where the finding returns. The first is the question of which branches there are: the wave can be divided into branches in many ways, and choosing the division is choosing which findings count as having happened. The second is the question of the odds. If every outcome occurs, what does it mean that one outcome has odds of nine in ten? Deutsch in 1999 and Wallace in 2012 argued that a rational agent in a branching world should weigh the branches by Born's rule. Their arguments are careful and widely studied. Whether they succeed is still debated among specialists. On this account the debate is the finding asserting itself: odds are odds of findings, and an interpretation without findings must rebuild them from inside evolution.

Bohm's Pilot Wave

The pilot-wave theory, proposed by Louis de Broglie in the 1920s and completed by David Bohm in 1952, takes geometric time and evolution time as basic. Every particle has a definite position at every moment, a path through the arena, and the wave guides the paths.

Its achievement is clarity. It is a complete and consistent account that reproduces every prediction of non-relativistic quantum theory, with no cut and no special role for instruments. It also showed that hidden variables, conditions beneath the quantum state that fix each result, are possible if they are not local, and it was this example that led John Bell to the theorem of Chapter 20.

What it does with the time of findings is absorb it into geometric time. An outcome is fixed by where the particles already were, together with our ignorance of where that was. But an irreversible finding is not the same as a reversible path about which one happens to be ignorant. The characteristic difficulty follows from the absorption. The wave guides each particle according to where all the others are at the same moment, so the theory needs a single slicing of space and time into moments, one surface of simultaneity for the whole world. Relativity's geometry supplies no such slicing, and no comparison registers one (Chapter 17). That is why the pilot wave extends to relativistic field theory only with difficulty.

Relational Quantum Mechanics

Relational quantum mechanics, proposed by Carlo Rovelli in 1996, takes the finding as basic and makes it relative. Any physical system can play the part of the comparer, and a finding is a fact relative to the system that makes it.

Its achievement is that it gives no special role to instruments or to minds. Every interaction is a comparison, which is exactly how this book uses the word. Its central claim, that a finding is a fact for the system that makes it, is close to the account of Chapter 17, in which something possible becomes manifest for that comparer.

What it sets aside is the role of structure: one shared layout in which every comparer's findings take their places together. The characteristic difficulty is the one that follows. If facts are relative to each comparer, how do two comparers come to agree? In 2023 Emily Adlam and Rovelli added a postulate to secure the agreement: the information a comparer holds is stored in its physical variables, and another comparer can measure those variables. On this account the added postulate is the role of structure returning, since a record, once made, reads the same for every comparer that consults it (Chapter 16).

QBism

QBism, developed by Christopher Fuchs, Rüdiger Schack and David Mermin, takes the finding as an agent's experience and the wave as that agent's degrees of belief about future experiences. The probability rule is a guide for how a rational agent should bet.

Its achievement is that the puzzles of influence at a distance disappear. In QBism a finding is local to the agent who has it, and nothing travels between distant experimenters except their reports, which arrive no faster than light. That agrees with Chapter 17: news of a distant finding arrives on the light cone.

What QBism sets aside is evolution time as a feature of the world. Possibility becomes belief. It also makes the finding depend on an agent, where the ground of this book takes a comparer to be any interaction. The characteristic difficulty follows from the first step. If the wave is each agent's belief, why must every agent's best beliefs follow the same rule and the same equation, and why do the frequencies that every agent records agree? The regularity that QBism must explain is the possibility it set aside, returning as a law that every agent obeys.

Collapse Models

Collapse models change the theory itself. In 1986 GianCarlo Ghirardi, Alberto Rimini and Tullio Weber added to Schrödinger's equation a rare, random process that narrows the wave of each particle to a small region. Philip Pearle in 1989, and Ghirardi, Pearle and Rimini in 1990, made the process smooth and continuous. Lajos Diósi, in 1987 and 1989, and Roger Penrose, in 1996, tied the rate of the process to gravity: on their view, alternatives of a mass in different places cannot persist for long, and the more mass is displaced the faster they end.

Their achievement is that they make outcomes part of physics. A single equation describes the atom and the instrument, with no cut and no special role for comparers, and a small object keeps its alternatives while a large one loses them quickly. They are also testable in a way the other interpretations

here are not, because they predict small departures from standard quantum theory. Experiments already set limits on them: in 2021 Sandro Donadi and his colleagues, working deep underground at Gran Sasso, ruled out the natural parameter-free version of the Diósi and Penrose model.

What collapse models set aside is the finding as an act distinct from evolution. They absorb the time of findings into a modified evolution time: the step to an outcome becomes a physical process inside the equation. Several consequences follow from that absorption. The process needs new constants of nature, for its rate and its size. It adds a faint, steady heating to all matter. And it makes the state change across a whole region, which a relativistic version must arrange with care, so that no slicing of the arena into moments is preferred.

The gravitational versions meet a sharper test. Chapter 12 found that each alternative of the matter carries its own geometry, and Chapter 13 described the experiment that follows: two small masses, each in two places, entangled by gravity alone. On Penrose's estimate, masses of that size lose their alternatives within a small fraction of a second, long before the entanglement could build. On the account of this book, gravity does not end the alternatives, and the entanglement appears with the phase of Chapter 13. So that experiment decides between them.

The Pattern

Set side by side, the six choices show one pattern.

Interpretation	What it takes as basic	What it sets aside or absorbs	The difficulty that follows
Copenhagen	the finding	evolution as a description of the world	no account between findings; no rule for the cut
Many worlds	evolution	the finding, absorbed into branching	which branches; what the odds mean
Pilot wave	geometry and evolution	the finding, absorbed into paths and ignorance	a slicing into moments that relativity does not supply
Relational	the finding, relative to each comparer	one shared layout of findings	how comparers agree
QBism	the finding, as experience	possibility in the world, made belief	why every agent's beliefs obey one law
Collapse models	a modified evolution	the finding as a distinct act	new constants; heating; a process spread across space

Each interpretation reduces three kinds of time to one or two, and each difficulty appears exactly where a set-aside time returns. That is what the third axiom predicts: the three roles are distinct, inseparable and not interchangeable, so a time can be neither removed nor turned into another without leaving a mark where it was. Chapter 20 states this as a general result about every interpretation.

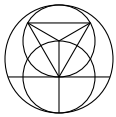
The account of this book keeps all three kinds of time, each in its own role. The finding is the act, as Copenhagen and the relational view hold. The wave describes possibility in the world, which is more than a tool and more than a belief. The geometry of each alternative is classical, as Chapter 12 found, and no finding changes it. The account adds nothing to the mathematics and

changes no prediction of quantum theory. On the experiment of Chapter 13, which separates it from the gravitational collapse models, it makes a definite prediction.

For the reader who wants the interpretations in technical form, with consistent histories added and the literature of each, the treatment is in *The Unquantized Arena*, Chapter 12, "The Interpretations as Modal Strategies".

> **Key idea.** Each interpretation of quantum theory is a choice about the three kinds of time. Copenhagen keeps the finding and makes the wave a tool, many worlds keeps only evolution, and the pilot wave keeps geometry and evolution. The relational view keeps the finding and sets aside the shared layout; QBism keeps the finding and makes possibility a belief; collapse models absorb the finding into evolution. Each has real achievements, and each meets its difficulty where the time it set aside returns. The Heisenberg cut is the boundary between the two levels of quantum theory, which is why it can be moved.

Physics has also found, by its own methods, results that no interpretation can escape. What do they show?



Chapter 20

The Walls Physics Has Already Found

Two Stations, One Pattern

In 2015 Bas Hensen, Ronald Hanson and their colleagues in Delft set up two stations 1.3 kilometers apart, each holding a single electron in a small crystal of diamond. The two electrons had been entangled, by way of light sent from each station to a point between them. In the sense of Chapter 13, the pair had a state of its own, and neither electron had a state of its own.

At each station, a random number generator chose, in each trial, which of two measurements to make on the electron there. The choice and the measurement at each station were finished before any signal from the choice at the other station, traveling at the speed of light, could have arrived. Each station recorded its setting and its result. Only afterward were the two records brought together and compared.

The comparison showed a pattern of agreement between the two records stronger than any account of a certain kind can produce. In such an account, each result is fixed in advance by conditions laid out locally, at the station where it is found. The same year, two groups using entangled light, one led by Marissa Giustina in Vienna and one by Krister Shalm in Boulder, found the same.

This chapter reads that result and two others that physics found by its own methods: results that no interpretation of quantum theory can escape. I find that all three are cases of one rule, the rule that Chapter 16 found in the three kinds of time.

Bell's Theorem

In 1964 John Bell showed that no local account of hidden variables can reproduce all the predictions of quantum theory.

A local account of hidden variables is one in which each result is fixed in advance, by conditions present at the place where the result is found, together with whatever setting is chosen there. Bell showed that every account of that kind limits how strongly the results at two distant stations can agree. John Clauser, Michael Horne, Abner Shimony and Richard Holt turned the limit in 1969 into a form that experiments can test: a certain combination of the agreements can be no larger than 2. Quantum theory predicts values up to about 2.8. The Delft experiment measured 2.42, with an uncertainty of 0.20.

An everyday case shows what kind of agreement a local account allows. Seal a red card in one envelope and a blue card in another, shuffle the envelopes, and send one to each of two cities. Whoever opens an envelope and finds red knows at that instant that the other card is blue. Nothing traveled between the cities: the agreement was laid out in advance, when the envelopes were sealed. Bell's theorem shows that the agreement between entangled particles is not of the envelope kind. No laying out in advance, at the two stations, can produce it.

Bell's theorem excludes local hidden variables. It does not exclude every hidden variable: the pilot wave of Chapter 19 has hidden positions, and it stands, because its guidance is not local.

Bell later gave the theorem a stronger form, built on a condition he called local causality. It excludes every account in which each single result is produced by conditions local to its station, even when those conditions give only the odds of the result. The account of this book meets that stronger form in one way. A finding is a fact for the comparer that makes it. A distant finding becomes a fact for a comparer here only when its record arrives, at the earliest on the light cone of the finding. So the joint pattern of the two results is a fact only where the two records meet, and no comparer has both results before the records meet.

On this account, then, each finding is made at its own station, by its own comparer. The pattern appears when the two records are brought together, at a meeting that lies in the future of both findings, and the records travel there no faster than light. Quantum theory itself guarantees that the choice of setting at one station changes nothing in the odds of the results at the other, so no message can be sent this way. No finding changes anything at a distance.

What Bell's theorem excludes is the demand that each finding be fixed by structure laid out in the arena in advance. In the terms of the three roles, in the focus of time of Chapter 16, that demand reduces the act to structure: the finding becomes a reading of a pattern that was already there. I read Bell's theorem as showing that the role of the act cannot be reduced to the role of structure.

Nine Boxes That Cannot Be Filled

In 1967 Simon Kochen and Ernst Specker showed that the measurable quantities of quantum theory cannot all have values fixed in advance, values that each measurement merely reveals, whatever else is measured alongside it.

David Mermin gave the simplest form of the argument in 1990. Take two particles, each of which can show only plus one or minus one in each of several measurements, and arrange nine joint measurements on the pair in a square of three rows and three columns. The three measurements in any row can be made together, and so can the three in any column. Quantum theory says that the three results in each row multiply to plus one. It says that the results in the first two columns multiply to plus one, and the results in the third column to minus one.

Now try to fill the nine boxes in advance, each with plus one or minus one. Multiply all nine numbers together, row by row: three rows, each giving plus one, give plus one. Multiply the same nine numbers column by column: plus one, plus one and minus one give minus one. The same nine numbers cannot multiply to both. So no filling of the boxes exists, and the values cannot be laid out before the measurements are made.

What the theorem of Kochen and Specker excludes is the demand that possibility be a fixed pattern of values laid out in advance. In the terms of the three roles, that demand reduces possibility to structure: the range of what could be found becomes a list of what is already there. I read the theorem as showing that the role of possibility cannot be reduced to the role of structure.

The State Is Not Ignorance

In 2012 Matthew Pusey, Jonathan Barrett and Terry Rudolph showed that the quantum state cannot be only a description of someone's knowledge about an underlying condition of the system.

Their argument tests that view on its own terms. On the view, two different preparations, each giving a different quantum state, could sometimes leave a system in the same underlying condition, so that the two states would differ only in what is known. Prepare two systems independently, each by one of the two preparations. Quantum theory then supplies a joint measurement on the pair in which each possible outcome is ruled out by one of the four combinations of preparations. Whenever both systems were in the shared condition, every outcome would be ruled out, and the measurement would have no possible result. Quantum theory gives that measurement a result every time. So, given that independent preparations give independent conditions, the quantum state is not only knowledge.

What the theorem of Pusey, Barrett and Rudolph excludes is the demand that possibility be moved out of the world and into the comparer's knowledge, so that what quantum theory describes as possible is only what the comparer does not know. In the terms of the three roles, that demand removes the role of possibility from the world. I read the theorem as showing that possibility in the world is not reducible to a lack of knowledge.

One Rule, Three Cases

The three results are three cases of one rule: the three roles of a comparison are distinct, inseparable and not interchangeable, so that none can be removed and none can be replaced by another.

Each theorem excludes a different attempt to remove a role or make one role do another's work.

Result	What it excludes	The reduction refused
Bell (1964)	results fixed by local conditions laid out in advance	the act reduced to structure
Kochen and Specker (1967)	values fixed in advance, whatever else is measured	possibility reduced to structure
Pusey, Barrett and Rudolph (2012)	the state as knowledge only	possibility removed from the world into the comparer's knowledge

Each theorem works inside its own mathematics, with its own assumptions, and none was derived from the ground of this book. What I claim is a reading: each shows, in its own setting, an instance of what the third axiom says in general. The third axiom gives the kind; the three theorems give three cases of it.

The reading also explains why the walls stand where they do. Bell's wall stands where an account tries to make the finding a reading of structure. The wall of Kochen and Specker stands where an account tries to make possibility a fixed list. The wall of Pusey, Barrett and Rudolph stands where an account tries to make possibility a lack of knowledge. Physics found each wall by working inside quantum theory. The ground of this book finds the same walls from outside it, in the structure of comparison itself.

The Single-Time Result

The three kinds of time give one further result, which applies to every interpretation alike. I call it the single-time result: no interpretation of quantum mechanics that operates within a single unified concept of time can be complete.

The reasoning is short. A complete interpretation must account for geometric time, the time of findings and evolution time, because quantum theory and its experiments use all three. The three kinds of time are distinct, inseparable and not interchangeable (Chapter 16). A single unified concept of time would hold all three in one description, and Chapter 16 showed that such a time needs both of the pairings that the Incommensuration Theorem forbids. So an interpretation built on one time must leave a kind of time out, or force one kind to do another's work, and in either case it is incomplete.

The result needs no assumption about hidden variables, about values fixed in advance, or about whether the state is knowledge. It rests on the three kinds of time alone. What it gives is a diagnosis of form. It says where each interpretation will meet its difficulty: at the place where its strategy sets a time aside or merges two times into one. Chapter 19 found each difficulty exactly there.

The account of this book is not built on one time. It keeps geometric time in the arena, as general relativity finds it; it keeps evolution time in the smooth mathematics of quantum theory; and it keeps the time of findings in the acts of comparison that every experiment ends in. It joins them through the probability rule and through the findings that both theories share, and it asks none of them to do another's work. That is why the single-time result, which limits every interpretation built on one time, does not limit it.

What the Walls Show

The walls that physics has found are the places where it tried to make one role of comparison do another's work.

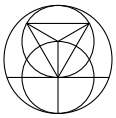
That is why they are walls and not gaps. A gap in knowledge closes when more is known. These walls are theorems about the structure of quantum theory, and experiments such as the one in Delft have confirmed the predictions that they rest on. They mark the same structure that the measurement problem marks: three kinds of time, each exact in its role, and no single time that holds them all.

This Part began with a question about why physics has trouble with time. The answer has now been given in full. Physics has three kinds of time, and its two great theories each hold two. The measurement problem is the residual of quantum theory, the place where its smooth mathematics meets the findings it describes. Decoherence brings the mathematics to that place and stops, and the interpretations are choices about which time to set aside. The theorems of Bell, of Kochen and Specker, and of Pusey, Barrett and Rudolph mark where each kind of reduction fails. The same structure that keeps the arena unquantized keeps the three kinds of time apart.

For the reader who wants the three theorems and the single-time result in technical form, the treatment is in *The Unquantized Arena*, Chapter 13, "Bell, Kochen and Specker, and PBR as Instances".

Key idea. Bell's theorem refuses to reduce the act to structure, the theorem of Kochen and Specker refuses to reduce possibility to structure, and the theorem of Pusey, Barrett and Rudolph refuses to move possibility out of the world into knowledge. All three are cases of one rule: the three roles of comparison are distinct, inseparable and not interchangeable. In Bell's experiments each finding is local, and the pattern appears when the records meet, no faster than light. The single-time result follows from the three kinds of time: no interpretation built on one unified time can be complete.

With the measurement problem understood, how should the programs that seek one theory of gravity and the quantum be judged?



PART VI

THE SEARCH REVISITED



Chapter 21

Four Questions for Any Theory

Judging a Design Before It Is Built

How can a program be judged before it is finished? By the commitment it makes at its first step, which the Incommensuration Theorem can examine as it stands. A research program is a long effort, shared by many people, that starts from one assumption and uses one family of methods to reach one goal. The goal of the programs of quantum gravity is a single theory that holds both general relativity and quantum theory. No program has reached it. Every program has made, from its first page, a choice about what gravity is, and that choice can be read today.

An engineer who knows that energy is conserved can judge a design for a perpetual-motion machine from its drawings. She does not need to wait for the machine to be built. She can even say where its trouble will show: at the part of the drawing where energy is asked to come from nothing. Her judgment does not depend on the inventor's skill, which may be great. It depends on a law that every machine must meet, and that this design does not meet.

The Incommensuration Theorem plays the part of that law here. It says that symmetry and continuity cannot both be perfected, simultaneously and fundamentally, in one comparison; and neither can asymmetry and discontinuity. It holds of every comparison, and so of every finding that any theory can make (Chapter 5, "Symmetry and Continuity Cannot Both Be Perfected", sets it out from the beginning). Put to a program, it yields four questions. The first three ask what the program commits itself to. The fourth asks whether the difficulties the program has met are the ones its commitments predict.

The First Question: Arena or Force?

Does the program treat gravity as the arena, or as one more force inside the arena? The arena is space and time together, the context in which everything that is measured is measured. The forces of particle physics belong to the content. Their charges can be counted, their pull can be canceled by charges of the opposite sign, and each force has its own strength.

Gravity carries the four marks of the arena instead. It acts on everything; nothing shields it; the mass that is pulled and the mass that resists being moved are one mass; and it has no charge of its own that comes in steps. (Chapter 9, "Four Marks of the Arena", derives each mark for any arena first and then finds it in gravity.) A program passes the first question when its foundations keep this difference. It fails the first question when it gives gravity a charge, a strength that changes with scale as the strengths of the other forces do, and a field to be quantized as theirs are.

Most programs that fail the first question fail it without deciding to. The method of quantizing a field worked for light and for the nuclear forces. They carry the same method over to gravity because it worked there, and the carrying over is itself the decision.

The Second Question: One Description With Both Characters?

Does the program ask one description to be both continuously asymmetric and discontinuously symmetric? General relativity finds a geometry that is smooth and that differs from place to place: continuous and asymmetric. Quantum theory finds outcomes that come in separate steps and repeat exactly: discontinuous and symmetric. The theorem allows each of these characters, and it forbids any single description whose findings carry both in the same respect.

A program can make this demand openly or quietly. It makes the demand openly when it gives the geometry of space a set of separate values and then asks that same geometry to be smooth at large scales. It makes the demand quietly when it starts from discrete quantum structure and expects that structure to fix one smooth geometry with nothing left over. In both cases the same arena is asked to carry both characters, and a description that does so would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both.

The Third Question: What Is Quantized?

What does the program quantize? To quantize a description is to exchange the characters of both of its levels at once. Its mathematics and its findings exchange their characters, and smooth findings become findings that come in steps (Chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It", shows this mode exchange on the electromagnetic field). The theorem leaves room for several uses of quanta near the arena, and the third question sorts a program's uses into the ones the theorem admits and the one it excludes.

The theorem admits the quanta of the content: matter and light, and everything else that moves through the arena. It allows a description of the small waves of the geometry about a smooth shape by quanta, as a description: such a quantum is a quantum of the description, never a quantum of the

arena, and it is not a step of space. It admits whole-number counts that no smooth stretching of a geometry can change, such as the number of holes in a surface. It excludes one thing: a quantum of the arena itself, at any scale, which would give space and time a least step.

A program that keeps to the admitted uses passes the third question. A program that quantizes the arena itself fails it, whatever else it does well.

The Fourth Question: Is the Difficulty Where It Should Be?

Has the program met the difficulty that its commitments predict? This question gives the survey its force. I reason from the theorem that a program built on a forbidden pairing will fall short of its goal, and I reason how the program will fall short, and where in its work the shortfall will appear. A program can ask the arena to be content in three ways, and each way has its own kind of difficulty.

Multiplicity

A program can start from the discrete and symmetric side and try to derive the smooth arena from it. I call the difficulty this predicts the difficulty of multiplicity. Discrete structure, however rich, does not fix one outcome that is continuous and asymmetric, because the two characters have no common perfection. So the program finds a great many arenas that fit its discrete starting point, and no principle within it that picks out the one we live in.

A musical score fixes which notes are played and in what order. It does not fix the room the music is played in, and the same score can sound in a cathedral, a kitchen or an open field. Anyone who tries to read the room from the score finds as many rooms as there are places to play.

Dynamics

A program can start from the smooth geometry of general relativity and quantize that geometry directly. I call the difficulty this predicts the difficulty of dynamics. Change is described as change across the arena: a before and an after, laid out in the continuous context. A program that makes the arena one

more quantum system has no continuous context left against which to say what changes. Its equation of change then resists being written in a form that gives back the world we see.

A ledger records the changes in an account, entry by entry, against the dates in its margin. Make the dates themselves into entries of the account, to be added and subtracted like the sums, and the ledger still holds every number. It no longer says which entry came first.

Recovery

A program can give up one character entirely and hope to recover it from the other. I call the difficulty this predicts the difficulty of recovery. The two characters are distinct, and neither can do the work of the other, so the character given up does not come back of itself. The program needs a further principle, not contained in its starting point, to bring it back.

A mosaic of square tiles can approximate a circle very closely. The tiles alone never decide that a circle is wanted, and most ways of laying the same tiles make no circle at all. The circle comes from the mosaicist, and not from the tiles.

The three ways, and the difficulty each one predicts, can be set side by side.

The program's first step	The difficulty predicted	Where it appears
Derive the smooth arena from discrete structure	Multiplicity	Many possible arenas, and no principle that selects ours
Quantize the smooth arena directly	Dynamics	An equation of change that will not give back the changing world
Give up one character and rebuild it from the other	Recovery	A further principle needed to bring the lost character back

The Questions Judge Commitments, Not People

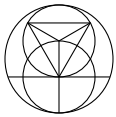
The four questions say nothing against the people who work in a program. The physicists who built these programs include some of the ablest minds of their century, and the next chapter credits what each program found before it names any difficulty. A difficulty that a founding commitment predicts cannot be removed by greater skill, because it lives in the commitment and not in the calculation. That is why the same kinds of difficulty appear in programs run by different people, with different mathematics, in different countries.

The questions also describe a program that answers all four of them well. Such a program keeps gravity as the arena, never asks one description to carry both characters, and quantizes only what the theorem admits. It therefore meets none of the three difficulties. Its remaining work is of another kind: work of completion, and not of repair. (The four questions are stated as criteria, with their grounds, in *The Unquantized Arena*, Chapter 14, "Four Criteria and Ten Programs".)

You can use the four questions yourself, on any proposal you read about. Ask what it takes gravity to be, whether it asks one description to be both smooth and stepwise, what it quantizes, and where its trouble lies. The answers will tell you more about the proposal's prospects than the size of its equations.

> **Key idea.** Every program of quantum gravity makes, at its first step, a choice about what gravity is, and the Incommensuration Theorem can judge that choice before the program is finished. Four questions follow: whether the program keeps gravity as the arena, whether it asks one description to carry both characters, what it quantizes, and whether its difficulties are the ones its choice predicts. Deriving the arena from discrete structure predicts multiplicity; quantizing the arena directly predicts a difficulty of dynamics; giving up one character predicts a difficulty of recovery.

The next chapter puts the four questions to ten programs, and credits each with what it found.



Chapter 22

What Each Program Found

"This perspective suggests that it may be no more appropriate to canonically quantize the Einstein equation than it would be to quantize the wave equation for sound in air."

Ted Jacobson, "Thermodynamics of Spacetime: The Einstein Equation of State" (1995)

The Search Was Not Wasted

Was the search wasted? No: every program of quantum gravity found something real, and much of what they found will last. They found new mathematics, exact results about black holes, new ways to compute how particles scatter, and a dictionary between geometry and quantum information. Each program also met a difficulty, and in each case the difficulty is the one that the program's founding commitment predicts.

The ten programs follow in turn. For each program, its achievement comes first. Then come the commitment the program made at its first step, and the difficulty that commitment predicts by the four questions of the last chapter. The ten fall into three groups. Four quantize the arena outright; three stand between; three leave the arena unquantized. The order of the groups is the gradient on which the chapter closes.

Four Programs That Quantize the Arena

String theory

String theory replaces each point-like particle with a tiny vibrating string, and reads each kind of particle as one way the string can vibrate. Its founding discovery, as John Schwarz recounts it (2007), was that string theories, in their consistent forms, are quantum theories of gravity, with the graviton among the string's vibrations. In 1996 Andrew Strominger and Cumrun Vafa used string theory to count the quantum states of a class of idealized black holes, and the count matched exactly the entropy that Jacob Bekenstein and Stephen Hawking had assigned to their horizons. These are two of the most striking results in modern physics.

The commitment at string theory's first step is to begin from quanta and to derive the arena from them. String theory's graviton sits in one list with the photon and the particles of the nuclear forces, set apart from them only by its labels, so the theory fails the first question. Where its gravitons describe small waves on a smooth background, they are quanta of a description, which the third question allows; they are never quanta of the arena. Its claim that the arena itself is built from string states is the one use of quanta the third question excludes.

The difficulty this commitment predicts is multiplicity, and it is the difficulty string theory has met. The theory admits an immense number of possible lowest-energy states, each with its own particles and forces. Each is called a string vacuum, and no principle inside the theory selects the string vacuum we live in. Leonard Susskind (2003) called their range "unimaginably large and diverse", and Michael Douglas (2003) set out to count them by statistics. A discrete starting point, however rich, has fixed no single smooth world; that is what multiplicity means.

String theory's own reply to this difficulty is called the swampland program, begun by Cumrun Vafa in 2005 and reviewed by Eran Palti in 2019. It looks for the conditions that any low-energy world must meet if it can come

from a consistent quantum theory of gravity, and so it narrows the range of worlds the theory admits. Its conditions are still conjectures, and none of them has yet selected one vacuum.

Loop quantum gravity

Loop quantum gravity takes the opposite path. It starts from the smooth geometry of general relativity, and it quantizes that geometry directly, without using any background space. Carlo Rovelli and Lee Smolin (1995) computed, in this framework, that the volume of every region and the area of every surface take separate values, labeled by the networks Roger Penrose had called spin networks. Thomas Thiemann (1998) then gave a well-defined form to the equation that governs how those networks change.

I read those separate values as results about the description the program chose, and not as steps of space. The program's starting insight, that gravity is geometry and not one more force, comes closer to passing the first question than string theory's starting point does. Its next step applies to the geometry the operation that belongs to content: it quantizes the context. It then asks the same geometry to have separate values and to be smooth at large scales, which is the second question's demand made openly.

The difficulty this commitment predicts is dynamics, and it is where the program's work has gathered. The equation of change, called the Hamiltonian constraint, has been written in several forms, and the program's central task is to show that the quantum geometry it governs gives back the smooth, changing geometry of general relativity at large scales. Recall the ledger whose dates were made into entries: every number is kept, and the order of change is what has to be won back.

Asymptotic safety

Asymptotic safety keeps the geometry smooth and treats it as a quantum field like the others. In a quantum field theory the strength of an interaction changes with the scale at which it is measured. A fixed point is a scale beyond which the strengths stop changing, so that the theory stays finite at the

shortest distances. Steven Weinberg proposed that gravity has such a point, and Martin Reuter (1998) gave the equation by which the flow of gravity's strengths can be computed.

Calculations of growing size have found the fixed point again and again. Mikhail Shaposhnikov and Christof Wetterich (2010) used it to predict a mass for the Higgs boson of about 126 billion electronvolts, two years before the particle was found near 125. That prediction stands among the program's achievements.

The commitment at the first step is to give gravity a strength that runs with scale, as the strengths of the other forces run, and to sum over every shape of the geometry. That fails the first question and the third. The difficulty this commitment predicts is dynamics, met where the quantized arena must be defined exactly, and that is where the program's evidence stops. The fixed point is found in truncations, calculations that keep a finite number of the theory's terms, and not in the full theory. For many years those calculations were made in a mathematical version of space and time in which time is turned into a fourth direction of space. Since 2023 they have also been made with time kept as time: Jannik Fehre, Daniel Litim, Jan Pawlowski and Manuel Reichert (2023), and Gabriel Assant, Daniel Litim and Manuel Reichert (2026), computed how the graviton of the program's description propagates in that setting and found the behavior the fixed point requires. I expect each larger truncation, with time turned into space or kept as time, to find the fixed point again, and the full theory with time kept as time to withhold it.

Causal set theory

Causal set theory, founded by Rafael Sorkin and his colleagues, replaces space and time with a scattering of separate points. The points carry one relation only: which point lies in the past of which, with the count of points in a region standing for its volume. Luca Bombelli, Joe Henson and Rafael Sorkin (2009) showed that points scattered at random, with uniform density, pick out no preferred direction of motion. So this kind of discreteness keeps the symmetry of relativity between frames in uniform motion, which no regular lattice can keep.

The commitment at the first step is to give up the continuous character entirely, and to recover it from the points. Content and context both become discrete, so the first question has no difference left to keep. The difficulty this commitment predicts is recovery, and it is where the program's work lies. Among all the ways of ordering a large number of points, almost all belong to a flat, three-layered kind that resembles no space and time at all, as Daniel Kleitman and Bruce Rothschild found when they counted such orders (1975). The theory therefore needs a further law, not contained in its points and their order, to favor the rare orders that look like a smooth arena. The program has found such a law in part. Peter Carlip, Steven Carlip and Sumati Surya showed in 2023 that the commonest of the orders that look like no space and time are strongly suppressed when the theory's own version of Einstein's law weighs each order, and in 2024 they extended the result to every dimension, on a condition about the size of the separation between points. That weighting, with that condition on the separation, is the further law that the difficulty of recovery calls for; neither comes from the points and their order alone.

Shuffle a deck of cards and almost every order you get looks like no pattern. The orders that run cleanly by suit and rank are a vanishing few, and nothing in the cards themselves prefers them. The preference has to come from somewhere else.

Three Programs That Stand Between

Causal dynamical triangulations

Causal dynamical triangulations, developed by Jan Ambjørn, Jerzy Jurkiewicz and Renate Loll, builds trial geometries from small flat pieces glued together, with one rule: every geometry must keep a consistent order in time. From the sum over such geometries, a world with four large dimensions emerges at large scales (Ambjørn, Jurkiewicz and Loll 2004). At short distances the effective dimension of that world falls to about two (2005).

The program stands between the groups for two reasons. Its pieces are a method of calculation, to be shrunk away in the end, and not a claim that space is made of pieces. Its rule of order keeps the one-way order of the

arena, which belongs to the arena's asymmetric character. It still sums over geometries, and so it still quantizes the arena. The difficulty this sum predicts is the one on which the program's success turns: whether the pieces can be shrunk away to leave a smooth theory, and whether that theory is general relativity.

ER=EPR

Two systems are entangled, as Chapter 13 set out, when their alternatives are joined so that neither has a state of its own. Juan Maldacena and Leonard Susskind (2013) proposed that a pair of entangled black holes and a bridge of geometry joining their interiors are one phenomenon in two descriptions. They named the proposal ER=EPR, joining the initials of Einstein and Rosen, who described the bridge, to those of Einstein, Podolsky and Rosen, who described the entangled pair. It grew from Mark Van Raamsdonk's argument (2010) that removing the entanglement between two parts of a quantum system pulls the matching regions of its geometry apart.

As a correspondence, the proposal is of the kind the theorem leads us to expect. It maps a description of the continuous and asymmetric kind, the bridge of geometry, onto a description of the discrete and symmetric kind, the entangled state. Its difficulty lies in how it is read. The usual reading is that geometry emerges from entanglement, which ranks the quantum side as the source and the geometry as its product. I accept the correspondence and decline the ranking: a map between two descriptions shows that they translate, and translation runs in both directions (the next chapter makes this argument for the largest such map).

The amplituhedron

A scattering amplitude is the number from which the probability of each outcome of a collision is computed. Nima Arkani-Hamed and Jaroslav Trnka (2014) found that the amplitudes of a highly symmetric relative of the theory of the strong force, in its simplest limit, are the volume of a geometric object they named the amplituhedron. In that picture the rules of locality and of conserved probability come out of the geometry, rather than being put in at the start.

The program's present work lies wholly within the content, where it raises nothing against the theorem. Its object is a shape in a continuous space, so its starting point is not discrete in the theorem's sense. Its founding commitment is that space and time are not fundamental and should come out of amplitude data. Amplitudes are data of the content, so an extension that derived the arena from them would be one more derivation of the context from the content, and would meet the difficulty of multiplicity. An extension that gave only a description of small waves on a smooth background by quanta would stay within what the third question admits. The program stands between the groups because its place depends on which extension it makes.

Three Programs That Leave the Arena Unquantized

Emergent gravity

Ted Jacobson (1995) derived Einstein's equation from two premises: that entropy is proportional to the area of a horizon, and that heat, entropy and temperature are related as thermodynamics says they are. He drew the conclusion in the sentence that opens this chapter. Erik Verlinde (2011) and Thanu Padmanabhan (2010) developed the thermodynamic reading of gravity further. Verlinde's account of Newton's law as an entropic force has met published objections: Archil Kobakhidze (2011) argued that neutrons held in quantum states by the Earth's gravity behave in a way that an entropic force cannot produce, and Matt Visser (2011) showed that the conservative character of gravity, which does no net work on a body carried around a closed path, leaves an entropic account of Newton's law very little room. The objections bear on Verlinde's derivation of Newton's law, and Jacobson's equation of state stands.

The commitment at the first step is that gravity is not a force to be quantized. That commitment passes the first three questions, and it predicts none of the three difficulties. What the program does not supply is a reason why gravity alone is thermodynamic, while the other forces are not; it takes that as its starting point. The ground of comparison supplies the reason: gravity is the arena, and the arena is the context of every finding, where the continuous

and asymmetric character belongs. I do not read the arena as emerging from the content; I read it as the context the content requires. The two readings agree on the step that matters here, which is that the arena is not to be quantized.

Twistor theory

Twistor theory, which Roger Penrose began in the 1960s, treats the events of space and time as derived objects. It encodes them in a complex space, called twistor space, in which the paths of light are the basic elements. Solutions of the equations of gravity and of the nuclear forces that have one handedness, which mathematicians call anti-self-dual, can be written in twistor space as data of a simpler kind. Michael Atiyah, Maciej Dunajski and Lionel Mason reviewed the theory's first fifty years in 2017. Edward Witten (2004) found that the scattering amplitudes of a theory closely related to the strong force take a simple form there, lying along curves in twistor space.

The commitment at the first step is to reformulate the geometry without quantizing it, and twistor methods quantize only the content. That commitment passes the first three questions, and it predicts none of the three difficulties. The work that remains is of completion: the curved geometries twistor space encodes most directly have one handedness, and the full geometry of general relativity has both. I notice that what twistors capture most readily is the light-cone structure of space and time, the part of the geometry that stays the same when every length is rescaled together, and so the part of symmetric character.

Non-commutative geometry

Non-commutative geometry describes a space through an algebra of quantities, including algebras in which the order of multiplying two quantities changes the result. Alain Connes's program of non-commutative geometry, developed with Ali Chamseddine and Matilde Marcolli, writes the whole Standard Model of particle physics, together with Einstein's gravity, as one action (Chamseddine and colleagues 2007). It builds that action from ordinary space and time joined to a small finite algebra that carries the particles'

charges. In that action the geometry of space and time is held classical. Since the arena stays unquantized, the four questions predict none of the three difficulties for this result.

The program's record holds one miss, and it belongs here. The action in its form of 2007 gave a mass for the Higgs boson near 170 billion electronvolts, and the particle was found in 2012 near 125. In 2012 Chamseddine and Connes restored agreement by adding one more field that the finite algebra supplies, which changes how the strengths of the forces run with scale. The miss and its repair concern the particles carried by the small algebra, and the classical geometry of space and time stands through both.

The Gradient

Set side by side, the ten programs run along a gradient, from the four that quantize the arena to the three that leave it alone.

Group	Program	First step	Difficulty predicted
Quantize the arena	Causal set theory	Give up the continuous character	Recovery
Quantize the arena	String theory	Derive the arena from quanta	Multiplicity
Quantize the arena	Loop quantum gravity	Quantize the geometry directly	Dynamics
Quantize the arena	Asymptotic safety	Treat the geometry as a field with a running strength	Dynamics (the full theory, with time kept as time)
Stand between	Causal dynamical triangulations	Sum over geometries that keep the order of time	The smooth limit
Stand between	ER=EPR	Map geometry onto entanglement	The reading of the map
Stand between	The amplituhedron	Derive locality from amplitude data	Multiplicity, if extended to derive the arena
Leave the arena alone	Twistor theory	Reformulate the geometry without quantizing it	None of the three
Leave the arena alone	Emergent gravity	Read the arena's law as an equation of state	None of the three
Leave the arena alone	Non-commutative geometry	Hold the geometry classical in one action	None of the three

The four programs of the first group share one structural error. Each gives the arena the treatment that belongs to content, and they differ only in the direction and the method of the attempt. String theory derives the arena from

quanta; loop quantum gravity quantizes the arena as it stands; causal set theory replaces the arena with points; asymptotic safety quantizes the arena as one more field. (The ten assessments are set out criterion by criterion, with the literature, in *The Unquantized Arena*, Chapter 14, "Four Criteria and Ten Programs".)

One Error, Four Expressions

The multitude of string vacua, the equation of change of loop quantum gravity, the scarcity of smooth-looking orders among causal sets, and the fixed point found only in truncations are four expressions of one structural impossibility. They are not four separate puzzles that happen to have resisted able people. A program that assumes, at its first step, that gravity can be quantized has committed itself to a forbidden pairing. Its difficulties downstream are that commitment, met again in the program's own mathematics.

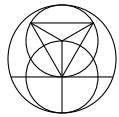
The other end of the gradient tells the same story from the other side. The programs closest to this book's thesis are the ones that do not try to quantize gravity. Jacobson reads Einstein's equation as an equation of state; Penrose reformulates the geometry without quantizing it; the action of non-commutative geometry keeps the geometry classical. Each avoided the difficulties that the programs of the first group have met, and each did so by the choice it made at its first step.

Nothing in this verdict takes back the credit given above. A program can be wrong about what gravity is and right about a great deal else. The count of black-hole states, the spectra of the geometric operators, the fixed-point equation, the triangulated sums, and the dictionary between entanglement and geometry are mathematics that will keep its value. Some of it already travels between fields: the networks that model the geometry of the correspondence of the next chapter grew from methods built to study quantum materials (Swingle 2012). What the gradient judges is one assumption, made at the start and shared across most of the search.

For ninety years the search has asked how gravity should be quantized. The survey asks a question that comes before that one. Why would we assume that gravity is quantizable?

Key idea. Every program of quantum gravity found something real, from black-hole entropy counted in string theory to the Higgs mass predicted by asymptotic safety and Einstein's equation derived from thermodynamics. Each program's difficulty is the one its founding commitment predicts: multiplicity for deriving the arena from quanta, dynamics for quantizing the geometry, recovery for giving up the continuum. The programs closest to the thesis are the ones that leave the arena unquantized.

The next chapter takes up the largest map between the two descriptions that physics has found: gravity in a region, translated exactly into a quantum theory on its boundary.



Chapter 23

Two Descriptions That Translate

The Strongest Objection

What about the result from string theory that gravity inside a region equals a quantum theory on its boundary? It is two descriptions of one system, one in each of the two characters the theorem allows, joined by an exact dictionary. Read that way, it is the clearest example of this book's thesis that physics has found. To see why, the objection has to be stated first, at its full strength.

The setting is a special kind of space and time, called anti-de Sitter space. It is curved with the same negative curvature everywhere, like a saddle at every point. It has an edge at infinite distance, and a ray of light can travel to that edge and back in a finite time. Physicists call the interior of this space the bulk, and its edge the boundary.

In 1997 Juan Maldacena proposed, on strong grounds, that string theory in such a space, gravity included, is equivalent to a quantum field theory without gravity that lives on the boundary. His paper was published in 1998. That boundary theory is a highly symmetric relative of the theory of the strong force, of the kind called conformal: it looks the same at every magnification. The equivalence is called the correspondence, and physicists also write it as AdS/CFT. Every comparison of the two sides that physicists have been able to compute since has given the same answer on both. I will call the string theory of the interior, gravity and all, the bulk theory. I will call its description by smooth classical geometry, with quantum fields living on that geometry, the bulk's geometric description.

Here is the objection. The boundary theory is a quantum theory in the full sense, and it is exactly equivalent to the bulk theory, gravity and all. So gravity, it seems, is a quantum theory after all, described in geometric language. If that were the right reading, the correspondence would be the very merger of the two characters that the Incommensuration Theorem forbids.

The objection has a stronger form. When the boundary theory has a finite number of kinds of field, it is still a complete quantum theory, and many physicists take it to define the bulk theory at all lengths, down to the Planck length and below. On that reading the boundary theory is a full quantum theory of gravity, defined through its boundary.

Each Side in Its Own Character

The first thing to check is the character of each side, by what each side finds. Everything the boundary theory can find is fixed by a list of numbers: how each of its quantities changes under magnification, and the coefficients with which pairs of its quantities combine into others. The list is discrete, and its entries sort the theory's quantities into families labeled by a symmetry that holds exactly. I identify the findings of the boundary theory as discontinuously symmetric, the character of quantum theory.

The boundary theory's mathematics is continuous. It is written on a smooth space and time, and its predictions are smooth functions of where its quantities are placed. Its findings and its mathematics are in opposite characters, as the two levels of every theory are (Chapter 10, "How a Theory Sees", checks this against six established theories).

The bulk side, in its geometric description, finds a geometry: smooth, and different from one region to the next, with no finite list of labels that sorts its possible shapes. I identify the findings of the bulk side as continuously asymmetric, the character of general relativity. Its mathematics is ruled by symmetries: its equations take the same form in every system of coordinates, and the empty space has a large group of exact symmetries of its own. Again the findings and the mathematics are in opposite characters.

So the two sides of the correspondence stand in the two characters the theorem allows, one on each side. A map between them is a map between conjugate descriptions. From the theorem I reason that such a map has three features. It translates systematically; it reduces neither side to the other; and it leaves, on each side, something that the other side does not carry. That something is the residual of Chapter 10, here seen on each side of a map: what one description of a system leaves out because its character cannot hold it. The literature on the correspondence shows all three features.

How Far the Dictionary Reaches

The dictionary reaches deep into the bulk, and it does not reach everywhere. Alex Hamilton, Daniel Kabat, Gilad Lifschytz and David Lowe (2006a) wrote a field at a point of the bulk as a spread-out sum of quantities on the boundary. In empty anti-de Sitter space their construction is exact, and they extended it to points behind horizons in simple cases. That is a long reach for a quantum theory with no gravity in it.

Ahmed Almheiri, Xi Dong and Daniel Harlow (2015) proposed that the dictionary works like a code that protects information against loss. The bulk plays the part of the message, and the boundary the part of the encoded record. A point deep in the bulk can be recovered from more than one region of the boundary, as a message spread across many records can be read from any of several subsets of them. The code is redundant, and nothing in it is lost. Fernando Pastawski, Beni Yoshida, Daniel Harlow and John Preskill (2015) built exact small models of such codes.

The code has a reach, and the reach has an edge. A code protects a chosen set of messages, and the bulk descriptions the code protects are those close to one smooth geometry. Inside a black hole, Kyriakos Papadodimas and Suvrat Raju (2013) built the interior's quantities from the boundary, and their construction depends on which state the boundary is in. To tell the hole's individual states apart from the boundary needs a precision of one part in a number that grows exponentially with the hole's entropy. Ben Freivogel and col-

leagues (2015) found regions near a black hole's horizon, and near a dense enough star, that the boundary's standard probes do not reach; they called them shadows.

Taken together, these results show that the boundary reaches the bulk densely but not exhaustively. A reach of this kind is what the theorem leads us to expect of any map from a description of one character into a description of the other. A finer map reaches further, and no map of this kind reaches everything. What the boundary reaches with ease is the large-scale shape of the geometry and the gentle excitations on it. What lies at the edge of its reach is the fine detail near horizons and inside black holes. That is the residual on the bulk side, and it is of the bulk's own character: the smooth geometry where it varies most.

A Bridge, Not a Derivation

The best-known entry in the dictionary is the formula of Shinsei Ryu and Tadashi Takayanagi (2006). Entanglement entropy is a measure of how strongly a region is entangled with everything outside it, computed from the probabilities in the region's quantum state. Ryu and Takayanagi found that the entanglement entropy of a region of the boundary equals the area of the smallest surface in the bulk that ends on the region's edge, in units fixed by Newton's constant. A quantity of the quantum side equals a quantity of the geometry.

The two sides of the formula are in the two characters. The entropy depends only on the list of probabilities, and it stays the same however the description inside the region is turned into an equivalent one; it is a quantity of the symmetric kind. The area is smooth: it changes continuously as the region or the geometry changes. The formula is a bridge between the two characters. It is a bridge of the same kind as the probability rule of quantum theory, which joins a smooth description to the odds of discrete findings (Chapter 17, "Why a Measurement Has an Outcome", treats that rule as the bridge between the two levels of quantum theory). Neither bridge turns one side into the other; each lets a quantity of one character be read off in the other.

Geometry has an older bridge of the same shape, the theorem of Gauss and Bonnet. Add up the curvature of a closed surface over the whole surface, and the total is always two pi times a whole number that is fixed by the number of the surface's holes. For a sphere the total is four pi; for the surface of a ring doughnut it is zero. Squash a sphere into any lumpy shape you like, and its total stays four pi. The bridge is exact. It is not invertible: the whole number cannot tell you which of the infinitely many shapes you started from.

Three features of the formula of Ryu and Takayanagi mark it as a bridge of this kind rather than a derivation of geometry from entanglement. First, the argument that establishes it, given by Aitor Lewkowycz and Juan Maldacena (2013), assumes the dictionary and works inside it. Second, the formula does not run backwards in full: the bulk's shadows are reached by no smallest surface, so geometries that differ only inside a shadow give every boundary region the same entropy. Third, the formula has a correction. Thomas Faulkner, Aitor Lewkowycz and Juan Maldacena (2013) found that, at the next order, the entropy is the area plus the entanglement of the quantum fields in the bulk across the surface.

The third feature says the most. The completed formula is a sum of two terms: a term of geometry, which is the area, and a term of content, which is the entanglement of the quantum matter that lives on that geometry. The dictionary's own best formula joins quantum matter and classical geometry side by side. The reading that geometry is "nothing but" entanglement takes one direction of a bridge that runs both ways.

Neither Side Comes First

The usual reading of the correspondence takes the boundary theory as the fundamental description, and the bulk theory, gravity included, as emerging from it. The reading has real motives: the boundary theory is a well-defined quantum theory, while no one has a definition of quantum gravity in the bulk that stands on its own. The correspondence between the boundary theory and the bulk theory, though, is exact in both directions. If the boundary theory fixes the bulk theory, the bulk theory fixes the boundary theory. A duality that is exact in both directions does not privilege either side.

Emergence is a one-way relation. If one thing emerges from another, the second is the more fundamental of the two. An exact equivalence gives no such direction, so it cannot by itself show that the bulk theory emerges from the boundary theory. What the correspondence shows is that one system admits two descriptions, one in each character, with a systematic translation between them. This is correspondence, not emergence.

The residuals point the same way. The bulk's geometric description, too, leaves a residual: it reaches only some of the boundary theory's states. Patrick Hayden, Matthew Headrick and Alexander Maloney (2013) found that every boundary state whose entanglement is given by the formula of Ryu and Takayanagi obeys a rule on how its parts can share information, a rule that quantum states in general need not obey. So the states that a smooth geometry describes are a special class. The rule is called monogamy. Roughly, it says that a region which shares a great deal of information with one neighbor has correspondingly less to share with a second, as a person who confides everything in one friend has little left to confide in another. The boundary theory holds every pattern of entanglement its states allow, and the bulk's geometric description covers only the patterns that pass the test.

The two residuals are complementary. What the boundary theory reaches least well about the bulk, the fine geometry near horizons, is what the bulk's geometric description gives in its own language. What the bulk's geometric description reaches least well about the boundary theory, the patterns of entanglement with no geometry behind them, is what the boundary theory describes in its own language. Each side's blind spot is the other side's native domain. Brian Swingle (2012), and Pastawski and his colleagues with their code models, built discrete networks that reproduce features of the bulk geometry. The networks approximate the smooth geometry; they do not replace it.

What the Correspondence Shows

By the theorem, a systematic correspondence between a description of the discrete and symmetric kind and a description of the continuous and asymmetric kind is a bridge between conjugates, not a merger of them. The corres-

pondence of the bulk and the boundary is such a correspondence. So it is a bridge, and the objection that opened this chapter rests on reading the bridge as a merger.

Look at the regime where the dictionary has been tested most. When the boundary theory has a great many kinds of field, Maldacena found that the bulk side is described by classical gravity, with quantum fields living on its geometry. The corrections to that picture include a description of the small waves of the geometry by quanta, which the theorem allows as a description; its quanta are never quanta of the arena. Nothing in the tested correspondence asks the arena itself to be quantized at any scale.

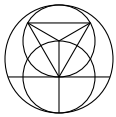
The stronger form of the objection concerns a finite number of kinds of field, where the boundary theory is complete and the bulk's geometric description holds only approximately. On this account, what the boundary theory defines there is the content description of the system, in its own character. Where the curvature of the bulk reaches the Planck curvature, the bulk's geometric description reaches the edge of its domain, as Chapter 15, "Where the Two Descriptions Meet", found for every smooth description, and the boundary theory supplies no geometry with steps in its place. For a finite number of kinds of field, the boundary theory describes the content completely, and it describes no quantum state of the arena itself.

Our universe has no edge of the anti-de Sitter kind, so the correspondence is a model rather than a description of our world. It is still the hardest test the thesis faces, because it is exact, and because its exactness is what makes the objection strong. Its two sides stand in the two characters; its dictionary reaches far and leaves a residual on each side; its central formula joins geometry and quantum matter side by side. (The technical form of these results, with the literature, is in *The Unquantized Arena*, Chapter 15, "AdS/CFT as Cross-Modal Correspondence".)

You have met correspondences of this kind before. A good translation of a poem carries its sense across, line by line, and still each language keeps what the other cannot hold: the rhyme in one, a turn of phrase in the other. Nobody concludes from a perfect translation that French is English in disguise.

Key idea. The correspondence between gravity in anti-de Sitter space and a quantum theory on its boundary joins two descriptions of one system, one continuously asymmetric and one discontinuously symmetric. Its dictionary reaches deep into the bulk and leaves a residual near horizons; its central formula is a bridge whose completed form adds the entanglement of quantum matter to an area of classical geometry. Because the equivalence runs exactly both ways, neither side is more fundamental: this is correspondence, not emergence.

If the two descriptions are joined and never merged, is there then no theory of everything? The next chapter answers.



Chapter 24

Joined, Not Merged

One World, One Account, Two Descriptions

Is there then no theory of everything? There is one account of everything, and it is made of two descriptions joined together. The Incommensuration Theorem permits a unity of that kind, in which two descriptions come from one source and stand side by side. It forbids only one kind of unity: a single description in which the directions of space and time themselves carry both characters together. The search for quantum gravity has looked for the forbidden kind.

The phrase "theory of everything" has two meanings, and the difference between them is the whole of this chapter. The first meaning is an account that leaves nothing out: every finding, of every kind, has its place in it. The second meaning is an account written as one description, with one character throughout, from which everything else follows as a special case. The theorem allows the first and forbids the second.

Two Kinds of Unity

I call the first kind of unity a common source. Two descriptions have a common source when both arise from the same act and apply to the same world, each perfected in one of the two characters a comparison allows. Joined side by side, they cover between them what each alone leaves out.

I call the second kind of unity a merger. A merger is one description in which the directions of the arena themselves carry the character of content at the foundation, while the same directions are asked to keep the arena's own character. A direction, here, is a direction of space or the direction of time: one of the independent respects in which the arena lays out the findings of every description. A merger asks the same direction to be smooth and different from place to place, and also to come in exact, repeating steps. Quantum electrodynamics is no merger. Its field is content laid on the arena, and the directions of space and time in which its photons are counted keep their own smooth character.

A railway timetable and the railway itself make a working pair of this kind. The timetable lists departures as separate entries that can be counted, and each entry repeats exactly from week to week. The track is one unbroken line through country that changes from mile to mile. Every train in the timetable runs on the track, and the two are joined at every station. Nobody asks the track to be made of timetable entries, or the timetable to be made of track. A single object that was, in the same respect, a list of separate entries and an unbroken, varying line would be no improvement on the pair; it would be an object that cannot be built.

Why the Search Expected a Merger

The search expected a merger because mergers had served physics well. Electricity and magnetism were found to be one field, and later the electromagnetic and weak forces were written as one theory. Each success joined one kind of content to another, and each made the next one seem likely. After those successes, gravity looked like the one force left outside, waiting to be brought in by the same method.

That expectation rests on the answer to the first of the four questions. It treats gravity as one more force inside the arena. Gravity carries the four marks of the arena instead, and so it is the context in which those other mergers took place. Bringing the arena into a merger with its own content would not be one more step of the same kind. It would ask one description to carry both characters, which is the step the theorem excludes.

What the Theorem Permits

The theorem permits a common source, and the ground of comparison asks for one. Every comparison has a content and a context, and *An Immanent Metaphysics* states their inseparability: "Where there is content, there must also be context. Where there is context there must also be content." The two laws of the same work give each its character: "Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context." So the two descriptions are inseparable, because content and context are; and they are distinct, because their laws are.

The source they share is the act of comparison itself. Quantum theory perfects what comparison finds as content: discrete, symmetric, countable findings. General relativity perfects what comparison finds as context: the continuous, asymmetric arena in which those findings are laid out. Each is complete in its own character, and each captures what the other must leave out.

Physics already uses the permitted unity, as one of its most productive tools. Quantum fields living on a classical, curved geometry make exactly such a pair, joined side by side. That framework, in its standard form, lays the quantum fields on one fixed classical geometry, and it is the framework in which black holes were found to radiate (Chapter 25, "Where Does the Information Go?", follows that result). This book's account extends the framework: each alternative of the matter carries its own classical geometry (Chapter 12, "One Geometry for Each Alternative", sets this out and shows why one averaged geometry fails).

The joined descriptions also meet at definite places, and those meetings are bridges rather than mergers. The probability rule joins quantum theory's smooth mathematics to its discrete findings. The formula of Ryu and Takayanagi joins an entropy of the quantum side to an area of the geometry, and its completed form adds the entanglement of the quantum matter to the area (Chapter 23, "Two Descriptions That Translate"). The Planck scale is where an object's gravitational size equals its quantum size, so that neither description dominates and both are relevant (Chapter 15, "Where the Two Descriptions Meet").

The theorem also has nothing against joining one content to another. Two kinds of content share one character, so one description can carry both without any forbidden pairing. The theorem is silent on whether nature joins further forces in that way, and this book takes no position on it.

What the Theorem Forbids

The theorem forbids a merger: one description in which the directions of space and time themselves carry both characters. A quantized arena is such a merger. General relativity needs the geometry along the directions of space and time to be continuous and asymmetric, and quantization would make the geometry along those same directions discontinuous and symmetric. Such a description would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. A quantized arena asks all four of the directions of space and time, so it would carry both: continuity with symmetry, and asymmetry with discontinuity, the two pairings the theorem excludes.

The word "themselves" carries the weight of the prohibition. Quantum fields on a curved geometry give one character to the fields' findings and the other to the directions of space and time, and nothing in them is asked to be both. A quantized arena gives both characters to the directions of space and time themselves. The first is a pair joined at every point; the second is a single thing that would have to be two things in one respect.

The prohibition is exact, and it is narrow. It excludes a quantum of the arena itself, at any scale, and with it any least length, area or duration of space and time. It does not exclude a description of the small waves of the geometry about a smooth shape by quanta, whose quanta are quanta of the description only. A detector that registers a gravitational wave in single clicks would not count against the account, and it would not show gravitation quantized, since a classical wave acting on a quantum detector gives the same clicks. It does not exclude matter in several alternatives at once, each alternative with its own smooth geometry; that is two descriptions joined in each alternative, and in no alternative merged.

Excluded in principle, and therefore in all practice: no cleverer mathematics will make a merger of this kind succeed, because the obstacle lies in what a comparison can perfect and not in the mathematics. That is why the difficulties of Chapter 22 recur in program after program. A program that seeks a merger meets the theorem in its own mathematics, in the form its first step predicts.

What Follows for the Search

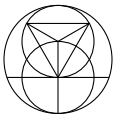
The work that follows is the study of the joints. Where do the two descriptions meet, and how does each hand its findings to the other? Much of that work is already done, though it was done under other names. The experiment described in Chapter 12 asked which geometry a quantum outcome carries, and the answer was the geometry of the outcome found. The calculation that Chapter 25 follows asked what quantum matter does on a classical geometry with a horizon, and the answer was that the hole radiates. The formula of Ryu and Takayanagi asked how an entropy of one description is read in the other. The joints also pose questions that experiment will answer. Two masses, each held in two places and related only by gravity, should become entangled, with a phase of about a third of a radian for the design proposed in 2017 (Chapter 13, "The Experiment Being Built", gives what the outcome would separate). Later designs use other masses, separations and times, and the same formula gives their phases. Free fall should stay the same for every kind of matter, and light of every energy, arriving from across the universe, should show space and time smooth.

Quantum matter on classical geometry, with each alternative of the matter carrying its own geometry, is not waiting to be replaced by something deeper. It is what depth looks like, when the structure of comparison itself is taken as the foundation. The two descriptions remain distinct, inseparable, and neither can do the other's work. (The general form of the distinction between a common source and a merger, with what it implies for each program, is set out in *The Unquantized Arena*, Chapter 16, "Common Source, Not Merger".)

The deepest claim of this book is positive. It is not that quantum gravity is impossible, as if the equations were too hard or more ingenuity were needed. It is that the relationship between general relativity and quantum theory is richer than unification would allow. They do not oppose each other, and they are not to be merged. They are two descriptions of one world, and every comparison you make uses both: the steps of what you find, and the smooth arena in which you find it.

> **Key idea.** The Incommensuration Theorem permits a unity of common source, in which two descriptions arise from the one act of comparison and are joined side by side, each perfect in its own character. It forbids a merger, a single description in which the directions of space and time themselves carry both characters, and a quantized arena is such a merger. Physics already works with the permitted unity: quantum matter on classical geometry, meeting at bridges such as the probability rule and the Planck scale.

Part VII puts the joined account to its hardest physical case: what happens to information that falls into a black hole.



PART VII

BLACK HOLES, OBJECTIONS AND TESTS



Chapter 25

Where Does the Information Go?

Does a black hole destroy what falls into it? While a black hole stands, it hides what falls into it from everything outside, and it destroys nothing. The famous puzzle about black holes and information rests on a premise, and the premise is the category error this book has traced through physics: it treats the geometry of space and time as one more item in the quantum state of the matter.

This chapter sets out four things in turn: the radiation that black holes give off; the puzzle that the radiation raises; the premise underneath the puzzle; and what the account says about black holes of two very different kinds. The first kind is small enough to evaporate completely. The second kind is large, the kind that stars leave behind.

What a Black Hole Is

A black hole is a region of space and time from which nothing, not even light, can return to the outside. Its boundary is called the horizon. The horizon is a feature of the shape of the geometry, and no material lies there. It is the surface beyond which every path into the future leads inward. General relativity describes a black hole completely, and its description is classical: a smooth geometry, with no steps in it.

Most of the black holes we know of were left behind by large stars that, at the end of their lives, could no longer hold themselves up against their own weight. Astronomers now find them in several independent ways: by the stars that orbit them, by the gas that heats as it falls toward them, and by the waves of geometry that two of them send out as they spiral together and merge. The LIGO and Virgo collaborations reported the first such waves in 2016, as Chapter 1 described.

In 1973 Jacob Bekenstein argued that a black hole has an entropy, and that its entropy is proportional to the area of its horizon. Entropy, in the sense used here, measures hidden variety. It counts how many different inner states a system could be in, given everything that can be found about the system from outside, and it counts them by the number of yes-or-no questions it would take to single out one of those states. I read the entropy of a black hole as a count made relative to a context: it counts the states of the interior that the outside world cannot tell apart, and the outside world is the context of that count.

Why should that count go by the area of the horizon, when everything that fell in lies inside? Because a count made from a place goes by the boundary. A region has two natural counts of its information. One takes the interior cell by cell, every small piece of it, and that count grows with the volume. The other takes what crosses the region's boundary through interaction, and that count grows with the area of the boundary.

The two counts need different vantages. This argument takes the focus of counting the information of a region, the last focus in the table of Chapter 6, and it asserts only the first and the third axioms. To count every cell of the interior, you would need the whole region in view together, laid out like a fin-

ished map, from a vantage that holds all of it. That vantage is the omniscient role of Chapter 6, "Three Roles and the Order of Finding": what is laid out as structure, able to be held in view as a whole. To count what crosses the boundary, you need only stand somewhere outside and register what reaches you: what falls in, the pull of the hole's mass, the glow it gives off. That count is made from a place, by interaction, and it is the immanent role, the act as it happens. Every finding in physics is made from a place inside the world.

The first axiom says: "The immanent is more fundamental than the omniscient and/or the transcendent." The transcendent named there is the third role, the stance with what is possible. So the count made from a place is the more fundamental of the two, and I take the information of a horizon to be counted by its area. The area law itself is established physics: Bekenstein found it, and Hawking fixed its exact proportion in 1975. What I add is the reason the law goes by area. The count is real. It counts the states of the interior that the outside cannot tell apart, with the outside as its context, and it is the count that this chapter calls the hole's capacity. (This reading of the area law, with the established law it rests on, is set out in *The Unquantized Arena*, Chapter 17, "Black-Hole Information and the Page Curve".)

The Radiation Hawking Found

In 1974 Stephen Hawking found that a black hole is not wholly black. It glows, very faintly, with the spectrum of a warm body, and the smaller the hole, the warmer its glow. A hole with the mass of the Sun would glow at about sixty billionths of a degree above absolute zero. Energy carried off by the glow is mass lost by the hole, and a hole of the Sun's mass would take about 10^{67} years to evaporate: a number of years written as a one followed by sixty-seven zeros.

How Hawking reached his result matters for everything that follows. He took the quantum fields of matter and light and laid them out on the classical geometry of a star falling inward to form a hole. He then compared two descriptions of the empty state of those fields: the description natural long before the hole formed, and the description natural long afterward, far away from the hole. The two descriptions disagree about what counts as empty.

What is empty in the first description contains, in the second, a steady thermal flow of quanta moving outward from the horizon. That flow is Hawking radiation.

Every step of the calculation keeps the geometry classical. At no step is space or time quantized, and no quantum of the geometry appears. The quantum fields are the content, the smooth geometry is their context, and the radiation follows from the two working together, each in its own character. A popular picture describes the radiation as pairs of particles that appear near the horizon, one falling in and one escaping. Hawking offered that picture as an aid to intuition. His calculation does not use it, and the account in this book does not use it either.

The Puzzle

In 1976 Hawking drew a startling conclusion from his own result. The radiation is thermal. Taken by itself, it is as featureless as the glow of a warm coal, and it carries no record of what fell into the hole. When the hole has evaporated completely, only the radiation is left.

To see why that is a puzzle, two words are needed. A quantum state is pure when it is as fully specified as quantum theory allows. A quantum state is mixed when it is less fully specified than that, as the state of one part of a larger whole usually is. Quantum theory has a rule about how states change: the evolution of a complete system keeps every distinction among its states, so that two different beginnings never lead to the same end. Physicists call this rule unitarity. Under it, a pure beginning stays pure.

Hawking's conclusion was that a black hole breaks unitarity. A star that began in a pure state would end, through the hole, as radiation in a mixed state. Two stars that differed in every detail of their make-up, but had the same mass, charge and spin, would end as the same radiation. The hole would have erased the difference between them. Most physicists refused that conclusion and held to unitarity, and the argument that followed, for more than forty years, is the black-hole information paradox. In one sentence: Hawking's calculation says that the final radiation is mixed, and quantum theory, applied to the whole system, says that it must be pure.

The Premise Underneath

The phrase "applied to the whole system" carries the weight of the paradox. The demand that the final radiation be pure treats the black hole, its geometry and its matter together, as one quantum system with one quantum state, whose evolution must keep every distinction. It then treats the radiation at the end as all that remains of that system. The premise is this: geometry and matter form one quantum state.

This book has already found the reason to reject that premise. Chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It", found that quantizing the arena of space and time would give it the character of content, and that placing the arena inside the quantum state of the matter is the category error itself. Chapter 12, "One Geometry for Each Alternative", found that each alternative of the matter carries its own classical geometry. The geometry is where the matter's state is laid out. It is not an entry in that state.

What, then, does quantum theory guarantee? It guarantees that the state of the matter, laid out on its geometry, evolves without loss. Hawking's own calculation shows exactly that. The quantum fields on the whole geometry of the hole, inside the horizon and outside it together, stay in a pure state at every stage the calculation describes. The radiation outside is entangled with the field inside the horizon: findings made on the radiation and findings made on the inside field would be correlated more tightly than any two separately specified things can be. The radiation alone is mixed because it is only part of what the field is doing. Its partner lies inside.

A coded message and its key make a fair picture. The message alone reads as noise, and the key alone reads as noise. The meaning is in the pair, and nothing of the meaning is lost while both exist, even when only one of them is in your hands.

Information Is Not Destroyed

Information is not destroyed in a black hole. Neither of the two descriptions destroys it. Quantum theory carries the matter's state forward without loss on its geometry. General relativity carries what falls in forward to the end

of the interior, where the smooth description reaches the edge of its domain. What the horizon does, while the hole stands, is hide the information: no signal from inside the horizon reaches the outside. The radiation looks thermal because it is a part, seen without its partner. An instrument accelerating through empty space registers a faint warmth for the same reason, as the next chapter describes: it too receives only part of the field.

With the premise removed, the paradox becomes a plainer question. Does the information, hidden behind the horizon, ever reach the outside world? The answer depends on the kind of hole, and the account separates two kinds: holes small enough to evaporate completely while an outside world remains around them, and holes formed from stars.

Small Holes and the Page Curve

Take first a hole far lighter than any star, light enough to evaporate completely while an outside world remains around it. For such a hole, the whole history of its radiation is gathered in one outside world, and a definite question can be asked of that history. If the whole formed by what fell in, the content inside the horizon and the radiation stays pure, as seen from that outside world, how does the entanglement between the radiation and the hole change over time? The whole here is the matter's state laid out on its geometries: what fell in, the content inside the horizon, and the radiation. The condition puts no geometry into the state.

Don Page gave the answer in 1993, and its shape is called the Page curve. The measure that matters is the entanglement entropy of the radiation, the measure of entanglement met in Chapter 23: here, how much the radiation and the hole share in quantum correlation. Three steps fix the curve.

The first step is the condition itself: the whole is pure. For a pure whole made of two parts, the two parts have the same entanglement entropy, and that one number measures how far the two parts are entangled.

The second step is that each part has a capacity. The radiation can share no more than its own entropy, the thermal entropy it has carried away, which grows as the hole radiates. The hole can share no more than its own entropy, the one Bekenstein found, which shrinks as the hole radiates. That bound on

the hole belongs to the condition, as the purity does: it takes the interior's states of content that the outside cannot tell apart to be no more in number than the hole's entropy allows.

The third step is that the smaller capacity is filled. Page showed in 1993 that, for nearly every pure state of a whole made of two parts, the entanglement entropy lies within half a unit of the smaller of the two capacities. I read "nearly every" as a symmetry: the states that no comparison made from outside can tell apart count alike.

A conversation between a person with a large memory and a person with a small one shows the third step at work. What the two share can be no larger than the smaller memory, and in a long conversation it fills the smaller memory.

So the entanglement entropy of the radiation follows the smaller of the two capacities. Early on, the radiation's capacity is the smaller one, and the entanglement rises with it. Later, the hole's shrinking capacity is the smaller one, and the entanglement falls with it, reaching nothing when the hole is gone. The moment at which the two capacities are equal is the turning point of the curve, and it is called the Page time. The curve runs to the last moments of the hole, when its mass approaches the Planck mass, the scale at which, as Chapter 15, "Where the Two Descriptions Meet", set out, the two descriptions meet and both are relevant.

The turning point comes before the hole has lost half its entropy. The reason is a law of heat. Radiating into empty space always produces at least as much entropy as the radiating body loses; for a black hole, this is the generalized second law that Bekenstein stated in 1974. If the radiation gained exactly as much entropy as the hole lost, the two capacities would be equal when the hole had lost exactly half. Because the radiation gains more, its capacity grows faster than the hole's shrinks, and the two meet earlier.

Page computed the numbers in 2013 for a hole that radiates light and the small waves of its geometry. He counted the small waves in the terms of their description by quanta, and the count is a count of that description. Chapter 14, "Waves Without Particles of Space", set out why such quanta are quanta of a description and never quanta of the arena. The radiation gains about one and a half times the entropy the hole loses, and the turning point comes when

the hole has lost about two-fifths of its entropy. Measured in time, the turning point comes at a little more than half of the hole's evaporation time. Less than half the entropy takes more than half the time because a black hole loses entropy slowly at first and quickly at the end: the smaller a hole becomes, the hotter it is and the faster it radiates. However much entropy the radiation gains, the turning point comes no later than about 65 per cent of the evaporation time. The computation, step by step, is set out in *The Unquantized Arena*, Chapter 17, "Black-Hole Information and the Page Curve".

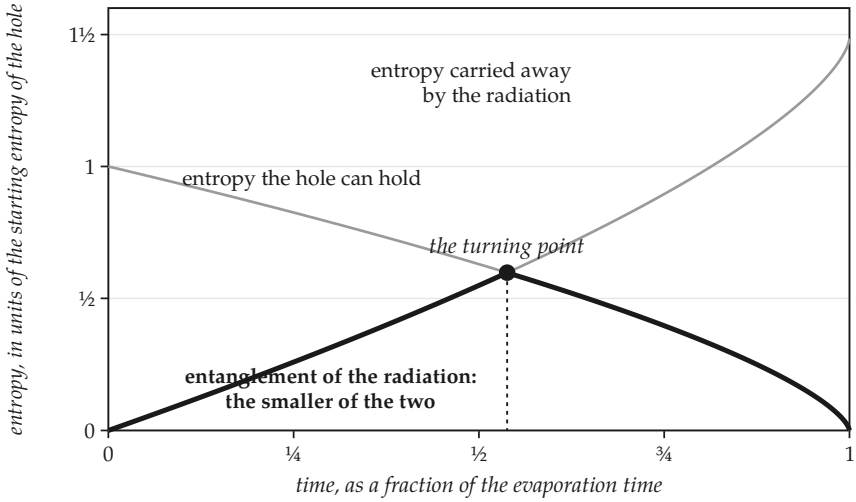


Figure 25.1. *The Page curve for a hole that radiates light and the small waves of its geometry. The entanglement of the radiation follows the smaller of two capacities, rising with the entropy the radiation has carried away and falling with the entropy the hole can hold, and it turns where the two are equal, at a little more than half the evaporation time.*

In 2019 two papers, one by Geoff Penington (published in 2020) and one by Ahmed Almheiri, Netta Engelhardt, Donald Marolf and Henry Maxfield, computed the same rise and fall with a different method, drawn from the program of quantum gravity. Their method is a sum over geometries, and the geometries that decide its answer are smooth classical ones. Their result is called the island formula, because after the turning point it counts a region inside the hole, an island, as belonging with the radiation. Past the turning

point, the curve requires correlations in the late radiation that Hawking's calculation on one fixed geometry does not track, and the island computation finds them by including further smooth classical geometries in its sum. I take the agreement as agreement in the shape of the curve. The three steps fix that shape, and they use no quantized geometry.

In 2013 Ahmed Almheiri, Donald Marolf, Joseph Polchinski and James Sully argued that, past the turning point, radiation that ends in a pure state, ordinary physics outside the hole and a smooth horizon cannot all hold, so that the horizon would have to become a wall of energy, a firewall, that burns whatever falls in. The account keeps the smooth horizon and ordinary physics outside, and gives up only the assumption that the partners inside the horizon are independent of the early radiation, which is what the island expresses by counting them with the radiation; a falling body then meets nothing special at the horizon, as the equivalence principle of Chapter 9 requires.

Holes Formed From Stars

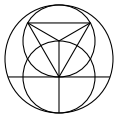
Now take a hole formed from a star. A hole of the Sun's mass needs about 10^{67} years to evaporate, and for that hole the account says exactly two things. The information that fell in is not destroyed. While the hole stands, the information is not accessible from outside. Both statements follow from what this chapter has already found: neither description destroys the information, and the horizon lets no signal out.

The account asserts neither that this information returns to the outside world nor that it is lost to the outside world. The thesis needs neither assertion. The paradox was a contradiction between Hawking's calculation and a demand, and the contradiction is removed by removing the premise of the demand, whatever becomes of the information afterward.

You may have heard that black holes threaten the foundations of quantum theory. They threaten only the premise that the arena of space and time is part of the quantum state of what it holds. Quantum theory keeps every distinction in the matter's state, and the horizon of general relativity, while the hole stands, hides part of that state from the outside. Each does its own work, as everywhere else in this book.

Key idea. Hawking radiation is real, and it comes from quantum fields on a classical geometry. The information paradox assumes that geometry and matter form one quantum state; without that premise, the state of the matter evolves without loss on its geometry, and information is not destroyed. For a small hole that evaporates while an outside world remains, if the whole stays pure and the hole's capacity is its entropy, the radiation's entanglement follows the smaller of two capacities. That curve turns before the hole has lost half its entropy, at a little more than half of its evaporation time. For a hole formed from a star, the information is not destroyed and is not accessible from outside while the hole stands, and the account asserts neither its return to the outside world nor its loss to it.

Chapter 26 sets out the strongest objections a physicist will raise against the whole account, beginning with Hawking radiation itself.



Chapter 26

What a Physicist Will Say

What are the strongest objections to the account? They come from physics that works. Each points to a real result that seems to place quantum effects inside gravity itself, and each deserves to be met at its full strength. For each one the question is the same: does the result need the arena of space and time to be quantized, or is it what quantum content on a classical geometry gives? I state each objection as a physicist would put it, and then I answer it.

"Hawking Radiation Is Quantum Gravity at Work"

The objection runs: "A black hole is made of nothing but geometry, and it radiates quanta. So quantum effects act within gravity itself. If gravity were purely classical, black holes could not radiate."

Black holes do radiate, and the radiation is real. Chapter 25, "Where Does the Information Go?", set out how Hawking found it: by laying quantum fields on the classical geometry of a black hole. The geometry in that calculation is never quantized, and no quantum of geometry appears in it.

William Unruh showed in 1976 how little the result needs. An instrument accelerating steadily through empty, flat space registers a warm bath of quanta, at a temperature set by its acceleration. Flat space is as classical as geometry gets. The warmth arises because the accelerating instrument has a horizon behind it, beyond which it can never receive a signal, so it receives only part of the field. Hawking radiation is the same kind of result, with the horizon of a black hole in place of the instrument's horizon.

A careful physicist will press further. Traced backward in time, the radiation leaving a black hole comes from waves squeezed near the horizon to wavelengths far shorter than the Planck length, where neither description holds alone. Unruh answered this too, in 1995. He studied sound in a fluid that flows faster than sound, which has a horizon for sound waves, and he changed how the sound behaves at very short wavelengths. The thermal glow came out unchanged: in his model, the glow does not depend on how the waves behave at the shortest wavelengths. I take Hawking radiation as real, and as independent of what happens beyond the scale where the two descriptions meet. Hawking radiation is the strongest single example of what this book describes: quantum content and a classical context, each in its own character, working together.

"The Entropy Has Been Counted"

The objection runs: "The entropy of a black hole is a quarter of its horizon area, measured in units of the Planck length squared. So a hole has an enormous number of inner states. String theory and loop quantum gravity have both counted those states from microscopic degrees of freedom of gravity. If gravity has inner states that can be counted, it has a discrete space of states, and so gravity is quantized."

The entropy of a black hole is real. The question is what is being counted.

In 1996 Andrew Strominger and Cumrun Vafa achieved the first exact count. For a special class of black holes in string theory, those with the largest charge their mass allows, they counted the states of a system of strings and membranes in the regime where gravity is too weak to form a hole at all. In that regime the geometry is flat and classical. The count is protected: it stays the same as the strength of gravity is turned up, until the same system has become a black hole. The number they found matched a quarter of the horizon area exactly.

I read the count as a count of content. What Strominger and Vafa counted are the states of the strings and membranes, and they counted them in a description whose geometry is classical. The match shows that the hole's entropy counts states of its content that the outside world cannot tell apart. The count exhibits no step of the arena.

In 1998 Abhay Ashtekar, John Baez, Alejandro Corichi and Kirill Krasnov counted the states of a horizon in loop quantum gravity. In that theory the horizon is pierced by the edges of a network, and each piercing carries a step of area. The count gives an entropy proportional to the area, as it should. The factor of one quarter appears when a free number of the theory, called the Immirzi parameter, is given the value that produces it. The count is built on steps of area, which are the least steps this account excludes, and its agreement with the known answer comes from choosing that free number. So the count does not show a step of the arena independently of the answer it was set to match. The law it reproduces, entropy in proportion to area, is a law this account keeps.

"Gravity Is Exactly Dual to a Quantum Theory"

The objection runs: "The correspondence that Juan Maldacena proposed in 1997, in a paper published in 1998, gives an exact match between gravity inside a region of a certain curved space and a quantum theory on that region's boundary. If gravity is exactly matched by a quantum theory, then gravity is a quantum theory that only looks classical from inside. This is the strongest evidence that gravity can be quantized."

The correspondence is one of the great results of modern physics, and Chapter 23, "Two Descriptions That Translate", set out what it is: two descriptions of one system, each in its own character. The boundary theory is a quantum theory of discrete, symmetric content. The interior is a smooth geometry with quantum fields on it. Where the dictionary between them is best understood, the interior geometry is classical, which is the very pairing this book describes.

The translation between the two descriptions has limits. Alex Hamilton, Daniel Kabat, Gilad Lifschytz and David Lowe showed in 2006 how to rebuild a field inside from data on the boundary, in the regime where the interior geometry is classical. Ahmed Almheiri, Xi Dong and Daniel Harlow showed in 2015 that the interior is written into the boundary in the way a code that corrects errors writes a message: spread out, and redundant. A given region inside can therefore be recovered only from a large enough part of the boundary. An exact correspondence would show that a quantum description and a geometric description can capture one system together. It would leave the geometry in the role of context. Neither side is more fundamental, and each captures what the other leaves out.

"The Singularities Demand a Quantum Gravity"

The objection runs: "General relativity predicts singularities, where density and curvature grow without limit and its equations cease to apply. Roger Penrose in 1965, and Stephen Hawking and Penrose in 1970, showed that singularities are generic, not artifacts of a special symmetry: they occur inside every black hole. Only quantum effects in gravity can remove them. If gravity cannot be quantized, the singularities stand, and physics has no coherent account of them."

The singularity theorems are among the deepest results of general relativity, and what they show is exact. Under conditions that ordinary matter meets, some paths of falling bodies or of light cannot be continued within the smooth geometry beyond a finite time. That is the content of a singularity theorem. It shows that the smooth description has an edge to its domain.

In the cases worked out in full, the curvature grows without limit along such a path, and before the path reaches its end the curvature that the smooth description gives reaches the Planck curvature, where, as Chapter 15, "Where the Two Descriptions Meet", set out, the two descriptions meet and the smooth description's domain ends. The infinity is what the smooth description gives when it is carried past the edge of its own domain. To ask for the geometry at the singularity is to apply a concept where the concept does not belong, which is a category error.

Physics has met this situation before. Around 1900, classical physics, applied to the radiation inside a hot oven, predicted that the radiation holds an infinite amount of energy at the shortest wavelengths. The prediction became known as the ultraviolet catastrophe. Max Planck's law of 1900, which has no such excess, rests on energy passing between the radiation and the walls of the oven in steps. That law put steps into the content, the energy exchanged between matter and light. It did not make space stepwise. The infinity had marked a description carried beyond its domain, and the cure was the right description of the content. The singularity is read the same way: it marks the edge of the smooth description, and it calls for no quantized arena. The account adds no third description beyond that edge, and it needs none.

"Gravitational Waves Must Come in Gravitons"

The objection runs: "Gravitational waves have been detected. Every other wave in physics has been quantized: light comes in photons, and sound in a solid comes in phonons. So gravitational waves come in gravitons, and if gravitons exist, gravity is quantized."

Gravitation is not quantized, and the step from light and sound to gravity takes the arena for one more content. Light is a field within the arena, and sound in a solid is a vibration of a held structure, a lattice whose bonds hold its atoms in place. A phonon is a quantum of the description of that vibration. It is not a piece of the space the solid sits in. The phonon is the nearest sense by which the word "graviton" can be made intelligible, and even that sense fails for the arena, because the arena is not a held structure. Small waves of geometry can be described by quanta, and John Donoghue showed in 1994 that general relativity, described in this way at low energies, gives definite and tiny quantum corrections. The account keeps that description as a description, and a graviton names at most a quantum of it, as Chapter 14, "Waves Without Particles of Space", set out.

In 2024 Germain Tobar, Sreenath Manikandan, Thomas Beitel and Igor Pikovski showed that a heavy resonator, cooled and watched closely, could register single clicks from a passing gravitational wave, which they read as single quanta taken from the wave. In the same year Daniel Carney, Valerie

Domcke and Nicholas Rodd showed that such clicks would be produced equally by a classical wave acting on a quantum detector, just as the photoelectric effect, by itself, does not show that light is made of photons. So a detection of that kind would not count against the account, and it would not show gravitation quantized: what the detector registers is a step of its own, content against the arena as its context. What the account excludes is a step of the arena itself, and a quantum of the arena at any scale. The description by quanta holds where the wave is small compared with the geometry it ripples, and it reaches its edge at the Planck curvature, where the ripple is as large as the geometry.

"The Early Universe Will Show Gravity Quantized"

The objection runs: "Many cosmologists hold that the universe, very early, expanded with extreme speed for a brief time, in an epoch called inflation. Small waves of geometry from that epoch would have left a faint pattern in the polarization of the oldest light in the sky, the cosmic microwave background. The strength of those waves depends on Planck's constant, so a smooth geometry with no quantum in it would produce none. If the pattern is seen, gravity is quantized." Lawrence Krauss and Frank Wilczek made this argument in 2014, and they wrote that seeing the pattern would firmly establish the quantization of gravity.

The argument is right about what the pattern would show. If those waves exist, their strength is computed from the quantum jitter of the small waves in their description by quanta. This book allows that description as a description, as Chapter 14, "Waves Without Particles of Space", set out: its quanta are quanta of the description, never quanta of the arena. In the calculation behind the objection, the expanding geometry of the universe is a smooth classical background at every step, and only the description of the small waves riding on it is given quanta. Planck's constant in the strength marks the strength as computed in a quantum description, and the quantum description here is the description of the small waves, with the background classical throughout. So a pattern left by those early waves would show that the description of small

waves by quanta gives their strength, and it would not show gravitation quantized. The answer is the same whether or not the pattern is found, and the account rests on neither outcome.

"Something New Must Happen at the Planck Scale"

The objection runs: "At the Planck scale, quantum effects and gravitational effects are both at full strength. Something new must happen there. A gravity that is never quantized cannot be squared with the existence of the Planck scale."

Both kinds of effect are at full strength at the Planck scale, and the account says exactly that. The Planck scale is where an object's gravitational size equals its quantum size, the two lengths of Chapter 15: the first grows with the mass, and the second shrinks as the mass grows. There, neither description dominates, and both are relevant. A single description that perfected both characters at that scale would need a pairing the Incommensuration Theorem forbids.

What is new at the Planck scale is the meeting itself. It is the one scale at which neither description can be set aside, and at which no single description can do the work of both, so the two work together, joined side by side, as Chapter 24, "Joined, Not Merged", described. The Planck length marks that meeting. The arena has no least step, at the Planck length or anywhere else. At every length far larger than the Planck length, the account predicts what general relativity and quantum theory already predict, so the experiments that test it are the ones in which it differs from its rivals. The size of the Planck length follows from Newton's constant, which is measured. The ground fixes the role of the Planck scale and not its size, because the axioms are structural, not dimensional.

"This Is Only Philosophy"

The objection runs: "The argument is philosophy, not physics. Physics is decided by experiment, not by reasoning about comparison. A theorem about concepts cannot tell nature what it may do."

Every measurement is a comparison, and that is why a result about comparison binds every finding physics can make. The Incommensuration Theorem has no physical premise. Its passage into physics is made by identifications, each stated in the first person, and each beside the observation that tests it. Gravity is identified as the arena by four marks, and each mark is an experimental fact that Chapter 9, "Four Marks of the Arena", set out.

Physics already lives with results of this form. John Bell showed in 1964 that no theory in which every outcome is fixed in advance by local causes can reproduce all the predictions of quantum theory. His result settled, before any experiment was run, what a local theory of that kind could predict, and experiments then decided between quantum theory and its local rivals. The argument of this book has the same shape: a structural result first, then predictions that can fail. The objection is right that experiment decides, and the account accepts that standard in full. It predicts that two masses related only through gravity will become entangled, that every material falls alike, and that space and time will show no least step, and instruments now being built will test each prediction.

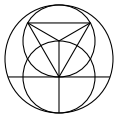
The Pattern

Taken together, the objections show one pattern. Every result cited as evidence that gravity must be quantized is one of four things. It is a quantum description on a classical geometry: quantum content, as Hawking radiation is, or the description of small waves by quanta, whose quanta belong to that description. It is a count of content, as the counts of a black hole's states are. It is a correspondence between two descriptions, as the correspondence of Maldacena is. Or it marks the edge of a description's domain, as the singularities and the Planck scale do. None of these results needs the arena quantized. Each needs the boundary between the two descriptions to be understood, and

the ground of comparison gives that boundary exactly. The full technical form of every objection, with its literature, is in *The Unquantized Arena*, Chapter 18, "Objections at Full Strength".

> **Key idea.** Each strong objection points to real physics, and none of that physics needs a quantized arena. Hawking radiation is quantum fields on a classical geometry, unchanged by what happens at the shortest scales. Waves of geometry from the early universe, if their pattern is found, are small waves whose strength a description by quanta computes; the account allows that description, and gravitation is not quantized. The counts of a black hole's states count content, and the correspondence of Maldacena joins two descriptions without merging them. The singularities and the Planck scale mark the edge of the smooth description's domain, as the ultraviolet catastrophe marked the edge of classical physics. The ground is a statement about comparison, and its application to physics makes predictions that experiment decides.

Chapter 27 states those predictions, with their present evidence and the experiments that will decide them.



Chapter 27

What the World Will Show

How will we know whether the account's consequences hold? By experiment, as in all of physics. This account makes predictions that the world can contradict. Each prediction below is stated first, then given with the evidence that bears on it now, and then with the experiments that will decide it. Two different methods that agree decide a great deal; one experiment at the edge of its precision decides nothing.

Gravity Will Entangle Two Masses

The prediction: two masses, each placed in two locations at once and related to each other only through gravity, will become entangled. Each pair of their alternatives will gather the phase that Newton's law of attraction gives it. Sougato Bose and his colleagues proposed the experiment in 2017. In their version, each mass is about a hundredth of a billionth of a gram, each is held in two places a quarter of a millimeter apart, and their centers are a little less than half a millimeter apart, for two and a half seconds. For the design proposed in 2017, the phase that decides the entanglement comes to about a third of a radian. Later designs use other masses, separations and times, and the same formula gives their phases. The calculation is set out in *The Unquantized Arena*, Chapter 8, "The Entanglement Test".

The reason is the one Chapter 12, "One Geometry for Each Alternative", gave. Each alternative of the matter carries its own classical geometry. So each pair of alternatives, one for each mass, has its own energy of attraction, and its own phase. Different phases for different pairs are what entanglement is made of here.

The evidence so far is indirect and points the same way. In 1981 Don Page and C. D. Geilker moved the masses of a sensitive gravity balance according to the outcome of a quantum random event, and the gravity they measured followed the outcome, not the average of the possible outcomes. They read the result as indirect evidence for quantum gravity. What it excludes is one geometry shaped by the average of the alternatives, and it agrees equally with one classical geometry for each alternative. The entanglement experiment itself, proposed by Bose and colleagues and by Chiara Marletto and Vlatko Vedral in 2017, is being developed by several groups and has not yet been performed.

What the result will separate is stated exactly. It separates this account from every account with one geometry for all the alternatives, which predicts no entanglement. It also separates this account from every account in which gravity itself ends a superposition. In the proposal of Roger Penrose in 1996, for example, superpositions of masses this large decay before entanglement can build. It does not separate this account from the quantum treatment of small waves of geometry, which gives the same phase at these scales. Marletto and Vedral argue that such entanglement shows gravity to be non-classical. A geometry for each alternative meets their condition, since the geometry differs from one alternative to the next and the alternatives keep their phases, so the result would confirm their conclusion as they state it. What it would not show is a quantum of the arena itself.

A different experiment, the detection of single clicks from a gravitational wave, decides nothing here. Chapter 26, "What a Physicist Will Say", gave the reason: such clicks come equally from a classical wave acting on a quantum detector. A detection of that kind agrees with the account, and the account does not rest on one.

Every Material Will Fall Alike

The prediction: every body falls in the same way, whatever it is made of, because the mass that gravity pulls on and the mass that resists acceleration are one quantity. This equality is called the equivalence principle. The reason for it is that gravity is the arena, and the arena couples to energy as such. The equality is the third of the four marks that Chapter 9, "Four Marks of the Arena", set out. General relativity also holds this equality exactly. The account gives the reason for it, and it stands with general relativity against theories in which the coupling depends on what a body is made of. Thibault Damour and Alexander Polyakov showed in 1994 that the dilaton of string theory, a field that would couple to matter alongside gravity, would produce small departures from general relativity of the kind that tests of free fall look for.

The evidence is strong. In 2022 the MICROSCOPE satellite reported its final result: titanium and platinum, falling together in orbit around the Earth, fall alike to within a few parts in a million billion. In 2023 the ALPHA-g experiment at CERN, described in Chapter 9, found that atoms of antihydrogen fall down, with an acceleration that agrees with the ordinary value within the precision of the experiment. Antimatter falls down, as a coupling to energy, which has one sign, requires.

The prediction that every material falls alike is the sharpest of the predictions, because the account allows no small difference: it predicts that the difference in free fall between any two materials is exactly zero, at every level of precision. Every test so far has agreed with it, and each finer test of free fall will measure that difference again.

Space and Time Will Stay Smooth

The prediction: no experiment will find a least length, area or duration in space and time, and light of every energy will travel at one speed through empty space. The second half is the rule of relativity called Lorentz invariance: the laws of physics are the same for every steadily moving instrument,

and the speed of light is the same for all of them. The reason is that the arena is the context of physics and is continuous; a step in the arena would give it the character of content.

The best evidence comes from the arrival times of light. A gamma-ray burst in a distant galaxy sends out, at nearly the same moment, photons of very different energies, and they travel for a very long time across intergalactic space before they reach a telescope. The simplest pictures of a grain in space and time, at about the Planck length, make light of higher energy travel at a slightly different speed, so that its photons arrive measurably earlier or later. In 2013 Vlasios Vasileiou and colleagues, using the Fermi telescope, found no delay in the light of several bursts, among them the burst of 2009 described in Chapter 15, and placed the energy scale of such an effect above seven times the Planck energy, the energy held in the Planck mass, about two billion joules. In 2024 the LHAASO collaboration, which runs the Large High Altitude Air Shower Observatory in China, observed the exceptionally bright burst of 9 October 2022, and placed the same energy scale above ten times the Planck energy. Those simplest pictures of a grainy space and time are therefore excluded.

Some pictures of a discrete space and time keep the speed of light the same at every energy, and photon arrival times do not reach them. The prediction reaches them too, because it is wider than any one test: no observation will find a least step in space or time. The same prediction covers areas and volumes: no discrete spectrum of area or volume will be observed, that is, no surface whose area takes only separate values with nothing between them, and no region whose volume does. Echoes of the gravitational waves from merging black holes are one place where a discrete spectrum of area or volume would show, and the account says that none will be found there or anywhere else. Discreteness in particle physics, in charges and in the kinds of particle, belongs to content, and the account predicts it.

The Correspondence Will Stay a Translation

The prediction: the dictionary of the correspondence that Chapter 23, "Two Descriptions That Translate", described will remain a translation between two descriptions, limited in both directions. Not everything about the interior will be rebuilt from the boundary, and not everything about the boundary will be written in the interior geometry. The reason is that the two sides are descriptions in the two characters that the Incommensuration Theorem keeps apart, and a translation between them carries the structure of each without turning either into the other.

The evidence is the state of the field. Interior fields are rebuilt from the boundary in the regime where the interior geometry is classical. A given region inside can be recovered only from a large enough part of the boundary, as the work on codes that correct errors showed. I read each advance in the subject the same way: it has made the translation more precise, and it has kept it a translation between two descriptions. This prediction is decided by mathematics rather than by instruments. The mathematics that decides this prediction treats a boundary theory of finite size, with every correction included. For that case the account predicts that a restriction on what can be translated remains, and so not every quantity of the interior can be rebuilt exactly from the boundary.

The Argument, Once More

Here is the whole argument, told once more from the beginning.

Everything physics knows, it found by comparison. A thermometer, a detector and a telescope each compare, and a comparison is the act of finding whether two things are the same or different, in some respect. Every comparison has the same six parts. It finds a sameness or a difference. Its finding is about a content, in a context, the respect in which the two things compared are two. And it has two sides: the side from which the comparison is made, and the side about which it is made. These six are not added to comparison. They are what comparison is.

From those parts come four characters that any finding can have: continuity, symmetry, discontinuity and asymmetry. The Incommensuration Theorem, the result of *An Immanent Metaphysics* on which this book rests, says that symmetry and continuity cannot both be perfected, simultaneously and fundamentally, in one comparison, and that neither can asymmetry and discontinuity. Of the ten ways to pair the characters, two survive: continuity with asymmetry, and symmetry with discontinuity. Every comparison perfected at its foundation is one of those two, and no third.

Every comparison also has three roles: the act as it happens; what is laid out as structure; and the stance, with the range of what is possible. The three axioms of *An Immanent Metaphysics* state how the roles stand to one another. The act comes first, and structure and stance are conjugate. The stance precedes the finding, the finding precedes the record, and the record shapes the next stance. The three roles are distinct, inseparable, and none can do the work of another. A theory has two levels in the same way, its mathematics and its findings, and the two levels always have opposite characters: smooth mathematics with discrete findings, or the reverse.

The two established theories of physics are the two survivors. General relativity is the continuously asymmetric theory: smooth, with no steps, and different from place to place. Quantum theory is the discontinuously symmetric theory: exact laws that hold everywhere, and findings that come in separate steps. Content is governed by symmetry, and context by continuity, so I identify quantum theory as the theory of content, and general relativity as the theory of its context. I identify gravity as the arena, the context of physics, by four marks found in experiment. Gravity couples to everything, and nothing shields it. The mass it pulls on is the mass that resists, and it has no charge that comes in steps.

Quantizing a field exchanges the characters of both levels of its theory together: its findings come to be held in exact steps, and its mathematics in a continuous space of states. Light can take that mode exchange, because light is content. The arena cannot take it, because the mode exchange would give the arena the character of content: quantized, it would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same

directions, both. To quantize the arena is therefore to apply a concept where the concept does not belong. That is the category error, and it is the error behind the search for one theory.

When the error is removed, what is left is definite. Each alternative of the matter carries its own classical geometry. Two masses related only through gravity will become entangled, with the phase Newton's law gives. Gravitation is not quantized: small waves of geometry can be described by quanta, those quanta belong to the description, and no step of the arena exists. The Planck scale is where the two descriptions meet, both relevant there and neither dominant. The measurement problem is what a smooth description leaves over when its findings are discrete, and each interpretation of quantum theory is a choice about time, which drops or absorbs one of the three kinds of time. Each program of quantum gravity found something real, and each met the difficulty that its founding commitment predicts. The correspondence of Maldacena translates between two descriptions without merging them. A black hole destroys no information, and while it stands it hides what fell in.

The two theories that never fail are not waiting to be merged into one. They are joined, side by side, as two descriptions of one world: each perfect in its own character, and each capturing what the other leaves out. Quantum matter, with a classical geometry for each of its alternatives, is the meeting of the two theories as each one stands.

The argument invites the test, and the instruments that will make it are being built.

> **Key idea.** The account makes predictions the world can contradict. Two masses related only through gravity will become entangled, with a phase of about a third of a radian for the design proposed in 2017. Every material will fall alike, exactly; the best test so far agrees to a few parts in a million billion. Space and time will show no least step, and light of every energy will travel at one speed; the arrival times of light from distant bursts already exclude the simplest grainy pictures, up to ten times the Planck energy. The correspondence of Maldacena will remain a translation, limited in both directions.

The pages that follow give every number in this book, with its source.

The Numbers

Every number this book prints is listed here, Part by Part, with its value, the way it is computed or the work it comes from, and the chapters that use it. The physical constants are the values of CODATA 2022 (Mohr, Newell, Taylor and Tiesinga 2025). Values computed from those constants are given to the precision the calculation supports. Numbers that appear only inside an illustration, such as the chance of rain in a weather forecast in Chapter 10 or the even odds of the illustrative superposition in Chapter 12, are not measurements, and they are not listed.

Part I: The Search

Quantity	As printed	Source or calculation	Chapters
The turning of Mercury's orbit	43 seconds of arc per century	The measured excess over Newtonian gravity and the pull of the planets, computed by Einstein (1915a)	1
The gain of a satellite clock of the Global Positioning System	about 38 millionths of a second a day	Net fractional rate 4.4647×10^{-10} (Ashby 2003), times 86,400 seconds: 3.86×10^{-5} seconds	1, 16
The drift without the correction	more than eleven kilometers a day	Light travels 11.6 kilometers in 38.6 millionths of a second (Ashby 2002)	1
The first gravitational wave: masses, dis-	about 36 and 29 solar masses; about 1.3 billion light years; one part in a thousand billion billion;	Abbott and others (2016): distance 410 megaparsecs; peak strain	1, 8, 14, 25

Quantity	As printed	Source or calculation	Chapters
tance, stretch, energy	about three solar masses radiated	1.0×10^{-21}; 3.0 solar masses	
The black hole of M87	about six and a half billion solar masses	Event Horizon Telescope Collaboration (2019)	1
The electron's magnetism	measured to about a tenth of a part in a trillion; agreement with theory to about one part in a trillion	Fan, Myers, Sukra and Gabrielse (2023): 0.13 parts per trillion	1, 10
The raised optical clock	33 centimeters; faster by about four parts in a hundred million billion	Chou, Hume, Rosenband and Wineland (2010): a shift of about 4×10^{-17}	1, 4, 8
The Planck mass	about twenty-two millionths of a gram	The square root of the reduced Planck constant times the speed of light over Newton's constant: 2.17643×10^{-8} kilograms	1, 15, 26
The span of the search	more than ninety years	From Rosenfeld (1930) to the present	1, 2, 11, 22
The Planck length	about 1.6×10^{-35} meters: a decimal point, thirty-four zeros, and then the digits 16	The square root of the reduced Planck constant times Newton's constant over the cube of the speed of light: 1.61626×10^{-35} meters	2, 15, 26

Part II: The Ground

Quantity	As printed	Source or calculation	Chapters
Twelve pure fifths	a little under 129.75	$(3/2)^{12} = 531441/4096 = 129.746$	5
Seven octaves	exactly 128	2^7	5
The Pythagorean comma	about a quarter of a semitone	$531441/524288 = 1.01364$, which is 23.46 cents, against 100 cents in a semitone	5
The narrowing of each fifth in equal temperament	about a fiftieth of a semitone	23.46 cents divided among twelve fifths: 1.96 cents	5
The ten pairings, three tests, two survivors	ten; three; two	Four compounds paired with themselves (four) and with one another (six): the enumeration of *An Immanent Metaphysics*, Technical Appendix	5, 27

Part III: The Diagnosis

Quantity	As printed	Source or calculation	Chapters
The step of electric charge	whole multiples of one third of the electron's charge	The charges of the quarks: established physics	7, 9
The proton's inertia	about 1836 times that of the electron	The proton-to-electron mass ratio, 1836.15 (CODATA)	9
The merger of two neutron stars	light and waves within two seconds of each other; speeds equal to within a few parts in a million billion	Abbott and others (2017): a delay of 1.74 seconds; the difference of speeds between -3×10^{-15} and $+7 \times 10^{-16}$ of the speed of light	8
Antihydrogen falling	downwards, at a rate consistent with ordinary matter	Anderson and others (2023): $0.75 \pm 0.13 \pm 0.16$ of the ordinary acceleration	9, 27
Titanium and platinum falling	alike to within a few parts in a million billion	Touboul and others (2022): $(-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$, consistent with zero	9, 27
The fall of a classical electron into its nucleus	about a hundred-billionth of a second	The standard calculation of the energy a circling charge radiates: 1.6×10^{-11} seconds	11

Part IV: What Is Left When the Error Is Removed

Quantity	As printed	Source or calculation	Chapters
The light time from the Moon	a little more than a second	The mean distance of the Moon, about 384,400 kilometers, over the speed of light: 1.28 seconds	12
			13, 27

Quantity	As printed	Source or calculation	Chapters
The mass of each diamond	about a hundredth of a billionth of a gram	10^{-14} kilograms (Bose and others 2017)	
The width of each diamond	about two thousandths of a millimeter	A diamond sphere of that mass, at 3510 kilograms per cubic meter, has a radius of 0.879 micrometers and a width of 1.76 micrometers	13
The separations of the design	two places a quarter of a millimeter apart; centers a little less than half a millimeter apart; near places about a fifth of a millimeter apart; far places about seven tenths	250, 450, 200 and 700 micrometers (Bose and others 2017; the last two follow from the first two)	13, 27
The time the diamonds are held	about two and a half seconds	2.5 seconds (Bose and others 2017)	13, 27
The entangling phase, for the design proposed in 2017	about a third of a radian: about eighteen degrees, or a twentieth of a full turn	0.314 radians, computed below from the parameters of Bose and others (2017); 0.3139 radians is 17.99 degrees	13, 19, 24, 27
The quantum corrections to Newton's law at one meter	about one part in a number seventy digits long	The square of the Planck length over the distance: 2.6×10^{-70} at one meter (the size of the correction of Donoghue 1994)	14, 15
The stretch of a LIGO arm	far less than the width of a proton	A strain of 1.0×10^{-21} on four kilometers is 4×10^{-18} meters, against a proton width of about 1.7×10^{-15} meters	14
	at most about one graviton in a century; a billion tril-	Rothman and Boughn (2006): 10^{-2} detections	14

Quantity	As printed	Source or calculation	Chapters
A graviton detector of Jupiter's mass	lion trillion or more neutrino events for each	a year; 10^{33} to 10^{34} neutrino events per graviton event	
Gravitational sizes	the Earth, about four and a half millimeters; the Sun, about one and a half kilometers	Newton's constant times the mass over the square of the speed of light: 4.44 millimeters and 1.48 kilometers	15
Quantum sizes	a proton, about an eighth of its own width; the Earth, sixty-seven zeros after the decimal point	The reduced Planck constant over the mass times the speed of light: 2.10×10^{-16} meters and 5.89×10^{-68} meters	15
The gap between the two sizes	for a proton, a factor thirty-nine digits long; for the Earth, sixty-five digits long	1.69×10^{38} and 7.53×10^{64}	15
A grain of sand of the Planck mass	a quarter of a millimeter across	A quartz sphere of 2.18×10^{-8} kilograms, at 2650 kilograms per cubic meter, is 0.250 millimeters across	15
The Planck length beside a proton	about a hundred billion billion times smaller	1.7×10^{-15} meters over 1.62×10^{-35} meters: 1.05×10^{20}	15
The limit of a light microscope	about half the wavelength of its light	The diffraction limit of standard optics	15
The burst of 2009	a photon of about 31 billion electronvolts, within about a second of the low-energy light; a limit above 1.2 times the Planck energy	Abdo and others (2009): arrival 0.829 seconds after the trigger	15
		6.67430×10^{-11} cubic meters per kilogram	15

Quantity	As printed	Source or calculation	Chapters
Newton's constant	known to about two parts in a hundred thousand	per second squared, with a relative uncertainty of 2.2×10^{-5} (Mohr, Newell, Taylor and Tiesinga 2025)	

Part V: The Measurement Problem

Quantity	As printed	Source or calculation	Chapters
The net rate of a satellite clock	about 4.46 parts in ten billion; about 38.6 millionths of a second a day	Ashby (2003): 4.4647×10^{-10}	16
The Delft test of Bell's theorem	stations 1.3 kilometers apart; 2.42, with an uncertainty of 0.20	Hensen and others (2015)	20
The local bound and the quantum value	no larger than 2; up to about 2.8	Clauser, Horne, Shimony and Holt (1969); the quantum maximum is twice the square root of two, 2.83	20

Part VI: The Search Revisited

Quantity	As printed	Source or calculation	Chapters
The Higgs mass predicted by asymptotic safety	about 126 billion electronvolts	Shaposhnikov and Wetterich (2010)	22
The Higgs mass found	near 125 billion electronvolts, two years later	ATLAS Collaboration (2012): 126.0; CMS Collaboration (2012): 125.3	22
The dimensions of causal dynamical triangulations	four at large scales; about two at short distances	Ambjørn, Jurkiewicz and Loll (2004; 2005)	22
The age of twistor theory	its first fifty years	Atiyah, Dunajski and Mason (2017)	22
The total curvature of a closed surface	two pi times a whole number; four pi for a sphere; zero for a ring	The theorem of Gauss and Bonnet: two pi times the Euler number, which is two for a sphere and zero for a torus	23
The precision needed to tell a hole's states apart	one part in a number that grows exponentially with the hole's entropy	Papadodimas and Raju (2013)	23

Part VII: Black Holes, Objections and Tests

Quantity	As printed	Source or calculation	Chapters
The temperature of a hole of the Sun's mass	about sixty billionths of a degree above absolute zero	Hawking's temperature for one solar mass: 6.17×10^{-8} kelvin, computed below	25
The evaporation time of a	about 10^{67} years		25

Quantity	As printed	Source or calculation	Chapters
hole of the Sun's mass		2.1×10^{67} years counting light alone, computed below	
The entropy of a black hole	a quarter of its horizon area, in units of the Planck length squared	Bekenstein (1973), with the coefficient fixed by Hawking (1975)	25, 26
How close a part's entanglement comes to the smaller capacity	within half a unit	Page (1993a)	25
The entropy the radiation gains	about one and a half times what the hole loses	The ratio 1.4847 for photons and the small waves of the geometry, with their grey-body factors (Page 2013)	25
The entropy lost at the turning point	about two-fifths; about 40 per cent	The hole keeps 0.598 of its entropy at the crossing, computed below	25
The time of the turning point	a little more than half of the evaporation time	0.538 of the evaporation time, computed below (Page 2013 gives 0.5381)	25
The black-body idealization	a ratio of four-thirds; a crossing at four-sevenths of the initial entropy; 0.568 of the evaporation time	Computed below	25
The latest possible turning point	no later than about 65 per cent of the evaporation time	0.646, computed below, from the generalized second law (Bekenstein 1974)	25
The length of the argument over information	more than forty years	From Hawking (1976) to the computations of 2019	25

Quantity	As printed	Source or calculation	Chapters
The Planck energy	about two billion joules	The Planck mass times the square of the speed of light: 1.956×10^{9} joules	27
The limits from the arrival times of light	above seven times the Planck energy; above ten times the Planck energy	Vasileiou and others (2013): 7.6 times; Cao and others, the LHAASO Collaboration (2024): 10 times, from the burst of 9 October 2022	27

The Calculations in Words

The Planck values. The Planck length is the square root of the reduced Planck constant times Newton's constant, divided by the cube of the speed of light, and it comes to 1.61626×10^{-35} meters. The Planck mass is the square root of the reduced Planck constant times the speed of light, divided by Newton's constant, and it comes to 2.17643×10^{-8} kilograms. An object's gravitational size is Newton's constant times its mass, divided by the square of the speed of light; its quantum size is the reduced Planck constant divided by its mass times the speed of light. Setting the two equal gives the Planck mass, and either size at that mass is the Planck length.

The entangling phase. In each joint alternative of the two diamonds, the phase is Newton's energy of attraction for that alternative's distance, multiplied by the time and divided by the reduced Planck constant. The factor common to all four is Newton's constant times the square of the mass, times the time, divided by the reduced Planck constant, which for 10^{-14} kilograms and 2.5 seconds is 1.582×10^{-4} meters. The combination that no adjustment of one diamond alone can change is that factor times the sum of one over 200 micrometers and one over 700 micrometers, less twice one over 450 micrometers. It comes to 0.314 radians.

The temperature and lifetime of a hole. Hawking's temperature is the reduced Planck constant times the cube of the speed of light, divided by eight pi times Newton's constant times the mass times Boltzmann's constant. For

one solar mass, 1.98892×10^{30} kilograms, it is 6.17×10^{-8} kelvin. The evaporation time, counting light alone, is 5120 pi times the square of Newton's constant times the cube of the mass, divided by the reduced Planck constant times the fourth power of the speed of light. For one solar mass it is 2.1×10^{67} years, and the book gives its order, 10^{67} years.

The turning point of the Page curve. Let the ratio be the entropy the radiation gains divided by the entropy the hole loses. The two capacities cross when the hole keeps the fraction ratio over one plus ratio of its initial entropy. The hole's entropy falls as the two-thirds power of the fraction of the evaporation time that remains, so the crossing comes at one minus that kept fraction raised to the power three halves. With the ratio 1.4847 that Page (2013) computes for photons and the small waves of the geometry, the hole keeps 0.598 of its entropy, having lost about 40 per cent, and the turning point comes at 0.538 of the evaporation time. A perfect black body radiating into empty space gives the ratio four-thirds, a crossing at four-sevenths, and 0.568 of the evaporation time; that value is an idealization. The generalized second law makes the ratio at least one, and a ratio of one gives a crossing at one half and the latest possible turning point, one minus two to the power minus three halves, which is 0.646 of the evaporation time.

The clocks of the satellites. A satellite clock of the Global Positioning System runs fast, net of the slowing from its speed and the quickening from its height, by 4.4647 parts in ten billion (Ashby 2003). Over the 86,400 seconds of a day that is 38.6 millionths of a second, and light travels 11.6 kilometers in that time.

The Ten Cases

This page lists the ten ways of pairing the four characters of comparison, with the verdict of each of the three tests of Chapter 5, so that you can check each row and tick it.

The four characters, each a pair of verdicts, with the stance of the comparing held the same:

Character	Verdict on the content	Verdict on the context
continuity	same	same
discontinuity	different	same
symmetry	same	different
asymmetry	different	different

The first test asks whether a character is paired with itself; such a pairing says nothing new. The second test asks whether a character is paired with its direct opposite; such a pairing contradicts itself. The third test asks whether a sameness appears with a difference among the two verdicts on the content, and again among the two verdicts on the context; a pairing that fails in either respect has lost the inseparability of sameness and difference.

The ten pairings, four of a character with itself and six of two different characters:

Pairing	Verdicts on the content	Verdicts on the context	First test	Second test	Third test	Result
1. continuity with asymmetry	same, different	same, different	passes	passes	passes	holds
			passes	passes	passes	holds

Pairing	Verdicts on the content	Verdicts on the context	First test	Second test	Third test	Result
2. symmetry with discontinuity	same, different	different, same				
3. continuity with symmetry	same, same	same, different	passes	passes	fails on the content	forbidden
4. discontinuity with asymmetry	different, different	same, different	passes	passes	fails on the content	forbidden
5. continuity with discontinuity	same, different	same, same	passes	fails	fails on the context	fails
6. symmetry with asymmetry	same, different	different, different	passes	fails	fails on the context	fails
7. continuity with continuity	same, same	same, same	fails	passes	fails on both	fails
8. discontinuity with discontinuity	different, different	same, same	fails	passes	fails on both	fails
9. symmetry with symmetry	same, same	different, different	fails	passes	fails on both	fails
10. asymmetry with asymmetry	different, different	different, different	fails	passes	fails on both	fails

Two pairings hold: continuity with asymmetry, and symmetry with discontinuity. They pass all three tests, and in each of them the second character is the exact complement of the first. A comparison perfected at its foundation is continuously asymmetric or discontinuously symmetric.

Two pairings are forbidden: continuity with symmetry, and discontinuity with asymmetry. They pass the first two tests and fail only the third, and the Incommensuration Theorem names them.

The other six fail as plain repetitions or contradictions: the four pairings of a character with itself fail the first test, and the two pairings of a character with its opposite fail the second. The third test fails all six as well, so the inseparability of sameness and difference alone leaves the same two pairings standing.

The Aphorisms Cited

These are the aphorisms of *An Immanent Metaphysics* that this book cites, given verbatim and in the order in which they stand in that work.

1.3-1. Where the foundations of any domain can be subsumed by three necessary and sufficient concepts, known as the modalities, so too does the relationships between these three have a consistent pattern and form. This form/pattern, common to the foundations of all domains/worlds, is described by the three Axioms.
All of the descriptive power of this metaphysics ultimately descends from the Axioms.

> **Axiom I**: The immanent is more fundamental than the omniscient and/or the transcendent.
> The omniscient and the transcendent are in conjugate relationship to one another.
>
> **Axiom II**: A class of the transcendent
> will precede an instance of the immanent.
> A class of the immanent will precede
> an instance of the omniscient.
> A class of the omniscient will precede
> an instance of the transcendent.
>
> **Axiom III**: The (classes/instances) of the immanent, omniscient, and transcendent are distinct,
> inseparable, and non-interchangeable.

This book cites it in Chapters 6, 8, 15, 16, 17, 18, 19, 20, 21, 25 and 27.

1.42-5. The Intrinsics of Comparison: The six concepts of sameness, difference, content, context, objectivity, and subjectivity, when taken together as a group.
This book cites it in Chapters 4, 7, 17 and 27.

1.42-4. The Root Tautology: Comparison is isomorphic with interaction and relation.
The concept of comparison is isomorphic with the concept of interaction and the concept of relation.
All relations are interactions and are comparisons. All comparisons are interactions and are relations.
This book cites it in Chapters 1, 4 and 8.

1.83-2. *Parallel Aspect; 1st:* That which is external to the boundary of self, which is objective (has the nature of existence), also has the nature of the omniscient modality. As such, the basis of the lawfulness of that which is the content of perception will be defined in terms of symmetry. The deep lawfulness of the content of perceptions will be defined by the nature of symmetry laws.
This book cites it in Chapter 8.

1.83-3. *Parallel Aspect; 2nd:* That which is internal to the boundary of self, which is subjective (having the nature of creativity/ creation), also has the nature of the transcendent modality. As such, the basis for the lawfulness of that which is the context of perception will be defined in terms of continuity. The deep lawfulness of the context of perceptions will be defined by the nature of continuity laws.
This book cites it in Chapter 8.

1.83-5. Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context.
This book cites it in Chapters 7, 9, 21, 24, 25 and 27.

1.41-2. The Theory of Practice:

Axiom III (as the being of Foundational Triplication) is an assertion that there *are* three modalities, or three terms/concepts, which together subsume the essence of a domain, and which fill (or have) the roles of the modalities of immanent, omniscient, and transcendent. Axiom III does not in itself identify which domain essential concept is associated with which modality, but only that such associations are always strongly definable.

Axiom I (as the essence of the Practice of Theory) distinguishes the immanent modality from the omniscient and transcendent modalities. The immanent modality is that term which (in a context which is both objective and non-temporal) is used as the necessary basis for the definition/description of both the omniscient and the transcendent terms, and/or that which can only be (completely/accurately) defined/described in terms of both the omniscient and the transcendent terms. Axiom I has the effect of naming/identifying the term with an immanent modal association, even though it does not identify which of the two remaining terms are to be associated with the omniscient and transcendent modalities.

Axiom II (as the essence of the Theory of Practice) distinguishes the omniscient modality from the transcendent modality. The omniscient modal term is identified by being that term which in practice (in a context which is both subjective and temporal) necessarily follows from the being of the term with an immanent modal association (that term already identified by Axiom I). The transcendent modal term is identified by being that term which in practice necessarily precedes the being of the term with an immanent modal association.
This book cites it in Chapters 6 and 7.

1R1. Where discussing the concept of comparison in the common usage terms (i.e., where holding the terms subjective and objective as implicit), it is apparent that exactly four other concepts are both necessary and sufficient for the formal consideration of the concept of comparison. These concepts are 'sameness', 'difference', 'content', and 'context'. Any process of comparison involves all four of these terms as intrinsics. Where there is comparison there must be sameness, difference, content, and context. No comparison can be defined without implicitly making reference to all six of these concepts. As such, the collection of these six abstract concepts will be hereafter referred to as 'the intrinsics of comparison'.

1R2. In consideration of the meanings of the intrinsics of comparison, certain statements of inseparability are apparent. For example, it is clear that the concepts of sameness and difference are inseparable. Whenever one appears, the other is implicit. Where there is sameness, there must also be difference. Where there is difference, there must also be sameness.

1R3. Further, the concepts of content and context are inseparable. Where there is content, there must also be context. Where there is context there must also be content.
This book cites it in Chapters 4, 7, 9, 12, 24, 25 and 27.

1R4. This inseparability of the concepts of content and context, and the concepts of sameness and difference, is assumed as being fundamental to, and irrevocably inherent in, all discussions of the concept of comparison, or of anything that is defined in terms of comparison.

1R5. Where the above concepts of comparison, sameness, difference, content, and context, are inseparable, it is also clear that they are distinct concepts with distinct meanings. For example, the meanings of these terms are not interchangeable in any formal statement, without changing or altering the fundamental meaning of that statement.

1R6. It is understood that similar aspects of distinctness and non-interchangeability will apply similarly to any terms that are defined in terms of these.

1R7. It is clear that some of the intrinsic concepts of comparison are applicable to one another. In particular, the concepts of sameness and difference can be applied to the concepts of content and context. Also, (the concepts of) content and context can be applied to (the concepts of) sameness or difference. As it turns out, there are four primary concepts which are formed from these specific applications (i.e., as second order compounds and conjugations).

1R8. Where there is a sameness of subjective context, the following definitions hold about the objective.

> **continuity** is a reference to a sameness of content where there is a sameness of context,
>
> **discontinuity** is a reference to a difference of content where there is a sameness of context,
>
> **symmetry** is a reference to a sameness of content where there is a difference of context,
>
> **asymmetry** is a reference to a difference of content where there is a difference of context,

This book cites it in Chapters 5 and 17.

1R9. Clearly the concepts of symmetry and continuity are not the same, for 1) there are discrete and non-continuous things which are symmetric, and 2) twisted continuous things that are non symmetric. For this reason, it is valid to consider the formation and inter-applicability of these compound concepts of the intrinsics of comparison, and their opposites, to one another. However, in considering the mutual compatibility and applicability of the secondary compounds to form tertiary conceptual compounds, certain conflicts become immediately apparent.

1R10. For example, some of these tertiary compound candidates can be rejected since the duplicate application of the same concept to itself is of no semantic value. In this way four potential candidates are discarded from the list of potentially valid mutual applications.

1R11. Another of these conflicts concerns the mutual applicability of the types of comparison to one another in the terms of 'opposite meanings'. Some of the pairs of these concepts are mutually incompatible as they are clearly opposites of one another. For example, the concept of continuity with the concept of discontinuity asserts that for the same context, something is both identically the same as and identically different than itself. Clearly something (in this case content) cannot both be the same and be different in the same way at the same time.

1R12. In the same way that the formation of a single compound concept out of 'sameness of content' and 'difference of content' would be invalid as it violates concept distinctness (as defined earlier). A compound that assumes that the context is both the same and different violates distinctness. For example, it would be problematic to assert that for the same content, its context is both the same and different. Thus, there is the limitation (in the formation of compound concepts of the four types of comparison) that something cannot be both the same and different in the same way at the same time.

1R13. Insofar as there are two pairs (of the four concepts which are available) which are opposites to one another, then clearly they will not be mutually compatible or applicable. In this way two additional potential third compound candidates can be dropped from the acceptable list of mutual applications. Other potential candidates will also be dropped where they violate the requirement of distinctiveness.

1R14. In addition, due to the inseparability and distinctiveness of the intrinsic concepts of comparison themselves, the formation of the compounds of using these are subject to certain limits. In particular, one of these limits is 1) where the concept of content appears, the concept of context must appear also, and 2) where the concept of context appears, the concept of content must also appear. This, however, is not a problem as the definitions of the terms always ensures that there will be an even matching of a content with a context and a context with a content, wherever these appear.

1R15. However, in a very similar manner (as in the immediately above paragraph), two more candidates for tertiary compound concepts must be rejected. This is a more subtle limitation. In the same manner that the concepts of content and context are inseparable, conceptually distinct, and non-interchangeable, the concepts of sameness and difference are also inseparable, distinct, and non-interchangeable. In particular, where the concept of sameness appears, the concept of difference must appear also, and vise versa. The 'number of appearances' of each concept must match the number of appearances of its opposite.

1R16. Fortunately, all of these limitations can be accounted for fairly easily by noting that when any one of the concepts in each of the concept axis domains appears, its mate must appear also. All that is needed to determine whether the third order compounds of the intrinsics of comparisons are valid is to check that all of the concepts 'add up' or 'cancel out' under the inseparability relationships. In this way, the four compounds formed by the four types of comparison can be checked rather readily.

1R17. In summary form, the above limitations could be organized into three basic sets: where [!x] indicates an invalid application to self, [!y] indicates an invalid mutual application of opposites, and where [!z] indicates an invalid assumption of the separability of sameness and difference. As such, the results of the consideration of all candidates for the formation of the third compounds of the intrinsics of comparison may be listed as follows.

Table of Tertiary Conjunctions:			
[!x]	continuity	and continuity	(ss & ss)
[!z]	continuity	and symmetry	(ss & sd)
[!y]	continuity	and discontinuity	(ss & ds)
	continuity	and asymmetry	(ss & dd)
[!x]	symmetry	and symmetry	(sd & sd)
	symmetry	and discontinuity	(sd & ds)
[!y]	symmetry	and asymmetry	(sd & dd)
[!x]	discontinuity	and discontinuity	(ds & ds)
[!z]	discontinuity	and asymmetry	(ds & dd)
[!x]	asymmetry	and asymmetry	(dd & dd)

This book cites it in Chapter 5.

1R18. From the above list, it is clear that there are only two remaining combinations which are mutually compatible; 1) continuity with asymmetry and 2) symmetry with discontinuity. All other combinations are, for various reasons, incompatible with one another.

1R19. Concept inseparability and the associations as defined, require that an eventity/comparison is either 'continuously asymmetric' **OR** it is 'discontinuously symmetric'. An eventity/comparison cannot be 'discontinuously asymmetric' nor 'continuously symmetric'. This constitutes the fundamental result of this essay, and is called the 'Incommensuration Theorem':

> *The concepts of symmetry and continuity cannot both be simultaneously and fundamentally applied to any eventity/comparison.*
>
> *The concepts of asymmetry and discontinuity cannot both be simultaneously and fundamentally applied to any eventity/comparison.*

1R20. Any absolute application of the concept of comparison must ultimately be continuous and asymmetric, **or** symmetric and discontinuous.
This book cites aphorisms 1R1 to 1R20 together, as the enumeration and statement of the Incommensuration Theorem, in Chapters 5, 7, 10, 15, 16, 21, 23, 24, 26 and 27.

1.46-12. All that is qualia is intercompositional, has the nature of non-local context, and is understood in terms of continuity.
All that is quanta is interexclusionary, has the nature of local content, and is understood in terms of symmetry.
This book cites it in Chapters 7, 9, 12 and 24.

Notes

Part I

Chapter 1. The perihelion of Mercury: Einstein (1915a). The bending of starlight at the eclipse of 29 May 1919: Dyson, Eddington and Davidson (1920). The shift in the frequency of light falling through a height: Pound and Rebka (1960). The clocks of the Global Positioning System: Ashby (2003), who gives the net correction to the rate of a satellite clock as 4.4647 parts in ten billion, which over a day of 86,400 seconds is about 38.6 millionths of a second; and Ashby (2002), who states that uncorrected clock errors would cause navigational errors of more than 11 kilometers in one day. The first observation of gravitational waves, with the masses, the distance (about 410 megaparsecs), the peak strain and the energy radiated: Abbott and others (2016). The shadow of the black hole in M87, with its mass: Event Horizon Telescope Collaboration (2019). The electron's magnetic moment, measured to 0.13 parts per trillion and agreeing with the prediction of the Standard Model to about one part in a trillion: Fan, Myers, Sukra and Gabrielse (2023). The optical clocks raised by 33 centimeters: Chou, Hume, Rosenband and Wineland (2010). The radiation of black holes: Hawking (1974), with the full calculation in Hawking (1975). The mass at which an object's gravitational size equals its quantum size is the Planck mass, about 2.18×10^{-8} kilograms, fixed by Newton's constant, Planck's constant and the speed of light. The proposals for an entanglement experiment: Bose and others (2017); Marletto and Vedral (2017). The use of "observer" for anything that makes a comparison follows *An Immanent Metaphysics*, in which comparison, interaction and relation are one act (the Root Tautology, aphorism 1.42-4; Chapter 4). Each pre-

diction of the account, with its present evidence and the experiments that will decide it, is set out in *The Unquantized Arena*, Chapter 19, "Predictions, Evidence and Decisive Experiments".

Chapter 2. The first description of the small waves of geometry by quanta: Rosenfeld (1930). Bronstein's extension, and his remark that a full treatment might require giving up Riemannian geometry: Bronstein (1936). The divergences of quantized gravity with matter: 't Hooft and Veltman (1974); of pure gravity at two loops: Goroff and Sagnotti (1986). Gravity as an effective theory, "a well behaved quantum field theory at ordinary energies": Donoghue (1994). The canonical quantization and the Wheeler–DeWitt equation: DeWitt (1967). String theory as a theory of gravity: Scherk and Schwarz (1974); anomaly cancellation: Green and Schwarz (1984); the counting of black-hole states: Strominger and Vafa (1996); the correspondence between gravity in a region and a quantum theory on its boundary: Maldacena (1998), first circulated in 1997. Loop quantum gravity: Ashtekar (1986); the discrete values of area and volume in that quantization: Rovelli and Smolin (1995). Asymptotic safety: Weinberg (1979); Reuter (1998). Causal sets: Bombelli, Lee, Meyer and Sorkin (1987). Causal dynamical triangulations: Ambjørn, Jurkiewicz and Loll (2004). Feynman's remarks at the Chapel Hill conference of 1957, on a small ball put into a superposition of two places and the gravitational field it would then have: DeWitt and Rickles (2011), the report of the conference. The argument that gravity must be quantized: Eppley and Hannah (1977). Accounts that keep the arena classical: Jacobson (1995); Penrose (1996); Oppenheim (2023). The Planck length is about 1.616×10^{-35} meters. The diagnosis of this book rests on the structure of comparison, and it does not rest on the failure of the direct method to be renormalizable; the weight of that failure is set out in *The Unquantized Arena*, Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale". The ten programs, with the criteria by which each is judged, are set out in *The Unquantized Arena*, Chapter 14, "Four Criteria and Ten Programs".

Chapter 3. The visitor who asks where the University is: Ryle (1949), Chapter 1. The term "category error" is used throughout in Ryle's philosophical sense: a question that applies a concept where the concept does not belong. The search for the motion of the Earth through the ether: Michelson and

Morley (1887); the electrodynamics that needs no ether: Einstein (1905). The seven sentences are set out as a chain of premises and steps, each with what it rests on, in *The Unquantized Arena*, Chapter 6, "The Category Error", which also states the exact scope of the claim: the arena has no least step, and nothing is a quantum of the arena itself, at any scale.

Part II

Chapter 4. The Root Tautology, "Comparison is isomorphic with interaction and relation": *An Immanent Metaphysics*, aphorism 1.42-4. The six intrinsics of comparison: *An Immanent Metaphysics*, aphorism 1.42-5. The inseparability of content and context, which this book calls the inseparability principle, is *An Immanent Metaphysics*, aphorism 1R3: "Where there is content, there must also be context. Where there is context there must also be content." The six intrinsics and the Root Tautology are stated as numbered premises in *The Unquantized Arena*, Chapter 1, "Comparison, the Intrinsics and the Incommensuration Theorem". The optical clocks: Chou, Hume, Rosenband and Wineland (2010). The ground of this chapter is set out at length in *Comparison; The Necessary Basis*, Chapter 1, "What Does It Mean to Compare?", Chapter 3, "One Act, Three Names", and Chapter 4, "The Six Intrinsics of Comparison".

Chapter 5. The Incommensuration Theorem, with the full enumeration of the pairings of the four compounds and the three tests: *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20, with the table of pairings at 1R17. The four compounds are defined with the stance of the comparing held the same, and each is a pair of verdicts on what is compared: "Where there is a sameness of subjective context, the following definitions hold about the objective" (*An Immanent Metaphysics*, aphorism 1R8). Symmetries and conservation laws: Noether (1918). The Pythagorean comma is arithmetic: twelve fifths give $(3/2)^{12} = 531441/4096$, about 129.746, against seven octaves, $2^7 = 128$; the ratio of the two is 531441/524288, about 1.01364, or 23.46 cents, where an equal-tempered semitone is 100 cents; equal temperament narrows each fifth by 23.46/12, about 1.96 cents. The theorem is set out at length for the general reader in *Incommensuration; The Theorem and the Three Axioms*,

Chapter 2, "Symmetry and Continuity Cannot Both Be Perfected", and proved step by step in its Chapter 6, "The Incommensuration Theorem: Statement and Proof".

Chapter 6. The three axioms are quoted in the wording of *An Immanent Metaphysics*, aphorism 1.3-1. Axiom III alone does not fix which pair of intrinsics fills which role: "Axiom III does not in itself identify which domain essential concept is associated with which modality, but only that such associations are always strongly definable" (*An Immanent Metaphysics*, aphorism 1.41-2). The same aphorism gives Axiom I as what identifies the immanent term, and Axiom II as what tells the omniscient term from the transcendent; which axioms each focus of this book asserts is set out in the chapter. The check of the two levels against six established theories is set out in *The Unquantized Arena*, Chapter 3, "The Cross-Modal Principle, Checked Against Six Theories". The three roles, their binding by focus and the order of finding are set out at length in *Comparison; The Necessary Basis*, Chapter 5, "Three Roles", Chapter 6, "Which Pair Takes Which Role", and Chapter 7, "The Order of Finding"; the derivation of the three axioms from the theorem is in *Incommensuration; The Theorem and the Three Axioms*, Chapter 8, "From the Theorem to the Axioms".

Part III

Chapter 7. The six intrinsics of comparison, among them content and context, are those of *An Immanent Metaphysics*, aphorism 1.42-5. The two laws (the law of symmetry applies to all that is of content; the law of continuity applies to all that is of context) are aphorism 1.83-5. The reasons for the two laws, with the reason that a context is smooth and not only joined, are given beside the premises in *The Unquantized Arena*, Chapter 1, "Comparison, the Intrinsics and the Incommensuration Theorem". The characterization of content as local and interexclusionary, and of context as non-local and intercompositional, is this book's reading of aphorism 1.46-12, which says: "All that is qualia is intercompositional, has the nature of non-local context, and is understood in terms of continuity. All that is quanta is interexclusionary, has the nature of local content, and is understood in terms of symmetry." The in-

separability principle (where there is content, there must also be context, and the reverse) is aphorism 1R3. The two surviving pairings and the two forbidden ones are found by the enumeration in the Technical Appendix, aphorisms 1R1 to 1R20. Which concept stands in which of the three roles depends on the focus of the argument being made, by the rule that Chapter 6 states, and Axiom III alone does not fix it (aphorism 1.41-2); the grid of six measures takes the focus of the measures themselves. The reasoning from the law of symmetry to findings in steps, and from the law of continuity to smooth findings, is set out as numbered propositions in *The Unquantized Arena*, Chapter 4, "What Quantization Is". The division of physics into content and context, with gravity on the side of context, is stated as premises in *The Unquantized Arena*, Chapter 5, "The Content and Context Partition". The two laws and the six measures are set out for the steps from comparison to physics in *Measurement; The Steps from Comparison to Physics*, Chapter 3, "Content and Context in Physics", and Chapter 9, "The Mode Assignment".

Chapter 8. The three axioms are given in the wording of *An Immanent Metaphysics*, aphorism 1.3-1; the plain glosses in the chapter are glosses, and the axioms themselves are quoted exactly in Part II. The Root Tautology, that comparison is one act with interaction and relation, is aphorism 1.42-4. The reason the arena bends rests on the three axioms and the Incommensuration Theorem alone, in the focus of the two laws, where the comparing, the content and the context fill the immanent, omniscient and transcendent roles, following aphorisms 1.83-2 and 1.83-3 of *An Immanent Metaphysics*, which give the objective side of a comparison the omniscient modality with the lawfulness of content in symmetry, and the subjective side the transcendent modality with the lawfulness of context in continuity; that binding is set out in *Comparison; The Necessary Basis*, Chapter 6, "Which Pair Takes Which Role". The theorem, being about a comparison, holds of the comparison as a whole; a symmetric content beside a continuous context is allowed because the two characters hold of two different things, as *Incommensuration; The Theorem and the Three Axioms*, Chapter 6, "The Incommensuration Theorem: Statement and Proof", sets out in its Section VIII. The form of the relation between curvature and energy is taken from general relativity as established, in Einstein's field equations (Einstein 1915b), and is derived nowhere in this book. The same ar-

gument is stated as a proposition, and marked as not used by the chain of the category error, in *The Unquantized Arena*, Chapter 5, "The Content and Context Partition"; and *Measurement; The Steps from Comparison to Physics*, Chapter 10, "Context and Its Markers", states it again and carries it on to where, and how much, the arena curves. The first detection of gravitational waves: Abbott and others (2016). The equal speeds of gravitational waves and light, from the neutron-star merger of 2017: Abbott and others (2017). The clocks raised by about a third of a meter: Chou, Hume, Rosenband and Wineland (2010). Newton's constant is measured; the axioms fix the role of the arena and no size within it. The partition of physics into content and context, on which this chapter's reasoning rests, is stated as premises, with the four marks of the arena and their evidence, in *The Unquantized Arena*, Chapter 5, "The Content and Context Partition".

Chapter 9. The inseparability principle is *An Immanent Metaphysics*, aphorism 1R3; the two laws are aphorism 1.83-5; the characterization of content and context is aphorism 1.46-12. Antihydrogen falling downwards: Anderson and others (2023), the ALPHA-g experiment, with an acceleration consistent with that of ordinary matter. Titanium and platinum falling alike to within a few parts in a million billion: Touboul and others (2022), the final results of the MICROSCOPE mission. The physical picture of why no arrangement of matter covers a source of gravity uses the equality of falling, the third mark, and so it is given as a picture that agrees with the structural argument from the nature of a region. The four marks as numbered propositions, each with its premises and its experimental evidence, are in *The Unquantized Arena*, Chapter 5, "The Content and Context Partition"; the equivalence principle as a prediction with its present evidence and the experiments to come is in *The Unquantized Arena*, Chapter 19, "Predictions, Evidence and Decisive Experiments". The four marks are derived again, with their evidence, in *Measurement; The Steps from Comparison to Physics*, Chapter 10, "Context and Its Markers".

Chapter 10. The first test of the Incommensuration Theorem, which removes every pairing of a character with itself, is part of the enumeration in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20; the cross-modal principle applies that test to a theory taken as a comparison of its

mathematics with its findings. Continuous symmetries of a theory's equations and the quantities they conserve: Noether (1918). The electron's magnetism, measured and predicted in agreement to about one part in a trillion: Fan, Myers, Sukra and Gabrielse (2023). Paths that cannot be continued, once the matter of a star has fallen in far enough to trap its own light: Penrose (1965). The principle, its two arrangements, and the six theories in one table, with the character of each theory's findings and mathematics stated in technical terms, are in *The Unquantized Arena*, Chapter 3, "The Cross-Modal Principle, Checked Against Six Theories". The principle is set out again, with the same six theories, in *Measurement; The Steps from Comparison to Physics*, Chapter 8, "The Cross-Modal Principle".

Chapter 11. The energy of radiation in steps: Planck (1901), the paper that followed his lecture of December 1900. The atom with separate levels: Bohr (1913). The classical electron falls into the nucleus in about 1.6×10^{-11} seconds, by the standard calculation of the energy it radiates. Quantizing as a mode exchange of both levels together, the classical theories of matter as descriptions of content in the conjugate character, and the check against content and context are set out in *The Unquantized Arena*, Chapter 4, "What Quantization Is". The chain of nine steps, with its exact scope (no least step of the arena; no quantum of the arena at any scale), is set out as propositions in *The Unquantized Arena*, Chapter 6, "The Category Error".

Part IV

Chapter 12. The step from content to context is the inseparability principle, in the words of *An Immanent Metaphysics*, aphorism 1R3: "Where there is content, there must also be context. Where there is context there must also be content." Its companion characterization, content local and understood in terms of symmetry, context non-local and understood in terms of continuity, is this book's reading of aphorism 1.46-12, which states it of quanta, as local content, and of qualia, as non-local context. The chapter applies the principle to each alternative of the matter: each alternative is a possible content, so each has a context, and the geometry is that context. That the alternatives have two contexts and not one shared context rests on the same characterization: con-

texts join one another (they are intercompositional), and they join only where their contents are found together, while alternatives exclude one another as contents. The same ground places the curvature of a wave, or of a hole formed from waves, in the context of the content that produced it (*The Unquantized Arena*, Chapter 7, "One Geometry for Each Alternative", gives the argument in its Questions). That the order of events can differ between the geometries of the alternatives, and can be put in superposition by a mass held in two places: Zych, Costa, Pikovski and Brukner (2019).

The averaged geometry, one geometry shaped by the probability-weighted average of the matter's energy: Møller (1962); Rosenfeld (1963). The torsion balance: Cavendish (1798). The experiment in which a Geiger counter's reading decided the positions of the large masses, and the balance turned with the actual positions: Page and Geilker (1981). That a single geometry shaped by the quantum state, together with the change of weights at a finding, sends signals faster than light: Bahrami, Großardt, Donadi and Bassi (2014).

The three reasons for a geometry for each alternative (inseparability, the light cone, and the experiment) are set out as propositions, with the limit in which the averaged and the per-alternative forms give the same geometry for sharply defined matter, in *The Unquantized Arena*, Chapter 7, "One Geometry for Each Alternative".

The answer to the other two horns of Eppley and Hannah's argument: an apparatus able to make their measurement would be massive enough to lie within its own Schwarzschild radius, the size at which a mass is a black hole: Mattingly (2006). A fuller treatment, with the presupposition of one classical field for all alternatives, is in *The Unquantized Arena*, Chapter 7, "One Geometry for Each Alternative".

Chapter 13. Feynman's ball in two places, with his conclusion that there must be an amplitude for its gravitational field: his remarks in the discussion recorded as Chapter 23, "The Necessity of Gravitational Quantization", of DeWitt and Rickles (2011), the report of the Chapel Hill conference of 1957. The experiment: Bose, Mazumdar, Morley, Ulbricht, Toroš, Paternostro, Geraci, Barker, Kim and Milburn (2017), with masses of about 10^{-14} kilograms, places about 250 micrometers apart, centers about 450 micrometers apart, and about 2.5 seconds of hold; micro-diamonds with an embedded ni-

trogen-vacancy spin are their candidate test masses. The argument that a go-between in one definite state cannot entangle two systems, and that entanglement through gravity would show gravity is not such a go-between: Marletto and Vedral (2017). The conducting plate between the two diamonds: van de Kamp, Marshman, Bose and Mazumdar (2020).

The phase of each joint alternative is Newton's energy of attraction for that alternative's distance, multiplied by the time and divided by the reduced Planck constant. The combination that no adjustment of one diamond alone can change, the near-near and far-far phases less the two mixed phases, comes to 0.314 radians for the parameters above, the design proposed in 2017; later designs use other masses, separations and times, and the same formula gives their phases (Bose and colleagues 2026); the calculation, the decoherence that the design must hold off, and the rival readings of the result are set out in *The Unquantized Arena*, Chapter 8, "The Entanglement Test".

The accounts that predict no entanglement: a geometry that is classical and the same for all alternatives, as in Oppenheim (2023); and the ending of superpositions by gravity, as in Penrose (1996). That linearized quantum gravity gives the same phase follows because, for slow masses at these distances, it uses the same Newtonian energy in each joint alternative.

Chapter 14. The first observation of a gravitational wave, on 14 September 2015, with a peak strain of about one part in 10^{21}: Abbott and others, the LIGO Scientific Collaboration and the Virgo Collaboration (2016). General relativity as an effective theory, with its leading quantum corrections to the interaction of two masses: Donoghue (1994); the quantum correction is of relative size about the square of the Planck length over the distance, about 10^{-70} at one meter. The full objection from the graviton, and its answer, are given again among the objections a physicist raises in Chapter 26, "What a Physicist Will Say".

On the detection of single gravitons: Dyson (2013), from his lecture of 2012, for the mirrors that would collapse into a black hole; Rothman and Boughn (2006), for a detector of Jupiter's mass close to a neutron star, with as many as 10^{-2} detections a year and 10^{33} to 10^{34} neutrino events for every graviton event without shielding; Tobar, Manikandan, Beitel and Pikovski (2024), for stimulated absorption in a massive acoustic resonator, read out by

continuous sensing of quantum jumps and correlated with a gravitational-wave detection; Carney, Domcke and Rodd (2024), for the point that a classical wave acting on a quantum detector gives the same clicks, as the photoelectric effect can be explained by a classical light wave or by photons.

That the account excludes a step of the arena itself, and a quantum of the arena at any scale, while it allows a description of small waves by quanta below the Planck curvature, whose quanta are quanta of the description only, is set out with the effective theory and the weight of its non-renormalizability in *The Unquantized Arena*, Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale".

Chapter 15. The units of length, mass and time built from the constants of nature: Planck (1899). The gravitational size is the length Gm/c^2, half the radius of the horizon of a black hole of that mass; the quantum size is the length $\hbar/(mc)$; the two are equal when the mass is the square root of $\hbar c/G$, the Planck mass of about 2.18×10^{-8} kilograms, and both then equal the Planck length of about 1.62×10^{-35} meter. The value of Newton's constant, 6.67430×10^{-11} in units of cubic meters per kilogram per second squared, with a relative uncertainty of about 2.2×10^{-5}: the CODATA 2022 value (Mohr, Newell, Taylor and Tiesinga 2025).

The reading of the Planck scale as the meeting of the two surviving characters of comparison rests on the Incommensuration Theorem, whose enumeration of the ten pairings is in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20, and on the three axioms, aphorism 1.3-1. That the axioms fix relations and not magnitudes, so that one measured constant sets the scale, is stated for the specialist, with the effective theory of gravity and the Planck scale as the meeting of the descriptions, in *The Unquantized Arena*, Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale".

The arrival of a photon of about 31 billion electronvolts from the gamma-ray burst GRB 090510 within about a second of the burst's low-energy emission, and the resulting lower limit of 1.2 times the Planck energy on the scale of a speed of light that varies linearly with energy: Abdo and others (2009).

Part V

Chapter 16. The rate of a clock in the orbit of the Global Positioning System, compared with a clock on the ground, differs by a net fraction of about 4.46 parts in ten billion, which comes to about 38.6 millionths of a second a day; the satellite clocks are offset before launch to allow for it: Ashby (2003). The three times are set out in *Measurement; The Steps from Comparison to Physics*, Chapter 12, "Time", which calls the time of findings by its name in physics, event time.

The identification of the three kinds of time with the three roles (geometric time with the omniscient, the time of findings with the immanent, evolution time with the transcendent) uses the roles as *An Immanent Metaphysics* gives them, and their relation (distinct, inseparable, non-interchangeable) is its Axiom III, aphorism 1.3-1, quoted in the chapter.

The direction of the time of findings is the one-way order of Axiom II (a class of the transcendent precedes an instance of the immanent; a class of the immanent precedes an instance of the omniscient; a class of the omniscient precedes an instance of the transcendent), *An Immanent Metaphysics*, aphorism 1.3-1. The identification of that order with the direction of time is made in the first person in the chapter.

The characters assigned to the three kinds of time follow the two levels of Chapter 10: in general relativity the geometry found is continuous and asymmetric, and the time-reversal symmetry of the field equations belongs to the law; in quantum theory the findings are discontinuous and symmetric, and the smooth change of the state belongs to the mathematics. The two forbidden pairings that a unified time would need are those of the Incommensuration Theorem, whose full enumeration is in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20.

The canonical quantization of gravity, which yields an equation for a quantum state of the geometry with no time variable: DeWitt (1967).

Chapter 17. The single-electron double-slit experiment, in which each electron arrives as one dot and the dots build up interference fringes: Tonomura, Endo, Matsuda, Kawasaki and Ezawa (1989).

Schrödinger's cat, the measurement problem stated on a living creature whose fate depends on one atom: Schrödinger (1935).

The residual, the part of a theory's findings that its mathematics approaches without containing, follows from the two levels of Chapter 10, whose characters are conjugate. The pairing of quantum theory's residual (the measurement problem) with general relativity's (the singularity) is the same principle applied to both theories.

The priority of the act over structure and possibility is Axiom I of *An Immanent Metaphysics*, aphorism 1.3-1: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another."

The experiment in which the gravitational field followed the actual outcome of a quantum random event, and not the weighted average of the outcomes: Page and Geilker (1981). Its authors read it as evidence that gravity should be quantized; the reading of this book, set out in Chapter 12, is that it excludes a single geometry sourced by the average and agrees with one geometry for each alternative.

The probability rule: Born (1926). The result that, in a space of possibilities of three or more dimensions, the only consistent assignment of odds that does not depend on what is measured alongside is the rule of Born: Gleason (1957). Its extension to general measurements, which holds in every dimension including two: Busch (2003).

The identification that the condition of Gleason's result is a symmetry (the same content across different contexts) uses the definition of symmetry among the four compounds of comparison set out in Part II (sameness of content where there is a difference of context), which is *An Immanent Metaphysics*, aphorism 1R8, built on the six intrinsics of aphorism 1.42-5. The three outcomes that Gleason's result needs belong to the space of possibilities; they are unrelated to the three roles of Axiom III.

The argument that one geometry for each alternative passes no signal faster than light, with its literature, is in *The Unquantized Arena*, Chapter 7, "One Geometry for Each Alternative".

Chapter 18. The experiment in which interference of carbon-seventy molecules fades as gas is let into the interferometer, in quantitative agreement with decoherence theory: Hornberger, Uttenthaler, Brezger, Hackermüller, Arndt and Zeilinger (2003).

The founding papers of decoherence named in the chapter: Zeh (1970); Zurek (1981); Joos and Zeh (1985). The review: Zurek (2003). The account of many copies of one record in the surroundings: Zurek (2009).

The review that sets out what decoherence settles, and finds its bearing on the problem of outcomes still debated: Schlosshauer (2005).

The openness of the future rests on Axiom II of *An Immanent Metaphysics*, aphorism 1.3-1 ("A class of the transcendent will precede an instance of the immanent"), read with the plain sense of class and instance: a class is a kind, standing ready as a whole; an instance is one particular case of a kind. The second ground is the Incommensuration Theorem: a description that fixed every finding in advance would perfect symmetry and continuity together.

The experiment whose decoherence budget is discussed, two masses each held in two places and related only through gravity, is the subject of Chapter 13, "The Experiment Being Built", and of *The Unquantized Arena*, Chapter 8, "The Entanglement Test".

Chapter 19. The movability of the boundary between the system described by the wave and the instrument that records the finding: von Neumann (1932), Chapter VI. The identification of the Heisenberg cut as the boundary between the two levels of quantum theory, the smooth description of possibility and the discrete record of the act, is made in the first person in the chapter.

The many-worlds interpretation: Everett (1957). The arguments that a rational agent in a branching world should weigh branches by Born's rule: Deutsch (1999); Wallace (2012).

The pilot-wave theory in its complete form: Bohm (1952), in two parts.

Relational quantum mechanics: Rovelli (1996). The postulate added to secure agreement between comparers: Adlam and Rovelli (2023).

QBism: Fuchs, Mermin and Schack (2014).

The collapse models: Ghirardi, Rimini and Weber (1986); Pearle (1989); Ghirardi, Pearle and Rimini (1990); Diósi (1987; 1989); Penrose (1996). The underground experiment that rules out the parameter-free model of Diósi and Penrose: Donadi, Piscicchia, Curceanu, Diósi, Laubenstein and Bassi (2021).

The pattern of the chapter, that each interpretation meets its difficulty where the time it sets aside returns, is Axiom III of *An Immanent Metaphysics*, aphorism 1.3-1, applied to the three kinds of time of Chapter 16. The priority given to Copenhagen's choice is Axiom I of the same aphorism.

The prediction that separates the account of this book from gravity-driven collapse, entanglement through gravity with a phase of about a third of a radian for the design proposed in 2017, is computed in *The Unquantized Arena*, Chapter 8, "The Entanglement Test". Later designs use other masses, separations and times, and the same formula gives their phases.

Chapter 20. The loophole-free test with electron spins in diamond, at stations 1.3 kilometers apart, with a measured value of 2.42 ± 0.20 against the local bound of 2: Hensen and others (2015). The tests with entangled light: Giustina and others (2015); Shalm and others (2015).

Bell's theorem: Bell (1964). The stronger form, which is built on the condition Bell called local causality and excludes every account in which each single result is produced by conditions local to its station, even when those conditions give only odds: Bell's later papers on the condition, collected in Bell (1987). The testable form with the bound of 2: Clauser, Horne, Shimony and Holt (1969). The quantum value can reach twice the square root of two, about 2.83.

The theorem of Kochen and Specker: Kochen and Specker (1967). The square of nine measurements: Mermin (1990).

The theorem that the quantum state is not only knowledge, given that independent preparations give independent underlying conditions: Pusey, Barrett and Rudolph (2012).

The one rule of which the three theorems are read as cases is Axiom III of *An Immanent Metaphysics*, aphorism 1.3-1: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-in-

terchangeable." The readings are identifications: each theorem is a result inside its own mathematics, and the axiom gives the kind of which each is a case.

Part VI

Chapter 21. The law that the four questions apply is the Incommensuration Theorem: symmetry and continuity cannot both be perfected, simultaneously and fundamentally, in one comparison; and neither can asymmetry and discontinuity. Its full enumeration of the ten candidate pairings, with the three tests that leave two survivors, is in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20.

The first question rests on the two laws of *An Immanent Metaphysics*, aphorism 1.83-5: "Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context." A program that gives gravity a charge and a running strength treats the context by the law of the content. The four marks by which gravity is identified as the arena are derived and checked in Chapter 9, "Four Marks of the Arena", and in *The Unquantized Arena*, Chapter 5, "The Content and Context Partition", with the experimental evidence.

The third question's list of admitted uses of quanta follows from what quantization is: a mode exchange of both levels of a description at once, which content can take and the arena cannot. A description of small waves by quanta is admitted as a description, and its quanta are quanta of the description only; a detector that registers a gravitational wave in single clicks would not count against the account, and would not show gravitation quantized. The specialist form is in *The Unquantized Arena*, Chapter 4, "What Quantization Is", and Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale".

The difficulty of recovery rests on the third axiom of *An Immanent Metaphysics*, aphorism 1.3-1, whose terms are "distinct, inseparable, and non-interchangeable": a character given up cannot be supplied by the other, because neither can do the other's work. The three predicted difficulties, multiplicity, dynamics and recovery, are stated as consequences of the criteria in *The Unquantized Arena*, Chapter 14, "Four Criteria and Ten Programs".

Chapter 22. The epigraph is the closing sentence of the abstract of Jacobson (1995), quoted exactly. Jacobson derived Einstein's equation by demanding that the relation of heat, temperature and entropy hold for every local horizon through every point, with entropy proportional to horizon area.

String theory's founding result is recounted by Schwarz (2007); the count of black-hole states is Strominger and Vafa (1996), for a class of extremal black holes. The range of its vacua is described by Susskind (2003) as "unimaginably large and diverse"; the statistical approach to counting them is Douglas (2003), with the counting formula of Ashok and Douglas (2004). The swampland program is Vafa (2005), reviewed in Palti (2019). The answer this book gives to what the count of black-hole states shows is in Chapter 26, "What a Physicist Will Say".

The discrete spectra of area and volume in loop quantum gravity are computed in Rovelli and Smolin (1995); the well-defined form of the equation of change, the Hamiltonian constraint, is Thiemann (1998). I read these spectra as properties of the quantized description, and the arena as having no least length, area or duration; the reason is the mode exchange of both levels that quantization makes, which the arena cannot take (Chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It").

The equation for the flow of gravity's strengths is Reuter (1998), solved there in a simple truncation; the Higgs-mass prediction is Shaposhnikov and Wetterich (2010), who give 126 billion electronvolts "with only a few GeV uncertainty"; the particle was found in 2012 at about 125 billion electronvolts by the ATLAS Collaboration (2012), which measured 126.0, and the CMS Collaboration (2012), which measured 125.3. The computations with time kept as time, in Lorentzian signature, are Fehre, Litim, Pawlowski and Reichert (2023) and Assant, Litim and Reichert (2026); the calculations with time turned into a fourth direction of space are in Euclidean signature. The four criteria and the Lorentzian difficulty are set out in *The Unquantized Arena*, Chapter 14, "Four Criteria and Ten Programs".

The theorem that random scattering preserves the symmetry between frames in uniform motion is Bombelli, Henson and Sorkin (2009). The count of partial orders, in which almost all orders of many elements fall into one special three-layered class, is Kleitman and Rothschild (1975). The suppres-

sion of those orders in the causal-set path integral is Carlip, Carlip and Surya (2023), extended to layered orders in every dimension in Carlip, Carlip and Surya (2024).

The emergence of four large dimensions in causal dynamical triangulations is Ambjørn, Jurkiewicz and Loll (2004); the fall of the spectral dimension to about two at short distances is Ambjørn, Jurkiewicz and Loll (2005).

ER=EPR is Maldacena and Susskind (2013); the argument that entanglement holds geometry together is Van Raamsdonk (2010). The amplituhedron is Arkani-Hamed and Trnka (2014), for the theory known as planar maximally supersymmetric Yang–Mills theory.

The thermodynamic reading of gravity is developed in Padmanabhan (2010) and Verlinde (2011); the objections to the entropic account of Newton's law are Kobakhidze (2011), from the quantum states of neutrons in the Earth's gravitational field, and Visser (2011), from the conservative character of gravity. Twistor theory over its first fifty years is reviewed in Atiyah, Dunajski and Mason (2017), including the encoding of anti-self-dual solutions in twistor space; the twistor-space form of gauge-theory amplitudes is Witten (2004). The action of non-commutative geometry for the Standard Model with gravity is Chamseddine and colleagues (2007), whose Higgs mass is near 170 billion electronvolts; the repair by a neutral scalar singlet is Chamseddine and Connes (2012).

The tensor networks that model holographic geometry, built from methods for the renormalization of quantum states of matter, are Swingle (2012).

Chapter 23. The correspondence is Maldacena (1998), first circulated in 1997, who conjectured that string theory on anti-de Sitter space is equivalent to a conformal field theory on its boundary, and found that at large numbers of fields the bulk side is supergravity, a classical theory of gravity.

The identification of the boundary's findings as discontinuously symmetric and the bulk's as continuously asymmetric applies the two-levels reading of every theory: each theory's findings and its mathematics are in conjugate characters (Chapter 10, "How a Theory Sees"; in technical form, *The Unquantized Arena*, Chapter 3, "The Cross-Modal Principle, Checked Against Six The-

ories"). The two characters are the two survivors of the Incommensuration Theorem, *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20.

The construction of bulk fields from boundary quantities is Hamilton, Kabat, Lifschytz and Lowe (2006a). The reading of the dictionary as a quantum error-correcting code is Almheiri, Dong and Harlow (2015); exact code models are Pastawski, Yoshida, Harlow and Preskill (2015), in which one bulk quantity is recoverable from several boundary regions. The interior construction, and the precision needed to tell a hole's states apart, are Papadodimas and Raju (2013); the regions no standard probe reaches are Freivogel and colleagues (2015).

The formula of Ryu and Takayanagi is Ryu and Takayanagi (2006); its derivation within the dictionary is Lewkowycz and Maldacena (2013); its first quantum correction, the entanglement of bulk fields across the surface, is Faulkner, Lewkowycz and Maldacena (2013). The Gauss–Bonnet theorem is classical differential geometry: for a closed surface the total curvature equals two pi times the Euler number, which is two for a sphere and zero for a torus.

The monogamy of holographic mutual information is the result of Hayden, Headrick and Maloney (2013): for three regions, the information one shares with the other two together is at least the sum of what it shares with each. The tensor-network models of the bulk geometry are Swingle (2012) and Pastawski, Yoshida, Harlow and Preskill (2015).

Chapter 24. The inseparability principle is quoted from *An Immanent Metaphysics*, aphorism 1R3, and the two laws from aphorism 1.83-5. The characterization of the two, this book's reading of aphorism 1.46-12 of the same work, gives each its own nature: content is local and understood in terms of symmetry; context is non-local and understood in terms of continuity.

The prohibition of a merger is the Incommensuration Theorem applied to one description: a description in which the same directions are both continuously asymmetric and discontinuously symmetric asks all four characters of those directions, and so would carry both forbidden pairings, continuity with symmetry and asymmetry with discontinuity, in one comparison (*An Imman-*

ent Metaphysics, Technical Appendix, aphorisms 1R1 to 1R20). The permission of a common source is the same theorem's two survivors, each perfected in its own description.

The exact scope of "not quantized" has two parts: the arena has no least step, and nothing is a quantum of the arena itself, at any scale. A description of the small waves of the geometry by quanta is admitted as a description, and its quanta are quanta of the description only; a detector that registers a gravitational wave in single clicks would not count against the account and would not show gravitation quantized.

The phase of about a third of a radian is the standard Newtonian entangling phase for the design proposed in 2017 and described in Chapter 13, "The Experiment Being Built"; its value and the calculation are in *The Unquantized Arena*, Chapter 8, "The Entanglement Test".

Part VII

Chapter 25. The reading of a black hole's entropy as a count made relative to a context rests on the inseparability of content and context: where there is content, there is context, and where there is context, there is content (*An Immanent Metaphysics*, aphorism 1R3). A count of the interior states that the outside world cannot tell apart is a count of content whose context is the outside world. The entropy Bekenstein proposed (1973), with the coefficient Hawking fixed (1975), is a quarter of the horizon area measured in squares of the Planck length.

The temperature of the glow is inversely proportional to the mass of the hole: in Hawking's formula, it is the reduced Planck constant times the cube of the speed of light, divided by eight pi times Newton's constant times the mass times Boltzmann's constant. For one solar mass this gives about 6.2×10^{-8} kelvin. The evaporation time grows as the cube of the mass; for one solar mass, counting light alone, it is about 2×10^{67} years, and the text gives the order, about 10^{67} years.

Hawking's derivation (1975) compares the description of the field's empty state natural before the hole formed with the one natural long afterward, far away; the second finds in the first a thermal flux at the horizon's temperature. Hawking describes the picture of pairs, one partner falling in and one escaping, as a heuristic aid; the computation does not depend on it, and neither does this book. The thermal response of an accelerating instrument in flat space is Unruh's (1976).

The paradox is Hawking's (1976): a pure state evolving, through formation and complete evaporation, into a mixed one. The diagnosis of its premise follows the category error of Chapter 11: the demand that the radiation end pure treats the geometry as part of the quantum state of the whole, whereas the geometry is the context of the matter's findings, and each alternative of the matter carries its own (Chapter 12).

The three steps behind the Page curve are stated as a conditional result: if the whole formed by what fell in, the content inside the horizon and the radiation stays pure, as seen from one outside world, and if the hole's capacity is its entropy, the entanglement entropy of the radiation follows the smaller of its two capacities. The whole is the matter's state laid out on its geometries, and the condition puts no geometry into the state. The bound on the hole's capacity belongs to the condition, as the purity does. The third step uses Page's result on the average entropy of a subsystem (1993a), which places the entanglement entropy of a part within half a unit of the smaller capacity for nearly every pure state of the whole. I read the equal weighting of states that no outside comparison can tell apart as a symmetry over those states; the law of symmetry applies to all that is of content (*An Immanent Metaphysics*, aphorism 1.83-5). The curve itself is Page's (1993b).

The time of the turning point follows from one ratio: the entropy the radiation gains, divided by the entropy the hole loses. The generalized second law (Bekenstein 1974) makes that ratio at least one. When the ratio is one, the capacities cross when the hole keeps half its entropy, at about 0.646 of the evaporation time; that is the latest possible turning point. A perfect black body radiating into empty space gives a ratio of four-thirds, a crossing at four-sevenths of the initial entropy, and about 0.568 of the evaporation time; that is an idealization. Page (2013), with the actual emission of photons and of the

small waves of the geometry, counted in the terms of their description by quanta, finds a ratio of about 1.4847, a crossing when the hole keeps about 0.598 of its initial entropy (it has lost about 40 per cent), and a turning point at about 0.538 of the evaporation time, the computed value the text gives as a little more than half. The fraction of time follows because the hole's entropy falls as the two-thirds power of the fraction of time remaining.

The island formula (Penington 2020; Almheiri, Engelhardt, Marolf and Maxfield 2019; reviewed by Almheiri, Hartman, Maldacena, Shaghoulian and Tajdini 2021) is obtained from a sum over geometries, whose deciding configurations are classical solutions. The book reads the agreement as agreement in the shape of the curve, which the three steps fix, and adopts no superposition of geometries.

The argument for a firewall is that of Almheiri, Marolf, Polchinski and Sully (2013): a late quantum of the radiation cannot be fully entangled both with the early radiation and with its partner behind the horizon, if that partner is independent of the early radiation. After the turning point the island, which contains the partners, is counted with the radiation, so the partners are not independent of it. The answer, with the argument it meets, is set out in *The Unquantized Arena*, Chapter 17, "Black-Hole Information and the Page Curve".

For holes formed from stars, the two statements the account makes rest on the two descriptions alone: quantum theory loses nothing of the matter's state on its geometry, general relativity carries what falls in to the end of the interior, where the smooth description reaches the edge of its domain, and the horizon passes no signal outward. Nothing in the thesis depends on where the information goes afterward.

Chapter 26. The thermal response of a uniformly accelerating instrument in flat space, at a temperature proportional to its acceleration, is Unruh's (1976). The insensitivity of the thermal flux to the behavior of waves at very short wavelengths was shown by Unruh (1995) in a fluid flowing faster than its own speed of sound, which has a horizon for sound.

Strominger and Vafa (1996) counted the degeneracy of protected bound states of strings and branes for a class of five-dimensional extremal black holes in string theory, at weak coupling, and matched it to a quarter of the ho-

rizon area. Ashtekar, Baez, Corichi and Krasnov (1998) obtained an entropy proportional to the horizon area in loop quantum gravity; its coefficient depends on the Immirzi parameter, and a suitable choice of that parameter gives the coefficient of one quarter. Both counts are read here as counts of content, the first in a description whose geometry is classical, the second on a spectrum of area that this account does not admit.

The correspondence is Maldacena's (1998). The reconstruction of interior fields from the boundary is that of Hamilton, Kabat, Lifschytz and Lowe (2006b); the reading of the correspondence as a code that corrects errors is that of Almheiri, Dong and Harlow (2015). The full treatment is in *The Unquantized Arena*, Chapter 15, "AdS/CFT as Cross-Modal Correspondence".

The singularity theorems are those of Penrose (1965) and Hawking and Penrose (1970); what they establish is that some paths of falling bodies or light cannot be extended beyond a finite time within the smooth geometry, under energy conditions that ordinary matter meets. The reading of the singularity as the edge of the smooth description's domain rests on the meeting of the two descriptions at the Planck curvature (Chapter 15). Planck's law of radiation, found in 1900 and published in 1901, has no infinity of the classical kind, and it rests on steps in the energy exchanged between matter and radiation.

General relativity treated as an effective theory at low energies, with its leading quantum corrections, is Donoghue's (1994). The proposal to register single clicks from a gravitational wave, read by its authors as single quanta, is that of Tobar, Manikandan, Beitel and Pikovski (2024); the result that such clicks are equally produced by a classical wave acting on a quantum detector is that of Carney, Domcke and Rodd (2024). Gravitational waves were first reported by the LIGO Scientific Collaboration and the Virgo Collaboration (Abbott and others 2016).

The argument from the early universe is that of Krauss and Wilczek (2014). The energy of the waves left by an epoch of rapid expansion can be built from Newton's constant, the rate of expansion, Planck's constant and the speed of light in one way only, and that way carries Planck's constant squared; they conclude that the waves are a quantum effect of gravity. In the standard calculation the expanding geometry is a classical background and

only the description of the small waves on it is given quanta, which is the description this book allows, as a description. The answer holds whether or not such waves are found. The technical answer is set out in *The Unquantized Arena*, Chapter 18, "Objections at Full Strength".

The gravitational size of an object of mass m is Newton's constant times m divided by the square of the speed of light; its quantum size is the reduced Planck constant divided by m times the speed of light. The two are equal when the square of the mass equals the reduced Planck constant times the speed of light divided by Newton's constant, which is the Planck mass, about 2.2×10^{-8} kilograms; the corresponding Planck length is about 1.6×10^{-35} meters. The axioms fix the role of this scale and not its size, because they are structural, not dimensional; its size is fixed by the measured value of Newton's constant.

The Incommensuration Theorem is stated with its full enumeration in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20. Bell's theorem (Bell 1964), as a structural result decided by experiment, is set out in Chapter 20, "The Walls Physics Has Already Found".

Chapter 27. Each prediction, with its present evidence and the experiments under way, is set out in technical form in *The Unquantized Arena*, Chapter 19, "Predictions, Evidence and Decisive Experiments".

The entanglement experiment is that of Bose and colleagues (2017) and of Marletto and Vedral (2017). For the design proposed in 2017, two masses of 10^{-14} kilograms, each held in two places 250 micrometers apart, with centers 450 micrometers apart, for 2.5 seconds, the phase that decides the entanglement is Newton's constant times the square of the mass times the time, divided by the reduced Planck constant, times the sum of the inverse separations of the two like pairs of places less twice the inverse separation of the unlike pairs. It comes to 0.314 radians. The prediction separates this account from accounts with one geometry for all alternatives and from collapse driven by gravity (Penrose 1996), and it does not separate it from the quantum treatment of small waves of geometry. The experiment of Page and Geilker (1981) excluded one geometry shaped by the average of the alternatives.

MICROSCOPE (Touboul and others 2022) found the Eötvös ratio of titanium and platinum to be $(-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$, consistent with zero. ALPHA-g (Anderson and others 2023) measured the gravitational acceleration of antihydrogen as $0.75 \pm 0.13 \pm 0.16$ of the ordinary value. The dilaton of string theory and its departures from general relativity are discussed by Damour and Polyakov (1994). The equality of the two masses as a mark of the arena rests on the identification of gravity as context (Chapter 9), and on the inseparability of content and context (*An Immanent Metaphysics*, aphorism 1R3).

The limits from the arrival times of light are those of Vasileiou and colleagues (2013), from the Fermi Large Area Telescope, above 7.6 times the Planck energy for an effect linear in energy, and of the LHAASO Collaboration (Cao and others 2024), from the burst GRB 221009A, above 10 times the Planck energy for the same kind of effect. That the arena has no least step follows from its continuity: the law of continuity applies to all that is of context (*An Immanent Metaphysics*, aphorism 1.83-5).

The limits of reconstruction in the correspondence are set out in *The Unquantized Arena*, Chapter 15, "AdS/CFT as Cross-Modal Correspondence", with the work of Hamilton, Kabat, Lifschytz and Lowe (2006b) and of Almheiri, Dong and Harlow (2015).

The three axioms are given in the wording of *An Immanent Metaphysics*, aphorism 1.3-1; the six intrinsics are those of aphorism 1.42-5; the Incommensuration Theorem, with its enumeration of the ten pairings, is in the Technical Appendix, aphorisms 1R1 to 1R20.

Key Terms

Alternative. One of the outcomes that quantum theory holds together for a system before a finding selects one; in each alternative the system is definite.

Anti-de Sitter space. A space and time curved with the same negative curvature everywhere, with an edge at infinite distance that light can reach and return from in a finite time.

Arena. Space and time together: the setting in which everything that is measured is measured, and the context of physics; its shape is gravity.

Asymmetry. A difference of content where there is a difference of context: nothing is invariant.

Averaged geometry. A single geometry for all the alternatives of the matter together, shaped by the average of their energies weighted by their probabilities; the proposal of Møller and Rosenfeld.

Axiom. A statement that holds of every comparison whatever.

Bell's theorem. The result that no local account of hidden variables can reproduce all the predictions of quantum theory.

Black hole. A region of space and time from which nothing, not even light, can return to the outside.

Black-hole information paradox. The contradiction between Hawking's calculation, which makes the final radiation of an evaporated hole mixed, and quantum theory applied to the whole system, which demands that it be pure.

Born rule. The probability rule: the odds of each outcome are the square of the size of the wave for that outcome; the bridge between quantum theory's two levels.

Boundary. The edge of anti-de Sitter space, where the equivalent quantum theory lives.

Bulk. The interior of anti-de Sitter space, where gravity acts. The bulk theory is the full theory there, gravity included; the bulk's geometric description is its description by smooth classical geometry with quantum fields on it.

Category. A kind of thing, together with the concepts that apply to things of that kind.

Category error. A question that applies a concept where the concept does not belong.

Causation. A relation with a direction between findings, in which the first makes a difference to the second.

Class. A kind, standing ready as a whole.

Collapse model. A modification of Schrödinger's equation that adds a random physical process narrowing the wave, so that large objects lose their alternatives quickly.

Common source. The unity of two descriptions that arise from the same act, apply to the same world, and are each perfected in one of the two characters a comparison allows, joined side by side.

Comparer. Whatever makes a comparison: a person, an instrument, or any physical system that registers one thing against another; the word names a position inside the world, and not a mind.

Comparison. The act of finding whether two things are the same or different, in some respect.

Compound. One of the four characters made by applying sameness or difference to content and to context, with the stance of the comparison held fixed: continuity, discontinuity, symmetry and asymmetry.

Conformal. Of a quantum field theory: looking the same at every magnification.

Conjugate. Of two things: each can be perfected only at the expense of the other, and neither is prior to the other, as symmetry and continuity are.

Content. What the verdict of a comparison is on: what a measurement finds.

Context. The respect in which the two things compared are two: the setting, such as two places or two moments, that makes one finding distinct from another.

Continuity. A sameness of content where there is a sameness of context: nothing is sudden.

Continuously asymmetric. Of a comparison: perfected as continuity with asymmetry; smooth, and different from place to place.

Correspondence. The exact equivalence, proposed by Maldacena, between gravity in anti-de Sitter space and a conformal quantum theory on its boundary; physicists also write it as AdS/CFT.

Cosmic microwave background. The oldest light in the sky, left from the early universe and now reaching us from every direction.

Cross-modal principle. The mathematics of a theory and its findings are always in conjugate characters; the reach of the mathematics into the findings is dense, and never exhaustive.

Curvature. The departure of the arena from flatness at a place: how its shape there differs from the shape of a flat arena.

Decoherence. The fading of interference between the alternatives of a system as its surroundings come to carry records of which alternative it is in; it follows from Schrödinger's equation applied to system and surroundings together.

Determinism. The claim that a complete specification of everything at one moment fixes everything at every other moment.

Difficulty of dynamics. The difficulty predicted for a program that quantizes the arena directly: with no continuous context left, its equation of change resists giving back the world we see.

Difficulty of multiplicity. The difficulty predicted for a program that derives the smooth arena from discrete structure: many arenas fit its starting point, and no principle within it picks out ours.

Difficulty of recovery. The difficulty predicted for a program that gives up one character: the character given up does not come back without a further principle.

Direction. A direction of space or the direction of time: one of the independent respects in which the arena lays out the findings of every description.

Discontinuity. A difference of content where there is a sameness of context: something is sudden.

Discontinuously symmetric. Of a comparison: perfected as symmetry with discontinuity; exactly invariant, and coming in countable steps.

Effective theory. A description that holds within a stated range, such as energies far below a stated limit, or gentle curvature and small waves, and whose corrections can be computed there.

Electronvolt. The energy an electron gains in crossing one volt; physicists measure the energies of particles in billions of electronvolts.

Entanglement. The joining of the alternatives of two systems so that neither has a state of its own; the findings on the two are then correlated more tightly than any exchange of ordinary signals could arrange.

Entanglement entropy. A measure of how strongly a region or a part of a whole is entangled with the rest, computed from the probabilities in its quantum state.

Entropy. A measure of hidden variety: how many inner states a system could be in, given everything found about it from outside, counted by the number of yes-or-no questions needed to single out one.

Equivalence principle. The equality of the mass that gravity pulls on and the mass that resists acceleration, so that every body falls in the same way; in its stronger form, over a small enough region, falling freely in gravity cannot be told apart from floating free of it.

Evolution time. The variable along which the quantum state changes smoothly between findings; the time of possibility.

Exist. To stand as structure: timeless, the same in every instance, able to be held in view as a whole.

External. Of a time: supplied from outside a description as a setting, and not described by it.

Field. A quantity spread through space and time, with a value at every place and moment.

Finding. What one act of comparison yields: one outcome selected from among its possibilities, by which something possible becomes manifest for the comparer; it never requires a mind.

Firewall. A wall of energy at the horizon that would burn whatever falls in, proposed by Almheiri, Marolf, Polchinski and Sully; this account keeps the horizon smooth.

Fixed point. A scale beyond which the strengths of a quantum field theory's interactions stop changing, so that the theory stays finite at the shortest distances.

Focus. What a comparison, or an argument about comparison, is taken to be about; which concept takes which of the three roles depends on it.

Forbidden pairings. Continuity with symmetry, and asymmetry with discontinuity: the two pairings the Incommensuration Theorem excludes.

Formalism. The physicists' word for the mathematics of a theory, its first level.

Four marks of the arena. It couples to everything; nothing shields it; the charge by which it holds a body and the body's inertia are one quantity; and it has no charge of its own that comes in steps.

Gamma-ray burst. A brief, extremely bright flash of high-energy light from a distant galaxy.

Geometric time. Time as a direction within the arena, the time of paths, intervals and light cones; as found, continuous and asymmetric.

Geometry. The shape of the arena of space and time in a region.

Gravitational size. The length that measures how strongly a mass curves the geometry around it; squeezed inside about twice this length, the mass would be a black hole. It grows with the mass.

Gravitational wave. A ripple in the geometry of space and time, which stretches distances in one direction and squeezes them in the other as it passes.

Graviton. At most a quantum of a description: in the description of small waves of geometry by quanta, one step of energy in one way a small wave can ripple, with no mass and twice the photon's spin. Never a quantum of the arena, a step in the extent of space and time, or a particle that carries gravity; gravitation is not quantized. The phonon gives the word its nearest sense, and the arena is not a held structure.

Hawking radiation. The faint thermal glow of a black hole, found by laying quantum fields on its classical geometry.

Heisenberg cut. The line drawn between the system described by the wave and the instrument that records the finding; it can be moved without changing any prediction, and I identify it as the boundary between the two levels of quantum theory.

Held structure. A structure whose parts are held in place by bonds, as the bonds of a crystal hold its atoms in a lattice; its vibrations are described by phonons. The arena is not a held structure.

Horizon. The boundary of a black hole: the surface beyond which every path into the future leads inward.

Identification. A stated step from the structure of comparison to a physical quantity, made in the first person, with the observation that would test it.

Immanent. The role of the act as it happens.

Immirzi parameter. A free number of loop quantum gravity that sets the size of its steps of area.

Incommensurable. Having no common measure.

Incommensuration Theorem. Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity; any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric.

Inflation. A proposed epoch, very early in the history of the universe, of extremely rapid expansion.

Inseparability principle. Where there is content, there must also be context, and where there is context, there must also be content.

Instance. One particular case of a kind, here and now.

Intercompositional. Of contexts: composing with one another into one.

Interexclusionary. Of contents: excluding one another from the same place.

Interferometer. An apparatus that splits a beam into paths and brings the paths back together, so that interference between the paths can be seen.

Interpretation. Of quantum theory: an account of what the theory describes, added to the mathematics and the probability rule that every interpretation shares.

Intrinsics. The six parts of every comparison: sameness, difference, content, context, the subjective and the objective.

Island formula. A computation of the Page curve by a sum over geometries, which after the turning point of the curve counts a region inside the hole as belonging with the radiation.

Isomorphic. Having the same form, so that each can be read as the other.

Kochen–Specker theorem. The result that the measurable quantities of quantum theory cannot all have values fixed in advance, whatever else is measured alongside them.

Law of physics. A statement of what a measurement keeps finding the same when its circumstances change: a sameness of content across a difference of context.

Light cone. The set of places and times that light from an event can reach.

Linear. Of an equation: carrying the sum of two starting states to the sum of the two results.

Linearized quantum gravity. The description of the small waves of gravity on a smooth background by quanta.

Local causality. Bell's condition that each single result is produced by conditions local to its station, even when those conditions give only odds; the stronger form of Bell's theorem shows that no account meeting the condition reproduces the predictions of quantum theory.

Local hidden variables. Conditions present at the place where a result is found which, with the setting chosen there, fix the result in advance; Bell's theorem excludes them.

Lorentz invariance. The rule of relativity that the laws of physics are the same for every steadily moving instrument, and that the speed of light is the same for all of them.

Many worlds. The interpretation in which the wave always changes by Schrödinger's equation and every outcome occurs in its own branch.

Measurement problem. The question of how quantum theory's smooth description, which carries every possible outcome with its weight, gives the single definite outcome that every run of an experiment shows.

Merger. One description in which the directions of space and time themselves carry the character of content at the foundation while keeping the arena's own character; the theorem forbids it. Quantum electrodynamics, a field of content laid on the arena, is no merger.

Mixed state. A quantum state less fully specified than quantum theory allows, as the state of one part of a larger whole usually is.

Modalities. The three roles in any comparison: the act as it happens; what is laid out as structure; and the stance and range of what is possible.

Newton's constant. The number that sets the strength of gravity; it is measured, and its value fixes the size of the Planck scale.

Objective. The side about which a comparison is made.

Observer. Anything that makes a comparison, as a comparer does: a person, an instrument, or an atom that absorbs a particle of light. The word never means a mind.

Omniscient. The role of what is laid out as structure, able to be held in view as a whole.

Page curve. The rise and fall of the entanglement entropy of a black hole's radiation, following the smaller of the two capacities, on the condition that the whole stays pure and that the hole's capacity is its entropy.

Page time. The moment at which the Page curve turns.

Perfected. Holding exactly, at the foundation of a description.

Phase. A turning quantity that each alternative carries, like the hand of a clock, turning at a rate set by the energy of that alternative.

Phonon. One step of energy in one way a crystal lattice can vibrate; a quantum of the description of the vibration of a held structure. The nearest sense in which the word "graviton" can be made intelligible, and even that sense does not make a graviton a quantum of the arena.

Pilot wave. The interpretation of de Broglie and Bohm in which every particle has a definite position at every moment and the wave guides the paths.

Planck curvature. The curvature at which the quantum corrections to Einstein's equations become as large as the effects they correct; the smooth description holds below it.

Planck energy. The energy held in the Planck mass, about two billion joules.

Planck length. The length that the gravitational size and the quantum size reach at the Planck mass, about 1.6×10^{-35} meters; the bound of every comparison made from a place inside the world, and not a least length, since the arena has none.

Planck mass. The mass at which the gravitational size and the quantum size are equal, about twenty-two millionths of a gram.

Planck scale. The meeting of the two descriptions, where an object's gravitational size equals its quantum size; there neither description dominates, and both are relevant.

Planck's constant. The constant that sets the size of every quantum step: the energy of one packet of light is Planck's constant multiplied by the light's frequency.

Pointer states. The alternatives of a system that survive repeated copying of information into its surroundings, and so appear as the sharp readings of a detector.

Primitive. Of a rule: laid down in a description without being derived from anything else in it.

Problem of time. The disappearance, in the canonical quantization of gravity, of the time in which a quantum state evolves: the equation for a quantum state of the geometry, the Wheeler–DeWitt equation, contains no time.

Pure state. A quantum state as fully specified as quantum theory allows.

Pusey–Barrett–Rudolph theorem. The result that, if independent preparations give independent underlying conditions, the quantum state cannot be only knowledge about an underlying condition of the system.

QBism. The interpretation in which a finding is an agent's experience and the wave is that agent's degrees of belief.

Quantize. To change the character of both levels of a theory together: its findings from continuously asymmetric to discontinuously symmetric, and its mathematics from symmetric to continuous, so that what it finds comes in exact, countable steps.

Quantum. A definite packet in which a quantity is found, such as the energy of light of a given color.

Quantum Darwinism. Zurek's account of how the surroundings of a system hold many copies of the same record, so that many comparers can consult it and find the same thing.

Quantum gravity. The program of a single theory in which the arena itself is quantized.

Quantum size. The length within which quantum theory will not let a mass be pinned down without spending enough energy to make new particles of its kind. It shrinks as the mass grows.

Quantum state. Quantum theory's description of a system, the wave of the theory, which describes a range of possible findings and gives each its odds.

Range of findings. The outcomes a theory says a measurement can have; the character of a theory's findings is the character of this range, and each finding is one member of it.

Real. Given in an actual act of comparison: here, now, with the future still open.

Relational quantum mechanics. Rovelli's interpretation, in which any physical system can be the comparer and a finding is a fact relative to the system that makes it.

Renormalizable. Of a quantum field theory: having infinities that can be absorbed into finitely many measured quantities.

Research program. A long effort, shared by many people, that starts from one assumption and uses one family of methods to reach one goal.

Residual. What a description leaves over where it meets the limit of its own character: the point it approaches as closely as it can and does not contain. For quantum theory it is the finding; for general relativity, the singularity; for each side of a correspondence, what only the other side describes.

Root Tautology. The statement of *An Immanent Metaphysics* that comparison is isomorphic with interaction and relation: one act, named three ways.

Sameness and difference. The two verdicts a comparison can give.

Scattering amplitude. The number from which the probability of each outcome of a collision is computed.

Shared assumption. The assumption, common to the programs of quantum gravity, that the arena itself is to be quantized as the other forces were.

Shield. An arrangement that makes a region from which a phenomenon is absent, as a metal box keeps out electric fields.

Single-time result. The result that no interpretation of quantum mechanics that operates within a single unified concept of time can be complete.

Singularity. Where the equations of general relativity, followed far enough, give a curvature that grows without limit along a path that cannot be continued; there the smooth description reaches the edge of its domain.

Singularity theorem. A result of general relativity showing that some paths of falling bodies or light cannot be continued within the smooth geometry beyond a finite time.

Six measures. Pattern, force and probability, the contents, and space, time and possibility, their contexts, in a grid of two rows and three roles.

Spin. The built-in rotation that a particle carries; in an electron, or in a flaw in a crystal, it makes a tiny magnet, which can point up, point down, or be held in both alternatives together.

Standard Model. The quantum theory of the electromagnetic, weak and strong forces and the particles they act on.

String vacuum. A lowest-energy state of string theory, with its own particles and forces.

Subjective. The side from which a comparison is made.

Superposition. A combination of alternatives that quantum theory holds together, with a definite relation between them, until a finding selects one.

Surface of simultaneity. A slice of the arena on which every finding would be settled for every comparer together; no comparison registers one.

Swampland program. String theory's search for the conditions that any low-energy world must meet if it can come from a consistent quantum theory of gravity.

Symmetry. A sameness of content where there is a difference of context: something is invariant.

Time of findings. The order of before and after that findings make; discrete and symmetric, with the one-way direction of the order of the three roles.

Transcendent. The role of the stance and of the range of what is possible.

Triple. Three concepts in the three roles, for which there is one identified focus in which all three axioms hold.

Truncation. A calculation that keeps a finite number of a theory's terms.

Twistor space. The complex space of twistor theory, in which the paths of light are the basic elements and the events of space and time are derived objects.

Two laws. The law of symmetry applies to all that is of content; the law of continuity applies to all that is of context.

Two levels of a theory. Its mathematics, which exists, and its findings, which are real.

Ultraviolet catastrophe. The prediction of classical physics, around 1900, that radiation in a hot oven holds infinite energy at the shortest wavelengths; Planck's law of 1900, which rests on energy exchanged in steps, has no such excess.

Unified time. One time in which geometric time, the time of findings and evolution time are all perfected together in one description; it would need both forbidden pairings.

Unitarity. The rule that the evolution of a complete quantum system keeps every distinction among its states.

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