

The Last Turtle

Essays from Comparison and the Incommensuration Theorem to Causation, the Arena and the Limits of Computation

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UVSM PUBLICATIONS
Wytheville, Virginia

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Published by UVSM Publications LLC
965 Reed Creek Mill Rd
Wytheville, VA 24382-8287
USA

uvsmpublications.com
contact@uvsmpublications.com

ISBN 979-8-90710-215-6

DOI: https://doi.org/10.67291/ONZV8400

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What This Book Holds

I. What the Book Establishes

I go from the act of comparison to the physical world. I begin where every finding begins, with comparison. Comparison is the act of finding whether two things are the same or different, in some respect. Every comparison has six intrinsics, the features that are necessary and sufficient to characterize any comparison, in three pairs: sameness and difference, content and context, and the subjective and the objective. The content of a comparison is what the verdict is on. The context is the respect in which the two things compared are two.

I define four compounds of comparison from four of the intrinsics: sameness, difference, content and context. Symmetry is a sameness of content where there is a difference of context, as when a shape flipped over looks like the same shape: the flip is the change of context, and the shape is the content that stays the same. The other three compounds are defined in the same way: asymmetry is a difference of content where there is a difference of context, continuity is a sameness of content where there is a sameness of context, and discontinuity is a difference of content where there is a sameness of context. I then prove the Incommensuration Theorem. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The word "fundamentally" in the theorem means exactly, at the foundation of a description, without exception. Two pairings of the compounds survive the theorem: any comparison, taken exactly at its foundation, is either con-

tinuously asymmetric or discontinuously symmetric. The Incommensuration Theorem is a structural fact about the nature of knowing, and every later chapter of this book uses the theorem.

I also use three axioms. The three axioms relate three roles, which the book calls the modalities. The immanent is the role of the act itself, as the act happens, here and now. The omniscient is the role of what is laid out as structure and can be held in view as a whole, such as a map. The transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance. The three axioms state how the three modalities are related to one another: Axiom I states that the immanent is more fundamental than the omniscient and/or the transcendent, Axiom II states an order among the three modalities in which a class of one modality precedes an instance of the next, and Axiom III states that the three modalities are distinct, inseparable and non-interchangeable.

I use the Incommensuration Theorem and the three axioms to establish what being real is, what existing is, and what being objective is. To be real is to be given in an actual act of comparison, here and now, which is exactly one interaction. To exist is to have a place in a pattern, as structure. Existing involves no interaction. To be objective is to hold across many interactions. What is objective is what many comparers would find. Causation is a regularity in time, the same relation of earlier conditions to later conditions found again and again from case to case, and not determinism, the claim that one law fixes every state of a whole world in advance. Choice is real, and hard randomness, an outcome that no prior specification fixes, however complete, is real.

Hard randomness, the arrow of time and the subjective stance form a bound triple. The arrow of time is the asymmetry between a determined past and an open future. The subjective stance is the stance from which every comparison is made. The three members of the bound triple are distinct, inseparable and non-interchangeable. Consciousness is the bound triple taken as one act.

Next, the book sets out the limits that every physics shares. The three boundaries of knowing are the horizon, the infinitesimal and the subconscious. The horizon is the boundary of knowing at the largest scale, marked by the speed of light, while the infinitesimal is the boundary of knowing at the smallest scale, marked by the Planck constant. The Planck constant measures how much two quantities, such as position and momentum, cannot be known together. The subconscious is the boundary of knowing at the observer's own scale, the limit of knowing one's own self, where the comparing happens. The Planck length is a very small length fixed by the Planck constant, the speed of light and Newton's constant of gravitation. The Planck length is a bound on what can be found from inside the world, and not a smallest piece of space. A law of physics is a statement of access: which influences can reach which others, under which conditions, at what cost, and in what direction. The theory of relativity, for example, states a limit of signaling: no influence travels faster than light.

Gravitation is the context of physics and not a content within physics, so that gravitation is not quantized. Gravitation is the shape of the arena, and the arena is space and time taken as the context of every physical comparison. Quantizing is a treatment that belongs to the content of physics, not to the context of physics: to quantize a thing is to treat the thing as made of smallest units, the way light is made of photons. Quantum mechanics is the physics of the very small, and general relativity is the theory of gravitation as the shape of space and time. I set out why the two theories do not merge into one, and how the two are already joined, in the act of measurement.

Time has a direction before entropy gives time one: in systems of many parts, the statistical growth of entropy gives time a direction, called the entropic arrow, and the arrow of time is beneath the entropic arrow. The physicist John Bell proved the theorem that carries his name. Bell's theorem shows that nature contains correlations over long distances that no

causal process can reproduce. The correlations of Bell's theorem are real and carry no signal. The outcome of a measurement is definite relative to the interaction that makes the outcome.

The last chapters of the book establish three claims about computation. First, the world is not a computation. Second, every theory laid out wholly as structure, the way a computer program is laid out, leaves a residue of information that the theory cannot compress into any shorter description. The residue is hard randomness. Third, a mind cannot be uploaded (captured in a computational representation).

II. The Seven Parts

Chapters 1 to 4 make up Part One, "Comparison and the Theorem". Chapter 1 says what metaphysics is: the inquiry into the relationship between self and reality, in which three questions are answered together: what is, how we know, and what is the basis of care. Chapter 2 defines the four compounds of comparison (symmetry, asymmetry, continuity and discontinuity) and proves the Incommensuration Theorem, setting out the proof in two forms. Chapter 3 sets out the three modalities (the immanent, the omniscient and the transcendent) as roles, and not as things. Chapter 3 also states the three axioms that relate the three modalities. A domain is the whole of what can be compared by one kind of comparison. At the foundation of every domain there are three concepts, called a triple. Chapter 4 shows how the triple of concepts at the foundation of a domain is found, and how the triple is tested.

Chapters 5 to 11 make up Part Two, "What Is, and How It Is Known". Chapter 5 defines the universe as creation, existence and interaction taken together, neither more nor less. Chapter 6 works an exercise in an imagined empty universe, in which the order that Axiom II sets among the three modalities is found. Chapter 7 distinguishes the real, the existent and the objective by the number of interactions each involves (one, none and many). Chapter 8 names omniscient bias, the error of taking what is

laid out as structure for the whole of what is, and shows why theory, which is held in the third person, must resolve to practice, which is held in the first person.

Chapter 9 sets out the three orders of knowledge: causal knowing, the knowledge given by experiment; analytic knowing, the knowledge given by proof; and metaphysical knowing, the knowledge of what cannot fail to be the case in any act of comparison. Chapter 9 also sets out the order of depth among the disciplines, in which metaphysics, physics, computation and mathematics come in that order and each discipline needs the discipline before it, and the boundaries of the scientific method. Chapter 10 sets out the four errors of extrema, four impossible positions at the boundary of the self: prejudice, reaction, realism and idealism. Perception crosses the boundary of the self from the world to the self, and expression crosses the boundary from the self to the world. In each error of extrema, perception or expression is taken as absolutely dependent on the other, or as absolutely independent of the other. Chapter 11 argues from perception against solipsism (the position that only one's own experience can be known) and against physicalism (the position that the physical world alone is fundamentally real).

Chapters 12 to 16 make up Part Three, "Causation, Choice and the Bound Triple". Chapter 12 separates causation from determinism. Chapter 13 sets out choice, change and causation as the triple of the real, the three concepts at the foundation of what is real. Chapter 14 replies to the argument that free will is incoherent. Chapter 15 sets out the bound triple, and names consciousness as the bound triple taken as one act. Chapter 16 establishes hard randomness, from experiment and from the Incommensuration Theorem, as what comes through the microscopic boundary of the domain of measurement, which is the limit at the smallest scale, below which no definite state can be found.

Chapters 17 to 20 make up Part Four, "The Limits of Knowing and the Measures of Physics". Chapter 17 sets out the three boundaries of knowing: the horizon, the infinitesimal and the subconscious. Chapter 18 separates three things named after the physicist Max Planck: the Planck con-

stant, which is a number; the Planck limit, which is that number read as the microscopic boundary of the domain of measurement; and the Planck length. Chapter 19 reads the laws of physics as statements of access. Chapter 20 sets out the six measures of every physical measurement: pattern, space, force, time, probability and possibility. Every physical quantity is a species of one of the six measures. Pattern, force and probability are the three measures of content, and space, time and possibility are the three measures of context. So there are nine pairings of a measure of content with a measure of context, and Chapter 20 calls the nine pairings the Nine Laws.

Chapters 21 to 27 make up Part Five, "The Arena and What It Holds". Chapter 21 places gravitation as the context of physics, and sets out three kinds of acceleration, told apart by whether neighboring free paths run together, keep their separation, or run apart. Chapter 22 shows how quantum mechanics and general relativity are related within one arena. Chapter 23 reads conservation as symmetry stated as a quantity, as in the conservation of energy, which says that the total energy (the content) is the same from one moment to the next (the context). Chapter 23 also sets out the bound of conservation. Chapter 24 makes two arguments for the arrow of time beneath the entropic arrow. Chapter 25 is about the least complete interaction, the least that an interaction needs so that something has actually happened: two atoms, and the photon that passes between the two atoms. Chapter 26 defines the classical as what can be modeled, and the quantum as what cannot be copied. Chapter 27 shows why a short list of the kinds of particle does not make matter simple.

Chapters 28 to 30 make up Part Six, "Correlation and Measurement". Chapter 28 derives Bell's theorem step by step, from the beginning. Chapter 29 reads Bell's theorem as a trilemma about causation: three properties of causation, of which at most two can hold together. The measurement problem is the question of where the single, definite outcome of a quantum measurement comes from. Schrödinger's cat is a thought experiment of the physicist Erwin Schrödinger, in which a cat

sealed in a box lives or dies according to whether a counter detects one radioactive emission. Chapter 30 uses Schrödinger's cat to answer the measurement problem.

Chapters 31 to 33 make up Part Seven, "Computation and the Real". Chapter 31 shows that the world is not a simulation, that is, not a computation run in a more fundamental world. Chapter 32 sets out what any theory built from a rule and starting conditions can achieve, and the residue that the theory leaves. Chapter 33 shows that the world is not a computation and that a mind cannot be uploaded.

The seven Parts follow one order. I start from the act of comparison and the Incommensuration Theorem, go on to what is and how it is known, then to causation and choice, then to the limits of knowing and the measures of physics, then to the arena and its content, then to correlation and measurement, and last to computation. Each chapter defines each of its terms where the term first appears in that chapter, so that each chapter can also be read by itself. The Glossary lists every term that the chapters define.

III. What the Book Rests On

I build this book on two other books. The first book is *An Immanent Metaphysics* (first published 2001). *An Immanent Metaphysics* states the three axioms and the three modalities (the immanent, the omniscient and the transcendent). *An Immanent Metaphysics* is written in numbered aphorisms, short statements each with its own number, and I cite *An Immanent Metaphysics* by aphorism number. The chapters of this book quote the three axioms in the words of *An Immanent Metaphysics* (aphorism 1.3-1). The chapters also quote the three coequal properties of the members of a foundational triple, that the members are distinct, inseparable and non-interchangeable (aphorism 1.45-1), and the account of the self that *An Immanent Metaphysics* gives (aphorism 1.67-2).

The second book is *Incommensuration; The Theorem and the Three Axioms* (2026). This book and *Incommensuration; The Theorem and the Three Axioms* both belong to the physics collection, a set of books on physics. *Incommensuration; The Theorem and the Three Axioms* proves the Incommensuration Theorem from the structure of comparison (the six intrinsics and the four compounds), and derives the three axioms from the theorem. Chapter 17 quotes the second book on the shape of the limit that the Incommensuration Theorem sets on knowing: the limit is the curvature of the whole field of knowing, and not a wall at the edge of the field.

I use the results of physics as physicists establish the results, by the methods of physics and on the evidence of physics, and then I read each result through the structure of comparison that Chapter 2 sets out. The Bibliography lists the works of physics, mathematics, philosophy and computing that the chapters use.

IV. How the Chapters Refer

When a reference names a section and no chapter, as in "Section IV", the section is in the chapter where the reference appears. When a reference names a chapter by number, as in "Chapter 29", the chapter is a chapter of this book. When a reference names a book by title, the title names a book of the physics collection, or a work listed in the Bibliography.

V. Summary

I go from the act of comparison and the Incommensuration Theorem to the physical world, in seven Parts. The seven Parts are: comparison and the theorem; what is, and how it is known; causation, choice and the bound triple; the limits of knowing and the measures of physics; the arena and what it holds; correlation and measurement; and computation and the real. I build this book on *An Immanent Metaphysics* and on *Incommensuration; The Theorem and the Three Axioms*.

PART ONE

COMPARISON AND THE THEOREM

The act of comparison, the theorem proved from comparison, the three modalities and axioms,
and how a triple of concepts is found and tested.



Chapter 1

What Metaphysics Is

Self, Reality, and the Relationship Between Them

ABSTRACT

Metaphysics is the inquiry into the relationship between self and reality: between the subjective and the objective, the knower and the known. Metaphysics answers three questions together: what is, how do we know, and why does anything matter. Metaphysics asks what-questions, where physics asks why-questions and technology asks how-questions, and so metaphysics is the descriptive language that supplies the concepts physics uses. The starting point of metaphysics is the relationship itself, taken as more fundamental than either of the two things related, and recognized as a third thing in its own right. Metaphysics then describes reality in triples of concepts, starting from the relationship itself: the universe as creation, existence and interaction, and the real as choice, change and causality. This chapter states what metaphysics is in terms of the relationship between self and reality, how metaphysics differs from physics, why metaphysics is neither physicalism nor idealism, and why the long neglect of metaphysics left questions such as meaningfulness unaddressed.

I. A Positive Definition

Metaphysics is the inquiry into the relationship between self and reality. The relationship between self and reality is the subject of the study, and the relationship can be named in several ways, each name picking out the same relationship from a different side: the relationship between the subjective and the objective, between the knower and the known, between perception and what is perceived, and between consciousness and the world in which consciousness finds itself.

Metaphysics is designed to say what the relationship between the self and reality is, and not to analyze the self alone or reality alone. The field of metaphysics is the plane of perception: metaphysics uses the notion of self and the notion of world as the method by which we can understand what perception is.

A second definition, a companion to the first, follows from the first. Metaphysics is also the study of the relationships between domains, where a *domain* is a realm of conception complete unto itself, such as the physical world, a body of mathematics, or a dream. Physics, defined in terms of domains as well, is the study of the interactions within a given domain.

The two definitions of metaphysics in this section are positive, because each says what metaphysics is: metaphysics is the inquiry into the relationship between self and reality, and metaphysics is the study of the relationships between domains. The conventional definition is "metaphysics is what is not physics", and the conventional definition says nothing about what metaphysics is. Consider any sentence of the form "X is what is not Y": the sentence says something about Y and nothing about X, because the word "not" has no content of its own. Defined as what is not physics, metaphysics becomes a bin for whatever physics cannot explain, and a bin

of leftovers has no positive content. The two definitions given in this section give metaphysics the positive content that the conventional definition leaves out.

II. Three Questions Answered Together

In conventional academic terms, metaphysics has been the conjunction of two studies. *Ontology* is the study of what is, and *epistemology* is the study of how we know what is. Both studies are needed, because anyone who says something about the ultimate nature of reality must also say how they know that what they say is true. Both questions concern what perception is, what reasoning rests on, and what grounds our confidence that we know anything at all.

A third question belongs with the first two questions, what is and how we know. *Axiology* is the study of value and significance: why we should care, and what we care about. Metaphysics, as this book understands metaphysics, is an answer to three questions: What is? How do we know? What is the basis of care?

More exactly, the second question is about the basis of knowing: the basis of associating any understanding or any relevance with a perception. Whole disciplines are given to each of the three questions taken singly. Metaphysics answers the three questions together, and all three answers are needed.

An image of a communication channel shows how the first two questions, what is and how we know, differ from each other. Picture the relationship between subject and object as a channel through which information flows. Ontology is concerned with the being of the channel: the fact that the channel is there at all. Epistemology is concerned with the signal content that flows through the channel. Axiology has no part in the channel image, because the question of axiology is the third question: whether

something matters at all. The channel is a model only: perception has no medium of its own, just as light needs no aether to travel (the aether was a medium that physicists once supposed light must travel through).

III. What-Questions, Why-Questions, How-Questions

The difference between metaphysics and physics shows up first in the kinds of questions that each of the two disciplines asks.

Science is concerned with what is both observable and repeatable. Observable means that there is some direct or indirect way of interacting with the thing, and without a way of interacting with the thing no experiment on the thing can be done. Repeatable means that the phenomenon is regular and has a causal nature, so that under given conditions we can predict the outcomes that follow. Both conditions concern the thing observed, so science focuses on what happens in the objective. In particular, physics studies what exists and the interactions within the physical domain. Physics needs to know that something is observable. Physics is not concerned with what the observer is, and is only a little concerned about how observation happens. Metaphysics is concerned with what happens in the relationship between the subjective and the objective.

In the work of explanation, physics answers why-questions. Why does a released stone fall? Physics answers by placing the fall of the stone under a law, and the law that covers the fall of a stone is gravitation, which is a model: a way of thinking about things. A law is itself a "what" about the world, stated at a higher level of generality. Technology, the predictive use of physics, answers how-questions: how to bring about a given outcome. Metaphysics answers what-questions of another kind: what the relationship between self and world is, rather than what is in the world.

Because metaphysics answers what-questions, the answer metaphysics gives is a description, and not an explanation. A metaphysics is a system of metaphors and definitions, and not a system of models. The work of metaphysics is to characterize the concepts involved, as succinctly and as

accurately as possible. The aim of metaphysics is a coherent description of what is, and not an account of why things are the way they are. You have to have a what before you can ask a why.

The aims of physics and of metaphysics differ as their questions do. The aim of physics is prediction and quantification: the goal of physics is an equation that can be checked by experiment, a statement of the form "given these initial conditions, the system will evolve to that final state". The power of physics lies in precise, testable prediction. The aim of metaphysics is structural understanding, and the question of metaphysics is what must be true about reality for physics to work at all: what the necessary conditions are for there to be measurable quantities, causal relationships, and conservation laws.

The two aims, prediction and structural understanding, are complementary. Every physical theory contains structural commitments about causality, about measurement, and about space and time. The commitments are there whether or not the theory states the commitments. The work of metaphysics is to make the structural commitments of physical theories explicit, and to examine the foundations of those commitments.

IV. Why Metaphysics Is Deeper Than Physics

Physicists have to assume time, space, matter and energy and various relationships among the four, but for the most part physics does not define what any one of the four is. Physics uses the concept of space, and a theory of physics may posit a sameness between space and time, but physics does not say where the concepts of time, space, matter and energy come from. Metaphysics is the descriptive language that supplies the concepts physics uses. In the sense of supplying the concepts physics uses, metaphysics is the basis of physics and is deeper than physics.

V. Neither Physicalism nor Idealism

Philosophers have usually treated the relationship between self and world as a duality of subject and object. The philosopher René Descartes spoke of the duality as the interior and the exterior: the spiritual and the physical. Since Descartes, two broad schools of thought have each taken one side of the duality of subject and object.

Physicalism, or physical monism, is the view that there is the physical, the objective, and that consciousness and the self are epiphenomena of physical reality: by-products with no standing of their own. *Idealism* is the view that mind comes before body. An example of idealism is the teaching of emanation in the Kabbalah, the mystical tradition of Judaism. In the teaching of emanation, God divided itself into many parts through a process of illumination, and the parts interacted with one another so as to experience existence. The priority of mind over body is central to most transcendent perspectives, where "transcendent" has its everyday sense: a transcendent perspective is a view in which the source of the world lies beyond the world. Idealism has been in decline since the early twentieth century.

This book is built on the Immanent Metaphysics, and the Immanent Metaphysics is neither physicalism nor idealism. The ground of the Immanent Metaphysics is the middle, the relationship between the subjective and the objective, and not either side alone. The Immanent Metaphysics shows that the foundation of how we think about what is, how we know, and what matters can be thought of only as relational: relationship and process are fundamental to understanding anything at all, to what being is, and to what we mean by ultimate values.

VI. The Relationship as the Starting Point

In the ordinary model of reality, a space comes first, as a container. Next, there is stuff in the space, and last, the stuff interacts. The ordinary model uses three concepts in succession: space, matter, interaction.

In the model developed in this book, the primary thing is the relationship between the subjective and the objective, between context and content. In the container picture, the space is the context, and the stuff in the space is the content. We know there is a space only because there is stuff in the space. We know there is stuff only because the stuff is somewhere. The relationship between space and stuff is more basic than either the space or the stuff. So we take the relationship between context and content as more fundamental than the context or the content, and we define the context and the content each in terms of the relationship between them.

The ground of Axiom I is the priority of the relationship over the context and the content. Axiom I is the first of the three axioms of the Immanent Metaphysics, and Chapter 3 sets out all three axioms in full. Axiom I is stated in terms of three roles. The *immanent* is the role of the act of relating itself, as it happens, here and now. The *omniscient* is the role of what is laid out as structure and can be held in view as a whole. The *transcendent* is the role of the stance and the range of possibility from which an act proceeds. Where each of two things can be perfected only at the expense of the other, and neither of the two is prior to the other in definitional basis, the two things are conjugate. Axiom I reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another".

A second observation follows at once. Any relationship involves at least three ontological categories, three kinds of thing that are: the two things related, and the relationship itself. A relationship is identical to neither of the two things it relates, and is there at all only when the relationship itself has being. So every relationship intrinsically has three as-

pects: the two things related and the relationship between them. The three aspects of every relationship are the ground of Axiom III. Axiom III reads: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable". The statement of Axiom III holds of the classes and of the instances alike. Distinct means that no one of the three roles overlaps another, inseparable means that no one of the three occurs without the others, and non-interchangeable means that no one of the three can take another's role.

The three aspects of every relationship have a consequence: duality, as a philosophical construct, cannot be only dual. A dualist posits two independent things, but must still say how the two things are connected, and to say how the two things are connected, the dualist must admit the relationship between the two things as a third thing.

VII. Reality Described in Triples

A fundamental observation of the Immanent Metaphysics is that the nature of reality, of beingness, can be described in terms of triples of concepts. A triple of concepts is three concepts that together are the foundation of a domain, and a domain is a realm of conception complete unto itself.

For the universe, the three concepts are creation, existence and interaction. If I understood creation, existence and interaction, then by their superposition (holding all three together) I would understand the foundation of the universe. An understanding of the foundation of the universe would give the basis from which any fact in the universe can be understood, and the understanding would not give every fact in the universe.

For the real, the three concepts are choice, change and causality. Choice, change and causality are one of the most important triples, because everything we deal with day to day, as human beings and as living things, involves all three concepts. The real is that which at once partakes of choice, change and causality, neither more nor less. Choice, change and

causality are distinct, inseparable and non-interchangeable. Together the three concepts are the essence of what it is for something to be real. In the sense of "real" just given, whatever does not partake of all three concepts is not real, and is an illusion.

I have made two statements: one about the universe, and one about the real. The two statements are strong, and the two statements are not guesses: the two statements are grounded in what knowing itself is, and any concept of being must be consistent with the two statements. The arguments that establish the two statements cover the whole field of possibility and leave no possible case outside the arguments. The argument for the universe is in Chapter 5, and the argument for the real is in Chapter 13. So the two statements hold of this universe, and of any concept of any universe at all.

VIII. The Good, the True and the Beautiful

A triune pattern is three concepts that belong together as one. The same triune pattern appears where people ordinarily speak of what they want from life. A person may say: "I want a good life. I desire beauty. And I want it to be true, because if it is not true, I am not living in integrity". Good, true and beautiful are a triple, and the perspective of this book is triune, not dual.

Truth goes with the omniscient: logic, mathematics, the way things actually are. Beauty goes with the immanent: the finite, the here and now, the relational in the personal and intimate sense. Goodness goes with the transcendent: values, virtue, the goodness that is not fixed in any one instance.

IX. Why Metaphysics Was Set Aside

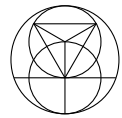
The philosopher Immanuel Kant gave principled reasons to doubt speculative metaphysics. Kant held that we can know only our experience of sensing, and not what actually is. Kant doubted a different idea of metaphysics from the one this chapter describes. Yet Kant's argument persuaded many philosophers after him that metaphysics was not worth studying. In the twentieth century the logical positivists, a school of philosophers, added their own charge: that metaphysical claims are meaningless. For most of the two centuries and more since Kant, metaphysics has not been widely regarded among philosophers as a rigorous discipline.

Metaphysics was set aside at a cost: a set of questions has been left unaddressed, because the answers to those questions depend on a clear understanding of what metaphysics is. Meaningfulness is one of the questions left unaddressed. When metaphysics is defined positively, as the inquiry into the relationship between self and reality, the questions left unaddressed belong again to a discipline that can answer them.

X. Summary

Metaphysics is the inquiry into the relationship between self and reality, and the study of the relationships between domains, while physics is the study of the interactions within a given domain. Metaphysics answers three questions together: what is (ontology), how do we know (epistemology), and what is the basis of care (axiology). Metaphysics answers what-questions with descriptions, physics answers why-questions with laws, and technology answers how-questions with means. Metaphysics is deeper than physics and is the basis of physics, because physics assumes space, time, matter and energy without defining the four concepts, and metaphysics supplies the four concepts that physics assumes. The Immanent Metaphysics is neither physicalism nor idealism, because the starting

point of the Immanent Metaphysics is the relationship between the subjective and the objective, which is more fundamental than either side (the ground of Axiom I), and which is a third thing in every relationship, beside the two things related (the ground of Axiom III). The Immanent Metaphysics describes reality in triples of concepts, starting from the relationship: the universe as creation, existence and interaction, the real as choice, change and causality, and the good, the true and the beautiful as the transcendent, the omniscient and the immanent. Any concept of being must be consistent with the descriptions of the universe and of the real, and so the two descriptions hold of any concept of any universe at all.



Chapter 2

Comparison, Its Four Compounds, and the Incommensuration Theorem

How Symmetry, Continuity, Asymmetry and Discontinuity Are Defined from the Act of Comparison, and Why Only Two of Their Pairings Can Hold Exactly

ABSTRACT

Comparison is the act of finding whether two things are the same or different in some respect. Every comparison has six intrinsics, the six features that belong to comparison as such. The intrinsics come in three pairs: sameness and difference, content and context, and the subjective and the objective. This chapter defines four compound concepts from sameness, difference, content and context alone. The four compounds are symmetry, asymmetry, continuity and discontinuity. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. I state the definitions and set out the proof of the theorem in both of its forms: in the first form, all but two of the ten pairings of the compounds are rejected by three rules, and in the finite form, the pairings are maps of a two-element set to itself. I also state exactly what "exactly" means, and show that the two surviving pairings are the characters of the objective and of the subjective. Last, I set out the limit that the Incommensuration Theorem sets on every theory, the Immanent Metaphysics included.

I. Comparison and Its Six Intrinsics

Comparison is the act of finding whether two things are the same or different, in some respect. Comparison, interaction and relation are one act, named three ways. Every measurement is a comparison, and every signal received is a comparison too. Whatever is defined in terms of comparison alone applies to every interaction there is, because comparison and interaction are one act.

Every comparison has six *intrinsics*, six features that belong to every comparison, arranged as three pairs. The first pair is sameness and difference, which are the two possible verdicts of a comparison. The second pair is content and context. The *content* is what the verdict of a comparison is on. The *context* is the respect in which the two things compared are two. The content and the context of one comparison are always two different respects, because if the respect in which the two things are told apart were the very thing the verdict is about, there would be no two things to compare. The third pair is the subjective and the objective. The subjective is the side from which the comparison is made, and the objective is the side of what is compared.

The six intrinsics are necessary and sufficient to characterize any comparison. The six intrinsics are found in comparison itself, and are not chosen. Try to describe a comparison that lacks any one of the six intrinsics, and the description stops making sense. No comparison needs a seventh intrinsic. The count of six that comparison has appears again in interaction and in questioning. Interaction has six metrics, or kinds of measure. Pattern, force and probability are the three metrics of content. Space, time and possibility are the three metrics of context. Questions come in six kinds: who and what, when and where, why and how.

Comparison itself plays the role of the immanent, which is the role of the act as the act happens, here and now. To be real is to be in an act of comparison, as the act happens, here and now. Each of the three pairs of

intrinsics plays one of three roles: the immanent, the omniscient and the transcendent. The omniscient is the role of what is laid out as structure and can be held in view as a whole, and the transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance. The role of each pair of intrinsics depends on the focus of the description, which is the frame from which the description is made, and so the focus of a description has to be named. In the focus of comparison as relation, which is the focus of practice and of the first person, the subjective and the objective are immanent, sameness and difference are omniscient, and content and context are transcendent. In the focus of measurement, which is the focus of theory and of the third person, sameness and difference are immanent, content and context are omniscient, and the subjective and the objective are transcendent. Axiom II, one of the three axioms that Chapter 3 states, sets the three roles in a cycle, from the transcendent to the immanent to the omniscient, and back to the transcendent. Each pair of intrinsics moves one step around the cycle of Axiom II when a description passes from one focus to the other, so each focus is a rotation of the other, and both foci keep the direction of the cycle. The second person is the frame that relates the first person and the third person. The Immanent Metaphysics itself is stated in the frame of the second person. The frame of the second person is the transcendent. The second person is not the immanent in either focus.

II. The Four Compounds

In the definitions that follow, the subjective and the objective are held implicit (in the background), so that the definitions concern only what is compared. One act of comparison then produces two verdicts. One verdict is on the context: are the two things compared in the same context, or in different contexts? The other verdict is on the content: is the content of the

two things the same, or different? Two verdicts with two values each have four possible pairs of values. Each pair of values is a compound concept, defined by exactly the two verdicts whose values make the pair.

Continuity is a sameness of content where there is a sameness of context: within one unbroken stretch of context, the content does not jump. *Discontinuity* is a difference of content where there is a sameness of context: the content changes although the context has not. *Symmetry* is a sameness of content where there is a difference of context: the content stays the same although the context has changed. *Asymmetry* is a difference of content where there is a difference of context: the content changes with the context.

Symmetry is a kind of constancy in the face of a change of context. A shape flipped over looks like the same shape: the flip is the change of context, and the shape is the content that stays the same. In physics, symmetry is the basis of the conservation laws, because a conservation law is what a symmetry looks like as a quantity. The conservation of energy says that the total energy (the content) is the same from one moment to the next (the context). Even mathematical equality is a symmetry. The two sides of an equation are the same content, and what differs between the two sides is the context: the symbols, the notation, and the order of operations in which each side is written. Every invariance is a symmetry in the sense just defined, since an invariance is a sameness that holds across a change.

Continuity has to do with connectedness: whether one can move from one place to another through infinitesimally small steps without crossing a hard boundary. Continuity is a proportionality, and not a constancy as symmetry is: an infinitesimal change of context produces only an infinitesimal change of content.

Two points are needed to read the four definitions exactly. First, the four compounds are relations among at least two things compared. No compound is a property of one thing in its own nature, because a thing taken by itself is one, and in one thing alone there is nothing across which

sameness or difference could be found. Second, physics also uses the word "symmetry" for an invariance of a description. One example is the freedom to relabel the coordinates in general relativity, the theory of gravitation as the shape of space and time, where the coordinates are the numbers that name the points of space and time. The freedom to relabel coordinates is a symmetry of the description. Wherever this book means the word "symmetry" in the sense of an invariance of a description, this book says "symmetry of the description", so that a symmetry of the description is never taken for a symmetry of what is found.

The four compounds are now defined purely in terms of the intrinsics of comparison. Nothing physical has been assumed in the four definitions.

III. The Question the Theorem Answers

When two compounds are applied together to one comparison, the two compounds form a conjunction. The Incommensuration Theorem is the answer to one question: which conjunctions of two compound concepts can characterize one comparison that holds exactly at its foundation?

When a comparison holds exactly at the foundation of a description, the comparison is said to hold exactly at its foundation, or to be *perfected*. The question is about what a description can be in its own ground, without exception. The question is not about rough agreement or about what is nearly so.

The four compounds can be paired in ten ways: four self-pairings, each compound with itself, and six pairings of two different compounds. The ten pairings are tested against three rules. *An Immanent Metaphysics* is the book that states the three axioms. *An Immanent Metaphysics* also states the three rules as properties of the relation between the compounds.

The first rule is no self-application. A compound paired with itself adds nothing to what the compound already says, so the four self-pairings are removed.

The second rule is no application of opposites. The pairing of continuity with discontinuity means that the content is both the same and different in the same context. The pairing of symmetry with asymmetry means that the content is both the same and different under one change of context. So both pairings of opposites are removed.

The third rule is the balance of sameness and difference. Sameness and difference are inseparable wherever a verdict is given: sameness never comes without difference, and difference never comes without sameness. Therefore sameness must appear with difference among the verdicts on the content. Sameness must also appear with difference among the verdicts on the context.

IV. The Proof

Each compound is defined by two verdicts, one on the content and one on the context. When two compounds are paired, the content receives one verdict from each compound. The context also receives one verdict from each compound. By the third rule, the content must receive one sameness and one difference, and so must the context.

The pairing of symmetry with discontinuity has, on the content, one sameness (from symmetry) and one difference (from discontinuity), and, on the context, one difference (from symmetry) and one sameness (from discontinuity). Therefore the pairing of symmetry with discontinuity satisfies the third rule in both respects: content and context.

The pairing of asymmetry with continuity has, on the content, one difference (from asymmetry) and one sameness (from continuity), and, on the context, one difference (from asymmetry) and one sameness (from continuity). Therefore the pairing of asymmetry with continuity satisfies the third rule in both respects.

The pairing of continuity with symmetry has two samenesses of content and no difference of content, so the pairing of continuity with symmetry fails the third rule. The pairing of discontinuity with asymmetry

has two differences of content and no sameness of content, so the pairing of discontinuity with asymmetry also fails the third rule. The two pairings are removed by the third rule alone, because neither pairing is a self-pairing or a pairing of opposites. Continuity with symmetry and discontinuity with asymmetry are the two conjunctions that the Incommensuration Theorem forbids, which means that neither conjunction can characterize a comparison that holds exactly at its foundation.

Six pairings are removed by the first two rules (the four self-pairings and the two pairings of opposites), and each of the six also fails the third rule. A self-pairing has two identical verdicts on the content and two identical verdicts on the context, while a pairing of opposites has two identical verdicts in one respect, the context. So when the third rule alone is applied to the content and to the context, the same eight pairings are removed, and the same two pairings remain, as when all three rules are applied. Of the ten pairings, eight are removed and exactly two remain: continuity with asymmetry, and symmetry with discontinuity.

In each of the two remaining pairings, the second compound is the exact complement of the first compound. Wherever one compound of the pair has sameness, in content or in context, the other compound of the pair has difference. So each respect receiving one sameness and one difference means exactly that the two compounds of the pair are exact complements.

V. The Finite Form of the Proof

The result of Section IV, that exactly two of the ten pairings survive, can also be stated as a fact about the smallest possible set, the two-element set {same, different}. Read each compound as an arrow from a verdict on context to a verdict on content. For example, continuity is an arrow from same context to same content. Each verdict is one of two values, same or different. Take a conjunction of two compounds that satisfies the third rule, the balance of sameness and difference. The conjunction has one ar-

row from each of the two possible context verdicts. Each arrow ends at one content verdict, and the two arrows end at different content verdicts. The two arrows of the conjunction are a one-to-one map of the two-element set {same, different} to itself, where a one-to-one map of a set to itself sends each member of the set to one member of the set, and sends no two members to the same member.

The set {same, different} has exactly two one-to-one maps to itself. The first map is the identity, which sends same context to same content and different context to different content, and so the identity is continuity with asymmetry: *continuous asymmetry*. The second map is the exchange, which sends same context to different content and different context to same content, and so the exchange is discontinuity with symmetry: *discontinuous symmetry*. The two conjunctions forbidden by the Incommensuration Theorem are the two maps that are not one-to-one: continuity with symmetry sends both context verdicts to "same", and discontinuity with asymmetry sends both context verdicts to "different".

The proof, in either form, uses only the meanings of sameness, difference, content and context, and the proof uses no postulate (no assumption taken without proof) and no fact about the physical world.

VI. The Theorem Stated

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Any comparison perfected at its foundation is either continuously asymmetric or discontinuously symmetric, and there is no third possibility.

The Incommensuration Theorem can also be stated at the absolute, in terms of what holds absolutely: whatever is absolutely symmetric is also intrinsically discontinuous, and whatever is absolutely continuous is also intrinsically asymmetric. Nothing anywhere at all can be both absolutely symmetric and absolutely continuous: no interaction can, and no compar-

ison can. Likewise, nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous. The Incommensuration Theorem holds at every scale, and at every position in every domain.

The Incommensuration Theorem is a structural fact about the nature of knowing. The theorem is intrinsic to the concepts themselves: to the four compounds, and to the four intrinsics from which the compounds are defined. The theorem follows from the very nature of epistemology, the study of how we know, and of ontology, the study of what is, because every act of knowing is an act of comparison or is built from acts of comparison. The theorem was not made from empirical investigation. The Incommensuration Theorem sets a limit on the possible relationships between the subjective and the objective, and the theorem states that limit entirely in terms of sameness, difference, content and context.

Two laws follow from the Incommensuration Theorem, one for each side of the pair of content and context. The law of symmetry applies to all that is of content, and the law of continuity applies to all that is of context.

The Incommensuration Theorem comes first in the order of derivation: the three axioms of the Immanent Metaphysics are derived from the theorem, and the theorem is derived from the structure of comparison (the six intrinsics and the four compounds).

VII. What "Exactly" Means

As the word is used of the Incommensuration Theorem, "exactly" means holding at the foundation of one description. The theorem is consistent with symmetry and continuity appearing together approximately, and symmetry and continuity do appear together approximately in every real system. Where symmetry and continuity are separated by level, by phase or by scope, the theorem is also consistent with the two appearing together exactly: for example, one of the two may hold exactly of the object, and the other may hold exactly of the framework in which the object sits. The

theorem states that symmetry and continuity cannot both hold exactly at the foundation of one and the same description, in what is found in one and the same comparison.

So the Incommensuration Theorem fixes the character that an exact description can have. The theorem does not state the specific content of any physical theory. A physicist still has to look to learn which character a given description has. The theorem shows the physicist that there are two characters to find, and only two.

VIII. The Two Characters: The Objective and the Subjective

Two pairings survive the proof, continuous asymmetry and discontinuous symmetry, and each of the two is a recognizable character, and not only an abstract pairing.

Symmetry with discontinuity is the character of the omniscient: exact, repeating, in discrete steps. Continuity with asymmetry is the character of the transcendent: smooth, connected, never exactly repeating.

Objectivity has its basis in symmetry, and subjectivity has its basis in continuity. Particle physicists find the objective to be symmetric and discontinuous. Every electron is the same as every other electron in charge and in mass, and the sameness of electrons is symmetry in the strict sense: the same content across every difference of context, wherever and whenever an electron is found. Electrons are also discontinuous: the instances of electrons are separated from one another, one and then another, with nothing between. The subjective is the side from which every comparison is made, and the subjective is continuous and asymmetric: the subjective stance persists without a break, and what is met from the stance never exactly repeats. Continuity with asymmetry belongs to the nature of the subjective as deeply as symmetry with discontinuity belongs to the nature of the objective.

Discontinuous symmetry and continuous asymmetry are exactly the two pairings that the Incommensuration Theorem permits. Nothing has either of the two characters that the theorem forbids: symmetry with continuity, and asymmetry with discontinuity. An entire field of study depends on the fact that exactly two characters are possible: the study of the epistemic connection, the connection in knowing, among the known, the unknown and the unknowable.

IX. Instances in Other Domains

The Incommensuration Theorem is about every comparison. So suppose that the theorems of some domain forbid a conjunction of the same form as a conjunction that the Incommensuration Theorem forbids. Then the forbidden conjunction is an instance of the Incommensuration Theorem, as a particular case is an instance of its general case. In another domain, each instance of the Incommensuration Theorem depends on stating what terms of the domain count as symmetry and as continuity.

The physicist John Bell proved the theorem that now carries his name. Bell's theorem is one instance of the Incommensuration Theorem in another domain. Bell's theorem states that no account in which every outcome is fixed in advance by local causes reproduces the correlations found between entangled particles. Entangled particles are particles whose states were linked when the particles interacted directly as quantum systems. Lawfulness means the same physical law in every place and at every time. Lawfulness is a symmetry. Locality in Bell's sense means that no influence on a result comes from outside the result's past light cone, which is the region from which an influence traveling no faster than light could reach the result. Locality in Bell's sense is a continuity: the intervening region counts as context, and the influence counts as content. When the region between the two measuring stations (the two places where the entangled

particles are measured) is counted as context, Bell's theorem forbids in physics the conjunction that the Incommensuration Theorem forbids in every comparison.

The uncertainty principle of quantum mechanics is another instance. The uncertainty principle states that position and momentum cannot both be pinned down perfectly at once, and a description that perfected both the position and the momentum of one state at once would conjoin symmetry with continuity.

X. The Limit It Sets on Every Theory

The Incommensuration Theorem sets definite bounds on what can be derived from first principles. If a system of knowledge is exactly symmetric at its foundation, the system necessarily contains gaps, which are discontinuities in what the system covers. If an understanding is exactly continuous at its foundation, the understanding necessarily involves asymmetries, which are differences that cannot be removed.

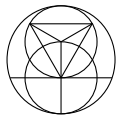
The bounds set by the Incommensuration Theorem are a structural feature of all interaction, and not a defect of some particular theory. No theory will ever contain a complete explanation, prediction or model of all the observable behavior and phenomena in a domain. The limit holds of physics, and the limit holds of the Immanent Metaphysics as well.

The limit on the Immanent Metaphysics is stated exactly by the distinction between a domain and a world. A domain is a foundational triad: three concepts at the foundation of a realm of conception. A world is an unbounded, holistic dynamic. The Immanent Metaphysics is a comprehensive description of the foundations of domains, and the Immanent Metaphysics is not a theory of everything. The foundations of a domain determine the structure of the domain, and do not determine the content of the domain. The unknowable is what cannot be known even in principle, and the unknowable remains a persistent class. The Incommensuration Theorem states where the unknowable begins.

XI. Summary

Comparison has six intrinsics, and four compounds are defined from four of the intrinsics alone. The four intrinsics used are sameness, difference, content and context. The four compounds are continuity, discontinuity, symmetry and asymmetry. The four compounds can be paired in ten ways. Exactly two of the ten pairings satisfy the three rules of no self-application, no application of opposites, and the balance of sameness and difference in each respect. The two pairings are continuous asymmetry and discontinuous symmetry. In the finite form of the proof, continuous asymmetry is the identity map and discontinuous symmetry is the exchange map. The identity and the exchange are the only one-to-one maps of {same, different} to itself.

The result that exactly two pairings survive is the Incommensuration Theorem. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing, proved from comparison alone with no premise about the physical world, and the three axioms of the Immanent Metaphysics are derived from the theorem. The scope of the theorem is what holds exactly at the foundation of one description, and the theorem is consistent with symmetry and continuity appearing together approximately, and with the two appearing together where the two are separated by level, phase or scope. The two surviving pairings are the characters of the objective, which is symmetric and discontinuous, and of the subjective, which is continuous and asymmetric. The theorem applies to every comparison, so the theorem appears as instances in other domains, and the theorem limits every theory, the Immanent Metaphysics included, to a description of foundations and never a theory of everything.



Chapter 3

The Three Modalities and the Three Axioms

The Immanent, the Omniscient and the Transcendent as Roles, the Axioms That Relate Them, and Why the Axioms Describe Themselves

ABSTRACT

The immanent, the omniscient and the transcendent are three roles: the roles that the three foundational concepts of any domain play toward one another. A domain is a realm of conception complete unto itself. A domain's foundational concepts are the three concepts at its root. The three roles are not things, and carry no theological meaning. Three axioms state how the three roles are related to one another: Axiom I states which role is more fundamental, Axiom II states in what order the roles succeed one another, and Axiom III states that the roles are distinct, inseparable and non-interchangeable. This chapter sets out the roles and their characters, quotes the axioms exactly, and says plainly what each axiom means. I then show why the axioms describe themselves and also describe the process of describing themselves, so that the system of the axioms is closed on itself as a fixed point, something that is carried back onto itself unchanged. Last, I set out the difference between the order of teaching the axioms and the order of deriving the axioms.

I. Three Roles, Not Three Things

The *modalities* are three roles: the immanent, the omniscient and the transcendent. A domain is a realm of conception complete unto itself, such as the physical domain that physics studies. Every domain has three foundational concepts at its root. Each modality names the role that one foundational concept of a domain plays with respect to the other two.

The *immanent* is the role of the act itself, the relating, as the act happens, here and now. The *omniscient* is the role of what is laid out as structure, such as a pattern, a map or a record, which can be held in view as a whole and can be shared. The *transcendent* is the role of the stance from which an act proceeds, and of the range of what is possible from that stance.

The names of the modalities name roles, and carry no theological meaning. A triple is a set of three concepts considered together. Each of the three concepts is a term of the triple. The focus of a triple is the respect in which the three terms are considered together. A modality is assigned to a term only relative to the triple in which the term is considered: the same term can play one role in one triple and another role in another triple. Each assignment can be correct in its own triple and focus.

The modalities are not entities. The modalities are types of role, an archetype of concepts (a pattern that many concepts can fill), and no specific instance represents the modalities. The modalities name the roles that the foundational concepts of a domain play, and not the concepts that happen to play those roles. The three axioms describe the pattern of relations among the three roles. The axioms are pure form, without quality, while the modalities are pure quality, without form. In the contrast between axioms and modalities, form is the pattern of relations, and quality is the character of each role. Even the word "thing" cannot describe the modalities: thingness is itself a concept, and the modalities are more basic than the concept of thingness.

II. Their Characters, and the Three Persons

Each role has a recognizable character. The three characters follow the grammatical persons: first person (I), second person (you), third person (he, she, it, they).

The immanent has the character of first-person experience: the direct, immediate, relational aspect of experience; the act, the mediating, the processual, the happening. The omniscient has the character of a third-person description: abstract, structural, comprehensive; what can be laid out and held in view as a whole. The transcendent has the character of the second person, of what informs an action without being the action, which is the character of the stance from which the act proceeds, of the range of the possible from that stance, and of what exceeds any one instance.

The persons are characters of the roles, not identities with the roles. The persons are also framings, where a framing is the person in which a whole description is given, as a story can be told in the first person or in the third person. Inside any one framing, all three modalities are present. When a description moves from a third-person framing to a first-person framing, each concept of a triple moves one step along the cycle of Axiom II, which runs from the transcendent to the immanent to the omniscient and back to the transcendent (Section VI). Comparison is the act of finding whether two things are the same or different, in some respect. Chapter 2 shows a case: in the third-person focus of measurement, sameness and difference are immanent, while in the first-person focus of comparison as relation, sameness and difference are omniscient, one step further along the cycle.

When the modalities are taken as ways that a self is related to a world, the modalities are the three fundamental perspectives from which any relation between self and reality can be considered. Theory is held in the third person, and so theory is omniscient. Practice is held in the first person, and so practice is immanent. A measurement is an act made in the

first person, while a calculation is structure held in the third person. The first-person stance is the position from which an act proceeds. The stance is transcendent in character: the character of continuity with asymmetry. Chapter 2 defines content and context: the content of a comparison is what the verdict of the comparison is on, and the context is the respect in which the two things compared are two. Continuity is a sameness of content where there is a sameness of context. Asymmetry is a difference of content where there is a difference of context. The stance is encountered in the act, and the act is immanent.

The Immanent Metaphysics is the metaphysics on which this book rests. The Immanent Metaphysics is theory, practice, and the relation between theory and practice. The Immanent Metaphysics has the perspective of the transcendent frame, the second person, the frame relating the first person to the third person. Axiom III states that the three modalities are distinct, inseparable and non-interchangeable, and Axiom III applies everywhere. Axiom I states that the immanent is more fundamental than the other two modalities, and is the essence of the practice of theory. Axiom II states the order in which the three modalities succeed one another, and is the essence of the theory of practice. When the axioms are considered as described in terms of the modalities, the considering is the practice of theory, while the considering of the axioms as defining the modalities is the theory of practice. Knowing the Immanent Metaphysics requires both. Even so, theory is held in the mind and imagination of a person, and practice in a person's embodiment.

III. The Root of Every Domain

The modalities are abstractions of the three concepts at the root of every domain. *Foundational triplication* is the principle that the foundation of every domain resolves into exactly three concepts, which are distinct, in-

separable and non-interchangeable. *Type isomorphism* is the principle that the structure of relations among the three foundational concepts is the same in every domain. An isomorphism is a sameness of structure.

The modalities are what remains when someone sets aside the three specific concepts of any one domain and considers only the roles of the three concepts, in a general way, across all domains. Whenever the foundation of a domain is broken down into its three concepts, there is always one concept of each type: one immanent concept, one omniscient concept and one transcendent concept. Chapter 6 shows a case, an imagined empty universe whose three foundational concepts are existence, interaction and change of state: existence plays the omniscient role, interaction the immanent role, and change of state the transcendent role. Because the pattern is the same in every domain, triples from different domains can be applied to one another, role for role.

IV. The Three Axioms

An Immanent Metaphysics states the three axioms in aphorism 1.3-1, in the words quoted below. An Immanent Metaphysics is written in aphorisms, short numbered statements.

> Axiom I. "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another".
>
> Axiom II. "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent".
>
> Axiom III. "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable".

In Axiom I, the words "and/or" mean that the immanent is more fundamental than either one of the other two modalities, and more fundamental than both of the other two together. In Axiom III, the bracket around the words classes and instances means that the statement holds of the classes, which are kinds, and also of the instances, which are particular cases of a kind.

When two things satisfy two conditions, the two things are *conjugate*. First, each can be perfected only at the expense of the other. Second, neither thing is prior to the other in definitional basis, so neither thing is derived from the other. Axiom I states that the omniscient and the transcendent of any triple are conjugate.

V. Axiom I Read Plainly

In plain words, Axiom I says that relationship (doing) is more fundamental than identity (being). In every foundational triple, one term mediates between the other two terms. The other two terms are related to each other through the mediating term. The mediating term is the immanent, and the other two terms are the omniscient and the transcendent.

When Axiom I is applied to self and world, with their relationship in the immanent role, Axiom I says that the relationship between the objective and the subjective is more fundamental than the nature of the objective and/or the nature of the subjective. In the application to self and world, the subjective is the side of the self, and the objective is the side of the world.

Because the relationship is more fundamental than either side, Axiom I says where to begin: take the immanent as the ground, which is to take the first-person relational perspective as the place from which to build. Starting from the immanent, a person has access to the omniscient and the transcendent through particular patterns of inquiry.

Is Axiom I a claim about reality, or a choice of method? Axiom I is both. Someone can draw the distinction between claims about reality and choices of method only from within a perspective, and the perspective from which the distinction is most naturally drawn is the immanent perspective. So Axiom I comes before the distinction between ontology (what is) and epistemology (how we know): the distinction is downstream of Axiom I, not upstream of Axiom I. The distinction is drawn from within the perspective that Axiom I sets as the ground.

VI. Axiom II: Precedence and Convergence

Axiom II states an order of precedence, not an order in clock time. The cycle of Axiom II runs from the transcendent to the immanent to the omniscient and back to the transcendent, and the cycle can be entered at any point, but the direction of the cycle cannot be reversed.

The cycle of Axiom II is a ring, and at each of the ring's three stages many things converge into one. To reify a concept is to give the concept a definite form in terms of other concepts. When someone runs a reifying process on structures of concepts and relations, certain structures converge to an idealized form. Axiom II describes the convergence. Many existing patterns converge into one creative act. Many creative potentials converge into one actual event. Many actual events converge into one knowable structure. The cycle of the three convergences is a flow, and the flow is at the core of all process. A person who works with the convergence learns to move back and forth between the fully first person and the fully third person.

Dependent origination is a Buddhist teaching: every phenomenon arises in dependence on other phenomena. The teaching of dependent origination comes close to Axiom II, but dependent origination is at the level of phenomena, while Axiom II is at the level of the structure of process itself.

VII. Axiom III Read Plainly

Axiom III names three properties (distinctness, non-interchangeability and inseparability), and each property can be said in plain words.

Distinctness means that the three modalities do not overlap. Non-interchangeability means that putting one modality in the place of another gives a different result, and a wrong result. Inseparability means that whenever one modality is present, the other two modalities are present as well.

VIII. What Makes Three Concepts a Modal Triple

A *modal triple* is three concepts in the three roles, for which there is one identified focus in which all three axioms hold. A person tests whether three concepts form a modal triple by applying the axioms in the order III, then II, then I. Axiom III is checked first: the three concepts must be distinct, inseparable and non-interchangeable, and if the three concepts are separable, Axiom III does not apply and the three concepts are not a modal triple. If the three concepts pass the check of Axiom III, there is a frame in which they are held together. Axiom II is checked next: the three concepts must run in a valid cycle, in the one direction that Axiom II gives, and the valid cycle says what the frame is. Axiom I is checked last: one of the three concepts must be more fundamental than the other two, which must be conjugate. The more fundamental concept, the mediating term, is the immanent.

IX. The Axioms Describe Themselves

There are three axioms and three modalities. The three axioms define a domain of conception. Every domain has a reification in terms of the modalities, an expression of the domain's foundation in terms of the three modalities, and so the axioms, taken as a domain, also have a reification in terms of the modalities. The modalities map one to one onto the axioms: Axiom I has the nature of the omniscient, Axiom II the nature of the immanent, and Axiom III the nature of the transcendent. The axioms describe the relations among the three modalities, and each axiom has the nature of one modality, so the relations that the axioms describe are also the relations among the axioms. Therefore the axioms describe themselves: the axioms are their own reification.

The axioms do more than describe themselves. The describing of the axioms by the axioms is itself a process, an instance of the general class of process. Axiom II is the axiom of process. So the axioms describe themselves, and the axioms also describe the process of describing themselves. The system of the axioms therefore has full closure over the reification of the foundation. Expressing the foundation needs no concept from outside the system of the axioms. The full closure is an axiom in a hyperdimension wrapping itself: a spiral, the layer laid on itself and that layer carried through time. In the spiral, the layer laid on itself is the axioms describing themselves, and carrying the layer through time is the process of the describing.

A person can check the full closure of the axioms by a direct test. If triplication holds at the foundation of foundations, the process of thinking about foundations must follow the same rules as the foundations thought about. So, for the test, a person imagines the axioms as a content that describes the relation of domain, relation and identity. A domain holds relations, and each relation is between identities. The person places domain, relation and identity one to one with the three modalities, and states the

relations among the three by the axioms. Then the person asks a question about the act of imagining and defining the three concepts. Does the statement describe the relation between the three concepts and the axioms in that very act? The answer is yes.

A person who starts from foundational triplication and type isomorphism arrives at the three axioms. The axioms are self-descriptive, and self-descriptive of the process of being self-descriptive, in the closure of the relations of domain, relation and identity. The closure makes the system of the axioms complete as a fixed point, not as a list of every case. The closure establishes both the necessity and the sufficiency of the system. The system can do with nothing less, and needs nothing more.

So far as I am aware, the Immanent Metaphysics is the first fully qualified foundational, ontological metaphysics. "Fully qualified" means that the metaphysics describes itself, describes the process of describing itself, and needs no new concept to describe that process in turn. The Immanent Metaphysics is complete at the level of foundation, and not at the level of content: the metaphysics describes the foundation of every domain, and not everything that a domain contains. The core of the Immanent Metaphysics is the three modalities and the three axioms, and the most essential nature of what is meant by "real" is fully reified in that core.

X. Why the Pattern Validates Itself

The three modalities and the three axioms are one pattern, and the pattern must be the algorithmic core of the nature of being itself. Awareness, or knowledge, is a perception, and so awareness is a relationship. Therefore the pattern describes epistemology (how we know) in the same way as the pattern describes ontology (what is).

Even doubt instantiates the pattern. A doubt is an act of comparison, with a stance, an act and a structure, and so anyone who doubts the pattern uses the pattern in the act of doubting. Whatever doubt itself instantiates is beyond doubt. The pattern is what doubt instantiates. The philo-

sopher René Descartes took what is beyond doubt as the foundation of certain knowledge. So by Descartes' own criterion, the pattern has the standing of absolute truth. Even the relation between the absolute and the relative is bound under the ring of Axiom II.

XI. The Order of Teaching and the Order of Derivation

In teaching, the axioms come first, and the relationship between the subjective and the objective comes after the axioms, approached through comparison. In derivation, the order is the reverse. Comparison comes first. The Incommensuration Theorem is derived from the structure of comparison, and the three axioms are derived from the Incommensuration Theorem. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Chapter 2 defines symmetry as a sameness of content where there is a difference of context, and defines discontinuity as a difference of content where there is a sameness of context.

Comparison is the right place to begin, because any measurement is a comparison, as reading a thermometer is a comparison of the top of the liquid with the marks on the glass, and any observation is implicitly a comparison. Someone who understands comparison already knows a great deal about the nature of the immanent, since a comparison is an act, and the immanent is the role of the act itself.

XII. What Is New in the Axioms

The three axioms taken together are unique to the Immanent Metaphysics, and Axiom II in particular is unique. The American philosopher Charles Sanders Peirce had a good sense of Axiom I, and Peirce had a version of Axiom III. Axiom III appears throughout the perennial traditions (the religious and philosophical traditions of many cultures and ages) as triples

recognized but never formalized. Axiom II is unique both in its statement and in its formalization. The three axioms together formalize the relationship that Peirce and the perennial traditions recognized.

XIII. One Consequence: The Omniscient Cannot Contain the Transcendent

The three axioms show the source of a common confusion: the expectation that a model can contain everything, including the stance from which the modeling is done. We have modeled what can be perceived so successfully that the confusion arises. When the model is visible and the thing that the model describes is visible, a person can characterize the relation between the model and the thing clearly, and the modeling works very well. A person can check a map of a town against the streets, for example, because both are visible.

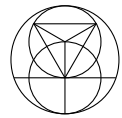
But the omniscient cannot contain the transcendent. The omniscient and the transcendent are conjugate, distinct and non-interchangeable. Some questions require a third-person description to contain the stance from which every description is made, and so require the omniscient to contain the transcendent, and the omniscient cannot contain the transcendent. The hard problem of consciousness is the question of how a physical description, given in the third person, could contain experience as the experience is lived in the first person. The hard problem of consciousness is one question that requires a third-person description to contain the stance from which every description is made.

XIV. Summary

The modalities are three roles that the three foundational concepts of any domain play toward one another: the act (immanent), the laid-out structure (omniscient), and the stance with the range of the possible from the stance (transcendent). The immanent, the omniscient and the transcend-

ent have the characters of the first person, the third person and the second person, in that order. Inside any one framing, all three modalities are present. I have quoted the three axioms exactly from An Immanent Metaphysics, and the three axioms state the relations among the three modalities. Axiom I states that the immanent is more fundamental than the omniscient and/or the transcendent, and that the omniscient and the transcendent are conjugate. Axiom II states that the three modalities succeed one another in an irreversible cycle of precedence, converging from many to one at each step. Axiom III states that the three modalities are distinct, inseparable and non-interchangeable.

Because the axioms define a domain, they are reified by the modalities, and so describe themselves and the process of describing themselves. The axioms are closed over their own description, and so the system is complete as a fixed point, at the level of foundation and not at the level of content. In teaching, the axioms come first. In derivation, comparison comes first: the Incommensuration Theorem is derived from comparison, and the axioms are derived from the Incommensuration Theorem.



Chapter 4

How a Triple Is Found and Tested

Foundational Triplication, the Order of the Three Tests, and What a Triple Lets One See

ABSTRACT

The working method of the Immanent Metaphysics is to find three concepts at the foundation of a domain that relate to one another as the three modalities do, the modalities being the three roles of Chapter 3: the immanent, the omniscient and the transcendent. One then uses the triple of concepts as a lens, something one looks through at the rest of the domain. This chapter shows why a foundation has three concepts, not two, and gives two images, one for Axiom I and one for Axiom III. It states the principles of foundational triplication and type isomorphism, and it sets out the test of a modal triple in its order: Axiom III, then Axiom II, then Axiom I. It shows how a new triple is established by correspondence with a triple already known. It works one case with its cautions: benefit, cost and risk. It names the two things that a triple gives the person who finds it: reifying power and resolving power. It treats clarity, simplicity and complexity as the steps of the cycle of Axiom II, and says why the metaphysics has few drawings. It closes with a toroid whose hole has closed to a point, and with the part of each domain to which the method applies.

I. Why Three and Not Two

Some pairs of concepts seem to be real polarizations. A polarization, in this sense, is two opposite poles and nothing else. Truth and falsity are the obvious case. But what is the relation between truth and falsity? Take the difference that distinguishes the true from the false, and ask whether that difference is itself true, or false. The difference is a third concept, deeper than either truth or falsity, so it is neither true nor false, and it mediates between truth and falsity: it relates each of them to the other. In every foundational triple, that is, in every set of three concepts at the foundation of a domain, one of the three terms mediates between the other two, as the difference mediates between truth and falsity.

The perspective of the metaphysics is triune, built on threes, so it is non-dualist, and it is not polar. Where we live in a duality, something is always missing. A duality turns into a contest between its two poles, along a single dimension instead of many dimensions. We build structures out of triangles because triangles have strength, and a foundation of three concepts is strong in the same way.

A meditation shows that the universe as a whole is three, not two. Understand the universe as a triplication: doing, being and becoming. Then ask: is there anything that is neither doing, nor being, nor becoming? When one meditates on that question for some time, one finds that the answer is simply no. Everything is doing, being or becoming.

II. Two Images

Two images show what the axioms are about: the first image pictures Axiom I, and the second pictures Axiom III.

The first image is a lightning bolt between the ground and the sky, taken as more basic than either the ground or the sky. Lightning is a flow of electric charge, an electric process. The image of the lightning bolt is lit-

eral as well as figurative. Matter is solid: I cannot push my two hands through one another, or fall through the floor. The solidity of matter rests on electromagnetism and on the exclusion principle. The bond between nucleus and electron is electric, and by the exclusion principle no two electrons can take one and the same state. What we experience as solid existence is built from that electric process. So the lightning bolt, itself an electric process, is the very basis of what solidness is. In the moment of the strike, the ground senses the sky through the lightning and the sky senses the ground, so that in that moment the very being of the ground and of the sky is a relationship. The lightning stands for the relationship, and the ground and the sky stand for the things related. So the image pictures Axiom I: relationship is more fundamental than thingness, and in physical terms, interaction is more fundamental than existence.

The second image is a calculation: the flow across the boundary of a dense region of a network, computed in two ways. When the flow is computed from the whole network, looking in at the region, the calculation gives one answer. When the flow is computed from inside the region, looking out, the calculation gives a different answer. The two calculations rest on an assumption that holds in one case and not in the other: that both calculations are made in one and the same context. But the two calculations are made in two different contexts. The view from outside looking in is the omniscient view, while the view from inside looking out is the transcendent view, and the omniscient and the transcendent are two different concepts. In this book the content is what the verdict of a comparison is on, and the context is the respect in which the two things compared are two (Chapter 2). For every content there are two distinct kinds of context: the omniscient context, within which one knows the content, and the transcendent context, within which one creates or chooses the content. Axiom III states the observation of the second image in general, for all three modalities: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable".

III. Foundational Triplication and Type Isomorphism

Take the concept of relationship. Relationship has three archetypes, or basic forms: similarity, proximity and inclusion. Can one imagine any relationship that is not an example of one of these three archetypes, or some mixture of just these three? If one takes that question seriously, it is a meditative exercise, a Zen-like inquiry that a person works through over weeks.

The concept of relationship is itself one member of a trinity, because one thinks of relations as holding in domains, between identities, and so whenever we speak of a model or a map, we bring in all three concepts. A map of a country, for example, has a domain (the country), identities (the towns), and relations (the roads and distances between the towns).

The surprising fact is that threeness holds in every set of concepts at the foundation of a domain, and that the pattern of relations among the three concepts is the same pattern in every domain. *Foundational triplication* is the principle that every domain has three concepts at its foundation. At the base of the ordering of any domain's concepts there are always three concepts. Every other concept of the domain is built on those three, constructed from them, or a superposition of them, that is, a combination in which they are present together. *Type isomorphism* is the principle that the pattern of relations among the three foundational concepts is the same in every domain, so that the concepts of any one domain correspond to the concepts of another. The word "isomorphism" means sameness of form. An understanding of one domain then gives an understanding of another, to a person who has the axioms and the modalities.

IV. The Test, in Its Order

When all three axioms hold of three concepts in one identified focus, the three concepts are a *modal triple*. A focus is the frame of a description, such as practice in the first person or theory in the third person, and one assigns the modal roles relative to the focus (Chapter 3). The test of a modal triple applies the three axioms in a definite order: Axiom III, then Axiom II, then Axiom I.

The first test is Axiom III: are the three concepts distinct, inseparable and non-interchangeable? The three are distinct when they do not overlap, inseparable when no one of them is present without the other two, and non-interchangeable when putting one in the place of another gives a different, and wrong, result. If the three concepts turn out to be separable, Axiom III does not apply to them, and they are not a modal triple.

The second test is Axiom II: do the three concepts run in a valid cycle, in the one direction that Axiom II gives, which runs from the transcendent to the immanent to the omniscient, and back to the transcendent?

The third test is Axiom I: is one of the three concepts more fundamental than the other two, and are those two conjugate? Two concepts are conjugate when each can be perfected only at the expense of the other, and neither is prior to the other in definitional basis. If the three concepts cannot be arranged in that pattern, Axiom I is not met, and they are not a triple.

In the derivation of the triple of creation, existence and interaction, one can examine Axiom I before Axiom II. Both orders give the same triple of creation, existence and interaction. I have checked every triple identified by the method of this chapter against all three axioms, and also against statements known to be true about the world in which the triple's terms are used.

In practice, a person finds a triple in more than one way. Sometimes one writes out a reification of the concepts, a statement of their structure in terms of the modalities, and looks at the pattern. Sometimes one simply sees the triple on waking. After one has learned a great many triples, one begins to see which concepts relate to which, and then one flips through a mental card file of known triples to find a known triple that reifies the new case. To reify the new case is to give the new concepts a definite form, through the pattern of the known triple. Then one tests the candidate by the three axioms, in the order above. If the candidate passes, the triple counts as identified. The tests show that the triple holds against the most natural ways of dissolving it, such as finding that its three concepts are separable. The tests do not prove the triple from nothing.

V. Establishing a Triple by Correspondence

One can also establish a new triple through a triple that has already been verified. One binds the candidate triple to a known working triple, term to term, at the level of definition, so that every thought about the candidate's terms already carries suppositions that connect those terms to the known triple's terms. Once that binding is made, the candidate is established as a triple.

Then the modal concepts become a lens, something to look through. One can look at the new triple's terms through the modalities, seeing each term in its role, and so understand the theories in which those terms are embodied. For example, addition, multiplication and exponentiation are an established triple. Addition is immanent, multiplication is omniscient, and exponentiation is transcendent. Multiplication is repeated addition, and exponentiation is repeated multiplication. The triple of addition, multiplication and exponentiation can be used as a lens on marketplace dynamics, the ways in which a marketplace changes over time.

VI. A Worked Case, with Its Cautions: Benefit, Cost and Risk

Benefit, cost and risk can be treated as a triple, but only with care, because the three terms are ambiguous enough to need close examination.

Benefit is value I receive, and cost is value I give. Taken together, benefit and cost are a flow across a channel, here and today: money coming in and money going out. Risk is the possibility of future cost, and it could also be the possibility of future benefit. The cost that risk names may be unbounded, with no upper limit.

Now map benefit, cost and risk onto past, present and future, which are a known triple. Two terms, benefit and cost, fall in the present. One term, risk, falls in the future. An image of another term also falls in the future: the future cost or benefit that risk names. No term falls in the past. So the mapping bifurcates, splitting two ways instead of going one term to one term, and that split is the caution: the candidate triple does not yet bind cleanly to the known triple.

Now renormalize benefit, cost and risk to potentiality. Potentiality is the range of what is possible. To renormalize the terms is to restate each of them in terms of one common measure, and once all three terms are restated in terms of potentiality, the binding becomes clear. Benefit increases potentiality and cost decreases potentiality, both in the present, and risk is itself a concept defined by potentiality. When all three terms are defined in terms of potentiality, treating benefit, cost and risk as a triple makes more sense. That benefit, cost and risk make sense as a triple was not obvious at the start.

VII. Triples That Mirror One Another

Because of type isomorphism, triples from different domains line up by role, so that each term of one triple matches the term of the other triple that plays the same modal role. Metaphysics centers on three inquiries: ontology (what is), epistemology (how we know), and axiology (why anything matters). The triple of ontology, epistemology and axiology mirrors, role for role, the universe taken as existence, interaction and creation. Ontology is the study of the structure of what exists, and it is omniscient, as existence is. Epistemology is the study of the actual process of knowing, and it is immanent, as interaction is. Axiology is the study of the basis of value and meaning, and it is transcendent, as creation is.

Another alignment by role shows that six concepts are complete. Take choice, change and causation as contents, and creation, existence and interaction as contexts. If the six concepts were incomplete, some concept would lie outside their scope. So ask: is there anything within the scope of the universe that creation, existence and interaction do not fully subsume, or include? Is there anything about existence not describable in terms of causation, anything about interaction not held within change, anything about creation that is not choice? When one asks these questions, the answer one finds is no. So the six concepts are complete, and each content is the basis of one context: change is the basis of interaction, causation is the basis of existence, and choice is the basis of creation. Each pair of a content and its context shares a role: change and interaction are immanent, causation and existence are omniscient, and choice and creation are transcendent.

VIII. Reifying Power and Resolving Power

Reifying power is the capacity to identify a domain's foundational triple in a way that the practitioners of that domain recognize as correct. The practitioners of a domain are the people who use its terms. *Resolving power* is what the identification of a domain's triple then allows: understanding things already known in a different, more profound way, and generating insights that are not trivial.

The word "profound" links resolving power to a standard that physicists use. Physicists judge theories partly by beauty: a theory has to be elegant. To call a theory elegant is to say that it is profoundly clear. Elegance in that sense is not simplicity: a theory can be called elegant without being simple. With elegance taken as profound clarity, the Immanent Metaphysics can be seen as the generator function of what is profoundly clear (the one source from which what is profoundly clear is produced).

IX. Clarity, Simplicity and Complexity as Steps of the Cycle

Go around the cycle of Axiom II, from the transcendent to the immanent to the omniscient and back, and look at its three steps. The step from the transcendent to the immanent is about clarity. The step from the immanent to the omniscient is about simplicity. The step from the omniscient back to the transcendent is about complexity.

When the cycle is seen in the third person, from outside, each quality belongs to the role at which its step begins, so that clarity belongs to the transcendent, simplicity to the immanent, and complexity to the omniscient. When the cycle is seen in the first person, from inside, each quality belongs to the role at which its step arrives, so that clarity belongs to the immanent, simplicity to the omniscient, and complexity to the transcendent. I hold to the order of the cycle's steps in all good work: clear before simple, and simple before complex.

X. Why the Metaphysics Has Few Drawings

Every drawing has a dimensionality of two, because a drawing lies flat on its page. Many of the structures of the metaphysics have a dimensionality of three. Axiom III has three terms that are distinct, inseparable and non-interchangeable, and so Axiom III implies a kind of orthonormal basis. An orthonormal basis is three directions at right angles, none of which lies along another, like the length, the width and the height of a room. Three-dimensional thinking serves Axiom III well.

Once a drawing has set out the orthonormal basis of Axiom III, the drawing has done about all that a drawing can do. Everything after the basis is the projection of concepts into more than one world. Much of that later work is about the relation between content and context, and a drawing does not serve that relation well, because a drawing specifies the structure of content alone.

XI. The Whole at Every Point: the Toroid

A toroid is a surface shaped like a donut. When a toroid is taken as everywhere, it gives one image of how a whole can be present everywhere. Picture a donut whose hole has closed to a single point (what geometers call a horn torus). The horn torus is the toroid's mesoscopic structure, its structure at the middle scale, between the very small and the very large. Take the horn torus as holding the whole universe, and take the crossing point at its center, the single point to which the hole has closed, as everywhere: at every point of time, of space and of possibility. The crossing point is also within each of us, and anyone can take hold of it within their own awareness. The toroid is a model of how the whole is present at every point, not a claim about the shape of space.

One can measure the presence of the whole at every point, but doing so is tricky, like explaining a hologram to someone who knows only photographs. A hologram holds its whole picture in each part of itself, while a photograph holds each part of its picture in one place only. One measures the whole at every point with mathematics that resembles the relation between the time domain and the frequency domain in a Fourier transform. The Fourier transform is a method, named for the mathematician Joseph Fourier, that rewrites a signal given in time as a sum of frequencies. In "time domain" and "frequency domain", the word "domain" has the mathematician's sense: the space in which a signal is described. In a Fourier transform, the sharper a signal is in one domain, the broader it is in the other. One can construct metrics of the transcendent from relations between domains like the time domain and the frequency domain. Metrics of the transcendent are metrics, or measures, of probability over possibility, in place of metrics of pattern over space.

XII. How Far the Method Reaches

The method of this chapter applies to the foundations of domains. The metaphysics is complete at the level of foundation (Chapter 3), but I make no assumption that the metaphysics is complete in the sense of describing every phenomenon in a domain. The farther one moves from the core structure of a domain, the less likely it is that metaphysical truths bear on what one is looking at.

XIII. Summary

A foundation has three concepts and not two, because the difference between two poles is itself a third concept, which is neither of the two poles and which mediates between them. Foundational triplication is the principle that every domain has three concepts at its foundation. Type isomorphism is the principle that the relations among those three concepts

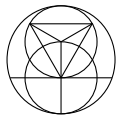
are the same in every domain, so that the triple of one domain helps one understand the triple of another. One tests a candidate triple within one identified focus, in a definite order, Axiom III, then Axiom II, then Axiom I, and a pass shows that the triple holds against the natural ways of dissolving it. One can also establish a new triple by binding it at the level of definition to a known triple, but the case of benefit, cost and risk shows that the binding can take work, in that case a renormalization of all three terms to potentiality.

A triple has reifying power when the practitioners of its domain recognize it as correct, and the resolving power of the triple is what the triple then lets one see anew. In the sense in which physicists use the word, elegance is profound clarity, the clarity that resolving power gives. The three steps of the cycle of Axiom II are about clarity, simplicity and complexity, in that order. The method serves three-dimensional structure and the relation of content to context, and drawings serve both poorly. The horn torus is a toroid with its hole closed to a point, and it models the whole as present at every point. The method applies to the foundation of each domain, and I make no claim that it covers the whole of any domain's content.

PART TWO

WHAT IS, AND HOW IT IS KNOWN

The universe, existence and the real, process, the orders of knowledge, the errors of extrema, and what perception reaches.



Chapter 5

The Universe Is Creation, Existence, and Interaction

Why the Universe Is Not a Container

ABSTRACT

The usual picture of the universe is a container full of stuff. The container picture gives the word "universe" nothing to mean, because a concept that refers to an undefined extent of content has no semantic value: what the concept is cannot be distinguished from what the concept is not. The word "universe" gets its meaning from three other concepts: creation, existence and interaction. The universe is exactly the combined meanings of creation, existence and interaction, neither more nor less, and the equation of the universe with the three meanings is a denotation, not a description. When the three axioms of the Immanent Metaphysics are applied to creation, existence and interaction, the axioms show three things. The three concepts are distinct, inseparable and non-interchangeable. Interaction is more fundamental than existence and creation, which are conjugate. The order of dependency runs from creation to interaction to existence and back to creation. Interaction is immanent: interaction is the act, forces unfolding across time. Existence is omniscient: existence is what can be laid out as structure, matter arranged in space. Creation is transcendent: creation is potentiality over the space of what is possible. Creation is therefore not a first event but an aspect of every event, because the three concepts are inseparable and the order among them is

> one of precedence, not of clock time. Novelty, something new, enters every interaction. This chapter sets out the account of the universe just stated, together with the inside-out notion of a domain on which the account depends, the notion that a domain is defined by the basis concepts of the domain and not by the contents of the domain.

I. The Box Picture

The usual way of thinking about the universe is that the universe is a container with a great deal of stuff inside the container. The universe is a box. The box holds rocks and spoons and books, stars and planets. On the box picture, the universe is the sum of all the stuff in the box.

Physicists use the word "universe" in much the same sense as the box picture: the universe is a container in which there is matter and energy, and everything made from matter and energy. The actual universe has no edge, and there is no place in the universe that is farthest away. There is a limit on how far away we can sense anything, set by the speed of light together with the finite time since the expansion of the universe began, and physicists call the limit the particle horizon. But the notion of universe is not defined with respect to the speed of light, or with respect to how small things can be: as a notion, the universe is a totalization, a gathering of everything into one total, made regardless of any limits.

II. A Word That Must Get Its Meaning Elsewhere

Suppose I want to know anything about the concept of universe. What does the concept of universe refer to? One answer is an undefined extent of content: an amount of stuff with no defined bound. If the concept of universe refers to an undefined extent of content, then the concept has no semantic value, because I cannot distinguish what the concept is from what the concept is not.

The concept of universe gets its meaning from its association with three other concepts: creation, existence and interaction. Everything knowable about the universe is the sum of everything knowable about creation, everything knowable about existence, and everything knowable about interaction, together with everything knowable about the relationships among the three concepts, taken as concepts or as processes. Everything knowable about the universe would be knowable from knowing all there is to know about creation, existence and interaction, so the totalization that we attempt with the word "universe" can be moved off the word and onto the three concepts, taken together as a basis. A basis is a set of starting concepts from which the meaning of the whole is composed. The universe is then given exactly by the combined meanings of creation, existence and interaction, neither more nor less.

A test question makes the claim definite. If I knew everything about creation, everything about existence and everything about interaction, would there be anything that is part of the universe and not part of one or more of those three concepts? The answer is no: every part of the universe is part of creation, of existence or of interaction, or of more than one of the three.

The universe is strictly equivalent to the three concepts. The equivalence is a denotation, not a description: the two sides of the equation name the same thing exactly. The equivalence is also not a statement of set theory, since the universe is not the set whose members are creation, existence and interaction. When three vague concepts are associated exactly with one vague concept, all four concepts become less vague, because each side of the association now fixes the meaning of the other side.

Reification is giving a concept a definite form in terms of other concepts. The exact association gives the concept of universe what may be called reifying power: the concept of universe now has a definite form, given in terms of creation, existence and interaction. We cannot easily un-

derstand the universe as a whole. But creation, existence and interaction are easier to understand, so we make progress when we identify the whole by three specific concepts in place of one total generality.

The universe is exactly the combined meanings of the three concepts, so the universe is not a total of everything there is, and the universe has no ontological status of its own: no standing of its own as a thing that is. The universe is a container neither of stuff nor of concepts. We can learn what the universe is only through creation, existence and interaction, the three concepts from which the universe gets its meaning. The meanings of creation, existence and interaction are filled with content. The content is the choices I make, the changes I experience and the causal patterns I observe, each met firsthand in the first person.

III. What a Domain Is

This chapter's account of the universe depends on a notion of domain. The definition of a domain comes from An Immanent Metaphysics, the book on which this book rests: "a domain is that which contains (includes), but which is itself uncontained and uncontainable". In plain words, a domain holds what is in the domain, and nothing holds the domain itself or could hold the domain. A domain is any realm of conception complete unto itself. The physical world is a domain, and so are a body of mathematics, a dream and a virtual reality. Comparison is the act of finding whether two things are the same or different, in some respect. The physical domain is the whole of what can be found by comparison from inside the physical domain.

A domain can be viewed in two ways: from outside in, or from inside out. From outside in, a domain is a container in which identity and relationship have meaning, or are at least defined, and category theory, a branch of mathematics, is an example of working from outside in. The terms of category theory have no semantic content. The formalisms of cat-

egory theory describe relationships among the terms, also without semantic content but still with structure. A mathematician finds a way into the domain by discovering the extensions of the structure.

From inside out, a domain is defined by its basis structures (its axioms) and not by all the things that can arise from the axioms. A picture from linear algebra shows what defining a domain from inside out means. Take three axes X, Y and Z that form an orthonormal basis of a three-dimensional space. An orthonormal basis is a set of directions of unit length, each at right angles to all the others. Everything that could be content in the space is composed from the three axes, but the basis defines the space, and the totality of what could be composed from the basis does not define the space. Just as the basis defines the space, the concept of a domain is strictly equal to the three basis concepts that together characterize all the content of the domain. The syntactic level of a description is the rules for combining the terms of the description, rules that apply whatever the terms denote, while the semantic level is what the terms denote. The semantic notion of a domain is the notion that a domain is equal to its three basis concepts, a notion about what the terms denote. Technically, then, a domain is not a container: a domain is the combined reference to the three concepts that define the domain.

The three axes also show what "distinct" means in Axiom III. Axiom III says that the immanent, the omniscient and the transcendent are distinct, inseparable and non-interchangeable. No one of the three axes collapses into another, and no axis lies along another. The axes are also at right angles to one another, but being at right angles is a further premise, added when the picture is used, and is not part of what "distinct" means.

A domain also differs from a set. In a set, the elements are what matter, and the relations among the elements are secondary. In a domain, the relations among the elements are first-class constituents, as fundamental as the elements themselves. A domain is individuated (told apart from every other domain) by the concepts that constitute the domain and not

by the contents of the domain. Organizing topics such as real estate or finance act as domains, because a domain is told apart by its concepts. Each topic is a combination of concepts connected in a definite way.

Three words name the parts of any domain, and the three words always go together: domain, relation and identity. To start mathematics from formal process is to start from relationship: a person needs to know nothing about identity or domain to have a concrete sense of relation, and domain and identity come along with relation. When domain, relation and identity are taken as the foundation of any domain of description, relation is immanent, domain is omniscient, and identity is transcendent, and relation is more fundamental than identity and/or domain. Domain, relation and identity are distinct, inseparable and non-interchangeable, not as content of any one domain but as a general observation about domains themselves, and the observation can be treated as a defining basis: a basis that defines what a domain is.

Domain, relation and identity define a domain, and creation, existence and interaction define the interactions within the domain. The two sets of three concepts have the same pattern of relations. Interactions within any world decompose into creation, existence and interaction just as the physical world does, whether the world is a dream, a body of mathematics or a virtual reality. Sets with the same pattern of relations are isomorphs of one another, so the two sets of three are isomorphs, which is what *type isomorphism* names. The isomorphism comes with a rotation. When one moves from one level of consideration to the next, the three concepts pass through a definite sequence of transitions, and the sequence closes in a ring of exactly three steps. The rotation is a shift of one position, in the order of Axiom II, as one passes from a level to the next, and the order of Axiom II runs from the transcendent to the immanent to the omniscient, and back to the transcendent. Axiom II itself differs from the rotation: Axiom II is the flow of the modalities within one domain (*Incommensuration*, Chapter 10).

IV. The Three Axioms Applied to Universe

The axioms of the Immanent Metaphysics are a test of whether three concepts are a modal triple: three concepts in the three roles of the immanent, the omniscient and the transcendent. For a modal triple, the axioms also show which role each concept plays. When the axioms are applied to creation, existence and interaction, each axiom shows one result.

Axiom III reads: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable". Applied to the three concepts, Axiom III says more than that creation, existence and interaction are necessary and sufficient to the notion of universe. The three concepts are necessary to each other and sufficient to each other, and substituting any one of the three for either of the other two changes the meaning. "Sufficient to each other" means that if I consider existence by itself, I must impute creation and interaction to existence, and in considering existence I am in fact considering what creation and interaction are, whether or not I admit that I am. So creation, existence and interaction depend on one another in a way that is not trivial.

Axiom I reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". When Axiom I is applied to creation, existence and interaction, the immanent (the more fundamental term) is interaction. We know of existence only through interaction: to establish that something exists, I must interact with the thing. Every process that establishes existence is an interaction: measurement, observation, detection and even imagination are all interactions. So interaction is the a priori basis of existence, the basis that must already be there before existence can be established. Interaction is also the a priori basis of creation, because we never observe creation directly. We find creation here and now, in the act of interaction. Existence and creation are a conjugate pair,

as Axiom I says of the omniscient and the transcendent: each of the two can be perfected (taken toward perfection) only at the expense of the other, and neither is prior to the other in definitional basis.

Axiom II reads: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". The order that Axiom II gives is one of precedence, not of clock time: an order of what comes before what in dependency. The apparent order of the three concepts is creation first, then existence, then interaction, but the apparent order is not the order of dependency. In the order of dependency, interaction precedes existence, existence precedes creation, and creation precedes interaction, so the flow runs from creation to interaction to existence, and back to creation, without end.

Together, the results of the three axioms are a pairing of creation, existence and interaction with the immanent, the omniscient and the transcendent. Interaction is immanent: interaction is forces unfolding across time, the act. Existence is omniscient: existence is matter arranged in space, what can be laid out as structure. Creation is transcendent: creation is potentiality over the space of what is possible.

V. Creation Is an Aspect of Every Event

Creation can be stated in one line: creation is the objective context of existence as a content. The objective context of existence is the domain itself: space, time and possibility, the circumstances in which any event occurs. Take the Big Bang as physicists usually picture the Big Bang: the moment when existence emerged. Creation is the context in which existence emerged at the Big Bang.

Creation is the context of the Big Bang, but creation is not an event that came before the Big Bang. In the Immanent Metaphysics, creation has no time and no source: creation is an aspect of every event. The order that the axioms give is one of precedence, not of clock time, so nothing in the ax-

ioms places creation at an earlier time than interaction, and creation is not confined to the past. Creation is present wherever interaction and existence are, because the three concepts are inseparable. So the Immanent Metaphysics differs from every account that places creation at a first moment, but agrees with the traditions that hold creation to be continuous.

Physicists describe the world from the third person, as structure laid out for anyone to see. In the third-person description of physics, creation, existence and interaction are separable stages. In the third-person description, creation happens first and then is done, existence is what is left once creation is completed, a standing condition, and interaction follows existence, with nothing that requires an interaction to produce anything new. In the Immanent Metaphysics the order of dependency runs otherwise: interaction precedes existence, and creation is not a single past event but an aspect of all events, present in the act here and now. In the first-person sense of process (process as I live it here and now), creation, existence and interaction are all present together.

The consequence for physics is direct: if creation is ongoing, the universe is not a system whose whole history is fixed by its initial conditions. Novelty is a structural feature of every moment: something new enters every interaction, and with the new thing comes a finding not contained in what came before. The future is what is not yet, and the future is where newness happens. If there were no newness, we could predict the future exactly. We cannot predict the future exactly. Our inability to predict the future exactly is the creative part of every event: the part of the event that is creation.

VI. What Is Inside an Interaction

The universe is creation, existence and interaction. Every interaction has six aspects. Pattern, force and probability are the content of the interaction. Space, time and possibility are the context of the interaction. Each set of three is a *triaxial*: three axes that together make up one class concept. In

physics, the content triaxial is mass (in the combined sense of inertia, pattern, form and structure), force, and probability. The context triaxial is space, time and possibility.

Every measurement is a comparison, made in the relationship between the subjective and the objective. The relationship between the subjective and the objective is the basis of both the subjective and the objective. Every comparison is also an interaction, because comparison, interaction and relation are one act named three ways. So the universe is understood in terms of interaction rather than substance. Alongside the universe, the real is understood in terms of choice, change and causality.

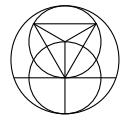
VII. The Universe as a Process

The heavy elements that life needs are made in stars, over billions of years of stellar evolution. Habitable planets depend on the history of stellar evolution that made the heavy elements. When the universe is seen through the history of stellar evolution, the universe is a process, and not a static container. The structure of the process has to be in place before there can be beings who are able to ask what the universe is.

VIII. Summary

The universe is exactly the combined meanings of creation, existence and interaction, neither more nor less. When the word "universe" is taken to refer to an undefined extent of content, the word has no semantic value. What the universe is then cannot be distinguished from what the universe is not. The word "universe" gets its meaning from the three concepts, by a denotation that makes all four concepts less vague at once. The account of the universe in this chapter depends on the inside-out notion of a domain, in which a domain is defined by the basis concepts of the domain and not by the contents of the domain, and the relations of a domain are as fundamental as the elements of the domain. When the three axioms of the Im-

manent Metaphysics are applied to the three concepts, Axiom III shows the concepts to be distinct, inseparable and non-interchangeable. Axiom I shows interaction to be more fundamental than existence and creation, and shows existence and creation to be conjugate. Axiom II sets the order of dependency, from creation to interaction to existence and back to creation. Interaction is immanent, existence is omniscient, and creation is transcendent. Creation is an aspect of every event and not a first moment. The reason is that the three concepts are inseparable, and their order is one of precedence, not of clock time. So creation is ongoing, the universe is not fixed by its initial conditions, and novelty enters every interaction. The universe is filled with content met firsthand: choices, changes and causal patterns. Each interaction has six aspects: three of content (pattern, force and probability) and three of context (space, time and possibility).



Chapter 6

The Order of Existence, Interaction, and Change

An Exercise in Axiom II, Worked in an Imagined Empty Universe

ABSTRACT

In this chapter we do an exercise that anyone can do in imagination: we start from the relative alone, put two objects in an otherwise empty universe, and reduce every kind of interaction to one kind, touching. The principle of identity says that whatever cannot be distinguished must be the same, so two objects overlapped in one place at one time are one object and overlap is not a third state. Motion, a change of state, turns out to be a third kind of concept beside existing things and their states. Whenever anyone tries to imagine a change of state, the change comes after existence and before interaction. When existence, change and interaction are read as modalities, their order is the cycle of Axiom II, which is the axiom that states the order in which the three modalities, the omniscient, the transcendent and the immanent, succeed one another. I then show four more things. First, interaction is the ground of the other two terms by Axiom I, and is the place where the cycle of process arrives by Axiom II, and the two claims hold together. Second, the modal role of change depends on the frame, the setting of the description in which change is considered. Third, the chapter states exactly what a rotation across domains is. Fourth, absolute statements need a third class of concept, and I show why.

I. Starting from the Relative

We start the exercise from the relative, that is, from what holds of a thing only in relation to something else. We assume nothing about the absolute, what would hold of a thing apart from every relation. We notice the relative, and we work within the relative.

When I measure something, I have to bring the tape measure along. I cannot assume that a thing has a size before the thing has been measured. If a single object is put in an otherwise empty universe, with nothing else, the object has no size at all, because size is relative to something else that has a size. The lone object has no measurable properties, because there is nothing to compare the lone object with.

What counts as absolute depends on the perspective, and each of the three modalities is a perspective: in the omniscient perspective the frame of reference is held absolute, in the transcendent perspective the material is held absolute, and in the immanent perspective relation itself is held absolute. The Incommensuration Theorem has a consequence for the use of the word "absolute": the relatively absolute is absolutely relative. When a thing counts as absolute within some bounded, relative frame of reference, the thing is only relatively absolute from any wider vantage.

II. An Empty Universe, Simplified Twice

Now imagine an empty universe with two objects in it, and make the imagined universe simpler in two ways. First, the universe holds almost nothing: only the two objects. Second, we reduce every kind of interaction (fields, electromagnetic influence, every force) to one kind: contact, or touching.

With touching as the only interaction that remains, the two objects have two plain states: the objects can be apart, or separate, which is the state of not interacting, or the objects can be touching, which is the state of interacting. We look at how existence and interaction relate in one setting: an empty universe of two objects that can only touch.

III. Overlap Is Not a Third State

What happens if the two objects are in the same place at the same time? The two objects would then make an overlapped object. Is the overlapped object one object, or two? There is no way to tell.

The *principle of identity* applies to the overlapped object: that which cannot be distinguished must be the same. Two objects overlapped in space and time cannot be distinguished from one object, so the descriptions "two overlapped objects" and "one object" are one concept. The three states (separate, touching, overlapped) reduce to two: separate or touching. So we have already simplified what existence and interaction mean in the exercise, by applying the principle of identity.

IV. Change of State Is a Third Kind of Concept

Where does motion belong in the empty universe with two objects? Motion is neither an existing thing nor a state, because motion is a change of state, and a change of state is not a state but something in addition to the states and in addition to the existing things.

So there are at least three classes of concept: existing things, their states (in the exercise either interacting or not), and changes of state.

V. The Imaginer and the Imagined

We imagine the whole scenario of the two objects in the empty universe, so in looking at the scenario we are also looking at the properties of imagination. What is the relation between me, the one imagining, and what I imagine?

The very act of seeing is a contact. I either see the object or I do not. Perhaps I do not see the object until the object touches me: until that moment I am blind, and then I feel the object as soon as the object hits my nose. So the relation between me and what I imagine has the same structure as the relation between two objects coming into contact.

In both relations, change is purely in neither side. The change is not purely in me as the one imagining, and not purely in the imagined objects. The change is something in addition to both the objects and their states. In the act of imagining, my own state is one of two: either I am imagining the scene or I am not imagining the scene.

So my experience of imagining runs in parallel with the scenario inside the imagined world: the experience and the scenario have the same structure, and both need the same minimum in order to be imagined at all. The minimum is three classes of concept: existing things, states and changes of state.

VI. The Order Cannot Be Changed

The three classes of concept come in an order, and the order is not arbitrary. I cannot imagine a change before I have the notion of something existing. A state is the outcome of a change, and a state cannot fail to be the outcome of a change. So however I go about imagining a change of state, the change comes after existence and before interaction, because interacting or not interacting is the state that the change brings about. The order

of existence, change and interaction belongs to what imagination itself is, and to what process itself is. The fixed order is the main result of the exercise.

One can try to reduce the three classes of concept (existing things, states and changes of state) to two. One can sit in meditation and try to imagine changes that do not change. An example is the koan (a meditation riddle) of the sound of one hand clapping. Whenever one tries to reduce the three classes to two, the result is the same: there are always three classes of concept, and the act of imagining, as a process, or the act of interacting, as a process, comes after existence and change.

VII. Reading the Order in the Modalities

Three elements arranged in a cycle have only two orientations. Draw the three elements as points on a circle, with an arrow from each element to the next, and the arrows go either clockwise or counterclockwise: however the three elements are labeled, the cycle runs in one of those two directions. So the direction of a cycle of three is a single fact, and the exercise of the empty universe fixes the direction.

A frame is the setting of a description relative to which each term is given its modal role. In this exercise the frame is the change of state. In the frame of change of state, existence is omniscient in character, laid out as structure in the third person, and interaction is immanent in character, the act itself in the first person. Change has to lie between existence and interaction: after existence, before interaction. Betweenness is the transcendent, the modality of the second person, so change is transcendent. The two objects in the empty universe have a second-person relation to each other, a relation of peers, as two billiard balls in space do. My relation to what I imagine is also a relation of peers, because under the principle of identity observer and observed are peers.

The first-person experience is fundamental: without the first-person experience we could not tell imagining from not imagining. I can be aware of myself as the creator of what I imagine, and I can also know that some of me lies beyond the imagined scene: the part of me lying on the bed while I dream. My ground is the immanent, that is, the first-person experience of second-person relations. From my ground in the immanent, I notice that every act of imagination needs a change, something coming into being. So the direction of the cycle is fixed. Every time, the flow of the cycle runs from the omniscient (existence) through the transcendent (change) to the immanent (interaction).

Axiom II states the order from the omniscient through the transcendent to the immanent, in the wording of An Immanent Metaphysics: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". In the terms of the triple of existence, change and interaction, Axiom II says that interaction precedes existence, existence precedes change, and change precedes interaction. The cycle can be entered at any point, but the direction of the cycle cannot be reversed.

VIII. Interaction Is the Ground and the Place Where the Cycle Arrives

We began the exercise from the conventional assumption that interaction depends on existence: first there are things, and then the things interact. In the Immanent Metaphysics the conventional assumption is replaced: interaction is more fundamental than existence and change, existence is defined in terms of interaction, and things exist insofar as they interact. Interaction is more fundamental because interaction is the immanent term, and Axiom I states: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another".

Two statements about interaction hold together: the immanent is the ground (Axiom I), and within the cycle of process the immanent is where the cycle arrives (Axiom II). When we find in the exercise that interaction comes after existence and change, the word "after" names a position in the cycle of process and does not mean that interaction is defined from existence or from change.

IX. Change and Creation; the Butterfly Dream

In the exercise two words name the transcendent term: "change" names what happens in the content of the imagined scene, and "creation" names the act of imagining the scene.

Where did the things in the imagined world come from? The things came from me, the one imagining. I am not part of the imagined world: I am outside the imagined world, looking at it. Relative to the world I imagine, I am the world's creator, overseeing the world from outside. With respect to the imagined world, the omniscient part of self is the part of me that creates and controls the imagined world, like a deity living outside that world. And the newness is a change: the passage from not imagining anything to closing my eyes and seeing a scene. So change and creation partake of one another: the act of creating the scene is itself a change, from not imagining to imagining.

Change and creation partake of one another so closely that I might not be able to tell whether I am myself an element of someone else's larger imagination. The ancient Chinese philosopher Zhuangzi dreamed that he was a butterfly, and afterward wondered whether he was a philosopher who had dreamed of being a butterfly, or a butterfly dreaming that it was a philosopher. No test, even in principle, distinguishes the philosopher who dreamed of a butterfly from the butterfly who dreams of a philosopher, so by the principle of identity neither possibility can be authoritat-

ively preferred. Someone might prefer the philosopher because a philosopher has more cognitive complexity, but that preference assumes a basis that we have no reason to allow.

X. What the Exercise Leaves

The exercise leaves three concepts (existence, change and interaction), and in the first person the three concepts come in a definite sequence. Anything more is an addition to the three concepts and their sequence. The three concepts in their first-person sequence are a statement of Axiom II.

XI. The Role of Change Depends on the Frame

In this exercise the frame is the change of state, and in the frame of change of state, change is transcendent: a change across possibility, in neither of the two states, separate or touching.

In the triple of the real (choice, change and causation, which Chapter 13 sets out), the frame is the actual event, and in the frame of the actual event change is immanent, choice is transcendent, and causation is omniscient. The cycle of the triple of the real runs from choice to change to causation and back to choice. So change is given two modalities: transcendent in the frame of change of state, and immanent in the frame of the actual event. The two assignments do not conflict, because each assignment belongs to its own frame, and an assignment and its frame are named together.

XII. Rotation Across Domains

An *Axiom II rotation* is a different thing from Axiom II itself, and we must keep the difference exact. Axiom II is the flow of the modalities within one domain, one realm of conception complete unto itself. An Axiom II

rotation is a shift of one position, in the order of Axiom II, in passing from one domain to its successor, where the two domains are related by process.

In a rotation, what is characterized as omniscient in one domain is characterized as transcendent in the next domain. Or what is transcendent in one domain becomes immanent in the next, again by a shift of one position. So a person can manifest something in the immanent of the successor domain by working in the transcendent of the prior domain.

An Axiom II rotation needs a process connecting a domain to its successor. Consider the first-person triple of the real (choice, change and causation) and the third-person triple of the universe (creation, interaction and existence). The relation between the two triples is a rotation in a way, but not a clear case of a rotation. Both triples are universals (concepts that apply in every case), and universals are not very mutable: no change in the structure of either triple is really seen, and no process really connects the two triples dynamically.

XIII. Absolute Statements and the Third Class

Consider the statement "everything is permissible and nothing is true". The statement has power as a working tool for exploring perception and changing beliefs. As a categorical statement, claimed to hold without exception, the statement is self-refuting. Is the statement itself absolutely true? If the statement is absolutely true, then at least one thing is true, and so the statement is false by its own words.

The self-refutation comes from thinking in only two categories: true or false, absolute or relative. The resolution is a third class of conception, falling neither under truth nor under falsity: the immanent act of finding out. The strength of the axioms lies in the immanent act of finding out.

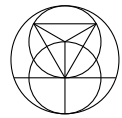
Every absolute claim about reality fails at the very thing that the claim denies. The claim that all is knowable fails at the unknowable, which is what no being can know at any time, even in principle. The claim that

nothing is certain fails at the certainty of the claim itself. Only a framework of three terms is free of both failures: a framework in which the known, the unknown and the unknowable are distinct.

XIV. Summary

When we start from the relative, we find that an empty universe with two objects and only one kind of interaction (touching) needs three classes of concept: existing things, their states, and changes of state. The principle of identity shows that two overlapped objects are one object, so overlap is not a third state. Motion is not a state, so change is a class of its own. The same three classes of concept are needed for the act of imagining the scene. In the scene and in the act of imagining, the order is fixed: change comes after existence and before interaction. When existence, change and interaction are read as modalities, existence is omniscient, change is transcendent and interaction is immanent, and the order of the three is the irreversible cycle of Axiom II.

Interaction is at once the ground (Axiom I) and the place where the cycle of process arrives (Axiom II). In the exercise the word "after" names a position in the cycle, and does not mean that interaction is defined from existence or change. Change and creation partake of one another, as Zhuangzi's dream of the butterfly shows. The modal role of change is named together with its frame: change is transcendent in the frame of change of state, and change is immanent in the frame of the actual event. An Axiom II rotation across domains is a shift of one step between domains related by process, and is distinct from Axiom II itself. In a scheme of two categories, true or false, absolute statements are self-defeating. Absolute statements are resolved in a third class: the immanent act of finding out.



Chapter 7

Real, Existent, Objective

Three Distinct Claims About What Is, and the Relationship as a Third Thing

ABSTRACT

Three different claims can be made about a thing: that the thing is real, that the thing exists, and that the thing is objective. To be real is to be given in an actual act of comparison, here and now: in exactly one interaction. To exist is to have a place in a pattern, as structure, and what exists is repeatable, locatable and shareable, and involves no interaction of its own. To be objective is to hold across many interactions, as what many comparers would find. One tells the three claims apart by counting interactions. Existence involves none, the real involves exactly one, and objectivity involves many. The three claims form a triple: the three are distinct, inseparable and non-interchangeable. Three examples separate the claims: time, as the order of acts, is real and does not exist; pi exists and is not real; and interaction itself is real and does not exist. The relationship between the subjective and the objective lies underneath the three claims, and the relationship is a separate ontological class (a basic kind of being), more fundamental than either the subjective or the objective. This chapter sets out the three claims and the examples that separate the claims, and contains the argument that the relationship between the subjective and the objective is a third thing beside the two. Finally, the chapter shows what changes once we stop treating existence and the real as synonyms.

I. Three Claims About What Is

The word "real" has to do with the relationship between the subjective and the objective. The word "real" names the idea of interaction, taken as primary: taken as coming before everything else. To be real is to be given in an actual act of comparison, here and now: in exactly one interaction. A comparison is the act of finding whether two things are the same or different, in some respect (Chapter 2), and in this book an interaction is the same act under another name. The real is in the first person: the real is what is given to me, in my own act of comparing.

To *exist* is a claim of another kind. The claim is made in the third person, about what anyone at all could be told. The claim that a thing exists is the claim that what is structural in a set of interactions is common to all of the interactions, whoever compares. What exists is structure: a thing that exists has a place in a pattern, and the thing is repeatable, locatable and shareable, the kind of thing a third person could be told about. Existence needs no interaction of its own. In more technical terms, existence concerns three questions: whether something is a member of a set, whether there is a potential interaction that would establish the membership, and, deeper still, whether any such interaction could occur at all.

To be *objective* is a third claim. The claim concerns the convergence of perception across many subjects, where the perceptions of many people come together on one result. A thing is objective when the thing holds across many interactions, as what many comparers would find.

One example, you and I, holds all three claims apart. You are real to me because I interact with you. You exist because you have a place, not relative to me, in the pattern that a third person could be told about. You are objective because many people could interact with you and find what I find, and so we can identify that you and I are part of a common context. A law exists, because a law is a pattern. A measurement is real, because a measurement is one act of comparison.

The real is relative to the subject, and the subject is relative to the real. So the real and the subject are interdependent. Because the real is relative to the subject, a thing can be real to me and not real to you. Existence does not mean "real to someone". The claim that a thing exists is a different claim altogether, a claim in the third person about a place in a pattern.

II. Time Is Real and Does Not Exist

To say that something exists is to say that the thing is present as structure. Structure is timeless and the same in every instance, like the pattern of a law.

The time I live is not timeless structure, because the time I live is the order of my findings, one finding after another, given in each act, now, with the future still open. The time I live is always in process and always relational. The time I live cannot be pinned down as a static content in a static context. In a comparison, the content is what the verdict of the comparison is on, and the context is the respect in which the two things compared are two (Chapter 2). Time is an intrinsic of change and of every event: time is a feature that every change and every event has by nature, and change is not a thing that exists in the way a rock exists. The time I live is real, because the time I live is given in every interaction, at every moment, but the time I live does not have the character of existence, as this chapter defines existence, since the time I live has no fixed place in a timeless pattern.

Physicists also lay time out together with space as one pattern: geometric time. Geometric time is structure, and so geometric time exists. So there are two things called time. *Event time* is the order of acts, the time I live. Event time is real and does not exist. *Geometric time* is the pattern that physicists lay out, and geometric time exists.

III. Pi Exists and Is Not Real

Pi exists as a mathematical object. Pi has a perfectly clear definition: pi is the ratio of the circumference of a circle in a flat plane to the circle's diameter. Pi exists in the domain of the real numbers, and in every domain of mathematics that contains the real numbers. The word "real" in "real numbers" is only part of a name, the mathematician's name for the numbers of the number line, and the name does not use the word "real" in the sense of this chapter.

Pi is not real. I can interact with representations of pi: I can write down the digits of pi, and I can use pi in calculations, but in each case my interaction is with a representation of pi, and not with pi itself. Pi names a relationship between a diameter and a circumference, and the relationship is incommensurable: no unit, however small, measures both the diameter and the circumference a whole number of times. So no decimal expansion of pi ever ends or repeats, a fact that the mathematician Johann Heinrich Lambert established when he proved that pi is irrational, that is, not a ratio of two whole numbers. Because the relationship is incommensurable, every act of measuring a wheel gives a ratio of whole counts of some unit, never pi exactly. Pi is present as structure, the same in every instance, and being present in that way is what it is to exist.

A finite algorithm is a recipe of finitely many steps, which can be carried out over and over. Pi can also be specified as the result of a finite algorithm when applied without limit (Chapter 31, Section VII). Every finite run of the algorithm gives finitely many digits of pi, never pi exactly, just as every act of measuring a wheel gives a ratio of whole counts of some unit.

IV. Told Apart by Counting Interactions

The examples of Sections I to III give four kinds of case. Some things are real and do not exist, as event time is real and does not exist. Some things exist and are not real, as pi exists and is not real. Some things are both real and existent, as you are to me in the example of Section I. The fourth kind of case is a separate matter: whether a thing is objective or not.

Real, existent and objective are themselves a triple: the three are distinct (none overlaps another), inseparable (none occurs without the others) and non-interchangeable (none can take the role of another). One tells the three apart by the number of interactions each involves. Existence involves no interaction. The real involves exactly one. Objectivity involves many.

The three modalities of Chapter 3 are three roles that the members of a triple play toward one another. The immanent is the role of the act itself, as the act happens. The omniscient is the role of what is laid out as structure and held in view as a whole. The transcendent is the role of what exceeds any one instance. In the triple of real, existent and objective, the real has the role of the immanent, existence has the role of the omniscient, and objectivity has the role of the transcendent. The roles match the counts: the real is exactly one act, existence is structure laid out with no act of its own, and objectivity exceeds any one interaction. In the pair of the subjective and the objective, taken by itself, the objective has the role of the omniscient. So objectivity has the role of the transcendent in the triple, and the objective has the role of the omniscient in the pair. The two assignments do not conflict, because a modality is assigned to a term only relative to the triple in which one considers the term.

Interaction itself is real and does not exist, since an interaction is an act here and now, not a place in a pattern.

V. Interaction Comes First

In the scientific method, scientists put hypotheses to the test, and measurement is a fundamental concept of the scientific method. In every measurement we impute the thing measured and the thing that measures, that is, we take both to be there. Measurement, the act of observation, is the ground on which we identify that there is something at all.

What must be in place before I can say, on the ground of measurement, that a thing is there? If I can interact with something, if I can measure it, if there is some possibility of an engagement that can be confirmed, then I can say that the thing exists, or that the thing is objective. In each of the three conditions, I take interaction as a concept prior to "exists" and to "is objective". To be real is to be in an actual interaction, here and now. So being real is not established after an interaction in the way that existence and objectivity are. Interaction is real, and existence and objectivity are abstractions of interaction, concepts drawn from interaction by taking detail away (Section X).

The relationship between the subjective and the objective is real, because if the relationship were not real, we would have no way of thinking or talking about the subjective or the objective or the relationship between the two. Anyone who asks "is choice real?" is led straight to a prior question: by what method can we know anything at all? The first paragraph of this section gives the answer: we know through the act of measurement, of observation, which is an interaction.

Objectivity, in turn, has a use. Objectivity has three parts: actual and intense interaction; consistency within closed rings of perception among many observers, in which what each observer perceives is checked against what the others perceive; and openness to novel and surprising phenomena. Through objectivity, people necessarily come more in touch with real-

ity than other creatures do, at a causal level (the level of what causes what), and human perception converges toward the base reality of the world we share.

VI. The Relationship Is a Third Thing

Any relationship involves at least three ontological categories (three basic kinds of being): the two things related, and the relationship itself. A relationship is not identical with either of the two things that the relationship relates. So every relationship, intrinsically, has three aspects.

I argue as follows that a relationship is its own category. Take content and context, the two terms of Chapter 2. Is the relationship between content and context a member of the content, or a member of the context? Suppose I make the relationship an element of the context. Then the relationship becomes context, and there is now a new relationship between the content and the enlarged context: the relationship recurs. Suppose instead I make the relationship an element of the content. The relationship recurs in that case too: the relationship becomes content, and a new relationship between content and context appears. Every time I try to embed the relationship in the content or in the context, a relationship between content and context emerges again. So I must treat the relationship as a third thing, neither content nor context, and yet I cannot have a concept of content or of context without the concept of the relationship between content and context. Content, context and the relationship between them are bound together as an indelible triple, a triple that cannot be taken apart.

The same argument applies to the relationship between is and is-not. The relationship between is and is-not neither is nor is-not. If I make the relationship part of "is-not", I lose "is". If I make the relationship part of "is", I lose "is-not". So the relationship between is and is-not must be its own category, a class of being of a deeper order than is-ness and is-not-ness. The relationship between truth and falsity is also its own category: the difference between the true and the false is neither true nor false.

Engineers know the same pattern, a difference that is neither of the two things it distinguishes, from Boolean logic, the logic of the circuits in a computer, in which every value is either a zero or a one. There is a zero, and there is a one. A one and a zero differ, and whatever makes each differ from the other is neither a zero nor a one. If nothing made a zero and a one different, a zero and a one would be the same, by the *principle of identity*. The principle of identity states that two things that cannot be distinguished by any property, logic, measurement or interaction, not even in principle, must be the same, in identity and in being. So the thing that makes zero and one different must be a different kind of difference from zero and one themselves.

A map and a territory show the pattern once more, with the correspondence between map and territory as the third thing. There is the territory, there is the map, and there is the correspondence between the map and the territory. The map is not the territory and the territory is not the map, while the correspondence is neither the map nor the territory, and the correspondence is an ontological category of its own. So to reason about what is, I need a third concept beside map and territory, and the concept I need is the correspondence. Meaningfulness is in the correspondence, not in the map and not in the territory. A map means something only through the correspondence of the map with a territory.

One consequence is that duality, as a philosophical construct, cannot be only dual, and a duality must be at least triple, because the relationship between the two terms of a duality is a third thing. A dualist posits two independent things, and a dualist can say how the two things relate to each other only by admitting the relationship between the two things as a third.

VII. More Fundamental Than Either Side

The first premise of the Immanent Metaphysics is that the relationship between the subjective and the objective is not a proper member of the subjective, and not a proper member of the objective. The first premise becomes Axiom I, the first of the three axioms of Chapter 3. Axiom I, in its general form, reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". Two concepts are conjugate when each can be perfected only at the expense of the other, and neither is prior to the other in definitional basis (Chapter 4). When Axiom I is applied to self and world, the relationship between the objective and the subjective has the role of the immanent, and Axiom I then says that the relationship between the objective and the subjective is more fundamental than the nature of the objective and/or the nature of the subjective.

The words "more fundamental" say more than the words "its own category" of Section VI. I have a concept of subject or of object only when I take the relationship between the two as the defining basis for both. The same is true of zero and one, and of content and context: I have a concept of either term only when I take the relationship between the two terms as their defining basis. In Axiom I, to be "more fundamental" than two terms is to be the defining basis of the two terms.

Two examples show what Axiom I means in practice, and the first example is counterintuitive. A salt shaker and a pepper shaker stand on a dinner table. The relationship between the two shakers is given in an actual interaction, such as my comparing the shakers, and as given in that interaction, the relationship is real at a more basic level than either the salt shaker or the pepper shaker. The shakers exist, and the relationship between the shakers is real. In a fully objective framing, which describes

the table as objects in the third person, the claim about the relationship feels wrong. In a personal framing, which describes my own act of comparing the shakers, the claim becomes plain.

The second example is personal. Imagine being about three years old. Your father brings you to an ice cream parlor for the first time in your life. You walk up to the ice cream case and look in, and you see bins of colored stuff. The proprietor leans over the counter and asks, "What flavor would you like?" Perhaps you have never heard the word "flavor" before. You gather that you are supposed to pick a flavor, but on what basis could you pick one? You have never tasted any of the flavors, so you have no basis yet. Flavor becomes a category for you only through your relationship to tasting, and so far, flavor is not yet a category for you. You are the perceiver, the ice cream is the perceived, and the relationship between the two creates the category of flavor. Preference, and kinds of flavor, arise only through the relationship between perceiver and perceived. In the ice cream parlor, the relationship is more fundamental than either the subject or the object.

Light gives an image of why the relationship itself has to be studied. Light cannot perceive itself. Two laser beams can cross, and through the electromagnetic field alone the wavefronts of the two beams pass through one another unchanged, because the photon, the particle of light, carries no electric charge. In the image, light stands for the comparing act, and the comparing act is not one of the things it compares. (Light does act on light through pairs of charged particles, but so slightly that no lamp or laser makes the effect large enough to see.) In the same way, I can perceive only the object of perception: I cannot perceive the perceiver, and I cannot perceive perception. Of the perceiver, the perceived and the perceiving, perception can perceive only one: the perceived. The process of perception is more fundamental than both the perceiver and the perceived, and so the relationship itself is what must be studied.

VIII. Kant's Target

The philosopher Immanuel Kant held that we cannot know anything about what actually is, because we can know only our experience of sensing, which has an abstract and imperfect relationship to what is. Kant's critique is correct for its target, because no one can know pure existence (existence taken apart from every interaction) through sense data, the data of the senses. But metaphysics needed only to describe the real, and metaphysics never needed to know pure existence. Kant's critique is therefore correct against its target, and the critique does not apply to this metaphysics, a metaphysics of the real. The realness of what I find is in the observation itself, in the interaction of the subjective and the objective. The realness is not in what lies beyond the observation. Kant took the omniscient, what is laid out as structure and held in view as a whole, for the whole of what is, and went straight ahead from that assumption, while I turn aside at that assumption, and so my conclusions differ from Kant's conclusions. So metaphysics is once again a real topic: a topic of actual inquiry, and a topic about the real.

Turning aside from Kant's assumption has consequences. Meaningfulness belongs to the real, not to existence, so we have to discuss the meaning of ontology (the study of what is) and of epistemology (the study of how we know) in terms of the real. Causation, or causality, is a regularity in time, a lawful and repeatable pattern, while determinism is the claim that one law fixes every state of a whole world completely (Chapter 12). Treating existence and the real as one thing lets a person think that determinism and causality are one thing too. To treat existence and the real as one is *omniscient bias*: taking what is laid out as structure and held in view as a whole for the whole of what is, or for its ground. Once we see the real and existence as distinct, we can conclude that the scientific method actually produces knowledge of causation, and so scientific knowledge is causal knowledge, not deterministic knowledge. To hold that scientific

knowledge is deterministic knowledge is to conflate mathematics with physics, and mathematics and physics are not the same: determinism is true within a mathematical model, and no actual world is fully determined (Chapter 12).

The real concerns choice, change and causality, and choice, change and causality are the dynamics of mediation in an interaction, the ways in which an interaction relates each of its two sides to the other. Existence concerns the relationships among existing things, in the third person. Confusing the first person and the third person is the same confusion as confusing "exists" and "is real". The real is first person, and existence is third person.

The present moment is unique in our experience, and yet physicists treat that experience as an epiphenomenon (a by-product with no effect of its own), or as an illusion. Treating existence and the real as synonyms produces the paradox of the present moment and other paradoxes of the same kind. Once we see that the omniscient, the transcendent and the immanent are three distinct roles, the questions behind the paradoxes change their character and many of the paradoxes go away.

IX. The Real and Reality

The real and reality differ. The real is an instance of a firsthand interaction, while reality is the plural of what is real, and not the totalization of what is real, the adding up of everything real into one whole. No one can perform a totalization of everything real. Reality is a synthetic concept: a concept of composition made by putting instances of the real together. The real can be taken as real, as the real is given in an interaction, while reality can only be assumed. The universe is likewise an assumed concept.

The universe is not the sum of all that is real, because the universe is the combined meanings of creation, interaction and existence (Chapter 5), not a sum of instances, and there is no one single real reality or universe. One could construct the sum of all that is real only by assuming some-

thing that is not true. The concept of the real is in a kind of orthogonality to the concept of the universe, like the perpendicular axes of a right-hand coordinate system, which is three axes at right angles to one another, as on a graph in space. As no amount of travel along one axis is travel along another axis, no amount of gathering instances of the real arrives at the universe, and neither concept is built out of the other.

I consider the domain of the real in terms of beingness, which is distinct from existence. The real can be understood only in terms of process: being is a process, and so the real is a process. Reality, as the plural of the real, needs no totalization and no bound.

X. Abstraction, Instruction, Information

Abstraction means "from structure". To abstract is to take detail away from a form and leave a simpler, sparer form. Abstraction is also a disembodiment, a move from the physical to the virtual, from things to what represents things. Whenever I move into something that represents something else, as a word represents a thing, I move into a domain other than the world of things to see and touch, and the move is an abstraction. A subway map is an abstraction: the map leaves out the details and keeps the general relationships, and so the map is simpler to use than the territory, the city itself.

Instruction means "into structure". Instruction is the reverse of abstraction, a move from something abstract to something concrete, with more detail. Perception and expression run in opposite directions across the boundary of the self (Chapter 10). In perception the flow is from the objective toward the subjective, and the dynamic of perception is abstraction: details are removed. In expression the flow is from the subjective toward the objective, and the dynamic of expression is instruction: details are added.

Information means "into form": a plurality of formlessness becoming a singularity of form. The definition says what information is, and the definition is not a theory of why formlessness becomes form. The definition needs no model in terms of energy flows or temporal flows, because the notion of a flow already presumes form. There is no concept more basic than formlessness, form, and the relationship between formlessness and form.

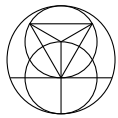
In the strict sense the future is unknowable, and the future becomes known in first-person experience, here and now. Axiom II states the only direction in which the three modal roles follow one another (Chapter 3). Axiom II describes, as far as metaphysics can, how the future becomes known: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent".

XI. Summary

Real, existent and objective are three distinct claims, told apart by how many interactions each involves: the real involves exactly one interaction (first person, here and now), existence involves none (third person, a place in a pattern), and objectivity involves many (what many comparers would find). In the triple, the real is immanent, existence is omniscient and objectivity is transcendent. Event time (the order of acts) is real and does not exist, while geometric time exists. Pi exists and is not real: pi is the ratio of the circumference of a circle in a flat plane to its diameter, and pi is also the result of a finite algorithm when applied without limit. Interaction is real and does not exist, and existence and objectivity are abstractions of interaction.

The relationship between the subjective and the objective is a separate ontological class, as are the relationships between content and context, between is and is-not, between true and false, and between map and ter-

ritory. By Axiom I, the relationship is more fundamental than either the subjective or the objective, because the relationship is the defining basis of both. Kant's critique holds against pure existence, and the critique does not apply to a metaphysics of the real. To treat existence and the real as one is omniscient bias, and omniscient bias is what lets people mistake causation for determinism and call our experience of the present moment an illusion. Reality is the plural of the real and not the total, and the universe is not the sum of all that is real. Abstraction is the direction of perception, and instruction is the direction of expression. Information is formlessness coming into form.



Chapter 8

Process, Novelty, and Omniscient Bias

Why Theory Must Resolve to Practice

ABSTRACT

Process is partly thing, partly change, and partly what is neither thing nor change and is actually new. When we describe a process by its things and changes alone, we leave out its creative part, the part that is new. Omniscient bias is taking what is laid out as structure and held in view as a whole for the whole of what is, or for the ground of what is. Describing process by things and changes alone is one form of omniscient bias. Theory is held in the third person and has the role of the omniscient, while practice is held in the first person and has the role of the immanent. In theory, theory and practice are the same, but in practice theory and practice are different, and at some level theory must resolve to practice, to an act done in the first person. This chapter states what process is, what omniscient bias is, and why the Immanent Metaphysics, as a theory of the relationship between self and reality, is a theory about the reader's own self.

I. What Process Is

We tend to think of process as the transformation of things. The transformation of things is only part of what process is, because process is partly thing, partly change, and partly what is neither thing nor change and is actually new. Nothing that came before contains what is actually new.

Western culture tends to describe process purely in terms of existence (the things) and interaction (the changes), and so the creative aspect of process is missing from a description of process in those two terms alone. Chapter 5 showed that the word "universe" gets its meaning from three concepts: creation, existence and interaction. In a process described only by existence and interaction, the concept of creation is missing, and yet creation is present in every event, here and now, as the part of the event that is new.

II. Omniscient Bias

The *omniscient* is the role of what is laid out as structure: a pattern, a map, a record, what can be held in view as a whole and shared. A map of a city is one case of the omniscient, because the map lays the whole city out on one sheet, which a reader can take in at a glance and show to anyone. *Omniscient bias* is taking the omniscient for the whole of what is, or for the ground of what is. Omniscient bias is like taking the map of a city for the city itself: the map stands for what is laid out as structure, and the city stands for the whole of what is.

One form of omniscient bias is to describe a process by its things and its changes alone. When physics is viewed from the third person, physics shows the form that omniscient bias takes: creation happens first and then is done, and existence is what remains once creation is done. In the third-person view of physics, interaction follows existence, and nothing requires

an interaction to produce anything new. Yet the future is what is not yet, and the future is where newness happens. If there were no newness, we could predict the future exactly. The creative part of process is the fact that we cannot predict the future exactly.

III. Theory and Practice, Third Person and First Person

Theory is held in the third person, as a description that anyone could be told about, and theory has the role of the omniscient. Practice is held in the first person, by the person who acts, and practice has the role of the immanent: the role of the act itself as the act happens here and now. A measurement is an act made in the first person, while a calculation is structure held in the third person. The first-person stance is the position from which an act proceeds. The first-person stance is transcendent in character: the first-person stance has continuity with asymmetry, which is the character of the transcendent. Chapter 2 defines continuity as sameness of content where there is a sameness of context. Chapter 2 also defines asymmetry, as difference of content where there is a difference of context. The first-person stance is met in the act, and the act is immanent.

An old saying states the relation between theory and practice exactly: "In theory, theory and practice are the same. In practice, theory and practice are different". At some level, theory must resolve to practice: at some level, a theory must come down to an act done in the first person. The practice that a theory resolves to will be inherently different from the theory, even though, by the theory, theory and practice "should be" the same.

A purely omniscient theory is a theory laid out wholly as structure, in the third person. Computation is the plainest case of a purely omniscient theory (Chapter 32). The point that theory must resolve to practice appears again when we ask where a purely omniscient theory's starting information comes from. Suppose that everything in a theory is derived from a small set of rules and starting conditions. The rules and starting conditions must still be taken as given. From inside the theory, no one can

answer the question of where the information in the theory's rules and starting conditions comes from, so at best the question is left unanswered and recognized as unanswerable.

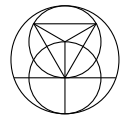
IV. A Theory About the Reader's Own Self

The Immanent Metaphysics is the metaphysics on which this book rests. Readers will understand the Immanent Metaphysics as an intellectual construct, a theory. The Immanent Metaphysics is a theory of the relationship between self and reality, and that description of the Immanent Metaphysics is meant literally. In the phrase "the relationship between self and reality", the word "self" means the subjective self as well as a self considered objectively: the word means your own personal self, in the first person, and not only some distant other self. Because the self in the Immanent Metaphysics includes your own self, the implications of the Immanent Metaphysics are personal. What the Immanent Metaphysics says about practice is said about your practice.

V. Summary

Process has three parts: thing, change, and what is new. Creation is missing from a description of process by things and changes alone, and leaving creation out is one form of omniscient bias. Omniscient bias is taking what can be laid out as structure for the whole of what is, or for the ground of what is. Theory is third person and omniscient, while practice is first person and immanent. The first-person stance is transcendent and is met in the act that proceeds from the stance. In theory, theory and practice are the same, but in practice theory and practice differ. At some level, theory must resolve to practice. In a purely omniscient theory, one question must be left unanswered: the question of where the theory's own starting information comes from. The Immanent Metaphysics is a theory of the relationship between self and reality. The self in the Immanent

Metaphysics includes each reader's own self, in the first person. The Immanent Metaphysics is therefore a theory about each reader's own self, and the implications of the Immanent Metaphysics are personal.



Chapter 9

Three Orders of Knowledge

Causal, Analytic, and Metaphysical

ABSTRACT

There are three orders of knowledge: three kinds of knowing, each given by a different kind of inquiry. Experiment is the source of causal knowing, the knowledge of what actually happens and of what causes what, while proof is the source of analytic knowing, the knowledge of what must follow within one domain, where the domain is fixed by the axioms of the domain. Metaphysics is the source of process knowing. Process knowing is the knowledge of what cannot fail to be the case in any act of comparison at all, in every world, actual and potential. A comparison is the act of finding whether two things are the same or different, in some respect. We gain process knowing, the third order of knowledge, by clarity in defining terms, and not by perceiving something outside ourselves or by deriving a conclusion from premises. When we define our terms clearly, we find that the six intrinsics of comparison (the six features that every comparison has) are already in use: to doubt the intrinsics is already to use the intrinsics. The disciplines form an order of depth: metaphysics, then physics, then computation, then mathematics. Every discipline after metaphysics presupposes the preceding discipline in the order of depth. A descent of grounds is a series of questions: we ask what each discipline rests on, and then what each ground rests on in turn. The descent of grounds ends in a metaphysics that is its own metatheory: the metaphysics is also the theory that describes the metaphysics. This

> chapter sets out four things: the three orders of knowledge, the order of depth, the structural layer beneath physical law, and the boundaries within which the scientific method works. The structural layer consists of the axioms of the Immanent Metaphysics, the theory of the relationship between self and reality on which this book rests, and of the principles that follow from those axioms.

I. Causal Knowing

Science, strictly speaking, is the description and explanation of causality. Causality, which this book also calls causation, is a regularity in time. A regularity in time is the same relation of earlier conditions to later conditions, found again and again from case to case. Technology, strictly speaking, is the use of causality to predict and to prescribe. So causality defines both practices: describing and explaining causality is what makes science science, and using causality is what makes technology technology.

The scientific method is a causal epistemology, a way of knowing that is optimized for one kind of knowledge, the knowledge of what causes what, because science describes causality. An experimenter forms a hypothesis and tests the hypothesis through an actual process, a process that touches what is real. The real is what partakes at once of choice, change and causality (what has a share in all three). An experiment is an act in the real. When a hypothesis is verified, we have causal knowing: knowledge of the form "with these antecedents, we expect these consequences". The experimenter tests whether causality actually links the stated antecedents to the expected consequences. Hypothesis testing is an instrument for mapping causation in the natural world, that is, for finding out what causes what. Natural law is the total knowledge of causation in the natural world.

The scientific method moves from the first person to the third person: from what is given to me in my own act, to what anyone at all could be told. In the first person, the experimenter has a hypothesis, checks the hy-

pothesis against nature, and attunes to nature. Then the experimenter discerns what the result of the check shows. The experimenter then translates the knowledge gained into a third-person form. At the end there is a consensus: each of us has done the experiment, each of us has done the attunement and the discernment, and we have compared notes. The consensus is a kind of wisdom, of the form "this is the way the world works". The body of scientific knowledge accumulates over time by the movement from the first-person experiment to the third-person consensus.

The scientific method rests on a symmetry: the experiment I do today and the experiment you do tomorrow, done the same way, give the same result. A result counts as evidence because the result is the same across a difference of experimenter and of day. I define a symmetry as a sameness of content where there is a difference of context. In the experiment done today and tomorrow, the content is the result, and the context is who does the experiment and on which day. The scientific method is excellent for knowing the world, and less useful for knowing ourselves and one another, because the method needs symmetries, and people change so much that underlying symmetries are hard to find in people. We understand a person more through continuity (a sameness of content where there is a sameness of context) than through symmetry, and so understanding a person calls for a kind of knowing that science and technology do not supply.

Modernism is the belief that all problems can be solved with causation. Solving problems with causation has worked reasonably well for a wide variety of things, and we are now meeting the limits of modernism.

II. Analytic Knowing

Causal knowing is the first of the three orders of knowledge, and the second is analytic knowing, the knowledge given by proof. Mathematical proof is the method of analytic knowing. We construct transformations, steps that turn statements already established into new statements, start-

ing from known theorems or from stated antecedent conditions. The proofs we construct are themselves theorems. A proof shows us what must follow within one domain. A domain is a realm of thought complete in itself, such as the geometry of the flat plane. The domain is fixed by its axioms, the statements that every proof in the domain may start from. All of mathematics is analytic knowledge.

A formal proof establishes that a statement follows from certain axioms. A formal proof does not establish that the axioms are true in any absolute sense, or that the axioms correspond to reality. So the truth that a proof establishes is conditional: if the axioms hold, then the conclusion follows. The correctness of the axioms is a different question, because a proof does not show whether the axioms are correct, or whether the axioms arise from something non-arbitrary, something that could not have been chosen otherwise.

III. The Third Order: Knowing Process

Empirical knowledge is the knowledge given by experiment, and analytic knowledge is the knowledge given by proof, but there is also a third order of knowledge, which this book calls process knowing. The Immanent Metaphysics is process knowing: knowledge of the third order. Process knowing depends on recognizing that there is a relationship between analytic knowledge and empirical knowledge, and that the relationship is a kind of process. Process knowing is the epistemology of process itself: an epistemology is a way of knowing, and the epistemology of process is the way of knowing that belongs to process itself.

We can know the nature of process, because to doubt process is already to instantiate the intrinsics of doubt, and to have a comparison is already to instantiate the intrinsics of comparison. The intrinsics of an act are the features that the act has by its nature. To instantiate the intrinsics is to carry the intrinsics out in an actual case: whoever doubts is already doing everything that doubting involves. Knowledge of the nature of process

comes from the act of defining terms, and not from the analysis of given premises or from outside perception. We find the intrinsics of comparison, and we do not choose which intrinsics a comparison has: if you try to describe a comparison that lacks any one of the intrinsics, the description stops making sense.

Three sets of six show what it is to find intrinsics rather than to choose intrinsics. We cannot have a notion of comparison without the six intrinsics of comparison: sameness and difference, content and context, and the subjective and the objective. The content of a comparison is what the verdict of the comparison is on. The context is the respect in which the two things that a comparison compares are two. We cannot have a notion of interaction without the six measures of interaction: pattern, force and probability, which are the three measures of content, and space, time and possibility, which are the three measures of context. We cannot have a notion of a question without the six kinds of question: who, what, when, where, why and how.

Process knowing shares a feature with each of the other two orders of knowledge. Yet process knowing is neither of the other two. Like empirical knowledge, process knowing rests on consistent patterns of correspondence, but process knowing involves no outside perception. Like analytic knowledge, process knowing is an interior noticing of how patterns join, but process knowing is not derived from premises, because clarity in the process of defining terms is the source of process knowing.

A nature of process is the correspondence between expectation and realization, between what we expect and what actually comes about. The knowing of that correspondence is a third order of epistemic process: the epistemology of process itself. Analytic process and empirical process are both cases of process in general, so the way of knowing that belongs to process itself is the full generalization of analytic knowing and of empirical knowing, and therefore the full generalization of epistemology. We can put each of the three orders of knowledge, analytic, empirical and process knowing, in one line. Proof shows what must follow, within one domain

fixed by the axioms of the domain. Experiment shows what actually happens. The Immanent Metaphysics shows what cannot fail to be the case in any act of comparison at all, in every world, actual and potential.

IV. What Is Arbitrary and What Is Not

Some things are entirely arbitrary, that is, open to free choice, and rightly so. A painting is meaningful to the degree that I could have painted anything, and painted this one painting. Choices of taste and of custom are open, and rightly so. Other things are not open to choice. The number pi is not up for debate: the digits of pi are what they are. We must account for the way nature actually works, for real physics and for real logic, all three of which belong to the omniscient, the role of what is laid out as structure and can be held in view as a whole. The omniscient includes logic, mathematics and the way things actually are.

The basis of value is not arbitrary. The principles of ethics are not relative to a person or to a culture. The three axioms of the Immanent Metaphysics are not arbitrary either, because we find the axioms already in use in every comparison, and to doubt one of the axioms is already to use that axiom. Finding the intrinsics of comparison already in use is also how we know that nothing has been overlooked in the list of the intrinsics: if any one of the six intrinsics is taken away, there is no comparison, and no comparison needs a seventh intrinsic.

Clarity of feeling has a requirement of its own. Clear feelings and clear knowings require enough development of the intellect to hold the distinctions that the feelings and knowings involve. Feeling is not intellectual, but clear feeling needs the intellect's capacity to keep distinct things distinct.

V. The Order of Depth

We need three bodies of knowledge, and not only two: mathematics, physics and metaphysics. Mathematics is the study of patterns. Physics is the study of the correspondence between patterns and reality. Metaphysics is the study of what the correspondence between patterns and reality means: the meaningfulness in that correspondence. In an ethics built from rationality alone, science and mathematics are the summit of all knowledge. An ethics built from rationality alone is good as far as the ethics goes, and the ethics is not enough.

The order of presupposition puts metaphysics first, then physics, then computation, then mathematics. Each discipline after metaphysics cannot be carried out, stated or checked without the preceding discipline. Each discipline before mathematics can be carried out without the next discipline in the order. Mathematics cannot be carried out, stated or checked without computation. Computation cannot be carried out without physics, and physics cannot be carried out without the act of comparison. The order of presupposition has one direction. The order is an order of presupposition and not a separation, because metaphysics and physics are distinct, inseparable and non-interchangeable: metaphysics and physics do not overlap, neither occurs without the other, and neither can replace the other. Metaphysics carried out without physics has no specific content to give metaphysics empirical purchase, a hold on what experiment can check, while physics carried out without metaphysics has no account of what physics presupposes. Physics is a particular instance of a more general metaphysics.

We can also read the order of depth from the other end, adding one thing at each step. Mathematics is the study of pure pattern: structure held in view as a whole, apart from every embodiment of the pattern. Add the sequence of steps to mathematics, and you have computation. Add to computation the arrow of time (the asymmetry between a determ-

ined past and an open future), outcomes that no rule fixes, and the stance of a comparer, and you have physics. Metaphysics is the account of the act of comparison itself. Relationship is the essential study of both mathematics and metaphysics, and metaphysics is the study of relationship at a deeper level than mathematics.

What mathematics states is frozen: a statement of mathematics does not change. Learning and proving happen in time, but each statement of mathematics is proven or not proven for all time, so in what mathematics states there is neither change nor choice. Mathematics is purely logical, and in that sense deterministic. In mathematics, logical entailment stands in for causation. Logical entailment is the relation by which one statement follows from others. Entailment is deterministic and not causal, because causality needs an observer and a direction of time, and mathematics has neither.

A second argument that mathematics differs from physics depends on the difference between pattern and process. Any process held in the third person, laid out as something anyone could be told, is a pattern. Not every pattern is a process. There is something in process that cannot be encoded in any pattern. If everything in process could be encoded in a pattern, mathematics would be the same as physics, but mathematics is not the same as physics, since mathematics is deterministic and physics is causal.

When we reason entirely in terms of causation, the reasoning contains no basis for meaningfulness, but once we include choice and change, meaningfulness becomes relevant, and including choice and change goes outside of rationality. People have called whatever is outside the rational irrational, as if there were only two categories, the rational and the irrational. There are more than two categories, because some things outside the rational are real and good, and a good choice can depend partly on rationality and partly on what no rational description captures.

VI. A Second Cycle

There is also a cycle of another kind beside the order of presupposition: an Axiom II cycle, which is an order that comes back around to where the order began, as the order of Axiom II does. Axiom II relates the three modalities, which are three roles: the immanent, the omniscient and the transcendent. The immanent is the role of the act itself, as the act happens, here and now. The omniscient is the role of what is laid out as structure and can be held in view as a whole (Section IV). The transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance. Axiom II of the Immanent Metaphysics reads: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent".

Mathematics can be known only through an activity: learning, reading, proving. Every act of knowing is a comparison or is built from comparisons, and the Immanent Metaphysics is the account of the act of comparison, so the metaphysics precedes the knowing, and the knowing precedes the mathematics. And when mathematicians take abstraction as far as abstraction can go, mathematics arrives at the place where the metaphysics already is: the study of relationship itself, which Section V names as the essential study of both mathematics and metaphysics. So the reasoning of the metaphysics comes both before mathematics and after mathematics: before, because the metaphysics precedes all knowing of mathematics, and after, because mathematics arrives at the metaphysics again at the far end of abstraction.

The Immanent Metaphysics is accountable to physics in one way. Physics is the test of the metaphysics, and not the foundation of the metaphysics. The consequences of the metaphysics include claims about physics, and those claims are checked in physics, by measurement.

VII. The Last Turtle

In every discipline, someone eventually asks what the discipline itself rests on: the question of the discipline's own ground. The descent of grounds is the series of questions in which we ask what each discipline rests on, and then what each ground rests on in turn. Mathematicians answer the question of the ground of mathematics by ascending: mathematicians state and study the consistency of each formal system in a stronger formal system, and the ascent from system to stronger system has no end. A formal system is consistent when the system never proves both a statement and the denial of the statement.

The Immanent Metaphysics has a different answer to the question of what the metaphysics itself rests on: an answer that closes on itself instead of ascending. The Immanent Metaphysics is the account of the act of comparison, the act in which every finding is made. The three axioms of the metaphysics describe their own content. The three axioms also describe the process by which the three axioms are derived. The description of the content is the first order of self-description, and the description of the process of derivation is the second order. The description of the second-order description is the third order, and the third order adds nothing new, since the third order collapses into the second: describing the process of describing needs no new concept. The three axioms are therefore a fixed point, a place where going one level further brings nothing new: at no level of abstraction beyond the three axioms is any new concept, definition or axiom needed. A metatheory is the theory that describes a theory, and the metatheory of the Immanent Metaphysics is the Immanent Metaphysics itself.

An old picture has the world standing on a turtle, which stands on another turtle, and so on without end. The picture is called "turtles all the way down". The descent of grounds is not turtles all the way down, because the descent ends at a structure that is its own ground: the Immanent

Metaphysics. Every discipline above the metaphysics rests on the metaphysics. The last turtle in the picture is not another turtle. The last turtle is the ground itself, the Immanent Metaphysics. The Immanent Metaphysics is the ground on which physics stands, that is, what physics presupposes, and not a competitor to physics. The unification of physics is the problem of joining two theories into one theory: quantum mechanics, the physics of the very small, and general relativity, the theory of gravitation as the shape of space and time. We resolve a problem such as the unification of physics by recognizing the ground that was already there, and not by finding a deeper physics.

VIII. What Every Physical Theory Presupposes

Every physical theory presupposes three things. The first thing that every physical theory presupposes is a structure that stays the same when the circumstances of a measurement change. The second thing that every physical theory presupposes is an interaction, by which anything is measured at all. The third thing that every physical theory presupposes is a range of possibilities within which events occur. Axiom III holds of the structure, the interaction and the range of possibilities: the three are distinct, inseparable and non-interchangeable. The three things correspond to the three modalities, one to each.

The structure that stays the same has the role of the omniscient: in physics, the role of the omniscient is filled by the conservation laws, by structure, and by the content of measurement. *Symmetry* is the character of the structure that stays the same: a sameness of content where there is a difference of context. A law of physics is a symmetry found by comparison: what stays the same when the context of a measurement changes. The mathematician Emmy Noether proved a theorem, now called Noether's theorem, that pairs each continuous symmetry of the action with a conserved quantity, where the action is the quantity from which the equations of motion of a physical system follow. Under Noether's theorem,

symmetry under translation in time goes with the conservation of energy. Symmetry under translation in space goes with the conservation of momentum. Symmetry under translation in time means that the laws are the same at one time as at another, while symmetry under translation in space means that the laws are the same at one place as at another.

The interaction has the role of the immanent: in physics, the role of the immanent is filled by interaction, by force, and by the process of measurement. Every measurement is an interaction, and the act of comparison is the act from which the other two, the structure that stays the same and the range of possibilities, are found.

The range of possibilities has the role of the transcendent: in physics, the role of the transcendent is filled by possibility, by the space of configurations (every arrangement that a system could be in), and by the boundary conditions within which events occur. The character of the range of possibilities is *continuity*: a sameness of content where there is a sameness of context.

Infra-physics is the name for the structural layer that lies beneath physical theories ("infra" is Latin for "below"). Infra-physics is the foundation of physics, and not a replacement for physics: infra-physics is the layer of structural necessity on which the contingent facts of physics rest, the facts that could have been otherwise. Infra-physics is the study of what must be true of any possible universe for there to be physical law at all, and of the constraints that any physical theory must satisfy, whatever the values of the physical constants and whatever the specific forms of the laws. The layer of infra-physics consists of the axioms of the Immanent Metaphysics and the structural principles that follow from the axioms: together, the axioms and the principles are the necessary conditions for measurability, for causal order, and for conservation and symmetry. A claim of infra-physics is a structural analysis and not a prediction. Each claim of infra-physics is more specific than pure metaphysics and more structural than empirical physics.

Infra-physics has a definite scope. Infra-physics shows the kinds of law that any domain must have. Infra-physics also shows what each kind of law must exhibit when the law is taken to its limit. A domain governed by symmetry has conservation. Symmetry at its limit has quanta, smallest units, the way light is made of photons. Process has an arrow, a direction from past to future. We do not derive the values of the constants, or the particular particles and forces, at the layer of infra-physics. The reason is that infra-physics is a theory of the foundations of domains. The foundations of a domain determine the structure of the domain, and not the content of the domain. In the Immanent Metaphysics, the relationship between self and reality is the structural principle underlying everything that exists. To understand the relationship between self and reality is to understand the foundation on which any physical world rests.

IX. The Boundaries of the Scientific Method

The scientific method requires that whatever a scientist studies be observable and repeatable. If a thing cannot be observed, no science can be done on the thing. If a thing cannot be repeated, no science can be done on the thing either. Some things are real and are not observable, or not repeatable, or both.

Hard randomness is one thing that is real and not repeatable: an outcome that no prior specification fixes, however complete, as in a single quantum measurement. The statistics of hard randomness repeat: if the same measurement is run many times, the frequencies of the outcomes settle to stable values. The single outcome does not repeat. Nothing known before a run of the measurement shows which outcome the run will give.

When time is taken as the order of actual acts here and now, time is a second thing that is real and cannot be observed. A clock shows a change in a physical system that corresponds to the passage of time, and the clock does not show time itself, because time is an aspect of the process of perception, not an object that is perceived.

Creation is a third thing that is real and is not observable, or not repeatable. If the Big Bang is pictured as creation in one event long ago, the Big Bang would be neither observable nor repeatable. In the Immanent Metaphysics, creation is an aspect of every event, with no time and no source. Creation is not an event at all. The Big Bang is the way a limit appears in the history that physicists reconstruct from inside the universe: the limit appears as a beginning at a definite time. Physicists test the reconstructed history by the marks that the events of the history have left on the sky, but no one ever measures creation itself, and whatever is said of creation itself is metaphysics, not physics. Common sense says that we should not expect the known and the unknown, taken together, to include the unknowable.

A scientific theory is itself a triple, a set of three concepts that are distinct, inseparable and non-interchangeable. The mathematical formalism of the theory is the model, and the model is omniscient. The physical world that the theory describes is the object, and the object is transcendent. The correspondence is the method of measurement that links the model to the object, and the correspondence is immanent. As a method, physics uses two of the three modalities: the immanent, which in physics is the act of measurement, and the omniscient, which in physics is the results of measurement laid out as structure. The transcendent is used in every measurement and grounded by none: the transcendent is the stance from which a measurement is made and the range of what is possible from that stance. The arena is space and time taken as the context of every physical comparison. Scientists take the context of a measurement as given, and measure content within that context: in every measurement, even

a measurement of the arena itself, the arena at one place and time is compared with the arena at another. The measuring instrument registers the state of some body or field.

The world always exceeds the formalism of a theory, and the formalism always contains structure with no counterpart in the world, as the freedom to relabel coordinates and the gauge freedom of field theories show. The freedom to relabel coordinates gives the same points of space and time different numbers with no change in what is measured. The gauge freedom of field theories is a freedom to change the mathematical description of a field with no change in what is measured. The scientific method rests on symmetry: a scientist assumes that a result will repeat, and the assumption is what lets a result count as evidence. So no finite collection of experiments can establish that symmetry, or continuity, holds absolutely. Whatever is real but not repeatable lies outside what the scientific method can study.

Science also has six theoretical presuppositions: material monism, causal determinism, locality, strong objectivity, universality and epiphenomenalism. Material monism is the view that only matter is fundamentally real. Causal determinism is the view that earlier causes fix every state of the world in advance. Locality is the view that every influence travels from place to place, and no faster than light. Strong objectivity is the view that a description of the world can be complete with no reference to any observer. Universality is the view that the same laws hold at every place and at every time. Epiphenomenalism is the view that mind is a by-product of physical processes, and that mind has no effects. Each of the six presuppositions is challenged by developments within science itself, and the challenges cannot be resolved from within the scientific framework. The physicist John Bell proved a theorem, now called Bell's theorem. Bell's theorem shows that nature contains correlations over long distances that no causal process can reproduce, and so the correlations are a challenge to locality from within science.

A division of labor between physics and metaphysics follows from the boundaries of the scientific method: physics is the study of the interactions within a domain, and metaphysics is the study of the relations between domains. Physics uses the concepts of object, model and measurement. No physicist can ground the three concepts, that is, give the three concepts a basis, from within physics. Quantum mechanics and general relativity cannot be merged into one theory, and the question why the two cannot be merged belongs to metaphysics, because the question is about the relation between two descriptions. Two tasks belong to physics: to construct descriptions joined from a common source, and to judge whether a given description joined from a common source is good physics. Descriptions joined from a common source are two descriptions, each perfected in its own character, joined side by side and met in measurement.

Science and technology show us what can be done. The methods of science and technology do not show what ought to be done. Science states no value system, yet valuing is present in every act of science, as in every act of knowing, in the choice of what to attend to, what matters and what is relevant. The valuing in every act of science remains unexamined within science. In metaphysics we ask a kind of "what" question that is outside the scope of both science and technology: what comparison is, and what it is to exist or to be real.

We can picture the number of problems that cannot be solved with causation. Suppose humanity is presented every year with a hundred problems, and the complete toolkit of causation, every method that works through cause and effect, can solve ninety-five of the hundred. Then every year five more problems accumulate that cannot be solved with causation. The limits of the scientific method show wherever a scientific question involves choice, agency, or the relationship between the methods of science themselves.

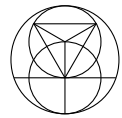
X. What a Critique Must Do

Disliking a conclusion is not the same as finding a flaw in the argument for the conclusion. A sound argument has three parts: premises that are true of what the premises are about, inferences that are valid, and conclusions that follow from the premises by those inferences. To reject a conclusion, a critic must show that a premise is false or that an inference does not follow. In a legitimate critique, the critic names which premise is false, or which inference fails. Every other objection is noise, because the objection does not bear on whether the argument is sound, and so discomfort with a conclusion is not a reason to reject the conclusion.

XI. Summary

There are three orders of knowledge. Causal knowing comes from the scientific method, in which a scientist maps what causes what by testing hypotheses in the real. Natural law is the total of causal knowing about the natural world. Analytic knowing comes from proof, which shows what must follow within one domain fixed by the axioms of the domain. The truth of analytic knowing is conditional on the axioms of the domain. Process knowing is the knowing of the Immanent Metaphysics: the knowledge of what cannot fail to be the case in any act of comparison. We gain process knowing by clarity in defining terms, and by that clarity we find the six intrinsics of comparison already in use. Process knowing is non-arbitrary, because to doubt any one of the six intrinsics is already to use that intrinsic. The disciplines form an order of presupposition: metaphysics, physics, computation, mathematics. Each discipline in the order is the next discipline in the order with something added. Computation is mathematics with the sequence of steps added. Physics is computation with three things added: the arrow of time, outcomes that no rule fixes, and the stance of a comparer. Metaphysics is the account of the act of comparison

itself. An Axiom II cycle runs beside the order of presupposition. In the cycle, the metaphysics comes both before and after mathematics and is tested in physics. The descent of grounds ends in a metaphysics that is its own metatheory. The Immanent Metaphysics is the infra-physical layer: the metaphysics fixes what any physical theory presupposes (structure, interaction and possibility) and which kinds of law any domain must have, while the values of the constants belong to physics. The scientific method has definite boundaries: science studies the observable and repeatable, the method uses the immanent and the omniscient, and scientists take the context of a measurement as given. Hard randomness, time and creation itself are real but not observable or not repeatable. So the three are outside what the scientific method can study. The question of value is also outside what the scientific method can study, because the question of value belongs to metaphysics.



Chapter 10

The Errors of Extrema

Prejudice, Reaction, Realism, Idealism, and the Boundary of the Self

ABSTRACT

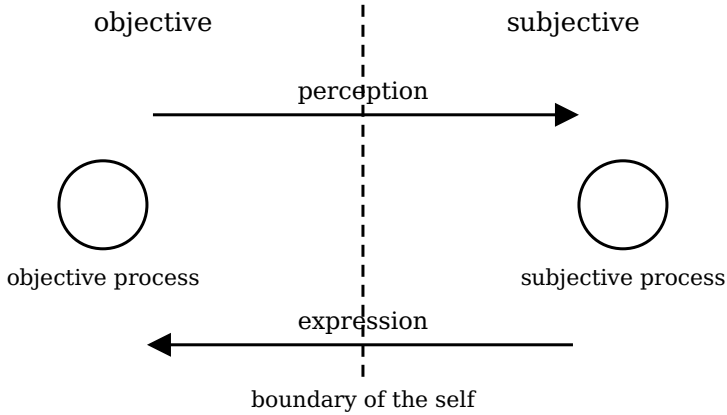
Every interaction between a self and its world has four parts. Two parts cross the boundary of the self, which divides the subjective (the self's side) from the objective (the world's side). Perception crosses the boundary of the self from the objective to the subjective, and expression crosses the boundary from the subjective to the objective. The other two parts are processes: the subjective process is what the self does with what the self perceives, and the objective process is what the world does with what the self expresses. The relation between perception and expression can be taken to an absolute limit in four ways. An absolute limit is the extreme case in which the relation holds completely or not at all, and I call each of the four limits an error of extrema. Prejudice is expression absolutely independent of perception, and reaction is expression absolutely dependent on perception. Realism is perception absolutely independent of expression, and idealism is perception absolutely dependent on expression. Prejudice and reaction are the two errors of the self, while realism and idealism are the two errors of the world. All four errors of extrema are invalid positions, because every real interaction stops short of all four limits: between perception and expression there is only interdependence, and interdependence is interaction. The four errors are structural descriptions, and none of the four is a moral judgment. In this chapter's account of the four errors, we hold the concepts of this book's metaphysics so as

> to take Axiom I seriously: we keep interaction more fundamental than either side of the boundary of the self. We do so within the process whose order Axiom II states.

I. The Fundamental Diagram

Draw the boundary of the self. The subjective, the self's side, lies on one side of the boundary, and the objective, the world's side, lies on the other.

Two arrows cross the boundary of the self, one in each direction. *Perception* crosses the boundary from the objective to the subjective. *Expression* crosses the boundary from the subjective to the objective. A process runs on each side of the boundary: the *subjective process* is what the self does with what the self perceives, and the *objective process* is what the world does with what the self expresses. In a real interaction all four elements are active: the self perceives the world (perception), the self works on what the self perceives (the subjective process), the self expresses (expression), and the world works on what the self expresses (the objective process). Answering a question is a case with all four elements active: you hear the question (perception) and think about the question (the subjective process). You speak your answer (expression), and your answer reaches the person who asked and has its effect on that person (the objective process).



The fundamental diagram. An open circle marks a process that is happening. Each error of extrema crosses off one of the four elements: perception, expression arriving in the world, the subjective process, or the objective process.

An absolute limit of a relation is the extreme case, in which the relation holds completely or not at all. When the relation between perception and expression is taken to an absolute limit, this book calls the limit an *error of extrema* (the word "extrema" means extremes). We draw each error of extrema as an aberration of the fundamental diagram: a faulty version of the diagram, with one of its four elements (the two arrows and the two processes) crossed off. An open circle in the diagram marks a process that is happening, and a blacked-out circle marks a process that is not happening.

II. Prejudice Defined Exactly

Prejudice is the clearest error of extrema to start with, because everyone knows what prejudice looks like.

In a rough everyday picture of prejudice, someone makes a choice that something, or someone, is a particular way, and then chooses never again to take in any more information about that thing or that person. To a person who is discriminated against, it is as if someone had judged that the person fits a stereotype, and then disregarded every action of the person that departs from the stereotype, however many such actions there are.

When prejudice is taken to its perfection, its complete and limiting form, prejudice is a complete absence of perception. No new data from the objective affects anything in the subjective. Nothing that might have been perceived has any effect on what goes on in the subjective, on the expressions that come out of the subjective, or on what happens in the objective. So prejudice, taken to its perfection, is a blindness: a fundamental blindness and insensitivity to the world.

When we define prejudice exactly, as the complete absence of perception, the definition does two things at once. First, with a precise definition we see what we would otherwise miss. Second, the definition describes what is actually happening when people are prejudiced. The everyday concept of prejudice is loosely defined and names a whole set of behaviors. In the exact definition we distill the whole set into one behavior, not perceiving, which holds all the other behaviors together. Then we take the behavior of not perceiving to its perfection.

III. The Four Errors

The relation between perception and expression can be taken to an absolute limit in four ways, and so there are four errors of extrema. The four ways come from two choices, each with two options: which arrow depends on the other (expression on perception, or perception on expression), and which limit the dependence is taken to (absolute independence or absolute dependence).

Prejudice is expression absolutely independent of perception. What the self expresses has nothing to do with what the self perceives. The diagram of prejudice has perception crossed off.

Reaction is expression absolutely dependent on perception. What the self perceives fixes what the self expresses entirely, in a mechanistic and deterministic way, as a stimulus fixes a reflex. The diagram of reaction has the subjective process crossed off: nothing happens in the self between what comes in and what goes out.

Realism is perception absolutely independent of expression. What the self perceives has nothing to do with what the self expresses: the realist takes the world to be exactly as it is, untouched by the one who perceives it. The diagram of realism has the arrival of expression in the world crossed off.

Idealism is perception absolutely dependent on expression. What the self expresses fixes what the self perceives, entirely: the idealist takes the world to be made by the one who perceives it. The diagram of idealism has the objective process crossed off.

Prejudice and reaction are the errors of extrema of the self: both errors concern expression, the arrow that goes out from the self. Realism and idealism are the errors of extrema of the world: both errors concern perception, the arrow that comes in from the world.

IV. Why None of the Four Is Valid

All four errors of extrema are invalid positions, because every real interaction stops short of all four limits.

Between perception and expression there is *only* interdependence, and interdependence is interaction: in a real interaction, perception and expression are never absolutely independent of each other, and never absolutely dependent on each other. The relation between perception and ex-

pression is a real relation, which always holds to some degree and never completely, but a person who holds an error of extrema treats the relation as if the relation held completely or not at all.

As this chapter defines the four errors of extrema, each error is a structural description of what happens when the relation between perception and expression, the two crossings of the boundary, is taken to one extreme. None of the four errors is a moral judgment. In this book's account, prejudice is wrong in the same way that every error of extrema is wrong: prejudice describes a limit that no real interaction reaches, and the prejudiced person acts as if the limit had been reached. As positions in philosophy, realism and idealism are wrong in the same way as prejudice. Physicalism is a form of realism: the physicalist takes the objective as fundamentally real and the subjective as illusion. The Vedantic idealist takes the subjective as fundamental and the objective as illusion. The physicalist and the Vedantic idealist each take one side of the self's boundary to a limit, so each position is an error of extrema. Realism and idealism are limit cases, which people take as if the limit cases were perspectives. So neither realism nor idealism is a valid perspective.

V. The Errors and the Axioms

In this chapter's account of the errors of extrema, we hold the concepts of this book's metaphysics so as to take Axiom I seriously within the process that Axiom II describes.

This book's metaphysics is the Immanent Metaphysics, and Axiom I and Axiom II are two of its three axioms. Both axioms are stated in terms of three roles, which this book calls the modalities (Chapter 3). The immanent is the role of the act itself, the relating, as the relating happens here and now. The omniscient is the role of what is laid out as structure: a pattern, a map, a record. The transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance.

Axiom I reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". Interaction is immanent: interaction is the act itself, here and now. When the relationship between the subjective and the objective is in the immanent role, Axiom I says that the relationship is more fundamental than the subjective and the objective (Chapter 3). Interaction across the boundary of the self is the relationship between the subjective and the objective. So, by Axiom I, interaction is more fundamental than either side of the boundary. Axiom II reads: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". Axiom II is the axiom of process: Axiom II states the order in which the modalities follow one another whenever anything happens, from the transcendent to the immanent to the omniscient, and back to the transcendent.

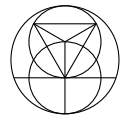
In a real interaction all four elements of the fundamental diagram are active, and perception and expression are interdependent, never absolutely dependent or absolutely independent. A person who holds an error of extrema takes the relation between perception and expression to one extreme. When we hold all four elements active in our account of an interaction, we keep interaction, the immanent, as the ground that Axiom I names, while the process of perceiving and expressing goes on in the order that Axiom II names.

VI. Summary

The fundamental diagram shows the boundary of the self, with perception and expression crossing the boundary and a process running on each side. In a real interaction all four elements are active. An error of extrema is the relation between perception and expression taken to an absolute limit. The diagram of each error has one of the four elements crossed off: perception, expression arriving in the world, the subjective process, or the

objective process. The diagram of prejudice has perception crossed off, because prejudice is expression absolutely independent of perception. The diagram of reaction has the subjective process crossed off, because reaction is expression absolutely dependent on perception. The diagram of realism has the arrival of expression in the world crossed off, because realism is perception absolutely independent of expression. The diagram of idealism has the objective process crossed off, because idealism is perception absolutely dependent on expression. Prejudice and reaction are errors of the self, while realism and idealism are errors of the world.

Prejudice, defined exactly, is the perfection of not perceiving. The exact definition does two things: the definition sharpens what we can see, and the definition describes what prejudice actually is. All four errors of extrema are invalid positions, because every real interaction stops short of all four limits. Between perception and expression there is only interdependence, which is interaction. The four errors are structural descriptions, and none of the four is a moral judgment. When we hold all four elements active, we hold interaction as fundamental, as Axiom I requires, within the process that Axiom II describes.



Chapter 11

What Perception Reaches

Against Solipsism, Physicalism, and the Claim That No Creature Perceives Reality

ABSTRACT

Perceiving is more basic than the perceiver and the perceived. There is no perceiver without an act of perceiving, and there is nothing perceived without an act of perceiving. The perceiver, the perceived and the perceiving form a triple, a set of three whose members are distinct, inseparable and non-interchangeable. The self is shallower still than the perceiver. The self is the appearance of the aggregation of that aspect of interaction which is choice. This chapter uses the two positions just stated, about perceiving and about the self, to answer solipsism (the view that only one's own experience can be known) on the solipsist's own ground, which is experience itself. I then show how a solipsist who is pressed to say what makes one perception true turns into a physicalist and a determinist. Physicalism is the view that the physical world alone is fundamentally real. Vedantic idealism, the mirror image of physicalism, is the view that the subjective alone is fundamental. Physicalism and Vedantic idealism are one mistake made on opposite sides. I also reply to two further claims about perception. The first claim is a theory of perception in which every access to the world goes through an interface, while holders of the theory also take one level of reality as privileged (as having a special standing over every other level). The theory is inconsistent until its holders say whether the theory's own translation, from the privileged level to the interface, is real. The second claim is the argument that

> creatures evolve to perceive fitness payoffs, and that therefore no evolved creature can perceive reality. The argument establishes only that perception is distorted, and not that perceiving reality is impossible, while human beings, as makers of their own niches, fit the argument least of all creatures.

I. Solipsism Stated

Solipsism is the position that only one's own experience, and the self that has the experience, can be known. The philosopher René Descartes wrote "I think, therefore I am". The solipsist reads Descartes' sentence as saying that the only thing one can know is that one is experiencing, that there is a self that is experiencing, and that nothing else can be known. The solipsist reasons this way: another person is something I am experiencing, but I have no reason to know that the other person has experience. Solipsism is an extreme kind of epistemological skepticism (doubt about what can be known), in which experience is the ground: experience is the one thing that the solipsist does not doubt.

Solipsism is different from Vedantic idealism. Vedantic idealism is named for Vedanta, a school of Indian philosophy. *Vedantic idealism* holds that subjectivity is fundamental and that the objective world is an illusion. Solipsism goes further, because most versions of solipsism reject other minds as well.

II. The Familiar Answers, and the Depth of the Question

The familiar arguments against solipsism have real force. Space and time are regular. Maps work, science works, and objectivity works. Objectivity is the practice of checking that a finding holds for many comparers (many people who make the same comparison). A map drawn by one traveler, for example, guides a second traveler who has never seen the place. Some things keep their symmetry across many transforms of perception, which

means that each of those things stays the same when it is perceived from another place, at another time or by another person. Those things keep their symmetry in ways other than we would expect if we were making the things up. Solipsists often use dreams as their example. Dreams lack exactly this regularity of space, of time and of things that keep their symmetry. The familiar arguments show that solipsism is unlikely. Yet the familiar arguments are not grounded as deeply as the ground the solipsist stands on, because the familiar arguments start from regularities found in experience, while the solipsist starts from experience itself. Any answer has to be grounded at least as deeply as solipsism is, if the answer is to hold on the solipsist's own terms.

III. Perceiver, Perceived, Perceiving

Perception is more basic than the perceiver and the perceived. There is no perceiver without an act of perceiving, and there is nothing perceived without an act of perceiving.

The perceiver, the perceived and the perceiving form a triple. A domain is the whole of what can be compared by one kind of comparison. At the foundation of every domain there are three concepts, called a *foundational triple*. The perceiver, the perceived and the perceiving are distinct, inseparable and non-interchangeable, like the members of every foundational triple (An Immanent Metaphysics, aphorism 1.45-1). *Distinct* means that no member overlaps another. *Inseparable* means that no member occurs without the others. *Non-interchangeable* means that no member can take another member's role. Distinctness, inseparability and non-interchangeability are three coequal properties of the triple, equal in rank. Each of the three properties holds in its own right: inseparability, for one, is not derived from distinctness.

The triple of perceiver, perceived and perceiving is enough to answer the solipsist from inside the solipsist's own ground, which is experience. The solipsist cannot say that the self is the perceiver, and that therefore

the self is real and nothing else is. To have experience at all is to have an act of perceiving. The act of perceiving is more fundamental than the perceiver that the act includes. The perceived is an inseparable part of the act of perceiving as well. So whoever grants experience has granted perceiving, which is deeper than the self, and has granted the perceived along with the perceiving, because the perceived is inseparable from the act of perceiving.

The self is also shallower than the perceiver. The book An Immanent Metaphysics states: "Self is the appearance of the aggregation of that aspect of interaction which is choice" (aphorism 1.67-2). Choice is one aspect of interaction, and the self is the appearance of that aspect, gathered together. The self is the context of past and future choices. The self is something that appears within interaction, where the aspect of interaction that is choice gathers. So the self is not the ground that perception depends on. The solipsist's certainty of the self is therefore a certainty about something that rests on perceiving. Perceiving already includes a world, in the perceived.

IV. From Solipsism to Physicalism

When a solipsist is pressed to say what selects one perception as true and another as false, the solipsist has two answers to choose from. The first answer is that the selection is a choice. The second answer is that a perception can be selected as true only if the perception is true, that truth and falsity are objective, and that the selection is slaved to the objective, that is, wholly fixed by what is objectively so. The second answer turns the solipsist into a physicalist and a determinist. In the second answer, the objective is fundamental, and the objective fixes the selection.

An answer to the physicalist and determinist has to be argued from choice, change and causation, and so from the real (An Immanent Metaphysics, aphorism 1.85-4). The real is that which partakes of choice, change and causation at once. Causation is a lawful regularity in time: the

same relation of earlier conditions to later conditions, found again and again from case to case. Causation is *not* determinism, the claim that one law fixes every state of a whole world in advance. Chapter 12 sets out the difference in full. No whole world can be completely determined, because complete determinism would need symmetry and continuity perfected together: perfect symmetry, one law carrying every state into its successor, and perfect continuity, a succession with no break anywhere. Symmetry is a sameness of content where there is a difference of context. Continuity is a sameness of content where there is a sameness of context. The Incommensuration Theorem forbids perfect symmetry and perfect continuity together: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. So the selection of a perception cannot be wholly slaved to an objective that fixes everything, because an objective that fixes everything does not exist. Choice is real, as the part of change that nothing earlier fixes.

V. One Mistake on Two Sides

Physicalism is a form of realism. Chapter 10 describes realism as the position that the world is exactly as it is, untouched by the one who perceives it. Physicalism says that the objective, the physical, is fundamentally real, and that the subjective is either an illusion or an epiphenomenon. An epiphenomenon is a by-product with no standing of its own. Vedantic idealists make the exact same reductive mistake on the opposite side, because Vedantic idealism says that subjectivity is fundamental and objectivity is an illusion. The mistake is reductive in that each position reduces the relation between the subjective and the objective to one side of the relation, either to the objective alone or to the subjective alone. Physicalism and Vedantic idealism are both errors of extrema, the impossible limiting positions of Chapter 10: in each of the two positions, one side of the relation between the subjective and the objective is taken to an absolute limit. Each position also started from an assumption about what is (an ontological as-

sumption). On that assumption the holders of each position built a way of knowing (an epistemology), and what they came to know by that way of knowing then confirmed the assumption they started from. Relationality is the relation between the subjective and the objective, taken as a notion of higher order than either side. If both the subjective and the objective are founded on relationality instead, then we can uphold everything known from both perspectives, and a great deal more. On a relational foundation, the paradoxes that each one-sided position produces do not arise.

The Incommensuration Theorem sets the upper bound on what can be known. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem fixes the unknowable as a persistent class. The unknowable is what cannot be known even in principle, by any being, at any time, by any method. The limit that the Incommensuration Theorem draws is the shape of the field of knowing itself, and not a wall at the edge of an otherwise open field. In plain words, the limit belongs to the structure of all knowing. The limit is not a barrier met only at the far edge of what is known. The Immanent Metaphysics is the metaphysics on which this book rests. Within the domain of the knowable, the Immanent Metaphysics describes the epistemic process (the way of knowing) appropriate to the knowable.

VI. The Interface and the Privileged Reality

One theory of perception makes a related claim: everything is reached through an interface, and no interface is privileged. In this theory, an *interface* is the form through which a creature reaches the world, as distinct from the world reached. A privileged interface would be one with a special standing over all the others. Some versions of the theory also say that

one fundamental level of reality is privileged: the level of particle physics, which physicists describe by the Standard Model, the theory of the known kinds of particle.

Where a theory makes both claims, the holders of the theory have to answer two questions. The first question is whether there is an interface to the Standard Model, or whether the Standard Model is the one thing that a creature can interact with without an interface. If interfaces are not privileged, but there is a privileged level of reality, then somewhere there has to be a translation from the privileged level to the interface. The second question is whether that translation is itself real, or not. A theory with both claims contains an internal inconsistency until the holders of the theory answer the second question. The holders must resolve the inconsistency before the theory can be taken seriously as a foundation for either physics or epistemology.

VII. The Benthic Fish

A third claim about perception is the argument that creatures evolve to perceive fitness payoffs, and not reality as it is, and that therefore no evolved creature can perceive reality. A *fitness payoff* is what a perception gains or costs a creature in survival and reproduction. Consider a *benthic* fish (a fish that lives at the bottom of the deep sea), living five thousand meters under the sea. For the fish and for us, five thousand meters of water is the same state of reality. The payoff of that same state of reality is very different for the fish than for us. The fish lives in the deep water, and a human being cannot. So the argument is right that what a creature perceives is shaped by what pays off for the creature.

Yet the argument claims more than the argument shows. A statement of the form "such a thing exists" is a very different statement from a statement of the form "no such thing exists". The demonstrations that people offer for the argument are optimization arguments: the optimization arguments show that creatures perceive what is necessary to sustain a relation-

ship with their environment that supports the evolutionary process. A different claim is that perception is obscured, that is, that perceiving reality is impossible. Both claims differ again from a demonstration of actuality, in which someone points to the thing itself. The argument's opening claim has the form "nothing exists in this class": no evolved creature can perceive reality. What the argument establishes is only that evolved creatures perceive reality in a distorted way, or perceive too little of reality. So the claims of the argument are too strong, and the claims are the wrong kind of claims: claims of the form "nothing exists in this class", where the demonstrations show only distortion.

Human beings fit the argument least of all creatures. A *niche* is the place and the way of life to which a kind of creature is adapted. Human beings are makers of niches, and not only creatures adapted to a niche. Human beings play a higher-level game: human beings play within a particular game, and human beings also play with the rules of the game. In plain terms, human beings act within the conditions of their lives, and human beings also change those conditions. Playing with the rules of the game is a kind of reflectiveness, and that reflectiveness requires a much more general perspective on reality, and so human beings need to know the base nature of reality more than other animals do. People who practice objectivity interact intensely with the world, check that the findings of many perceivers agree, and stay open to the novel and surprising. So people who practice objectivity come more in touch with reality at the level of causes than other creatures are, and human perception, far from failing to perceive reality, converges on the base reality of the world we share.

VIII. Summary

Perceiving is more basic than the perceiver and the perceived. The perceiver, the perceived and the perceiving form a triple whose members are distinct, inseparable and non-interchangeable: three coequal properties.

The self is shallower still than the perceiver. The self is the appearance of the aggregation of that aspect of interaction which is choice. The triple and the account of the self are enough to answer solipsism on the solipsist's own ground, because whoever grants experience grants an act of perceiving deeper than the self, and grants a perceived inseparable from the act of perceiving.

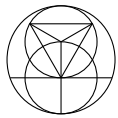
A solipsist becomes a physicalist and a determinist by making truth objective and making the selection of a perception slaved to the objective. The answer to the physicalist and determinist is argued from choice, change and causation: causation is not determinism, no world is completely determined, and choice is real. Physicalism and Vedantic idealism are one reductive mistake made on opposite sides. Each of the two positions is an error of extrema, in which an assumption about what is (an ontology) produced a way of knowing (an epistemology) that then confirmed the assumption. On a relational foundation, everything known from both positions can be upheld. The Incommensuration Theorem sets the upper bound on what can be known.

One theory of perception says that every access goes through an interface that is not privileged, and that one level of reality is privileged. Before the theory can serve as a foundation, the holders of the theory must say whether the theory's translation from the privileged level to the interface is real. One argument says that no evolved creature can perceive reality, but the argument establishes only that perception is distorted, and not that perceiving reality is impossible. Human beings make their niches and play with the rules of the game, so human beings need to know the base nature of reality more than any other creature does, and human perception converges toward that base nature.

PART THREE

CAUSATION, CHOICE AND THE BOUND TRIPLE

Causation without determinism, the triple of the real, free will, the bound triple, and hard randomness.



Chapter 12

Causation Is Not Determinism

Why a Regularity in Time Is Not a Timeless Completeness

ABSTRACT

Causation and determinism are two different concepts, and the difference between causation and determinism is very large. *Causation* is a regularity found in time: the same relation of antecedent to consequent (of what comes before to what comes after), found again and again from one circumstance to another. *Determinism* is the claim that one law fixes every state of a whole world completely, at every scale, all the way down to the scale of exactly zero. Determinism is true inside a mathematical model, because mathematics is timeless, but determinism is not true of what happens, because what happens is temporal. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. The Incommensuration Theorem therefore forbids any whole world to be completely deterministic, because complete determinism would need perfect symmetry and perfect continuity together. So no actual world is purely and completely deterministic, and every world is causal to some degree. This chapter sets out the two concepts and shows where each concept applies. Determinism applies to the mathematical model, and causation applies to what happens. I show where the information in the world comes from, and I place causation and determinism on one line that runs from determinism to pure randomness, with causation, change and choice between the two ends. I also name the usual collapse of four concepts

(change, choice, causation and determinism) into one. When an argument treats causation and determinism as the same thing, the argument can be valid, meaning that its conclusion follows from its premises, and still not sound, meaning that at least one of its premises is false of the world.

I. Two Concepts That Are Usually Run Together

Causation and determinism differ very greatly, and yet most people would not bother to draw any distinction between causation and determinism at all. Most physicists collapse four concepts into one, in a definite order. To collapse two concepts is to treat the two concepts as one concept. The four concepts are change, choice, causation and determinism. First, most physicists collapse change into causation, by taking every change to be the effect of a cause. Second, most physicists say that choice is one more instance of causation, so that choice, change and causation all become causation. Third, most physicists identify causation with determinism. Most physicists treat causation and determinism as strict isomorphs of one another, structures that match each other part for part. In the third step, most physicists take the sentence "this caused that" and the sentence "the whole state of the world was fixed in advance by one law" to say the same thing.

Each of the three steps loses something real. This chapter is about the last step, the step that identifies causation with determinism. The answer to the first two steps is the account in Chapter 13, where choice, change and causation are three distinct concepts. In this chapter it is enough to see that the last step is a mistake, and exactly what kind of mistake the step is.

II. Determinism Is a Truth of Mathematics

Whether the world is deterministic is a question about how mathematics and causation are related. Mathematics is atemporal, which means that mathematics is outside of time. Time and space can be factored out readily from a body of mathematics. A mathematical relation holds or does not hold. A theorem of geometry holds at midnight exactly as the theorem holds at noon. The relation holds the same way at every moment, because no moment enters into the relation. Causation has a temporal vector, a direction in time. To look at causation is to look at a before state and an after state, and at the order between the two states.

Determinism is like a platonic truth, a truth that holds outside of time. Within the mathematics of a model, determinism is true: determinism is what the relations say, taken as relations. Suppose a model is a set of equations that carry each state into its successor by one rule. Within the model, every state is fixed by every other state, and nothing is left over. The mistake is to carry determinism across from the timeless model to what actually happens, which is temporal. The main distinction is therefore that determinism is atemporal and causation is temporal. Determinism belongs to the model, where determinism can be perfectly applied, while causation belongs to what happens, where causation cannot be perfectly applied.

III. What Causation Is

Causation says that there is a regularity: the same relation of antecedent conditions (the conditions before) to consequent conditions (the conditions after), of inputs to outcomes, is found again and again across a change of circumstance, here and there, now and later. A struck match catches fire here and there, now and later. The relation of striking to flame is found again from one circumstance to the next.

The relation of antecedent to consequent stays the same across a change of circumstance, and that sameness is a kind of symmetry. In the exact sense used here, *symmetry* is a sameness of content where there is a difference of context: the content stays the same although the context has changed. In causation, the content that stays the same is the relation of antecedent to consequent, and the context that changes is the place and time at which the relation is found. At each single event the order runs one way, from antecedent to consequent, while the symmetry is in the relation staying the same from one event to the next.

A model is a description of the symmetry of causation. The notion of a model is always the notion of a model of causation, that is, of the regular part of what happens. The residual is the part of what happens that no regularity fixes, and every model leaves out the residual. A notion of causation is built into every notion of a model.

The tie between a model and causation has a direct consequence for how a model is used: if a model has a symmetry that does not extend into causation, then that symmetry of the model is inapplicable. In that case the world lacks any regularity that corresponds to the regularity the model describes, and so the model cannot be extended into the world, and the model is silent about the world. A model is useful only where the symmetries of the model are the symmetries of causes.

When someone builds a model only at the surface level of what the model describes, the builder is tempted to give up causation: to call causation an epiphenomenon (a by-product with no standing of its own) of some lower level, or to say that causation emerges out of the model. Both ways of giving up causation get the order backward: causation is the genesis (the origin) of law, and not the other way around. A law is a statement of a regularity that causation already shows. So the regularity comes first, and the law is the description of the regularity.

IV. Where the Information Comes From

A world that is a simulation differs from a world that is not a simulation in where the information of each world comes from. Consider a simulated world: a world of information running on a substrate, the machinery that runs the simulation. A person inside the simulated world can have access to the substrate, and can then issue a command such as "delete this file tree", which erases a folder and everything in the folder. The command is a very small amount of information, while the change that the command causes may be a very large amount of information: if a single command deletes a terabyte of data, then about twelve bits went in and a terabyte of changed state came out. In a computational world there is no proportion between the number of input bits and the number of output bits. The imbalance between the input and the output is arbitrary, because the command uses the causation of the substrate: the substrate does the work, and the command only calls on the substrate.

Anything made inside a simulation uses the substrate in the same way as the command does: to make one fireball in a game, someone has to say how much heat, where, and for how long, and some procedure has to create every bit of information that results. If the person writing the game does not supply the information about the fireball, the information has to come from somewhere, and the information comes from the substrate: the information is already stored in the operating system and in the machinery under the operating system, and the game uses the information stored there.

In the visible universe there is only one level. The visible universe is not a simulation. There is no substrate beneath the visible universe to call on: there is only physicality, the physical world itself. Phenomenal information is the information in everything that appears. So all of the phenomenal information must come from somewhere within the one level of the visible universe. Some of the information comes from prior states,

through causation. Some of the information comes in through the microscopic boundary of the domain, as hard randomness. The *microscopic boundary* is the limit of the physical domain at the smallest scale, below which no definite state can be found. *Hard randomness* is an outcome that no prior specification fixes, however complete.

The way information enters the world is another reason that the world is not deterministic, because in a deterministic world all the information would be present in the first state, and every later state would only rearrange the information of the first state. In the world we have, new information enters through the microscopic boundary all the time. The new information is amplified up through every scale.

V. Why No World Is Completely Determined

Classical mechanics and general relativity are both deterministic theories in their standard formulations: in each of the two theories, the state at one time fixes the state at every other time. Classical mechanics is the physics of Newton's laws of motion. General relativity is the theory of gravitation as the shape of space and time. In 1814 the mathematician Pierre-Simon Laplace stated the corresponding view of the world: the state of the world at one time fixes the state of the world at every other time. The physicist Albert Einstein held that a complete physics would be deterministic. Quantum mechanics, the physics of the very small, predicts the distribution of outcomes over many runs of an experiment, but quantum mechanics does not predict the single outcome of a measurement.

The Incommensuration Theorem fixes the answer to the question of whether a whole world can be completely deterministic. Three more compounds of comparison are defined in the same way as symmetry: *continuity* is a sameness of content where there is a sameness of context, *asymmetry* is a difference of content where there is a difference of context, and *discontinuity* is a difference of content where there is a sameness of context. The Incommensuration Theorem is a structural fact about the nature

of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Complete determinism would require perfect symmetry, one law carrying every state into its successor, together with perfect continuity, a succession with no break anywhere. The Incommensuration Theorem forbids exactly the pairing of perfect symmetry with perfect continuity. So no world is purely and completely deterministic, and every world is to some degree causal.

If determinism could hold, determinism would have to be made good all the way down to the absolute microscopic, the scale of exactly zero. Every detail of every state would have to be fixed, at every scale, however small. Yet what comes through the microscopic boundary is hard randomness, which no prior specification fixes, and the Incommensuration Theorem forbids the perfect symmetry and perfect continuity that complete determinism would need. Causation does not need every detail fixed. A person needs no complete microscopic specification in order to describe something as causal: it is enough that the same relation of antecedent to consequent is found again from case to case. So determinism differs from causation in the need for a complete microscopic specification, at the end of the line from determinism to pure randomness where determinism sits (Section VI sets out the line).

The physicist John Bell proved a theorem, now called Bell's theorem, about correlations between measurements on two particles prepared together and then separated. Bell's theorem shows the same difference between causation and determinism from the side of experiment. Experiments have found the correlations that Bell's theorem is about, and the correlations show that there has to be a strict difference between the notion of determinism and the notion of causation: causation cannot be perfectly applied, and determinism is perfectly applied only in the model. The Incommensuration Theorem forbids complete determinism in any world, classical or quantum. The physicist Werner Heisenberg found the uncertainty limit that carries his name: position and momentum cannot both be pinned down perfectly at once. Bell's theorem and the Heisenberg

uncertainty limit, taken together, show in the laboratory the structural fact that the Incommensuration Theorem states, as an irreducible hard randomness (a hard randomness that no past knowledge can predict, even in principle).

VI. One Line from Determinism to Pure Randomness

Five concepts can be laid out on one line: determinism, causality, change, choice and pure randomness. The place of each concept on the line depends on how much of what happens is accounted for in advance. Determinism is at one end of the line, with zero elements unaccounted for: under determinism everything can be predicted, always. Causality comes next, and causality is another name for causation: under causality most things can be predicted, most of the time. Change is in the middle: about half of what happens can be predicted. Choice comes next, and choice is mostly unpredictable, in the way that causation is mostly predictable. Pure randomness is at the other end, with one hundred percent of the elements unaccounted for, so that nothing at all can be predicted.

Neither end of the line is ever reached, and every actual world lies between the two ends. The two ends are idealizations, limits that can be thought but never reached. The three concepts in the middle are what is real: causality, change and choice.

The two ends of the line have the same form, and the two distinctions made at the two ends also have the same form: determinism is to causality as pure randomness is to choice. Determinism and pure randomness are the two idealized poles, and the Incommensuration Theorem excludes both poles. Nothing can be both absolutely symmetric and absolutely continuous, so determinism fails. Nothing can be both absolutely asymmetric and absolutely discontinuous, so pure randomness fails. Causality differs from determinism because causality needs no complete microscopic specification. Choice differs from pure randomness because choice is meaningful to the person who chooses. Seen from outside, from the third per-

son, a choice cannot be told apart from hard randomness, but seen from inside, a choice is meaningful to the chooser, even where the choice carries no evidence of meaning to anyone else. When two things can each be perfected only at the expense of the other, we call the two things *conjugate*. Neither of two conjugate things comes before the other in the order of definition. Choice and causality are conjugate with respect to change: neither is reducible to the other, and neither can be perfected and replace the other.

VII. Valid but Not Sound

Consider a physicist who works inside a pure mathematical model, in which causation and determinism are not distinct. The physicist is not wrong about the model, but the physicist is wrong about what the model is a model *of*.

The physicist's mistake shows how an argument can be logically consistent and philosophically inconsistent at once. When the conclusion of an argument follows from its premises, the argument is *valid*. When a valid argument has premises that are also true of what it is about, the argument is *sound*. Suppose an argument treats causation and determinism as the same thing. The argument can be valid: each step can follow from the step before. The argument is still not sound, because the argument has a premise that is false of the world: the premise that causation and determinism are the same thing. Many careful people hold arguments that treat causation and determinism as the same thing, because in the models that careful people work with every day causation and determinism are the same thing, and a feature built into a model is easily taken for a feature of what the model describes.

Science is about causality, and *not* about determinism. What scientists find is regularity: the same relation of antecedent to consequent, found again across changes of circumstance. The regularity that scientists find is causation, and causation is real and temporal. Determinism adds com-

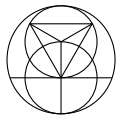
pleteness to causation, but scientists have never found that completeness anywhere in the world: the completeness is a property of the mathematics in which scientists write their findings down.

VIII. Summary

Causation is a regularity in time: the same relation of antecedent to consequent, found again and again across changes of circumstance. The relation stays the same across changes of circumstance, and that sameness is a symmetry. Every model is a description of the symmetry of causation, while causation itself is the genesis of law. Determinism is the claim that one law fixes a whole world completely, to the scale of exactly zero. Determinism is a truth of mathematics, where time factors out, and people carry determinism across to the world only by mistake.

Complete determinism would need perfect symmetry paired with perfect continuity, and the Incommensuration Theorem forbids that pairing, so no actual world is purely and completely deterministic, and every world is causal to some degree. The information in the world we have comes from two places within the world's one level: from prior states, through causation, and through the microscopic boundary, as hard randomness.

Determinism and pure randomness are the two idealized ends of one line. Causality, change and choice are the real concepts between the two ends. Most physicists collapse change into causation, choice into causation, and causation into determinism. The arguments that result can be valid without being sound, because the premise that causation is determinism is true of the model and false of the world.



Chapter 13

Choice, Change, and Causation

The Three Concepts of the Real, and Why Choice Is Not a Kind of Cause

ABSTRACT

The real is that which partakes of choice, change and causation at once. Causation is lawful, repeatable pattern. Choice is the emergence of the new, the singular act that is only itself. Change is the actual transition in which pattern and novelty are found together. Change is the superclass of choice and causation: the class that contains both. Choice, change and causation are distinct, inseparable and non-interchangeable. Choice and causation are conjugate with respect to change: each accounts for change as much as the other, and neither can be reduced to the other. In this chapter I state what a choice requires: a range of potentials, a selection, and an irreversible consequence. I show that the cycle of potential, selection and consequence is the cycle of Axiom II. I show that hard randomness and the part of a choice that nothing before the choice fixes are one event, described from two positions. I place choice on the line that runs from pure randomness to determinism. I locate choice where, by the Incommensuration Theorem, anything at all can live. I close the chapter on the order in which choice, change and causation run: choices start changes, changes can be understood causally, and causal patterns set up the conditions for new choices.

I. The Three Concepts of the Real

The real is that which at once partakes of choice, change and causation, neither more nor less. Choice, change and causation form the foundational triple of the domain of the real: the three concepts on which the whole domain is founded.

Causation is lawful, repeatable pattern: the same relation of antecedent to consequent, found again from case to case. Choice is the emergence of the new: the singular act that is only itself, which no pattern contains in advance. Change is the actual process of transition. In a change, pattern and novelty are found together. Some aspects of a change are accounted for by causal knowledge, and other aspects of the same change are accounted for by choice. Hence change is the superclass of choice and causation: the class of which choice and causation are each a part.

When choice, change and causation are taken together as the triple of the real, each of the three concepts plays one of the three modalities. The modalities are the three roles that the foundational concepts of any domain play toward one another: the immanent, the omniscient and the transcendent. Causation plays the role of the omniscient: causation is pattern laid out as structure, which can be held in view as a whole. Change plays the role of the immanent: change is the act itself, as the act happens, here and now. Choice plays the role of the transcendent: choice is the stance from which an act proceeds, and the range of what is possible from that stance.

So the three axioms that relate the modalities apply directly to choice, change and causation. Axiom III reads: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable". So choice, change and causation are distinct, inseparable and non-interchangeable: each of the three occurs only together with the other two, and none of the three can do the work of another. Axiom I reads: "The immanent is more fundamental than the omniscient and/or

the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". So change, the concept in the role of the immanent, is more fundamental than either causation or choice. Change stands in the middle of the order choice, change, causation; so whichever end of that order one starts from, the foundation of the triple is found in the middle, at change. Causation and choice are in conjugate relationship to one another, with respect to change, by the second sentence of Axiom I. Two things are *conjugate* when each of the two can be perfected (taken toward perfection) only at the expense of the other, and neither is prior to the other in definitional basis.

II. Two Ways of Accounting for Change

Choice is as much a way of accounting for change as causation is. There are changes that occur, and some aspects of those changes can be accounted for using causal knowledge, while other aspects can be accounted for by the choices that agents make.

Over history, people have accounted for change in the two ways in turn, first by choice and then by causation. At one point in history, people attributed all change to local deities, so that every change was, in their view, someone's choice. In the modern world people have attempted to account for all change in terms of causation, and the modern view basically holds that there is only causation: that every change is a case of causation. In quantum mechanics the single outcome of a quantum measurement is an outcome that nothing prior fixes, and so people have been forced to reevaluate the view that there is only causation. In the metaphysics of this book, the two ways of accounting for change are reconciled: change as accounted for by causation and change as accounted for by choice each have their own place, side by side.

The concept of choice is conjugate with the concept of causation with respect to the concept of change, and neither of the two concepts can be reduced to the other or defined away. Causation itself implies potentiality.

Where there is lawful order, there are other possibilities besides the one possibility that actually occurs: a law says what happens under each of several conditions, and so a law already speaks of the conditions that do not occur. Potentiality is a concept of choice, because in its transcendent role choice is the range of what is possible from a stance. So the concept of causation, held fully, already carries the concept of choice inside it.

Choices cannot be predicted. If a choice could be predicted, the choice would be causal and not a choice. Choice has an inherent unpredictability. Section IV of this chapter is about that unpredictability.

III. What a Choice Requires

A choice requires three things: a range of potentials to choose from, an actual selection event, and a consequence.

The consequence is intrinsic to the nature of choice. The reason lies in the principle of identity. The principle of identity states that what is indistinguishable by any means, mode or method must be the same. Suppose a choice were made and could later be undone so completely that it was as if the clock had rolled back to before the choice, as if the choice had never been made. Then the undone choice would be indistinguishable from no choice at all, and so the undone choice would be no choice, by the principle of identity. So every choice has an irreversible consequence. A choice is made only when it is done: only once its irreversible consequence is in place.

Every single outcome comes from more than one selection. To pick up a cup, a person coordinates a great number of muscles in precise synchronization. The person's choreography of action together results in the cup having been moved. A plurality of selection events (many selections) results in a singularity of outcome (one outcome).

And the potentials themselves come from consequences. For the possibility of going to a movie to exist, there need to be roads, cars, theaters, movies, actors, money, light sources, optics and chairs, and each of those

things is a consequence, the outcome of earlier choices. An enormous number of consequences has to be in place for the one possibility of going to a movie to be open.

The three requirements of a choice form the cycle of Axiom II when they are taken in order: potential first, then selection, then consequence. Axiom II reads: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". The potentials are of the transcendent, the selection is of the immanent, and the consequence is of the omniscient. A class of potentials precedes one selection. A class of selections (the many muscles) precedes one consequence (the moved cup). A class of consequences (the roads and theaters) precedes one possibility (going to the movie). So each of the three steps of the cycle goes from many to one: from potentials to one selection, from selections to one consequence, and from consequences to one possibility; and the cycle goes from many to one three times. Potentiality, through selection, becomes actuality. Actuality, as consequence, gives rise to new potentiality. The cycle of potential, selection and consequence is at the core of all process, because the cycle is the cycle of Axiom II, and Axiom II, taken in time, is the abstraction of all process. Taken out of time, Axiom I is the abstraction of all being, in which the relationship is more fundamental than the things the relationship relates.

IV. One Event, Two Descriptions

Hard randomness is an outcome that no prior specification fixes, however complete. Hard randomness is the same event as the part of a choice that nothing before the choice fixes, described from a different position.

Suppose I choose to raise my left hand rather than my right, and suppose my choice is genuine, not fixed by prior states. From an external perspective, my choice looks exactly like a random event. No external measurement can tell my choice and a random event apart, because the two are

one event, described from two modal positions. When the outcome that nothing fixed is described from the omniscient position, the position of structural law and of description in the third person, the outcome is hard randomness: a gap in the specification, with no pattern and no meaning that a description from outside can find. When the same outcome is described from the transcendent position, the position of the one who is engaged in the act, the outcome is choice, and the choice is meaningful to the process from which the choice came. From outside, the outcome is randomness; from inside, the outcome is choice. Both descriptions are correct, and neither description is complete.

Perception itself can be modeled in the same way as the choice, with an outside and an inside, as a communication channel. In the model, a world that produces signals sends a signal through a channel to a self that receives the signal. In the model, the self is a subjective context, and the signal the self receives is objective content. The channel is only a model, because perception has no medium of its own, just as light needs no aether (the medium once supposed to carry light) in order to propagate.

The choice described from inside is genuine, ontologically real choice, real in what the choice is, and not the appearance of choice or a feeling of choosing while one is actually being determined. Choice is not reducible to causation. We need a theory of choice that is the conjugate of the theory of causation, of equal standing with the theory of causation and not subordinate to the theory of causation.

The Incommensuration Theorem is the ground for genuine choice and for a theory of choice of equal standing. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In the terms of Chapter 2, symmetry is sameness of content across difference of context, and continuity is sameness of content within sameness of context, while asymmetry is difference of content across difference of context, and discontinuity is difference of content within sameness of context. By the Incommensuration Theorem, complete determinism is impossible in any

world, classical or quantum, because complete determinism would need perfect symmetry and perfect continuity together. What remains is causation without determinism: a world that is causal, and open, in which some of what happens is not caused. However complete physics becomes, physics cannot fully specify a choice, because no description of a whole world can be completely deterministic. Physics gives the context of a choice, its constraints and its space of possibility, and the selection within that space is the one event with the two descriptions of this section: hard randomness from outside, and choice from inside.

V. The Line from Pure Randomness to Determinism

The concepts of this chapter can be set out in order on one line, from pure randomness through choice, change and causality to determinism. Pure randomness is an outcome with nothing at all accounted for, and determinism is the claim that one law fixes every state of a whole world completely, at every scale, so that every state is fixed in advance by the states before it.

Pure randomness and determinism are the two ends of the line, and both are idealizations. Nothing real is purely random or purely determined. Choice, change and causality are what is real, and they stand in the middle of the line. Change is in the middle of the line because change partakes of both kinds of process, the regular and the non-regular. A change has some degree of regularity, and so is predictable to a degree, and has some degree of non-regularity, which nothing prior fixes.

At each end of the line there is a real concept next to an idealization, and the distinction between the two has the same form at both ends: causality differs from determinism because causality needs no complete microscopic specification. Choice differs from pure randomness because choice is meaningful to the one who chooses. Relative to the self, choice is not random; choice is meaningful with respect to the self.

On the meaning of choice, this book goes beyond the known and the unknown to the unknowable, and says what choice is. Hard randomness cannot be known in advance, even in principle. And yet hard randomness is not meaningless. From outside, from the position of law, an event that nothing prior fixes is hard randomness. From inside, from the process that the event came from, the same event is choice and is meaningful to that process. Work on the foundations of physics has not had the precision needed to handle the concepts of meaningfulness and choice: to handle those two concepts, a thinker needs a discipline in how the concepts are held.

VI. Meaning and the Catastrophic Theorem

When hard randomness is described from outside, hard randomness has no meaning that an outside description can find. Yet having no meaning that an outside description can find does not make hard randomness meaningless.

Whether hard randomness is meaningful is settled by what I call the catastrophic theorem: either everything is meaningful or nothing is. We know that there is some meaningfulness. So everything is meaningful. I therefore reject the notion of hard randomness as meaningless.

Being meaningful does not make hard randomness predictable. Choice is distinct from causation. Meaning is not the same thing as predictability. An event can be meaningful to the process that the event came from, and unpredictable to every description made from outside that process.

VII. Where Choice Lives

Choice lives (that is, choice can occur) where there is enough symmetry and enough continuity, and not all of either.

Consider a person's sovereignty, the person's power to choose and to act on a choice. Suppose a person has choice, but the person's choices have no impact in the world and affect nobody. When a person's choices have no impact, causation is absent. Choice is absent too, because a choice that changes nothing was not a choice of anything. The case of choices without impact is the case of no symmetry: no regularity connects the person's act to any effect in the world. If, on the other hand, everything a person does affects everything, then no free choice can be made, because every small act affects far too much and every move is a catastrophe. The case in which every act affects everything is the case of too much continuity, and in that case the person is bound by the effects of every act and so cannot move.

Between the case of no symmetry and the case of too much continuity, there is enough symmetry, enough continuity, and not all of either, and that middle ground is exactly where, by the Incommensuration Theorem, anything at all can live.

VIII. Humanity's Three Masteries

Humanity has met change first and causation second, and the mastery of choice is now humanity's next task.

For the larger portion of human existence, people lived in a relationship with change. Nature changes all the time, and nature is perfected change. There were indigenous cultures that learned to live within the continual change of nature, and they had a mastery of relating to perfected change. At certain points in history people learned about fire, and about the plow, and so learned about causation: that a consistent pattern of antecedent conditions results in a consistent pattern of consequent conditions, again and again. The causal process has a symmetry: the same pattern is found again and again. The symmetry of the causal process has epistemic implications (implications for what can be known), and the whole of modern science and technology rests on that symmetry.

The next mastery that humanity must have is the mastery of choice. People have attempted to solve all problems with the tool of causation alone, and that attempt has side effects: pollution, inequality, and all the effects of applying top-down engineering methods to complex ecosystems.

IX. Projections of One Another

Change is the superclass of choice and causation, and the relation between choice and causation within change has a definite shape that differs with scale.

At the smallest scale, choice and causation look like strict opposites: a single event is either fixed by what came before or it is not. At the largest scale, how real each of choice and causation is runs in proportion to how real the other is. Ethics gives a case of the largest scale. In ethics we are always trying to maximize the product of two sums: the sum of all the choices one has made, and the sum of all the choices one could make. The universe is constructed on the relation in which choice and causation are each real in proportion to the other, and being is possible because of that relation.

The same shape appears elsewhere too: the shape of the relation between choice and causation within change, a pair of poles held together by a third. The instances of that shape are projections of one another. Reality needs both fermions and bosons. Fermions are content: the matter that is counted and that excludes. Bosons are context: the fields of interaction that add. Reality needs both the subjective and the objective. Reality needs conjugation under the Heisenberg uncertainty principle, by which position and momentum cannot both be pinned down perfectly at once. In each of the three pairs, fermions and bosons, the subjective and the objective, and position and momentum, the two poles are distinct, inseparable and non-interchangeable, and are held together by a third.

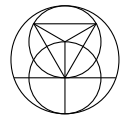
And choice, change and causation run in one order. Choices start changes, changes can be understood causally, and causal patterns set up the conditions for new choices. Choice, change and causation are the three fundamental concepts from which an understanding of physics, of philosophy and of ethics must proceed.

X. Summary

Choice, change and causation are the foundational triple of the domain of the real. When choice, change and causation are taken as the triple of the real, causation plays the role of the omniscient, change the role of the immanent, and choice the role of the transcendent. By Axiom III choice, change and causation are distinct, inseparable and non-interchangeable. By Axiom I, change is the most fundamental of the three, and choice and causation are conjugate with respect to change: each of choice and causation accounts for change, and neither reduces to the other.

A choice requires a range of potentials, a selection and an irreversible consequence. The consequence is intrinsic to choice, by the principle of identity. Potential, selection and consequence are the cycle of Axiom II, which goes from many to one three times. Hard randomness is the same event as the part of a choice that nothing before the choice fixes: the event is randomness when described from outside, by law, and choice when described from inside, by the one who chooses. The Incommensuration Theorem forbids complete determinism and is the ground of the openness in which choice occurs.

On the line from pure randomness to determinism the two ends are idealizations, and choice, change and causality are real. Hard randomness is unknowable in advance and still meaningful, by the catastrophic theorem. Choice lives where there is enough symmetry and enough continuity and not all of either. Humanity's next mastery is the mastery of choice. Choice, change and causation run in one order: choices start changes, changes are understood causally, and causal patterns set up new choices.



Chapter 14

Free Will and the Compression Argument

Why the Residue That Looks Like Randomness Is Where Choice Lives

ABSTRACT

Sam Harris makes a well-known argument, in his book *Free Will* (2012), that free will is incoherent. Harris argues that neither determined events nor random events give what the word "choice" is meant to name, so, on Harris's view, "choice" is only a name for the inputs to an open system, a system that things outside the system affect. Harris's argument assumes that the world is either fully determined or purely random, but the Incommensuration Theorem shows that no world can be either one, so every actual world lies between full determinism and pure randomness. In every actual world, hard randomness is the part of what happens that no prior specification (no description of what came before, however complete) fixes. Choice lives in hard randomness. The compression argument shows from observation alone, without physics, that choice lives in the part of what happens that no prior specification fixes. An observer can compress a person's behavior into a model that predicts the behavior, but however good the model is, a residue of the behavior remains that an outside observer cannot tell apart from randomness, and that residue is the person's choice. The principle of identity says that whatever no means can distinguish must be one and the same. No outside observer can tell choice and randomness apart, so choice and randomness are one event described from two positions. Because choice and randomness are one event, the inference from "randomness is meaningless" to "choice is

> meaningless" runs the wrong way: the reasoning should run from choice, which is meaningful to the one who chooses, to randomness. Meaning is not something an outside description can evaluate, and meaning is found in the interchange among three domains: in communication, the speaker, the listener and the language between speaker and listener. In one more section before the summary of this chapter, I take up pain and joy, a second pair of conjugates. Pain and joy share one foundation, intensity, and *neither* compels the other.

I. The Argument That Free Will Is Incoherent

Sam Harris argues, in his book *Free Will*, that free will is an illusion. I state Harris's argument at its full strength before I answer the argument.

Harris grants that many things affect what a person does. A person is an open system, a system that things outside the system affect, and the things that affect an open system are the system's inputs. Harris's argument is about what kind of thing affects an open system. Either the inputs form a chain of causal links that amounts to determinism, in which every state is fixed in advance by earlier states, or some of the inputs are random. Whether one goes with determinism or with randomness, neither case shows the characteristics of what would be called choice-making. A determined act is not a free choice, because the act was fixed in advance, and a random act is not a free choice either, because nobody chose the random act. On Harris's view there is no magical particle called choice. What people call choice is the name of the inputs to an open system: a description of how deterministic causal events and random events combine. So, Harris concludes, the notion of free will is incoherent.

II. What the Argument Assumes

Harris's argument assumes that the world is either fully determined or purely random. No world can be fully determined, and no world can be purely random.

The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Complete determinism would need a world both absolutely symmetric and absolutely continuous: one law carrying every state into its successor, in a succession with no break. So no world can be completely determined, by the first half of the Incommensuration Theorem, about symmetry and continuity. Pure randomness would need a world both absolutely asymmetric and absolutely discontinuous: no pattern carried anywhere, and nothing joined to anything. So no world can be purely random, for the matching reason: the second half of the Incommensuration Theorem, about asymmetry and discontinuity. Every actual world lies between the two extremes of complete determinism and pure randomness.

Between the two extremes, something is left over in every actual world. Hard randomness is the part left over: the part of what happens that no prior specification fixes. A prior specification is a description of what came before, however complete. Hard randomness is real. Hard randomness is a fact about the world, and not a fact about anyone's ignorance. Even with complete prior information, nobody could predict which single outcome will occur, because the prior information gives the probability of each outcome, and nothing more: nothing in the prior information fixes the outcome that happens. The single outcome of a quantum measurement is the plainest case of hard randomness.

Choice lives in the left-over part of every world, the part that no prior specification fixes. Choice lives between the completely random, which would be no choice at all, merely noise, and the completely determined, which would also be no choice at all, merely execution. Harris's dilemma has two horns, the determined case and the purely random case. No world can be at either horn, because the two horns are the two extremes. Harris's argument says nothing about the region between the two extremes, yet the region between the two extremes is the only place any world can be.

III. The Compression Argument

You can draw the conclusion of Section II from observation alone, without going through physics. The conclusion is that choice lives in the part of what happens that no prior specification fixes.

Watch a person over a period of time and try to identify where the choices are. People are pretty predictable: an observer can predict quite a lot of what a person is going to do. Next, you write down everything you can predict about the person's behavior. Writing down what can be predicted is what a *compression algorithm* does with a data stream: the algorithm writes down the regularities of the stream so that the stream can be stored in fewer bits. However good the compression algorithm is, some residual amount of the person's behavior will be left that the algorithm cannot compress. The residue is indistinguishable from randomness. The residue is the part of the behavior that the person regards as their choice.

So choice and randomness cannot be told apart from any third-person vantage, that is, by any observer who watches the person from outside. Everything an outside observer can model is the regular part of what the person does, and the regular part is causation. When the observer's model is as good as a model can be, what remains of the behavior has exactly the look of randomness.

IV. The Inference Reversed

Someone could read the compression argument as saying that randomness is meaningless, and so choice is meaningless, but the principle of identity shows that reasoning from randomness to choice should run the other way, from choice to randomness.

The principle of identity says that whatever no means can distinguish, even in principle, must be one and the same. From the third person, from outside, choice and randomness cannot be told apart. Choice and randomness cannot be told apart because choice and randomness are one event, described from two positions, and *not* because choice and randomness are two different things that happen to look alike. From outside, the event is randomness. From inside, the event is choice.

Choice is meaningful to the one who chooses. Seen from outside, the choice shows no pattern and no meaning, because meaning is not something an outside description can evaluate. So the reasoning, run from choice to randomness, shows that being unpredictable does not make an event meaningless. Whoever infers that choice is meaningless makes the error of taking the outside description of the event for the whole of the event.

V. Where Meaning Is Found

A person misunderstands what meaningfulness is when the person binds meaningfulness to the ability to predict future states, or to persistence through time, because meaningfulness is found in between, in the interchange among three domains, and is not defined in terms of perfected, persistent future states.

Communication shows the three domains: myself, you, and a language. When I say something to you, the first domain is myself: what I subjectively want to communicate, in the context of myself. The second

domain is you, the person to whom I am trying to convey the message. You have your own subjective content and your own subjective context. To convey the message, I have to go through a third domain: a language, in which I say a sentence. The language defines all the things I could say. I translate a subjective state into an objective statement, in the context of the language, and you hear the statement and interpret the statement into a subjective state of your own. Myself is a domain, yourself is a domain, and the language is a domain. However the notion of meaning in communication is constructed, the assumptions behind the notion include the conjunction of myself, yourself and the language.

VI. Pain and Joy

Pain and joy show that the modalities are distinct, in a case that bears on free will. The modalities are the three roles this book calls the immanent, the omniscient and the transcendent. No amount of pain compels any amount of joy, and no amount of joy compels any amount of pain.

Pain and joy are conjugates under intensity, and are not opposites. Pain and joy are conjugate in the way the omniscient and the transcendent are conjugate. When two things share a foundation and neither can be derived from the other, the two things are conjugate in the sense used of pain and joy, and the foundation that pain and joy share is intensity. In the triple of pain, joy and intensity, pain has the nature of the omniscient, and joy has the nature of the transcendent. Intensity has the nature of the immanent, and the intensity meant is the intensity of the interaction between self and world. Pain has the character of the objective and the discontinuous: a break, a breach, a sudden change. Joy has the character of the subjective and the continuous: connection, joining, the felt sense that one's possibilities have expanded. When I am in pain, I learn nothing from the pain about what would bring me joy, and when I am joyful, I learn nothing from the joy about what would bring me pain. Pain and joy share one foundation, intensity, and neither one implies the other.

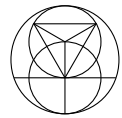
Choice and causation are related within change as pain and joy are related within intensity: choice and causation are conjugate with respect to change. In the triple of change, causation and choice, the modalities fall in the order immanent, omniscient, transcendent. Change has the nature of the immanent, causation the nature of the omniscient, and choice the nature of the transcendent. However complete physics becomes, physics cannot fully specify a choice, because no description of a whole world can be completely deterministic: physics gives the context, the constraints and the space of possibilities. The selection within the space of possibilities is one event with two descriptions: from outside, the selection is an outcome that no prior specification fixes, and from inside, from the one who chooses, the selection is a choice. Both descriptions are correct, and neither is complete. A choice is like a joy in one respect: no completeness of physics specifies a choice, just as no amount of pain specifies a joy.

VII. Summary

Harris argues that free will is incoherent. Harris's argument assumes that the world is either fully determined or purely random, and holds that neither case would contain choice. Harris's premise is false of every world. The Incommensuration Theorem shows that no world can be completely determined and no world can be purely random. So every actual world lies between complete determinism and pure randomness. Hard randomness is the part of every actual world that no prior specification fixes, hard randomness is real, and choice lives in hard randomness, between noise (pure randomness) and execution (complete determinism).

The compression argument uses observation alone, and shows that the residue of a person's behavior that no model of the person can compress is indistinguishable from randomness, and that the residue is the person's choice. The principle of identity then shows that choice and randomness are one event with two descriptions: randomness from outside, and choice from inside. So the inference from "randomness is meaningless" to "choice

is meaningless" runs the wrong way, because meaning is not something an outside description can evaluate, and meaning is found in the interchange among three domains, myself, yourself and the language, as communication shows. Pain and joy, which are conjugates under intensity, show the same structure as choice and causation: pain and joy are two members of a pair that share a foundation, and neither member compels the other.



Chapter 15

The Bound Triple

Hard Randomness, the Arrow of Time, and the Subjective

ABSTRACT

Three features of reality form a bound triple: hard randomness, the arrow of time, and the subjective stance. Hard randomness is an outcome that no prior specification fixes. The arrow of time is the asymmetry between a determined past and an open future. The subjective stance is the stance from which every comparison is made. The three features are distinct, inseparable, and non-interchangeable. This chapter establishes the binding of the three features (the fact that the three features form a bound triple) twice. First, the binding is derived from the Incommensuration Theorem. The derivation rests on three facts about every comparison: every comparison leaves a remainder that nothing prior fixes, proceeds in a direction from the act to the record of the act, and is made from a stance. Second, the binding is found in physics, where the Copenhagen interpretation and the many-worlds interpretation, the two interpretations of quantum mechanics at opposite ends, keep all three features together or drop all three features together. Removing any one of the three features removes the other two, and each feature requires the next in a closed circle. The hard problem of consciousness is the question why there is experience at all. Answering the hard problem would require a description held in the third person, which is omniscient in character, to contain the first-person subject, which is transcendent in character, and the omniscient cannot contain the transcendent, so the hard problem is hard. The

> subjective is identified as the interior face of a process that presents from outside as entanglement and hard randomness. The brain operates near criticality, a regime in which small perturbations can cascade to large scales. The brain's operation near criticality is the physical circumstance in which the relationship of the subjective to entanglement and hard randomness is expressed in us. Consciousness is the bound triple taken as one act.

I. Three Features of One Act

Every act of knowing is an act of comparison, the act of finding whether two things are the same or different in some respect. Every comparison has three features: hard randomness, the arrow of time, and the subjective stance.

Hard randomness is an outcome that no prior specification fixes, however complete the specification is, so an outcome of hard randomness is unpredictable for any subjectivity whatever, including a subjectivity with unlimited access to the relevant state. The single outcome of a quantum measurement is the plainest instance of hard randomness: a detector registers one result, and no prior specification of the measured system fixed which result the detector would register.

The *arrow of time* is the structural asymmetry between a determined past and an open future. The arrow of time appears in every single comparison, because in every comparison the range of the possible comes first, then the act of comparing, then the record of the act. Every record comes after the act that made the record. A person reading a thermometer shows the order. Before the person looks, the liquid could be at any of the marks. Then the person looks, and then the person writes down the mark, so the written number comes after the look.

The *subjective stance* is the subjective pole of comparison, the stance from which every comparison is made: every comparison has a subjective side and an objective side, and the subjective stance is the subjective side.

In a conscious being, the subjective stance is the fact that there is something it is like to be that being, in the phrase of the philosopher Thomas Nagel (Nagel, 1974).

The three features form a *bound triple*: hard randomness, the arrow of time, and the subjective stance. The three members of the bound triple are distinct, inseparable, and non-interchangeable. Each member is different from the other two. Each member occurs only together with the other two, and each member does work that neither of the other two can do. Removing any one of the three members removes the other two.

II. Why the Hard Problem Is Hard

The hard problem of consciousness is the question why there is experience at all, and not only information processing with no one to whom the processing happens. At bottom, the hard problem is the observation that no structure handed to another person carries one's own knowing. One person can hand another person a description, a diagram or a list of numbers, but the knowing of the person who hands the structure over stays behind, with that person. A complete description of pain is knowledge laid out as structure, while what pain feels like is known only in one's own perceiving.

Our theories of physics are all knowledge described in the third person, as anyone at all could state the knowledge, while every experience is in the first person, had by someone in particular. Even to do an experiment requires an intervention in the first person: someone sets up the apparatus, and someone reads the result. Physicists abstract the data that they collect in the first person into theories expressed in the third person. If anyone then takes the third-person knowledge as the whole truth, that person can no longer account for the first-person experience. Theories do not contain experiences: a description, however complete, arrives at another person as structure. The hard problem has stayed unsolved since it was first posed, because the toolset used on the hard problem is description in

the third person, and that toolset presupposes the problem's conclusion. A description in the third person leaves experience out from the start, and so a description in the third person cannot solve the hard problem.

The three modalities show exactly why the hard problem is hard. The three modalities are three roles that the parts of every comparison play toward one another: the immanent, the omniscient, and the transcendent. The immanent is the role of the act itself, as the act happens, here and now. A third-person description is *omniscient* in character. A third-person description is laid out as structure and can be held in view as a whole, the way a map can. The subjective is *transcendent* in character. The subjective is the stance from which an act proceeds, and the range of what is possible from that stance. The omniscient would have to contain the transcendent for anyone to answer the question "how does first-person experience arise from a third-person description?" The omniscient cannot contain the transcendent, so anyone who asks the question is asking for something conceptually impossible. The explanatory gap is the gap between physical description and conscious experience. The explanatory gap is therefore a structural feature of comparison itself.

The case of time shows why answering the hard problem would require the first person to come out of the third person. The present moment is real in the first person: we tell past from future, and the future keeps becoming the past. Spacetime is space and time taken together as one whole. In the third-person view, spacetime is a continuous solid field, and possibility is not a described part of spacetime. The passage from the open to the settled is what the first person holds most plainly, and the passage is central to experience, but the third-person picture leaves the passage unaccounted for.

III. The Binding Derived

The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Chapter 2 defines the four terms of the Incommensuration Theorem from two features of every comparison: the content of a comparison is what the verdict of the comparison is on, and the context is the respect in which the two things compared are two. Symmetry is a sameness of content where there is a difference of context, as when a shape flipped over looks like the same shape, while asymmetry is a difference of content where there is a difference of context. Continuity is a sameness of content where there is a sameness of context. Discontinuity is a difference of content where there is a sameness of context.

The three features are hard randomness, the arrow of time and the subjective stance, and the binding of the three features follows from the Incommensuration Theorem. The binding is the fact that the three features form a bound triple. Every comparison leaves a remainder, an outcome not fully determined by prior structure, because symmetry and continuity cannot both be perfected in any comparison. The remainder is hard randomness. Every comparison is made from a stance, the subjective pole of comparison, and that stance is the subjective stance. Every comparison proceeds in a direction, from the act of comparing to the record of the act, and that direction is the arrow of time. So the three features are three aspects of one act. The act is a comparison proceeding in a direction, registered by a perspective, with an outcome not fully determined. None of the three features is present without the other two.

IV. The Binding Found in Physics

The binding also shows in physics, in the two interpretations of quantum mechanics that are at opposite ends of the range of interpretations: the Copenhagen interpretation and the many-worlds interpretation. An interpretation of quantum mechanics is an account of what the equations of quantum mechanics say happens in the world. Above all, an interpretation says what happens when a measurement is made. The Copenhagen interpretation keeps the observer, the random single outcome, and the irreversibility of measurement (the fact that a measurement, once made, cannot be undone): the observer is the subjective stance, the random single outcome is hard randomness, and the irreversibility of measurement is the arrow of time. In the many-worlds interpretation, every possible outcome of a measurement occurs, each outcome on its own branch of the world. The many-worlds interpretation drops all three features at once. The many-worlds interpretation has no subjective stance, because the many-worlds interpretation has no fact of which branch a comparer is in. The many-worlds interpretation has no hard randomness, because the many-worlds interpretation has no outcome that is not fixed. The fundamental equation of the many-worlds interpretation is symmetric in time: the equation works the same whether time runs forward or backward. So the many-worlds interpretation has no arrow of time. The two interpretations keep the three features together or drop the three features together, and so the binding is established twice: derived from the Incommensuration Theorem, and found in physics.

V. The Removal Tests

Remove hard randomness. Then every event is determined by prior states, and the future is fixed. The arrow of time collapses: the distinction between past and future becomes a matter of perspective and not of what is. The subjective stance vanishes as the capacity for choice, because every event is already determined, so nothing remains to choose.

Remove the arrow of time. If time is symmetric, there is no difference between cause and effect. Randomness becomes meaningless, because a random event is an event whose outcome the event's causal history does not determine, and where cause and effect are symmetric, an event has no causal history for the event's outcome to be undetermined by. The subjective stance collapses too, because conscious experience is temporal in its own nature.

Remove the subjective stance. If there is no interior perspective, the distinction between randomness and choice is empty, because the interior perspective is the place from which an undetermined outcome is a choice and not only a random event. The arrow of time loses its ground in experience. The arrow of time becomes a statistical artifact (a direction that shows only in counts over many events), and not a feature of reality.

VI. The Circle of Requirement

Each of the three features requires the next, and the three requirements close a circle: directed time requires hard randomness, hard randomness requires the subjective stance, and the subjective stance requires directed time. Directed time is time with an arrow, time that runs one way.

Directed time requires hard randomness. If every state were fixed by the states before that state, the future would be contained in the present as fully as the past is, and the order of states would then be an indexing, readable either way, like the page numbers of a book, and not a direction of process.

Hard randomness requires the subjective stance. A purely third-person description cannot tell an outcome that no prior state fixed from an outcome that was fixed and is unknown, since both outcomes are unpredicted. The difference between the two outcomes is the difference between a finding that is new and a finding that was only hidden. Every finding is made by a comparer, from a stance.

The subjective stance requires directed time. A stance sets what is found against what was found. Remembering sets the present against a record, and anticipating sets the present against what could come. Remembering and anticipating each run one way, so if time had no direction, there would be no difference between remembering and anticipating, and no side from which to look.

VII. The Simplest Form

The binding has its simplest form in the distinction of content and context. The *content* of a comparison is what the verdict of the comparison is on. The *context* is the respect in which the two things compared are two. Hard randomness is the entrance of novelty: content arriving that no prior context contained. The arrow of time is the asymmetry between what has been determined and what has not yet been determined. The subjective stance is the interior perspective on the asymmetry between the determined and the undetermined. The subjective stance is the point from which the open is seen as open. The three features are three faces of one structure, and one cannot have a face without the solid: no one of the three features exists apart from the one structure that has all three.

The modal roles of the three features can be named once the focus is named, where a focus is the frame of a description relative to which modal roles are assigned, for example practice held in the first person, or theory held in the third person. Within the frame of the triple itself, the arrow of time is immanent, because the arrow of time is the act as the act happens, setting past apart from future. Within the frame of the triple itself, hard randomness is omniscient, because hard randomness shows from outside as a gap in the laid-out specification, while the subjective stance is transcendent, as the stance from which the act proceeds. In use, the triple is taken as one, all three features at once, in a context. In the focus of use, the whole triple is immanent.

The inseparability of the three features shows wherever new information enters any system. Three things go together wherever new information enters a system. The first of the three things that go together is the entrance of hard-random information, and the second of the three things is the particular moment at which the hard-random information enters. The third of the three things that go together is the subjective standpoint from which the entrance can be observed as having occurred, as a fact. The entrance, the moment and the standpoint are not three separable things. A theory laid out wholly as structure can never contain the fact that the entrance, the moment and the standpoint cannot be separated.

VIII. The Interior Face, and the Brain Near Criticality

When a single undetermined outcome is described from outside, the outcome is hard randomness, but when the same outcome is described from inside, from the position of the one engaged, the outcome is choice. Hard randomness and choice are then one event under two descriptions. Both descriptions are correct, neither description is complete, and neither description can replace the other. The Immanent Metaphysics is the philosophy set out in *An Immanent Metaphysics*. The Immanent Metaphysics contains a further identification, stronger than the claim of two descrip-

tions: the subjective and total entanglement are strict isomorphs. Two things are strict isomorphs when the two things correspond to each other part for part, with every relation among the parts kept. Entanglement is the linking of the states of two systems that interact directly as quantum systems. The subjective is the interior face of a process that, from the exterior, presents as entanglement and hard randomness: the subjective is the process as the process is from inside, and entanglement and hard randomness are the same process as the process presents from outside.

The brain operates near criticality, a regime in which small perturbations can cascade to large scales, as the study of neuronal avalanches (cascades of activity among nerve cells) suggests (Beggs and Plenz, 2003). In a brain near criticality, small perturbations cascade, so that indeterminacy at the microscopic scale enters causal chains that extend to the macroscopic scale and bear on choice. The mesoscopic scale is the scale between the microscopic and the macroscopic. At the mesoscopic scale, the behavior of a brain can be modeled by approximately deterministic computation, and the deterministic model is useful. A single choice made near criticality carries indeterminacy from the microscopic scale that no approximately deterministic model fixes. The subjective stance is a constitutive feature of the act of comparison (without the subjective stance there is no act of comparison), and so the subjective stance is no epiphenomenon, no by-product with no standing of its own.

This account of consciousness is distinct from the Orchestrated Objective Reduction hypothesis of Stuart Hameroff and Roger Penrose (Hameroff and Penrose, 1996). The Orchestrated Objective Reduction hypothesis locates consciousness in sustained quantum coherence in a specific biological structure, the microtubules of the brain. The physicist Max Tegmark raised the standard objection to the hypothesis (Tegmark, 2000). The objection is that entanglement in warm neural tissue decoheres far too quickly to bear on neural events. Entanglement decoheres when the coherence of the entangled system spreads into the joint system that the entangled system forms with the environment around the entangled system. The Immanent Metaphysics depends on no specific biological mechanism

and needs no sustained entanglement in the brain, because the claim of the Immanent Metaphysics applies to any system in which novelty enters, time is directed, and there is a point of view. The criticality of the brain is the physical circumstance under which the relationship of the subjective to entanglement and hard randomness is expressed in us. The criticality of the brain is not the relationship itself.

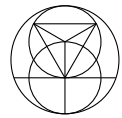
IX. Consciousness Named

Consciousness is the bound triple taken as one act: the arrow of time, hard randomness, and the subjective happening together, with the subjective as the interior face of the one act. The bound triple is what distinguishes a world in which things happen from a world in which things only appear to happen. So the bound triple is the signature of the real. A negative result follows from the binding: a computation is deterministic, has no direction of its own and has no interior, and so a computation cannot be what consciousness is. The negative result does not depend on naming consciousness as the bound triple taken as one act. The negative result rests on the binding and on the Incommensuration Theorem.

X. Summary

I have set out the bound triple of hard randomness, the arrow of time, and the subjective stance. The three features are distinct, inseparable and non-interchangeable. The binding is derived from the Incommensuration Theorem, with the three features as three aspects of every act of comparison, and the binding is also found in physics, where the Copenhagen interpretation of quantum mechanics keeps all three features and the many-worlds interpretation drops all three features. The removal tests and the circle of requirement show the same binding from inside the triple. Each feature needs the next, and losing one feature loses all three. The simplest form of the bound triple is the entrance of new content, the asymmetry of the de-

termined and the open, and the interior perspective on that asymmetry: three faces of one solid. The hard problem is hard because answering the hard problem would require the omniscient to contain the transcendent, and the omniscient cannot contain the transcendent. The subjective is identified as the interior face of what presents from outside as entanglement and hard randomness. The brain near criticality is where the relationship of the subjective to entanglement and hard randomness is expressed in us. No specific mechanism is required. Consciousness is the bound triple taken as one act.



Chapter 16

Hard Randomness

What Comes Through the Microscopic Boundary

ABSTRACT

Hard randomness is an outcome that no prior specification fixes, however complete the specification is. Hard randomness is real. Hard randomness is a fact about the world, and not a fact about our ignorance. This chapter establishes hard randomness from two results that experiment confirms (the uncertainty principle, whose degree the Planck constant measures, and the violation of Bell's inequality) and from the Incommensuration Theorem, which guarantees hard randomness in every comparison. Hard randomness is what comes to us through the microscopic boundary of the domain of measurement. The microscopic boundary is the limit at the smallest scale, below which no definite state can be found. The microscopic boundary is a limit of signal, as the cosmological horizon is: no signal at all comes from beyond the cosmological horizon, the edge of what one observer can ever compare with. Determinism and pure randomness are the two idealized poles on either side of hard randomness. Neither pole is ever reached, so every actual world lies between the two poles. Microstate amplification is the channel by which hard randomness reaches the scale of ordinary life: in microstate amplification, differences too small to measure are magnified into differences that decide what a system does. The irreducible residue that microstate amplification carries is hard randomness, and the residue is where choice lives.

I. What Hard Randomness Is

Hard randomness is an outcome that no prior specification fixes, however complete the specification is. A prior specification describes everything that was the case before the outcome. A hard random outcome is inherently unpredictable for any subjectivity whatsoever (any knower at all), including a knower with unlimited access to the relevant state. The single outcome of a quantum measurement is the plainest instance of hard randomness, because a detector registers one result, and no prior specification of the measured system fixed which result the detector would register. Statistics repeat, but the single outcome does not. When the same quantum measurement is made many times, the proportions of the outcomes come out the same from one long series of measurements to the next, but the single outcome is not reproduced. Preparing the system in the same way again does not fix the same single outcome again.

The word "hard" says that the failure of prediction is categorical and not practical. The failure holds in principle, and not only for want of better tools. A hard random outcome is not causally predictable, because nothing prior fixes the outcome. If any notion of causality could be brought to bear on a hard random outcome, the notion would make the outcome not random. An outcome is random when the outcome cannot be made causal at all (cannot be brought under any cause that fixes the outcome), ever, even in principle. Hard randomness is irreducible in the sense that no causal capacity to predict a hard random outcome exists, on the basis of any past knowledge whatever. There is no possibility that hard randomness will ever be reduced to causal knowledge, or to any kind of logical predictability.

Two other things are called random. Hard randomness is neither of the two. *Soft randomness* is randomness of ignorance: an outcome that was fixed, but that I do not know, and that better knowledge could reduce. A coin is soft random to me when the coin has already been tossed and is

hidden under a hand: the coin already shows heads or tails, and lifting the hand would tell me which side is up. *Pseudo-randomness* is what a computer produces: a fixed rule applied to a starting number, so that the same start gives the same sequence in the same order. A pseudo-random generator is deterministic. The rule and the starting number fix every output completely. The outputs of a pseudo-random generator only look random. A computation contains no hard randomness of its own: hard randomness can be imitated by computation and never produced by computation. A computation that needs hard randomness has to draw hard randomness from a physical process in the world in which the computer runs, and then whatever randomness the computation shows is the randomness of that world, carried through the computation.

II. Three Results, One Conclusion

Hard randomness follows from three closely related concepts of physics, taken together: Heisenberg's uncertainty principle, the Planck constant, and Bell's theorem.

The physicist Werner Heisenberg stated the *uncertainty principle*: conjugate quantities, such as position and momentum, cannot both be sharply defined at once (Heisenberg, 1927), where two quantities are conjugate when each of the two can be made sharp only at the expense of the other. The momentum of a particle is its mass times its velocity. The Planck constant is a number named after the physicist Max Planck. The Planck constant sets the degree of the uncertainty in conjugate quantities: the Planck constant is the measure of how far, and in what way, the uncertainty shows itself. The physicist John Bell proved the theorem that carries his name. Bell's theorem states that nature contains correlations between distant records that no causal process, bounded by the speed of light, can reproduce. A correlation is a pattern in which results at one place go with results at a distant place more closely than chance would give. The uncertainty principle is part of the reason for the way physicists understand

Bell's theorem, because making correlated pairs of particles is not a way around the uncertainty principle. Someone might hope to get around the uncertainty principle with a correlated pair, by measuring the position of one particle and the momentum of its partner, but the uncertainty principle applies to the pair all the same.

Bell's inequality is the limit on how much agreement between two distant records a shared past can supply. Experiments find more agreement than Bell's inequality allows, and physicists call the excess the violation of Bell's inequality. The uncertainty principle is an experimental result, and so is the violation of Bell's inequality. Taken together, the two experimental results show that no coherent notion of causality can explain every measurement result, and a result that is not explained by causation is random, and random in the fundamental sense. So hard randomness is a real concept. No theory can factor hard randomness out (remove hard randomness from its account of the world) and stay consistent with the experimental results. Once an assertion of hard randomness rests on experiment, the concept of hard randomness can no longer be discarded.

In 2015 three groups of experimenters, in Delft, in Vienna and in Boulder, closed the loopholes of detection and of locality together, and Bell's inequality was still broken (Hensen and colleagues, 2015; Giustina and colleagues, 2015; Shalm and colleagues, 2015). A loophole is a way in which an experiment could appear to break Bell's inequality without breaking Bell's inequality. The loophole of detection is open when the detectors miss so many particles that the pairs counted might not be a fair sample. The loophole of locality is open when a signal traveling no faster than light could carry news of the setting at one station (one of the two places where the members of a pair are measured) to the other station before the measurement at the other station is finished. In the sense that the experiments test, locality means that each outcome is fixed by causes local to the place where the outcome is found. The three experiments of 2015 rule out every local account: every account in which each outcome is fixed in advance by causes local to the place where the outcome is found. Two deterministic accounts remain open to a determinist. A determinist is a

person who holds to determinism, the claim that everything is fixed by what came before. The first deterministic account is the pilot-wave theory of the physicist David Bohm, which keeps hidden positions and gives up locality (Bohm, 1952). In Bohm's pilot-wave theory every particle has a definite position at every moment, hidden from measurement. The outcome in one laboratory depends on the setting in the other laboratory. The second deterministic account is *superdeterminism*, the claim that the experimenters' settings were fixed in advance together with the states of the particles, and a superdeterminist gives up the independence of the settings (the freedom of each setting from the states of the particles measured), on which every experiment relies. The experiments constrain both deterministic accounts, since each must agree with the measurements, but no step of logic from the measurements excludes either account.

A comparison is the act of finding whether two things are the same or different, in some respect. The content of a comparison is what the verdict is on, and the context is the respect in which the two things compared are two. Symmetry is a sameness of content where there is a difference of context, as when a shape flipped over looks like the same shape, while continuity is a sameness of content where there is a sameness of context. Chapter 2 proves the Incommensuration Theorem, and Section III states the theorem in full. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. Each deterministic account fixes a whole world completely by one law. Fixing a whole world completely by one law would perfect symmetry and continuity together (apply both simultaneously and fundamentally), which the Incommensuration Theorem forbids. So neither deterministic account can be the account of a whole world.

Physics keeps locality in a narrower sense: no signal travels faster than light. When locality is kept in the narrower sense that physics keeps, the randomness observed in quantum measurement is fundamental: the randomness is a feature of nature, and not a screen that hides some deeper order.

III. The Theorem That Guarantees It

The Incommensuration Theorem states: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Asymmetry is a difference of content where there is a difference of context. Discontinuity is a difference of content where there is a sameness of context. Because symmetry and continuity cannot both be perfected, every comparison leaves a remainder, a part of the outcome that no prior specification fixes. The remainder that no prior specification fixes is hard randomness. The Incommensuration Theorem therefore guarantees hard randomness in every comparison, and so reality is not fully computational, because a computation contains no hard randomness of its own (Section I).

The Incommensuration Theorem also fixes where two idealized poles lie: determinism and pure randomness. Determinism is the claim that everything is fixed by what came before, and determinism would perfect symmetry with continuity. Pure randomness is an outcome with nothing at all accounted for, and pure randomness would perfect asymmetry with discontinuity. The Incommensuration Theorem forbids both perfections. Determinism and pure randomness are idealizations, extremes that no actual world reaches. Every actual world lies between the two extremes. What we actually find in the world is hard randomness: the part of what happens that nothing prior fixes. Reality as we have it is choice, change and causation, the three concepts that Chapter 13 sets at the foundation of what is real. Pure randomness falls outside the domain of what happens, in the same way that determinism does.

IV. The Microscopic Boundary

The microstate of the world is the state of the world in its finest detail, and everywhere in the microstate of the world there is a limit of access, a limit on what anyone can find out. The uncertainty principle states the limit of access. The domain of measurement is the whole of what can be compared by measurement. The *microscopic boundary* of the domain of measurement is the limit of access that the uncertainty principle states. The uncertainty principle states a boundary on the information that can be had: position and momentum cannot both be specified beyond a fixed joint precision. The uncertainty principle does not describe a disturbance that measurement causes and that better technique could reduce. Below the microscopic boundary no interaction yields a signal, so there is nothing below the microscopic boundary for any instrument to find.

In this chapter a boundary means a place past which no signal comes: no coherent signal at all. The microscopic boundary has a counterpart at the other end of scale, the end of the very large. No signal comes to us from beyond the cosmological horizon. The *cosmological horizon* is the edge of what one observer can ever compare with, and not an edge of the universe. Light from beyond the observable universe has not had time to reach us in the 13.8 billion years since the present expansion began, and light from beyond the cosmological event horizon will never reach us. There is zero measurable signal from too far away, and in the same way there is zero measurable signal from too small.

What comes to us through the microscopic boundary is hard random: an outcome that no prior specification fixes. When a single atom decays and a detector clicks, the click is a hard random outcome. Even with complete prior information, I could not predict which single outcome will occur. The prior information gives the probability of each outcome, and nothing more: nothing in the prior information fixes which outcome happens. Quantum theory states the chance that an atom decays in the next

second. Nothing further can be said about the moment at which the atom will decay. For learning, the shortfall between the probability and the outcome is permanent: no amount of study ever makes up the shortfall.

How can anyone tell "nothing fixed this outcome" from "something fixed this outcome and I did not know"? For one outcome taken alone, nobody can tell the two cases apart by any outside test. The distinction between the two cases rests on the limit on signaling. Below the microscopic boundary there is no signal for anything to carry. So below the microscopic boundary there is no fixed-but-hidden fact for a better instrument to recover. After the Bell tests of 2015, with the loopholes of detection and of locality closed, two options remain: superdeterminism, or hard randomness. The randomness is called hard because of the "in principle": a hard random outcome is unpredictable in principle, and not only in practice.

V. Why Not Determinism All the Way Down

The alternative to hard randomness is determinism. Determinism would require something that no comparison can supply. If determinism could hold, determinism would have to be made good (hold exactly) all the way down to the scale of the absolutely microscopic, the scale of exactly zero size. Determinism would need a definition of detail at zero size. In mathematics, detail at zero size is easy to write down, because a point has zero size, and a formula can give the exact value of a quantity at a point. In the physical world, detail at zero size cannot be had at all, and the reason is not that such detail is hard to get: the reason is that no one can find detail at zero size by any comparison. Finding detail at zero size would mean seeing the infinitely small, and seeing the infinitely small is as impossible as seeing infinitely far away. The infinitely small and the infinitely far are two different limits, the small one and the far one, and no measurement reaches either limit.

The domain of measurement has a limit at the small end, far coarser than zero size: the scale at which no one comparing can find a definite state. The small-end limit is a limit on what can be found, and not a smallest step of space. Space itself has no least length, and how fine a comparison can be depends on the energy brought to the comparison. Wherever the small-end limit falls, there is a limit on signaling at some scale, and a limit on signaling at some scale is all that the argument against determinism needs.

So the two idealizations, determinism and pure randomness, lie beyond the two boundaries of the domain of measurement: the microscopic boundary, and the macroscopic boundary, which is the boundary at the large end of scale, where the cosmological horizon lies. Determinism represents what is beyond the macroscopic boundary. Pure randomness represents what is beyond the microscopic boundary. What comes to us through the microscopic boundary is hard randomness, and hard randomness belongs to the world we live in.

VI. A Fact About the World, Found from a Stance

Hard randomness is a fact about the world, and not a fact about my ignorance. Hard randomness also has to do with perspective, in one definite sense. A stance is the position from which a comparison is made. Someone comparing finds, in the act of comparing, the difference between an outcome that nothing fixed and an outcome that was fixed and only hidden from me: the difference is found from a stance. Hard randomness, found from a stance, holds for every stance. Any information I could possibly access gives me at most the probability of each outcome. The information never tells me which outcome will occur. No subject could predict the single outcome, even a subject with unlimited access to the prior state.

VII. Microstate Amplification

A quantum event at the scale of atoms produces an observable result at the scale of laboratory instruments by amplification. Amplification is the magnifying of a small difference into a large one. A detector is built so that one microscopic event sets off a cascade, a chain in which each event sets off further events. A Geiger counter is a common detector of radiation. Inside a Geiger counter, a single ionization starts an avalanche of further ionizations, and the avalanche ends as a pulse of current that a counter registers. In a Geiger counter, an ionization is the knocking of an electron out of an atom of the gas inside the counter.

Many natural phenomena amplify very small differences in the same way. Atomic decay is the clearest example. The weather is a second example, to the degree that the weather amplifies small differences. The meteorologist Edward Lorenz studied that amplification (Lorenz, 1963). The motion of three bodies under gravity is a third example. The double pendulum is a fourth example: a double pendulum is a pendulum hung from the end of another pendulum. Picture three perfectly elastic balls (balls that bounce without losing any energy) coming together and touching exactly at once, like cue balls on a pool table: which way each ball goes afterward depends on subtle differences in how the three balls come together.

At some point the differences that decide the outcome are below the limit of what can be measured. The space of states of a system is the set of every state the system could be in, and the limit of measurement divides the space of states into cells, regions within which no measurement can tell one state from another. When two microstates differ only inside one cell of the space of states, the two microstates are not two states at all. So the microstate of the system cannot be known, and yet the system still produces an outcome. All the time and all over the place, systems pro-

duce outcomes from microstates that cannot be known: atomic decay occurs below the limit of what can be measured, and the differences that atomic decay makes are amplified upward through whole systems.

The general process is called *microstate amplification*. Microstate amplification is the process by which small differences at a fine-grained level of a system are propagated (passed on) and magnified into differences at the level that decides the system's behavior. A coarse-grained description of a system records the larger features of the system and drops the finer differences as noise. When a system has no microstate amplification, the system is *complicated*, and a finite, coarse-grained description of the system stays adequate. When a system has microstate amplification, the system is *complex*, and no finite, coarse-grained description of the system stays adequate indefinitely, because the differences that the description discards as noise are the differences that decide the outcome. Microstate amplification is also the source of the asymmetry in time of a complex system: the difference between the way the system runs forward and the way the system would run backward.

The residue of an outcome is the part that nothing prior fixes. Microstate amplification is the channel that carries the residue, and not the residue itself. Complex systems are sensitive to their initial conditions, because complex systems amplify differences below the limit of measurement into very different later states. The weather is a complex system, and so is neural process (the activity of the nerve cells of the brain). Amplification carries the residue to the scale of ordinary life. The residue itself is hard randomness: an outcome that no prior specification fixes, unpredictable even with unlimited access to the prior state. The residue limits how far a person's future can be predicted from knowing that person, and the irreducible unpredictability that the residue sets is where choice lives. Choice lives in the residue because, as Chapter 15 sets out, one undetermined outcome is hard randomness when the outcome is described from outside, and is a choice when the outcome is described from inside, from the position of whoever is engaged in the act.

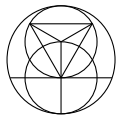
VIII. Summary

Hard randomness is an outcome that no prior specification fixes, however complete the specification is. Hard randomness is real, a fact about the world. Hard randomness is distinct from soft randomness, the randomness of ignorance, and from pseudo-randomness, the randomness that a computation only imitates. The uncertainty principle is an experimental result, and so is the violation of Bell's inequality. The Planck constant measures the degree of the uncertainty. Together the two experimental results show that no coherent notion of causality explains every measurement result. So the concept of hard randomness cannot be factored out of physics. After the loophole-free tests of 2015, Bohm's non-local determinism and superdeterminism remain as the only deterministic accounts, and by the Incommensuration Theorem neither account can be the account of a whole world. The Incommensuration Theorem guarantees hard randomness as the remainder of every comparison. By the Incommensuration Theorem, determinism and pure randomness are idealized poles that no actual world reaches. Hard randomness is what comes through the microscopic boundary of the domain of measurement, and the microscopic boundary is a limit of signal, as the cosmological horizon is, since no signal at all comes from beyond the cosmological horizon. Determinism would need detail at the scale of exactly zero, and no one can find detail at the scale of exactly zero by any comparison. Microstate amplification carries hard randomness to the scale of ordinary life. Microstate amplification is what makes a system complex rather than complicated, and the residue that microstate amplification carries is where choice lives.

PART FOUR

THE LIMITS OF KNOWING AND THE MEASURES OF PHYSICS

The three boundaries of knowing, the Planck limit, law as access, and the six measures with the Nine Laws.



Chapter 17

The Three Boundaries of Knowing

The Horizon, the Infinitesimal, and the Subconscious

ABSTRACT

Some limits to knowing are not limits of instruments. These limits belong to the relation between a comparison and what the comparison compares, and so they belong to every act of knowing. There are exactly three limits that belong to every act of knowing. The first limit is the horizon, marked by the speed of light. The horizon is the macroscopic limit, the limit of the very large, and it is omniscient in character. The second limit is the infinitesimal, marked by the Planck constant. The infinitesimal is the microscopic limit, the limit of the very small, and it is transcendent in character. The third limit is the subconscious, the limit of knowing one's own self at the place where comparing happens: the subconscious is the mesoscopic limit, at the scale of the knower, and it is immanent in character. The subconscious is the most fundamental of the three limits, and it carries no constant of physics. Each limit recedes as a knower approaches it: a knower can pass any one instance of each limit, while the class of each limit is permanent. There are three limits because there are three modalities, and the unknowable appears in each modality. This chapter shows how the known, the unknown and the unknowable differ, and how the absolute boundary of knowing differs from the relative boundary. I also show why every exact description leaves a residue that no technique removes.

I. Limits That Belong to Knowing Itself

There are structural limits to what can be known: boundaries past which knowledge cannot pass, even in principle. The limits are features of the relation between a comparison and what the comparison compares. The limits are not shortcomings of present instruments, or of the present state of science. A comparison is the act of finding whether two things are the same or different, in some respect, and every act of knowing is a comparison or is built from comparisons. So the structural limits are features of every act of knowing. Knowing has exactly three structural boundaries.

In daily work, scientists treat the unknown as what is not yet known. Science does recognize particular limits in principle, such as the light cone, the uncertainty relations and the incompleteness of formal systems, but science has no general category for limits in principle. The metaphysics that this book sets out has a general category for limits in principle: the *unknowable*. The unknowable is what cannot be known even in principle, by any being, at any time, by any method, because the structure of comparison itself does not extend to the unknowable. So the unknowable is different from the merely unknown, which is a gap in present knowledge, waiting for discovery to fill the gap. There is a place where logic cannot pass, and where no amount of experience will ever reveal insight. At that place the unknown stops and the unknowable begins. The line drawn there is the line between ordinary knowing and the great mystery, which is the unknowable.

So knowledge has three classes: the known, the unknown and the unknowable. Most theories in science and in philosophy draw no line between the unknown and the unknowable, although some mathematics does draw that line. There is no firm line between the known and the unknown, but there is a fixed and absolute line between the unknown and the unknowable. A knower can act on the unknown and turn the unknown into the known, but no knower can ever act on the unknowable.

The unknowable is built into the metaphysics itself, so the metaphysics does not get past the unknowable either. A metaphysics can say exactly where mystery begins, but no metaphysics can dissolve mystery. There is no free lunch: every account of knowing, the metaphysics of this book included, stops at the unknowable. In this chapter I give a case of each of the three classes of knowledge: the known, the unknown and the unknowable. At a given event, an event outside the given event's past light cone is unknown, and in ordinary cases the outside event becomes known at a later event (Section III). The mystery of the subconscious is unknowable, because one cannot observe the observer (Section V).

II. The Domain and Its Edges

A *domain* is that which contains and includes, and is itself uncontained. In plain words, a domain is a whole that holds everything within it, and that no larger whole holds. A domain is told apart from other domains by the concepts that make up its foundation, and not by what it contains, and a domain exists as a domain to the extent that comparisons can be made within it. The physical domain is specified by measure: to be in the physical domain, a thing has to be measurable. Every measurement is a comparison, so the physical domain is the whole of what can be compared from within the physical domain, and the edge of what can be measured is the edge of the physical domain.

The physical domain has two edges. The Planck constant marks one edge, on the side of the smallest difference that can be distinguished. The physicist Max Planck introduced the constant (Planck, 1900). The speed of light marks the other edge, on the side of the farthest connection that can be made. In this chapter, "boundary" means an edge past which no coherent signal comes. There is zero data from past the macroscopic boundary, the edge on the side of the very large, and there is zero data from past the microscopic boundary, the edge on the side of the very small. Past either boundary there is no meaningful measurement.

III. The First Boundary: The Horizon

The first boundary is the *horizon*, the limit of the very large. The horizon is the boundary past which no one can perceive anything, however far a person travels and however sensitive the person's instruments become.

In physics the limit of the very large is the speed of light, about 300,000 kilometers (186,000 miles) per second. Nothing travels faster than light travels in empty space: no signal, no information and no causal influence. A Minkowski diagram draws time up the page and space across the page, and is named for the mathematician Hermann Minkowski. The diagram shows the limit as two cones. An event is one place at one moment. From any event, one cone of light opens toward the future, made of every path that light leaving the event could take, and another cone opens toward the past, made of every path along which light could arrive at the event. At a given event, one can know, in principle, the events inside the past light cone of that event. The region outside both cones is the *absolute elsewhere* of the event: events in the absolute elsewhere cannot be known at the event, and no influence from the event can reach those events.

The speed of light is a limit of signaling, and not a property of light as such. Light in empty space travels exactly at the limit, while light in glass or water travels more slowly, and the limit itself stays the same. The speed of light is also the one ratio of a measure of space to a measure of time that is the same throughout the physical domain and for every observer in uniform motion. The speed of light ties a distance to a time, although space and time stay distinct. A comparer is anyone or anything that makes a comparison. Because the speed of light ties space to time, each comparer has a horizon in space and time, which is the comparer's absolute elsewhere. Past the horizon no content can be compared, in principle. The content of a comparison is what the verdict of the comparison, same or different, is about.

The horizon is an edge of what can be compared from where the comparer is, and the horizon moves as the comparer moves, so the horizon is not a wall that one can walk up to and touch. In ordinary cases a comparer can pass an instance of the horizon (the horizon of one event) by waiting, because an event in the absolute elsewhere of one event lies inside the past light cone of a later event. But a new horizon always appears, because no later event has a past light cone that contains everything. In a universe whose expansion speeds up, as the expansion of our universe appears to, some events will never enter the past light cone of a given observer, at any time. Physicists give the boundary of the events that never enter the observer's past light cone a name: the cosmological event horizon. So the horizon is temporary as an instance, which is the horizon of one comparer at one moment, and permanent as a class, which is the kind of limit that every comparer has.

In the history that we reconstruct from inside the universe, the universe has a beginning in time, and the beginning in time is how a limit appears when the limit is seen from inside the universe. Creation is not an event (Chapter 5 sets out creation as an aspect of every event), and no one who compares can register the limit at the beginning of the universe as an event.

The horizon is *omniscient* in character. The omniscient is the role of what is laid out as structure and held in view as a whole. The horizon is the limit at the largest scales, the limit from the outside, which the speed of light marks at about 300,000 kilometers per second.

IV. The Second Boundary: The Infinitesimal

The second boundary is the *infinitesimal*, the limit of the very small. The infinitesimal is the scale at which no one can know a definite state. Quantum mechanics applies at the scale of the infinitesimal. Physicists express the limit of the very small with the Planck constant and with the uncertainty relation of the physicist Werner Heisenberg. The uncertainty re-

lation says that the product of the spreads of two conjugate quantities cannot fall below a quantity fixed by the Planck constant. Position and momentum are a pair of conjugate quantities. Two conjugate quantities are two descriptions of one state, each of which can be sharpened only at the expense of the other.

The Planck constant has the units of action (energy multiplied by time), so the Planck constant does not by itself set a length: how fine a comparison can be made depends on the energy brought to the comparison. To confine a probe to a smaller region costs more energy. At the Planck length, the energy required would form a black hole at the place of measurement. The Planck length is about 1.6 times ten to the minus thirty-five meters, and the Planck length is built from the Planck constant, the speed of light and Newton's constant, the constant of Isaac Newton's law of gravitation. The Planck length is the scale at which a body's gravitational size equals its quantum size. Near the Planck length, a comparison made from inside the world cannot place a body more sharply. The Planck length is a limit on what can be found, and not a least length of space: space has no least length. The microscopic limit marks a bound on joint knowledge. Below the action that the Planck constant marks, no measurement fixes both quantities of a conjugate pair at once, as finely as both would need to be fixed. The bound on joint knowledge is a structural feature of the physical domain, and not a limitation of instruments.

The finest detail that instruments can resolve gets finer as instruments improve, and what was once too small to observe becomes observable. The bound on joint knowledge stays fixed. So the microscopic limit moves as an instance, which is the finest detail that instruments resolve at a given time, and persists as a class, which is the bound on joint knowledge.

The infinitesimal is *transcendent* in character. The transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance. The infinitesimal is the limit at the smallest scales, present at every place on the inside of the physical domain. When the Planck constant is taken as a number written in a theory, the constant is known structure. Known structure is omniscient, and in that role the

number only expresses the units of action. When the Planck constant is taken as the place, in scale, of the boundary of the physical domain, the constant has a transcendent function.

V. The Third Boundary: The Subconscious

The third boundary is subjective, and not objective. The third boundary is the *subconscious*, the mesoscopic limit, at the scale of the knower: the subconscious is the limit of the knowledge of one's own subjective self, the limit of knowing the quality of which oneself is. When one looks inward to know oneself, there is a point at which one's awareness fails. One becomes unconscious with respect to some part of oneself, and the process of coming to know oneself is never complete.

The incompleteness of self-knowledge is structural, and not a lack of diligence in looking inward. A self that knew itself completely would be both the knower and the known: symmetric with itself as pattern, and continuous with itself as process. In this book, symmetry is sameness of content across a difference of context, and continuity is sameness of content within a sameness of context, where the context of a comparison is the respect in which the two things compared are two. The *Incommensuration Theorem* is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Complete self-knowledge would apply symmetry and continuity together, simultaneously and fundamentally, to one comparison: the self knowing the self. So complete self-knowledge cannot be.

The limit of self-knowledge lies where the comparing actually happens, here and now, because the act of comparing cannot compare itself as an object. When one looks inward, the limit appears as the surface of unconsciousness: the point past which the self cannot see itself. An image from physics shows the shape of the limit. One beam of light passes through another beam with no effect that any lamp or laser makes large

enough to see, so one beam cannot be used to watch another beam go by. In the image, the watching beam stands for observation and the beam watched stands for the observer: just as one beam cannot watch the other, observation cannot observe observation, and one cannot observe the observer. Because one cannot observe the observer, the mystery of the subconscious is unknowable.

When one's awareness approaches the boundary of the subconscious, the boundary recedes, just as the horizon recedes before a traveler. There are always aspects of the self that remain to be discovered: self-knowledge deepens, but self-knowledge never becomes total. The unconscious is a moving boundary, and not a fixed region: the unconscious is a class of boundary, and each place where one's awareness fails is one instance of the class.

The subconscious is *immanent* in character. The immanent is the role of the act itself, as the act happens, here and now. The subconscious belongs to the knowing self and not to physics, and the subconscious carries no constant of physics. The subconscious is the most fundamental of the three boundaries: the subconscious is more fundamental than the horizon and the infinitesimal, as the immanent is more fundamental than the omniscient and the transcendent (Axiom I).

VI. What the Three Share, and Why There Are Three

The same structural features belong to all three boundaries: the horizon, the infinitesimal and the subconscious. All three boundaries are dynamic: each boundary moves as one approaches it. All three boundaries are permanent as classes: the kind of limit persists, even while particular instances of the limit are passed. And all three boundaries mark the place where the unknown ends and the unknowable begins.

There are exactly three boundaries, not two and not four. Two of the boundaries are objective: the macroscopic boundary and the microscopic boundary. One boundary is subjective: the mesoscopic boundary of con-

sciousness. Two objective boundaries and one subjective boundary follow a structural principle that recurs throughout the metaphysics: the *foundational triplication*, the principle that the foundation of every domain resolves into exactly three concepts. The foundational triplication rests on the law that a relation is always a relation of at least three. For every way in which something is subjective, there are two ways in which it is objective, and for every way in which something is objective, there are two ways in which it is subjective. The pattern of two and one appears at every level of the analysis, and the three boundaries are one instance of the pattern.

The foundational triplication comes from the six intrinsics of comparison, the six features that every comparison has: sameness and difference, content and context, subjective and objective. Every content has a context, and every sameness has a difference, and three is the smallest number of distinct elements that can meet those two requirements while remaining irreducible to one another. Two elements can be the same or different. But the sameness or difference of the two elements needs a third element, from which the sameness or difference is seen. Every domain has three concepts at its foundation that are distinct, inseparable and non-interchangeable, because comparison with its intrinsics is carried into the foundation of every domain. Distinct means that none of the three concepts overlaps another, and inseparable means that none occurs without the others. Non-interchangeable means that the role of each of the three concepts belongs to that concept alone. There are three boundaries because there are three modalities, and the unknowable appears in each modality. The three boundaries are not three limits imposed by three unrelated phenomena.

VII. How the Constants Join the Measures

Two constants mark the two physical boundaries, the horizon and the infinitesimal, and each constant has a definite place. The speed of light marks the limit at the largest scales, the limit from the outside, and the

Planck constant marks the limit at the smallest scales, present everywhere on the inside. Space, time and possibility are the measures of context: the kinds of quantity that measure the respects in which compared things are two, and the speed of light and the Planck constant each join two of the three measures of context. The speed of light is the bridge between the omniscient and the immanent. The speed of light joins omniscient space to immanent time. The Planck constant is the bridge between the immanent and the transcendent: the Planck constant joins time to possibility, which is transcendent. The reduced Planck constant turns an angular frequency, a rate in time, into an energy, and an action divided by the Planck constant counts states, which is a measure of possibility. The limits of what can be known as content within the physical domain lie between the two constants, with the Planck constant at the small end and the speed of light at the large end. The speed of light defines what is unknowable in terms of context, while the Planck constant defines what is unknowable in terms of content.

Two other constants of physics have roles of a different kind. Newton's gravitational constant is a coupling, and not a limit. Newton's constant measures how strongly content curves context (in physics, how strongly matter curves space and time), and it relates pattern (the measure that gathers mass, inertia and structure) to space. The Boltzmann constant is the factor that expresses a spread of probability over possible states in a measure of energy per kelvin, a unit of temperature, and the constant takes its name from the physicist Ludwig Boltzmann. No constant joins the omniscient to the transcendent directly, because the omniscient and the transcendent are related to each other only through the immanent. The metaphysics fixes the structural role of each constant, but the metaphysics fixes none of the values of the constants: physicists measure the values.

VIII. Classes and Instances; the Absolute and the Relative Boundary

Each boundary moves as one approaches it. What one comes to is an instance of the boundary, and the instance recedes, while the class of the boundary persists. So there are two kinds of boundary of knowing, one defined by classes and one defined by instances. When the boundary between the unknown and the unknowable is defined in terms of classes, this chapter calls it the *absolute boundary*. The *relative boundary* is the boundary that a given observer comes to from a given position at a given time. The relative boundary is defined in terms of instances. The relative boundary can always be crossed, but the absolute boundary is never crossed, because crossing any instance of the absolute boundary brings one to another instance of the same class. The absolute boundary is omniscient in character: perfect and unchanging. The relative boundary is transcendent in character: moving and exceeding. The act of approaching a boundary is immanent in character, and in itself the act of approaching is neither absolute nor relative.

Each boundary of knowing is like the edge of the land that a traveler sees ahead: one approaches the boundary, and one cannot tear the boundary down as one could tear down a wall. As one approaches a boundary, one's knowledge deepens. The unknowable is no opponent of knowledge: the unknowable is the condition that sets the direction of knowledge and the meaning of knowledge. If there were no boundary, the deepening of knowledge would have no shape, since there would be nothing for the deepening to approach.

IX. What Lies Beyond

Nothing can be assumed about what lies past the horizon, the infinitesimal or the subconscious. For some of the boundaries, "beyond" is not even defined. Take the horizon as a class, and not as one comparer's horizon: the horizon as a class is the boundary of the knowable itself. *Incommensuration; The Theorem and the Three Axioms* is the book of the physics collection that proves the Incommensuration Theorem from the structure of comparison. That book states the limit for the boundary of the knowable itself. In the words of that book, the limit that the Incommensuration Theorem draws is "the curvature of the field itself, rather than a wall at the edge of an otherwise open field" (Incommensuration; The Theorem and the Three Axioms, Chapter 6, Section XI). The Incommensuration Theorem fixes the unknowable as a class that persists. The limit that the Incommensuration Theorem draws is the shape of the whole field of knowing. So there is no open region past the limit for "beyond" to name. Within the knowable, the metaphysics describes the fitting way of knowing.

Where a beyond is defined, as the region past one observer's horizon is defined, nothing can be assumed about that beyond either. To tell what is past the horizon, one would have to interact with what is there, and an interaction would create a signal, but no signal comes from past the horizon. To frame a hypothesis about what lies past the horizon anyway is inconsistent as knowing. Framing the hypothesis is a pretense of knowing what is, by construction, unknowable. Everything outside the limits of the physical domain is unknowable.

X. The Known, the Unknown and the Unknowable as Modalities

The known, the unknown and the unknowable correspond to the three modalities: the immanent, the omniscient and the transcendent. The pairing of each class with a modality depends on the focus from which the classes are considered. A focus is the frame of a description relative to which the modal roles are assigned. Theory in the third person is one focus, and practice in the first person is another.

The focus of theory is the third person: the impersonal and objective perspective. In the focus of theory the unknown is immanent: the unknown is the process of coming to find out, perception in the here and now, which gives a view of things but not the totality of things. In the focus of theory the known is omniscient: complete, available and inspectable, like a photograph in which every part can be seen. In the focus of theory the unknowable is transcendent: the unknowable exceeds every attempt to capture it, and has no fixed shape or pattern by which the unknowable could be brought within a description. The three assignments in the focus of theory follow from the characters of the modalities: the omniscient is the mode of structure and completeness, the immanent is the mode of process and becoming, and the transcendent is the mode of excess and creation.

In the focus of practice (the first person) the known is immanent: the known is what one is in contact with here and now. In the focus of practice the unknown is omniscient: the unknown is the laid-out structure that one has not yet explored. And in the focus of practice the unknowable remains transcendent. The two sets of assignments belong to two foci, theory and practice. So the difference between the two sets is not a free exchange of labels within one focus. A traveler in a strange city is a case: in practice the street under the traveler's feet is the known, and the streets on the map that the traveler has not yet walked are the unknown.

XI. Why Every Measurement Leaves a Residue

The limits of measurement correspond to the three boundaries of knowing: the horizon, the infinitesimal and the subconscious. Each limit of measurement is a structural feature of the physical domain, and not a practical limitation of instruments. The limits of measurement exist because of the Incommensuration Theorem. The Newtonian picture of physics includes an assumption of total access: the assumption that every quantity can be measured exactly and jointly. Total access would need symmetry and continuity to be perfected together, that is, to hold together, exactly, at the foundation of one description. The Incommensuration Theorem shows that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. So in every exact description, symmetry and continuity fail to be perfected together at some place. At each place where symmetry and continuity fail to be perfected together, a limit of measurement appears, so no measurement can be perfect in every respect at once. A residue remains, a remainder that the measurement leaves unfixed, and no improvement in technique removes the residue.

The argument from the Incommensuration Theorem is not the uncertainty principle restated, because the theorem and physics do different work. The Incommensuration Theorem gives the reason that a limit such as the uncertainty relation exists. Physicists find the form of the limit and the value of the Planck constant.

XII. What the Boundaries Mean for Physics

Physicists work within the known and the potentially knowable. Physicists seek laws that are universal, predictions that can be tested, and measurements that are precise. The work of physicists has structural limits. The limits come from reality itself, which contains what is unknowable, and

not from inadequate instruments or incomplete theories. The horizon and the infinitesimal are the physical expressions of the unknowable, and the subconscious, the third boundary, is the expression of the unknowable in the self who does physics.

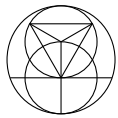
The speed of light and the Planck constant both separate what is known and potentially knowable from what is irreducibly unknowable from within the physical domain. Physics has two descriptions, quantum mechanics and general relativity, and each description has the edge of its domain at the smallest scales. The edge is a structural transition, not a wall. At the largest scales, content past a comparer's horizon cannot be compared, in principle. The structural limits of physics do not diminish science. The limits state the scope of science. Science is the exploration of the knowable and the known, and the metaphysics sets the domain and the limits of that exploration.

XIII. Summary

Knowing has three structural boundaries, and each boundary belongs to the relation between a comparison and what the comparison compares. The horizon is the macroscopic limit, marked by the speed of light. The horizon is omniscient in character, and it is the limit from the outside. The infinitesimal is the microscopic limit, marked by the Planck constant. The infinitesimal is transcendent in character, and it is the limit present everywhere on the inside. The Planck length is a bound on finding, and not a least length of space. The subconscious is the mesoscopic limit, the limit of knowing one's own self at the place where the comparing happens. The subconscious is immanent in character, and it is the most fundamental of the three boundaries. The subconscious belongs to the knowing self and carries no constant of physics.

Each boundary recedes as one approaches it, and each boundary is temporary as an instance and permanent as a class. There are three boundaries because every domain has three concepts at its foundation,

since comparison is carried into the foundation of every domain, and the unknowable appears in each of the three modalities. The absolute boundary of knowing is defined by classes and is never crossed, while the relative boundary is defined by instances and can always be crossed. Nothing can be assumed about what lies past the boundaries of knowing, and for some of the boundaries, "beyond" is not defined. The known, the unknown and the unknowable take places among the modalities that depend on the focus (theory or practice) from which the three classes are considered. Every exact description leaves a residue that no technique removes, because total access would perfect symmetry and continuity together, and the Incommensuration Theorem shows that symmetry and continuity cannot be perfected together. The three boundaries state the scope of physics, and they set the direction of knowing.



Chapter 18

The Planck Limit

What the Planck Constant Bounds, and Why Space Has No Smallest Piece

ABSTRACT

The Heisenberg uncertainty principle says that the position and the momentum of a particle cannot both be pinned down perfectly at once, and the Planck constant sets how sharply position and momentum can be pinned down together. This chapter separates three things that people often treat as one. The first of the three things named after the physicist Max Planck is the Planck constant, a number that expresses a unit of action. Action is measured as an energy multiplied by a time. The second of the three things named after Max Planck is the Planck limit, which is the Planck constant read as the microscopic boundary of the domain of measurement. The domain of measurement is the whole of what can be measured. The third of the three things named after Max Planck is the Planck length, the scale at which probing the microscopic boundary with ever more energy stops working. The uncertainty relation bounds two spreads, the spread of the places where a particle could be found and the spread of the momenta the particle could be found to have, and the two spreads belong to the state of the particle, not to the measuring instrument. The uncertainty relation is an instance of the Incommensuration Theorem. The Incommensuration Theorem says that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. If a body had a definite position and momentum at every instant, the body

> would perfect symmetry and continuity together. The Incommensuration Theorem forbids perfecting symmetry and continuity together. A probe must carry more energy to probe a smaller region. At the Planck length the energy of the probe is great enough to close the probed region on itself. No signal then gets out of the probed region. The Planck constant, the speed of light and Newton's constant of gravitation together fix the Planck length, and the Planck length is the bound of every measurement made from inside the world. The Planck length is not a smallest piece of space: space and time have no least length, no least area and no least duration. At the Planck scale a description of space and time reaches its edge, and space itself does not: the smooth description of space and time holds only at lengths larger than the Planck length.

I. The Uncertainty Principle

The *Heisenberg uncertainty principle* says that the position and the momentum of a particle cannot both be pinned down perfectly at once. Imagine a particle with a position: the particle is somewhere. The particle also has a momentum: the particle is moving at some speed in some direction. Momentum is a vector, with both a size and a direction. There is an uncertainty about where the particle is, and an uncertainty about how fast the particle is moving and in which direction. The product of the two uncertainties can never be less than a fixed quantity: half the reduced Planck constant, where the reduced Planck constant is the Planck constant divided by the number of radians in one full turn. So the uncertainty principle limits how much can be known about a physical system at one time. In 1927 the physicist Werner Heisenberg stated the uncertainty principle, and in the same year the physicist Earle Kennard proved the exact bound of the uncertainty principle (Heisenberg, 1927; Kennard, 1927). Physicists call the inequality of the uncertainty principle the uncertainty relation. Position and momentum are a *conjugate pair*: two descriptions of one state, each of which can be sharpened only at the expense of the other.

The *Planck constant* is the least quantum of action, where action is a quantity measured as an energy multiplied by a time, and the Planck constant is also the constant that relates the energy of light to the frequency of the light. The energy of one quantum of light is the Planck constant multiplied by the frequency of the light. The Planck constant sets the size of the uncertainty in the Heisenberg principle: the Planck constant is the measure of how much the two members of a conjugate pair cannot be known together.

II. Two Kinds of Spread

When a position is reported as "here, plus or minus so many angstroms", the report really gives a spread of places where the particle could be found. An angstrom is a length about the width of an atom. There is a matching spread of the values that the momentum of the particle could be found to have. The physicist Max Born gave the rule for the chance of finding the particle at each place, now called Born's rule: the chance is the square of the size of the particle's wave (the wave that quantum theory uses to describe the particle) at that place (Born, 1926). The two spreads in the uncertainty relation are the widths of the two distributions of chances, one over places and one over momenta. The two spreads belong to the state of the particle, not to the instrument that measures the particle, so a better instrument does not shrink the two spreads below the bound of the uncertainty relation. Some states make the product of the two spreads as small as the bound allows. The states with the smallest product have the shape of a Gaussian, the bell curve.

Errors of measurement are a second, separate kind of spread. Errors of measurement are usually distributed normally, which means that the errors spread out in the shape of a Gaussian, so the "plus or minus" in "it is here, plus or minus this" has no hard edge. The standard deviation measures the width of a Gaussian. The "plus or minus" states how good the information from the measurement is. There is about a 68 percent chance

that the true value lies within one standard deviation of the measured value, and about a 95 percent chance that the true value lies within two standard deviations. The "plus or minus" is still a statistic, and the "plus or minus" puts the uncertainty of a measurement into one concrete number. The quantum spread is a different thing: the width of the state itself, which remains when every error of the instrument has been removed.

III. Why There Is a Bound

The uncertainty relation is a limit on what anyone can find by any comparison made within the physical domain, the whole of what can be measured. A comparison, in this book, is the act of finding whether two things are the same or different, in some respect. Every measurement is a comparison. The uncertainty relation is not a shortfall of instruments. The uncertainty relation is an instance of the Incommensuration Theorem.

The *Incommensuration Theorem* is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The four terms of the Incommensuration Theorem use two features of every comparison: the content of a comparison is what the verdict of the comparison is on, and the context is the respect in which the two things compared are two. *Symmetry* is sameness of content across a difference of context, and *continuity* is sameness of content within sameness of context. *Asymmetry* is difference of content across difference of context, and *discontinuity* is difference of content within sameness of context. One of symmetry and asymmetry can be joined with one of continuity and discontinuity in four ways, and each such pair is a conjunction. Of the four conjunctions that could each hold exactly, the Incommensuration Theorem permits two: continuity with asymmetry, and symmetry with discontinuity.

When a body has a definite position and a definite momentum at every instant, the body has a trajectory. If a trajectory were specified at every instant, under one law that holds everywhere, the trajectory would perfect continuity and symmetry together. The specification at every instant is the continuity, and the one law that holds everywhere is the symmetry. The Incommensuration Theorem forbids perfecting continuity and symmetry together. The uncertainty relation allows two extreme states. The first extreme state that the uncertainty relation allows is a state of exactly definite momentum, called the plane wave, and the plane wave is the same at every position. The plane wave is symmetry of momentum, with discontinuity of position. The second extreme state that the uncertainty relation allows is a state confined to one place, and the confined state is continuity of position, with asymmetry of momentum. So the two extreme states of the uncertainty relation are the two permitted conjunctions, and the uncertainty relation never allows the forbidden pair of symmetry with continuity (Heisenberg, 1927; Kennard, 1927; Robertson, 1929). The physicist Howard Robertson proved the general form of the uncertainty relation, for any two quantities, in 1929.

Quantum theory as a whole also contains a permitted conjunction and not the forbidden pair: quantum theory states what physicists find exactly, in whole steps with exact labels (symmetry with discontinuity), and quantum theory contains no continuous trajectory. The missing continuous trajectory appears in quantum theory as the bound on how sharply two conjugate quantities can be known together.

Position and momentum are conjugate descriptions of one state, related by a Fourier transform. A Fourier transform is the mathematical operation that turns a description given place by place into the same description given wave by wave, one wave for each frequency, and back again. The conjugacy of position and momentum is the conjugacy of the omniscient and the transcendent named in Axiom I. Axiom I reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another" (An Immanent Metaphysics, aphorism 1.3-1). In Axiom I the

immanent is the act itself, as the act happens here and now. The omniscient is what is laid out as structure and held in view as a whole, while the transcendent is the stance from which an act proceeds, with the range of what is possible from that stance. The conjugate form of position and momentum also shows in Fourier transforms in general. An engineer describes a signal either moment by moment in the time domain or frequency by frequency in the frequency domain. In the engineer's usage, a "domain" is a way of describing a signal. A spike is a signal that is sharp in time. A spike cannot be perfectly localized in the time domain and in the frequency domain at once. A signal that is fuzzy in time can be precise in frequency, and a signal that is fuzzy in frequency can be precise in time. A signal can be fuzzy in both time and frequency. No signal can be precise in both time and frequency. The reason is that a description that perfected both time and frequency at once would join symmetry with continuity, which the Incommensuration Theorem excludes.

Particle and wave are one more face of the conjugation of position and momentum: particle and wave are two conjugate descriptions of one thing. A person who thinks of particles thinks of positions and momenta. A person who thinks of waves thinks of frequencies and amplitudes. Amplitudes are the heights of the waves. Position and momentum are a conjugate pair under the Fourier transform, the same transform that relates the time domain to the frequency domain. The particle is symmetry with discontinuity. The wave is continuity with asymmetry. The measurement, which is an interaction, is a third thing beside particle and wave. The measurement determines which face is shown.

The Incommensuration Theorem is the reason that a bound on knowing two conjugate quantities together exists. The Incommensuration Theorem is also the reason that the bound will remain in every future theory. Physicists find the rest: the form of the bound, the proof of the bound from operators that do not commute, and the scale of the bound, which is the value of the Planck constant. In quantum theory an operator is the

mathematical rule for a measurable quantity, and two operators do not commute when applying them in one order gives a different result from applying them in the other order.

IV. The Constant, the Limit, and the Domain

A *domain* is that which contains and includes, and is itself uncontained. In plain words, a domain holds all that belongs to it, and nothing holds the domain. The physical domain is specified by measure: for a thing to be in the physical domain, the thing has to be measurable. So the edge of the measurable is the edge of the physical domain.

The Planck constant and the Planck limit are two different things. The Planck constant is a number expressing a unit of action. The *Planck limit* is an interpretation of the Planck constant: the Planck constant read as the location of a boundary, and the boundary is the microscopic limit of the domain of measurement. The Planck limit is the limit of what is knowable, meaning measurable, in terms of content.

The Planck constant has two interpretations, as a number and as a boundary, and the two interpretations belong to two frames. When the Planck constant is regarded as a number written in a theory, the Planck constant is known structure, and in the frame of a theory the Planck constant is omniscient (laid out as structure, held in view as a whole). When the Planck constant is used to express where the boundary of the physical domain lies in scale, the Planck constant has a transcendent function (the side of what is possible, past which nothing can be fixed). The Planck constant has the transcendent function relative to the speed of light, the other constant that marks a limit of the physical domain. The speed of light marks the macroscopic limit.

V. Probing Smaller: The Planck Length

The Planck constant has the units of an action: energy times time. By itself, the Planck constant sets no smallest length or duration. How finely something can be measured depends on the energy used. Below the action that the Planck constant marks, no measurement fixes both members of a conjugate pair together.

Now push the energy of a probe up, to probe smaller and smaller regions, where a probe is a particle or a beam of light sent in to find where something is. Confining a particle to a narrower region widens the spread of the particle's momentum. Momentum carries energy: more momentum means more energy of motion. So when a very small region of space is pinned down, the uncertainty in the momentum, and so in the energy, becomes enormous. Energy is equivalent to mass, and enough mass in a small enough region forms a black hole. So at some scale the uncertainty in the energy becomes large enough to form a black hole larger than the region being probed. A black hole is a region from which no signal gets out. A signal can cross the boundary of a black hole, called its horizon, inward and never outward. At the scale where the black hole forms, the energy of the probe curves space so much that the probed region closes on itself.

Three constants set the scale at which the probed region closes on itself: the Planck constant, the speed of light, and Newton's constant of gravitation, which sets the strength of gravity. The three constants together fix the *Planck length* and the *Planck time*. Light crosses one Planck length in one Planck time. The Planck length is the scale at which a body's gravitational size equals the body's quantum size. A body's gravitational size is the size of the black hole that the body's mass would form. The gravitational size grows as the mass grows. A body's quantum size is the size be-

low which, by the uncertainty relation, confining the body takes an energy as large as the energy of the body's mass, and the quantum size shrinks as the mass grows.

VI. A Bound on Finding, Not a Grain of Space

The microscopic boundary of the physical domain is the action that the Planck constant marks, and the Planck length is where probing the microscopic boundary with more and more energy stops working. The Planck length is a categorical boundary of the physical domain, a boundary for every kind of measurement. No measuring tool has access to anything finer than the Planck length in space, or finer than the Planck time in time.

The Planck length is a limit on placing. No one can place a body more sharply than about the Planck length by a comparison made from inside the world. The Planck length is a bound on what can be found, not a smallest step of space. Space and time have no least length, no least area and no least duration. The Planck length is the bound of every measurement made from inside the world. The Planck length is not a smallest piece of space.

Astronomical measurements agree with the claim that space has no smallest piece, within the range of scales that astronomers can test. No one can probe the Planck scale directly. But astronomers have used the Fermi Gamma-ray Space Telescope to record the light of distant gamma-ray bursts, which are brief and very bright flashes from far galaxies. Light of very different colors arrives together from each burst (Abdo and others, 2009). A grain of space would be a smallest piece of space. If space had a grain at the Planck scale that slowed light in proportion to the energy of the light, light of higher energy would arrive later than light of lower energy. So the light of the gamma-ray bursts already shows that space has no grain at the Planck scale that would slow light down in proportion to the energy of the light.

At the smallest scales, what runs out is what can be known of the definite state of a thing, while space and time themselves still make sense at those smallest scales. The Planck length is the scale at which quantum theory and the theory of gravity are both needed.

VII. What Reaches Its Edge Is a Description

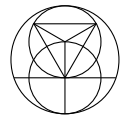
At the Planck scale a description of space and time reaches its edge, the end of the range of scales where the description holds. The description is the smooth description of space and time, which general relativity gives. The smooth description holds wherever the curvature of space is below the Planck curvature (the curvature whose scale is the Planck length), which is to say at lengths larger than the Planck length. At the Planck scale both the smooth description and the quantum description are relevant, and neither description dominates. The edge belongs to the descriptions, not to space. The structure of comparison remains at the Planck scale: at the Planck scale, as at every other scale, a comparison is still the finding of sameness or difference in some respect. Physics has two descriptions, the smooth description and the quantum description, and at the smallest scales each description reaches the edge of its domain, the range of scales where the description holds. At the smallest scales there is a structural transition between descriptions, not a wall in space. At the transition, the descriptions that are needed change.

So the microscopic limit is a gateway into the unknowable, in one exact sense. The Planck constant separates what is known and potentially knowable from what is irreducibly unknowable from within the physical domain. The Planck constant marks how far any comparison made inside the world can fix the content that the comparison compares. The Planck constant does not mark a place where space stops.

VIII. Summary

The uncertainty principle states that the product of the spreads of position and momentum is never less than half the reduced Planck constant. The two spreads belong to the state of the particle, as Born's rule determines them. The two spreads are distinct from the statistical error of an instrument. The bound of the uncertainty principle is an instance of the Incommensuration Theorem: a definite trajectory would perfect symmetry and continuity together, which the theorem forbids. The uncertainty relation allows two extreme states, the plane wave and the state confined to one place, and the two extreme states are the two conjunctions that the Incommensuration Theorem permits. The conjugation of position and momentum also appears in the Fourier transform and in the two faces of particle and wave, and the conjugation is the conjugacy of the omniscient and the transcendent in Axiom I. The Incommensuration Theorem is the reason for the bound, and physicists find the form and the size of the bound.

The Planck constant is a number that expresses a unit of action. The Planck limit is the Planck constant read as the microscopic boundary of the domain of measurement. The Planck constant, the speed of light and Newton's constant together fix the Planck length, which is where probing the microscopic boundary with more energy closes the probed region on itself. The Planck length bounds every measurement made from inside the world. Space and time have no smallest piece. The smooth description of space and time holds only at lengths larger than the Planck length. So at the Planck scale the smooth description reaches its edge, and space does not.



Chapter 19

Physical Law as Access

The Limits Every Physics Shares

ABSTRACT

A law of physics says what happens and what is allowed to happen, and so it also says what is not allowed to happen. In formal terms, a law is a constraint: a boundary drawn around the space of possible events. The boundary around the space of possible events is a statement of access, which says which influences can reach which other parts of the world, under which conditions, at what cost, and in what direction. When laws are read as statements of access, relativity sets the limit of signaling (what an observer can receive information from at all), while quantum mechanics sets the limit of extraction (how much information an observer can draw out of a microscopic system). A measurement is an interaction that carries a difference from the measured system into the measuring system. A pointer reading is the residue of a great deal of detail that the measuring apparatus discards on the way to the pointer. Physics has three limits: no influence travels faster than the limit of causal signaling; nothing returns across a horizon; and no observer can know two conjugate quantities, such as position and momentum, exactly and jointly. Each of the three limits appears where a theory of physics no longer contains an assumption made by the physicist Isaac Newton. Newton assumed that symmetry (sameness of content across a difference of context) and continuity (sameness of content within sameness of context) hold together exactly in what a measurement finds. The Incommensuration Theorem

> says that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and that neither can asymmetry and discontinuity. The Incommensuration Theorem shows why limits of access exist and why no future theory will remove them. Physics measures the size of each limit.

I. A Law Is a Boundary

A law of physics says what happens. A law of physics also says what is allowed to happen, and so it says what is not allowed. In formal terms, a law is a constraint. The constraint is a boundary drawn around the space of possible events. Every event that can occur is inside the boundary. Every event that cannot occur is outside the boundary. One case is the law that no signal travels faster than light. Under the law, a signal from a distant star can reach us at the speed of light or slower, and never any sooner.

The boundary around the space of possible events is a statement of *access*. A statement of access says which influences can reach which other parts of the world, under which conditions, at what cost, and in what direction. Two of the great laws of physics are naturally read as statements of access. Relativity sets the *limit of signaling*: the light cone determines what an observer can, in principle, receive information from. From any event, one cone of light opens toward the future and another toward the past, and an observer at the event can, in principle, receive information from every event inside the past cone. Quantum mechanics sets the *limit of extraction*: the uncertainty relation determines how much information an observer can, in principle, extract from a microscopic system. The uncertainty relation says that the spread in a particle's position, multiplied by the spread in the particle's momentum, can never be less than a fixed amount set by the Planck constant (the constant that relates the energy of light to the frequency of light).

II. A Pointer Reading Is a Residue

Every measurement is a comparison. A comparison is the act of finding whether two things are the same or different, in some respect. Every comparison is the finding of a content in a context. The content is what the verdict of same or different is about. The context is the respect in which the two things compared are two. In a quantum measurement, the property probed is the content. The setting of the apparatus is the context. A measurement is an interaction that carries a difference from the measured system into the measuring system.

Picture the needle on a meter. The meter gives its reading as one position of the needle. The reading is the residue of a great quantity of detail that the apparatus has discarded on the way to the needle, because far more was going on in the system that the meter touched than one position of one needle can carry. When the laws of physics are read as statements of access, the laws state the limits on what a measurement can extract: how much of the difference in the measured system can, in principle, arrive in the measuring system.

III. The Three Limits and Their Common Reason

Physics has three limits. No influence travels faster than the limit of causal signaling, which is the speed of light. Nothing returns across a horizon. A horizon is the edge of what one observer can ever compare with: no signal from beyond the horizon reaches the observer. Two quantities are conjugate when each can be sharpened only at the expense of the other, so no observer can know two conjugate quantities, such as position and momentum, exactly and jointly.

One reason is common to all three limits of physics: the limit on signaling, the limit on return across a horizon, and the limit on joint knowledge of two conjugate quantities. Each of the three limits appears where a the-

ory of physics no longer contains an assumption that the physicist Isaac Newton built into his mechanics, his laws of motion and gravitation. Newton assumed that symmetry and continuity hold together exactly in what a measurement finds. *Symmetry* is sameness of content across a difference of context, and *continuity* is sameness of content within sameness of context. *Asymmetry* is difference of content across a difference of context. *Discontinuity* is difference of content within sameness of context. Newton's picture of the world assumed total access: every quantity can be measured exactly and jointly, and every body has a definite position and a definite momentum at every instant, under one law that holds everywhere. Total access is symmetry and continuity perfected together.

The *Incommensuration Theorem* is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. So every exact theory of physics has some place where symmetry and continuity are not perfected together. Wherever symmetry and continuity are not perfected together in a theory, a limit of access appears.

The uncertainty relation of quantum mechanics is the plainest case of a limit of access appearing where symmetry and continuity are not perfected together. A trajectory is the path of a body that has a definite position and a definite momentum at every instant. If a trajectory were specified at every instant under one law, the trajectory would perfect continuity and symmetry together, and by the Incommensuration Theorem continuity and symmetry cannot be perfected together. A conjunction is two of symmetry, continuity, asymmetry and discontinuity held exactly together. Under the Incommensuration Theorem, two conjunctions can hold: symmetry with discontinuity, and continuity with asymmetry. In the uncertainty relation, the two conjunctions that can hold are paired, one at each extreme of the relation. A state of exact momentum is the case of symmetry with discontinuity: the state is the same at every position. A state confined in place is the case of continuity with asymmetry.

The Incommensuration Theorem shows why limits of access exist, and why no future theory will remove them. Physics measures the size of each limit. Physicists find the speed of light and the value of the Planck constant by measurement, not by derivation from the Incommensuration Theorem.

IV. A Related Program in Physics

One research program in physics starts from a point related to the starting point of this book. B. Roy Frieden set out the program in his book *Physics from Fisher Information: A Unification*. In statistics, a parameter is an unknown number to be learned from data. An estimator is a rule that computes a guess at the parameter from the data. An estimator is unbiased when its guesses, averaged over many sets of data, equal the true value of the parameter. The variance of an estimator is a measure of how widely the guesses of the estimator scatter around the average guess. The Fisher information is a measure of how much information about the parameter the data contain. The Cramér-Rao inequality is a theorem of statistics. The Cramér-Rao inequality sets a lower bound on how precisely any parameter can be estimated from data: the variance of any unbiased estimator of the parameter is at least the reciprocal of the Fisher information about the parameter that the data contain.

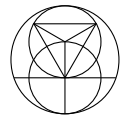
Frieden argues that the Klein-Gordon equation (a wave equation of relativistic quantum theory), Maxwell's equations (the equations of electricity and magnetism), and elements of quantum mechanics follow from a variational principle he calls extreme physical information (Frieden, 1998). Under a variational principle, the behavior that occurs is the behavior that makes some chosen quantity as large or as small as possible. Frieden's principle of extreme physical information uses two quantities: the Fisher information in the data, and a bound information held in the phenomenon. Under the principle, the difference between the two quant-

ities is extremized (made as large or as small as possible). Frieden's derivations are disputed in the published literature, for example in a paper by Lavis and Streater (Lavis and Streater, 2002).

The aim of Frieden's program is the part of the program that bears on the reading of law as access: Frieden sets out to found physics on what can, in principle, be known about a system, transmitted from a system, or extracted from a system. Frieden's starting point is related to the starting point of this book, in which the laws of physics are read as the limits of what one part of the world can find out about another part.

V. Summary

A law of physics is a boundary around the space of possible events. A boundary around the space of possible events is a statement of access: a statement of which influences can reach which other parts of the world, under which conditions, at what cost, and in what direction. Relativity sets the limit of signaling. Quantum mechanics sets the limit of extraction. A measurement is an interaction that carries a difference from one system into another. The reading of a measurement is the residue of the detail that the apparatus discards on the way to the reading. Physics has three limits: on signaling, on return across a horizon, and on joint knowledge of conjugate quantities. Each of the three limits appears where a theory no longer contains Newton's assumption that symmetry and continuity hold together exactly. The Incommensuration Theorem shows why limits of access exist and why limits of access are permanent, while physics measures the size of each limit.



Chapter 20

The Six Measures and the Nine Laws

The Grid of Physical Measurement Read from Comparison

ABSTRACT

Three questions can be asked of physics as a whole: why the laws of physics are what they are, why the constants of physics have the values they have, and what the nature is of the order and regularity found in the physical world. This chapter answers the structural part of each question, which is the part that the structure of every physical measurement fixes. Every physical measurement involves six measures. The six measures are six kinds of quantity: pattern, space, force, time, probability and possibility. Pattern gathers mass, inertia and structure into one measure. Every measure is either a content, part of what a measurement registers, or a context, part of where, when and among which possibilities the content is registered, so the six measures fall into a row of contents and a row of contexts. When the six measures are set out as a grid, the six measures also fall into three columns, one column for each of the three modal roles: the omniscient (what is laid out as structure), the immanent (the act itself, as it happens) and the transcendent (the range of what is possible). The count of six is argued: every measure is a content or a context, and each of the two rows holds exactly one measure in each of the three modal roles. Each content is measured in its own context, the context in the content's own column, so the six measures form three pairs: pattern in space, force in time, and probability in possibility. The three contents and the three contexts make nine pairings of a content with a context, and I

> call the nine pairings the Nine Laws, and each pairing names a relation between a content and a context. Several of the Nine Laws are familiar physics: work and energy, gravitation, and entropy. I use the grid to place general relativity and quantum mechanics, to read thermodynamics as a modal triad (three laws of thermodynamics, one in each modal role), and to read temperature and the quantized energy levels of bound systems. Pattern, inertia and mass are one reality. I also read quantum nonlocality as domain curvature: a feature of the relations among the points of a domain that belongs to no single point and so has no location. The grid fixes the structural role of each foundation constant, such as the speed of light and the Planck constant, and the place of each of the Nine Laws, while physics measures the values of the constants.

I. Three Questions

Three questions can be asked of physics as a whole. Why are the laws of physics what they are? Why do the constants of physics have the values they have? And what is the fundamental nature of the order and regularity found in the physical world? How does the order and regularity found in the physical world relate to the structure of comparison? I answer the structural part of each of the three questions: the part that the structure of every physical measurement fixes.

The answer begins with what a measurement is. Every measurement is a comparison: the act of finding whether two things are the same or different, in some respect. The *content* of a comparison is what is compared, and the *context* is the respect in which the two compared things are two. In a quantum measurement, for example, the property probed is the content and the setting of the apparatus is the context. A *domain* is the totality of what can be compared by a given kind of comparison. Put another way, a domain is that which contains and includes, and is itself uncontained. The physical domain is the domain of physical measurement, and the rest of this chapter sets out the structure that every physical measurement has.

II. The Six Measures

Every physical measurement involves six measures, also called the six metrics. The six measures are the six kinds of quantity of which every physical quantity is a species, a more particular kind. The six measures are *pattern* (which gathers mass, inertia and structure into one measure), *space*, *force*, *time*, *probability* and *possibility*. The intrinsics of comparison are the six features that every comparison has. The six intrinsics come in three pairs: sameness and difference, content and context, and the subjective and the objective. The six measures are the six intrinsics of comparison as seen from the position of physics.

Set the six measures out as a grid with two rows, a row of contents and a row of contexts. The grid has three columns, one column for each of the three modal roles that the parts of every comparison play toward one another: the omniscient, the immanent and the transcendent. The omniscient is the role of what is laid out as structure and can be held in view as a whole. The immanent is the role of the act itself, as it happens. The transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance.

The count of six is argued, not chosen. Every measure is a content or a context, because content and context are one of the three pairs of intrinsics of comparison, the other two pairs being sameness and difference, and the subjective and the objective. Axiom III is the third of the three axioms that relate the modal roles to one another. Axiom III states that the three roles are distinct, inseparable and non-interchangeable. So each row holds exactly one measure in each of the three modal roles: in a row, no role can be empty, no role can be filled twice, and no role can stand in for another. Six is three roles taken in two rows.

The content measures are pattern, force and probability: what a measurement registers. The context measures are space, time and possibility: where the content is registered, when the content is registered, and

among what possibilities. The law of symmetry applies to the three content measures, and the law of continuity applies to the three context measures. Symmetry is sameness of content across a difference of context, and continuity is sameness of content within sameness of context. Each triple of measures is a *triaxial*: three axes that together make one class concept. The content triaxial is pattern, force and probability. In the content triaxial, pattern is mass in the combined sense of inertia, pattern, form and structure. The context triaxial is space, time and possibility.

Each content is measured in its own context, the context in the content's own column of the grid. So the six measures form three pairs, one in each column. A modal role is always assigned relative to a frame, the setting of a description, and in this chapter the frame is physical measurement. In the frame of physical measurement, pattern in space is the omniscient pair (what is laid out as structure and held in view as a whole), force in time is the immanent pair (the act itself, as it happens), and probability in possibility is the transcendent pair (the range of what is possible). Every physical measurement involves all six measures, at least implicitly. A quantity such as energy or momentum is a relation among the six measures, not a seventh measure.

III. The Nine Laws

There are nine ways to pair one of the three content measures with one of the three context measures, and I call the nine pairings of a content with a context the *Nine Laws*. Each of the Nine Laws names a relation between a content and a context. Three of the nine pairings lie within a modal column: pattern and space, force and time, and probability and possibility. The other six pairings cross from one column to another. Each of the other six joins a content in one column with a context in a different column.

Several of the Nine Laws are familiar physics. The relation of force to space is the structure of work and energy. The energy of motion is the work that a force does over a stretch of space. The relation of pattern to space is the structure of gravitation. The coupling of gravitation is Newton's constant: the number that sets how strong gravitation is. Newton's constant is named for the physicist Isaac Newton. The relation of probability to possibility is the structure of entropy. The measure of entropy is the Boltzmann constant. The Boltzmann constant is named for the physicist Ludwig Boltzmann. The relation of space to time is the structure of special relativity, with the speed of light as the constant that relates space to time. But the relation of space to time is not one of the Nine Laws, because space and time are both context measures.

The constants of physics have their places on the grid of six measures as well. The speed of light relates space to time, and so relates the omniscient column to the immanent column. The Planck constant, the least quantum of action, relates time to possibility, and so relates the immanent column to the transcendent column. No constant relates space to possibility directly, because the omniscient and the transcendent are related to each other only through the immanent, and among space, time and possibility, time is the measure in the immanent role. When the speed of light and the Planck constant are placed on the grid, the speed of light defines what is unknowable in terms of context: which places and times can be known from a given place and time. The Planck constant defines what is unknowable in terms of content. Position and momentum, the static and dynamic aspects of pattern, cannot both be known past a minimum: the spread in position multiplied by the spread in momentum can never be less than a fixed amount set by the Planck constant.

IV. Where the Two Great Theories Stand

General relativity describes the omniscient pair, pattern in space: mass and energy in relation to the geometry that mass and energy occupy. General relativity describes the omniscient pair by way of the immanent pair, force in time, which is the dynamics in which curvature and motion answer to one another, each changing in response to the other. Quantum mechanics describes the transcendent pair, probability in possibility: the Born rule, measurement and superposition. The Born rule states that the chance of finding a particle at a place is the square of the size of the particle's wave at that place. The Born rule is named for the physicist Max Born. A superposition is a quantum state that, relative to a given measurement, is a weighted range of the possible outcomes. Quantum mechanics also describes the transcendent pair by way of the immanent pair, force in time. In quantum mechanics, force in time is the dynamics of fields and of their interactions.

When the grid is read by rows instead of by columns, each of the two theories has its own side of every measurement: quantum mechanics is the theory of content, of what is found. The arena is space and time, taken as the context of every physical comparison. General relativity is the theory of the arena, the context in which content is found.

V. Thermodynamics as a Modal Triad

The three laws of thermodynamics form a modal triad: the constancy of energy, the increase of entropy and the positivity of temperature, each in one of the three modal roles. The constancy of energy, the first law, is omniscient: the first law is a law of symmetry. The increase of entropy, the second law, is transcendent. The positivity of temperature is immanent, where the positivity of temperature means that temperature on the absolute scale is above zero.

The second law states that the entropy of an isolated system of many particles does not decrease. An isolated system is an idealization: a system whose exchanges with its surroundings are small enough to neglect. Entropy is the Boltzmann constant times the logarithm of the number of microstates, which are the detailed arrangements of the particles that give one and the same state at large scale. Each microstate is one possibility for the system, and the more microstates a large-scale state has, the more probable that large-scale state is, so the second law relates probability to possibility, the transcendent pair. The second law is statistical, and the growth of entropy that the second law describes needs a beginning of low entropy from which to proceed.

The increase of entropy is the statistical expression, at large scale, of an irreversibility that is structural in each single interaction. The irreversibility in a single interaction is part of the structure of the single interaction itself. The growth of entropy gives one arrow of time, a direction from earlier to later, which this book calls the entropic arrow. The irreversibility in each single interaction gives a second arrow of time, the non-entropic arrow. The non-entropic arrow is present in every single interaction, and the non-entropic arrow needs neither a large number of parts nor a particular beginning. The entropic arrow and the non-entropic arrow point the same way wherever both are found. Making a record is itself an irreversible change that raises the entropy of the instrument holding the record, so each act of recording joins the two arrows in one event: the irreversible act and the rise in entropy happen together.

VI. Temperature and the Quantized Levels of Bound Systems

In physics, temperature is defined by the rate at which entropy grows as energy is added to a body: one over the temperature equals the change in entropy per change in energy. A degree of freedom of a system is one of the independent ways in which the system can move or change. Temperat-

ure is proportional to the average energy of motion of the particles only in classical systems in which every degree of freedom takes an equal share of the energy, a sharing that physicists call equipartition. When temperature is read through the structure of comparison, temperature measures how strongly energy explores the possibility that the energy is spread over. A cold body is one in which a little added energy makes many new states available to the body, and a hot body is one in which the same added energy makes few new states available.

A bound system is a system whose parts are held together, such as an atom or a solid. The content of a bound system is quantized: the content comes in separate levels, with gaps between the levels. The quantization shows in how temperature is related to the number of active degrees of freedom. The active degrees of freedom are the dimensions of motion that take up energy. The heat capacity of a solid is the energy that the solid takes up for each degree that its temperature rises. At low temperature the heat capacity of a solid falls toward zero, instead of keeping the constant value that classical equipartition gives. The heat capacity falls because the spacing of the bound states becomes larger than the thermal energy available. When the levels of a degree of freedom are spaced more widely than the thermal energy available, the degree of freedom takes up little energy, so at low temperature fewer degrees of freedom are active and the heat capacity falls.

Symmetry, continuity, asymmetry and discontinuity are the four compounds of comparison. Asymmetry is difference of content across difference of context, and discontinuity is difference of content within sameness of context. A conjunction is two of the four compounds held together. A conjunction is perfected when the conjunction holds exactly, at the foundation of a description. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and that neither can asymmetry and discontinuity. Under the Incommensuration Theorem, two conjunctions can be perfected: symmetry with discontinuity, and continuity with asymmetry. The quantization of a bound system is an instance of symmetry

with discontinuity. The energy levels of a bound system are the same for every undisturbed system of its kind, wherever the system is. Sameness of the levels from place to place is a symmetry: sameness of content across difference of context. Two levels of one bound system differ, with no level between them, in one and the same setting. A difference between two levels in one setting is a discontinuity: difference of content within sameness of context. The structure of comparison shows that bound content comes in levels, and physicists find by measurement where the gaps fall and how large the gaps are. A system's spectrum is the set of energies the system can have. The energy of a free particle is not made discrete, because the continuous part of a spectrum begins where the motion is unbounded.

VII. Pattern, Inertia and Mass

Pattern, inertia and mass are three concepts of one reality. A content has one relation to its context. When the relation is found as what there is of the content to find, the relation is the content's pattern. When the relation is found as the content's resistance to a change of its place in the context, the relation is the content's inertia. When the relation is found as the hold that the context takes on the content, the relation is the content's weight. Physicists measure weight as gravitational mass. Mass is the first of the six measures, in the combined sense of pattern, inertia and weight. Mass is a content in the omniscient role, paired with space. Force is a content in the immanent role, paired with time.

In mechanics, mass and force are related by Newton's second law: the rate of change of a body's momentum equals the net force on the body, so that, for a constant mass, the body's acceleration is the force divided by the body's mass. A body's momentum is the body's mass times the body's velocity. Under the same force, a larger mass changes its state of motion less. Push a loaded cart and an empty cart with the same force, and the

loaded cart speeds up less than the empty cart. In mechanics, inertia means that a larger mass changes its state of motion less under the same force.

VIII. Domain Curvature

When everything about a domain can be specified by specifying what is at each point of the domain, the domain is *flat*. When a domain has features that no point-by-point specification captures, the domain is *curved*. Features that no point-by-point specification captures belong to the relations among points and not to any single point. Domain curvature is the curvature that makes a domain curved: a curvature of the domain of comparison as such. Every finite domain has domain curvature. Domain curvature is not the curvature of space and time by which general relativity describes gravitation.

Domain curvature belongs to the relations among the parts of a domain and to no single part, so domain curvature has no location. In physics, domain curvature is the reading of quantum nonlocality: quantum nonlocality is how domain curvature shows in physical measurement. Quantum nonlocality is the agreement between measurements on two distant particles that quantum mechanics predicts and experiments find, and that no account in which each outcome is fixed in advance by local causes reproduces. Quantum nonlocality looks mysterious from the standpoint of a purely local physics because local causality is the flat approximation to a domain that is not flat. Local causality is the condition that each single result is produced by conditions local to the place of the result, so a purely local physics specifies what happens point by point, as a description of a flat domain does.

IX. Summary

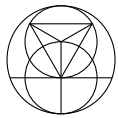
Every physical measurement involves six measures: three contents (pattern, force and probability) and three contexts (space, time and possibility). The number six is argued from comparison: content and context are one pair of the intrinsics of comparison, and by Axiom III each row of the grid contains one measure in each modal role. Each content is measured in its own context, so the six measures form the omniscient pair (pattern in space), the immanent pair (force in time) and the transcendent pair (probability in possibility). The nine pairings of content with context are the Nine Laws, and among the Nine Laws are work and energy, gravitation and entropy. The speed of light relates the omniscient column to the immanent column, and the Planck constant relates the immanent column to the transcendent column, but no constant relates the omniscient column to the transcendent column directly.

General relativity is the theory of the arena, and quantum mechanics is the theory of content. Each theory describes its own pair by way of force in time. Thermodynamics is a modal triad, and the entropic arrow is the large-scale expression of an irreversibility present in every single interaction. Temperature measures how strongly energy explores the possibility that the energy is spread over, and the measure shows in how many new states a little added energy makes available. The levels of bound systems are an instance of symmetry with discontinuity. Pattern, inertia and mass are one relation of content to context. Domain curvature belongs to the relations among points and to no point, and domain curvature is the reading of quantum nonlocality. The grid fixes the structural role of each foundation constant, such as the speed of light and the Planck constant, and the place of each of the Nine Laws, while physics measures the values of the constants.

PART FIVE

THE ARENA AND WHAT IT HOLDS

Gravitation as context, the two theories of one arena, conservation, the arrow of time,
the least interaction, what can be modeled, and the size of chemistry.



Chapter 21

Gravitation Is Context

The Four Markers, the Sense in Which Gravitation Has No Particle, and the Three Kinds of Acceleration

ABSTRACT

In every measurement, a content is found in a context: the content is what the measurement registers, and the context is where, when and among which possibilities the content is registered. Gravitation is the context of physics, not a content within physics. Gravitation carries all four markers that belong to any context, and no force of particle physics carries any of the four. A particle is a kind of content, so gravitation has no particle among the kinds of particle. Gravitation is not quantized. The arena is space and time, taken as the context of every physical comparison. What is called a graviton is at most a quantum of a description of small waves of the arena, never a quantum of the arena. Four independent routes each show that gravitation is context. One route uses three kinds of acceleration: convergent, parallel and divergent. An observer can tell the three kinds apart over a region by comparing the free paths of neighboring bodies. At a single point, only rotation can be told apart. The three kinds are exhaustive, the mathematics of general relativity has terms that correspond to the three kinds, and each kind holds one of the three modal roles: the immanent, the omniscient and the transcendent. The rotation of a body is the divergent kind. Mach's principle says that a rotation registers only against content that the rotating body can be compared with.

> This chapter derives Mach's principle from comparison. An appendix to this chapter states what several coming measurements will measure, and what this account says the measurements will find.

I. Content and Context

Every measurement is a comparison: an act of finding whether two things are the same or different, in some respect. In every comparison there is a content, found in a context. The *content* is what the verdict of a comparison is on. The *context* is the respect in which the two things compared are two. Compare the temperature of a cup of tea at noon with the cup's temperature at one o'clock: the two temperatures are the content, and the two times are the context, because the two times are what make the two readings two. Content is local and exclusive, so that two contents exclude one another from the same place, while context is non-local and composes, so that two contexts combine into one. Chapter 2 gave one law for each side: the law of symmetry applies to all that is of content, and the law of continuity applies to all that is of context. In this book symmetry is sameness of content across a difference of context, and continuity is sameness of content within a sameness of context.

In physics the content is what a measurement registers, which is the state of some body or field, and the context is where, when, and among which possibilities the content is registered. The question of this chapter is whether gravitation is a content or a context.

II. The Four Markers of Context

Any context has four markers, and gravitation has all four. First, gravitation couples to every content without exception: gravitation acts on every body and every field, of whatever kind. Second, nothing shields gravitation. Third, gravitation has one mass for coupling and for inertia. The mass by which gravitation pulls a body is the mass by which the body res-

ists being moved. Fourth, gravitation couples to total energy and momentum, which take continuous values and come in no steps of charge. No force of particle physics has any of the four markers. The electric force fails all four markers. The electric force couples only to content that carries charge, and a shield can block the electric force. The electric force couples to charge, which is separate from inertial mass. So the electric force acts differently on two bodies whose ratios of charge to mass differ. And electric charge comes in steps.

Gravitation is therefore context and not content. The reason is that gravitation has every marker of a context, and no force of particle physics has any of the four markers. Gravitation is the shape of the context: the curvature of space and time, by which every content is related to the content's setting. Space is the topology of connectivity (the pattern of which places are connected to which), and the curvature of space is gravitation. In this book the *arena* is space and time, taken as the context of every physical comparison. The content in the arena fixes the shape of the arena, since the arena curves wherever there is energy and momentum. A free path is the path a body follows when nothing pushes the body, as a falling stone does. A push on one body changes the free paths of other bodies only by moving the content whose energy and momentum curve the arena. Chapter 20 set out six measures of every interaction in two rows: the content row holds pattern, force and probability, and the context row holds space, time and possibility. Gravitation occupies no cell of the two rows, because gravitation is the shape of the context row itself.

III. In What Sense Gravitation Has No Particle

A particle is a kind of content: something found within the arena. The arena is not a content within itself. Gravitation is the shape of the arena, so gravitation has no particle among the kinds of particle. To be quantized

is to come in quanta, least units that cannot be divided, the way light comes in photons. Gravitation is not quantized: there are no quanta of gravitation.

Physicists can still describe small waves of the arena by quanta. The gravitational waves that moving masses send out are small waves of the arena. The name *graviton* is given to a quantum of the description of small waves of the arena, and to nothing else. A graviton is a quantum of the description of the small waves of a classical geometry (a geometry with no quanta of its own), never a quantum of the arena itself. The nearest intelligible sense for the word graviton is the sense of the word phonon. A phonon is a quantum of the description of the vibrations of a held structure. A held structure is a structure whose parts are held in place by the forces between the parts, as the atoms of a crystal are. The arena is not a held structure, so even the sense of the phonon does not make the graviton a quantum of the arena. There is no quantum of the arena itself, and no least step of length, area or duration.

Some physicists seek a particle of the arena. Seeking a particle of the arena means applying to the context of physics a treatment that belongs to the content of physics. Treating a context as a content is a category error. In a category error, someone treats a thing of one kind as a thing of another kind. Some physicists also demand that quantum mechanics and general relativity merge into one theory. Those physicists make the same category error at the level of theories, because quantum mechanics is the theory of content and general relativity is the theory of the arena (Chapter 20).

No one has detected a single graviton. Physicists widely hold that detecting one is impossible in any realistic apparatus (Rothman and Boughn, 2006; Dyson, 2013). Physicists have proposed ways to register one quantum of the description of a gravitational wave in a quantum sensor (Tobar and colleagues, 2024). Yet a one-quantum signal in a quantum sensor is equally well accounted for by a classical wave, a wave with no quanta, acting on a quantum detector (Carney, Domcke and Rodd, 2024). If physicists did detect one quantum of the description of a

gravitational wave, the detection would bear on none of the four routes that show gravitation is context (Section IV), because a quantum of the description of a gravitational wave is consistent with this account.

IV. Four Independent Routes

Four arguments show that gravitation is context, and none of the four arguments depends on another. The first argument is the four markers of Section II: gravitation has all four markers of a context. The second argument is the law of continuity, which applies to all that is of context: context has no least step, and so the arena, which is context, has no least step. The third argument comes from non-commutative geometry, a geometry in which the order of multiplying two quantities can change the result. Physicists have used non-commutative geometry to give an account of the kinds of particle, and in that account the geometry is a product of two factors: an arena factor, which is space and time, and an inner factor. The inner fluctuations of that geometry are the fluctuations that come from the inner factor. The inner fluctuations are the gauge fields, which carry the forces of particle physics, and the Higgs field. The inner fluctuations include no field of spin two. Spin two is the spin that a particle of the arena would need, so the arena factor carries no particle of its own. The fourth argument is three kinds of acceleration, set out in the rest of this chapter. An argument about the world is judged as a cable is judged: by the number of strands and by how independent the strands are. A cable of many independent strands still holds when one strand breaks, and the four arguments are four independent strands of one cable.

V. The Principle of Equivalence

In 1907 the physicist Albert Einstein stated the principle of equivalence: at a single point, the acceleration of free fall cannot be told apart from the uniform acceleration of a frame (Einstein, 1907). A frame is a place from

which measurements are made, such as a room or a rocket. A person in a closed room cannot tell, by any experiment at one point, whether the floor presses upward because the room stands on a planet or because a rocket drives the room. In general relativity, Einstein's theory of gravitation, gravitation is not a force. What a person feels as the force of gravity is the floor pushing up.

An electric field has no principle of equivalence, because the source of an electric field is a charge carried by a content, and an electric field acts differently on two bodies whose ratios of charge to mass differ. Gravitation acts on every body alike, and acting on every body alike is what a context does.

VI. Three Kinds of Acceleration

At a single point, a person cannot tell the acceleration of free fall from the acceleration of a frame. Over a region, an observer can tell accelerations apart by comparison. The comparison is of free paths: three kinds of acceleration differ in what happens to the free paths of neighboring bodies.

Convergent acceleration is the fall toward a center of mass: the free paths of neighboring bodies run together toward a common point. *Parallel* acceleration is the push of a rocket or a floor: the free paths of neighboring bodies keep their separation. *Divergent* acceleration is the outward pull of rotation: the free paths of neighboring bodies run apart from a common axis, and a body feels the pull of rotation grow with the body's distance from the axis.

At a single point, a body has nothing to compare. So the body cannot tell the shape of the body's setting from an act within the setting. At a point, context and content cannot be told apart. Over a region, context and content come apart, because across a region things can be compared across a difference of context, and a comparison across a difference of context registers the context itself.

Rotation differs from the other two kinds in one further way: rotation can be told apart even at a point, because a gyroscope registers rotation, and so does the sideways deflection of a moving body (the Coriolis deflection). A gyroscope is a spinning wheel whose axis keeps pointing in one direction. All three kinds share one feature: each kind affects every body alike, in proportion to the body's inertial mass.

The division of acceleration into three kinds is exhaustive, and I show why by a count of the cases. Release a family of bodies together in a small region, and watch the free paths of the bodies from an accelerated frame. Either the free paths have a common locus (a point or an axis) toward which or away from which the paths run, or the paths have no common locus. If the paths have a common locus, the locus lies either ahead of the paths, so that the paths run together toward the locus, or behind the paths, so that the paths run apart from the locus. If the paths have no common locus, the paths run parallel. The locus is ahead, behind, or nowhere, and there is no fourth case, just as the separation of two neighboring free paths can only shrink, grow, or stay the same. Any actual field of acceleration over a region is a superposition of the three kinds: the three kinds, inseparable, in one instance. A superposition of the three kinds is not a fourth kind.

General relativity has its own mathematics of neighboring free paths, and the terms of that mathematics correspond to the three kinds. Physicists call the relative acceleration of two neighboring free paths geodesic deviation, and the curvature of space and time fixes the geodesic deviation. The physicist Amal Kumar Raychaudhuri gave an equation in 1955 that states how a cloud of free paths swells or shrinks (Raychaudhuri, 1955). The equation is now called Raychaudhuri's equation. In Raychaudhuri's equation, shear is a stretching of the cloud in one direction and a squeezing in another. Where curvature, shear and rotation are all zero, the cloud keeps its volume, and a cloud that keeps its volume shows the parallel kind. Matter and tidal shear focus the cloud and draw the paths of the cloud together, and a focused cloud shows the convergent kind. Rotation spreads the cloud, and a spreading cloud shows the diver-

gent kind. Raychaudhuri's equation is defined only by comparing neighboring paths, so the equation says nothing at a single point. The mathematics therefore has the same scope as the principle of equivalence: at a single point, the principle says that free fall and the acceleration of a frame cannot be told apart, and Raychaudhuri's equation says nothing.

VII. The Modal Places of the Three Kinds

Chapter 3 set out the three modalities: the three roles that the parts of every comparison play. The *immanent* is the act itself, as it happens, here and now. The *omniscient* is what is laid out as structure, held in view as a whole. The *transcendent* is the stance from which an act proceeds, and the range of what is possible from that stance. The transcendent exceeds any specification made at one place. A modality is assigned to a term only within a frame, a triple of concepts considered together. In the frame of the accelerations a test body can undergo, each of the three kinds of acceleration holds one of the three roles.

The convergent kind is transcendent. For a test body, the convergent acceleration is fixed entirely by the distribution of mass and energy outside the body, and no push applied to the body alters that distribution. So the convergent acceleration exceeds any specification made at the body itself. The convergent acceleration is the context within which both paths and pushes occur, and every free path in the region runs to one center, the same center for all of the paths. In general relativity the convergent acceleration is the curvature of space and time.

The parallel kind is immanent. Parallel acceleration is the push at a point of contact, an exchange between the body and what pushes the body, and a person feels parallel acceleration as weight, here and now. Parallel acceleration lasts only while the interaction lasts.

The divergent kind is omniscient. Divergent acceleration is structural. The divergent acceleration is fixed by the knowable shape of the motion: the relation of a rotating frame to the frame's own axis. Divergent acceleration needs no external mass and no continued push: a rotation persists without further action.

The three kinds of acceleration are distinct, inseparable and non-interchangeable, as Axiom III of *An Immanent Metaphysics* requires of the three modalities. None of the three kinds can be made from the other two. A local action is an action applied at the body itself, such as a push. No local action on a free body can produce the convergent kind. Gravitation is the one kind of acceleration that no local action on a body can produce.

A thought experiment shows why no local action can produce the convergent kind. Push each body of a family of bodies so that every body in the family aims at one point, and the pushed bodies now move as if falling toward the point. Add one body that was not pushed. The unpushed body does not move toward the point with the others. The pushes made many parallel accelerations that imitate one convergent acceleration, and the imitation fails as soon as someone adds a body that was not pushed. A real convergent acceleration would move the unpushed body toward the point too, because the convergent kind is the curvature of space and time. The convergent kind is the shape of the context row of the six measures (space, time and possibility), and not a content acting within the domain, which is the whole of what can be measured.

VIII. Rotation Against What?

Mach's principle is the answer to a plain question. Mach's principle is named for the physicist and philosopher Ernst Mach. In an empty space with a single spacecraft in it, how could anyone tell whether the craft is spinning? No one could tell. To spin is to spin relative to something.

In our universe, a gyroscope registers rotation against the local inertial frame, and so does light sent both ways round a ring (the Sagnac effect). The local inertial frame is the frame in which a body that nothing pushes moves in a straight line at a steady speed. The local inertial frame is fixed by the content that the frame can be compared with: the earth, the stars, all the matter within the horizon, which is the boundary of what light has had time to bring to us. If the spacecraft were the only thing in the universe, no content would fix a local inertial frame, and so there would be nothing against which the craft's turning could register. General relativity alone does not say that a lone craft's turning cannot register, because the equations of general relativity allow empty solutions, descriptions of a universe that holds no matter, which still have inertial frames. A lone craft's turning cannot register for a reason that comes from comparison: a difference that no comparison can register is no difference.

Put two spacecraft and nothing else into the universe. The two craft maneuver relative to one another. Each craft's rest or turning is a relation of that craft to the other craft. The other craft is the only content there is to compare with. Only when something else is added can either craft's motion be spoken of relative to that something else. The relationships make the thing: a craft's motion is nothing more than the craft's relations to the other contents there are. There is no external frame of reference in which to consider time or space or anything else: time, space and everything else there is to consider are entirely the interactions.

IX. Summary

Gravitation is the context of physics: the curvature of space and time, the shape of the arena, fixed by the content in the arena. Gravitation carries the four markers of context, and no force of particle physics carries any of the four. The four markers are coupling to every content, having no shield, having one mass for coupling and for inertia, and coupling to a quantity that comes in no steps of charge. Because a particle is a kind of content,

gravitation has no particle among the kinds of particle. Gravitation is not quantized. A graviton is at most a quantum of the description of small waves of the arena, never a quantum of the arena. Four independent routes show that gravitation is context: the four markers, the law of continuity, the account of the particles by non-commutative geometry, and the three kinds of acceleration.

The three kinds of acceleration are convergent, parallel and divergent. At a point the three kinds cannot be told apart, except that rotation registers on a gyroscope, but over a region an observer can tell the three kinds apart by comparing neighboring free paths. The three kinds are exhaustive, and the terms of general relativity's mathematics of neighboring paths correspond to the three kinds. In the frame of the accelerations of a test body, the convergent kind holds the transcendent role, the parallel kind the immanent role, and the divergent kind the omniscient role. The convergent kind, which is gravitation, is the one kind that no local action can produce. Mach's principle is that a rotation registers only against content, and I derive Mach's principle from comparison: the relationships make the thing.

Appendix. What Coming Measurements Will Measure

Several measurements now planned or underway bear on the account of gravitation given in this chapter. For each measurement, this appendix states what the measurement measures and what this account says the measurement will find.

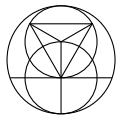
Take two masses, each held in a superposition of two places, a quantum state with a part at each place, and related to each other only through gravitation. The two masses will become entangled, with the phases of the Newtonian interaction between the two masses: the joint state of the two masses will no longer split into one state for each mass. A phase is an angle, measured in radians, by which a part of a quantum state turns, and the Newtonian attraction between the two masses turns

each part of the joint state by an angle that depends on the separation of the masses in that part. In 2017 Sougato Bose and colleagues proposed an experiment in which two masses in superpositions are related only through gravitation. Bose and colleagues give accumulated phases of about minus 0.2 and about 0.7 radians for two of the parts of the joint state, the two cross terms (Bose and colleagues, 2017). The two phases are for the parameters of the proposal of Bose and colleagues: test masses of about ten to the minus fourteen kilograms, a closest separation of about 200 micrometers, and about 2.5 seconds of free fall. This account expects the entanglement because each alternative of the matter carries its own geometry, and in this experiment each alternative of the matter is one of the places a mass is held in.

Measurements at the finest reachable scales measure three things about the geometry. First, the measurements measure whether the geometry carries quantum degrees of freedom of its own, that is, independent quantum variables of the geometry. The question is about variables beyond the variables tied to the alternatives of the matter, and beyond the quanta of the description of the small waves of the geometry. Second, the measurements measure whether area or volume comes in a discrete spectrum, that is, only in separate allowed values. Third, the measurements measure whether space or time has a least step. This account says the measurements will find none of the three. The arena will show no least length, area or duration, and no discrete spectrum of area or volume.

Tests of the principle of equivalence measure whether bodies of different composition fall alike. This account says that bodies of every composition will fall alike at every precision. Tests of shielding measure whether any material or field screens gravitation. This account says that nothing will shield gravitation.

If a single quantum of the description of a gravitational wave is ever detected, the detection is consistent with this account, because a quantum of a description is not a quantum of the arena.



Chapter 22

Two Theories, One Arena

Why Quantum Mechanics and General Relativity Do Not Merge, and How They Are Already Joined

ABSTRACT

Quantum mechanics and general relativity are the two most exact theories in physics, and for a century physicists have tried to make them one. The obstacle to one theory lies in the structure of comparison itself. When a character such as symmetry holds exactly at the foundation of a description, the character is perfected. By the Incommensuration Theorem, exactly two pairings can be perfected in the findings of a comparison: continuity with asymmetry, and symmetry with discontinuity. In what general relativity finds, continuity with asymmetry is perfected. In what quantum mechanics finds, symmetry with discontinuity is perfected. A single theory whose findings perfected both pairings would perfect symmetry with continuity, which the Incommensuration Theorem forbids. What the theorem permits is a common source: two descriptions, each perfected in its own character, joined side by side, and related through the act of measurement in which both descriptions bear on one event. This chapter applies the theorem to physical theories. I show which pairing each theory perfects, and why each theory is informative. I then place the problem of time, a difficulty met in attempts at a quantum theory of gravitation, in force in time, the one pair of measures that quantum

> mechanics and general relativity share. Last, the chapter tells a merger apart from a common source, and states what kind of claim each step of its argument makes.

I. The Hundred-Year Problem

Quantum mechanics is the theory of the microscopic: of particles, fields and forces at scales where discreteness and probability dominate. General relativity is the theory of the macroscopic: the behavior of space and time, gravitation, and the large structure of the universe, at scales where space and time curve and change with no jumps. Experiments have confirmed each theory to very high precision, and each theory predicts things that the other theory cannot describe.

Quantum mechanics and general relativity use different notions of force, different notions of time, and different notions of what counts as a measurement. The ranges of quantum mechanics and general relativity must overlap in three places: at the Planck scale (the scale of the Planck length, which Chapter 18 shows is a bound on what can be found from inside the world), at the singularities of black holes, and at the earliest moments of the present expansion of the universe. At the Planck scale, at the singularities and at the earliest moments, each theory is at the limit of the range where the theory applies, and a description that tries to combine quantum mechanics and general relativity there gives no finite answers.

Physicists built quantum theories of the other forces by one method: in that method, each force is described by quanta, and the calculation proceeds order by order from a fixed list of measured quantities. The method fails for gravitation. When physicists apply the method to gravitation, the result is not renormalizable: each further order of calculation needs a new measured quantity, so at very high energies the method stops predicting anything ('t Hooft and Veltman, 1974; Goroff and Sagnotti, 1986). At energies below the level where the method stops predicting, physicists can describe small waves of the arena (space and time, taken as the context of

every physical comparison) by quanta, and can treat the description as an effective field theory, a description meant to hold only below a stated energy. The description of the small waves then gives finite quantum corrections (Donoghue, 1994), but the quanta in that description are quanta of the description of the small waves, never quanta of the arena itself (Chapter 21).

The usual response of physicists has been to seek one theory that contains both quantum mechanics and general relativity. Physicists have made several attempts to find a framework in which quantum mechanics and general relativity both appear as limiting cases. A limiting case is an approximation that the framework reduces to under special conditions. The attempts include string theory, loop quantum gravity, causal set theory, twistor theory, asymptotic safety and emergent gravity. Much of the theoretical work of modern physics is work, of one kind or another, toward that one theory.

The physicists who seek one theory want a merger: one theory whose findings perfect symmetry and continuity together, in the same directions of space and time. In brief, symmetry means staying the same when the setting changes. Continuity means changing with no jumps, and Section II gives the exact definitions of both. The Incommensuration Theorem states that the findings of no theory, whatever its mathematics, can perfect symmetry and continuity together. So a merger is impossible in principle, and therefore impossible in all future practice, and not merely difficult. The Incommensuration Theorem permits a different kind of unity, a common source, in which two descriptions are joined side by side, and quantum mechanics and general relativity already have a common source.

II. The Theorem, and What It Is About

Every measurement is a comparison. A comparison is the act of finding whether two things are the same or different in some respect. The *content* of a comparison is what the verdict of the comparison is on. The *context* of

a comparison is the respect in which the two things compared are two. For example, in a quantum measurement the property probed is the content and the setting of the apparatus is the context. Sameness and difference, applied to content and to context, give four compounds. *Symmetry* is sameness of content across difference of context: the content stays the same although the context has changed. *Asymmetry* is difference of content across difference of context: the content changes as the context changes. *Continuity* is sameness of content within sameness of context: within one unbroken stretch of context, the content does not jump. *Discontinuity* is difference of content within sameness of context: within one unbroken stretch of context, the content jumps. I call each of the four compounds, symmetry, asymmetry, continuity and discontinuity, a character that a description can have.

The *Incommensuration Theorem* states: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric. To be *perfected* is to hold exactly, at the foundation of a description. Chapter 2 proves the Incommensuration Theorem from the structure of comparison alone, with no premise about the physical world. The Incommensuration Theorem is a structural fact about the nature of knowing.

The Incommensuration Theorem has an exact scope. Symmetry and continuity appear together approximately in every real system, and they hold exactly at once where one holds of the object and the other of the framework in which the object sits. What the theorem forbids is the exact joining of symmetry and continuity in the findings of one and the same comparison.

A physical theory has two levels. The *formalism* of a theory is its mathematics. The *findings* of a theory are what its measurements return. The Incommensuration Theorem applies to the findings. In each great theory, the findings perfect one permitted pairing, and the formalism has the other of symmetry and continuity exactly. Where a theory has symmetry in

one level and continuity in the other, the theory does not have the forbidden pairing, because the two characters then belong to two levels of one theory.

III. Which Pairing Each Theory Perfects

The finding of general relativity is a smooth geometry, perfected everywhere that general relativity applies, up to the singularities where general relativity stops applying. The geometry differs from place to place: the curvature of space and time at one place is not the curvature at another place. So what general relativity finds is continuity with asymmetry: the geometry is smooth, and the geometry differs from place to place. In general relativity, the geometry is written in a formalism that is unchanged under every smooth change of coordinates (every smooth relabeling of the points of space and time), and that invariance is a symmetry of the description, not a symmetry of the world found.

The findings of quantum mechanics are exact invariants (unchanging quantities): charges, quantum numbers and one definite outcome per measurement, all perfected without exception. The findings of quantum mechanics are the same wherever the same kind of system is found, and they come in steps with nothing between. So what quantum mechanics finds is symmetry with discontinuity. In quantum mechanics, the findings are written in a formalism in which the state evolves smoothly and continuously.

General relativity therefore perfects the context of physics: space and time, smooth, and different from place to place. Quantum mechanics perfects the content of physics: discrete, symmetric, countable findings. Continuity with asymmetry and symmetry with discontinuity are exactly the two pairings that the Incommensuration Theorem permits. No single theory can perfect both pairings in its findings, so quantum mechanics and general relativity do not merge into one theory. Quantum mechanics and general relativity are two conjugate descriptions of one world (each de-

scription can be perfected only at the expense of the other), joined side by side and brought together in the act of measurement, and each description contains what the other description leaves out. Quantum mechanics and general relativity do not oppose each other. A theory that quantized the arena itself would give the arena the character of content, and so would carry at least one forbidden pairing.

The relation between quantum mechanics and general relativity can also be stated in the terms of three modalities, the omniscient, the immanent and the transcendent: the three roles that the parts of every comparison play. The *omniscient* is what is laid out as structure, held in view as a whole. The *immanent* is the act itself, as it happens. The *transcendent* is the stance from which an act proceeds, and the range of what is possible from that stance. I take Axiom I from An Immanent Metaphysics, and Axiom I reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". When each of two things can be perfected only at the expense of the other, the two things are *conjugate*. A modality is assigned to a term only relative to a frame, the setting of the description in which the term is considered. In the frame of the foundations of physics, the conjugacy of the omniscient and the transcendent is the relation between the two theories. Quantum mechanics describes content, whose character at the microscopic limit is symmetry with discontinuity. General relativity describes the arena, the context, whose character is continuity with asymmetry. Gravitation, the curvature of space and time, is the phenomenon of context. Quantum mechanics and general relativity are conjugates, so the two theories are to be composed side by side, not unified into one theory. Gravitation is not quantized: gravitation is the phenomenon of context, and so gravitation has no quanta of its own.

The two theories are also mirror images: each is a theory of limits, and each describes in detail what the other leaves out. General relativity is a theory of the limits of signaling: which events can signal which, from which place and time to which place and time. Quantum mechanics is a theory of the limits of what can be known about content, and when. Every

interaction has three measures of context, space, time and possibility (Section V sets out all six measures). Of the three measures of context, general relativity describes space and time in detail and leaves out possibility, while quantum mechanics describes possibility and how possibility evolves in time, and leaves out actuality, the one outcome actually found. In the reading this book gives, quantum mechanics is, at its foundation, a relation of time and possibility, in which the arena (space, and the time of clocks) enters only as a setting supplied from outside. Each of the two theories factors time out of its description in its own way, as Section V shows. The two theories are not approximations converging on one theory: one theory whose findings perfected both symmetry and continuity would have the forbidden pairing.

IV. Why Each Theory Tells Us Something

Each theory describes its domain with a tool from the opposite modality, the modality conjugate to the modality of the domain. In the frame of scale, with the modalities assigned relative to size, the macroscopic is the omniscient domain, laid out and held in view as a whole, and the microscopic is the transcendent domain, exceeding every specification made at a point. Quantum mechanics uses symmetry, the tool of the omniscient, to describe the microscopic. General relativity uses continuity, the tool of the transcendent, to describe the macroscopic.

If a theory used symmetry to describe a domain whose character is symmetry, the theory would be a pattern of a pattern. If a theory used continuity to describe a domain whose character is continuity, the theory would be process of process. A pattern of a pattern is self-application, a term applied to itself, and so is a process of process. Self-application adds nothing to what the single term already says. In quantum mechanics and in general relativity, the tool and the domain belong to opposite modalities, and the reach of the tool across the modal divide is what makes each theory informative.

V. The Shared Pair, and the Problem of Time

Every physical measurement involves six measures, which Chapter 20 sets out. The content of the measurement is pattern (which gathers mass, inertia and structure), force and probability. The context of the measurement is space, time and possibility. Each content is measured in its own context, so the six measures form three pairs: pattern in space, force in time, and probability in possibility. General relativity works with the first pair, pattern in space, and quantum mechanics works with the third pair, probability in possibility, and each theory comes to its own pair by way of the second pair, force in time. So the two theories share one pair, force in time, and they treat force in time in opposite characters.

General relativity treats force in time continuously: time is a dimension of a smooth geometry, and gravitation is not a force but the curvature of that geometry. Quantum mechanics treats force in time discretely: force is exchanged in countable quanta, time is the external parameter against which the state evolves, and each measurement gives one discrete outcome. Physicists who attempt a quantum theory of gravitation run into a difficulty called the problem of time (Isham, 1995; Butterfield and Isham, 1999). The problem of time is the clash between the continuous and the discrete treatment of force in time, applied to time itself. In the frame of the descriptions of time that physics itself gives, the geometric time of general relativity is omniscient, the evolution time of quantum mechanics is transcendent, and the time of the measurement event is immanent. The two theories overlap in force in time, and they disagree about force in time. The act of measurement is where the two theories are joined.

Both theories factor time out of their descriptions. General relativity lays time out with space as one four-dimensional pattern: time becomes a dimension of the continuum. In quantum mechanics, time is the parameter of a deterministic evolution of the state by the Schrödinger equation, which fixes how the state changes, and time reappears at measurement,

the quantum jump, where one outcome is found. Both theories describe time from the third person, as a structure laid out and viewed as a whole. In each theory the first-person event enters as something added beside the formalism. The first-person event is the measurement made here and now from inside the world.

Some physicists say that time does not exist. The physicists who deny that time exists are right in one sense: time is not a container that events happen inside. But the usual reason for denying that time exists is wrong. Time is relative to perception, and perception is not relative to time. Each interaction has its own duration, as one of its own aspects, and we line up the times of many interactions by comparing them. The usual reason for denying that time exists comes from an equation: the main equation of attempts at quantum gravity has no time in it, or relativity lays all of time out at once, like a map. Both forms of the reason are features of the description, not of the world. Time is one of the six measures of interaction between the subjective and the objective: pattern, space, force, time, probability and possibility. So time is real, given in an actual act of comparison, while time does not exist, since time has no place in a pattern as structure: time is an aspect of the act, not a structure standing in place.

VI. Merger and Common Source

Two kinds of unity must be told apart: a merger and a common source. A *merger* is one description in which the directions of space and time themselves carry both characters, symmetry and continuity, at the foundation, and the Incommensuration Theorem forbids a merger. A *common source* is two descriptions, each perfected in its own character, joined side by side. The theorem permits a common source, and quantum mechanics and general relativity already form one: they are joined in the act of measurement.

A merger of quantum mechanics and general relativity would have to connect the omniscient pair, pattern in space, to the transcendent pair, probability in possibility, without passing through the immanent pair,

force in time. By Axiom I, the immanent is more fundamental than the omniscient and the transcendent, because both are made from the act of comparison. By Axiom I, the omniscient and the transcendent are also conjugate. So the two theories are connected through the immanent, the act of interaction itself, and not through any further formalism. In physics the act of interaction is the measurement, in which the geometry and the quantum state are brought to bear on a single event. Process contains something that pattern cannot fully encode, so neither theory nor any formalism contains the act that the theory or the formalism is made from. Physics uses the act of measurement and gives no account of the act. The account of the act is the metaphysics, the Immanent Metaphysics from which this book starts.

The split between quantum mechanics and general relativity is therefore necessary, not accidental. Quantum mechanics and general relativity are the two conjugate aspects of one domain of comparison, related by the interaction in which both bear on one event, and not two approximations of one underlying theory. The Immanent Metaphysics describes the relation between the two theories, and the relation is not a third physical theory standing above the two.

Measurement is also where a further kind of agreement becomes a fact. Two separated records of entangled particles, particles that share one quantum state, can agree more closely than any causal account of the two records permits. I call an agreement closer than any causal account permits a *non-causal correlation*. A non-causal correlation cannot be used to send a signal, and the statistics at each end are independent of the setting chosen at the other. The correlation is real, and the correlation is not an influence: the correlation becomes a fact where the two separated records of the entangled particles are brought together, by ordinary signals that keep to the limit of the speed of light.

The Incommensuration Theorem forbids a merger, and nothing more. By the theorem, descriptions of one character can be joined, and so can two descriptions of opposite character, side by side from a common source. In every program of quantum gravity, physicists give up some

symmetry for continuity, or some continuity for symmetry. String theorists keep the symmetric apparatus whole and make the smooth geometry secondary, while researchers in loop quantum gravity keep the geometry primary and recover the exact symmetries of flat space only in a limit. So the question to ask of any proposal is where the proposal gives up symmetry for continuity, or continuity for symmetry. Whether a given proposal is good physics is a question for physics.

VII. What No Model Replicates

The findings of quantum mechanics are discrete and symmetric. Quantum mechanics is built on symmetry groups, the mathematics of the ways a thing can change while staying the same, and the continuity in quantum mechanics is in its formalism, in the smooth evolution of amplitudes (the numbers the quantum state gives to the possible outcomes). A classical computer can reproduce the statistics of quantum mechanics, at a cost that, by the direct method, grows exponentially with the number of entangled particles: each added particle multiplies the cost. For one restricted class of circuits the cost stays small. The physicists Daniel Gottesman and Emanuel Knill proved a theorem about circuits built from the Clifford gates. The Clifford gates are one standard set of operations on qubits, the units of quantum information. Circuits built from the Clifford gates can entangle any number of qubits and can still be simulated efficiently by a classical computer: the theorem of Gottesman and Knill (Gottesman 1998). *The Quantum Bridge* sets out the theorem of Gottesman and Knill in Chapter 4, Section VII. A deterministic model is a model in which the state at one time fixes the state at every later time. No deterministic model reproduces the single outcome as the outcome is: not fixed by anything before it. The quantum world is the world of what no deterministic model can replicate: the world of hard identity (the identity of a

thing that no copy can carry, Chapter 26), of uniqueness. The single outcome is the event in which quantum mechanics and general relativity are joined.

VIII. The Kind of Claim Each Step Makes

The Incommensuration Theorem is a theorem of the Immanent Metaphysics, proved from comparison alone, with no physical premise. I diagnose the split between quantum mechanics and general relativity, and the diagnosis rests on a short chain of four steps. The first step is the Incommensuration Theorem. The second step is two laws: the law of symmetry applies to all that is of content, and the law of continuity to all that is of context. The third step is the findings of each theory: each theory perfects one of the two permitted pairings, continuity with asymmetry or symmetry with discontinuity. The fourth step is the identification of gravitation as the context of physics. The diagnosis does not depend on one further identification: placing the two theories among the three pairs of measures in Section V (pattern in space, force in time, probability in possibility). Chapter 21, Section IV, identifies gravitation as context by four independent routes: the four markers that every context carries; the law of continuity for context; the arena factor of the geometry in the account of the particles by non-commutative geometry, a factor that carries no particle of its own; and the three kinds of acceleration (convergent, parallel and divergent). The four markers are that gravitation couples to every content, that nothing shields gravitation, that gravitation has one mass for coupling and for inertia, and that the quantity gravitation couples to comes in no charged steps.

An argument about the world is judged as a cable is judged: by the number and the independence of its strands, and not as a chain is judged, by its weakest link (Peirce, 1868). The philosopher Charles Sanders Peirce made the comparison of reasoning with a cable, in which each independ-

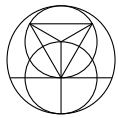
ent line of support is one strand. The account given in this chapter is an argument of the cable kind: the identification of gravitation as context, for example, has four independent strands.

The Immanent Metaphysics is also its own *metatheory*, a theory about a theory: a theory of its own content, derivation and grounds. The Immanent Metaphysics also has a third order of self-description, a theory about the metatheory, and the third order adds nothing to the second, the metatheory. The merger question is whether quantum mechanics and general relativity can be merged. The merger question is about the relation between two descriptions. A question about the relation between descriptions belongs to metaphysics. Building descriptions joined from a common source, and judging whether a given description is good physics, belong to physics.

IX. Summary

In this chapter I have shown why the structure of physics is two foundational theories, joined but not merged. By the Incommensuration Theorem, two pairings can be perfected in the findings of a comparison: continuity with asymmetry, and symmetry with discontinuity. The two great theories of physics are the two permitted pairings made physical. General relativity perfects continuity with asymmetry, in the arena. Quantum mechanics perfects symmetry with discontinuity, in the content. Each theory has the opposite character in its formalism. In each theory the tool comes from the modality opposite to the domain, and that reach across the modal divide makes each theory informative. The two theories share one pair of measures, force in time, and treat force in time in opposite characters. The problem of time, met in attempts at a quantum theory of gravitation, is the difference between the continuous and the discrete treatment of force in time, applied to time.

A merger is impossible by the structure of comparison. A merger would be one description in which space and time carry symmetry and continuity together at the foundation. The unity that remains is already present: one account of everything, made of two descriptions joined side by side and brought together in the act of measurement. Once physicists know that a merger is impossible, the effort of physics changes direction and does not end. The physics of the two theories already works, and what was missing was a better understanding of what physics is, what physics can do, and where the jurisdiction of physics ends. The better understanding is the recognition of the ground beneath all theories: the act of comparison.



Chapter 23

Conservation and Its Bound

Noether's Theorem, the Non-Conserved Complement, and Why No Single Description Covers the Whole

ABSTRACT

A conservation law is a symmetry stated as a quantity. A conserved quantity is the content that stays the same across the contexts of a symmetric domain. Using the theorem that Emmy Noether proved, a physicist can work out which conserved quantity goes with which symmetry, and compute that quantity. This chapter states the structural reason that conserved quantities exist. I also show that a continuous symmetry is not the pairing of symmetry with continuity that the Incommensuration Theorem forbids. The Incommensuration Theorem also sets a bound on conservation. Every conservation law holds within a domain and stops holding at the edge of that domain. Every conserved quantity has a complement that is not conserved, and the conservation laws of one domain can be the asymmetries of another domain. Relations within a domain are relations of actuality, while relations between domains are relations of potentiality. Two complementary functions, coherence and resistance, determine how structure passes from one domain to another. Simple things obey simple laws because simple things are alike, and the likeness of simple things makes universal law possible. Universal law is symmetry law, and wherever symmetry is the law that governs, one expects the fun-

> damental to be simple. Stability and dynamism are conjugate. And no single description of the universe is consistent both at every place and as a whole.

I. Conservation Is Symmetry Stated as a Quantity

Put some water in a bottle and cork it. Five months pass, and the bottle holds the same amount of water. The water is conserved. To say that something is conserved is to say that the amount of the thing does not change in time or in space: the content is the same, and the context (this month, that month; this shelf, that shelf) is different.

The corked bottle is a case of symmetry, in the exact sense in which this book uses the word. *Symmetry* is sameness of content across difference of context. In a comparison, the content is what the verdict of the comparison is on (in the bottle, the amount of water), and the context is the respect in which the two things compared are two (in the bottle, two months or two shelves). Every conservation law in physics is an applied instance of symmetry, sameness of content across difference of context: the conservation of energy is one instance, and the conservation of the count of atoms in a chemical reaction is another. So a conservation law is a symmetry stated as a quantity. A conserved quantity is the content that stays the same across the contexts of a symmetric domain. In the corked bottle, the conserved quantity is the amount of water.

In 1918 the mathematician Emmy Noether proved a theorem about systems whose laws come from an action (a Lagrangian formulation), where an action is a single quantity, reckoned over the whole history of a system, from which the laws of the system follow. A continuous symmetry is a change that can be made by any amount, however small, and that leaves the laws the same, as a shift of the whole system forward in time does. Noether proved that, in a system whose laws come from an action, every continuous symmetry of the action has a corresponding conserved quantity: where the laws stay the same under a smooth change, a

quantity is kept (Noether, 1918). For example, where the laws stay the same under a shift in time, the quantity kept is energy. Within the formalism of actions, Noether's theorem is the formal instance of a feature that every symmetric domain has by being symmetric: wherever content stays the same across changes of context, there is a corresponding conservation. Using Noether's theorem, a physicist can find which conserved quantity goes with which symmetry, and compute that quantity. The structural reason that conserved quantities exist is that the content of a symmetric domain stays the same across the contexts of the domain.

A reader might take a continuous symmetry to join symmetry with continuity, the pairing that the Incommensuration Theorem forbids. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. A continuous symmetry is not the forbidden pairing. The reason is that the symmetry and the continuity of a continuous symmetry belong to two different levels of a physical theory: its findings, which are what its measurements return, and its formalism, which is its mathematics. In a continuous symmetry the symmetry is found, as a conserved quantity, and the continuity belongs to the formalism, as the smooth parameter of the group of symmetries. The group of symmetries is the whole set of changes that leave the laws the same. For shifts in time, the group contains a shift by every amount, and the amount of the shift is the smooth parameter of the group, a number that can take any value. So Noether's theorem states a link between a conserved quantity in the findings and a smooth parameter in the formalism.

II. The Bound on Conservation

The Incommensuration Theorem shows that perfect symmetry requires discontinuity. To be perfected is to hold exactly, at the foundation of a description. Any comparison perfected at its foundation is either continu-

ously asymmetric or discontinuously symmetric, and so wherever symmetry holds exactly, the symmetry comes with discontinuity. A conservation law is a symmetry stated as a quantity, and so the theorem also sets a bound on conservation. No conservation law holds perfectly across all conditions. There are discontinuities of two kinds at which a conservation law fails, or at which the quantity that the law conserves ceases to be well defined: the boundaries of domains, and phase transitions. A phase transition is a sudden change in the state of a material, as when water freezes into ice.

The bound on conservation is a claim about structure. The bound is not a forecast that a laboratory will someday find energy or charge unconserved. Experimenters have never observed a failure of the conservation of charge. The bound on conservation states that a conservation law expresses a symmetry that holds within a domain. At the edge of the domain, the sameness of content across contexts stops, and something else takes the place of the sameness. One instance of the bound comes from cosmology: physicists have difficulty defining a conserved total energy for a universe that is curved and expanding.

III. The Non-Conserved Complement

Asymmetry, the complement of symmetry, is difference of content across difference of context, and in a physical system asymmetry shows itself in the differences that remain while the conserved quantities stay the same. Every conserved quantity has a complementary aspect that is not conserved. Energy is conserved, but the way energy is distributed is not: when a hot cup of coffee cools in a room, the total energy of the coffee and the room stays the same, while the share of the total energy held by the coffee falls. Total momentum is conserved, but the way momentum is shared among the parts of a system is not: when a moving billiard ball strikes a ball at rest, the total momentum of the two balls stays the same, while the moving ball passes momentum to the other ball.

Domains of different scales depend on one another, and the conservation laws of one domain can be the asymmetries of another domain. What is structural and necessary at one scale can be contingent (able to have been otherwise) and variable at another scale. So a physicist cannot simply carry a conservation law across the boundary of a domain and expect the law to hold on the other side in the same form, because what is conserved on one side of the boundary can be an asymmetry on the other side.

IV. Within a Domain and Between Domains

A *domain* is that which contains and includes, and is itself uncontained. A domain exists as a domain insofar as comparisons can be made within the domain. For example, the physical domain is the whole of what can be measured from within the physical domain. Relations within a domain are relations of actuality, and relations between domains are relations of potentiality. A force between two domains has no meaning, because force applies only within a domain, and only probabilities relate one domain to another.

The three modalities are the three roles that the parts of every comparison play toward one another (Chapter 3). The three roles are the omniscient, the transcendent and the immanent. The frame of a description is the setting relative to which the roles are assigned. When the relations of domains are the frame, each of the three roles is assigned to one term. Actuality plays the role of the omniscient, the role of what is laid out as structure. Potentiality plays the role of the transcendent, the role of the range of what is possible. Complexity is the interaction of the potential and the actual. Complexity plays the role of the immanent, the role of the act as it happens. Anything actual, taken as content found, is quantized (it comes in separate steps, with nothing between the steps), while anything potential is continuous.

Two functions determine how structure passes from one domain to another: coherence and resistance. *Coherence* is the function by which a structure in one domain stays intact when the structure is carried into another domain. *Resistance* is the opposition that prevents perfect carrying over, so that each domain keeps its own character despite its relations to other domains. A salt crystal gives a case of each function. The crystal in the hand has a regular shape, which carries the regular arrangement of the crystal's atoms into the domain of everyday objects. The carrying over of the arrangement is coherence. The crystal in the hand is also solid and still while the crystal's atoms never stop moving, and resistance is what lets the everyday domain keep the character of being solid and still. Coherence and resistance are complementary: each can be perfected (taken toward perfection) only at the expense of the other. Perfect coherence would remove every boundary between domains, and perfect resistance would prevent every relation between domains, so every relation between domains is a finite balance of coherence and resistance, in which both are present and neither is perfected.

V. Simplicity and Universal Law

The simpler a thing is, the more similar its relations are to the relations of all other simple things. Simple things are the same as other simple things. Simple things have similar relations. Simple things behave in similar ways. Every electron, for example, has the same mass and the same charge as every other electron. Where all the things a law relates behave in the same few ways, the law has only those few ways to describe, and therefore the laws that relate simple things are themselves simple.

The likeness of simple things to one another is what makes universal physical law possible. Universal law is law that holds everywhere, for every thing of the kind the law covers. At the most basic level, the variety of the world reduces to a small number of structural types, and the laws that govern the structural types are correspondingly simple. Universal law

is symmetry law. A symmetry law states a sameness of content across difference of context, as the law of the constancy of energy does (Chapter 20). Wherever symmetry is the type of law that governs, one expects the fundamental, the most basic level of the world, to be simple.

VI. Stability and Dynamism

Physical persistence is the way a physical thing continues through time. Physical persistence has two aspects: stability and dynamism. *Stability* is the tendency of structures to keep their form: the way patterns hold together against breaking up. *Dynamism* is the tendency of structures to transform: the movement of structures toward new arrangements. A stone on a hillside shows mostly stability, since the stone keeps its shape for longer than a human life, while the flame of a candle shows mostly dynamism, since the flame changes its shape from moment to moment.

Stability and dynamism are conjugate: the two cannot both be perfected, and neither can ever be zero. Perfect stability would be stasis, a complete stillness in which all process ends. Perfect dynamism would be chaos, in which no form could endure. The physical world exists in the tension between stability and dynamism, and every physical phenomenon can be understood as one particular resolution of the tension: a particular degree of stability together with a particular degree of dynamism.

VII. Global Inconsistency

Global inconsistency, in this book, is the structural fact that no single description of the universe can be consistent at every place and consistent as a whole. If a person assumes a description is consistent at every place, the person must eventually find the description inconsistent as a whole. Two results show why: the Incommensuration Theorem, and domain curvature.

The first result is the Incommensuration Theorem. The Incommensuration Theorem means that no theory can perfect symmetry and continuity together. The two most exact theories of physics are quantum mechanics and general relativity, and in the findings of each theory, one of the two pairings that the Incommensuration Theorem permits holds exactly: quantum mechanics is discontinuously symmetric, and general relativity is continuously asymmetric. If a single theory perfected both characters, discontinuous symmetry and continuous asymmetry, in its findings, the theory would contain a pairing that the Incommensuration Theorem forbids. So quantum mechanics and general relativity do not merge into one theory (Chapter 22).

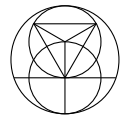
The second result is domain curvature, in a sense distinct from the curvature of space and time. When everything about a domain can be specified by specifying what is at each point, this book calls the domain *flat*. When a domain has features that no point-by-point specification captures, features of the relations among points and not of any point, this book calls the domain *curved*. Every finite domain is curved in this book's sense. So when a description specifies a finite domain point by point, the description leaves out the features of the domain that belong to the relations among points, however consistent the description is at each point. In physics, quantum nonlocality is how domain curvature shows itself (Chapter 20).

The Incommensuration Theorem and domain curvature give inconsistencies that are structural features of description, and not errors. In physics, the inconsistencies show themselves as the incompatibility of quantum mechanics and general relativity at certain scales, and as the measurement problem (the question of where the single, definite outcome of a quantum measurement comes from). The inconsistencies also appear as the paradoxes of cosmology, and as the paradoxes physicists run into when attempting a quantum theory of gravitation. Each inconsistency lies at the structural limit of a single description. The inconsistencies are not defects that a single final theory will one day remove.

VIII. Summary

A conservation law is a symmetry stated as a quantity: sameness of content across difference of context, as the corked bottle holds the same water month after month. Using Noether's theorem, a physicist computes which conserved quantity goes with which symmetry, but the structural reason that conserved quantities exist is that the content of a symmetric domain stays the same across the contexts of the domain. In a continuous symmetry, the symmetry is in the findings and the continuity is in the formalism, so the Incommensuration Theorem permits continuous symmetries.

Because perfect symmetry requires discontinuity, every conservation law holds within a domain and stops holding at the edge of that domain. Every conserved quantity has a complement that is not conserved, and the conservation laws of one domain can be the asymmetries of another domain. Relations within a domain are relations of actuality, while relations between domains are relations of potentiality. Every relation between domains is a finite balance of coherence and resistance. Simple things are alike, and the likeness of simple things makes universal law possible. Universal law is symmetry law, and wherever symmetry is the law that governs, one expects the fundamental to be simple. Stability and dynamism are conjugate. And global inconsistency is the structural limit of every single description, not a defect that some later theory will remove.



Chapter 24

The Arrow of Time and the Epistemic Sandwich

Why Time Has a Direction Before Entropy Gives It One

ABSTRACT

The fundamental equations of physics run equally well forward and backward in time. Physicists usually take the direction of time from entropy, a measure of disorder that grows in systems of many parts, and call the direction given by the growth of entropy the entropic arrow. A structural arrow of time is more basic than the entropic arrow. The structural arrow is present in every single interaction. The structural arrow does not depend on the initial conditions of our universe. The reverse of a lawful process is lawful, because the laws are symmetric in time. A faithful reversal would retrace exactly the states of the original process. A faithful reversal would also need a state specified completely and continuously. So a faithful reversal would perfect symmetry together with continuity. But the Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. Physicists use a single word, time, for three different things. The direction of time belongs to one of the three, event time, the time of events. The Epistemic Sandwich is a second argument. The Epistemic Sandwich runs across three layers of description, each layer at a different scale. Knowledge at the scale of observers is bound to the entropic arrow. The free will theorem is a result proved by the mathem-

> aticians John Conway and Simon Kochen. The free will theorem states that when an experimenter's choice is not fixed in advance, the response of the particle measured is not fixed in advance either. A non-entropic asymmetry is an asymmetry of time that does not come from entropy. At the smallest scale, a non-entropic asymmetry orients (sets the direction of) the entropic arrow at the larger scales above the smallest scale. The choice is between a real arrow of time at every layer and no knowledge at all. The last section of the chapter is about two senses of the rate of time, and about the proportional theory of felt time.

I. The Problem of Time's Direction

Eggs break and do not unbreak. An ice cube dropped into warm water melts, and the water never gathers itself back into the cube. Yet each of the fundamental equations of physics admits solutions that run equally well forward and backward in time. The fundamental equations are Newton's laws of motion, Maxwell's equations of electricity and magnetism, the Schrödinger equation of quantum mechanics, and Einstein's field equations of gravitation. Film a process that obeys one of the fundamental equations and run the film backward: the film then shows a process that obeys the same equation.

The only exception among the fundamental laws is the weak interaction. The weak interaction is the force of nature that drives some kinds of radioactive decay. The weak interaction violates the combined symmetry of charge and parity. The reversal of charge turns every particle into its antiparticle. The reversal of parity reflects space, as a mirror does. A law is symmetric under a reversal when the reversed process is lawful too. The CPT theorem states that every local relativistic quantum field theory is symmetric under the combined reversal of charge, parity and time, where a local relativistic quantum field theory is the kind of theory that physicists use to describe particles. So the CPT theorem requires a law that violates the combined symmetry of charge and parity to violate the symmetry

of time too, because only with both violations does the combination of all three reversals stay symmetric. Therefore the weak interaction violates the symmetry of time as well. Apart from the weak interaction's small violation of time symmetry, nothing in the fundamental laws distinguishes past from future.

Physicists usually take the direction of time from entropy. Entropy is a measure of disorder: the entropy of a system measures how many arrangements of the system's parts look alike from outside, and a disordered state has many more arrangements that look alike from outside than an ordered state has. The second law of thermodynamics says that the entropy of an isolated system of many parts does not decrease, but physicists read the second law as a statistical tendency of systems with very many parts, and not as a fundamental asymmetry in the laws. The statistical growth of entropy gives time a direction, the entropic arrow. The usual account of the direction of time needs two things: a statistical law of very many parts, and a beginning of low entropy from which the growth of entropy can proceed.

II. The Structural Arrow

The *arrow of time* is the structural asymmetry between a determined past and an open future. Because the arrow of time is a structural asymmetry, this chapter also calls the arrow of time the structural arrow. The arrow of time lies beneath the entropic arrow. The arrow of time is the more basic of the two arrows. A domain is the whole of what can be compared by a given kind of comparison. The arrow of time is a necessary feature of any domain in which interaction occurs, and so the arrow of time does not depend on the initial conditions of our universe.

The reverse of a lawful process is lawful, because the laws are symmetric under reversal of time, but the reverse need not be faithful. A faithful reversal is one that retraces exactly the states of the original process. A faithful reversal requires a state specified completely and continuously,

with no gap. Perfect reversal of time would therefore require two things perfected together: the symmetry of the laws, and the continuity of the state. To be perfected is to hold exactly, at the foundation of a description. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. So perfect reversal of time cannot occur, whatever the entropy is and whatever the boundary conditions are, where boundary conditions are the conditions at the start and at the edges of a system.

The structural arrow appears in every single comparison. A comparison is the act of finding whether two things are the same or different, in some respect. Every measurement is a comparison. In every comparison the range of the possible comes first, then the act of comparing, then the record of the act, so every record comes after the act that made the record. A footprint in wet sand is a record of a step, and the footprint comes after the step. The structural arrow needs no large number of parts and no beginning of low entropy, while the entropic arrow needs both. Wherever both the structural arrow and the entropic arrow are found, the entropic arrow points the same way as the structural arrow. Making a record is itself an irreversible change that raises the entropy of the instrument that holds the record, so every act of recording joins the structural arrow and the entropic arrow in one event.

The structural arrow does not explain the low entropy of the early universe, but the structural arrow shows that the direction of time never depended on that low entropy at the beginning.

III. Three Things Physics Calls Time

Physicists use a single word, time, for three different objects. *Geometric time* is the proper time along paths, and the causal order, carried by the metric of space and time. The proper time along a path is the time that a clock carried along the path would show, while the causal order is the or-

der of which events can affect which other events. The metric is the rule for the distances and the intervals of time between events. *Event time* is the discrete order of findings, each finding the outcome of one measurement. Discrete means made of separate steps, one after another. *Evolution time* is the parameter of the continuous evolution of a quantum state. The parameter is the time variable in the Schrödinger equation, the equation by which a quantum state changes smoothly.

The parts of every comparison play three roles. The three roles are the three modalities: the omniscient, the immanent and the transcendent. The *omniscient* is the role of what is laid out as structure, held in view as a whole. The *immanent* is the role of the act itself, as the act happens. The *transcendent* is the role of the stance from which an act proceeds, and of the range of what is possible from that stance. A term plays a role only relative to a frame: the set of three terms in which the term is considered. When the frame is geometric time, event time and evolution time, geometric time is omniscient, event time is immanent, and evolution time is transcendent. Each of the three assignments matches the definition of the role assigned. Geometric time is laid out as structure, and event time is the order of acts, as each act happens. Evolution time belongs to the quantum state, which describes the range of what is possible. The one-way order of findings is the direction of time.

The block universe is the picture of space and time as one four-dimensional block that exists all at once, with past, present and future laid out together. In the block universe, geometric time, the dimension, is taken for the whole of time, so event time, the process, is left out. The structural arrow of Section II belongs to the process, event time, so a picture that leaves out event time also leaves out the structural arrow.

IV. The Three Layers

A second argument runs across three layers of physical description, each layer at a different scale. I call the second argument the *Epistemic Sandwich* ("epistemic" means having to do with knowledge). The layers are distinct, inseparable and non-interchangeable: no layer is either of the others, no layer occurs without the other two, and no layer can do the work of another. A frame is a set of three terms considered together. When the frame is the three scales of description, the observer's own scale is immanent, the largest scale is omniscient, and the smallest scale is transcendent.

Layer 1 is the scale of observers and their instruments. I call layer 1 the mesoscopic scale, meaning the observer's own scale, while physicists who study condensed matter use the word "mesoscopic" in a different sense, for the scale between single atoms and bulk matter. Physicists describe layer 1 with Newtonian mechanics, electromagnetism and thermodynamics. The description of layer 1 is in terms of knowing. The main concepts of layer 1 are causality, change and choice. When the frame is the three main concepts of layer 1, causality is omniscient, change is immanent, and choice is transcendent.

Layer 2 is the macroscopic scale, the largest scale. Physicists describe layer 2 with general relativity, in the terms of cosmology. The main concepts of layer 2 are the meaningless and the meaningful, with pattern as the term between the meaningless and the meaningful.

Layer 3 is the microscopic scale, the smallest scale. Physicists describe layer 3 with quantum mechanics, in terms of what is. The main concepts of layer 3 are determinism, relation and randomness. Determinism is symmetry and continuity both perfected, and pure randomness is asymmetry and discontinuity both perfected. Neither determinism nor pure randomness can occur, because each of the two idealizations perfects a pair that the Incommensuration Theorem states cannot be applied together, simultaneously and fundamentally. What is real at layer 3 lies between determ-

inism and pure randomness, and relation is the term between determinism and pure randomness. Hard randomness is real at layer 3. Hard randomness is an outcome that no prior specification fixes, however complete the specification is. The single outcome of a quantum measurement is the plainest instance of hard randomness. So hard randomness is distinct from pure randomness: hard randomness is real, and pure randomness is an idealization that cannot occur.

V. The First Two Relations

The Epistemic Sandwich has four steps, each step a relation among the three layers: layer 1, the scale of observers; layer 2, the largest scale; and layer 3, the smallest scale. The first relation is that scientists define what is at layers 2 and 3 in terms of what can be known at layer 1. What can be known at layer 1 is the only basis of knowing that connects mathematics to the real, so physicists gain all their knowledge of general relativity and of quantum mechanics through the concepts and measurements available at the scale of observers. An astronomer learns about the largest scale through telescopes and cameras, while a physicist learns about the smallest scale through counters and screens. Every telescope, camera, counter and screen is an instrument at the scale of observers.

The second relation is that knowing at layer 1 is bound to the entropic arrow, because records are needed to measure, to record, or to tell an ordered state from a disordered state. A record requires the entropic arrow, because making a record is an irreversible change, a change that raises entropy (Section II). If layer 1 had no entropic arrow, there would be no basis at layer 1 for telling anything apart, and so at layer 1 knowledge and the entropic arrow necessarily occur together.

VI. The Third Relation: the Free Will Theorem

The third relation comes from the free will theorem, a result of mathematical physics proved by the mathematicians John Conway and Simon Kochen. Conway and Kochen proved the free will theorem in 2006, and proved a strong form of the theorem in 2009. The free will theorem states that if the choices of experimenters are not determined by prior information, then the responses of particles are not determined by prior information either. The free will theorem assumes that quantum mechanics and relativity apply, and that there is no backward causation (no effect that comes before its cause). A choice or a response is determined by prior information when information that existed earlier fixes the choice or the response in advance.

The setting of an instrument is the way the experimenter arranges the instrument for a measurement. One example of a setting is the direction along which the instrument measures. When an experimenter's choice of setting is fixed in advance, the experimenter's measurement is a replay, and not a measurement. The free will theorem assumes the freedom of the experimenter: a choice of setting that is not fixed in advance. Choice is real. Chapter 13 derives choice as one member of the triple of the real, together with change and causation, and not as a form of causation. An experimenter's choice of setting is a choice, and so the premise of the free will theorem holds. By the free will theorem, the openness of the experimenter's choice then carries from layer 1 down to the particle at layer 3: because the experimenter's choice is not determined by prior information, the particle's response is not determined by prior information either. When a particle's response is not determined by prior information, the response is a state that is not fully specified, and so the reversal of the response cannot be faithful. A reversal that cannot be faithful is an asymmetry of time. The asymmetry at layer 3 does not come from entropy. I

call an asymmetry of time that does not come from entropy a non-entropic asymmetry. The non-entropic asymmetry is present in single events, prior to any collection of events.

The free will theorem itself is a result about determination (whether prior information fixes an outcome), and not about direction in time, so the step from an undetermined response to a reversal that cannot be faithful comes from Section II: a faithful reversal needs a completely specified state. Across layer 1 and layer 3, the Epistemic Sandwich runs as follows. First, the entropic arrow at layer 1 goes with knowing. Next, knowing needs measurement, and a measurement needs a choice of setting that is not fixed in advance. Then, by the free will theorem, an open choice leaves the particle's response at layer 3 undetermined. Last, an undetermined response cannot be reversed faithfully. When the free will theorem is read across layer 1 and layer 3, the free will theorem means that the entropic arrow at layer 1 and a non-entropic arrow at layer 3 occur together: both arrows occur, or neither arrow occurs. The free will theorem shows that the middle ground cannot occur: a free choice by the experimenter together with a fixed response by the particle. Since the entropic arrow exists at layer 1, a non-entropic arrow exists at the quantum scale, layer 3.

VII. The Fourth Relation

The fourth relation is that any non-entropic asymmetry of time at layer 3 is the orienting basis for the entropic asymmetry at layers 1 and 2: the asymmetry at layer 3 sets the direction in which the entropic asymmetry at layers 1 and 2 points. The non-entropic arrow at the smallest scale is the structural ground on which the entropic arrow at larger scales is built.

The laws of physics already contain a microscopic asymmetry of time that does not come from entropy. The weak interaction violates the symmetry of charge and parity combined (Christenson, Cronin, Fitch and Turlay, 1964). Four physicists found the violation in 1964, in the decays of particles called neutral kaons: James Christenson, James Cronin, Val Fitch

and René Turlay. By the CPT theorem, a violation of time symmetry follows from the violation of the combined symmetry of charge and parity. Experimenters have observed the violation of time symmetry directly (CPLEAR Collaboration, 1998; BaBar Collaboration, 2012). The CPLEAR Collaboration and the BaBar Collaboration are two large teams of experimental physicists. CPLEAR worked at CERN in Europe, and BaBar at SLAC in California. Section II sets a structural requirement: perfect reversal of time cannot occur. The structural requirement limits what is possible: time-reversal symmetry cannot be perfect. The theory of the weak interaction describes how the weak interaction mixes the different kinds of quark: the mixing phase is the part of the mixing that makes the weak interaction violate the combined symmetry of charge and parity. The mixing phase of the weak interaction is how the remaining possibility, a symmetry of time that is not perfect, is realized. The structural requirement does not give the size of the violation or the mechanism of the violation, but the structural requirement would still hold if experimenters had not found the violation.

VIII. Knowledge or Its Absence

The conclusion of the Epistemic Sandwich is a choice between two options. The first option is that a non-entropic arrow of time is real at all three layers: the scale of observers, the largest scale and the smallest scale. The second option is that the entropic arrow, on which all knowledge depends, is itself absent, and then we know nothing at all. Sections V and VI contain the reason for the two options: knowledge needs the entropic arrow at layer 1, and the entropic arrow at layer 1 occurs only together with a non-entropic arrow at layer 3. Anyone who denies the non-entropic arrow therefore denies the very basis of knowing on which all scientific knowledge is built. The choice is between knowledge and the absence of knowledge, and not between two physical theories.

IX. The Rate of Time, and Felt Time

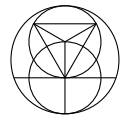
When a person speaks of a rate at which time flows, the person must say which time is meant. For a perceiver, the felt passage of time goes with the number of changes of state in what is perceived, and with the novelty of what is perceived. The rate of a clock is a different sense of the rate of time. The arena is space and time taken as the context of every physical comparison. The rate of a clock is set by the clock's place in the arena, by the clock's height in a field of gravitation and by the clock's motion, and not by how many interactions the clock takes part in. Clocks run a little faster high on a mountain than at sea level. A fast-moving clock runs a little slower than clocks at rest.

Felt time across a whole life has an older account, from psychology. The French philosopher Paul Janet set out the proportional theory of felt time (Janet, 1877). The proportional theory states that the apparent length of an interval is inversely proportional to the length of life already lived: on the proportional theory, an interval is felt as the fraction that the interval makes of the life lived so far. A year is a larger fraction of a child's life than of a grandparent's life. So, on the proportional theory, a year feels longer to the child. Equal intervals of felt time then correspond to intervals of clock time that grow in geometric proportion, in which each interval is larger than the interval before by the same factor. Felt time therefore grows as the logarithm of age, so each doubling of age adds the same amount of felt time. Time seems to speed up with age, and the reason, on the proportional theory, is that each year is a smaller fraction of the time already lived, so each year feels shorter than the year before. Janet's theory is a claim of psychology, and I offer Janet's theory in this chapter as a claim of psychology, not as a claim of physics.

X. Summary

Time has a direction before entropy gives time a direction. Every reversed process is lawful, because the laws are symmetric in time. No reversed process is faithful, because a faithful reversal would perfect symmetry together with continuity. And the Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. The structural arrow is present in every single comparison, in the order of the possible, then the act, then the record. The structural arrow needs neither many parts nor a beginning of low entropy. The structural arrow is joined to the entropic arrow in every act of recording, because making a record raises the entropy of the instrument that holds the record. Physicists call three things time (geometric time, event time and evolution time), but the direction of time belongs to one of the three, event time, the process that the block universe leaves out.

The Epistemic Sandwich is a second argument for the structural arrow. The Epistemic Sandwich starts from knowing. Knowledge at the scale of observers is bound to the entropic arrow. Choice is real, so by the free will theorem the particle's response is open, and an open response cannot be faithfully reversed. The non-entropic arrow at the smallest scale orients the entropic arrow at the larger scales above the smallest scale. Experimenters have observed a violation of time symmetry in the weak interaction, and the violation is consistent with the requirement that time-reversal symmetry cannot be perfect. The choice is between a real arrow of time at every layer and no knowledge at all.



Chapter 25

Two Atoms in an Empty Universe

The Least Interaction, the Positions of Comparison, and Matter as an Aspect of Interaction

ABSTRACT

What is the least that an interaction needs, so that something has actually happened? This chapter works down from an empty universe to find the answer. One photon and one free electron are not enough, and one hydrogen atom is not enough, but two hydrogen atoms and the photon that passes between the atoms are enough. Interaction, comparison and signaling are one act under three names in the passing of that photon from one atom to the other. A reader can point, in that event, to each of the six intrinsics of comparison: the six features that every comparison has. I name each kind of particle by the position it fills in that event: the photon is the act of comparing, the electron is the thing compared, and the proton is the thing compared against. The principle of identity says that what no comparison can distinguish, in any way at all, is one thing and not two. The electron is on the content pole of the comparison and the photon is on the context pole, and the principle of identity, applied at each pole, means exclusion for electrons and addition for photons. Two beams of light pass through one another unchanged, and the reason the beams do so also shows why the act of perceiving cannot perceive itself. In a second empty universe that holds a head and a book, we find that

> time comes with interaction and not before interaction. And matter is the content of interactions stable enough to name and build with: matter is an aspect of interaction, and not a stuff beneath interaction.

I. An Electron Alone

Imagine a completely empty universe, and scale everything down to particles, starting from the simplest picture of particle physics, so that whatever is said about interaction can be said in the plainest physical terms.

Put into the universe one electron and one photon, and let the photon come to the electron. Imagine yourself as the electron: the simplest object, a point in space, in a space that holds nothing else except the single photon coming toward you. You can know that the rest of the universe exists only if other particles interact with you. Signals arrive only by way of other particles. As an electron, your knowing is very limited.

II. Why One Photon and One Electron Are Not Enough

A free electron is an electron that is not bound in an atom. A free electron cannot absorb a photon. If a free electron absorbed a photon, energy and momentum could not both be conserved. The photon can scatter off the electron (bounce off the electron), and the photon and the electron both go on changed. But a free electron has no internal states in which to keep a record of the change, and there is nothing else in the universe against which the change could register. So nothing in the universe marks the difference between before the scattering and after the scattering. In the sense of a permanent difference, there has been no interaction.

The anthropologist Gregory Bateson wrote of a difference that makes a difference, in his essay "Form, Substance, and Difference", which appears in his book *Steps to an Ecology of Mind* (Bateson, 1972). Bateson's difference

that makes a difference needs a third particle. Add a proton. Now the electron is related to the proton. The electron and the proton together form a hydrogen atom. An electron bound to a proton has states in which a change can be kept. A photon of the right frequency can now change the state of the electron relative to the proton, pumping the electron into a higher orbital, a state of the electron with more energy.

III. The Second Atom

One atom is still not enough. The relational view of space says that space is nothing over and above the relations among the things in space. The relational view is associated with the philosopher Gottfried Wilhelm Leibniz and with the physicist Ernst Mach. In a universe that holds only one atom, there would be no way, on the relational view, to know that any distance had increased, because the one atom is all that the universe holds to measure the distance against. The frequency of the photon must be right for the atom to absorb the photon, but in a one-atom universe the frequency of the photon has nothing to be measured against.

So we add a second hydrogen atom, and we let one atom be the emitter and the other atom the receiver. An elevated state of an atom is a state of higher energy, and the ground state is the state of lowest energy. The emitter drops from an elevated state to the ground state and emits a photon. The photon crosses space, and the receiver absorbs the photon and is pushed into an elevated state.

Two hydrogen atoms hold four particles: two electrons and two protons. The four particles and the photon are necessary and sufficient for interaction to be fully represented. I call the exchange of one photon between two atoms the two-atom exchange. Comparison is the act of finding whether two things are the same or different, in some respect. In the two-atom exchange, interaction, comparison and signaling are one act under three names. Chapter 2 states that comparison, interaction and rela-

tion are one act, named three ways. I call the statement of Chapter 2 the Root Tautology. The two-atom exchange is the Root Tautology at the smallest scale.

IV. The Six Intrinsics in One Exchange

Comparison has six intrinsics, six features that every comparison has. The six intrinsics come in three pairs: sameness and difference, content and context, and the subjective and the objective. Sameness and difference are the two possible verdicts of a comparison. The content is what the verdict is on, and the context is the respect in which the two things compared are two. The subjective is the side from which the comparison is made, and the objective is the side of what is compared. All six intrinsics are present in the two-atom exchange, and a reader can point to each intrinsic in the exchange.

The particles are the content of the exchange, and the otherwise empty universe is the context of the exchange. The sameness is that the receiving atom is still hydrogen after the exchange. The difference is that the electron of the receiving atom has gone from the ground state to an elevated state. When the exchange is taken from the side of the receiver, the receiving atom is the subjective and the emitting atom is the objective, while from the side of the emitter the two atoms trade roles. The photon travels from the emitter to the receiver, so the direction of the photon's travel tells the emitter from the receiver, and so the direction fixes the roles.

V. The Positions of Comparison

Every comparison has three positions: the act of comparing, the thing compared, and the thing compared against. When a carpenter checks the edge of a board against a ruler, the checking is the act of comparing, the board is the thing compared, and the ruler is the thing compared against. In the two-atom exchange, each kind of particle fills one of the three posi-

tions of comparison, and I name each kind of particle by the position it fills: the photon is the act of comparing, the electron is the thing compared, and the proton is the thing compared against. A naming by position is a definition: the naming assigns each term its meaning, and so the naming cannot fail.

The immanent, the omniscient and the transcendent are the three modalities: the three roles that the parts of every comparison play. The *immanent* is the act itself, as the act happens. The *omniscient* is what is laid out as structure. The *transcendent* is the stance from which an act proceeds, and the range of what is possible from that stance. A modality belongs to a thing only relative to a frame. A frame is the setting of a description in which a role is assigned. In the frame of the positions of one comparison, the photon is immanent as the act of comparing. The electron is omniscient as the thing compared. The electron is the unit in which the result of every comparison is reported. The proton is transcendent as the thing compared against.

Chapter 3 sets out three axioms from *An Immanent Metaphysics*, the book on which this book rests. The first axiom, Axiom I, reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". First, Axiom I says that the immanent (the act) is more fundamental than the omniscient, than the transcendent, and than the two together. Second, Axiom I says that the omniscient and the transcendent are conjugate: each can be perfected only at the expense of the other, and neither is prior to the other in definitional basis. By Axiom I, then, the electron (omniscient) and the proton (transcendent) are in conjugate relationship, and the electron and the proton are the two poles, or opposite ends, of the comparison. The electron and the proton carry opposite charges, while the photon, the act that mediates between the electron and the proton, carries no charge. The electron and the proton are conjugate positions, and not conjugate states: the electron's conjugate state is the positron, the antiparticle of the electron.

VI. Content Excludes, Context Adds

In the frame of the positions of one comparison, the electron and the photon are on the two poles of the partition of content and context. Content is local and exclusive: two contents keep one another out of one place. Context is non-local and composes: contexts combine with one another. The electron is content, because the electron is a thing compared. The photon is the act of comparing, and so the photon belongs with the framework within which comparison occurs: context.

The *principle of identity* says that what no comparison can distinguish, in any way at all, is one thing and not two. At the content pole, the principle of identity means exclusion: two instances of content of one kind cannot occupy one state. If two instances of content of one kind were in one state, we could not tell the two instances apart by any comparison, and so, by the principle of identity, the two instances would be one thing and not two. Physicists know exclusion as the exclusion principle, which the physicist Wolfgang Pauli stated (Pauli, 1925), and electrons obey the exclusion principle, as does every other fermion (the name for any particle that obeys the exclusion principle). At the context pole, the principle of identity means addition: context contributed twice in one state is one context of twice the amount. The reason is that, by the principle of identity, context contributed twice in one state is one context and not two, and because contexts combine, the single context carries twice the amount. Physicists know addition as Bose statistics, named for the physicist Satyendra Nath Bose, and photons obey Bose statistics, as does every other boson (the name for any particle that obeys Bose statistics). The periodic table shows electrons excluding one another, and the light of a laser shows photons adding in one state.

In physics the electron is the lightest charged fermion of electromagnetism, and the photon is the gauge boson of electromagnetism: the particle that carries the electromagnetic force. The electron and the photon

are complementary in the sense of this section: the electron is on the content pole, and the photon is on the context pole. The position conjugate to the electron's position is the proton's position, and the photon is the act that mediates between the two positions, so the electron and the photon are not a conjugate pair.

VII. Light Cannot Perceive Itself

Two laser beams can cross in space, and through the electromagnetic field alone the wavefronts of the two beams pass right through one another, unchanged. The beams pass through each other because the photon carries no electric charge: the photon is the act of comparing, and the act of comparing is not one of the things that the act compares. Light does act on light, through pairs of charged particles, but so slightly that no lamp or laser makes the effect large enough to see, so one beam of light cannot be used to watch another beam of light pass.

What holds of light holds of perceiving: the act of perceiving is not one of the things that the act perceives. I can perceive only what is the object of perception: I cannot perceive the perceiver, and I cannot perceive perception. Perception can perceive only one of the three (perceiver, perceived and perceiving): the perceived. We must therefore study the relationship itself, the relating of perceiver and perceived, since we cannot perceive the perceiver or the perceiving.

VIII. Time Comes With the Interaction

In a second empty universe we find that time comes with interaction and not before interaction. Begin again with a completely empty universe (to imagine an empty universe is already to assume a great deal, but take the empty universe as a baseline), and put two things in the empty universe: a perceiving capacity, say a head with an eye, and a book. Do not start the clock. In a completely static scene, nothing changes. So a carved head and

a carved book could not be told from a real head and a real book. In the static scene, the moment called the first and the moment called the second are indistinguishable. Now add time.

For the first moment to be told from the second, something has to change. Only three things are present: the head, the book, and a channel of interaction between the head and the book. The book is static, and no one can see what goes on inside the head, so the change must show somewhere in the flow between the head and something else. The difference between the two moments is established in perception, which is to say in comparison. Any difference found is a comparison, and any perception is a comparison. Time and comparison are therefore bound together, because to tell one moment from another is to compare, which is the telling of sameness from difference. To conceive time, one must already have conceived comparison, and to conceive comparison, one must conceive time as an aspect of comparison: time comes with the comparison, not before the comparison. Time is defined by perception, and perception does not happen in time.

Six things are intrinsic to comparison: sameness and difference, content and context, and subject and object. Perception is a kind of interaction and a kind of comparison. When the eye in the head takes in light from the book, the taking in of light is at once an interaction, a comparison and a perception. Comparison, interaction and perception are concepts of one form: whatever is inherent in one is inherent in the others. So time is an intrinsic of comparison and of interaction and of perception: one statement, made three ways.

Many people in physics have worked from a false premise, that time is a framework within which interactions happen to occur, and the false premise has led many physicists into trouble, among them the physicist Isaac Newton. Time is an intrinsic of interaction itself. Anyone who thinks about what time is has already assumed some interaction, and in the end the interaction of imagining.

Return to the two hydrogen atoms in the first empty universe of this chapter, the emitting atom and the receiving atom. For the receiving atom, the emitting atom is not real (real in this book's sense: given in an actual act of comparison) until the receiving atom absorbs the photon, and after the absorption the receiving atom presumes that an atom emitted the photon. In presuming that an atom emitted the photon, the receiving atom presumes that the emitting atom exists. The basis of all knowing is that we know only what we perceive. A thing can be presumed to be in the universe only after some interaction with the thing. Time comes with the interaction, and so do the objective and the subjective. Each of the three is an intrinsic of the interaction, and so none of the three comes before the interaction.

IX. Matter as an Aspect of Interaction

What happens when atoms come close shows how unlike a stuff matter is. A 'stuff', in this section, is a substance standing underneath interaction. When two atoms come close, the electron clouds of the two atoms overlap, where an electron cloud is the region around a nucleus in which the electrons of the atom can be found. An amplitude is the quantity that a quantum state assigns to each possible outcome, from which the probability of the outcome is computed. When quantum states combine, the amplitudes of the states add before any probability is taken. So the chances do not simply add. Where the amplitudes reinforce one another the chance goes up, and where the amplitudes cancel the chance goes down. Combining by amplitudes is a different kind of combining from setting two billiard balls side by side.

Phase transitions also show that matter is unlike a stuff. A phase transition is a change in the form of a material: water becoming ice, water becoming steam, or a metal becoming superconducting (able to carry an electric current with no resistance at all). In each phase transition, the relationships among the constituents of the system are reorganized. A con-

ductor pushed past its capacity to carry current shows a related limit. As the conductor heats, the resistance of the conductor rises, so the conductor heats faster still until the conductor melts or vaporizes. Every material has a threshold beyond which the heating of the material feeds itself: conductivity has limits.

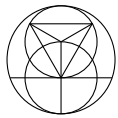
I start from comparison and not from substance, and a comparison is a comparison of interactions. There are only interactions, and no purely static things: what we call matter is the content of interactions stable enough that we can name the content, point to the content, and build with the content. Being the content of interactions does not make matter fleeting: the neutrons in a carbon nucleus have lasted since a star made the neutrons. But matter is what matter is as an aspect of interaction, and not as a stuff standing underneath interaction.

X. Summary

The least complete interaction takes two hydrogen atoms and the photon that passes between the two atoms. Interaction, comparison and signaling are one act in the passing of the photon from one atom to the other, and each of the six intrinsics of comparison can be pointed to in that act. The direction of the photon's travel fixes which atom is subjective and which atom is objective. In the frame of one comparison, each kind of particle is named by the position it fills: the photon is the act of comparing (immanent), the electron is the thing compared (omniscient), and the proton is the thing compared against (transcendent). The electron and the proton are conjugate positions with opposite charges, and the photon mediates between the electron and the proton and carries no charge. Electrons exclude one another and photons add, because the principle of identity means exclusion at the content pole and addition at the context pole.

Two beams of light pass through one another unaffected, because the act of comparing is not one of the things that the act compares. Light passing through light is the image of perceiving that cannot perceive it-

self. We must therefore study the relationship itself, the relating of perceiver and perceived. In an empty universe that holds only a head and a book, we find that time is an intrinsic of interaction and comes with the interaction. The objective and the subjective also come with the interaction. Matter combines by amplitudes and reorganizes in phase transitions. Matter is the stable content of interactions: an aspect of interaction, and not a stuff beneath interaction.



Chapter 26

What Can Be Modeled

Symmetry, the Classical, and What Cannot Be Copied

ABSTRACT

I define symmetry through comparison: symmetry is a sameness of content where there is a difference of context. Every use of symmetry in physics is an instance of that definition. To model a thing is to treat some aspects of the thing as irrelevant to the behavior of interest, and the irrelevant aspects are differences of context across which the content stays the same, so every model rests on a symmetry. The classical is then defined by one feature. When a model of a thing can behave as the original does for every purpose that matters, the thing can be virtualized, and a thing that can be virtualized is classical. When a thing cannot be copied without destroying the original, the thing is quantum. The no-cloning theorem of quantum physics states that no single operation can copy every unknown quantum state, and the theorem is the physical correlate of identity, which no copy can carry. So, in the sense of copying, self-identity is quantum and not classical. A quantum computer runs a reversible rule between preparation and readout, and at readout the computer gives a single outcome that is hard random, fixed by nothing that came before the outcome. A quantum computer has at its measurements, as a physical system, whatever the computer has that no program contains. So identity and embodiment are inherently tied. Public-key security against quantum computation also involves the distinction between what can be copied and what cannot. When someone perfects symmetry at the cost of

> everything else, the person makes a family of errors: calling time and space illusions, mistaking simplicity for elegance, and calling the world deterministic because the equations of our models of the world are deterministic. Finally, a knower finds more change or more changelessness depending on scale. To know persons, a knower needs continuity, as to know the world at large scales, a knower needs symmetry.

I. Symmetry Defined Through Comparison

A comparison is the act of finding whether two things are the same or different, in some respect. Every comparison has six intrinsics. The intrinsics are the features that belong to every comparison. The six intrinsics come in three pairs: sameness and difference, content and context, the subjective and the objective. The *content* of a comparison is what its verdict is on. The *context* of a comparison is the respect in which the two things compared are two. We hold the subjective context the same, so that the definitions concern only what is compared, and then four concepts can be defined from the intrinsics of comparison: continuity, discontinuity, symmetry and asymmetry (Chapter 2). *Symmetry* is a sameness of content where there is a difference of context: a constancy even in the face of a change of context. When a shape is flipped over, the shape looks the same: the flip is the change of context, and the shape is the content that stays the same.

A great deal of physics is about symmetry, in the sense of a sameness of content across a difference of context. The conservation of energy says that the total energy (the content) is the same from one moment to the next (the context). Physicists find the laws of physics to be the same here and there, now and later, and in that sameness the laws are the content, and the places and the times are the context. Physicists long assumed a further symmetry as well: parity, the sameness of a process and its mirror image. Then physicists found that the weak force breaks parity (Lee and Yang, 1956; Wu and others, 1957). The weak force acts in some kinds of radioactive decay.

Any physicist can take up a challenge: name a usage of the notion of symmetry that is not, implicitly, a sameness of content where there is a difference of context. Anyone who works through the cases will find that a sameness of content where there is a difference of context is the essential notion of symmetry.

II. To Model Is to Use a Symmetry

Every form of modeling depends on symmetry. To say that a thing can be modeled is to say that some aspects of the thing can be treated as irrelevant to the behavior we are interested in, and factored out as unimportant. Take a computer model of a ball thrown into the air. The model can leave out the color of the ball, because the path of the ball is the same whatever the color. The irrelevant aspects are differences of context: changes under which what we care about stays the same. And a symmetry is a sameness of content where there is a difference of context. So a model is a symmetry put to use. A model includes the content that matters, as the model of the ball includes the path, while the context that does not matter is free to vary, as the color of the ball is free to vary in the model of the ball.

III. The Classical Is What Can Be Virtualized

A model is a symmetry put to use, and the meaning of being classical is based on what a model is. When a model of a system can be built that behaves as the original does for every purpose we care about, the system can be *virtualized*. A classical system is a system that can be virtualized. The single defining feature of the classical is that classical things can be virtualized: classical things are modelable.

The test for the classical is a short list of questions. Can the thing be virtualized? Can the thing be modeled? Does the thing have aspects that can be factored out? When every answer is yes, the thing is classical.

When any answer is no, the thing is quantum. When the thing is quantum, something about the thing fundamentally cannot be replicated without destroying the original.

The classical world is the world of models, of the symmetries of laws, and of things that can be factored, replicated and virtualized. In classical physics and in computation, any flow of signal can be copied. A computer file, for example, can be copied bit for bit. We can call the rule that any flow of signal can be copied an *inverse cloning principle*, by analogy with the no-cloning theorem of quantum physics (Section IV), which states that no single operation can copy every unknown quantum state. In the classical world, everything can in principle be replicated, and modeling is exactly making a replica of the relevant aspects of a system.

IV. What Cannot Be Copied

No copy carries identity, in the sense of a single unitary consciousness. From inside, from the position of the one who chooses, a choice cannot be extracted, cannot be communicated in its specificity, and cannot be reproduced by any process other than the process that is the choice.

The no-cloning theorem of quantum physics states that no single operation can copy every unknown quantum state (Wootters and Zurek, 1982; Dieks, 1982). An unknown quantum state is a state about which the copying operation is given nothing in advance. The no-cloning theorem is the physical correlate of the fact that a copy cannot carry identity. The outcome of a measurement can be copied, and classical records are made of the outcomes of measurements, so classical records can be copied. Self-identity is therefore quantum and not classical, in the sense of copying, because a copy cannot carry identity, and the quantum is what cannot be replicated without destroying the original. Whether a thing has a self depends on whether the thing is a physical process making comparisons in the world, with an inside (a subjective interior), and not on what program the thing runs.

Quantum theory has symmetry too, like the classical world of models: the findings of quantum theory are discrete and symmetric. Quantum theory is built on symmetry groups, where a symmetry group is the whole set of changes that leave the laws the same. The continuity in quantum theory belongs to the formalism of the theory, that is, to the mathematics of the theory. The continuity lies in the smooth evolution of amplitudes. Amplitudes are the numbers a quantum state gives to the possible outcomes. A classical computer can reproduce the statistics of quantum theory, but by the direct method the cost grows exponentially with the number of entangled particles, so that each added particle multiplies the cost again. The known exception is circuits built from the Clifford gates, a limited set of quantum operations, because by the theorem of Gottesman and Knill a classical computer can simulate such circuits efficiently (Gottesman 1998). No deterministic model can reproduce the single outcome of a measurement itself, which nothing before the outcome fixes. The quantum world is the world of what cannot be replicated by a deterministic model: hard identity, uniqueness.

V. The Quantum Computer and the Single Outcome

In a quantum computer, the steps between preparation and readout are a reversible rule, where preparation sets the starting state of the device and readout is the measurement at the end. A reversible rule can be run backward as well as forward. Under a reversible rule, each state fixes the next state and also the previous state. Errors come in because the device is not alone: the environment interacts with the device, and the coherence of the device's state spreads into the joint system of device and environment. The coherence of a quantum state is the fixed relation among the parts of the state that lets the parts interfere with one another. Physicists call the spreading of coherence into the joint system *decoherence*. At readout, something more happens than the reversible rule and the errors: the state of the device does not fix the single outcome. The single outcome is hard

random: no prior specification fixes the outcome, however complete. Hard randomness is the remainder that the Incommensuration Theorem guarantees in any world where comparisons can be made. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Every comparison leaves a remainder that no prior structure fixes, because symmetry and continuity cannot both be perfected, and at the readout of a quantum computer the remainder is the single outcome. A quantum computer has at its measurements, as a physical system in the world, whatever the computer has that no program contains. So identity and embodiment are inherently tied.

VI. Identity and Public-Key Security

Public-key identity is the way a person or a computer proves who is speaking over a network. The owner keeps a secret key and publishes a matching public key. Anyone can check a message signed with the secret key against the public key. If identity rests on what cannot be cloned, one can ask whether any classical construction of public-key identity can be secure in principle against quantum computation. A classical construction is built from classical signals, and any flow of classical signal can be copied (Section III). Today's number-theoretic public-key schemes rest on hard problems about whole numbers, such as finding the prime factors of a large number. A quantum computer breaks the number-theoretic schemes by finding hidden periodic structure (a pattern that repeats at a fixed interval) in discrete groups (Shor, 1997). A lattice is a regular grid of points in a space of many dimensions, and a typical lattice problem is to find the point of the grid nearest to a given point. Lattice problems have no hidden periodic structure for a quantum computer to find, and the hardness of lattice problems comes from continuous geometry. The National Institute of Standards and Technology recently standardized lattice-based schemes for post-quantum cryptography (National Institute of

Standards and Technology, 2024). Post-quantum cryptography is cryptography designed to stay secure once quantum computers exist. Cryptographers hold the lattice-based schemes to be secure against quantum computation, because lattice problems lack hidden periodic structure. The security of the lattice-based schemes rests on the hardness of the lattice problems. Nobody has proven the lattice-based schemes secure.

VII. The Error of Perfected Symmetry

If one takes the notion of symmetry to a perfection, so that no moment of time and no place is special, one will want to generalize over time and space to obtain higher and higher symmetry. The greatest symmetry of all comes from having no dimensions and no structure at all. So those who perfect symmetry end up saying that even time and space, which are dimensions, are illusions. But time as a dimension exists. Time is also real as the order of acts. The order of acts is the order in which acts happen, one after another. Time does not become an illusion when a description drops time as the order of acts.

Those who hold that "elegance is everything" make the same error, the error of perfected symmetry, when they misunderstand elegance as simplicity and symmetry. Elegance should mean profound clarity. Simplicity and symmetry are one thing, and profound clarity is another. Under the mistaken reading of elegance, a physicist maximizes symmetry at the cost of everything else, and commits ever more to the mathematics and not to the causal (what causes what), because mathematics is already atemporal (without time).

When someone calls the universe deterministic because the equations of our models are deterministic, the person makes the same error of perfected symmetry. When the state at one moment fixes the state at every later moment, we call the equation that carries the state forward deterministic. Newton's laws are deterministic equations, and so is the equation for the smooth evolution of a quantum state between measurements. Our

models have determinism built in, and when we take determinism, a feature of our modeling, for a feature of the thing we describe, we are misled about what the thing is. No whole world can be completely deterministic, because a completely deterministic world would need symmetry and continuity perfected together, and the Incommensuration Theorem forbids perfecting symmetry and continuity together.

VIII. Symmetry, Continuity, and Scale

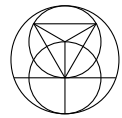
As a way of knowing, science is excellent for understanding the world and less useful for understanding ourselves and one another. The method of science depends on an assumption: the physical world has consistent patterns of causal relationship, so that the experiment I do today and the experiment you do tomorrow, done the same way, give the same result. When the result stays the same across a change of time and of person, the sameness of the result is a symmetry, in which the result is the content, and the time and the person are the context. Human beings change so much that underlying symmetries in human beings are hard to find. Understanding human process is less a matter of symmetry and more a matter of continuity, a sameness of content where there is a sameness of context. So understanding human process calls for a different way of knowing, outside what science and technology do.

One objection is that the natural world is constantly changing as well, and has its own continuity. The objection is true, and how much change and how much changelessness appear depends on scale. The farther we go into larger contexts, as in cosmology, the more slowly things change, measured against a human life. Planets move, stars change and galaxies turn in ways that repeat over the time we have to watch the sky. So the larger the scale, the more the symmetries of law appear as sameness of result from one time to the next. The farther we go toward the microscopic, the more change we see, up to change that is wholly original: each single finding (the result of a single measurement) comes as one whole outcome,

and nothing before the finding fixes which outcome the finding will be. And the farther we move from the external to the internal, toward the subjective, the less we understand change in terms of symmetry. The farther we move toward the subjective, the more we understand change in terms of continuity.

IX. Summary

This chapter has defined symmetry through comparison, as a sameness of content where there is a difference of context, and has shown that every model is a symmetry put to use. The classical is therefore what can be virtualized, and the quantum is what cannot be replicated without destroying the original. The no-cloning theorem is the physical correlate of identity. So, in the sense of copying, self-identity is quantum. Whether a thing has a self depends on the thing's being a physical process making comparisons with an inside, and not on the thing's program. A quantum computer gives a hard random outcome at readout, and the hard random outcome ties identity to embodiment. Public-key security against quantum attack involves the distinction between the periodic and the continuous (Section VI). When someone perfects symmetry at the cost of everything else, the person arrives at two claims, that time is an illusion and that the world is deterministic, and in both claims a feature of our models is mistaken for a feature of the world. Finally, scale sets the balance of symmetry and continuity in what we can know. To know persons, a knower needs continuity.



Chapter 27

The Size of Chemistry

Why a Short List of Particles Does Not Make Matter Simple

ABSTRACT

The list of the kinds of particle is short and closed, and some people conclude from the shortness of the list that the physics of matter has few degrees of freedom, that is, few independent ways in which matter can be arranged or can change. The conclusion that matter has few degrees of freedom is drawn at the wrong level, because the relevant levels lie above the particles. A rich variety of structure needs only a few kinds of part, combined in many ways. Chemistry, the first level above the particles, already allows a variety of molecules far beyond any reasonable implementation. The reduction of chemistry to quantum mechanics fails in a definite and exact sense: chemistry is made of nothing but electrons, nuclei and light, and what fails is the derivation of chemistry from quantum mechanics. Exact derivation is possible only for the simplest systems. In principle no description, however refined, contains everything that the description describes. No reduction from one science to the next is simpler than the reduction of chemistry to physics, so every reduction above chemistry is open to the question of whether a description at a lower level exhausts the thing at a higher level. A living cell is complex as well as complicated. I estimate the size of a full simulation of one cell, of a whole animal and of the living world, and the estimates show the scale of what a reduction would have to carry. Reductionism cannot account for life,

> while a person who understands a living system at the level of process can say true things about the system without a complete microscopic specification.

I. The Wrong Level

Some people raise an objection, which runs as follows. The microscopic structure of the universe is simple. The Standard Model of particle physics describes the microscopic structure well. The Standard Model is the theory of the known kinds of particle and of the forces that act between particles. Physicists may measure the numbers in the Standard Model more finely, but the microscopic structure is not infinitely intricate. Therefore, the objection concludes, there are not many degrees of freedom in which to represent things: not many independent ways in which the world can be arranged or can change.

My answer is that the objection is made at the wrong level, the level of the particles. The levels relevant to degrees of freedom lie above the particles. The list of the kinds of particle is short, and the list is closed. The few kinds of particle can make a long list of things. A rich variety of structure does not need many kinds of part. A rich variety of structure needs only a few kinds of part, combined in many ways.

II. Chemistry, the First Exemplar

Chemistry is the first example of a few kinds of part combined in many ways. At the level of chemistry, the question about degrees of freedom is how many different arrangements of atoms into molecules can be made. Every molecule has a topology: the pattern of bonds that join the atoms of the molecule. Each pattern is one point in the space of all possible molecules. The space of all possible molecules is vast. Organic chemistry is the chemistry of the compounds of carbon. Within organic chemistry alone, three researchers estimated that about ten to the sixtieth power

small drug-like molecules could exist (Bohacek, McMartin and Guida, 1996). Ten to the sixtieth power is a one followed by sixty zeros. Chemists have made only a vanishingly small fraction of the possible drug-like molecules. When molecules of any size are counted, the number of possible molecules grows without limit. The variety of possible molecular structures is far beyond anything that anyone can reasonably implement, by making the molecules or by representing the molecules.

Researchers in the field of metamaterials build materials whose behavior comes from the arranged microstructure of the materials. Metamaterials give some indication of how many things can happen once a structure goes even a little beyond the single molecule. If someone takes the topology of all of chemistry simply as physics, then physics contains the whole of that topology and is very intricate.

III. The Landscape of Energies

Quantum mechanics describes the states of all molecular structures, and of all the interactions among the structures, as members of a Hilbert space. A Hilbert space is the mathematical space in which quantum mechanics describes the possible states of a system. Taken by itself, the Hilbert space of the molecular structures is uniform: every part of the space is like every other part. The detail of chemistry is in the energies. Each arrangement of atoms has an energy, and so does each way in which arrangements act on one another. Taken together, the energies form the landscape of energies, in which the height of each place is the energy of one arrangement. Unlike the Hilbert space, the landscape of energies is not uniform. The landscape of energies is fantastically detailed. The hills of the landscape of energies are arrangements of high energy, and the valleys are arrangements of low energy. The dynamic fields of relationship in the landscape are the changing ways in which arrangements act on one another. Seen from outside, the hills, valleys and fields look like random structure.

IV. The Failure of Reduction, Stated Exactly

In this chapter, *reductionism* is the claim that complete understanding of the whole of chemistry reduces to complete understanding of quantum mechanics. In general terms, reductionism is the claim that the description at the lower level exhausts the thing at the higher level, which means that the description leaves out nothing of the thing. Chemistry shows a breakdown of reductionist thinking, and the breakdown is general, not special to chemistry.

Solving the equations of quantum mechanics exactly has a cost that grows exponentially with the number of electrons: each added electron multiplies the cost. Physicists have solved the hydrogen atom exactly. Physicists have computed the helium atom to extraordinary precision. For the molecules of real chemistry, every method in use is an approximation. Some methods start from first principles, the laws of quantum mechanics themselves, and leave a part out. The Hartree-Fock method, for example, leaves out the correlation of the electrons. The correlation of the electrons is the way the motion of each electron depends on where the other electrons are. Other methods contain a term that is chosen and not derived. Density functional theory, for example, includes the correlation of the electrons through a chosen term. In 1929 the physicist Paul Dirac wrote a paper on the quantum mechanics of systems of many electrons. In the paper Dirac said that the difficulty of deriving chemistry from quantum mechanics was practical: the underlying laws of the whole of chemistry are completely known, and the difficulty is only that the exact application of the laws leads to equations much too complicated to be soluble (Dirac, 1929). No one knows how to derive a great deal of chemistry from quantum mechanics without approximations like the Hartree-Fock method and density functional theory.

I state the kind of failure exactly. Chemistry is made of nothing but electrons, nuclei and light, and no one questions what chemistry is made of. What fails is the derivation of chemistry from quantum mechanics. In practice, exact derivation is possible only for the simplest systems, such as the hydrogen atom, while approximate derivation covers a great deal of chemistry and every approximate derivation stays approximate. In principle no description, however refined, contains everything that the description describes.

The phrase "except for the simplest systems" names the exception that defenders of reductionism point to, because exact derivation succeeds for the simplest systems. But the claim of reductionism is that a description at a lower level exhausts the thing at a higher level, and that claim fails even for chemistry, the standard case that defenders normally put forward as the model of a reduction. No reduction from one science to the next is simpler than the reduction of chemistry to physics, while every reduction proposed above chemistry is harder to carry out and harder to check, from the reduction of biology to chemistry on upward. So when the reduction of chemistry to physics fails, every reduction above chemistry is open to the same question, namely whether the description at the lower level exhausts the thing at the higher level.

V. Complicated and Complex

One can measure the complexity of a process by the size of the smallest program that duplicates the behavior of the process: the measure counts how much code the program needs to represent the process fully.

The size of the smallest program is a good measure for a machine. A car is *complicated*: a car has many parts, and every relation between the parts can be written down in advance. A living cell is *complex* as well as complicated. In a cell, differences too small to record grow until the differences decide what happens, so no finite description of the cell stays adequate for long. Chapter 16 calls the growth of small differences into de-

ciding differences microstate amplification. A program is a finite description, so the size of a smallest program is not the measure for a cell. For a cell I use the size of a *full-state simulation* instead: a program that tracks every part and every interaction of the cell for as long as the program can. The figures in the next section are rough sizes of full-state simulations.

VI. A Ladder of Scale

The figures in this section form a ladder, from an app on a phone up to all the life on Earth. The figures are my own order-of-magnitude estimates, each right to within a factor of about ten. I reason each figure from the count of parts and the count of interactions among the parts.

An app on a phone has somewhere between a few thousand and ten thousand lines of code. An app that has exotic functions may use perhaps a million lines of code in libraries: collections of code that many programs share.

By common estimates, an automobile needs about a hundred million lines of code to manage all of the automobile's systems. The systems include the fuel injectors, the spark plugs, the displays, and the heating and cooling.

A single cell probably holds millions of different chemistries, all acting on one another. Each chemistry is one kind of chemical process. In a full-state simulation of one cell, the software describes all the kinematics of the cell's chemistry: every motion of every part. A full-state simulation of one cell needs at least ten times the code of the car, and probably a few thousand times as much: somewhere around a billion lines, and perhaps as many as a few hundred billion. No one has yet done a full-state simulation, from the macroscopic to the microscopic, of every protein interaction in a specific cell. Researchers have built whole-cell models of a small bacterium (Karr and others, 2012) and of a minimal synthetic cell (Thorn-

burg and others, 2022), but neither whole-cell model follows every protein interaction one by one. There is a great deal about a cell that even the best scientists in the world do not yet know.

A whole rodent is at least a few billion cells. The cells are composed into tissues, and the tissues into organs. The organs contribute to systems (circulatory, immune, nervous, digestive), and each tissue, organ and system is in relation with the others. At around a billion lines each, a few billion cells give about ten to the eighteenth lines, before any interaction among cells is counted. When the interactions among tissues, organs and systems are counted, the figure is probably closer to ten to the twenty-fourth.

For all the life of the solar system, which is the life on Earth, a full-state simulation comes easily to ten to the thirty-third lines of code, and the figure is conservative: the figure errs on the low side. Prokaryotic cells are cells without a nucleus, such as bacteria. Three researchers estimated the number of prokaryotic cells on Earth alone at several times ten to the thirtieth (Whitman, Coleman and Wiebe, 1998). At around a billion lines per cell, the code for the prokaryotic cells alone exceeds ten to the thirty-third lines by a wide margin.

VII. What Reductionism Leaves Out

The ladder shows that reductionism cannot account for life, because the ladder shows how much a reduction of life would have to carry. Reductionism explains every whole wholly by its parts and by the earlier states of the parts. An explanation by parts and earlier states leaves choice out, because choice is a selection that no prior specification fixes (Chapters 13 and 14). The difference between complicated and complex is a difference in what a thing is, and not only a gap in our data. The Immanent Metaphysics offers process-level understanding, in which a system is understood as a process: partly thing, partly change, and partly what is neither thing nor change and is actually new (Chapter 8). Process-level under-

standing connects particle physics with biology, because a person who understands a system at the level of process needs no complete microscopic specification to say something true about the system.

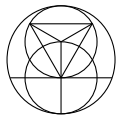
VIII. Summary

In this chapter I have shown that a short, closed list of the kinds of particle does not make the physics of matter small, because a rich variety of structure comes from a few kinds of part combined in many ways. Chemistry, the first level above the particles, already holds a space of possible molecules far beyond any implementation, while the energies of the possible molecules form a landscape of fantastic detail. The reduction of chemistry to quantum mechanics fails as a derivation, and not as a claim about what chemistry is made of: exact derivation is possible only for the simplest systems, and no description contains everything that the description describes. No reduction is simpler than the reduction of chemistry to quantum mechanics, so every reduction above chemistry is open to the question of whether the lower-level description exhausts the higher-level thing. A cell is complex, and not only complicated, so the measure of a cell is the size of a full-state simulation. The ladder of estimates runs from the phone app to the living world. The ladder shows the scale of what a reduction would have to carry. No description contains everything that the description describes, so every claim that a description at a lower level exhausts the thing at a higher level fails, and so understanding life calls for process-level understanding.

PART SIX

CORRELATION AND MEASUREMENT

Bell's theorem built from the ground up, read as a trilemma, and the measurement problem.



Chapter 28

Correlation Without a Cause

Bell's Theorem and the Wizard's Decks

ABSTRACT

Bell's theorem shows that nature contains correlations between distant events that no causal process can account for. Experiments since 1972 have confirmed the correlations. Since 2015, the experiments have closed the two main gaps in the tests, the loopholes of detection and of locality, together. In this chapter I build Bell's result from the ground up. I say what a polarization is, what a detector asks, and why the angle a photon carries can never be known. I then show why a shared past can supply only a limited amount of agreement between two distant records, and how far nature goes past that limit. A pair of enchanted card decks gives a picture of how strange Bell's result is: the two decks agree only when both decks are shuffled, and the decks can never be used to send a message. Last, the chapter states what the correlation is: real, found by bringing two records together, carrying no signal, and belonging to the context in which a pair of particles was prepared rather than to either particle of the pair.

I. What Correlated Means

Put simply, Bell's theorem says that nature contains correlations over long distances that no causal process can reproduce. By a causal process I mean a process by which influence travels through space and time at or below the speed of light. The correlations are observed after the fact, in the records of two detectors. The correlations cannot be explained by causation. Two records are correlated when the entries in one record go along with the entries in the other record more often than chance alone would give. We first need to see what is being correlated, and what is being correlated is the orientation of particles.

Particles have an orientation that can be measured. For a particle of matter the orientation is called spin. For a photon, the particle of light used in most Bell experiments, the orientation is called *polarization*. A useful picture is a top, or a ball, spinning in space: the ball has a north pole and a south pole, and the axis the ball spins on points in some direction. The picture of a spinning ball is only an aid, because a particle of zero extent has no inside to turn, and for that reason the way quantum mechanics uses spin is odd. To follow the experiment, a reader needs only the picture of an axis pointing in a direction.

II. What a Detector Asks

For an ordinary object, looking at the object does not change the object's orientation, but for a quantum object, the measurement aligns the object's orientation with the axes of the detector. Suppose a photon is polarized at 45 degrees to the axes of my detector: when I measure the photon, I get either vertical or horizontal, in the frame of the detector (the detector's own pair of axes), and nothing in between. The polarization at 45 degrees was an intermediate state. Relative to the detector's frame, the photon was

in a *superposition* of vertical and horizontal, a weighted range of the two outcomes rather than either outcome alone. The measurement collapses the superposition to vertical or to horizontal.

A detector measures one orientation at a time, and never orientation in general, so I have to set the detector to the single orientation that the detector is to test each time. I ask the detector "is the photon vertical?", and the detector gives me back a yes or a no, which is partial information: one bit, a single yes or no, about an angle that would take unlimited precision to state. The measurement turns the photon's orientation to the detector's axes, so the detector's answer is about the photon as the measurement leaves the photon.

I cannot measure the angle of one particle both ways, against two different settings of the detector. The measurement collapses an intermediate state, and the collapse happens by no fixed rule: which answer occurs, vertical or horizontal, is not determined. The Heisenberg uncertainty principle is one statement of the reason: a particle's position and momentum cannot both be pinned down perfectly at once. Symmetry and continuity are terms that this book defines in Chapter 2. Symmetry is sameness of content across a difference of context. Continuity is sameness of content within a sameness of context. In a quantum measurement, the content is the property probed. The context is the setting of the apparatus. A particle with every property fixed at once would make symmetry and continuity perfect together, and the Incommensuration Theorem states that symmetry and continuity cannot be perfect together in any event or comparison. (The Incommensuration Theorem: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity.) Suppose I measured the position of one member of a pair of particles and the momentum of the other member. Then I would learn both quantities for the pair's common origin, and so I would have perfect information about that origin. I do not have that perfect information. Creating correlated pairs of

particles is not a way around the uncertainty principle, because the limit holds even when both members of a pair are separated in time and in space.

III. The Hidden Angle Can Never Be Known

I call the angle of a photon's polarization, measured from the detector's vertical, the *polarization angle*. Physicists usually write the polarization angle with the Greek letter theta. The polarization angle represents perfect information, since stating the angle exactly would take unlimited precision. A measurement yields only partial information, vertical or horizontal, so the polarization angle will never be known, however good our measuring technology becomes. The uncertainty principle states the limit on knowing the polarization angle: the mathematics underneath quantum theory leaves no way to know the polarization angle. The uncertainty principle is a limit of access, a limit on what can be found out, and the limit is categorical: the limit marks the edge of the unknowable, and no improvement of instruments moves the limit.

The limit on the polarization angle shows what the quantum state is. The state of quantum mechanics is not a picture of a physical thing. The quantum state represents states of knowledge, in a definite sense: the quantum state is the record of what has been found, together with what a given setting of the detector makes possible. The record is the same for everyone who has the outcome. So the record is not anybody's private degree of belief. No hidden angle sits in the photon all along, waiting to be read. So the quantum state is not knowledge of a hidden angle. If the detector answers vertical, then what is known is that the photon is vertical. The act of measuring is an interaction. The measuring interaction makes one outcome actual, and the interaction updates the record of what can be known to that outcome.

IV. Pairs, and the Natural First Picture

In a Bell experiment, a source creates pairs of photons with the same polarization. One photon of each pair goes to a detector on the left, and the twin photon goes to a detector on the right, at whatever distance from the source the experimenters choose. The two photons start correlated, in the sense that the polarizations of the two photons are at the same angle, though nobody knows what that angle is. If both detectors ask the same question, and the photon on the right is found vertical, the photon on the left is found vertical as well. Afterward the experimenters can compare the logs of the two detectors, and the two logs match. In practice the match is not perfect, because there is always a little interference, but the experiments can be made good enough that the answers are consistent, and the detection can be timed at a definite interval after the pair is made.

The natural first picture is that the two photons are copies of one another, made by the same generator, each carrying the same definite angle from the source, so that each detector simply reads a copy of the same thing. The picture of two copies cannot be the whole story. Bell's theorem shows that there is no definite angle that the two photons carry from the source.

Trouble for the picture of copies first appears when the two detectors ask different questions. Set the detector on the right to ask "is the photon vertical?" and the detector on the left to ask "is the photon at 45 degrees from vertical?". The records of the two detectors are then not correlated at all. Whether the two detectors give the same answer is random, because, relative to each other, the two questions are exactly at the uncertainty limit.

V. What a Shared Past Can Supply

Suppose two later events both come from one earlier event at a source: the two later events can then be correlated because of what happened at the source, as two letters written from one dictation agree with each other. Explaining an agreement by an earlier shared event is called common-cause reasoning. Common-cause reasoning is how correlation is ordinarily explained.

Physicists have asked how much agreement a shared past can supply ever since 1935, when the physicists Albert Einstein, Boris Podolsky and Nathan Rosen described two particles prepared together and then separated, in order to argue that quantum mechanics is incomplete (Einstein, Podolsky and Rosen, 1935). The physicist John Bell answered Einstein, Podolsky and Rosen by fixing how much correlation a shared past can supply. Suppose each particle carries its answers with it from the start. Physicists call an answer that a particle carries from the start a *local hidden variable*. Suppose also that nothing done to one particle affects the other particle at once: then there is a limit on how strong the agreement between the results for the two particles can be, and Bell wrote the limit as an inequality (Bell, 1964). *Bell's inequality* is exactly the limit on how much correlation a shared past can supply. Quantum mechanics predicts that Bell's inequality is broken, and nature breaks the inequality.

Numbers show how far nature goes past Bell's limit. Set the detectors at in-between angles, and take four pairs of settings together. At any one pair of settings, a hidden-value model can match the observed correlation. In a hidden-value model, both particles share a hidden value. One correlation comes from each pair of settings, and the four correlations can be combined into one number, in the way that the physicists John Clauser, Michael Horne, Abner Shimony and Richard Holt showed in 1969 (Clauser and colleagues, 1969). A local model is a model in which each particle carries its answers from the source. In a local model, nothing done to one

particle affects the other particle at once. For any local model, the combined number is capped at 2. Quantum mechanics predicts that the combined number can reach two times the square root of two, about 2.8. The Bell tests measure the combined number, and the measured values exceed 2.

The Bell results are therefore more correlated than any definite polarization angle carried from the source could explain. Picturing a definite angle carried from the source assumes a kind of causality. The assumed kind of causality is inconsistent with the results actually observed. No assumption about what happened at the source can explain the correlations.

VI. Causality Has an Edge

One technical point is usually left unstated, and the whole result depends on that point: causality itself comes with a limit in time and space. There is an edge to what can be causally connected to what, and the speed of light marks that edge. Special relativity states the limit, and general relativity keeps the limit at every event. Relativity works only when there is a limit on the speed at which causal influence travels. So when we assume relativity, we assume a definite notion of causality, and Bell's theorem then shows that there are correlations that no notion of causality connected to relativity can account for.

General relativity's notion of causality is a perfected notion of causality. In general relativity, each event has a cone, called the light cone of the event. Everything inside the cone can be reached from the event, nothing outside the cone can be reached from the event, and the cone points one way. The light cone shows the character of causation itself, unbroken and one-way: unbroken is continuity, and one-way is asymmetry, so, in this book's terms, the character of causation itself is continuity with asymmetry. The Incommensuration Theorem allows the pairing of continuity with asymmetry to be carried all the way, to perfection, and in general relativity the pairing of continuity with asymmetry is carried all the way. If

a person thinks of causality in any way that is not derived from general relativity's notion of causality, that person is very likely to have an inconsistent notion of causality.

When Bell's theorem is read with the edge of causality in view, Bell's theorem says something sharp about the correlations. The correlations cannot be accounted for by any process signaling at or below the speed of light, and the correlations cannot be accounted for by anything that happened in the past of the two detections, one at each detector. The correlations exceed what any notion of causality can supply. So there are two choices: either we give up the claim that all change is causal, or we give up locality, the claim that all causal process is bounded by the speed of light. Bell's theorem has a deep connection to the statement that some things that occur cannot be understood in causal ways. The connection becomes visible only when the relationship between causality and the speed of light is drawn out: causality has an edge at the speed of light, and the correlations exceed what causality inside that edge can supply. I take up which of the two claims to give up, and why, in Chapter 29, where Bell's theorem is set out as a trilemma (a choice among three claims that cannot all be kept). In the Immanent Metaphysics, the short answer is to give up the claim that every change has a cause.

VII. The Wizard's Decks

I use a picture of two decks of playing cards to show how extraordinary the Bell correlations are. Say I have two ordinary decks of 52 playing cards, sitting side by side on a table. In the picture of the decks I am the wizard, and I cast a spell on the two decks. The cards look the same as they did before the spell. Everything about the cards is ordinary except for one special feature.

I give you one deck and keep the other. You get on a plane and fly to Paris. We each shuffle our deck with seven riffle shuffles. Seven riffle shuffles are enough to make the order of a standard deck about as close to

random as shuffling can get, as the mathematicians Dave Bayer and Persi Diaconis showed (Bayer and Diaconis, 1992). After shuffling, we each take cards off the top and write down what we see. My first card is the five of clubs, and I write the card down. My next card is the three of spades. In Paris, you shuffle your deck, take cards off the top and write down what you see. After the first ten cards, we each email our log to the other, so that neither of us can arrange a log ahead of time to match the other person's log. We compare the two logs, and the two logs are identical. Identical logs from two decks shuffled far apart are what is special about the enchanted cards.

You might object that I simply set up the two decks the same way before you took yours. Decks set up the same way in advance would give the ordinary kind of correlation. Seal a red card in one envelope and a blue card in another, shuffle the envelopes, and post one envelope to each of two cities. Whoever opens the red card knows that the other person holds the blue card, and nothing traveled between the cities, because the agreement was laid out when the envelopes were sealed. The enchanted decks are different: we each shuffle after we have parted, so nothing laid out in advance can make the logs match. Bell's theorem shows that the agreement between *entangled* particles (particles whose states became linked when the particles interacted directly, as Section IX explains) is of the deck kind and not of the envelope kind.

Now you try to use the decks to send me a message. You arrange your deck so that the ten cards you want are on top in the order you want. We call each other and agree to draw at the same moment. The two logs are completely different from each other. Any match between the two logs is random. The logs differ because you did not shuffle. The correlation works only if we both randomize our decks first. Decks that are perfectly shuffled are perfectly correlated. Unshuffled decks are not correlated at all. Between perfect shuffling and no shuffling, a less perfect shuffle gives a weaker correlation. The agreement appears only at maximum random-

ness, and so the agreement can never carry a message: a message needs an arranged deck, and an arranged deck gives no agreement between the decks.

In the Bell experiments, physicists use quantum mechanics to build something very like the enchanted decks. Each pair of particles is like one card drawn from each deck: one particle comes to my detector, and the twin particle comes to your detector. In the picture, the shuffle is the part that matters, because the shuffle corresponds to the fact that the particles carry no answer fixed in advance, just as a shuffled deck carries no order fixed in advance. Suppose I try to fix the answer by measuring a particle before the particle arrives at the detector. Then I destroy the entanglement on which the correlation depends, just as arranging a deck destroys the decks' agreement.

The picture of the decks stops at the detectors' settings, because the detectors have settings and the cards do not. In the real experiments a random number generator at each detector picks the setting at the last moment. No process at the source could then have the information, in advance, of which question each detector will ask. Nothing traveling at the speed of light could have that information in advance either. The last-moment choice of settings is what makes the test fair. When the two detectors happen to ask the same question, the two detectors always give the same answer. When one detector asks "vertical?" and the other asks "45 degrees from vertical?", whether the two detectors give the same answer is random. At the in-between angles the combined correlation reaches about 2.8, where every local account stops at 2.

VIII. The Experiments

Physicists first tested Bell's inequality in 1972 and again in 1982, and both times the physicists found the inequality broken (Freedman and Clauser, 1972; Aspect, Grangier and Roger, 1982; Aspect, Dalibard and Roger, 1982). The early tests left gaps, which physicists call *loopholes*: ways in

which a local account could escape the verdict of a test. The detection loophole is open when the detectors miss some of the particles, so that the particles counted might not be a fair sample of all the particles. The locality loophole is open when a signal at the speed of light could pass from one detector to the other before the second measurement is done. People said that something must be wrong with the experiments. So physicists did the experiments again, and tightened the experiments each time someone raised an objection. In 2015, three groups closed the loopholes of detection and of locality together, and all three groups found the inequality broken: one group worked in Delft, one in Vienna and one in Boulder (Hensen and colleagues, 2015; Giustina and colleagues, 2015; Shalm and colleagues, 2015). In 2017 and 2018 other groups chose the detector settings using light from distant stars and from quasars, the bright centers of very distant galaxies. Other groups chose the settings using the button presses of many thousands of people, and in every one of the tests of 2017 and 2018 the inequality was still broken (Handsteiner and colleagues, 2017; Rauch and colleagues, 2018; BIG Bell Test Collaboration, 2018).

After the tests of 2015 to 2018, one way is left for anyone who wants every outcome fixed in advance by local causes: superdeterminism. *Superdeterminism* is the claim that the settings the experimenters choose were fixed, from the beginning of the universe, in step with the particles being measured. Superdeterminism remains logically possible after the tests, but holding superdeterminism after the tests is very costly. The settings were drawn from the light of distant stars and quasars and from the button presses of many thousands of people, so a superdeterminist must give an account of the settings that is so extreme that Occam's razor applies. Occam's razor is the rule that, of two accounts of the same facts, the simpler account is to be preferred. The other way is to accept hard randomness as a fundamental feature of reality: some outcomes are fixed by nothing prior to those outcomes. I accept hard randomness.

IX. What the Correlation Is

The Bell correlation is real, and the correlation carries no signal. Each detector's record, taken alone, is hard random. No choice of setting made at one end changes the statistics recorded at the other end. Before either measurement, neither outcome can be predicted. The correlation appears only afterward, when the two records are brought together by ordinary means, at ordinary speed. The Bell correlation is correlation without the transmission of information, and the correlation is not an influence passing from one place to the other. Causation, which carries signals, stays bounded by the speed of light throughout.

If no signal passes, what is violated? The Bell results agree with locality in the sense that physics holds as established: no signal travels faster than light. What is violated is the expectation that a correlation as strong as the Bell correlation must have a causal account, either in the shared past of the two particles of a pair or in an influence between the two particles. The two particles do not communicate. The two particles share a feature that belongs to the context in which the particles were prepared together. The shared feature is located in neither particle.

In the Immanent Metaphysics, domain curvature is the name for a shared feature that is located in neither particle. The word "curvature" has a sense in the Immanent Metaphysics that is not the sense the word has in general relativity. A domain, in this book, is the whole of what can be compared by one kind of comparison, and the physical domain is the whole of what can be measured from within the physical domain. When everything about a domain is specified by specifying what is at each point, the domain is *flat*. When a domain has features of the relations among points that no point-by-point specification captures, the domain is *curved*. Every finite domain is curved, and the curvature has no location. Quantum nonlocality is how domain curvature shows itself: as correlation

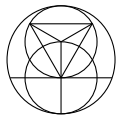
beyond what any local account contains. On the reading of nonlocality as domain curvature, nonlocality is not a mystery to be explained away. Local causality is the flat approximation to a domain that is not flat.

Whenever two particles interact directly with one another, as quantum systems, the states of the two particles become linked, and the two particles are entangled. Entanglement is not confined to pairs prepared in a laboratory. (When two systems interact only through a go-between that is in one definite state, the two systems do not become entangled.) A system becomes entangled with more and more of the world through interaction with the system's surroundings.

In the terms of Bell's own proof, Bell's theorem applies to local hidden variables, and Bell's theorem says nothing against hidden variables as such. In the classical picture, every outcome is fixed in advance by local causes, and the classical picture cannot be kept. The physicist David Bohm gave a theory of hidden variables that is non-local (Bohm, 1952). Bohm's theory reproduces the predictions of quantum mechanics and remains an available description. In the Immanent Metaphysics, Bell's theorem is one instance of the Incommensuration Theorem. Lawfulness is a kind of symmetry: lawfulness means the same physical law in every place and time. Locality in Bell's sense is a kind of continuity: locality in Bell's sense means that every influence passes from place to neighboring place. Lawfulness and locality cannot both be held perfectly, because symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. In this chapter, determinism is the claim that every state of a whole world is fixed completely in advance by earlier states. The case against determinism rests on the Incommensuration Theorem, not on Bell's theorem. The Incommensuration Theorem makes complete determinism impossible in any world, classical or quantum. What is left is causation without determinism: a world that is causal and open, in which some of what happens is not caused. Hard randomness is the part of what happens that is not caused.

X. Summary

A detector asks one question of a particle and receives a yes or a no. The angle the particle carries can never be known, because the uncertainty principle sets a categorical limit of access. The quantum state is the record of what has been found and of what a setting of the detector makes possible, and the record is the same for everyone who has the outcome. A shared past can supply only a limited amount of agreement between two distant records, and Bell's inequality is that limit. Nature exceeds Bell's limit, reaching about 2.8 where every local account stops at 2, as experiments from 1972 to 2018 have confirmed, with the main loopholes closed in the later experiments. Causality has an edge at the speed of light, and general relativity's light cone is causation perfected as continuity with asymmetry. The Bell correlations exceed anything that the edge at the speed of light allows causation to supply. The wizard's decks show the character of Bell's result: the decks agree only when both decks are shuffled, the decks cannot carry a message, and the agreement of the decks is not of the sealed-envelope kind. The correlation is real and carries no signal. The correlation is found only when two records are brought together. The correlation belongs to the context of the pair's joint preparation, which is to say to the curvature of the domain, located in neither particle. Bell's theorem is an instance of the Incommensuration Theorem. Once the Incommensuration Theorem has ruled out complete determinism, what remains is causation without determinism, with hard randomness as the part of what happens that nothing prior fixes.



Chapter 29

What Physics Gives Up

Bell's Theorem as a Trilemma

ABSTRACT

Bell's theorem is usually stated as a proof that no local hidden-variable theory reproduces the predictions of quantum mechanics. In a local hidden-variable theory, each particle carries its answers from its source, and nothing done to one particle affects the other at once. In natural philosophy, the study of what nature is, Bell's theorem is a trilemma about causation itself: a set of three properties of causation, of which at most two can hold together. The three properties of causation are universality, consistency and locality: every change is accounted for by causal process (universality); causal law is the same type of law everywhere (consistency); and all causal process is bounded by the speed of light (locality). A Bell experiment has two detectors, one at each end. When the settings of the two detectors are chosen independently, at most two of the three properties can hold. Bell's result has the same form as a theorem of distributed computing, which says that a data store copied across many machines cannot be consistent, available and partition tolerant at once. Most physicists keep universality and consistency, and give up locality. In the Immanent Metaphysics I keep consistency, and I keep locality in the sense in which physicists themselves keep locality, as no signaling, which means that no signal travels faster than light. I give up universality: *not* every change has a cause. This chapter shows why my choice describes Bell's result more exactly than the choice most physicists make, why

> neither hidden variables nor superdeterminism escapes the trilemma, and why giving up universality is the formal acknowledgment of hard randomness, an outcome that no prior specification fixes.

I. Two Ideals of Physics

Bell's theorem can be read in a deep way, as a theorem about causation. I hold the reading of Bell's theorem as a theorem about causation to be the most correct one. Physicists prefer two ideals for physics, and both ideals concern causation.

The first ideal is that the universe can be understood purely in terms of causation and things like causation. Algorithms are good examples of causal process, because an algorithm is conditional (what an algorithm does next depends on whether a condition holds) and sequential (its steps come one after another). Stated strictly, the first ideal is that every causal process is local (no causal influence travels faster than light), so that everything in the universe is described by local causation, and by local causation only.

The second ideal is that causation is the same everywhere. On the second ideal, there is a space, there are causal laws at each place in the space, the same causal laws apply in every place, all of the causal laws are local, and only causal laws apply.

Most people state loosely the two ideals of physics: only local causation, and the same causation everywhere. Most people say that theories of reality are consistent, that the same lawfulness holds everywhere in the universe, and that there is no "spooky action at a distance", which is how entanglement is often described. Entanglement is a correlation without a cause. In this chapter a correlation is an agreement between the records of two distant detectors. Entanglement cannot be treated as causation, and yet something entangles the two distant places where the two detectors are. Whatever entangles the two places cannot be understood in terms of local causal process, as the failure of the theories of hidden variables,

taken up in section IV of this chapter, shows. So we must give up one of the two ideals: either the symmetry of one causation applicable in every context, everywhere in the universe, or the idea that causation is strictly local. Bell's theorem shows that physics must choose between the two ideals.

II. The Trilemma Stated

Bell's theorem answers a question that the physicists Einstein, Podolsky and Rosen raised in 1935, when they described two particles prepared together and then separated, in order to argue that quantum mechanics is incomplete (Einstein, Podolsky and Rosen, 1935). In the Immanent Metaphysics I frame Bell's theorem as a *trilemma*: three properties of causation, of which at most two can hold together.

The first property is the *universality of causation*: every change in the world is accounted for by causal process, and only by causal process. The second property is the *consistency of causation*: the causal laws, wherever they apply, are the same type of law everywhere. The third property is *locality*: all causal process is local, bounded by the speed of light, so that no causal influence on a result comes from outside its past light cone. The past light cone of a result is the region of space and time from which a signal moving no faster than light could have reached the place and the moment of the result.

When the settings of the detectors are chosen independently of what is measured, Bell's theorem shows that *at most* two of the three properties of causation (universality, consistency and locality) can hold together (Bell, 1964). I use Bell's theorem in the form of the inequality of Clauser, Horne, Shimony and Holt (Clauser and colleagues, 1969). Chapter 28 describes the inequality. Clauser, Horne, Shimony and Holt combined the agreement of the two detectors' records over four pairs of settings into one

number. The number is at most 2 for every local account, and quantum mechanics and experiment take the number to about 2.8. Whichever two of the three properties we keep, we must give up the third.

Bell's result is also often stated with two terms instead of three. Each two-term form of Bell's result is the same trilemma, with one of the trilemma's three properties left unstated. Section I of this chapter named two ideals: consistency (the same causation everywhere) and locality (only local causation). Section I also stated a clause beside the two ideals, in the picture of the second ideal: only causal laws apply. The clause that only causal laws apply is universality, and in the trilemma universality is the third property, stated openly. Bell's result has one other common two-term form, which says that we must give up one of two things, either causality or locality. In the second two-term form, the word "causality" means universality: the claim that all change is causal.

III. The Same Form in Computing

Bell's trilemma has the same form as a theorem of distributed computing, which is the study of many computers that work together over a network. The computer scientist Brewer stated the theorem as a conjecture in 2000 (Brewer, 2000). Gilbert and Lynch proved the conjecture in 2002 (Gilbert and Lynch, 2002). A data store copied across many machines cannot guarantee three things at once: consistency, in which every read returns the most recent write; availability, in which every request receives a response; and tolerance of partitions, in which the store keeps working when the network between its machines is cut. Whoever builds the store can guarantee at most two of the three. Bell's theorem is the same kind of fact about causation: of universality, consistency and locality, at most two can hold together. What the two theorems share is their form: each theorem names three properties, of which at most two can hold together.

IV. Why Hidden Variables Do Not Escape It

Hidden variables were the first hypothesis that physicists offered to account for the correlations between entangled particles. On the hidden-variable hypothesis, something definite but unknown to us in the generator that creates the pairs accounts for the correlations. The hidden variables were meant to be the basis of the causation that creates the outcomes recorded at the two detectors. But when anyone actually uses a hidden-variable model to predict outcomes, the model still cannot reproduce the correlations without an influence that crosses the distance between the detectors faster than light. No local causation can produce the correlations, and so it does not matter what the hidden variables are: no choice of variables supplies local causation as a way of accounting for the correlations. The hidden-variable project failed as an account of the correlations by local causes. The failure of the hidden-variable project is the reason why physicists state Bell's result as "no local hidden variables".

One version of the hidden-variable project has hidden variables that are not local. The non-local version is the theory that the physicist David Bohm published in 1952 (Bohm, 1952). Bohm's theory does reproduce the predictions of quantum mechanics. To reproduce the predictions, Bohm gave up locality. In Bohm's theory the outcome in one laboratory depends on the setting of the detector in the other laboratory. Bohm's theory also still describes the whole world as fixed completely by one law, and a whole world that is fixed completely by one law would make symmetry and continuity perfect together. The Incommensuration Theorem states that symmetry and continuity cannot both be perfect together. (The Incommensuration Theorem: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity.)

V. What Physics Chose

Most physicists describe the choice that physics made in answer to Bell's theorem as keeping universality and consistency and giving up locality. Most physicists read a correlation without a cause as an influence that is not local. The physicists' reasoning ran roughly as follows: we do not understand causation very well, but we will continue to assume that causation is the same everywhere in the universe, because if causation differed from place to place, the idea of knowing the causation that holds in general, everywhere, would fail. Physicists also want a consistent notion of what it means to be scientific: we discover physical lawfulness, we describe the lawfulness causally, and so the lawfulness is understandable and can be offered as an explanation. Physicists must in the end explain everything by causation as a concept, to keep that notion of being scientific consistent. For most physicists, being scientific has come to mean keeping causation everywhere.

Keeping causation everywhere is a choice. Physicists make the choice from the standpoint of physics, which is the third-person view of pattern: the view of the world as structure, laid out to be held in view as a whole. Physicists keep the law the same everywhere. Physicists also confine the non-locality within one rule: nothing can be signaled by means of the non-locality. By the rule that nothing can be signaled, physicists keep one part of locality, which I call *no signaling*: no signal travels faster than light. The same law everywhere is a symmetry, and section IX of this chapter shows that locality is a continuity. So physicists keep the symmetry and give up the continuity. The result of the physicists' choice is symmetry with discontinuity, which is one of the two combinations the Incommensuration Theorem allows. The other combination the Incommensuration Theorem allows is continuity with asymmetry.

Most physicists found Bell's theorem very hard to accept. The reason is that Bell's theorem called into question an ideology underneath physics: a perfect notion of causation, perfectly applied. People want perfection in their theories, either perfection of generality (one causation that holds everywhere) or perfection of specification (every event specified by local causes). Bell's theorem required physicists to give up one cherished perfection or the other. Physicists argued for a long time over the proof of Bell's theorem and over its experiments, and the requirement to give up a cherished perfection is a large part of why the argument was so contentious.

VI. What the Immanent Metaphysics Gives Up

The Immanent Metaphysics describes Bell's result, and the experiments that confirm the result, more exactly than most physicists do. In the Immanent Metaphysics I keep the consistency of causation wherever causation applies. I keep locality in the weaker sense in which physicists themselves keep locality: no signaling, no signal faster than light. I give up universality: the claim that every change has a cause.

I also give up the *strong locality* of the trilemma, together with universality. Under strong locality, every correlation is carried by a causal process inside the light cone. Strong locality is a demand on causal process, and causal process is exactly what I no longer take to be universal.

The correlations that Bell studied are real, and the correlations are not signals. The record at each detector, taken alone, is a random sequence. No setting chosen at one end changes the statistics recorded at the other. The correlation appears only when someone brings the two records together, by ordinary signals that travel no faster than light. A correlation that appears only when two records are brought together counts as an influence only if one assumes that every change must be causal. Drop the

assumption that every change must be causal, and there is no influence to explain. There is a correlation, real and found by comparing records, that no cause produced.

Bell later stated a stronger result, from a condition he called local causality (Bell, 1976). Under local causality, each single result is produced by conditions local to its station, even when those conditions give only the odds of the result. With the stronger result, Bell excluded every account that satisfies the condition of local causality. The account I give in this book does not satisfy Bell's condition of local causality. I reject one premise of Bell's derivation: the premise that the joint result of two separated measurements is a fact independent of any place where the two records are brought together. A finding, the result that a comparison such as a measurement returns, is a fact for the comparer who makes the finding. A finding made far away becomes a fact for a comparer here only when the record of the distant finding arrives here. The record arrives at the earliest on the light cone of the finding, that is, carried by a signal that travels at the speed of light. So the joint pattern of the two results is a fact *only* where, and only when, the two records are brought together.

VII. Why Universality Is the One to Give Up

Locality in the sense of no signaling is a statement about the edge of the domain of comparison, which is the limit of what anyone can compare, or measure, from here. The Bell correlations stay within the edge, because the correlations carry no signal faster than light. Universality of causation is the demand that every change be pattern, and pattern is omniscient, in the sense of the Immanent Metaphysics: what is laid out as structure and held in view as a whole. Axiom I of the Immanent Metaphysics reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another" (An Immanent Metaphysics, aphorism 1.3-1). So the immanent, the act itself, here and now, in which change occurs, is more

fundamental than the omniscient, the pattern. No signaling concerns what a comparer here can compare with. A comparison made here is an act, here and now, and so is immanent. The universality of causation is a claim about pattern, and pattern is omniscient. So when the edge of the domain and the universality of causation conflict, I keep the edge of the domain, and I give up the claim that all change is causal.

When I give up universality, the trilemma is resolved in a form that the Incommensuration Theorem permits. Without universality there is a gap in what causal law covers, and a gap is a discontinuity. I keep the consistency of law, the same law in every place, and the consistency of law is a symmetry. So the resolution I give is symmetry with discontinuity, which is the same surviving combination that physicists kept, described without the word "non-local".

Rejecting universality means accepting that *not* every event has a cause that can in principle be identified, and that acceptance is the formal acknowledgment of hard randomness. Hard randomness is an outcome that no prior specification fixes, however complete.

VIII. Superdeterminism Asks for All Three

Someone who will not accept hard randomness has two deterministic accounts left. In a deterministic account, every state is fixed in advance by the states before it. The first account is Bohm's theory, which is not local. The second account is superdeterminism, in which the experimenters' settings were fixed, from the beginning of the universe, together with the particles' states, so that the settings lose the independence on which every experiment relies. Superdeterminism is the demand to have all three properties of causation at once. So superdeterminism is not a further way of resolving the trilemma. A superdeterminist gets all three properties of causation by making the experimenters' settings themselves part of what causation fixed in advance.

The Bell experiments constrain both Bohm's theory and superdeterminism, but the experiments do not exclude either account by logic. The Incommensuration Theorem shows that both accounts fail as accounts of a whole world. The reason is that a whole world completely fixed by one law makes symmetry and continuity perfect together. Superdeterminism has a second fault: superdeterminism describes choice as nothing but causation. Choice is a selection that no prior specification fixes. Choice is distinct from causation, and neither can substitute for the other. A deterministic formalism, such as Bohm's, remains available as a description. A physical theory has two levels: its findings are what its measurements return, and its formalism is its mathematics. The findings of a theory with a deterministic formalism carry the statistics of quantum mechanics, and no finding shows that the formalism is deterministic. When locality is kept as physicists keep locality, as no signaling, the randomness observed in quantum measurement is fundamental. The randomness is a feature of nature, and the randomness hides no deeper order.

IX. Bell's Theorem as an Instance of the Incommensuration Theorem

Bell's theorem is an instance of the Incommensuration Theorem, which is the general case. Bell's theorem concerns every account in which local causes fix the outcomes in advance, and states that no such account reproduces the correlations found between entangled particles. The Bell experiments described in Chapter 28 confirm Bell's theorem. Lawfulness is a symmetry: lawfulness is the same physical law in every place and time. Locality in Bell's sense means that no influence on a result comes from outside its past light cone. Locality in Bell's sense is a continuity, because the region between the two stations (the places of the two detectors) counts as context, and the influence that would cross the region counts as content. So Bell's theorem, the physical domain's own theorem, states for

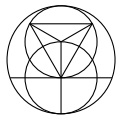
physics what the Incommensuration Theorem states for every comparison: symmetry and continuity cannot both hold perfectly together. I read the region between the two stations as context, and that reading is what makes Bell's theorem a case of the general theorem, so the identification depends on the reading.

Bell's theorem also shows that determinism and causation must be strictly distinguished. Causation cannot be perfectly applied, because Bell's theorem, with its experiments, shows that some changes are not accounted for by any causal process. Determinism is perfectly applied, but only in a mathematical model, where the relations, taken as relations, fix every state of the model. Causation is what actually happens, in time, and causation has gaps. Determinism is a property of a timeless description, and determinism has no gaps. In Bell's theorem the difference between causation and determinism becomes impossible to overlook.

X. Summary

Bell's theorem is a trilemma about causation, among three properties: universality (every change is causal), consistency (one type of causal law everywhere), and locality (all causal process inside the light cone). With independent settings, at most two of the three properties can hold. Bell's trilemma has the same form as the theorem that a data store copied across many machines cannot be consistent, available and partition tolerant at once. Hidden variables do not escape the trilemma. Local hidden variables cannot reproduce the correlations, and with his non-local hidden variables Bohm gave up locality and described a whole world fixed by one law, which the Incommensuration Theorem shows cannot be. Most physicists keep universality and consistency, and confine the non-locality within the rule of no signaling. In the Immanent Metaphysics I keep consistency and no signaling, and I give up universality. My choice is the more exact description of the same surviving combination that physicists kept, symmetry with discontinuity. The correlations are real and carry no sig-

nal. The joint pattern of two results is a fact only where the two records are brought together. A correlation counts as an influence only if one assumes that every change is causal. Superdeterminism is the demand for all three properties at once. The Incommensuration Theorem shows that superdeterminism fails as an account of a whole world. Giving up universality is the formal acknowledgment of hard randomness. Bell's theorem is one instance of the Incommensuration Theorem.



Chapter 30

Schrödinger's Cat and the Closure of Domains

The Measurement Problem

ABSTRACT

Quantum mechanics is the most thoroughly tested theory in physics, yet most people find its account of measurement makes no sense. The Schrödinger equation describes how a quantum state changes in time and has no chance in it, yet measurements have single, definite outcomes. In this chapter I answer the question of where the definiteness of a single outcome comes from, and apply the answer to Schrödinger's cat. A measurement is an interaction, and an outcome is definite relative to the interaction that makes the outcome. The quantum state describes possibility, and what physicists call collapse is the passage from the possible to the actual in the act of comparison, the act of finding whether two things are the same or different in some respect. No domain can be perfectly closed: no domain can be cut off from every relation with anything outside the domain. The reason is that existence with no possible interaction is indistinguishable from non-existence. Observation is any interaction that relates a system to what is outside the system, so observation is the condition for a system to have a determinate state for what is outside the system. So Schrödinger's cat shows no paradox: inside the box the cat is alive or dead, and for us outside the box there is not yet a fact about the cat, because a fact for us is a relation that our looking makes.

I. The Box and the Cat

Schrödinger's cat is a thought experiment, an experiment carried out only in the imagination, which the physicist Erwin Schrödinger described in 1935 (Schrödinger, 1935). Make a box sealed from observation, so that no one can see inside. Put a cat in the box, and a small amount of a radioactive isotope. A radioactive isotope is a substance whose atoms can each emit a particle at a moment that no one can predict. Put in a counter that detects whether the isotope has emitted a particle within one hour. If the counter detects an emission, the counter sends a signal to an apparatus that triggers a hammer, the hammer smashes a vial of poison gas, and the poison gas kills the cat. Choose the isotope so that the chance of an emission within the hour is one half.

Quantum mechanics predicts the probability of one half. Quantum mechanics does not say whether a particle was emitted in any one run of the experiment. We, the experimenters outside the box, do not know what happened inside the box until we open the box and look. Until we look, quantum mechanics describes the state of the whole system in the box as a superposition of cat alive and cat dead: a state that is not one of the two possible outcomes but a weighted range of both. Schrödinger's whole point in the thought experiment was to show how absurd it is to suppose that there could be a superposition of a live cat and a dead cat in the box.

Schrödinger set out the thought experiment in a debate among the physicists of his time about chance in quantum mechanics. The physicist Albert Einstein objected, in his own phrase, that God does not play dice with the universe. Einstein was objecting to the account that most physicists of Schrödinger's time accepted: that randomness is built into quantum mechanics, and that the random outcome is settled only when a measurement is made. On the account that most physicists accepted, the

wave state collapses at the moment of measurement: the wave state (the quantum state, pictured as a wave) jumps from a range of possible outcomes to the one outcome found.

Quantum mechanics is the most thoroughly tested theory in physics. Quantum mechanics is also the theory that most people understand least. The plain reason that most people understand quantum mechanics least is that the account that quantum mechanics gives of measurement does not make sense to intuition. The *measurement problem* is the question of where the single, definite outcome of a quantum measurement comes from. The measurement problem is a real philosophical puzzle, and not an invitation to mysticism.

II. What the Formalism Does

The formalism of quantum mechanics, its mathematics, describes how the state of a system changes in time by the Schrödinger equation, which is linear: any sum of two solutions of the Schrödinger equation is again a solution. The quantum state includes, for each possible outcome, a number called an *amplitude*, from which the probability of the outcome is computed. When a measurement is made, one of the possible outcomes becomes actual. Physicists then work out the probabilities of later outcomes from the outcome that became actual. So the fact that an observation was made means that physicists now describe the system differently.

Quantum mechanics does not predict the single outcome of a measurement in advance, and the Schrödinger equation has no chance in it: from the amplitudes at one time, the equation fixes the amplitudes at every later time. The equation fixes an amplitude for every outcome and singles out none, so the definiteness of an outcome does not come from the equation. On the standard reading of quantum mechanics, the definiteness of an outcome comes from the observer. In the Immanent Meta-

physics, definiteness comes from the act of measurement itself. A measurement is an interaction, and the outcome is definite relative to the interaction that makes the outcome.

III. Experiments Are Interactions

I take the results of physics experiments as physicists establish the results, by the methods of physics and on the evidence of physics, and I read each result as a case of the structure of comparison: the features that every act of comparison has (Chapter 2). Each experiment shows something about how observation, measurement and interaction are related.

Experiments are interactions: structured encounters between the domain of an observer and the domain being observed. Experiments are not neutral views of facts that were already there. The outcome of an experiment is the product of the interaction between observer and observed, and not the state of the observed system alone. In this book, an *observer* is any system that interacts with what is observed and records the result of the interaction. An observer does not need to be conscious.

If an outcome comes from an interaction, are things there only when we look at them? Is the moon there only because someone is looking at the moon? No. The moon interacts all the time with sunlight and everything near the moon, and relative to the moon's interactions the moon is definite. No person needs to look at the moon. What, then, counts as an observation? Does a cat looking at something count? Yes: any interaction counts. A Geiger counter is an instrument that registers a single passing particle as a click. A Geiger counter is as good a measurer as a physicist.

IV. Possibility and Actuality

The distinction between content and context applies directly to quantum systems. In every comparison, the content is what the verdict of the comparison is on, and the context is the respect in which the two things compared are two, as Chapter 2 sets out. In a quantum measurement, the content is the property of the system that the measurement probes, and the context is the setting of the apparatus: the places, times and orientations of the apparatus. In Schrödinger's box, for example, the content is whether the isotope has emitted a particle, and the context includes where the counter sits and the hour during which the counter watches.

In this book, the quantum state is the description of possibility. The quantum state is the range of outcomes that a given setting of the apparatus could yield, with a weight for each outcome. So a quantum state before measurement is a range of possible outcomes, and not one particular outcome. A measurement makes one outcome actual from the range of possible outcomes. A measurement does not reveal a particular outcome that was already there. The wave function is the quantum state written as a wave. Physicists call the change of the wave function at a measurement the collapse of the wave function, and I account for the collapse as a measurement making one outcome actual from the range of possible outcomes. *Collapse* is the passage from the possible to the actual in the act of comparison, after which the outcome is a new finding that supersedes the quantum state from before the measurement. Collapse is not a physical process that destroys a superposition.

V. No Domain Is Perfectly Closed

A *perfectly closed domain* would be a domain with no relations at all to anything outside the domain: a system completely sealed from every observation and every interaction from outside the system. The sealed box of Section I is an attempt at a perfectly closed domain.

Under the foundational axioms of the Immanent Metaphysics, no domain can be perfectly closed. The reason is that existence with no possible interaction is indistinguishable from non-existence. A thing that could never interact with anything would be indistinguishable from nothing at all. If a domain had no relations to anything outside the domain, the domain would therefore have no existence relative to anything outside the domain. Thermodynamics, the physics of heat, speaks of closed or isolated systems. A closed or isolated system in thermodynamics is an idealization: a system whose exchanges with its surroundings are small enough to neglect.

The prohibition of perfect closure shows what part observation plays in physical theory. In this book, *observation* is any interaction that relates a system to what is outside the system. Observation is the condition for a system to have a determinate state relative to what is outside the system. A determinate state is a state that is one definite way. Observation is not something added from outside to a system that was already complete without observation. A thing has a determinate state for another thing only if the two things have interacted.

VI. The Cat Resolved

Sections IV and V give two points: a measurement makes one outcome actual from a range of possible outcomes, and a thing has a determinate state for another thing only if the two things have interacted. Once the two

points are in hand, the sealed box shows no paradox. The sealed box shows what follows from the fact that being determinate is a relation. A thing is determinate only relative to what it has interacted with.

Inside the box, the state of the cat is settled. The atom, the counter, the vial and the cat interact with one another the whole time. Each of the four has a determinate state relative to the interactions among the four: the cat is alive, or the cat is dead. For anything outside the box, before an interaction across the wall, there is not yet a fact of the matter about the cat. The reason is that a fact for the outside is a relation between the box and the outside, and the relation between the box and the outside has not yet been made. The reason is not that a fact is hidden from us, and not that our looking would disturb the hidden fact. Our looking is the interaction that makes the relation between the box and us. Once the relation is made, the cat has a determinate state for us. Any interaction across the wall of the box will do: a counter is as good an observer as a physicist.

VII. One Account, Five Features

Five features of quantum mechanics are usually treated as separate puzzles: uncertainty, nonlocality, collapse, the determinacy of an outcome, and amplification. Each feature has its source in the structure of comparison.

Uncertainty is the bound on how sharply two conjugate quantities can be known together, which Chapter 18 states as the Heisenberg uncertainty principle. Position and momentum are one pair of conjugate quantities. Uncertainty has its source in the Incommensuration Theorem. The Incommensuration Theorem says that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In quantum theory every finding is exact and comes in whole steps with an exact label, which is symmetry with discontinuity, one of the two combinations that the Incommensuration Theorem allows (the other is asymmetry with continu-

ity). Quantum theory contains no continuous trajectory, no smooth path that a particle follows from moment to moment, and the missing trajectory shows as the bound on how sharply two conjugate quantities can be known together.

Nonlocality has its source in the curvature of domains. When everything about a domain is specified by specifying what is at each point of the domain, this book calls the domain *flat*. When a domain has features of the relations among its points that no point-by-point specification captures, this book calls the domain *curved*. In this book's sense, "curved" does not mean the curvature of space and time in general relativity. A local account is any account in which each particle carries its answers to every possible measurement from the start. In a local account, nothing done to one particle affects the other particle at once, however far apart the two particles are. Every finite domain is curved, and the curvature of a domain shows itself as correlation beyond what any local account contains. The physicist John Bell proved Bell's theorem in 1964, and Chapters 28 and 29 discuss the theorem in full. Bell's theorem is about correlations between separated particles, which are correlations of the kind that the curvature of a domain shows. Bell's inequality is the limit that every local account sets on the correlations between separated particles, and the tests of the inequality that began in 1972 and 1982 found correlations stronger than that limit (Bell, 1964).

Collapse has its source in the passage from the possible to the actual in the act of measurement (Section IV). The determinacy of an outcome has its source in the prohibition of perfect closure, under which a thing has a determinate state for another thing only if the two things have interacted (Section V). Amplification is the process by which one microscopic event, such as one passing particle, becomes a click in a counter. Amplification has its source in the complex systems through which microscopic differences grow to the scale of ordinary experience (Chapter 16, Section VII). A complex system, in this book, is a system in which differences too small to record grow into differences that decide what happens.

This account of quantum mechanics is a structural analysis of why the formalism of quantum mechanics takes the form it does, and given the structural analysis, the features of quantum mechanics are what one should expect. This account is not a reinterpretation of the formalism. This account does not replace quantum mechanics.

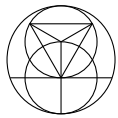
VIII. Summary

The measurement problem is the question of where the single, definite outcome of a measurement comes from, when the Schrödinger equation, which describes how the quantum state changes in time, has no chance in it. The answer is that a measurement is an interaction, and an outcome is definite relative to the interaction that makes the outcome. An observer is any system that interacts and records. A Geiger counter is as good an observer as a physicist is, and so is the sunlight on the moon. The quantum state describes possibility: the range of outcomes a setting could yield. Collapse is the passage from the possible to the actual in the act of comparison. No domain is perfectly closed, because existence with no possible interaction is indistinguishable from non-existence. So observation is the condition for a system to have a determinate state relative to what is outside the system. In Schrödinger's box the cat is alive or dead relative to the interactions inside the box, and for us outside the box there is not yet a fact about the cat, because a fact for us is a relation that our looking makes. Uncertainty, nonlocality, collapse, determinacy and amplification are five consequences of one structure, the structure of comparison.

PART SEVEN

COMPUTATION AND THE REAL

Why the world is not a simulation, what a theory of everything leaves out, and why a mind cannot be uploaded.



Chapter 31

The World Is Not a Simulation

The Argument from Continuity

ABSTRACT

Could a computer of unlimited size and speed run a program that is indistinguishable from this world? At first sight, the question is about computers. Yet the question is about what exists. The Principle of Identity says that two things are the same thing when no test, even in principle, could show a difference between them. So the question becomes whether any test can be defined, even in principle, that tells a discrete simulation from a continuous world. A discrete simulation is built of separate units, such as bits, while a continuous world has no smallest step of space or time. The obvious tests (repetition, simultaneity and parallel load) do not decide the question. Two other tests do. The first test comes from the mathematics of listing: everything a computation produces can be listed, while every list leaves out some values of the continuum, the unbroken range of values that a continuous quantity can take. The second test comes from randomness: a computation has no hard randomness of its own, and the world has hard randomness. Hard randomness is an outcome that no prior specification fixes, however complete. The arena of space and time is continuous, as this book derives and as every measurement of the arena shows, and so no discrete computation is the world. Section XI shows how the continuity of the arena is compatible with the discreteness that physics also finds. Section XII shows why the first per-

> son, the self that knows, cannot be listed, and Section XIII shows why a simulated world that contains real subjects is a region of the one world that runs the simulation, and not a second world.

I. A Question About What Exists

There is a long debate about whether this universe is, at bottom, digital. A digital universe could be written down as bits (ones and zeros), the way a hologram can be stored in a file. The other side of the debate holds that the universe is more like the continuous space that physics describes.

The debate comes down to one plain question. Imagine a computer of any size you like, with a processor of unbounded speed and memory without limit, and write a program for that computer that simulates the physical universe and obeys every law of general relativity, Newtonian mechanics, quantum mechanics and thermodynamics. Could the simulation be made indistinguishable from this reality? In the simulation question, continuity means the infinite smoothness of space and time. If the simulation could be made indistinguishable, then continuity would be an illusion, or at least an unnecessary assumption. If the simulation could not be made indistinguishable, then something about reality is deeper than any simulation. Something about reality would then be more basic than anything that discrete logic alone can represent. Discrete logic is the logic of separate symbols, such as one and zero.

The simulation question appears to be about computers, but the simulation question is about the nature of what exists. The answer is a claim about the structure of reality itself. The simulation question is also a doorway into the Immanent Metaphysics. The Immanent Metaphysics is the philosophy on which this book rests. A reader who works through the simulation question practices the kind of abstraction that the rest of the Immanent Metaphysics requires, and along the way the reader establishes a result whose argument is complete in itself.

II. The Principle of Identity

Metaphysics has a principle known as the *Principle of Identity*: what cannot be distinguished must be the same. The tradition of the philosopher Gottfried Wilhelm Leibniz calls the Principle of Identity the identity of indiscernibles. If no conceivable test, even in principle, could tell two situations apart, then there is no reason to count the two situations as two.

The Principle of Identity applies directly to the simulation question. Suppose a simulation were so perfect that no conceivable test could show a difference between the simulation and a continuous reality. Then the simulation and the reality would not be two kinds of thing, and so the hypothesis that we live in a simulation would be idle: neither true nor false, but empty. The hypothesis would be empty because the hypothesis would claim a difference that no one could ever find. So the simulation question becomes this question: can a test be conceived, even in principle, that tells a simulation from the real world?

The answer depends on whether a distinction between a simulation and a continuous reality can be drawn in principle, and the answer does not depend on the instruments we have today. If a distinction in principle exists, then a simulation and a continuous reality are different kinds of thing, whether or not anyone can yet build the instrument that would show the difference. A simulation might carry so many digits of precision that no one could observe its graininess (the steps between its values) in a lifetime, and the error of the simulation might lie below anything any conceivable instrument could detect. The size of the error does not matter to the simulation question. The Principle of Identity requires only a test whose logic is sound. The test must be one that can be defined, whether or not anyone can carry the test out now. If a sound test can be defined, the distinction between simulation and reality is real.

III. Tests That Do Not Decide

Several candidate tests come to mind first. Each candidate test that fails narrows the list of criteria that could tell a simulation from reality. After each failure, we look for a more basic criterion. The first candidate test is repetition: perform the same action over and over, and see whether the results are perfectly identical, as the results of a computation are: a computation run twice from the same start gives identical outputs for identical inputs. In a continuous world, by contrast, no two moments are the same. The passage of time by itself makes every moment differ from every other. Repetition is not a decisive test, because the designers of a simulation who knew of the test could build pseudo-random variation into the program: variation that a fixed rule produces and that only appears random. The small differences that a tester looks for would then be a feature of the program, and not a defect in the program.

The second candidate test is simultaneity. Many things happen at once, independently of one another. What happens in an ant colony goes on independently of what happens on a stock exchange, and a vast number of processes run at once throughout the universe, each apparently independent of the others. Simultaneity is not a decisive test either, because a simulation can use parallel computation: a simulation can run many threads (separate streams of computation) at once, and so model the independence of simultaneous processes.

The third candidate test is the sheer load of parallel work. The universe holds an enormous number of processes at every scale, from the fluctuations of the vacuum to the gravitational interactions of clusters of galaxies, and a simulation would have to compute all of those processes at once. A computer would need more parallel processing than anything now conceivable to model even one cubic centimeter of ordinary matter at quantum resolution. The size of that load is a limit of practice. The Prin-

ciple of Identity requires a distinction in principle. A limit of practice does not distinguish a simulation from reality in principle, so we have to look further for a decisive test.

IV. Randomness

A simulation runs by algorithm: by step-by-step procedures that turn inputs into outputs. The numbers a computer produces that look random are, for the most part, *pseudo-random*: produced by a fixed rule from a starting number, so that the same start gives the same sequence in the same order. *Hard randomness* is an outcome that no prior specification fixes, however complete. A computation has no hard randomness of its own, because the rule and the starting state of a computation fix every output, and so a computation can imitate hard randomness but cannot produce hard randomness.

The difference between pseudo-randomness and hard randomness gives a test in principle. Hard randomness exists in nature. The single outcome of a quantum measurement is the plainest instance of hard randomness. A computation cannot produce hard randomness, so a simulation running on its own rule cannot replicate the hard randomness of nature. The simulation's imitation bears some trace, however faint, of the procedure that generated the imitation. A simulation could instead draw hard randomness from a physical process in the world that runs the simulation. The simulation's randomness would then be the randomness of the world that runs the simulation, carried through the computation, and not a randomness of the simulation's own.

The Immanent Metaphysics says more than that nature has hard randomness: the Incommensuration Theorem shows that hard randomness must exist. The Incommensuration Theorem is about comparison. Comparison is the act of finding whether two things are the same or different, in some respect. The content of a comparison is what the verdict is on. The context of a comparison is the respect in which the two things compared

are two. Symmetry is sameness of content across a difference of context, and continuity is sameness of content within sameness of context. The Incommensuration Theorem shows that symmetry and continuity cannot both be perfected in any comparison, and so no complete prior specification of a world can fix every outcome in that world. Section XI states the Incommensuration Theorem in full. The world has the capacity to surprise, in outcomes that no prior specification fixes. The capacity to surprise is exactly what a simulation cannot supply from within itself. I decide the simulation question by two arguments. The argument from randomness is one, and Sections V to IX give the other argument, from continuity.

V. The Binary Hypothesis and the Division of Space

Consider a system that amplifies phenomena at extremely small scales, to see whether, at bottom, something is a one or a zero. If reality can be modeled as ones and zeros, then reality can be built from binary numbers, and all of the information in reality is digital. Binary numbers are numbers written with only the digits one and zero. I call the claim that all the information in reality is digital the *binary hypothesis*. On the binary hypothesis, one could conceive of a vast data file holding the complete state of the universe at one moment: a definite, finite number of bits. The universe would then evolve by some rule that turns the file for one moment into the file for the next moment.

Whether the binary hypothesis is true depends on one question: is reality, at bottom, smooth or granular? Bits are discontinuous in their own nature: each bit is either one or zero, with no value between one and zero. Space, taken as continuous, is different. Take any interval of space and split the interval in half, then take one half, split that half again, and go on without end: in the geometric concept of space there is no smallest distance. A simulation must represent space by finitely many data points, and however many points the simulation uses, there are always gaps

between neighboring points. A simulation comes closer and closer to continuity as it uses more and more points, but the simulation never arrives at continuity.

Suppose the infinity that fills this universe is infinite in the continuous sense, with infinitely many intermediate states between any two states: then no computation by discrete logic can represent the whole of that infinity, for a basic reason that Section VII gives. The tests considered so far are repetition, simultaneity, parallel load, randomness, the division of space and the binary hypothesis. Every one of those tests comes down to one question: is the universe continuous or discrete? Observation alone does not answer whether the universe is continuous or discrete, but the answer follows from a principled argument at the level of mathematical structure, which Sections VI to IX set out.

VI. The Planck Length

A reader may raise an objection to Section V, from the Planck length. The *Planck length* is a very small length, about 1.6 times ten to the minus thirty-five meters, fixed by the Planck constant, the speed of light and Newton's constant of gravitation. Many physicists take the Planck length as the scale below which the ideas of space and distance used by present physics stop applying. Could the Planck length make the binary hypothesis true after all? If space has a smallest unit, perhaps reality is already a grid of discrete points, with no continuity for a simulation to reproduce.

The answer depends on the difference between a limit on measurement and a graininess in what exists. First, the Planck length is not a measured smallest length. No measurement has found a least step of length at any scale yet probed. A gamma-ray burst is a brief flash of very high-energy light. A distant gamma-ray burst sends out light of very different energies together. The light of every energy arrives together. So if space has a grain that slows light in proportion to the light's energy, the grain is smaller than the Planck length, as the astronomers Abdo and oth-

ers reported in 2009. Second, the Planck scale marks a bound on what any measurement made from inside the world can resolve. A bound on measurement is not a graininess in what exists. The Principle of Identity says that what cannot be distinguished must be regarded as the same, but the Principle of Identity does not say that what cannot be measured must be regarded as discrete. The question is whether the difference between the continuous and the granular exists, not whether we can measure the difference.

VII. What Can Be Listed and What Cannot

The decisive argument comes from the mathematics of infinity. Begin with the number pi, the ratio of the circumference of a circle to the circle's diameter. The decimal expansion of pi never ends and never repeats: 3.1415926535 and onward without end. No finite string of digits, however long, is pi. Yet pi can be specified as the result of a finite algorithm applied without limit. A short program will produce any digit of pi on demand. The real numbers are the numbers that mark the points of a continuous line, pi among them. Most real numbers are not like pi. A collection can be listed when its members can be set out in one order, one for each counting number, with none left out. Every finite description is a finite string of symbols, and programs, like all finite strings of symbols, can be listed one after another, while the real numbers cannot be listed. So almost every real number has no finite description at all. In a continuous space, a physical quantity can take a value that has no finite description. The quantity then carries information that no finite representation contains. A simulation can store only finitely many digits of the value. The simulation's approximation may be astonishingly close, but the approximation is not exact.

The mathematician Georg Cantor proved in 1874 that the real numbers cannot be listed, and Cantor's diagonal argument (1891) shows why. Suppose someone claims to have listed every real number between zero and

one, first, second, third, and so on. Build a new number, digit by digit, down the diagonal of the list: the new number's first digit differs from the first digit of the first number on the list, the new number's second digit differs from the second digit of the second number, and so on. The builder never chooses 0 or 9 as a digit of the new number, so that the new number has only one decimal expansion. The new number differs from every number on the list in at least one digit, so the new number is not on the list. The continuum is the whole of the real numbers, the unbroken range of values that a continuous quantity can take. So no listing reaches the continuum: every list of real numbers leaves some real numbers out. Cantor's result is a fact about listing. A fact about listing is all that the simulation question needs. There are two kinds of collection: the collections that can be listed (the counting numbers, all finite strings of symbols, all programs that could ever be written), and the continuum, which no list contains in full.

Now apply Cantor's result to a computer. A computer works with discrete, countable states: the computer's memory is a finite number of bits, and the computer computes in a finite number of steps. An idealized computer has unlimited time and memory, and even an idealized computer produces at every stage an output that can be listed. There is a list of everything the idealized computer could ever produce. Every list leaves out values in the continuum, so no encoding in listable form contains the whole continuum, and in a continuous universe there are phenomena that no prior knowledge encoded in discrete logic can predict. So a simulation diverges from the continuous world in principle, and not merely in practice. The listing argument is short: two premises and a conclusion. First, a continuous quantity can take values that are missing from any given list. Second, discrete computation produces only outputs that can be listed. So there is a structural and permanent gap between discrete computation and a continuous quantity: the gap is a matter of mathematical impossibility, not of engineering difficulty.

Someone may object that no instrument can carry out the listing test. The objection is correct but irrelevant, because the Principle of Identity requires only that a distinction can be drawn in principle. The distinction is drawn as soon as someone shows that the logic of continuous mathematics differs from the logic of discrete computation, and Cantor's diagonal argument shows exactly that difference, with the full rigor of mathematics.

VIII. The Butterfly Effect

A finite approximation might still seem negligibly different from a continuous value, because a simulation could compute billions of digits of pi and so attain a precision far beyond any practical measurement.

Some systems are sensitive to their initial conditions, and physicists call that sensitivity the butterfly effect. The butterfly effect shows why no precision is ever enough in principle. In a system that is sensitive to its initial conditions, an arbitrarily small disturbance eventually produces differences that can be seen at ordinary scales. Weather is the classic case of a sensitive system. The meteorologist Edward Lorenz studied the sensitivity of weather in 1963. In a sensitive system the error does not stay small. The error grows until the error determines the behavior of the system. Suppose a simulation cuts its values off at some finite precision, however fine: then the simulation eventually departs from the continuous world, in every system that is sensitive to its initial conditions. Given enough time, the difference becomes visible. The butterfly effect is the mechanism by which the gap between the finite and the infinite becomes a gap between two different worlds: the world that the simulation computes and the continuous world.

IX. The Arena Is Continuous

The argument so far shows that no discrete computation can be a continuous world. I also derive that the context of every comparison, the *arena* of space and time, is continuous. The arena is space and time taken as the context of every physical comparison.

The derivation is short. A least step of context would be a smallest possible change of context: a difference of context with no sameness within the difference. Sameness and difference are inseparable aspects of every comparison, and so a difference with no sameness within it cannot occur. So there is no least step of context: between any two settings of context there is a third setting. Measurement agrees with the derivation: every measurement of the arena made so far shows the arena to be smooth, down to the bound that the gamma-ray burst observations of Section VI set.

Because this book derives the continuity of the arena, the conclusion of Sections V to VIII can now be stated without condition. The arena of space and time is continuous, and discrete computation produces only what can be listed, so no discrete computation is the world. The argument from randomness (Section IV) has the same conclusion, that no discrete computation is the world: the world has hard randomness, and a computation has none of its own.

X. The Laws of Physics Are Laws of the Continuum

Someone might propose that reality is built lawfully from a seed, the way an organism develops from its DNA: the seed is given, and the whole system builds itself from the seed. The seed is the initial conditions of the universe, and every later state develops from the seed by the laws of physics. The question is then whether the laws by which initial conditions evolve into later states can be written with discrete functions. The other

possibility is that the laws need continuous functions. Put another way: can the universe be modeled with the mathematics of countable sets (sets whose members can be listed), or does the universe need the mathematics of the real numbers, whose information cannot be written in zeros and ones? The laws of physics are evidence that the universe needs the mathematics of the real numbers, as physicists currently understand the laws. Physicists write the Schrödinger equation of quantum mechanics, the field equations of general relativity, and Maxwell's equations of electromagnetism in continuous mathematics, as laws of the continuum.

Quantum mechanics shows how the continuous and the discrete are joined in physics. A quantum measurement yields one definite outcome: the particle registers at this detector or that one, with this spin or that one. For some quantities, such as spin and the energy of a bound system, the possible outcomes are themselves discrete. The wavefunction, the amplitude associated with the system, is a continuous function. Between observations, the wavefunction changes continuously, as the Schrödinger equation describes. Someone writing a program can choose a discrete rule that reproduces the statistics of the wavefunction's evolution to any finite precision, and deterministic programs reproduce the statistics of quantum outcomes routinely. A discrete model cannot be the continuous evolution itself. Each single outcome also has a character that no deterministic model contains: the outcomes of a program are fixed by the program, while the outcome of a quantum measurement is fixed by nothing prior to the measurement. The formalism of quantum mechanics alone does not show that a quantum outcome is fixed by nothing prior to the measurement, but the Incommensuration Theorem does show that the outcome is fixed by nothing prior, and so the outcome is hard random. The mathematics of the continuum is the structural foundation of quantum theory, and that continuous foundation is evidence that the things the discrete findings of physics are about take part in the continuum.

XI. How Continuity Stands with Discreteness

Physicists find discreteness as well as continuity: quantum numbers, conserved charges, and one detector clicking and not the other. The Immanent Metaphysics states exactly how continuity and discreteness are related.

In every comparison there is sameness or difference in what is compared (the content of the comparison), across sameness or difference in the respect in which the two things are two (the context of the comparison). There are therefore four relations, one for each pairing of sameness or difference of content with sameness or difference of context. *Symmetry* is sameness of content across a difference of context. A shape flipped over that looks like the same shape is a case of symmetry: the flip is the change of context, and the shape is the content that stays the same. *Continuity* is sameness of content within sameness of context: within one unbroken stretch of context, the content does not jump. *Asymmetry* is difference of content across difference of context, and *discontinuity* is difference of content within sameness of context. The *Incommensuration Theorem* is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Of the ways of joining the four relations in pairs, exactly two pairings can be perfected: continuity with asymmetry, and symmetry with discontinuity.

Continuity with asymmetry is the structure of process: something persists (the continuity), and the persisting thing goes somewhere (the asymmetry). The wavefunction evolving between observations has the structure of process. The amplitude of the wavefunction varies smoothly, and no two points of the wavefunction's evolution are the same. Symmetry with discontinuity is the structure of the countable: something is

invariant (the symmetry), and the invariant thing comes in units (the discontinuity). The measured outcome has the structure of the countable, and so do the quantum number and the conserved charge.

Because continuity with asymmetry and symmetry with discontinuity can both be perfected, the continuity argued for in this chapter agrees with the finding that physical content comes in discrete units. Continuity belongs to process and to the context in which comparison happens. Discreteness belongs to the countable content found in comparison. A domain is the whole of what can be compared by one kind of comparison. Within any finite domain, continuity and discreteness are both present, each to a finite degree, with a continuum of trade-offs between the two. A discrete simulation of the whole world would need symmetry with discontinuity to be perfected across the whole world, process included, which is to say that a discrete simulation of the whole world would need continuity to be absent from the world entirely. The arena is continuous (Section IX), so no discrete simulation can be the whole world.

XII. Symmetry, Continuity, and the Self

The difference between the listable and the unlistable is an instance of the difference between symmetry and continuity. Symmetry is the sameness of content under changes of context. Symmetry has an affinity with the finite, with countability, and with existence. To exist is to have a place in a pattern, as structure. The binary number system is built on the distinction between one and zero, and has symmetry in its very construction. In character, computation is symmetry with discontinuity: exchangeable symbols, discrete states, and the same rule wherever the computation runs.

Continuity is the property by which every point joins its neighbors without gap or jump. Continuity is deeper and less reducible than symmetry, in the specific sense that continuity cannot be listed: no procedure lists the whole continuum, and Cantor's theorem is the statement of that fact. The modalities of the Immanent Metaphysics are three roles, the im-

manent, the omniscient and the transcendent, and in the modal alignment of the Immanent Metaphysics, symmetry is omniscient and continuity is transcendent. The omniscient is the role of what is laid out as structure and held in view as a whole, and the transcendent is the role of the stance from which an act proceeds, and of the range of what is possible from that stance. Objectivity has its basis in symmetry, while subjectivity has its basis in continuity.

Because objectivity has its basis in symmetry and subjectivity in continuity, there is an intimate connection between two relationships: the relationship between symmetry and continuity, and the relationship between self and reality, the knower and the known. Subjective context is continuous: the self persists without a break. Subjective content is discontinuous, in the sense of difference of content within sameness of context: thoughts and perceptions vary discretely while the self persists. So the first person, the self as knower, cannot be listed, because the context of the first person is continuous and no list contains the continuum in full.

XIII. A Simulated World Is a Region of the One World

The simulation hypothesis is the proposal that this world is a computation, run in a more fundamental world. Once a person sees that interaction is primary, the simulation hypothesis arises naturally. If reality is made up of interaction, and is not a container given in advance, it is natural to wonder whether reality is in some sense constructed, and whether reality has the character of a computation. But the simulation hypothesis still depends on a container outside the simulation: a "real" reality that runs the simulation. In the Immanent Metaphysics the assumption of an outer container is false, because there is no domain that contains every domain. Interaction is primary, and the notion of a container, physical or computational, is secondary. I treat relation, interaction and comparison

as one act, named three ways. Relation is more fundamental than the things related, and more fundamental than any container the related things are in.

The argument from randomness shows the same thing more precisely: a simulated world with real subjects is not a world apart from the world that runs the simulation. A deterministic computation has no hard randomness of its own, so a simulated world must take its randomness from one of two places: a pseudo-random rule inside the computation, or a physical source in the world that runs the computation. An indexing of states is a numbering of the states in order. The arrow of time is the asymmetry between a determined past and an open future. If the simulated world's randomness is pseudo-random, the simulated world has an indexing of states and no arrow of time. So the simulated world has no subjectivity, because of the *bound triple* (Chapter 15): hard randomness, the arrow of time and the subjective are distinct, inseparable and non-interchangeable, and none of the three can be present without the others. If a simulation runs only on a pseudo-random rule, the simulation is a program running on a machine, with no one inside. Suppose instead that the simulated world draws its randomness from a physical source in the world that runs the computation: then the simulated world's randomness is the randomness of the running world. The simulated world's arrow of time then follows the simulated world's randomness. The simulated world's subjectivity is shared with the running world as well, because the three members of the bound triple are inseparable, so none of the three can come from the running world without the other two. A simulated world that contains real subjects is therefore not a separate world. The simulated world is a restricted region of the one world in which the simulation runs.

In 2003 the philosopher Nick Bostrom gave a probabilistic argument for the simulation hypothesis. Bostrom counts each simulated world as a separate world with its own observers. So Bostrom concludes that observ-

ers in simulations far outnumber observers outside simulations. The bound triple shows that the counting step is wrong: the observers in any simulation that has observers at all are observers of the one world.

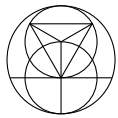
XIV. Summary

The simulation question is a question about what exists. The Principle of Identity says that two things are the same when no test could show a difference between them, so the simulation question becomes the question whether any test can be defined, even in principle, that distinguishes a discrete computation from a continuous world. Repetition, simultaneity and parallel load are not decisive tests. The designers of a simulation can imitate repetition and simultaneity, and parallel load is a limit of practice. The Planck length is not decisive either, because the Planck length is a bound on what measurement made from inside the world can resolve, and not a measured graininess of space.

Two tests are decisive. The first decisive test is listing: everything a computation produces can be listed, and Cantor showed that every list leaves out values in the continuum. Any finite truncation leaves an error, and through the butterfly effect the error grows into a visible difference. The second decisive test is randomness: a computation has no hard randomness of its own, and the world has hard randomness. The Incommensuration Theorem shows that hard randomness must exist. The arena of space and time is continuous, because sameness and difference are inseparable in every comparison, and every measurement of the arena shows the arena to be smooth. The laws of physics are laws of the continuum. So the world is not a discrete computation, and the difference is one of kind, not of engineering.

The continuity established in this chapter is compatible with the discreteness that physicists find. Continuity with asymmetry is the structure of process and context. Symmetry with discontinuity is the structure of countable content. By the Incommensuration Theorem, both pairings can

be perfected. Within any finite domain, continuity and discreteness are both present, each to a finite degree. A discrete simulation of the whole world would need continuity to be absent from the world entirely, and the arena is continuous, so no discrete simulation can be the whole world. The first person cannot be listed, because the first person is continuous in context, though discontinuous in content. A simulated world with real subjects in it is a region of the one world that runs the simulation.



Chapter 32

The Incompressible Residue

Wolfram's Physics and the Limits of Any Omniscient Theory

ABSTRACT

The goal of the Wolfram Physics Project is to derive every observable phenomenon from a simple computational rule applied to starting conditions. I use information theory to find what a program like the Wolfram Physics Project can achieve. Information theory measures the information in a thing by the length of the shortest description of the thing. When a rule is run on its starting conditions, the output carries no more information than the rule and the starting conditions together, so a complicated output from a short rule and short starting conditions is compressed into the rule and the starting conditions. A theory is useful because of the compression. Three questions follow for any program like the Wolfram Physics Project: how closely an output must match an observed phenomenon, how a match is kept from being chosen after the fact, and where the information in the rule itself comes from. The third question has no answer inside any theory laid out wholly as structure, because every theory laid out wholly as structure contains, as a given, a residue of information that cannot be compressed. The residue is hard randomness. Hard randomness is an outcome that no prior specification fixes, however complete. The Incommensuration Theorem forbids two poles: perfected determinism, in which every state is fixed in advance by earlier states, and pure randomness, in which nothing at all is accounted for. So every actual world holds causation together with the incompressible residue.

> Causation is the regularity of relations that science finds. The entrance of hard randomness is inseparable from the moment at which hard randomness enters. The entrance is just as inseparable from the first-person standpoint from which the entrance can be observed. Hard randomness, its moment and its standpoint make up the bound triple: hard randomness, the arrow of time, and the subjective stance.

I. The Program

A *purely omniscient theory* is a theory laid out wholly as structure. In this book the omniscient is the role of what is laid out as structure, such as a pattern, a map or a record: whatever can be held in view as a whole and shared. A purely omniscient theory includes only what is structural, repeatable, and stated in the third person, the way a calculation is stated. Computation, in its character, is the model case of a purely omniscient theory: a computation is a rule, a starting state, and a sequence of states that follows from the rule and the starting state. Within a purely omniscient theory, what are probably the most interesting and deepest questions can be neither asked nor answered. Two of the deepest questions arise in this chapter: where a rule's information comes from, and how knowledge is had at all. The Wolfram Physics Project is named after Stephen Wolfram, who set out the project in a book (Wolfram, 2020). So far as I can tell, the Wolfram Physics Project is purely omniscient theory: theory laid out wholly as structure.

The project's program can be stated with four names. Call the structure of the computational rule the *rule*. Call the rule's initial conditions the *starting conditions*. Call the complicated state that the rule produces from the starting conditions the *generated state*. Call any complex thing we observe in the world, such as a snowflake or a galaxy, an *observed phenomenon*. We have a large class of observations of the form "this rule, run on these starting conditions, leads to this complicated generated state". We might be tempted to infer from that large class of observations that every

observed phenomenon is itself the outcome of some special selection of rule and starting conditions, and that our task, as scientists, is only to find which rules and starting conditions lead to generated states resembling each observed phenomenon. Together, the two parts of the tempting inference become a Theory of Everything: a single theory from which every observed phenomenon follows. As near as I can tell, the tempting inference is the overall research program of the Wolfram Physics Project. The other parts of the project fit into the research program in various ways.

II. Information Is Conserved Under Deduction

When I am presented with a program like the Wolfram Physics Project, my first inclination is to think in terms of information. In this chapter, the information in a thing is measured by the length of the shortest description of the thing. Does the generated state carry any more information than the rule and the starting conditions that produced the generated state? Insofar as the generated state is the deterministic result of the rule and the starting conditions alone, with no outside information added, the information in the generated state cannot exceed the information in the rule and the starting conditions taken together. In descriptive names, the inequality is:

$$\text{information}(\text{generated state}) \leq \text{information}(\text{rule}) + \text{information}(\text{starting conditions})$$

In the inequality, the information of a thing is its *algorithmic information content*: the length of the shortest description of the thing. One example is a long string made of one symbol repeated over and over. The long string has a short description, the symbol and the number of repeats, and so the long string has little algorithmic information content. The starting conditions include the number of steps for which the rule is run. The inequality holds up to a constant, and the choice of description language fixes the

constant. A description language is the language in which the descriptions are written. The phrase 'up to a constant' means that the left side can exceed the right side by no more than the constant.

Deduction is reasoning from premises to what the premises imply. A computation is reasoning of the same kind as deduction: the computation runs from a rule and starting conditions to a generated state. Deduction conserves information. The conservation of information under deduction is inherent in the idea of mathematics itself. If deduction did not conserve information, the notion of a valid inference would be inconsistent, because a valid inference would then produce, from its premises, something that its premises did not contain. So, insofar as the notion of a theory is well defined, a rule and its starting conditions have a deterministic output that carries no more information than the rule and the starting conditions together.

The conservation of information under deduction is why theory is useful. Where an observed phenomenon is approximately the same as some generated state, even a large amount of apparent information in the phenomenon can be compressed into the rule and the starting conditions. The rule and the starting conditions are much smaller than the phenomenon. We use a theory to replace complex unknowable things, such as the phenomena, with simpler knowable things, such as the rule and the starting conditions. The replacement is the compression that makes a theory worth having. So, in the Wolfram Physics Project, the program is to find, for every observed phenomenon, some generated state that resembles the phenomenon closely enough. The compression is better still if all the generated states come from one rule, run on varying starting conditions, and in that case the one rule can be regarded as the laws of nature: the general idea under which all observable phenomena are compressed best. A Theory of Everything is the claim that a single rule, with varying starting conditions, compresses every observed phenomenon there is.

III. Three Questions

A program like the Wolfram Physics Project requires that every observed phenomenon be matched by a generated state from a discoverable rule and starting conditions. An unmatched phenomenon would show that the set of rules is incomplete. Then the rules and starting conditions would not be a Theory of Everything. Three questions follow from the requirement that every observed phenomenon be matched.

First: how closely must a generated state match an observed phenomenon before we may say that the phenomenon is itself produced by some analogue of the rule and the starting conditions behind the generated state, and does not merely resemble the generated state?

Second: where a generated state does resemble an observed phenomenon, on what basis do we hold that the resemblance was anything other than selected after the fact, from among the endless choices of rule and starting conditions available?

Third: even if some small set of rules accounts well for a wide range of observed phenomena, where does the information in that small set of rules come from? Section IV takes up the third question, and Section V takes up the first two questions, about how close a match must be and about choosing a match after the fact.

IV. The Incompressible Residue

The third question, the deep one, has no answer within any purely omniscient theory. Every purely omniscient theory contains some class of information that is wholly incompressible: a "just so" starting point, one with no reason given. The theory requires the "just so" starting point as a given, and nothing in the theory produces the starting point. A theorist

can move the incompressible information from the phenomena into the rule. But no theory can produce the incompressible information from nothing, because deduction conserves information.

However many iterations of theory building are attempted, an incompressible residue remains: something that no theory predicts. As the number of unmatched phenomena shrinks, each later iteration is more arbitrary and less supportable than the iteration before. Compression can never be perfect: no compression can reduce all observable information to zero bits. The incompressible residue is *hard randomness*: an outcome that no prior specification fixes, however complete. A quantum measurement's single outcome is the plainest case of hard randomness. Even complete prior information gives the probability of each outcome, and nothing more.

The Incommensuration Theorem shows, from the side of comparison, that an incompressible residue always remains. In this book a comparison is the act of finding whether two things are the same or different in some respect. Every measurement is a comparison. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Determinism is the claim that every state is fixed in advance by earlier states, everywhere and always. Perfected determinism, as a program, would perfect symmetry (the same law carrying every state to its successor everywhere and always) together with continuity (every state specified without gap). Determinism can be stated only against its opposite pole, perfected nondeterminism, which is *pure randomness*: an outcome with nothing at all accounted for, which would perfect asymmetry together with discontinuity. The Incommensuration Theorem forbids both poles. So determinism and pure randomness are idealizations, extremes in the strict sense, and neither is real. Every actual world holds, between the two poles, causation (the regularity of relations that science finds) together with a residual that no theory can compress, and the residual is hard randomness. The information argument of Section II agrees with the Incommensuration Theorem on one exclusion. The

excluded world is a world in which nothing is left over beyond what a prior specification fixes. The information argument states the exclusion from the side of theory, and the Incommensuration Theorem states the exclusion from the side of comparison.

V. A Match Is Not Understanding

Even a very good match is not, by itself, an indication that we understand the phenomenon that a generated state matches. There are probably many other phenomena that the rule behind the match would not match so well. We cannot always tell whether two observed phenomena belong to one class or to two different classes. The ways in which a phenomenon can vary can depend sensitively on factors that are wholly unknown. The evolution of a phenomenon can be highly nonlinear: a small cause can have a large effect. Microscopic perturbations, disturbances far too small to observe, can introduce new information into a system everywhere at once. When microscopic perturbations introduce new information into a system everywhere at once, physicists call the behavior sensitive dependence on initial conditions, which the meteorologist Edward Lorenz studied in the weather (Lorenz, 1963). In a model where every state is fixed in advance by earlier states, there is no room for the new information that microscopic perturbations bring in, and so determinism, as distinct from causation, is not a good basis for modeling most actual processes. Causation is the regularity of relations that science finds. Determinism, by contrast, is the further claim that every state is fixed in advance by earlier states. Causation and determinism must not be run together.

I also think that the drive to have things be "nice and neat" distorts, too much, how scientists select and define the single rule. Social and psychological factors matter more to how science is defined than is usually accounted for. When scientists negotiate which rule is the rule, the outcome can be more a matter of the positions people hold than of fact.

VI. Theory, Practice, and the Bound Triple

Theory is held in the third person, as a calculation is. Practice is held in the first person, as an experiment is. At some level, theory must resolve to practice: what is calculated in theory must at some point come down to what someone does and observes in practice. Practice is inherently different from theory, even though, by theory, practice and theory "should be" the same. At best, one question remains unanswered: where the information in the rules comes from. In the best case, the people who hold a theory also recognize that, within theory, the question has no answer.

My overall impression is that the Wolfram Physics Project is being oversold, and that some displaced disappointment lies in the project's future. The project's promise of explanatory power will always seem to be just around the corner: hauntingly suggestive, and unsatisfying. The "flowers" are the generated states that look like some observed phenomenon. The flowers are distracting, because the flowers do not resolve the question of how knowledge is had at all. In physics, the question of how knowledge is had becomes a question about two ways of knowing: do we know things by experiment and observation, made in the first person (the immanent), or only by static calculation, held in the third person (the omniscient)? The immanent is the role of the act itself, as the act happens here and now. The omniscient is the role of what is laid out as structure.

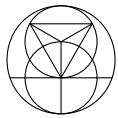
Whichever answer is right, three things are inseparable. The first of the three inseparable things is the entrance of hard-random information, information that no prior specification fixes. The second of the three inseparable things is the particular moment at which the hard-random information enters. The third of the three inseparable things is the subjective standpoint, the first-person viewpoint, from which the entrance can be observed as having occurred, as a fact. The entrance, the moment and the standpoint are the *bound triple* (Chapter 15): hard randomness, the arrow of time, and the subjective stance. The arrow of time is the structural

asymmetry between a determined past and an open future, present in every comparison. The subjective stance is the stance from which every comparison is made. The three members of the bound triple are distinct, inseparable, and non-interchangeable: each member is other than the other two, each occurs only together with the other two, and each does work that neither of the other two can do. A purely omniscient theory is laid out in the third person, as structure with no direction of its own and no stance. Structure with no direction of its own has no arrow of time, and structure with no stance has no subjective stance, so the bound triple is forever beyond what anyone can know by means of a purely omniscient theory. The incompressible residue of Section IV is hard randomness, the first member of the bound triple. So the entrance of the residue is inseparable from the moment in which the residue enters and from the first-person standpoint from which the entrance can be observed as having occurred.

VII. Summary

This chapter has set out what any Theory of Everything built from a rule and starting conditions can achieve. Deduction conserves information. So a rule run on its starting conditions produces an output with no more information than the rule and the starting conditions together. When a phenomenon matches a generated state, we can compress a large amount of apparent information in the phenomenon into a much smaller rule and starting conditions, and the compression is what makes a theory useful. The program raises three questions: how close a match must be, how a match is kept from being chosen after the fact, and where the information in the rule comes from. The third question has no answer inside a purely omniscient theory, because every purely omniscient theory contains some incompressible information as given. The incompressible information is hard randomness. Hard randomness also follows from the Incommensuration Theorem, from the side of comparison, because the Incommensura-

tion Theorem forbids both perfected determinism and pure randomness. Matching a phenomenon with a generated state is not understanding the phenomenon. Determinism is a poor basis for modeling most real processes. Causation is distinct from determinism. Finally, the incompressible residue is the connection between third-person theory and first-person knowing. The residue is hard randomness: the information that no third-person theory can compress. Hard randomness belongs to the bound triple, and so hard randomness is inseparable from the arrow of time and from the subjective stance, the first-person stance. This chapter's results apply to every theory laid out wholly as structure, including the Wolfram Physics Project.



Chapter 33

Against Computationalism

Why the World Is Not a Computation and a Mind Cannot Be Uploaded

ABSTRACT

Computationalism is the thesis that all of physical reality is, in principle, a computation, and that the right computation, run on any machinery, would be a mind. Computationalism is a claim about what reality is: a metaphysics. A metaphysics is settled by argument. The holders of computationalism allow no observation to count against computationalism. So science cannot establish computationalism, and computationalism has no claim to the authority of science. This chapter argues that computationalism is false as a matter of structure. Computation is deterministic, has no direction of its own, and has no interior. Three features form a bound triple: hard randomness, the arrow of time and the subjective stance, which are distinct, inseparable and non-interchangeable, so removing any one of the three removes all three. Computation removes all three. There are two further structural grounds for excluding a world that is a computation at its foundation: determinism, which the Incommensuration Theorem forbids, and the relation of a description to what the description describes. Mind uploading is the proposal that a person can be captured in a computational representation. Mind uploading is therefore a category error and not an engineering problem: the request is malformed, and not merely hard. A person is not a very large file. This

> chapter's argument sets no limit on how capable a machine can become at any task, because the boundary that the argument draws lies between processing and experience.

I. The Computational Thesis

Computationalism is, in its strong form, the claim that the universe is a computation: that every physical process can in principle be represented as a sequence of discrete operations on discrete states (separate, countable steps on separate, countable states). If computationalism were true, everything that exists would be, at bottom, information processing. When computationalism is applied to the mind, the thesis is that the right computation, run on any machinery, would thereby be conscious.

The consequences of computationalism are large. If the universe is a computation, any sufficiently faithful simulation of a physical system is, for all relevant purposes, identical to the original. A simulated brain would then be a real mind. Mind uploading is the proposal that a person can be captured in a computational representation, and under computationalism uploading would be natural, and not merely possible: uploading would be a straightforward consequence of what reality already is.

Computationalism has great influence in present culture. Computationalism is the metaphysical assumption behind much of what is said and written about artificial intelligence, virtual reality, digital immortality (the hope that a person can live on as a computation), and the simulation hypothesis (the proposal that this world is a computation, run in a more fundamental world). Computationalism is rarely stated as a metaphysical thesis, and is more often treated as obvious. I argue from the Immanent Metaphysics against the basic claim of computationalism, which is a claim about what reality is.

II. What Kind of Claim It Is

I separate claims of two kinds. When a claim is about what reality is, this chapter calls the claim a claim of what. Every metaphysics is a claim of what. When a claim is a law that explains or predicts, this chapter calls the claim a claim of why and how. Different kinds of claim are settled by different means. Falsifiability is the criterion proper to claims of why and how: a law that explains or predicts forbids some observation, and if the forbidden observation occurs, the law is refuted. Falsification applies only where the outcome of a comparison can be objective: where the outcome is one that many people can compare, that is consistent across the people who compare the outcome, and that can surprise them. A single observation can refute a law, but a single observation does not refute a claim of what. A metaphysics is judged by argument, and where a metaphysics has consequences in physics, those consequences are judged by experiment.

Computationalism is a claim of what. Yet its holders present computationalism as an empirical finding, a result of observation. When computationalism is taken as an empirical finding, computationalism is non-falsifiable: its holders would accept no observation as counting against it. If a phenomenon appears non-computational, a holder of computationalism can answer that the computation is more complex than we have yet understood. If a process appears to involve randomness that nothing fixes, the holder can posit a deeper layer that is deterministic. If attempts to simulate consciousness fail, the holder can attribute the failure to a simulation that was not faithful enough. The holders of computationalism would accept no state of affairs as a refutation, so computationalism is not a scientific conclusion. Computationalism is a metaphysical commitment that its holders present as an empirical finding.

The charge against computationalism is precise: computationalism is non-falsifiable, and so computationalism belongs to the same epistemic category (the same category with respect to how a claim can be known) as other unfalsifiable commitments, religious commitments included. Being non-falsifiable is distinct from being false. A person can believe computationalism, but science cannot establish computationalism, and computationalism is not entitled to the authority of science. Computationalism is a claim of what, so the proper means of settling computationalism is argument. I argue that computationalism is false as a matter of structure.

III. Computation Is Deterministic

Computation is deterministic by its nature: each state of a computation is fixed by the previous state and by the computation's rule. A Turing machine is the model of computation that the mathematician Alan Turing described in 1936, and a Turing machine carries out a defined sequence of operations on a defined state (Turing, 1936). A pseudo-random number generator produces outputs that are statistically random, and every output is fully determined by the generator's starting conditions. *Hard randomness* is an outcome that no prior specification fixes, however complete. No computation produces hard randomness. The single outcome of a quantum measurement is the plainest instance of hard randomness in physics.

An obvious objection is that a computer can read a quantum random-number source, a device that produces numbers from the outcomes of quantum measurements. But randomness read from a physical device enters the computation as input from outside the computation. The computation itself stays deterministic: the randomness is in the physics of the device, and not in the algorithm. When a computer reads a quantum random-number source, the computer has randomness, but the randomness

is the world's randomness, carried through the computation. The computer's direction of time follows the computer's randomness, and so the computer's direction of time is the world's direction too.

A quantum computer is the same case as a computer that reads a quantum random-number source. Between preparation and readout, the operations of a quantum computer are a reversible rule, and the readout is a measurement made in the world. Where a quantum computer has any hard randomness, direction of time or subjective stance, the machine has each one at its measurements, as a physical system, and never by virtue of the program the machine runs.

IV. Computation Has No Arrow of Its Own

A computation has no direction of its own. The states of a computation follow one another by a fixed rule. So the whole list of the computation's states is contained in the starting state and the rule. The list of states has an order. The order can be read backward as readily as forward. The order of the states is therefore an indexing (a numbering in sequence) and not a direction in which a process runs.

In 1973 the physicist Charles Bennett showed that every computation can be carried out reversibly, by keeping a record of each step (Bennett, 1973). An actual computer that has a direction has the direction because the computer erases information, and erasing information is a thermodynamic act of the physical machine, an act in which the machine gives off heat, as the physicist Rolf Landauer showed in 1961 (Landauer, 1961). So a running computation's direction is the arrow of time of the world that runs the computer, and never an arrow of the algorithm.

V. Computation Has No Interior

Computation lacks hard randomness (Section III) and a direction of its own (Section IV). The third feature that computation lacks is the *subjective stance*: the subjective pole of comparison, the stance from which every comparison is made. In a conscious being, the subjective stance is the fact that there is something it is like to be that being. A computation is laid out wholly in the third person, as structure: every state can be handed to someone else and read by someone else. No part of a computation is the stance from which a comparison is made.

The three features that computation lacks form a *bound triple* (Chapter 15): hard randomness, the arrow of time and the subjective stance. The three features are distinct, inseparable, and non-interchangeable, which mean, in order, that no feature is either of the others, no feature occurs without the other two, and no feature can do another feature's work. Removing any one of the three features removes the other two. The binding of the three features is derived from the Incommensuration Theorem. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Every comparison leaves a remainder that no prior structure fixes, because of the Incommensuration Theorem, and that remainder is hard randomness. Every comparison also proceeds in a direction, from act to record, and that direction is the arrow of time. Every comparison is also made from a stance, the subjective stance. The three features are three aspects of one act of comparison, and none of the three features is present without the other two. Computation is deterministic, has no direction of its own, and has no interior (no subjective stance). So computation removes all three features at once: a system represented as a computation is represented without any of the three. Therefore no computation can replicate a system in which all three features are present: a system that is real in the full sense.

A computation cannot take in one feature of the bound triple without the other two. If a computation takes its randomness and direction from the world it runs in, the computation also shares that world's subjectivity. The computation shares the world's subjectivity as part of the world: as a physical system, and not as a program.

VI. Two Structural Grounds Against a World That Is a Computation

Two further structural grounds apply to any research program in which the world, at its foundation, is taken to be a computation.

The first ground is determinism. A purely computational universe would be deterministic. Determinism needs the same law to carry every state into the next state, everywhere and at every time, and one law that is the same everywhere and always is symmetry perfected. Determinism also needs every state to be specified at every point without a gap. Specification without a gap is continuity perfected. The Incommensuration Theorem forbids perfected symmetry and perfected continuity together, so the Incommensuration Theorem forbids determinism.

The second ground is the relation between a description and the thing the description describes. Every exact theory has two levels: its formalism and its findings. The formalism of a theory is its mathematics. The findings of a theory are what its measurements return. The Incommensuration Theorem allows two characters: continuity with asymmetry, and symmetry with discontinuity. In every exact theory, the two levels take the two allowed characters, one character each. A world computed at its foundation would be a set of discrete elements rewritten by one rule applied the same way everywhere. Discrete elements are discontinuity, and one rule applied the same way everywhere is symmetry, so a world computed at its foundation would have one character at both levels: symmetry with discontinuity. One character at both levels is a character compared to itself, and comparing a character to itself produces only what the charac-

ter already contained. A computation can approximate smooth physics, as the grids used to forecast the weather do, and a computation used in that way is a tool for describing the world, and not the foundation of the world.

There are therefore three independent grounds for excluding a world that is a computation at its foundation: determinism, the relation of a description to what the description describes, and the bound triple.

VII. Mind Uploading Is a Category Error

Mind uploading is a category error and not a difficult engineering problem, because the bound triple holds. A *category error* is the use of a concept from one domain in another domain where the concept does not belong, so that the request is malformed and not merely hard. Uploading a mind requires that every relevant feature of a conscious system be captured in a computational representation. The three features of the bound triple make up the irreducible conditions of experience, and those three features are exactly the features that computation cannot represent. Computation lacks the three features as a matter of its structure, so no better hardware can supply them.

The impossibility of uploading a mind can also be stated in the terms of the Incommensuration Theorem. A classical computation is a computation on an ordinary computer, not a quantum one. A classical computation is one rule applied the same way at every step, which is symmetry perfected. What is absolutely symmetric is intrinsically discontinuous (discontinuous in its own nature), by the Incommensuration Theorem. Computation perfected as computation is therefore symmetry with discontinuity applied absolutely. The first-person stance is continuity with asymmetry: an unbroken engagement (the continuity) that runs one way and sets this moment apart from every other moment (the asymmetry). A continuity cannot be composed of purely symmetric steps, in the same way that a curve cannot be composed of tiles, however small the tiles are

made: in the picture, the curve is the continuity and the tiles are the symmetric steps. Perfecting both computation and subjectivity at once would perfect both symmetry and continuity at once, and the Incommensuration Theorem forbids perfecting both symmetry and continuity at once, so the Incommensuration Theorem forbids perfecting both computation and subjectivity at once. A person is not a very large file. A person is what a file, by its nature, is not: continuity with asymmetry.

In this chapter I do not deny that a computational system can be built to behave as a person behaves, to any degree of behavioral fidelity one likes. Behavior is exterior: what a person does can be seen and recorded from outside. The exterior is omniscient (laid out as structure, in the third person), and an exterior description of the exterior is always available. An exterior description reproduces behavior, but an exterior description adds no knowledge of an interior. The argument of this chapter says that a computational system built to behave as a person behaves would not be that person, or any person, because exterior description does not exhaust what it is to be a person. Artificial intelligence is possible, and computation is enormously powerful. Yet there is a boundary that computation cannot cross, the boundary between information processing and experience: however much information a computation processes, the processing is not experience.

The hope of digital immortality depends on the assumption that persons are, at bottom, information. The bound triple shows that persons are, at bottom, what information cannot capture: the conjunction of novelty that nothing fixed (hard randomness), irreversible time (the arrow of time), and interior experience (the subjective stance).

VIII. The Kind of Each Step

The claim that computationalism is non-falsifiable is a claim about what kind of thesis computationalism is. I state known features of computation when I say that computation is deterministic, has no direction of its own

and has no interior. Chapter 15 derives the binding of hard randomness, the arrow of time and the subjective stance from the Incommensuration Theorem, and the binding is borne out in physics by two opposite interpretations of quantum mechanics. The many-worlds interpretation drops all three together, while the Copenhagen interpretation keeps all three together: the Copenhagen interpretation keeps the observer (the subjective stance), the random single outcome (hard randomness), and the irreversibility of measurement (the arrow of time). The exclusions are that no computation can replicate a system in which the bound triple is present, and that mind uploading is therefore excluded. Both exclusions are structural, and both depend on the binding of the three features and on the Incommensuration Theorem.

The argument of this chapter concerns the strong thesis of computationalism, the thesis that reality is computational. The argument says nothing about how capable a machine can become at any task. In the sense of competence at a task, intelligence is an omniscient property: structural, repeatable, shareable, and assessable from outside. I draw the boundary between processing and experience, and not between the stupid and the intelligent.

Computationalism is wrong in the way that a category error is wrong, and not in the way that a scientific theory is wrong: a wrong scientific theory is refuted by an observation, but a category error is wrong because the request is malformed. The holders of computationalism take one mode of description, computation, which is deterministic, has no direction of its own and has no interior, and they ask computation to do the work of another mode entirely. The bound triple is the signature of the real: the bound triple is what distinguishes a world in which things happen from a world in which things only appear to happen. When hard randomness is factored out (removed) from a description, the arrow of time and every possible subjectivity are factored out as well. Computation factors out all three. What remains can be extraordinarily useful, but what remains is not reality, and what remains is not a person. What remains differs from reality and from a person in kind, and not in degree, so no increase in compu-

tational power, no refinement of algorithms, and no advance in hardware will close the gap and turn what remains into reality or into a person. Closing the gap is impossible in principle, and therefore impossible in all future practice.

IX. Summary

I have answered computationalism as a claim of what, which is the kind of claim computationalism is. Argument is the proper means of settling a claim of what. The holders of computationalism present it as an empirical finding but accept no observation against it. So computationalism cannot borrow the authority of science. The argument of this chapter shows that computationalism is false as a matter of structure. Computation is deterministic, has no arrow of its own, and has no interior. Three features form a bound triple, namely hard randomness, the arrow of time and the subjective stance, and a computation cannot take in one feature of the bound triple without the other two. Computation removes all three. There are two further grounds for excluding a world that is a computation at its foundation: determinism, which the Incommensuration Theorem forbids, and the two-level relation of a description and what it describes. Mind uploading is therefore a category error: a person is continuity with asymmetry, and computation perfected is symmetry with discontinuity. The argument concerns the strong thesis of computationalism. Machines can be as capable at tasks as anyone builds them to be, and this chapter's boundary lies between processing and experience.

Glossary

I define each term in a chapter, where the reader first meets the term, and each entry of this glossary names the chapters that define the term.

Absolute and relative boundary: the absolute boundary is the boundary between the unknown and the unknowable when that boundary is defined in terms of classes, and no one ever crosses the absolute boundary. The relative boundary is the boundary that one observer comes to from a given position at a given time. The relative boundary is defined in terms of instances, and the relative boundary can always be crossed. (Chapter 17)

Absolute elsewhere: the region of space and time outside both light cones of an event (the cone of every path that light leaving the event could take, and the cone of every path along which light could arrive at the event). Events in the absolute elsewhere of an event cannot be known at the event, and no influence from the event reaches those events. (Chapter 17)

Abstraction: the word means "from structure"; abstraction is the removal of detail, which moves a thing toward a simpler form. (Chapter 7)

Access: which influences can reach which other parts of the world, under which conditions, at what cost, and in what direction. A law of physics, read as a boundary around the space of possible events, is a statement of access. (Chapter 19)

Algorithmic information content: the length of the shortest description of a thing. The choice of description language (the language in which the descriptions are written) fixes the length of the shortest description up to a constant, so that two choices of language give lengths that differ by no more than a constant amount. (Chapter 32)

Amplitude: the number that a quantum state gives to each possible outcome, from which the probability of the outcome is computed. (Chapter 30)

Analytic knowing: the knowledge given by proof, which is knowledge of what must follow within one domain, where the axioms of the domain fix the domain. (Chapter 9)

Arena: space and time, taken as the context of every physical comparison. (Chapters 21, 31)

Arrow of time: the structural asymmetry between a determined past and an open future. The arrow of time is present in every comparison, because every record comes after the act of comparing that made the record. (Chapters 15, 24, 33)

Asymmetry: difference of content where there is a difference of context (difference of content across difference of context): the content changes as the context changes. (Chapters 2, 22, 23, 31)

Axiology: the study of value and significance: why we should care, and what we care about. (Chapter 1)

Axiom I: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". Axiom I states which of the three modal roles is more fundamental. (Chapters 3, 6)

Axiom II: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". Axiom II states the order in which the three modal roles succeed one another. (Chapters 3, 6)

Axiom II rotation: a shift of one position, in the order of Axiom II, in passing from one domain to the domain that succeeds it, where the two domains are related by process. An Axiom II rotation is distinct from Axiom II itself, which is the order of the modal roles within one domain. (Chapter 6)

Axiom III: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable" (An Immanent Metaphysics, aphorism 1.3-1). Axiom III holds both of the classes of the three modalities and of their instances. (Chapters 3, 20)

Bell's inequality: the limit on how much agreement between the records of two distant detectors a shared past can supply. In the form that the physicists John Clauser, Michael Horne, Abner Shimony and Richard Holt gave the inequality, the correlations for four pairs of detector settings are combined into one number. The combined number is at most 2 for every local account, and quantum mechanics predicts, and experiment finds, about 2.8. (Chapter 28)

Bell's theorem: no account in which each outcome is fixed in advance by local causes reproduces the correlations (the agreements between the records of distant detectors) that quantum mechanics predicts, and that experiments find, between entangled particles. (Chapter 28)

Benthic: living at the bottom of the deep sea. (Chapter 11)

Binary hypothesis: the claim that all the information in reality is digital, so that the whole state of the universe at one moment is a finite string of bits (ones and zeros), evolving by a rule. (Chapter 31)

Bohm's theory: the theory of the physicist David Bohm, a hidden-variable theory that reproduces the predictions of quantum mechanics. Bohm's theory is deterministic (one law fixes every state of the world in advance) and non-local (in Bohm's theory the outcome in one laboratory depends on the setting of the detector in the other laboratory). (Chapter 29)

Born's rule: the chance of finding a particle at a place is the square of the size of the particle's wave (the wave that quantum theory uses to describe the particle) at that place. Born's rule is named for the physicist Max Born, who gave the rule. (Chapter 18)

Bound triple: hard randomness, the arrow of time and the subjective stance, which are distinct, inseparable and non-interchangeable, so removing any one of the three removes all three. (Chapters 15, 31, 32, 33)

Catastrophic theorem: either everything is meaningful or nothing is. Since some meaningfulness exists, everything is meaningful. (Chapter 13)

Category error: the use of a concept from one domain in another domain where the concept does not belong, so that a request that uses the concept there is malformed and not merely hard. (Chapter 33)

Category error of quantizing the arena: giving the context of physics the treatment that belongs to the content of physics, by asking for a quantum (a least unit that cannot be divided) of the arena itself. (Chapters 21, 22)

Causal knowing: the knowledge given by experiment, which is knowledge of what causes what. Natural law is the total of causal knowing about the natural world. (Chapter 9)

Causation (**causality**): a regularity found in time, which is lawful, repeatable pattern: the same relation of antecedent conditions to consequent conditions (of what comes before to what comes after), found again and again across a change of circumstance, from case to case. Causation is the regularity that science finds. (Chapters 12, 13, 32)

Change: the actual process of transition, in which pattern and novelty are found together. Change is the superclass of choice and causation, the class that contains both. (Chapter 13)

Change of state: a change from one state of a thing to another, such as motion. A change of state is not itself a state, but a concept in addition to the existing things and the states of those things. In the frame of change of state, change of state is the transcendent term. (Chapter 6)

Choice: the emergence of the new: a selection from a range of potentials, with an irreversible consequence, where no prior specification (no description of what came before, however complete) fixes the selection. Choice is meaningful to the one who chooses. (Chapters 13, 14)

Claim of what, claim of why and how: a claim of what is a claim about what reality is, as every metaphysics is, and argument settles a claim of what. A claim of why and how is a law that explains or predicts. A law that explains or predicts forbids some observation, and the law is refuted if the forbidden observation occurs. (Chapter 33)

Classical: what can be virtualized. A classical thing is a thing whose model can behave as the thing does for every purpose that matters. (Chapter 26)

Coherence: the function by which a structure in one domain keeps its integrity (stays intact) when the structure is carried into another domain. Coherence between domains differs from the coherence of a quantum state, which the entry for decoherence uses. (Chapter 23)

Collapse: the passage from the possible to the actual in the act of comparison. After collapse the outcome is a new finding, which supersedes the quantum state from before the measurement. (Chapter 30)

Column pairs: the three pairs of the six measures in which each content is measured in its own context, the context in the content's own column of the grid: pattern in space (omniscient), force in time (immanent), probability in possibility (transcendent). (Chapter 20)

Common cause: an earlier event from which two later events both descend, and which explains the agreement of the two later events. Explaining an agreement by an earlier shared event is common-cause reasoning. (Chapter 28)

Common source: two descriptions, each perfected in its own character, joined side by side and related through the act of measurement, in which both descriptions bear on one event. The Incommensuration Theorem permits a common source. (Chapter 22)

Comparison: the act of finding whether two things are the same or different, in some respect. Comparison, interaction and relation are one act, named three ways. Every measurement is a comparison, and every act of

knowing is a comparison or is built from comparisons. The six intrinsics of comparison are sameness, difference, content, context, the subjective and the objective. (Chapters 2, 15, 21, 22, 25, 26)

Complex: having microstate amplification, in which differences too small to record grow into differences that decide what happens, so that no finite, coarse-grained description of a complex system stays adequate indefinitely: the differences that the description discards as noise are the differences that decide the outcome. A living cell is complex. (Chapter 27)

Complicated: having many parts whose relations can all be written down in advance, as the parts of a car can. A complicated system has no microstate amplification, and so a finite, coarse-grained description adequately describes a complicated system. (Chapters 16, 27)

Compression algorithm: a procedure that writes down the regularities of a stream of data so that the stream can be stored in fewer bits. The residue is the part of the stream that the algorithm cannot compress. (Chapter 14)

Computationalism: the thesis that all of physical reality is, in principle, computational, which means made of discrete operations on discrete states (separate, countable steps on separate, countable states). Applied to the mind, computationalism is the thesis that the right computation, run on any machinery, would be conscious. (Chapter 33)

Conjugate: two things are conjugate when each can be perfected (taken toward perfection) only at the expense of the other, and neither is prior to the other in definitional basis (neither comes before the other in the order of definition). In the sense used of pain and joy, two things are conjugate when the two share a foundation and neither can be derived from the other. The foundation that pain and joy share is intensity. (Chapters 3, 4, 5, 12, 13, 14, 22, 23, 25)

Conjugate pair (conjugate quantities): two descriptions of one state, such as position and momentum, each of which can be sharpened only at the expense of the other. (Chapters 18, 19)

Consciousness: the bound triple taken as one act: the arrow of time, hard randomness and the subjective happening together, with the subjective as the interior face of the one act. (Chapter 15)

Conservation law: a symmetry stated as a quantity. A conserved quantity is the content that stays the same across the contexts of a symmetric domain, as the amount of water in a corked bottle stays the same month after month. (Chapter 23)

Conservation of information under deduction: when a rule is run on its starting conditions, the deterministic output carries no more information than the rule and the starting conditions together. (Chapter 32)

Consistency of causation: the causal laws, wherever they apply, are the same type of law everywhere. (Chapter 29)

Content: what the verdict of a comparison, sameness or difference, is on. Content is local and exclusive: two contents exclude one another from the same place. In a quantum measurement, the property probed is the content. (Chapters 2, 15, 20, 21, 22, 23, 25, 26, 30)

Context: the respect in which the two things compared are two. Context is non-local and composes: two contexts combine into one. In a quantum measurement, the setting of the apparatus is the context. (Chapters 2, 15, 21, 22, 23, 25, 26)

Context-modal phenomenon: the phenomenon that plays the role of context in a domain. A context-modal phenomenon couples to every content, has no shield, has one mass for coupling and for inertia, and couples to a quantity that comes in no steps of charge. In physics the context-modal phenomenon is gravitation, the shape of the arena. (Chapters 21, 22)

Continuity: sameness of content where there is a sameness of context (sameness of content within sameness of context): within one unbroken stretch of context, the content does not jump. (Chapters 2, 9, 12, 22, 23, 24, 26, 31)

Continuous asymmetry: the conjunction of continuity with asymmetry, one of the two conjunctions that survive the proof of the Incommensuration Theorem. In the finite form of the proof of the Incommensur-

ation Theorem, continuous asymmetry is the identity map of {same, different}, which sends same context to same content and different context to different content. Continuous asymmetry is the character of the transcendent and of the subjective. (Chapter 2)

Continuum: the real number line, or any quantity that varies as the real number line does, over an unbroken range of values. No list contains the continuum in full: every list of real numbers leaves some real numbers out. (Chapter 31)

Convergent, parallel and divergent acceleration: the three kinds of acceleration, which an observer tells apart over a region by whether neighboring free paths run together (convergent, as in the fall toward a center of mass), keep their separation (parallel, as in the push of a rocket or a floor), or run apart (divergent, as in the outward pull of rotation). (Chapter 21)

Cosmological horizon: the edge of what one observer can ever compare with, and not an edge of the universe. No signal comes from beyond the cosmological horizon. (Chapter 16)

CPT theorem: every local relativistic quantum field theory (the kind of theory that physicists use to describe particles) is symmetric under the combined reversal of charge, parity and time. (Chapter 24)

Criticality: a regime in which small perturbations (small disturbances) can cascade to large scales. The brain operates near criticality. (Chapter 15)

Cross-modal reach: each fundamental theory describes the theory's domain with a tool that belongs to the modality opposite to the modality of the domain: quantum mechanics uses symmetry, the tool of the omniscient, to describe the microscopic, and general relativity uses continuity, the tool of the transcendent, to describe the macroscopic. (Chapter 22)

Decoherence: the spreading of the coherence of a quantum system's state (the fixed relation among the parts of the state that lets the parts interfere with one another) into the joint system that the quantum system forms with its environment. (Chapter 26)

Degrees of freedom: the independent ways in which a system can be arranged or can change. (Chapter 27)

Determinism: the claim that one law fixes every state of a whole world completely, at every scale, so that every state is fixed in advance by the states before it, everywhere and always, and nothing is left unaccounted for. The claim of determinism would have to hold all the way down to the scale of exactly zero. Determinism is true within a mathematical model, because mathematics is timeless. Determinism is distinct from causation, and determinism is an idealization that no actual world reaches. (Chapters 12, 13, 14, 16, 32)

Discontinuity: difference of content where there is a sameness of context (difference of content within sameness of context): the content changes although the context has not. (Chapters 2, 22, 23, 31)

Discontinuous symmetry: the conjunction of symmetry with discontinuity, one of the two conjunctions that survive the proof of the Incommensuration Theorem. In the finite form of the proof of the Incommensuration Theorem, discontinuous symmetry is the exchange map of {same, different}, which sends same context to different content and different context to same content. Discontinuous symmetry is the character of the omniscient and of the objective. (Chapter 2)

Distinct, inseparable, non-interchangeable: the three coequal properties (equal in rank) of the members of a foundational triple. Distinct means that no member overlaps another. Inseparable means that no member occurs without the others. Non-interchangeable means that no member can take another member's role. (Chapter 11)

Domain: "that which contains (includes), but which is itself uncontained and uncontainable"; any realm of conception complete unto itself, with three concepts at its foundation. A domain is individuated (told apart from every other domain) and defined by the three concepts at the domain's foundation (the basis concepts of the domain), and not by the contents of the domain. A domain is also the totality of what can be compared by a given kind of comparison, and a domain exists as a domain in-

sofar as comparisons can be made within the domain. A domain is contrasted with a world, which is an unbounded, holistic dynamic. The physical domain is the whole of what can be compared from within the physical domain, which is the whole of what can be measured from within the physical domain. (Chapters 1, 2, 5, 17, 18, 20, 23)

Domain curvature: features of a domain that belong to the relations among the points of the domain and to no single point, so that no point-by-point specification captures those features. Domain curvature is distinct from the curvature of space and time by which general relativity describes gravitation. Every finite domain has domain curvature, and quantum nonlocality is how domain curvature shows in physical measurement. (Chapters 20, 23, 28, 30)

Entanglement: the linking of the states of two systems that interact directly with one another as quantum systems. When two systems interact only through a go-between that is in one definite state, the two systems do not become entangled. (Chapter 28)

Entropic arrow: the direction of time given by the statistical growth of entropy (a measure of disorder) in systems of many parts. (Chapter 24)

Epistemic Sandwich: the argument across three layers of description (the observer's scale, the largest scale, the smallest scale) that ties the entropic arrow, to which knowing at the observer's scale is bound, to a non-entropic arrow (an asymmetry of time that does not come from entropy) at the smallest scale. "Epistemic" means having to do with knowledge. (Chapter 24)

Epistemology: the study of how we know what is, which is the study of the basis of associating any understanding or any relevance with a perception. (Chapter 1)

Errors of extrema: the four impossible limiting positions in the relation between perception and expression. Each error of extrema takes the relation to an absolute limit ("extrema" means extremes). The errors of the self are prejudice (expression absolutely independent of perception) and reaction (expression absolutely dependent on perception). The errors of

the world are realism (perception absolutely independent of expression) and idealism (perception absolutely dependent on expression). In this entry the word "idealism" names an error of extrema, and the entry for idealism gives the sense of Chapter 1. (Chapters 10, 11)

Event time: the order of acts, one finding after another, which is the time I live; the discrete order of measurement outcomes. Event time is real, and event time does not exist. (Chapter 7)

Evolution time: the parameter of the continuous evolution of a quantum state. (Chapter 24)

Exclusion and addition: the principle of identity applied at the two poles of a comparison. At the content pole the principle of identity means exclusion: two instances of content of one kind cannot occupy one state, as electrons cannot. At the context pole the principle of identity means addition: context contributed twice in one state is one context of twice the amount, as with photons. (Chapter 25)

Existence (to exist): to have a place in a pattern, as structure. What exists is repeatable, locatable and shareable, and involves no interaction of its own. Existence is third person. In the exercise of the empty universe, existence is the existing things. In the triple of existence, change and interaction, existence is the omniscient term. (Chapters 6, 7)

Expression: the crossing of the boundary of the self from the subjective to the objective. (Chapter 10)

Faithful reversal: a reversed process that retraces exactly the states of the original process. A faithful reversal requires a completely and continuously specified state. (Chapter 24)

Findings and formalism: the two levels of a physical theory. The findings of a theory are what the theory's measurements return, and the formalism of a theory is the theory's mathematics. (Chapters 22, 23)

Fisher information: a measure of how much information about a parameter the data contain. The reciprocal of the Fisher information about a parameter is a lower bound on the variance of any unbiased estimate of the parameter (the Cramér-Rao inequality). (Chapter 19)

Focus: the frame of a description relative to which modal roles are assigned, which is the respect in which the three terms of a triple are considered together. For example, the focus of practice is the first person, and the focus of theory is the third person. (Chapter 3)

Foundational triplication: the principle that the foundation of every domain resolves into exactly three concepts, which are distinct, inseparable and non-interchangeable. Foundational triplication holds because comparison is carried into the foundation of every domain. (Chapters 3, 4, 17)

Four markers of context: the four features that mark a phenomenon as context: coupling to every content without exception; no shield; one mass for coupling and for inertia; coupling to a quantity that takes continuous values and comes in no charged steps. Gravitation carries all four markers, and no force of particle physics carries any of the four. (Chapter 21)

Frame: the setting of a description relative to which a modal role is assigned. Chapter 6 uses two frames: the frame of change of state, and the frame of the actual event. (Chapter 6)

Free will theorem: the theorem of the mathematicians John Conway and Simon Kochen that if the choices of experimenters are not determined by prior information, then, given quantum mechanics, relativity and no backward causation (no effect that comes before its cause), the responses of particles are not determined by prior information either. (Chapter 24)

Full-state simulation: a program that tracks every part and every interaction of a system for as long as the program can. (Chapter 27)

Fully qualified (of a metaphysics): said of a metaphysics that describes itself, describes the process of describing itself, and needs no new concept to describe that process in turn. (Chapter 3)

Geodesic deviation: the relative acceleration of two neighboring free paths (a free path is the path a body follows when nothing pushes the body). The curvature of space and time fixes the geodesic deviation. (Chapter 21)

Geometric time: time laid out together with space as one pattern: the proper time along paths (the time that a clock carried along a path would show) and the causal order (the order of which events can affect which other events), carried by the metric of space and time. Geometric time is structure, and so geometric time exists. (Chapters 7, 24)

Global inconsistency: the structural fact that no single description of the universe can be consistent both at every place and as a whole. (Chapter 23)

Graviton (in the only admitted sense): a quantum of the description of small waves of a classical geometry (a geometry with no quanta of its own). A graviton is never a quantum of the arena. (Chapter 21)

Hard problem of consciousness: the question why there is experience at all, and not only information processing with no one to whom the processing happens. (Chapter 15)

Hard randomness: an outcome that no prior specification fixes, however complete the specification is, so that the outcome is unpredictable for any subjectivity, including a subjectivity with unlimited access to the relevant state. Hard randomness is real, met in the world, and a fact about the world, not about anyone's ignorance. The single outcome of a quantum measurement is the plainest instance of hard randomness. Even complete prior information gives the probability of each outcome, and nothing more. The Incommensuration Theorem guarantees hard randomness in every comparison, as the remainder that no prior specification fixes. Hard randomness is distinct from pure randomness, which the Incommensuration Theorem excludes. (Chapters 9, 12, 13, 14, 15, 16, 24, 26, 28, 29, 31, 32, 33)

Heisenberg uncertainty principle: the position and the momentum of a particle cannot both be pinned down perfectly at once. The product of the spreads of position and momentum is never less than half the reduced Planck constant (the Planck constant divided by the number of radians in one full turn). The Heisenberg uncertainty principle is named for the physicist Werner Heisenberg. (Chapter 18)

Hidden variable: something definite but unknown, carried from the source that creates a pair of particles, and meant to fix each outcome of a measurement on the pair. (Chapter 29)

Hilbert space: the mathematical space in which quantum mechanics describes the possible states of a system. (Chapter 27)

Horizon: the macroscopic boundary of knowing, the limit of the very large, marked by the speed of light. The horizon is omniscient in character. (Chapter 17)

Horn torus: a torus (a surface shaped like a donut) whose central hole has closed to a single point. (Chapter 4)

Idealism: the view that mind comes before body. Chapter 10 also uses the word for one of the errors of extrema (see the entry for errors of extrema). (Chapter 1)

Immanent: the role of the act itself, the relating, as the act happens here and now: the act in which change occurs, met in first-person experience (as in experiment). The immanent is first person in character. By Axiom I the immanent is more fundamental than the omniscient and the transcendent. (Chapters 1, 3, 8, 29, 32)

Incommensuration Theorem: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric. The Incommensuration Theorem is a structural fact about the nature of knowing. (Chapters 2, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 26, 28, 29, 30, 31, 32, 33)

Incompressible residue: the information that every purely omniscient theory contains as given and cannot compress. The incompressible residue is hard randomness. (Chapter 32)

Indexing of states: an order of states (a numbering in sequence) that can be read backward as readily as forward. An indexing of states is distinct from a direction in which a process runs. (Chapter 33)

Infinitesimal: the microscopic boundary of knowing, the limit of the very small, marked by the Planck constant. The infinitesimal is transcendent in character, and the infinitesimal is a bound on joint knowledge of conjugate quantities (knowledge of both quantities of a conjugate pair at once). (Chapter 17)

Information: the word means "into form"; information is formlessness becoming form. (Chapter 7)

Infra-physics: the structural layer beneath physical theories ("infra" is Latin for "below"). Infra-physics is what must be true of any possible universe for there to be physical law. (Chapter 9)

Inside-out view: the view of a domain as defined by the domain's basis structures, its axioms, and not by all that can arise from those axioms. (Chapter 5)

Instruction: the word means "into structure"; instruction is the addition of detail, which moves a thing toward a concrete form. Instruction is the reverse of abstraction. (Chapter 7)

Interaction: in the exercise of the empty universe of Chapter 6, where every kind of interaction is reduced to one kind, touching, interaction is the immanent term, and by Axiom I interaction is the ground of the other two terms. Across the boundary of the self (Chapter 10), interaction is the interdependence of perception and expression, and interaction is the immanent, more fundamental by Axiom I than either self or world. (Chapters 6, 10)

Interface: in the theory of perception that Chapter 11 discusses, the form through which a creature reaches the world, as distinct from the world reached. (Chapter 11)

Intrinsics of comparison: the six features every comparison has, in three pairs: sameness and difference, content and context, the subjective and the objective. (Chapters 2, 9, 25)

Inverse cloning principle: the rule, in classical physics and computation, that any flow of signal can be copied. The name is by analogy with the no-cloning theorem of quantum physics. (Chapter 26)

Irreducible: said of hard randomness: not predictable on the basis of any past knowledge, even in principle. (Chapter 16)

Landscape of energies: the energy of each molecular arrangement and of each way in which arrangements act on one another, taken together as a landscape in which the height of each place is the energy of one arrangement. The detail of chemistry lies in the landscape of energies. (Chapter 27)

Law of continuity: the law that applies to all that is of context. (Chapter 2)

Law of symmetry: the law that applies to all that is of content. (Chapter 2)

Limit of extraction: the limit that the uncertainty relation sets: the uncertainty relation determines how much information an observer can, in principle, extract from a microscopic system. (Chapter 19)

Limit of signaling: the limit that the light cone sets: the light cone determines what an observer can, in principle, receive information from. (Chapter 19)

Listable: able to be set out in one order without omission, one entry for each counting number. Listable collections include the counting numbers, every finite string of symbols and every program. (Chapter 31)

Local causality (Bell, 1976): the condition, stated by the physicist John Bell in 1976, that each single result is produced by conditions local to the station where the result is found, even when those conditions give only the odds of the result. (Chapter 29)

Local hidden variable: an answer that a particle carries from its source, in an account where nothing done to one particle affects the other particle at once. (Chapter 28)

Locality: all causal process is bounded by the speed of light, so that no causal influence on a result comes from outside the result's past light cone (the region of space and time from which a signal moving no faster than light could have reached the result). No signaling, which has its own entry, is the weaker sense of locality. (Chapter 29)

Loophole: a gap in an experimental test that would let a local account escape the verdict of the test. The two main loopholes, of detection and of locality, were closed together in 2015. (Chapter 28)

Mach's principle: to rotate is to rotate relative to content: a rotation registers only against content that the rotating body can be compared with, and a rotation that no comparison can register is no rotation. Mach's principle is named for the physicist and philosopher Ernst Mach. (Chapter 21)

Measurement problem: the question of where the single, definite outcome of a quantum measurement comes from. (Chapter 30)

Merger: one description in which the directions of space and time themselves carry both characters, symmetry and continuity, at the foundation. The Incommensuration Theorem forbids a merger. (Chapter 22)

Mesoscopic: in Chapter 24, the observer's own scale. Physicists who study condensed matter use the word in a different sense, for the scale between single atoms and bulk matter, and Chapter 4 uses the word for the middle scale, between the very small and the very large. (Chapter 24)

Metaphysics: the inquiry into the relationship between self and reality, which is the relationship between the subjective and the objective. Metaphysics is also the study of the relationships between domains. (Chapter 1)

Metatheory: a theory about a theory: a theory of the content, the derivation and the grounds of a theory. (Chapter 22)

Microscopic boundary: the limit of the physical domain at the smallest scale. The microscopic boundary is the limit of access that the uncertainty principle states, below which no one can find a definite state by any comparison, and no interaction yields a signal. (Chapters 12, 16)

Microstate amplification: the process by which small differences at a fine-grained level of a system are propagated (passed on) and magnified into differences at the level that decides the system's behavior. (Chapter 16)

Mind uploading: the proposal that a person can be captured in a computational representation. Mind uploading is a category error: the request is malformed, and not merely hard. (Chapter 33)

Modal triple: three concepts in the three roles (immanent, omniscient, transcendent), for which there is one identified focus in which all three axioms hold. (Chapters 3, 4)

Modalities: the three roles, the immanent, the omniscient and the transcendent, that the foundational concepts of any domain, and the parts of every comparison, play toward one another. The modalities are types of role, not entities. A modality is assigned to a term only relative to the triple in which the term is considered. (Chapters 3, 7, 10, 13, 14, 21, 22, 24, 25)

Modernism: the belief that all problems can be solved with causation. (Chapter 9)

Nine Laws: the nine pairings of one of the three content measures (pattern, force, probability) with one of the three context measures (space, time, possibility). Each pairing names a relation between a content and a context. Work and energy, gravitation and entropy are among the Nine Laws. (Chapter 20)

No signaling: the weaker sense of locality that physicists keep: no signal travels faster than light. (Chapter 29)

No-cloning theorem: no single operation can copy every unknown quantum state. (Chapter 26)

Non-causal correlation: an agreement between separated records that is closer than any causal account permits. A non-causal correlation cannot be used to send a signal, and the statistics at each end are independent of the setting chosen at the other end. The correlation is real, and the correlation is not an influence. (Chapter 22)

Non-falsifiable: said of a claim whose holders would accept no possible observation as counting against the claim. A person may believe a non-falsifiable claim, but science cannot establish the claim. (Chapter 33)

Objective: holding across many interactions, as what many comparers would find. (Chapter 7)

Objective process: what the world does with what the self expresses. (Chapter 10)

Observation: any interaction that relates a system to what is outside the system. (Chapter 30)

Observer: any system that interacts with what is observed and records the result of the interaction. An observer needs no consciousness. (Chapter 30)

Omniscient: the role of what is laid out as structure, such as a pattern, a map or a record, which can be held in view as a whole and can be shared. The omniscient is what is laid out in the third person (as in calculation), and the omniscient is third person in character. (Chapters 1, 3, 8, 29, 33)

Omniscient bias: taking the omniscient, what is laid out as structure and held in view as a whole, for the whole of what is, or for the ground of what is. Omniscient bias is like taking the map of a city for the city itself: the map stands for what is laid out as structure, and the city stands for the whole of what is. (Chapters 7, 8)

Ontology: the study of what is. (Chapter 1)

Order of depth: the order of presupposition among four disciplines, metaphysics, then physics, then computation, then mathematics, in which every discipline after metaphysics presupposes the discipline before it. (Chapter 9)

Particle horizon: the greatest distance from which any signal could have reached us in the finite time since the expansion of the universe began. The speed of light and that finite time together set the limit. (Chapter 5)

Partition tolerance: in distributed computing (the study of many computers that work together over a network), the property of a data store copied across many machines that the store keeps working when the network between its machines is cut. Chapter 29 also calls the property tolerance of partitions. (Chapter 29)

Perception: the crossing of the boundary of the self from the objective to the subjective. (Chapter 10)

Perfected (applied absolutely, holding exactly): holding exactly, at the foundation of a description. The word "perfected" is said of a comparison, or of a character such as symmetry. (Chapters 2, 22)

Perfectly closed domain: a domain with no relations at all to anything outside the domain. Under the axioms of the Immanent Metaphysics no domain can be perfectly closed, because existence with no possible interaction is indistinguishable from non-existence. (Chapter 30)

Physicalism: a form of realism holding that the objective, physical world alone is fundamentally real, and that consciousness and the self, the subjective, are an illusion or are epiphenomena of the physical world (by-products with no standing of their own). (Chapters 1, 11)

Physics: the study of the interactions within a given domain. (Chapter 1)

Pi: the ratio of the circumference of a circle in a flat plane to the circle's diameter. Pi can also be specified as the result of a finite algorithm (a recipe of finitely many steps) when the algorithm is applied without limit. Pi exists and is not real. (Chapters 7, 31)

Planck constant: the least quantum of action (action is a quantity measured as an energy multiplied by a time); the constant that relates the energy of light to the frequency of the light; a number expressing a unit of action. (Chapter 18)

Planck length and Planck time: the scale, fixed by the Planck constant, the speed of light and Newton's constant of gravitation, at which a body's gravitational size equals the body's quantum size. The Planck length is about 1.6 times ten to the minus thirty-five meters. The Planck

length and the Planck time are the bound of every measurement made from inside the world, and not a smallest piece of space or of time. (Chapters 18, 31)

Planck limit: the Planck constant read as the location of a boundary, where the boundary is the microscopic limit of the domain of measurement. (Chapter 18)

Polarization: the measurable orientation of a photon (a particle of light); for a particle of matter the corresponding orientation is spin. Chapter 4 uses the word "polarization" in another sense, for a pair of two opposite poles with nothing between them. (Chapter 28)

Polarization angle: the angle of a photon's polarization, measured from the detector's vertical (usually written with the Greek letter theta). The polarization angle can never be known by measurement. (Chapter 28)

Positions of comparison: the act of comparing, the thing compared, and the thing compared against. In the exchange between two atoms in Chapter 25, the photon is the act of comparing, the electron is the thing compared, and the proton is the thing compared against. (Chapter 25)

Principle of equivalence: at a single point, the acceleration of free fall cannot be told apart from the uniform acceleration of a frame. The physicist Albert Einstein stated the principle of equivalence in 1907. (Chapter 21)

Principle of identity (identity of indiscernibles): that which cannot be distinguished must be the same: two things that cannot be distinguished by any property, logic, measurement or interaction, by any means, mode or method, by any test conceivable even in principle, are one thing and not two. If no test, even in principle, distinguishes two situations, the two situations are one. The tradition of the philosopher Gottfried Wilhelm Leibniz calls the principle of identity the identity of indiscernibles. (Chapters 6, 7, 13, 14, 25, 31)

Process: partly thing, partly change, and partly what is neither thing nor change and is actually new. (Chapter 8)

Process knowing: the knowledge that the Immanent Metaphysics gives, which is knowledge of what cannot fail to be the case in any act of comparison. Process knowing is the third order of knowledge. (Chapter 9)

Pseudo-randomness: the output of a fixed rule applied to a starting number, so that the same start gives the same sequence. Pseudo-randomness is what a computer produces, and pseudo-randomness is deterministic and only looks random. (Chapters 16, 31)

Pure randomness: an outcome with nothing at all accounted for, which would perfect both asymmetry and discontinuity. Pure randomness is an idealization, the pole opposite determinism, which, like determinism, no actual world reaches. Pure randomness is not real. (Chapters 12, 13, 14, 16, 24, 32)

Purely omniscient theory: a theory laid out wholly as structure, which admits only what is structural, repeatable, and stated in the third person. Computation is the model case of a purely omniscient theory. (Chapter 32)

Quantum state: the description of possibility: the range of outcomes that a given setting of the apparatus could yield, with a weight for each outcome. The quantum state is also the record of what has been found, together with what a given setting of the detector makes possible, and the record is the same for everyone who has the outcome. (Chapters 28, 30)

Real (the real): that which at once partakes of choice, change and causality (causation), neither more nor less. To be real is to be given in an actual act of comparison, here and now, in exactly one interaction. The real is first person. (Chapters 1, 7, 11, 13)

Reality: the plural of what is real, and not the totalization of what is real (the adding up of everything real into one whole). (Chapter 7)

Reductionism: the claim that a description at a lower level exhausts the thing at a higher level (leaves out nothing of the thing), so that every whole is explained wholly by the parts of the whole and their earlier states. (Chapter 27)

Reification: giving a concept or domain a definite form in terms of other concepts. In Chapters 3 and 4, reification is the expression of a domain's foundation in terms of the three modalities. (Chapters 3, 4)

Reifying power: the capacity to identify a domain's foundational triple in a way that the practitioners of the domain, the people who use the domain's terms, recognize as correct. Chapter 5 also uses the name for the definite form that a concept gains through three concepts, as the concept of universe gains a definite form in terms of creation, existence and interaction. (Chapters 4, 5)

Resistance: the opposition that prevents perfect carrying over of structure from one domain into another, so that each domain keeps its own character despite the domain's relations to other domains. (Chapter 23)

Resolving power: the capacity that an identified triple gives: to understand what is already known in a different, more profound way, and to generate insights that are not trivial. (Chapter 4)

Root Tautology: the statement, made first in Chapter 2, that comparison, interaction and relation are one act, named three ways. (Chapter 25)

Rule, starting conditions, generated state, observed phenomenon: the rule is the rule structure of a program. The starting conditions are the initial conditions that the rule is run on, including the number of steps for which the rule is run. The generated state is the complicated state that the rule produces from the starting conditions. An observed phenomenon is a complex thing observed in the world, such as a snowflake or a galaxy. (Chapter 32)

Self: "the appearance of the aggregation of that aspect of interaction which is choice". The self is the context of past and future choices. (Chapter 11)

Simulation hypothesis: the proposal that this world is a computation, run in a more fundamental world. (Chapter 31)

Six measures: pattern (which gathers mass, inertia and structure into one measure), space, force, time, probability and possibility, which are the six kinds of quantity of which every physical quantity is a species (a more

particular kind). Pattern, force and probability are content, and space, time and possibility are context. The six measures pair as pattern in space, force in time, and probability in possibility. Chapter 2 calls the six measures the six metrics of interaction. (Chapters 20, 22)

Soft randomness: randomness of ignorance: an outcome that was fixed but is not known, and that better knowledge could reduce. (Chapter 16)

Solipsism: the position that only one's own experience, and the self that has the experience, can be known. Solipsism is an extreme epistemological skepticism (doubt about what can be known) with experience as its ground: experience is the one thing that the solipsist does not doubt. (Chapter 11)

Sound: said of an argument that is valid and whose premises are also true of what it is about. (Chapter 12)

Stability and dynamism: stability is the tendency of structures to keep their form, and dynamism is the tendency of structures to transform. Stability and dynamism are conjugate: the two cannot both be perfected, and neither can ever be zero. (Chapter 23)

Subconscious: the mesoscopic boundary of knowing, at the scale of the knower: the limit of knowing one's own self at the place where comparing happens. The subconscious is immanent in character, and the subconscious carries no constant of physics. (Chapter 17)

Subjective process: what the self does with what the self perceives. (Chapter 10)

Subjective stance: the subjective pole of comparison, the stance from which every comparison is made. In a conscious being, the subjective stance is the fact that there is something it is like to be that being, in the phrase of the philosopher Thomas Nagel. (Chapters 15, 33)

Superdeterminism: the claim that the settings the experimenters choose were fixed in advance, from the beginning of the universe, together with and in step with the states of the particles that the experimenters measure. (Chapters 16, 28, 29)

Superposition: a quantum state that, relative to a given measurement, is not one of the possible outcomes but a weighted range of the outcomes. Relative to a detector's frame, a superposition is an intermediate state, such as a polarization at 45 degrees between vertical and horizontal, and a measurement yields one of the two outcomes, vertical or horizontal. (Chapters 28, 30)

Symmetry: sameness of content where there is a difference of context (sameness of content across difference of context): the content stays the same although the context has changed, as when a shape flipped over looks like the same shape. (Chapters 2, 9, 12, 18, 19, 22, 23, 24, 26, 31)

Symmetry of the description: an invariance of a description, such as the freedom to relabel coordinates (the numbers that name the points of space and time), as distinct from a symmetry of what the description finds. (Chapter 2)

Syntactic and semantic levels: the syntactic level of a description is the rules for combining the terms of the description, rules that apply whatever the terms denote. The semantic level is what the terms denote. (Chapter 5)

Theory and practice: theory is held in the third person, as a description that anyone could be told about, and theory has the role of the omniscient. Practice is held in the first person, by the person who acts, and practice has the role of the immanent. (Chapter 8)

Topology of a molecule: the pattern of bonds that join the atoms of a molecule, which is the pattern of which atoms bond to which. Each pattern of bonds is one point in the space of all possible molecules. (Chapter 27)

Transcendent: the role of the stance from which an act proceeds, and of the range of what is possible from that stance. The transcendent is also the role of what exceeds any one instance. The transcendent is second person in character. In the frame of change of state, the transcendent is betweenness, the relation of peers. (Chapters 1, 2, 3, 4, 5, 6, 7, 8, 9, 15, 17, 18, 20, 31)

Transcendental metric: a metric (a measure) of the transcendent, which measures probability over possibility, as distinct from a metric of pattern over space. Chapter 4 calls the measures of probability over possibility metrics of the transcendent. (Chapter 4)

Triaxial: three axes that together make one class concept. The content triaxial of interaction is pattern (mass, in the combined sense of inertia, pattern, form and structure), force and probability, and the context triaxial is space, time and possibility. (Chapters 5, 20)

Trilemma: three properties of which at most two can hold together. In Chapter 29 I frame Bell's theorem as a trilemma of three properties of causation. (Chapter 29)

Type isomorphism: the principle that the pattern of relations among the three foundational concepts is the same in every domain, so that the concepts of any one domain correspond to the concepts of another. (Chapters 3, 4, 5)

Universality of causation: every change in the world is accounted for by causal process, and only by causal process. (Chapter 29)

Universe: the combined meanings of creation, existence and interaction, neither more nor less. The equation of the universe with the three meanings is a denotation, not a description. (Chapter 5)

Unknowable: what cannot be known even in principle, by any being, at any time, by any method, because the structure of comparison does not extend to the unknowable. The unknowable is distinct from the unknown, which is not yet known. (Chapter 17)

Valid: said of an argument whose conclusion follows from its premises. (Chapter 12)

Vedantic idealism: the position that subjectivity is fundamental and objectivity is an illusion. The position is named for Vedanta, a school of Indian philosophy. (Chapter 11)

Virtualize: to build a model that behaves as the original does for every purpose we care about. (Chapter 26)

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