

The Concept Branches

Fifteen Side Channels of the Concept Sequence

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UVSM PUBLICATIONS
Wytheville, Virginia

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Published by UVSM Publications LLC
965 Reed Creek Mill Rd
Wytheville, VA 24382-8287
USA

uvsmpublications.com
contact@uvsmpublications.com

ISBN 979-8-90710-124-1

DOI: https://doi.org/10.67291/ONRX3888

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Preface

What This Book Is

The Concept Sequence traces, in numbered steps, how physics follows from the act of comparison. Comparison is the act of finding sameness and difference; every measurement is an act of comparison, and so is every act of knowing. The sequence runs from comparison itself, through the Incommensuration Theorem and the three axioms of the Immanent Metaphysics, to the forces, the kinds of particle and the cosmos. Each step of the sequence is an entry with a number that names its chapter: entry 13.14 is an entry of Chapter 13.

This book holds the branches of that sequence. A branch follows one subject further than the main line of the sequence needs to go: the spectral geometry in which the masses are written, the masses and mixings themselves, the couplings, the dark matter and the dark energy, the early universe, the formal rendering of the axioms, and others. There are fifteen branches, lettered A to O, and each branch is one chapter of this book. Each chapter opens with an overview that names the chapters of *The Concept Sequence* on which it rests, and each chapter closes with a Summary of what it traces.

The book is a map, and it owns no result. Every result it states is established in another book of the collection: the book in which the argument for that result is made. That book is the result's owner. Each entry states one result, names its owner by chapter and section, and gives the result the standing that its owner gives it. Where the wording of an entry and the current edition of its owner differ, the owner rules, and the entry follows the owner. The arguments are in the owners; this book shows where each result sits, what it rests on, and where its argument is read.

How to Read an Entry

Every entry has four parts. The number names the chapter by its letter and the entry by three digits: entry F-012 is an entry of Chapter F. The title names the concept or the result. The statement follows, written as one or more clauses that begin with the word "That"; each clause makes one assertion. Below the statement, a note gives the entry's sources and its standing.

The sources are cited in four ways. A book of the collection is cited by its title, chapter and section, at the place where its argument is made. *The Concept Sequence* is cited by chapter and entry number. A work by another author is cited by its authors and year, and its full entry is in the Bibliography. An entry of another chapter of this book is cited by its number, beside a plain statement of what that entry carries.

Many entries are followed by a box. A box explains the entry above it: a piece of standard mathematics or physics in plain words, a worked example, or the picture behind a definition. Each chapter can be read on its own, once the chapters of *The Concept Sequence* that its overview names have been read.

The Standings

The standing of a result names the kind of step that reaches it. Each standing is a different relation between a result and the world, and each claims something different of its result; none of them is a degree of confidence. This book uses eight standings, in the words in which the collection defines them (*The Strengths of the Kinds*, Chapter 1, Section I, defines the standings of a number).

A result is *derived* when an argument gives it; a count that the structure of comparison fixes is derived.

A result is *derived, given a stated identification*, when it follows by argument once one identification is made. The note names the identification, and the identification carries its own question: whether it is true of the world.

A value is *a measured input* when it is taken from measurement and used as an input.

A value is *a measured agreement* when it agrees with measurement and the argument does not state why it agrees. A measured agreement carries that question, why the value lands, and it is never presented as derived.

A result stands at *a fork that a named measurement decides* when two arguments for it are carried side by side, and the note names the measurement that will decide between them.

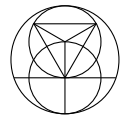
A step is *an identification* when it places a physical quantity on a part of the structure of comparison. The author makes each identification in the first person, and the note names it as such.

An entry is *a definition* when it fixes the meaning of a term.

A result is *a theorem* when it is proved from stated premises; a theorem of mathematics is named together with its source.

Where an entry states several results, its note gives each result its own standing and names, in parentheses, the result to which each standing belongs.

Two further distinctions run through the book. The Incommensuration Theorem is a structural fact about the nature of knowing: it is inherent in every act of comparison, and so in every act of knowing, and the entries that rest on it state it plainly. A proof checked by a machine shows that its formal statement follows from its formal premises; that is its validity. A checked proof settles a claim about the world when its soundness is shown as well: the formal statement truly renders the claim, and the premises hold. Chapter K keeps the two apart wherever it speaks of a checked proof.



Chapter A

The Spectral Infrastructure

CHAPTER OVERVIEW

This chapter sets out the mathematics on which the collection's account of the spectral action rests. The spectral action is the principle of Chamseddine and Connes that the action of physics, the quantity from which the equations of its fields follow, depends only on the spectrum of one operator, the Dirac operator (A-033). In this chapter I begin with the language of geometry: manifold, metric, curvature, connection and fiber bundle. I then take up the Dirac operator, with the spin-c form of the Dirac operator on $\mathbb{C}P^2$, and after that the heat kernel and the Seeley-DeWitt expansion. Next comes the finite spectral triple of the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, and the product of that triple with spacetime. A spectral triple is an algebra, a Hilbert space on which the algebra acts, and a Dirac operator. The algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ joins the complex numbers, the quaternions and the three-by-three complex matrices. I end the chapter with the spectral action and the four named inputs of the spectral action, the moduli of the finite Dirac operator, and the account of the values of the masses. *The Measures of the Kinds* gives the account of the values of the masses separately from the spectral action (A-022). Each term named in this overview is defined in plain words in the entry of this chapter that bears the term's name. Each entry states either a standard definition or theorem of geometry and spectral theory, with the outside work that is its source, or a result of the collection, with the book of the collection that

owns the result. The chapter rests on ten chapters of *The Concept Sequence*: Chapter 13, The Incommensuration Theorem; Chapter 17, Quantization and the Partition; Chapter 18, The Three Forces; Chapter 24, The Algebra of Comparison; Chapter 25, The Internal Space in Its Two Instances; Chapter 26, The Two Kernel Integers and the Kernel Triangle; Chapter 27, The Context Instance: Radii, Volume and the Count 72; Chapter 28, The Two Measured Anchors and the Span to the Planck Mass; Chapter 29, The Couplings: Rank, Running, Weights and Crossings; and Chapter 30, The Spectral Action and the Form of the Standard Model.

Two facts apply to every part of the chapter. First, the internal space $\mathbb{C}P^2 \times S^2$ has two instances. The internal space is the product of the complex projective plane $\mathbb{C}P^2$ with the two-sphere S^2 (A-006), and its two instances are the one space taken once as content and once as context. On the content instance the internal space is a space of states, with no size, no radius and no fields. On the context instance the internal space is the space of possibility of *The Gravitational Passage*, the collection's book on gravitation, with radii $\sqrt{6}\, l_{Pl}$ and l_{Pl}, where l_{Pl} is the Planck length (*The Measures of the Kinds*, Preface; *The Kinds of Particle*, Chapter 3, Section III; *The Gravitational Passage*, Chapter 9, Section IX). Each instance is a range, of states or of possibilities, and neither instance is a place: nothing travels through either instance, and the particles are not vibration modes of either instance. Second, the spectral action fixes the form of the Lagrangian of the Standard Model coupled to gravity, and the values in that Lagrangian come from other routes or from measurement. The Lagrangian is the expression from which the equations of the fields follow, and the Standard Model is the established theory of the known particles and of their three forces. *Non-Commutative Geometry and the Immanent Metaphysics* states the limit of the spectral action in these words: "The derivation stops at the moduli, the entries of the finite Dirac operator, and it derives no value of any coupling or mass" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V).

A-I. The Language of Geometry

A-001 Manifold. That a manifold of dimension n is a space that is like a piece of ordinary n-dimensional space near each of its points. That around every point of a manifold there is a coordinate chart: a smooth one-to-one labeling of a neighborhood of the point by n real numbers. That where two charts overlap, the change from one labeling to the other is smooth. You know flat surfaces, such as a table or a sheet of paper, and curved surfaces, such as the surface of a sphere or the surface of the Earth. A manifold generalizes both kinds of surface. A manifold may have any number of dimensions, and near any one of its points you can lay down a coordinate grid like ordinary graph paper. The global shape of a manifold may be curved and may close on itself, and its topology (the features of its shape that survive any stretching without tearing) may be quite different from the topology of flat space. The internal space $\mathbb{C}P^2 \times S^2$ is a manifold (*The Concept Sequence*, Chapter 25): the internal space is compact (closed and bounded, like the surface of a ball) and smooth, and its real dimension is six, four dimensions from $\mathbb{C}P^2$ and two from S^2.

Sources: a standard definition; for the internal space, *The Kinds of Particle*, Chapter 3, Section I, and *The Gravitational Passage*, Chapter 9, Section VII. Standing: a definition.

> ***Why "locally like flat space" matters.***
>
> The surface of the Earth is curved, but a small enough patch of ground looks flat to an engineer laying a foundation. In mathematical terms the patch of ground is a neighborhood of a point, and a manifold supplies coordinates near every point. When a manifold also has a metric, a rule for distances (A-002), you can choose coordinates at any point so that the metric agrees with the flat metric at that point. In coordinates that agree with the flat metric at a point, the metric departs from the flat metric only at second order in the distance from the point, that is, by an amount that shrinks like the square of the distance. Curvature (A-003) is the measure

of that second-order departure. The flatness of each small neighborhood is what makes calculus possible on curved spaces: you do the local calculations in a chart and carry the results across the overlaps of the charts. The internal space has six dimensions, and every point of the internal space has a neighborhood with six coordinates in which derivatives can be computed.

A-002 Metric. That the metric is the rule that measures distances on a manifold. You know the Pythagorean theorem for flat space: the distance between two nearby points is the square root of the sum of the squares of their coordinate differences. A metric generalizes the Pythagorean theorem: a metric is a local rule for computing the distance between nearby points, and on a curved manifold the rule can vary from place to place. On a flat surface in ordinary coordinates, the rule of the metric is the same everywhere. On a curved surface, such as the surface of a sphere described by latitude and longitude, the rule of the metric changes with position: a degree of longitude covers a greater distance near the equator than near a pole. The metric fixes the distances, angles, areas and volumes on the manifold.

Sources: a standard definition (Riemann 1854). Standing: a definition.

The metric in symbolic form.

In flat two-dimensional space, take two points separated by coordinate differences dx and dy: the distance ds between the two points is given by $ds^2 = dx^2 + dy^2$ (the Pythagorean theorem). On a curved surface, the Pythagorean formula generalizes to $ds^2 = g_{11}\,dx^2 + 2g_{12}\,dx\,dy + g_{22}\,dy^2$, where the coefficients g_{11}, g_{12}, g_{22} are functions of position. The coefficients record how much the coordinate grid is stretched and sheared at each place, and the coefficients, taken together, are the metric. On a manifold of dimension n the metric is a symmetric $n \times n$ array, and it has $n(n+1)/2$ independent components at each point. On a six-dimensional manifold such as $\mathbb{C}P^2 \times S^2$, the number of components is six times seven divided by two, which equals twenty-one.

A-003 Curvature. That curvature is the quantity that measures how much a manifold departs from flatness. You know that a sphere is curved and a table is flat, and curvature puts a number on that difference. A geodesic is a straightest path on a surface. On a sphere, two geodesics that start out parallel converge and meet at the poles: two meridians, for instance, are parallel where they cross the equator, and they meet at the poles. On a saddle-shaped surface, two initially parallel geodesics diverge. On a flat surface, two initially parallel geodesics remain parallel. The curvature of a surface records which behavior of initially parallel geodesics occurs: the curvature is positive where initially parallel geodesics converge (as on a sphere), negative where they diverge (as on a saddle), and zero where the surface is flat.

Sources: standard definitions (Riemann 1854; Ricci and Levi-Civita 1900). Standing: a definition.

> ***Three kinds of curvature.***
>
> On a manifold of more than two dimensions, there are several distinct measures of curvature. The most detailed measure is the *Riemann curvature* (the full curvature tensor, an array of numbers at each point), which gives the curvature of every plane through every point. The *Ricci curvature* of a unit direction is the sum of the curvatures of the planes that the direction spans with the other members of an orthonormal frame, a set of unit directions at right angles to one another. The Ricci curvature measures how the volume of a small ball of geodesics differs from its flat value. The *scalar curvature* is the trace of the Ricci curvature, the sum of the Ricci curvatures of the directions of an orthonormal frame: a single number at each point. When the complex projective plane $\mathbb{C}P^2$ carries its Fubini-Study metric (the metric that transition probability fixes, A-007), normalized so that its Ricci curvature is six times the metric, the scalar curvature of $\mathbb{C}P^2$ is twenty-four. For a two-sphere of radius r, the scalar curvature is $2/r^2$. Entry A-008 gives the values on the two instances of the internal space.

A-004 Connection. That a connection is the rule for transporting a direction from one point to another on a curved manifold. On a flat surface, you can slide an arrow from one point to another without changing its direction or its length, and the arrow arrives the same whatever path it takes. On a curved manifold, an arrow carried around a closed path returns rotated, and the amount and direction of the rotation depend on the path. A connection specifies how the arrow changes at each point, for each direction in which the arrow is carried. On a manifold with a metric (A-002), exactly one connection preserves lengths and angles under transport and has no torsion (a small parallelogram drawn by carrying one short step along another always closes): the Levi-Civita connection, which the metric fixes uniquely. The uniqueness of the Levi-Civita connection is the fundamental theorem of Riemannian geometry.

Sources: a standard definition (Levi-Civita 1917). Standing: a definition.

> ***Connections in the physics of the kinds.***
>
> Connections on fiber bundles (A-005) describe the three forces of the kinds of particle (*The Concept Sequence*, Chapter 18). Comparison has three positions: the act of comparing (the immanent), the thing compared (the omniscient) and the thing compared against (the transcendent). *The Story of the Particles* names the three forces in its frame of measurement, the frame in which the comparing act is light (*The Story of the Particles*, Level 7, Section I, and Level 9, Section III). In that frame of measurement the omniscient force acts among the three colors, with the group SU(3) and eight gluons. The transcendent force acts between the two sides of a doublet (a pair of particle states, such as the left-handed up and down quarks), with the group SU(2) and three weak bosons. The immanent force acts on the phase, with the U(1) of hypercharge and one hypercharge boson, and the hypercharge boson mixes with the neutral weak boson to give the photon and the Z. A gauge field carries one of the three forces, and a gauge field is a connection: the gauge field specifies how the internal state of a field is compared from one point of spacetime to the next. The curvature of a gauge connection is the field strength.

> The same mathematical notion, a connection, describes the arena (curved spacetime, in which the particles are found): the Levi-Civita connection of spacetime governs how directions turn under transport in curved spacetime. The forces and the arena share a mathematical language, and nothing more. Gravity is the arena: gravity is context and not content, and gravity is not a fourth force (*The Story of the Particles*, Level 7, Section VI). *Measurement*, the collection's book on measurement, states how the two descriptions are related: the description of the arena and the description of its content "are composed side by side and not merged" (*Measurement*, Chapter 10, Section IX).

A-005 Fiber Bundle. That a fiber bundle is a mathematical structure in which a space of the same kind, the fiber, is attached at each point of a manifold, the base. That over a small region of the base the bundle is a product of the region with the fiber. Picture a fiber bundle this way: attach to every point of a surface a set of possible configurations, such as all the arrows at that point. The fiber is the set attached at one point, and the bundle is the collection of all fibers over all points, joined smoothly. A matter field of the Standard Model is a section of a fiber bundle over spacetime: the section picks one value in the fiber at each point of spacetime. For a matter field the fiber is a vector space on which the gauge group, the group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ of the three forces, acts. The labels color, weak isospin and hypercharge say how the gauge group acts on the fiber of a matter field (*The Concept Sequence*, Chapter 33). A field takes each of its values at a position in the base, and the internal labels of each value lie in the fiber.

Sources: a standard definition. Standing: a definition.

> ***The hair-on-a-head analogy.***
>
> Think of a person's head as the base and the hair at each point as an arrow in the fiber. The collection of all hairs over the scalp is a section of a bundle, the bundle of directions tangent to the surface. If you try to comb all the hairs on a sphere so that they all lie flat and point smoothly, you will always find at least one point where the hair sticks up or where a

crown forms. The point where the hair sticks up is a topological obstruction, a defect that no smooth combing removes, because the defect comes from the shape of the sphere as a whole. In mathematical terms the combed hair is a tangent field, and its zeros are the points where the hair sticks up or a crown forms. The Poincaré-Hopf theorem states that the indices of the zeros of a tangent field add up to the Euler characteristic of the surface, and for the sphere the Euler characteristic is 2. Each zero has an index, which counts how many times the field turns as you go once around the zero. The Euler characteristic is the number of vertices minus the number of edges plus the number of faces, for any division of the surface into polygons.

The internal space $\mathbb{C}P^2 \times S^2$ carries fiber bundles as well. Theorems of topology give the invariants of the internal space and of its bundles. Among the invariants are the Euler characteristics $\chi(\mathbb{C}P^2) = 3$ and $\chi(S^2) = 2$, and the first Chern coefficient $c_1(\mathbb{C}P^2) = 3$, the integer that measures how the tangent bundle of $\mathbb{C}P^2$ is twisted. The counts of the particles rest on the structure of comparison: the three generations of particles (the three repeating families of the matter particles) are the three rotations of the content (A-023). The number of generations equals $c_1 = 3$ because both numbers count the positions of comparison (*The Kinds of Particle*, Chapter 7, Section III). The count of generations is set neither by the index of the Dirac operator nor by the Chern class of the space of states (*The Kinds of Particle*, Chapter 7, Section II). *Non-Commutative Geometry and the Immanent Metaphysics* states a mechanism of topology for the equality of the two numbers as a hypothesis, its Conjecture 10.2.

A-006 The Internal Space as a Manifold. That the internal space $\mathbb{C}P^2 \times S^2$ is a compact smooth manifold of real dimension six (*The Concept Sequence*, Chapter 25). The complex projective plane $\mathbb{C}P^2$ is the set of complex lines through the origin of $\mathbb{C}^3$: two non-zero triples of complex numbers name the same point when one triple is a non-zero complex multiple of the other. The plane $\mathbb{C}P^2$ is a compact manifold of real dimension four. The two-sphere S^2 is the surface of a ball, and S^2 is also the complex projective line $\mathbb{C}P^1$, the set of complex lines through the origin of $\mathbb{C}^2$. The

product $\mathbb{C}P^2 \times S^2$ pairs every point of $\mathbb{C}P^2$ with every point of S^2. The algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ fixes the internal space: the two factors of the space are the state spaces of the two non-commutative summands of the algebra, the summands in which the order of multiplication matters (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 18, Section II). A pure state is a state that is not a mixture of other states. The pure states of $M_3(\mathbb{C})$ form $\mathbb{C}P^2$, and the ray space of $\mathbb{H}$ forms S^2. *The Gravitational Passage* derives the same internal space by a second route that uses no algebra: "Given the identifications of Chapter 9, the internal space is the only compact homogeneous product that meets them" (*The Gravitational Passage*, Chapter 20, Section I). The internal space has two instances. On the content instance the internal space is a space of states, with no size, no radius and no fields. On the context instance the internal space is the space of possibility, six of the ten dimensions of context (the other four are the dimensions of spacetime), with radii $\sqrt{6}\, l_{Pl}$ for $\mathbb{C}P^2$ and l_{Pl} for S^2. Each instance is a range, and neither instance is a place: nothing travels through either instance.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Section III, and Chapter 18, Section II; *The Gravitational Passage*, Chapter 9, Sections VI and VII, and Chapter 20, Section I; *The Measures of the Kinds*, Preface; *Measurement*, Chapter 11, Sections VIII and XII. Standing: a theorem (that $\mathbb{C}P^2 \times S^2$ is a compact smooth manifold of real dimension six); derived (the place of $\mathbb{C}P^2 \times S^2$ as the internal space, as the product of the state spaces of the two non-commutative summands of the derived algebra); derived, given a stated identification (the same space, reached in *The Gravitational Passage* as the only compact homogeneous product that meets the identifications of its Chapter 9).

A-007 The Metric of the Internal Space, on Its Two Instances. That each factor of the internal space carries a metric fixed up to its scale: the Fubini-Study metric on $\mathbb{C}P^2$ and the round metric on S^2. That the scales are fixed in different ways on the two instances. On the content instance, transition probability (the probability that a system in one state is found

in the other) fixes the metric: the distance d between two states a and b satisfies $\cos^2 d = |\langle a|b\rangle|^2$, so a distance is a transition angle and has no length. In the transition unit the radius of $\mathbb{C}P^2$ is 1 and the radius of the doublet sphere S^2 is 1/2. The product metric of the content instance is not an Einstein metric, in the sense that the box below explains (*The Kinds of Particle*, Chapter 3, Section III). On the context instance, *The Gravitational Passage* takes the Ricci curvature of every unit internal direction to be the Planck curvature $1/l_{Pl}^2$. The Planck curvature is the curvature at the modal boundary, and the modal boundary is the bound of every comparison that a located comparer makes. Once every internal Ricci curvature is identified with the Planck curvature, the internal metric is an Einstein metric, with one Ricci constant on both factors. Write R_O for the radius of $\mathbb{C}P^2$ and R_T for the radius of S^2 (the subscripts are labels only). The Fubini-Study metric has Ricci constant $6/R_O^2$, and the round two-sphere has Ricci constant $1/R_T^2$. So the identification sets $6/R_O^2 = 1/R_T^2 = 1/l_{Pl}^2$, which gives $R_O/R_T = \sqrt{6}$, $R_T = l_{Pl}$ and $R_O = \sqrt{6}\, l_{Pl}$. The Planck length l_{Pl} in this entry is the Planck length formed from the measured Newton's constant.

Sources: *The Kinds of Particle*, Chapter 3, Sections I and III; *The Gravitational Passage*, Chapter 9, Sections VIII and IX, Chapter 10, Section V, and Chapter 11, Section IV. Standing: an identification (the curvature of each internal direction at the modal boundary, on the context instance); derived, given a stated identification (the ratio of the radii and the radii themselves, given that identification).

> ***What "Einstein metric" means in plain terms.***
>
> An Einstein metric is a metric whose Ricci curvature (A-003) is the same multiple of the metric in every direction at every point: in the Ricci sense, every direction is curved by the same amount. The Fubini-Study metric on $\mathbb{C}P^2$ is Einstein on its own, and so is the round metric on S^2. The product of the two metrics is Einstein only when the two Ricci constants are equal, and the two constants are equal at exactly one ratio of the radii. In the normalization of *The Gravitational Passage*, every projective line of

$\mathbb{C}P^2$ is a round sphere of radius $R_O/2$. Without a normalization, the Einstein condition says that a projective line of $\mathbb{C}P^2$ has 3/2 times the area of S^2. The Einstein condition is a field equation of the context instance. The content instance carries no Einstein condition, because nothing is solved on a space of states.

A-008 The Curvature of the Internal Space. That when $\mathbb{C}P^2 \times S^2$ carries a product metric, the curvature invariants of its two factors fix the curvature invariants of the product. That the scalar curvature of the product is the sum of the scalar curvatures of the factors. When $\mathbb{C}P^2$ carries the Fubini-Study metric normalized so that its Ricci curvature is six times the metric, the scalar curvature of $\mathbb{C}P^2$ equals twenty-four and its volume equals $\pi^2/2$. In the same normalization the squared norm of the Ricci tensor equals one hundred and forty-four, and the squared norm of the Riemann tensor equals one hundred and ninety-two. For a two-sphere of radius r, the scalar curvature equals $2/r^2$, the squared norm of the Ricci tensor equals $2/r^4$, the squared norm of the Riemann tensor equals $4/r^4$, and the area equals $4\pi r^2$. On the context instance (A-007), each of the six internal directions has Ricci curvature $1/{l_{Pl}}^2$, the scalar curvature is $6/{l_{Pl}}^2$, and the volume is $72(\pi {l_{Pl}}^2)^3$. *The Gravitational Passage* uses the scalar curvature in its Kaluza-Klein reduction, which reads the ten-dimensional geometry as the geometry is seen from the four open dimensions of spacetime. The bare curvature term of that reduction depends on the internal metric only through the scalar curvature, and at the derived radius the bare curvature term is $-3/(8\pi)\ {M_{Pl}}^4$, where M_{Pl} is the Planck mass. On the content instance there is no length, and there is no volume in a radius unit. The volume of $\mathbb{C}P^2$ on the content instance is $\pi^2/2$, in the Fubini-Study measure.

Sources: *The Gravitational Passage*, Chapter 9, Sections VIII and IX, and Chapter 11, Section III; *The Kinds of Particle*, Chapter 3, Section III; *The Strengths of the Kinds*, Chapter 5, Section VII. Standing: a theorem.

The curvature invariants, written symbolically.

For $\mathbb{C}P^2$ (Fubini–Study, Ric $= 6g$): $R = 24$, $|\mathrm{Ric}|^2 = 144$, $|\mathrm{Riem}|^2 = 192$, $\mathrm{Vol}(\mathbb{C}P^2) = \pi^2/2$.

For S^2 of radius r: $R = 2/r^2$, $|\mathrm{Ric}|^2 = 2/r^4$, $|\mathrm{Riem}|^2 = 4/r^4$, $\mathrm{Area}(S^2) = 4\pi r^2$.

On a two-dimensional manifold the Riemann tensor is fixed by the scalar curvature alone, so that $|\mathrm{Riem}|^2 = R^2$ and $|\mathrm{Ric}|^2 = R^2/2$. The plane $\mathbb{C}P^2$ is a symmetric space, the quotient $\mathrm{SU}(3)/\mathrm{U}(2)$. In the normalization Ric $= 6g$ the curvatures of the planes through a point of $\mathbb{C}P^2$ lie between 1 and 4, so the Riemann tensor of $\mathbb{C}P^2$ carries more information than its Ricci tensor. The values for $\mathbb{C}P^2$ satisfy the Chern-Gauss-Bonnet formula, which gives the Euler characteristic from the curvature: $\chi = (1/32\pi^2) \int (|\mathrm{Riem}|^2 - 4|\mathrm{Ric}|^2 + R^2)\ \mathrm{dvol}$. With the values above, $(1/32\pi^2)(\pi^2/2)(192 - 576 + 576) = 3 = \chi(\mathbb{C}P^2)$.

On the context instance, ${R_O}^2 = 6\,{l_{Pl}}^2$ and ${R_T}^2 = {l_{Pl}}^2$, so $\mathrm{Vol}(\mathbb{C}P^2) = (\pi^2/2)\,{R_O}^4 = 18\pi^2\,{l_{Pl}}^4$, $\mathrm{Area}(S^2) = 4\pi\,{l_{Pl}}^2$, and the product has volume $72\pi^3\,{l_{Pl}}^6 = 72(\pi {l_{Pl}}^2)^3$.

A-II. The Dirac Operator in Detail

A-009 Spectrum of an Operator. That the spectrum of a self-adjoint operator is the set of numbers λ for which the operator minus λ times the identity has no bounded inverse. That when the operator acts on a finite-dimensional space, or is an elliptic self-adjoint differential operator on a compact manifold, the spectrum consists of eigenvalues only, each of finite multiplicity, which can be listed in increasing order. An eigenvalue of an operator is a number λ for which some non-zero vector is simply multiplied by λ when the operator acts on the vector, and the multiplicity of the eigenvalue is the number of independent vectors of that kind. A self-adjoint operator is an operator equal to its own adjoint, and its spectrum

consists of real numbers. The spectrum is to an operator what the list of resonant frequencies is to a musical instrument. In a spectral triple (A-027) the spectrum of the Dirac operator is the datum from which the spectral action is computed (A-033). *Non-Commutative Geometry and the Immanent Metaphysics* reads the spectrum of the Dirac operator as the catalogue of the scales of comparison (Chapter 4, Section IV).

Sources: a standard definition; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 4, Section IV. Standing: a definition; an identification (the spectrum of the Dirac operator read as the catalogue of the scales of comparison).

> ***"Can one hear the shape of a drum?"***
>
> The mathematician Kac (1966) asked whether the spectrum of a drum, its complete list of resonant frequencies, determines the shape of the drumhead. In general the spectrum does not determine the shape. Milnor (1964) gave two sixteen-dimensional flat tori (spaces shaped like the surface of a doughnut, but with no curvature) with the same spectrum and different shapes, and Gordon, Webb and Wolpert (1992) gave two plane drums of different shape with the same spectrum. So the spectrum alone does not carry the whole geometry. A spectral triple carries more than the spectrum of its Dirac operator: a spectral triple carries the algebra and its representation (the way the algebra acts on the Hilbert space) as well. From the algebra, its representation and the Dirac operator together, the geometry can be recovered (A-027).

A-010 The Dirac Operator. That on a manifold with a metric and a spin structure, the Dirac operator D is the first-order differential operator acting on spinor fields that is built from the Levi-Civita connection (A-004) and Clifford multiplication, the multiplication of spinors by the matrices γ^μ below. That in local terms $D = \Sigma\, \gamma^\mu \nabla_\mu$, where the γ^μ are matrices satisfying $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (up to a sign convention). That the square of D is a Laplace-type operator (A-012). The Laplacian generalizes the second derivative to curved spaces. The Dirac operator differs from the Laplacian in that the Dirac operator acts on spinor fields: fields whose values carry

an internal orientation that changes sign under a full turn of the frame. The Dirac operator, and not the Laplacian, is the operator of the equation of motion of the fermions (the particles of half-integer spin), because fermions are described by spinor fields.

Sources: Dirac (1928); Atiyah and Singer (1963); a standard definition. Standing: a definition.

> ***Spinors and handedness.***
>
> A scalar field assigns a single number to each point of the manifold: a temperature, a density, a potential. A spinor field assigns to each point a value that transforms in a special way under rotations of the frame at that point. When the frame is turned through one full turn, the spinor returns to its negative, and the spinor returns to itself only after two full turns. The return after two turns, and not after one, is called the double-covering property, and the double-covering property is the mathematical mark of half-integer spin. In even dimensions the spinors at a point split into two halves of opposite chirality (handedness), the left-handed and the right-handed, and the Dirac operator maps each half into the other.

A-011 The Spin-c Structure. That $\mathbb{C}P^2$ admits spin-c structures and admits no spin structure. That a Dirac operator on $\mathbb{C}P^2$ is therefore a spin-c Dirac operator, coupled to a $U(1)$ connection on a line bundle (a bundle whose fiber is a single complex line). A spin structure is a consistent rule for how spinors turn when the frame turns, and the box below explains what a spin-c structure adds. The obstruction to a spin structure is the second Stiefel-Whitney class. For $\mathbb{C}P^2$ the second Stiefel-Whitney class is the reduction modulo 2 of the first Chern class $c_1(\mathbb{C}P^2) = 3h$, where h is the class of a projective line, and the class is not zero because 3 is odd. Every oriented four-manifold admits a spin-c structure. As a complex surface, $\mathbb{C}P^2$ has a canonical spin-c structure, whose determinant line bundle is the anticanonical bundle $O(3)$, with first Chern class $3h$. The coefficient 3 equals the Euler characteristic $\chi(\mathbb{C}P^2) = 3$. The two-sphere S^2 is a spin manifold. The phase enters the content instance through the circle of

phase that comes with the spin-c structure: "the phase is the symmetry of that bundle, and the hypercharge boson comes from it" (*The Story of the Particles*, note 5; *The Kinds of Particle*, Chapter 4, Section IV). On the side of the algebra, the $U(1)$ of the circle of phase comes from the summand $\mathbb{C}$ of $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$. The summand $\mathbb{C}$ has no inner automorphisms, and it acts on the finite Hilbert space through the two-sided action (A-052). An inner automorphism is a map $a \mapsto uau^*$ given by a unitary u of the algebra itself, and on the commutative $\mathbb{C}$ every such map is the identity. The number of generations is set by the structure of comparison, and not by the spin-c structure or by $c_1 = 3$ (*The Kinds of Particle*, Chapter 7, Section II).

Sources: *The Kinds of Particle*, Chapter 3, Section IV, Chapter 4, Section IV, and Chapter 7, Section II; *The Story of the Particles*, note 5; *The Gravitational Passage*, Chapter 8, Section IV, and Chapter 12, Section VIII; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 14, Section II; Lawson and Michelsohn (1989), Appendix D. Standing: a theorem ($\mathbb{C}P^2$ is spin-c and not spin); an identification (the hypercharge boson from the circle of phase).

> ***What a spin-c structure does.***
>
> A spinor needs, at each point, a consistent rule for how the spinor turns when the frame turns (A-010). On $\mathbb{C}P^2$ no consistent rule exists for spinors alone: carried around certain loops of frames, the spinors come back with the wrong sign. You remove the defect by coupling the spinor to a circle of phase at every point: the spinor and the phase are each ambiguous by a sign, and the product of the spinor and the phase is not. The pair of spinor and phase is a spin-c structure, and the circle of phase, with its connection, is the $U(1)$ that comes with the spin-c structure. On the content instance the spin-c structure is a property of the topology of $\mathbb{C}P^2$, and no field or flux lives on $\mathbb{C}P^2$ (*The Kinds of Particle*, Chapter 3, Section IV).

A-012 Lichnerowicz Formula. That the square of the Dirac operator equals the connection Laplacian (the operator $\nabla^*\nabla$, which measures how fast a spinor field varies from point to point) plus a curvature term. For the Dirac operator of a spin manifold the curvature term is one quarter of

the scalar curvature: $D^2 = \nabla^*\nabla + R/4$. For a spin-c Dirac operator coupled to a connection A on the determinant line bundle, a further term appears: one half of the curvature F_A of the connection A, acting on spinors by Clifford multiplication. The heat-kernel coefficients of D^2 (A-018) are computed from the curvature of the manifold and of the bundle by means of the Lichnerowicz formula. The Lichnerowicz formula also gives a theorem of Lichnerowicz: on a compact spin manifold of positive scalar curvature, D has no zero eigenvalue.

Sources: Schrödinger (1932); Lichnerowicz (1963); Lawson and Michelsohn (1989), Appendix D. Standing: a theorem.

> ***The Lichnerowicz formula, written symbolically.***
>
> For a spin manifold: $D^2 = \nabla^*\nabla + R/4$, where R is the scalar curvature.
>
> For a spin-c structure with connection A on the determinant line bundle: $D_A{}^2 = \nabla^*\nabla + R/4 + \tfrac{1}{2}\, c(F_A)$, where $c(F_A)$ denotes the Clifford action of the curvature two-form F_A (an imaginary-valued two-form in the usual convention) on spinors.

A-013 Decomposition on a Product Manifold. That on a product of two even-dimensional manifolds, each with a spin or spin-c structure, the Dirac operator of the product is $D = D_1 \otimes 1 + \gamma_1 \otimes D_2$, where γ_1 is the chirality grading of the first factor, the operator that tells the left-handed spinors from the right-handed. That $D^2 = D_1{}^2 \otimes 1 + 1 \otimes D_2{}^2$, because D_1 anticommutes with γ_1. That the eigenvalues of D^2 are therefore the sums of an eigenvalue of $D_1{}^2$ and an eigenvalue of $D_2{}^2$, with eigenvectors built from products of eigenvectors of the factors. The product form of this entry also joins spacetime to the finite triple in the spectral action (A-028). On the content instance of $\mathbb{C}P^2 \times S^2$ the mathematics of this entry is used for one count only. A harmonic spinor is a spinor that the Dirac operator sends to zero. The cohomology of the line bundle $O(k)$ gives the harmonic spinors of the spin-c Dirac operator on $\mathbb{C}P^2$ twisted by $O(k)$. There is one state for $k = 0$ and for $k = -3$, there are three for $k = 1$ and for $k = -4$,

and there are none for $k = -1$ and $k = -2$. *The Kinds of Particle* uses the count of harmonic spinors for the representation content of one generation, sixteen spinor states. The count is a count of spinors and representations, and it is not a set of modes of a field: the count gives the kinds within one generation, and it gives no multiplicity of generations.

Sources: a standard result (Lawson and Michelsohn 1989); *The Kinds of Particle*, Chapter 3, Sections IV and VII, and Chapter 7, Section II. Standing: a theorem; an identification (the use of the count for the kinds of one generation, *The Kinds of Particle*, Chapter 3, Section IV).

A-014 The Quantum Numbers Come from the Representation. That the quantum numbers of the fermions are outputs of the representation of the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ on the finite Hilbert space and of the unimodularity condition. That the quantum numbers are not labels of the eigenvalues of a Dirac operator on the internal space. The quantum numbers of the fermions are color, weak isospin and hypercharge. The unimodularity condition is the condition that each unitary of the algebra act on the finite Hilbert space with determinant one (A-052). One generation contains six multiplets, each written as (color, weak isospin, hypercharge) under $SU(3) \times SU(2) \times U(1)$, in the convention $Q = T_3 + Y$, where Q is the electric charge, T_3 the third component of weak isospin and Y the hypercharge. The six multiplets are the left-handed quark doublet $(3, 2, 1/6)$, the right-handed up-type quark $(3, 1, 2/3)$, the right-handed down-type quark $(3, 1, -1/3)$, the left-handed lepton doublet $(1, 2, -1/2)$, the right-handed charged lepton $(1, 1, -1)$, and the right-handed neutrino $(1, 1, 0)$. With the three colors counted, the six multiplets hold sixteen states. No quantum number is assigned by hand.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 8, Section II (Proposition 8.3), and Chapter 14, Section IV; *The Story of the Particles*, Level 9, Section III. Standing: a theorem (a computation on the finite triple, given the representation and the unimodularity condition, *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 14).

A-015 Discreteness of a Spectrum, and the Discreteness of Content. That when an elliptic self-adjoint differential operator, such as a Dirac operator or a Laplacian, acts on a compact manifold, its spectrum is discrete. That the eigenvalues of the operator have finite multiplicities and no finite accumulation point, and that their number below a bound grows by Weyl's law (the box below). The discreteness of a spectrum is a theorem about operators, and the discreteness of content has another ground. The ground of the discreteness of content is the Incommensuration Theorem: "When something is absolutely symmetric, it must also be intrinsically discontinuous" (*Incommensuration*, Chapter 6, Section VII). The content of comparison is governed by symmetry, and so the content of comparison is quantized (*The Gravitational Passage*, Chapter 7, Section IV; *The Concept Sequence*, Chapter 17). The content instance of the internal space has no size and carries no tower of modes. A field on an internal space with a size has Kaluza-Klein modes, the modes in which the field can vibrate around that internal space, and the Kaluza-Klein modes belong to the context instance (*The Kinds of Particle*, Chapter 3, Section III).

Sources: Weyl (1911); Minakshisundaram and Pleijel (1949); *Incommensuration*, Chapter 6, Section VII; *The Gravitational Passage*, Chapter 7, Section IV; *The Kinds of Particle*, Chapter 3, Sections III and VII. Standing: a theorem (the discreteness and Weyl's law); derived, given a stated identification (the quantization of content, from the Incommensuration Theorem, given the identification of the content of physics as what the law of symmetry governs).

> ***Weyl's law.***
>
> For the Laplacian on a closed manifold M of dimension n, the number $N(\lambda)$ of eigenvalues not exceeding λ, counted with multiplicity, satisfies $N(\lambda) \sim \omega_n \, \mathrm{Vol}(M) \, \lambda^{n/2} / (2\pi)^n$ as λ grows, where ω_n is the volume of the unit ball in $\mathbb{R}^n$. So the count of the high eigenvalues measures the dimension and the volume of the manifold.

A-III. The Heat Kernel and Spectral Geometry

A-016 The Heat Equation. That for a non-negative self-adjoint operator P, such as the Laplacian or the square of a Dirac operator, the heat equation $\partial u/\partial t = -Pu$ describes how an initial distribution spreads and smooths over time on a manifold. In this chapter the heat equation is a tool for drawing geometric information out of a spectrum, and it makes no statement about physical heat. When the initial distribution is written as a sum of eigenvectors of P, the part along an eigenvector with eigenvalue λ decays as $e^{-\lambda t}$: parts with larger eigenvalues decay faster. So the solution of the heat equation carries the eigenvalues, and the behavior of the solution at short times carries the local geometry, without any eigenvalue being computed one by one.

Sources: a standard definition (Fourier 1822). Standing: a definition.

> ***Heat on a curved surface.***
>
> Imagine heating a single point on the surface of a sphere and watching the heat spread. At very short times the heat has not spread far, the sphere looks flat over the warmed patch, and the heat spreads in a round patch as it would on a plane. The first departure from the flat behavior comes from the curvature near the point. So when the total heat is summed over the whole surface, its short-time behavior gives first the area (or, in more dimensions, the volume) and then, term by term, integrals of the curvature. The Seeley-DeWitt expansion (A-018) is the systematic form of that series of terms. At long times the heat settles into the states of zero eigenvalue, and the number of states of zero eigenvalue is a topological quantity. For the Laplacian acting on functions on a connected manifold, the number of states of zero eigenvalue is one: the only state of zero eigenvalue is the constant function.

A-017 The Heat Kernel. That for an operator P, the heat kernel $K(t, x, y)$ is the solution of the heat equation at the point y and time t when a unit of heat is placed at the point x at time zero. That the heat kernel is the sum over n of $e^{-\lambda_n t}\, \varphi_n(y)\, \varphi_n(x)^*$, written in terms of the eigenvalues λ_n and orthonormal eigenvectors φ_n of P. The heat kernel joins the spectrum of an operator (A-009) to the geometry of the manifold (A-001 to A-008). The trace of the heat kernel is the integral of $K(t, x, x)$ over the manifold, and the trace depends only on the spectrum. At short times the trace is given by integrals of local curvature invariants, called the heat invariants. The heat invariants do not in general determine the manifold, because manifolds of different shape can share a spectrum (A-009).

Sources: a standard definition; Minakshisundaram and Pleijel (1949). Standing: a definition.

> ***The heat kernel in symbolic form.***
>
> $K(t, x, y) = \Sigma_n\, e^{-\lambda_n t}\, \varphi_n(y)\, \varphi_n(x)^*$.
>
> The trace of the heat operator is
>
> $\mathrm{Tr}\, e^{-tP} = \int K(t, x, x)\, \mathrm{dvol}(x) = \Sigma_n\, e^{-\lambda_n t}$,
>
> where each eigenvalue is counted as often as its multiplicity. The trace of the heat operator is a function of the time t alone, and its expansion in powers of t at small t is the Seeley-DeWitt expansion.

A-018 The Seeley–DeWitt Expansion. That for a Laplace-type operator P acting on sections of a vector bundle of rank N over a closed manifold of dimension d, the trace of the heat operator has, as t tends to zero, the asymptotic expansion $\mathrm{Tr}\, e^{-tP} \sim \Sigma_k\, t^{(k-d)/2}\, a_k(P)$, the sum running over even $k \geq 0$ (the odd coefficients vanish on a manifold without boundary). That each Seeley-DeWitt coefficient a_k is the integral of a local invariant built from the curvature of the manifold, the curvature of the bundle connection, and the endomorphism E of P. The rank N of the bundle is the dimension of its fiber, and the endomorphism E is the part of P that acts at

each point without taking any derivative. The coefficient a_0 measures the volume times the rank, a_2 measures the integrated scalar curvature and the trace of E, and a_4 measures the integrated squares of the curvatures. The fluctuated Dirac operator is the Dirac operator with the gauge fields and the Higgs field added to it (A-051). In the spectral action (A-033), the coefficients a_0, a_2 and a_4 of the square of the fluctuated Dirac operator give the terms of the action in turn. The coefficient a_0 gives the cosmological term, the term that acts as a constant energy of empty space. The coefficient a_2 gives the Einstein-Hilbert term (the action of general relativity, the integrated scalar curvature) with the mass term of the Higgs field. The coefficient a_4 gives the gauge, Higgs and curvature-squared terms (A-020).

Sources: DeWitt (1965); Seeley (1967); Gilkey (1975); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III. Standing: a theorem.

> ***The first three coefficients.***
>
> Write the operator as $P = -(g^{ij}\nabla_i\nabla_j + E)$, acting on a bundle of rank N with connection curvature Ω over a closed manifold of dimension d, with R the scalar curvature and $|Ric|^2$, $|Riem|^2$ the squared norms of the Ricci and Riemann tensors. Then (Gilkey 1975):
>
> $a_0 = (4\pi)^{-d/2} \int N \, dvol$
>
> $a_2 = (4\pi)^{-d/2} (1/6) \int tr(6E + R) \, dvol$
>
> $a_4 = (4\pi)^{-d/2} (1/360) \int tr[60RE + 180E^2 + 30\Omega_{ij}\Omega^{ij} + (5R^2 - 2|Ric|^2 + 2|Riem|^2)] \, dvol,$
>
> where terms that are total derivatives, and so integrate to zero on a closed manifold, are omitted. For the square of the Dirac operator of a spin manifold, the Lichnerowicz formula (A-012) gives $E = -R/4$ in this sign convention.

A-019 The Coefficients on the Product Geometry. That in the spectral action the Seeley-DeWitt coefficients are the coefficients of the square of the fluctuated Dirac operator D_A of the product geometry $M \times F$ (A-028, A-051), where M is a four-dimensional spin manifold and F the finite spectral triple. That the trace is taken over the spinors on M and over the finite Hilbert space $\mathscr{H}_F$. On the product geometry the endomorphism E of ${D_A}^2$ contains the Higgs field, and the bundle curvature Ω contains the field strengths of the gauge fields. The leading coefficient is proportional to the volume of M times $\dim \mathscr{H}_F = 96$. In the expansion of the spectral action the volume coefficient is $48 f_4 \Lambda^4/\pi^2$, and the coefficient of the Euler density (the curvature density whose integral gives the Euler characteristic) is $11 f_0/(60\pi^2)$, the Euler weight being $11/(5760\pi^2)$ for each Dirac spinor. The symbols f_4 and f_0 are moments of the cutoff function (A-034), and Λ is the energy scale of the spectral action. The coefficients of the expansion of the spectral action, other than the volume coefficient and the coefficient of the Euler density, depend on conventions of normalization. Chamseddine, Connes and Marcolli (2007) give the remaining coefficients. The internal space $\mathbb{C}P^2 \times S^2$ enters the product geometry as the space of states of the non-commutative summands of the finite algebra (A-029), and the internal space is not a factor of the product. The spectral action contains no heat-kernel coefficient of a Dirac operator on $\mathbb{C}P^2 \times S^2$.

Sources: Chamseddine, Connes and Marcolli (2007); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III, and Chapter 16, Section I; *The Kinds of Particle*, Chapter 3, Section V; *The Strengths of the Kinds*, Chapter 5, Section III; *The Connes Correspondence*, Chapter 6, Section III. Standing: a theorem of Chamseddine, Connes and Marcolli (2007), reported.

A-020 What Each Coefficient Gives. That in the asymptotic expansion of the bosonic spectral action in the cutoff Λ, $\mathrm{Tr}\, f(D_A/\Lambda) \sim \Sigma_k\, f_k\, \Lambda^k\, a_{4-k}({D_A}^2) + O(\Lambda^{-1})$, the three terms give the form of the physics set out below (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III). The bosonic spectral action is the first term of the spectral

action (A-033). The term in Λ^4 is a cosmological term: it carries the moment f_4 and the coefficient a_0, and it is proportional to the volume of spacetime and to dim $\mathcal{H}_F$. The term in Λ^2 carries f_2 and a_2, and it gives the Einstein-Hilbert action and a mass term for the Higgs field. The term independent of Λ carries f_0 and a_4, and it gives the Yang-Mills action of the three gauge groups (the action of the gauge fields), the kinetic and quartic terms of the Higgs field (the terms for its motion and for its interaction with itself), the coupling of the Higgs field to curvature, and a Weyl-curvature term. The expansion fixes the form of the three terms. The values in the terms come from elsewhere: the couplings (the numbers that set the strengths of the forces) and the masses. At the scale Λ the expansion imposes relations among the couplings (A-037), and the values at any one scale are measured, or are given by the accounts set out in entries A-022 and A-046 to A-049.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III; *The Connes Correspondence*, Chapter 6, Sections I and III; Chamseddine, Connes and Marcolli (2007). Standing: a theorem of Chamseddine, Connes and Marcolli (2007), reported.

> ***The expansion and its three terms.***
>
> The bosonic spectral action is Tr $f(D_A/\Lambda)$, with Λ an energy scale and f a positive cutoff function (A-034). The moments of f are $f_0 = f(0)$ and $f_k = \int_0^\infty f(v)\, v^{k-1}\, dv$ for $k > 0$.
>
> Tr $f(D_A/\Lambda) \sim f_4\Lambda^4\, a_0 + f_2\Lambda^2\, a_2 + f_0\, a_4 + \ldots$
>
> The first term is the cosmological term. The second term is the Einstein-Hilbert term together with the mass term of the Higgs field. The third term is the Yang-Mills action, the Higgs kinetic and quartic terms, the coupling of the Higgs field to curvature, and the Weyl-curvature term. Three terms of the expansion give the form of the bosonic Lagrangian, and the values in the bosonic Lagrangian come from measurement or from the accounts of the values.

A-IV. Masses, Generations and the Count 72

A-021 The Masses Belong to the Finite Dirac Operator. That the rest masses of the particles are carried by the entries of the finite Dirac operator D_F of the finite spectral triple, the Yukawa matrices (A-032), which are the matrices of the couplings of the fermions to the Higgs field. *Non-Commutative Geometry and the Immanent Metaphysics* states the point in these words: "Rest mass is a Yukawa eigenvalue" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section XIII). The entries of D_F are the moduli of the finite geometry, the parameters that its constraints leave free, and the derivation of the algebra stops at the moduli (A-040). The values of the masses are given by the mass ladder of *The Measures of the Kinds* (A-022). So the masses are not eigenvalues of a Dirac operator on $\mathbb{C}P^2 \times S^2$, and the particles are not vibration modes of the internal space. On the content instance the internal space has no size, no radius and no fields, and no tower of modes is formed on the internal space. A harmonic form is a form that the Laplacian sends to zero. On the content instance the harmonic forms are invariant under SU(3) and uniform, so no preferred locus in the space distinguishes three generations, and the space of states supplies no hierarchy of masses (*The Kinds of Particle*, Chapter 3, Sections IV and VIII).

Sources: *The Kinds of Particle*, Chapter 3, Sections III, IV, VII and VIII; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V, Chapter 10, Section V, and Chapter 19, Section XIII; *The Connes Correspondence*, Chapter 7, Section II. Standing: a definition (the masses as entries of D_F, in the finite triple); a theorem (the content instance has no length, and its harmonic forms are uniform).

A-022 The Mass Ladder. That *The Measures of the Kinds* places every charged fermion by one formula, the step formula $n = (\Delta + gP + g^2d)\ s$. In the step formula, n is the count of halvings of mass below the top quark, where a halving is a factor of two in mass. The symbol g is the gen-

eration index, 0 for the heaviest generation and 2 for the lightest. In the step formula a sector is one family of charged fermions: the up-type quarks, the down-type quarks or the charged leptons. The symbol Δ is the offset of the sector, P its step cost and d its curvature, and s is the step factor shared by every sector. The form of the step formula follows from comparison. One comparison has two outcomes, so one halving is a factor of 2 in mass. Three generations give three points per sector, and three points fix a quadratic in g with no cubic term. The step costs are $P_q = 6$ halvings per generation for the quarks and $P_l = 9/4$ for the charged leptons. The curvatures are $d_{up} = 1$, $d_{dn} = -7/12$ and $d_{lep} = 16/9$. The offsets are $\Delta_{up} = 0$, $\Delta_{dn} = 53/10$ and $\Delta_{lep} = 391/60$. The step factor is $s = 1 + \varepsilon - c_2\varepsilon^2 = 1.0128448$, with $\varepsilon = 11/(80\pi^2) = 0.013932$ and $c_2 = 28/5$. On the mass ladder the electron lies 18.361 halvings below the top quark. The hierarchy of the masses is placed by the mass ladder, and not by a spacing of eigenvalues.

Sources: *The Measures of the Kinds*, Chapter 7, Sections I to XI and Summary, and Chapter 13, Section X. Standing: derived (the form of the ladder, the step costs and the curvatures); a measured input (the bottom and the tau, from which the two offsets are found by a condition that every true instance of the ladder meets); a measured agreement (the kernel writings of the offsets, each carried with the question why it lands). The positions of the masses carry the standings given in entry A-049.

> ***The nine charged fermions on the ladder.***
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> The undressed counts are the counts computed from the step formula alone. With the constants applied exactly as stated and none adjusted, the undressed counts in halvings below the top are: top 0, charm 7.090, up 16.206; bottom 5.368, strange 10.854, down 15.159; tau 6.600, muon 10.680, electron 18.361 (*The Measures of the Kinds*, Chapter 7, Table XI-1). The order that the ladders give is the measured order, including the down quark heavier than the up quark.

> *The Measures of the Kinds* states where the test of the formula lies (Chapter 7, Section I). The formula has eight constants (P_q, P_l, d_{up}, d_{dn}, d_{lep}, Δ_{dn}, Δ_{lep} and s), as many as the eight charged-fermion masses above the electron, and for any eight masses there are eight constants that reproduce those masses. So the test of the formula lies in the fractions of 3, 4 and 5 that the constants take, and in the identities among the constants, and not in the agreement of the undressed formula with the masses alone.

A-023 Three Generations. That there are three generations because there are three rotations of the content around the cycle of Axiom II, and a fourth rotation is the first rotation again. Axiom II is the axiom that orders the three positions of comparison in a directed cycle: transcendent to immanent, immanent to omniscient, omniscient to transcendent ($T \rightarrow I \rightarrow O \rightarrow T$). In the focus of comparison as relation, the first generation has the immanent pair foremost (objective/subjective; force/time), the second generation the omniscient pair (sameness/difference; inertia/space), and the third generation the transcendent pair (content/context; probability/possibility). The direction of the cycle bounds the count of generations from above, because an exchange of two pairs gives no instance. Axiom III bounds the count from below, because no pair can take the place of another (*The Kinds of Particle*, Chapter 7, Section I). On the content instance the index of the Dirac operator gives one, and not three. The Chern class gives a threefold multiplicity, and that multiplicity is a color triplet, and not three generations (*The Kinds of Particle*, Chapter 7, Section II). The number of generations equals $c_1 = 3$ because both numbers count the positions of comparison (*The Kinds of Particle*, Chapter 7, Section III). *Non-Commutative Geometry and the Immanent Metaphysics* fixes the equality of the number of generations with $c_1 = 3$ under its postulate P-Rep (Theorem 12.1), and again under its postulates P-Gen and P-Cl (Corollary 19.11). P-Rep is the postulate that the form of what is found is the form of finding. P-Cl is the closure postulate, that $c_1^2 + N^2$ is the square of a positive integer H, with $N = 4$ the real dimension of the quaternions $\mathbb{H}$. P-Gen sets the space of the copies of one generation as the

regular representation of the cyclic group of order three generated by the rotation of Axiom II. *Non-Commutative Geometry and the Immanent Metaphysics* states a mechanism of topology for the same equality as a hypothesis, its Conjecture 10.2. A second route also gives three generations. The strong coupling is the strength of the strong force, and the strong coupling "runs": its value changes with the energy at which it is measured. The strong coupling runs at the one-loop rate -7, which the quarks of exactly three generations give, and the measured running agrees with the rate -7 (*Geeze, Wublets, and the Higgs*, Level 5, Section VI; *The Connes Correspondence*, Chapter 8, Section V). Measurements of the width of the Z boson give the number of light neutrino species as 2.9963 ± 0.0074 (Janot and Jadach 2020). The measured number of species is half a standard deviation from three (*The Kinds of Particle*, Chapter 7, Section VII).

Sources: *The Kinds of Particle*, Chapter 7, Sections I to IV and VII; *The Story of the Particles*, Level 5; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 10, Section IV (Conjecture 10.2), Chapter 12 (Theorem 12.1), and Chapter 19, Section VII (P-Gen, Corollary 19.11); *Geeze, Wublets, and the Higgs*, Level 5, Section VI; *The Connes Correspondence*, Chapter 8, Sections IV and V; Janot and Jadach (2020). Standing: derived (three rotations of the content, from the cycle of Axiom II); a theorem, given the postulates of *Non-Commutative Geometry and the Immanent Metaphysics*, each an identification (the equality of the number of generations with $c_1 = 3$); a measured agreement (the number of light neutrino species).

A-025 The Count 72 and the Context Instance. That measurement gives the span from the electron mass to the reduced Planck mass, counted in halvings: $E = \log_2(M_{Pl,red}/m_e) = 72.0132112 \pm 0.0000162$. The reduced Planck mass is the mass M at which $8\pi GM^2/(\hbar c)$ equals one, formed from the measured Newton's constant G. The electron mass is the measured unit of content. That the integer of the span is identified with the count $72 = 8 \times 9 = n_C{}^3 c_1{}^2$, written in the two kernel integers. The kernel integers are $c_1 = 3$, the count of the positions of comparison, and $n_C = 2$, the count of the conjugate pair, content and context. The count 72 is

eight first-order states (two outcomes at each of three positions) times nine joins (three positions set against three across the pair). The identification is an assumption, and the measured span is its test. The remainder is $r = E - 72 = 0.0132$. The first order of the remainder is $\varepsilon = 11/(80\pi^2) = 0.013932$, and $E = 72 + \varepsilon(1 - k\varepsilon)$. The lepton ladder gives $k = 3.7160 \pm 0.0012$, against $k = 3.7120 \pm 0.0835$ from the measured span. The question why the integer lands on 72 is carried with its clues. One clue is that on the context instance the same 72 is a volume: with $R_O^2 = 6\, l_{Pl}^2$ and $R_T^2 = l_{Pl}^2$, the volume of $\mathbb{C}P^2 \times S^2$ is $72(\pi l_{Pl}^2)^3$ (A-008). Newton's constant is measured, not predicted.

Sources: *The Strengths of the Kinds*, Chapter 5, Sections I to IV and VII; *The Kinds of Particle*, Chapter 2, Sections I and VII; *The Measures of the Kinds*, Chapter 7, Section X and Summary, and Chapter 13, Table X-1; *The Gravitational Passage*, Chapter 9, Section IX; CODATA 2022 for the electron mass and Newton's constant. Standing: an identification (the integer of the measured span with the count 72), assumed, with the measurement as its test; a measured agreement (the remainder's form $\varepsilon(1 - k\varepsilon)$ with k about 3.71, and the same 72 as the volume of the context instance, each carried with the question why it lands).

The chain, with the kind of each step.

The Root Tautology is the statement that comparison, interaction and relation are one act, named three ways. The Root Tautology gives the six intrinsics of comparison, the three pairs named in A-023. The Incommensuration Theorem is proved from the six intrinsics by finite elimination (a theorem), and the three axioms are derived from the Incommensuration Theorem. Each clause of the axioms is given the form of a condition on a spectral triple, and each resulting condition is a postulate and an identification. The algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ follows from the postulates by theorems of algebra (A-030). When the spectral action is applied to the product of spacetime with the finite triple, with its four inputs stated (A-035), the spectral action gives the form of the Standard Model coupled to gravity (a theorem of Chamseddine, Connes and Marcolli 2007, reported). The derivation stops at the moduli, the entries of the finite Dirac operator. The values of the masses come from the mass ladder (A-022), with

their stated standings (A-049). The couplings are measured at one scale, and the structure of comparison fixes their rank order. Gravitation is derived in *The Gravitational Passage*, with Newton's constant measured. The electron mass and Newton's constant are the two measured anchors.

A-V. The Spectral Triple

A-026 Hilbert Space. That a Hilbert space is a complex vector space with an inner product $\langle \psi, \varphi \rangle$, in which every sequence whose terms come arbitrarily close to one another converges. The inner product $\langle \psi, \varphi \rangle$ is a complex number given to each pair of vectors ψ and φ: it is conjugate-linear in one slot and linear in the other, and it is positive when $\psi = \varphi \neq 0$. That in a spectral triple the algebra and the Dirac operator act on a Hilbert space. For spacetime the Hilbert space is $L^2(M, S)$, the square-integrable spinor fields on M, which are the spinor fields whose squared size, integrated over the whole of M, is finite. For the finite triple of the kinds the Hilbert space is $\mathscr{H}_F = \mathbb{C}^{96}$ (A-032).

Sources: a standard definition (von Neumann 1929); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Section IV, and Chapter 4, Section III; Born (1926). Standing: a definition.

The Hilbert space as a space of states.

A Hilbert space contains every state that a system can be in, including every superposition (combination) of states, and the Hilbert space has a geometry of overlap between states. Two states are compared by their inner product. If the inner product of two states is zero, the two states are orthogonal (at right angles), and they share nothing. If one state is a multiple of the other, the two states are the same state. The transition probability between two normalized states ψ and φ, states of length one, is $|\langle \psi, \varphi \rangle|^2$ (Born 1926). An operator D acts on the Hilbert space: D takes a state ψ to a state $D\psi$. The eigenvalue equation $D\psi = \lambda\psi$ picks out the states that D only rescales, by the number λ. *Non-Commutative Geometry and the*

> *Immanent Metaphysics* identifies the Hilbert space of a comparison triple, the spectral triple that the book builds from the structure of comparison (A-030), with the context in which a comparison is evaluated (Chapter 4, Section III).

A-027 Spectral Triple. That a spectral triple $(A, \mathscr{H}, D)$ consists of an algebra A represented faithfully on a Hilbert space $\mathscr{H}$ and a self-adjoint operator D on $\mathscr{H}$, the Dirac operator. The algebra is represented faithfully when each element of A acts on $\mathscr{H}$ as an operator and no two elements act alike. That a spectral triple is even when it carries a grading γ ($\gamma = \gamma^*$, $\gamma^2 = 1$, γ commuting with A, and $D\gamma = -\gamma D$). The grading splits $\mathscr{H}$ into two parts, because $\gamma^2 = 1$: the states on which γ is $+1$ and the states on which γ is -1. That a spectral triple is real, of KO-dimension n modulo 8, when it carries an antilinear isometry J of $\mathscr{H}$, the real structure, with $J^2 = \varepsilon$, $JD = \varepsilon' DJ$ and $J\gamma = \varepsilon'' \gamma J$, where the three signs ε, ε' and ε'' are fixed by n. An antilinear isometry is a map that keeps every length and turns each complex factor into its complex conjugate. A compact Riemannian spin manifold M has a canonical triple, $(C^\infty(M), L^2(M, S), \not{D})$: the smooth functions on M, the square-integrable spinor fields on M, and the Dirac operator of M. For the canonical triple, the geodesic distance between two points x and y is recovered as the supremum (the least upper bound) of $|a(x) - a(y)|$ over the functions a with $\|[D, a]\| \leq 1$ (Connes 1994). The reconstruction theorem (Connes 2013) states that a commutative spectral triple comes from a manifold when the triple meets Connes's conditions. The supremum formula of the canonical triple also defines a spectral distance between states in any spectral triple, finite spectral triples included.

Sources: Connes (1994; 1996; 2013); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Section IV (Definitions 3.10 to 3.12), and Chapter 4. Standing: a definition (the triple); a theorem (the distance formula and the reconstruction theorem, Connes).

What the three ingredients carry.

In Riemannian geometry a space is given by its points, its distances and its curvature. In a spectral triple the algebra plays the part of the functions on the space, and the Hilbert space is the space of states on which those functions act. The Dirac operator measures how fast the functions change from place to place. The distances and the curvature of the space can be recovered from the algebra, the Hilbert space and the Dirac operator taken together. The language of spectral triples also describes spaces that are not manifolds. The finite triple of the kinds describes a space that is not a manifold: its algebra is $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, its Hilbert space is $\mathbb{C}^{96}$, and its geometry falls into separate metric components, pieces that lie at no finite distance from one another (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 4, Section VIII). The spectral action is evaluated on the product of the finite triple with spacetime (A-028).

Non-Commutative Geometry and the Immanent Metaphysics pairs the components of a real even triple with features of comparison: the algebra with composition; the Hilbert space with the context of evaluation; the Dirac operator with sameness and difference, the metric of comparison; the real structure J with the two terms of a state, the conjugate pair of Axiom I; and the grading γ with content and context. Each pairing is an identification, and that book states each pairing as an identification in its Chapter 4.

A-028 The Product Geometry. That the spectral action is evaluated on one spectral triple, the product of the canonical triple of a four-dimensional spin manifold M with the finite triple F: $A = C^\infty(M) \otimes A_F$, $\mathscr{H} = L^2(M, S) \otimes \mathscr{H}_F$, $D = \not{D}_M \otimes 1 + \gamma_5 \otimes D_F$, with $J = J_M \otimes J_F$ and $\gamma = \gamma_5 \otimes \gamma_F$. The KO-dimensions of the two factors add, so the product has KO-dimension $4 + 6 = 10 \equiv 2$ modulo 8. In KO-dimension 2 modulo 8 the bilinear form $(\xi, \eta) \mapsto \langle J\xi, D\eta \rangle$ is antisymmetric on the even part of $\mathscr{H}$, the part on which the grading γ is $+1$. The fermionic action is defined through this bilinear form, and the fermionic action so defined removes the doubling of the fermionic degrees of freedom, by which the product Hilbert space holds the states of each fermion twice over (Connes 2006; Barrett 2007). In

the construction of Chamseddine, Connes and Marcolli the manifold M is Riemannian: its metric counts every direction alike, as a direction of space. The product also has a Lorentzian form, whose metric counts one direction, time, with the sign opposite to the three directions of space (Barrett 2007). *Measurement* derives that pattern of signs, one sign for time against three for space (*Measurement*, Chapter 11, Section IX).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Sections I and V; *The Gravitational Passage*, Chapter 8; *Measurement*, Chapter 11, Section IX; Connes (2006); Barrett (2007); Chamseddine, Connes and Marcolli (2007). Standing: a definition (the product); a theorem (the KO-dimension of the product and the removal of the doubling).

The two modes side by side.

The product geometry composes the two modes that the Incommensuration Theorem permits: the continuous and asymmetric mode, and the discontinuous and symmetric mode. The manifold factor carries the arena, curved spacetime, in which the particles are found. The manifold factor is continuous in its geometry, because its points form one metric component, each point at a finite distance from every other. The manifold factor is asymmetric, because a generic metric admits no isometry (no motion of the space that keeps every distance). The finite factor carries the content. The finite factor is discontinuous in its geometry, because it falls into separate classes of comparison, and the finite factor is symmetric, because its inner symmetries act exactly. The two modes are joined by scope, as the Incommensuration Theorem allows: the product keeps the continuous, asymmetric character in the manifold factor and the discontinuous, symmetric character in the finite factor, and no single geometry perfects both continuity and symmetry (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section V).

The Gravitational Passage works with a product of the same two modes, $M^4 \times \mathbb{C}P^2 \times S^2$. In the product of *The Gravitational Passage* the internal factor is the context instance, the six dimensions of possibility, with radii.

In the product $M \times F$ of the spectral action the internal factor is the finite triple, and $\mathbb{C}P^2 \times S^2$ appears only as the space of states of the non-commutative summands of the finite algebra, which is the content instance.

A-029 The Finite Algebra. That the finite algebra is the direct sum $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ of the complex numbers, the quaternions and the three-by-three complex matrices. That in the frame of measurement of *The Story of the Particles*, the frame in which the comparing act is light, $\mathbb{C}$ carries the immanent, $\mathbb{H}$ the transcendent and $M_3(\mathbb{C})$ the omniscient. Each summand gives one factor of the gauge group (A-052): $\mathbb{C}$ gives the $U(1)$ of hypercharge, the force on the phase; $\mathbb{H}$ gives the $SU(2)$ of the weak force, which acts on left-handed doublets; and $M_3(\mathbb{C})$ gives the $SU(3)$ of color. The algebra itself is a list of three summands, and not a triple that carries the three modalities, the three positions of comparison (the immanent, the omniscient and the transcendent): the modal labeling is brought to the algebra (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 19.1). *Non-Commutative Geometry and the Immanent Metaphysics* labels the same summands by an admissible rotation of the frame of measurement: $\mathbb{C}$ omniscient, $\mathbb{H}$ immanent, $M_3(\mathbb{C})$ transcendent.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Sections II and III, Chapter 14, and Chapter 19, Section I (Propositions 19.1 and 19.2); *The Story of the Particles*, Level 9, Section III. Standing: a definition (the algebra); an identification (the modal labels, in the frame named).

The three summands of the finite algebra.

(1) The complex numbers $\mathbb{C}$ are the numbers $a + bi$ with $i^2 = -1$. When $\mathbb{C}$ is taken as a real algebra with its conjugation, $\mathbb{C}$ has exactly one state, and the ray space of $\mathbb{C}$ is a single point. $\mathbb{C}$ has no inner automorphisms, the maps $x \mapsto uxu^{-1}$ by a unitary u of the algebra, because $\mathbb{C}$ is commutative and every such map leaves each number as it is. The $U(1)$ of $\mathbb{C}$ still acts on the Hilbert space, through the two-sided action of the algebra (A-052).

(2) The quaternions $\mathbb{H}$ are the numbers $q = a + bi + cj + dk$ with real a, b, c, d and $i^2 = j^2 = k^2 = ijk = -1$, introduced by the mathematician William Rowan Hamilton (Hamilton 1844). The quaternions form a real algebra of dimension four, and $\mathbb{H}$ acts on $\mathbb{C}^2$ as the 2×2 complex matrices of the form $[[\alpha, \beta], [-\bar{\beta}, \bar{\alpha}]]$. As a real algebra $\mathbb{H}$ has exactly one state. The ray space of $\mathbb{H}$ is $\mathbb{C}P^1 \cong S^2$, the space of complex lines in $\mathbb{C}^2$, and the unit quaternions act transitively on $\mathbb{C}P^1$, carrying any point to any other, through $SU(2) \to SO(3)$.

(3) The three-by-three complex matrices $M_3(\mathbb{C})$ form an algebra of complex dimension nine, and $M_3(\mathbb{C})$ is non-commutative: the order of multiplication matters. The pure states of $M_3(\mathbb{C})$ are the vector states $m \mapsto \langle \xi, m\xi \rangle$ for unit vectors ξ in $\mathbb{C}^3$, and the pure states of $M_3(\mathbb{C})$ form $\mathbb{C}P^2$.

The symbol $\oplus$ denotes the direct sum: an element of A_F is a triple (z, q, m) with z complex, q a quaternion and m a 3×3 matrix, and sums and products are taken component by component. The two non-commutative summands give the two factors of the internal space: $\mathbb{H}$ gives S^2, and $M_3(\mathbb{C})$ gives $\mathbb{C}P^2$ (A-006).

A-030 Derivation of the Finite Algebra. That the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is derived from the structure of comparison. The Incommensuration Theorem is proved from comparison, and the three axioms are derived from the Incommensuration Theorem. Each clause of the axioms is then given the form of a condition on a comparison triple. A comparison triple is a real even finite spectral triple of KO-dimension 6, and its conditions are named CT1 to CT6, with P-KO (A-031). Two postulates give $A \cong \mathbb{C} \oplus \mathbb{H} \oplus M_n(\mathbb{C})$ with $n \geq 2$. A carrier is a summand of the algebra that carries one of the three modalities (A-029). The postulate P-III states that there is one carrier for each modality, and that the three carriers are of three distinct characters. The postulate P-I states that the carriers of the conjugate pair are of complex type. P-III and P-I give the algebra by the structure theory of finite-dimensional algebras (the Artin-Wedderburn theorem) and by the theorem of Frobenius (1878) that the real division algebras of finite dimension are $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{H}$. The statement that P-III and P-

I give $\mathbb{C} \oplus \mathbb{H} \oplus M_n(\mathbb{C})$ is the Three-Carrier Theorem (*Non-Commutative Geometry and the Immanent Metaphysics*, Theorem 7.3). In the labeling of *Non-Commutative Geometry and the Immanent Metaphysics*, the Three-Carrier Theorem fixes $\mathbb{H}$ as the carrier of the immanent. In the frame of measurement the same summand $\mathbb{H}$ carries the transcendent, and the two labelings are one structure seen from two standpoints (Proposition 19.2). The postulate P-Rep states that the form of what is found is the form of finding. P-Rep gives $n = c_1 = 3$, the structural constants $(c_1, N, H) = (3, 4, 5)$, which satisfy $3^2 + 4^2 = 5^2$, and three generations (Theorem 12.1). Each postulate is an identification: a stated step from comparison to a condition on the triple. The steps from the postulates to the algebra are theorems. Chamseddine and Connes (2007; 2008) obtained the same algebra by a classification under their own hypotheses: irreducibility, KO-dimension 6, quaternion linearity, a non-trivial grading, the choice of the simplest remaining case, and a Majorana coupling of the right-handed neutrino, a coupling of the right-handed neutrino to its own antiparticle. The derivation from comparison and the classification of Chamseddine and Connes have one intermediate result in common, KO-dimension 6, and each arrives at KO-dimension 6 for its own reasons.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 4, Chapter 7 (Theorem 7.3), Chapter 9, Section V, Chapters 10 to 12 (Theorem 10.1, reported, and Theorem 12.1), Chapter 18, Section I, and Chapter 19, Section I (Proposition 19.2); *The Connes Correspondence*, Chapter 1, Section 10; Frobenius (1878); Chamseddine and Connes (2007; 2008). Standing: a theorem, given the postulates, each of which is an identification.

A-031 Real Structure, Grading and the Order Conditions. That two order conditions tie the components of a real even spectral triple (A-027) to one another. The order conditions are written with the right action $b^\circ = Jb^*J^{-1}$ of an element b of the algebra. That the order-zero condition is $[a, b^\circ] = 0$ for all a and b in A, which says that the left action of A and its right action through J commute, giving the same result in either order. That the order-one condition is $[[D, a], b^\circ] = 0$ for all a and b in A. Under

the order-one condition the inner fluctuations of D, the changes of D that the algebra itself produces, take the linear form $D + A + JAJ^{-1}$ (A-051; Chamseddine, Connes and van Suijlekom 2013). For the finite triple of the kinds the signs of the real structure are $J^2 = +1$, $JD = DJ$ and $J\gamma = -\gamma J$. *Non-Commutative Geometry and the Immanent Metaphysics* states the three signs as its postulate P-KO, and in words P-KO says three things: exchanging the two terms of a state twice returns the state; exchanging the two terms exchanges content and context; and the act of comparison is indifferent to the exchange. *Non-Commutative Geometry and the Immanent Metaphysics* proves that the three signs hold exactly when the KO-dimension is 6 modulo 8 (Proposition 4.2).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Section IV (Definition 3.11), and Chapter 4, Sections V to VIII (Proposition 4.2); Connes (1996); Chamseddine, Connes and van Suijlekom (2013). Standing: a definition (the order conditions); a theorem (Proposition 4.2 of *Non-Commutative Geometry and the Immanent Metaphysics*); an identification (the postulate P-KO).

> ***The order conditions in plain language.***
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> The order-zero condition says that the algebra acts on the Hilbert space from two sides at once, on the left directly and on the right through J, and that the two actions can be applied in either order with the same result. A bimodule is a space on which an algebra acts from both sides, with the two actions giving the same result in either order, so the order-zero condition makes the Hilbert space a bimodule. *Non-Commutative Geometry and the Immanent Metaphysics* identifies the two sides of the action with the two terms of a state, and that book identifies J, which exchanges the two sides, with charge conjugation, the exchange of each particle with its antiparticle. The order-one condition says that the commutators $[D, a]$ commute with the right action, and the commutators play the part of one-forms, the objects that record how fast a function changes in each direction. When the algebra is commutative, the order-one condition says that D is a first-order differential operator, an operator that takes only first derivatives. In the finite triple of the kinds, the order-one condition restricts

> the finite Dirac operator D_F to a definite block form. The three carriers $\mathbb{C}$, $\mathbb{H}$ and $M_3(\mathbb{C})$ are the corners of a triangle, and an edge of the triangle joins two carriers. A sector is one of the four groups of particle states listed in A-032. D_F changes at most one index of a sector at a time, and every change that D_F makes to a carrier index is along the one edge between $\mathbb{H}$ and $\mathbb{C}$ (Proposition 8.4; A-032).

A-032 The Finite Hilbert Space and the Finite Dirac Operator. That the finite Hilbert space is $\mathscr{H}_F = \mathbb{C}^{96}$, with $96 = 2 \times 16 \times 3$. The count is sixteen particle states in each generation, the right-handed neutrino included, and their sixteen antiparticles (J exchanges each particle state with its antiparticle), in each of three generations. In one generation the particle states fall into four sectors: the lepton doublet (2 states), the quark doublet in three colors (6), the right-handed neutrino and electron (2), and the right-handed up and down quarks in three colors (6) (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 8.3). The finite Dirac operator D_F is made of five 3×3 matrices in generation space, the space that labels the three generations. Four of the five matrices of D_F are the Yukawa matrices Y_ν, Y_e, Y_u and Y_d, the couplings of the fermions to the Higgs field, and each Yukawa matrix couples left-handed doublets to right-handed singlets, the right-handed states that the weak force does not pair. The fifth matrix is the Majorana matrix M_R, which would couple the right-handed neutrino to its antiparticle. $M_R = 0$, because a Majorana mass would join a state to its own reversal, and no state of content is its own reversal, since the two terms in which every state is held are non-interchangeable (Axiom III) (*The Branches of the Kinds*, Chapter 1, Section II). So the neutrinos are Dirac particles, whose mass joins the left-handed and right-handed components of one state. D_F crosses one edge of the triangle of carriers, the edge between $\mathbb{H}$ and $\mathbb{C}$, and D_F commutes with the color carrier $M_3(\mathbb{C})$ (Proposition 8.4). The entries of D_F are the moduli of

the finite geometry, the parameters that its constraints leave free. The entries of D_F carry the masses and mixings of the fermions, and the derivation of the algebra fixes none of the entries (A-040).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 8 (Propositions 8.3 and 8.4), Chapter 10, Sections I and V, Chapter 19, Section X, and Appendix C; *The Branches of the Kinds*, Chapter 1, Sections II and III; *The Story of the Particles*, Level 4, Section I, and Level 9, Section III. Standing: a theorem (the count and the block form, checked on the explicit triple in Appendix C of *Non-Commutative Geometry and the Immanent Metaphysics*); derived (the zero Majorana matrix, from Axiom III, which *Non-Commutative Geometry and the Immanent Metaphysics* states as its postulate P-Terms).

> ***How D_F carries the masses.***
>
> The off-diagonal blocks of D_F contain the Yukawa matrices, the entries that join left-handed states to right-handed states. When the Higgs field takes its vacuum value v, its value in the state of lowest energy, the mass matrix of each mass sector (the neutrinos, the charged leptons, the up quarks or the down quarks) is the Yukawa matrix of that sector times v (up to a factor fixed by convention). The rest masses of a mass sector are the singular values of its mass matrix, the amounts by which the matrix stretches its principal directions. The unitary changes of basis that make the up and down mass matrices diagonal give the quark mixing matrix. The neutrino and charged-lepton mass sectors give the lepton mixing matrix in the same way, through the unitary changes of basis that make their mass matrices diagonal.
>
> Chamseddine and Connes arrange the 16 states of one generation in the finite space as a 4×4 array (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 8, Section II). The array has four states of weak isospin and chirality by four 'colors', with the lepton counted as a fourth color.

A-051 Inner Fluctuations: the Gauge Fields and the Higgs Field. That the Dirac operator of the product geometry is fluctuated by the algebra: $D \mapsto D_A = D + A + JAJ^{-1}$, with $A = \Sigma\, a_i[D, b_i]$ a self-adjoint one-

form. To fluctuate D is to add to D the terms A and JAJ^{-1} that the algebra itself produces, and the fluctuated operator takes the linear form $D + A + JAJ^{-1}$ under the order-one condition (A-031). The gauge fields are the fluctuations along the directions of the manifold M: one-forms with values in the Lie algebra (the infinitesimal generators) of the group of unitaries of A_F, the elements u with $u^*u = 1$. The fluctuations along the finite directions are the commutators $[D_F, a]$. Because D_F acts along the one edge between $\mathbb{H}$ and $\mathbb{C}$, the commutators of D_F with the algebra form one field, the Higgs field: a doublet under the $SU(2)$ of $\mathbb{H}$, with hypercharge $1/2$ and no color. No scalar field arises along the other two edges, and no fluctuation carries color (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 19.10). *The Story of the Particles* states that one field arises, on the one edge that D_F crosses, in these words: "One edge is crossed; one edge carries a field" (*The Story of the Particles*, Level 8, Section II). The Higgs field is a scalar, of spin zero, because the Higgs field lives on an edge of the triangle of carriers and not along a direction of the manifold. The Higgs field is a fluctuation of the Dirac operator, and the Higgs field leaves unchanged the algebra whose ray spaces form $\mathbb{C}P^2 \times S^2$. The Higgs field is not a fluctuation of the radius of the two-sphere.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section II, and Chapter 19, Sections IV and VI (Propositions 19.7 and 19.10); *The Story of the Particles*, Level 8, Section II; *The Kinds of Particle*, Chapter 3, Section VI; *The Connes Correspondence*, Chapter 6, Section I; Chamseddine, Connes and Marcolli (2007). Standing: a theorem (Proposition 19.10 of *Non-Commutative Geometry and the Immanent Metaphysics*, on the finite triple); a theorem of Chamseddine, Connes and Marcolli (2007), reported (the decomposition of the fluctuations).

A-052 The Gauge Group from the Unitaries of the Algebra. That the gauge group is computed from the unitaries of the finite algebra acting on both sides of the finite Hilbert space, and reduced by the unimodularity condition, a condition on the overall phase. A unitary of A_F is $u = (\lambda, q, m)$, with λ in $U(1)$, q a unit quaternion and m in $U(3)$, so the unitary

group is $U(1) \times SU(2) \times U(3)$. The inner automorphisms, the unitaries counted modulo the center (the unitaries that commute with every element), form the group $SO(3) \times PU(3)$, of dimension $3 + 8 = 11$, and $\mathbb{C}$ contributes nothing to that group. $SO(3) \times PU(3)$ is the identity component of the isometry group of $\mathbb{C}P^2 \times S^2$, the part of that group that can be reached continuously from the identity (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 18.1). A unitary u acts on $\mathscr{H}_F$ through $Ad(u) = uJuJ^{-1}$. Under $Ad(u)$ the phase λ of $\mathbb{C}$ does act, because λ acts on the left of some sectors and on the right of others. The unimodularity condition is $Det\ \pi(u) = (\lambda \det m)^{4N_{gen}} = 1$, and the condition ties the phase of the color summand to the phase of $\mathbb{C}$. In the unimodularity condition, $\pi(u)$ is the action of u on the finite Hilbert space, and N_{gen} is the number of generations. The unimodularity condition leaves $U(1) \times SU(2) \times SU(3)$, up to a finite group, of dimension 12. The hypercharges of A-014 are the phases by which the remaining $U(1)$ acts on the multiplets, the groups of states that the gauge group turns into one another. The gauge group has one dimension more than the isometry group of $\mathbb{C}P^2 \times S^2$, and the extra dimension is the $U(1)$ of $\mathbb{C}$. So the gauge group is not the isometry group of $\mathbb{C}P^2 \times S^2$. *Non-Commutative Geometry and the Immanent Metaphysics* takes unimodularity as an input, following Chamseddine, Connes and Marcolli. *The Story of the Particles* gives the ground of unimodularity in Axiom I, as "the condition on the overall phase" (Level 9, Section III).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 14, Sections I to V, and Chapter 18, Section II (Proposition 18.1); *The Story of the Particles*, Level 9, Section III; *The Connes Correspondence*, Chapter 3, Sections 3 and 5; Chamseddine, Connes and Marcolli (2007). Standing: a theorem (a computation on the finite triple, given the unimodularity condition, *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 14).

> ***The hypercharges as phases.***
> Take $u = (\lambda, 1, \lambda^{-1/3}1_3)$ with $\lambda = e^{i\theta}$. The action of the unitary u on each multiplet is a phase $e^{i\theta k}$, and the hypercharge of the multiplet is $Y = k/2$: left-handed quark doublet $k = 1/3$, $Y = 1/6$; right-handed up quark $k = 4/3$, $Y = 2/3$; right-handed down quark $k = -2/3$, $Y = -1/3$; left-handed lepton doublet $k = -1$, $Y = -1/2$; right-handed electron $k = -2$, $Y = -1$; right-handed neutrino $k = 0$, $Y = 0$ (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 14, Section IV). The hypercharges follow from the representation and the unimodularity condition, and none of the hypercharges is put in by hand.

A-VI. The Cutoff Function and the Spectral Action

A-033 The Spectral Action. That the spectral action principle of Chamseddine and Connes (1997) states that the physical action depends only on the spectrum of the Dirac operator, the set of its eigenvalues. That in the form of Chamseddine, Connes and Marcolli (2007) the action is $S = \mathrm{Tr}\, f(D_A/\Lambda) + \tfrac{1}{2}\langle J\xi, D_A\xi\rangle$. In the formula for S, D_A is the fluctuated Dirac operator of the product geometry (A-028, A-051), f is a positive cutoff function (A-034), and Λ is an energy scale. The trace is taken over $\mathcal{H}$, and ξ runs over the Grassmann vectors in the even part of $\mathcal{H}$: vectors whose components are anticommuting variables, as fermion fields require. The first term of S is the bosonic action, the action of the fields that carry the forces, of the Higgs field and of gravity. The bosonic action depends only on the spectrum of D_A, and it is invariant under the unitaries of A. The second term of S is the fermionic action, the action of the matter particles. The spectral action has four named inputs (A-035). *Non-Commutative Geometry and the Immanent Metaphysics* states what follows from the four inputs: "Given these, no term of the Lagrangian is chosen separately: the gauge group, the representation of the fermions, the Higgs doublet and its

potential, and the coupling to gravity all follow from the spectral action" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV). The values of the couplings and the masses come from other routes or from measurement.

Sources: Chamseddine and Connes (1997); Chamseddine, Connes and Marcolli (2007); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Section V (Definition 3.14), and Chapter 13; *The Connes Correspondence*, Chapter 6, Sections I and III. Standing: a definition (the principle, applied with its four inputs stated); a theorem of Chamseddine, Connes and Marcolli (2007), reported (the form of the Lagrangian that follows).

> ***The spectral action principle.***
>
> The bosonic action is $S_b = \mathrm{Tr}\, f(D_A/\Lambda)$, the sum, over the eigenvalues of D_A, of the cutoff function evaluated at each eigenvalue divided by Λ. The fermionic action is $S_f = \tfrac{1}{2}\langle J\xi, D_A\xi\rangle$. In the KO-dimension of the product, $10 \equiv 2$ modulo 8, the form of the fermionic action removes the doubling of the fermions (A-028). The total action is $S_b + S_f$.
>
> The spectral action principle has a reason. When an action depends only on the spectrum of D, the action depends only on what D encodes, the distances of the geometry and its invariants (A-027). If an action depended on further data, the action would need assumptions beyond the spectral triple. The expansion of the spectral action contains the Einstein-Hilbert term (A-020). *The Gravitational Passage* derives the Einstein-Hilbert term by a second route, by Kaluza-Klein reduction of the ten-dimensional context and by Lovelock's theorem (*The Gravitational Passage*, The Chain at a Glance; *The Connes Correspondence*, Chapter 6, Section III; Chapter D, entry D-006, which states Lovelock's theorem, and entry D-016, which states the Kaluza-Klein reduction).

A-034 The Cutoff Function and Its Three Moments. That the cutoff function f is a positive function that sets how strongly each eigenvalue of D_A contributes to the bosonic action, and that f suppresses the eigenvalues far above Λ. That in the asymptotic expansion of the bosonic action, the expansion in powers of Λ that holds when Λ is large, f enters only

through three numbers, its moments: $f_0 = f(0)$, $f_2 = \int_0^\infty f(v)\, v\, dv$, and $f_4 = \int_0^\infty f(v)\, v^3\, dv$. The moment f_4 multiplies the cosmological term (with Λ^4), f_2 the Einstein-Hilbert term and the mass term of the Higgs field (with Λ^2), and f_0 the gauge, Higgs kinetic and quartic, and curvature-squared terms (A-020). The cutoff function is an input of the spectral action: neither *Non-Commutative Geometry and the Immanent Metaphysics* nor the comparison route, the derivation of the algebra from the structure of comparison (A-030), fixes f or any of its moments (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV; *The Connes Correspondence*, Chapter 6, Section IV).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Sections III and IV, and Chapter 16, Section I; *The Connes Correspondence*, Chapter 6, Section IV; Chamseddine, Connes and Marcolli (2007). Standing: a definition.

> ***The three moments and what they multiply.***
>
> $\mathrm{Tr}\, f(D_A/\Lambda) \sim f_4\Lambda^4\, a_0 + f_2\Lambda^2\, a_2 + f_0\, a_4 + O(\Lambda^{-1})$.
>
> • $f_4\Lambda^4\, a_0$: the cosmological term, proportional to the volume of spacetime and to $\dim \mathscr{H}_F = 96$. The coefficient of the cosmological term is $48 f_4\Lambda^4/\pi^2$ (A-019). The natural scale of the cosmological term is set by the cutoff. The observed vacuum energy density is about 2.5×10^{-47} GeV^4, which is roughly 10^{-111} of the fourth power of a cutoff near 10^{16} GeV, about 10^{-120} of the fourth power of the reduced Planck mass, and about 10^{-123} of the fourth power of the full Planck mass (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 16, Section I).
>
> • $f_2\Lambda^2\, a_2$: the Einstein-Hilbert term and the mass term of the Higgs field.
>
> • $f_0\, a_4$: the Yang-Mills terms of the three gauge groups, the terms of the three force fields, the Higgs kinetic and quartic terms, the coupling of the Higgs field to curvature, and the Weyl-curvature term. f_0 and the trace over $\mathscr{H}_F$ give the three gauge kinetic terms a common normalization, and the common normalization yields the relations at Λ of A-037.

A-035 The Four Inputs of the Spectral Action. That the spectral action has four named inputs. The first input is the product form $M \times F$ of spacetime with the finite space. The second input is the finite triple F: its algebra is derived (A-030), its number of generations is fixed by a postulate (P-Rep, or P-Gen), and its moduli, the entries of D_F, are not fixed. The third input is the cutoff function f, which enters through its three moments f_0, f_2 and f_4. The fourth input is the scale Λ (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV; *The Connes Correspondence*, Chapter 6, Section I). The comparison route fixes none of the moments. *The Gravitational Passage* gives the value of the cosmological constant as the share of the vacuum energy accessible to an observer inside the vacuum, and the value does not come from a moment of f (*The Gravitational Passage*, Chapter 13, Section II). Newton's constant is the strength of the Einstein-Hilbert term, and Newton's constant is measured (*The Gravitational Passage*, Preface). Measurement supplies the values of the gauge couplings at any one scale.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV, Chapter 19, Section XIV, and Appendix B; *The Connes Correspondence*, Chapter 6, Sections I and IV; *The Gravitational Passage*, Preface, and Chapter 13, Section II. Standing: a definition (the inputs, each classed as an input in the postulate ledger of *Non-Commutative Geometry and the Immanent Metaphysics*, Appendix B).

A-037 What the Spectral Action Fixes and What It Leaves. That at the scale Λ the expansion of the spectral action imposes relations among the couplings: the three gauge couplings satisfy $g_3^2 = g_2^2 = (5/3)g_1^2$; the Higgs quartic coupling is tied to the gauge coupling and the Yukawa couplings; and the sum of the squared Yukawa couplings is tied to the gauge coupling (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Section II). The first relation is the equality of the three traces over the finite space of states, one trace for each gauge group. The weak angle θ_W sets how the hypercharge boson and the neutral weak boson mix to give the photon and the Z. With the three traces equal, $\sin^2\theta_W =$

$\mathrm{Tr}(T_3^2)/\mathrm{Tr}(Q^2) = 2/(16/3) = 3/8$, where the traces are taken over one generation. The relations all come from one step, the single trace of the spectral action over the whole product, and the single trace ties the couplings to one common measure. In the Connes program, the program of Connes and his collaborators in non-commutative geometry, the relations are read as conditions of unification at the cutoff. When the value 3/8 is run down to the mass of the Z boson with the content of the Standard Model alone, 3/8 gives about 0.21, against the measured 0.2312. The original spectral model gave a Higgs mass of about 170 GeV, and the measured mass of about 125 GeV excludes that value (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Sections III and IV). In the comparison route the equality of the traces is a relation among the weights of the three forces in the structure of comparison, as *The Story of the Particles* states: "The equality the structure asserts is not an equality of measured strengths" (*The Story of the Particles*, note 19). The running weak angle equals 3/8 at the first crossing of the couplings, the scale where the hypercharge coupling crosses the weak coupling, near 10^{13} GeV, 54.259 halvings above the electron. The running weak angle equals 3/8 at no other scale. The running started from 3/8 at the first crossing returns $\sin^2\theta_W = 0.2314$ at the Z mass (*The Strengths of the Kinds*, Chapter 3, Sections III and IV). The measured couplings meet at no scale (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Section III), and the structure of comparison imposes no grand-unification relation, no relation that makes the three couplings equal at one high scale (*The Strengths of the Kinds*, Chapter 3, Section IV). The weak angle at the Z mass is measured as 0.23122 ± 0.00006 (Particle Data Group 2026). The weak angle at the Z mass is a measured agreement, and the agreement restates the place of the first crossing. Measurement supplies the values of the couplings at any one scale.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV, and Chapter 15, Sections II to IV; *The Connes Correspondence*, Chapter 5, Sections II and III, and Chapter 6, Section III; *The Strengths of the Kinds*, Chapter 3, Sections III and IV; *The Kinds of Particle*, Chapter 5, Section V; *The*

Story of the Particles, note 19; *The Measures of the Kinds*, Chapter 13, Tables X-1 and X-3; Chamseddine, Connes and Marcolli (2007); Particle Data Group (2026). Standing: a theorem (the relations at Λ, Chamseddine, Connes and Marcolli 2007, and the trace ratio 3/8 over one generation); derived ($\sin^2\theta_W = 3/8$ at the first crossing, an identity of the definitions at that scale); a measured agreement (the weak angle at the Z mass).

A-038 The Connes Program and the Comparison Route. That the Connes program in non-commutative geometry and the comparison route agree on the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, the gauge group $U(1) \times SU(2) \times SU(3)$, the sixteen fermions of a generation with their charges, the single Higgs doublet, the relation of the couplings at the finest resolution, and the stability of the proton. The resolution is the separation at which a comparison is made (Chapter C, entry C-002), and the finest resolution is the smallest separation, reached at the highest energy. That the Connes program and the comparison route differ on four points, and each difference has its source in a hypothesis or in the reach of a route, in what one route derives and the other does not. First, the comparison route derives three generations, and in the Connes program the number of generations is an input. Second, the neutrino is a Dirac particle in the comparison route and a Majorana particle in the Connes program. Neutrinoless double beta decay is a decay of a nucleus that emits two electrons and no neutrino, and it can occur only if the neutrino is its own antiparticle. So neutrinoless double beta decay occurs in the Connes program and does not occur in the comparison route. Third, Chamseddine and Connes arrive at the measured Higgs mass with the field σ, the fluctuation of the Majorana entry (Chamseddine and Connes 2012). The comparison route has no field σ, and in the comparison route the Higgs mass, 125.17 GeV, is a measured agreement. Fourth, in the comparison route the cosmological constant has a value, and the spectral action leaves that value unfixed. On gravity the Connes program and the comparison route arrive at the Einstein-Hilbert term by different steps: the spectral action contains the Einstein-Hilbert term, and *The Gravitational Passage* derives the Einstein equations without the spectral action. Each difference is decided by a measure-

ment: the search for neutrinoless double beta decay decides the neutrino, and the measured values decide the other differences (*The Connes Correspondence*, Chapter 28, Section IV). The Connes program and the comparison route start from different hypotheses and agree on one specific algebra, and that agreement is the evidence proper to the identification of the algebra of comparison with the algebra of the content of the particles (*The Connes Correspondence*, Chapter 1, Section 10).

Sources: *The Connes Correspondence*, Chapter 1, Section 10, Chapter 18, Sections I and II, and Chapter 28, Section IV; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 10, Chapter 15, Section IV, and Chapter 18; *The Branches of the Kinds*, Chapter 1, Section VI; Chamseddine and Connes (2007; 2008; 2012); Chamseddine, Connes and Marcolli (2007). Standing: an identification (the algebra of comparison with the algebra of the content of the particles), for which the convergence of the two routes is the evidence proper to it; a fork that a named measurement decides (Dirac or Majorana neutrinos, decided by the search for neutrinoless double beta decay).

> ***The two programs in one view.***
>
> Algebra: in both, $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$; selected in the Connes program with a Majorana coupling, and reached in the comparison route without it. Gauge group: in both, $U(1) \times SU(2) \times SU(3)$; unimodularity imposed in the Connes program, and set by Axiom I in the comparison route. Generations: an input in the Connes program; three, the rotations of the content, in the comparison route. Neutrino: Majorana masses, the seesaw (by which a heavy Majorana mass makes the light neutrino masses small) and neutrinoless double beta decay in the Connes program; Dirac masses, no heavy neutrino and no neutrinoless double beta decay in the comparison route. Higgs: one doublet in both; about 170 GeV, then 125 GeV with the field σ, in the Connes program; 125.17 GeV with no field σ, a measured agreement, in the comparison route. Cosmological constant: the form of the term with no value in the Connes program; the accessible share, about 1.7 percent above the observed density, in the comparison route. Couplings: equal traces at the cutoff, as the relation of grand unification, in the Connes program; equal traces as weights, 3/8 at the first crossing, and no grand-unification relation, in the comparison route.

Masses and mixings: parameters of the finite geometry in the Connes program; entries of the finite geometry, with values in *The Measures of the Kinds*, in the comparison route. Gravity: the Einstein-Hilbert term of the spectral action in the Connes program; Kaluza-Klein reduction and Lovelock's theorem, with gravity the arena, in the comparison route. (*The Connes Correspondence*, Chapter 18, Section II.)

A-VII. The Moduli of the Finite Dirac Operator

A-039 Moduli Space. That a moduli space is the space of all configurations that satisfy a given set of constraints, counted up to the changes of description that leave a configuration the same. Each point of a moduli space is one configuration, and moving through the moduli space means deforming the configuration while keeping every required property.

Sources: a standard definition. Standing: a definition.

A moduli space pictured.

Consider all ellipses, counted up to position and orientation, so that two ellipses of the same shape and size count as one. Each ellipse is then fixed by two numbers, the lengths of its two axes, so the space of all ellipses is a two-dimensional surface. Each point on the surface of ellipses is one ellipse, and sliding along the surface deforms the ellipse continuously. Requiring a fixed area reduces the space of ellipses to a curve, and requiring a fixed eccentricity as well (a fixed degree of flattening, set by the ratio of the two axes) reduces the space to a single point.

The moduli of the finite Dirac operator (A-040) form a moduli space, as the ellipses do. Each point of the space of moduli is one finite Dirac operator, as each point of the surface of ellipses is one ellipse, and each point is one assignment of the masses and the mixings of the fermions. The derivation of the algebra picks out no point of the space of moduli, as *Non-Commutative Geometry and the Immanent Metaphysics* states: "The deriva-

> tion stops at the moduli, the entries of the finite Dirac operator, and it derives no value of any coupling or mass" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V).

A-040 The Moduli of the Finite Dirac Operator. That the finite Dirac operators compatible with the finite triple of the kinds are given by the five matrices Y_ν, Y_e, Y_u, Y_d and M_R (A-032). A finite Dirac operator is compatible with the triple when it is self-adjoint, anticommutes with the grading, commutes with J, and satisfies the order-one condition. That the moduli of the finite geometry are the five matrices Y_ν, Y_e, Y_u, Y_d and M_R, counted up to the changes of basis in generation space that leave the triple the same. Ćaćić (2011) studied the moduli of finite spectral triples in general. The moduli carry the masses and the mixings of the fermions, and neither the derivation of the algebra nor the classification of Chamseddine and Connes determines the values of the moduli (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 10, Section V). With the Majorana matrix zero (A-032), the moduli carry twenty physical parameters: six quark masses, three charged-lepton masses and three neutrino masses, four parameters of quark mixing (three angles and one phase) and four of lepton mixing (three angles and one phase) (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Section I).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V, Chapter 10, Section V, Chapter 15, Sections I and II, and Chapter 19, Section XIV; *The Branches of the Kinds*, Chapter 1, Section II; Ćaćić (2011); Chamseddine, Connes and Marcolli (2007). Standing: a theorem (the moduli and their count); derived (the zero Majorana matrix, from Axiom III, *The Branches of the Kinds*, Chapter 1, Section II).

> ***The twenty parameters, and what the bosonic action sees of them.***
>
> Charged masses: up, charm and top; down, strange and bottom; electron, muon and tau. Neutrino masses: three. Quark mixing: three angles and one CP-violating phase, the phase by which quark mixing tells matter from its mirror image under charge conjugation and parity (CP). Lepton mixing: three angles and one Dirac CP phase. Total: $9 + 3 + 4 + 4 = 20$.

In the asymptotic expansion of the bosonic spectral action, the finite Dirac operator enters through a few trace invariants of the Yukawa matrices, among them the color-weighted sums of Tr(Y*Y) and of Tr((Y*Y)²) over the four mass sectors (the neutrinos, the charged leptons, the up quarks and the down quarks) (Chamseddine, Connes and Marcolli 2007). Each of the trace invariants is unchanged by a change of basis within one mass sector, so the trace invariants depend on the masses and not on the mixing angles or phases.

The values of the twenty parameters are the subject of *The Measures of the Kinds*, the collection's book on the values of the masses and mixings, which gives each parameter with its standing (A-046 to A-049).

A-VIII. Form and Values: Where the Masses Come From

A-046 Form from the Triple, Values from the Measures. That the masses are given in two parts, the form and the values. The finite geometry fixes the form: a Yukawa matrix for each mass sector as an entry of D_F (A-032), the Higgs field on the crossed edge (A-051), the form of the Higgs potential from the spectral action (A-020), and the form of the mass terms after the Higgs field takes its vacuum value. The values are the work of *The Measures of the Kinds*, which counts each mass in halvings below a reference (A-022) and gives each of the twenty-six numbers of the Standard Model its standing (*The Measures of the Kinds*, Chapter 13, Table X-3). *The Measures of the Kinds* sets side by side two accounts of what measurement supplies, and both accounts are true. In the first account, the working account, the measured inputs are the electron mass, the strength of light α_{em}, the strength of color α_s, the vacuum value v, and the two offsets of the charged ladders. Each offset is found by a condition from a measured mass: the offset of the down ladder from the bottom, and the offset of the lepton ladder from the tau. In the second account, the closure account, the step on the interval from the tau to $\sqrt{2}$ v is written as

a product, and the chain of steps runs from the electron alone to the vacuum value, the top and the Higgs mass. The rule of the product step in the closure account is that the orders of a reading multiply across two domains and add within one domain. The rule is argued: addition, multiplication and exponentiation are a triple; bringing two domains into relation is not addition; and exponentiation is excluded because the axioms close without totalizing. *The Measures of the Kinds* derives the placement of the factors of the product step, given a stated identification: the first-order reading of a relation of two domains is carried by the domain that holds the two domains (*The Measures of the Kinds*, Chapter 7, Section IX). In both accounts α_{em} and α_s are measured inputs.

Sources: *The Measures of the Kinds*, Preface, Chapter 7, Sections I, VIII and IX, and Chapter 13, Sections VIII and X (Tables X-1 and X-3); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V; *The Connes Correspondence*, Chapter 6, Section I, and Chapter 9, Sections II to IV. Standing: a theorem (the form, on the finite triple); a measured input (the inputs of each account); derived, given a stated identification (the placement of the factors of the product step in the closure account, whose domain rule is argued).

A-047 The Constants of the Step Formula. That the constants of the step formula (A-022) are written in the kernel triangle, and that the constants are not spacings of the eigenvalues of a Dirac operator on $\mathbb{C}P^2$. The kernel triangle is the right triangle with legs $O = c_1 = 3$ and $N = 4$ and hypotenuse $H = 5$, and its sides are written in the two kernel integers $c_1 = 3$ and $n_C = 2$ (*The Kinds of Particle*, Chapter 2, Sections I and II). The quark step cost is $P_q = 6 = c_1 n_C$, because a rotation of the content (the step from one generation to the next, A-023) resolves each of the six intrinsics of its new instance once, the six verdicts (the outcomes of resolving the six intrinsics) are independent, and six comparisons are six halvings. The lepton step cost is $P_l = 9/4 = 6 \times 3/8$, from the share rule: the charged leptons hold the share 3/8 of one generation's charge squared. The sector rule states that the step cost of a charged sector depends on the color of the sector alone. The three curvatures close in one identity, $O\ d_{lep} +$

$N\,d_{dn} = O\,d_{up}$, with $d_{up} = 1 = 1^2$, $d_{lep} = 16/9 = (4/3)^2$ and $d_{dn} = -7/12$: $3 \times 16/9 - 4 \times 7/12 = 16/3 - 7/3 = 3$. The offsets $\Delta_{dn} = 53/10$ and $\Delta_{lep} = 391/60$ are found by a condition that every true instance of the ladder meets, and the offsets are written as unit forms over $V = ONH = 60$. The step factor $s = 1 + \varepsilon - c_2\varepsilon^2$ carries the one leftover ε (A-048).

Sources: *The Measures of the Kinds*, Chapter 7, Sections I and IV to IX, and Summary; *The Kinds of Particle*, Chapter 2, Sections I and II. Standing: derived (P_q, P_l and the curvatures, at first order); a measured agreement (the kernel writings of the offsets and the second-order coefficient $c_2 = 28/5$, each carried with the question why it lands).

A-048 The One Mismatch ε and the Euler Weight of the Spectral Action. That the leftover $\varepsilon = 11/(80\pi^2) = 0.013932$ has three writings, and the three writings are writings of one number. *The Strengths of the Kinds* calls ε the one mismatch between a counted measure and its continuous reading. The first writing is in kernel integers: $(c_1^2 + n_C)/(N^2H\pi^2)$. The second writing is in the gauge self coefficient of color, $(11/3)(O/H)/(16\pi^2)$, where 11 is the one-loop coefficient with which the carriers of color, the gluons, antiscreen: they make the color force stronger at larger distances. The third writing is in the Euler weight of the spectral action, $72 \times 11/(5760\pi^2)$: seventy-two Dirac spinors, each with the Euler weight $11/(5760\pi^2)$ of the scale-free term (A-019). The third writing restates the arithmetic $72/360 = 1/H$, and it is a writing of ε, and not a derivation of ε from the spectral action. The existence of the mismatch is derived from the Incommensuration Theorem: every reading of a count by a continuous whole leaves a residue. An argument from the product geometry fixes where the powers of π come from: every factor π comes from the manifold factor, and every rational number comes from the finite factor. On the span to the Planck mass (A-025), the measured remainder is 0.948 ε. The first-order span $72 + \varepsilon$ stands 44.5 standard deviations above the measured span, so the measurement is precise enough to resolve a second order, a term beyond the first order in ε.

Sources: *The Strengths of the Kinds*, Chapter 5, Section III (Boxes III-1 to III-3); *The Kinds of Particle*, Chapter 2, Section VII, and Chapter 3, Section V; *The Measures of the Kinds*, Chapter 7, Section IX; *The Connes Correspondence*, Chapter 6, Section III. Standing: a theorem (the equality of the three writings, by arithmetic); derived (the existence of the mismatch, from the Incommensuration Theorem, and the power of π in it, from the product geometry); an identification (the rule that every small leftover of a reading counted in halvings is this one mismatch at first order), tested by the measured span and the measured masses.

A-049 The Masses and Their Standings. That each mass of the charged fermions and of the Higgs sector carries a stated standing (*The Measures of the Kinds*, Chapter 13, Table X-3). The electron mass is the measured input, the unit of content: 0.51099895069(16) MeV (CODATA 2022). The muon mass is derived from the electron mass on the ladder, to first order. At its full precision the muon mass is a measured agreement: the computed ratio of the muon mass to the electron mass is 206.7682862, against the measured 206.7682827(46). The tau mass is derived from the electron and muon masses by Koide's relation (Koide 1983), given a stated identification: that each part of the amplitude vector is measured by its share of the squared size. Koide's relation states that the sum of the electron, muon and tau masses is two thirds of the square of the sum of their square roots. The derived tau mass is 1776.969 MeV, against the measured 1776.93 $\pm$ 0.09 MeV, and the measured tau mass supplies the offset of the lepton ladder. The up, down, strange, charm and bottom masses are measured agreements, as positions on the step formula, and the measured bottom mass supplies the offset of the down ladder. The coupling of the top quark to the Higgs field is written y_t. The value $y_t = 1$ is fixed by the structure of the kinds, and the scale of the scheme in which y_t is stated is a setting, a choice made in stating the coupling. Every value of the top quark's mass is a measured agreement. The Higgs self-coupling at tree level (the lowest order, before loop corrections) is $\lambda = 1/8$, derived given a stated identification: $\lambda = a^2/2$, which identifies the self-coupling with half the square of the doublet's half, $a = 1/2$. The Higgs mass is computed with its loop

term from the measured vacuum value. The computed Higgs mass is 125.17 GeV, against the measured 125.13 ± 0.11 GeV (Particle Data Group 2026): a measured agreement. The vacuum value v is a measured input in the working account.

Sources: *The Measures of the Kinds*, Chapters 7 to 9, and Chapter 13, Sections II to IV, VI and X (Tables X-1 and X-3); Koide (1983); CODATA 2022; Particle Data Group (2026). Standing: a measured input (the electron mass, and v in the working account); derived (the muon at first order, and $y_t = 1$, fixed by the structure of the kinds); derived, given a stated identification (the tau, given that each part of the amplitude vector is measured by its share of the squared size, and $\lambda = 1/8$, given $\lambda = a^2/2$); a measured agreement (the quark masses, the muon at full precision, the top mass, the Higgs mass).

A-050 The Spectral Route and the Route of the Measures. That for the masses, the spectral route and the route of the measures draw the same line between form and values. The spectral route is the route through the spectral action, and the route of the measures is the route of *The Measures of the Kinds*. The finite geometry fixes the form of the masses, as entries of the finite Dirac operator, and the values of the masses are the work of a further account. In the Connes program the entries of the finite Dirac operator are parameters of the finite geometry: input data that specify which finite spectral triple is chosen. The algebraic derivation of the comparison route stops at the same line between form and values (*The Connes Correspondence*, Chapter 9, Sections II and III). The account of the values takes its own route, separate from the spectral action: the step formula counts halvings below a reference, and the constants of the step formula are written in the kernel triangle.

Sources: *The Connes Correspondence*, Chapter 9, Sections II and III; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V, and Chapter 10, Section V; *The Measures of the Kinds*, Chapters 7 and 13. Standing: a theorem (the algebra is derived from the postulates, each an identification, and its derivation leaves the entries of D_F unfixed). The values carry the standings given in entry A-049.

What each route supplies.

The spectral action supplies the form: the gauge group, the representation of the fermions, the single Higgs doublet and the form of its potential, the coupling to gravity, and the relations among the couplings at its cutoff. *The Measures of the Kinds* supplies the values of the masses and mixings, each with one of these standings: derived; derived, given a stated identification; a measured input; a measured agreement; or a fork that a named measurement decides (the neutrino masses, the lepton mixing angles and the lepton CP phase). *The Gravitational Passage* supplies gravitation, with Newton's constant measured. *The Gravitational Passage* also supplies the value of the cosmological constant as the accessible share, the share of the vacuum energy accessible to an observer inside the vacuum (A-035). The accessible share is an identification with a computed value compared with measurement: 2.56×10^{-47} GeV⁴, about 1.7 percent above the observed 2.52×10^{-47} GeV⁴ of the Planck 2018 parameters (Planck Collaboration 2020a), the parameters of the universe that the team of the Planck satellite published from its data (*The Gravitational Passage*, Chapter 13, Sections II to IV).

Summary

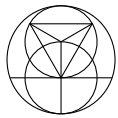
In this chapter I lay out, entry by entry, the mathematics on which the spectral account of the particles rests. The first three parts of the chapter state the standard language that the spectral account uses: manifold, metric, curvature, connection and fiber bundle; the Dirac operator, its square by the Lichnerowicz formula, and its spin-c form on $\mathbb{C}P^2$; and the heat kernel, whose Seeley-DeWitt coefficients turn a spectrum into integrals of curvature. Each item of the standard language is a definition or a theorem of geometry, given with its outside source. On the internal space $\mathbb{C}P^2 \times S^2$ the same language is read in two ways, one for each instance of the internal space. On the content instance, the space of states of *The Kinds of Particle*, distances are transition angles with no length. The kinds and

charges of the content are read on the content instance, which has its Chern classes and its symmetry group, and no field lives on the content instance and no mode is formed on it. On the context instance of *The Gravitational Passage*, one identification fixes the radii $\sqrt{6}\ l_{Pl}$ and l_{Pl} and the volume $72(\pi l_{Pl}^2)^3$: the identification that every internal direction carries the Planck curvature.

In the middle of the chapter I set out the spectral triple and the spectral action in the form that the spectral triple and the spectral action take in two books, *Non-Commutative Geometry and the Immanent Metaphysics*, and *The Connes Correspondence*. The finite algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is a theorem, given postulates, and each postulate is an identification of a clause of the axioms. The classification of Chamseddine and Connes reaches the same algebra as the derivation, from other hypotheses. The convergence of the derivation and the classification on one algebra is the evidence proper to identifying the algebra of comparison with the algebra of the content of the particles. When the product of spacetime with the finite triple is fluctuated by the algebra, the fluctuations give the gauge fields and the one Higgs doublet on the one crossed edge. The gauge group and the hypercharges come from the unitaries of the algebra, with the condition on the overall phase. Given the four named inputs of the spectral action, the single trace of the spectral action gives the form of the Standard Model coupled to gravity. The single trace also gives the relation of the couplings at the cutoff of the spectral action. In the comparison route, the relation of the couplings at the cutoff is read as the equality of the weights of the three forces.

The last parts of the chapter mark the line where the form ends and the values begin. The entries of the finite Dirac operator are the moduli of the finite geometry, twenty physical parameters with the Majorana matrix zero, and the derivation of the algebra stops at the moduli. The values are the work of *The Measures of the Kinds*. The mass ladder counts each charged fermion in halvings below the top, with constants written in the kernel triangle. The one mismatch ε is the leftover in the step from counts

to continuous measures. Each value that *The Measures of the Kinds* gives has the standing that The Measures of the Kinds states for that value. The electron mass is a measured input, and the tau mass and the Higgs self-coupling $\lambda = 1/8$ are derived, given a stated identification. The quark masses and the Higgs mass are measured agreements. The three generations are the three rotations of the content. The count 72 is identified with the measured span to the Planck mass, and the measured span tests the identification. None of the results that I list in this chapter is owned by this chapter; in this chapter I gather in one place the mathematics on which the results rest and the books in which each result is argued.



Chapter B

The Mass Spectrum

CHAPTER OVERVIEW

This chapter traces the values of the masses and mixings of the fermions, the particles of matter, as *The Measures of the Kinds* carries them: the mass ladder and its step formula, the nine charged fermion masses, the dressing of the rungs, what the finite geometry fixes and what it leaves, the quark and lepton mixing, the two routes of the neutrino masses, and the composites computed from the quark masses. It rests on *The Concept Sequence*, Chapter 26, The Two Kernel Integers and the Kernel Triangle; Chapter 28, The Two Measured Anchors and the Span to the Planck Mass; Chapter 29, The Couplings: Rank, Running, Weights and Crossings; Chapter 30, The Spectral Action and the Form of the Standard Model; Chapter 34, Quarks and Leptons; Chapter 35, The Lepton Sector; Chapter 38, The Higgs Boson; and Chapter 40, The Mixing Matrices. Chapter E, The Higgs Potential and Electroweak Phase Transition, states the potential whose self-coupling at tree level entry B-021 uses.

The internal space $\mathbb{C}P^2 \times S^2$ has two instances. The content instance of the internal space is a space of states with no size, no radius and no fields. The context instance of the internal space has radii, and the context instance belongs to *The Gravitational Passage*. Every statement of this chapter about the internal space is made on its content instance: no mass in this chapter is an eigenvalue of a Dirac operator on that space, and no particle is a vibration mode of it. Each number is given with its measurement, its

source, its distance from measurement and its standing, as *The Measures of the Kinds*, Chapter 13, Section X, Table X-3, gives the standings. A distance from measurement is written as a pull: the stated value less the measured value, divided by the combined uncertainty, in standard deviations (σ).

B-I. Yukawa Couplings and the Language of Mass

B-001 Yukawa Coupling. That a Yukawa coupling is a number measuring how strongly a fermion couples to the condensate of the Higgs field, and therefore how much mass the fermion has. The condensate is the constant value the Higgs field takes throughout space, and the Higgs field is the field on the one crossed edge of the finite space, the edge between the two sides of the weak doublet (*The Concept Sequence*, Chapter 30, entry 30.13). That every such coupling is the ratio of a part to its whole, so it lies between zero and one, and no particle of matter is heavier than $v/\sqrt{2} = 174.10$ GeV, where v is the vacuum value of the Higgs field, the size of the condensate. That the top quark's coupling is one, $y_t = 1$: in the frame of measurement of *The Story of the Particles* the up-type sector stands at the immanent position, the comparing act turned upon its own content, whose extent is the identity, and only its third-generation member, the top, meets the whole.

Sources: *The Measures of the Kinds*, Chapter 13, Sections I to III; *The Story of the Particles*, Level 6, Section V, and Level 8, Section IV. Standing: derived ($y_t = 1$, a value the structure sets, with the scheme in which it is read a setting fixed by stating it); a definition (the Yukawa coupling); derived (the ceiling $v/\sqrt{2}$, from a coupling being the ratio of a part to its whole).

The Yukawa coupling in symbolic form.

The mass of a fermion is related to its Yukawa coupling y_f by $m_f = y_f\, v\, /\, \sqrt{2}$, where $v = 246.21967 \pm 0.00006$ GeV is the vacuum value, read from the Fermi constant G_F by $v = (\sqrt{2}\, G_F)^{-1/2}$. The factor $\sqrt{2}$ is the conven-

tional normalization of the Higgs doublet. At $y_t = 1$ the ideal top lies at $v/\sqrt{2}$, about 174.1 GeV; the coupling inferred from the measured top mass is about 0.99. For the tau the coupling is about 0.0102, and for the heaviest neutrino, whose mass is a Dirac mass of the same form, about 3×10^{-13}.

B-002 Tree-Level Prediction. That a tree-level prediction is the first approximation to a physical quantity, computed from the classical equations of the theory without the effects of quantum fluctuations. The term "tree level" comes from the diagrams of quantum field theory, in which the simplest calculations correspond to branching diagrams that look like trees and contain no closed loops. A tree-level prediction is like computing the path of a thrown ball while leaving out the resistance of the air: it carries the basic physics, and systematic corrections are needed for agreement with observation at high precision.

Sources: standard definition. Standing: a definition.

B-003 Running Mass and the Reading Rule. That the mass of a quark, as quoted from experiment, depends on the energy scale and on the scheme at which it is read, because the cloud of virtual particles around a quark changes with the energy of the probe; this dependence is called the running of the mass. That a quark's self-reading scale is the scale at which its running mass equals that scale, and its mass there is written $m(m)$. That *The Measures of the Kinds* reads each quark at one stated scale: the top at its direct mass, measured from the kinematics of its decays; the charm and the bottom each at its own self-reading scale; and the up, the down and the strange together at the pair floor $2m_p = 1.8765$ GeV, twice the proton's mass, the least mass at which a whole of color is made as content. That the light quarks' measured values are quoted at 2 GeV, and are carried to the floor by the running factor 1.01849 ± 0.00058 (four-loop running, with the strong coupling α_s matched at the bottom mass at two loops).

Sources: *The Measures of the Kinds*, Chapter 7, Section II, and Chapter 9, Section I and Table I-1; *The Strengths of the Kinds*, Chapter 6, Section IV (the floor). Standing: a definition (the running mass and the self-reading scale, and the reading scale of each quark, a setting fixed by stating it); derived (the reading rule, from the three cuts of the quark ladder).

B-004 Radiative Correction. That a radiative correction is a systematic adjustment to a tree-level prediction (entry B-002) that accounts for the effects of virtual particles, particles that momentarily appear and disappear as the uncertainty principle allows. These corrections are computed as a series of increasingly precise approximations called loop orders. A one-loop correction includes the processes in which one virtual particle circulates once through the interaction; a two-loop correction includes the processes with two such circulations. Each further loop improves the precision of the prediction and increases the difficulty of the computation. The names refer to the shape of the diagrams: a one-loop diagram contains one closed cycle, and a two-loop diagram contains two.

Sources: standard definition. Standing: a definition.

B-005 Pull and Information. That the pull of a stated value is its distance from measurement: the stated value less the measured value, divided by the combined uncertainty, in standard deviations; a placement passes when its pull lies within two standard deviations. That the information an output carries is written in bits: the base-two logarithm of the measured value over the larger of its measured uncertainty and the distance between the stated and the measured values, which is the number of binary digits of the measured value that the output gets right. That an output resting on a branch of a fork is marked with its branch, and an output with no measured value of its own (a mass sum against a bound, an identity at a scale) carries no count of bits.

Sources: *The Measures of the Kinds*, Preface; Chapter 9, Chapter Overview (the terms); Chapter 13, Section X. Standing: a definition.

B-II. The Step Formula

B-006 The Mass Ladder. That a comparison has two outcomes, same or different, exhaustive and exclusive, so one comparison divides the open configurations in two; one comparison is one halving, a factor of two in mass, and the base of the mass ladder is 2. That a sector is one family of fermions of one kind (the up quarks, the down quarks, the charged leptons or the neutrinos), and a ladder is the set of places of one sector's three generations, each counted in halvings below the top quark. That the generation index g counts from the heaviest generation: $g = 0$ is the heaviest, $g = 1$ the middle and $g = 2$ the lightest. That a fermion n halvings below the top has the mass $m_t \times 2^{-n}$, so that the electron, 18.361 halvings below the top on the undressed ladder, and the top span more than five orders of magnitude in about eighteen halvings.

Sources: *The Measures of the Kinds*, Chapter 7, Sections I and II, and Section XI, Table XI-1. Standing: derived (the base 2 and the form of the count).

> ***The ladder in symbolic form.***
>
> $m = m_{ref} \times 2^{-n\,s}$, where n is the count of a member in units of the step, $s = 1.0128448$ is the step factor (entry B-009), and m_{ref} is the mass at which the ladder is placed. The quark masses are read as powers of two below the top, and the charged-lepton masses as powers of two in ratio to the electron, whose mass is the unit of mass (entry B-014).

B-007 The Step Formula. That each charged sector has three generations, three points determine exactly one polynomial of degree at most two, and so the count of halvings of a sector is a quadratic in g with exactly three coefficients and no cubic term: $n = (\Delta + gP + g^2 d)\,s$. That Δ is the sector's offset (the zero, written by adding), P its step cost (the unit, written by multiplying) and d its curvature (the extent, written by a power), and that s is one step factor shared by every sector and every

rung. That the constants of the three charged sectors, as rows (Δ, P, d), are up (0, 6, 1), down (53/10, 6, −7/12) and charged lepton (391/60, 9/4, 16/9). That eight constants (P_q, P_l, d_{up}, d_{dn}, d_{lep}, Δ_{dn}, Δ_{lep} and s) are as many as the eight charged-fermion masses above the electron, so the test of the formula lies in the fractions of 3, 4 and 5 that the constants take and in the identities they satisfy among themselves (entries B-022 and B-042), and the agreement of the undressed formula with the masses alone is not the test.

Sources: *The Measures of the Kinds*, Chapter 7, Section I (Box I-1), Section VII and Section XI. Standing: derived (the form of the formula, from three generations and the base 2).

> ***The places the formula gives.***
>
> In units of s below the top, the brackets $\Delta + gP + g^2d$ are: top 0, charm 7, up 16; bottom 53/10, strange 643/60, down 449/30; tau 391/60, muon 949/90, electron 3263/180. Multiplied by s = 1.0128448 they give the undressed counts in halvings: top 0, charm 7.090, up 16.206; bottom 5.368, strange 10.854, down 15.159; tau 6.600, muon 10.680, electron 18.361. The order these give is the measured order of the nine, including the down quark above the up quark in the first generation, which the existence of hydrogen requires.

B-008 The Step Costs and the Sector Rule. That the quark step cost is six halvings per generation: a generation is a rotation of the content, which resolves each of the six intrinsics of comparison of its new instance once; the six verdicts are independent and two-valued, so they are six comparisons and six halvings, $P_q = 6 = c_1 n_C$, where $c_1 = 3$ counts the three positions of comparison and $n_C = 2$ counts the conjugate pair content and context (the two kernel integers). That the charged-lepton step cost is the quark step weighted by the charged leptons' share of one generation's charge squared: over one generation, with three colors and both chiralities, the sum of the squared charges is 16/3, of which the charged leptons carry 2, a share of 3/8, so $P_l = 6 \times 3/8 = 9/4$ (the share rule: the

one content ladder is the whole, and each context ladder pays its share of it). That the sector rule states that the step cost of a charged sector depends on its color alone, so both quark sectors share $P_q = 6$. That the step cost is reached by further routes kept side by side: a joint estimate from the four lighter quark masses gives $P_q = 5.9989 \pm 0.0044$ (bare, with the light quarks at 2 GeV), and the down sector alone, through the combination $n_d - 4n_s + 3n_b = -2P_q$, which uses no top, gives $P_q = 5.991 \pm 0.027$ with the strange and the down read at the floor, with 6 at $+0.33\sigma$.

Sources: *The Measures of the Kinds*, Chapter 7, Sections III to V (Boxes IV-1 and V-1); Chapter 9, Section IV (Box IV-3). Standing: derived.

B-009 The Step Factor and the One Mismatch. That at first order the step factor exceeds one by a small leftover ε, the mismatch between a counted measure and its continuous reading, $\varepsilon = 11/(80\pi^2) = 0.013932$, with $11 = c_1^2 + n_C$ and $80 = N^2H$ in the kernel triangle, the right triangle with legs $O = 3$ and $N = 4$ and hypotenuse $H = 5$ that the two kernel integers fix. That the Incommensuration Theorem makes that residue nonzero at every reading of a count by a continuous measure, and that the power of π in ε is argued, while the rational $11/5 = 16\pi^2\varepsilon$ is taken as the kernel triangle writes it. That ε is a pure number of the content instance of the internal space, which has no size; ε has no radius in it, and the step factor is the same on every rung because the space of states is homogeneous. That on the rungs the step factor is written as a sum, $s = 1 + \varepsilon - c_2\varepsilon^2 = 1.0128448$, with second-order coefficient $c_2 = 28/5 = H + O/H$, and that on the interval from the tau to $\sqrt{2}$ v the step factor is written as a product, $(1 + \varepsilon)(1 - c_2\varepsilon^2) = 1.0128296$. That the rule placing the two writings, that orders multiply across two domains and add within one, is argued from the three operations of arithmetic: bringing elements into one domain is addition, bringing two whole domains into relation is a different act and so not addition, and exponentiation is excluded because the axioms close without totalizing, so multiplication is the one operation left. That the placement of the product step's factors is derived, given a stated identification: the first-order reading of a relation of two domains is one kind of

reading, carried by the domain that holds them, so the interval carries one first-order mismatch ε, and the two domains it holds enter at the next order. That four leftovers measured at four places, resting on different measured quantities, lie within 0.15ε of ε: 0.922ε on the step from one generation to the next, 0.916ε on the top quark's shortfall below $v/\sqrt{2}$, 0.85ε at the light quarks' floor, and 0.948ε on the span from the electron to the reduced Planck mass. That the Z mass, through the step, asks $c_2 = 5.606$, and the muon, which does not enter that determination, asks $c_2 = 5.600016 \pm 0.000021$ (with the muon's dressing constant held at $7/9$), where $28/5$ stands at -0.76σ.

Sources: *The Measures of the Kinds*, Chapter 7, Section IX (Boxes IX-1 and IX-2, Table IX-1); *The Strengths of the Kinds*, Chapter 5, Section III (the form of ε); *The Kinds of Particle*, Chapter 3, Section V. Standing: derived (the power of π in ε, and the one step factor on every rung of a homogeneous space of states); an identification (the rational $11/5 = 16\pi^2\varepsilon$, taken as the kernel triangle writes it, which the four measured leftovers meet within 0.15ε); derived (the rule that orders multiply across two domains and add within one); derived, given a stated identification (the placement of the product step's factors, the domain that holds two domains carrying their one first-order mismatch); a measured agreement ($c_2 = 28/5$, whose question is why it lands on H + O/H).

B-010 The Top and the Condensate's Whole. That with $y_t = 1$ the ideal top lies at the condensate's whole read by content, $v/\sqrt{2} = 174.10$ GeV, and the measured top, 172.60 ± 0.27 GeV (direct mass, Particle Data Group 2026), falls 0.0125 halvings short of it, which is the one mismatch ε at first order. That six readings of the top against that whole, each resting on a different measured input, lie within 0.7σ of the measured top: the span reading (Newton's constant and the electron) 172.5166 GeV; the relation reading (the Higgs mass and v) 172.657 GeV; the tau ladder at second order 172.570 GeV; the step top (v and the step) 172.5604 GeV; the electron alone with the product step 172.5622 GeV (-0.14σ); and the nested offset with the foot 172.63 ± 0.30 GeV. The measured top, at its error of 0.27 GeV, does not separate them. That a top measured to about ± 0.016 GeV separates the span reading from the second-order tau ladder at three standard

deviations, and a top measured to ± 0.05 GeV separates the product step from the chain that takes the top one bare ε below $v/\sqrt{2}$ with the tau ladder at first order.

Sources: *The Measures of the Kinds*, Chapter 13, Sections III and VII to X (Table VII-1; Section X, Table X-2). Standing: derived ($y_t = 1$, with its scheme a setting fixed by stating it); a measured agreement (every value of the top's mass).

B-041 The Offsets. That the up offset is $\Delta_{up} = 0$, because the top is the reference, and that the down and lepton offsets, $\Delta_{dn} = 53/10$ and $\Delta_{lep} = 391/60$, are found by a condition that every true instance of the ladder meets, since of the three coefficients the offset is the one that no formula reaches. That, found by condition, each offset enters the ladder from measurement: the down offset from the measured top and bottom, and the lepton offset from the measured top and tau. That measured, $\Delta_{dn} = 5.29767 \pm 0.00302$, and $53/10$ stands at $+0.77\sigma$. That measured from the top and the tau, $\Delta_{lep} = 6.51818 \pm 0.00223$, and $391/60$ stands at -0.68σ. That measured from the tau and v, $\Delta_{lep} = 6.516777 \pm 0.000233$, and $391/60$ stands at -0.48σ. That the difference of the two offsets Δ_{dn} and Δ_{lep}, tested without the top, stands at -1.88σ. That each offset has a writing in the kernel triangle: $\Delta_{lep} = V/O^2 - O^2/V = 20/3 - 3/20 = 391/60$, with $V = ONH = 60$, and $\Delta_{dn} = V/(ON) - ON/V + 1/2 = 24/5 + 1/2 = 53/10$, equivalently $N + O/2 - r/H = 4 + 3/2 - 1/5$, with the inradius $r = 1$.

Sources: *The Measures of the Kinds*, Chapter 7, Sections I and VIII (Table VIII-1); Chapter 13, Section X. Standing: a measured input (the two offsets, in the working account, each carrying the question why its kernel writing lands on its measured placement); a theorem (each kernel writing equals its offset exactly).

B-042 The Curvatures and Their Closures. That the three charged curvatures are 1 for the up quarks, $16/9$ for the charged leptons and $-7/12$ for the down quarks. That the up curvature is the identity: the up sector, the reference, is comparison measuring itself, whose extent, what the act reaches, is exactly what it acts on, of value one, $d_{up} = r = 1$. That the

lepton curvature is $d_{lep} = (N/O)^2 = 16/9$, and the down curvature is $d_{dn} = H/(ON) - 1 = -7/12$, the down ladder being the up ladder read in reverse from its light end, rescaled by $O/H = 3/5$, with the placement 53/10 added and the term $r^2/V = 1/60$ in the power slot. That the three charged extents close as $O\, d_{lep} + N\, d_{dn} = O\, d_{up}$: $3 \times 16/9 + 4 \times (-7/12) = 3$; the step-weighted lepton curvature is the long leg, $P_l d_{lep} = 4 = N$; and the quark doublet's step-weighted extent is the circumradius, $P_q(d_{up} + d_{dn}) = 5/2 = H/2$.

Sources: *The Measures of the Kinds*, Chapter 7, Sections I (Box I-2), VI (Box VI-1) and VII. Standing: derived (the curvatures, from the kernel triangle); a theorem (the closures, identities among the derived values).

B-III. The Nine Charged Fermion Masses

B-011 The Up-Type Quark Masses. That the up ladder has its head at the top, with $\Delta_{up} = 0$, $P_q = 6$ and $d_{up} = 1$, so that the top, the charm and the up lie 0, 7 and 16 steps below the top: $n = g(6 + g)$. That with the top at its direct mass, 172.60 $\pm$ 0.27 GeV (Particle Data Group 2026), the ladder places the charm at its self-reading scale at 1.2670 GeV bare, against the measured $m_c(m_c) = 1.2729 \pm 0.0045$ GeV (Particle Data Group 2026), at -1.21σ, and at 1.274 GeV with the first-order dressing of the rungs (entry B-017), at $+0.12\sigma$. That it places the up at the floor at 2.284 MeV bare, at $+1.18\sigma$ against the measured 2.16 $\pm$ 0.07 MeV at 2 GeV (Particle Data Group 2024, the same value in 2026) carried to the floor (2.200 MeV), and at 2.278 MeV dressed, at $+1.09\sigma$. That a measurement of the up mass with the same central value and an uncertainty of about 0.04 MeV at the floor would place the ladder's up at two standard deviations.

Sources: *The Measures of the Kinds*, Chapter 9, Sections II and IV (Boxes IV-1 and IV-2, Table IV-1); Chapter 13, Section X, Table X-3. Standing: a measured agreement.

B-012 The Down-Type Quark Masses. That the down ladder has its head at the bottom, 53/10 steps below the top, with the same step cost $P_q = 6$ and the curvature $d_{dn} = -7/12$, so that the bottom, the strange and the down lie 53/10, 643/60 and 449/30 steps below the top, and the strange and the down lie 65/12 and 29/3 steps below the bottom. That placed from the measured top, the bottom is $m_b(m_b) = 4.1792$ GeV, at -0.77σ from the measured 4.186 ± 0.006 GeV (Particle Data Group 2026); this agreement is the offset's own question, since the bottom is the datum by which 53/10 is measured (entry B-041). That placed from the measured bottom, the strange at the floor is 93.38 MeV bare (-1.70σ) and 93.87 MeV dressed (-1.03σ), against the measured 92.9 ± 0.7 MeV at 2 GeV (Particle Data Group 2026) carried to the floor (94.62 MeV); and the down at the floor is 4.726 MeV bare (-0.85σ) and 4.713 MeV dressed (-1.02σ), against the measured 4.70 ± 0.07 MeV at 2 GeV (Particle Data Group 2024, the same value in 2026) carried to the floor (4.787 MeV). That the down quarks stand at the boundary between the unknown (the up quarks) and the known (the charged leptons) among the four positions of knowing.

Sources: *The Measures of the Kinds*, Chapter 7, Section III; Chapter 9, Sections II to IV (Boxes III-1, IV-1 and IV-2, Table IV-1); Chapter 13, Section X, Table X-3. Standing: a measured agreement.

B-013 The Charged Lepton Masses. That the charged-lepton ladder has the offset 391/60, the step cost 9/4 and the curvature 16/9, so that the tau, the muon and the electron lie 6.600, 10.680 and 18.361 halvings below the top on the undressed ladder. That within the ladder the muon lies (91/12)s and the tau (209/18)s halvings above the electron. That the electron's mass, 0.51099895069(16) MeV (CODATA 2022, the values recommended by the Committee on Data of the International Science Council), is the unit of mass and is never derived. That from the electron alone the ladder places the muon and the tau within one percent, and with its dressing (the turnover: strength $\kappa = \varepsilon/6$ and second-order constant $B = 7/9$) it places the muon at $m_\mu/m_e = 206.7682862$, against the measured

206.7682827(46) (CODATA 2022), at $+0.75\sigma$. That the tau follows from the measured electron and muon by Koide's relation (entry B-043) at 1776.969 MeV, against the measured 1776.93 $\pm$ 0.09 MeV (Particle Data Group 2024), at $+0.43\sigma$.

Sources: *The Measures of the Kinds*, Chapter 7, Sections II and XI; Chapter 8, Sections III, V and VI; Chapter 13, Section X, Tables X-1 and X-3. Standing: derived (the muon from the electron, to first order); a measured agreement (the muon at its full precision); derived, given a stated identification (the tau, by Koide's relation, the identification being that each part of the amplitude vector is measured by its share of the squared size); a measured input (the electron, the unit).

> ***Why the charged leptons carry so much information.***
>
> The charged leptons carry no color and receive no corrections from the strong force, so they are the cleanest test of the step formula. Counted in bits (entry B-005), the muon's agreement carries 25.4 bits, the most of any output of *The Measures of the Kinds*, and the tau's carries 14.3 bits. The muon's agreement has the standing of a measured agreement, carried with the question why the muon lands on its measured value. Toward that reason the book points to its second-order constant, which lands on $7/9 = (c_1^2 - n_C)/c_1^2$, whose numerator 7 the down quarks' curvature $-7/12$ shares.

B-014 What Measurement Supplies: Two Accounts. That *The Measures of the Kinds* states what measurement supplies in two accounts, and holds both true. That in the working account the measured inputs are the electron's mass, which is the unit, the strength of light α_{em} ($1/\alpha_{em} = 137.035999177(21)$, CODATA 2022), the strength of color $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ (Particle Data Group 2024) and the vacuum value $v = 246.21967 \pm 0.00006$ GeV, together with the two offsets of the charged ladders, found from the measured bottom and tau (entry B-041). That in the closure account the step on the interval from the tau to $\sqrt{2}\,v$ is written as a product, its domain rule (orders multiply across two domains and add within one) is argued, with the factor each domain supplies a stated identification

(entry B-009), and with it the electron alone carries the chain to the vacuum value, at 246.21966 GeV (-0.16σ, 22.0 bits), the top, at 172.5622 GeV (-0.14σ against 172.60 $\pm$ 0.27 GeV, Particle Data Group 2026), and the Higgs mass, at 125.172 GeV. That the chain rests on no bridge from the electron to Newton's constant: Newton's constant is the measured input of the side of gravitation, and the ground fixes its role and not its magnitude.

Sources: *The Measures of the Kinds*, Chapter 7, Sections IX and XII; Chapter 13, Sections VIII and X (Table X-1). Standing: a measured input (the inputs of the working account); derived (the domain rule of the product step); derived, given a stated identification (the values of the closure account, the identification being the factor each of the two domains supplies to the product step).

B-043 Koide's Relation and the Tau. That the three charged-lepton masses, read through their square roots, divide their total equally between the part the three share and the part in which they differ: $Q = (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ (Koide 1983; the balance of sameness and difference). That the balance is derived, given a stated identification: the three square roots form the amplitude vector, its common part and its differing part carry equal squared size, and each part is measured by its share of the squared size. That measured, Q stands 0.43σ from 2/3, inside its two-standard-deviation window, in which 2/3 is the one simple form, and the balance carries 15.0 bits before any form is chosen. That from the measured electron and muon the balance gives the tau at 1776.969 MeV ($+0.43\sigma$), and fixes Brannen's phase at 0.22222205, 1.75×10^{-7} below 2/9. That the ladder's own reading of the tau, with its turnover series as written, is 1777.146 MeV ($+2.4\sigma$), 0.177 MeV above the balance's, and that a tau mass measured to $\pm$0.045 MeV (the precision stated for the Belle II experiment) separates the ladder's 1777.146 MeV from the balance at 3.9 standard deviations, and the ladder with the head factor at $h = 0$, 1777.169 MeV, from the balance at 4.4.

Sources: *The Measures of the Kinds*, Preface; Chapter 8, Sections I to IV and VI (Table VI-1), and Summary; Koide (1983); Brannen (2006). Standing: derived, given a stated identification (Koide's balance 2/3 and the tau from the measured electron and muon, the identification being that each part of the amplitude vector is measured by its share of the squared size).

B-IV. The Dressing of the Rungs and the Nine Masses against Measurement

B-015 Lattice QCD. That lattice quantum chromodynamics (lattice QCD) is a computational method for the strong force (the omniscient force in the frame of measurement; *The Concept Sequence*, Chapter 18, entry 18.11). It replaces spacetime by a grid, a regular lattice of points separated by a fixed spacing, and computes the behavior of quarks and gluons on that grid by numerical simulation. Lattice QCD computes how quarks bind together into composite particles (such as protons, neutrons and pions) from the theory of the strong force, and it gives the running masses of the light quarks that the Particle Data Group and the Flavour Lattice Averaging Group (FLAG) quote.

Sources: *The Matter We Know*, Volume One, Chapter 2, Sections VI and VII; *The Strengths of the Kinds*, Chapter 6, Section I. Standing: a definition.

B-016 Hadron. That a hadron is a colorless whole of quarks held together by gluons, the carriers of the strong force. The proton (two up quarks and one down quark) and the neutron (two down quarks and one up quark) are the two hadrons that compose all atomic nuclei. The pion and the kaon are lighter hadrons, pairings of a quark with an antiquark. The mass of a hadron is mostly the energy of its binding, and only a small part the sum of its quarks' masses: the proton's mass, 938.27208943(29) MeV (CODATA 2022), is about 99 percent the energy of the color field and of the quarks' motion. That a hadron is a composite, a combination of the kinds, and adds no new kind.

Sources: *The Matter We Know*, Volume One, Chapter 2, Sections II and VII; *The Story of the Particles*, Level 9, Section I. Standing: a definition (the hadron); a measured input (the proton's mass); derived (a hadron adds no new kind).

B-017 The Dressing of the Rungs. That at first order in ε the second and third rungs of every charged ladder are dressed: the mass at $g = 1$ is multiplied by $1 + (10/27)\varepsilon$ and the mass at $g = 2$ by $1 - (5/27)\varepsilon$, and the dressing vanishes at every head. That the pair is the factor $1 + \kappa\Phi(g)$ with strength $\kappa = \varepsilon/6$ and rung function $\Phi(g) = \lambda_g(9 - \lambda_g)/9$, $\lambda_g = g(g + 3)$, which on the lepton ladder *The Measures of the Kinds* calls the turnover. That applied to all three charged ladders the dressing moves every place that moves closer to its measured value: the χ^2 (the sum of the squared pulls) over the eight places below the top falls from 15.88 to 3.65, with the order of the nine unchanged. That the dressing moves the charm and the strange up by 0.52 percent and the up and the down down by 0.26 percent, and that the quark masses pass bare as well as dressed, so their agreement does not rest on the dressing. That this dressing is distinct from the running of a quark mass between scales (entry B-003), which is the standard running of quantum chromodynamics.

Sources: *The Measures of the Kinds*, Chapter 7, Section X; Chapter 8, Section V; Chapter 9, Section IV. Standing: a measured agreement (whose question is why the first-order pair $(10/27, -5/27)\varepsilon$ meets the measured rungs within one ε^2, toward which the book points to $\kappa = \varepsilon/6$ and to a rung function that saturates at $c_1^2 = 9$).

B-019 The Nine Charged Masses against Measurement. That the nine charged fermion masses stand against measurement as follows. The top: the electron alone with the product step gives 172.5622 GeV against 172.60 $\pm$ 0.27 GeV (Particle Data Group 2026), -0.14σ; six readings lie within 0.7σ (entry B-010). The bottom: placed from the top, 4.1792 GeV against 4.186 $\pm$ 0.006 GeV (Particle Data Group 2026), -0.77σ. The charm: 1.2670 GeV bare (-1.21σ) and 1.274 GeV dressed ($+0.12\sigma$) against 1.2729 $\pm$ 0.0045 GeV (Particle Data Group 2026). The strange, at the floor: 93.38 MeV bare (-1.70σ) and 93.87 MeV dressed (-1.03σ). The down, at the

floor: 4.726 MeV bare (-0.85σ) and 4.713 MeV dressed (-1.02σ). The up, at the floor: 2.284 MeV bare ($+1.18\sigma$) and 2.278 MeV dressed ($+1.09\sigma$). The tau: 1776.969 MeV by Koide's relation against 1776.93 ± 0.09 MeV (Particle Data Group 2024), $+0.43\sigma$. The muon: $m_\mu/m_e = 206.7682862$ against 206.7682827(46) (CODATA 2022), $+0.75\sigma$. The electron: the unit, 0.51099895069(16) MeV (CODATA 2022). That every quark placement passes, bare and dressed, at the floor and at 2 GeV, and with the top measured or reached from the tau.

Sources: *The Measures of the Kinds*, Chapter 8; Chapter 9, Sections III, IV and VII; Chapter 13, Section X, Tables X-1, X-2 and X-3. Standing: a measured input (the electron); derived, given a stated identification (the tau, by Koide's relation, each part of the amplitude vector measured by its share of the squared size); derived (the muon to first order); a measured agreement (the muon at its full precision, the six quark masses and every value of the top's mass).

> ***What these numbers mean.***
>
> The nine masses span more than five orders of magnitude, from the top at about 172.6 GeV to the electron at 0.511 MeV, a count of about eighteen halvings. Eight constants of the step formula are as many as the eight masses above the electron, so the agreement of the formula with the masses is not by itself the test; the test lies in the constants' being simple fractions of 3, 4 and 5 and in the identities among them (entries B-022 and B-042). The quark masses carry the larger uncertainties of their measurement, at the level of one to a few percent, and they stand as measured agreements; among the charged leptons the muon's agreement carries the most information of any output of *The Measures of the Kinds*.

B-V. What the Finite Geometry Fixes and What the Ladder Carries

B-020 The Form and the Values. That in the finite geometry the masses and mixings of the fermions are entries of the finite Dirac operator D_F (the operator of the finite spectral triple, an algebra with the space of

states on which it acts and a Dirac operator, from which a geometry is read): the four Yukawa matrices, with the Majorana matrix zero in this account. That the finite geometry fixes the gauge group, the representation and the one Higgs doublet, and the Yukawa matrices are moduli of D_F, entries that the finite geometry does not determine. That the program of Chamseddine and Connes treats these entries as parameters of the finite geometry, and the algebraic derivation of this account stops at the same line, deriving no value of any coupling or mass from the algebra. That the values are carried by *The Measures of the Kinds*, which counts each mass in halvings below a reference (Parts B-II and B-III of this chapter) and states each with its standing. That rest mass is a Yukawa eigenvalue of the finite geometry, and exchange mass and confinement mass are not concepts of the finite geometry.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Section II, and Chapter 19, Section XIII; *The Connes Correspondence*, Chapter 5, Section VI. Standing: a theorem (what the finite geometry fixes and what it leaves as moduli).

B-021 The Higgs Self-Coupling and the Higgs Mass. That the Higgs self-coupling at tree level is $\lambda = 1/8$, fixed by the form route, $\lambda = a^2/2$ with $a = 1/2$ the doublet's half (the weak isospin of each side of the doublet), at every number of colors, and by the share route at three colors, as the down quark's share of one generation's charge squared, $(1/3)/(8/3) = 1/8$. That the step $\lambda = a^2/2$ identifies the self-coupling with half the square of the doublet's half. That the spectral action's own relations at its cutoff tie λ to the gauge coupling and the couplings to the condensate, and they do not give $1/8$ (entry B-031, the cutoff relations of the spectral action). That at $\lambda = 1/8$ the Higgs mass at tree level is $v/2 = 123.11$ GeV, -18.4σ from the measured mass. That the measured on-shell self-coupling, $\lambda = m_H^2/(2v^2) = 0.12914 \pm 0.00023$, exceeds $1/8$ by $(0.980 \pm 0.054) \times 1/(24\pi^2)$, and the mass formula $m_H = v\sqrt{(2(1/8 + 1/(24\pi^2)))} = 125.1715$ GeV, with the measured v and no input from the top, stands at $+0.38\sigma$ from the measured 125.13 ± 0.11 GeV (Particle Data Group 2026). That

the loop term $1/(24\pi^2)$ is the doublet's half times the photon's one-loop unit $\kappa_H = 1/(12\pi^2)$. Chapter E, entry E-005, states the same self-coupling at tree level within the Higgs potential, and entry E-005 also gives three routes to the doublet's half.

Sources: *The Measures of the Kinds*, Chapter 13, Sections II, IV and VI (Box VI-1, Table VI-1), and Section X, Table X-1. Standing: derived, given a stated identification ($\lambda = 1/8$ at tree level, the identification being $\lambda = a^2/2$); a measured agreement (the Higgs mass 125.17 GeV, whose question is why the measured excess over $1/8$ agrees with $1/(24\pi^2)$).

B-022 The Constants as Invariants of the Kernel Triangle. That each constant of the step formula is an exact invariant of the kernel triangle (legs O = 3, N = 4, hypotenuse H = 5, inradius r = 1, area A = 6, circumradius R = 5/2, triple product V = ONH = 60): $d_{up} = r = 1$; $P_q = A = 6$; $P_l = (O/n_C)^2 = 9/4$; $d_{lep} = (N/O)^2 = 16/9$; $d_{dn} = H/(ON) - 1 = -7/12$; $\Delta_{dn} = N + O/n_C - r/H = 53/10$; $\Delta_{lep} = N + R + r/V = 391/60$; $c_2 = H + O/H = 28/5$; and $\varepsilon\pi^2 = (12 - 1)/(N^2H) = 11/80$. That the map from the invariants to the slots of the formula is one to one and respects type: each invariant sits in the slot of its own arithmetic type (sum, product or power), and no invariant appears in two slots. That the matrix of the charged constants, rows (Δ, P, d), has determinant $-6391/60 = -(7 \times 11 \times 83)/60$, with $7 = c_1^2 - n_C$, $11 = c_1^2 + n_C$, $83 = c_1^4 + n_C$ and $60 = V$.

Sources: *The Measures of the Kinds*, Chapter 7, Sections I and VII. Standing: a theorem (the writings and the determinant, given the constants).

B-VI. The Quark Mixing Matrix

B-023 Flavor Mixing. That the states that the weak force's charged current joins differ from the mass eigenstates of the quarks, the states with definite masses (the weak force is the transcendent force in the frame of measurement; *The Concept Sequence*, Chapter 18, entry 18.13). The charged current turns a mass state of one kind into a combination of the three

mass states of the other kind. The matrix that describes this transformation is the quark mixing matrix (the Cabibbo-Kobayashi-Maskawa matrix): a three-by-three unitary matrix whose entry V_{ij} is the amplitude with which the charged current joins the up-type mass state i (u, c or t) to the down-type mass state j (d, s or b). That after the phases of the quarks are redefined, it holds three angles and one phase, the count of angles being $c_1(c_1 - 1)/2 = 3$. That the one phase is how CP violation, the difference between a process and its mirror under charge and parity together, is realized: the Incommensuration Theorem's requirement limits where a process and its mirror can differ, and the one phase that three generations allow is how the remaining possibility is realized. That mixing can arise only in the Yukawa couplings, because the strong force and the gauge carriers treat every generation alike.

Sources: *The Measures of the Kinds*, Chapter 10, Section I; *The Story of the Particles*, Level 7, Section III; Cabibbo (1963); Kobayashi and Maskawa (1973). Standing: a definition (the matrix); a theorem (its count of three angles and one phase, for three generations); derived (CP violation through the one phase, as *The Story of the Particles* states it).

> ***What the quark mixing matrix is, physically.***
>
> Consider the top quark undergoing a weak decay. It can turn into a bottom, a strange or a down quark, with probabilities given by the squares of the magnitudes of the corresponding entries. The top turns almost always into a bottom; the measured magnitudes $|V_{ts}| = 0.0415 \pm 0.0009$ and $|V_{td}| = 0.0086 \pm 0.0002$ (Particle Data Group 2024, from the mixing of B mesons) make the turns to a strange and to a down rare and rarer. A transition of one step between generations is more probable than a transition of two steps, and the measured matrix has this pattern. The mixing values are pure numbers of a kind that a law fixes, and their standings are stated in entries B-024 to B-026.

B-024 The Texture Zero and the Mass Textures. That when the lighter of two copies of a quark kind has a zero diagonal entry in the mass matrix (a texture zero), its mass comes only through its joining to the heavier

copy, and the angle between the two is about the square root of their ratio of masses, $\theta \approx \sqrt{(m_{light}/m_{heavy})}$ (Gatto, Sartori and Tonin 1968). That a single zero in the first diagonal entry stays consistent with the data, and textures with further zeros, as in Fritzsch's full texture, give a third generation that measurement does not bear out. That zeros in both (1,1) entries are a choice of weak basis, so the content of the texture zero lies in the signed spectrum of each sector, the decoupling of the 1-2 block and the relative phase of the two rotations. That the mass textures are held as a hypothesis, with their outputs stated as values; the texture of the masses alone stands 17σ from the measured $|V_{ub}|/|V_{cb}|$.

Sources: *The Measures of the Kinds*, Chapter 10, Sections II, VII and IX (Boxes II-1, II-2 and VII-1); Gatto, Sartori and Tonin (1968); Fritzsch (1977). Standing: a theorem (the angle of a copy with a zero diagonal entry, and the zeros of both (1,1) entries as a choice of weak basis); an identification (each mass texture, held as a hypothesis, with its outputs stated as values).

B-025 The Right Angle and the Quark CP Phase. That unitarity makes the d column and the b column of V orthogonal, and the three routes between them, one per generation, close the unitarity triangle with angles α, β and γ. That the kernel triangle's right angle is carried to the unitarity triangle, between the routes of the first and third generations, with the second generation's route as the hypotenuse: $\alpha = 90°$, at $+1.59\sigma$ of the direct $\alpha = 84.1° +3.7° -3.0°$ (Particle Data Group 2026). That given the right angle the quark CP phase is a law of the three mixing angles, $\cos \delta_q = \cot \theta_{12} \cot \theta_{23} \sin \theta_{13}$, which gives $\delta_q = 67.15° \pm 0.70°$ on the 2026 angles, against the measured $66.12° \pm 1.43°$ (Particle Data Group 2026), at $+0.65\sigma$, twice as precise as its measurement. That given the right angle the triangle has one free angle, β, and three independent arguments, each with its own identification, read $\beta = \pi/8$ and $\gamma = 3\pi/8$, at -0.26σ of the measured $\beta = 22.62° \pm 0.45°$ (Particle Data Group 2026). That the reading $\delta_q = \arctan c_1 = \arctan 3 = 71.57°$ stands at $+3.80\sigma$ of the measured phase and is kept as a hypothesis with its pull.

Sources: *The Measures of the Kinds*, Chapter 10, Sections III to V and IX; Chapter 13, Section X, Table X-1; Jarlskog (1985). Standing: derived, given a stated identification (the right angle, the identification being that each generation's route lies on the side of the kernel triangle whose position that generation has foremost); derived, given a stated identification (the quark CP phase as a law of the three mixing angles, given the right angle); derived, given a stated identification ($\beta = \pi/8$ and $\gamma = 3\pi/8$, by three arguments with three identifications); an identification ($\delta_q = \arctan 3$, kept as a hypothesis with its pull).

B-026 The Mixing Values against Measurement. That the Cabibbo angle is read from the down ladder in two laws, kept side by side. The first takes the down rotation alone: $|V_{us}| = \sqrt{(m_d/m_s)} = 2^{-(17/8)s} = 0.22495$, at -0.32σ of the global fit of sin θ_{12}, 0.22517 ± 0.00068 (Particle Data Group 2026), and at $+0.76\sigma$ of the value from kaon decays, 0.22431 ± 0.00085 (Particle Data Group 2024, the same in 2026). The second adds the up rotation in quadrature at the right-angle phase: 0.22410 at the floor, at -0.25σ of the kaon value and -1.58σ of the global fit. At most one of the two laws is exact, and a measurement of $|V_{us}|$ to ± 0.00043 decides between them. That the counted value $|V_{cb}| = 2^{-(68/15)s} = 0.04148$ stands at -0.60σ of sin $\theta_{23} = 0.04189\ {}^{+0.00081}_{-0.00069}$ (Particle Data Group 2026). That with the right angle, $\beta = \pi/8$, the two-sector $|V_{us}|$ and the measured $|V_{cb}|$, $|V_{ub}| = 0.003684 \pm 0.000066$, at -0.75σ of sin $\theta_{13} = 0.003763\ {}^{+0.000088}_{-0.000083}$, and the Jarlskog invariant $J = (3.112 \pm 0.111) \times 10^{-5}$, at -0.31σ of $3.16\ {}^{+0.13}_{-0.11} \times 10^{-5}$ (Particle Data Group 2026).

Sources: *The Measures of the Kinds*, Chapter 10, Sections II, VI and IX (Boxes II-3, II-4 and VI-1, Tables II-1 and II-2); Chapter 13, Section X, Table X-3. Standing: derived, given a stated identification ($|V_{us}|$, the identification being the texture zero at the down quark of the first generation, with the right angle also for the second law); a measured agreement (the counted $|V_{cb}|$, and $|V_{ub}|$ as one of the quark mixing angles, computed in Box VI-1 from the premises named in the entry, with J computed from those angles and the quark CP phase).

The d/s width of the down ladder.
The d/s column of the down ladder has the width $P_q + 3d_{dn} = 6 - 7/4 = 17/4$ units of the step, so $m_d/m_s = 2^{-(17/4)s} = 0.050605$; the bottom mass cancels from the ratio, which stands at $+1.47\sigma$ of the Particle Data Group's 2026 lattice ratios and at $+0.06\sigma$ of its 2026 masses at 2 GeV. Half the width in the exponent gives the first Cabibbo law. A third writing, independent of the step s, is the Pythagorean triangle (9, 40, 41) that Euclid's formula makes from (H, N): read as the tangent of the down rotation, $9/40 = O^2/(2NH)$ gives $m_d/m_s = 81/1600 = 0.050625$, and 9/40 differs from $2^{-(17/8)s}$ by 4.6×10^{-5}.

B-VII. The Lepton Mixing Matrix and the Neutrino Masses

B-027 Lepton Mixing. That the lepton sector has its own mixing matrix, the Pontecorvo-Maki-Nakagawa-Sakata matrix, whose entry $U_{\alpha i}$ is the amplitude of the mass state i in the flavor α (the electron, muon or tau neutrino). That because the neutrino is a Dirac particle the matrix has three angles and one phase, every one observable in oscillation, and no Majorana phases. That the quark mixing matrix lies close to the identity while two lepton angles are large, and four routes bear on the difference, kept side by side: wide separation of masses goes with small mixing and close masses with large mixing; the quarks are the content-level and the leptons the context-level re-projection of one generation; on the content instance the context factor S^2 goes with the large lepton angles and the content factor $\mathbb{C}P^2$ with the small quark angles; and the quark block is anchored at the identity, the lepton block at the sameness frame of the three generations, which gives the tribimaximal values $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$ and $\theta_{13} = 0$ before correction (Harrison, Perkins and Scott 2002). That the anchor at the sameness frame is held as a hypothesis.

Sources: *The Measures of the Kinds*, Chapter 11, Sections I and II (Boxes I-1 and II-1); *The Branches of the Kinds*, Chapter 1, Section VII; Harrison, Perkins and Scott (2002). Standing: a definition (the matrix); derived (its count of three angles and one phase, on the Dirac neutrino); an identification (the anchor at the sameness frame, held as a hypothesis).

B-028 The Neutrino Masses: Two Routes. That each neutrino mass is a Dirac mass, joining the left-handed neutrino to its right-handed partner through the Higgs field, $m_\nu = y_\nu v/\sqrt{2}$, first order in the condensate like every charged fermion mass; no further mass scale enters and no heavy neutrino exists, since no state of content is its own reversal and so there is no Majorana mass and no seesaw scale; the lightness of the neutrinos is the smallness of y_ν, about 3×10^{-13} for the heaviest. That the neutrino is the fourth sector of the step formula, with its own offset and curvature, and its three masses are carried as a fork of two routes, decided by the mass sum. That on the count route the step cost is the charged lepton's $9/4$, the curvature is $-41/60$ (the quark curvatures weighted by their shares of charge squared, $4:1$, read reversed), and the offset is the nested product $\Delta_{\text{lep}}(1 + \Delta_{\text{dn}}) = 41.055$ units of s below the top; placed from the measured tau it gives $m_1, m_2, m_3 = 15.149, 17.433, 52.366$ meV, with a sum of 84.95 meV. That on the Koide route the neutrinos are in the scope of Koide's relation with the root of the lightest mass negative; it takes both splittings as inputs and gives 0.370, 8.726 and 50.211 meV, with a sum of 59.31 ± 0.25 meV; on its ladder the step cost is $3/2$ and the curvature 1. That for the constants of the neutrino's ladder the two routes are one selection: the power at which the ladder reads the number $O/n_C = 3/2$.

Sources: *The Measures of the Kinds*, Chapter 12, Sections I to IV, VI (Table VI-1) and VIII; *The Branches of the Kinds*, Chapter 1, Sections II and V. Standing: a fork that a named measurement decides (the three masses, decided by the measured sum of the neutrino masses); derived (the Dirac kind of the masses and the absence of a seesaw scale).

B-029 The Neutrino Splittings against Measurement. That oscillation measures differences of squared masses (splittings), because mass enters every propagation phase as its square. That the count route, placed from the tau alone, takes no splitting as input and predicts both: $\Delta m^2_{21} = 7.441 \times 10^{-5}$ eV², at -0.94σ of $(7.60 \pm 0.17) \times 10^{-5}$ eV² (Particle Data Group 2026), and $\Delta m^2_{31} = 2.5127 \times 10^{-3}$ eV², at -0.36σ of $(2.521 \pm 0.023) \times 10^{-3}$ eV² (Particle Data Group 2026, normal order). That the count route's curvature $-41/60$ meets the curvature the splittings ask, -0.68361 ± 0.00106, at $+0.26\sigma$. That the Koide route takes both splittings as inputs. That, with the lightest mass of the Koide route fixed by the hydrogen relation $m_{\nu 1} = \alpha^4 m_e/4 = 0.36226$ meV and with Δm^2_{31} as input, Koide's relation predicts $\Delta m^2_{21} = (7.470 \pm 0.040) \times 10^{-5}$ eV². That the predicted Δm^2_{21} stands at -0.74σ of the Particle Data Group's 2026 value and at -0.24σ of the JUNO value $(7.50 \pm 0.12) \times 10^{-5}$ eV².

Sources: *The Measures of the Kinds*, Chapter 12, Sections I, III, VI and VIII; Chapter 13, Section X, Table X-1; JUNO Collaboration (2026). Standing: a measured agreement (on the count branch of the neutrino fork, the two splittings, and on the Koide branch, Δm^2_{21} with the hydrogen relation).

B-030 The Lepton Mixing Angles and Phase. That two families of laws correct the anchor and fix the lepton phase δ_ℓ, carried side by side as a fork. That the carry family carries the right angle of the quark unitarity triangle to the lepton block, $\cos \delta_\ell = \cot \theta_{12} \cot \theta_{23} \sin \theta_{13} > 0$, and with the correction TM1, which keeps the first column of the anchor ($|U_{e1}|^2 = 2/3$), and $\sin^2\theta_{13} = 1/45$ it gives $\sin^2\theta_{12} = 0.3185$ ($+0.96\sigma$ of 0.307 ± 0.012, Particle Data Group 2026), $\sin^2\theta_{23} = 0.5444$ ($+0.63\sigma$ of 0.533 ± 0.018) and $\delta_\ell = 281.37°$ ($+1.86\sigma$ of $217.8° +34.2° -30.6°$), in the fourth quadrant. That $\sin^2\theta_{13} = 1/45$ stands at $+0.75\sigma$ of 0.0217 ± 0.0007 (Particle Data Group 2026) and within 1σ of three measurements. That the segment family ties the lepton phase to the quark phase, $\delta_q + \delta_\ell = 3\pi/2$, which puts δ_ℓ at 203.88°, in the third quadrant. That the quadrant of δ_ℓ decides between the

families, with the octant of θ_{23} and the size of the lepton Jarlskog invariant (about 0.033 against about 0.013) following. That a texture built with the kernel number of the lepton block, $\varepsilon_{\nu} = c_1/(c_1^2 + 1) = 3/10$ (a number distinct from the one mismatch ε), real by its own assumption and so outside the fork, gives $\theta_{12} = 33.12°$, $\theta_{23} = 49.17°$ and $\theta_{13} = 8.571°$, with the masses of the boundary placement of the neutrino's ladder; θ_{23} and θ_{13} entered none of its constants.

Sources: *The Measures of the Kinds*, Chapter 11, Sections III to VII (Tables I-1, IV-1 and V-1); Chapter 12, Section VII; Chapter 13, Section X, Tables X-1 and X-3; Esteban and others (2024). Standing: a fork that a named measurement decides (the lepton angles and phase, decided by the quadrant of δ_{ℓ}); a measured agreement ($\sin^2\theta_{13} = 1/45$ and the texture's angles).

B-VIII. The Cutoff Relations of the Spectral Action

B-031 The Cutoff Relations. That at its cutoff scale the spectral action (the action read from the spectrum of the Dirac operator up to a highest energy, the cutoff) fixes the ratio of the three gauge couplings, $g_3^2 = g_2^2 = (5/3)g_1^2$, a relation between the Higgs quartic coupling, the gauge coupling and the Yukawa couplings, and a relation between the sum of the squared Yukawa couplings and the gauge coupling. That the three relations follow from the spectral action's single trace over the whole finite space, which ties the couplings to one common measure. That in the program of Chamseddine and Connes the three relations hold at a cutoff near the scale of grand unification, and are conditions of unification at that scale. That the three relations, imposed at the scale of grand unification, gave a Higgs mass near 170 GeV (Chamseddine, Connes and Marcolli 2007), which the measured mass of about 125 GeV excludes, and that the three relations do not give the self-coupling 1/8. That in this account the equality of the three traces is a relation among the weights that the structure gives the three forces. That the equality of the three traces is not an equality of measured strengths and not a condition of unification. That

the single trace reaches the same relation among the weights as a second route. That the measured couplings meet at no scale, and that no coupling runs from a scale of the internal space. That no mass of this chapter rests on the relation between the squared Yukawa couplings and the gauge coupling: the Yukawa matrices are moduli of the finite geometry (entry B-020), and their values are those of Parts B-II, B-III and B-VII.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Sections II to IV; *The Connes Correspondence*, Chapter 5, Sections II and III; *The Kinds of Particle*, Chapter 5, Section IV; *The Strengths of the Kinds*, Chapter 2, Section VI, and Chapter 3, Sections III and IV; *The Measures of the Kinds*, Chapter 13, Section IV; Chamseddine, Connes and Marcolli (2007). Standing: a theorem (the relations, as properties of the spectral action at its cutoff in its own setting); derived (the equality of the three traces as a relation among weights).

B-IX. Lattice QCD and the Composites

B-035 From Quarks to Hadrons. That the quark masses of the ladder are properties of the quarks, and the quarks other than the top are never at rest and never alone, so their masses are read as exchange mass at stated scales (entry B-003), while the masses measured most directly in the laboratory are those of the hadrons (entry B-016), the proton, the neutron, the pion and the kaon. That the bridge between the quark masses and the hadron masses is quantum chromodynamics, computed on the lattice (entry B-015), and that the hadrons are composites, so a computation of them tests the quark masses used as inputs and leaves the list of kinds untouched.

Sources: *The Measures of the Kinds*, Chapter 7, Section II (the three meanings of mass), and Chapter 9, Section VI; *The Matter We Know*, Volume One, Chapter 2. Standing: a definition.

B-036 Quark Mass Ratios and the Six-Mass Relation. That on the step constants the ratio of the up and down masses is $m_u/m_d = 2^{-(31/30)s} = 0.4841$, at $+0.65\sigma$ of the Particle Data Group's 2026 lattice ra-

tio 0.473 ± 0.017 and at +0.80σ of FLAG 2024's four-flavor average 0.465 ± 0.024; the scale does not matter for a ratio of two light masses. That among the six masses the ratio $m_u m_t m_s/(m_d m_b m_c)$, each mass at the scale at which it is quoted, agrees with $\sqrt{2}$: its base-two logarithm, the h-sum, is 0.4678 ± 0.0529 halvings measured (Particle Data Group 2026), and the $\sqrt{2}$ law (one half) stands at +0.61σ. That the step constants give the same ratio the value $2^{(11/20)s} = 1.471$, at +1.69σ, so two laws stand at one measured number, 0.057 halvings apart. That a measurement of the up mass to ±1 percent would leave the two laws 2.0 standard deviations apart, because the other five masses, at their present errors, hold 0.025 halvings of the h-sum's uncertainty; to separate them at three standard deviations, those five must also be measured to about half their present errors.

Sources: *The Measures of the Kinds*, Chapter 9, Sections VI and VII (Box VI-1, Table VI-1); Aoki and others (2026), the FLAG Review 2024. Standing: a measured agreement.

B-037 The Proton from the Ladder's Quark Masses. That the quark masses of the ladder, with their corrections, carried through lattice QCD with chiral perturbation theory (the expansion of quantum chromodynamics in the light quark masses) and the separation of the effects of quantum chromodynamics and quantum electrodynamics published by the Budapest-Marseille-Wuppertal collaboration, give the proton's mass at 939.7 MeV, against the measured 938.27 MeV, 0.15 percent above it. That the proton's mass is about 99 percent the energy of the color field and of the quarks' motion (entry B-016), so it is little sensitive to the light quark masses.

Sources: *The Measures of the Kinds*, Chapter 9, Section VI (the heavy triple); Borsanyi and others (2015). Standing: a measured agreement (a standard computation with the ladder's quark masses as inputs).

B-038 The Neutron–Proton Mass Difference. That the neutron is heavier than the proton by 1.29333251 ± 0.00000038 MeV (CODATA 2022), the quantity that lets hydrogen last and sets which nuclei can exist.

That the sign follows from the ladder's constants: the down lies $449/30 = 14.967$ steps below the top and the up 16, so $m_d > m_u$, the order that the existence of hydrogen requires. That the difference is a balance of two opposed parts: a part from the down-minus-up quark mass difference, which makes the neutron heavier, and an electromagnetic part, which makes the proton heavier. With the measured quark masses the lattice computation gives about $+2.52$ MeV and about -1.00 MeV, a total of 1.51 $\pm$ 0.28 MeV, in agreement with the measured difference (Borsanyi and others 2015). That with the ladder's quark masses the same standard computation gives 1.37 to 1.38 MeV, 6 to 7 percent above the measured 1.293 MeV and within the lattice computation's uncertainty of 0.28 MeV: the ladder's $m_d - m_u$, 2.442 MeV bare and 2.435 MeV dressed, scales the quark-mass part to 2.38 and 2.37 MeV.

Sources: *The Measures of the Kinds*, Chapter 9, Sections V and VI (Box V-1); *The Matter We Know*, Volume One, Chapter 3, Section II (Box 3.1); Borsanyi and others (2015). Standing: derived (the sign, from the step constants); a measured agreement (the size computed from the ladder's quark masses, within the lattice computation's uncertainty).

> ***Why the difference is sensitive.***
>
> The neutron-proton mass difference is a small difference of two larger, opposed parts, about $+2.5$ MeV from the quark masses and about -1.0 MeV from electromagnetism. A small shift in the down-minus-up quark mass difference therefore moves the total by a large fraction of itself, which is why this observable is the most sensitive hadron-level probe of m_u/m_d. A measurement of the up mass to ± 1 percent (entry B-036) bears on the same ratio.

B-X. The Mass Spectrum on the Content Instance

B-040 The Mass Spectrum on the Content Instance. That every statement of this chapter about the internal space $\mathbb{C}P^2 \times S^2$ is made on its content instance, the space of states of the two non-commutative summands of the algebra, with no size, no radius and no fields; its context instance, with radii $\sqrt{6}\, l_{Pl}$ and l_{Pl}, belongs to *The Gravitational Passage* and enters no value of this chapter. That no particle is a vibration mode of the internal space, and no mass is an eigenvalue of a Dirac operator on $\mathbb{C}P^2 \times S^2$; a rest mass is a Yukawa eigenvalue of the finite Dirac operator D_F, whose values the finite geometry does not fix. The finite geometry fixes the form of the masses (entry B-020), the kernel integers $c_1 = 3$ and $n_C = 2$ and the kernel triangle write the constants of the step formula (entry B-022), and the values are carried by the step formula, Koide's relation and the condensate's whole, each with the standing stated at its entry. That the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ from which the finite geometry is read is the algebra that Chamseddine and Connes reach by their own route, and the two programs stand at one line on the masses: the finite geometry fixes their form, and their values are the work of a further account.

Sources: *The Measures of the Kinds*, Preface, and Chapter 7, Section XII; *The Kinds of Particle*, Chapter 3, Sections III and VIII; *The Branches of the Kinds*, Chapter 1, Section III; *The Connes Correspondence*, Chapter 4, Section VII, and Chapter 5, Section VI. Standing: a definition (the content instance and its scope).

Summary

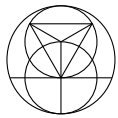
This chapter is the branch of the map that carries the sizes of the particles of matter. Its ground is *The Measures of the Kinds*, which places every charged fermion on one formula, the step formula $n = (\Delta + gP + g^2 d)s$, and I state that formula and its values at the standings that The Measures of the Kinds gives them.

The form of the ladder is derived before any number. Comparison gives the base 2, since one comparison is one halving; three generations give a quadratic with a zero, a unit and an extent; six independent verdicts make the quark step six halvings; and the charged leptons' share $3/8$ of one generation's charge squared makes their step $9/4$. The curvatures 1, $16/9$ and $-7/12$ follow from the kernel triangle, the right triangle with sides 3, 4 and 5 that the two kernel integers fix, and every constant of the formula is an exact invariant of that triangle, each in the slot of its own arithmetic type. The step factor s carries the one mismatch $\varepsilon = 11/(80\pi^2)$ of the particle side; it is written as a sum on the rungs and as a product on the interval from the tau to $\sqrt{2}\,v$, by the rule, argued in *The Measures of the Kinds*, Chapter 7, Section IX, that orders multiply across two domains and add within one, with the placement of the product step's factors derived, given a stated identification.

What measurement supplies is stated in two accounts, and both hold. In the working account the measured inputs are the electron's mass, which is the unit, the strengths of light and of color, the vacuum value v and the two offsets of the charged ladders. In the closure account the electron alone carries the chain to the vacuum value, the top and the Higgs mass, the vacuum value at the precision of its measurement. Against these inputs stand the values. The top's coupling $y_t = 1$ is derived from the identity extent of the up-type sector, with its scheme a setting, and every value of the top's mass is a measured agreement. Koide's balance $2/3$ and the tau's mass from it are derived, given the stated identification that each

part of the amplitude vector is measured by its share of the squared size. The Higgs self-coupling $\lambda = 1/8$ at tree level is derived, given the stated identification $\lambda = a^2/2$, and the Higgs mass with its loop term, 125.17 GeV, is a measured agreement. The muon is derived to first order and is a measured agreement at its full precision; the six quark masses, read by the rule that fixes each quark's reading scale, are measured agreements; the step's second-order coefficient 28/5 and the counted $|V_{cb}|$ are measured agreements; the quark CP phase, the Cabibbo angle and the angle $\beta = \pi/8$ are derived, each given its stated identification. Two forks are carried with the measurements that decide them: the three neutrino masses, between the count route and the Koide route, decided by the sum of the masses, and the three lepton angles with the lepton phase, between the carry family and the segment family, decided by the quadrant of the phase.

The parts of the chapter fit as form and value. The finite geometry of *Non-Commutative Geometry and the Immanent Metaphysics* fixes the gauge group, the representation and the one Higgs doublet, and leaves the Yukawa matrices as moduli; the program of Chamseddine and Connes stops at the same line; the step formula, Koide's relation and the condensate's whole carry the values. The quark mixing matrix carries the kernel triangle's right angle, and its one phase is how CP violation is realized, as *The Story of the Particles* states; the lepton mixing matrix, on the Dirac neutrino of *The Branches of the Kinds*, has three angles and one phase. The composites test the quark masses through standard computation, and the ladder's quarks return the proton's mass within 0.15 percent. Every statement about the internal space is made on its content instance, a space of states with no size, so the whole spectrum is a spectrum of counts and their readings, and no mass in it is a mode of a space with radii.



Chapter C

The Interaction Strengths

CHAPTER OVERVIEW

This chapter traces the three strengths of interaction: what measurement supplies and what the structure fixes, how the strengths change with the resolution at which they are read, where the running strengths cross one another, the weak angle and the masses of the W and Z bosons, and the two scales on which the strengths bear, the span from the electron to the reduced Planck mass and the binding scale of the proton. Its owner throughout is *The Strengths of the Kinds*; the carriers' law is owned by *The Measures of the Kinds*, and the spectral action's single trace by *The Kinds of Particle*, *The Connes Correspondence* and *The Strengths of the Kinds*. The spectral action is the action computed from the spectrum of the geometry's Dirac operator.

Chapter C rests on nine chapters of *The Concept Sequence*. Chapter 18, The Three Forces, states that comparison acts on structured content in exactly three ways, one for each position of comparison. Chapter 18 also states that gravitation is the arena in which the three ways act. Chapter 24, The Algebra of Comparison, gives the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ and the gauge group of that algebra, $U(1) \times SU(2) \times SU(3)$, with unimodularity. Chapter 25, The Internal Space in Its Two Instances, gives the internal space $\mathbb{C}P^2 \times S^2$, whose content instance is a space of states with no size. Chapter 26, The Two Kernel Integers and the Kernel Triangle, gives $c_1 = 3$ and $n_C = 2$, and the right triangle with sides 3, 4 and 5. The other five

chapters are Chapter 28, The Two Measured Anchors and the Span to the Planck Mass; Chapter 29, The Couplings: Rank, Running, Weights and Crossings; Chapter 30, The Spectral Action and the Form of the Standard Model; Chapter 37, The Gauge Bosons; and Chapter 44, No Grand Unification. The kernel integers are $c_1 = 3$, the count of the three positions of comparison (the omniscient, the immanent and the transcendent), and $n_C = 2$, the count of the conjugate pair, content and context. The triangle's sides are written, as *The Strengths of the Kinds* writes them, O = 3 (the short leg), N = 4 (the long leg) and H = 5 (the hypotenuse), with inradius r = 1 (the radius of the circle drawn inside the triangle, touching all three sides).

C-I. The Couplings and What Is Measured

C-001 Coupling Constant. That each of the three gauge interactions (the three forces, each acting through its own group of transformations, its gauge group) has its strength at a given resolution characterized by one number g, called its coupling. The three are the three ways in which comparison transforms a state: color, the weak doublet and the phase, whose carriers are the gluons, the W bosons and the hypercharge boson. That the strength $\alpha = g^2/(4\pi)$ is a pure number with no unit, and the inverse strength is $1/\alpha$; and that the chance of the interaction acting in one encounter grows with the coupling, roughly as its square. Light's coupling, the fine-structure constant α_{em}, is a function of the weak coupling and the hypercharge coupling, so it adds no fourth strength (entry C-017 writes light's inverse strength as the weighted mean of the two). At low energy the strengths stand in the order strong, then weak, then electromagnetic. The structure fixes this rank order from what each force acts on and from whether its carriers act on one another, and the values of the strengths at any scale are read from measurement.

Sources: *The Strengths of the Kinds*, Chapter 2, Section I; *The Story of the Particles*, Level 7, Section V. Standing: a definition (the coupling and its strength); derived (the rank order); a measured input (the values).

> ***What a coupling constant measures.***
>
> Think of a coupling constant as a volume knob for a force. Turn it up, and the force gets stronger: particles scatter more, bind more tightly, or decay more quickly. Turn it down, and the force weakens. Each of the three interactions has its own knob. The strengths are written α_{em} for light (the electromagnetic strength), α_2 for the weak doublet and α_s (also α_3) for color. At zero momentum $1/\alpha_{em} = 137.035999177(21)$ (CODATA 2022), the digits in parentheses being the standard uncertainty in the last digits. A strength that changes with energy is quoted in a stated scheme; the MS-bar scheme is the standard convention for removing the infinite parts of loop calculations at a chosen mass. At the mass of the Z boson (about 91 GeV), in the MS-bar scheme, $1/\alpha_{em} = 127.951 \pm 0.009$ (Particle Data Group 2022, electroweak review), $1/\alpha_2 = 29.5848 \pm 0.0080$ (from the measured α_{em} and the measured weak angle, entry C-005), and $\alpha_s = 0.1180 \pm 0.0009$ (Particle Data Group 2024).

C-002 Running and the Beta Function. That the strength of each interaction changes with the resolution at which it is read, the resolution being the separation at which a comparison is made, or equivalently the energy that reaches that separation; that this change is called the running of the coupling; and that the equation giving the rate of the change is called the beta function of that interaction. Running is a change of the resolution at which the three ways are seen. The structure that fixes the kinds stays the same under the running, and the kernel triangle stays fixed with the structure.

Sources: *The Strengths of the Kinds*, Chapter 2, Section I. Standing: a definition.

Why couplings run.

The apparent color of a distant mountain changes as the light and the air between you and it change. The mountain itself stays as it is; what you see depends on the medium in between. In the same way the effective strength of an interaction depends on the cloud of virtual particles that surrounds every source: pairs of particles and antiparticles that appear and vanish again. The terms used here are those of *The Strengths of the Kinds*. A turning is the transformation that one of the three ways performs; a label is the charge a particle carries in a turning (its color, its weak isospin or its hypercharge); a content is a particle of matter. Pairs of charged contents near a source take up part of its label and cover it (in the standard term, screening), so that the strength seen from farther away is smaller. Carriers that carry the label of their own turning (the W bosons and the gluons) take up part of the label and carry it outward (spreading; in the standard term, antiscreening), which acts against covering. At higher energies, equivalently at shorter distances, a probe reaches inside more of this cloud, and the effective coupling changes.

C-003 Loop Order. That the beta function is computed as a series of successive approximations called loop orders. The first approximation, called one-loop, includes only the simplest virtual-particle processes, a single closed loop of virtual particles in each diagram. The next approximation, called two-loop, includes processes with two loops, capturing interactions among the virtual particles themselves. Each higher loop order adds a correction that is typically smaller than the preceding one, so that the one-loop approximation captures the dominant behavior of the running. In this chapter the one-loop running gives the form of each law, and the values are stated from the running at two and three loops.

Sources: *The Strengths of the Kinds*, Chapter 2, Sections III and V; Chapter 3, Section I. Standing: a definition.

C-004 The Three Couplings and Their Groups. That the three interactions act through the three factors of the gauge group $U(1) \times SU(2) \times SU(3)$ (*The Concept Sequence*, Chapter 18, entry 18.18, and Chapter 24,

entry 24.15): the phase, U(1) of hypercharge, with coupling g'; the weak doublet, SU(2), with coupling g_2 (also written g); and color, SU(3), with coupling g_3 and strength $\alpha_3 = \alpha_s$. In the focus of comparison as relation (the binding of the three positions of comparison that the frame of measurement of *The Story of the Particles* uses), color is omniscient, the weak doublet transcendent and the phase immanent, and each row keeps its position on both sides of the Higgs condensate, the value the Higgs field keeps throughout the vacuum. The hypercharge coupling has two normalizations. In the normalization Y, with electric charge $Q = T_3 + Y$, the strength is $\alpha_Y = g'^2/(4\pi)$. In the unification normalization the strength is $\alpha_1 = (5/3)\,\alpha_Y$, and α_1 with no normalization named means the unification normalization. The unification normalization is a convention of bookkeeping, and it asserts no unifying group. Each coupling is conventionally written as its inverse strength $1/\alpha$, because at one loop the inverse strength changes linearly with the logarithm of the resolution.

Sources: *The Strengths of the Kinds*, Chapter 2, Section I; Chapter 3, Section II; and the note to Chapter 2, Section III; *The Kinds of Particle*, Chapter 4, Section III. Standing: a definition.

C-005 What Measurement Supplies. That the strength of light and the strength of color are measured inputs. Light's inverse strength at zero momentum is $1/\alpha_{em}(0) = 137.035999177(21)$ (CODATA 2022), and in the MS-bar scheme at the Z mass it is $1/\hat{\alpha}_{em}(M_Z) = 127.951 \pm 0.009$ (Particle Data Group 2022, electroweak review), the hat marking a value in the MS-bar scheme. Color's strength at the Z mass is $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ (Particle Data Group 2024). The two electroweak inverse strengths at the Z mass follow from the measured $1/\hat{\alpha}_{em}(M_Z)$ and the measured weak angle $\sin^2\theta_W(M_Z) = 0.23122 \pm 0.00006$ (MS-bar; Particle Data Group 2026) by the electroweak definitions $1/\alpha_2 = \sin^2\theta_W/\alpha_{em}$ and $1/\alpha_1 = (3/5)\cos^2\theta_W/\alpha_{em}$: $1/\alpha_2(M_Z) = 29.5848 \pm 0.0080$ and $1/\alpha_1(M_Z) = 59.0197 \pm$

0.0062. The weak angle at the Z mass has its own standing, a measured agreement: entry C-025 reaches it by running from the value 3/8 at the first crossing.

Sources: *The Strengths of the Kinds*, Chapter 1, Section VII and Table VII-1; Chapter 2, Section II and Box II-1. Standing: a measured input.

C-006 Why Light's Strength Stays Measured. That light's strength α_{em} is one comparison that each of six kinds of measurement registers (force, inertia, time, space, probability and possibility), so that an argument fixing it from one kind of measurement fixes it for all six and has to agree with each; and that its independence of comparison is derived for the comparisons the argument examined, with a stated hypothesis. The gauge group of light's phase is the circle U(1), and for a compact group (one that is closed and bounded, as the circle is) the symmetry leaves exactly one positive number undetermined: the coupling. Charge quantization, charge conservation, the kinds and the possible verdicts are the same at every value of α_{em}. The hypothesis is the circle's rule: a value is fixed by concepts alone only when one form supplies all the terms the value relates. The terms of α_{em} stand on all three pairs of the six intrinsics of comparison (sameness and difference, content and context, the subjective and the objective), and no single form supplies the terms of α_{em} together. So under the circle's rule no concept or set of concepts fixes α_{em}. α_{em} is therefore a measured input. *The Strengths of the Kinds* holds that no pure number among the particles is a brute constant (a number with no reason). *The Strengths of the Kinds* holds further that a number that is not derived carries either the law that would fix the number or, as α_{em} does, the argument that no law fixes the number.

Sources: *The Strengths of the Kinds*, Chapter 1, Sections IV and VI, and Box VI-1. Standing: derived, given a stated identification: the circle's rule, which *The Strengths of the Kinds* states as its hypothesis (the independence of α_{em}, for the comparisons the argument examined); a measured input (the value of α_{em}).

C-007 Light's Strength Read at the Proton's Mass. That a comparison's strength, in a relation that reads the strength of that comparison as a whole, is read at the mass of what the comparison is made against, and that light's strength in such a relation is therefore read at the proton's mass m_p: in the opening triple of *The Story of the Particles* (the frame of measurement) the photon is the act of comparing, the electron the thing compared and the proton the thing compared against. The scale at which a running strength is read is a setting, fixed by stating it. Light's MS-bar inverse strength at the proton's mass is $T(m_p) = 134.157 \pm 0.12$: the MS-bar value 134.07 ± 0.12 at 1.035 GeV, as *The Strengths of the Kinds* gives it (Chapter 2, Table II-1), carried down 0.142 halvings to m_p with the electron, the muon and the up, down and strange quarks active. A halving is a factor of two in mass.

Sources: *The Strengths of the Kinds*, Preface; Chapter 1, Section VII; Chapter 2, Sections II and VIII. Standing: a definition (the reading scale is a setting, fixed by stating it).

C-008 No Size Fixes the Strengths. That the internal space, on the instance on which the kinds of particle are read, is a space of states with no size, so that it supplies no size from which a strength could be computed and no scale from which a coupling could run. The internal space $\mathbb{C}P^2 \times S^2$ has two instances (*The Concept Sequence*, Chapter 25, entries 25.11 and 25.12). On the content instance of the internal space, the instance on which the kinds of particle are read, the internal space is the space of states of the color label and the doublet label, with no size, no radius and no fields. The content instance carries no scale from which a coupling could run, and no tower of Kaluza-Klein modes is formed on the content instance. Its context instance, the space of possibility, has radii ($\sqrt{6}\, l_{Pl}$ and l_{Pl}, with l_{Pl} the Planck length) and is the internal space of *The Gravitational Passage* (Chapter 9, Section IX). *The Strengths of the Kinds* makes every statement about the internal space on the content instance. The rates of running change only at the masses of known contents and of the W and Z

bosons (entry C-013 states these thresholds). *The Story of the Particles* states the same: the structure gives no computation of the values of the strengths, because the finite space of states on which it rests has no size.

Sources: *The Strengths of the Kinds*, Preface; Chapter 1, Section VI; Chapter 2, Section VI; *The Kinds of Particle*, Chapter 3, Sections III and VII; *The Story of the Particles*, note 19. Standing: derived.

> ***Why there is no scale to run from.***
>
> A coupling runs between two masses: running is a change of resolution, and resolution is measured as a mass or an energy. A space of states with no size supplies no mass at which a running could start. The masses at which a rate of running changes are the masses of contents that are found (the quarks, the leptons, the W and Z bosons and the Higgs boson), and each content enters the running only at energies above its own mass.

C-II. The One-Loop Rates

C-009 The Three One-Loop Rates. That at one loop each inverse strength changes linearly with the logarithm of the resolution, $1/\alpha_i(\mu) = 1/\alpha_i(M_Z) - (b_i/2\pi)\ \ln(\mu/M_Z)$ for $i = 1, 2, 3$ and a mass μ, and that the three one-loop rates are $b_1 = 41/10$ for the hypercharge in the unification normalization ($41/6$ in the normalization Y), $b_2 = -19/6$ for the weak doublet and $b_3 = -7$ for color. In this convention a negative rate makes the strength fall at shorter distance, which is asymptotic freedom (the weakening of a force at short distance). The rates are fixed by structure: the closed list of contents (three generations of the sixteen states of matter, and the one Higgs doublet) supplies every term of the one-loop count, and no free number enters at that order. A change of a rate at any mass other than that of a known content would reopen the closed list, so the rates test the list. The derived rates agree in form with the measured running of light's strength from zero momentum to the Z mass and of color's strength from the tau mass to the TeV scale.

Sources: *The Strengths of the Kinds*, Chapter 2, Section III; *The Story of the Particles*, note 19. Standing: derived (the rates); a measured agreement, stated in form (the measured running agrees with the rates).

> ***The sign of the rate.***
> In the convention of entry C-009 a negative rate means that the strength grows toward lower energy (coarser resolution) and falls toward higher energy (finer resolution): color and the weak force behave so. A positive rate means that the strength falls toward lower energy and grows toward higher energy: the hypercharge behaves so. Two effects make a rate. Carriers that carry the label of their own turning spread that label, by (11/3)N for the group SU(N): 22/3 for the weak doublet and 11 for color; spreading makes the strength fall at short distance. Each sense (handedness) of a content that answers a turning (carries a label in it) covers by 2/3 of the square of its label, counting a member of a set as one half, and the Higgs doublet covers by 1/3 of the square of its label, which comes to 1/6 in the weak doublet and 1/6 in the phase in the normalization Y (entry C-010 gives the count); covering makes the strength grow at short distance. The carrier of the phase carries no label and does not spread, so the phase strength only grows toward short distance. In the opposite convention, used in the literature of color, color's rate is written $b_0 = -b_3 = 7$.

C-010 Two Routes to the Rates, and a Third for Color. That the three rates are reached by two routes that rest on different premises, and color's rate by a third. The first route counts the contents of *The Story of the Particles*. Per generation the sum of Y^2 over the Weyl states (the states of one handedness each) is 10/3; over three generations, times 2/3, it gives 20/3, and the Higgs doublet adds 1/6, so $b_1 = 41/6$ in the normalization Y, which is 41/10 in the unification normalization. The W bosons spread by 22/3, and the twelve left-handed doublets of three generations cover by 4, with the Higgs doublet's 1/6, so $b_2 = -19/6$. The gluons spread by 11, and the six kinds of quark, each in both senses, cover by 4, so $b_3 = -7$. The second route is the standard count of sources: the gauge bosons contribute $(0, -22/3, -11)$, three generations of fermions $(4, 4, 4)$, and one Higgs

doublet (1/10, 1/6, 0). The third route, for color alone, writes the magnitude of color's rate in the kernel integers: $11 - 2n_f/3 = 7 = c_1^2 - n_C$, with $n_f = 6$ kinds of quark, $c_1 = 3$ and $n_C = 2$. The right-handed neutrino, the sixteenth state of each generation, carries none of the three labels and does not enter the running.

Sources: *The Strengths of the Kinds*, Chapter 2, Section III, Boxes III-1 and III-2, and Table III-1. Standing: derived.

C-011 The Fermions Do Not Separate the Forces. That each complete generation adds 4/3 to each of the three rates, so that the fermions' part of every difference $b_i - b_j$ is zero for any number of complete generations, and that the gauge bosons and the one Higgs doublet separate the three couplings, the matter loading all three equally. The statement is arithmetic about the charges of one generation, which fill complete multiplets $5 + 10$ of the group SU(5) (a multiplet being a set of states that a group's transformations turn into one another), with the hypercharge in the unification normalization. It is a property of this content: it asserts no SU(5) gauge group of the theory, and anomaly freedom (the cancellation of the quantum effects that would break a force's symmetry) does not by itself give it. The equal loading of the three forces by one generation holds because there are three colors.

Sources: *The Strengths of the Kinds*, Chapter 2, Section III and Box III-3; *Geeze, Wublets, and the Higgs*, Level 5, Section VI. Standing: derived.

C-012 Light's Own Rate. That light has its own rate in the unbroken phase (the phase before the Higgs condensate), read as the same weighted sum that composes its inverse strength (entry C-017): $b_{em} = (H\, b_1 + O\, b_2)/O = (5/3)(41/10) - 19/6 = 11/3$ exactly. This rate equals $(11/3)\, r$ with $r = 1$ the inradius of the kernel triangle. The charged matter loads light's rate by $11 = O^2 + n_C$, and the pair of charged weak carriers takes away $(11/3)\, n_C = 22/3$. The identity $N(H + O) + r = O(O^2 + n_C) = 33$ behind this writing holds, along the triangle family with sides c, $c + 1$, $c + 2$ and unit inradius, only at $c_1 = 3$.

Sources: *The Strengths of the Kinds*, Chapter 2, Section III and Box III-4. Standing: derived.

C-013 The Thresholds. That the rates change only at thresholds: the masses of the contents of the closed list, and the masses of the W and Z bosons, which the Higgs condensate gives mass. A content of mass m contributes to a rate only at energies above its mass (decoupling, the theorem of Appelquist and Carazzone, 1975), and below the electron's mass light's coupling stops running, so that its value at zero momentum, about 1/137.036, is a single definite number. From zero momentum to the Z mass, $1/\alpha_{em}$ falls by about 8 to 9 units, about 4.3 of them from the charged leptons; the remainder comes from the quarks and is computed from measured data or from lattice calculation, both of which take color's strength as given.

Sources: *The Strengths of the Kinds*, Chapter 2, Sections II and IV. Standing: a theorem (decoupling, Appelquist and Carazzone 1975), applied to the closed list of contents.

C-014 Beyond One Loop, and the Matching at the Top. That beyond one loop the rates take in the strengths themselves and the couplings of the top quark and the Higgs field, so that they are fixed in form and take the values of the strengths as inputs. When the three gauge couplings are run at three loops from the published couplings at the top quark's mass, with the three-loop coefficients of Mihaila, Salomon and Steinhauser (2012), the three gauge couplings reproduce, within rounding, the published couplings at the unreduced Planck mass, 1.2209×10^{19} GeV. At the unreduced Planck mass the run gives $g_1 = 0.61537$, $g_2 = 0.50546$ and $g_3 = 0.48732$, against the published 0.6154, 0.5055 and 0.4873 (Buttazzo and others, 2013). The matching at the top is the step that fixes the MS-bar strengths at the top quark's mass from measured quantities. With the running taken to three loops, the matching sets the precision of the running, and two published matchings are carried side by side without a choice between them: that of Martin and Robertson (their program SMDR,

Standard Model in Dimensional Regularization, 2019), which returns the W mass as an output, and that of Buttazzo and others (2013), which takes the W mass as an input.

Sources: *The Strengths of the Kinds*, Chapter 2, Section V and Box V-1. Standing: derived, from measured inputs by the standard three-loop running.

C-015 The Two-Loop Matrix. That at two loops the rates form a three-by-three matrix of coefficients in which each entry describes how the running of one coupling is affected by the value of another, so that the running of each coupling depends on the values of all three (and, through further terms, on the top quark's coupling), whereas at one loop each coupling runs on its own. Because the three runnings are joined in this way, the one-loop crossing places differ from the three-loop places of entry C-022 by -0.10, $+0.52$ and $+1.84$ halvings, and every law of place in this chapter is stated of the three-loop places.

Sources: *The Strengths of the Kinds*, Chapter 2, Section V; Chapter 3, Section I and Table I-1. Standing: a definition.

C-016 The Running Counts Three Generations. That color's one-loop rate -7 requires exactly three generations. Each unit of a family's self-action (the action of its carriers on one another) contributes $-11/3$, each one-handed state carrying the family's basic label $+1/3$, and each scalar pair $+1/6$. So color's $-7 = -11 + 12 \times (1/3)$ counts twelve one-handed colored states. One generation supplies four (the two sides of the left-handed quark doublet and the two right-handed quarks), so twelve is three generations; two kinds of quark doublet per generation would give eight per generation, and twelve is no multiple of eight. The weak rate $-19/6$, with the one Higgs doublet, counts twelve left-handed two-sided states: the same three generations, with one lepton doublet in each. The measured running of color agrees with the rate -7. The count of states in the one-loop rates (color's -7 and the weak $-19/6$) is a second route to the count of three generations. The other route is the three rotations of the

content round the cycle of Axiom II, the directed cycle in which the three positions of comparison precede one another (*The Concept Sequence*, Chapter 22, entry 22.16, and Chapter 25, entry 25.24).

Sources: *Geeze, Wublets, and the Higgs*, Level 5, Section VI; *The Story of the Particles*, Level 5, Sections I, II and V; *The Kinds of Particle*, Chapter 7, Section I. Standing: derived.

C-III. Relations Among the Couplings

C-017 Light's Inverse Strength as a Weighted Mean. That light's inverse strength is, at every scale, the 5 : 3 weighted mean of the two electroweak inverse strengths: $1/\alpha_{em} = 1/\alpha_2 + (5/3)(1/\alpha_1) = (H/\alpha_1 + O/\alpha_2)/O$, the standard electroweak matching with the hypercharge in the unification normalization. The factor $5/3 = H/O$, the kernel triangle's hypotenuse over its short leg, is the trace ratio $\mathrm{Tr}\, Y^2/\mathrm{Tr}\, T_3^2$ over one generation, the quark charge squared over the lepton charge squared. In the shares of one generation's charge squared (quark share $S_q = 5/8$, lepton share $S_\ell = 3/8$) the same relation reads $1/\alpha_{em} = (S_q/\alpha_1 + S_\ell/\alpha_2)/S_\ell$, and where $\alpha_1 = \alpha_2 = \alpha$ it reduces to $1/\alpha_{em} = (8/3)/\alpha$.

Sources: *The Strengths of the Kinds*, Chapter 2, Sections II and VII, and Box VII-1; *The Kinds of Particle*, Chapter 5, Section V. Standing: a definition (the electroweak matching); derived (the writing of 5/3 as H/O and as the trace ratio).

C-018 Candidate Relations That Do Not Hold. That the relation the structure asserts among the couplings is the equality of the three traces, a relation among weights (entry C-024); and that five candidate relations among the running couplings themselves are tested against measurement and do not hold, each with its pull or its miss stated. The pull of a value is the stated value less the measured value, divided by the combined uncertainty, in standard deviations (written σ). First, the three running couplings set equal at about 9.7×10^{16} GeV (67.4 halvings above the electron's mass) and carried down at one loop give $1/\alpha_{em}(M_Z) = 145.6$ against the

measured 128.0, a miss of fourteen percent, and $\sin^2\theta_W = 0.203$ against 0.231; the same tension is known for the minimal spectral model and for minimal unification. Second, the trace of the three inverse strengths at the end of the measured span, $1/\alpha_1 + 1/\alpha_2 + 1/\alpha_3 = 133.628 \pm 0.064$, set equal to light's inverse strength at the proton's mass, 134.157 ± 0.12, has a pull of -3.9 standard deviations at two loops (-3.1 to -4.5 by scheme). Third, $1/\hat{\alpha}_{em}(M_Z) = 2^7 = 128$ has a pull of $+5.4$ standard deviations. Fourth, the geometric mean $\alpha_2^2 = \alpha_1\alpha_3$ is 32 percent off at the Z mass. Fifth, the product form $\alpha_2 \sim \alpha_1\alpha_3$ misses by a factor of about 40 where each strength is near 1/40. The first candidate tests an equality of the running strengths, which is a different equality from the equality of the traces.

Sources: *The Strengths of the Kinds*, Chapter 2, Section VII and Table VII-1. Standing: a fork that a named measurement decides (each candidate is a stated hypothesis, and the measurement named in it has decided against it, with its pull or miss given).

C-019 Light at the Proton: the Restated Relation. That light's inverse strength at the proton's mass equals 3π times the gap between the two electroweak inverse strengths at the end of the measured span: $1/\hat{\alpha}_{em}(m_p) = 3\pi\,(1/\alpha_2 - 1/\alpha_1)(E)$, with $E = \log_2(M_{Pl,red}/m_e) = 72.0132112 \pm 0.0000162$ halvings the measured span from the electron's mass to the reduced Planck mass (the mass M at which $8\pi GM^2/(\hbar c)$ equals one, G being Newton's constant); equivalently, the measured unit $U = T(m_p)/\Delta_{21}(E)$ equals $3\pi = 9.42478$, where $\Delta_{21}(E)$ is the gap $1/\alpha_2 - 1/\alpha_1$ read at E. Across the five computations beyond one loop the pull of 3π is $+1.00$ and -2.04 at three loops (the matchings of Martin and Robertson and of Buttazzo and others), -0.44 and -1.60 at two loops, and $+0.06$ at two loops with the matching at tree level; at one loop $U = 9.376$ and 3π lies at $+4.2$, so the relation is a relation of the running beyond one loop. The unit 3π is a measured agreement. Three writings of 3π stand beside it as clues to the law that fixes it: $3\pi = 4\pi/b_e$, with $b_e = 4/3$ the one-loop coefficient of one Dirac fermion of unit charge; $3\pi = 6 \times \pi/2$, the

Gauss measure of six unit points; and $3\pi = 2\pi \times 3/2$, one full turn of phase times the half-twist 3/2 by which a fermion's label is shifted on the space of states. Read the other way, the relation returns light's strength at the proton's mass from α_2 and α_1, and, solved for the span, it returns the span's integer from the measured couplings alone (entry C-032).

Sources: *The Strengths of the Kinds*, Chapter 2, Section VIII, Table VIII-1 and Box VIII-1. Standing: a measured agreement.

C-IV. Where the Couplings Cross

C-020 The Law of Place. That a crossing is a scale at which two running inverse strengths are equal; that its place is counted in halvings above the electron's mass, $h = \log_2(\mu/m_e)$, one halving being one doubling of energy; and that at one loop the crossing of forces i and j lies at $X_{ij} - X_Z = 2\pi\, D_{ij}(M_Z)/((b_i - b_j)\ \ln 2)$ halvings, where $D_{ij} = 1/\alpha_i - 1/\alpha_j$ is the gap between the two inverse strengths and X_Z is the place of the Z mass. The law of place is a form: the measured gaps at the Z mass enter only when a place is computed from it. Since $D_{13} = D_{12} + D_{23}$ at every scale, the three distances $L_{ij} = X_{ij} - X_Z$ obey one linear relation, $(b_1 - b_3)L_{13} = (b_1 - b_2)L_{12} + (b_2 - b_3)L_{23}$: two of the three crossing places are free, and the third is fixed by them. At one loop the middle place, $X_{13} = (218\, X_{12} + 115\, X_{23})/333$, is the weighted mean of the other two for every value of the couplings.

Sources: *The Strengths of the Kinds*, Chapter 3, Sections I and III, Boxes I-1 and III-1. Standing: derived.

C-021 The Spacing Ratio. That the spacing ratio of the three crossings, $(X_{23} - X_{13})/(X_{13} - X_{12})$, equals at one loop the ratio of rate differences $(b_1 - b_2)/(b_2 - b_3) = (109/15)/(23/6) = 218/115 = 1.8957$, for any couplings at which the three crossings are not all at one place. The gauge bosons alone give rate differences in the proportion 2 : 1 : 3 and a spacing ratio of exactly 2; the generations cancel from every difference (entry

C-011); the one Higgs doublet, adding (1/10, 1/6, 0) to the three rates, lowers the ratio to 218/115, a departure of 5 percent from 2. In the kernel integers, $218/115 = n_C(1 - 1/110)/(1 + 1/22)$, with $110 = n_C\ H\ (O^2 + n_C)$ and $22 = n_C(O^2 + n_C)$; the standard writing and the kernel writing are one count written twice. Since 218/115 is a standard one-loop result of the Standard Model's content, the agreement of the measured crossings with it is consistency with that content, and it holds equally for the Standard Model and for the structure. At three loops the spacing ratio is 1.8581 $\pm$ 0.0006, 2 percent from the one-loop value; the difference is the content of the two-loop and three-loop terms.

Sources: *The Strengths of the Kinds*, Chapter 3, Section II and Box II-1. Standing: derived.

C-022 Where the Crossings Are. That the places of the crossings are values read from the measured couplings run upward. At three loops, with the couplings of the Particle Data Group 2024 compilation, they lie at 54.259 ($\alpha_1 = \alpha_2$), 58.200 ($\alpha_1 = \alpha_3$) and 65.521 ($\alpha_2 = \alpha_3$) halvings above the electron's mass; at one loop they lie near $10^{13.0}$, $10^{14.4}$ and $10^{17.0}$ GeV (54.16, 58.72 and 67.36 halvings). The order $\mu_{12} < \mu_{13} < \mu_{23}$ follows from the measured couplings: the first crossing lies below the last exactly when D_{12}/D_{23} at the Z mass is smaller than 218/115, and the measured ratio is 29.4/21.1 = 1.39.

Sources: *The Strengths of the Kinds*, Chapter 3, Sections I and III, Tables I-1 and III-1. Standing: derived, from the measured couplings (measured inputs) run upward.

C-023 The Measured Couplings Meet at No Scale. That the gauge group at every scale is the product U(1) × SU(2) × SU(3), with its twelve gauge bosons and no others, so that no simple Lie group (no single group containing the three) is the gauge group at any scale; and that the measured couplings, run upward with the Standard Model's own rates, draw close to one another and meet at no scale. Run from their measured values at the Z mass to 5.3×10^{18} GeV with the one-loop rates, the three inverse

strengths are about 34 (hypercharge, unification normalization), 49 (weak) and 51 (color), and they are not equal. The rank order of the strengths at low energy needs the three only nearly equal at the finest resolution, within about twenty units of inverse strength, and the Standard Model's values meet that condition. The account predicts that no gauge boson beyond the twelve is found; the measurement that decides it is the search for the gauge bosons of a unifying group.

Sources: *The Strengths of the Kinds*, Chapter 3, Section III; *The Story of the Particles*, Level 7, Section V, and note 19; *The Kinds of Particle*, Chapter 4, Section I; *The Concept Sequence*, Chapter 44, entries 44.11 and 44.12. Standing: derived (the product group and the absence of a meeting); a measured input (the values, run upward).

C-024 The Equal Traces and the First Crossing. That the structure fixes a relation among the weights of the three forces (as distinct from an equality of their measured strengths): over the finite space of states the three forces have equal weights as traces, $g_3^2 = g_2^2 = (5/3)\, g'^2$, with g' the hypercharge coupling in the normalization Y ($g_3^2 = g_2^2 = g_1^2$ in the unification normalization). Two routes give the equal traces. The structure gives no force a greater weight than another, because the three positions are distinct and none stands before the others in the cycle of Axiom II; and the spectral action's single trace over the whole finite space ties the couplings to one common measure at its cutoff, the result that Chamseddine, Connes and Marcolli (2007) obtain at the cutoff. The spectral action is the action computed from the spectrum of the geometry's Dirac operator. At the equal traces the weak angle has $\sin^2\theta_W = \mathrm{Tr}\, T_3^2/\mathrm{Tr}\, Q^2 = 2/(16/3) = 3/8$, the traces taken over one generation; 3/8 is also O/(O + H), the leptons' share of one generation's charge squared. The trace ratio and the kernel triangle's ratio are one function of the number of colors, $N_c/(2(N_c + 1))$, and the right angle of the triangle with sides c, c + 1, c + 2 selects $N_c = 3$. In unified theories 3/8 is the boundary value of the weak angle at a unification scale; here 3/8 is a relation among weights, and the structure imposes no grand-unification relation. Since $1/\alpha_2 - 1/\alpha_1 = (8/5)(\sin^2\theta_W$

$- 3/8)/\alpha_{em}$ at every scale, the running weak angle equals 3/8 at one scale only: the first crossing, where $\alpha_1 = \alpha_2$, 54.259 halvings above the electron's mass, near 10^{13} GeV; there $\tan^2\theta_W = O/H = 3/5$.

Sources: *The Kinds of Particle*, Chapter 5, Sections IV and V; *The Connes Correspondence*, Chapter 5, Sections II and III; *The Strengths of the Kinds*, Chapter 2, Section VII and Box VII-1; Chapter 3, Section IV and Box IV-1; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV, and Chapter 15, Section III (the mathematics of the spectral action); *The Story of the Particles*, Level 7, Section V, and note 19; *The Concept Sequence*, Chapter 29, entries 29.10 and 29.11. Standing: derived.

C-025 The Weak Angle at the Z Mass. That the running started from 3/8 at the first crossing returns $\sin^2\theta_W = 0.2314$ at the Z mass, and that this value restates the place of the crossing: the first crossing is defined by $\alpha_1 = \alpha_2$, where the angle is 3/8, so the value at the Z mass and the place of the crossing are one fact. The measured weak angle at the Z mass, $\sin^2\theta_W(M_Z) = 0.23122 \pm 0.00006$ (MS-bar; Particle Data Group 2026), is therefore a measured agreement. The equal traces are a relation among weights, so they set no value of the running angle at a unification scale. The minimal spectral model reads 3/8 as such a value: run from 3/8 at the scale of unified theories with the Standard Model content alone, the angle comes to about 0.21 at the Z mass, against the measured 0.2312. That result is a known tension of the minimal spectral model, which Chamseddine, Connes and Marcolli (2007) state (the coupling constants "do not exactly meet at unification scale"), and it is no agreement with experiment.

Sources: *The Strengths of the Kinds*, Chapter 3, Section IV; *The Connes Correspondence*, Chapter 5, Section II; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Section III; *Geeze, Wublets, and the Higgs*, Level 6, Section VII. Standing: a measured agreement.

C-026 The Level and the Place of the First Crossing. That the level of the first crossing, the common inverse strength $1/\alpha_1 = 1/\alpha_2$ there, is read from measurement: 42.25163 ± 0.00390 at three loops (SMDR matching from the Particle Data Group 2024 inputs, with top mass 172.57 GeV,

Higgs mass 125.20 GeV and $\Delta\alpha_{had} = 0.02783$, the contribution of hadrons to the running of light's strength up to the Z mass, and with the W mass taken as the Standard Model's output, 80.3498 GeV), with 42.26925 under the matching of Buttazzo and others and 42.24 at two loops. The value $169/4 = 42.25$ stands at -0.42σ against the level. The laws of scale are three laws taken together: light's relation at the proton's mass (entry C-019), a law of the ratio of the gaps at the span's end (entry C-027), and color's law of binding, $1/\alpha_3(E) = (7 \ln 2/2\pi)(E - x_p + H)$, which places the proton $x_p = \log_2(m_p/m_e)$ halvings above the electron by color's running (entry C-033 gives its form); a closure is a joint solution of these laws, and the second closure (entry C-027) takes 169/4 as its law of the level, a stated hypothesis. At the first crossing the fine-structure strength is $1/\alpha_{em} = (8/3)/\alpha = 112.671 \pm 0.010$. The place of the first crossing has two routes that rest on different measurements. The measured couplings at the Z mass, run upward at three loops, give $X_{12} = 54.259 \pm 0.014$ halvings, a displacement of $+0.259 \pm 0.014$ above $54 = (3/4) \times 72$. Light's relation of entry C-019, with light's strength measured near the proton's mass and the measured span, gives $X_{12} = 54.268 \pm 0.016$. The two routes to the place of the first crossing give displacements above 54 that agree at 0.4σ.

Sources: *The Strengths of the Kinds*, Chapter 3, Sections IV and V, Table V-1 and Box V-1. Standing: derived, from the measured couplings run upward (the level); a measured agreement (the two routes to the place, and the value 169/4 against the measured level, which the second closure takes as a stated hypothesis).

C-027 The Gaps at the Span's End. That at the span's end, $E = 72.013$ halvings, the three inverse strengths leave two splits, $\Delta_{21} = (1/\alpha_2 - 1/\alpha_1)(E)$ and $\Delta_{32} = (1/\alpha_3 - 1/\alpha_2)(E)$, whose ratio, written $\rho = \Delta_{21}/\Delta_{32}$ (a symbol distinct from the ρ parameter of entry C-028), is measured as 5.093 ± 0.128 at three loops (Particle Data Group 2024 couplings; the uncertainty comes mainly from α_s); and that a law of this ratio is a law of the two crossing places measured from the span's end. Two values of ρ are carried side by side with their pulls, and neither is chosen: $H = 5$, the kernel tri-

angle's hypotenuse, at -0.73σ against the three-loop ratio, and the count $21/4$ at $+1.23\sigma$. The count $21/4 = \dim \mathrm{K} \times (\mathrm{D} - 3)/(\mathrm{D} - 2)$ at $\mathrm{D} = 10$, with $\dim \mathrm{K} = 6$ the dimension of $\mathbb{C}\mathrm{P}^2 \times \mathrm{S}^2$ and $\mathrm{D} = 10$ the dimension of its product with spacetime, equals $c_1(2c_1 + 1)/(c_1 + 1)$ and also $|b_3|/b_e = 7/(4/3)$; it is a count of the product and uses no radius, and no statement of *The Strengths of the Kinds* identifies the count $21/4$ with ρ. $\rho = \mathrm{H}$ passes as a test of the measured ratio alone, and used as an exact law in the first closure of the laws of scale it fails in every matching scheme considered; so $\rho = \mathrm{H}$ is carried as a hypothesis with both outcomes stated. The second closure, with the level $169/4$ and $\rho = 21/4$, gives $\alpha_s(M_Z) = 0.11888 \pm 0.00011$, $+0.97\sigma$ against 0.1180 ± 0.0009 (Particle Data Group 2024).

Sources: *The Strengths of the Kinds*, Chapter 3, Section VI, Tables VI-1 and VI-2, and Box VI-1. Standing: a measured agreement (the measured ratio against H and against $21/4$, each with its pull, with $\rho = \mathrm{H}$ carried as a stated hypothesis whose two outcomes are stated).

C-V. The Weak Angle and the Masses of W and Z

C-028 The Angle and the Masses at Tree Level. That the weak angle θ_W is the angle of the rotation by which the neutral weak boson W^3 and the hypercharge boson B combine into the photon, $A = \sin\theta_W W^3 + \cos\theta_W B$, and the Z, $Z = \cos\theta_W W^3 - \sin\theta_W B$, with $\tan\theta_W = g'/g$; that the electric coupling is $e = g\sin\theta_W = g'\cos\theta_W$; and that $\sin^2\theta_W = \alpha_{em}/\alpha_2$ is the share of the electric strength in the weak strength. At tree level (the leading order of the calculation, before any correction from virtual states), one Higgs doublet, whose two halves have equal standing, gives $m_W = g\,v/2$ and $m_Z = \sqrt{(g^2 + g'^2)}\,v/2$, with v the vacuum value of the Higgs field, so that $m_W = m_Z\cos\theta_W$, the parameter $\rho = m_W^2/(m_Z^2\cos^2\theta_W)$ equals one, and the Z is heavier than the W. The exact relation $m_W = m_Z\cos\theta_W$ at tree level stands, and the Z mass itself is read from measurement, 91.1880 ± 0.0020 GeV (Particle Data Group 2024). The structure asserts

equal weights, not equal strengths (entry C-024): equal weights taken as equal strengths at the finest resolution would put the Z mass near 89.2 GeV.

Sources: *The Strengths of the Kinds*, Chapter 4, Sections I and II, Boxes I-1 and II-1; *Geeze, Wublets, and the Higgs*, Level 6, Sections IV and VII. Standing: derived.

C-029 The Departure from $\rho = 1$. That the departure of ρ from one has its form from the way each carrier is corrected by the fermions that answer its turning: the W is corrected by an upper and a lower partner of a doublet together, the Z by each member with itself, so that ρ stays one where partners have equal mass and departs from one where they differ, most for top and bottom. The same form of the departure of ρ from one is reached by a calculation within the Standard Model itself (Veltman, 1977), and the Standard Model calculation and the route of this entry are kept side by side. The size is measured. The tree relation with the measured m_Z and the MS-bar angle 0.23122 gives $m_W = 79.95$ GeV, 0.51 percent below the measured 80.3625 ± 0.0077 GeV (Particle Data Group 2026); the measured m_W asks $\hat{\rho} = 1.0103$, against the Standard Model value $\hat{\rho} = 1.01016 \pm 0.00009$ (Particle Data Group 2024, electroweak review), almost all of it the top's loop. With the top and bottom correction removed, $\rho_0 = 1.00031 \pm 0.00019$ (Particle Data Group 2024, electroweak review), and the tree value 1 stands at -1.6σ against it.

Sources: *The Strengths of the Kinds*, Chapter 4, Section III, Box III-1 and Table III-1. Standing: derived (the form); a measured agreement (the tree value 1 against the measured ρ_0, at -1.6σ).

C-030 The Carriers' Law. That the squared masses of the W and Z bosons share the square of the vacuum value by the carriers' law $m_W^2 + m_Z^2 = v^2(1/4 - 1/(16\pi^2))$, with $v = 246.21967$ GeV measured through the Fermi constant. At the top's coupling $y_t = 1$, which the structure fixes, and the self-coupling $\lambda = 1/8$, derived, given the stated identification $\lambda = a^2/2$ with $a = 1/2$ the doublet's half, the square of the vacuum value divides

into the top's share 1/2, the Higgs's share 1/4 and the carriers' share 1/4. Relative to its ideal share 1/4, the measured carriers' share is displaced by -2.990 ± 0.010 in units of $\kappa_H = 1/(12\pi^2)$, the Higgs's loop factor, where the law gives $-3 = -O$, a pull of -0.98σ (Particle Data Group 2026 masses, m_Z from Particle Data Group 2024). With the measured v and m_Z, the law gives $m_W = 80.3547 \pm 0.0023$ GeV, -0.97σ against the measured 80.3625 ± 0.0077 GeV (Particle Data Group 2026). The quarter is a statement about shares of v^2, and it is no value that the couplings take at an argued scale.

Sources: *The Measures of the Kinds*, Chapter 13, Section V, Boxes V-1, V-2 and V-3, and Table V-1; Chapter 13, Sections III and IV (the top's coupling and the self-coupling). Standing: a measured agreement.

C-031 The Weak Angle at Measured Scales. That at tree level every pole asymmetry of Z decays (an asymmetry measured at the peak of the Z resonance) is a function of $\sin^2\theta_W$ alone, the ρ parameter canceling from it; and that light's relation of entry C-019, with light's strength read at the proton's mass, gives $\sin^2\theta_W(M_Z) = 0.23127$ to 0.23130 against the measured 0.23122 ± 0.00006 (Particle Data Group 2026), a residue of $+0.00005$ to $+0.00008$, which is 0.83σ to 1.33σ. The residue lies within the uncertainty of the reading scale itself, which is known within ± 0.19 halvings about 10.98 halvings above the electron's mass (the proton's mass lies at 10.842). The closure that joins the carriers' law (entry C-030) to light's relation gives a second value through the same relation, 0.231164 ± 0.000036, at -0.80σ. The two extractions of the angle from single asymmetries, 0.23223 ± 0.00029 from the forward-backward asymmetry of the bottom quark and 0.23098 ± 0.00026 from the lepton asymmetry measured at SLD (the SLAC Large Detector), differ from each other by 3.2σ (the combination of the LEP and SLD experiments, 2006; LEP being the Large Electron-Positron Collider).

Sources: *The Strengths of the Kinds*, Chapter 4, Section IV and Box IV-1; *The Measures of the Kinds*, Chapter 13, Section V and Table V-2 (the closure's value). Standing: a measured agreement.

C-VI. The Span and the Binding Scale of the Proton

C-032 The Span, and Its Second Route through the Couplings. That the span from the electron's mass to the reduced Planck mass, $E = \log_2(M_{Pl,red}/m_e) = 72.0132112 \pm 0.0000162$ halvings, is measured from Newton's constant and the electron's mass (CODATA 2022); that its integer is the count $72 = 8 \times 9 = n_C{}^3 c_1{}^2$, eight first-order states (two outcomes at each of the three positions) times nine joins (three positions against three across the pair); and that the identification of the span with the count is assumed, the measurement being its test (*The Concept Sequence*, Chapter 28, entries 28.13 and 28.18). A second route reaches the span through the couplings, with no input from Newton's constant: light's relation of entry C-019, solved for E with the measured couplings and two-loop running, returns $E = 72.0120 \pm 0.0221$, -0.05σ from the measured span; over the matching schemes it spreads from 72.0120 to 72.0204, and at one loop it returns 71.920, -4.2σ. The reach of this route is the integer: its uncertainty is larger than the remainder $r = E - 72 = 0.0132$, which an uncertainty of about ± 0.005 would resolve. Newton's constant is the measured input of the side of gravitation: the ground fixes its role and not its magnitude (*The Unquantized Arena*, Proposition 9.4).

Sources: *The Strengths of the Kinds*, Chapter 5, Sections I, II and VI, and Table VI-1; *The Concept Sequence*, Chapter 28, entry 28.22. Standing: a measured input (the span); an identification (the span's integer with the count 72), which the measured span tests; a measured agreement (the second route's return of the integer).

C-033 The Binding Scale of the Proton. That color's strength, a pure number, sets a mass by its running: read downward from a fine resolution M, color's strength reaches order one at a scale Λ with $\ln(M/\Lambda) = 2\pi/(b_0\, \alpha_s(M))$ at one loop, $b_0 = -b_3 = 7$, and the proton's mass is a fixed multiple of Λ (dimensional transmutation: a strength with no unit fixing a

mass); so m_p/m_e is the exponential of an inverse strength of color times numbers fixed by structure. About ninety-nine percent of the proton's mass is the energy of the color field and of the motion of the quarks and gluons; the quarks' own masses supply about one percent. The measured strength of color, run upward at one loop, is about 1/50 at the reduced Planck mass, and $2\pi \times 50/7 = 44.9$ natural-log units is of the same size as the 42.4 units from the proton's mass to the reduced Planck mass. Color's strength and m_p/m_e are one value: a law for either is a law for both. To give m_p/m_e to its measured digits, the inverse strength of color would have to be known to about twelve significant figures, so those figures come from a law, and a value of color's strength chosen to agree with the measured ratio cannot supply them. Color's strength is carried as a measured input, and the routes of *The Strengths of the Kinds*, Chapter 6, return α_s and m_p/m_e each from the other as values that agree with measurement (color's way asks $\alpha_s(M_Z) = 0.11898$, $+1.1\sigma$, under SMDR matching).

Sources: *The Strengths of the Kinds*, Chapter 6, Sections I, II and V, Boxes II-1 and II-2, and Summary. Standing: derived (the form of m_p/m_e); a measured input (α_s); a measured agreement (the routes that return each from the other).

C-034 The Numbers the Strengths Carry. That of the twenty-six numbers of the Standard Model with Dirac neutrinos, five are carried with the strengths, and each has its own standing. The five numbers carried with the strengths are the electron's mass, which is the unit (a measured input); the strength of light α_{em} (a measured input); the weak angle (3/8 at the equal traces, derived; the value at the Z mass, a measured agreement); the strength of color α_s (a measured input); and the strong angle θ. The strong angle θ is the angle by which the strong force could distinguish matter from the mirror image of matter. The strong force's own angle is zero by structure, and the phase from the quark masses is carried as zero, a measured agreement, since measurement places θ below about one part in ten billion. The structure fixes the form of the couplings: their rank order, their one-loop rates, the law of place of their crossings and the weight

3/8; measurement supplies their values. The other twenty-one numbers, the masses and the mixings, are carried by *The Measures of the Kinds* (*The Concept Sequence*, Chapter 30, entry 30.16).

Sources: *The Strengths of the Kinds*, Chapter 1, Section VII and Table VII-1; *The Branches of the Kinds*, Chapter 3, Section V (the strong force's own angle). Standing: a measured input, derived and a measured agreement, as stated for each number.

Summary

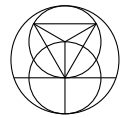
This chapter is the branch of the map that carries the three strengths of interaction, and its owner is *The Strengths of the Kinds*. The structure fixes the form of the couplings, and measurement supplies their values. The rank order of the strengths at low energy is derived from what each force acts on; the one-loop rates $41/10$, $-19/6$ and -7 are derived from the closed list of contents, by two routes for all three and by a third for color; the fermions load the three rates equally, so the gauge bosons and the one Higgs doublet set their differences; and light's own rate is $11/3$.

The strengths of light and of color are measured inputs, and the independence of light's strength is derived for the comparisons the argument examined, given the circle's rule. The internal space read on its content instance has no size, so no coupling runs from it, and the rates change only at the masses of known contents. The law that places each crossing is derived and uses no measured value; the spacing ratio $218/115$ is fixed by the content, and the places themselves are read from the measured couplings run upward. The measured couplings meet at no scale, and the gauge group at every scale is the product of its three factors.

The equality the structure asserts is the equality of the three traces, a relation among weights that two routes reach: the equal standing of the three positions of comparison, and the spectral action's single trace, as *The Kinds of Particle*, *The Connes Correspondence* and *The Strengths of the Kinds* carry it. At the equal traces the weak angle is $3/8$, and the running angle

meets 3/8 at the first crossing alone, so its value at the Z mass is a measured agreement that restates the place of that crossing. Light's relation at the proton's mass, with its unit 3π, is a measured agreement beyond one loop, and from it come the second route to the span's integer, the second route to the place of the first crossing and the weak angle at the measured scales.

At tree level one Higgs doublet gives $m_W = m_Z \cos \theta_W$; the departure of the ρ parameter from one is derived in form and measured in size; and the carriers' law of *The Measures of the Kinds* shares v^2 between the W and the Z as a measured agreement. The span from the electron to the reduced Planck mass is measured, and its integer is identified with the count 72, which the measurement tests. Color's strength sets the proton's binding scale by dimensional transmutation, so color's strength and the ratio m_p / m_e are one value. Together the parts make one account of the five numbers the strengths carry: the forms are the structure's, the values are measurement's, and each agreement between them is stated with its pull.



Chapter D

The Action Principle and Spectral Action

CHAPTER OVERVIEW

This chapter traces the action principle, from which the field equations of physics follow, and the spectral action of Ali Chamseddine and Alain Connes, which gives the form of the Standard Model coupled to gravitation. It covers the stationary action principle; general covariance and Lovelock's theorem; the spectral action, its asymptotic expansion and the form of each of its terms; and gravitation as *The Gravitational Passage* derives it, by the Kaluza-Klein reduction of the ten-dimensional context with the internal space held, with Lovelock's theorem giving the form of the four-dimensional equations.

In this chapter I rely on *The Concept Sequence*, Chapter 30, The Spectral Action and the Form of the Standard Model, and Chapter 39, Gravity Is the Arena: No Graviton. I also draw on Chapter 24, The Algebra of Comparison (the algebra $\mathbb{C} \oplus \mathbb{H} \oplus \mathrm{M}_3(\mathbb{C})$ and the gauge group); Chapter 25, The Internal Space in Its Two Instances; Chapter 27, The Context Instance: Radii, Volume and the Count 72; Chapter 28, The Two Measured Anchors and the Span to the Planck Mass (the electron mass and Newton's constant); and Chapter 29, The Couplings: Rank, Running, Weights and Crossings. It uses five tools that Chapter A of this book, The Spectral Infrastructure, sets out: the spectral triple, the product geometry, the inner fluctuations, the Seeley-DeWitt coefficients and the cutoff function. Each is stated again in plain words where this chapter first uses it.

Two routes in the collection reach the Einstein-Hilbert term, the term of the action whose variation gives Einstein's field equations. *The Gravitational Passage* derives the Einstein equations from the structure of comparison with no spectral action; the spectral action of the product geometry contains the Einstein-Hilbert term as a second appearance of the same term. "That is a unification in the sense of a common source, and the Incommensuration Theorem permits it. What the theorem excludes for the product is a merger" (*The Gravitational Passage*, Chapter 8, Section IV). Newton's constant is the one measured constant of gravitation.

D-I. The Stationary Action Principle

D-001 The Action as a Single Number. That the action is a single number computed from a complete history of the fields (every configuration of matter, radiation and geometry over the region of space and time considered) by integrating, over the space of the description, a quantity called the Lagrangian density, built at each point from the fields and their derivatives: the first derivatives of the matter fields, and, for gravitation, the curvature of the metric, which carries the metric's second derivatives. In mechanics the Lagrangian is the kinetic energy (the energy of motion) minus the potential energy (the energy of configuration); in field theory the Lagrangian density is the corresponding expression for each field and each interaction. The space of integration depends on the description: in *The Gravitational Passage* the gravitational action is integrated over the ten-dimensional context $M^4 \times \mathbb{C}P^2 \times S^2$, four dimensions of spacetime and six of possibility (entry D-008), and in the spectral action the trace runs over the spinors on four-dimensional spacetime and over the finite Hilbert space (entry D-009).

Sources: standard definition (Lagrange 1788; Hamilton 1834); *The Gravitational Passage*, Chapter 11, Section I; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III. Standing: a definition.

> ***The action in everyday language.***
>
> The reader knows from engineering that energy comes in two forms: kinetic (the energy of a moving object) and potential (the energy stored by virtue of position, as in a ball held above the ground or a compressed spring). In mechanics the Lagrangian is the difference between these two: kinetic minus potential, at each moment. The action is the total of this difference, accumulated over the whole history considered. It is one number for a whole history of the system.

D-002 The Stationary-Action Principle. That the history a physical system actually follows (the paths of particles, the evolution of fields, the curvature of spacetime) is one that makes the action stationary: unchanged to first order under every small variation of the fields that vanishes at the boundary of the region considered. If the fields are altered by a small amount, the action changes by an amount of second order in the size of the alteration. For a body whose action is many times the quantum of action ħ, *The Quantum Passage* derives the principle from the sum over paths: the phases of neighboring paths cancel, except near the path whose action does not change to first order, so the body is found along the path of stationary action.

Sources: standard physics (Hamilton 1834); *The Quantum Passage*, Chapter 9, Section VII. Standing: derived (for a body whose action is many times ħ, as the limit of the sum over paths); a measured agreement (the field equations that follow from the principle, entry D-003, agree with measurement).

> ***Stationary, not minimum.***
>
> The reader may know the phrase "principle of least action", which suggests the action is always minimized. The action need only be stationary, and a saddle point or a maximum of the action also satisfies the condition of stationarity. The requirement is that the first variation of the action vanish, as a hilltop and a valley floor both have zero slope. Whether the action is a minimum, a maximum or a saddle depends on the second-order behavior, which does not enter the equations of motion.

D-003 Equations of Motion from the Action. That the condition of stationarity produces one equation of motion for each field in the action, the Euler-Lagrange equation of that field, and that the fundamental field equations of classical and quantum field theory (Maxwell's equations, the Yang-Mills equations, Einstein's field equations, the Dirac equation) each follow from an action by this one variational procedure.

Sources: standard (Euler 1744; Lagrange 1788; Hilbert 1915 for the Einstein equations). Standing: a theorem (the Euler-Lagrange equations of the calculus of variations).

> ***One action, many equations.***
>
> Suppose the action depends on several fields: the metric of spacetime (gravitation), the gauge fields (the three forces of the kinds), and the fermion fields (matter). Varying the metric and demanding stationarity gives Einstein's equations. Varying the gauge fields gives the Yang-Mills equations. Varying the fermion fields gives the Dirac equations. No equation is postulated separately; each follows from the requirement that the action be stationary.

D-II. General Covariance and the Lovelock Theorem

D-004 General Covariance. That general covariance is the requirement that the laws of physics take the same form in every system of coordinates (that is, that every result of comparison be unchanged under every smooth, invertible change of the labels assigned to the events of spacetime), and that *The Gravitational Passage* identifies this requirement with what the omniscient modality requires of a description: what is compared must not depend on how it is described. The omniscient modality is the role of what is known, as structure and pattern. The arena is continuous, so a change of labels is a smooth invertible map. The arena is also asymmetric, since a generic geometry has no isometry and singles out no fam-

ily of labels. So the changes of description that must leave every result unchanged are all the smooth changes of coordinates, not only the changes of a fixed group such as the Lorentz transformations. Covariance restricts the action when the action is built from the metric alone, with no field fixed in advance; covariance alone restricts nothing, since any theory can be given a covariant form (Kretschmann 1917).

Sources: *The Gravitational Passage*, Chapter 11, Section I and Questions, and Chapter 12, Section VII; Einstein (1916); Kretschmann (1917). Standing: an identification (general covariance with what the omniscient modality requires of a description).

> ***Why coordinate labels cannot matter.***
>
> The reader knows from engineering that the choice of units (meters or feet, Celsius or Fahrenheit) does not change the physics. General covariance extends this idea from the units to the coordinate system itself: the whole labeling of the events of spacetime is chosen by the one who describes, and the comparison does not find it. A law that depended on a particular choice of coordinates would be a law about labeling, and what is compared would not enter it.

D-005 The Second-Order Condition. That the field equations derived from the action are required to contain at most second derivatives of the fields (the fields themselves, how fast they change, which is the first derivative, and how fast that rate of change itself changes, which is the second derivative, and no third or higher derivative), so that the initial data are the fields and their first time derivatives at one moment; and that field equations of higher order carry extra degrees of freedom, which generically include states of negative energy (Ostrogradsky 1850).

Sources: *The Gravitational Passage*, Chapter 12, Section VII; Ostrogradsky (1850). Standing: a definition (the second-order condition); a theorem (Ostrogradsky's result that equations of higher order carry extra degrees of freedom, which generically include states of negative energy).

Why higher derivatives are excluded.

Newton's second law, F = ma, is a second-order differential equation: it involves acceleration (the second derivative of position with respect to time) but not the rate of change of acceleration (the third derivative). Specifying the position and velocity of a particle at one instant determines its later motion. An equation with higher derivatives would need more initial data, and the extra freedom it carries generically includes motions whose energy is unbounded below. Curvature-squared terms may still appear as corrections to an effective theory; their size relative to the Einstein term is the curvature times the square of the Planck length, negligible far below the Planck curvature (*The Gravitational Passage*, Chapter 12, Section VII).

D-006 The Lovelock Theorem. That Lovelock's theorem (Lovelock 1971; 1972) states that in four dimensions the most general symmetric, divergence-free tensor built from the metric and its first and second derivatives is a linear combination $A\,G_{\mu\nu} + B\,g_{\mu\nu}$ of the Einstein tensor and the metric, with A and B constants; so that, with the stress-energy of matter as source and $A = 1$, $B = \Lambda$, the gravitational field equations are the Einstein equations with a cosmological constant, and within the theorem's conditions there is no other equation. In terms of the action, the gravitational Lagrangian is the Einstein-Hilbert term plus a cosmological term, up to terms that do not contribute to the field equations (in four dimensions the Gauss-Bonnet term, a fixed combination of the squares of the curvature, is a total derivative).

Sources: Lovelock (1971; 1972); *The Gravitational Passage*, Chapter 12, Section VII. Standing: a theorem (Lovelock's theorem).

The Lovelock theorem in symbolic form.

In four dimensions, the gravitational action satisfying general covariance and the second-order condition is, up to terms that do not change the field equations,

$S_{grav} = (1/16\pi G_N) \int d^4x \sqrt{(-g)}\,(R - 2\Lambda)$,

where R is the scalar curvature (the measure of how far the geometry departs from flatness at a point; Chapter A, entry A-003, defines curvature), Λ is the cosmological constant, g is the determinant of the metric (negative for a Lorentzian metric), and $\sqrt{(-g)}\, d^4x$ is the volume element. Varying the metric gives $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$.

The theorem is a uniqueness result: within its conditions, this form is the only one. The conditions are the inputs: covariance, an action of the metric alone, and equations of second order. The left-hand side must have zero divergence because the stress-energy of the matter is conserved; the Einstein tensor meets this identically, by the contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$.

D-007 The Lovelock Theorem in Higher Dimensions. That in more than four dimensions Lovelock's theorem allows further terms beyond the Einstein-Hilbert term, each generally covariant and each giving field equations of second order: the Gauss-Bonnet term, quadratic in curvature, in five or more dimensions, the cubic Lovelock term in seven or more, and the quartic in nine or more; so that in ten dimensions the Gauss-Bonnet, cubic and quartic terms are all allowed. The Einstein-Hilbert term is the leading term, the only one linear in curvature.

Sources: Lovelock (1971); *The Gravitational Passage*, Chapter 11, Section I. Standing: a theorem (Lovelock's theorem in more than four dimensions).

Why the higher terms leave the four-dimensional form unchanged.

On the ten-dimensional context $M^4 \times \mathbb{C}P^2 \times S^2$, each higher Lovelock term, integrated over the internal space, adds to three quantities: the four-dimensional Planck mass, the bare curvature term of the reduction (entry D-021), and the couplings of the moduli, the fields that describe the size and shape of the internal space (entry D-017). With the moduli fixed, the four-dimensional theory is generally covariant and of second order in the four-dimensional metric, and Lovelock's theorem in four dimensions fixes its form (entry D-006). The four-dimensional Gauss-Bonnet term that the reduction also produces is a total derivative in four dimensions, and with the moduli fixed it adds nothing to the field equations. The form of the

four-dimensional equations therefore holds whatever the coefficients of the higher terms are. A large ratio of scales does not suppress those coefficients, since the internal radius is only a few Planck lengths; their effects enter the values of the constants, and the form of the equations stays the same (*The Gravitational Passage*, Chapter 11, Section I and Questions).

D-008 The Ten-Dimensional Action. That on the ten-dimensional context $M^4 \times \mathbb{C}P^2 \times S^2$ of *The Gravitational Passage*, in which the internal factor is the context instance (the internal space read with radii, $R_O = \sqrt{6}\, l_{Pl}$ for $\mathbb{C}P^2$ and $R_T = l_{Pl}$ for S^2, with l_{Pl} the Planck length: the six dimensions of possibility, whose metric Chapter A, entry A-007, gives), the gravitational action is built from the metric alone, and its leading term is the ten-dimensional Einstein-Hilbert action $S_{10} = \int d^{10}x \sqrt{(-g_{10})}\, R_{10}$, with the higher Lovelock terms allowed beside it (entry D-007). The product form of this space is identified from the Incommensuration Theorem: the continuously asymmetric factor M^4 carries the arena, the discontinuously symmetric factor $\mathbb{C}P^2 \times S^2$ is the content factor relative to the exterior (*The Gravitational Passage*, Appendix C, Section VIII), in the sense of *The Gravitational Passage* (Chapter 9, Section I) that the content's comparisons across the six metrics are realized in it, while the factor itself is the space of possibility, six of the ten dimensions of context; and the theorem excludes their merger into one geometry. The content's fields are fields over M^4; their kinds and charges are read on the internal space taken as a space of states with no size, the content instance, on which no field lives. *The Gravitational Passage* derives the context (the field equations, the cosmological constant and the matter fraction) and makes no claim about the content: its spectra, charges and representations. The form of the content's Lagrangian is given by the spectral action of the product of spacetime with the finite triple (entries D-009 and D-015), and the content enters the gravitational equations only through its stress-energy tensor.

Sources: *The Gravitational Passage*, Chapter 8, Sections I to IV, Chapter 9, Section I, Chapter 11, Section I, Chapter 12, Section V, and Appendix C, Section VIII; *The Kinds of Particle*, Chapter 3, Section III. Standing: an identification

(the product, and its two factors as the arena and the content factor, with the separation of the factors argued from the Incommensuration Theorem); a theorem (the leading term, from general covariance and the second-order condition by Lovelock's theorem).

> ***What the ten-dimensional description holds, and what it leaves to the content.***
>
> The ten-dimensional action of *The Gravitational Passage* is the action of the metric alone. Three facts about the content are named there so that the boundary is plain, and I make no claim on them: the isometry group of $\mathbb{C}P^2 \times S^2$ is locally $SU(3) \times SU(2)$, with no $U(1)$ factor; on an internal space that admits spinors, a smooth Kaluza-Klein theory with gravity alone gives no chiral fermions (Witten 1981; 1985); and $\mathbb{C}P^2$ is not a spin manifold, only spin-c, so a spinor field on it needs a $U(1)$ connection (Chapter A, entry A-011: $\mathbb{C}P^2$ admits no spin structure and admits spin-c structures). The gauge group of the kinds, with its $U(1)$ factor, is computed from the finite algebra (Chapter A, entry A-052: the unitaries of the algebra acting on both sides of the finite Hilbert space, reduced by the unimodularity condition, the condition on their overall phase).

D-III. The Spectral Action

D-009 The Chamseddine–Connes Spectral Action. That the spectral action, introduced by Ali Chamseddine and Alain Connes in 1997, expresses the bosonic part of the action of the product geometry $M \times F$ (the product of four-dimensional spacetime M with the finite spectral triple F; Chapter A, entry A-028) as the trace of a function of its fluctuated Dirac operator, and that in the form of Chamseddine, Connes and Marcolli (2007) the whole action is $S = \mathrm{Tr}\, f(D_A/\Lambda) + \tfrac{1}{2}\langle J\xi, D_A\xi\rangle$, with $D_A = D + A + JAJ^{-1}$ the Dirac operator with its inner fluctuations (the gauge fields and the Higgs field, which enter as fluctuations of the Dirac operator by the algebra; Chapter A, entry A-051). A spectral triple is an algebra, a Hil-

bert space on which the algebra acts, and a Dirac operator on that space (Chapter A, entry A-027). The spectral triple supplies the Dirac operator, the Hilbert space over which the trace is taken, and the algebra whose unitaries give the gauge group. The action has four named inputs: the product form $M \times F$; the finite triple F, whose algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is derived from the structure of comparison (Chapter A, entry A-030), whose number of generations is fixed at three by a postulate, P-Gen, and whose moduli, the entries of its Dirac operator, are not fixed; the cutoff function f, through three of its moments; and the scale Λ (Chapter A, entry A-035, lists the four inputs).

Sources: Chamseddine and Connes (1997); Chamseddine, Connes and Marcolli (2007); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 3, Section V, and Chapter 13, Sections III and IV; *The Connes Correspondence*, Chapter 6, Section I; Chapter A, entries A-033 to A-035 (the spectral action, the cutoff function and its three moments, and the four inputs). Standing: a definition (the spectral action principle, applied with its four inputs stated).

The spectral action formula.

The bosonic action is

$S_b = \mathrm{Tr}\, f(D_A/\Lambda)$,

where D_A is the fluctuated Dirac operator of the product geometry, Λ is an energy scale (the cutoff), and f is a positive cutoff function (Chapter A, entry A-034) that sets how strongly each eigenvalue of D_A contributes.

The fermionic action is

$S_f = \tfrac{1}{2}\langle J\xi, D_A\xi\rangle$,

where J is the real structure (an antilinear map of the Hilbert space that exchanges particles and antiparticles) and ξ runs over the Grassmann vectors in the even part of the Hilbert space (vectors whose components anticommute, as the fields of fermions do). The KO-dimension of a real spectral triple is a number modulo 8 fixed by the signs with which J relates to the Dirac operator and the grading; in the KO-dimension of the product, $10 \equiv 2$ modulo 8, this form removes the doubling of the fermi-

ons. The full action is $S = S_b + S_f$. "Given these, no term of the Lagrangian is chosen separately" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV). The outputs are forms; the values of the couplings and masses come from measurement and from the routes that entry D-024 sets out.

D-010 Asymptotic Expansion. That an asymptotic expansion of a quantity in a large parameter is a series of terms, each smaller than the one before as the parameter grows, such that, as the parameter grows, the error of every truncation becomes small compared with the last term kept; the series need not converge, and its use is that a few terms approximate the quantity well when the parameter is large. The bosonic spectral action (entry D-009) has such an expansion in the cutoff Λ, obtained from the heat-kernel expansion of $D_A{}^2$, whose coefficients, the Seeley-DeWitt coefficients, are integrals of local invariants built from the curvature and the fields (Chapter A, entries A-018 and A-019), and the form of the physics can be read from it term by term.

Sources: standard definition (Poincaré 1886); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III; Chapter A, entries A-018 to A-020 (the Seeley–DeWitt expansion, its coefficients on the product geometry, and what each coefficient gives). Standing: a definition.

Why the expansion works.

The reader knows from engineering that many quantities can be approximated by keeping the first few terms of a series. A Taylor series of a smooth function, for instance, approximates it by a polynomial near a point. An asymptotic expansion is used in the same way, with one difference: it is an expansion in a large parameter, and adding more and more terms need not make it converge; what is guaranteed is that, for a fixed number of terms, the approximation improves as the parameter grows.

For the spectral action the expansion is

$$\mathrm{Tr}\, f(D_A/\Lambda) \sim \Sigma_k\, f_k\, \Lambda^k\, a_{4-k}(D_A{}^2) + O(\Lambda^{-1}) = f_4\Lambda^4 a_0 + f_2\Lambda^2 a_2 + f_0 a_4 + O(\Lambda^{-1}),$$

where $f_0 = f(0)$ and $f_k = \int_0^\infty f(v)\, v^{k-1}\, dv$ for $k > 0$ are the moments of the cutoff function (Chapter A, entry A-034), and the a_j are the Seeley-DeWitt coefficients of $D_A{}^2$ on the product geometry (Chapter A, entry A-019). The notation $O(\Lambda^{-1})$ stands for terms that vanish at least as fast as $1/\Lambda$ as Λ grows.

D-011 The Three Terms of the Expansion. That the asymptotic expansion of the bosonic spectral action (entry D-010) has three leading terms, distinguished by the power of Λ they carry, and that together they give the form of the bosonic Lagrangian: the term in Λ^4 is a cosmological term; the term in Λ^2 gives the Einstein-Hilbert action and a mass term for the Higgs field; and the term independent of Λ gives the Yang-Mills action of the three gauge groups, the kinetic and quartic terms of the Higgs field, the coupling of the Higgs field to curvature, and a Weyl-curvature term.

Sources: Chamseddine, Connes and Marcolli (2007); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III; *The Connes Correspondence*, Chapter 6, Section I; Chapter A, entry A-020 (what each coefficient gives). Standing: a theorem (the expansion and its terms, proved by Chamseddine, Connes and Marcolli 2007 for the product triple).

D-012 The First Term: The Cosmological Term. That the first term of the expansion, the product of the moment f_4, the fourth power of Λ, and the coefficient a_0, is a cosmological term: an energy density of empty space, proportional to the volume of spacetime and to the dimension of the finite Hilbert space, $\dim \mathcal{H}_F = 96$, and independent of D_F, of the carriers and of the classes of comparison. The spectral action fixes the form of this term and no value for it: its natural size, set by the cutoff, exceeds the observed density by roughly 111 orders of magnitude (against a cutoff near 10^{16} GeV) to roughly 123 (against the full Planck mass). *The Gravitational Passage* gives the value of the cosmological constant by another route, with no spectral action: "I identify the cosmological constant with the share of the vacuum energy that is accessible to an observer inside the vacuum", $\rho_\Lambda = c_1\, f\, M_{Pl}{}^4/S$, with $c_1 = 3$, the count of the channels through

which a measurement registers, $f = 1 - 3/(8\pi)$, the factor by which the bare curvature of the internal space lowers the vacuum's energy at the Planck scale, M_{Pl} the full Planck mass formed from the measured Newton's constant, and $S = \pi R^2/l_{Pl}^2$ the count of the observer's particle horizon of radius R (*The Gravitational Passage*, Chapter 13, Section II). The predicted density, 2.56×10^{-47} GeV⁴, lies about 1.7 percent above the observed 2.52×10^{-47} GeV⁴ of the Planck 2018 parameters (Planck Collaboration 2020a), within the ± 2.8 percent that the measured uncertainty of the matter fraction gives the ratio.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III, and Chapter 16, Sections I and II; *The Connes Correspondence*, Chapter 6, Section III (3.4); *The Gravitational Passage*, Chapter 12, Section IV, and Chapter 13, Sections II to V; Planck Collaboration (2020a). Standing: a theorem (the form of the term, proved by Chamseddine, Connes and Marcolli 2007); an identification (the cosmological constant as the share of the vacuum energy accessible from inside, with its value computed and compared with measurement).

The physical meaning of the cosmological term.

The cosmological constant is the energy density of empty space itself, which drives the observed acceleration of the expansion of the universe. In the spectral action it is the term that counts the whole Hilbert space without distinguishing anything within it: its coefficient depends on the dimension of the finite Hilbert space alone (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 16, Section II). Its natural size is enormous, and the spectral action does not by itself explain the observed ratio (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 16, Section I).

The Gravitational Passage gives the value on the context instance: the observed constant is the share of the vacuum's energy, at the scale of the modal boundary, that is accessible from inside, about 10^{-123} of the Planck density. The modal boundary is the Planck scale: the bound of every comparison that a located comparer makes, fixed by the measured Newton's constant (*The Gravitational Passage*, Chapter 17). Combined with

the Friedmann equation the share gives the identity $I^2\Omega_\Lambda = 8 - 3/\pi$, where I is the radius of the particle horizon in units of the Hubble radius; the Hubble constant and the Planck mass cancel from it, and it predicts the matter fraction $\Omega_m = 0.311$, against the measured 0.315 ± 0.007; the equation of state is $w = -1$ exactly (*The Gravitational Passage*, Chapter 13, Sections III to V).

D-013 The Second Term: Einstein–Hilbert Gravity, a Second Appearance. That the second term of the expansion, the product of the moment f_2, the second power of Λ, and the coefficient a_2, gives the Einstein-Hilbert action (the term whose variation yields Einstein's field equations) together with a mass term for the Higgs field; and that, in the comparison route, this Einstein-Hilbert term is a second appearance of the term that *The Gravitational Passage* derives without the spectral action, by the Kaluza-Klein reduction of the ten-dimensional context with the internal space held, with Lovelock's theorem giving the form (Part D-IV, entries D-016 to D-022). In the spectral action the strength of the term is set by $f_2\Lambda^2$, an input; in both programs its strength, Newton's constant, is measured: $G_N = 6.67430 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$ (CODATA 2022), and the Planck mass is formed from it. The appearance of the gravitational term beside the gauge terms, from one trace, is a unification in the sense of a common source, which the Incommensuration Theorem permits; what the theorem excludes is a merger of the arena and its content into one geometry.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III; *The Connes Correspondence*, Chapter 6, Sections II (2.2) and III (3.2); *The Grand Synthesis*, Chapter 10, Section VI (the same statement); *The Gravitational Passage*, Preface, The Chain at a Glance, Chapter 8, Section IV, and Chapter 12, Section III; CODATA 2022. Standing: a theorem (the term in the expansion, proved by Chamseddine, Connes and Marcolli 2007); a measured input (Newton's constant).

> ***Gravity is the arena.***
>
> A force of the Standard Model is a transformation of content: it acts on the labels a state carries, and each of the three forces comes from one summand of the algebra. Gravity acts on no label. "Gravity is not a fourth force with a carrier in this sequence. It is the arena. It is context, not content" (*The Story of the Particles*, Level 7, Section VI). Gravitation is not quantized. The small waves of the metric can be described by quanta, and a quantum of that description is a step in the energy of one mode of the wave, never a step in the extent of the geometry. The word "graviton" names at most such a quantum of a description, never a quantum of the arena, and no graviton is among the kinds. The phonon, a quantum of the description of the vibrations of a held structure such as a solid, is only the nearest sense in which such a quantum can be made intelligible; the arena is not a held structure, so even that sense does not make the quantum a quantum of the arena. Nothing is a quantum of the arena itself, at any scale (*The Unquantized Arena*, Chapter 6, Section IV, and Chapter 9, Section I; *The Gravitational Passage*, Chapter 12, Questions). The arena and its content stand together as a product: "The relationship between them is therefore a product, not a unification" (*The Gravitational Passage*, Chapter 6, Section III).

D-014 The Third Term: Gauge, Higgs and Weyl. That the third term of the expansion, the product of the moment f_0 and the coefficient a_4, independent of Λ, contains the Yang-Mills kinetic terms of the three gauge groups, the kinetic and quartic terms of the Higgs field, the coupling of the Higgs field to curvature, and a Weyl-curvature term, and that at the scale Λ it imposes relations among the couplings: $g_3^2 = g_2^2 = (5/3)g_1^2$, a relation tying the Higgs quartic coupling to the gauge coupling and the Yukawa couplings, and a relation tying the sum of the squared Yukawa couplings to the gauge coupling. The relations come from one step, the single trace of the spectral action over the whole product, which ties the couplings to one common measure. The spectral action fixes the form of

the Higgs potential; the values of the gauge couplings at any one scale are measured, and the Higgs values come by the route of *The Measures of the Kinds* (inset below).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Sections III and IV, and Chapter 15, Sections II to IV; *The Kinds of Particle*, Chapter 5, Section IV; *The Connes Correspondence*, Chapter 6, Section I (1.3); *The Strengths of the Kinds*, Chapter 2, Section VII; Chapter A, entry A-037 (what the spectral action fixes and what it leaves). Standing: a theorem (the terms, and the relations at Λ, proved by Chamseddine, Connes and Marcolli 2007).

What each part of the third term does.

The Yang-Mills kinetic terms govern the dynamics of the gauge fields: the eight gluons, the three weak bosons and the hypercharge boson, the last of which mixes with the neutral weak boson to give the photon and the Z. The equality of the three traces over the finite space of states gives $\sin^2\theta_W = \mathrm{Tr}(T_3^2)/\mathrm{Tr}(Q^2) = 3/8$, the traces taken over one generation. In the comparison route this is a relation among the weights that the structure gives the three forces (*The Kinds of Particle*, Chapter 5, Section IV); it asserts no equality of measured strengths. The running weak angle equals 3/8 at the first crossing of the couplings, near 10^{13} GeV, and at no other scale, and the measured couplings meet at no scale (*The Strengths of the Kinds*, Chapter 3, Sections III and IV). The running of the weak angle, started from 3/8 at the first crossing, returns 0.2314 at the Z mass, against the measured 0.23122 ± 0.00006 (Particle Data Group 2026). The weak angle at the Z mass is a measured agreement (*The Measures of the Kinds*, Chapter 13, Section X, Table X-3).

The Higgs quartic term gives the shape of the potential of the Higgs field, whose curvature at its minimum sets the mass of the Higgs boson. The original spectral model, with its relations imposed at a scale of unification, gave a Higgs mass of about 170 GeV (Chamseddine, Connes and Marcolli 2007), against the measured 125.13 ± 0.11 GeV (Particle Data Group 2026). *The Measures of the Kinds* gives the Higgs values by another route. The self-coupling 1/8 at tree level is derived, given a stated identification: $\lambda = a^2/2$, the self-coupling identified with half the square of the

doublet's half, a = 1/2. The mass of 125.17 GeV that follows from it with its loop term, $1/(24\pi^2)$, and the measured vacuum value is a measured agreement (*The Measures of the Kinds*, Chapter 13, Sections II, IV and VI).

The Weyl-curvature term is a curvature-squared term. Its size relative to the Einstein term is the curvature times the square of the Planck length: of order one at the Planck curvature, and negligible at every curvature that experiment reaches (*The Gravitational Passage*, Chapter 12, Section VII).

D-015 The Fermionic Action. That the fermionic part of the action, which governs the matter fields, is given separately from the bosonic trace Tr $f(D_A/\Lambda)$, by $\frac{1}{2}\langle J\xi, D_A\xi\rangle$, where ξ carries the fermion fields with values in the finite Hilbert space, and that it yields the fermion kinetic terms, their couplings to the gauge fields, and the Yukawa couplings to the Higgs field, which give the fermions their masses when the Higgs field takes its vacuum value. In the comparison route the Majorana matrix is zero and the neutrinos are Dirac particles. A Majorana mass would join a state of content to its own reversal, its own conjugate, making the state's two terms interchangeable, which Axiom III forbids; so no Majorana mass exists, for either chirality. In the construction of Connes the Majorana matrix of the right-handed neutrinos is not zero, and its terms give light neutrino masses by the seesaw mechanism, in which a heavy Majorana mass suppresses the observed neutrino mass.

Sources: Chamseddine, Connes and Marcolli (2007); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III; *The Branches of the Kinds*, Chapter 1, Sections I, II and V; *The Story of the Particles*, Level 4, Section I; Chapter A, entry A-032 (the finite Hilbert space $\mathbb{C}^{96}$ and the finite Dirac operator). Standing: a theorem (the terms, proved by Chamseddine, Connes and Marcolli 2007); derived (the zero Majorana matrix, from Axiom III, which *Non-Commutative Geometry and the Immanent Metaphysics* carries as its postulate P-Terms).

The full action assembled.

$S = S_b + S_f = \mathrm{Tr}\, f(D_A/\Lambda) + \frac{1}{2}\langle J\xi, D_A\xi\rangle.$

> The bosonic part, through its expansion (entries D-010 to D-014), gives the form of the cosmological term, the Einstein-Hilbert term, the gauge terms and the Higgs potential. The fermionic part gives the form of the matter terms, including the Yukawa couplings. Given the four inputs of the spectral action, the bosonic part and the fermionic part together give the form of the Lagrangian of the Standard Model coupled to gravity. The values in that Lagrangian come from measurement and from the routes that entry D-024 sets out.

D-IV. The Kaluza–Klein Reduction

D-016 Kaluza–Klein Reduction. That the Kaluza-Klein reduction is the standard technique by which an action written on a product of four extended dimensions with a compact internal space is reduced to an effective action on the four extended dimensions, by integrating over the internal dimensions, and that *The Gravitational Passage* applies it to the ten-dimensional gravitational action (entry D-008) on the context $M^4 \times \mathbb{C}P^2 \times S^2$ with the internal space held: "The Kaluza–Klein reduction of the ten-dimensional action, with the internal space held, gives the Einstein equations in four dimensions, and Lovelock's theorem gives their form as the only one" (*The Gravitational Passage*, The Chain at a Glance). The reduction gives the four-dimensional Einstein-Hilbert term with the Planck mass set by the volume of the internal space, the moduli that describe the size and shape of the internal space, and a bare curvature term. With the reduction I derive the context, and I make no claim about the content.

Sources: Kaluza (1921); Klein (1926); *The Gravitational Passage*, The Chain at a Glance, Chapter 8, Section IV, and Chapter 11, Sections I to V. Standing: a definition (the Kaluza-Klein reduction, a standard technique); an identification (the product $M^4 \times \mathbb{C}P^2 \times S^2$ to which *The Gravitational Passage* applies it).

The reduction as a view from the four open dimensions.

A Kaluza-Klein reduction reads the geometry of ten dimensions from the four open ones: it extracts four-dimensional physics from the ten-dimensional structure (*The Gravitational Passage*, Chapter 8, Section IV). The six internal dimensions are the dimensions of possibility: they are not spatial, and they are never traversed (*Measurement*, Chapter 11, Section XII). What the reduction yields in four dimensions is gravitation: the Einstein equations, the strength of gravity set by the internal volume, and the contribution of the internal curvature to the vacuum energy. The kinds of the particles of the Standard Model come from the finite algebra and its representation, and they are not modes of this reduction (Chapter A, entry A-014: the quantum numbers of the fermions are outputs of the representation of the algebra on the finite Hilbert space; and Part A-V of that chapter, The Spectral Triple).

D-017 Decomposition of Metric Fluctuations. That the small fluctuations of the ten-dimensional metric about the product background fall into three classes, each giving a different kind of four-dimensional field. Fluctuations along M^4 are the four-dimensional metric perturbation $h_{\mu\nu}$, with two physical polarizations, the two polarizations of gravitational waves; they satisfy the four-dimensional Einstein equations. Mixed fluctuations, with one index along M^4 and one along the internal space, are four-dimensional vector fields associated with the symmetries of the internal space; they belong to the internal space read with radii, the context instance, and they are not the gauge fields of the content: they couple to the matter and the gauge forces only through gravity, and *The Gravitational Passage* does not use their particular form or couplings. Fluctuations within the internal space are scalar moduli: the volume modulus, which sets the overall size of the internal space and thereby the four-dimensional Planck mass, and the shape modulus, which sets the ratio of the two radii. The moduli are held: each internal direction stands at the modal boundary, which is fixed by the measured Newton's constant and is the same throughout the exterior, so $R_O = \sqrt{6}\, l_{Pl}$ and $R_T = l_{Pl}$ at every point of the

exterior. One mechanism, the least motion of the metric with the least extent as its local term, holds the internal space there at first order, with moduli masses of 2.25 and 3.09 Planck masses, so that the force the moduli carry has a range shorter than the Planck length. The hold rests on the modal boundary, and this mechanism gives its stiffness.

Sources: *The Gravitational Passage*, Chapter 8, Section IV, Chapter 11, Section II and Questions, and Appendix D; *The Branches of the Kinds*, Chapter 2, Section III. Standing: a theorem (the decomposition of the small fluctuations of a metric on a product); derived, given a stated identification (the hold, given that each internal direction stands at the modal boundary, where its curvature is the Planck curvature); derived (the masses of the moduli, computed at first order for the one mechanism named).

The symmetries of the internal space, and the gauge group.

The identity component of the isometry group of $\mathbb{C}P^2 \times S^2$, with the Fubini-Study and round metrics, is $PU(3) \times SO(3)$: eight generators from $\mathbb{C}P^2$ and three from S^2, eleven in all, with no $U(1)$ factor. This group equals the group of inner automorphisms of the finite algebra (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 18.1). The gauge group of the kinds, $U(1) \times SU(2) \times SU(3)$ up to a finite group, has one dimension more, the $U(1)$ of the summand $\mathbb{C}$, which acts through the two-sided action of the algebra on the finite Hilbert space (Chapter A, entry A-052). "The gauge group is not the isometry group of $\mathbb{C}P^2 \times S^2$" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 18, Section II). The seven-dimensional internal spaces of Witten (1981), circle bundles over $\mathbb{C}P^2 \times S^2$, have isometry group $SU(3) \times SU(2) \times U(1)$; the isometry group of the six-dimensional $\mathbb{C}P^2 \times S^2$ has no $U(1)$ factor.

The fluctuations along M^4 are the gravitational waves. Their description by quanta is the effective field theory of gravity, which holds below the Planck curvature (*The Unquantized Arena*, Chapter 9, Section I). A quantum of that description is a step in the energy of one mode of a wave, and the word "graviton" names at most such a quantum of a description, never a quantum of the arena: nothing is a quantum of the arena itself, at

any scale. The description ends at the Planck scale, and no description takes over past it (*The Unquantized Arena*, Chapter 9, Section IV; *The Gravitational Passage*, Chapter 12, Questions).

D-018 The Product Form of the Dirac Operator. That on a product of four-dimensional spacetime with an internal factor, the Dirac operator takes the product form $D = \not{D}_4 \otimes 1 + \gamma_5 \otimes D_{\text{int}}$, where γ_5 is the chirality grading of spacetime (which tells left-handed from right-handed spinors) and D_{int} is the Dirac operator of the internal factor, and that this form follows from the product structure (Chapter A, entry A-013: on a product of two even-dimensional manifolds the Dirac operator is $D_1 \otimes 1 + \gamma_1 \otimes D_2$). In the spectral action the internal factor is the finite triple, and $D = \not{D}_M \otimes 1 + \gamma_5 \otimes D_F$ (Chapter A, entry A-028). When the internal factor is the manifold $\mathbb{C}P^2 \times S^2$, the Dirac operator of $\mathbb{C}P^2$ is the Dirac operator of the spin-c structure of $\mathbb{C}P^2$ (Chapter A, entry A-011).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section I; *The Connes Correspondence*, Chapter 6, Section I (1.1); Chapter A, entries A-011, A-013 and A-028 (the spin-c structure of $\mathbb{C}P^2$, the Dirac operator on a product, and the product geometry). Standing: a theorem (the product form of the Dirac operator).

> ***The decomposition formula.***
>
> $D = \not{D}_4 \otimes 1 + \gamma_5 \otimes D_{\text{int}}$.
>
> The first term measures how a spinor field varies across spacetime; the second measures its variation in the internal factor, multiplied by the chirality grading γ_5, which takes the value $+1$ on spinors of one handedness and -1 on the other. Because $\not{D}_4$ anticommutes with γ_5, the square of D is the sum of the squares of the two parts (Chapter A, entry A-013).

D-019 Zero Modes and the Fermions. That the kinds of fermion come from the representation of the finite algebra on $\mathscr{H}_F$, and their chirality from the grading of the finite triple (Chapter A, entries A-014 and A-032: the quantum numbers are outputs of the representation, and $\mathscr{H}_F = \mathbb{C}^{96}$

holds sixteen particle states and their sixteen antiparticles in each of three generations); and that the fermions of the Standard Model are therefore not zero modes of a Dirac operator on a ten-dimensional space, and the three generations do not correspond to harmonic forms of $\mathbb{C}P^2$. The ten-dimensional product admits no spin structure, since $\mathbb{C}P^2$ admits none, so no spinor field of ten dimensions is defined on it; the content's fields of half-integer spin are spinors of M^4 (*The Gravitational Passage*, Chapter 12, Questions). On the content instance of the internal space, zero-mode counting is a spinor and representation count on the geometry of the space of states, and no mode of a field: it gives the kinds within one generation and no multiplicity of generations (*The Kinds of Particle*, Chapter 3, Section VII). The index of the Dirac operator on the content instance gives one, not three (*The Kinds of Particle*, Chapter 7, Section II). The three generations are the three rotations of the content around the cycle of Axiom II (Chapter A, entry A-023). On an internal space that admits spinors, a smooth Kaluza-Klein theory with gravity alone gives no chiral fermions (Witten 1981; 1985).

Sources: *The Kinds of Particle*, Chapter 3, Sections IV and VII, and Chapter 7, Sections I and II; *The Gravitational Passage*, Chapter 8, Section IV, and Chapter 12, Questions; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapters 8 and 14; Witten (1981; 1985); Chapter A, entries A-013, A-014, A-023 and A-032. Standing: a theorem (the index on the content instance, the absence of a spin structure on $\mathbb{C}P^2$, and Witten's result); derived (the count of three generations, from the three rotations of the content).

D-020 Massive Modes Belong to the Context Instance. That the massive Kaluza-Klein modes of a field on an internal space with a size, with masses set by the non-zero eigenvalues of its operators, belong to the context instance, and that the content instance has none (*The Kinds of Particle*, Chapter 3, Key Concepts). On the context instance, whose radii are a few Planck lengths, the levels of the metric are of the order of the Planck mass. The moduli of the hold lie at 2.25 and 3.09 Planck masses at first order (entry D-017). The dark matter is a conserved excitation of the

six dimensions of possibility, made at the turn (the transition from one epoch to the next): a vibration of context, and not an item of content. The dark matter is made of gyrations: a gyration is a conserved excitation of the six held internal dimensions, and physically a massive excitation of the metric of those dimensions; a gyration is felt only by its weight, and it is no quantum of the arena and no quantum of gravitation. The lightest persisting level is the lightest one that changes sign under the antipodal map of S^2, the map that sends each point of the sphere to the opposite point: about $\sqrt{2}$ Planck masses at the classical level and about 0.6 Planck masses at first order. None of these levels is a particle of the Standard Model, and no dark-matter species is among the kinds.

Sources: *The Kinds of Particle*, Chapter 3, Key Concepts; *The Gravitational Passage*, Chapter 11, Section II; *The Weight of Possibility*, A Note to the Reader, Chapter 1, Questions, and Chapter 4, Sections III and V. Standing: an identification (the dark matter as a conserved excitation of the six held internal dimensions, the identification of *The Weight of Possibility*, Chapter 1, Section IV); derived (the masses of the levels, computed at the classical level and at first order).

> ***The levels and their scale.***
>
> The internal space of the context is a few Planck lengths across, so the levels of the metric on it lie near the Planck mass, about 10^{19} GeV, far above any energy reached in experiment. The levels of the metric on the internal space are levels of the context, and no kind of content is among those levels. The persistence of the lightest odd level rests on the antipodal parity of the sphere: the waves of the open geometry are even under it, so an odd level cannot decay into them (*The Weight of Possibility*, Chapter 4, Section III). Its ratio to the ordinary matter is the identity $\Omega_c/\Omega_b = 6 - 2/\pi = 5.3634$, against the measured 5.370 ± 0.048 (*The Weight of Possibility*, Chapter 3, Section V).

D-V. The Complete Four-Dimensional Action

D-021 The Four-Dimensional Gravitational Lagrangian. That integrating the ten-dimensional gravitational action over $\mathbb{C}P^2 \times S^2$ gives a four-dimensional Lagrangian whose gravitational part has three terms: first, the Einstein-Hilbert term, with the four-dimensional Planck mass set by the ten-dimensional scale and the volume of the internal space, $M_{Pl}^2 = M_{10}^8 \, \mathrm{Vol}(K)$, with K the internal space and M_{10} the ten-dimensional scale; second, the kinetic and potential terms of the moduli; and third, a bare cosmological term from the curvature of the internal space, of the order of the Planck density, entering with the sign of a negative cosmological constant, the sign of anti-de Sitter space, and equal to $-3/(8\pi)\, M_{Pl}^4$ at the derived radius. The bare term is part of the external value of the vacuum energy (the total vacuum energy density at the scale of the modal boundary), of which the observed constant is the share accessible from inside (entry D-012); the bare term is therefore not the observed cosmological constant. The content sector adds its own terms, whose form is given by the spectral action (entries D-009 to D-015), and its stress-energy enters the right-hand side of the four-dimensional field equations.

Sources: *The Gravitational Passage*, Chapter 11, Sections III to V, Chapter 12, Sections I to V, and Chapter 13, Section II. Standing: derived (the four-dimensional Lagrangian, by the reduction, a standard technique, applied to the product); derived, given a stated identification (the bare term $-3/(8\pi)\, M_{Pl}^4$, at the radii that follow from placing every internal direction at the modal boundary, Chapter A, entry A-007).

D-022 The Planck Mass and the Internal Volume. That the reduction relates the strength of gravity to the internal space, $M_{Pl}^2 = M_{10}^8 \, \mathrm{Vol}(K)$, with M_{Pl} the full Planck mass, $G_N = \hbar c / M_{Pl}^2$, and M_{10} formed in the same way from the ten-dimensional coupling, and that this relation fixes the strength of gravity in terms of the ten-dimensional scale and the in-

ternal volume, and fixes neither of them on its own. *The Gravitational Passage* reads the relation in the other direction: the measured Newton's constant fixes the Planck mass, and the Planck mass sets the scale at which the modal boundary lies, where the internal space is held (entry D-017). Newton's constant is the one measured constant of gravitation; the axioms fix its role, and measurement fixes its magnitude. At the radii $R_O = \sqrt{6}\, l_{Pl}$ and $R_T = l_{Pl}$, the volume of the context instance is $72\pi^3 l_{Pl}{}^6 = 72(\pi l_{Pl}{}^2)^3$: counted in Planck polydiscs (one disc of Planck radius in each of the three complex directions of the internal space), it is 72 (*The Gravitational Passage*, Chapter 9, Section IX; Chapter A, entry A-008, gives the curvature and volume of the internal space on its two instances). That volume carries the same integer, 72, as the count that *The Strengths of the Kinds* reads in the span from the electron mass to the reduced Planck mass, seventy-two halvings, eight first-order states of comparison times nine joins. The two forms of the number, a volume on the side of context and a count on the side of content, give the same integer at three positions of comparison, $c_1 = 3$, and only there (*The Strengths of the Kinds*, Chapter 5, Section VII; Chapter A, entry A-025, the count 72 and the context instance).

Sources: *The Gravitational Passage*, Preface, Chapter 9, Section IX, Chapter 11, Section III, Chapter 12, Section III, and Chapter 17, Section III; *The Unquantized Arena*, Chapter 9, Section VI (Proposition 9.4); *The Strengths of the Kinds*, Chapter 5, Sections II and VII; Chapter A, entries A-008 and A-025. Standing: derived (the relation $M_{Pl}{}^2 = M_{10}{}^8\, \mathrm{Vol}(K)$, from the reduction); a measured input (Newton's constant); derived, given a stated identification (the volume 72 in Planck polydiscs, at the radii that place every internal direction at the modal boundary).

D-023 The Gauge Couplings: Relations and Values. That the gauge couplings enter the spectral action through the term $f_0\, a_4$ (entry D-014), which at the scale Λ imposes the relation $g_3{}^2 = g_2{}^2 = (5/3)g_1{}^2$, equivalent to $\sin^2\theta_W = 3/8$ as the ratio of the traces $\mathrm{Tr}(T_3{}^2)/\mathrm{Tr}(Q^2)$ over one generation, and that the values of the couplings are measured. In the Connes program the relation is a condition of unification at the cutoff. Run down

to the Z mass with the content of the Standard Model alone, 3/8 gives about 0.21 against the measured 0.2312. The gap between 0.21 and 0.2312 is a tension of the minimal spectral model that Chamseddine and Connes acknowledge. In the comparison route the relation is one among the weights the structure gives the three forces, and it holds at the finest resolution by two routes: the structure gives no force a greater weight than another, because the three positions of comparison are distinct and none stands before the others in the cycle of Axiom II; and the single trace of the spectral action ties the couplings to one common measure. The rank order of the strengths, strong, then weak, then electromagnetic, is fixed by structure; the running weak angle equals 3/8 at the first crossing (hypercharge with weak), near 10^{13} GeV, and at no other scale; the measured couplings meet at no scale; and the gauge group is the product of its three factors, with no unifying group. "No coupling runs from a scale of the internal space", since the content instance, on which the couplings are read, has no size, no radius and no scale (*The Strengths of the Kinds*, Chapter 2, Section VI).

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section IV, and Chapter 15, Section III; *The Kinds of Particle*, Chapter 5, Section IV; *The Connes Correspondence*, Chapter 5, Section III, and Chapter 6, Section III (3.1); *The Story of the Particles*, Level 7, Section V; *The Strengths of the Kinds*, Chapter 2, Sections VI and VII, and Chapter 3, Sections III and IV; *The Measures of the Kinds*, Chapter 13, Section X, Tables X-1 and X-3; Chapter A, entry A-037. Standing: a theorem (the relation at Λ and the trace ratio); derived (the running weak angle equals 3/8 at the first crossing); a measured input (the values of the couplings, read as the strength of light and the strength of color); a measured agreement (the weak angle at the Z mass).

D-024 What the Action Fixes and What Measurement Supplies. That, given its four inputs, the spectral action fixes the form of the Lagrangian of the Standard Model coupled to gravity (the gauge group, the representation of the fermions, the single Higgs doublet and the form of its potential, the coupling to gravity, and the relations among the couplings at Λ), and that the values of the couplings and the masses come from else-

where: "The derivation stops at the moduli, the entries of the finite Dirac operator, and it derives no value of any coupling or mass" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V). *The Measures of the Kinds* sets out the values, number by number, with their standings (Chapter 13, Section X, Table X-3). The two measured anchors are the electron mass, the unit of content, and Newton's constant, the measured input of gravitation. In the working account of the particle side the measured inputs are the electron mass, the strength of light α_{em}, the strength of color α_s, the vacuum value v of the Higgs field, and the two offsets of the charged ladders. In the closure account the step of the mass ladder is written as a product, and the electron alone carries the chain to v, the top and the Higgs mass. The rule of that product, that orders multiply across two domains and add within one, is argued in *The Measures of the Kinds*, Chapter 7, Section IX (addition, multiplication and exponentiation form a triple; bringing two domains into relation is not addition; exponentiation is excluded because the axioms close without totalizing), and the placement of the factors of the product step is derived, given a stated identification there.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V, Chapter 13, Section IV, and Chapter 15, Section I; *The Connes Correspondence*, Chapter 6, Sections I (1.3) and IV; *The Measures of the Kinds*, Preface, Chapter 7, Section IX, and Chapter 13, Sections VIII and X, Table X-3; Chapter A, entries A-046 to A-049 (form from the triple and values from the measures, the constants of the step formula, the one mismatch, and the masses with their standings). Standing: a theorem (the form, given the four inputs, proved by Chamseddine, Connes and Marcolli 2007); a measured input (the two anchors, and the inputs of the working account); derived, given a stated identification (the placement of the factors of the product step).

> ***The parameters of the Standard Model, and their standings.***
>
> The Standard Model with massless neutrinos has nineteen parameters: three gauge couplings, six quark masses, three charged-lepton masses, four parameters of quark mixing (three angles and one phase), two of the Higgs sector (its mass and its vacuum value), and the strong CP angle θ.

With Dirac neutrino masses one adds three masses and four mixing parameters, for twenty-six (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 15, Section I; *The Measures of the Kinds*, Chapter 13, Section X).

The Measures of the Kinds gives each of the twenty-six numbers its standing (Chapter 13, Section X, Table X-3). The electron mass, α_{em} and α_s are measured inputs, and so is v in the working account. The tau mass is derived, given a stated identification (each part of the amplitude vector measured by its share of the squared size), from the electron's and the muon's masses by Koide's relation: the sum of the three charged-lepton masses is two thirds of the square of the sum of their square roots. The muon mass is derived from the electron's on the mass ladder to first order, and at its full precision it is a measured agreement. The Higgs self-coupling 1/8 at tree level is derived, given a stated identification ($\lambda = a^2/2$, half the square of the doublet's half), and the Higgs mass with its loop term is a measured agreement. The top's coupling of one is set by the structure: *The Story of the Particles* fixes it, and the scale of the scheme in which it is read is a setting; every value of the top's mass is a measured agreement. The masses of the five lighter quarks are measured agreements, places on the step formula of the mass ladder. The three quark mixing angles are measured agreements, and the Cabibbo angle among them is derived, given a stated identification (the texture zero at the down quark of the first generation). The quark CP phase is derived from the three measured angles, given a stated identification (the right angle of the unitarity triangle, carried from the right angle of the kernel triangle {3, 4, 5}). That phase is how the weak force realizes the one difference between a process and its mirror-and-charge-conjugate that the Incommensuration Theorem leaves possible (*The Story of the Particles*, Level 7, Section III). The weak angle is 3/8 at the equal traces, and its value at the Z mass is a measured agreement. The strong angle is zero: the strong force's own angle is zero by structure, and the phase from the quark masses, carried as zero, is a measured agreement. The neutrino masses are a fork that the sum of the neutrino masses decides; the lepton mixing angles and the lepton CP phase are a fork that the quadrant of the lepton CP phase decides.

D-025 The Chain from Comparison to the Action. That the collection reaches the action by two chains from one ground, each step of a named kind. The ground: the Root Tautology (comparison, interaction and relation are one act, named three ways) gives the six intrinsics of comparison; the Incommensuration Theorem, a structural fact about the nature of knowing, is proved from them by finite elimination (a theorem); the three axioms are derived from the theorem. The chain of content: each clause of the axioms is given the form of a condition on a spectral triple (postulates, each an identification); the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ follows by theorems of algebra (Chapter A, entry A-030, derives the finite algebra); the gauge group and the hypercharges follow by computation, given unimodularity (Chapter A, entry A-052, computes the gauge group from the unitaries of the algebra); I identify the representation of the algebra with the observed kinds of particle (*The Connes Correspondence*, Chapter 6, Section II; *The Grand Synthesis*, Chapter 10, Section I); the spectral action, applied with its four inputs, gives the form of the Lagrangian of the Standard Model coupled to gravity (entries D-009 to D-015; a theorem of Chamseddine, Connes and Marcolli 2007); and the derivation stops at the moduli, whose values *The Measures of the Kinds* gives with their standings. The chain of context: the two surviving modes of the theorem are identified with the context and the content of physics, and placed in the two factors, in the sense of entry D-008, of the product $M^4 \times \mathbb{C}P^2 \times S^2$ (an identification); the internal space is the only compact homogeneous product meeting the identifications of *The Gravitational Passage*, Chapter 9 (a classification theorem, given the identifications); the radii follow from one identification, that every internal direction stands at the modal boundary (Chapter A, entry A-007; *The Gravitational Passage*, Chapter 9, Section IX); the ten-dimensional action is reduced with the internal space held (a standard technique, with the hold derived from the same identification); and Lovelock's theorem gives the form of the four-dimensional equations, $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$, with Newton's constant measured and Λ given by the information deficit, the share of the vacuum's energy accessible from in-

side (an identification, with its value computed). The chain of content and the chain of context meet in the Einstein-Hilbert term, which the chain of context derives and which the spectral action contains a second time. That meeting is a common source, which the Incommensuration Theorem permits, but the Incommensuration Theorem excludes a merger.

Sources: *Incommensuration*, Chapters 6 and 8; *The Gravitational Passage*, The Chain at a Glance, Chapters 1 to 4, Chapter 8, Section IV, Chapter 9, and Chapters 11 to 13; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 9, Section V, Chapter 12, and Chapter 13; *The Connes Correspondence*, Chapter 6, Section II; Chapter A, entries A-007, A-025, A-030 and A-052. Standing: a theorem (the Incommensuration Theorem, the algebra from the postulates, the classification of the internal space given its identifications, Lovelock's theorem, and the expansion of the spectral action); an identification (the postulates, the representation as the kinds, the product, the conditions on the internal space, the radii, and the cosmological constant); a measured input (Newton's constant).

> ***The two chains, compressed.***
>
> Comparison (Root Tautology; six intrinsics)
>
> ↓ Incommensuration Theorem (theorem); three axioms derived
>
> Content: postulates on a spectral triple (identifications) → $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ (theorem) → spectral action on $M \times F$ with four inputs → form of the Standard Model coupled to gravity; values from *The Measures of the Kinds*, with their standings.
>
> Context: product $M^4 \times \mathbb{C}P^2 \times S^2$ (identification) → Kaluza–Klein reduction with the internal space held → Lovelock's theorem → $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$, with G_N measured and Λ the accessible share.
>
> The Einstein-Hilbert term appears in both: derived in the chain of context, and appearing a second time in the spectral action.

Summary

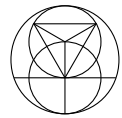
This chapter is the map of where the action of physics comes from in the collection, and of what each part of it fixes. Its first two parts set out the tools that every later step uses. A history of the fields is given one number, its action; the history that is followed makes the action stationary, and every field equation of physics follows from that one condition by the calculus of variations. General covariance, which *The Gravitational Passage* identifies with what the omniscient modality requires of a description, together with an action of the metric alone and equations of second order, leaves by Lovelock's theorem one form of the gravitational equations in four dimensions: Einstein's equations with a cosmological constant.

The third part traces the spectral action of Chamseddine and Connes, one trace of a function of the Dirac operator on the product of spacetime with the finite triple. Given its four inputs (the product, the finite triple, three moments of the cutoff function and the scale), its expansion gives the form of the Standard Model coupled to gravitation: a cosmological term, the Einstein-Hilbert term with a mass term for the Higgs field, and the gauge, Higgs and Weyl terms, with relations among the couplings at the cutoff. Those relations come from the single trace, which ties the couplings to one common measure, as *The Kinds of Particle*, *The Connes Correspondence* and *The Strengths of the Kinds* carry it; in the comparison route the same equality is a relation among the weights the structure gives the three forces, reached by a second route. The spectral action gives forms; the values come from measurement and from the routes that *The Measures of the Kinds* sets out, each number with its standing.

The fourth and fifth parts trace gravitation on its own ground. *The Gravitational Passage* reduces the action of the metric on the ten-dimensional context, spacetime times the six held dimensions of possibility, and the reduction gives the Einstein-Hilbert term with the Planck mass set by the internal volume, the moduli of the internal space, held at the modal

boundary, and a bare curvature term that enters the external value of the vacuum energy. Newton's constant is the one measured constant of gravitation, and the cosmological constant is the share of the vacuum's energy accessible from inside. The small waves of the metric can be described by quanta, and those quanta belong to the description; nothing is a quantum of the arena itself, at any scale, and past the Planck scale no description takes over. The massive levels of the metric belong to the context instance, and the gyrations of the dark matter are among them, each an excitation of the metric of the six held internal dimensions, no quantum of the arena and no quantum of gravitation.

The two chains, of content and of context, start from one ground, the Incommensuration Theorem and the three axioms derived from it, and meet in the Einstein-Hilbert term: derived in the chain of context, and appearing a second time in the trace of the spectral action. The meeting of the two chains in the Einstein-Hilbert term is a common source, which the Incommensuration Theorem permits. The product $M^4 \times \mathbb{C}P^2 \times S^2$ keeps the arena and the content in separate factors, as the Incommensuration Theorem requires.



Chapter E

The Higgs Potential and Electroweak Phase Transition

CHAPTER OVERVIEW

This chapter traces the Higgs field and its potential: the field on the one crossed edge of the finite space, the form of its potential as the spectral action gives it, the values the structure fixes and the values measurement supplies, the mass of the Higgs boson, the character of the electroweak transition and of the strong transition in the early universe, and what is stated of the vacuum. It rests on *The Concept Sequence*, Chapter 24, The Algebra of Comparison; Chapter 25, The Internal Space in Its Two Instances; Chapter 26, The Two Kernel Integers and the Kernel Triangle; Chapter 29, The Couplings: Rank, Running, Weights and Crossings; Chapter 30, The Spectral Action and the Form of the Standard Model; Chapter 37, The Gauge Bosons; Chapter 38, The Higgs Boson; Chapter 41, The Completeness of the Kinds; and Chapter 45, Further Predictions. Chapter B, The Mass Spectrum, carries the masses that the couplings of the fermions to the condensate give, and Chapter H, Baryogenesis, carries the share of matter that the sphalerons leave when they stop.

The internal space $\mathbb{C}P^2 \times S^2$ has two instances. The content instance of the internal space is a space of states with no size, no radius and no fields; the context instance of the internal space has radii, and the context instance belongs to *The Gravitational Passage*. Every statement of this chapter about the internal space is made on its content instance. The Higgs field

> is a field over four-dimensional spacetime and not a change of the space of states, on which no field lives, and the transitions of the early universe are computations of four-dimensional physics to which the internal space adds no term. A distance from measurement is written as a pull: the stated value less the measured value, divided by the combined uncertainty, in standard deviations.

E-I. The Higgs Potential

E-001 The Higgs Field: the Field on the Crossed Edge. That the Higgs field is the field on the one crossed edge of the structure. The two sides of every doublet (the up quark and the down quark, the neutrino and the electron) are distinct, inseparable and non-interchangeable (Axiom III of *The Story of the Particles*), and the weak force rotates one side into the other, so a comparison across the edge between the two sides is required. The partition of every comparison into content and context operates at three scales, each with an edge: fermion against boson, the two sides of a doublet, and quark against lepton. A field lives on an edge when a transformation crosses it, and of the three edges only the edge between the two sides of a doublet is crossed, by the weak force. One edge is crossed, and one edge carries a field. That this field, the Higgs field, is a scalar (spin zero) because an edge has no direction. That the Higgs field is a doublet of the weak force SU(2), with hypercharge 1/2 (the charge of the U(1) factor of the forces), with two internal components and no color. That the Higgs field is one field for quarks and leptons alike, and that the Higgs field is the only scalar. In noncommutative geometry, which builds a geometry from an algebra in place of a set of points, the same field is the fluctuation [D_F, a] of the finite Dirac operator D_F across the one edge that D_F crosses, between the summands $\mathbb{H}$ and $\mathbb{C}$ of the algebra. The Higgs field is a fluctuation of the Dirac operator and not of the two-sphere: on its

content instance the internal space has no radius, and the condensate is the value that the Higgs field keeps throughout space, not a change of the space of states.

Sources: *The Story of the Particles*, Level 8, Sections I, II and V; *The Branches of the Kinds*, Chapter 4, Section II (Box 4.2); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section II, and Chapter 19, Section VI (Proposition 19.10); *The Kinds of Particle*, Chapter 3, Section VI; *The Concept Sequence*, Chapter 38, entries 38.10 and 38.13. Standing: derived.

> ***The ball-in-a-valley picture.***
>
> Imagine a ball sitting in a landscape of hills and valleys. The ball rolls to the lowest point it can find, a valley. If the valley is shallow, it takes little energy to push the ball uphill; if the valley is steep, it takes a great deal. The value of the Higgs field is the ball. The potential energy surface is the landscape. The curvature of the valley at its bottom sets the Higgs boson's mass. The distance of the ball from the center of the landscape is the vacuum value. Against the vacuum value, each particle that takes mass from the Higgs field has a mass in proportion to the particle's own coupling to the field. The spectral action fixes the shape of the landscape, which is the form of the potential (E-006). The distance of the valley from the center is the measured vacuum value (E-004). The curvature of the valley at tree level is set by the self-coupling 1/8 (E-005).

E-002 The Mexican Hat Potential. That the potential energy surface of the Higgs field has a raised center surrounded by a circular valley, the shape of the bottom of a wine bottle, or of a Mexican hat seen from above. That at the raised center the Higgs field has the value zero, and no particle has a mass from the field. That at any point in the surrounding valley the Higgs field has a nonzero value, and every particle that couples to it has a mass from it. That the valley is a continuous ring and not a single point: the field can roll from the center into the valley in infinitely many directions, all equally low in energy. *The Story of the Particles* reads the settling of the field through the Incommensuration Theorem, that no one compar-

ison can be perfected as both symmetric and continuous: the underlying structure treats both sides of the doublet alike, and the ground state, the condensate, picks a side.

Sources: *The Story of the Particles*, Level 8, Section III; *The Coming of the Kinds*, Note 2 to Chapter 6. Standing: a definition.

> ***Why a hat and not a bowl.***
>
> A Mexican hat potential has its minimum away from the center: the field rolls off the raised center and settles into the valley. The raised center is an unstable equilibrium: like a ball balanced on top of a hill, any small disturbance sends it rolling. A bowl-shaped potential, with steep sides curving down to a single minimum at the center, would leave the Higgs field at zero and give no particle a mass from it. The ring of the valley is the set of configurations where the energy is lowest. The system settles at one point on the ring, though all points are equivalent. This settling is what breaks the symmetry: the $SU(2) \times U(1)$ gauge symmetry that rotates the Higgs field around the ring is reduced to the single $U(1)$ symmetry that leaves the chosen point unchanged. That unbroken $U(1)$ is electromagnetism, generated by $Q = T_3 + Y$, the electric charge as the sum of the weak isospin label T_3 and the hypercharge Y.

E-003 Vacuum Expectation Value. That the value of the Higgs field at the bottom of the valley, the value it settles to in the lowest-energy state, is called the vacuum expectation value, or vacuum value, and the settled value that fills all of space is called the condensate. That each particle that takes its mass from the condensate has a mass equal to the vacuum value times its own coupling to the condensate: a fermion f has m_f = y_f v/$\sqrt{2}$, and the W boson has m_W = g v/2, with g the weak coupling. That a carrier of force becomes massive when the condensate tells apart the states it rotates among, and stays massless when it does not. The W and Z bosons become massive; the gluons remain massless because the condensate carries no color, and the photon remains massless because the condensate sits on the side of the doublet whose electric charge is zero (*The Concept Sequence*, Chapter 37, entries 37.11 and 37.18).

Sources: *The Measures of the Kinds*, Chapter 13, Section I; *The Story of the Particles*, Level 8, Sections III and IV. Standing: a definition (the vacuum value and the condensate); derived (which carriers take mass from the condensate).

E-004 The Measured Vacuum Value and the Ceiling. That the vacuum value is measured. It is read from the Fermi constant, the strength of the weak interaction at low energy as measured in the decay of the muon, G_F = 1.1663785(6) × 10^{-5} GeV^{-2} (Particle Data Group 2026; CODATA 2022 gives 1.1663787(6) × 10^{-5} GeV^{-2}), by v = (√2 G_F)^(−1/2) = 246.21967 ± 0.00006 GeV, a relative precision of 0.24 parts per million. *The Measures of the Kinds* states what measurement supplies in two accounts, set side by side. In the working account of The Measures of the Kinds, v is a measured input. In its closure account, the step of the mass ladder is written as a product, by the domain rule that orders multiply across two domains and add within one, argued in its Chapter 7, Section IX, with the factor each domain supplies a stated identification; there the chain from the electron alone reaches v = 246.21966 GeV, −0.16 standard deviations from its measurement. That every coupling to the condensate is a ratio of a part to its whole and lies between 0 and 1, so that no particle of matter is heavier than $v/\sqrt{2}$ = 174.1036 GeV. The top quark stands near this ceiling: its coupling to the condensate is one, y_t = 1, a value the structure sets (the up-type sector is comparison turned upon its own content, whose extent is the identity), with the scheme in which it is read a setting fixed by stating it; its measured mass is 172.60 ± 0.27 GeV (Particle Data Group 2026).

Sources: *The Measures of the Kinds*, Chapter 13, Sections I, III, VIII and X, and Chapter 7, Section IX; *The Story of the Particles*, Level 6, Section V, and Level 8, Section IV. Standing: a measured input (v, in the working account); derived, given a stated identification (v in the closure account, the identification being the factor each of the two domains supplies to the product step); derived (the ceiling $v/\sqrt{2}$, and $y_t = 1$ with its scheme a setting).

E-005 The Tree-Level Quartic Coupling. That the shape of the Mexican hat potential near its minimum is controlled by the quartic coupling, or self-coupling, λ, the coefficient of $(\Phi^\dagger\Phi)^2$ in the energy of the Higgs doublet Φ, and that at tree level (the lowest order of the calculation, before quantum corrections) the Higgs mass is m_H = $\sqrt{(2\lambda)}$ v. That the self-coupling at tree level is $\lambda = 1/8$. It rests on the doublet's half $a = 1/2$, the weak isospin of each side of the weak doublet, reached by three routes that are kept side by side: the isospin projections $\pm 1/2$ on the two poles of the doublet's sphere of states, read as a space of states; the ratio of the quark charges, $|Q_d|/Q_u = (N_c - 1)/(N_c + 1)$, which is 1/2 only at three colors; and the kernel writing $a = c_1/(2(n_C + 1)) = 3/6$, in the kernel integers $c_1 = 3$ (the three positions of any comparison) and $n_C = 2$ (the conjugate pair, content and context). The form route gives $\lambda = a^2/2 = 1/8$ at every number of colors; the step $\lambda = a^2/2$ is a stated identification, of the self-coupling with half the square of the doublet's half. The share route reads λ as the down quark's share of one generation's charge squared, $(1/3)/(8/3) = 1/8$, which holds only at three colors, and equivalently as $\lambda = \sin^2\theta_W(\text{traces}) - 1/4 = 3/8 - 1/4$, with $\sin^2\theta_W = 3/8$ the weak angle at the equal traces. That at $\lambda = 1/8$ the tree mass is exactly $v/2 = 123.11$ GeV, which lies 1.61 percent and -18.4 standard deviations below the measured 125.13 ± 0.11 GeV (Particle Data Group 2026); the quantum correction carries the rest (E-008).

Sources: *The Measures of the Kinds*, Chapter 13, Sections II and IV (Boxes II-1, IV-1 and IV-2), and Section X (Table X-3). Standing: derived (the doublet's half $a = 1/2$); derived, given a stated identification ($\lambda = 1/8$ at tree level and the tree mass $v/2$, the identification being $\lambda = a^2/2$, the self-coupling as half the square of the doublet's half).

> ***Written symbolically.***
>
> The doublet's half: a = 1/2 (three routes, E-005). The self-coupling at tree level: $\lambda = a^2 / 2 = (1/2)^2 / 2 = 1/8 = 0.125$. The tree mass: $m_H = \sqrt{(2\lambda)}\, v = v/2 = 246.21967/2 = 123.11$ GeV. The tree mass of 123.11 GeV is the tree-level value, the value before quantum corrections.

E-006 The Form of the Potential from the Spectral Action. That the spectral action gives the form of the Higgs potential, $V = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$, and that the values in it come from measurement and from the doublet's half: the vacuum value is measured (E-004), and the self-coupling at tree level is 1/8 (E-005). The spectral action is the action built from the spectrum of the Dirac operator of the product of space and time with the finite space, read up to a highest energy, the cutoff Λ. Its bosonic part, the trace Tr f(D_A/Λ), has an expansion in powers of the cutoff: the leading coefficient gives a cosmological term, the next gives the Einstein-Hilbert term and the mass term of the Higgs field, and the last gives the terms of the three gauge forces and the kinetic and quartic terms of the Higgs field. So the potential has a negative quadratic term at tree level, and its minimum breaks $SU(2) \times U(1)$ to the $U(1)$ of electromagnetism, with the vacuum value set by the ratio of the two coefficients. At the cutoff the single trace of the spectral action over the whole product ties λ to the gauge couplings and to the couplings to the condensate; those relations do not give 1/8, and the value 1/8 is reached by the form route of E-005. The Higgs mass is read from the measured vacuum value and the self-coupling, $m_H = \sqrt{(2\lambda)}\, v$, and not from the cutoff.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Sections III and IV; *The Measures of the Kinds*, Chapter 13, Section IV; *The Connes Correspondence*, Chapter 5, Section II; Chamseddine, Connes and Marcolli (2007). Standing: derived (the form of the potential).

> ***The potential and its two coefficients.***
>
> $V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$. Its minimum lies at $|\Phi|^2 = \mu^2/(2\lambda)$, that is, at $v^2 = \mu^2/\lambda$ with $|\langle\Phi\rangle| = v/\sqrt{2}$, and the mass of the radial excitation is $m_H^2 = 2\lambda v^2 = 2\mu^2$. The spectral action fixes the form: one negative quadratic term and one positive quartic term in the one doublet. The vacuum value v is measured (E-004); the self-coupling at tree level is 1/8 (E-005).

E-007 Quantum Corrections to the Potential. That quantum fluctuations, the virtual particles that couple to the Higgs field, add to the tree-level potential a further energy that depends on the field's value, called the one-loop effective potential (Coleman and Weinberg, 1973). That in this account the tree potential already carries its negative quadratic term (E-006), so its minimum lies away from zero before any quantum correction, and the vacuum value is measured (E-004). That in the Standard Model the top quark's contribution to the quartic coupling, its Coleman-Weinberg term, is $3y_t^4/(16\pi^2)$ times logarithms and is negative; the term by which the measured self-coupling exceeds 1/8 is a different thing (E-012).

Sources: *The Measures of the Kinds*, Chapter 13, Sections IV and VI; Coleman and Weinberg (1973). Standing: a definition (the one-loop effective potential); derived (the quadratic term of the tree potential).

E-008 The Higgs Mass beyond Tree Level. That the mass of the Higgs boson is set by the curvature of the potential at the bottom of the valley. In the on-shell definition $\lambda = m_H^2/(2v^2)$, which reads λ from the measured pole mass (the physical mass of the particle) and the measured v, the measured self-coupling is 0.12914 ± 0.00023, exceeding its tree value 1/8 by $0.004136 \pm 0.000227 = (0.980 \pm 0.054) \times 1/(24\pi^2)$ with the Particle Data Group's 2026 mass. That the mass formula $m_H = v\sqrt{(2(1/8 + 1/(24\pi^2)))} = 125.1715$ GeV, with the measured v and no input from the top quark, lies $+0.38$ standard deviations from the measured 125.13 ± 0.11 GeV (Particle Data Group 2026) and -0.26 from 125.20 ± 0.11 GeV (Particle Data Group 2024). The value 125.17 GeV is a measured agree-

ment. Two writings of the loop term are kept side by side: $y_t^4/(8\pi^2 c_1)$ at $y_t = 1$ and $c_1 = 3$, with the factor $1/c_1$ a premise of its own; and the field's clause of the division of v^2, $m_H^2 = (v/2)^2(1 + N\kappa_H)$ with $N = 4$ and $\kappa_H = 1/(12\pi^2)$, the photon's one-loop unit per unit charge squared, so that the loop term is the doublet's half times κ_H. Within $m_H^2 = (v^2/4)(1 + n\kappa_H)$ with n a whole number, the measured $n = 3.919 \pm 0.215$ fixes $n = 4$ uniquely among the integers from -8 to 12. A Higgs mass measured to ± 0.03 GeV would decide between the formula's 125.17 GeV and the midpoint relation $m_H^2 = m_Z\, m_t$ (125.455 $\pm$ 0.098 GeV, +2.21 standard deviations), and a Higgs mass measured to about ± 0.005 GeV would separate the forms of the second order.

Sources: *The Measures of the Kinds*, Chapter 13, Section VI (Boxes VI-1 and VI-2, Table VI-1), and Section IX; *The Connes Correspondence*, Chapter 9, Section VI. Standing: a measured agreement (the Higgs mass with its loop term, 125.17 GeV); a fork that a named measurement decides (the formula against the midpoint relation, decided by a Higgs mass measured to ± 0.03 GeV).

The Higgs mass formula.

$m_H = v\sqrt{(2\lambda)}$, with $\lambda = 1/8 + 1/(24\pi^2) = 0.129222$: $m_H = 246.21967 \times \sqrt{0.258443} = 125.1715$ GeV. Tree value alone: $v/2 = 123.11$ GeV. Measured: 125.13 $\pm$ 0.11 GeV (Particle Data Group 2026). A writing of the Higgs mass from the top quark, $m_H = m_t\sqrt{((3\pi^2 + X)/(6\pi^2))}$, carries a coefficient X; with the measured top it requires $X = 3/(2(1 - 2\kappa_H)) = 1.5258$, and the measured $X = 6\pi^2(m_H^2/m_t^2 - 1/2)$ is 1.515 $\pm$ 0.112 (Particle Data Group 2026 masses). The value $X = 6/5$ gives 124.50 GeV, -5.8 standard deviations from the measured mass.

The original spectral model.

With the relations of the spectral action imposed at a scale of unification, the original spectral Standard Model gave a Higgs mass of about 170 GeV (Chamseddine, Connes and Marcolli, 2007), which the measured 125 GeV excludes. Chamseddine and Connes (2012) reached a mass near 125 GeV through a real scalar field σ made from the Majorana entry of the finite Dirac operator. In this account the neutrinos are Dirac and the Majorana

> entry is zero (*The Branches of the Kinds*, Chapter 1, Sections III to V), so that route is closed here. The self-coupling $\lambda = 1/8$ imposed at a high scale and run down gives m_H = 149 to 152 GeV, more than 200 standard deviations from the measurement: the relation $\lambda = 1/8 + 1/(24\pi^2)$ is a statement at the weak scale in the on-shell definition, not a boundary value at a high scale.

E-009 The Goldstone Bosons. That the Higgs field has four real components, two complex components each with a size and a phase, corresponding to the four real directions in which the field can be displaced. That when the field settles into the valley, three of these four components are Goldstone modes, the massless modes that a broken continuous symmetry leaves, corresponding to motion along the valley floor (around the ring of the hat) rather than up the valley walls. That these three are absorbed by the three weak bosons, the W^+, the W^- and the Z (*The Concept Sequence*, Chapter 37, entry 37.18), as their third state of turning (the longitudinal polarization), giving each a mass. That the fourth component, the radial direction perpendicular to the valley floor, is the physical Higgs boson (*The Concept Sequence*, Chapter 38, entry 38.10), whose mass is set by the curvature of the potential in the radial direction (E-008). That the photon remains massless because the U(1) generated by $Q = T_3 + Y$ is unbroken: the condensate carries $Q = 0$. Counted before and after the settling, the electroweak carriers and the Higgs field hold twelve states (four massless carriers with two each and four Higgs components; three massive carriers with three each, the photon with two and the Higgs boson with one): the states are rearranged, and none is lost or gained.

Sources: *The Branches of the Kinds*, Chapter 4, Section II (Box 4.3); *The Coming of the Kinds*, Chapter 6, Section III (Box 6.1); *The Story of the Particles*, Level 8, Section V. Standing: derived.

E-II. What Is Fixed and What Is Measured

E-010 The Higgs Self-Coupling. That the Higgs boson interacts with itself: two Higgs bosons can scatter into two Higgs bosons, and three Higgs bosons can interact to produce other Higgs bosons; the strength of this self-interaction is the self-coupling λ. That the structure fixes the self-coupling at tree level, $\lambda = 1/8$ (E-005), and with it the tree mass $v/2$; the on-shell self-coupling read from the measured mass and the measured vacuum value is 0.12914 ± 0.00023, and $1/8 + 1/(24\pi^2) = 0.129222$ agrees with it (E-008). That in the modified minimal-subtraction scheme (MS-bar), the standard convention in which running couplings are quoted, the self-coupling at the top quark's mass is $\lambda(M_t) = 0.12603$ (from the fit of Buttazzo and others, 2013, evaluated at the measured masses), and $\lambda(\mu) = 1/8$ is reached near 182 GeV (181.8 GeV at two loops on the 2026 inputs; 183.2 ± 3.6 GeV by a second determination). The on-shell and the MS-bar values are one measured fact read in two schemes, and the scheme of the scale at which $\lambda(\mu) = 1/8$ is a setting, fixed by stating it. The rate at which the self-coupling changes with energy is fixed in form, and it uses the values of the top quark's coupling and the gauge couplings.

Sources: *The Measures of the Kinds*, Chapter 13, Sections IV, VI (Boxes VI-3 and VI-4) and IX; Buttazzo and others (2013). Standing: derived, given a stated identification ($\lambda = 1/8$ at tree level, the identification being $\lambda = a^2/2$); a measured agreement (the loop term).

E-011 The Hierarchy. That in the Standard Model the Higgs boson is an elementary scalar whose mass receives quantum corrections from every energy scale up to the scale at which the description is cut off; the question why the Higgs mass is about 125 GeV and not near the Planck mass is called the hierarchy problem. That the ratio of the vacuum value to the reduced Planck mass (the Planck mass divided by $\sqrt{(8\pi)}$, the mass fixed by Planck's constant, the speed of light and Newton's constant), $v/M_Pl,red$, is a pure number, and this account carries it as measured: in the working

account v is a measured input (E-004), and Newton's constant is the measured input of the side of gravitation. That the Higgs field is the field on the crossed edge and the only scalar (E-001), a fluctuation of the Dirac operator and not a mode of the geometry of the internal space; the internal space on its content instance has no size, so no scale of that space enters the Higgs mass. What the structure fixes in this sector is the self-coupling at tree level, 1/8, and with it the tree mass v/2 as a ratio to the measured v (E-005).

Sources: *The Strengths of the Kinds*, Chapter 5, Section I; *The Measures of the Kinds*, Chapter 13, Sections I and IV; *The Kinds of Particle*, Chapter 3, Sections III and VI; *The Branches of the Kinds*, Chapter 4, Section II. Standing: a definition (the hierarchy problem); a measured input (the ratio $v/M_{Pl,red}$, from the measured v and the measured Newton's constant); derived, given a stated identification (the self-coupling 1/8 at tree level and the tree mass v/2, the identification being $\lambda = a^2/2$).

E-012 The Loop Term Is Not the Top Quark's Loop. That the term $1/(24\pi^2)$ by which the measured self-coupling exceeds 1/8 is a count of its own, and a different thing from the Standard Model's loop of the top quark. The top's Coleman-Weinberg term in the quartic coupling is $3y_t^4/(16\pi^2)$ times logarithms and is negative; $1/(24\pi^2)$ is positive, carries no logarithm, and is 2/9 of $3/(16\pi^2)$; the top's term equals it only at the scale $\mu = m_t\, e^{(1/9)} = 192.9$ GeV. The Higgs boson's own loop is 0.28 of the term, and it carries a logarithm. That the term $1/(24\pi^2)$ is the residue of a count read on the space of states, which has no size. The term $1/(24\pi^2)$ is not produced by the running of a coupling. No tower of Kaluza-Klein modes (the massive modes a field would have on an internal space of finite size) enters the term $1/(24\pi^2)$, since no such tower is formed on the content instance. It is read at the field's own mass shell, in the on-shell definition of λ.

Sources: *The Measures of the Kinds*, Chapter 13, Section VI; *The Kinds of Particle*, Chapter 3, Section VII. Standing: a measured agreement.

E-013 What Is Fixed and What Is Measured in the Higgs Sector. That in the Standard Model as first written the Higgs sector has two adjustable parameters, the Higgs mass and its vacuum value (equivalently the quadratic coefficient and the quartic coupling), both taken from measurement. That in this account the field, its quantum numbers, the form of its potential, the breaking of the symmetry and the form of the mass terms follow from the structure (E-001, E-006); the vacuum value is a measured input in the working account (E-004); the self-coupling at tree level, $\lambda = 1/8$, is derived, given the stated identification $\lambda = a^2/2$ (E-005); the top quark's coupling $y_t = 1$ is a value the structure sets, with the scheme in which it is read a setting (E-004); and the Higgs mass with its loop term, 125.17 GeV, is a measured agreement (E-008). The masses of the W and Z bosons follow at tree level from v and the couplings, $m_W = g\,v/2$ and $m_Z = \sqrt{(g^2 + g'^2)}\,v/2$, so that $m_W = m_Z \cos\theta_W$ (Chapter C, entry C-028, which states these relations of the weak angle and the two masses at tree level). The strengths of light and of color are measured inputs, and the weak angle at the Z mass is a measured agreement (Chapter C, entry C-025, which reads the measured angle there as the place of the first crossing of the running couplings).

Sources: *The Measures of the Kinds*, Chapter 13, Sections I to IV, VI, IX and X; *The Strengths of the Kinds*, Chapter 1, Section VII, and Chapter 4, Section II; *The Story of the Particles*, Level 8, Section IV. Standing: derived (the field, its quantum numbers, the form of the potential and of the mass terms, and $y_t = 1$); a measured input (v in the working account, and the strengths of light and of color); derived, given a stated identification ($\lambda = 1/8$ at tree level, the identification being $\lambda = a^2/2$); a measured agreement (the Higgs mass with its loop term, and the weak angle at the Z mass).

E-III. The Electroweak Phase Transition

E-014 Phase Transition Defined. That a phase transition is a change in the state of a physical system, like water freezing into ice or boiling into steam. That in each case the system changes from one organized state to another, and the change involves a rearrangement of the microscopic constituents. That in the early universe the Higgs field underwent a change of this kind as the temperature dropped: from holding no settled value (no particle has a mass from it) to holding a nonzero value (the particles that couple to it have masses). That this cooling through the scale of the weak force, in which the Higgs field takes its value and the joined electroweak force separates into the weak force and electromagnetism, is called the electroweak transition.

Sources: *The Coming of the Kinds*, Chapter 4, Section IV, and Chapter 6, Sections I to III. Standing: a definition.

E-015 First-Order Transition and Crossover. That phase transitions come in two different kinds. That a first-order transition is abrupt: there is a sharp boundary between the old state and the new, the system releases a definite amount of energy (the latent heat) at the transition, and in the early universe the transition proceeds through the forming and growth of bubbles of the new phase within the old, like bubbles of steam in boiling water; at a bubble's wall the matter is out of balance. That a crossover is smooth: there is no sharp boundary, no latent heat, no bubbles and no walls; the system changes steadily from the old state to the new, like honey thickening as it cools, with no temperature at which the system jumps, and at every moment the system is settled in its state. That the distinction matters for cosmology: the expanding and colliding bubbles of a first-order transition drive matter out of balance and are a source of gravitational waves, and a crossover does neither.

Sources: *The Coming of the Kinds*, Chapter 4, Section IV. Standing: a definition.

E-016 When the Electroweak Transition Occurred. That the center of the electroweak crossover lies at a temperature of 159.5 ± 1.5 GeV (D'Onofrio and Rummukainen, 2016), at an age of the universe of about 9.1 × 10^{-12} seconds, counted in the history reconstructed from inside. That above it the Higgs field holds no settled value: the field is present but its value is zero, the two sides of each doublet are still distinct but the vacuum does not mark which is which, the four carriers of the joined force (three weak bosons and the hypercharge boson) are massless, and the quarks and leptons have no mass from the field. That through and below the center of the crossover the field's value grows smoothly toward its present value, v = 246.21967 GeV. As the field's value grows, the masses of the W and Z bosons, the quarks, the charged leptons and, through a far weaker coupling, the neutrinos switch on in proportion to their couplings, while the photon and the gluons stay without mass. That as the temperature falls through the tens of GeV, the four heaviest kinds (the top quark, the Higgs boson, the Z and the W) fall away in that order, and the count of states in balance falls from 106.75 to 86.25.

Sources: *The Coming of the Kinds*, Chapter 6, Sections II to V (Table 6.1, Box 6.2); *The Story of the Particles*, Level 8, Section III; D'Onofrio and Rummukainen (2016). Standing: derived (computations of standard physics from measured inputs: the temperature on the lattice at the measured Higgs mass, the age from the expansion rate of a universe filled with radiation, and the count of states from the list of kinds).

E-017 The Finite-Temperature Effective Potential. That at high temperatures the potential energy surface of the Higgs field is modified by the thermal motion of the particles in the hot plasma. That the modified potential, called the finite-temperature effective potential, includes the zero-temperature potential with its quantum corrections, the thermal correction from the bosons and fermions in the plasma, and a further correction called ring resummation that accounts for the screening of long-wavelength bosonic modes by the hot plasma. That at very high temperatures the thermal correction restores the symmetry (Kirzhnits and Linde,

1972): the field returns to zero. That outside the narrow crossover region the perturbative results agree well with the non-perturbative ones (D'Onofrio and Rummukainen, 2016); within it, the character of the transition is decided by lattice computation, the solution of the equations of the full theory on a grid of points by computer (E-021, E-022).

Sources: *The Coming of the Kinds*, Chapter 4, Section IV, Chapter 6, Section II, and Note 1 to Chapter 6; Kirzhnits and Linde (1972); D'Onofrio and Rummukainen (2016). Standing: a definition.

> ***The effective potential at high temperature.***
>
> In the high-temperature limit, the perturbative effective potential is commonly written as a polynomial in the Higgs field φ:
>
> $V(\varphi, T) \approx D(T^2 - T_0^2)\,\varphi^2 - E\,T\,\varphi^3 + (\lambda_T / 4)\,\varphi^4$
>
> where D sets the thermal mass, T_0 is the temperature at which the curvature at $\varphi = 0$ changes sign, E is the cubic coefficient, and λ_T is an effective quartic coupling. The coefficient D controls how quickly the thermal correction restores the symmetry at high temperature. The cubic term is the term that can make a barrier between the symmetric phase ($\varphi = 0$) and the broken phase in this perturbative writing; whether the full theory has such a barrier is decided on the lattice.

E-018 The Cubic Coefficient. That the cubic term in the high-temperature effective potential, the term proportional to the temperature times the cube of the Higgs field, comes from the bosons of the plasma. In the one-loop thermal correction each boson whose mass m depends on the field adds a term proportional to $-m^3T$, and the fermions add no odd power of the mass (Quirós, 1999); the weak bosons, whose masses grow in proportion to the field, give the term in φ^3. That the ring resummation changes the size of this term: it includes the screening of the long-wavelength modes of the bosons by the plasma, whose infrared contributions are of the same order as the one-loop cubic term (Carrington, 1992). That in perturbation theory this term makes a barrier between the symmetric phase

(where the Higgs field is zero) and the broken phase, and that whether the transition of the full theory is first-order or a crossover is decided by lattice computation (E-021).

Sources: *The Coming of the Kinds*, Chapter 4, Section IV, and Note 5 to Chapter 4; Quirós (1999); Carrington (1992). Standing: a definition (the cubic coefficient); derived (its origin in the bosons' one-loop thermal terms, standard physics).

E-021 The Lattice Endpoint. That lattice computations of the Standard Model show that the electroweak transition has bubbles and walls, that is, is first-order, only for a Higgs mass below an endpoint of 72.4 $\pm$ 1.7 GeV (Csikor, Fodor and Heitger, 1999), the Higgs mass at which the line of first-order transitions ends. Kajantie, Laine, Rummukainen and Shaposhnikov (1996) found that "there is no hot electroweak phase transition at large Higgs masses" and placed the endpoint between 70 and 95 GeV. Above the endpoint the transition is a crossover.

Sources: *The Coming of the Kinds*, Chapter 4, Section IV (Box 4.3), and Note 5 to Chapter 4; *The Spectral Chain*, Chapter 22, Section VIII; Csikor, Fodor and Heitger (1999); Kajantie, Laine, Rummukainen and Shaposhnikov (1996). Standing: derived (lattice computations of the Standard Model).

E-022 The Electroweak Transition Is a Crossover. That with the measured Higgs mass, 125.13 $\pm$ 0.11 GeV (Particle Data Group 2026), far above the endpoint, the electroweak transition is a crossover: a smooth change with no jump, no bubbles and no walls. D'Onofrio and Rummukainen (2016) locate its center from the maximum of the susceptibility of the Higgs condensate (the size of the fluctuations of the field's value, largest where the value changes fastest with the temperature), T_c = 159.5 $\pm$ 1.5 GeV, and report that "While the cross-over is smooth, it is very well defined with a width of only approximately 5 GeV". In *The Coming of the Kinds* the crossover is the field's whole smooth settling toward its present value, steepest within a few GeV of its center and continuing below it. That the character of the transition follows from the measured Higgs mass

by computation; the Higgs mass with its loop term in this account, 125.17 GeV (E-008), lies on the same side of the endpoint, so this account and the Standard Model give the transition one character.

Sources: *The Coming of the Kinds*, Chapter 4, Section IV (Box 4.3), Chapter 6, Section III, and Note 1 to Chapter 6; *The Measures of the Kinds*, Chapter 13, Section VI; D'Onofrio and Rummukainen (2016). Standing: derived (a lattice computation of the Standard Model at the measured Higgs mass); a measured input (the Higgs mass).

E-025 No Gravitational-Wave Signal from the Electroweak Transition. That a crossover has no bubbles and no walls, so the electroweak transition is not a source of the gravitational waves that the expanding and colliding bubbles of a first-order transition would produce. That the list of kinds is closed, with one Higgs doublet as the only scalar (*The Concept Sequence*, Chapter 41, entries 41.10 and 41.16), so no further field can make the transition first-order. A detection, by a space-based observatory such as LISA (the Laser Interferometer Space Antenna, which observes gravitational waves at millihertz frequencies), of a stochastic background of gravitational waves from a first-order electroweak transition would need a field beyond that closed list. That a null result at those frequencies agrees with this account and with the Standard Model alike.

Sources: *The Coming of the Kinds*, Chapter 4, Section IV; *The Branches of the Kinds*, Chapter 4, Section II; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Sections IV and VI (Propositions 19.7 and 19.10). Standing: derived (from the crossover and the completeness of the kinds).

E-031 The Sphalerons Leave Balance During the Crossover. That a sphaleron passage is a passage of the weak field from one of its equal lowest states to the next; it is the one process among the kinds that changes the baryon number, by three units together with three units of lepton number, keeping $B - L$, the baryon number less the lepton number. That the passages run faster than the universe expands from about 10^{12} GeV down through the steepest part of the crossover. That they fall behind the expansion gradually at 131.7 ± 2.3 GeV (D'Onofrio, Rummukainen and

Tranberg, 2014), at about 1.3×10^{-11} seconds, 27.8 GeV below the crossover's center, while the field is still growing toward its present value. That no bubble wall drives the gathering (the hot matter of the early universe) from balance while the sphalerons still run, so the third of Sakharov's conditions for making an excess of matter, a departure from balance in the processes that change the baryon number while they run, is not met at the transition, and electroweak baryogenesis is excluded in the Standard Model with the measured Higgs mass. The baryon number is fixed from then on at the share of $B - L$ that balance gave it at 131.7 GeV (Chapter H, entry H-019, which gives that frozen share as about one third of $B - L$).

Sources: *The Coming of the Kinds*, Chapter 4, Sections II and IV (Boxes 4.1 and 4.3), and Chapter 6, Section IV (Table 6.1); *The Branches of the Kinds*, Chapter 2, Section II; D'Onofrio, Rummukainen and Tranberg (2014). Standing: derived (from a lattice computation of the Standard Model at the measured Higgs mass and the closed list of kinds).

E-IV. The Strong Transition

E-026 Confinement Defined. That the property called confinement is the fact that quarks cannot appear alone: they are always bound inside composite particles called hadrons (such as protons and neutrons). That the structure gives the ground of confinement: each quark carries one of three colors, which are labels for the three positions within the content; a comparison requires all three positions; so only combinations of quarks that form a complete comparison, with no net color, can be observed: three quarks, one of each color (a baryon), or a quark with an antiquark (a meson). That at temperatures above about 156 MeV the quarks and gluons are crowded so densely that they move among one another nearly as free particles, in a quark-gluon plasma, and that as the gathering cools

and thins they are bound in hadrons; the change from the one state to the other in the early universe is the strong transition, the transition of quantum chromodynamics (the quantum theory of color).

Sources: *The Story of the Particles*, Level 6, Section III; *The Spectral Chain*, Chapter 22, Sections I and II; *The Coming of the Kinds*, Chapter 8, Section III. Standing: derived, given a stated identification (the ground of confinement, from Axiom III, the identification being the three colors as the three positions of comparison held inside the content); derived (the temperature, a lattice computation, E-028).

E-027 Center Symmetry and the Polyakov Loop. That the symmetry group SU(3) of the strong interaction has a center, the subgroup of three elements that commute with everything in the group, written $\mathbb{Z}_3$, which acts on states of charge k by ω^k with ω = e^(2πi/3). That the center is related to confinement through a quantity unchanged by gauge transformations, called the Polyakov loop, whose thermal average measures the free energy of an isolated static quark. That in the theory of gluons alone (with no quarks), the Polyakov loop has a thermal average of zero in the confined phase (an isolated quark has unbounded free energy) and a nonzero thermal average in the deconfined phase (an isolated quark has finite free energy), where the center symmetry is spontaneously broken. That on the content instance of the internal space the action of the center $\mathbb{Z}_3$ is a statement about which labels the space of states carries, and the confinement criterion (the Polyakov loop) is decided by four-dimensional gauge dynamics.

Sources: *The Kinds of Particle*, Chapter 4, Section VI; *The Spectral Chain*, Chapter 22, Sections III and IV; *The Matter We Know*, Volume One, Chapter 2, Section IV. Standing: a definition.

> ***The Polyakov loop as a thermometer for confinement.***
>
> Imagine trying to place a single static quark in a box filled with hot gluons. The Polyakov loop measures the energy cost of doing so. If the cost is unbounded, the quark cannot exist alone: confinement. If the cost is finite, the quark can exist alone: deconfinement. In the theory of gluons

alone, the thermal average of the Polyakov loop $\langle \ell \rangle$ is therefore an order parameter for the transition, a quantity that is zero in one phase and nonzero in the other phase: $\langle \ell \rangle = 0$ in the confined phase, and $\langle \ell \rangle \neq 0$ in the deconfined phase. Quarks of finite mass break the center symmetry explicitly, and with them $\langle \ell \rangle$ is not a strict order parameter.

E-028 The Strong Transition at the Physical Point. That the nature of the strong transition, first-order, second-order or a crossover, depends on the masses of the quarks. That the dependence is summarized in a diagram called the Columbia plot, which maps the order of the transition as a function of the light quark masses and the strange quark mass. That in the theory of gluons alone (all quark masses infinite) the transition is first-order. That at the physical quark masses, with the up quark at about 2 MeV and the down quark at about 5 MeV, the transition is a crossover, a smooth change with no jump, like the settling of the Higgs field. The HotQCD Collaboration (2019) reports, for equal numbers of quarks and antiquarks, T_c(0) = 156.5 ± 1.5 MeV, and the gathering of the early universe, whose quarks exceeded its antiquarks by about four parts in a billion, passed it between about 1.2 and 2.4 × 10^{-5} seconds. That the proton gains most of its mass at this crossover: below it the quarks are bound in threes, and each whole of three carries the energy of the color field that binds it, about ninety-nine percent of the proton's 938 MeV.

Sources: *The Coming of the Kinds*, Chapter 8, Sections III and IV (Box 8.1), and Note 1 to Chapter 8; *The Spectral Chain*, Chapter 22, Sections IV to VI; *The Strengths of the Kinds*, Chapter 6, Section I; HotQCD Collaboration (2019). Standing: derived (lattice computations of quantum chromodynamics at the physical quark masses, and the age from the expansion rate).

The Columbia plot.

The Columbia plot has the light quark mass m_l on one axis and the strange quark mass m_s on the other axis. In the upper-right corner (all masses large, approaching the theory of gluons alone) the transition is first-order. At the lower-left corner, where the quark masses go to zero (the chiral limit), the order of the transition is set by the chiral symmetry

> of the massless quarks. The physical quark masses place the universe in the interior of the plot, within the region of crossover, and the nature of the change at the physical point does not depend on the order at that corner.

E-029 What the Structure Gives the Strong Transition. That the structure grounds confinement in the comparison of the three colors (E-026) and carries the strength of color as a measured input; the binding scale of the proton follows from color's strength by its running (Chapter C, entry C-033, which gives the proton's mass as a fixed multiple of the scale at which color's strength reaches order one). That the masses of the light quarks are values on the mass ladder of *The Measures of the Kinds*, standing as measured agreements. That the character of the strong transition at the physical point, a crossover at 156.5 ± 1.5 MeV, is a computation of quantum chromodynamics on the lattice from these measured inputs, and the internal space adds no term to that computation: its content instance has no size, no fields and no modes.

Sources: *The Story of the Particles*, Level 6, Section III; *The Spectral Chain*, Chapter 22, Section VII; *The Strengths of the Kinds*, Chapter 1, Section VII, and Chapter 6, Sections I and II; *The Measures of the Kinds*, Chapter 9, Section VII, and Chapter 13, Section X; *The Coming of the Kinds*, Chapter 8, Section III. Standing: a measured input (the strength of color); a measured agreement (the masses of the light quarks); derived (the crossover, a lattice computation of quantum chromodynamics from these inputs, with no term from the internal space).

E-V. The Vacuum

E-030 What Is Stated of the Vacuum. That three statements are made of the vacuum. First, the Higgs potential is bounded below, by the rule that every way of varying has a least state, so that energy has a floor. Second, there is one Higgs doublet and it is the only scalar, so the potential is the potential of one doublet, with no second scalar direction in

which a further minimum could lie. Third, the internal space $\mathbb{C}P^2 \times S^2$ is unique, reached by two routes. One route reaches the internal space as the product of the state spaces of the two non-commutative summands of the one algebra. The other route reaches the internal space as the only product of compact homogeneous spaces that meets the stated conditions on the symmetries and the Euler number of that product. So there is one internal space, $\mathbb{C}P^2 \times S^2$, with no landscape of alternatives from which a vacuum would be selected. That the weak field has a row of equal lowest states, between which the sphaleron passages run (E-031); in the cold universe of today the field reaches the next of these only by tunneling through the ridge between them, at a rate that carries a factor of about 10^{-161}, so the baryon number does not change today.

Sources: *The Measures of the Kinds*, Chapter 13, Section VI; *The Branches of the Kinds*, Chapter 2, Section III (Box 2.4), Chapter 3, Section III (Box 3.4), and Chapter 4, Section II; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section VI (Proposition 19.10); *The Kinds of Particle*, Chapter 3, Sections I and II; *The Coming of the Kinds*, Chapter 4, Section II (Box 4.1), and Chapter 6, Section IV. Standing: derived.

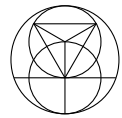
Summary

This chapter is the branch of the map that carries the Higgs field, its potential and the two transitions through which the early universe cooled. Its first part derives the field from the structure. The two sides of every doublet are distinct and inseparable, the weak force crosses the edge between them, and a field lives on the one edge that a transformation crosses: *The Story of the Particles* and *The Branches of the Kinds* derive that field, a colorless doublet of hypercharge one half and spin zero, and *Non-Commutative Geometry and the Immanent Metaphysics* finds the same field as the fluctuation of the finite Dirac operator across the one edge it crosses. The spectral action gives the potential its form, a negative quadratic term

and a positive quartic term in the one doublet, and with that form the condensate settles away from zero and leaves the photon and the gluons without mass.

The second part sets out what the structure fixes and what measurement supplies, as *The Measures of the Kinds* gives them. The vacuum value is a measured input of the working account, reached in the closure account from the electron alone, given the stated identification of the product step. Every coupling to the condensate is a ratio of a part to its whole, so no particle of matter is heavier than $v/\sqrt{2}$, and the top quark's coupling of one is a value the structure sets. The self-coupling at tree level is $1/8$, derived, given the stated identification $\lambda = a^2/2$ with $a = 1/2$ the doublet's half, and it gives the tree mass $v/2$. The measured excess of the self-coupling over $1/8$ agrees with $1/(24\pi^2)$, the doublet's half times the photon's one-loop unit, a count of its own and not the top quark's loop, and with it the Higgs mass is 125.17 GeV, a measured agreement that a Higgs mass measured to ± 0.03 GeV separates from the midpoint relation.

The third and fourth parts follow the early universe through the scale of the weak force and through the binding of color, as *The Coming of the Kinds* and *The Spectral Chain* report them from lattice computation. With the measured Higgs mass the electroweak transition is a smooth crossover centered at 159.5 GeV, and the sphalerons stay in balance through its steepest part and leave balance at 131.7 GeV, so the baryon number is set at its balance share and no excess is made at the transition. The strong transition at the physical quark masses is a crossover at 156.5 MeV, at which the proton gains nearly all of its mass. The structure supplies the kinematics of both: the one scalar of the closed list, and the color singlets as the only states found. The internal space, a space of states with no size on its content instance, adds no term to the computation of either transition, the electroweak or the strong. The last part states what is said of the vacuum: a potential bounded below, one doublet, one internal space, and a weak field whose passages between its lowest states are effectively absent today.



Chapter F

Dark Matter

CHAPTER OVERVIEW

This chapter traces the account of the dark matter that *The Weight of Possibility* gives, with its companion *The Weight of the Unseen*, on the context that *The Gravitational Passage* fixes. On that account the dark matter is a conserved excitation of the metric of the six held internal dimensions, the dimensions of possibility, made at the turn from one epoch of the universe to the next: a vibration of context, felt only by its weight, which is no quantum of the arena and no quantum of gravitation. In this chapter I set out what is observed, and why a dark component is required on that account. I then set out how the ratio of the dark component to the ordinary matter is fixed, why the dark component persists, and what mass the gyrations of the dark component have. Last, I set out what the measurements say, and why a dark fire, a chain that turns ordinary matter into dark matter, cannot happen within an epoch. Each prediction is stated with the measurement that decides it.

The characterization of the dark matter rests on the hold that *The Gravitational Passage* secures, "the internal space has the same size and the same shape at every point of the exterior", and that hold rests on the modal boundary, the Planck scale (entry F-004). In this chapter I rely on these chapters of *The Concept Sequence*: Chapter 16, The Two Laws and the Content/Context Partition (the grid of the six measures); Chapter 18, The Three Forces, and Chapter 39, Gravity Is the Arena: No Graviton (gravity is the arena, not a fourth force); Chapter 25, The Internal Space in Its Two

Instances; Chapter 27, The Context Instance: Radii, Volume and the Count 72; Chapter 28, The Two Measured Anchors and the Span to the Planck Mass; Chapter 41, The Completeness of the Kinds; and Chapter 45, Further Predictions, whose entry 45.12 states the dark matter in brief. Chapter G traces the identity for the cosmological constant and the matter fraction 0.311 that this chapter uses, and Chapter I collects the whole budget of the universe.

F-I. What Is Observed, and What the Dark Matter Is

F-001 What Is Observed. That the universe contains a component of matter that has never been seen to do anything except gravitate. It was first inferred from the speeds of the galaxies in the Coma cluster, too fast for the visible galaxies to hold together (Zwicky 1933). The outer parts of the Andromeda galaxy turn about as fast as its inner parts, so that the galaxy's mass keeps growing far from its center (Rubin and Ford 1970). The bending of light weighs the component without any assumption about how the matter moves. When two clusters of galaxies collide, their gas is slowed and heated, and the component passes through without measurable collision. In the colliding cluster 1E 0657-56 (the Bullet Cluster), the mass mapped by gravitational lensing lies with the galaxies, apart from the hot gas that holds most of the ordinary matter (Clowe and others 2006), and the same collision bounds the scattering of the component on itself below about one square centimeter per gram (Markevitch and others 2004). The amount of the component has not changed since the microwave background (the light left from the early universe, which reaches us from every direction) was released: a decay into anything unseen would have changed the expansion since then, and the measurements bound such a decay to lifetimes beyond about 160 billion years (Poulin, Serpico and Lesgourgues 2016). By mass the component is about 5.4 times the ordinary matter, and that ratio is the same everywhere the microwave background can measure it. The early universe departed from

uniformity in two possible kinds: adiabatic departures, in which every component departs together, and isocurvature departures, in which one component departs against another. In models that mix the two kinds, a separate pattern of departures in the cold dark matter (an isocurvature mode) carries less than 1.3 percent of the observed variance of the microwave background's temperature, at 95 percent confidence (Planck Collaboration 2020b). The facts to be accounted for are therefore three: a component that persists, that acts only through gravitation, and that stands to the ordinary matter in a fixed ratio.

Sources: *The Weight of Possibility*, Chapter 1, Section I; *The Weight of the Unseen*, Chapter 1, questions 1.1 to 1.4. Standing: a measured input (the observations the account takes as the facts it has to explain).

F-002 No Dark-Matter Species among the Kinds. That the list of the kinds of particle is closed (*The Concept Sequence*, Chapter 41, The Completeness of the Kinds, entries 41.10 to 41.19). A kind of particle is told from every other kind by its labels, its generation and its field kind, so a new kind would need a new label (a fourth summand of the algebra), an old label held toward the wrong term of a state, a fourth generation, or a fourth field kind, and the structure of comparison has none of these. What the programs that posit a dark-matter particle ask of it is mass with no coupling to light and no coupling to the other two forces: a spinor with no labels but the phase, at zero. That kind is occupied, once in each generation, by the right-handed component of the neutrino, and that component plays no part in what is attributed to the dark matter. So no dark-matter species exists among the kinds of content: no weakly interacting massive particle, no axion, no sterile neutrino beyond the right-handed component of each neutrino, and no other proposed species. On the finite spectral triple (the algebraic description of the internal structure of the particles in non-commutative geometry) the same closure is proved in *Non-Commutative Geometry and the Immanent Metaphysics*: no field beyond the spinor fields, the gauge fields and the Higgs doublet (Proposition 19.7), no supersymmetry (Proposition 19.8), the right-handed neutrino as the only

total singlet (Proposition 19.9), and the Higgs field as the only scalar (Proposition 19.10). The gravitational effects attributed to the dark matter are real, and they belong to the context. A gyration (entry F-006) is a vibration of the six held internal dimensions, a vibration of context, which the list of the kinds of content does not count; nothing is a quantum of the arena itself, at any scale.

Sources: *The Story of the Particles*, Level 9, Section IV; *Non-Commutative Geometry and the Immanent Metaphysics*, Propositions 19.7 to 19.10; *The Coming of the Kinds*, Chapter 1, Section V; *The Weight of Possibility*, Chapter 1, Questions; *The Concept Sequence*, entry 41.18. Standing: derived; a theorem (*Non-Commutative Geometry and the Immanent Metaphysics*, Propositions 19.7 to 19.10).

F-003 The Turn and the Epoch Defined. That the turn is the transition from one epoch of the universe to the next, where one epoch ends inside its black holes and the next begins, and that an epoch is the span of the universe's history between two turns. *The Gravitational Passage* holds that the collapse inside the black holes of one epoch and the origin of the next "are one transition in a cycle, approached as a limit from its two sides", and that no comparison registers the turn between the two sides as an event (*The Gravitational Passage*, Chapter 18, opening). In the words of *The Weight of the Unseen*, "The turn is the limit of the time of every history: each history of an epoch runs back toward it and does not pass it" (Chapter 2, question 2.5). A component that gives energy to nothing after it is made, and takes energy from nothing, can have exchanged energy with the rest of the world only where conditions were extreme enough for its kind of exchange to occur. At the turn, and only there, the six internal dimensions and the three open dimensions of space re-form together. *The Weight of Possibility* takes the turn to be the only place where the dark component is made.

Sources: *The Gravitational Passage*, Chapter 18, opening, and Sections VII and VIII; *The Weight of the Unseen*, Chapter 2, question 2.5; *The Weight of Possibility*, Chapter 1, Section II, and Glossary ("epoch", "the turn"). Standing: a definition.

F-004 The Six Dimensions of Possibility, and the Hold. That the ten dimensions of context are one of time, three of space and six of possibility, and that the six of possibility are the context instance of $\mathbb{C}P^2 \times S^2$, the complex projective plane times the two-sphere. $\mathbb{C}P^2$ is held at a radius of $\sqrt{6}$ Planck lengths and S^2 at a radius of one Planck length, where the Planck length, about 1.6×10^{-35} meters, is the length formed from Newton's constant, the quantum of action and the speed of light (*The Gravitational Passage*, Chapter 9, Section IX). The Passage identifies the internal space of its reduction with the space of possibility: "$\mathbb{C}P^2$ is the space of possibilities of a system with three alternatives, and S^2, which is $\mathbb{C}P^1$, is that of a system with two. The internal space is a space of possibility in the plain mathematical sense", and "I identify the two sixes as one set" (*The Gravitational Passage*, Appendix C, Section VIII). The internal space cannot shrink to nothing, since it is the context of possibility and Axiom III excludes content without context. A size held at zero with no spread is excluded by the uncertainty relation, so the internal space has a floor: a least extent, with a least motion that cannot be removed. The internal space is held. Each of its six directions stands at the modal boundary, the Planck scale, where the smooth description of space and time and the stepwise description of content each reach the edge of its domain, and where the curvature each direction sees is the Planck curvature. That boundary is fixed by the measured Newton's constant, which has one value in this domain, so "the internal space has the same size and the same shape at every point of the exterior" (*The Gravitational Passage*, Chapter 11, Section II). The size and the shape of the internal space (its moduli) are therefore fixed at these radii throughout the exterior. The floor is one mechanism of the hold: at first order it holds the internal space at the derived radii as a stable minimum, and the moduli then have masses of 2.25 and 3.09 Planck masses, so the force they carry has a range shorter than the Planck length (*The Gravitational Passage*, Chapter 11, Section II, and Appendix D). No comparison traverses the six dimensions of possibility as it traverses space, and so they present to a located comparer as compact. The context instance is dis-

tinct from the content instance of the same schema, which is a space of states with no size and no radius, on which the kinds and charges of the content are read and no field lives (*The Concept Sequence*, Chapter 25, The Internal Space in Its Two Instances, entries 25.11 to 25.13).

Sources: *The Gravitational Passage*, Chapter 9, Section IX; Chapter 11, Section II; Chapter 17, Sections II and V; Appendix C, Sections V to VIII; Appendix D; *The Weight of Possibility*, Chapter 1, Section III; *The Concept Sequence*, Chapter 25 and Chapter 27, entry 27.10. Standing: an identification (the two sixes as one set, *The Gravitational Passage*, Appendix C, Section VIII); derived, given a stated identification (the radii, from the identification that every internal direction reaches the Planck curvature, *The Gravitational Passage*, Chapter 9, Section IX); derived (the hold, from the modal boundary and the measured Newton's constant, *The Gravitational Passage*, Chapter 11, Section II).

F-005 The Identification. That the dark matter is a conserved excitation of the six dimensions of possibility, made at the turn, with no exit afterward. An excitation of the internal space is a distortion of a context, not an item of content. The dark matter is made of gyrations, and a gyration is a vibration of the internal geometry, as a wave of sound in a crystal is a vibration of its lattice: in its role it is a vibration of context. The comparison with a lattice holds because the internal space is held: it has the same size and the same shape at every point of the exterior (entry F-004), as a crystal keeps its lattice, and so each of its modes of vibration has a definite energy. The open dimensions, the arena, are not held in that way: gravitation, the distortion of space, is not quantized, and nothing is a quantum of the arena itself, at any scale. The excitation has energy, so the excitation gravitates; the excitation is not content, so nothing that registers content registers the excitation. Those two facts give the observed signature of the dark matter: gravitation, and nothing else.

Sources: *The Weight of Possibility*, A Note to the Reader; Chapter 1, Section IV; Note 5; *The Weight of the Unseen*, Chapter 3, question 3.1; *The Concept Sequence*, Chapter 39. Standing: an identification (the identification of *The Weight of Possibility*, Chapter 1, Section IV, argued there).

> ***A vibration of the setting, not a thing in it.***
>
> A wave of sound in a crystal is a vibration of its lattice: a vibration of the crystal itself, not an extra object placed inside the crystal. On this account the dark matter is made of gyrations, and a gyration is a vibration of the internal geometry in the same way. The internal space is held at one size, as a crystal holds its shape, and that is why each of its modes of vibration has a definite energy, as a crystal's modes do. Ordinary space is not held at one size as the internal space is, and gravitation, the bending of ordinary space, does not come in units. A vibration carries energy, and all energy pulls, so the dark matter has weight. Light, detectors and every instrument register content, and the dark matter is a vibration of a context, so no instrument registers it except through its weight.

F-006 What a Gyration Is. That a gyration is a vibration of context and not an item of content, so it is not a particle: in *The Weight of Possibility* "a particle is an item of content", and on the identification of entry F-005 the dark matter is not content. Physically a gyration is a massive excitation of the metric of the six held internal dimensions, a state of the Kaluza and Klein kind: a massive vibration of the metric of compact extra dimensions, of the kind that theories in the line of Kaluza and Klein study (Salam and Strathdee 1982; Hinterbichler, Levin and Zukowski 2014). A gyration is a vibration of a held structure, as the vibrations of a crystal are; it is no quantum of the arena and no quantum of gravitation. My claim concerns the role of a gyration, and in that role a gyration is a vibration of context. Physically a gyration is the state of the Kaluza and Klein kind just named, which theories of that kind call a particle. These states belong to the context instance, at about the Planck mass (1.22×10^{19} GeV, the mass formed from the same three constants as the Planck length); the content instance, which has no size, carries no tower of such states (no ladder of heavier vibrations).

Sources: *The Weight of Possibility*, Chapter 1, Questions; Chapter 4, Section II; Note 2; *The Weight of the Unseen*, Chapter 3, questions 3.1 and 3.2; *The Concept Sequence*, Chapter 25, entries 25.11 to 25.13; *The Grand Synthesis*, Chapter 13,

Section III, which carries the same statement of the tower. Standing: a definition (of the gyration, and of the sense of the word particle in which a gyration is not one).

F-007 Why Context Must Distort. That the Incommensuration Theorem applies to each of the three columns of context: time, space and possibility. The theorem states that "Any comparison, perfected at its foundation, must be either continuously asymmetric or discontinuously symmetric", and it "applies to any domain in which comparison occurs" (*The Gravitational Passage*, Chapter 3, Section IV); it is a structural fact about the nature of knowing. Content is discontinuously symmetric, and context, which carries the continuity, is continuously asymmetric. Each column of context is a domain of comparison in its own right, so no column of context can be perfectly uniform: a perfectly uniform context would perfect continuity and symmetry together, which the theorem forbids. An asymmetry of context is a difference of content across a difference of context, and in a geometry it is a departure from uniformity, which *The Weight of Possibility* calls a distortion. The three axioms of the modalities (the three roles, immanent, omniscient and transcendent, that the columns of the grid of measures carry; entry F-008) then place the asymmetry in every column, each by a distortion of its own. The three contexts are inseparable, so none can be undistorted while another is distorted; they are non-interchangeable, so no column's distortion can stand in for another's; and they are distinct, so the distortions are three. Each column therefore carries a distortion that is not zero: in space, gravitation; in time, the cosmological constant; in possibility, the dark matter. On this account the dark matter is required: a world with a continuous context of possibility carries it, as it carries gravitation and a cosmological constant. How much of it there is relative to the ordinary matter is the registered ratio, $6 - 2/\pi$ (entry F-019).

Sources: *The Weight of Possibility*, Chapter 2, Section I; Appendix A, Section IX; *The Gravitational Passage*, Chapter 2, Section II, and Chapter 3, Section IV. Standing: derived, given a stated identification (the distortion of the context of possibility is the dark matter).

F-008 The Three Distortions and the Three Edges. That in the grid of the six measures each of the three distortions of context binds to its own column and meets the edge of reality that bounds that column's context. The grid has two rows and three columns: pattern, force and probability in the row of content; space, time and possibility in the row of context; pattern and space omniscient, force and time immanent, probability and possibility transcendent (*The Gravitational Passage*, Chapter 7, Section III; *The Concept Sequence*, Chapter 16, entries 16.12 to 16.16). A content is what a measurement registers, and a context is where the measurement registers it. The three edges are the three limits of what actual comparison made from inside the world can reach: the cosmological horizon, which bounds comparison in extent; the turn, which bounds it in time; and the Planck scale, which bounds it in scale. The binding is by role: the Planck scale is a length, and each of its roles in the account (the size of the six dimensions of possibility, the bound of the comparison a located comparer makes, and the limit of perfected continuity) is a role of the transcendent column.

Sources: *The Weight of Possibility*, Chapter 2, Sections II to IV; Appendix A, Sections I to V; *The Weight of the Unseen*, Chapter 2, questions 2.5 and 2.6. Standing: an identification.

> ***The binding, column by column.***
>
> Omniscient column: content pattern; context space; distortion of the context, gravitation; edge of the context, the horizon.
>
> Immanent column: content force; context time; distortion of the context, the cosmological constant; edge of the context, the turn.
>
> Transcendent column: content probability; context possibility; distortion of the context, the dark matter; edge of the context, the Planck scale.
>
> The pairing is tightest in the transcendent column: the dark matter is a distortion held exactly at its column's edge, the Planck scale. In the immanent column the cosmological constant is fixed through the time

> elapsed since the turn (Chapter G, entries G-017 and G-018, which trace the Passage's identity and the matter fraction it gives). In the omniscient column gravitation acts within the horizon.

F-009 The Three Axioms Applied to the Three Distortions. That the three distortions satisfy the three axioms as a triple: three terms that stand to one another as the three modalities stand. By Axiom III they are distinct: the cosmological constant is uniform and never clumps; gravitation is local and follows content; the dark matter clumps and never dissipates or collides. They are inseparable: in a flat epoch the shares of the cosmological constant, of the matter and of the radiation sum to one in the one equation of the expansion, and the dark matter, acting only through gravitation, cannot be registered apart from it. The three distortions are non-interchangeable: no amount of the cosmological constant makes a halo, and no amount of dark matter accelerates the expansion. Gravitation is the response of the arena to content, and not a component of the budget at all. By Axiom II, bound to the edges, the three distortions run in the order of the cycle of Axiom II, transcendent before immanent, immanent before omniscient, and omniscient before transcendent. The internal space is released from the Planck scale (transcendent) and the turn follows (immanent); the cosmological constant, fixed by the time since the turn, fixes the radius of the horizon of the emptied epoch (omniscient); within the horizon, gravitation clumps matter into black holes, whose interiors reach the Planck scale again. The reverse order (immanent before transcendent, omniscient before immanent, and transcendent before omniscient) matches no step of the account of The Weight of Possibility. By Axiom I the cosmological constant is the clear immanent of the three (the term that plays the immanent role most plainly), since it is the share of the vacuum's energy accessible to an observer inside the vacuum (Chapter G, entry G-012, the accessible share). Gravitation and the dark matter are then the conjugate pair, the omniscient and the transcendent terms that Axiom I sets in conjugate relation: gravitation binds to the horizon, the limit of perfected symmetry, and the dark matter to the Planck scale, the limit of perfected

continuity. The two meet in one way only, the dark matter registering as a distortion of space by gravitating, which is the one way the dark matter is observed.

Sources: *The Weight of Possibility*, Chapter 2, Sections V and VI; Appendix A, Sections VI to VIII and X; *The Weight of the Unseen*, Chapter 2, questions 2.7 and 2.8. Standing: an identification.

F-010 The Budget Read by the Grid. That the Friedmann equation of a flat epoch (the equation of the expansion of a uniform universe) divides the energy into shares, and each share has a cell of the grid of measures. In the row of content, what the turn emits: pattern in space is the ordinary matter; force in time is the radiation; probability in possibility is the form of the primordial departures from uniformity. In the row of context, what is distorted: space by gravitation, whose large-scale term, the spatial curvature, is zero in a flat epoch; time by the cosmological constant; possibility by the dark matter. The new epoch begins spatially flat (*The Gravitational Passage*, Chapter 18, Section IX), and the measured spatial curvature is consistent with zero (Planck Collaboration 2020a). So the distortion of space is all local (gravitation is structure and has no share of its own in the budget). The distortion of time is all global (the cosmological constant is uniform). The distortion of possibility is both global and local: global in its amount, which the registered ratio fixes, and local in its halos. The dark matter carries the primordial departures first: it does not oscillate with the radiation and the ordinary matter, so the departures grow first in the dark component, and structure forms in its wells.

Sources: *The Weight of Possibility*, Chapter 2, Section VII; *The Gravitational Passage*, Chapter 18, Sections IX and XI. Standing: an identification.

F-II. The Registered Ratio

F-011 Two Residues. That the turn sends nearly all of its energy into radiation, and two residues remain: an excess of ordinary matter over its antimatter, and the dark matter. Both are conserved in number, and since the

helium of the first minutes formed both have diluted as matter, so their ratio has not changed since then. Before then the binding of helium lowered the mass of the ordinary matter by about 0.18 percent; the ratio of this chapter concerns the two densities as they stand after helium formed, which are what the microwave background measures. The neutrinos are content and a residue of neither kind: they left the turn as radiation, and their density relative to that of the ordinary matter kept changing until they slowed, long after the ordinary matter was cold.

Sources: *The Weight of Possibility*, Chapter 3, Sections I and V. Standing: a definition.

F-012 A Ratio Fixed at Registration. That the ratio of the dark matter to the ordinary matter is fixed at registration, as an identity of the present measurement. Registration is the act by which a measurement made inside the world fixes what it registers. *The Gravitational Passage* fixes the cosmological constant in the same way: "So Λ is fixed once, by the present measurement, as the constant term of one geometry, and the reconstructed history carries that value at every epoch, before the present and after it" (*The Gravitational Passage*, Chapter 13, Section VI). The ground of that step is Axiom I, which "makes the immanent act prior to the structure reconstructed from it" (the same section). The gyrations are made at the turn; their amount relative to the ordinary matter is what the identity fixes, and the reconstructed history carries that value at every time since helium formed. An identity whose terms are all fixed by structure, with no count of the particle horizon (which changes with the epoch), can govern only a quantity that is constant in time, and of the ratios of the matter budget the ratio of the dark matter to the ordinary matter is the one that has stayed constant since helium formed. The identity that fixes the ratio at registration therefore asks the turn for no dark mass per baryon, and it requires nothing of the proton's mass or of the excess of baryons. The identity fixes the ratio everywhere a registration reaches, so the account predicts that the dark component carries the same pattern of departures

from uniformity as the ordinary matter, with no separate pattern of its own; the measurements find none, to about a percent of the variance (entry F-001).

Sources: *The Weight of Possibility*, Chapter 3, Section I, and Questions; *The Gravitational Passage*, Chapter 13, Section VI. Standing: an identification (the ratio as an identity of the present measurement, on the ground of Axiom I).

F-013 The Ordinary Residue Is History. That the amount of the ordinary residue, counted as the number of baryons for every photon, is a fact of content and a number of the universe's history, and not a number fixed by the structure that fixes the kinds. The Story of the Particles states the same ruling: "The existence of antiparticles is structural: comparison requires them. The excess of matter is not structural" (*The Story of the Particles*, Level 4, Section IV). *The Coming of the Kinds* carries the excess as a value of $B - L$ (the baryon number less the lepton number), a residue the turn leaves, kept by every change after it: no process among the kinds makes it, and the kinds fix only the share of it held as baryons (*The Coming of the Kinds*, Chapter 5, Section IV, and Chapter 7, Section VI). The amount is taken as measured: $\eta = (6.13 \pm 0.04) \times 10^{-10}$ baryons for every photon, read from the Planck 2018 baryon density $\Omega_b h^2 = 0.02237 \pm 0.00015$ (the density of baryons in units of the critical density, times the square of the Hubble constant in units of 100 kilometers per second per megaparsec; Planck Collaboration 2020a) through the conversion $\eta = 274 \times 10^{-10}\ \Omega_b h^2$ that the Particle Data Group gives in its review of nucleosynthesis in the early universe (Fields, Molaro and Sarkar, in Particle Data Group 2026); *The Coming of the Kinds* sets out the step in its Chapter 1, Box 1.1. The registered ratio then fixes the amount of the dark residue relative to it.

Sources: *The Story of the Particles*, Level 4, Section IV; *The Coming of the Kinds*, Chapter 1, Section VI and Box 1.1; Chapter 5, Section IV; Chapter 7, Section VI; *The Strengths of the Kinds*, Chapter 1, Section I; *The Weight of Possibility*, Chapter 3, Section I. Standing: a measured input (a number of history, not of structure).

F-014 The Count: Six against One. That a registration counts kinds, not holders of energy, and counts what it registers by independent classes, as *The Gravitational Passage* counts its channels: "The count of channels is therefore the Euler characteristic of $\mathbb{C}P^2$, the number of its independent classes" (*The Gravitational Passage*, Chapter 13, Section II). An independent class is one member of a basis of a space's cohomology, the algebra that counts the independent cycles of a space dimension by dimension. When every class lies in even degree, as on $\mathbb{C}P^2$, S^2 and their product, the number of independent classes is the Euler number, a theorem of topology: three for $\mathbb{C}P^2$, two for S^2, six for their product, and one for a ball. The rule of counting is one statement: a registration counts every class of a factor over which the counted thing ranges, and one class of a factor at one class of which the counted thing is fixed. The channels of the registering act take every class of $\mathbb{C}P^2$ and one class of S^2, $3 \times 1 = 3$. The dark residue is an excitation of the whole internal space and ranges over both factors, $3 \times 2 = 6$; to range over a factor is to be a distortion of the whole factor, and not of one class of it. The count counts the classes of the space the residue distorts, and not the harmonics of a gyration, so it is the same whichever odd state of entry F-027 is the lightest. The ordinary residue is registered in the open space within the particle horizon, "the only horizon a first-person measurement holds" (*The Gravitational Passage*, Chapter 13, Section II); that region is a ball and has one class, so the ordinary residue counts 1, and, having no extent in possibility, it takes no class of the internal space. That a registration counts residues by classes, and that the dark residue ranges over both classes of S^2 (the row of content and the row of context), is the account's principle. Two facts support it: the dark residue is not content, so nothing fixes it to the row of content; and the antipodal map of S^2, the parity that keeps the gyrations (entry F-024), acts on the two classes of S^2 with opposite signs, $+1$ on the class of degree zero and -1 on the class of degree two.

Sources: *The Weight of Possibility*, Chapter 3, Section II; *The Gravitational Passage*, Chapter 9, Sections I and IV; Chapter 13, Section II. Standing: an identification (the account's principle that a registration counts residues by independent classes).

> ***Why no count of holders of energy gives six to one.***
> A wave of the ten-dimensional metric has 35 independent polarizations. Counted by direction, the internal dimensions and the open dimensions stand 6 to 3; counted by polarization, 33 to 2 or 21 to 2; counted by the massless modes, 12 to 1 or 11 to 1. None of these is six to one. The count of six to one is a count of classes: the three classes of $\mathbb{C}P^2$ times the two classes of S^2 for the dark residue, and the one class of a ball for the ordinary residue. On $\mathbb{C}P^2$, on S^2, on their product and on a ball, the number of independent classes over any field, the Euler number, the number of cells and (on the first three) the number of points a generic rotation fixes all agree, so the way of counting is not a free choice.

F-015 The Count of One, and the Short Loop. That the count of one for the open space holds for every registration in the universe, since the identity fixes the ratio wherever a registration is made (entry F-012). So the region within the horizon of every location is a ball, and no closed path anywhere in space that goes around the universe, and so cannot be shrunk to a point, is shorter than the diameter of the horizon. *The Weight of Possibility* calls such a path, shorter than the diameter of the horizon, a short loop. A short loop would change the count, and with it the ratio. A region that holds some number of independent classes counts that number for the ordinary residue and six times that number for the dark residue, while the abiding of $\mathbb{C}P^2$ (the reduction by the share of its curvature, $2/\pi$; entry F-016) is one term of its own, taken once, so the ratio is $6 - 2/(\pi \times \text{the number of classes})$. For two classes the ratio is $6 - 1/\pi = 5.682$, which the combined measurement of the microwave background excludes (entry F-019: 5.370 ± 0.048). Searches of the microwave background find that every closed path through our location that goes around the universe is longer than 98.5 percent of the diameter of the last

scattering surface, the sphere from which the microwave background comes (Petersen and others 2023), so the region within the horizon is shown to be a ball out to 96.5 percent of the horizon's radius.

Sources: *The Weight of Possibility*, Chapter 3, Section II; *The Weight of the Unseen*, Appendix, question A.16. Standing: derived, given a stated identification (the rule of counting of entry F-014).

F-016 The Share of a Curvature. That the open dimensions are flat, so the class registered in the open carries no curvature and stands whole, while the internal dimensions are curved, each at the Planck curvature, and a count registered there is reduced by the share of its curvature. The curvature of the internal space enters the four-dimensional equations as a bare term (*The Gravitational Passage*, Chapter 11, Section III): an energy density of magnitude $R/(16\pi)$ in Planck units, where R is the scalar curvature, 6 for the whole internal space, 4 for $\mathbb{C}P^2$ and 2 for S^2. A density ρ has, through the Friedmann equation, a curvature radius L with $1/L^2 = 8\pi\rho/3$, and a horizon of that radius has the count $S_L = \pi L^2$ in the measure the Passage uses for the particle horizon, the count $A/4$ of Bekenstein and Hawking for a sphere of area A (Bekenstein 1973; Hawking 1975). The share of a curvature is three channels among that count, c_1/S_L, with $c_1 = 3$, the count of the channels through which a measurement registers. For the whole internal space L is one Planck length, $S_L = \pi$ and the share is $3/\pi$: the $3/\pi$ of the Passage's identity $I^2\Omega_\Lambda = 8 - 3/\pi$, in which I is the distance light has traveled over the whole expansion in units of the Hubble radius and Ω_Λ is the share of the cosmological constant in the energy (Chapter G, entries G-017 and G-018). For $\mathbb{C}P^2$ alone $S_L = 3\pi/2$ and the share is $2/\pi$; for S^2 alone $S_L = 3\pi$ and the share is $1/\pi$. *The Weight of Possibility* calls the share of a factor's curvature that holds fast in a registration, as a term of its own by which it reduces a count of classes, the abiding of that factor.

Sources: *The Weight of Possibility*, Chapter 3, Section III, and Glossary ("abiding"); *The Gravitational Passage*, Chapter 11, Section III; Chapter 13, Section III. Standing: an identification (the measure of the reduction, read from the Passage's identity).

F-017 Which Curvature Abides. That a count that includes a factor's Euler number is not reduced again by that factor's curvature. By the theorem of Gauss and Bonnet, the Euler number of a surface is its whole linear curvature, $(1/4\pi)\int R\, dA$, at any radius, so the two classes of S^2 hold the whole curvature of S^2; the Euler number of $\mathbb{C}P^2$, three, comes wholly from an integral of the squared curvature and includes none of its ordinary curvature (theorems of geometry). In the count of the dark residue, 3×2, which includes the Euler number of S^2, the curvature of S^2 therefore does not abide, and the count is reduced only by the abiding of $\mathbb{C}P^2$, $2/\pi$. In the Passage's identity the count takes one class of S^2 and does not include the Euler number of S^2, so the curvature of all six internal directions abides there, $3/\pi$; exempting S^2 there too would give the matter fraction 0.300 in place of 0.311. One statement gives both numbers. That statement is a principle of bookkeeping: it states what a count already contains, and it is not a statement about the dynamics of the turn. Four further considerations agree with it. The Passage already takes $\mathbb{C}P^2$ apart from S^2 in its count of channels. The trace of a factor's own Einstein tensor is its scalar curvature times one less half its dimension, which is -4 for $\mathbb{C}P^2$ and zero for S^2, so a surface exerts no stress on what it holds; read from the accelerations of the motion instead, the abiding would be $3/\pi$ and the ratio 5.045, which the measurement excludes. S^2 is the factor that carries the two rows, content and context, between which the split of the two residues falls, and the curvature of the factor that does the dividing abides on neither side. And the abiding falls on the distortions of context and not on registered content (entry F-018).

Sources: *The Weight of Possibility*, Chapter 3, Section IV; Chapter 2, Section VIII. Standing: an identification (the account's principle of bookkeeping); a theorem (that of Gauss and Bonnet, which supports the choice of factor).

F-018 The Abiding Falls on Context. That the reduction of a count by the share of a curvature appears in exactly two places in the budget: in the cosmological constant, the distortion of time, and in the dark matter, the distortion of possibility, the two distortions of context that carry a share of their own. It does not appear in the ordinary matter, which is content. The channels through which the cosmological constant is registered are counted, and are not reduced. The cosmological constant and the dark matter are fixed by different identities and by one measure, c_1/S_L, which is why they share one coefficient (entry F-020).

Sources: *The Weight of Possibility*, Chapter 2, Section VIII; Chapter 3, Section IV. Standing: an identification.

F-019 The Registered Ratio. That the dark residue is the count of six classes less the abiding of $\mathbb{C}P^2$, and the ordinary residue is the one class of the open: $\Omega_c/\Omega_b = 6 - 2/\pi = 5.3634$. The share of the ordinary matter in the cold matter is $1/(7 - 2/\pi) = 0.15715$. With the Passage's matter fraction, 0.3108 (rounded to 0.311; Chapter G, entry G-018), and the neutrinos of the smallest masses that the oscillation experiments allow ($\Omega_\nu = 0.0014$), the cold matter is 0.3094, and the two densities are $\Omega_b = 0.0486$ and $\Omega_c = 0.2608$, each rounded from a pair whose ratio is exactly $6 - 2/\pi$; the share of the ordinary matter in all the matter is then 0.15644. Planck alone gives 5.364 ± 0.065 (Planck Collaboration 2020a); the combination of the South Pole Telescope, Planck and the Atacama Cosmology Telescope gives 5.370 ± 0.048 (Camphuis and others 2026). Both ratios are computed in *The Weight of Possibility* from the two published densities, treated as uncorrelated; a correlation of -0.4 between the densities would enlarge each uncertainty by about 15 to 18 percent, and the neighbors of the rule within its form, $6 - 1/\pi$ and $6 - 3/\pi$, would still lie more than five standard deviations from the combined measurement. Every analysis of the microwave background alone lies within 1.3 standard deviations of the prediction. The microwave background measures the two densities

from the heights of its acoustic peaks, which depend on the dark matter only through its gravitation and through its taking no part in the sound of the plasma of photons and ordinary matter.

Sources: *The Weight of Possibility*, Chapter 3, Section V; Chapter 5, Section I; *The Weight of the Unseen*, Appendix, question A.4. Standing: a measured agreement (the value $6 - 2/\pi$ follows from the count of entry F-014 and the abiding of entry F-017, which are identifications, and stands within 0.14 standard deviations of the combined measurement).

> ***The same form as the identity for the cosmological constant.***
>
> The rule has the form of the Passage's identity: a whole set by the structure, less an abiding. In the identity $I^2\Omega_\Lambda = 8 - 3/\pi$, the whole is 8: "The 8π of the Einstein equations, divided by the π of the horizon entropy, gives 8; the three channels, $c_1 = 3$, cancel the 3 of the critical density" (*The Gravitational Passage*, Chapter 13, Section III). The abiding there is that of the whole internal space, $3/\pi$. In the registered ratio the whole is the count of six classes, and the abiding is that of $\mathbb{C}P^2$, $2/\pi$.

F-020 The Shared Coefficient and the Shape of the Internal Space. That *The Gravitational Passage* places each internal direction at a curvature of one over the square of the Planck length, identifies the coefficient of that curvature as one (Chapter 9, Section IX), and names it among three coefficients of order one that its comparison with the matter fraction does not set (Chapter 13, Questions). Written k, that coefficient enters the identity and the rule in the same way: $I^2\Omega_\Lambda = 8 - 3k/\pi$ and $\Omega_c/\Omega_b = 6 - 2k/\pi$. So two observables measure one coefficient: the ratio measures it at 0.99 ± 0.08, two to three times more closely than the matter fraction does, and the two agree with the microwave background alone. The rule also reaches the shape of the internal space, since it depends on the curvature of $\mathbb{C}P^2$ alone. With the whole internal curvature held at 6, and with the ratio of the radius of $\mathbb{C}P^2$ to the radius of S^2 written x, the rule reads $\Omega_c/\Omega_b = 6 - 36/(\pi(12 + x^2))$, which equals $6 - 2/\pi$ exactly at the Passage's ratio $\sqrt{6}$. Read through the rule, the combined measurement of entry F-019

gives the scalar curvature of $\mathbb{C}P^2$ as 3.96 ± 0.30, against the 4 of the Passage's Einstein condition, and the ratio of the radii as 2.49 ± 0.28, against $\sqrt{6} = 2.45$. The shape and the choice of the rule are one measurement. At a ratio of the radii of 2 the rule equals $6 - 9/(4\pi)$, which is exactly 6f, where $f = 1 - 3/(8\pi) = 0.8806$ is the Passage's external value (the vacuum's energy at the Planck scale in Planck units, less the bare curvature term); the rule is stated at the Passage's ratio $\sqrt{6}$, so a measured ratio can confirm the rule at the Passage's shape, or fit the rule at another shape, but it cannot do both (*The Weight of Possibility*, Chapter 5, Section IV).

Sources: *The Weight of Possibility*, Chapter 3, Section VI; Chapter 5, Sections III and IV; *The Gravitational Passage*, Chapter 9, Section IX; Chapter 13, Section II and Questions. Standing: derived, given a stated identification (the rule of entry F-019); a measured agreement (the coefficient, 0.99 ± 0.08, against the Passage's 1).

F-021 The Family and the Weight of the Match. That the rule is selected from a small family of expressions of one form (a count of six less an abiding, chosen in a way the Passage's quantities allow), and that twelve expressions are examined in all; *The Weight of Possibility* gives each of the twelve with its distance from the measurements of entry F-019. Within the rule's own form, a count of six less an abiding, no abiding (6), the abiding of S^2 ($6 - 1/\pi = 5.6817$) and the abiding of the whole internal space ($6 - 3/\pi = 5.0451$) lie between 4.9 and 13.2 standard deviations from those measurements, and are excluded. Two other forms built from the Passage's quantities lie within two standard deviations: six shares of the external value, $6f = 6(1 - 3/(8\pi)) = 5.2838$, and $(6 - 3/\pi)/((1 + f)/2) = 5.3653$. The form $(6 - 3/\pi)/((1 + f)/2)$ differs from the rule by 0.036 percent, and the rule needs one step fewer than that form. The simple fraction $16/3 = 5.3333$, the value that a relaxation of the masses of the axion and the proton gives for a gauge group of eight colors (Banerjee, Brzeminski and Hook 2026), lies 0.56 percent below the rule. A ratio measured to half a percent separates the rule from 6f at about three standard deviations,

and a ratio measured to about 0.2 percent separates it from 16/3. The weight of the rule rests on its form, on the exclusion of its neighbors within that form, and on the coefficient it shares with the Passage's identity.

Sources: *The Weight of Possibility*, Chapter 3, Section VII; Chapter 5, Sections I and VIII. Standing: derived (the values of the twelve expressions, computed); a measured agreement (their distances from the measurements of entry F-019); a fork that a named measurement decides (a ratio measured to half a percent, and to about 0.2 percent).

F-III. Why the Dark Persists, and Its Gyrations

F-022 What Persistence Requires. That a state persists when no exit from it is open, an exit being a lower state that the state can reach through a coupling that exists. The dark matter has three possible exits: into the content of the world, into the waves of the open geometry, and into evaporation.

Sources: *The Weight of Possibility*, Chapter 4, Section I; Chapter 1, Section IV; *The Weight of the Unseen*, Chapter 4, question 4.1. Standing: a definition.

F-023 No Exit into Content. That an excitation of the internal geometry and an item of actual content are not interchangeable: the geometry of possibility is re-formed only where the comparison itself re-forms, at the turn. So no process within an epoch turns an excitation of the internal geometry into an item of actual content. Physically a gyration is a massive excitation of the metric of the six held internal dimensions, and in its role it is a vibration of context, not an item of content. On the identification of entry F-005, a gyration is a vibration of context and not an item of content. The fact that a gyration is a vibration of context, and not an item of content, is the ground of the observed silence of the dark matter. The dark matter is silent in every search through an interaction other than gravity, because the dark matter couples to the content through gravity alone.

Sources: *The Weight of Possibility*, Chapter 4, Section II; Chapter 1, Section IV; *The Weight of the Unseen*, Chapter 4, question 4.2. Standing: derived, given a stated identification (the identification of entry F-005).

F-024 No Exit into the Waves of the Open Geometry: the Antipodal Parity. That the action of the geometry contains the metric alone and no orientation of the internal space, so it is unchanged when the sphere S^2 is mapped to its antipodes (each point to the point opposite it), and every state of the metric is either even or odd under that map. The sign of a harmonic of degree l on the sphere under the antipodal map is $(-1)^l$, as for the parity of the spherical harmonics in quantum mechanics. The waves of the open geometry are constant over the internal space, so those waves are even. A single odd excitation cannot decay into even states alone. The lightest odd state, having no lower odd state to fall to, cannot decay at all. A symmetry of this kind is known as a parity of the Kaluza and Klein kind: a reflection of the internal space that the action respects, under which the levels of the reduction alternate in sign and odd states couple only in pairs (Cheng, Matchev and Schmaltz 2002; Servant and Tait 2003). In the parity that protects the gyrations, the reflection is the antipodal map of the sphere itself, which leaves no point fixed, and the sphere is not identified with its antipodes. Were the sphere identified with its antipodes, the internal factor would be the real projective plane, on which the odd states do not exist. A gyration, which has about the Planck mass, is protected only by an exact symmetry, and a gauged discrete symmetry is exact (Krauss and Wilczek 1989). *The Weight of Possibility* takes the antipodal parity as gauged, on the ground of the principle of identity: two configurations of the whole geometry that differ only by the antipodal map cannot be told apart by any comparison, so they are one configuration. The parity is then exact, and it protects the gyrations on the account that gravitation is not quantized and on the mainstream account alike. Taking the parity as gauged has three consequences. Geometries in which the sphere returns to its antipodes around a closed path of the open space must be admitted, and within the ball that a registration holds such a re-

turn can be undone by a change of coordinates. In a closed space the odd gyrations number an even total. And the content couples to nothing that the antipodal map reverses: nothing in the world couples to the orientation of S^2, which is a prediction about the content. The weak interaction agrees with that prediction, since it distinguishes left from right in the open spacetime, and the antipodal map of the internal sphere leaves the orientation of the open spacetime unchanged.

Sources: *The Weight of Possibility*, Chapter 4, Section III; Chapter 1, Section IV; *The Weight of the Unseen*, Chapter 4, questions 4.3 and 4.4; Appendix, question A.11. Standing: derived, given a stated identification (the antipodal parity is gauged, on the ground of the principle of identity).

F-025 The Lowest Motion, the Breathing Excitations, and the Settling. That the lowest motion of the internal space, the motion of its floor, cannot radiate: "That motion is already the lowest state, so it cannot radiate or dissipate" (*The Gravitational Passage*, Chapter 11, Section II). The even excitations of the breathing kind (the swelling and shrinking of the internal volume), which lie above the lowest motion, do not persist. Their coupling to the waves of the open geometry vanishes at the order the Passage computes, but the term of Gauss and Bonnet, which the ten-dimensional action admits with a coefficient the Passage leaves free (*The Gravitational Passage*, Chapter 11, Section I), couples the breathing of the internal volume to those waves, and no symmetry forbids that coupling. The term of Gauss and Bonnet is the first of the Lovelock terms, the terms of higher order in the curvature that keep the field equations of second order (Lovelock 1971). For a coefficient of natural size the lowest breathing excitation decays within about a Planck time; a lifetime above 10^{11} years would need the coefficient below about 8.8×10^{-33} Planck areas, and the floor's own energy at one loop is expected to produce a larger effective coefficient even where the coefficient of the action is zero. So the breathing excitations made at the turn have long since decayed, and the persistent dark matter is the lightest odd excitation alone. As the internal space settles into its held size at the turn, the energy of its motion leaves as gravitation-

al waves of the open dimensions, which the microwave background registers as extra radiation in the effective number of light species (the count by which it measures the radiation beside the photons). The account requires that those waves stay within the measured bound, about three percent of the content's radiation at the time when the content had about a hundred species (Planck Collaboration 2020a; Calabrese and others 2025; Goldstein and Hill 2026): either the internal space settles gently, or a later stage of the content releases enough radiation to outweigh the waves (entry F-031).

Sources: *The Weight of Possibility*, Chapter 4, Section III; Chapter 6, Section I; *The Gravitational Passage*, Chapter 11, Sections I and II; *The Weight of the Unseen*, Chapter 4, question 4.6. Standing: derived, given a stated identification (the coefficient of the term of Gauss and Bonnet is of natural size); a measured input (the bound on the effective number of light species, which the settling meets).

F-026 No Exit into Evaporation. That an excitation of the Planck scale has no lower level into which to evaporate: the next scale at which evaporation could act lies some thirty orders of magnitude away, and the floor of the internal space is the lowest state of its kind. Remnants of black holes at the Planck mass have been held stable for the same reason (Chen and Adler 2003; Rovelli and Vidotto 2018). A gyration is a level of vibration of the internal geometry, with a definite energy and a definite sign under the antipodal map, and not a collapsed region of the open geometry.

Sources: *The Weight of Possibility*, Chapter 4, Section IV, and Questions; Chapter 1, Section IV; *The Weight of the Unseen*, Chapter 3, question 3.4, and Chapter 4, question 4.5. Standing: derived.

F-027 The Lightest Odd Level. That the lowest level of the internal space that holds odd states is the first harmonic of the sphere: a function of degree one on S^2, constant over $\mathbb{C}P^2$. Its Laplacian eigenvalue is twice the Planck curvature, the value the Passage gives as the slowest variation the internal space admits: "the first eigenvalue of its Laplacian, $12/R_O{}^2 = 2/R_T{}^2$, is twice the Planck curvature" (*The Gravitational Passage*, Chapter

17, Section V), where R_O and R_T are the radii of $\mathbb{C}P^2$ and S^2. The internal space whose levels these are is held by the Passage: "the internal space has the same size and the same shape at every point of the exterior" (*The Gravitational Passage*, Chapter 11, Section II). At the classical level four odd multiplets share the lowest mass, a squared mass of 2 in Planck units, which is $\sqrt{2}$ Planck masses, about 1.7×10^{19} GeV: a spin-two triplet, $(1, 3)$ under $SU(3) \times SO(3)$ (the isometries of $\mathbb{C}P^2$ and the rotations of S^2); two spin-one octet-triplets, $(8, 3)$; and a spin-zero octet-triplet, $(8, 3)$. These classical masses are those of the operators of the reduction, with the higher Lovelock terms leaving the operators unchanged and the floor acting as an ordinary source. At one loop the floor's local term, the least extent, lowers the squared masses of all four by the same amount, at first order from 2 to about 0.4, and so the mass from $\sqrt{2}$ to about 0.6 Planck masses; the rest of the floor, read locally, lifts the spin-zero octet-triplet and leaves the other three where they are. The candidates for the lightest odd state are therefore the spin-two triplet and the two spin-one octet-triplets. The term of Gauss and Bonnet can split these three candidates already at the classical level, and whichever candidate is lightest persists, with its own mass. A gyration of 0.6 to $\sqrt{2}$ Planck masses has about 10^{19} times the mass of a proton.

Sources: *The Weight of Possibility*, Chapter 4, Section V; *The Weight of the Unseen*, Chapter 3, question 3.2; *The Gravitational Passage*, Chapter 11, Section II; Chapter 17, Section V. Standing: derived (the spectrum of the reduction on the held internal space, at the classical level and with the floor's local term at first order).

F-028 Collisionless. That the gyrations collide neither with the content nor with one another. They act on the content only by their weight (entry F-023). Two odd gyrations can annihilate together into even states, which the parity allows; at the density of the dark matter near the Sun, with a cross section of the order of the Planck area, that happens about once in 10^{78} seconds for each gyration, which is no exit at all. Their scattering on one another through gravity alone, at the speeds of matter in galaxies and

in clusters (a few hundred to a few thousand kilometers per second), has a cross section per unit mass for the transfer of momentum between about 10^{-45} and 10^{-50} square centimeters per gram, at least forty-five orders of magnitude below the bound of about one square centimeter per gram that colliding clusters set (entry F-001).

Sources: *The Weight of Possibility*, Chapter 4, Section III; *The Weight of the Unseen*, Chapter 3, question 3.5. Standing: derived.

F-029 The Fields of the Internal Symmetries. That the rotations of S^2 and the isometries of $\mathbb{C}P^2$ (the maps that keep every distance) are symmetries of the internal geometry, and in the reduction they give fields over the open spacetime under which the odd triplet is charged (the standard result of the reduction; Salam and Strathdee 1982). Nothing in the content carries the charges of these fields, so their quanta couple to the matter and the gauge forces only through gravity (*The Gravitational Passage*, Chapter 11, Section II; *The Branches of the Kinds*, Chapter 2, Section III). Computed in *The Weight of Possibility* by Weinberg's prescription (Weinberg 1983) at the Passage's radii, the coupling is about 6 for the rotations of S^2 and about 16/3 for the isometries of $\mathbb{C}P^2$, in the units in which electromagnetism has 1/137; these two values are that book's computation, and Weinberg gives the prescription. The fields are therefore strongly coupled at the Planck scale and may confine there; confined, they dress each gyration into a singlet that is still odd and acts at long range only through gravity. The account requires that these fields not reach the open spacetime as a long-range force on charges that do not cancel: the tidal streams of the Sagittarius dwarf galaxy disfavor a difference of more than ten percent between the accelerations of dark and ordinary matter (Kesden and Kamionkowski 2006), and a force acting on the dark matter alone is bounded at about five percent of gravitation (Bean, Flanagan, Laszlo and Trodden 2008). These couplings are about 300 times the weak and strong couplings at the Planck scale (Buttazzo and others 2013), so the isometries of the

context instance are not the forces of particle physics, which belong to the algebra of the content (*The Concept Sequence*, Chapter 18, The Three Forces, and Chapter 24, The Algebra of Comparison).

Sources: *The Weight of Possibility*, Chapter 4, Section V; Chapter 6, Section II; *The Gravitational Passage*, Chapter 11, Section II; *The Branches of the Kinds*, Chapter 2, Section III. Standing: derived (the couplings, computed by Weinberg's prescription at the Passage's radii); a measured input (the bounds on a force acting on the dark matter alone).

F-030 The Number of the Gyrations. That with the registered ratio there is about one gyration for every 1.5 to 3.4 × 10^{18} baryons (for a gyration of 0.6 to $\sqrt{2}$ Planck masses), and about 2 to 4 × 10^{-28} gyrations for every photon of the microwave background. In the neighborhood of the Sun, at the measured local density of dark matter, there is about one gyration in every few tens of thousands of cubic kilometers. No detector yet built registers a single gyration through gravity alone: arrays of sensors proposed for the purpose (Carney, Ghosh, Krnjaic and Taylor 2020) would need about 80 decibels of reduction of quantum noise (Qin and others 2025). Even taken as made independently of one another, the gyrations leave by their number alone a departure of about 5 to 8 × 10^{-27} in a region twenty megaparsecs across, against measured departures of about 10^{-5}, so the finite number of the gyrations adds no measurable noise.

Sources: *The Weight of Possibility*, Chapter 4, Section VI; Chapter 3, Section I; *The Weight of the Unseen*, Chapter 5, question 5.10. Standing: derived.

F-031 Made Only at the Turn: the Conditions on the Turn. That anything that gravitates can be made in pairs by a hot plasma through gravity alone, and that the account requires this channel to stay closed after the turn. If the plasma of the new epoch formed at once, the measured amount of dark matter requires that the plasma never have been hotter than about one thirtieth of the mass of a gyration (the divisor runs from about twenty-eight to about thirty-four with the strength of the coupling): about 5 × 10^{17} GeV for a gyration of $\sqrt{2}$ Planck masses, and about 2 × 10^{17} GeV for 0.6 Planck masses. The divisor is computed in *The Weight of Possib-*

ility from the method and cross sections of Garny, Sandora and Sloth (2016) and of Garny, Palessandro, Sandora and Sloth (2018). If the turn's energy reached the plasma through a stage that behaved as matter, that stage must end below about one hundredth of the mass of a gyration. The gyrations made directly as the internal space re-forms must number about 10^{-27} of what an abrupt re-forming would make, which requires the re-forming, and the capture of the internal space by the floor at its end, to be slow and gentle compared with the period of the lightest odd level. These limits are conditions on the turn: the Passage does not fix "the temperature at which the reconstructed history of the new exterior begins" (*The Gravitational Passage*, Chapter 18, Section XI). The measured amount of dark matter is the measurement that decides them: an epoch that began with a hot plasma near the Planck temperature makes the gyrations about as numerous as the photons (Kolb and Slansky 1984), far beyond that amount.

Sources: *The Weight of Possibility*, Chapter 4, Section VI; Chapter 6, Section I; *The Gravitational Passage*, Chapter 18, Section XI; *The Weight of the Unseen*, Chapter 6, question 6.7. Standing: derived (the limits, for couplings of the gyrations to the plasma suppressed by the Planck mass); a measured input (the amount of dark matter, which decides the conditions).

F-032 The Gyrations of the Previous Epoch. That the gyrations do not decay, and need not. On the Passage's account of the turn every gyration, like all other matter, either ends in the interior of a black hole or is carried away alone, and in both cases the internal space of which it is an excitation de-compactifies. At the turn the Passage "defines no field there, of any spin" (*The Gravitational Passage*, Chapter 18, Section VII), and "no count is carried across the turn, for the matter or for the geometry, since each count is relative to the context that ends" (Chapter 18, Section VIII). What returns is the law that protects the gyrations: "In the new epoch the product re-forms with the same internal topology" (Chapter 18, Section VII), so the new epoch has the same odd levels and the same antipodal parity. Each epoch's gyrations are therefore made at its own turn, and the

ratio registered in an epoch counts all the gyrations present in it. In conformal cyclic cosmology the dark matter, also of about the Planck mass and made at the transition between epochs, must decay within each epoch (Penrose 2018).

Sources: *The Weight of Possibility*, Chapter 6, Section III; Chapter 4, Questions; *The Gravitational Passage*, Chapter 18, Sections VII and VIII; *The Weight of the Unseen*, Chapter 6, questions 6.4 and 6.5. Standing: derived.

F-IV. The Measurements

F-033 Equation of State Defined; Perfectly Cold. That the equation-of-state parameter w of a component that fills the universe is the ratio of its pressure to its energy density: $w = 0$ for non-relativistic matter, which gravitates and exerts no pressure, $w = 1/3$ for radiation, and $w = -1$ for a cosmological constant. A gyration of about the Planck mass has no motion from heat that matters, and the momentum of each gyration falls as the universe expands, so the dark matter is perfectly cold whatever speed the turn gives the gyrations. A gyration moving with the speed it would have in a plasma at one thirtieth of its mass travels about two centimeters in the whole history of the universe, and a gyration that left the turn with a hundred times its mass in momentum travels less than a meter. On every scale astronomers resolve, the gyrations therefore cluster exactly as standard cold dark matter does, with $w = 0$.

Sources: *The Weight of Possibility*, Chapter 5, Section VI; *The Weight of the Unseen*, Chapter 3, question 3.6, and Appendix, question A.12. Standing: derived; a definition (of the equation-of-state parameter).

> ***Why w matters.***
>
> The equation-of-state parameter fixes how a component's density changes as the universe expands. Matter with $w = 0$ dilutes as the volume grows: double the volume, halve the density. Radiation with $w = 1/3$ dilutes faster, because each wavelength is stretched as well. A cosmological constant, $w = -1$, does not dilute at all. The microwave background requires

> the non-baryonic component to be pressureless, to take no part in the sound of the plasma of photons and ordinary matter, and to be present before the background was released; a perfectly cold, collisionless component made at the turn meets all three requirements.

F-034 Rotation Curve Defined. That a rotation curve is a graph of how fast stars and gas orbit the center of a galaxy as a function of their distance from the center. In Newtonian gravity the orbital speed depends on the mass enclosed within the orbit, so beyond the visible disk, where the luminous mass does not grow further with distance, the speed should fall. The observed speed stays approximately constant, "flat", to the outermost measured radius.

Sources: *The Weight of Possibility*, Chapter 1, Section I (Rubin and Ford 1970). Standing: a definition.

> ***Why flat rotation curves imply unseen mass.***
>
> If the orbital speed at some distance from the center of a galaxy stays constant as that distance grows, the enclosed mass must grow in proportion to the distance. The visible light of the galaxy falls off exponentially beyond a few kiloparsecs, so the mass that is seen is concentrated in the inner regions, while the mass implied by the rotation keeps growing far beyond the edge of the visible disk. On this account that unseen mass is the dark matter of entry F-005, gathered in a halo.

F-035 The Radial Acceleration Relation and the Modified Dynamics. That the radial acceleration relation is measured at 2693 points in 153 disk galaxies of the SPARC sample of 175 (the sample of Spitzer Photometry and Accurate Rotation Curves; Lelli, McGaugh and Schombert 2016; McGaugh, Lelli and Schombert 2016). At each radius the acceleration found from the rotation is one function of the acceleration that the visible stars and gas alone would produce. The radial acceleration relation has a scatter of about 0.13 dex (a factor of about 1.35), and most of that scatter comes from errors of measurement. The function follows the visible matter where its acceleration is large, and departs from it more and

more as that acceleration falls to about 1.2×10^{-10} meters per second per second and below, the acceleration scale proposed by Milgrom (1983). The account places that relation where cold dark matter places it, in the formation of galaxies under general relativity, since its dark matter clusters exactly as cold dark matter does (entry F-033). Changing the law of gravity instead, as the modified Newtonian dynamics does, still leaves clusters of galaxies short of mass (Sanders 2003; Famaey, Pizzuti and Saltas 2025), and without dark matter it predicts a third peak of the microwave background lower than the one measured (McGaugh 2004; Planck Collaboration 2020a). With the field equations exact throughout the domain of the smooth description, below the Planck curvature, as *The Gravitational Passage* derives them, a galaxy falling freely in the field of a larger one feels that field only through its tide, as general relativity requires, and not through the external field effect that the modified dynamics predicts.

Sources: *The Weight of Possibility*, Chapter 5, Section V; *The Weight of the Unseen*, Chapter 1, question 1.7, and Appendix, question A.13. Standing: derived (a galaxy falling freely feels a larger galaxy's field only through its tide, from the exact field equations of *The Gravitational Passage*); a measured input (the radial acceleration relation).

F-036 Small Scales. That a perfectly cold, collisionless component meets every present bound that separates cold dark matter from its warmer or wavier alternatives. A combined analysis of the counts of the faintest satellite galaxies and of the substructure seen by strong lensing places the half-mode mass (the mass scale at which a cut-off would halve the amplitude of the primordial departures relative to cold dark matter) below about ten million solar masses at 95 percent confidence (Nadler and others 2021). The forest of hydrogen absorption in the light of distant quasars (the Lyman-alpha forest) places the mass of a thermal relic of warm dark matter above 5.3 keV at 95 percent confidence (Palanque-Delabrouille and others 2020). On small scales the dark matter of this account is the cold dark matter of standard cosmology, so the three standing questions that

cold dark matter meets there (cores at the centers of dwarf galaxies where simulations give cusps, fewer faint satellite galaxies than small halos, and no dense, massive satellites of the kind called too big to fail) are answered here as within cold dark matter. The answers most studied there are the feedback of stars on the gas of small galaxies, the suppression of star formation in the smallest halos after reionization, and the faint satellites that deep surveys continue to find (Bullock and Boylan-Kolchin 2017). I predict no cut-off in the structure of the dark matter on small scales.

Sources: *The Weight of Possibility*, Chapter 5, Section V; *The Weight of the Unseen*, Appendix, question A.12. Standing: a measured agreement (with the present bounds on small-scale structure).

F-037 A Second Kind of Measurement. That routes that do not use the microwave background measure the share of the ordinary matter in all the matter, for which the prediction, with the lightest neutrinos, is 0.15644 (entry F-019). The baryon density from primordial nucleosynthesis (Schöneberg 2024) with the baryon acoustic oscillations (the imprint of the sound waves of the early plasma on the spacing of galaxies) of the Dark Energy Spectroscopic Instrument (DESI Collaboration 2025) gives 0.1587 ± 0.0066; the same baryon density with the full-shape analysis of that instrument (Forero-Sánchez and others 2026) gives 0.1542 ± 0.0063; the gas of galaxy clusters gives 0.156 ± 0.034 (Mantz and others 2022). Each agrees with the prediction; the most precise of them is about five times less precise than the microwave background, and the cluster gas about thirty times less.

Sources: *The Weight of Possibility*, Chapter 5, Section II; *The Weight of the Unseen*, Appendix, question A.7. Standing: a measured agreement.

F-038 Analyses That Add the Baryon Acoustic Oscillations. That I hold the matter fraction at 0.311 and the cosmological constant at one value (*The Gravitational Passage*, Chapter 13, Sections III and VI). So I expect the pull of the baryon acoustic oscillations toward a lower matter fraction to recede as the data on those oscillations grow. A result counts as settled when it is established at five standard deviations by two independ-

ent kinds of measurement, since a single kind can carry an error of its own that its stated uncertainty does not include (*The Weight of the Unseen*, Appendix, question A.3). The analyses that add the oscillations of the second data release of the Dark Energy Spectroscopic Instrument lie lower: the two densities those analyses infer give 5.199 ± 0.035 (Louis and others 2025, Table 5), 4.7 standard deviations from the prediction, or about 4.0 with a correlation of -0.4 between the two densities. The oscillations do not measure the dark density: they measure the matter fraction, and they lower the dark density the analyses infer where they prefer a lower matter fraction. Read through the shared coefficient (entry F-020), in the most precise of these analyses the ratio asks for a coefficient of 1.26 ± 0.06 and the matter fraction for 0.68 ± 0.11, 4.7 standard deviations apart, so the two observables pull the coefficient in opposite directions. Both figures rest on one kind of measurement, and within the standard model the oscillations of that instrument and the microwave background differ by 2.3 standard deviations (DESI Collaboration 2025), so a combination of the two measurements measures that difference along with the ratio.

Sources: *The Weight of Possibility*, Chapter 5, Section I; *The Weight of the Unseen*, Appendix, questions A.3 and A.6; *The Gravitational Passage*, Chapter 13, Sections III and VI. Standing: a measured input (the ratio from the combined analyses named).

F-039 The Amplitude of Clustering. That the account fixes how strongly matter has clumped by today, once the primordial amplitude is taken from the microwave background, which the account does not predict. The measure is $S_8 = \sigma_8(\Omega_m/0.3)^{1/2}$, where σ_8 is the root-mean-square departure of the density of matter in spheres eight megaparsecs across in units of the Hubble constant, and Ω_m is the matter fraction. With the Passage's matter fraction 0.311, the registered ratio, and the primordial amplitude and tilt of Planck 2018, the amplitude of clustering today is $S_8 = 0.823 \pm 0.006$, computed at the matter density that keeps the angular size of the acoustic scale the microwave background measures; with the

Passage's Hubble constant at its central value, h = 0.678, it is 0.827 ± 0.007. The standard model, with its matter fraction taken from the same measurement, gives 0.832 ± 0.013 (Planck Collaboration 2020a); the account's value is lower, and its uncertainty half as large, because the account fixes the matter fraction where the standard model takes it from the measurement. The lensing of the microwave background agrees with the account within one standard deviation (Qu and others 2026; Omori and others 2026), as do the complete Kilo-Degree Survey (Wright and others 2025) and the growth rate measured from the motions of galaxies (Alam and others 2021). The final galaxy clustering and weak lensing of the Dark Energy Survey lies 2.5 standard deviations below the account (Abbott and others 2026a), and the clusters counted by eROSITA lie 3.2 standard deviations above it (Ghirardini and others 2024); those two measurements differ from each other by 4.5 standard deviations. The amplitude of clustering depends on the matter fraction: a value confirmed away from 0.823 bears first on the identity for the cosmological constant, unless the matter fraction is confirmed at 0.311 by a separate measurement.

Sources: *The Weight of Possibility*, Chapter 5, Section VI; *The Weight of the Unseen*, Appendix, question A.14. Standing: derived (from the Passage's matter fraction, the registered ratio, and the primordial amplitude and tilt of Planck 2018, taken as measured inputs); a measured agreement (with the lensing of the microwave background, the Kilo-Degree Survey and the growth rate).

F-040 **The Searches.** That on the identification of entry F-005 every search for the dark matter through any interaction but gravity finds nothing, at any sensitivity. A detector registers content: light, charge, the recoil of a nucleus. The direct searches (XENON, LUX, PandaX, CDMS, CRESST and others) look for the recoil of a nucleus or an electron struck by the particle of dark matter that those searches posit; each is correctly built and operated, and each records what the account predicts, since the dark matter acts on a nucleus only by its weight. "No detector will ever catch the dark matter by any means other than its weight" (*The Weight of the Unseen*, Appendix, question A.10). That no detector will ever catch the dark matter by any means other than its weight is a prediction that follows from

the identification of entry F-005. Other accounts of dark matter that interacts only through gravity make the same prediction for these searches (Feng, Rajaraman and Takayama 2003), so a lasting null result separates this account from particles of weak interaction; the ratio, the coefficient it shares with the Passage's identity and the absence of a short loop separate it from those other accounts (entry F-041).

Sources: *The Weight of Possibility*, Chapter 5, Section VII; *The Weight of the Unseen*, Appendix, question A.10; *The Grand Synthesis*, Chapter 13, Section II, which carries the same statement of the direct searches. Standing: derived, given a stated identification (the identification of entry F-005).

F-041 The Predictions. That the account's own prediction of number is the ratio $6 - 2/\pi = 5.3634$, and that with the Passage's matter fraction it also gives the share of the ordinary matter in the cold matter, 0.15644, and the amplitude of clustering, 0.823, which rest on the two books together. Besides that prediction of number, I make six predictions of kind. The first three predictions are that the dark matter acts only through gravity, that the dark matter does not decay, and that the dark matter is perfectly cold, with no cut-off in its structure on small scales. The last three predictions are that the dark matter is made of gyrations of about the Planck mass, that no short loop exists anywhere in space, and that the dark component carries no separate pattern of primordial departures. The ratio, the coefficient it shares with the Passage's identity, and the absence of a short loop are the account's alone; the first four predictions of kind it shares with other accounts of a stable, cold dark matter that acts only through gravity, and the sixth with every account in which the dark matter and the ordinary matter vary together from place to place.

Sources: *The Weight of Possibility*, Chapter 5, Section IX; *The Weight of the Unseen*, Appendix, question A.1. Standing: a definition (of the predictions the account names).

F-042 The Dark Fire Closure. That a dark fire, a chain in which ordinary matter is turned into dark matter atom by atom, each conversion setting off the next, "cannot happen on this account, in a collider, in a star or

anywhere else within an epoch" (*The Weight of the Unseen*, Chapter 3, question 3.10). Three grounds rest on what the dark matter is, and that characterization rests on the hold that *The Gravitational Passage* secures: a gyration is an excitation of the metric of the six held internal dimensions (entries F-005 and F-006), and the internal space has the same size and the same shape at every point of the exterior, a hold that rests on the modal boundary (entry F-004). The first ground is what the dark matter is: it is a vibration of the space of possibility, not an item of content, and it couples to content through gravity alone. Within an epoch the one way content can make dark matter is the making of pairs through gravity alone. The making of pairs through gravity alone needs a gas hotter than about one thirtieth of the mass of a gyration (entry F-031). No other process within an epoch turns an item of content into a vibration of possibility. The second ground is energy: one gyration holds the energy of the mass of about ten billion billion protons, while the largest particle collider, colliding two heavy nuclei head on, brings together about a million times the energy of a proton's mass, trillions of times too little to make one gyration. The most energetic particles that reach the Earth from space each carry less than a ten-millionth of the energy one gyration holds, and gravity alone can make gyrations in pairs only in a gas hotter than about one thirtieth of the mass of a gyration, which nothing in our epoch has been. The third ground is the link a fire needs: each conversion would have to pass something on to the next atom, and the gyrations act on ordinary matter only by their weight, which at the scale of atoms is far too weak to pass anything on. A fourth ground, the completeness of the kinds (entry F-002), covers every new kind of elementary particle, so a collider makes no new kind to be stable, aggregative and dangerous. These grounds reach the processes that the argument from cosmic rays does not reach, such as a coherent collision of heavy nuclei with no counterpart among cosmic rays (*The Grand Synthesis*, Chapter 13, Section IV).

Sources: *The Weight of the Unseen*, Chapter 3, questions 3.9 and 3.10; *The Story of the Particles*, Level 9, Section IV; *The Gravitational Passage*, Chapter 11, Section II; *The Grand Synthesis*, Chapter 13, Section IV, which carries the same four grounds and the gap in the argument from cosmic rays. Standing: derived.

F-043 What the Account Claims. That the dark matter is a conserved excitation of the six dimensions of possibility, made at the turn and kept by the antipodal parity of the sphere (entries F-005 and F-024); that it is one of the three distortions of context, required by the Incommensuration Theorem applied to each column of context (entry F-007); and that its ratio to the ordinary matter is an identity of the present measurement, $6 - 2/\pi = 5.3634$, against the measured 5.370 ± 0.048, which shares its one coefficient with the Passage's identity for the cosmological constant (entries F-019 and F-020). It predicts, in kind, that no search through any interaction but gravity will find it, that it does not decay, that its structure has no cut-off on small scales, that its gyrations are of the Planck mass, that no short loop exists anywhere in space, and that it carries no separate pattern of primordial departures (entry F-041). By what it is, it cannot be made from ordinary matter within an epoch, so no dark fire can happen (entry F-042).

Sources: *The Weight of Possibility*, Chapter 6, Section IV; *The Weight of the Unseen*, Chapter 3, question 3.10. Standing: an identification (the dark matter as a conserved excitation of the context instance); a measured agreement (the registered ratio); derived, given a stated identification (the predictions of kind, from the identification of entry F-005).

Summary

In this chapter I set out the account of the dark matter that *The Weight of Possibility* gives, with the companion book *The Weight of the Unseen*. That account is built on the context that *The Gravitational Passage* fixes. The dark matter is a conserved excitation of the metric of the six held internal dimensions, the context instance of $\mathbb{C}P^2 \times S^2$, which the Passage identifies

with the dimensions of possibility; it is made at the turn, and it is a vibration of context, which is no quantum of the arena and no quantum of gravitation. Its characterization rests on the hold that the Passage derives from the modal boundary: the internal space has the same size and the same shape at every point of the exterior.

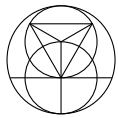
In the first part of this chapter I place the dark matter among the three distortions of context, as the distortion of the context of possibility. The Incommensuration Theorem, a structural fact about the nature of knowing, applies to each column of context, and the three axioms give each column a distortion of its own: gravitation in space, the cosmological constant in time, and the dark matter in possibility. The three stand as one triple, bound to the three edges of reality, with the cosmological constant as the clear immanent and gravitation and the dark matter as the conjugate pair; the list of the kinds of content, which *The Story of the Particles* and *Non-Commutative Geometry and the Immanent Metaphysics* close, holds no dark-matter species, and the gyrations of the dark matter are vibrations of context that the list does not count.

The second part fixes the amount. The ratio of the dark matter to the ordinary matter is an identity of the present measurement, as the Passage's identity fixes the cosmological constant: a registration counts six classes for the dark residue and one for the ordinary residue, and the abiding of $\mathbb{C}\mathrm{P}^2$ reduces the six by $2/\pi$, so the ratio is $6 - 2/\pi = 5.3634$, a measured agreement with 5.370 ± 0.048. The ordinary residue itself is a number of history, the residue the turn leaves, as *The Coming of the Kinds* rules, and it is taken as measured; the rule shares one coefficient with the Passage's identity and reaches the shape of the internal space.

The third part keeps the dark matter. The three exits of the dark matter are closed. No process within an epoch turns a vibration of possibility into content. The gauged antipodal parity keeps the lightest odd level from passing the energy of that level to the even waves of the open geometry. The Planck scale offers no lower level into which the dark matter could

evaporate. The lightest odd level has $\sqrt{2}$ Planck masses at the classical level and about 0.6 at first order, so the dark matter is perfectly cold and collisionless, and each epoch's gyrations are made at its own turn.

The fourth part sets the predictions beside their measurements: the ratio, a second kind of measurement of the ordinary share, small scales, the amplitude of clustering, the silence of every direct search, and six predictions of kind, each with the step it bears on. From what the dark matter is, I end this chapter on the dark fire closure. The fact that the dark matter is a vibration of possibility and not content, the energy that one gyration holds, and the absence of a link each rule out the conversion of ordinary matter into dark matter within an epoch. The completeness of the kinds rules out every new kind of particle that a collider could make.



Chapter G

Dark Energy and the Cosmological Constant

CHAPTER OVERVIEW

This chapter traces the cosmological constant, the constant term of Einstein's field equations that drives the accelerating expansion of the universe. It follows *The View from Inside* and *The Gravitational Passage*, Chapter 13, from the problem of measuring empty space from inside it to the numbers that measurement tests: the identification of the cosmological constant with the share of the vacuum's energy that an observer inside the vacuum can take in, the formula and the identity that follow from that share, why the constant is one number in one history, and the measurements that decide it.

The chapter rests on these chapters of *The Concept Sequence*: Chapter 13, The Incommensuration Theorem; Chapter 15, The Three Axioms (Axiom I is used directly here); Chapter 16, The Two Laws and the Content/Context Partition, and Chapter 20, The Grid of Measurement, for the six measures; Chapter 18, The Three Forces (entry 18.19), and Chapter 39 (entry 39.10), for gravitation as the arena; Chapter 25, The Internal Space in Its Two Instances, with the Euler characteristic $\chi(\mathbb{C}P^2) = 3$ (entry 25.20); Chapter 26, The Two Kernel Integers and the Kernel Triangle, for $c_1 = 3$ (entry 26.10); Chapter 27, The Context Instance: Radii, Volume and the Count 72, for the radii of the internal space; Chapter 28, The Two Measured Anchors and the Span to the Planck Mass, for Newton's constant as the measured input of gravitation (entry 28.11) and the unre-

duced Planck mass (entry 28.14); and Chapter 45, Further Predictions, whose entry 45.15, The Cosmological Constant, this chapter extends. Chapter F takes from this chapter the matter fraction 0.311 and the coefficient of the internal curvature that the identity shares with the ratio of dark to ordinary matter. Chapter I collects the whole budget of the universe's energy.

G-I. The Cosmological Constant Problem

G-001 The Cosmological Constant. That the cosmological constant, written Λ, is the constant term of Einstein's field equations, and that the vacuum energy density it carries, written ρ_Λ, is related to it by $\Lambda = 8\pi G_N \rho_\Lambda$, with G_N Newton's constant. That Einstein introduced the term in 1917 (Einstein 1917), and that observations of distant supernovae showed in 1998 that the expansion of the universe is accelerating (Riess and others 1998; Perlmutter and others 1999), which a positive constant term produces. That the observed density is $\rho_{\Lambda,\mathrm{obs}} = 2.52 \times 10^{-47}\ \mathrm{GeV}^4$, computed from the parameters of the Planck satellite's 2018 results used as one set (Planck Collaboration 2020a): the Hubble constant in units of 100 kilometers per second per megaparsec, $h = 0.674$; the matter fraction $\Omega_m = 0.315$ and the radiation fraction $\Omega_r = 9 \times 10^{-5}$ (both defined in entry G-015); and a flat universe.

Sources: *The View from Inside*, Chapter 1, and Notes, Part I; *The Gravitational Passage*, Chapter 13, Section IV. Standing: a definition (Λ and ρ_Λ); a measured input (the observed density).

> ***On energy density units.***
>
> Physicists measure energy in electron volts (eV); one GeV (giga-electron-volt) is one billion electron volts, roughly the energy contained in the mass of a single proton. An energy density is an amount of energy per unit volume. In natural units, where the speed of light and the reduced

> Planck constant are both set equal to one, an energy density has the dimensions of an energy raised to the fourth power, GeV^4. In these units the observed vacuum density is $\rho_{\Lambda,\text{obs}} = 2.52 \times 10^{-47}\ \text{GeV}^4$.

G-002 Vacuum Energy and the Planck Density. That quantum field theory gives empty space an energy: every quantum field has a ground state with a nonzero energy, and summed over all fields up to the Planck scale that energy density is about the Planck density, $M_{\text{Pl}}{}^4 = 2.222 \times 10^{76}$ GeV^4. That the Planck mass used here is the full Planck mass, $M_{\text{Pl}} = \sqrt{(\hbar c/G_N)} = 1.221 \times 10^{19}$ GeV, formed from Newton's constant, the speed of light and Planck's constant alone, and fixed by the measured G_N, the one measured constant of gravitation (*The Concept Sequence*, Chapter 28, entry 28.11). That the full Planck mass marks the modal boundary: the scale at which an object's gravitational radius equals its quantum wavelength, where the smooth description of the arena of space and time (general relativity) and the stepwise description of content (quantum mechanics) exchange dominance. The boundary is a scale formed from the three constants alone, so the full Planck mass marks it; the 8π of the reduced Planck mass, $M_{\text{Pl}}/\sqrt{(8\pi)}$, belongs to the coupling of the field equations, $8\pi G_N$.

Sources: *The View from Inside*, Chapter 10 ("Back to the Numbers"), and Appendix C ("The Inputs"); *The Gravitational Passage*, Chapter 17, Sections II and III. Standing: a definition (the Planck mass and the Planck density, formed from G_N, c and ħ); a measured input (G_N).

G-003 The Mismatch. That the vacuum energy density that quantum field theory gives at the Planck scale is about 10^{123} times the observed density of the cosmological constant (2.222×10^{76} GeV^4 against 2.52×10^{-47} GeV^4). With the reduced Planck mass the ratio is about 10^{120}, smaller by $(8\pi)^2 \approx 632$; I use the full Planck mass throughout, because the modal boundary is a scale formed from the constants alone (entry G-002). That this mismatch is the cosmological constant problem as physicists state it (Weinberg 1989), widely regarded as the worst quantitative pre-

diction in the history of science: a successful theory missing a measurable quantity by the largest margin of any prediction. That no calculation within quantum field theory alone has closed it, and that some physicists have turned to the anthropic principle: the argument that Λ varies from region to region of a multiverse, and that observers find themselves where it takes a value that permits them.

Sources: *The View from Inside*, Chapter 1 ("The Number That Shouldn't Exist" and "What Went Wrong?"), Chapter 2 ("The 123 Orders of Magnitude"), and Notes, Part I. Standing: a definition (the cosmological constant problem, as the ratio of the Planck density to the measured density); a measured input (the observed density).

> ***The scale of the mismatch.***
>
> The Planck density is $M_{Pl}{}^4 \approx 2.22 \times 10^{76}$ GeV^4. The observed vacuum density is $\rho_{\Lambda,obs} \approx 2.52 \times 10^{-47}$ GeV^4. The ratio of the Planck density to the observed density is about 10^{123}.

G-004 Why the Usual Strategies Share One Assumption. That the strategies tried against the mismatch are four: to find a symmetry that forces the cancellation; to accept a landscape of values and select ours anthropically; to modify the theory that connects the vacuum energy to gravity; and to compute the vacuum energy differently. That each treats the cosmological constant as a property of the vacuum in the absence of observers, instruments or measurements, and asks one question only: what the vacuum energy density is. The account of this chapter begins instead from where the observer stands: every measurement of the vacuum is made from inside the vacuum it measures.

Sources: *The View from Inside*, Chapter 3 ("The Shape of the Problem" and "The Pattern"). Standing: a definition (the four strategies, and the starting point of the account).

G-005 The Problem of Being Inside. That a measurement of the vacuum energy density is made by an observer inside the universe whose vacuum it measures, so the observer, the apparatus and the vacuum are parts of

one system: there is no laboratory outside the universe, and no control sample of empty space to compare with. The observer's own degrees of freedom (the independent ways in which something can hold energy or vary) are excitations of the same quantum fields whose ground-state energy the observer is trying to measure, and they are entangled with the vacuum: part of what is true of each is held only in the relation between them. That this entanglement leaves an information deficit which no better instrument removes: some of the information about the vacuum is held in the correlation between the observer and the vacuum, and the observer cannot reach it. A system cannot fully measure itself. The Incommensuration Theorem, a structural fact about the nature of knowing, states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity (*The Concept Sequence*, Chapter 13). *The Gravitational Passage* identifies the deficit as an instance of that theorem: "I identify this deficit as an instance of the Incommensuration Theorem at the cosmological scale" (Chapter 13, Section I).

Sources: *The Gravitational Passage*, Chapter 13, Section I; *The View from Inside*, Chapter 11 ("Inside, and Joined"). Standing: an identification (the information deficit as an instance of the Incommensuration Theorem).

> ***The fish and the water.***
>
> A fish that sets out to measure the water it swims in cannot take a sample of water to a bench outside the water, because there is no outside. Its instruments are wet. Every measurement it makes is made through water, by an instrument made partly of water. The gap between what the water holds and what the fish can extract comes from the measurer being part of the measured, and no skill of the fish and no quality of its instruments closes it.

G-006 What Curves Space, and Why the Total Curves Nothing. That the arena of space and time bends in proportion to what comparison finds, because the arena carries the strain between a continuous comparing and a symmetric content (*The View from Inside*, Chapter 8), so what no

comparison finds places no strain on the arena. That the vacuum's total energy is found by no comparison, for two reasons. First, a measurement receives the vacuum's energy only through the degrees of freedom it engages, and no measurement made from inside engages them all. Second, the total, taken as a whole, is the same at every place, the same for every comparer in uniform motion, and without a break anywhere: it would be perfectly symmetric and perfectly continuous at once, the pairing that the Incommensuration Theorem excludes for anything found. In the words of *The View from Inside*: "What curves space is what a comparison can find. The vacuum's total energy is taken in as a whole by no comparison, and taken as a whole it would be perfectly symmetric and perfectly continuous at once, which no comparison finds". So the total curves nothing, whatever its size, and the factor of 10^{123} needs no cancellation. The energy of the vacuum is real and of the size quantum field theory gives; the account changes what that energy does to space.

Sources: *The View from Inside*, Chapter 10 ("What Curves Space", "A Cup from the Sea" and "The Shape That No Comparison Finds"), and Chapter 8. Standing: derived, given a stated identification (that what a measurement receives of the vacuum's energy is what is held by the degrees of freedom it engages).

> ***A cup from the sea.***
>
> A cup dipped into the sea holds real sea water, from which a great deal can be learned, but no cup ever holds the sea. A measurement of the vacuum is in that position, and more so: the instrument is made of the very fields whose energy it measures, so it is a cupful of the sea trying to know the rest of the sea. The share it takes in is found as a count of whole degrees of freedom, a step at a time, and is the same for every comparer: it is discontinuously symmetric, one of the two characters the Incommensuration Theorem allows. The total, taken whole, is not found at all.

G-II. The Information Deficit Formula

G-007 The Identification. That *The Gravitational Passage* identifies the cosmological constant with the share of the vacuum's energy that an observer inside the vacuum can reach: "I identify the cosmological constant with the share of the vacuum energy that is accessible to an observer inside the vacuum, not with the total" (Chapter 13, Section II). The total is the external value, the vacuum energy density at the scale of the modal boundary; the observed share is the internal value; the internal value divided by the external value is the accessible share; and what the share leaves out is the information deficit, the price of being inside.

Sources: *The Gravitational Passage*, Chapter 13, Section II; *The View from Inside*, Chapter 11 ("The Share We Find"). Standing: an identification.

G-008 The External Value. That the external value has a ground-state part and a classical part. The ground-state part is the vacuum energy of the quantum fields at the scale of the modal boundary, taken as M_{Pl}^4, the Planck density. The internal space, the six small dimensions $\mathbb{C}P^2 \times S^2$ held a few Planck lengths across (*The Concept Sequence*, Chapter 25), has a floor: a least extent, with a least motion that cannot be removed (*The Gravitational Passage*, Chapter 11, Section II). The energy of that floor is a zero-point energy, and it is counted within the ground-state part. The classical part is the bare curvature term of the Kaluza-Klein reduction, the step that integrates the ten-dimensional action over the internal space to give the four-dimensional equations (*The Gravitational Passage*, Chapter 11, Section III). That term carries no $\hbar$ and has the sign of anti-de Sitter space, the sign of a constant term that draws space together; at the derived radius, where each internal direction sees the Planck curvature (*The Gravitational Passage*, Chapter 9, Section IX), it is $-3/(8\pi)\ M_{Pl}^4$. The external value is therefore $f\ M_{Pl}^4$, with $f = 1 - 3/(8\pi) = 0.881$. The reduction on a positively curved internal space produces the bare curvature term, so the

bare curvature term belongs to the external value as a consequence of the reduction, and including the bare curvature term in the external value is no further choice.

Sources: *The Gravitational Passage*, Chapter 13, Section II and Questions; Chapter 11, Sections II and III; Chapter 9, Section IX; *The View from Inside*, Appendix A, Section II. Standing: derived (f, from the bare curvature term at the derived radius); an identification (the ground-state part as exactly M_{Pl}^4, and the floor's energy counted within it).

G-009 Entropy and the Count of a Horizon. That entropy, in physics, is a measure of the number of microscopically different arrangements compatible with the same macroscopic description, and equivalently of the information needed to specify the microscopic state. That the Bekenstein-Hawking entropy of a horizon of area A is $S = A/(4l_{Pl}^2)$, with l_{Pl} the Planck length (Bekenstein 1973; Hawking 1975), which for a sphere of radius R is $S = \pi R^2/l_{Pl}^2$, the count of the independent degrees of freedom within the horizon. The count is stated in nats, the natural unit of information (about 1.44 bits), the measure in which $dE = T\,dS$ holds with Boltzmann's constant equal to one.

Sources: *The Gravitational Passage*, Chapter 13, Section II; *The View from Inside*, Chapter 12 ("Information Written on a Surface"), and Appendix A, Section III. Standing: a definition.

> ***Why area, not volume?***
>
> In ordinary experience a bigger box holds more information. For a black hole the count is set by the boundary: packing more information into a region than its boundary area permits forms a black hole, whose entropy is a quarter of its horizon area in Planck units. For a sphere of radius R the area is $4\pi R^2$, so the count is $S = 4\pi R^2/(4l_{Pl}^2) = \pi R^2/l_{Pl}^2$.

G-010 The Particle Horizon. That the radius R in the count is the radius of the observer's particle horizon, the bound of everything that has reached the observer since the expansion began. The particle horizon is the only horizon a first-person measurement holds; the event horizon, the

bound of everything that will ever reach the observer, is defined by the future, which no present measurement contains. The particle horizon is not a thermodynamic horizon in the sense of Gibbons and Hawking (a horizon with its own temperature, such as the event horizon of a universe of vacuum energy alone; Gibbons and Hawking 1977), and *The Gravitational Passage* assigns it the Bekenstein-Hawking count by an identification: "the bound of what has been compared is the bound of the information available to the comparison" (Chapter 13, Section II). With the Planck 2018 parameters (entry G-001) the particle horizon is 46.1 billion light years, and its count is $S = 2.29 \times 10^{123}$.

Sources: *The Gravitational Passage*, Chapter 13, Sections II and IV; *The View from Inside*, Chapter 12 ("Four Edges" and "The Edge of What Has Reached Us"). Standing: an identification (the particle horizon carries the Bekenstein-Hawking count); a measured input (its radius and count, from the Planck 2018 parameters).

G-011 The Three Channels. That a measurement registers through three channels, one for each content it registers in the three columns of the grid of measures: "What is measured is content; the arena in which it is measured is context. Each content metric is measured in its own context: pattern in space, force in time, probability in possibility" (*The Gravitational Passage*, Chapter 7, Section I, as quoted in Chapter 13, Section II). The content row has exactly three members, one in each modal role (the omniscient, the immanent and the transcendent, the three positions of comparison), so the channels are three. Each column is counted once, by its content: in each column the context is the arena in which the measurement registers the content, and the same measurement registers no second thing there. In the internal space the three channels are the three classes of $\mathbb{C}P^2$ taken with one class of S^2, the class of the content row: "The classes that share one class of S^2 number three, whichever class of S^2 it is: $\chi(\mathbb{C}P^2) \times 1 = 3$" (Chapter 13, Section II). The count of channels is the Euler characteristic $\chi(\mathbb{C}P^2) = 3$, the number of independent classes of $\mathbb{C}P^2$ (*The Concept Sequence*, entry 25.20). It is written c_1 because on every complex

projective space the count of alternatives, the Euler characteristic and the coefficient of the first Chern class (a topological invariant of a complex space) are equal (Milnor and Stasheff 1974); the count is the count of the three positions of comparison (*The Concept Sequence*, Chapter 26, entry 26.10).

Sources: *The Gravitational Passage*, Chapter 13, Section II and Questions; Chapter 7, Sections I and III; Chapter 9, Sections I and IV; *The View from Inside*, Chapter 12 ("Why Three"). Standing: an identification (the channels as the three contents a measurement registers); a theorem ($\chi(\mathbb{C}P^2) = 3$, equal to the coefficient of the first Chern class of $\mathbb{C}P^2$).

G-012 The Accessible Share. That the accessible share of the external value is c_1/S, with $c_1 = 3$: one degree of freedom for each channel, among the S degrees of freedom the horizon counts. The share rests on three steps. First, the S degrees of freedom hold the external value in equal parts: the external value is a density, the same at every place, and the count is proportional to the area of the horizon with one constant, so degrees of freedom that nothing distinguishes hold equal parts. Once the two premises (the energy within the horizon is the sum of their parts, and nothing distinguishes one from another) are granted, that any k of them hold exactly k/S of the external value is a theorem, whatever the correlations among them. Second, the channels are three (entry G-011). Third, the comparer is one stance, so the degrees of freedom a measurement engages are counted once, and each channel engages a different degree of freedom, since the three contents are distinct (Axiom III: the three modal roles are distinct, inseparable and non-interchangeable). A measurement then engages three of the S degrees of freedom and receives c_1/S of the external value. The correlation between observer and vacuum (entry G-005) is the reason a measurement receives only the parts held by the degrees of freedom it engages; the number of those degrees of freedom sets the size of what it receives. The share is of a count, not a fluctuation in one: a share of $1/\sqrt{S}$ would be the relative size of a statistical fluctuation in a count of S.

Sources: *The Gravitational Passage*, Chapter 13, Section II and Questions; *The View from Inside*, Chapter 11 ("Tiles on the Horizon" and "A Count, Not a Coin Toss"). Standing: an identification (each of the three steps); a theorem (the ratio k/S, on the two premises stated).

G-013 The Information Deficit Formula. That the vacuum density accessible to an observer inside the vacuum is $\rho_\Lambda = c_1 \, f \, M_{Pl}^4 \, / \, S$, and $\Lambda = 8\pi G_N \rho_\Lambda$. The formula uses two numbers from the structure of comparison, $c_1 = 3$ and $f = 1 - 3/(8\pi)$, and two from measurement: Newton's constant, which fixes the Planck mass, and the horizon. The suppression of the observed density relative to the Planck density is about 10^{-123}, the reciprocal of the horizon count, because the comparer is one stance inside the horizon, engaging the S degrees of freedom of the horizon's count through three channels, a different one through each.

Sources: *The Gravitational Passage*, Chapter 13, Section II, Equation (13.3); *The View from Inside*, Chapter 11 ("The Equation, Piece by Piece"). Standing: an identification.

> ***Written symbolically.***
>
> $\rho_\Lambda = c_1 \, f \, M_{Pl}^4 \, / \, S$, with $S = \pi R^2 / l_{Pl}^2$, $f = 1 - 3/(8\pi)$, $c_1 = 3$; $\Lambda = 8\pi G_N \rho_\Lambda$.

G-014 The Formula Evaluated. That with the Planck 2018 parameters used as one set (entry G-001), the formula gives $\rho_\Lambda = 3 \times 0.881 \times (2.222 \times 10^{76} \text{ GeV}^4) \, / \, (2.29 \times 10^{123}) = 2.56 \times 10^{-47} \text{ GeV}^4$, against the observed $2.52 \times 10^{-47} \text{ GeV}^4$ from the same parameters (Planck Collaboration 2020a). The predicted density lies about 1.7 percent above the observed one, within the ± 2.8 percent that the measured uncertainty of Ω_m, ± 0.007, gives the ratio. With the ground-state part of the external value alone ($f = 1$) the predicted density is $2.91 \times 10^{-47} \text{ GeV}^4$, about fifteen percent above the observed one. With one set of parameters the ratio of the predicted density to the observed one is exactly $8f/(I^2\Omega_\Lambda)$, with I the radius of the particle horizon in units of the Hubble radius (entry G-016), evaluated at

the measured Ω_m, and Ω_Λ the vacuum fraction (entry G-015); so this comparison and the comparison of the matter fraction in entry G-018 are one test, seen two ways.

Sources: *The Gravitational Passage*, Chapter 13, Section IV, Equations (13.9) and (13.10); *The View from Inside*, Chapter 13 ("The Same Number, Seen from Other Sides"), and Appendix C ("Results"). Standing: a measured agreement.

G-III. The Self-Consistency Identity

G-015 Density Parameters. That cosmologists describe the energy content of the universe by fractions of the critical density, the total density that a spatially flat universe expanding at the measured rate H_0 must hold, $3H_0^2/(8\pi G_N)$: the matter fraction Ω_m (ordinary and dark matter together), the radiation fraction Ω_r (photons and neutrinos while relativistic; $\Omega_r \approx 9 \times 10^{-5}$ at present), the vacuum fraction Ω_Λ, and the curvature term Ω_K. In a spatially flat universe $\Omega_m + \Omega_r + \Omega_\Lambda = 1$. The measured matter fraction is 0.315 ± 0.007 (Planck Collaboration 2020a).

Sources: *The Gravitational Passage*, Chapter 13, Section III; *The View from Inside*, Chapter 13 ("The Second Way: The Astronomers' Way"). Standing: a definition (the density parameters); a measured input (the matter fraction 0.315 ± 0.007).

G-016 The Conformal Time Integral. That the dimensionless conformal time integral is $I(\Omega_m) = \int_0^\infty dz \,/\, E(z)$, with $E(z) = \sqrt{(\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4)}$ the dimensionless Hubble function and z the redshift (light reaching us from redshift z has had its wavelength stretched by the factor 1+z on the way). I is the comoving distance light has traveled over the whole expansion, each stretch of the path counted at its size today, in units of the Hubble radius c/H_0, so the particle horizon is $R = I/H_0$ in units with $c = 1$.

Sources: *The Gravitational Passage*, Chapter 13, Section III, Equation (13.4); *The View from Inside*, Appendix C ("The Identity"). Standing: a definition.

> ***Reading the integral.***
> The integral adds up the path of light era by era, from today ($z = 0$) back to the beginning ($z \to \infty$). The expression under the root says how fast the universe was expanding at redshift z compared with today: the vacuum's share stays the same; the matter's share grows as $(1+z)^3$, because the universe was then $1+z$ times smaller in each of three directions; the radiation's share grows as $(1+z)^4$, one factor more, because light is also stretched.

G-017 The Identity. That the formula and the Friedmann equation must agree: the Λ the formula gives must reproduce the expansion history from which the horizon is computed. With $S = \pi R^2 M_{Pl}^2$ and $R = I/H_0$ the formula gives $\rho_\Lambda = c_1 f M_{Pl}^2 H_0^2/(\pi I^2)$; the Friedmann equation, the equation of general relativity for a uniform expanding universe (Friedmann 1922), gives $\rho_\Lambda = \Omega_\Lambda \times 3H_0^2 M_{Pl}^2/(8\pi)$. Equating the two, the Hubble constant and the Planck mass cancel, and $I^2\Omega_\Lambda = 8c_1 f/3 = 8f = 8 - 3/\pi$. The 8π of the Einstein equations, divided by the π of the horizon entropy, gives 8; the three channels, $c_1 = 3$, cancel the 3 of the critical density; and $3/\pi$ is the share of the bare curvature of all six internal directions, which Chapter F, entry F-016, reads as three channels among the count π of a horizon one Planck length in radius. With $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$ for a spatially flat universe (entry G-027 gives the flatness), the identity is one equation in one unknown, Ω_m.

Sources: *The Gravitational Passage*, Chapter 13, Section III, Equations (13.5) to (13.7); *The View from Inside*, Chapter 13 ("Setting the Two Equal"). Standing: derived, given a stated identification (the accessible share, entries G-007 to G-013).

G-018 The Matter Fraction. That the solution of the identity is $\Omega_m = 0.3108$ (stated as 0.311) and $\Omega_\Lambda = 0.689$, against the measured 0.315 ± 0.007 (Planck Collaboration 2020a), 0.6 standard deviations away. The baryon acoustic oscillations (a ripple of fixed size in how galaxies are spread

through space, left by sound waves in the early universe and used as a ruler), as measured by the Dark Energy Spectroscopic Instrument alone, give 0.2975 ± 0.0086 in the same model (DESI Collaboration 2025); the prediction lies 1.6 standard deviations from that value, on the other side, so it lies between the two measurements. The measured radiation fraction, from 7×10^{-5} to 10^{-4}, moves the solution from 0.3119 to 0.3103, so this input gives $\Omega_m = 0.311 \pm 0.001$. With the ground-state part alone ($f = 1$) the identity is $I^2\Omega_\Lambda = 8$, with solution $\Omega_m = 0.280$. Read the other way, the comparison with 0.315 ± 0.007 measures the coefficient of the ground-state part as 0.99 ± 0.02 of M_{Pl}^4. The identity fixes the total matter fraction and says nothing of what the matter is made of; the split between dark and ordinary matter is the registered ratio $\Omega_c/\Omega_b = 6 - 2/\pi$ of Chapter F, entry F-019.

Sources: *The Gravitational Passage*, Chapter 13, Sections III, IV and VII, Equation (13.8); *The View from Inside*, Chapter 13 ("One Equation, One Unknown"), and Appendix C ("The Identity" and "Sensitivities"); *The Gravitational Companion*, "Comparisons at Single Steps", Section VIII. Standing: a measured agreement.

> ***The numbers of the identity.***
> $I^2\Omega_\Lambda = 8 - 3/\pi = 7.045$. At the solution, $\Omega_\Lambda = 0.689$ and $I = \sqrt{(7.045/0.689)} = 3.20$. With the Planck 2018 parameters themselves, $I = 3.181$, the particle horizon is 46.14 billion light years, the de Sitter radius $r_\Lambda = \sqrt{(3/\Lambda)}$, the radius of a universe holding the vacuum energy alone, is 17.53 billion light years, and the horizon count is $S = 2.292 \times 10^{123}$.

G-019 The Same Test, Seen from Other Sides. That the identity can be stated without Planck units: it says that the present particle horizon is $2\sqrt{(2f)} = 2.65$ times the de Sitter radius, against $46.1/17.5 = 2.63$ from the measured parameters; and that the age of the universe is 0.792 de Sitter times ($1/H_\Lambda$, with $H_\Lambda = \sqrt{(\Lambda/3)}$), against 0.787 ± 0.009 from the measured parameters. With the matter density in physical units that the microwave background measures, $\Omega_m h^2 = 0.1430 \pm 0.0011$ (Planck Collaboration 2020a), the identity gives $h = 0.678 \pm 0.003$; Chapter I, entry I-014,

carries this Hubble constant with the radiation fraction solved together. Each of these three statements is the matter-fraction test of entry G-018, stated through another measurement.

Sources: *The Gravitational Passage*, Chapter 13, Sections III, IV and VII; *The View from Inside*, Chapter 13 ("The Same Number, Seen from Other Sides"), and Appendix C ("Results"); *The Gravitational Companion*, "Comparisons at Single Steps", Section VIII. Standing: a measured agreement (the same test as entry G-018).

G-020 What the Other Choices Give. That the formula rests on five choices, each with a reason that does not refer to the comparison with measurement: the particle horizon (entry G-010), the count of three channels (entry G-011), the classical part of the external value (entry G-008), the floor's energy counted within the ground-state part (entry G-008), and the full Planck mass (entry G-002). With four horizons (the Hubble radius, the event horizon, the de Sitter radius and the particle horizon), five integers in place of c_1 (1, 2, 3, 4 and 6), the ground-state part alone or with the classical part, the floor's energy counted within the ground-state part or outside it, and the full or the reduced Planck mass, there are 160 combinations; one of them gives a density within the measured interval ($\pm$2.8 percent in the ratio), the combination the reasons select, at 1.017 times the observed density; the next nearest is the same combination with the floor's energy counted outside, at 1.10. In *The View from Inside* I also count the integer 5, which gives $4 \times 6 \times 2 \times 2 \times 2 = 192$ combinations. Among the 192 combinations, the same one combination lands within the measured interval (The View from Inside, Appendix A, Section IV). Other counts give other matter fractions: six channels give 0.163, five give 0.197, two give 0.418, and one bit for each channel in place of one nat gives 0.407, against the 0.311 of three channels counted in nats.

Sources: *The Gravitational Passage*, Chapter 13, Section II and Questions; *The View from Inside*, Appendix A, Sections III and IV. Standing: derived (the value each other choice gives, computed from the formula and the identity); an identification (the choices themselves, entries G-002 and G-008 to G-012).

Predicted over observed density, with the ground-state part alone, for each horizon and integer.

Hubble radius: 3.9 (1), 7.8 (2), 11.7 (3), 15.6 (4), 19.5 (5), 23.4 (6).

Event horizon: 2.9 (1), 5.9 (2), 8.8 (3), 11.8 (4), 14.7 (5), 17.7 (6).

De Sitter radius: 2.7 (1), 5.3 (2), 8.0 (3), 10.7 (4), 13.3 (5), 16.0 (6).

Particle horizon: 0.38 (1), 0.77 (2), 1.15 (3), 1.54 (4), 1.92 (5), 2.31 (6).

The entry the reasons select, the particle horizon with 3, is 1.15, and the classical part of the external value brings it to 1.017. The reduced Planck mass in place of the full one divides every entry by $(8\pi)^2 \approx 632$.

G-021 The Measurement That Decides the Share. That the matter fraction is the measurement that decides the accessible share. The account predicts $\Omega_m = 0.311 \pm 0.001$, and the coming surveys (the Dark Energy Spectroscopic Instrument, the Euclid space telescope, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope) will narrow the measured value toward an uncertainty of about 0.001. The identification of the accessible share has four parts: the count S of the particle horizon, the equal parts of the external value, the count of three channels, and one degree of freedom for each channel (*The Gravitational Passage*, Chapter 13, Section III). The prediction of the matter fraction takes three coefficients of order one, each at a stated value. The ground-state part of the external value is taken as exactly M_{Pl}^4, and the comparison with 0.315 ± 0.007 reads it back as 0.99 ± 0.02. The internal curvature is taken as exactly the Planck curvature (*The Gravitational Passage*, Chapter 9, Section IX); the registered ratio of dark to ordinary matter shares this coefficient, and Chapter F, entry F-020, reads it from that ratio as 0.99 ± 0.08. The bare curvature term is taken from the Einstein-Hilbert term of the reduction, the one term of the ten-dimensional action linear in curvature; the classical energy of the higher Lovelock terms (the terms of that action of higher order in curvature) would shift it, and f with it (*The Gravitational Passage*, Chapter 11, Section I). A change of one percent in f moves Ω_m by ∓ 0.0025, so the measured ± 0.007 corresponds to about ± 3 percent in f. The predic-

tion uses the identification that counts the floor's energy within the ground-state part: counted outside, its first-order energy, $+0.071\ M_{Pl}^4$, would give $f = 0.951$ and $\Omega_m = 0.292$, about three standard deviations below the Planck value.

Sources: *The Gravitational Passage*, Chapter 13, Sections III, IV and VII, and Questions; Chapter 11, Section I; *The View from Inside*, Appendix B ("The Matter Fraction"), and Appendix C ("Sensitivities"). Standing: a fork that a named measurement decides (the matter fraction, measured by the coming surveys); an identification (the three coefficients at their stated values, and the floor's energy counted within the ground-state part).

G-IV. One Constant, One History

G-022 One Geometry, One Constant. That the field equations themselves require Λ to be one number. In $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ the divergence of the Einstein tensor $G_{\mu\nu}$ vanishes by the contracted Bianchi identity, an identity of the curvature of every geometry; the divergence of the metric $g_{\mu\nu}$ vanishes; and the stress-energy $T_{\mu\nu}$, the density and flow of the content's energy and momentum, is conserved. The divergence of the left side is then the gradient of Λ, and the equations set that gradient to zero, so within one geometry Λ does not vary, from place to place or from epoch to epoch. That Λ does not vary within one geometry is a theorem of the geometry, and the theorem uses no measurement and no value of w. A vacuum energy that is the same for every comparer in uniform motion has a stress-energy proportional to the metric, so the accessible share enters the field equations as that constant term, the one constant term that Lovelock's theorem allows: in four dimensions the only field equations of second order built from the metric alone are Einstein's, with one constant term (Lovelock 1971).

Sources: *The Gravitational Passage*, Chapter 13, Section VI; Chapter 12, Section VII; *The View from Inside*, Chapter 15 ("One Map, One Scale"). Standing: a theorem.

G-023 The Comparison That Fixes Λ. That the comparison that fixes Λ is the present measurement, at the particle horizon of that measurement. To say that something exists is to say that it stands as structure; to say that something is real is to say that it is given in an actual act, occurring now (*The Gravitational Passage*, Chapter 5, Section II). Λ, the constant term of the law of the one geometry, exists; the value of the formula at the particle horizon of a measurement is given in that act, so it is real. Axiom I makes the immanent act, the comparison occurring now, prior to the structure reconstructed from it: the history, with its epochs, is reconstructed from the impression the past makes on the present, and the values the formula would give at the particle horizons of other epochs exist as structure, are not real, and fix nothing. "So Λ is fixed once, by the present measurement, as the constant term of one geometry, and the reconstructed history carries that value at every epoch, before the present and after it" (*The Gravitational Passage*, Chapter 13, Section VI). An observer at redshift 1 appears in the reconstructed history with the records she made, and her records carry the one constant, Λ. The formula, applied on paper to her particle horizon, gives 7.0 times Λ, a number computed from the structure of the history that no record carries as a finding. The formula is therefore a statement about the one act that is real, and the identity is a statement about the epoch of that act.

Sources: *The Gravitational Passage*, Chapter 13, Sections V and VI; Chapter 5, Section II; *The View from Inside*, Chapter 14, and Chapter 15 ("Which Measurement Fixes It" and "An Astronomer Long Ago"). Standing: derived (from Axiom I and the definition of the real, with no further identification).

G-024 Why the Particle Horizon and No Other. That only the particle horizon can carry the value that one geometry carries, and the same reasoning fixes nothing on the Hubble radius or on the event horizon. In a history with a positive constant and with matter and radiation of positive

density, the expansion rate exceeds H_Λ at every epoch, so the Hubble radius and the event horizon are each smaller than the de Sitter radius at every epoch; on either of them the value of the formula exceeds $8f = 7.05$ times the constant at every epoch, and no measurement could find on them a value that one geometry carries. Only the particle horizon grows past the de Sitter radius. This second reason for the particle horizon rests on the coefficients ($8c_1f/3 = 8f$ exceeds 1), and it stands apart from the first reason, that the particle horizon is the only horizon a first-person measurement holds (entry G-010).

Sources: *The Gravitational Passage*, Chapter 13, Section VI; *The Gravitational Companion*, "Comparisons at Single Steps", Section VII; *The View from Inside*, Chapter 12 ("A Second Reason, Quite Separate"). Standing: derived.

G-025 Why the Present Epoch. That with one Λ, $I^2\Omega_\Lambda = (R/r_\Lambda)^2$ at every epoch, and R only grows, so the identity holds at one epoch only, and the identity predicts that the present is that epoch. The question why the present falls there has three parts, answered each in its own way. The ratio of the present age to the de Sitter time, 0.792, is given by the identity and tested by measurement (entry G-019). The absolute epoch, about 10^{61} Planck times after the beginning of the reconstructed history, is a question about when comparers exist, and so about the content, which the account takes from the measured particle horizon. Why the real act stands at the one epoch where the identity holds, of all the epochs at which comparers can stand, is answered from Axiom I: the question takes the presents of all epochs as given together, and Axiom I does not grant that; the step rests on Axiom I for that part alone. On this account the nearness of the vacuum and matter densities at the present, a long-standing puzzle of cosmology, is the identity, holding at the epoch of the real act.

Sources: *The Gravitational Passage*, Chapter 13, Section VI and Questions; *The View from Inside*, Chapter 15 ("Why Now?"). Standing: derived (the ratio of the present age to the de Sitter time, from the identity, and the third part, from Axiom I); a measured input (the absolute epoch, from the measured particle horizon).

G-026 Equation of State. That the equation of state of the cosmological constant, w, the ratio of its pressure to its energy density (Chapter F, entry F-033, defines w for every component), is $w = -1$ exactly: the formula fixes Λ once, from the horizon of the present measurement, as the constant term of one geometry, which the reconstructed history carries at every epoch; no dynamical field, no quintessence (a slowly changing field proposed as the source of the acceleration) and no evolving scalar drives the acceleration. Planck 2018, combined with baryon acoustic oscillations and Type Ia supernovae, gives $w = -1.03 \pm 0.03$ (Planck Collaboration 2020a). The final analysis of the Dark Energy Survey with its own supernovae, galaxy clustering and weak lensing, combined with the Dark Energy Spectroscopic Instrument and the microwave background, gives $w_0 = -0.82 \pm 0.05$ and $w_a = -0.63$ $(+0.21/-0.18)$, about three standard deviations from a cosmological constant (Abbott and others 2026b); without the supernovae the same analysis finds 2.6 standard deviations. Standard cosmology also takes $w = -1$; I fix $w = -1$ exactly, by structure, and so my account parts from every account in which the vacuum's energy drifts. I predict that the reported departure from a cosmological constant will not be confirmed. The coming surveys decide whether the reported departure is confirmed, and I count a result as settled at five standard deviations by two independent kinds of measurement.

Sources: *The Gravitational Passage*, Chapter 13, Section V; Chapter 18, Section XIII; *The View from Inside*, Chapter 15 ("Exactly Constant"), and Appendix B ("The Constancy of the Vacuum's Energy"). Standing: derived ($w = -1$); a fork that a named measurement decides (the reported departure, by the coming surveys).

G-027 Spatial Flatness. That the homogeneous model of the epoch is spatially flat. The turn is the limit between one epoch of the universe and the next, which the history reconstructed from inside the new epoch finds as its Big Bang (*The Gravitational Passage*, Chapter 18, Sections VII and VIII). A radius of spatial curvature is a length, and the restoration of the context at the turn leaves the new epoch two lengths of its own: the length

of the turn, of the order of the Planck length, and the de Sitter radius that the reconstructed history carries. A radius of the Planck length would close the new epoch within a few Planck times or leave the new epoch dominated by curvature. Neither the closed epoch nor the epoch dominated by curvature produces the structure that the cycle of epochs requires. A radius equal to the de Sitter radius leaves a curvature of about 10^{-64} today. "The homogeneous model of the new epoch is therefore spatially flat" (*The Gravitational Passage*, Chapter 18, Section IX), and the geometry found is curved only by its departures from uniformity, at their own size, about 10^{-5}. The measured curvature agrees: $\Omega_K = 0.001 \pm 0.002$ (Planck Collaboration 2020a, with the baryon acoustic oscillations). The identity of entry G-017 uses this flatness.

Sources: *The Gravitational Passage*, Chapter 18, Sections IX and XIII; Chapter 13, Section III. Standing: derived (the flatness of the homogeneous model); a measured agreement (Ω_K).

G-028 Horizon Proposals Compared. That the idea that the vacuum energy is set by the size or entropy of a horizon has a literature: Fischler and Susskind (1998), on holographic bounds in cosmology; Cohen, Kaplan and Nelson (1999), who bounded the vacuum energy by the size of the region; holographic dark energy (Hsu 2004; Li 2004), in which an energy that tracks a horizon through time is proposed; Padmanabhan and Padmanabhan (2013; 2017), who fix Λ by a count of 4π units of information at the horizon; Banks (2001) and Bousso (2000), on Λ as the inverse of the number of states; and Sorkin's everpresent Λ, which reaches the same order of magnitude by a fluctuation of order $M_{Pl}^4/\sqrt{N}$, with N the four-volume of the universe in Planck units (Sorkin 1997; Ahmed, Dodelson, Greene and Sorkin 2004), and in which Λ fluctuates from epoch to epoch. I take the coefficient of the information deficit formula from the three channels of comparison. I fix Λ once, by the horizon of the measurement, and I do not carry Λ through time. Allowed to track the particle horizon, the formula would be holographic dark energy on that horizon, $\rho = 3c^2M^2/L^2$ in the form of Li (2004), with M the reduced Planck mass, L the

radius of the horizon and $c^2 = 8f$; it would give $w = -1/3 + 2/(3I)$, about -0.12 today (-0.124 with the measured $I = 3.181$, and -0.125 at the identity's solution), which the data exclude, and since its w exceeds $-1/3$ it would never make the expansion accelerate. Allowed to follow the event horizon, the formula would give $w = -0.54$.

Sources: *The Gravitational Companion*, "Comparisons at Single Steps", Section VII; *The View from Inside*, Chapter 3 ("A Clue at the Horizon"), Chapter 15 ("Exactly Constant"), Appendix A, Section V, and Notes, Part I. Standing: derived (the values of w for the tracking forms).

G-029 What Being Inside Withholds. That I give two answers, at different depths, to the mismatch of entry G-003. The structural answer: the total energy of the vacuum curves nothing, since no comparison finds it whole, so the factor of 10^{123} needs no cancellation (entry G-006). The numerical answer: an observer inside the vacuum engages three of the S degrees of freedom that hold its energy, and that share enters Einstein's equations as their one constant term, Λ (entry G-013). The constant is small because the observer is one stance inside a large horizon, and its smallness is the measure of what being inside withholds. The observed vacuum density is set by four things: the number of roles in a comparison supplies the three channels, $c_1 = 3$, and the curvature of the internal space supplies the trim f, the factor by which that curvature lowers the vacuum's total, both from the structure of comparison; the strength of gravity sets the vacuum's total, and the size of the particle horizon sets the count, both measured. The matter fraction, which the identity gives, depends on neither measured quantity, since both cancel from it.

Sources: *The View from Inside*, Chapter 16 ("Two Answers" and "What Being Inside Withholds"); *The Gravitational Passage*, Chapter 13, Section VII. Standing: an identification (the accessible share, and the channels as the three contents a measurement registers); a theorem ($\chi(\mathbb{C}P^2) = 3$, which gives c_1); derived (f); a measured input (G_N and the horizon); a measured agreement (the predicted density and the matter fraction).

Summary

This chapter is the branch of the map that carries the cosmological constant. Its results belong to *The View from Inside* and to *The Gravitational Passage*, Chapter 13, with the flatness of the epoch from Chapter 18 of the Passage and the comparison with other horizon proposals from *The Gravitational Companion*.

Its first part states the problem and the ground on which the answer stands. Quantum field theory gives empty space an energy near the Planck density, about 10^{123} times the density the sky shows. A measurement of the vacuum is made from inside the vacuum, by instruments made of the same fields and entangled with them, and *The Gravitational Passage* identifies the information deficit this leaves as an instance of the Incommensuration Theorem. *The View from Inside* derives, given its stated identification, that the vacuum's total energy curves nothing: what curves space is what a comparison can find, and no comparison finds the total whole. The 10^{123} therefore needs no cancellation.

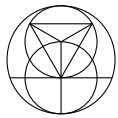
The second part gives the number. The cosmological constant is identified with the accessible share: three degrees of freedom, one for each channel of a measurement, among the S that the particle horizon counts, taken of an external value f M_{Pl}^4 in which the bare curvature of the internal space lowers the Planck density by $3/(8\pi)$. Each choice in the formula has its reason before any number is computed, and of the 160 combinations of the alternatives (192 with the integer 5) one lands within the measured interval, the one the reasons select. With the Planck 2018 parameters the formula gives 2.56×10^{-47} GeV^4 against the observed 2.52×10^{-47}.

The third part closes the circle. The share must agree with the Friedmann equation, and in agreeing the Hubble constant and the Planck mass cancel, leaving the identity $I^2\Omega_\Lambda = 8 - 3/\pi$, one equation in pure numbers. Its solution, a matter fraction of 0.311 ± 0.001, stands against the

measured 0.315 ± 0.007, and the same test reads, from other sides, as the ratio of the particle horizon to the de Sitter radius, the age of the universe in de Sitter times and a Hubble constant of 67.8. The matter fraction is the measurement that decides the accessible share, and the coming surveys will measure the matter fraction to about 0.001.

The fourth part says why the constant is one number. The field equations require Λ to be one number in one geometry, a theorem that uses no measurement. The present measurement is the one real act, so it fixes Λ, and the reconstructed history carries that value at every epoch; the particle horizon is the one horizon on which that value can be carried at all. From this follow an equation of state of exactly -1 and the answer to why the matter and vacuum densities are comparable now, and the restoration at the turn gives the flatness the identity uses.

Taken together, the chapter shows the smallness of the cosmological constant as the measure of the observer's place: two numbers from the structure of comparison, $c_1 = 3$ and f, and two from measurement, Newton's constant and the horizon, set the energy of empty space, and the matter fraction, from which both measured quantities cancel, is the pure number by which measurement tests the account.



Chapter H

Baryogenesis

CHAPTER OVERVIEW

This chapter traces the excess of matter over antimatter and what fixes that excess. The kinds of particle are the species of particle that the structure of comparison gives, each the same wherever and whenever it is found, and their list is closed. The amount of matter is a residue left from the earliest moments, kept by every change after them: no process among the kinds makes it, and the kinds fix only the share of it held as baryons (*The Coming of the Kinds*, Chapters 5 and 7; *The Story of the Particles*, Level 4, Section IV). In this chapter I follow that result about the amount of matter in five parts. The first states what the excess is and how it is measured. The second sets the kinds beside the three conditions that Andrei Sakharov set out for an excess to develop from none. The third shows that every process among the kinds keeps the baryon number less the lepton number, written $B - L$, so that the excess present today requires a value of $B - L$ present from the earliest moment our history reaches. The fourth reads the share of that residue that the kinds fix as baryon number. The fifth follows the first minutes, in which the light nuclei were made and the number of baryons per photon is measured a second time.

In this chapter I rely on *The Concept Sequence*, Chapter 35, The Lepton Sector; Chapter 36, Antiparticles; Chapter 38, The Higgs Boson; Chapter 40, The Mixing Matrices; Chapter 41, The Completeness of the Kinds; Chapter 42, Proton Stability; and Chapter 44, No Grand Unification.

> Chapter E treats the Higgs field and its transition, and Chapter L treats the turn, the limit that our history, traced backward, takes for its beginning.

H-I. The Excess of Matter over Antimatter

H-001 The Matter-Antimatter Asymmetry. That every particle that carries electric charge, or another additive quantum number (a label that comes in whole units and adds up when things are put together), has an antiparticle distinct from itself, with the same mass and every such number reversed: for every electron a positron, for every quark (a particle of matter that carries color, the label of the strong force) an antiquark, for every proton an antiproton. That the existence of antiparticles is structural, since the structure of comparison requires them: a state of matter is held toward the thing compared and toward the thing compared against, and the same state held the other way round is its antiparticle; the photon, the Z boson and the Higgs boson carry no such number and are each their own antiparticle. That when a particle meets its antiparticle, the particle and the antiparticle annihilate, and the energy of the pair is carried off, most often as two photons. That the observed universe is made of matter: the stars, planets, gas clouds and galaxies are made of matter, and regions of matter and antimatter in contact would annihilate along their boundaries and fill the sky with gamma rays beyond what is observed. That this excess of matter over antimatter is called the baryon asymmetry of the universe, and the account of how it came about is called baryogenesis.

Sources: *The Story of the Particles*, Level 4, Sections I and III; *The Coming of the Kinds*, Chapter 1, Section IV, and Chapter 5, Section VI; *The Branches of the Kinds*, Chapter 1, Section II, and Chapter 3, Section III; Cohen, De Rújula and Glashow (1998). Standing: derived (the existence of antiparticles, from the structure of comparison); a measured input (the dominance of matter).

H-002 Baryon Defined. That a baryon is a whole made of three quarks, one of each color, the proton (two up quarks and a down quark) and the neutron (one up quark and two down quarks) being the baryons of every atomic nucleus. That the baryon number, written B, counts each quark as one third and each antiquark as minus one third, so that each baryon counts plus one, each antibaryon minus one, and every lepton, every carrier of force and the Higgs boson zero. That the lepton number, written L, counts each lepton (a particle of matter that carries no color: the electron, the muon, the tau and their three neutrinos) as plus one and each antilepton as minus one. That the baryon asymmetry of the universe is the statement that the baryon number of the observable universe is positive: there are more baryons than antibaryons.

Sources: *The Coming of the Kinds*, Chapter 1, Section II, and Chapter 4, Sections I and II. Standing: a definition.

> ***How small is the excess?***
>
> Early in our history, when the gathering of particles and light that filled the universe was far hotter than the mass of a proton, quarks and antiquarks were made and unmade in pairs, in nearly equal numbers. For about every billion antiquarks there were about a billion and one quarks. As the gathering cooled, the matched pairs annihilated, and the small surplus remained. That surplus, about one part in a billion, is every proton and every neutron in every star and planet. With no surplus at all, the annihilation would have run until the survivors were too spread out to meet, and would have left about one baryon for every 10^{18} photons, hundreds of millions of times fewer than are measured.

H-003 The Baryon-to-Photon Ratio, a Number of History. That the excess is stated as one number: the number of baryons less antibaryons for every photon of the microwave background (the light left from the early universe, which reaches us from every direction), written η_B (or η) and called the number of baryons per photon. That its value read from the microwave background is $\eta = (6.13 \pm 0.04) \times 10^{-10}$, about six baryons for

every ten billion photons, from the baryon density $\Omega_b h^2 = 0.02237 \pm 0.00015$ measured by the Planck satellite (the density of the baryons in units of the density at which space is flat, times the square of the present expansion rate in units of 100 kilometers per second per megaparsec) and the conversion $\eta = 274 \times 10^{-10}\ \Omega_b h^2$; and that its value read from the light nuclei left from the first minutes is $(6.04 \pm 0.12) \times 10^{-10}$ (entry H-023). That η_B is a number of the history of this universe: it is not an attribute of any kind of particle, since a kind has the same attributes wherever and whenever it is found, and no law of the kinds sets it. That the structure of the kinds is unchanged when the three reversals C, P and T (charge conjugation, parity and time reversal; entries H-005 and H-006) are taken together, so that a gathering in balance (the settled state in which every reaction runs as often as its reverse) that holds no excess of any kept quantity (a total that no reaction among the kinds changes) holds each kind and its antiparticle in equal numbers; the structure by itself gives no excess. That I therefore take η_B from measurement. What the structure of the kinds fixes is the share held as baryons of the residue that the earliest moments leave (entries H-014 and H-018 to H-020).

Sources: *The Story of the Particles*, Level 4, Section IV ("The existence of antiparticles is structural: comparison requires them. The excess of matter is not structural. It belongs to the initial conditions, to the history of the universe, to the facts that the zoo does not fix"); *The Strengths of the Kinds*, Chapter 1, Section I ("A number of this universe's history is not an attribute of any kind of particle, since a kind of particle has the same attributes wherever and whenever it is found"); *The Measures of the Kinds*, Chapter 10, Section I ("a history is a number such as the excess of matter over antimatter, a residue left from the earliest moments"); *The Coming of the Kinds*, Chapter 1, Section VI and Box 1.1, and Chapter 4, Section V; Planck Collaboration (2020a), Table 2; Particle Data Group (2026), review "Big-Bang Nucleosynthesis". Standing: a measured input (η, a number of the history of this universe).

H-II. The Three Sakharov Conditions

H-004 The Sakharov Conditions. That in 1967 Andrei Sakharov set out three conditions that must hold together for an excess of baryons to develop in a universe that starts with none. That the three conditions are: first, some process must change the baryon number; second, the reactions that make the excess, taken together, must treat matter and antimatter differently, which requires that they break C (charge conjugation) and also CP (charge conjugation and parity taken together); and third, the process must run while the universe is out of thermal balance (a departure from equilibrium), so that the reverse reaction does not undo what it makes. That an excess develops from none only where all three hold together.

Sources: Sakharov (1967); *The Story of the Particles*, Level 4, Section IV, and its Note 13; *The Coming of the Kinds*, Chapter 4, Section I, and its Note 1 to Chapter 4. Standing: a theorem (of standard physics: Sakharov 1967).

> ***Why all three are needed.***
>
> Each condition does one job, and no other condition can do it. The first gives a process that changes the baryon number at all: where the baryon number is exactly kept, no process creates or destroys baryons. The second makes matter and antimatter come out differently: where C and CP are exact symmetries, every process that makes a baryon has a CP image that makes an antibaryon at exactly the same rate, and the net production is zero. The third keeps the reverse reaction from undoing the work: in thermal balance every reaction runs as often as its reverse, so any excess of baryons that one process makes is undone by the reverse process. A ratchet behaves in the same way: a toothed wheel with a catch, jostled by a gas, goes nowhere on average while the catch and the gas are at one temperature, however its teeth are cut.

H-005 Charge Conjugation Defined. That the operation called charge conjugation, written C, replaces every particle with its antiparticle: every electron with a positron, every quark with an antiquark, every neutrino

with an antineutrino. That a law is called C-symmetric if it is unchanged under this replacement. That only the weak force breaks a reversal by its form (the way it couples to the particles): the strong force and the force on the phase (the force that acts on the phase every charged particle carries, which is electromagnetism once the Higgs field takes its value) break none by their form, and in particular keep C, so the force between two quarks is the same as the force between two antiquarks. That the weak force breaks C by its form: the weak force acts only on the left-handed component of every particle (the component that turns one way about the particle's direction of motion) and on the right-handed component of every antiparticle. So the image of a left-handed neutrino under C, a left-handed antineutrino, is not acted on by the weak force.

Sources: *The Branches of the Kinds*, Chapter 3, Section II (Rule 3 and Box 3.3) and Section III; *The Coming of the Kinds*, Chapter 4, Section III. Standing: a definition (C); derived (that only the weak force breaks a reversal by its form: Rule 3 of *The Branches of the Kinds*, argued in its Box 3.3); a measured agreement (the breaking of C by the weak force).

H-006 Parity and CP Defined. That the operation called parity, written P, exchanges left and right, the two senses of a turning: parity is the operation of looking at the world in a mirror. That the weak force violates parity as fully as a force can, since it acts on only the left-handed component of every fermion (a particle of matter, a quark or a lepton) and the right-handed component of every antifermion; the decay of the cobalt-60 nucleus shows that preferred handedness. That CP is the combined operation of charge conjugation and parity, and that the form of the weak force keeps CP: the image of a left-handed neutrino under CP is a right-handed antineutrino, which the weak force does act on. That the Incommensuration Theorem, the structural fact about the nature of knowing that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, limits where a process and its image under CP can differ. The strong force and the force on the phase, whose acts stay wholly within content (what a comparison finds), treat a process and its image under CP alike. For the weak force, which alone carries a

state across from content to context (the setting in which content is found), a difference between a process and its image under CP is left possible. That this remaining possibility is realized by one phase: with three generations (the three sets of quarks and leptons, each repeating the first with the same charges), the mass terms of the quarks (the joins, which join each left-handed state to a right-handed one) carry a phase that no choice of description removes (Kobayashi and Maskawa 1973), and the breaking of CP observed in the decays of kaons and B mesons is that phase, read through the weak transitions.

Sources: *The Story of the Particles*, Level 7, Section III ("The theorem limits where a process and its mirror-and-charge-conjugate can differ, and the one phase that three generations allow is how the two come to differ"); *The Branches of the Kinds*, Chapter 3, Chapter Overview ("CP and T fail wherever the joins (the mass terms, which join each left-handed state to a right-handed one) carry an irremovable phase"), Section II ("The weak way's form keeps T and CP") and Section III; *The Coming of the Kinds*, Chapter 4, Section III; Kobayashi and Maskawa (1973); *The Concept Sequence*, Chapter 40, entry 40.11 (the quark mixing matrix and its one phase), and Chapter 45, entry 45.11 (the strong force's own angle is zero). Standing: a definition (P and CP); derived (where CP can be broken, and that the one phase of quark mixing realizes that possibility: *The Story of the Particles*, Level 7, Section III, and *The Branches of the Kinds*, Chapter 3, Section II); a measured agreement (the breakings of P and CP).

H-007 Baryon Number Violation: The First Sakharov Condition. That every ordinary reaction among the kinds keeps the baryon number and the lepton number, each on its own, because no carrier of force carries both color and the weak charge (the label of the weak force), so no single transformation turns a quark into a lepton. That one collective process among the kinds changes the baryon number all the same: the passage of the field of the weak force from one of its lowest states to the next, called a sphaleron transition (entry H-008). That a sphaleron transition changes the number of particles in each of the twelve left-handed doublets of the three generations (a doublet being a pair of left-handed states that the

weak force turns into each other: nine quark doublets, three colors in each of three generations, and three lepton doublets) by one unit, all in the same direction, through the anomaly of the weak force (the quantum effect by which a count that every ordinary reaction keeps is changed when a force's field, as a whole, passes from one of its lowest states to the next; 't Hooft 1976); so it changes B by three units and L by three units together, changes $B + L$ by six, and keeps $B - L$. That in the cold universe of today the passage can occur only by tunneling through the barrier, with a rate that carries a factor of about 10^{-161}, so that the baryon number is kept in practice, and no single proton can make the passage, since a passage needs many particles at once and a temperature above the scale of the weak force. That in the hot early universe the passages run faster than the universe expands from about 10^{12} GeV down to 131.7 ± 2.3 GeV (a GeV is a billion electronvolts, an electronvolt being the energy an electron gains across one volt, and a temperature is given as the typical energy of motion it gives a particle), and in that stretch the first condition is supplied by the kinds.

Sources: *The Branches of the Kinds*, Chapter 2, Sections II and III, and Box 2.4; *The Coming of the Kinds*, Chapter 4, Section II and Box 4.1, and its Notes 2 and 3 to Chapter 4; 't Hooft (1976); Kuzmin, Rubakov and Shaposhnikov (1985); D'Onofrio, Rummukainen and Tranberg (2014). Standing: derived (from the anomaly of the weak force and the lattice computation of the rate, both standard physics, and, for the absence of every other process that changes the baryon number, from the closed list of kinds).

H-008 Sphaleron Defined. That a sphaleron is a configuration of the field of the weak force, and not a particle: the configuration at the top of the lowest pass over the barrier between two neighboring lowest states of that field. That the lowest states on the two sides of the barrier differ in baryon number by three units and in lepton number by three units: a passage over the barrier in one direction makes nine quarks and three leptons, and a passage in the other direction removes nine quarks and three leptons. That at zero temperature the only way to cross the barrier is by tunneling, which is suppressed by a factor of order $\exp(-4\pi/\alpha_W) \approx$

10^{-161}, with α_W the weak fine-structure constant (the strength of the weak force, about 1/29.6 at the scale of the weak force). That in a gathering hot enough the particles jostle the field over the pass again and again, and the rate of passages is set by the temperature and by the height of the barrier; the barrier rises as the Higgs field takes its value and the carriers of the weak force take their masses.

Sources: *The Coming of the Kinds*, Chapter 4, Section II, Figure 2 and Box 4.1, and Chapter 6, Section IV; *The Branches of the Kinds*, Chapter 2, Section III and Box 2.4. Standing: a definition (the sphaleron); derived (standard physics: the change of B and L, and the rate at zero temperature).

> ***The mountain pass picture.***
>
> Picture a row of equal valleys separated by ridges. Each valley is a lowest state of the field of the weak force, and neighboring valleys differ in baryon number by three units. The sphaleron is the configuration at the pass, the lowest point of a ridge. In a cold universe the field sits in one valley and can reach the next only by tunneling through the ridge, a passage whose rate carries a factor of about 10^{-161}: baryon number is kept in practice. In a hot universe the jostling of the particles carries the field over the passes again and again, and the baryon number changes by three units with each passage, together with the lepton number. As the Higgs field takes its value the ridges rise, and the passages grow rarer.

H-009 C and CP Violation: The Second Sakharov Condition. That the second condition is supplied by the kinds: the weak force breaks C by its form (entry H-005), and the joins of the quarks carry the one phase that breaks CP (entry H-006). That the size of the CP breaking of the quarks has one measure that does not depend on any choice of description, the Jarlskog invariant J (Jarlskog 1985), measured as $J = (3.16\ {}^{+0.13}_{-0.11}) \times 10^{-5}$ (Particle Data Group 2026). That the quark phase acts only through the differences of the masses of quarks of the same charge, and vanishes whenever two of them are equal; so its strength at a temperature T, written as a pure number, is J times the differences of the squared masses of the quarks of each charge, divided by the twelfth power of T

(Cline 2006). That at the temperature of the electroweak crossover (the smooth cooling through the scale of the weak force, entry H-010), 159.5 GeV, this strength is about 4.4×10^{-22}, more than a trillion (10^{12}) times smaller than the measured excess of baryons per photon that it would have to drive; so the quark phase is far too small for an excess made at the electroweak transition. That the value of the quark phase itself is a separate result: *The Measures of the Kinds* derives the phase $\delta_q = 67.15 \pm 0.70°$ as a law of the three mixing angles, given the right angle of the unitarity triangle (the triangle that the three routes of quark mixing close) between the routes of the first and third generations, against the measured $66.12 \pm 1.43°$.

Sources: *The Coming of the Kinds*, Chapter 4, Section III and Box 4.2, and its Note 4 to Chapter 4 (a second route: Gavela, Hernández, Orloff and Pène 1994); Cline (2006), Section 4.1; Jarlskog (1985); *The Measures of the Kinds*, Chapter 10, Chapter Overview and Sections I and IV; Particle Data Group (2026), review "CKM Quark-Mixing Matrix"; *The Concept Sequence*, Chapter 40, entry 40.15 (the quark phase from the three angles). Standing: a measured input (J); derived (standard physics: the strength of the quark phase at the crossover); derived, given a stated identification (δ_q, whose identification is the right angle of the unitarity triangle between the routes of the first and third generations, carried from the triangle {3, 4, 5} of the two integers that fix the algebra of the kinds).

> ***Why the quark phase is so faint at the transition.***
>
> Three sisters seen across a wide field look alike: the differences between their faces are still there, but they are too small against the distance to be seen. The quark phase acts only through differences of exactly that kind, the differences between the masses of quarks of the same charge. At a temperature near 160 GeV every quark except the top is light against the temperature, and the differences of their masses, measured against the temperature, are tiny. So the phase is present, but it has almost nothing to act through: its strength at the transition is about 4.4×10^{-22}, against a measured excess of about 6×10^{-10}.

H-010 Departure from Equilibrium: The Third Sakharov Condition. That the third condition requires that the process changing the baryon number run while the universe is out of balance, so that it is not undone by its reverse. That the electroweak transition is the one event in the cooling of the kinds at which the only process that changes the baryon number stops: it is the cooling through the scale of the weak force, in which the Higgs field (the field present everywhere in space whose settled value gives the W and Z bosons, the quarks and the leptons their masses) takes its value and the carriers of the weak force take their masses. That a transition of this kind is either first-order, going by bubbles whose walls are out of balance, or a crossover, a smooth change with no bubbles and no walls. That lattice computations of the Standard Model (the physicists' account of the kinds and their three forces, computed by solving the equations of the weak force and the Higgs field on a grid of points) give a first-order transition only for a Higgs mass below an endpoint of 72.4 $\pm$ 1.7 GeV (Csikor, Fodor and Heitger 1999; Kajantie, Laine, Rummukainen and Shaposhnikov 1996 bound it between 70 and 95 GeV), and the measured Higgs mass is 125.13 $\pm$ 0.11 GeV; so the electroweak transition is a crossover. That the crossover is the whole smooth settling of the Higgs field toward its present value, centered at T_c = 159.5 $\pm$ 1.5 GeV and steepest over about 5 GeV (D'Onofrio and Rummukainen 2016).

Sources: *The Coming of the Kinds*, Chapter 4, Section IV and Box 4.3, and its Note 5 to Chapter 4, and Chapter 6, Section III; D'Onofrio and Rummukainen (2016); Csikor, Fodor and Heitger (1999); Kajantie, Laine, Rummukainen and Shaposhnikov (1996); Particle Data Group (2026) (the Higgs mass). Standing: derived (standard physics: the lattice computation, with the measured Higgs mass as a measured input).

H-011 No Excess Is Made at the Electroweak Crossover. That through the steepest part of the crossover the sphalerons stay in balance: at the center, 159.5 GeV, their passages still run far faster than the universe expands. That they fall behind the expansion at T_* = 131.7 $\pm$ 2.3 GeV, 27.8 GeV below the center and partway through the settling of the field, gradu-

ally and with the baryon number at its balance value; and the crossover supplies no bubble wall at which the gathering would be driven from balance while they still run. That the sphaleron is the only process among the kinds that changes the baryon number at the transition, so the third condition has no place there. That electroweak baryogenesis, the making of the excess at the electroweak transition, is therefore excluded in the Standard Model with the measured Higgs mass, and the kinds of particle are the kinds of the Standard Model. That the kinds supply the first two conditions (entries H-007 and H-009), and at the transition the sphalerons stay in balance; in balance the structure, unchanged under C, P and T taken together, holds each kind and its antiparticle in equal numbers wherever no kept quantity carries an excess (entry H-003); so the kinds make no excess at the transition, and the excess is carried by what the kinds keep (entries H-012 to H-014).

Sources: *The Coming of the Kinds*, Chapter 4, Sections IV and V and Box 4.3, and Chapter 6, Section IV; D'Onofrio, Rummukainen and Tranberg (2014); *The Branches of the Kinds*, Chapter 2, Section II, and Chapter 3, Section III and Box 3.4 (the CPT theorem); *The Kinds of Particle*, Chapter 6, Section IV (the kinds are those of the Standard Model). Standing: derived (from the lattice computations, the measured Higgs mass and the closed list of kinds).

H-III. What Every Change Keeps

H-012 Every Process Among the Kinds Keeps B − L. That the ordinary reactions (the three forces and the couplings of the quarks and leptons to the Higgs field) keep B and L each on its own, at every order of perturbation theory (the standard expansion of a reaction in powers of its coupling). That beyond perturbation theory B and L change only through the anomaly of the weak force, which is the same for both, so the sphaleron changes B and L by equal amounts; the windings of the color field (its passages from one lowest state to the next) change neither, because the strong force acts on the left-handed and right-handed components of a quark alike and B and L count the two components alike. That a process

changing B alone would need a carrier with both color and the weak charge, and the kinds hold none ("no X or Y boson at any scale"); that a process changing L alone would need a Majorana mass (a mass that joins a particle to its own antiparticle), and the kinds hold none ("No Majorana mass exists, for either chirality", chirality being a component's being left-handed or right-handed), because no state of content is its own antiparticle; and that the list of kinds is closed, so there is no further process. That every process among the kinds of particle therefore keeps B − L, at every temperature.

Sources: *The Coming of the Kinds*, Chapter 5, Section II and Box 5.1; *The Branches of the Kinds*, Chapter 1, Chapter Overview and Section II, Chapter 2, Sections II, III and V and Box 2.3, and Chapter 4, Chapter Overview; 't Hooft (1976); Kuzmin, Rubakov and Shaposhnikov (1985); *The Concept Sequence*, Chapter 41, entry 41.13 (the right-handed neutrino as a component of the Dirac neutrino, with no Majorana mass), and Chapter 42, entries 42.11 to 42.13 (the windings of the strong force, no carrier joining quark to lepton, and baryon number kept by every process built from single transformations). Standing: derived (from the closed list of kinds and the anomaly of the weak force in the Standard Model).

> ***Two tally counters.***
>
> Picture a wall with two tally counters, one counting baryons and one counting leptons, with every machine of a large factory wired to them. Most machines touch neither counter. One machine clicks both forward by three together, or both back by three together. No machine clicks the baryon counter without also clicking the lepton counter, or the lepton counter without also clicking the baryon counter. The two counts change, but the difference between them never changes: whatever it is today, it was the same when the factory opened. The difference is B − L.

H-013 What Balance Erases. That while the sphalerons run in balance they move B + L until their passages in the two directions run equally often, and leave B − L alone. That in balance, with the total hypercharge of the gathering zero (hypercharge being the charge that, before the elec-

troweak transition, plays the part electric charge plays after it), the baryon number settles at a fixed share of B − L, a share that the kinds and their charges fix. That an excess of baryons with B − L equal to zero, put into the gathering by any process at any temperature above 131.7 GeV, is therefore erased once the sphalerons are in balance, together with its matching excess of leptons, since the fixed share of zero is zero. That an excess of baryons survives the sphalerons only where B − L is not zero (Harvey and Turner 1990: "it is necessary that the Universe possess a nonzero value of B−L prior to the epoch of fermion-number violation if baryon and lepton asymmetries are to survive").

Sources: *The Coming of the Kinds*, Chapter 5, Section III and Box 5.2; Kuzmin, Rubakov and Shaposhnikov (1985); Harvey and Turner (1990); Laine and Shaposhnikov (2000). Standing: derived (standard physics of balance, with the kinds of the closed list).

H-014 The Excess Is a Residue Present from the Start. That baryons exist today and outnumber antibaryons; that nothing has changed the baryon number since the sphalerons stopped at 131.7 GeV; that at that moment the baryon number stood at its balance value, a fixed share of B − L; so B − L was not zero when the sphalerons stopped. That B − L is kept by every process (entry H-012), so it had that same value at every earlier moment, back to the earliest moment our history reaches. That the excess of matter over antimatter is therefore carried as a value of B − L, a residue that the turn leaves, kept by every change after it; the turn is the limit that our history, traced backward, takes for its beginning (Chapter L traces it), and how the turn leaves the residue belongs to the account of the turn. That the amount of the residue is a number of history, as the excess is (entry H-003): it is left from the earliest moments, no process among the kinds makes it, and the kinds fix only the share of it held as baryons.

Sources: *The Coming of the Kinds*, Chapter 2, Section III, Chapter 5, Section IV, and Chapter 14, Section IV; *The Story of the Particles*, Level 4, Section IV ("How much of each remains is a residue left from the earliest moments, kept by every change after them; no process among the kinds makes it, and the kinds fix only the share of it held as baryons"); *The Weight of Possibility*, Chapter 3, Section I

("The turn sends nearly all of its energy into radiation. Two residues remain: an excess of ordinary matter over its antimatter, and the dark matter. Both are conserved in number"); *At the Edge of Reality*, Chapters 16 and 17. Standing: derived (that the excess present today requires a value of B − L present from the earliest moment our history reaches); a measured input (the amount of the residue, a number of history, read from the measured η through the share of entry H-019, as entry H-020 gives it).

H-015 The Three Routes Physicists Use. That physicists use three routes to make the excess in the course of the universe's history, and each rests on one thing more than the kinds hold. That the first route, through grand unification (theories that join the three forces into one at very high energy; Yoshimura 1978), needs heavy carriers with both color and the weak charge whose decays change the baryon number directly. The kinds hold no such carrier. That the second route, leptogenesis (Fukugita and Yanagida 1986), needs heavy neutrinos with Majorana masses whose decays out of balance leave an excess of leptons, part of which the sphalerons carry into baryons; the quarks and leptons of the kinds hold Dirac masses only (a Dirac mass joins the left-handed component of a particle to the right-handed component of the same particle). That the third route, electroweak baryogenesis, needs a departure from balance at the electroweak transition; the transition is a crossover through whose steepest part the sphalerons stay in balance (entry H-011). That none of the three routes has a starting point in the list of kinds, which is the result of entry H-014 seen from the side of the routes.

Sources: *The Coming of the Kinds*, Chapter 5, Section V, and its Notes 1 and 2 to Chapter 5; Yoshimura (1978); Fukugita and Yanagida (1986); *The Branches of the Kinds*, Chapter 1, Chapter Overview, and Chapter 2, Section V; *The Concept Sequence*, Chapter 41, entry 41.13 (no Majorana mass), Chapter 42, entry 42.12 (no carrier joins quark to lepton), and Chapter 44, entries 44.10 and 44.11 (no unifying group). Standing: derived (from the closed list of kinds).

H-016 One Sign Everywhere. That the departures from evenness with which our history starts are adiabatic: a place that holds a little more of one component of matter and radiation holds a little more of every com-

ponent, in the same proportion, so the number of baryons per photon is the same from place to place at any one time. That the residue is kept by every change after the start, and the kinds fix the share of it held as baryons alike in every place, so the excess has one sign everywhere: nowhere in the observable universe do antibaryons outnumber baryons. That measurement finds the sky this way by two independent routes: departures of one component against another, of the kind uncorrelated with the main departure, account for no more than 1.55 percent of the variation of the temperature of the microwave background (Planck Collaboration 2020b, Table 14, the cold-dark-matter mode); and the gamma-ray sky excludes regions of matter and antimatter in contact (Cohen, De Rújula and Glashow 1998: "a matter-antimatter symmetric universe is empirically excluded").

Sources: *At the Edge of Reality*, Chapter 19 ("The account predicts adiabatic departures and no isocurvature"); *The Coming of the Kinds*, Chapter 5, Section VI, and its Notes 3 and 4 to Chapter 5; Planck Collaboration (2020b); Cohen, De Rújula and Glashow (1998). Standing: derived (from the adiabatic departures that *At the Edge of Reality*, Chapter 19, derives from the turn); a measured agreement (the bound on departures of one component against another, and the gamma-ray sky).

H-IV. The Share Kept as Baryons

H-017 The Count of States in Balance. That the number of states in balance at a temperature T, each fermion state weighted by seven eighths of a boson state (a boson being a carrier of force or the Higgs boson), is written g_*; it sets the energy density of the gathering, $\rho = (\pi^2/30)\, g_*\, T^4$, and with it the expansion rate, $H = 1.66\, g_*^{1/2}\, T^2/M_{Pl}$, with $M_{Pl} = 1.22 \times 10^{19}$ GeV the Planck mass (the mass formed from the constant of gravitation, Planck's constant and the speed of light). That above the mass of the top quark, with every kind of the Standard Model light against T, the count is $g_* = 28 + (7/8) \times 90 = 106.75$: 28 bosonic states and 90 fermionic

states. That the right-handed neutrino is left out of the count: it reaches the other kinds only through the neutrino's mass, with a coupling of about 3×10^{-13}, and its reactions run less than 4×10^{-10} as fast as the expansion at every temperature above the electroweak crossover, so it never comes into balance. That *The Kinds of Particle* writes the same count in the two kernel integers (the two integers, $c_1 = 3$ and $n_C = 2$, that fix the algebra of the kinds), $g_* = (2c_1^2 + 10) + (7/8)\, c_1(8c_1 + 6)$, and states that the kernel writing adds nothing to the standard count: it is a count of states from the list of kinds, not a measured number.

Sources: *The Coming of the Kinds*, Chapter 3, Sections IV and V, Boxes 3.2 and 3.3, and its Note 1 to Chapter 3; *The Kinds of Particle*, Chapter 6, Section II and Box II-1; Particle Data Group (2026), review "Big-Bang Cosmology", Eqs. (22.42) and (22.47) and Section 22.3.2 ($4N(T) = 427$). Standing: derived (a count over the closed list of kinds).

> ***Counting the states.***
>
> Before the Higgs field takes its value, each of the twelve carriers of force has two states of turning: the eight gluons give 16, the three weak bosons 6, and the hypercharge boson 2; the Higgs doublet adds 4, two components each with a size and a phase. Total bosonic: 28. After the field takes its value the same 28 are rearranged: the W^+, W^- and Z have 3 states each, 9 in all, the photon 2, the Higgs boson 1, and the gluons 16.
>
> Each quark has 3 colors × 2 components (left-handed and right-handed) = 6 states, and 6 more for its antiquark, 12 per kind; with 6 kinds of quark, 72. Each charged lepton has 2 components and its antiparticle, 4 states, 12 in all; each neutrino is counted with one component and its antineutrino, 2 states, 6 in all. Total fermionic: 90.
>
> With the weighting of seven eighths: $g_* = 28 + (7/8) \times 90 = 28 + 78.75 = 106.75$.

H-018 The Share Before the Higgs Field Has a Value. That in a gathering in which the sphalerons, the forces and the couplings to the Higgs field all run in balance, and which carries no net hypercharge, color or

weak charge, each reaction ties together the chemical potentials (the measures of how far each kind leans toward particle over antiparticle), and the count over N_f generations and N_H Higgs doublets gives $B = (8N_f + 4N_H)/(22N_f + 13N_H) \times (B - L)$ (Harvey and Turner 1990). That the kinds supply $N_f = 3$ and $N_H = 1$, with the right-handed neutrino out of balance, so that while the sphalerons run in balance before the Higgs field has a value, $B = (28/79)(B - L) = 0.3544\ (B - L)$ and $L = -(51/79)(B - L)$. That deep in the broken phase (the state of the gathering once the Higgs field has its value), where the value of the Higgs field is large against the temperature, the share is $12/37 = 0.3243$, and for every value of the field in between it lies strictly between 12/37 and 28/79, falling as the field grows against the temperature (Laine and Shaposhnikov 2000). That the share is a count: it comes from the numbers of quarks, leptons and Higgs doublets and from their charges, which the structure fixes.

Sources: *The Coming of the Kinds*, Chapter 7, Sections II to IV, Boxes 7.1 and 7.2, and its Notes 1 and 2 to Chapter 7; Harvey and Turner (1990); Laine and Shaposhnikov (2000); *The Kinds of Particle*, Chapter 7 (three generations); *The Branches of the Kinds*, Chapter 4, Section II (one Higgs field). Standing: derived (standard physics of balance, with the counts of the kinds).

> ***An estate divided by a will.***
>
> The size of an estate is a matter of a family's history; the shares are named by the will, written beforehand, as fractions of whatever the estate turns out to contain. The residue $B - L$ is the estate: its size is history. The will is the list of kinds, with their charges and couplings, and it names the share that goes to the baryons.

H-019 The Share That Froze. That the share that holds for the universe is the one at which the sphalerons let go of the baryon number, at $T_* = 131.7 \pm 2.3$ GeV, while the Higgs field is still growing toward its present value; so it lies between 28/79 and 12/37. That a computation that follows the baryon number through the crossover, as the value of the field and the sphaleron rate change together with the falling temperature, gives the

frozen share as $C_{sph} = 0.3328 \pm 0.0005$ of $B - L$ (Benso and others 2026). That about one third of the residue $B - L$ was therefore held as baryon number when the sphalerons stopped, and the leptons carry the rest with the opposite sign, an excess of antileptons over leptons about twice the excess of baryons. That from then on neither number changes: below 131.7 GeV every process among the kinds keeps B, and at every temperature every process keeps $B - L$.

Sources: *The Coming of the Kinds*, Chapter 6, Section IV, Chapter 7, Section V and Table 7.1, and its Note 3 to Chapter 7; Benso and others (2026); D'Onofrio, Rummukainen and Tranberg (2014). Standing: derived (a computation of standard physics across the crossover, with the kinds of the closed list).

H-020 The Measured Amount and the Size of the Residue. That the amount of matter has two parts: a residue left from the earliest moments, which is history and is taken as measured; and the share of it held as baryons, which the kinds fix. That the measured amount is $\eta = (6.13 \pm 0.04) \times 10^{-10}$ baryons per photon of the microwave background (entry H-003), a measured input; their number, baryons less antibaryons, was set at 131.7 GeV, and the later annihilation of baryons with antibaryons removes them in pairs and leaves that number as it was. That read through the frozen share, the measured amount gives the size of the residue: $B - L = \eta / C_{sph} = (1.84 \pm 0.01) \times 10^{-9}$ units for every photon of today, 3.005 times the baryon number, with the lepton number $L = -2.005\ B$, that is, about -1.23×10^{-9} per photon. That the same number of baryons per photon is measured a second time in the first minutes (entry H-023), and the two measurements agree.

Sources: *The Coming of the Kinds*, Chapter 7, Section VI and Box 7.3, and Chapter 14, Section IV; *The Story of the Particles*, Level 4, Section IV; *The Strengths of the Kinds*, Chapter 1, Section I. Standing: a measured input (η); derived (the size of the residue, from the measured η through the frozen share).

H-V. Big Bang Nucleosynthesis: The Second Measurement

H-021 Big Bang Nucleosynthesis Defined. That Big Bang nucleosynthesis is the making of the first atomic nuclei in the first minutes of our history. That the first nucleus on the way, the deuteron (one proton and one neutron, bound by 2.2246 MeV, an MeV being a million electronvolts and the binding energy being the energy needed to break a nucleus into its protons and neutrons), is broken by the light as fast as it forms while the photons able to break it outnumber the baryons, since the photons outnumber the baryons by more than a billion to one; that this lasts until the temperature falls to about 0.1 MeV, about one twenty-first of the binding energy, about two minutes into our history. That from then on the deuterons last, and reactions through helium-3 and the triton (one proton and two neutrons) carry nearly every neutron into helium-4, the most tightly bound of the light nuclei, beyond which no nucleus of five or eight nucleons (protons and neutrons) lasts; the amounts of the light nuclei are essentially fixed by about 180 seconds. That the products are hydrogen, about three quarters of the mass of the ordinary matter; helium-4, about a quarter; and traces of deuterium and helium-3 (a few parts in a hundred thousand of the hydrogen by number) and of lithium-7 (about one part in ten billion). That the kinds enter these minutes in two places, the count of states, which sets the expansion rate, and the count of neutrons (entry H-022); with the closed list of kinds and the sea of neutrinos counted as $N_{eff} = 3.044$ kinds of radiation beside the light (N_{eff} being the effective number of kinds of neutrino in the radiation), these are the inputs of the standard computation.

Sources: *The Coming of the Kinds*, Chapter 12, Chapter Overview, Sections II, III and VI, and Box 12.1, and Chapter 9, Section V; *The Alphabet of Particles*, the entries for the deuteron, the triton and the alpha particle; Particle Data Group (2026), review "Big-Bang Nucleosynthesis", Sections 24.2 and 24.3. Standing: a definition (Big Bang nucleosynthesis); derived (standard physics, with the kinds of the closed list).

> ***Why the deuteron waits.***
>
> Picture a lottery in which each ticket has one chance in a billion of winning, with ten billion tickets sold every week: someone wins nearly every week. The photons are the tickets. Even when only one photon in a billion carries more than the 2.22 MeV that breaks a deuteron, more than one such photon stands beside every baryon, and a deuteron that forms is broken again almost at once. Near 0.1 MeV the photons able to break a deuteron fall below one for each baryon, and from then on the deuterons last.

H-022 The Neutrons Are Counted. That the weak force turns neutrons into protons and back, through reactions with the electrons, positrons and neutrinos of the gathering, and while those reactions run in balance the ratio of neutrons to protons is $\exp(-\Delta m/T)$, with $\Delta m = 1.29333251 \pm 0.00000038$ MeV the amount by which the neutron outweighs the proton (Particle Data Group 2026). That the reactions fall behind the expansion gradually, near the temperature at which the neutrinos go free; the ratio stops following its balance value as the temperature falls through about 0.8 MeV, about one second into our history, and is left at about one neutron for every six protons, the balance value at about 0.72 MeV. That after that only the decay of the free neutron changes the count, one way, into a proton, with a mean life of 878.3 ± 0.4 seconds (Particle Data Group 2026); so by the time nuclei can hold together, at about 0.1 MeV, about one neutron is left for every seven protons: one for every 7.1 protons at 131 seconds and one for every 7.6 at 180 seconds.

Sources: *The Coming of the Kinds*, Chapter 11, Sections II, III and V, Table 11.1 and Boxes 11.1 and 11.2; Kolb and Turner (1990), Chapter 4; Particle Data Group (2026), review "Big-Bang Nucleosynthesis" ("$n/p = e^{-Q/T_{fr}} \simeq 1/6$"; "the neutron fraction dropped to $n/p \simeq 1/7$"), and the neutron listing. Standing: derived (standard physics, with the measured mass difference and mean life as measured inputs).

H-023 The Light Elements. That with nearly every neutron bound in helium-4 and one proton going with each, the helium-4 holds a share $2x/(1 + x)$ of the mass of the baryons for x neutrons per proton: one quarter for one neutron in seven protons, and 0.233 to 0.247 for the neutrons counted at 180 and 131 seconds; the measured primordial helium mass fraction (the share of the mass of ordinary matter in helium-4 before any star changed it), $Y_p = 0.245 \pm 0.003$ (Particle Data Group 2026), lies in that range. That a little deuterium escapes the chain, and the more baryons there are for each photon, the less is left; so the deuterium left measures the number of baryons per photon. That the measured primordial deuterium is $D/H = (2.508 \pm 0.029) \times 10^{-5}$ (Particle Data Group 2026; Cooke, Pettini and Steidel 2018 give $(2.527 \pm 0.030) \times 10^{-5}$), and the light elements read through the reactions of the first minutes give $\eta = (6.04 \pm 0.12) \times 10^{-10}$. That the microwave background gives $\eta = (6.13 \pm 0.04) \times 10^{-10}$; the two differ by 0.09×10^{-10}, 0.7 of their combined uncertainty, so the two measures, which rest on different physics, agree, and the share of the residue held as baryons is measured twice and found the same.

Sources: *The Coming of the Kinds*, Chapter 12, Sections III and IV, Boxes 12.2 and 12.3, and its Notes 1 and 2 to Chapter 12; Particle Data Group (2026), review "Big-Bang Nucleosynthesis", Eqs. (24.1) to (24.6); Cooke, Pettini and Steidel (2018); Planck Collaboration (2020a). Standing: derived (the helium fraction from the neutron count); a measured input (D/H and the two values of η); a measured agreement (the two values of η, and the helium fraction).

H-024 Lithium-7 and the Oldest Stars. That the standard computation of the first minutes gives lithium-7 at $(4.72 \pm 0.7) \times 10^{-10}$ of the hydrogen (Particle Data Group 2026, review "Big-Bang Nucleosynthesis", Eq. (24.10)). That the lithium-7 found in the atmospheres of the old, metal-poor stars of the halo of the Milky Way (the sparse, roughly round cloud of old stars around the disk of the galaxy, whose stars are poor in the elements heavier than helium), where it forms a nearly uniform plateau called the Spite plateau, is $(1.45 \pm 0.25) \times 10^{-10}$ of their hydrogen (the same review, Eq. (24.4)), about a third of the computed amount. That

physicists call this difference the cosmological lithium problem. That the amount of lithium-7 seen in those stars depends on nuclear rates and on its later destruction in the stars themselves, by diffusion, rotational mixing and burning in young stars before they settle (the same review, Section 24.5); the list of kinds, which gives the first minutes their standard inputs, changes nothing in it.

Sources: *The Coming of the Kinds*, Chapter 12, Section V, and its Note 3 to Chapter 12; Particle Data Group (2026), review "Big-Bang Nucleosynthesis", Eqs. (24.4) and (24.10) and Section 24.5. Standing: a measured input (the lithium-7 of the halo stars); derived (the computed amount, by standard physics with the kinds of the closed list).

> ***Why the lithium lies with the stars.***
>
> The deuterium left from the first minutes fixes the number of baryons per photon, and the microwave background, read hundreds of thousands of years later, fixes the same number by other physics. The two agree. That agreement holds the number of baryons per photon in place, so the lithium seen today is set by the nuclear rates and by what the stars have done to their lithium since.

H-025 What the First Minutes Left. That the presence of the light nuclei was made by the cooling, at one temperature, as the presence of every kind before them was: the first wholes of more than one nucleon came into being as the gathering fell through 0.1 MeV. That the kinds enter the first minutes through the count of states and the count of neutrons; with the closed list of kinds and $N_{eff} = 3.044$ these are the standard inputs, and the helium follows from the neutrons that the kinds and the cooling left. That the dark matter, made at the turn and not among the kinds, acts on the first minutes only through gravity, and radiation beyond the three kinds of neutrino, such as gravitational waves released at the turn, stays within the bound that the measured N_{eff} sets (Chapter L carries that bound). That the number of baryons per photon is measured by the deu-

terium of the first minutes and by the microwave background, and the two agree: the amount of matter is a residue the turn leaves, kept by every change after it, and the kinds fix the share of that residue held as baryons.

Sources: *The Coming of the Kinds*, Chapter 9, Section V, Chapter 12, Section VI, and Chapter 14, Sections IV to VI; *The Weight of Possibility*, Chapter 4, Section III. Standing: derived (standard physics, with the kinds of the closed list as inputs).

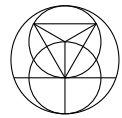
Summary

In this chapter I trace the excess of matter over antimatter from the measurement of that excess to what fixes that excess. In this chapter I rely on one ruling of *The Coming of the Kinds*, which *The Story of the Particles* states in the same words. The ruling is that how much matter remains is a residue left from the earliest moments, kept by every change after them. No process among the kinds makes that residue, and the kinds fix only the share of the residue held as baryons. The excess is measured as about six baryons for every ten billion photons of the microwave background, $(6.13 \pm 0.04) \times 10^{-10}$, a number of the history of this universe and not an attribute of any kind of particle.

The first two parts of the chapter set the kinds of particle beside Sakharov's three conditions. The kinds supply a process that changes the baryon number, the sphaleron, which moves three units of baryon number and three of lepton number together at each passage; and they supply the breakings of C and CP, the first by the form of the weak force, the second by the one phase of the quark joins, which is how the possibility that the Incommensuration Theorem leaves open is realized (*The Story of the Particles*, Level 7, Section III). With the measured Higgs mass the electroweak transition is a smooth crossover; the sphalerons stay in balance through its steepest part and let go at 131.7 GeV, and the quark phase there is more than a trillion times too faint, so no excess is made at the transition.

The third part gives the reason the excess is a residue. Every process among the kinds keeps B − L, the baryon number less the lepton number, and while the sphalerons run in balance they erase any excess that comes with B − L equal to zero; so the excess present today requires a value of B − L present from the earliest moment our history reaches, left by the turn and kept by every change after it. The three routes physicists use to make the excess each need one thing more than the kinds hold, and the residue, set with adiabatic departures, has one sign everywhere.

The fourth part reads the share. The count over three generations and one Higgs doublet gives 28/79 before the Higgs field has a value and 12/37 deep in its settled state, and the share that froze when the sphalerons let go is 0.3328, about one third (*The Coming of the Kinds*, Chapter 7). Read through that share, the measured amount puts the residue at $(1.84 \pm 0.01) \times 10^{-9}$ units of B − L for every photon of today. The fifth part follows the first minutes: the neutrons left by the weak force and by the decay of the free neutron go into helium-4, a quarter of the mass, and the deuterium left over measures the number of baryons per photon a second time, $(6.04 \pm 0.12) \times 10^{-10}$, in agreement with the microwave background. So in this chapter I give one account in two halves. The first half is that the residue is history, measured twice and found the same. The second half is that the share of the residue held as baryons is structure, fixed by the kinds.



Chapter I

The Cosmic Budget

CHAPTER OVERVIEW

This chapter assembles the budget of the universe that the books of the collection give, and gives each value its standing, which is one of these: fixed by the structure of comparison, taken from measurement, reached by an identification and found by measurement (a measured agreement), or carried as a fork that a named measurement decides. It draws on three chapters: Chapter F (the dark matter as a vibration of context, and the registered ratio $\Omega_c/\Omega_b = 6 - 2/\pi$), Chapter G (the cosmological constant as the share of the vacuum's energy accessible to an observer inside the vacuum, and the identity $I^2\Omega_\Lambda = 8 - 3/\pi$), and Chapter H (how much matter remains, a residue left from the earliest moments). Every value stands beside the measurements the owner books cite: Planck 2018 (Planck Collaboration 2020a), the combination of the South Pole Telescope, Planck and the Atacama Cosmology Telescope (Camphuis and others 2026), and the second data release of the Dark Energy Spectroscopic Instrument (DESI Collaboration 2025).

In this chapter I rely on these chapters of *The Concept Sequence*: Chapter 28, The Two Measured Anchors and the Span to the Planck Mass (Newton's constant as the measured input of gravitation); Chapter 35, The Lepton Sector (the neutrino masses and their order); Chapter 45, Further

Predictions (the dark matter and the cosmological constant, entries 45.12 and 45.15); and Chapter 46, Predictions and the Measurements That Test Them (entry 46.15, the cosmos and its measurements).

I-I. The Cosmological Parameters

I-001 Critical Density Defined. That the critical density is the density at which the universe is spatially flat: neither curved like the surface of a sphere nor curved like a saddle, but flat like an infinite plane. That this density depends on the expansion rate: a faster-expanding universe needs a greater density to be flat. That cosmologists express every density as a fraction of the critical density, written Ω: the ratio of the density of a given component to the critical density.

Sources: *The Coming of the Kinds*, Chapter 1, Box 1.1; *The Gravitational Passage*, Chapter 13, Section III. Standing: a definition.

> ***Why fractions of critical density?***
>
> The Friedmann equation, the equation that governs the expansion of a uniform universe, takes a simple form when densities are written as fractions of the critical density: $\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_K = 1$, where Ω_m is the matter fraction, Ω_r the radiation fraction, Ω_Λ the vacuum fraction, and Ω_K the curvature term. In a flat universe $\Omega_K = 0$.

I-002 The Parameters of the Budget Defined. That the standard cosmological model, ΛCDM (a cosmological constant Λ with cold dark matter), describes the composition of the universe by a small set of density fractions. That Ω_m is the total matter fraction, ordinary and dark matter together, with the neutrinos once they are slow. That Ω_b is the fraction of the ordinary matter, the baryons (protons and neutrons, with their electrons). That Ω_c is the fraction of the cold dark matter, which acts only through gravity. That Ω_ν is the fraction of the neutrinos. That Ω_Λ is the vacuum fraction, contributed by the cosmological constant. That $\Omega_m = \Omega_b$

$+ \Omega_c + \Omega_\nu$, and in a spatially flat universe $\Omega_m + \Omega_r + \Omega_\Lambda = 1$. That in ΛCDM these fractions are inferred from the data, while on this account two dimensionless numbers of the budget are fixed by identities, the matter fraction Ω_m (Chapter G, entries G-017 and G-018) and the ratio Ω_c/Ω_b (Chapter F, entry F-019), and the amount of ordinary matter is a number of history, taken as measured (entry I-009).

Sources: *The Gravitational Passage*, Chapter 13, Sections III and VII; *The Weight of Possibility*, Chapter 3, Sections I and V. Standing: a definition.

I-003 The Hubble Constant. That the Hubble constant, written H_0, measures the present expansion rate of the universe: the speed at which a distant galaxy recedes, for each unit of its distance. That it is expressed in kilometers per second per megaparsec, where one megaparsec is about 3.26 million light-years. That the reduced Hubble parameter is $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, so that $H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ corresponds to $h = 0.678$. That many cosmological quantities are written as products with h^2, for example $\Omega_m h^2$, because the microwave background measures that combination more tightly than either factor alone.

Sources: *The Gravitational Passage*, Chapter 13, Section IV; *The View from Inside*, Appendix C. Standing: a definition.

> ***Why $\Omega_m h^2$ is the quantity the microwave background measures.***
>
> The microwave background constrains the physical matter density, the amount of matter in each unit of volume, more tightly than it constrains the matter fraction of the critical density. The physical density is proportional to $\Omega_m h^2$, because the critical density itself is proportional to h^2. Planck 2018 gives $\Omega_m h^2 = 0.1430 \pm 0.0011$ (Planck Collaboration 2020a). On this account the identity of Chapter G fixes Ω_m, so the measured $\Omega_m h^2$ then gives h (entry I-014).

I-004 Spatial Curvature. That the curvature term Ω_K measures how far the geometry of space departs from flatness on the largest scales: $\Omega_K = 0$ for a spatially flat universe, $\Omega_K > 0$ for a negatively curved (open) one,

and $\Omega_K < 0$ for a positively curved (closed) one. That the measured value is $\Omega_K = 0.001 \pm 0.002$ (Planck Collaboration 2020a, with the baryon acoustic oscillations). That on this account the homogeneous model of the epoch is spatially flat, and space is curved only by its departures from uniformity, at their own size, about 10^{-5} (entry I-015; Chapter G, entry G-027, which traces the flatness of the new epoch).

Sources: *The Gravitational Passage*, Chapter 18, Section IX; *The Grand Synthesis*, Chapter 13, Section I, which carries the same measured value. Standing: a definition (of the curvature term); a measured input ($\Omega_K = 0.001 \pm 0.002$); derived (the flatness of the epoch, *The Gravitational Passage*, Chapter 18, Section IX).

I-005 The Temperature of the Microwave Background. That the microwave background, the light left from the early universe and released when the universe cooled enough for electrons to bind to nuclei, has a present temperature of $T_0 = 2.72548 \pm 0.00057$ kelvin (Fixsen 2009). That every cubic centimeter of space holds about 411 of its photons, from the number density of photons in thermal balance, $n_\gamma = (2\zeta(3)/\pi^2)T^3$. That this measured temperature converts the measured baryon density $\Omega_b h^2$ into the number of baryons for every photon, η (entry I-009).

Sources: *The Coming of the Kinds*, Chapter 1, Section VI and Box 1.1 (with the Particle Data Group's reviews "Big-Bang Cosmology" and "Big-Bang Nucleosynthesis", Particle Data Group 2026). Standing: a measured input.

I-006 The Primordial Departures: Amplitude, Tilt and Kind. That the primordial departures from uniformity are the small variations of density laid down at the beginning of the epoch, which later grew into galaxies and clusters. That their spectrum is described by an amplitude A_s, the strength of the departures at a reference scale, and a spectral index n_s, the tilt: $n_s = 1$ means equal strength at every scale, and $n_s < 1$ means the larger scales are slightly stronger. That I take the amplitude and the tilt of the primordial departures from the microwave background. *The Gravitational Passage* identifies the departures with the form of the content carried

across the turn, states that the amplitude "is set by the count of states over which the form of the content is shared" and that a tilt is a reference to a length of the new epoch, and takes both values from measurement; the measured index, about 0.97, is a weak reference of the departures to such a length, and its value, sign included, is taken as measured. That the account fixes the kind of the departures: they are departures of density, one form for every component of the content, so the dark component carries no separate pattern (Chapter F, entry F-012); and the primordial departures of the metric are scalar, so that primordial gravitational waves arise only at second order, with a ratio of tensor to scalar power far below 10^{-6}.

Sources: *The Gravitational Passage*, Chapter 18, Sections IX, XI and XIII; *The Weight of Possibility*, Chapter 5, Section VI; *The View from Inside*, Appendix B, "The Scope of the Predictions". Standing: a measured input (A_s and n_s); derived (the scalar, adiabatic kind of the departures).

I-007 What the Account Fixes, and What It Takes from Measurement. That in ΛCDM six parameters are inferred from the microwave background: the baryon density $\Omega_b h^2$, the cold dark matter density $\Omega_c h^2$, the angular size of the sound horizon $100\theta_*$, the optical depth of reionization τ (how much of the background's light the gas ionized again by the first stars scatters), the scalar amplitude $\ln(10^{10} A_s)$, and the spectral index n_s. That the account fixes two dimensionless numbers of the budget: the matter fraction $\Omega_m = 0.311$, by the identity $I^2\Omega_\Lambda = 8 - 3/\pi$ (Chapter G, entries G-017 and G-018), and the ratio $\Omega_c/\Omega_b = 6 - 2/\pi$, by the registered ratio (Chapter F, entry F-019). That it also fixes two parameters that extended models leave free: the equation of state of the cosmological constant, $w = -1$ exactly, and the spatial curvature, flat. That the account takes from measurement: Newton's constant G_N, the one measured constant of gravitation; the present horizon, through the measured expansion history (used for the vacuum density, not for the identity); the radiation fraction $\Omega_r \approx 9 \times 10^{-5}$; the amount of ordinary matter, η, a number of history (entry I-009); the physical matter density $\Omega_m h^2$, from which it ob-

tains h (entry I-014); and the amplitude and tilt of the primordial departures (entry I-006). That I take the optical depth of reionization from measurement, as the analyses of the microwave background give that optical depth. That the neutrino mass sum, a parameter of extended models, is carried as a fork of two values (entry I-016).

Sources: *The Gravitational Passage*, Chapter 13, Section VII; *The Weight of Possibility*, Chapter 3, Section I, and Chapter 5, Section VI; *The View from Inside*, Appendix B, "The Scope of the Predictions", and Appendix C, "The Inputs". Standing: a definition.

I-II. The Budget

I-008 The Total Matter Fraction. That the identification of the cosmological constant with the share of the vacuum's energy accessible to an observer inside the vacuum (Chapter G, entries G-007 to G-013), combined with the Friedmann equation, gives an identity from which the Hubble constant and the Planck mass both cancel: $I^2\Omega_\Lambda = 8 - 3/\pi$, where I is the radius of the particle horizon in units of the Hubble radius, the distance light has traveled over the whole expansion (Chapter G, entry G-016). That for a flat universe its solution is $\Omega_m = 0.311$ (0.3108; $\pm$ 0.001 from the measured radiation density) and $\Omega_\Lambda = 0.689$, against the measured 0.315 $\pm$ 0.007 (Planck 2018), 0.6 standard deviations away; the baryon acoustic oscillations of the Dark Energy Spectroscopic Instrument alone give 0.2975 $\pm$ 0.0086 (DESI Collaboration 2025), 1.6 standard deviations away on the other side. That the identity fixes the total matter fraction and says nothing of what the matter is made of.

Sources: *The Gravitational Passage*, Chapter 13, Sections III, IV and VII; *The View from Inside*, Appendix C, "Results"; Chapter G, entries G-017 and G-018. Standing: a measured agreement.

I-009 The Amount of Ordinary Matter Is History. That the existence of antiparticles is structural, and the excess of matter over antimatter belongs to history: "It belongs to the initial conditions, to the history of the

universe, to the facts that the zoo does not fix" (*The Story of the Particles*, Level 4, Section IV). That the excess is a residue left from the earliest moments, carried as a value of B − L (the baryon number less the lepton number), which every change after the turn keeps: no process among the kinds makes it, and the kinds fix only the share of it held as baryons (*The Coming of the Kinds*, Chapter 5, Section IV; Chapter H traces it). That the amount is therefore taken as measured: $\Omega_b h^2 = 0.02237 \pm 0.00015$ (Planck 2018 results VI, Table 2, from the temperature, polarization and lensing data; Planck Collaboration 2020a), which with T_0 (entry I-005) and the conversion $\eta = 274 \times 10^{-10}\ \Omega_b h^2$ that the Particle Data Group gives (Fields, Molaro and Sarkar, in Particle Data Group 2026) gives $\eta = (6.13 \pm 0.04) \times 10^{-10}$, about six baryons for every ten billion photons. That what the kinds fix is the share of the residue held as baryon number when the sphalerons (the transitions of the electroweak field that change baryon and lepton number together) left balance at 131.7 GeV: between $28/79 = 0.3544$ (Harvey and Turner 1990) and $12/37 = 0.3243$ (Laine and Shaposhnikov 2000), and computed across the crossover as 0.3328 ± 0.0005 (Benso and others 2026). That, read through that share, the measured η puts the residue at $(1.84 \pm 0.01) \times 10^{-9}$ units of B − L for every photon of today. "The residue is history; the share is structure" (*The Coming of the Kinds*, Chapter 7, Summary).

Sources: *The Story of the Particles*, Level 4, Section IV; *The Coming of the Kinds*, Chapter 1, Section VI and Box 1.1; Chapter 5, Section IV; Chapter 7, Sections V and VI, Box 7.3 and Summary; *The Strengths of the Kinds*, Chapter 1, Section I. Standing: a measured input (η, a number of history and not of structure); derived (the share of B − L held as baryon number, fixed by the kinds and their charges, with its two limits 28/79 and 12/37 and the computation across the crossover, 0.3328 ± 0.0005, by Benso and others 2026).

> ***The conversion.***
> $\eta = 274 \times 10^{-10} \times \Omega_b h^2 = 274 \times 10^{-10} \times 0.02237 = 6.13 \times 10^{-10}$, with uncertainty $274 \times 10^{-10} \times 0.00015 = 0.04 \times 10^{-10}$. The factor 274 carries the number density of photons at the measured temperature of the microwave background, the mass of the baryons and the critical density.

I-010 The Ratio of the Dark Matter to the Ordinary Matter. That the ratio is an identity of the present measurement, of the same form as the identity for the cosmological constant: $\Omega_c/\Omega_b = 6 - 2/\pi = 5.3634$ (Chapter F, entry F-019). That Planck 2018 gives $\Omega_c h^2 = 0.120 \pm 0.001$ and $\Omega_b h^2 = 0.0224 \pm 0.0001$, and that *The Weight of Possibility* computes from the published densities, treated as uncorrelated, 5.364 ± 0.065 from Planck alone and 5.370 ± 0.048 from the combination of the South Pole Telescope, Planck and the Atacama Cosmology Telescope (Camphuis and others 2026). That analyses adding the baryon acoustic oscillations of the Dark Energy Spectroscopic Instrument lie lower, 5.199 ± 0.035 (Louis and others 2025, Table 5; Chapter F, entry F-038, which states the account's prediction for them and the rule of decision).

Sources: *The Weight of Possibility*, Chapter 3, Section V; Chapter 5, Section I. Standing: a measured agreement.

I-011 The Shares of the Budget. That with the matter fraction 0.3108 (entry I-008), the ratio $6 - 2/\pi$ (entry I-010), and the neutrinos of the smallest masses the oscillation experiments allow ($\Omega_\nu = 0.0014$, for a mass sum of about 59 meV), the cold matter is 0.3094, and the two cold densities are $\Omega_b = 0.0486$ and $\Omega_c = 0.2608$. That the share of the ordinary matter in the cold matter is $1/(7 - 2/\pi) = 0.15715$, and its share in all the matter is 0.15644. That routes that do not use the microwave background agree with that share: 0.1587 ± 0.0066 (nucleosynthesis with the baryon acoustic oscillations of the Dark Energy Spectroscopic Instrument), 0.1542 ± 0.0063

(nucleosynthesis with the full-shape analysis of that instrument), and 0.156 ± 0.034 (the gas of galaxy clusters) (Chapter F, entry F-037). That the vacuum fraction is $\Omega_\Lambda = 0.689$ (entry I-008).

Sources: *The Weight of Possibility*, Chapter 3, Section V; Chapter 5, Section II; *The Measures of the Kinds*, Chapter 12, Sections I and VI. Standing: derived (from the matter fraction, the registered ratio and $\Omega_\nu = 0.0014$ for the smallest neutrino masses, and with the count route's neutrino sum of entry I-016, $\Omega_\nu = 0.0020$ and a baryon share of all the matter of 0.1561); a measured agreement (the routes free of the microwave background).

I-012 The Cosmological Constant. That the vacuum density accessible to an observer inside the vacuum is $\rho_\Lambda = c_1 \, f \, M_{Pl}{}^4/S$, with $c_1 = 3$ the count of the channels through which a measurement registers, $f = 1 - 3/(8\pi) = 0.881$ the vacuum's energy at the Planck scale less the bare curvature of the internal space, M_{Pl} the full Planck mass, and $S = \pi R^2/l_{Pl}{}^2$ the count of the particle horizon of radius R (Chapter G, entry G-013). That with the Planck 2018 parameters used as one set ($h = 0.674$, $\Omega_m = 0.315$, flat), $S = 2.29 \times 10^{123}$ and the formula gives $\rho_\Lambda = 2.56 \times 10^{-47}$ GeV^4, against the observed 2.52×10^{-47} GeV^4 from the same parameters: about 1.7 percent above, within the ± 2.8 percent that the measured uncertainty of Ω_m gives the ratio. That the matter fraction of entry I-008 and the comparison of the predicted vacuum density with the observed density are one test, seen two ways.

Sources: *The Gravitational Passage*, Chapter 13, Sections II and IV; Chapter G, entry G-014. Standing: a measured agreement (the predicted density); derived ($c_1 = 3$ and f, fixed by the structure of comparison); an identification (the accessible share); a measured input (Newton's constant and the horizon).

I-013 The Equation of State. That the equation of state of the cosmological constant is $w = -1$ exactly: the constant is fixed once, by the present measurement, as the constant term of one geometry (Chapter G, entries G-022 to G-026). That Planck 2018 with baryon acoustic oscillations and Type Ia supernovae gives $w = -1.03 \pm 0.03$, and that the final analysis of

the Dark Energy Survey, combined with the Dark Energy Spectroscopic Instrument and the microwave background, reports $w_0 = -0.82 \pm 0.05$ and $w_a = -0.63$ $(+0.21/-0.18)$, about three standard deviations from a constant (Abbott and others 2026b). That I predict that the reported departure from a constant will not be confirmed.

Sources: *The Gravitational Passage*, Chapter 13, Section V; Chapter 18, Section XIII; Chapter G, entry G-026. Standing: derived.

I-014 The Hubble Constant. That with the matter fraction of the identity, $\Omega_m = 0.3108$, and the physical matter density the microwave background measures, $\Omega_m h^2 = 0.1430 \pm 0.0011$ (Planck 2018, temperature, polarization and lensing), $h = \sqrt{(0.1430/0.3108)} = 0.678 \pm 0.003$, a Hubble constant of 67.8 ± 0.3 km s^{-1} Mpc^{-1}. That the radiation fraction itself depends on h ($\Omega_r h^2 = 4.18 \times 10^{-5}$ for the photons and three light neutrinos), and solved together $\Omega_r = 9.1 \times 10^{-5}$, which leaves the result unchanged at this precision. That this is the same test as Ω_m, stated through one more measurement; the inference of $\Omega_m h^2$ is made in a spatially flat universe with $w = -1$, the two features the account also gives.

Sources: *The Gravitational Passage*, Chapter 13, Sections IV and VII; *The Gravitational Companion*, "Comparisons at Single Steps", Section VIII; *The View from Inside*, Chapter 13, and Appendix C, "Results". Standing: a measured agreement (the same test as Ω_m).

I-015 Flatness. That the homogeneous model of the epoch is spatially flat, because the only lengths the new epoch has of its own are the length of the turn, of the order of the Planck length, and the de Sitter radius, and neither gives a measurable curvature (Chapter G, entry G-027). That the curvature measured within one horizon is that of the departures, of order 10^{-5}, and the measured value is $\Omega_K = 0.001 \pm 0.002$.

Sources: *The Gravitational Passage*, Chapter 18, Sections IX and XIII. Standing: derived.

I-016 The Neutrino Mass Sum: a Fork. That the neutrino masses are Dirac masses, since no state of content is its own reversal (*The Measures of the Kinds*, Chapter 12, Section I; *The Concept Sequence*, Chapter 35, The Lepton Sector, entry 35.22), so the effective mass of neutrinoless double beta decay is $m_{\beta\beta} = 0$ on both routes below, and that the masses are carried as a fork of two routes that a named measurement decides. That the count route, the full mass ladder placed from the measured tau, gives $m_{\nu 1}$, $m_{\nu 2}$, $m_{\nu 3}$ = 15.149, 17.433, 52.366 meV and Σm_{ν} = 84.95 meV, and predicts both differences of the squared masses that oscillations measure (the splittings). That the Koide route, under the rule of the sign with the 2026 splittings, gives 0.370, 8.726, 50.211 meV and $\Sigma m_{\nu} = 59.31 \pm 0.25$ meV, and takes both splittings as inputs. That both routes give the normal order of the masses on a premise they share, so a measurement of the order does not decide between them, and an inverted order would exclude both together. That the Koide route's sum reproduces the budget of entry I-011 exactly ($\Omega_{\nu} = 0.0014$), and the count route's sum gives $\Omega_{\nu} = 0.0020$ and a baryon share of all the matter of 0.1561. That the beta-decay mass (the effective mass of the electron neutrino that beta decay measures), a measurement free of any cosmological analysis, is about 17.6 meV on the count route and about 9 meV on the Koide route (8.93 meV); KATRIN bounds it below about 0.45 eV (KATRIN Collaboration 2025).

Sources: *The Measures of the Kinds*, Chapter 12, Sections I, V and VI, Tables VI-1 and VI-2. Standing: a fork that a named measurement decides (the mass sum taken at the Passage's matter fraction and h, with $w = -1$, or the beta-decay mass).

I-III. The Microwave Background and the Other Probes

I-017 The Power Spectrum of the Microwave Background Defined. That the microwave background carries a pattern of temperature variations, regions slightly hotter or cooler than the average, imprinted by sound waves in the plasma of photons and ordinary matter before the

universe became transparent. That this pattern is decomposed by angular scale into a power spectrum: the strength of the variations as a function of the angular scale on which they appear. That the spectrum shows a series of acoustic peaks, standing sound waves frozen at the moment the photons were released, whose heights carry the densities of the ordinary and the dark matter and whose positions carry the angular size of the sound horizon.

Sources: *The Weight of Possibility*, Chapter 5, Section I. Standing: a definition.

> ***A chord and its notes.***
>
> A musical chord can be decomposed into its component frequencies, each contributing a certain amount of power, and the map of the microwave background can be decomposed in the same way into components at different angular scales. Each component (a multipole, labeled ℓ) corresponds to one angular scale on the sky, and the height of the spectrum at each multipole measures how much temperature variation exists at that scale. The acoustic peaks are the fundamental mode and its overtones, frozen in place when the universe became transparent.

I-018 The Sound Horizon and the Baryon Acoustic Oscillations Defined. That the sound horizon r_d is the greatest distance a sound wave could travel in the early plasma before the baryons decoupled from the photons. That it is imprinted both in the microwave background and in the distribution of galaxies, as a ripple of fixed size, and that surveys of the baryon acoustic oscillations, such as the Dark Energy Spectroscopic Instrument, use it as a standard ruler: comparing its apparent size at different redshifts with its physical size gives the expansion history and, from it, the matter fraction.

Sources: *The View from Inside*, Appendix B, "The Matter Fraction"; *The Weight of Possibility*, Chapter 5, Section I. Standing: a definition.

> ***The standard ruler.***
> If the physical size of a circle is known and its apparent angular size is measured, its distance follows. The sound horizon is such a circle on the sky: its physical size is fixed by the physics of the early plasma, and its apparent size at different redshifts records how the universe expanded.

I-019 What the Microwave Background Tests. That the heights of the acoustic peaks are set by general relativity, by sound in the plasma of photons and ordinary matter, by the scattering of light on free electrons and by the forming of hydrogen atoms, and depend on the dark matter only through its gravitation and through its taking no part in that sound; so the microwave background measures the ratio of the two densities, which tests the registered ratio (entry I-010). That the dark matter of the account is perfectly cold and collisionless, pressureless and present before the background was released, and so behaves in the background as the cold dark matter of ΛCDM does (Chapter F, entry F-033). That I take the angular size of the acoustic scale as measured: the amplitude of clustering of entry I-023 is computed at the matter density that keeps that angular size.

Sources: *The Weight of Possibility*, Chapter 5, Sections I and VI; *The Weight of the Unseen*, Chapter 3, question 3.6. Standing: a measured agreement (the ratio); derived (the cold, collisionless behavior).

I-020 The Kind of the Primordial Departures. That the account fixes the primordial departures as departures of density, the same for every component of the content, so that the dark component carries no separate pattern: in models that mix the two kinds of departure, an isocurvature pattern in the cold dark matter (one component departing against another) carries less than 1.3 percent of the observed variance of the temperature of the microwave background, at 95 percent confidence (Planck Collaboration 2020b). That primordial gravitational waves arise only at second order, with a ratio of tensor to scalar power far below 10^{-6}; the Si-

mons Observatory, with a forecast uncertainty on that ratio of about 1.2×10^{-3} (Simons Observatory Collaboration 2026), will on this account find nothing.

Sources: *The Gravitational Passage*, Chapter 18, Sections IX and XIII; *The Weight of Possibility*, Chapter 1, Section I, and Chapter 3, Section I; *The Compactification*, Chapter 6, Section V. Standing: derived.

I-021 The Baryon Acoustic Oscillations. That I hold the matter fraction at 0.311 and the cosmological constant at one value. So I expect the pull of the baryon acoustic oscillations toward a lower matter fraction to recede as the data on those oscillations grow. I also expect the coming surveys (the Dark Energy Spectroscopic Instrument, the Euclid space telescope, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope) to converge on 0.311. That the baryon acoustic oscillations of the Dark Energy Spectroscopic Instrument alone give $\Omega_m = 0.2975 \pm 0.0086$ (DESI Collaboration 2025), 1.6 standard deviations below the account's 0.311, while the microwave background gives 0.315 ± 0.007, 0.6 standard deviations above it. That within ΛCDM the oscillations of that instrument and the microwave background differ by 2.3 standard deviations (DESI Collaboration 2025). That the oscillations do not measure the dark density: they measure the matter fraction, and analyses that add them lower the dark density they infer where they prefer a lower matter fraction (entry I-010).

Sources: *The Gravitational Passage*, Chapter 13, Section III; *The Weight of Possibility*, Chapter 5, Section I; *The View from Inside*, Appendix B, "The Matter Fraction". Standing: a measured input (the matter fraction from the oscillations alone); a fork that a named measurement decides (the matter fraction measured to about 0.001 by the coming surveys).

I-022 The Budget with Its Standings. That the budget of this chapter, value by value, with each value's standing, is as the inset sets out. That the ground behind it is this: two numbers fixed by the structure of comparison ($c_1 = 3$ and $f = 1 - 3/(8\pi)$); one identification, the accessible share of the vacuum's energy (*The Gravitational Passage*, Chapter 13, Section II); a

second identification, that of *The Weight of Possibility*, Chapter 1, Section IV, that the dark matter is a conserved excitation of the six dimensions of possibility; the principles of counting that *The Weight of Possibility*, Chapter 3, Sections II and IV, states as the account's own (a registration counts by classes, six against one; a count that includes a factor's Euler number is not reduced again by that factor's curvature); measured inputs (G_N, the horizon, Ω_r, η, $\Omega_m h^2$, and the amplitude and tilt of the primordial departures); and one rule, the abiding of $\mathbb{C}P^2$ alone, which gives the registered ratio, which is selected from a small family of expressions of one form with twelve expressions examined in all, and whose neighbors within its form the measurement excludes (Chapter F, entry F-021). That the independent quantitative tests are two: the matter fraction (with which the vacuum density, h, the age in de Sitter times and the ratio of the particle horizon to the de Sitter radius are one test), and the ratio of the dark to the ordinary matter. That these two share one coefficient, the internal curvature in Planck units (Chapter F, entry F-020). That the equation of state, the flatness and the kind of the primordial departures are separate predictions of kind, and the neutrino mass sum is a fork.

Sources: *The Gravitational Passage*, Chapter 13, Sections II and VII; *The Weight of Possibility*, Chapter 1, Section IV; Chapter 3, Sections II and IV; Chapter 5, Sections VIII and IX; *The View from Inside*, Appendix C. Standing: a measured agreement (the matter fraction, with the vacuum fraction, the vacuum density, h and the age as one test, the registered ratio, and the baryon share); derived ($w = -1$, the flatness, and the amplitude of clustering from the measured primordial amplitude and tilt); a measured input (η, A_s and n_s, Newton's constant, the horizon, Ω_r and $\Omega_m h^2$); a fork that a named measurement decides (the neutrino mass sum).

The budget, value by value.

Matter fraction Ω_m: 0.311 from $I^2\Omega_\Lambda = 8 - 3/\pi$; measured 0.315 ± 0.007 (Planck 2018); a measured agreement.

Vacuum fraction Ω_Λ: 0.689; the same test as Ω_m.

Vacuum density ρ_Λ: 2.56×10^{-47} GeV⁴; observed 2.52×10^{-47} GeV⁴; a measured agreement, the same test as Ω_m.

Reduced Hubble constant h: 0.678 ± 0.003 from Ω_m and the measured $\Omega_m h^2$; measured 0.674 ± 0.005 (Planck 2018); the same test as Ω_m.

Age in de Sitter times $t_0 H_\Lambda$: 0.792; measured 0.787 ± 0.009; the same test as Ω_m.

Ratio of dark to ordinary matter Ω_c/Ω_b: $6 - 2/\pi = 5.3634$; measured 5.370 ± 0.048 (Camphuis and others 2026); a measured agreement.

Baryon share of all the matter: 0.15644 (with $\Omega_\nu = 0.0014$); measured by routes free of the microwave background 0.1587 ± 0.0066, 0.1542 ± 0.0063, 0.156 ± 0.034; a measured agreement.

Amount of ordinary matter η: $(6.13 \pm 0.04) \times 10^{-10}$; a measured input (a number of history).

Equation of state w: -1 exactly; measured -1.03 ± 0.03; derived.

Spatial curvature: flat, curved only by the departures at about 10^{-5}; measured $\Omega_K = 0.001 \pm 0.002$; derived.

Amplitude of clustering S_8: 0.823 ± 0.006; derived, from the primordial amplitude and tilt of Planck 2018 taken as measured inputs.

Neutrino mass sum Σm_ν: 84.95 meV (count route) or 59.31 ± 0.25 meV (Koide route); a fork that a named measurement decides.

Primordial amplitude A_s and tilt n_s: measured inputs.

I-023 The Amplitude of Clustering. That with the matter fraction 0.311, the registered ratio, and the primordial amplitude and tilt of Planck 2018, the amplitude of clustering today is $S_8 = \sigma_8(\Omega_m/0.3)^{1/2} = 0.823 \pm 0.006$ (0.827 ± 0.007 with $h = 0.678$ at its central value), where σ_8 is the root-mean-square departure of the density of matter in spheres eight megaparsecs across in units of the Hubble constant; ΛCDM, with its matter fraction taken from the same measurement, gives 0.832 ± 0.013 (Planck Collaboration 2020a). That the lensing of the microwave background, the complete Kilo-Degree Survey and the growth rate measured from the mo-

tions of galaxies agree with the account within one standard deviation, the final galaxy clustering and weak lensing of the Dark Energy Survey lies 2.5 standard deviations below it, and the clusters counted by eROSITA lie 3.2 standard deviations above it (Chapter F, entry F-039, which names each measurement). That the amplitude of clustering depends on the matter fraction: a value confirmed away from 0.823 bears first on the identity for the cosmological constant, and, once the matter fraction is confirmed at 0.311 by a separate measurement, it bears on the clustering of the dark matter, which on these scales clusters exactly as standard cold dark matter does.

Sources: *The Weight of Possibility*, Chapter 5, Section VI. Standing: derived (from the matter fraction, the registered ratio, and the primordial amplitude and tilt of Planck 2018, taken as measured inputs); a measured agreement (with the lensing of the microwave background, the Kilo-Degree Survey and the growth rate).

I-IV. Tensions

I-024 The Hubble Constant beside Its Measurements. That I take the early-universe side of the disagreement over the Hubble constant (the Hubble tension), and I say that the dispute will settle near 67.8. That the Hubble tension is the disagreement between the value of the Hubble constant inferred from the early universe and the value measured with the local distance ladder. That the account's 67.8 ± 0.3 km s^{-1} Mpc^{-1} (entry I-014) lies 0.8 standard deviations from the microwave background's own fit within ΛCDM, 67.4 ± 0.5 (Planck Collaboration 2020a), and 4.9 standard deviations from the Cepheid distance ladder, 73.04 ± 1.04 (Riess and others 2022), with the two uncertainties added in quadrature. That, with the microwave background's $\Omega_m h^2$, a Hubble constant of 73 would require $\Omega_m = 0.268$, which the identity excludes; a Hubble constant outside 65.6 to 70.2 would put the matter fraction more than three standard deviations of the measured matter fraction (0.007) from 0.311, so the identity

excludes 73 and does not exclude 70. That the value from the tip of the red giant branch, calibrated with the Hubble and James Webb space telescopes, is 70.39 $\pm$ 1.22 (statistical) $\pm$ 1.33 (systematic) (Freedman and others 2025), 1.4 standard deviations from 67.8.

Sources: *The Gravitational Companion*, "Comparisons at Single Steps", Section VIII; *The Gravitational Passage*, Chapter 13, Section IV; *The View from Inside*, Chapter 13; Appendix B, "The Matter Fraction"; and Appendix C, "Results". Standing: a measured agreement (with the microwave background's value, 67.4 $\pm$ 0.5); a fork that a named measurement decides (the Hubble constant as the distance ladders settle it).

I-025 The Neutrino Mass Sum against the Cosmological Bounds. That "The cosmological inference of the mass sum is a test under an analysis, the cosmological model together with the prior on the masses, and not a value" (*The Measures of the Kinds*, Chapter 12, Section VI). That the second data release of the Dark Energy Spectroscopic Instrument with the microwave background (Elbers and others 2025) gives, at 95 percent confidence, $\Sigma m_\nu < 64.2$ meV in flat ΛCDM with the prior $\Sigma m_\nu > 0$ and three equal masses; < 101 meV with the prior of the normal order; < 105 meV with the prior $\Sigma m_\nu \geq 59$ meV; < 133 meV with the prior of the inverted order; and < 163 meV in w_0w_aCDM (the model whose dark energy evolves in time through two parameters), and 152 meV marginalized over histories of the dark energy. That with other likelihoods of the Planck data the flat-ΛCDM bound is 69.1 meV (plik) and 77.4 meV (PR4 L-H), and that within ΛCDM the limit computed by the method of Feldman and Cousins is 53 meV. That the bound of 64.2 meV lies between the two routes: the count route's 84.95 meV lies above it, and the Koide route's 59.31 meV below it; with the prior of the normal order (101 meV) and in w_0w_aCDM (163 meV) both sums lie below the bound. That of the four freedoms that relax the bound in the published analyses (a dark energy that evolves in time, dark matter that decays into dark radiation, an exchange of energy between dark matter and dark energy, and the removal of the late-universe distances), the collection's own cosmology closes three: the Passage

fixes $w = -1$ exactly and a constant cosmological constant, which, as *The Measures of the Kinds* reads it, leaves no exchange of energy between dark matter and dark energy, and the dark matter of *The Weight of Possibility* does not decay. That the measurement that settles the fork is the bound on the mass sum taken at the Passage's Ω_m and h, with $w = -1$ and no exchange; the fork is carried until the sum is measured there.

Sources: *The Measures of the Kinds*, Chapter 12, Sections I and VI, Table VI-2. Standing: a fork that a named measurement decides (the mass sum taken at the Passage's matter fraction and h, with $w = -1$ and no exchange).

I-027 Lithium-7 from the First Minutes. That the standard computation of the first minutes gives lithium-7 at $(4.72 \pm 0.7) \times 10^{-10}$ of the hydrogen, and the lithium-7 found in the metal-poor stars of the Galactic halo is $(1.45 \pm 0.25) \times 10^{-10}$ of their hydrogen, about a third of the computed amount (the Particle Data Group's review "Big-Bang Nucleosynthesis", Fields, Molaro and Sarkar, in Particle Data Group 2026). That the amount of lithium-7 depends on nuclear rates and on its later destruction in stars, and the list of kinds, which gives the first minutes their standard inputs, changes nothing in it.

Sources: *The Coming of the Kinds*, Chapter 12, Section V and note 3. Standing: a measured input (the computed and the observed amounts of lithium-7, as the review gives them).

I-V. Scope and the Coming Measurements

I-028 The Coming Measurements. That each coming measurement below is given with what I say the measurement will find and the step of my account that the measurement tests. The Euclid space telescope, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope, measuring the expansion history and the growth of structure, will find $w = -1$ exactly, at every era (the cosmological constant fixed once as the accessible share). The Dark Energy Spectroscopic Instrument and Euclid, measuring the matter fraction and the curvature of space toward an un-

certainty of about 0.001 in Ω_m, will find $\Omega_m = 0.311$ and a flat space (the identity and the flatness of the new epoch). The Simons Observatory, measuring the twist in the polarization of the microwave background that primordial gravitational waves would leave, will find nothing (waves of the geometry only at second order); with the Planck data it will measure the densities of the dark and the ordinary matter, and find $\Omega_c/\Omega_b = 6 - 2/\pi = 5.3634$, with the ratio of the radius of $\mathbb{C}P^2$ to the radius of S^2 measured to about 8 percent, which is the ratio Ω_c/Ω_b to about 0.6 percent (the registered ratio). The direct searches (LZ and others) will find nothing, at any sensitivity (the dark matter as a vibration of context). A measurement of the neutrino mass sum at the Passage's Ω_m and h, with $w = -1$, will find 84.95 meV or 59.31 meV, deciding the fork; the beta-decay mass, about 17.6 or about 9 meV, decides it without any cosmological analysis; JUNO, with T2K and NOvA, is expected to reach about three standard deviations on the order of the masses, for which both routes give the normal order.

Sources: *The View from Inside*, Appendix B; *The Gravitational Passage*, Chapter 18, Section XIII; *The Compactification*, Chapter 6, Section V; *The Weight of Possibility*, Chapter 5, Section IV; *The Weight of the Unseen*, Appendix, question A.10; *The Measures of the Kinds*, Chapter 12, Sections I and VI; *The Concept Sequence*, entry 46.15. Standing: a definition (of the tests, each with the step it bears on).

I-029 The Scope of the Cosmological Account. That the identity for the cosmological constant fixes the matter fraction, the constancy of the vacuum's energy and the flatness of the epoch. That the composition of the matter is supplied by the registered ratio of *The Weight of Possibility*; the particles, their masses and their charges belong to the account of the content; the magnitude of Newton's constant is taken from measurement; and the expansion rate apart from the matter fraction, the amplitude and tilt of the primordial departures, and the temperature at which the reconstructed history of the new epoch begins are outside the identity's scope. That the amount of ordinary matter, η, is a number of history and is taken as measured.

Sources: *The View from Inside*, Appendix B, "The Scope of the Predictions"; *The Gravitational Passage*, Chapter 13, Section VII; Chapter 18, Section XI; *The Coming of the Kinds*, Chapter 1, Section VI. Standing: a definition (of the scope of the identity).

I-030 The Budget in Brief. That the budget of this account is: a vacuum fraction of 0.689 and a matter fraction of 0.311, fixed by the identity $I^2\Omega_\Lambda = 8 - 3/\pi$, from the share of the vacuum's energy accessible to an observer inside the vacuum; within the matter, a ratio of dark to ordinary matter of $6 - 2/\pi = 5.3634$, fixed by the registered ratio, giving $\Omega_c = 0.2608$ and $\Omega_b = 0.0486$ with the lightest neutrinos; an amount of ordinary matter that is history, $\eta = (6.13 \pm 0.04) \times 10^{-10}$, taken as measured; an equation of state of exactly -1; a flat space; a Hubble constant of 67.8 ± 0.3 km s^{-1} Mpc^{-1} from the measured $\Omega_m h^2$, the same test as the matter fraction; and a neutrino mass sum carried as a fork, 84.95 or 59.31 meV. That the two quantitative tests, the matter fraction (0.311 against 0.315 ± 0.007) and the ratio (5.3634 against 5.370 ± 0.048), are measured agreements, each stated with the measurement that decides it (Chapter G, entry G-021; Chapter F, entry F-041).

Sources: *The Gravitational Passage*, Chapter 13, Sections III, IV and VII; *The Weight of Possibility*, Chapter 3, Section V, and Chapter 6, Section IV; *The Coming of the Kinds*, Chapter 1, Section VI; *The Measures of the Kinds*, Chapter 12, Section VI. Standing: a measured agreement (the matter fraction and the registered ratio); derived ($w = -1$ and the flatness); a measured input (η); a fork that a named measurement decides (the neutrino mass sum).

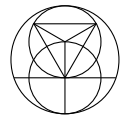
Summary

In this chapter I give the budget of the universe as the books of the collection give that budget. I give each value with its owner and its standing. Two dimensionless numbers are fixed by identities: the matter fraction 0.311, from the identity $I^2\Omega_\Lambda = 8 - 3/\pi$ that *The Gravitational Passage* derives from the share of the vacuum's energy accessible to an observer in-

side the vacuum, and the ratio of the dark to the ordinary matter, $6 - 2/\pi = 5.3634$, from the registered ratio of *The Weight of Possibility*. Both are measured agreements, 0.311 against 0.315 ± 0.007 and 5.3634 against 5.370 ± 0.048, and they share one coefficient, the internal curvature in Planck units.

With these two numbers the budget divides: a vacuum fraction of 0.689; cold matter of 0.3094 with the lightest neutrinos, of which 0.0486 is ordinary and 0.2608 dark; and an ordinary share of all the matter, 0.15644, which the routes free of the microwave background also find. The vacuum density, the Hubble constant of 67.8 ± 0.3 km s^{-1} Mpc^{-1} and the age in de Sitter times are the same test as the matter fraction, seen from other sides; the account agrees with the early-universe value of the Hubble constant.

The amount of ordinary matter is a number of history, as *The Coming of the Kinds* rules: a residue left from the earliest moments, carried as B − L and kept by every change after the turn, of which the kinds fix only the share held as baryons, 0.3328; the measured $\eta = (6.13 \pm 0.04) \times 10^{-10}$ is taken as measured. The equation of state, exactly −1, the flatness of space, and departures of one kind for every component, with primordial gravitational waves only at second order, are derived predictions of kind, each with the measurement that decides it. The neutrino mass sum is a fork that *The Measures of the Kinds* carries, 84.95 meV on the count route and 59.31 meV on the Koide route, decided by the sum taken at the Passage's matter fraction and h, or by the beta-decay mass.



Chapter J

Gravitation and the Content/Context Bridge

CHAPTER OVERVIEW

This chapter traces gravitation on the ground of comparison, from the account that *The Unquantized Arena* and its companion for the general reader, *Two Theories, One World*, set out. Every finding of physics is an act of comparison, and every comparison has a content and a context: the content is what the verdict of a comparison is on, and the context is the respect in which the two things compared are two. Quantum theory describes the content of physics, and general relativity describes its context, the arena of space and time in which everything measured is measured. Gravitation is not quantized: the arena has no least step, and nothing is a quantum of the arena itself, at any scale. In this chapter I follow the result that gravitation is not quantized through what remains when the error of quantizing the arena is removed: one classical geometry for each alternative of the matter; the experiment that entangles two masses through gravity; what the word "graviton" can name, and where the Planck scale lies; the three kinds of time and the measurement problem; the correspondence between gravity in anti-de Sitter space and a conformal field theory on its boundary; the formula of Ryu and Takayanagi as a bridge between two descriptions; the proposal called ER = EPR; black-hole information and the Page curve; the objections and the predictions; the reach of computation and simulation; and the structural arrow of time. An identification, a word the entries use often, is a stated step from the

structure of comparison to a physical quantity, made in the first person, with the observation that bears on it; the premises numbered P1 to P23 are those of *The Unquantized Arena*, each named beside its number.

The chapter rests on these chapters of *The Concept Sequence*: Chapter 13, The Incommensuration Theorem; Chapter 14, The Three Modalities; Chapter 15, The Three Axioms; Chapter 16, The Two Laws and the Content/Context Partition; Chapter 17, Quantization and the Partition; Chapter 18, The Three Forces, whose entry 18.19 states that gravity is the arena; Chapter 25, The Internal Space in Its Two Instances; and Chapter 39, Gravity Is the Arena: No Graviton.

J-I. Why the Arena Is Not Quantized

J-001 Quantum Gravity and the Diagnosis. That "quantum gravity" names the program of constructing a theory that quantizes the gravitational field in the way that quantum electrodynamics quantizes the electromagnetic field. So in such a theory the geometry of space and time is treated as a field whose findings are the eigenvalues of operators on a Hilbert space. A Hilbert space is the space of states of quantum theory, on which each measurable quantity is an operator whose eigenvalues are the possible findings of that quantity. That quantum theory describes the content of physics: what a measurement finds, in discrete classes with exact labels; and that general relativity describes the context of physics: the arena of space and time in which every content is found, smooth, and different from place to place. That to quantize the arena is to apply to the context the treatment that belongs to the content, and that this is a category error: a question that applies a concept where the concept does not belong. That "category error" is meant in the philosophical sense, a claim about which concepts apply to which kind of thing, made on the ground of comparison; it borrows nothing from the mathematical theory of categories. That gravitation is therefore not quantized.

Sources: *The Unquantized Arena*, Preface, What This Book Claims and How It Is Set Out, Section I, and Chapter 6, Sections I and II (Proposition 6.2); *Two Theories, One World*, Chapter 3, What a Category Error Is, and Chapter 11, What Quantizing Does, and Why the Arena Cannot Take It; *The Concept Sequence*, Chapter 17, entry 17.13, Chapter 18, entry 18.19, and Chapter 39, entry 39.10. Standing: derived, given a stated identification (P18, the identification of gravity as the context of physics).

> ***What "quantization" means in plain language.***
>
> To quantize a field is to pass from a description in which the field takes smooth values of any size to a description in which what is found comes in discrete steps with exact labels. When the electromagnetic field was quantized, the result was the photon: light is exchanged in indivisible packets of energy. That step was made within an arena of space and time that stayed as it was: the classical field and the quantized field are laid out in the same space and time. To quantize gravity would be to make the same step for the arena itself. The arena admits no such step. A quantized arena would carry at least one of the two pairings of characters that the Incommensuration Theorem forbids, and no further arena would remain in which its two descriptions could be compared (entry J-007, the arena admits no mode exchange).

J-002 The Incommensuration Theorem. That the four compounds of comparison are pairs of verdicts on content and on context: continuity (same content, same context), symmetry (same content, different context), discontinuity (different content, same context) and asymmetry (different content, different context). That the Incommensuration Theorem states: "symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity". That the theorem is a structural fact about the nature of knowing: it is about comparison, the act in which every finding is reached, and it has no physical premise. That the proof is by finite elimination over the ten pairings of the four compounds (six pairings of distinct compounds and four self-pairings): the rule !x (no self-application)

removes the four self-pairings; the rule !y (no opposite pairing) removes continuity with discontinuity and symmetry with asymmetry; and the rule !z (the balance of sameness and difference within each respect, which their inseparability requires) removes continuity with symmetry and discontinuity with asymmetry. That two pairings survive: continuity with asymmetry (the continuously asymmetric mode) and symmetry with discontinuity (the discontinuously symmetric mode); any comparison perfected at its foundation, that is, held exactly at the foundation of a description, is in one of these two modes.

Sources: *Incommensuration*, Chapter 5, Sections IV to VI, and Chapter 6, Sections V and VII; *The Unquantized Arena*, Chapter 1, Sections V to VII (P8, the four compounds; P9, the theorem); *The Concept Sequence*, Chapter 12, and Chapter 13, entries 13.11 to 13.14. Standing: a theorem.

> ***The two modes in the two theories.***
>
> A physical theory has two levels: its formalism, which exists, and its findings, which are real; the two levels are in conjugate modes (entry J-004, the cross-modal principle). General relativity finds a smooth metric and a continuous curvature, and a generic spacetime has no isometry (no transformation that carries it exactly onto itself): its findings are continuously asymmetric. Its formalism is symmetric: invariance under diffeomorphisms (smooth changes of coordinates), local Lorentz invariance, and the transformation laws of tensors. Quantum mechanics finds discrete eigenvalues of bound systems, exact quantum numbers, and one outcome from among the alternatives at each measurement: its findings are discontinuously symmetric. Its formalism is continuous: a Hilbert space, unitary evolution in continuous time, and complex amplitudes that vary smoothly.

J-003 The Mode Assignment. That the findings of content, at their foundation, are discontinuously symmetric, and the findings of context, at their foundation, are continuously asymmetric; this follows from the two laws (the law of symmetry governs content; the law of continuity governs context) and from the Incommensuration Theorem. That these are the

only two mode pairs of a fundamental physical description: discontinuously symmetric findings with a continuous formalism, and continuously asymmetric findings with a symmetric formalism. That P16, an identification, takes the findings of quantum mechanics and of quantum field theory as discontinuously symmetric, and the findings of Newtonian mechanics, classical electrodynamics, general relativity and thermodynamics as continuously asymmetric; the observable features on which P16 rests (the discrete spectra of quantum theory; the smooth metric of general relativity) serve also as a check of the two propositions above, which reach the same modes from the two laws. That a context-modal phenomenon is one that is the context of every physical comparison, and that any such phenomenon has four markers, derived before any phenomenon is named: it couples to every content; nothing shields it; its coupling to each content equals that content's inertia; and it has no discrete charge. That P18, an identification, takes gravity as the context-modal phenomenon of physics, the arena, by the match of all four markers. That P18 is the one identification in the chain from the ground to the category error, and the one that experiment tests directly: it predicts that free fall stays universal for matter and antimatter at every sensitivity yet to be reached, and that no arrangement of matter shields gravity.

Sources: *The Unquantized Arena*, Chapter 3, Section III (P16), Chapter 4, Sections II to IV (Propositions 4.1 to 4.3, Table 4.1), and Chapter 5, Sections II to VII (Propositions 5.1 to 5.5, P18); *Measurement*, Chapter 9, The Mode Assignment, and Chapter 10, Context and Its Markers; *Two Theories, One World*, Chapters 7 to 9; *The Concept Sequence*, Chapter 16, entries 16.12 and 16.15, and Chapter 17, entries 17.10 and 17.11. Standing: derived (Propositions 4.1 to 4.3, the modes of the findings of content and of context and the two mode pairs, and Propositions 5.1 to 5.4, the four markers); an identification (P16, the modes of the findings of six theories); an identification (P18, gravity as the context of physics).

J-004 The Cross-Modal Principle. That any investigation is a comparison of a framework (the structure by which the investigation is carried out) with a content (what the verdicts of the investigation are on). That frame-

work and content are always in conjugate modes: a symmetric framework probes continuous content, and a continuous framework probes symmetric content; a framework in the same mode as its content would apply that mode to itself, which the rule of no self-application (!x) excludes. That the reach of a framework into its content is dense (it approaches every finding of the content as closely as wished) and never exhaustive (it does not contain the content), so that every framework meets a residual in its content. That for a physical theory the framework is the formalism and the content is the set of findings, and that each of six established theories (Newtonian mechanics, classical electrodynamics, general relativity, quantum mechanics, quantum field theory and thermodynamics) has its two levels in conjugate modes.

Sources: *The Unquantized Arena*, Chapter 3, Sections II to IV (Proposition 3.1, Table 3.1); *Measurement*, Chapter 8, Sections II to IV (Proposition 8.1); *Category Theory and the Immanent Metaphysics*, Chapter 18, The Cross-Modal Principle; *Two Theories, One World*, Chapter 10, How a Theory Sees. Standing: derived (Proposition 3.1, from the Root Tautology, that comparison is isomorphic with interaction and relation, the Incommensuration Theorem, the three axioms, the distinction between what exists and what is real, and Proposition 2.2), and the six established theories of Table 3.1, each read in its standard form, bear it out.

> ***Why the cross-modal principle matters for quantum gravity.***
>
> Each of the two fundamental theories seems at first to contain both characters: quantum mechanics has continuous wavefunctions and discrete spectra, and general relativity has a smooth metric and invariance under smooth changes of coordinates. The cross-modal principle sorts these features by level. In quantum mechanics the findings are discrete and the formalism is continuous; in general relativity the findings are continuous and the formalism is symmetric. Each theory is conjugate at its two levels, and the principle states that every description exact at its foundation is so. The principle has content: it excludes, at the foundation of physics, a theory whose formalism and findings carry one character, such as a cellular automaton whose states are discrete and whose rule is the same at every site and every step.

J-005 The Two Residuals. That each of the two fundamental theories, quantum mechanics and general relativity, meets a residual where the character of the theory's findings exceeds what the mode of the theory's formalism can carry. That the residual of quantum mechanics is the measurement problem: the discontinuity of a definite finding, which the continuous formalism approaches as closely as wished and does not contain. That the residual of general relativity is the singularity: under the conditions of the singularity theorems, some geodesics (the paths of free fall and of light) cannot be extended. Along those geodesics, in the cases worked out in full, the curvature grows without bound, and the symmetric formalism does not carry that unbounded growth past the edge of its domain. That the smooth description holds below the Planck curvature, the curvature whose radius is the Planck length, so the residual of general relativity is met where the curvature reaches that value. That inside the horizon of a black hole that does not rotate, the direction toward the center is the direction of time, so the end that the interior reaches is a moment and not a place. That the two residuals are conjugate: the residual of quantum mechanics is a discontinuity met by a continuous formalism, and the residual of general relativity is an asymmetry without bound met by a symmetric formalism.

Sources: *The Unquantized Arena*, Chapter 3, Section V (Proposition 3.2, P17); Penrose (1965); Hawking and Penrose (1970); *At the Edge of Reality*, Chapter 1, What Happens at the Bottom of a Black Hole? Standing: derived, given a stated identification (Proposition 3.2, given P16, the modes of the findings of the two theories); a theorem (the singularity theorems, P17, established physics).

J-006 Quantization as a Mode Exchange. That quantization is a mode exchange: in one step, the findings of a description pass from continuously asymmetric to discontinuously symmetric, and the character that its formalism holds exact against those findings passes from symmetry to continuity; the two levels are conjugate before the step and after it. That the formalism must change with the findings, because a change of the findings alone would leave discontinuously symmetric findings in a sym-

metric formalism, which would apply a compound to itself, and the rule ! x excludes that. That classical and quantum electrodynamics describe one domain in the two mode pairs: the classical theory finds continuous electric and magnetic fields with a symmetric formalism (U(1) gauge invariance, Lorentz covariance), and the quantum theory finds whole numbers of photons and a charge in fixed units with a continuous formalism (Fock space, the path integral, the smooth running of its couplings). That both formalisms keep the gauge group and the Lorentz group, and the mode exchange concerns which character each formalism holds exact against its findings: in the classical theory exact symmetries of the equations place findings that vary smoothly, and in the quantum theory a continuous space of states gives the odds of findings that come in exact steps. That the mode exchange agrees with canonical quantization and with deformation quantization and adds no procedure of its own; what it adds is the reason why the formalism must change when the findings change. That the mode exchange of a content is made within an arena that stays as it is: the classical field and the quantized field are laid out in one space and time.

Sources: *The Unquantized Arena*, Chapter 4, Sections V and VI (Proposition 4.4, Table 4.2); *Measurement*, Chapter 9, Section VII, Quantization as a Mode Exchange; *Two Theories, One World*, Chapter 11; *The Concept Sequence*, Chapter 17, entry 17.13. Standing: derived (Proposition 4.4, from the cross-modal principle and the rule of no self-application).

J-007 The Arena Admits No Mode Exchange. That the program of quantum gravity makes two demands of one description of the arena: quantization (discontinuously symmetric findings with a continuous formalism, the mode pair of content) and recovery of general relativity (continuously asymmetric findings with a symmetric formalism, the mode pair of context). That recovery is asked only in a limit, and that a program which makes the arena quantum at its foundation and recovers smooth geometry only as a limit inverts which character the arena has perfected: the arena is the context of every finding, the findings of quantum electrodynamics included, and its perfected character is continuous asymmetry. That the arena admits no mode exchange, for two reas-

ons. From its findings: one description of the arena would hold the continuity and the asymmetry of the context together with the discontinuous symmetry of a quantized finding, and so would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. From its role: the mode exchange of a content is a passage between two descriptions that are compared within one arena that stays as it is, and a mode exchange of the arena would need a further setting in which its two descriptions could be compared; that setting would then be the context of physics in the arena's place, which contradicts the identification of the arena as the context. That for the arena the mode exchange is incoherent, and not only difficult. That to quantize the arena is therefore a category error.

Sources: *The Unquantized Arena*, Chapter 6, Sections I and II (Propositions 6.1 and 6.2, Table 6.1); *Two Theories, One World*, Chapter 11; *The Concept Sequence*, Chapter 17, entry 17.13. Standing: derived, given a stated identification (P18, gravity as the context of physics).

J-008 The Exact Scope. That "the arena is not quantized" means exactly two things: the arena has no least step (no least length, area or duration), and nothing is a quantum of the arena itself, at any scale. That gravitation is not quantized, and the claim holds at every scale: the arena has no step at any scale, and at the Planck scale, where the two descriptions meet, no single description carries both characters. That four things are left allowed. First, small waves of geometry about a smooth background can be described by quanta: a quantum of that description is a step in the energy of one mode of the wave, and the lengths, areas and durations of the four-dimensional geometry keep no least value; the effective field theory of gravity (a quantum field theory that describes a system below some energy, with every effect of higher energies summarized in local terms) computes the effects of the small waves in this description below the Planck curvature (Donoghue 1994). The phonon is the nearest sense of such a quantum: a phonon is a quantum of the description of the vibrations of a held structure, a solid whose bonds hold its atoms in a lattice; the arena is

not a held structure, so even that sense does not make a quantum of the description a quantum of the arena. Second, a detector that registers a gravitational wave in single clicks would not count against the account, and it would not show gravitation quantized (entry J-016, what the word "graviton" names). Third, the integers that characterize the global form of a manifold, such as its Euler characteristic and its signature, are discrete and do not change under a smooth change of the metric; they are discontinuously symmetric already, and a sum over them, of the kind topological quantum field theories make, gives the arena no step. Fourth, the matter in the arena is quantized in full, and each alternative of the matter carries its own classical geometry (entry J-013, one geometry for each alternative).

Sources: *The Unquantized Arena*, Chapter 6, Section IV (Proposition 6.3), and Chapter 9, Section I (Proposition 9.1); *The Gravitational Passage*, Chapter 6, Section II (Marker 3), and Chapter 12, the Questions closing Section VIII; *Two Theories, One World*, Chapter 11 and Chapter 14, Waves Without Particles of Space; *The Concept Sequence*, Chapter 39, entry 39.12. Standing: derived, given a stated identification (Proposition 6.3, given P18, gravity as the context of physics); derived (the description of small waves by quanta in the effective field theory of gravity, Donoghue 1994, established physics); a theorem (the topological invariants of a manifold are unchanged by a smooth change of its metric); an identification (one classical geometry for each alternative of the matter, Proposition 7.1, entry J-013).

J-009 The Two Instances of the Internal Space. That the context of physics has ten dimensions: one of time, three of space and six of possibility; the four of time and space are traversed, and the six of possibility are not directions of space and are never traversed: they are met only as what could be found. That the internal space $\mathbb{C}P^2 \times S^2$ (the product of the complex projective plane and the two-sphere) has two instances, and each part of the physics is read on its own instance. Its context instance is the space of possibility, six of the ten dimensions of context, with radii $\sqrt{6}\, l_{Pl}$ ($\mathbb{C}P^2$) and l_{Pl} (S^2) in the normalization of *The Gravitational Passage*, where l_{Pl} is the Planck length; gravitation, the cosmological constant and the

dark matter are read on it. The content instance of the internal space is a space of states, with no size, no radius and no fields, and the kinds and charges of the content are read on the content instance. Neither instance is a place, and nothing travels through either. That the internal space in its context instance is a held structure: it is held at the same size and the same shape at every point of the exterior, with each of its six directions at the modal boundary, where the curvature it sees is the Planck curvature; the arena of space and time is not a held structure. That the dark matter is made of gyrations, which are vibrations of that held structure, as the vibrations of a crystal are vibrations of its lattice: a gyration is a massive excitation of the metric of the six held internal dimensions, a state of the Kaluza-Klein kind, and it is no quantum of the arena and no quantum of gravitation. That the content instance is a space of states, not a quantized geometry, and the counts of the kinds of particle rest on the structure of comparison, not on a spectrum of the internal space. That the two senses of entry J-008 hold of the arena of space and time throughout: it has no least step, and nothing is a quantum of the arena itself, at any scale.

Sources: *The Measures of the Kinds*, Preface; *Measurement*, Chapter 11, Sections XI and XII; *The Gravitational Passage*, Chapter 9, Section IX (the radii), and Chapter 11, Section II (the hold); *The Weight of Possibility*, Chapter 1, Questions, and Chapter 4, Section II; *The Concept Sequence*, Chapter 25, entries 25.11 to 25.14. Standing: an identification (the two instances, *The Measures of the Kinds*, Preface); derived, given a stated identification (the radii and the hold, given that each internal direction stands at the modal boundary); an identification (the dark matter as gyrations, excitations of the metric of the six held internal dimensions, *The Weight of Possibility*).

J-010 Canonical Gravity: The Arena Put Into the State. That canonical quantum gravity applies the mode exchange to general relativity directly: it writes the theory in Hamiltonian form, with the metric h_{ij} of a spatial slice as its configuration variable, and promotes the constraints of the theory to operators acting on a wave functional $\Psi[h_{ij}]$, a quantum state that assigns an amplitude to each spatial metric (DeWitt 1967). That the Hamiltonian constraint becomes the Wheeler-DeWitt equation, $\hat{H}$

$\Psi[h_{ij}] = 0$, which contains no time parameter, so that the quantum state has nothing in which to evolve: the problem of time (Isham 1993). That *The Unquantized Arena* reads the problem of time as the category error in its plainest form: the context in which things happen has been put into the quantum state, instead of serving as the arena in which the state evolves. That a continuous formalism needs a context in which its smooth evolution is laid out; once the arena is itself the state, the formalism has no context left, and its evolution has no parameter. That each proposed resolution that chooses some degree of freedom to serve as a clock restores, within the description, a part of the context that the quantization removed. That the Hamiltonian form of gravity and the algebra of constraints, on which every later canonical approach builds, are achievements of the canonical program, and they stand.

Sources: *The Unquantized Arena*, Chapter 6, Section VI, Chapter 10, Section IV, and Chapter 14, Section III (Proposition 14.1(b)); DeWitt (1967); Isham (1993). Standing: derived (the Wheeler-DeWitt equation contains no time parameter, DeWitt 1967); derived (Proposition 14.1(b), the difficulty of dynamics met by a program that quantizes the arena directly); an identification (the problem of time as the arena put into the state).

J-011 Common Source, Not Merger. That the Incommensuration Theorem permits a common source: two descriptions, one continuously asymmetric and one discontinuously symmetric, each perfected in its own directions, joined side by side and related by a systematic translation; and that the inseparability of content and context requires that both descriptions be present wherever either is. That the theorem forbids a merger: one formalism in which the directions of the arena themselves (its coordinates, lengths, areas and durations, and its degrees of freedom) carry the discontinuous symmetry of content at the foundation, so that the same directions carry both characters. That the merger is excluded in principle, and therefore in all practice, whatever the mathematics of the formalism. That general relativity and quantum theory are conjugates in the sense of Axiom III: distinct, since each perfects a different surviving conjunction; inseparable, since every finding has a content and a context; and

non-interchangeable, since neither can do the other's work. That *The Unquantized Arena* reads their relation as one of mutual amplification: a richer symmetric structure of content asks for a richer arena, and a more complex geometry hosts a richer content. That the theorem forbids no interaction between quantum matter and geometry: the energy of the matter shapes the geometry, and the geometry guides the matter, in every alternative. That a union of descriptions of one character (Minkowski's union of space and time; the electroweak theory) is neither forbidden nor required by the theorem.

Sources: *The Unquantized Arena*, Chapter 6, Section V (Proposition 6.4), and Chapter 16, Sections I to III (Propositions 16.1 and 16.2, Table 16.1); *Two Theories, One World*, Chapter 24, Joined, Not Merged; *The Concept Sequence*, Chapter 18, entry 18.19, and Chapter 39, entry 39.11. Standing: derived (Propositions 6.4, 16.1 and 16.2); an identification (mutual amplification).

J-II. What Remains: One Geometry for Each Alternative

J-012 The Semi-Classical Equation and Its Two Forms. That the semi-classical equation (Møller 1962; Rosenfeld 1963) keeps one geometry for the whole quantum state of the matter, sourced by the expectation value of the stress-energy (the average of the density and flow of energy and momentum over the alternatives of the state, each weighted by its probability), and that in the Newtonian limit it becomes the Schrödinger-Newton equation (Diósi 1984). That the equation has two forms: in the first, the quantum state is never reduced; in the second, the state is reduced at each finding. That the first form is excluded by the experiment of Page and Geilker (1981): a radioactive decay selected one of two placements of the large masses of a Cavendish torsion balance; the first form predicts that the two pulls cancel and that the bar stays where it hangs on every run; the bar turned on every run, in the direction set by the actual placement of the masses. That the experiment excludes exactly one geometry

sourced by the expectation value in a state that is never reduced, and every account in which the field follows the outcome passes it. That Kuo and Ford (1993) reach a matching conclusion within the semi-classical approach: an equation sourced by the expectation value holds only where the fluctuations of the stress-energy about that value are small, and a mass held in two placements is a state for which they are large. That the second form signals faster than light: a geometry sourced by the reduced weights here would change with a distant choice of measurement (Gisin 1990; Bahrami, Großardt, Donadi and Bassi 2014). That where the matter is sharply one alternative, as for a star, a planet or a black hole of definite mass, the expectation value is the stress-energy of that one alternative, and the equation is Einstein's equation for it; every computation of astrophysics, and of quantum fields on a given background, is made in that case and stands.

Sources: *The Unquantized Arena*, Chapter 7, Sections I, II and IV (Proposition 7.3, Table 7.1); Page and Geilker (1981); Kuo and Ford (1993); *Two Theories, One World*, Chapter 12, One Geometry for Each Alternative. Standing: a fork that a named measurement decides (the experiment of Page and Geilker, 1981, decided it against the unreduced form); derived (the signaling of the reduced form, from Gisin 1990 and Bahrami, Großardt, Donadi and Bassi 2014); derived (the semi-classical equation is Einstein's equation where the matter is sharply one alternative).

> ***The semi-classical equation.***
>
> The equation is: $G_{\mu\nu}[g] = (8\pi G / c^4) \langle\psi| \hat{T}_{\mu\nu} |\psi\rangle$. In words: the curvature of the one geometry g, on the left, equals Einstein's constant times the expectation value of the stress-energy operator of the matter in its quantum state ψ, on the right. The geometry is sourced by the average of the matter's alternatives, each weighted by its probability. In the experiment of Page and Geilker, that average is right about the mean over many runs and wrong about every single run.

J-013 One Geometry for Each Alternative. That each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by the stress-energy of that alternative alone, and that the relative phases of the alternatives are kept; *The Unquantized Arena* calls this configuration per-alternative geometry. That an alternative of the matter is one term of the decomposition of its quantum state into states of definite distribution of stress-energy, at the resolution at which the geometry is sourced. That *The Unquantized Arena* makes this identification (Proposition 7.1) on these premises: where there is content, there is also context (the inseparability principle); each alternative of the matter is a possible content; the geometry is the context of the matter (P18); contexts compose only where their contents are found together, and alternatives are never found together, so no context is the context of two alternatives; and Einstein's equations fix the geometry from its source. That per-alternative geometry quantizes nothing: within each alternative the geometry is continuous, classical and without a least step, and the label of each geometry belongs to an alternative of the matter, not to a state space of geometries. That a finding changes no geometry, here or anywhere: a finding selects among alternatives that already carry their geometries; news of a distant finding enters a comparer's record only at an event on or within the future light cone of the finding, in the geometry of the alternative that the finding selects; and no comparison registers a surface of simultaneity. That per-alternative geometry passes the experiment of Page and Geilker: in each alternative the large masses sit in one placement and source the field of that placement.

Sources: *The Unquantized Arena*, Chapter 7, Sections III and IV (Propositions 7.1 and 7.2); *Two Theories, One World*, Chapter 12; Eberhard (1978). Standing: an identification (Proposition 7.1), which the experiment of Page and Geilker supports by excluding the averaged geometry; derived (Proposition 7.2, a finding changes no geometry).

> ***One geometry for each alternative, in symbols.***
> $|\Psi\rangle = \Sigma_i\, c_i\, |\psi_i\rangle$, $G_{\mu\nu}[g^{(i)}] = (8\pi G / c^4)\, T^{(i)}_{\mu\nu}$ for each i. In words: the state of the matter is a sum of alternatives ψ_i with complex coefficients c_i; each alternative has its own geometry $g^{(i)}$, whose curvature is sourced by the stress-energy $T^{(i)}$ of that alternative alone; and the coefficients, and so the relative phases, are kept. Where the matter is sharply one alternative, the sum has one term, and the equation is the semi-classical equation of entry J-012.

J-014 The Entanglement Test. That per-alternative geometry predicts that two masses, each held in two places and related only by gravity, become entangled. That, with n and f naming the nearer and the farther place of each mass, and d_{ab} the separation of the masses in the joint alternative (a, b), the phase of each joint alternative after a time t is $\varphi_{ab} = Gm^2t / (\hbar\, d_{ab})$, and the entangling phase is $\Delta\varphi = \varphi_{nn} + \varphi_{ff} - \varphi_{nf} - \varphi_{fn} = (Gm^2t / \hbar)(1/d_{nn} + 1/d_{ff} - 2/d_{nf})$, the one combination of the four phases that no local operation alters. That for the parameters that Bose and colleagues proposed in 2017 (masses of about 10^{-14} kg; separations $d_{nn} = 200$ μm, $d_{nf} = d_{fn} = 450$ μm and $d_{ff} = 700$ μm; a time of 2.5 s), the entangling phase is 0.314 rad, and the concurrence of the state (a measure of the entanglement of two two-level systems, zero for every unentangled state and at most one) is $|\sin(\Delta\varphi/2)| = 0.156$; for any later design, the same formula gives the phase from that design's masses, separations and time. That the outcome separates every account with one geometry for all the alternatives (the semi-classical equation; classical stochastic gravity, Tilloy and Diósi 2016 and Oppenheim 2023) and gravity-induced collapse (for which the superposition ends within about 0.012 s, against the 2.5 s the phase needs) from every account in which each alternative has its own geometry; linearized quantum gravity predicts the same phase as per-alternative geometry, and the outcome leaves the two together. That a null result bears on per-alternative geometry only in an experiment in

which the coherence of each interferometer is verified to hold over the protocol, so that ordinary decoherence is excluded as the cause of the missing entanglement.

Sources: *The Unquantized Arena*, Chapter 8, Sections II to V (Propositions 8.1 and 8.2, equations 8.1 to 8.3, Table 8.1); Bose, Mazumdar, Morley, Ulbricht, Toroš, Paternostro, Geraci, Barker, Kim and Milburn (2017); Marletto and Vedral (2017); *Two Theories, One World*, Chapter 13, The Experiment Being Built. Standing: derived, given a stated identification (Propositions 8.1 and 8.2, given per-alternative geometry, Proposition 7.1, entry J-013); a fork that a named measurement decides (the experiment of Bose and colleagues and its later designs).

> ***What the outcome decides, and what it does not.***
>
> The experiment decides whether each alternative of the matter has its own geometry. The arena is not quantized in either of the two senses of entry J-008, and that claim concerns the Planck scale and beyond, which the experiment does not reach. Marletto and Vedral argue that a mediator of entanglement must have at least two observables that do not commute. Per-alternative geometry meets that criterion: the geometry takes a different state in each alternative of the matter, with the relative phases kept. A positive result would therefore agree with their conclusion as they state it, and it would show neither a least step of the arena nor any quantum of the arena itself, at any scale.

J-015 Gravity-Induced Collapse and Gravitational Decoherence. That Diósi (1987) and Penrose (1996) proposed that gravity itself ends a superposition of masses, with a lifetime $\tau_{DP} \approx \hbar \,/\, E_G$, where E_G is the gravitational self-energy of the difference between the mass distributions of the two alternatives. That gravity-induced collapse adds a process to quantum dynamics, and per-alternative geometry adds none: on this account a superposition ends only through a finding, an interaction that registers which alternative is the case, made by an instrument, an environment or any other comparer; gravitation alone ends none. That an underground search for the faint radiation that the collapse noise would make

charged particles emit (Donadi, Piscicchia, Curceanu, Diósi, Laubenstein and Bassi 2021) rules out the natural version of the Diósi-Penrose model, the version with no free parameter; the versions with a larger effective size of the mass density are tested in a new regime by the experiment of entry J-014, for whose masses and placements the lifetime $\hbar$ / E_G is far shorter than the time the experiment needs. That gravity produces decoherence of the ordinary kind, and per-alternative geometry describes it directly: time dilation decoheres the position of a composite body held in a superposition of heights, because the proper time of its internal motion runs at a different rate in each alternative (Pikovski, Zych, Costa and Brukner 2015). That per-alternative geometry predicts no decoherence beyond the ordinary kind; models that make decoherence a fundamental consequence of a classical gravity (Tilloy and Diósi 2016; Oppenheim 2023) predict an extra decoherence and no entanglement through gravity; the experiment of entry J-014 separates the two, and it also bounds any extra channel of decoherence directly.

Sources: *The Unquantized Arena*, Chapter 7, Sections V and VI; Diósi (1987); Penrose (1996); Donadi, Piscicchia, Curceanu, Diósi, Laubenstein and Bassi (2021); *Two Theories, One World*, Chapter 12. Standing: a fork that a named measurement decides (the underground search of Donadi and colleagues, 2021, decided it against the natural version of the Diósi-Penrose model, and the experiment of entry J-014 decides it for the other versions); derived, given a stated identification (no decoherence beyond the ordinary kind, given per-alternative geometry, Proposition 7.1).

J-016 What the Word "Graviton" Names. That gravitation is not quantized. That the effective field theory of gravity quantizes a description: the description of the small waves of a classical geometry about its background, which holds below the Planck curvature; its quanta are quanta of that description at most, and never quanta of the arena, of space and time, or of the geometry. That a quantum of that description is one step in the energy of one mode of a small wave, and not a step in the extent of the geometry. That the word "graviton" names at most a quantum of that description, never a quantum of the arena; it names no particle, no carrier of

gravitation and no kind of matter. That the phonon is the nearest sense by which the word can be made intelligible: a phonon is a quantum of the description of the vibrations of a held structure, a solid whose bonds hold its atoms in a lattice; the arena is not a held structure, since it has no parts held in place by bonds and its curvature is set by the content in it, so even the phonon's sense does not make the graviton a quantum of the arena. That a detector that registers a gravitational wave in single clicks would not count against the account, and it would not show gravitation quantized: the clicks are quantum jumps of the detector, which is content, and they are produced equally by a classical wave acting on a quantum detector (Carney, Domcke and Rodd 2024), as the photoelectric effect, taken alone, is produced equally by a classical light wave acting on quantized atoms. That the kinds of particle are content, and gravity is the arena, their context, not a fourth force among them; so no graviton is among the kinds of particle.

Sources: *The Unquantized Arena*, Chapter 9, Sections I to III (Propositions 9.1 and 9.2), and Chapter 18, Section V; *The Gravitational Passage*, Chapter 6, Section II (Markers 3 and 4), and Chapter 12, the Questions closing Section VIII; *Two Theories, One World*, Chapter 14, Waves Without Particles of Space; *The Story of the Particles*, Level 7, Section VI; Donoghue (1994); Carney, Domcke and Rodd (2024); *The Concept Sequence*, Chapter 39, entries 39.10, 39.12 and 39.13. Standing: derived, given a stated identification (Propositions 9.1 and 9.2, given P18, gravity as the context of physics, and Proposition 7.1, one classical geometry for each alternative).

J-017 The Planck Scale and Newton's Constant. That the Planck length, about 1.6×10^{-35} m, is the length at which an object's gravitational size (the length at which its own gravity becomes strong) equals its quantum size (the length within which it cannot be confined without creating further particles). That the Planck scale is where the two descriptions meet: there neither description dominates, and both are relevant; the Planck length is not a least length, area or duration of the arena; and the smooth description holds below the Planck curvature. That each existing description reaches the edge of its domain at the Planck scale, and no de-

scription takes over there: the account posits no third description beyond the Planck scale, and no new geometry there. That the non-renormalizability of quantized general relativity (its need for infinitely many independent constants to absorb its divergences at high energy; 't Hooft and Veltman 1974) carries less weight than the structural point: effective field theory handles theories that are not renormalizable, non-renormalizability marks the scale at which the corrections of the effective theory become of order one, and the diagnosis rests on entry J-007, the arena admits no mode exchange, not on renormalizability. That, because the axioms are structural and not dimensional, the ground fixes the role of Newton's constant and measurement fixes the magnitude of Newton's constant. The role of Newton's constant is the strength of the coupling of every content to its context, universal, unshieldable and the same for every kind of matter. The magnitude of Newton's constant is $G = 6.67430(15) \times 10^{-11}$ m^3 kg^{-1} s^{-2} (CODATA 2022).

Sources: *The Unquantized Arena*, Chapter 6, Section VII, and Chapter 9, Sections IV to VII (Propositions 9.3 and 9.4); *The Gravitational Passage*, Chapter 17, Sections II and IV; *Two Theories, One World*, Chapter 15, Where the Two Descriptions Meet; 't Hooft and Veltman (1974); CODATA 2022; *The Concept Sequence*, Chapter 28, entries 28.11 and 28.16. Standing: derived (Proposition 9.3, the meeting of the two descriptions, from the two sizes of a mass and the Incommensuration Theorem); derived, given a stated identification (Proposition 9.4, the role of Newton's constant, given P18 and its four markers); a measured input (the value of Newton's constant).

J-III. Three Times and the Measurement Problem

J-018 Three Kinds of Time. That physics uses the word "time" for three mathematical objects of different kinds. That geometric time is the structure of time that the metric of spacetime carries: the proper time along each timelike worldline and the causal order of events; what a comparison finds of it is continuous and asymmetric. That event time, the time of find-

ings, is the order of findings for a comparer: a discrete sequence of outcomes, each the label of one element of a measurement, ordered by precedence; what is found of it is discontinuous and symmetric. That evolution time is the parameter t of the strongly continuous one-parameter group of unitary operators $U(t) = \exp(-iHt/\hbar)$ that carries the quantum state (Stone 1932); it belongs to the formalism of quantum mechanics, and it is continuous. That P19, an identification made in the focus of time (the frame of an argument about time, in which all three axioms hold), takes geometric time as the omniscient modality (what is laid out as structure and held in view as a whole), event time as the immanent modality (the act itself) and evolution time as the transcendent modality (the possibility from which an act proceeds), and takes the one-way order of findings, the order of Axiom II, as the direction of time. That the three are distinct, inseparable and non-interchangeable, the three conditions of Axiom III; general relativity holds geometric time and event time, and quantum mechanics holds event time and evolution time.

Sources: *The Unquantized Arena*, Chapter 10, Sections II to IV (Definitions 10.1 to 10.3, P19, Table 10.1, Proposition 10.1); *Measurement*, Chapter 12, Sections II and III; *Two Theories, One World*, Chapter 16, Three Kinds of Time; *The Concept Sequence*, Chapter 14, entries 14.11 to 14.15. Standing: a definition (the three objects); an identification (P19); derived (Proposition 10.1).

J-019 No Unified Time. That no unified time exists: a time found both as geometric time and as event time would carry at least one of the two forbidden pairings, and a unified time, which asks all four characters of one time, would carry both. That any formalism that holds two of the three times natively holds the third only as external (a background parameter, not a dynamical variable), as primitive (a rule given without derivation within the formalism), or not at all.

Sources: *The Unquantized Arena*, Chapter 10, Section V (Propositions 10.2 and 10.3); *Two Theories, One World*, Chapter 16. Standing: derived (Proposition 10.2, from the Incommensuration Theorem and Definitions 10.1, 10.2 and 10.4); derived, given a stated identification (Proposition 10.3, given P19).

J-020 Decoherence. That decoherence is the spreading of the relative phase of two alternatives into an environment that registers which alternative is the case, after which the alternatives no longer interfere in any measurement made on the system alone; in the density matrix of the system, written in the basis of the records, the off-diagonal elements decay toward zero. That decoherence fixes the basis of the records and makes interference among the alternatives unobservable, and that it does not select an outcome: every product of decoherence, the diagonal reduced state included, is an operator of the formalism given at a value of evolution time, while a finding is one outcome selected in event time.

Sources: *The Unquantized Arena*, Chapter 7, Section VI, and Chapter 11, Section III (Proposition 11.2); *Two Theories, One World*, Chapter 18, How Close a Smooth Theory Comes. Standing: a definition (decoherence); derived, given a stated identification (Proposition 11.2, given P19).

> ***Decoherence in everyday terms.***
>
> A small ball held in a superposition of two places behaves like one wave passing through two slits: while nothing registers which place, the two parts of the wave interfere. When stray molecules of air, or light, scatter from the ball, each part of the wave leaves its own mark on them. The two marks differ, so the two parts no longer interfere in any measurement made on the ball alone, and the fringes fade. Decoherence explains why the fringes fade. The finding of one place is a further act, a comparison that selects one outcome: the reduced description that decoherence leaves still lists both places with their probabilities.

J-021 The Measurement Problem as a Cross-Modal Residual. That the measurement problem is the residual of quantum mechanics: unitary evolution, in evolution time, approaches the finding as closely as wished and does not contain it; the finding belongs to event time, and it is not a state of the formalism. That in the focus of time the finding stands in the immanent role and the state stands in the transcendent role. By Axiom I the immanent is more fundamental than the transcendent, so the finding,

given in the act, is prior to the state. The state describes the range of possibility from which the act selects, and the state exists, as the description of possibility. That the demand for a mechanism that carries the state into the outcome within one time rests on the premise of a single unified time, which entry J-019 excludes. So the account adds no mechanism.

Sources: *The Unquantized Arena*, Chapter 11, Sections II and V (Proposition 11.1); *Two Theories, One World*, Chapter 17, Why a Measurement Has an Outcome; *The Quantum Passage*, Chapter 9, The Measurement Problem Dissolved; *The Concept Sequence*, Chapter 13, entry 13.20, and Chapter 15, entry 15.10. Standing: derived, given a stated identification (Proposition 11.1, given P19).

J-022 The Born Rule as the Bridge. That, given the Hilbert-space formalism of quantum mechanics, the Born rule is the only bridge from the formalism to the findings that assigns each element of a measurement the same probability in every measurement to which it belongs (Gleason 1957, for projectors, the operators of the outcomes of a sharp measurement, on separable Hilbert spaces of dimension at least three; Busch 2003, for effects, the operators of the outcomes of a general measurement, in every dimension, dimension two included). That *The Unquantized Arena* identifies that condition as a symmetry of the bridge: a projector that belongs to two different measurements is one content found in two contexts, and symmetry is sameness of content across a difference of context.

Sources: *The Unquantized Arena*, Chapter 11, Section IV (Proposition 11.3); Gleason (1957); Busch (2003). Standing: a theorem (Gleason 1957, Busch 2003); an identification (the condition as a symmetry of the bridge).

J-IV. AdS/CFT, the Ryu–Takayanagi Formula and ER = EPR

J-023 The Holographic Principle. That the holographic principle is the conjecture that the information a region of space can hold grows with the area of its boundary, measured in Planck units, and not with its volume, in the way that a hologram records a three-dimensional image on a two-

dimensional surface. That it was proposed by 't Hooft (1993) and by Susskind (1995), on the ground of the Bekenstein-Hawking entropy of a black hole, which is one quarter of the area of its horizon in units of the Planck area (Bekenstein 1973; Hawking 1975), and that Fischler and Susskind (1998) applied it to cosmology. That the holographic principle is a conjecture of physics: *The Gravitational Companion* says of a program that starts from it that "the holographic principle it starts from is itself a conjecture".

Sources: *The Gravitational Companion*, Other Programs That Obtain General Relativity, Section III; *The View from Inside*, Chapter 3, Why Nothing Else Has Worked; *The Unquantized Arena*, Chapter 17, Section III (the Bekenstein–Hawking entropy, equation 17.2); 't Hooft (1993); Susskind (1995); Fischler and Susskind (1998). Standing: a definition (of a conjecture of physics).

J-024 AdS/CFT: Two Descriptions in Conjugate Modes. That Maldacena (1998) conjectured that type IIB string theory on the product of five-dimensional anti-de Sitter space with a five-sphere is equivalent to four-dimensional maximally supersymmetric Yang-Mills theory with gauge group SU(N), and that the equivalence is made precise by an equality of generating functions (Gubser, Klebanov and Polyakov 1998; Witten 1998). That anti-de Sitter space is the maximally symmetric spacetime of constant negative curvature, with a timelike boundary at infinity, and that the correspondence between gravity in anti-de Sitter space (AdS) and a conformal field theory (CFT) on its boundary is called AdS/CFT. That the findings of the boundary theory (its conformal data: the scaling dimensions and spins of its primary operators, and the coefficients of their operator product expansion) are discontinuously symmetric, and its formalism is continuous; the findings of the bulk geometry (the metric and its curvature) are continuously asymmetric, and its formalism is symmetric; so the two sides are descriptions in conjugate modes. That neither side is more fundamental than the other: a duality exact in both directions privileges neither side, emergence is an asymmetric relation, and each description leaves a residual that the other description contains. That the exact duality relates two theories, and the exact duality gives the arena no quantum of its own. That the philosophy of dualities reaches the same

verdict on its own ground (Teh 2013; Dieks, van Dongen and de Haro 2015). That any systematic correspondence between a discontinuously symmetric description and a continuously asymmetric description is a bridge between conjugate descriptions and not a unification; AdS/CFT is such a correspondence, and it shows no quantum of the arena. That the claim of the account, that the arena has no quantum, concerns the actual world, which has no timelike boundary at infinity. That the AdS/CFT correspondence is the strongest test case of that claim, and not the ground of that claim.

Sources: *The Unquantized Arena*, Chapter 15, Sections I, II, VI and VII (Propositions 15.1, 15.4 and 15.5, Table 15.1); *Two Theories, One World*, Chapter 23, Two Descriptions That Translate; Teh (2013); Dieks, van Dongen and de Haro (2015). Standing: derived, given a stated identification (Proposition 15.1, given P16, the modes of the findings of the two sides); derived (Propositions 15.4 and 15.5, correspondence and not emergence).

J-025 Quantum Error Correction and Code Distance. That quantum error correction is a method for protecting quantum information from errors: a quantum error-correcting code encodes k logical qubits (the information to be protected) into n physical qubits (the hardware), and its code distance d is the least number of qubits on which an error must act to change the encoded information while keeping the state within the code space (an error that no check of the code detects). That a code of distance d corrects every error that acts on at most $\lfloor (d-1)/2 \rfloor$ qubits, the greatest integer not exceeding $(d-1)/2$.

Sources: Knill and Laflamme (1997); *The Quantum Bridge*, Chapter 4, Section II; Wootters and Zurek (1982) and *The Quantum Passage*, Chapter 10, Section III (the no-cloning theorem of the inset below). Standing: a definition (a quantum error-correcting code and its distance); a theorem (the errors a code of distance d corrects).

> ***Error correction by analogy.***
>
> Imagine writing a message on a chalkboard that is occasionally smudged by a careless passer-by. One strategy is to write the message three times. If one copy is smudged, the other two still agree, and the original message can be recovered. The more copies there are, the more smudges the message survives. Quantum error correction works by the same logic, with one difference: an unknown quantum state cannot be copied (the no-cloning theorem), so the encoding spreads the information across entangled states instead of duplicating it.

J-026 Reconstruction and Its Reach. That the bulk effective field theory about a given geometry is a code subspace of the boundary Hilbert space (the subspace of states in which the encoded information lies): the encoding is an isometry (a map that keeps every inner product) and is redundant, and an operator at the center of the bulk can be reconstructed on each of several boundary regions, so that it survives the erasure of any one of them (Almheiri, Dong and Harlow 2015); and every bulk operator in the entanglement wedge of a boundary region (the bulk region between that region and its extremal surface) can be reconstructed on that region (Dong, Harlow and Wall 2016). That the reach of reconstruction has three limits, each drawn from the literature. First, the code subspace about one geometry is small compared with the boundary Hilbert space. Second, behind a horizon the reconstruction depends on the state (Papadodimas and Raju 2013). Third, at finite N the number of independent localized operators within a bulk volume reaches the holographic bound (Hamilton, Kabat, Lifschytz and Lowe 2006). That *The Unquantized Arena* reads these limits as the reach and the residual of the cross-modal principle (entry J-004): the boundary description reaches the local geometric findings of the bulk densely and not exhaustively, and the residual lies where the bulk is most continuous in character: inside horizons, and at the scale where the bulk description meets its own edge.

Sources: *The Unquantized Arena*, Chapter 15, Section III (Proposition 15.2); Almheiri, Dong and Harlow (2015); Dong, Harlow and Wall (2016); Papadodimas and Raju (2013); Hamilton, Kabat, Lifschytz and Lowe (2006). Standing: derived (the results of reconstruction, in the literature cited, within the dictionary of the correspondence); an identification (the limits as the reach and residual of the cross-modal principle, Proposition 15.2).

J-027 The Ryu–Takayanagi Formula. That the formula of Ryu and Takayanagi (2006) gives the entanglement entropy of a boundary region A of a holographic conformal field theory as the area of the minimal bulk surface γ_A anchored on the edge of A and homologous to it, divided by four times Newton's constant, in units with $\hbar = c = 1$. That Hubeny, Rangamani and Takayanagi (2007) gave its covariant form, with an extremal surface in place of the minimal one; that Lewkowycz and Maldacena (2013) derived it by carrying the replica construction of the boundary entropy into the bulk gravitational path integral, assuming the dictionary of the correspondence; that Faulkner, Lewkowycz and Maldacena (2013) added the entanglement entropy of the bulk quantum fields across the surface; and that Engelhardt and Wall (2015) extended the prescription beyond first order by extremizing the generalized entropy, the area term plus the entropy of the bulk fields. That the entropy on the left side is divergent in the continuum, and the formula gives its universal part once the area is cut off near the boundary.

Sources: *The Unquantized Arena*, Chapter 15, Section IV (equations 15.2 and 15.3); Ryu and Takayanagi (2006); Hubeny, Rangamani and Takayanagi (2007); Lewkowycz and Maldacena (2013); Faulkner, Lewkowycz and Maldacena (2013); Engelhardt and Wall (2015). Standing: derived (within the AdS/CFT correspondence, from its dictionary, Lewkowycz and Maldacena 2013).

> ***The formula in plain language.***
>
> Imagine one system described in two ways: a boundary description (a quantum theory living on a surface) and a bulk description (a gravitational theory living in the interior). The formula says: pick a region on the boundary; find the smallest surface in the interior whose edge matches

the edge of that region; measure the area of that surface in Planck units; divide by four. The result is the entanglement entropy of the region, the amount of quantum correlation between the chosen region and the rest of the boundary. The formula joins a quantity of the boundary description (an entropy) to a quantity of the bulk description (an area).

The formula and its corrections in symbolic form.
$S(A) = \text{Area}(\gamma_A) \,/\, (4\, G_N)$, where $S(A)$ is the entanglement entropy of boundary region A, γ_A is the minimal bulk surface homologous to A, and G_N is Newton's constant. With the corrections of the bulk quantum fields: $S(A) = \text{ext}_\chi \,[\, \text{Area}(\chi) \,/\, (4\, G_N) + S_{\text{bulk}}(\Sigma_\chi) \,]$, the extremum, over bulk surfaces χ anchored on the edge of A, of the area term plus the entropy of the bulk fields in the region Σ_χ between χ and A.

J-028 The Ryu–Takayanagi Formula as a Bridge. That the two sides of the formula differ in character: the entropy depends only on the spectrum of the reduced density matrix of A, and is the same under every unitary change of basis on A, so it is symmetric in character; the area of a surface in a smooth geometry varies smoothly with the region and with the metric, so it is continuous in character. That the formula, with its corrections, is a bridge: an exact, mediated and non-invertible relation between a symmetric quantity of the boundary description and a continuous quantity of the bulk description; it derives neither geometry from entanglement nor entanglement from geometry. That three results support the bridge reading: the derivation of Lewkowycz and Maldacena assumes the dictionary; the relation is not invertible, since the entropy data are one real number for each boundary region, and different bulk states with equal entropies give equal data; and the corrected formula holds in any quantum error-correcting code with complementary recovery (Harlow 2017). That the geometric side reaches the further entropies of the spectrum, the Rényi entropies, only by a different geometry for each index (Dong 2016). That the Gauss-Bonnet theorem shows the same three features in elementary form: for a compact surface M with boundary, $\int_M K\, dA + \int_{\partial M} \kappa_g\, ds =$

$2\pi\,\chi(M)$, an exact relation between a continuous integral of curvature and an integer, mediated by compactness, and not invertible. That the formula is a bridge of the same type as the Born rule (entry J-022), which takes the continuous state of the formalism to the probabilities of discrete findings.

Sources: *The Unquantized Arena*, Chapter 15, Section IV (Proposition 15.3, equation 15.4); *The Quantum Bridge*, Chapter 1, Section V; Harlow (2017); Dong (2016). Standing: derived (Proposition 15.3, resting on the derivation of Lewkowycz and Maldacena, the difficulty of multiplicity of Proposition 14.1(a), and the result of Harlow); a theorem (the Gauss-Bonnet theorem).

J-029 Holographic Monogamy. That entropies given by the formula of Ryu and Takayanagi satisfy the monogamy of mutual information, $I(A{:}B) + I(A{:}C) - I(A{:}BC) \leq 0$ for three boundary regions A, B and C, while in general quantum systems the left side can take either sign (Hayden, Headrick and Maloney 2013); and that the holographic entropy cone is bounded, for up to four regions, by strong subadditivity and monogamy, with further families of inequalities from five regions on (Bao, Nezami, Ooguri, Stoica, Sully and Walter 2015). That *The Unquantized Arena* reads these results as the residual on the side of the bulk: areas carry only the patterns of entanglement that lie inside the holographic entropy cone, and the patterns outside it belong to the boundary description, which no smooth geometry carries; so the residuals of the two sides are complements.

Sources: *The Unquantized Arena*, Chapter 15, Section V (equation 15.5); Hayden, Headrick and Maloney (2013); Bao, Nezami, Ooguri, Stoica, Sully and Walter (2015). Standing: a theorem (the monogamy of holographic mutual information, and the holographic entropy cone); an identification (the residual reading).

J-030 Wormhole (Einstein–Rosen Bridge). That a "wormhole", more precisely called an Einstein-Rosen bridge, is a geometric structure in general relativity in which two separate regions of spacetime are connected by a narrow throat (Einstein and Rosen 1935). That the simplest example arises from the maximal extension of the Schwarzschild solution (the

solution describing a non-rotating black hole): the extended geometry contains two asymptotic regions joined by a bridge that passes through the black hole interior. That this bridge is non-traversable: no signal or object can pass through it from one region to the other. That a bridge of this kind is a feature of a smooth geometry, a description of the arena whose findings are continuously asymmetric (entry J-003, the mode assignment).

Sources: Einstein and Rosen (1935); *The Unquantized Arena*, Chapter 14, Section IV, program 8, ER=EPR; *The Quantum Bridge*, Chapter 1, Section I. Standing: a definition (of a structure of general relativity).

> ***Wormholes in plain language.***
>
> Imagine a sheet of paper with two distant points marked on it. Now fold the paper so the points touch, and punch a hole connecting them. The hole is a "wormhole": a short passage between two regions that are far apart along the surface. In general relativity, the "paper" is the fabric of spacetime, the "fold" is the curvature produced by extreme gravity, and the "hole" is the Einstein-Rosen bridge. An Einstein-Rosen bridge pinches shut before anything can cross, and that is how it differs from the wormholes of science fiction: it is non-traversable.

J-031 Entanglement and Entanglement Entropy. That two systems are entangled when their joint state cannot be written as a product of a state of each: a pure joint state is entangled when it is not of the form $|\psi_1\rangle \otimes |\psi_2\rangle$, and a mixed joint state is entangled when it is not a convex combination of product states; in plain words, their alternatives are joined so that neither has a state of its own. That measuring one gives information about the other at any separation, and that no signal passes between them: every local statistic of one system is the same whatever measurement is made on the other (Eberhard 1978). That the entanglement entropy of one part of a system in a pure joint state is the von Neumann entropy $S = -\mathrm{Tr}(\rho \log \rho)$ of the reduced density matrix ρ of that part, obtained by tracing over the other part; it is zero for a product state, and it is larger the more the two

parts are entangled. That entanglement is content: in the account of *The Quantum Passage* it is content that belongs to the joint comparison and to neither of the comparisons of the parts alone, and the entangled state is a description whose findings are discontinuously symmetric.

Sources: *The Quantum Passage*, Chapter 8, Section II; *Two Theories, One World*, Chapter 13 and Chapter 22; *The Unquantized Arena*, Chapter 7, Section IV; *The Quantum Bridge*, Chapter 1, Section I; Eberhard (1978). Standing: a definition (entanglement and entanglement entropy); a theorem (no signal passes between entangled systems, Eberhard 1978); an identification (entanglement as content).

> ***Entanglement entropy in everyday terms.***
>
> Suppose two coins are prepared so that whenever one shows heads the other shows tails, and nobody knows which coin shows which. Each coin alone is completely uncertain, and so is the pair: the pair is either heads-tails or tails-heads, one bit of uncertainty. For classical coins the uncertainty of the whole is never less than the uncertainty of either part. An entangled pair in a pure state breaks that rule. The pair as a whole is in one definite state, with zero entropy, while each member taken alone is maximally uncertain. The entanglement entropy measures this: it is the uncertainty of one part when the whole is pure. No arrangement of classical coins shows it.

J-032 The ER = EPR Proposal. That the eternal two-sided black hole in anti-de Sitter space has a dual description as two copies of the boundary theory in an entangled state (Maldacena 2003). That from that case Van Raamsdonk (2010) argued that removing the entanglement between two parts of a quantum system pulls the matching regions of its geometry apart, so that connected spacetime is built from entanglement. That Maldacena and Susskind (2013) proposed that a pair of entangled black holes and a bridge of geometry joining their interiors are one phenomenon in two descriptions, and that every entangled pair is joined by a bridge of

geometry. That the proposal is called ER = EPR, after the bridges of Einstein and Rosen (entry J-030) and the entangled pairs of Einstein, Podolsky and Rosen (entry J-031).

Sources: *The Unquantized Arena*, Chapter 14, Section IV, program 8; *The Quantum Bridge*, Chapter 1, Section I; *Two Theories, One World*, Chapter 22, What Each Program Found; Maldacena and Susskind (2013); Van Raamsdonk (2010); Maldacena (2003); Einstein, Podolsky and Rosen (1935). Standing: a definition (what the proposal states).

J-033 ER = EPR as a Correspondence. That the founding commitment of ER = EPR is a correspondence between a continuously asymmetric description, the connecting geometry, and a discontinuously symmetric description, the entangled state. That as a correspondence the proposal agrees with the content/context partition, and it is of the kind the Incommensuration Theorem leads one to expect: a map between two descriptions shows that they translate, and translation runs in both directions. That where both descriptions of a system exist, the relation that ER = EPR asserts between a connecting geometry and an entangled state is a bridge between conjugate descriptions. That this bridge is exact, mediated and not invertible, and derives neither description from the other (*The Quantum Bridge*, Proposition 1.1). That the reading usually drawn from the proposal, that geometry emerges from entanglement and that gravity is quantum at bottom, ranks the quantum side as the source and the geometry as its product, and does not agree with the partition; *Two Theories, One World* states: "I accept the correspondence and decline the ranking: a map between two descriptions shows that they translate, and translation runs in both directions". That, read as a correspondence, the proposal meets none of the three difficulties that the account predicts for programs of quantum gravity. That, read as a derivation of geometry from entanglement, the proposal meets the difficulty of multiplicity, since the entanglement data do not fix one geometry (entry J-028). That the account supplies no Einstein-Rosen geometry to a single entangled pair of particles: each alternative of the pair carries the classical geometry its own stress-en-

ergy sources, and for two electrons that geometry has no horizon and no second exterior (*The Quantum Bridge*, Proposition 1.2). That its place among the programs of quantum gravity is therefore in the middle of the gradient, between the programs that quantize the arena and those that keep it classical, and that place depends on how the proposal is read.

Sources: *The Unquantized Arena*, Chapter 14, Sections III to VI (Proposition 14.1; program 8; the summary table; the gradient); *The Quantum Bridge*, Chapter 1, Section III (Propositions 1.1 and 1.2); *Two Theories, One World*, Chapter 22. Standing: an identification (the correspondence reading); derived (the difficulty of multiplicity for the derivation reading, from Proposition 14.1(a)); derived (*The Quantum Bridge*, Proposition 1.1); derived, given a stated identification (*The Quantum Bridge*, Proposition 1.2, given per-alternative geometry, Proposition 7.1).

> ***The two readings.***
>
> Two readings of ER = EPR are available, and the account separates them. On the first, entanglement is fundamental and geometry emerges from it; that reading asks the discrete side to fix the continuous one, and it meets the difficulty of multiplicity, since many geometries fit the same entanglement data. On the second, the geometric description and the quantum description are two descriptions of one phenomenon, related by a translation that runs both ways; that reading agrees with the content/context partition, and it needs no quantum of the arena. The mathematics of the proposal is the same on both readings; the readings differ in what they say the mathematics shows.

J-V. Black Holes, Objections and Predictions

J-034 Black-Hole Information: The Premise Removed. That Hawking (1975) computed, for quantum fields on the classical geometry of a collapsing star, that a black hole radiates with a thermal spectrum at the temperature $T_H = \hbar c^3 / (8\pi G M k_B)$, about 6.2×10^{-8} K for a hole of one solar

mass, and that Hawking (1976) concluded that a hole which evaporates completely turns a pure state (a quantum state whose von Neumann entropy is zero) into a mixed one (a state whose von Neumann entropy is positive). That the paradox rests on two further premises: (W) that the classical geometry describes the whole history, its singularity included; and (Q) that geometry and matter together form one quantum state whose evolution is one unitary map. That premise (Q) applies the concept of a quantum state to the arena, which is the category error of entry J-007; and that the loss in Hawking's conclusion is located at the singularity, where the classical description reaches the edge of its domain, so that it is not a result of either description. That information, in the sense of the form of the content, carried with the matter wherever the matter goes, is not destroyed in a black hole: no step of either description destroys it. That for a hole formed from a star, with a temperature below 6.2×10^{-8} K and an evaporation time longer than 10^{67} years, the account requires only that the information is not destroyed and is not accessible from outside while the hole stands, which is established general relativity.

Sources: *The Unquantized Arena*, Chapter 17, Sections I to III and VII (P20, P21; Propositions 17.1 to 17.3); *Two Theories, One World*, Chapter 25, Where Does the Information Go?; Hawking (1975); Hawking (1976). Standing: derived (Hawking radiation and evaporation, quantum fields on a classical geometry, P20 and P21, established physics); derived, given a stated identification (Propositions 17.1 to 17.3, given P18 and per-alternative geometry, Proposition 7.1).

J-035 Why Area, Not Volume. That the Bekenstein-Hawking entropy, $S_{BH} = k_B c^3 A / (4G\hbar)$, one quarter of the area A of the horizon in units of the Planck area, is established physics (Bekenstein 1973; Hawking 1975), and that *The Unquantized Arena* reads it as a count relative to a context: the number of states of the hole that no comparison from outside tells apart. That a bounded region has two natural counts of its information: one taken cell by cell through the interior, which grows with the volume, and one taken through the interactions that cross the boundary, which grows with the area; in the focus of counting the information of a region, *The Un-*

quantized Arena identifies the first count as omniscient and the second as immanent. That by Axiom I the immanent is more fundamental than the omniscient, so the count made from a place, through interaction, is the more fundamental of the two, and *The Unquantized Arena* identifies the dependence of the entropy on area as that count; the identification makes no claim on the coefficient one quarter.

Sources: *The Unquantized Arena*, Chapter 17, Sections III and IV; Bekenstein (1973); Hawking (1975); Bombelli, Koul, Lee and Sorkin (1986); Srednicki (1993). Standing: derived (the area law, Bekenstein 1973 and Hawking 1975, established physics); an identification (the area law as the immanent count).

J-036 The Page Curve as a Conditional Result. That for a hole small enough to evaporate while an outside world remains, the Page curve, the entanglement entropy of the radiation outside plotted against time, follows as a conditional result under two conditions and two premises: (C) the whole formed by what fell in, the content inside the horizon and the radiation began pure and stays pure, as described from one outside world, with no geometry put into the state; (H) the entanglement entropy of the content inside the horizon cannot exceed S_{BH}; P22, the entanglement entropy of the radiation cannot exceed its coarse-grained entropy S_{emit}, the thermal entropy computed from its spectrum alone; and P23, the joint state is one of very many that no comparison from outside tells apart, each counted alike, with Page's result on the average entropy of a part of a random pure state (Page 1993a). That under these, the entanglement entropy of the radiation is the smaller of the two capacities, to within half a nat: $S_{rad}(t) = \min [S_{emit}(t), S_{BH}(t)]$. That, with r the ratio of the entropy emitted to the Bekenstein-Hawking entropy lost, the generalized second law gives $r \geq 1$ (Bekenstein 1974), and Zurek (1982) showed that r exceeds one, so the turn of the curve comes before the hole has lost half its entropy, and no later than 0.646 of the evaporation time. That with the greybody factors of photons and of the small waves of the geometry included ($r = 1.4847$; Page 2013), the turn comes at 0.538 of the evaporation time, when the hole has lost 40.25 per cent of its entropy, which are Page's

values. That Page counts the small waves in the terms of their description by quanta, and the count is a count of that description, so the count quantizes nothing of the arena.

Sources: *The Unquantized Arena*, Chapter 17, Section V (Propositions 17.4 and 17.5, Table 17.1); Page (1993a); Page (1993b); Page (2013); Bekenstein (1974); Zurek (1982). Standing: derived, given a stated identification (Propositions 17.4 and 17.5, given the conditions (C) and (H), the established bound P22, and P23, which includes the identification of the equal weighting as a symmetry).

J-037 The Objections at Full Strength. That nine objections offered as evidence for a quantum theory of geometry are each answered by what their phenomenon is on the account: Hawking radiation is quantum fields on a classical geometry; microstate counting is a count of states of content, or a count resting on a chosen spectrum of area; AdS/CFT is a correspondence between two descriptions of one system; the singularity is the edge of the classical description's domain; gravitons are quanta of a description of the small waves of a classical geometry; the primordial gravitational waves, should they be found, are small waves of a classical geometry whose amplitude is computed in a description by quanta; nonrenormalizability marks the scale where the corrections of the effective theory become of order one; the Planck scale is the meeting of the two descriptions; and the objection that the argument is "only philosophy" is answered by its observable consequences (entry J-038, five predictions). That every phenomenon offered as evidence for quantum gravity is, on the account, one of four things: a result for content on a classical context, a count of the degrees of freedom of content, a correspondence between two descriptions, or the edge of a description's domain; none of the nine exhibits any quantum of the arena itself, at any scale.

Sources: *The Unquantized Arena*, Chapter 18, Sections I to X (Table 18.1); *Two Theories, One World*, Chapter 26, What a Physicist Will Say. Standing: an identification (what each phenomenon is on the account).

J-038 Five Predictions. That the account makes five predictions, and that each prediction is stated with the measurement that decides the prediction and with the step on which the prediction bears. First, two masses related only through gravity become entangled with the Newtonian phase (entry J-014); the experiment of Bose and colleagues and its later designs decide it, and it bears on per-alternative geometry. Second, gravitation is not quantized: no effect will show the geometry carrying quantum degrees of freedom of its own, beyond those tied to the alternatives of the matter, and a description of the small waves of the geometry by quanta holds below the Planck curvature, its quanta being quanta of that description, never quanta of the arena. A scattering of small waves that departs from the effective theory where its corrections are large, or a superposition of macroscopically different geometries that no superposition of the matter sources, would decide it, and it bears on the exact scope of entry J-008. Third, the Eötvös parameter (twice the difference of the free-fall accelerations of two bodies divided by their sum) is zero for every pair of materials at every precision, antimatter falls as matter does, and nothing shields gravity; torsion balances, the MICROSCOPE satellite and its proposed successor STEP, and the free fall of antihydrogen decide it, and it bears on the identification of gravity as the context (P18). Fourth, space and time have no least length, area or duration, so the speed of light does not depend on energy through any step of spacetime; the arrival times of high-energy photons from distant sources and the speed and polarizations of gravitational waves decide it, and it bears on the first sense of entry J-008. Fifth, the dictionary of AdS/CFT holds within limits that no construction removes; a theorem at finite N giving an exact map, independent of the state, from every boundary state to a smooth bulk geometry would decide it, and it bears on the reading of the correspondence as a map between conjugate descriptions (entry J-024). That a result counts, by the rule of decision of *The Unquantized Arena*, when it is established at five standard deviations by at least two independent kinds of measurement.

Sources: *The Unquantized Arena*, Chapter 19, Sections I to VII (Predictions 19.1 to 19.5, Table 19.3); *Two Theories, One World*, Chapter 27, What the World Will Show; *The Concept Sequence*, Chapter 46, entry 46.15. Standing: a fork that a named measurement decides (each of the five); derived, given a stated identification (Prediction 19.1, given per-alternative geometry, Proposition 7.1, and Predictions 19.2 to 19.4, given P18 through Propositions 5.1 to 5.4 and 6.3); derived (Prediction 19.5, from the reading of the correspondence, Propositions 15.4 and 15.5).

J-VI. Computation and Simulation

J-039 Classical Computation. That a classical computation is a deterministic algorithm: a step-by-step procedure whose next state follows necessarily from its current state by a fixed rule, carried out by digital machinery in definite steps. That one rule applied the same way at every step is symmetry perfected, and by the first clause of the Incommensuration Theorem what is absolutely symmetric is intrinsically discontinuous; a continuity cannot be composed of purely symmetric steps. That a classical computation therefore has no hard randomness of its own: everything in a classical computation is already fixed by the rule and the current state. That a classical computation also has no arrow of time of its own: its steps come in an order that adds nothing the rule did not already fix. The order of the steps of a classical computation is an indexing of states, which is geometric time and not event time. That a computation, as a symmetric framework, can approach the continuous as closely as one likes and never be it; this is the cross-modal principle (entry J-004) applied to computation.

Sources: *Causality Without Determinism*, Chapter 5, Sections II and VI; *The Basis of Physics*, Essay 19, Against Computationalism, Sections III to V. Standing: derived.

J-040 A Quantum Computer. That a quantum computer is a physical device whose operations, between the preparation and the final readout, are a reversible rule: every step can be undone exactly, so the sequence runs as well backward as forward, which is symmetry without an arrow. That the space of states of its register is the Hilbert space of quantum mechanics, which belongs to the continuous formalism of that theory, and that its findings, the outcomes of its readout, are discrete (entry J-002, the inset on the two modes). That its readout is a measurement, a physical comparison in the world, with a single undetermined outcome and an irreversible record; so a quantum computer has, at its measurements, what every measuring instrument has, and its program, as a program, is a symmetric rule like any other.

Sources: *Causality Without Determinism*, Chapter 5, Section VII; *The Quantum Bridge*, Chapter 7, Section II; *The Unquantized Arena*, Chapter 3, Table 3.1. Standing: derived.

J-041 The Continuum Argument Against Simulation. That the continuum holds what no discrete computation reaches, so no discrete computation can capture a continuous reality, and the difference is one of kind, not of engineering. That every computation, however large, and even an idealized machine with unbounded time and unbounded tape, produces only outputs that can be listed, and the totality of what it could ever produce can itself be listed; by Cantor's diagonal argument, the continuum holds what no listing reaches. That by the Principle of Identity (what no conceivable test can distinguish must be regarded as the same) the question whether the world is a simulation is a question about a test in principle, and the diagonal argument defines such a test. That sensitivity to initial conditions is the mechanism that amplifies the gap between what can be listed and the continuum into a difference between two histories. That the context of every comparison is continuous, as the Incommensuration Theorem requires and as physics finds, with the smoothness of the arena at every scale yet probed as the agreeing evidence; so there is something about what exists that no computation can capture. That a simula-

tion of the arena on any computing device, classical or quantum, reaches the findings of the arena densely and never exhaustively, and the boundary is the same for both kinds of device.

Sources: *The Basis of Physics*, Essay 18, The Continuum Argument Against Simulation, Sections II, V, VI and IX; *Incommensuration*, Chapter 5, Section III; *The Quantum Bridge*, Chapter 7, Section IV (Proposition 7.1); *The Concept Sequence*, Chapter 15, entry 15.11; Cantor (1891). Standing: derived (the exclusion, given the continuity of the context, which is itself derived from the law of continuity); a theorem (Cantor's diagonal argument).

J-042 What a Deterministic Simulation Cannot Reproduce. That a deterministic simulation can reproduce the statistics of quantum outcomes to any precision chosen in advance (entangled pairs are simulated on classical computers every day, at a cost that grows exponentially with the number of particles entangled), and that it cannot reproduce their ontic character: the outcomes of the simulation are determined by the program, while the outcomes of a quantum system are not determined by prior information. That hard randomness (an outcome that no prior specification fixes, however complete), the arrow of time and the subjective are bound together, distinct, inseparable and non-interchangeable (the bound triple), so that a process that lacks two of them lacks the third. That a classical computation lacks hard randomness and the arrow, and so lacks the subjective; and that whether a system has the bound triple depends on whether it is a physical process making comparisons in the world, with an inside, and never on which program it runs.

Sources: *Causality Without Determinism*, Chapter 3, and Chapter 5, Sections III, IV, VII and VIII; *Incommensuration*, Chapter 8, Section X; Conway and Kochen (2006); Conway and Kochen (2009). Standing: derived (from the free will theorem of Conway and Kochen, under its stated conditions, and the Incommensuration Theorem).

J-043 The Simulation Hypothesis. That the simulation hypothesis proposes that our world is a computation running in a more fundamental world, and that we, and everything we find, are states of that computation

(Bostrom 2003). That I answer the simulation hypothesis with three arguments. First, a deterministic simulation cannot be what the universe is, because its outputs are determined and the universe's are not (entry J-042). Second, the hypothesis asks for a description of the whole world from outside it, and such a description has no comparison behind it, because the whole has no outside to compare it with. Third, a simulation fed true randomness from its host world has the host's randomness and the host's arrow, so a simulated world containing real subjects is a restricted region of the one world in which it runs, and not a separate world. That the hypothesis in the form that matters, that we are real subjects in a world that is not real, is therefore excluded by the structure of comparison, in principle and therefore in all future practice.

Sources: *Causality Without Determinism*, Chapter 5, Section VIII; *Measurement*, Chapter 2, Section II, and Chapter 14, Section X; *The Quantum Bridge*, Chapter 7, Section VIII; Bostrom (2003). Standing: derived.

J-044 Approximate Simulation. That approximate simulation is possible and useful: a computation can approach a continuous process as closely as wished, which is the dense reach of a symmetric framework into a continuous content, and it never contains that process. That the behavior of a system at the mesoscopic level, where its relevant causal processes are described by approximately deterministic models, can be simulated, as the behavior of a brain at that level can; its constituent particles cannot be simulated as what they are, since they have no prior state that fixes their outcomes, although their statistics can be reproduced to any precision chosen in advance. That the errors of a simulation are of three kinds: the error of content computed on a context held fixed, which letting the context respond reduces; the error of a finite stage of refinement, which refinement makes as small as wished; and the residual error, met where the description computed reaches the edge of its domain or where a computation on one geometry is asked for a quantity of the whole, which refinement does not reduce. That a whole-universe simulation that is a world of its own is impossible in principle, and not only beyond present technology.

Sources: *Causality Without Determinism*, Chapter 5, Sections VI and VIII; *The Unquantized Arena*, Chapter 3 (Proposition 3.1); *The Quantum Bridge*, Chapter 7, Section IV (Proposition 7.1), and Chapter 8, Section I (Definitions 8.1 to 8.3). Standing: derived; a definition (the three kinds of error).

J-VII. The Direction of Time

J-045 The Structural Arrow of Time. That the Incommensuration Theorem gives an arrow of time that is not entropic: a symmetry of the laws guarantees that the reverse of a lawful process is lawful, and does not guarantee that the reverse is faithful, retracing the original exactly; faithful reversal would require a continuously specified state together with perfected symmetry, which is the conjunction of symmetry and continuity that the theorem forbids; so perfect time reversal is precluded. That this arrow follows from the intrinsics of comparison alone and is present in any single interaction, not only in ensembles; it depends on no entropy, no boundary condition, no Past Hypothesis (the hypothesis that the early universe had very low entropy) and no particular formalism. That *The Basis of Physics* identifies it with the precedence of Axiom II applied to the physical domain, which is the one-way order of findings (P19, entry J-018). That the entropic arrow of thermodynamics is joined to the structural arrow in every act of recording, since making a record raises the entropy of the instrument that holds it, and that the entropic arrow needs further premises, statistical mechanics and the Past Hypothesis among them. That the Past Hypothesis is, on this account, a statement about initial conditions, which fixes how the structural arrow is expressed in the history of this universe, and not the source of the arrow: the direction of time was not waiting on it.

Sources: *The Basis of Physics*, Essay 11, The Structural Arrow of Time, Abstract and Sections III, IV and VIII; *Measurement*, Chapter 12, Section VII, and Chapter 14, Section III; *The Unquantized Arena*, Chapter 10, Section II (P19); *The*

Concept Sequence, Chapter 15, entry 15.13. Standing: derived (the preclusion of perfect time reversal); an identification (the arrow with the precedence of Axiom II).

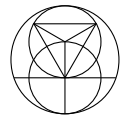
Summary

This chapter is the branch of the map on which gravitation stands, and its result is the one that *The Unquantized Arena* and *Two Theories, One World* establish: gravitation is not quantized. The Incommensuration Theorem, a structural fact about the nature of knowing, allows a comparison perfected at its foundation only two characters, continuous asymmetry and discontinuous symmetry. The two laws assign the first to context and the second to content, so the findings of the content of physics come in discrete classes with exact labels, the findings of its context are smooth and different from place to place, and every theory holds its findings and its formalism in conjugate modes, which the cross-modal principle states and six established theories bear out. Quantization is a mode exchange of both levels of a description together, made within an arena that stays as it is. Gravity has the four markers of any context, and with that identification, P18, the arena is the context of physics; it admits no mode exchange, and to quantize it is a category error. The exact scope is two senses: the arena has no least step, and nothing is a quantum of the arena itself, at any scale.

What remains is quantum matter on a classical geometry, one geometry for each alternative of the matter, the identification of *The Unquantized Arena* that the experiment of Page and Geilker supports. A finding changes no geometry, two masses related only through gravity become entangled with the Newtonian phase, and a superposition ends only through a finding, never through gravitation alone. The word "graviton" names at most a quantum of the description of the small waves of a classical geometry, never a quantum of the arena; the phonon is the nearest sense of it, and the arena is not a held structure. The Planck scale is where the two descriptions meet, and no description takes over there, as *The Unquantized Arena* and *The Gravitational Passage* state. The six internal dimen-

sions of the context instance are held, at the same size and shape at every point of the exterior, and the dark matter is made of gyrations, each an excitation of their metric, no quantum of the arena and no quantum of gravitation.

The same ground of comparison reaches the finding and the correspondences of physics. Physics has three times, distinct, inseparable and non-interchangeable; no unified time exists, and the measurement problem is the residual where evolution time meets event time, with the Born rule as the bridge between the two levels. AdS/CFT, the formula of Ryu and Takayanagi and ER = EPR are bridges between conjugate descriptions, exact, mediated and not invertible, with neither side more fundamental; information is not destroyed in a black hole, and the Page curve follows under its stated conditions. Each objection offered for a quantum theory of geometry is answered by what its phenomenon is, and five predictions, each with the measurement that decides it, carry the account to experiment. Computation is symmetry perfected: it reaches the continuous densely and never contains it, so a simulation is a process within the one world and never a world of its own. Last, the Incommensuration Theorem gives time a structural arrow, present in every single interaction, beneath the arrow of entropy. The parts fit as one account: the arena is context, the matter is content, and each is described exactly in its own character and joined to the other side by side.



Chapter K

Formal Axiomatization and Machine Verification

CHAPTER OVERVIEW

This chapter traces the renderings of the Incommensuration Theorem and of the three axioms of the Immanent Metaphysics in mathematics, and what a machine check of a rendering shows. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. It is proved from comparison, before any mathematics, by a finite examination of the ways the four compound concepts can be paired (*The Concept Sequence*, entries 13.11 to 13.14). A rendering is a statement of that theorem, or of the axioms, in a chosen mathematical setting, and each rendering is true in that setting in the form "if these premises, then this". In this chapter I state the renderings in type theory, in homotopy type theory and in cohesive toposes, say where higher-categorical structure enters those renderings and where it does not, and report what a proof assistant has checked. A machine-checked proof shows validity: every step follows from its premises by the rules of the formal system. A claim is settled when soundness is shown as well: the formal statement truly renders the claim, and the premises hold. The theorem sets the scope of each rendering, and no rendering sets the scope of the theorem.

The chapter rests on *The Concept Sequence*, Chapter 13, The Incommensuration Theorem (entries 13.14 and 13.21); Chapter 14, The Three Modalities (entries 14.12 to 14.14); and Chapter 15, The Three Axioms (entries 15.10, 15.13, 15.15, 15.20 and 15.21). From those chapters it takes the finite form of the theorem (the admissible comparisons are the group Aut(2) of order two: the identity and the exchange of same and different) and the derivation of the axioms from the theorem. The results of this chapter are owned by *The Formal Foundations*, *Category Theory and the Immanent Metaphysics* and *Incommensuration*, and each concept of type theory, homotopy type theory, category theory, cohesion and machine checking is defined where it is first used.

K-I. Renderings of the Theorem and the Axioms, and Their Machine Checks

K-001 Type Theory. That a type theory is a formal system in which every term has a type, written t : A ("t is a term of type A"). That under the correspondence of propositions with types, a proposition is read as a type and a proof of the proposition is a term of that type, so that a proposition is proved when its type has a term (is inhabited). That in this correspondence a proof is exhibited, and not only asserted: the term that inhabits the type is the proof, and checking the proof is checking that the term has the type. That such a check shows the validity of the proof: the proof follows from its premises by the rules of the system. That the check does not reach the soundness of the proof, which is whether the type truly states the claim that the type is meant to state, and whether the premises of the proof hold.

Sources: *The Formal Foundations*, Chapter 7, Section I ("Agda is a proof assistant based on dependent type theory, in which propositions are types and proofs are their elements"); *Category Theory and the Immanent Metaphysics*, Chapter 3, Section II (validity and soundness); Howard (1980); Martin-Löf (1984). Standing: a definition.

> ***Why types matter for formalizing the axioms.***
>
> In a conventional logical system, one states an axiom and asserts that it is true. The axiom is a sentence; its truth is a yes-or-no judgment. In type theory, a statement is a type, and the question that the type raises is whether anything inhabits the type: whether a term exists that has the structure the type demands. A type with such a term is inhabited; a type with none is empty.
>
> The proof of the Incommensuration Theorem from comparison is an examination (*The Concept Sequence*, entry 13.14): the four compound concepts pair in ten ways, the three rules !x, !y and !z (no application to self, no application of opposites, no separation of sameness and difference) remove eight, and two survive. In finite form it is a theorem about the two-element set of verdicts, whose admissible comparisons are exactly the identity (continuous asymmetry) and the exchange (discontinuous symmetry) (*Category Theory and the Immanent Metaphysics*, Theorem 5.4). A type-theoretic statement of that finite form is a rendering of the examination, and a term that inhabits it is a written record of the same finite check.

K-002 Homotopy Type Theory. That homotopy type theory is a type theory in which types are read as spaces (homotopy types), terms as points of those spaces, and the terms of an identity type a = b, for two terms a and b of one type, as paths from a to b. That two terms of one identity type may be distinct paths between the same endpoints, that paths between paths are again terms of identity types, and that this structure of paths, paths between paths, and so on, is information that a two-valued logic of propositions does not record. That the univalence axiom states, for any two types A and B in a universe of types (a type whose terms are types), that the canonical map from the identifications A = B to the equivalences A ≃ B is itself an equivalence: an equivalence of types is the same as an identification of them.

Sources: *The Formal Foundations*, Chapter 3, Section I ("univalence states that an equivalence of types is an identification of them"); Voevodsky (2010); Univalent Foundations Program (2013). Standing: a definition.

> ***Paths, loops, and higher structure.***
>
> Consider two points in a space. In homotopy type theory, a path from one to the other is an identification of them. If there are two different paths, one may ask whether one path can be continuously deformed into the other. If it can, the two identifications are themselves identified; if it cannot, they are two different identifications of the same two points.
>
> In a logic in which a proposition is only true or false, this distinction is not recorded: if a proposition is true, all its proofs are treated alike. In homotopy type theory, the space of terms of a type may have its own internal structure: connected components, loops, and loops between loops. In a set, by contrast, two elements are either equal or not, and there is nothing further to record.
>
> The univalence axiom extends this to types themselves: two types that are equivalent, with the same points, the same paths, and the same higher paths up to equivalence, are identified. In this sense the axiom makes structure decide the identity of types, whatever their labels.

K-003 Higher Category Theory, and the One-Level Rendering. That higher category theory is the mathematics of categories (a category being a collection of objects with maps between them, called morphisms, that compose) that have, besides objects and morphisms, 2-morphisms (maps between morphisms), 3-morphisms (maps between 2-morphisms), and so on: n-categories for every natural number n, and ∞-categories with k-morphisms for every k. That the categorical rendering of the Incommensuration Theorem and the three axioms in *Category Theory and the Immanent Metaphysics* uses 1-categorical structures only: cohesive toposes are 1-toposes, the quadruple adjunction that defines cohesion is a chain of ordinary functors (a functor being a map between categories that keeps composition), the three modalities are ordinary endofunctors (functors from a category to itself), the ternary cycle of Axiom II runs between or-

dinary subcategories, and no 2-category or ∞-category is required at any step (cohesion and its adjunctions are defined in entry K-004). That this is a fact about the rendering, and it matches what is rendered: the structure of the theorem involves three modalities, a finite set of relationships among them, and a cycle that closes, and nothing in it requires an unbounded hierarchy. That the same book identifies the proliferation of higher categories, in which each level captures structure the previous level missed and generates structure that requires a level above it, with the mathematical form of the theorem's own limit: a framework of perfected uniformity that reaches for the whole of its content keeps falling short by one level. That a rendering in a higher setting, such as homotopy type theory, is a further rendering: the higher structure it carries belongs to that setting, and it adds nothing to the content of the theorem. That in the standard model of cohesive homotopy type theory, the ∞-topos of sheaves on cartesian spaces, the types that are both discrete and codiscrete are exactly the propositions (the types that are empty or contractible, that is, have at most one point up to paths), at every level of the hierarchy, so the higher levels hold no survivor that the level of sets does not.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 3, Section VI; Chapter 4, Section 4.4; Chapter 14, Sections 14.9 to 14.11; *The Formal Foundations*, Chapter 3, Section VI (Proposition 3.2) and Section IX; *The Concept Sequence*, entries 13.14, 15.10, 15.13 and 15.15. Standing: a definition (higher categories); a theorem (that the categorical rendering uses 1-categorical structures only, and Proposition 3.2 of *The Formal Foundations* in the standard model); an identification (the proliferation of higher categories as the form of the theorem's limit, stated in the first person in *Category Theory and the Immanent Metaphysics*, Chapter 3, Section VI, and Chapter 14, Section 14.11).

> ***Why one level suffices.***
>
> Category theory starts from postulated concepts, object, morphism and composition, which are not themselves of the kind they describe: the concept of a morphism is a morphism only inside a higher category, which needs a still higher one to describe it. That difference of type drives the ascent from level to level.

> The Immanent Metaphysics starts from the Root Tautology, that comparison, interaction and relation are one act, and the act of comparing is itself a comparison. Its description closes at three levels: the six intrinsics (sameness and difference, content and context, the subjective and the objective) describe comparison, the theorem describes the constraints on comparison, and the recognition that the theorem is itself an act of comparison closes the loop. The categorical rendering reflects this closure: it lives in the part of category theory that has closed, toposes with finitely many modalities and a cycle among them that returns to its start.

K-004 The Modal Operators in Cohesion. That a topos is a category that behaves like the category of sets: it has products, objects of maps, and an object Ω of truth values, and so a logic of its own. That a cohesive topos $\mathscr{E}$ over a base topos $\mathscr{S}$ is a topos joined to $\mathscr{S}$ by a chain of four adjoint functors, $\Pi \dashv \text{Disc} \dashv \Gamma \dashv \text{CoDisc}$, in which Γ extracts the points (global sections) of an object, Disc re-embeds a set with no connection between its points (discrete), CoDisc re-embeds a set with every point joined to every other (codiscrete), and Π extracts the connected pieces. That an adjunction $L \dashv R$ is a natural bijection between the maps $L(A) \to B$ and the maps $A \to R(B)$, so that each functor is determined by the other. That the three modalities of *The Concept Sequence*, the omniscient (entry 14.12), the transcendent (entry 14.13) and the immanent (entry 14.14), are rendered as three endofunctors of $\mathscr{E}$: the omniscient modality as the flat modality $\flat = \text{Disc} \circ \Gamma$, which keeps the points of an object and none of their connections; the transcendent modality as the sharp modality $\sharp = \text{CoDisc} \circ \Gamma$, which keeps the points and joins every point to every other; and the immanent modality as the identity functor Id, which returns each object with the connections it actually has. That a fourth endofunctor, the shape $\int = \text{Disc} \circ \Pi$, presents each connected piece of an object as one point, and enters the ternary cycle of Axiom II (entry K-008). That, with a probe (a map from a connected object) identified as one comparison, the discrete objects are absolutely symmetric and absolutely discontinuous (the discontinuously symmetric mode) and the codiscrete objects are absolutely

continuous and absolutely asymmetric (the continuously asymmetric mode). That in the 1-categorical setting of this rendering the shape records only the connected components of an object; in a higher-categorical cohesive setting the shape records the full homotopy type. That cohesion has also been developed inside homotopy type theory, where the shape, ♭ and ♯ are modal operators of the type theory: ♯A can be formed from every type A, and ♭A only from a type that depends on crisp variables alone (variables used only through their points; entry K-006).

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 2, Section 7 (Definitions 7.1 and 7.2: cohesion and its modalities); Chapter 5, Section 5.7 (the probe as one comparison, Propositions 5.13a and 5.14); Chapter 7, Section 7.6; Chapter 8, Definition 8.1; Chapter 9, Remark 9.2a; Chapter 18, Example 18.5 (the shape in the 1-categorical setting); *The Formal Foundations*, Chapter 3, Section II; Shulman (2018); *The Concept Sequence*, entries 14.12 to 14.14. Standing: a definition (cohesion and the modal endofunctors); an identification (the probe as one comparison, the single-comparison form of the bridge from comparison to cohesion, Identification 12.1b, stated in the first person); a theorem (in the setting: the discrete objects realize one mode and the codiscrete objects the other, Proposition 5.14).

> ***What "flat", "sharp" and "shape" do to an object.***
>
> Think of an object of a cohesive topos as a landscape: a set of points, with paths that join some of them. The flat operator ♭ keeps every point and removes every path: each point stands alone. What ♭ returns is the discrete image, in which every point is separated from every other.
>
> The sharp operator ♯ keeps every point and joins every point to every other: nothing separates any point from any other. What ♯ returns is the codiscrete image.
>
> The shape ∫ replaces each connected piece of the landscape by a single point: it counts the pieces, and forgets which points lie in each.
>
> The identity leaves the landscape as it is, with the paths it actually has. All three images, ♭X, X and ♯X, have the same points; they differ in how those points stand to one another. In this rendering the formal counterpart of the Incommensuration Theorem is the statement that no object

with two or more points is both discrete and codiscrete. No landscape with two or more points is at once a set of isolated points and a single wholly joined whole.

K-005 Axiom III in Cohesion: Distinctness. That Axiom III reads, in the wording of *An Immanent Metaphysics*, aphorism 1.3-1: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable". That in finite form the distinctness clause says that the set of modalities {T, I, O} (the letters naming the transcendent, the immanent and the omniscient) has three elements. That in cohesion the distinctness clause is the statement that the three modal endofunctors are distinct: ♭ is not isomorphic to Id, Id is not isomorphic to ♯, and ♭ is not isomorphic to ♯ (two functors being isomorphic when a natural two-way correspondence carries one to the other). That the last of these is the non-degeneracy condition ND1: there is a set S for which Disc(S) is not isomorphic to CoDisc(S). That over the base $\mathscr{S}$ = Set (the topos of sets), with the Nullstellensatz condition C5 (every connected piece has a point), ND1 holds at every set with two or more elements: no object with two or more points is both discrete and codiscrete. That in a degenerate cohesive topos, in which Disc = CoDisc, the discrete and the codiscrete images of a set coincide, every connected object has exactly one point, and the four compound concepts collapse into one another.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 5, Definition 5.9; Chapter 7, Definition 7.1 and Section 7.2.1; Chapter 12, Theorem 12.1 and Proposition 12.1a; Chapter 13, Theorem 13.1(e); Appendix F.3, Proposition F. 3.3; *The Formal Foundations*, Chapter 3, Section III (distinctness as the type-theoretic form of ND1); *The Concept Sequence*, entry 15.15. Standing: a theorem (in the setting); an identification (Identification 12.1b, which places comparison in that setting).

What degeneracy means.

The topos of sets is a degenerate example of cohesion: Γ is the identity, and Disc = CoDisc = Id. Every set is at once discrete and codiscrete, and the "wholly separated" and "wholly joined" versions of a set are the same

> set. In such a setting there is no difference between structure and connection, and the vocabulary in which the Incommensuration Theorem is stated is missing. A degenerate topos does not contradict the Incommensuration Theorem, because a degenerate topos cannot state the Incommensuration Theorem. Non-degeneracy is what lets a topos carry the theorem.

K-006 Axiom III in Cohesion: Inseparability and Non-Interchangeability. That in finite form the inseparability clause of Axiom III (*The Concept Sequence*, entry 15.15) says that the three modalities form a single orbit under the cyclic order T → I → O → T, and the non-interchangeability clause says that the admissible symmetries of the triad are the three rotations, so that no transposition of two modalities is admitted. That in cohesion the inseparability clause is the statement that every object has all three modal images, stated as a schema over objects: for each object X, if X has a point, then ♭X and ♯X have a point. That in cohesion the non-interchangeability clause is the statement that ♭ is not isomorphic to ♯ (ND1), with ♭ a non-trivial comonad (an operation with a natural map from its image back to the object, here ♭X → X) and ♯ a non-trivial monad (an operation with a natural map from the object into its image, here X → ♯X), while Id is trivially both. That in the topos what makes ♭ and ♯ non-interchangeable is ND1: in a degenerate cohesive topos the single functor ♭ ≅ ♯ is both a comonad and a monad, so the difference between comonad and monad does not by itself keep them apart. That in the type theory of cohesion non-interchangeability is carried by the rules themselves: ♭ is comonadic and ♯ monadic, ♭A is formed only from crisp data and ♯A from any data, and no rule exchanges the two.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 5, Definition 5.9; Chapter 8, Proposition 8.1 and Remark 8.2; Chapter 10, Propositions 10.6 and 10.7; Chapter 13, Theorem 13.1(e); *The Formal Foundations*, Chapter 3, Section III; *The Concept Sequence*, entry 15.15. Standing: a theorem (in the setting); an identification (Identification 12.1b).

> ***The crisp and the cohesive.***
> In the type theory in which Shulman develops cohesion, the rules for ♭ and ♯ refer to a separate context of crisp variables, variables on which a term depends only through their points and not through their cohesion. The rules give ♭ and ♯ different forms: ♭A is formed only from crisp data and ♯A from any data, and no rule carries one to the other. The rules state what ♭ and ♯ are, and they are also satisfied when every type is at once discrete and codiscrete, as in the topos of sets. So the difference between some ♭A and the matching ♯A, the type-theoretic form of ND1, is a further condition, in the type theory as in the topos.

K-007 Axiom I in Cohesion: Primacy and Conjugacy. That Axiom I reads, in the wording of *An Immanent Metaphysics*, aphorism 1.3-1: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". That in finite form the conjugacy is the transposition $\tau = (O\ T)$, which exchanges the omniscient and the transcendent and fixes the immanent; τ reverses the orientation of the cycle, so the omniscient and the transcendent are exchanged by a symmetry of the unoriented triad and by no symmetry of the oriented triad: they are conjugate and not interchangeable, and the priority of the immanent is its position as the one term fixed by the conjugacy. That in cohesion the conjugacy is the construction of ♭ and ♯ from the one functor Γ, composed with its two adjoint partners: $\flat = \mathrm{Disc} \circ \Gamma$ with Disc the left adjoint of Γ, a comonad; $\sharp = \mathrm{CoDisc} \circ \Gamma$ with CoDisc the right adjoint of Γ, a monad. That in cohesion the priority of the immanent is definitional: ♭ and ♯ are defined as composites that pass through the base, while Id is defined on $\mathscr{E}$ alone. That every natural transformation (a map between two functors compatible with every morphism) from ♭ to ♯ factors through the identity, and the canonical comparison $\theta : \flat \Rightarrow \mathrm{Id} \Rightarrow \sharp$ is the only one: every comparison of the discrete presentation with the codiscrete presentation passes

through the object itself. The factoring through the identity is the categorical form of the statement that the omniscient and the transcendent relate to each other only through the mediation of the immanent.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 5, Definition 5.9 and Proposition 5.10; Chapter 8, Propositions 8.1, 8.2 and 8.2a; Chapter 13, Theorem 13.1(c); *The Formal Foundations*, Chapter 3, Section IV, and Chapter 6, Proposition 6.1; *The Concept Sequence*, entries 13.14, 15.10 and 15.20. Standing: a theorem (in finite form, Proposition 5.10, and in the setting, Propositions 8.1, 8.2 and 8.2a); an identification (Identification 12.1b, for the reading of the setting as comparison).

> ***What the chain $\flat X \to X \to \sharp X$ means.***
>
> For any object X (any "landscape"), the flat image $\flat X$ is the discrete version (the points standing alone), the sharp image $\sharp X$ is the codiscrete version (the points wholly joined), and X itself sits between them. The counit $\flat X \to X$ takes each separated point to the same point of X. The unit $X \to \sharp X$ takes each point of X to the same point of the wholly joined version.
>
> The composite $\flat X \to \sharp X$ compares the discrete version with the codiscrete version, and it always passes through X. When X has two or more points, this composite is not an isomorphism: the discrete and the codiscrete versions of two or more points are different objects. When X has at most one point, the composite can be an isomorphism, because there is nothing to separate or to join. For an object that is neither discrete nor codiscrete, neither the counit nor the unit is an isomorphism: that is how an actual object stands between the two extremes.

K-008 Axiom II in Cohesion: The Directed Cycle. That Axiom II reads, in the wording of *An Immanent Metaphysics*, aphorism 1.3-1: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". That the cycle it states is $T \to I \to O \to T$. That in cohesion the three modalities determine three categories: T, the codiscrete objects (the objects X for which $X \to \sharp X$ is an isomorphism); I, the whole topos $\mathscr{E}$; and O, the discrete objects (the

objects X for which ♭X → X is an isomorphism). That three adjunctions join them in a cycle of left adjoints T → I → O → T: from T to I by Disc ∘ Γ, from I to O by the shape ∫ = Disc ∘ Π, and from O to T by CoDisc ∘ Γ. That the cycle closes on T and on O: the composite of the three left adjoints, begun at T or at O, is isomorphic to the identity there; begun at I, it returns an object as its shape ∫X, and the composite of the right adjoints returns it as ♭X. That the direction of the cycle is fixed before the formalization, by the order in which the pairs of intrinsics engage in one act of comparison and by the conjugacy of Axiom I; the formalization identifies which formal cycle carries it. That being a cycle of left adjoints leaves the orientation open, since a reverse cycle of left adjoints O → I → T → O also closes on T and on O; over Set with C5 the forward cycle alone has a step that sends a class of points to a single point, the step from I to O, which acts on points as the map from points to connected pieces. That I identify this points-to-pieces map with the clause "A class of the immanent will precede an instance of the omniscient": each connected piece is a class of points, and the step presents it as one point.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 9: Definition 9.1; Propositions 9.3, 9.4, 9.6, 9.8 and 9.9; Section 9.5 (Proposition 9.12 and Section 9.5.3); Chapter 13, Theorem 13.1(d); *The Formal Foundations*, Chapter 3, Section V; *Incommensuration*, Chapter 8, Sections III and IV (the direction of Axiom II, from the theorem); *The Concept Sequence*, entry 15.13. Standing: a theorem (in the setting: the cycle, its closure, and Proposition 9.12); an identification (the points-to-pieces map as the class-to-instance step, stated in the first person).

> ***The cycle keeps the content and changes the mode.***
>
> Begun at T or at O, a full turn of the cycle returns the same underlying set of points, presented in turn as a codiscrete object, as a discrete object in the whole topos, as a discrete object among the discrete objects, and as a codiscrete object again. The mode of presentation changes; the content stays the same.

> Begun at I, a full turn returns the object as the set of its connected pieces, $\int X$, and not as the object itself. The categories T and O are each equivalent to the base, so a turn through them carries the pieces and the points of a genuinely cohesive object, and the rest of its cohesion stays behind. The cohesion left behind by a turn through T and O is the categorical form of the claim that the immanent contains more than the passage through the two derived modalities carries.

K-009 The Incommensuration Theorem in Cohesion. That the Incommensuration Theorem (*The Concept Sequence*, entry 13.14) is proved from comparison by finite elimination, and that in finite form it needs no topos: the admissible comparisons are exactly the identity on {same, different} (continuous asymmetry) and the exchange of its two elements (discontinuous symmetry). That the Incommensuration Theorem has a rendering in a topos cohesive over Set with C5 and ND1, with a probe (a map from a connected object) identified as one comparison. In that rendering, no object with two or more points is both absolutely symmetric and absolutely continuous, and no object with two or more points is both absolutely asymmetric and absolutely discontinuous. That both prohibitions fall to one witness, the codiscrete object on two points, CoDisc(2), which is connected and joins two points in every object that admits every assignment of points. That in this setting the objects that are both discrete and codiscrete are exactly the empty object 0 and the one-point object 1. That the ten pairings of the four absolute properties fare as in the proof from comparison: the four self-pairings are degenerate, four pairings are realized by no object, and exactly two are realized, symmetry with discontinuity by the discrete objects and continuity with asymmetry by the codiscrete objects. That this rendering is a theorem in its setting, and the proof from comparison is prior to it.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 5, Theorem 5.4 and Section 5.7 (Propositions 5.15 to 5.17, Theorem 5.18); Chapter 13, Theorems 13.0 and 13.1(b), Remark 13.2; Appendix F.3, Proposition F.3.3; *Incommensuration*, Chapter 6; *The Concept Sequence*, entry 13.14. Standing: a theorem (in

finite form, Theorem 5.4, and in the setting, Propositions 5.15 to 5.17, Theorem 5.18 and Proposition F.3.3); an identification (a probe is one comparison, *Category Theory and the Immanent Metaphysics*, Section 5.7.2).

> ***One witness for both prohibitions.***
>
> The codiscrete object on two points is the wholly joined object: nothing separates its two points. Non-degeneracy, with C5, makes it connected, so a map from it is a probe. If an object with two distinct points x and y admits every assignment of points, the assignment that sends the two points of CoDisc(2) to x and y is made by a probe, and that probe is not constant and places two different points at two positions. So the object is neither absolutely symmetric nor absolutely discontinuous. In a degenerate topos, the discrete and codiscrete objects on two points coincide, every probe is constant, and the one object Disc(2) = CoDisc(2) has all four absolute properties at once. Non-degeneracy is exactly what separates the two modes.

K-010 Self-Description. That the three axioms describe themselves: in finite form, each axiom is a consideration in one modality of the nature of another, and the three pairs (consideration → nature) are exactly the three edges of the cycle, O → T, T → I and I → O. That in cohesion the transcendent modality is internal to the topos, as the Lawvere-Tierney topology j♯ on the object of truth values Ω (an operation on truth values that closes each proposition; its sheaves here are the codiscrete objects), and the clauses of the axioms about ♯ are propositions of the internal language (the logic the topos has through Ω) that are true in the topos: the topos verifies them with the logic it has. That the omniscient modality ♭ is a comonad and not an operator on Ω, so the clauses about ♭ are theorems stated outside the internal language, or schemata over objects: the topos knows the transcendent through its logic and the omniscient through its objects. That the self-enacting property of the theorem, that any denial of it instantiates it, has its categorical form in Lawvere's fixed-point theorem (where a map from an object A onto all the maps from A to Y exists, every

map from Y to itself has a fixed point). That the self-description of the axiom system is a fixed point that closes, and not an ascending sequence of descriptions of descriptions.

Sources: *Category Theory and the Immanent Metaphysics*, Chapter 5, Proposition 5.11; Chapter 10, Propositions 10.6 and 10.7, Remarks 10.5a and 10.8, Section 10.6; Chapter 13, Theorem 13.1(f); Chapter 14, Section 14.10; *Incommensuration*, Chapter 16, Section V (the fixed point); *The Concept Sequence*, entries 15.10, 15.13, 15.15 and 15.21. Standing: a theorem (in finite form, Proposition 5.11, and in the setting, Proposition 10.7 for the ♯-clauses and Remark 10.5a for the external ♭-clauses).

> ***"True of" and "true in".***
>
> A mathematician can check, from outside a cohesive topos, that its functors satisfy the conditions of non-degenerate cohesion. When the functors of a cohesive topos satisfy the conditions of non-degenerate cohesion, as a mathematician checks from outside the topos, the Incommensuration Theorem is true of the topos. A reasoner whose logic is the internal logic of the topos, whose propositions are the elements of Ω, can state and verify the clauses about ♯ with the resources of the topos alone. When the clauses about ♯ can be stated and verified in the internal logic of the topos, with the resources of the topos alone, the Incommensuration Theorem is true in the topos. For the clauses about ♯ the two agree. The clauses about ♭ are true of the topos, and inside it they are stated as schemata over its objects.

K-011 Proof Assistant and Machine Checks. That a proof assistant is a computer program that checks whether a proof is valid by verifying every step against the rules of a formal system. That checking a proof is a computation, a sequence of steps each of which compares a line with its rule and with the lines before it. That the Incommensuration Theorem and the three axioms in finite form have been stated in the proof assistant Agda and checked, with no postulate and no imported library, under the option that keeps the development consistent with homotopy type theory (--without-K) and the option that refuses every postulate (--safe); the checks

were made with Agda 2.6.3, and the sources are printed in full in *The Formal Foundations*, Appendix A. That the checks are these: every bijection of the two-element set of verdicts is the identity or the exchange, and both are their own inverses; of the ten conjunctions of the four compound concepts, exactly continuity with asymmetry and symmetry with discontinuity satisfy the three criteria; the three modalities are distinct and form one orbit of the cycle, every map that commutes with the cycle is a rotation, and no transposition is a symmetry; and the conjugacy $\tau = (\mathrm{O}\ \mathrm{T})$ is an involution that fixes only the immanent and reverses the orientation of the cycle. That the development constructs these structures, so it exhibits a model of the axioms in finite form, and a set of statements that has a model is consistent. That a second development, with its postulates stated at its head, checks that every negation is a proposition, and that, in a rendering whose setting postulates non-degeneracy in a universal form, every type that is both discrete and codiscrete satisfies the double negation of being a proposition (it is not the case that it fails to be a proposition); in that rendering the setting's postulate already states what the derivation concludes, so the content of the rendering comes from a model of the setting, and *The Formal Foundations* proves the statement in the standard model of cohesion (its Proposition 3.2). That a machine-checked proof shows validity: each step follows from the definitions and postulates by the rules of the system, relative to the correctness of the checker. That a machine-checked proof settles a claim when the soundness of the proof is shown as well: the formal statement truly renders the claim, and the premises of the proof hold. That for the finite form the premises are definitions with no postulate, and *Category Theory and the Immanent Metaphysics*, Chapter 5, proves the statements; the steps that join the finite form to comparison are identifications: that a comparison returns a verdict on context and a verdict on content, each same or different, is the one modeling step of that chapter, and that the three modalities form a cycle in the stated order is Axiom II, derived in *Incommensuration*, Chapter 8. That such a step is made in the first person, with the observation that bears on it, and no proof assistant checks it, since it is the choice of the formal sys-

tem and not a derivation within it. That the results about the cohesive topos are proved in the books that state them, and not in Agda: *Category Theory and the Immanent Metaphysics*, Propositions 5.14 to 5.17 and 9.9, and *The Formal Foundations*, Propositions 3.2 and 6.2; for the categorical rendering, the topos of reflexive graphs and the topos of smooth sets are models, each cohesive over Set with C5 and non-degenerate, and so that setting is consistent. That a machine check of a rendering is itself made of comparisons from end to end, so it lies within the scope of the theorem it renders, and the theorem sets the scope of the check.

Sources: *The Formal Foundations*, Chapter 7, Sections I to V (Checked 7.1 to 7.5), and Appendix A; Chapter 3, Sections VI and VII (Propositions 3.2 and 3.3); *Category Theory and the Immanent Metaphysics*, Chapter 2, Section 7 (C5, reflexive graphs, smooth sets, Example 7.5); Chapter 5 (Theorem 5.4, Corollary 5.6, Definition 5.9, Proposition 5.10); Chapter 22, Section 22.8; *Physics Is Not Math*, Chapter 7, Section VII ("Every proof is checked by a computation"), and Chapter 21, Sections III and IV; *The Concept Sequence*, entries 13.14, 13.21, 15.10, 15.13, 15.15 and 15.20. Standing: a definition (a proof assistant); a theorem (Checked 7.1 to 7.3 of *The Formal Foundations*: the finite form, valid as checked with no postulate, and sound as a statement of the finite form, which *Category Theory and the Immanent Metaphysics*, Chapter 5, proves and the constructed model shows consistent); an identification (the steps from comparison to the finite form and to the cohesive setting, which no proof assistant checks).

> ***What the machine checks.***
>
> A proof checked by a reader can contain gaps: unstated assumptions, or steps that feel correct and have not been checked. A machine-checked proof has no such gap relative to its rules and to the correctness of the checker, because every step must be justified explicitly by those rules. If the machine accepts the proof, the derivation is valid relative to the rules and the postulates it was given. What the machine checks is validity.
>
> Soundness asks two further things: whether the formal statement says what the claim says, and whether the postulates hold. Whether the postulates render the concepts of comparison, modality and incommensuration is decided by the identification that joins the two, which is stated in the first person and examined by its reasons, and the machine does not de-

> cide it. The theorem itself is held in the act of comparison, and a check of it in any formal system is one more act of comparison that the theorem governs.

K-012 The Conjugacy of Axiom I and the Real-Cohesive Axiom. That in Shulman's real-cohesive homotopy type theory, the word "real" refers to the real line $\mathbb{R}$. The real-cohesive axiom states that a crisp type is discrete if and only if the type is $\mathbb{R}$-null, that is, if and only if every map from $\mathbb{R}$ into the type is constant. So the shape $\int$ is the localization at $\mathbb{R}$ (the operation that makes every map from the real line into a type constant). That this axiom chooses which cohesive settings are meant, those whose connectedness is detected by maps from the real line, as in smooth and topological spaces; it contains no involution and no exchange of $\flat$ with $\sharp$. That the rendering of the three axioms holds in every topos cohesive over Set with C5 and non-degenerate, so it does not depend on that choice of setting. That the conjugacy of Axiom I (*The Concept Sequence*, entry 15.10) is, in finite form, the transposition $\tau = (\mathrm{O}\ \mathrm{T})$, which reverses the orientation of the cycle of Axiom II, so the omniscient and the transcendent are conjugate and not interchangeable (Axiom III). That in cohesion $\flat$ is a comonad and $\sharp$ a monad built from one functor Γ with its two adjoint partners, ND1 keeps them apart, and no autoequivalence of a non-degenerate cohesive topos carries every discrete object to a codiscrete object, so the conjugacy cannot be an automorphism of the topos; it is carried by operators that reverse an order or a phase, such as τ on the triad and charge conjugation, which exchanges every particle with its antiparticle, on fields. That the exchange that does belong to the theorem is the exchange of the roles of content and context: both admissible comparisons, the identity and the exchange of {same, different}, are their own converses, so the theorem is unchanged when content and context exchange roles.

Sources: Shulman (2018); *Category Theory and the Immanent Metaphysics*, Chapter 2, Bibliographic Note; Chapter 5, Proposition 5.10 and Corollary 5.6; Chapter 8, Proposition 8.1 and Remark 8.2; Chapter 13, Theorem 13.1; *The Formal Foundations*, Chapter 3, Section II, and Chapter 6, Propositions 6.1 and

6.2 and Section IV; *The Concept Sequence*, entries 15.10 and 15.20. Standing: a definition (the real-cohesive axiom, which chooses the setting); a theorem (the conjugacy in finite form, Proposition 5.10, the exchange of content and context, Corollary 5.6, and the absence of any exchange of the discrete and codiscrete objects in the topos, Proposition 6.2 of *The Formal Foundations*).

> ***Conjugate, and not interchangeable.***
> Draw the three modalities as the corners of a triangle, with arrows T → I → O → T. Swapping O and T while keeping I in place turns the arrows into O → I → T → O: the same triangle, with the cycle running the other way. So the swap is a symmetry of the triangle without its arrows, and it is not a symmetry of the triangle with its arrows. Taken together, "conjugate" and "non-interchangeable" mean that the swap of the omniscient and the transcendent keeps the triangle and reverses its arrows. The omniscient and the transcendent are each other's complement, and putting one in the place of the other reverses the order that Axiom II fixes. The immanent is the one corner the swap leaves in place.

K-013 The Theorem and Its Renderings. That the Incommensuration Theorem is a structural fact about the nature of knowing, proved by finite elimination from the parts of comparison, with every step of the proof an act of comparison and no step using an axiom of any mathematical system. That its renderings in mathematics, in cohesive toposes and in finite spectral triples (the algebraic setting of *Non-Commutative Geometry and the Immanent Metaphysics*) among them, are theorems in their settings, each exact as mathematics and each true in the form "if these premises, then this". That the four disciplines stand in one order of depth, the order of presupposition: metaphysics, physics, computation, mathematics; the proof from comparison belongs to the first, and every rendering to the last. That every rendering is made of comparisons from end to end, so it lies within the scope of the theorem, while the theorem lies outside the scope of any rendering: dependence runs one way, and the theorem sets the scope of the rendering. That a checked rendering is valid in its setting, and its soundness as a rendering of the theorem rests on the identification

that joins its setting to comparison. That, in the words of *Physics Is Not Math*: "A formalization exists. The theorem, held in the act, is real". That the formal renderings of this chapter keep every power that mathematics has: each can be checked line by line, and each carries the theorem into a branch of mathematics where its consequences can be worked out in that branch's own terms.

Sources: *Physics Is Not Math*, Preface; Chapter 5; Chapter 21, Sections III and IV; *Category Theory and the Immanent Metaphysics*, Preface, Chapter 3, Section II, Chapter 13, Remark 13.2, and Chapter 14, Section 14.5 (validity and soundness); *Incommensuration*, Chapter 6; *The Concept Sequence*, entries 13.14, 13.21, 15.10, 15.13 and 15.15. Standing: derived (from the Root Tautology and Axiom I: *Physics Is Not Math*, Chapter 21, Section IV).

> ***A map and its territory.***
>
> A rendering of the theorem is like a map of the Earth: it is exact in its own medium, and anyone can check it line by line. The map is drawn from the territory, and the territory is not drawn from the map. Where a map falls short of the territory, the shortfall belongs to the map. In the same way, where a mathematical rendering does not reach a part of the theorem, that is a property of the mathematics, and where a rendering and the theorem seem to disagree, the place to search is the translation between them. A rendering that set conditions on the theorem it renders would put the omniscient before the immanent, and so reverse the first axiom, which is itself derived from the theorem.

Summary

This chapter is the account of how the Incommensuration Theorem and the three axioms are written out in mathematics, and of what a machine check of such a writing shows. The theorem is a structural fact about the nature of knowing, proved from comparison by a finite examination, and

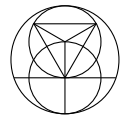
every rendering of it is a theorem in its own setting, true in the form "if these premises, then this" (*Category Theory and the Immanent Metaphysics*, Remark 13.2; *Physics Is Not Math*, Chapter 21).

The first entries define the languages. In type theory a proof is a term and checking it is checking its type; homotopy type theory adds paths and univalence; higher category theory adds maps between maps without end. The categorical rendering needs none of the higher levels: it uses 1-categorical structures only, as the closed structure of the theorem leads one to expect, and in the standard model of cohesive homotopy type theory the higher levels hold no survivor that the level of sets does not (*Category Theory and the Immanent Metaphysics*, Chapter 14; *The Formal Foundations*, Chapter 3).

The central entries give the rendering in cohesion. The omniscient, the immanent and the transcendent are rendered as the flat modality, the identity and the sharp modality of a cohesive topos. Axiom III is rendered as the distinctness of the three, with non-degeneracy keeping flat and sharp apart; Axiom I as their construction from one functor and the passage of every comparison of the extremes through the object itself; Axiom II as a cycle of three adjunctions that closes on the discrete and the codiscrete objects; and the theorem as the statement that no object with two or more points is both absolutely symmetric and absolutely continuous, or both absolutely asymmetric and absolutely discontinuous, with one witness for both. Each of these is a theorem in the setting, and the step from comparison to the setting is an identification stated in the first person (Identification 12.1b).

In the last entries I say what a machine check is. A proof assistant has checked the theorem and the axioms in finite form with no postulate, and the development is itself a model that shows them consistent (*The Formal Foundations*, Chapter 7). A machine-checked proof shows validity. A claim is settled when soundness is shown as well, the formal statement truly rendering the claim and the premises holding. The steps that join a formal system to comparison are identifications, which no proof assistant checks. The conjugacy of Axiom I is the orientation-reversing transposition of the

finite form, and no automorphism of the topos can carry it. So the chapter is one picture seen from two sides: the renderings are exact and checkable, and they keep every power mathematics has; the theorem, held in the act, sets the scope of every one of them.



Chapter L

The Turn and the Early Universe

CHAPTER OVERVIEW

This chapter traces how the product geometry of four large dimensions and six held small ones comes to hold at the beginning of an epoch of the universe, and what the beginning of an epoch is. It follows *The Gravitational Passage*, Chapters 18 and 19 and Appendix B, *At the Edge of Reality* and *The Compactification*: the hold on the six internal dimensions and the conditions under which it holds; the failure of those conditions at the end of every black-hole interior; the Kasner curve, on which the end of an interior and the opening of an epoch share one arc; the turn between two epochs, which the history of the new epoch finds as its Big Bang; the hot, even start of our history; and the sky the turn leaves, with the measurements that decide each prediction.

The chapter rests on these chapters of *The Concept Sequence*: Chapter 13, The Incommensuration Theorem; Chapter 15, The Three Axioms (Axiom III, the inseparability of content and context, carries the restoration of context); Chapter 24, The Algebra of Comparison, and Chapter 25, The Internal Space in Its Two Instances, for the one internal space; Chapter 27, The Context Instance: Radii, Volume and the Count 72, for its radii; Chapter 39, entry 39.10, Gravity Is the Arena; and Chapter 45, Further Predictions, entries 45.12 (the dark matter) and 45.15 (the cosmological

constant). Chapter F carries the dark matter made at the turn, Chapter G the cosmological constant, and Chapter H the residue of matter over anti-matter.

L-I. The Ten Dimensions and the Hold on the Six

L-001 The Six Dimensions of Possibility. That the contexts of comparison have ten independent ways of differing, in three kinds: one dimension of time, three of space and six of possibility. "Time has one dimension, space has three, and possibility has six: one and three and six make ten" (*Measurement*, Chapter 11, Section XI). The four of time and space are traversed by located comparers; the six of possibility are met only as the range from which each finding is one selection, and no located comparer travels through them as it travels through space. *The Gravitational Passage* identifies the six of possibility with the six dimensions of the internal space $\mathbb{C}P^2 \times S^2$ (the product of the complex projective plane and the two-sphere), in its context instance, the instance that has radii (Appendix C, Section VIII). On that instance the radius of the sphere S^2 is one Planck length, l_{Pl}, and the radius of the round five-sphere whose quotient by the circle of phases is $\mathbb{C}P^2$ is $\sqrt{6}\, l_{Pl}$, about 2.45 Planck lengths (*The Gravitational Passage*, Chapter 9, Section IX). The four dimensions of time and space are large; the six are compact and small, a few Planck lengths across, and because no comparison travels through them they present to a located comparer as compact (*The Gravitational Passage*, Chapter 17, Section V). Their extent shows through its effects, among them the strength of gravity, since the four-dimensional Planck mass is set by the volume of the internal space (*The Gravitational Passage*, Chapter 11, Section III).

Sources: *Measurement*, Chapter 11, Sections VIII, XI and XII; *The Gravitational Passage*, Chapter 9, Section IX, Chapter 11, Section III, Chapter 17, Section V, and Appendix C, Section VIII; *The Compactification*, Chapter 1, Sections II and III; *At the Edge of Reality*, Chapters 3, 7 and 8; *The Concept Sequence*, Chapter 25 and Chapter 27 (entries 27.10 and 27.11). Standing: derived (the count of ten,

from the structure of comparison); an identification (the six of possibility with the six internal dimensions, and the radii, from one identification at the modal boundary).

> ***The grain of the stone.***
>
> A sculptor's block of stone has three directions you can measure with a tape. It also has a grain, present at every point, which decides how the stone can be shaped, and which no tape measures. The six dimensions of possibility are present in that way, everywhere: no one walks along them, and they are met only as the forms that matter can take.

L-002 The Moduli of the Internal Space, Held. That the moduli are the size and shape of the internal space, read in the four-dimensional description as scalar fields: the volume modulus, which sets the overall size of the internal space and thereby the four-dimensional Planck mass, and the shape modulus, which sets the ratio of the two internal radii. The internal space cannot shrink to nothing: it is the context of possibility, Axiom III excludes content without context, and a size held at zero with no spread is excluded by the uncertainty relation; so it has a floor, a least extent with a least motion that cannot be removed. The hold: while the large world of space and time is the context of the compact factor, each of the six internal directions stands at the modal boundary (the scale at which the smooth description of the arena and the stepwise description of content exchange dominance), where the curvature it sees is the Planck curvature, and the boundary is fixed by the measured G_N, which has one value in this domain, so the moduli are fixed at $R_O = \sqrt{6}\, l_{Pl}$ and $R_T = l_{Pl}$ throughout the large world. The hold rests on that argument. *The Gravitational Passage*, Appendix D, computes one mechanism for the hold, and the hold does not rest on it: the least motion of the metric, with the least extent as its local term, holds the internal space at the derived radius and ratio as a stable minimum at first order (the stabilization by zero-point energy of Candelas and Weinberg 1984, with the metric itself as the field). At first order the masses of the moduli are 2.25 and 3.09 Planck masses, so

the force they carry has a range shorter than the Planck length, some thirty orders of magnitude below the shortest range any test of gravity reaches, and four-dimensional gravity is general relativity; that bound needs only the order of the masses.

Sources: *The Gravitational Passage*, Chapter 11, Section II and Questions, and Appendix D, Section III; *The Compactification*, Chapter 1, Section IV, and Chapter 2, Section V; Candelas and Weinberg (1984). Standing: derived (the floor, from Axiom III and the uncertainty relation, and the hold, from the modal boundary and the measured G_N); derived (the masses of the moduli in the first-order mechanism, a computation).

> ***The floor.***
>
> The curvature of the internal space pulls it toward a smaller size; its least extent and least motion resist. That least motion is already the lowest state, so it cannot radiate or dissipate. In the first-order mechanism of *The Gravitational Passage*, Appendix D, the floor holds the internal space in a well about one twentieth of the Planck density deep (*The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I).

L-003 The Compactification Conditions. That the internal space is the content factor of the product $M^4 \times \mathbb{C}P^2 \times S^2$ and the large factor M^4 is its context factor, and that the product is the geometric form of Axiom III's inseparability of content and context: the compact factor is held as content relative to the large factor as its context. The conditions that hold the six compact, called the compactification conditions, are these: the large world is present as a context for the compact factor, as a domain of the continuously asymmetric description of space and time (the smooth description of general relativity, below the Planck curvature), with an inertial reference fixed in it. The second condition rests on inertia being a comparison: to be at rest, or to move uniformly, or to rotate, is to do so relative to something, and a reference for inertia is fixed by the content of the world within reach (the Mach principle in its most conservative formulation). Every part of the world we know meets both conditions.

Sources: *The Gravitational Passage*, Chapter 18, Sections IV and VI; *The Compactification*, Chapter 1, Section IV; *At the Edge of Reality*, Chapter 8. Standing: derived (from Axiom III and from inertia as a comparison), at the standing of those principles.

L-II. The End of an Interior and the Kasner Curve

L-004 The Conditions Fail at the End of Every Interior. That the end of a black-hole interior belongs to no moment of the time of the outside world. That all interiors are one at their end, on the principle that a difference no comparison can register is no difference. That every observer's comparable outside world empties, with the matter of that outside world ending inside horizons or carried away alone. That the inertial reference unpins where nothing is left to compare against. That the continuously asymmetric description reaches the edge of its domain at the end of every interior, at the Planck scale. At the end of every interior both compactification conditions therefore fail: the large factor ceases to serve as the context in which the compact factor is held as content, and the compact form of the internal space cannot persist as it was.

Sources: *The Gravitational Passage*, Chapter 18, Sections I to VI, and Chapter 17, Section IV; *The Compactification*, Chapter 1, Section V; *At the Edge of Reality*, Chapters 9 to 13 and 15. Standing: derived (the first step from general relativity, and the identity of the interiors, the unpinning of the reference and the failure of the conditions from principles of comparison, each at the standing of its principle).

L-005 The Kasner Form. That near a spacelike singularity the geometry of empty space takes, to leading order, the form found by Kasner (1921): $ds^2 = -d\tau^2 + \Sigma_i \, \tau^{2p_i} \, dx_i^2$, where τ is the proper time measured from the singularity, at $\tau = 0$, and each spatial direction x_i has a scale factor τ^{p_i}, with p_i its Kasner exponent. The Einstein equations in empty space require two rules: the exponents add to one, and the squares of the expo-

nents add to one. Three facts follow. The sign rule: as τ falls toward zero, a direction with a positive exponent shrinks and a direction with a negative exponent grows, and the two rules require at least one negative exponent except where a single exponent is one and the others are zero. The volume rule: the volume of a region goes as τ raised to the sum of the exponents, which is one, in any number of dimensions. The rule of the direction of time: reversing the direction of time leaves every exponent unchanged; read toward a singularity in the future a direction with a positive exponent shrinks, and read away from a singularity in the past the same direction grows.

Sources: Kasner (1921); *The Gravitational Passage*, Appendix B, Section I; *The Compactification*, Chapter 1, Section VI; *At the Edge of Reality*, Chapter 14 and Appendix A. Standing: a theorem (of general relativity, Kasner 1921, with the three facts that follow from its two rules).

> ***The ordinary interior.***
>
> Inside the horizon of a black hole without rotation, near the center, the three directions of space have the exponents minus one third, two thirds and two thirds: one direction is stretched and two are squeezed, the tidal stretching of a falling body along one axis. Minus one third plus two thirds plus two thirds is one, and one ninth plus four ninths plus four ninths is one. Placed in the ten-dimensional geometry with the six internal exponents zero, the three exponents of the ordinary interior still meet both of Kasner's rules (the exponents sum to one, and their squares sum to one). So the ordinary interior is a valid Kasner point in which the internal space holds its size.

L-006 The Grouping of the Nine Directions. That the product $M^4 \times \mathbb{C}P^2 \times S^2$ groups the nine directions of space as three large directions, the four directions of $\mathbb{C}P^2$ and the two directions of S^2. The Kasner rules hold for any grouping of the nine directions, and in pure higher-dimensional Einstein theory no group of three directions is preferred in the expansion (Freund 1982); the grouping of three, four and two comes from the product geometry (*The Concept Sequence*, Chapter 25), and the rules then

give the curve of entry L-007. The ordinary interior of entry L-005, whose three directions of space have unequal exponents, lies off the family in which each group moves together.

Sources: *The Gravitational Passage*, Appendix B, Sections II and III, and Questions; *The Compactification*, Chapter 1, Section VI; Freund (1982). Standing: a theorem (given the grouping that the product geometry supplies).

L-007 Equation (The Kasner Curve for Three, Four and Two Directions). That with the three large directions sharing the exponent p, the four directions of $\mathbb{C}P^2$ the exponent q and the two of S^2 the exponent s, the Kasner rules become $3p + 4q + 2s = 1$ and $3p^2 + 4q^2 + 2s^2 = 1$. Eliminating s gives the quadratic $24q^2 + (24p - 8)q + (15p^2 - 6p - 1) = 0$, whose discriminant is $-32(3p + 1)(9p - 5)$; so a real solution exists exactly when p lies between $-1/3$ and $5/9$, and the solutions form a closed curve, the intersection of a plane with an ellipsoid. At each end of the curve q equals s, and the six internal directions move as one block. At the lower end $p = -1/3$ and $q = s = 1/3$ (check: $-1 + 2 = 1$; $3/9 + 6/9 = 1$). At the upper end $p = 5/9$ and $q = s = -1/9$ (check: $15/9 - 6/9 = 1$; $75/81 + 6/81 = 1$). The exponent of S^2 is written s here, so that r keeps its usual meaning, the tensor-to-scalar ratio of entry L-023.

Sources: *The Gravitational Passage*, Appendix B, Section III (which writes r for the exponent of S^2); *The Compactification*, Chapter 1, Section VI, Equations (1.5) to (1.7); *At the Edge of Reality*, Chapter 14 and Appendix A. Standing: a theorem.

L-008 The Arc at the Upper End. That at the upper end of the curve both internal exponents are negative, and they stay negative along a short arc through it, which ends where the S^2 exponent reaches zero, at $p = (1 + 2\sqrt{2})/7 \approx 0.5469$, and where the $\mathbb{C}P^2$ exponent reaches zero, at $p = (3 + 2\sqrt{6})/15 \approx 0.5266$. At the upper end, read toward a singularity in the future, each of the three large directions shrinks as $\tau^{5/9}$ and each of the six internal directions grows as $\tau^{-1/9}$; the volume of the three goes as $\tau^{5/3}$, that of the six as $\tau^{-2/3}$, and that of all nine as τ. The growth of the six is slow: over a billionfold fall in the time remaining, each of the three

shrinks a hundred thousand times and each of the six grows ten times, so the growth is still under way when the curvature reaches the Planck scale. Read away from a singularity in the past, the same exponents describe the opposite history: each of the three grows as $\tau^{5/9}$ and each of the six contracts as $\tau^{-1/9}$. The lower end has neither behavior.

Sources: *The Gravitational Passage*, Appendix B, Section IV; *The Compactification*, Chapter 1, Section VI ("The arc" and "Volumes, and the two directions of time"); *At the Edge of Reality*, Chapter 14, Appendix A, and Appendix C (Part III). Standing: a theorem.

> ***Three shrink while six grow.***
>
> Picture a room shrinking in length, width and height together, while six further directions, everywhere in it and far too small to see, slowly swell. By the time the room has shrunk a hundred thousand times in each direction, the six have grown ten times. Run the same film backward, and a room opens out in three directions while six close up.

L-009 Where the Interior and the Origin Are Placed. That reversing the direction of time takes each point of the Kasner curve to itself. So the end of an interior, read toward a singularity, and the origin of an epoch, read away from a singularity, share one arc of the curve and differ only in the direction of travel. That the sharing of one arc follows from Kasner's two rules. The arc is selected by the behavior each history has: on it the interior collapses while the six grow, and the origin opens in three directions while the six contract. *The Gravitational Passage* places both at the end point of the arc, where the six move as one block. The statement that collapse and origin are one transition, approached from its two sides in opposite directions of time, uses only the shared arc, and at the level of the dimensional schema it holds at every point of the arc. In ten dimensions empty-space gravity near a generic spacelike singularity passes through an endless sequence of Kasner forms (Demaret, Henneaux and Spindel 1985); a single form with each group moving together is consistent within the product geometry, where the curvature of the internal space falls away

against the rate of collapse, and the action of the metric alone has no form field threading the internal space, which would also drive that sequence (Damour and Henneaux 2000).

Sources: *The Gravitational Passage*, Appendix B, Sections IV and VII, and Questions; *The Compactification*, Chapter 1, Section VI ("The placement"); Demaret, Henneaux and Spindel (1985); Damour and Henneaux (2000). Standing: a theorem (the sharing of one arc, from the Kasner rules); an identification (the placement of both histories at the end point of the arc).

L-III. The Turn

L-010 The Restoration of Context. That when the compactification conditions fail, the content does not cease: at the Planck scale each existing description reaches the edge of its domain, and no description takes over; the edge belongs to the descriptions, and it is not an ending of the content. Axiom III excludes content without context, and the only structure from which a new context can form is the ten-dimensional geometry that remains, so that geometry restores a large outside context. Nothing then holds the six compact, and the internal space expands. The expansion of the internal space is the release of the six. The distinction between the large and the small dimensions then re-forms: a new four-dimensional spacetime re-forms as the large factor of the product, with a new cosmological horizon and, as its matter spreads, a new inertial reference, with the same internal shape $\mathbb{C}P^2 \times S^2$, and with three large directions of space, the number of directions of space of the context factor. The new four-dimensional geometry has no prior identity: its large directions are neither the old small ones nor the old large ones.

Sources: *The Gravitational Passage*, Chapter 18, Section VII, and Chapter 17, Section IV; *The Unquantized Arena*, Chapter 9, Section V; *The Compactification*, Chapter 1, Section V; *At the Edge of Reality*, Chapter 16. Standing: derived (from Axiom III), at the standing of the principles of entry L-004.

> ***What the restoration is.***
>
> The restoration is a transition in a cycle: the product geometry re-forming its large factor, because the conditions that dissolved the old outside world also make a new one necessary. The restoration is therefore neither a creation from nothing, nor a fluctuation in a vacuum that was already there, nor a breakdown at which the equations fail and the description falls silent.

L-011 The Turn and the Big Bang. That the collapse inside the black holes of one epoch and the origin of the next are one process, approached as a limit from its two sides, in opposite directions of time; and that the limit between the two sides is called the turn. To know the turn as an event would need a comparison that registers it, a comparison of a before with an after made across the turn; the outside world of the old epoch has no moment at the end of an interior, so no comparison registers the turn as an event. Duration, temperature and rate apply to the approach on each side and to the history reconstructed from inside the new epoch; none of them applies to the turn. The past is known by the impression it makes on the present, and the history reconstructed from that impression reports what it finds as located events; traced backward, that history meets the limit and renders it as a point in time. *The Gravitational Passage* identifies the restoration of the outside context with the Big Bang of the new epoch: the Big Bang of the equations of cosmology is the turn as it appears in a history reconstructed from inside, which *The Gravitational Passage*, Chapter 19, Section III, calls the projection. So the question of what came before the Big Bang has an answer: the collapse of the interiors of the black holes of the epoch before ours, where "before" names the order of the cycle and not a time measured across the limit.

Sources: *The Gravitational Passage*, Chapter 18, Sections VII, VIII and XIII, and Chapter 19, Section III; *The Compactification*, Chapter 1, Section V; *At the Edge of Reality*, Chapters 2, 16 and 17. Standing: an identification (the restoration with the Big Bang).

L-012 The Compactification on Our Side of the Turn. That on our side of the turn the arc of entry L-008 is read away from the limit: at its upper end each of the three large directions grows as the time counted from the limit, in the new epoch's own history, raised to the power 5/9, and each of the six internal directions contracts as that time raised to the power $-1/9$. *The Gravitational Passage* calls this history the cosmological compactification; the six were never directions of space, and "compactification" means the six held compact as content of the large factor taken as their context. Kasner-type cosmologies in which three directions expand while the others contract are known: a vacuum solution in five dimensions (Chodos and Detweiler 1980), and cosmologies of ten- and eleven-dimensional supergravity with three directions preferred (Freund 1982). The Kasner account gives the direction of the change and its powers; the contraction ends where the floor captures the internal space at its held radii (entry L-002).

Sources: *The Gravitational Passage*, Chapter 18, Section VII, and Appendix B, Section IV; *The Compactification*, Chapter 1, Sections II, VI and VII; *At the Edge of Reality*, Chapter 16; Chodos and Detweiler (1980); Freund (1982). Standing: a theorem (the powers on the arc); an identification (the placement of the origin of our epoch on the arc, entry L-009).

L-013 The Cycle Without a First Turn. That each epoch makes structure, structure makes black holes, the interiors of the black holes are one at their end, and the turn restores a large world, which is the next epoch; that the turns of successive epochs are distinct, so the epochs follow one another in a single sequence and not as a branching population; and that the cycle has no first iteration. The cosmological constant does not prevent the cycle: it belongs to the description of the outside world, and at the end of an interior it has no purchase. The structure the account derives is the same in every epoch: the shape $\mathbb{C}P^2 \times S^2$ of the internal space, the ratio and the sizes of its radii in Planck units, and the count $c_1 = 3$. G_N is the one scale the account takes from measurement; a comparison of G_N in one epoch with G_N in the next would be a comparison across the turn,

and no comparison is made across it, so the account assigns no value of G_N to another epoch. What a comparer in any epoch measures is a ratio, such as the mass of a particle to the Planck mass, and every such ratio is found within one epoch.

Sources: *The Gravitational Passage*, Chapter 18, Section XII and Questions; *The Compactification*, Chapter 1, Section IX, and Chapter 2, Section VII; *At the Edge of Reality*, Chapter 20. Standing: derived, given a stated identification (the restoration as the Big Bang, entry L-011).

L-IV. Where Our History Starts

L-014 Hot, Even and Close to Balance. That on our side of the turn the reconstructed history starts in a definite state: "The matter at the beginning of our reconstructed history is hot and close to equilibrium, the settled state in which nothing further changes on average, so its entropy is already high" (*At the Edge of Reality*, Chapter 18). The start is even, up to small departures, about one part in a hundred thousand, which the Incommensuration Theorem requires (entry L-021). Nearly all of the energy is radiation, light and every particle moving nearly as fast as light, and two residues remain, each conserved in number. The first residue is the excess of matter over antimatter, carried as a value of $B - L$ (the baryon number less the lepton number). The first residue is left by the turn and kept by every change after the turn, and the kinds of particle fix only the share of the first residue that is held as baryons (Chapter H). The second residue is the dark matter, made at the turn (Chapter F). As the universe expands, the gathering of matter and light cools, the temperature of the gathering falling as the inverse of the scale of distances, and that falling temperature is the clock of the early history.

Sources: *At the Edge of Reality*, Chapter 18; *The Coming of the Kinds*, Chapter 1, Section VI, and Chapter 2, Sections II to IV; *The Weight of Possibility*, Chapter 3, Section I ("The turn sends nearly all of its energy into radiation. Two residues

remain: an excess of ordinary matter over its antimatter, and the dark matter. Both are conserved in number"); *The Compactification*, Chapter 1, Section VIII. Standing: a measured input (the hot start close to balance, as the reconstructed history finds it, and the amounts of the two residues); derived (the departures, which the Incommensuration Theorem requires).

L-015 How Hot. That the measured amount of dark matter sets a ceiling on how hot the gathering of our epoch ever was. The dark matter is made of gyrations, each an excitation of the metric of the six held internal dimensions, no quantum of the arena and no quantum of gravitation, made only at the turn (Chapter F). Anything that gravitates can be made in pairs by a hot plasma through gravity alone, so a gathering hotter than about one thirtieth of the mass of one gyration would have made more dark matter than is measured: about 5×10^{17} GeV for a gyration of $\sqrt{2}$ Planck masses, its mass at the classical level, and about 2×10^{17} GeV for 0.6 Planck masses, its mass at first order. If the energy of the turn reached the gathering through a stage that behaved as matter and decayed into the content, the end of that stage lies below about one hundredth of the mass of a gyration. The ceiling, 2 to 5×10^{17} GeV, is 1.2 to 2.9×10^{15} times the mass of the top quark, so the cooling of the gathering passes every mass of the kinds of particle.

Sources: *The Weight of Possibility*, Chapter 4, Sections V and VI, and Chapter 6, Section I; *The Coming of the Kinds*, Chapter 2, Section IV and Box 2.1; *The Compactification*, Chapter 1, Section VIII. Standing: derived, given a stated identification (the dark matter as gyrations made only at the turn, Chapter F), a ceiling from the measured amount of dark matter.

L-016 Balance, and Falling Out of It. That in a gathering in balance every reaction runs as often forward as backward, and the number of each kind is set by its mass, its count of states, the temperature and the quantities every reaction keeps. While radiation fills the gathering, the expansion rate is $H = 1.66\, {g_*}^{1/2}\, T^2/M_{Pl}$, with $g_* = 106.75$ the count of states of the relativistic kinds above the mass of the top quark. A process whose rate exceeds the expansion rate holds the quantity it changes at its balance value;

a reaction whose rate falls below the expansion rate lets the quantity go, and the quantity stays at the value the reaction last held it at. The history of the kinds after the start is made of events of three sorts, met in order of falling temperature: a kind falls away as the temperature passes below its mass; a reaction falls behind the expansion; and a settling or a binding waits until the temperature falls below a scale of its own, as when the Higgs field settles near 159.5 GeV, the quarks are bound near 156.5 MeV, and the first nuclei hold together near 0.1 MeV. Chapter H, Part H-IV, follows these events for the share of the residue of matter held as baryons.

Sources: *The Coming of the Kinds*, Chapter 3, Section V and Box 3.3, and The Story in Brief ("The Map of the Cooling"); Kolb and Turner (1990), Chapters 3 and 5; Particle Data Group (2026), review "Big-Bang Cosmology", Equation (22.47). Standing: derived (standard physics, with the closed list of kinds).

L-V. The Sky the Turn Leaves

L-017 The Horizon and Flatness Problems, and Inflation. That the microwave background has one temperature across the sky to about a part in a hundred thousand, although in the standard history regions more than about a degree apart never exchanged a signal before the background was released: the horizon problem. That space on the largest scales is very nearly flat, although in the standard history any small curvature at the beginning would have grown enormously: the flatness problem. That the standard answer to both is inflation (Guth 1981): a phase of very rapid expansion early in the history, in which a region small enough to reach one temperature is stretched far beyond everything we can see, flattening any curvature it had; inflation has been developed in many forms.

Sources: *At the Edge of Reality*, Chapter 2; *The Gravitational Passage*, Chapter 18, Questions; Guth (1981). Standing: a measured input (the one temperature of the sky and the flatness of space); a definition (inflation, as standard cosmology states it).

L-018 One Domain: The Sameness of the Sky. That on this account the sameness of the sky has a common origin: the interiors are one at their end (entry L-004), and the new outside world re-forms from that one domain with a geometry that has no prior identity, so it inherits no large-scale anisotropy or inhomogeneity from the old outside world. Regions that no signal within the new epoch has connected are alike because they come from one domain, not because a signal passed between them.

Sources: *The Gravitational Passage*, Chapter 18, Section VIII and Questions; *The Compactification*, Chapter 6, Section III; *At the Edge of Reality*, Chapter 19. Standing: derived, given a stated identification (the restoration as the Big Bang, entry L-011, with the identity of the interiors, entry L-004).

L-019 Spatial Flatness. That a radius of spatial curvature is a length, and no length of the old epoch passes the turn, since the new geometry has no prior identity and no comparison of the new epoch registers the mass of the old interiors. The lengths left are the two of the new epoch: the length of the turn, of the order of the Planck length, and the de Sitter radius that the reconstructed history carries (the radius of a universe holding the vacuum energy alone). A radius of the Planck length would close the new epoch within a few Planck times or leave it dominated by curvature, and neither epoch produces the structure the cycle requires; a radius equal to the de Sitter radius leaves a curvature today of about 2.5×10^{-64} as a share of the energy of a flat universe. The homogeneous model of the new epoch is therefore spatially flat, and the geometry found is curved only by its departures, at their own size, about 10^{-5}. The measured curvature is 0.001 ± 0.002 from the microwave background with galaxy surveys (Planck Collaboration 2020a), and 0.0023 ± 0.0011 from

the Dark Energy Spectroscopic Instrument with the microwave background (DESI Collaboration 2025), 2.1 standard deviations from flat. The account predicts a curvature of the size of the departures.

Sources: *The Gravitational Passage*, Chapter 18, Sections IX and XIII; *The Compactification*, Chapter 6, Section VI; *At the Edge of Reality*, Chapter 19, and Appendices B and C; Planck Collaboration (2020a); DESI Collaboration (2025). Standing: derived, given a stated identification (entry L-011).

L-020 What the Account Says of Inflation. That the account needs no inflationary phase to explain the sameness of the sky or the flatness of space: the sameness comes from one domain (entry L-018), and the flatness from the lengths that remain (entry L-019). That, on the identification of the departures from uniformity with the form of the content carried through the turn (entry L-021), the account excludes an inflationary phase long enough to stretch the lengths observed today from within the early horizon to beyond it, since in such a phase the departures on those lengths would be those of the inflationary vacuum. That a shorter phase, which leaves the observed lengths outside the early horizon throughout, is neither required nor excluded, and the account does not fix the shorter phase. That after a long inflation of the usual kind, the bound on primordial gravitational waves would keep the temperature far below the ceiling of entry L-015, and dark matter of about the Planck mass could not be made in the measured amount; on this account that dark matter is made at the turn.

Sources: *The Gravitational Passage*, Chapter 18, Questions; *The Compactification*, Chapter 1, Section VIII, and Chapter 6, Section VIII; *At the Edge of Reality*, Chapter 19; *The Weight of Possibility*, Chapter 4, Section VI. Standing: derived, given a stated identification (the restoration as the Big Bang, entry L-011, and the departures as the form of the content, entry L-021).

L-021 The Departures from Uniformity. That the new metric departs from perfect sameness, because perfect sameness in every direction and at every place would make symmetry and continuity both perfect, which the Incommensuration Theorem forbids. That *The Gravitational Passage* identi-

fies the departures with the form of the content carried from one side of the turn to the other, the old distinctions averaged and not erased. The vacuum of the new epoch adds almost nothing to them: a length observed today lay, near the turn in the reconstructed history, about 10^{27} times beyond the horizon of that time, and on a wave so far beyond the horizon the vacuum carries a power below 10^{-51}, against the observed power of about 2×10^{-9}. Because the interiors are one at their end, the form that passes is correlated across the whole new outside world, with a spectral index near one; independent pieces, averaged, would leave white noise, with a spectral index of 4 in the curvature. It is one form for every component of the content, so the departures are adiabatic (every component departs from uniformity together) and no isocurvature mode (one component departing against another) separates one component from another; the microwave background bounds such a mode below 1.3, 1.7 and 1.7 percent of the variation of its temperature for the three kinds of mode, at 95 percent confidence (Planck Collaboration 2020b). A departure of density sources a scalar departure of the metric, and a wave of the metric needs an anisotropic stress that content close to balance does not have at first order, so the primordial departures of the metric are scalar, and waves of the metric arise from them only at second order.

Sources: *The Gravitational Passage*, Chapter 18, Sections VIII, IX and XIII; *The Compactification*, Chapter 6, Sections II to V; *At the Edge of Reality*, Chapters 18 and 19, and Appendices B and C; Planck Collaboration (2020b). Standing: an identification (the departures with the form of the content); derived, given a stated identification (their correlation, adiabatic form and scalar character, given that identification).

L-022 The Spectral Index and the Amplitude. That the spectral index, written n_s, measures how the strength of the departures depends on length: an index of exactly one means the same strength at every length, a pattern called scale invariant, and an index below one means a pattern tilted slightly toward larger lengths. The measured index is $n_s = 0.965 \pm 0.004$ (Planck Collaboration 2020a; Planck Collaboration 2020b), and the

measured power of the departures is about 2×10^{-9}. On this account a spectrum that refers to no length is exactly scale invariant, and a tilt is a weak reference of the departures to a length of the new epoch: the length of the turn or the de Sitter radius, between which every observed length lay near the turn. The account takes the amplitude and the tilt, with its sign, from measurement; the measured index, below one, is a measured input of the account. For a reference that weakens as a fixed power of the logarithm of the ratio of lengths, the running of the index (its change from one length to another) is about 6×10^{-4} for a reference to the length of the turn and about 4×10^{-4} for the de Sitter radius; the measured running is consistent with zero, with an uncertainty of about 6×10^{-3} (Planck Collaboration 2020b; Calabrese and others 2025).

Sources: *The Gravitational Passage*, Chapter 18, Sections IX and XI, and Questions; *The Compactification*, Chapter 6, Section VII; *At the Edge of Reality*, Chapter 19 and Appendix C; Planck Collaboration (2020a); Planck Collaboration (2020b); Calabrese and others (2025). Standing: a measured input (the index, its sign and the amplitude); derived (the tilt as a reference to a length, and the running for the stated form of the reference).

L-023 No Primordial Gravitational Waves. That the tensor-to-scalar ratio, written r, is the power of the primordial gravitational waves (waves of the metric present from the beginning of an epoch) divided by the power of the departures of density. That, since the primordial departures of the metric are scalar and waves arise from them only at second order (entry L-021), the account predicts r far below 10^{-6}. That the measured bound is $r < 0.032$ at 95 percent confidence (Tristram and others 2022). The Simons Observatory forecasts an uncertainty $\sigma_r = 1.2 \times 10^{-3}$ (Simons Observatory Collaboration 2026), and LiteBIRD forecasts a total uncertainty $\delta r < 0.002$ (LiteBIRD Collaboration 2026). The account predicts that the Simons Observatory and LiteBIRD will find no primordial gravitational waves.

Sources: *The Gravitational Passage*, Chapter 18, Sections IX and XIII; *The Compactification*, Chapter 6, Section V; *At the Edge of Reality*, Chapter 19, and Appendices B and C; Tristram and others (2022); Simons Observatory Collaboration (2026); LiteBIRD Collaboration (2026). Standing: derived, given a stated identification (the departures as the form of the content, entry L-021); a fork that a named measurement decides (the Simons Observatory and LiteBIRD).

L-024 Entropy at the Turn and the Past Hypothesis. That the Past Hypothesis is the assumption that the universe began in a state of very low entropy, which the statistical argument for the growth of entropy requires as a boundary condition, and which Albert (2000) argues should be admitted as a law of nature in its own right. That the matter at the beginning of our reconstructed history is hot and close to balance, so its entropy is already high; the low entropy at the start is in the gravitational part: the interiors of black holes carry an enormous entropy of gravity, about 10^{77} for one solar mass and about 10^{104} for all black holes in the observable universe (Egan and Lineweaver 2010), while the new epoch begins with its geometry even and the part of its curvature that distorts shapes near zero. That on this account an entropy is a count of the states a context can distinguish, reckoned relative to that context; the entropy of an interior counts the states the outside world cannot tell apart, and at the turn that outside world has emptied, so the count has no reference left; the new epoch reckons its counts relative to its own context, and that is the reset. That the reset meets the objection Tolman (1934) raised against cyclic histories, since no count is carried across the turn and each epoch begins its own count. That the structural arrow of time, which the Incommensuration Theorem requires of every interaction, does not wait on the Past Hypothesis; the Past Hypothesis states how that arrow is expressed in the particular history of this universe.

Sources: *The Gravitational Passage*, Chapter 18, Section VIII, and Appendix B, Section VI; *The Compactification*, Chapter 3, Sections I, II and V to VIII; *At the Edge of Reality*, Chapter 18 and Appendix C; *The Basis of Physics*, Essay 11, The Structural Arrow of Time, Section IV; Albert (2000); Tolman (1934); Egan and

Lineweaver (2010). Standing: derived, given a stated identification (the restoration as the Big Bang, entry L-011), from the principle that every entropy is a count of the states a context can distinguish.

L-025 Penrose's Weyl Curvature Hypothesis. That the part of spacetime curvature that distorts shapes, as distinct from the part that changes volumes, is called the Weyl curvature, and that Penrose (1979) proposed, as a separate principle, that the Weyl curvature was near zero at the Big Bang, to account for the low entropy of gravity at the beginning. That on this account the new four-dimensional metric re-forms from a geometry in which it had no separate identity, so it inherits no large-scale anisotropy or inhomogeneity from the old outside world and begins conformally flat (with its Weyl curvature near zero), up to the departures the Incommensuration Theorem requires. The Weyl curvature hypothesis follows from the restoration, instead of being a separate postulate.

Sources: *The Gravitational Passage*, Chapter 18, Section VIII; *The Compactification*, Chapter 3, Section VI; *At the Edge of Reality*, Chapter 18; Penrose (1979). Standing: derived, given a stated identification (the restoration as the Big Bang, entry L-011).

L-VI. Vacuum Structure

L-026 One Internal Space, No Landscape. That the internal space is reached by two routes that meet on one space: on the route of the algebra, $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is the one algebra of a comparison triple meeting the postulates of *Non-Commutative Geometry and the Immanent Metaphysics* (Theorem 7.3 and Theorem 12.1), and $\mathbb{C}P^2 \times S^2$ is the product of the state spaces of its two non-commutative summands; on the route of *The Gravitational Passage*, which uses no algebra, $\mathbb{C}P^2 \times S^2$ is the only compact homogeneous product that meets the identifications of its Chapter 9. Its shape is fixed up to scale, its radii are fixed in Planck units by the modal boundary, and its moduli are held (entry L-002). The action is the action of the met-

ric alone, so no flux threads the internal space and no flux integer can be chosen. At the order computed there is no second minimum to tunnel to, and the hold ends only where the conditions of the hold fail, at the end of an interior. So there is one held configuration of the internal space, and no landscape of possible internal shapes and no selection problem among them. In string theory, by contrast, the six compact dimensions can be folded into a very large number of possible shapes, each giving a different world of particles and forces.

Sources: *The Compactification*, Chapter 2, Sections VI, VII, IX and XI; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 7 (Theorem 7.3) and Chapter 12 (Theorem 12.1); *The Gravitational Passage*, Chapter 9, Sections VI, VII and IX, and Chapter 11, Section II; *At the Edge of Reality*, Chapter 3; *The Concept Sequence*, Chapter 24 (entries 24.10 and 24.11) and Chapter 25 (entry 25.18). Standing: a theorem (the algebra, at the standing of its postulates, each an identification); derived (the uniqueness of the compact homogeneous product, by theorems of topology at the standing of the identifications of *The Gravitational Passage*, Chapter 9, and the one held configuration).

L-VII. Gravitational Waves from the Early Universe

L-027 Stochastic Gravitational Wave Background. That a stochastic gravitational wave background is a faint, diffuse field of gravitational radiation composed of many unresolved sources, analogous to the microwave background but in waves of the metric in place of light. That gravitational waves are small oscillations of the context, the arena of space and time, with two polarizations and a speed equal to the speed of light, as general relativity gives them; the arena is not quantized: it has no least step, and nothing is a quantum of the arena itself, at any scale. The merger GW170817 bounds the difference between the speed of gravitational waves and the speed of light to between -3×10^{-15} and $+7 \times 10^{-16}$ of the speed of light (Abbott and others 2017). That waves present from the be-

ginning of an epoch would add to the radiation of the early universe and are counted by the microwave background as radiation beyond the light and the three kinds of neutrino.

Sources: *The Gravitational Passage*, Chapter 16, Section V; *The Unquantized Arena*, Chapter 6, Proposition 6.3; *The Compactification*, Chapter 7, Section II; *The Coming of the Kinds*, Chapter 9, Section V; Abbott and others (2017). Standing: a definition (the background); a measured agreement (the speed bound).

L-028 The Settling of the Internal Space and the Effective Number of Light Species. That as the internal space settles into its held size at the turn, the energy of its motion goes into the breathing excitations of the internal volume (the even excitations of its size above its lowest motion), which decay within about a Planck time, for a coefficient of natural size of the Gauss-Bonnet term of the ten-dimensional action, into gravitational waves of the open dimensions. That the microwave background registers such waves as extra radiation, counted in the effective number of light species, N_{eff}, whose standard value for the three kinds of neutrino is 3.044 (Bennett and others 2021); the measured values are $N_{eff} = 2.99 \pm 0.17$ (Planck Collaboration 2020a) and 2.86 ± 0.13 (Calabrese and others 2025). That the measured N_{eff} allows those waves less than about three percent of the radiation of the content at the time when the content had about a hundred species (Goldstein and Hill 2026), so the account requires of the turn either that the internal space arrive at its held size displaced by no more than about one part in a thousand of its radius, or that a stage that behaves as matter carry at least as much energy as the waves and decay into the content, so that the radiation it releases outweighs them. That the account requires the waves to stay within the measured bound, and the account names no particular excess in N_{eff}.

Sources: *The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I; *The Compactification*, Chapter 1, Section VII, and Chapter 7, Section III; *The Coming of the Kinds*, Chapter 9, Section V; Goldstein and Hill (2026); Bennett and others (2021); Pagano, Salvati and Melchiorri (2016); Planck Collaboration (2020a); Calabrese and others (2025). Standing: derived (the decay of the

breathing excitations, an estimate for a Gauss-Bonnet coefficient of natural size); a measured input (N_{eff}); a fork that a named measurement decides (the requirement on the turn, by the effective number of light species).

L-029 What the Coming Surveys Will Measure. That the account's predictions for the early universe are these: no primordial gravitational waves, r far below 10^{-6} (entry L-023); a flat space, with a curvature of order 10^{-5} (entry L-019); one form for every component, with no isocurvature mode (entry L-021); and one pattern across the whole sky, nearly the same strength at every length, which is not the pattern of independent pieces (entry L-021). That the electroweak transition, with the measured Higgs mass, is a crossover with no bubbles and no walls (Chapter H, entry H-010), so it supplies no source of gravitational waves from colliding bubble walls. That a result counts as confirmed when it is established at five standard deviations by at least two independent kinds of measurement (*The Gravitational Passage*, Chapter 20, Questions).

Sources: *At the Edge of Reality*, Appendix B; *The Gravitational Passage*, Chapter 18, Section XIII, and Chapter 20, Questions; *The Coming of the Kinds*, Chapter 4, Section IV. Standing: a definition (of the predictions the account names for the early universe, each with the standing of its own entry, given the identifications of entries L-011 and L-021); derived (the absence of a bubble-wall source, standard physics of a crossover).

Summary

This chapter is the branch of the map that carries the turn between two epochs of the universe and the early history that follows it. Its results belong to *The Gravitational Passage*, Chapters 11 and 16 to 19 and Appendix B, to *At the Edge of Reality* and *The Compactification*, which set the same account out for a general reader and in the order in which an epoch begins, and, for the start of our history, to *The Coming of the Kinds* and *The Weight of Possibility*.

The first part sets out what is held. The contexts of comparison have ten dimensions, one of time, three of space and six of possibility, and the six are identified with the internal space $\mathbb{C}P^2 \times S^2$, with radii of $\sqrt{6}$ and one Planck length. While the large world is their context, the six stand at the modal boundary and are held there, the same at every point; a first-order mechanism computes the hold's stiffness, and the hold rests on the boundary.

The second part follows the end of an interior. The conditions of the hold fail at the end of every black-hole interior. On the Kasner curve for three, four and two directions, an exact consequence of the Einstein equations in empty space, the upper end describes three directions shrinking as the time to the power $5/9$ while each of the six grows as the time to the power $-1/9$, and the same exponents read the other way describe the opening of an epoch.

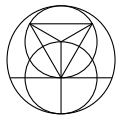
The third part is the turn. The content does not cease at the Planck scale, where each description reaches the edge of its domain and no description takes over; content needs a context, so the ten-dimensional geometry restores one. *The Gravitational Passage* identifies that restoration with the Big Bang of the new epoch, the turn as a history reconstructed from inside renders it, and the cycle of epochs has no first turn.

The fourth part is where our history starts: hot, even and close to balance, nearly all radiation, with two residues kept in number, the excess of matter over antimatter and the dark matter made at the turn, and with a ceiling on its temperature of 2 to 5×10^{17} GeV set by the measured dark matter.

The fifth part reads the sky the turn leaves. The sameness of the sky comes from one domain and the flatness of space from the lengths that remain, so no inflationary phase is needed, and a long one is excluded. The departures are the form of the content carried through the turn: correlated across the sky, one form for every component, and scalar, so primordial gravitational waves lie far below anything measurable. The low gravita-

tional entropy at the start follows from the reset of every count at the turn, which gives Penrose’s Weyl curvature hypothesis as a consequence and meets Tolman’s objection to cycles.

In the last parts I state that the internal space has one held configuration and no landscape, and that the gravitational waves of the settling must stay within the measured count of light species. Each prediction stands with the measurement that decides it, under the rule of five standard deviations by two kinds of measurement, and the coming surveys will measure each of them.



Chapter M

Structural Proofs

CHAPTER OVERVIEW

This chapter traces four structural results, each stated with the premises it rests on and with the standing it has: why the context of space and time has one dimension of time and three of space, within a context of ten dimensions; why the internal space is $\mathbb{C}P^2 \times S^2$; how the exact symmetry of the three colors stands with the non-interchangeability of the three positions of comparison; and how the charges of one generation are fixed and the gauge anomalies cancel. Each result is a theorem from stated premises, an identification (a stated step from the structure of comparison to a physical or geometric structure, made in the first person), or a result derived from them, and each entry says which. Where I give a proof, I keep two things apart: the validity of the proof and the soundness of the proof. The validity of the proof is what the proof shows: that the result follows from the premises. The soundness of the proof is the further condition under which the result settles a claim about the world: the statement truly renders that claim, and the premises hold.

The chapter rests on *The Concept Sequence* through Chapter 26: Chapter 13, The Incommensuration Theorem; Chapter 14, The Three Modalities (with the binding of the modalities by focus); Chapter 15, The Three Axioms (with the directed cycle T → I → O → T, from the transcendent to the immanent to the omniscient and back); Chapter 16, The Two Laws and the Content/Context Partition (with the grid of the six measures); Chapter 22, The Quarks: the Six Intrinsics Re-projected (with confine-

ment and the fractional charges); Chapter 23, Three Colors and Anomaly Cancellation; Chapter 24, The Algebra of Comparison (the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ and the gauge group); Chapter 25, The Internal Space in Its Two Instances (with the ten dimensions of context, one of time, three of space and six of possibility); and Chapter 26, The Two Kernel Integers and the Kernel Triangle (the count $c_1 = 3$ of the three positions of comparison and the count $n_{\mathbb{C}} = 2$ of the poles, content and context). It uses the frame of measurement of *The Story of the Particles*, in which $\mathbb{C}$ is immanent, $\mathbb{H}$ transcendent and $M_3(\mathbb{C})$ omniscient.

M-I. Definitions and Method

M-001 Theorem, Identification and Derivation. That I reach the results of this chapter by kinds of step, and that I name the kind of each step where I take that step. A theorem is a statement proved from stated premises; a result of mathematics used by its standard statement, such as the value of an Euler characteristic or a classification of spaces, is a theorem, and it is named. An identification is a stated step from the structure of comparison to a physical or geometric structure, made by the author in the first person ("I identify X with Y"), with its reasons given. A result is derived when it follows by argument from the structure of comparison, and derived, given a stated identification, when it follows once a named identification is made. That the kinds of step are kinds, and not grades of claim: a result that rests on an identification holds at the standing of that identification, and a reader who rejects the identification rejects the results that rest on it. That a proof shows validity: the result follows from the premises. A proved result settles a claim about the world when its soundness is shown as well: the formal statement truly renders the claim, and the premises hold. In this chapter the identifications are the places where the premises meet the world, and each entry names them.

Sources: *Measurement*, Chapter 6 (identifications and kinds of step); *The Measures of the Kinds*, Preface (the standings of a number); *The Gravitational Passage*, The Chain at a Glance, and Chapter 20, Questions. Standing: a definition.

M-002 Gauge Anomaly. That an anomaly is a quantum effect that breaks the conservation law of a symmetry in a fixed, computable way. That for a gauge symmetry the requirement that quantum corrections keep the symmetry is the requirement that certain sums, taken over the hypercharges and other charges of the fermions of one generation, vanish; when they vanish, the description of the forces is consistent, and a sum that does not vanish makes it mathematically inconsistent. That the requirement that these sums vanish is called anomaly cancellation. That anomaly cancellation is a different condition from unimodularity, the condition that the determinant of each unitary's action on the fermion states is one, which fixes the abelian factor of the gauge group.

Sources: *The Kinds of Particle*, Chapter 5, Chapter Overview and Section I; *The Strengths of the Kinds*, Terms (anomaly); *The Story of the Particles*, Level 6, Section IV; *The Concept Sequence*, Chapter 23. Standing: a definition.

M-003 Homogeneous Space and the Classification. That a compact homogeneous space is a quotient G/H of a compact Lie group G by a closed subgroup H, with G acting transitively by isometries: every point is carried to every other by an element of G, and no point is distinguished from another. That the coset G/H is the set of positions to which one point can be carried by G, with H the subgroup that holds that point fixed. That the compact homogeneous spaces of low dimension are classified: the connected compact homogeneous surfaces are the sphere S^2, the real projective plane and the torus, and the homogeneous Riemannian spaces of four dimensions are classified by Ishihara (1955).

Sources: *The Gravitational Passage*, Chapter 9, Sections II and VI; Ishihara (1955). Standing: a definition (homogeneous space); a theorem (the classification of compact homogeneous spaces of dimensions two and four).

M-II. The Dimensional Identity

M-004 The Two Appearances of Three. That the integer three appears twice in the geometry of physics. The first appearance: the context of space and time has four dimensions, of which three are dimensions of space. The second appearance: the factor $\mathbb{C}P^2$ of the internal space has Euler characteristic three, with three cells in its standard decomposition (of dimensions zero, two and four), and the coefficient of its first Chern class (a topological invariant of its complex structure, counted in units of the class of a projective line) is also three, $c_1 = 3$. That both are the one triad of the three positions of comparison, carried by each factor in its own way: I identify the triad, in the four-dimensional context, with three directions of extension, and, in $\mathbb{C}P^2$, with three independent classes, $\chi(\mathbb{C}P^2) = 3$. That these identifications have the same standing as the translation of the grid of the six measures into the internal geometry (entry M-009): they are grounded in the axioms and pass through a geometric reading.

Sources: *The Gravitational Passage*, Chapter 9, Section VII, and Chapter 10, Section IV; *The Kinds of Particle*, Chapter 2, Section I ($c_1 = 3$ as the count of the three positions); *The Concept Sequence*, Chapters 15, 25 and 26. Standing: an identification (the triad as three directions and as three classes); a theorem ($\chi(\mathbb{C}P^2) = 3$ and $c_1 = 3$).

M-005 What a Dimension Is. That the dimension of a context is the number of its independent ways of differing: the number of independent ways in which two findings placed in that context can differ, where a way of differing is independent of others when no combination of differences in the others yields it. That this is the definition mathematics uses for a space: the dimension of a smooth space is the number of independent directions in which two nearby points can differ. That at the level of the ground a way of differing is a way in which comparison can register a difference, so the dimension of a context can only be a number the structure of comparison supplies. That the sets of elements in the structure of com-

parison have five sizes and no others: one (the comparing, a single stance), two (the members of any pair), three (the pairs: the verdict, the respect and the stance), four (the followable intrinsics: sameness, difference, content and context), and six (the intrinsics). That the minimality principle bounds the count from above: of the numbers comparison supplies, a context takes the least that meets every requirement comparison places on it, because a further way of differing would be a difference no comparison registers, and a difference that no comparison can register is no difference.

Sources: *Measurement*, Chapter 11, Sections I to IV; *At the Edge of Reality*, Chapter 5; *The Concept Sequence*, Chapter 25. Standing: a definition (dimension); derived (the five numbers, and the minimality principle from the principle that a difference no comparison can register is no difference).

> ***Why composed ways do not count.***
>
> On a map, "north-east" is a way of differing, but it is composed from east and north, so it adds no dimension. In the same way, the three pairs of intrinsics, each taken in one of its two members, can be combined in eight ways; those eight are composed from three choices of two, so eight is not a number comparison supplies as a dimension. Only the sizes of the sets of elements count: one, two, three, four and six.

M-006 The First Road: One Dimension of Time, Three of Space. That time has one dimension: time is the context of the order in which findings are found, the comparing is one stance, and from one stance any two findings stand in one relation of order, earlier and later. That space has three dimensions, the least number comparison supplies that meets every requirement on space. Space has more than one: space and time are distinct and non-interchangeable (the triple condition, from Axiom III), and a context has no feature but its form of difference, so space must differ from time in its number of ways. Space has more than two: every difference is placed twice, by a comparing and in a context (the doubling), and the distinction between two ways of a context is placed in that context only by a third way not composed from them (the placing rule), so a context of two

ways leaves the distinction between them placed only once. Three, four and six all meet the doubling, and by the minimality principle (entry M-005) space takes the least of them: three. That I identify the space so counted with the familiar directions of the large world, the directions in which things extend.

Sources: *Measurement*, Chapter 11, Sections V and VI; *At the Edge of Reality*, Chapter 6; *The Concept Sequence*, Chapters 15 and 25. Standing: derived, given a stated identification (space as the directions of extension), with the doubling, the placing rule and the minimality principle as its premises.

> ***The placing rule, pictured.***
>
> On a flat page you can tell east from north, but nothing on the page stands apart from both. Lift your eyes off the page, and the third direction, straight up, is the one from which the page's two directions can be told apart. With three ways, each way places exactly one of the three distinctions, the one between the other two. Three facts agree with the count, and the count does not depend on them: in three dimensions, and in no other number, each plane has exactly one perpendicular line; three is the only number of ways equal to the number of pairs of those ways; and time and space together have four dimensions, the number of the followable intrinsics.

M-007 The Second Road, and the Ten Dimensions of Context. That the same counts of time and space are reached by a second road that shares no step with the first. That the cycle of Axiom II has three transitions, from the transcendent to the immanent, from the immanent to the omniscient, and from the omniscient to the transcendent, each an independent relationship between two roles; I identify each, in the large world, with an independent direction of extension, because each transition joins two roles that stand apart and are held together, and space is the context in which things that stand apart are held together: the three transitions give three directions of space. That the cycle also has one direction, the order of its return, and I identify that direction with the one direction of time. That Axiom I gives the same count of one for time by a further route:

the immanent is more fundamental than the other two, in the context row of the grid of the six measures the immanent context is time, so time is the single prior direction, the cut that divides before from after. That possibility, the third context, has six dimensions: the range of what a comparison could find spans all six intrinsics, and possibility stays bound to the stance, so it cannot be confined to the four followable intrinsics. That one, three and six make ten: one dimension of time, three of space and six of possibility. That the four of time and space are traversed by located comparers, and the six of possibility are never traversed and are not directions of space: they are met only as the range from which each finding is one selection.

Sources: *Measurement*, Chapter 11, Sections VII, VIII, X and XI; *At the Edge of Reality*, Chapter 7; *The Gravitational Passage*, Chapter 10, Sections I, II and IV; *The Concept Sequence*, Chapters 15 and 25. Standing: an identification (the three transitions as three directions of space, and the direction of the cycle as the direction of time); derived (the six dimensions of possibility, and the total of ten).

> ***Written symbolically.***
>
> Time: 1. Space: 3. Possibility: 6. Total: $1 + 3 + 6 = 10$. The number of dimensions of space, the Euler characteristic of the complex projective plane, and the count of the three positions of comparison are one integer, three: $d_{space} = c_1 = \chi(\mathbb{C}P^2) = 3$. The first equality rests on the identifications of entries M-006 and M-007, the second is a theorem (the Euler characteristic and the first Chern class of $\mathbb{C}P^2$), and the joining of the two is the identification of entry M-004. The three counts form a progression: time has the comparing, which is one; space adds the pair, which is two; possibility adds the triad of pairs, which is three.

M-008 Consequence: The Lorentzian Signature. That the metric of space and time has one sign for time against three for space, $(-, +, +, +)$. That the ground is the directed precedence of Axiom II: a class of the transcendent precedes an instance of the immanent, a class of the immanent precedes an instance of the omniscient, and a class of the omniscient precedes an instance of the transcendent; precedence runs one way. That I

identify that precedence, in the context of space and time, with the order of events, and that the order has two features. That it has a limit: I identify the precedence of a class of the immanent over an instance of the omniscient with a span of time before every placing of a finding in space, and the macroscopic boundary of the domain with one ratio of spatial to temporal measure, the same for every comparer in uniform motion, the speed c; homogeneity, isotropy, the principle of relativity and the group law fix the transformations between frames up to one constant, the boundary makes that constant nonzero, and the cone of precedence makes it positive, equal to $1/c^2$. That the limit is carried by the metric itself, as a cone at each event bounded by the directions of zero interval, so the metric is indefinite. That the order has a direction: what precedes is distinct from what follows, so the cone at each event falls into a before and an after, and the cone falls into two halves for exactly one direction of time; with two or more, the cone is one connected piece, and before cannot be told from after. That the signature therefore has exactly one negative sign.

Sources: *Measurement*, Chapter 11, Section IX; *The Gravitational Passage*, Chapter 10, Section III; *At the Edge of Reality*, Chapter 6; Lévy-Leblond (1976); Tegmark (1997); Craig and Weinstein (2009); *The Concept Sequence*, Chapters 15 and 25. Standing: derived, given a stated identification (precedence as the order of events, and the macroscopic boundary of the domain as the speed c).

> ***Agreement from the field equations.***
>
> Analyses of the field equations agree with the count of one. With two or more directions of time the wave equation is ultrahyperbolic, and its initial value problem is ill posed for data given on a surface that contains no direction of time (Tegmark 1997; Craig and Weinstein 2009). Precedence is a difference of content, what precedes and what follows, where there is a difference of context, the two events, and it varies continuously from event to event: it is the continuously asymmetric conjunction, realized as the causal order of the context.

M-III. The Uniqueness of the Internal Space

M-009 Two Routes to One Space. That the internal space $\mathbb{C}P^2 \times S^2$ is reached by two routes that share no step. That the first route is the algebra of comparison: the summand $M_3(\mathbb{C})$ of $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ has the space of its pure states $\mathbb{C}P^2$, the summand $\mathbb{H}$ has the space of its rays $\mathbb{C}P^1 \cong S^2$, and the summand $\mathbb{C}$ has a single point; in the frame of measurement used in this sequence, $\mathbb{C}P^2$ is the state space of the omniscient summand and S^2 that of the transcendent summand (*Non-Commutative Geometry and the Immanent Metaphysics* labels the same summands by an admissible rotation, with $\mathbb{H}$ immanent and $M_3(\mathbb{C})$ transcendent). That the second route is that of *The Gravitational Passage*, which uses no algebra and translates the grid of the six measures into geometry by three identifications: one dimension for each measure; a product of a factor that carries the three modalities and a factor that carries the two rows, content and context, each connected and of positive dimension; and each factor carrying its terms as its independent classes, each the class of a cell of even dimension; with each factor taken to be homogeneous. From these, the classification of compact homogeneous spaces selects $\mathbb{C}P^2 \times S^2$ as the only compact homogeneous product that meets them (entries M-010 to M-012). That the identifications of the second route refer to the grid of the six measures alone: they name neither the particles to be reproduced nor a symmetry group the internal space should have. That the internal space has two instances, the content instance and the context instance. The content instance is a space of states, with no size, no radius and no fields, on which the kinds and charges of the particles are read. The context instance is the space of possibility, six of the ten dimensions of context, with radii. Neither instance is a place, and nothing travels through either: the six dimensions of possibility are never traversed.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Definition 3.9 and Chapter 18, Section II; *The Gravitational Passage*, Chapter 9, Sections I, II and VII; *The Connes Correspondence*, Chapter 1, Sections 8 and 9; *The Measures of the Kinds*, Preface; *Measurement*, Chapter 11, Sections VIII and XII; *The Concept Sequence*, Chapters 24 and 25. Standing: a theorem (the state spaces of the summands); an identification (the three conditions and the homogeneity of the second route); a theorem (the selection, given those identifications).

> ***A space of possibilities.***
>
> Think of a quantum system that has three alternatives. Its possible states are all the ways of weighting and phasing those three, counted without regard to overall size or overall phase, and the space of all those states is the complex projective plane. In the same way, the space of states of a system with two alternatives is the sphere. Wootters (1981) showed that the natural distance on such a space measures how well one state can be told apart from another. So each factor of the internal space is a space of possibilities in the plain mathematical sense.

M-010 The Hopf-Samelson Theorem and the Euler Characteristic. That the Euler characteristic χ of a space is the alternating sum of its Betti numbers, the numbers of its independent cohomology classes in each degree, and that when every independent cohomology class lies in even degree, χ is the total number of independent classes. That for a compact homogeneous space G/H, every independent class lies in even degree exactly when χ is positive, which holds exactly when H has the same rank as G (the rank of a compact Lie group being the dimension of its maximal torus) (Hopf and Samelson 1941). That the third condition of the second route (entry M-009) places every independent class of each factor in even degree and fixes their number, so it reads $\chi(K_M) = 3$ for the factor that carries the three modalities and $\chi(K_R) = 2$ for the factor that carries the two rows. That the Euler characteristic of a product is the product of the Euler characteristics, so $\chi(K) = 3 \times 2 = 6$, the number of the six measures.

Sources: *The Gravitational Passage*, Chapter 9, Section IV; Hopf and Samelson (1941); *The Concept Sequence*, Chapter 25. Standing: a theorem (the Hopf-Samelson theorem, and the product rule for the Euler characteristic); an identification (the counts of terms as the Euler characteristics of the factors).

M-011 The Factor of Dimension Two: the Sphere. That both factors have non-zero Euler characteristic, and a compact manifold of odd dimension has Euler characteristic zero, so both factors have even dimension; with a total of six, the possible dimensions of the two factors (K_M, K_R) are $(6, 0)$, $(4, 2)$, $(2, 4)$ and $(0, 6)$. That each factor has positive dimension, which excludes $(6, 0)$ and $(0, 6)$, and a connected compact surface has Euler characteristic at most two, so no surface has $\chi = 3$, which excludes $(2, 4)$. That the split is therefore $(4, 2)$: the factor that carries the modalities has dimension four, and the factor that carries the two rows has dimension two. That the connected compact homogeneous surfaces are the sphere S^2, the real projective plane and the torus, with Euler characteristics 2, 1 and 0, so the only one with $\chi = 2$ is S^2.

Sources: *The Gravitational Passage*, Chapter 9, Section VI; *The Concept Sequence*, Chapter 25. Standing: a theorem (given the identifications of entry M-009).

M-012 The Factor of Dimension Four: the Complex Projective Plane. That the connected compact homogeneous four-manifolds with positive Euler characteristic are finite quotients of simply connected ones, each a quotient G/H in which H has the same rank as G (entry M-010), and the simply connected ones are three: the four-sphere S^4 ($\chi = 2$), the complex projective plane $\mathbb{C}P^2$ ($\chi = 3$), and $S^2 \times S^2$ ($\chi = 4$). That a quotient by a free action of a finite group of order k has Euler characteristic χ/k, so the non-trivial quotients of S^4 and of $S^2 \times S^2$ have Euler characteristic 1 or 2, and $\mathbb{C}P^2$ admits no free action of a non-trivial finite group, since by the Lefschetz theorem every self-map of $\mathbb{C}P^2$ has a fixed point. That the only connected compact homogeneous four-manifold with $\chi = 3$ is therefore $\mathbb{C}P^2$, and the internal space is $K = \mathbb{C}P^2 \times S^2$, of dimension $4 + 2 = 6$ and Euler characteristic $3 \times 2 = 6$. That homogeneity is the decisive condition: there

are four-manifolds with $\chi = 3$ that are not homogeneous, and the condition of homogeneity excludes those four-manifolds. That the product condition excludes the six-dimensional flag manifold $SU(3)/(U(1) \times U(1))$, which has Euler characteristic six and is a single homogeneous space, not a product; and the triple product $S^2 \times S^2 \times S^2$ has Euler characteristic eight. That the classification is a proof, and it shows validity: given the conditions of entry M-009, $\mathbb{C}P^2 \times S^2$ is the only space that meets them. That it selects the internal space of physics rests as well on the soundness of those conditions: on the identifications that translate the grid of the six measures into geometry, and on homogeneity, which *The Gravitational Passage* states as identifications with their reasons.

Sources: *The Gravitational Passage*, Chapter 9, Sections I, II, VI and VII, and The Chain at a Glance; Ishihara (1955); *The Concept Sequence*, Chapter 25. Standing: a theorem (given the identifications of entry M-009).

> ***Three appearances of the integer three.***
>
> The integer three that the classification meets in $\chi(\mathbb{C}P^2) = 3$ appears in three places. It is the count of the three positions of comparison, $c_1 = 3$ (an identification of the kernel integer with that count). It is the Euler characteristic of $\mathbb{C}P^2$ and the coefficient of its first Chern class (theorems). And it is the one positive root of the selection equation of the kernel triangle: with $n_C = 2$, the right-angle condition $c^2 + (c+1)^2 = (c+2)^2$ gives $(c-3)(c+1) = 0$, so $c = 3$, the triangle with sides 3, 4 and 5 (arithmetic). The three appearances of the integer three are agreements on one integer; the selection of $\mathbb{C}P^2$ rests on the classification of entries M-011 and M-012, given the identifications of entry M-009.

M-013 What the Identified Geometry Carries. That the first Chern class of $\mathbb{C}P^2$ is three times the class of a projective line, so its coefficient equals the Euler characteristic of $\mathbb{C}P^2$: $c_1 = \chi(\mathbb{C}P^2) = 3$. That $\mathbb{C}P^2$ with the Fubini-Study metric and S^2 with the round metric have everywhere positive Ricci curvature, and the first Chern class of $\mathbb{C}P^2 \times S^2$ does not vanish, so the internal space is not a Calabi-Yau space (a space whose first Chern

class vanishes, which admits a Ricci-flat metric) and admits no Ricci-flat Kähler metric. That the shape and scale of the metric of the context instance are fixed by one identification: I take the modal boundary to be one curvature, the Planck curvature $1/l_{Pl}^2$, for every internal direction, and each internal direction to reach it, so the internal metric is an Einstein metric, and the radii are $R_O = \sqrt{6}\, l_{Pl}$ for $\mathbb{C}P^2$ and $R_T = l_{Pl}$ for S^2, with ratio $R_O/R_T = \sqrt{6}$ in the normalization of *The Gravitational Passage*. That these radii belong to the context instance only: the content instance has no size and no radius, the particles are read on it by their kinds and charges, and they are not vibration modes of the internal space. That the isometries of $\mathbb{C}P^2 \times S^2$ are, in their identity component, $SO(3) \times PU(3)$, the inner automorphisms of the algebra, of dimension eleven; the gauge group $U(1) \times SU(2) \times SU(3)$ has one dimension more: its $U(1)$ comes from the summand $\mathbb{C}$, acting through the bimodule (the two-sided action of the algebra on the fermion states), so the gauge group is not the isometry group of $\mathbb{C}P^2 \times S^2$. That the number of generations is three because the content turns three ways around the cycle of Axiom II, one way for each pair of intrinsics held foremost, and a fourth turn is the first again. The index of the Dirac operator on the space of states gives one, not three, and the threefold multiplicity that the first Chern class gives is a color triplet; the index and the Chern class do not set the count of generations. The equality $N_{gen} = c_1 = \chi(\mathbb{C}P^2)$ therefore holds as the equality of two derived integers: three generations from the cycle, and $c_1 = 3$ from the count of the positions.

Sources: *The Gravitational Passage*, Chapter 9, Sections VII to IX; *Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 18.1, and Chapter 19, Section VII (Corollary 19.11); *The Kinds of Particle*, Chapter 3, Section III, Chapter 5, Section III, and Chapter 7, Sections I and II; *The Story of the Particles*, Level 5, Section I; *The Concept Sequence*, Chapters 24 and 25. Standing: a theorem (c_1, the positive curvature, the isometry group); derived, given a stated identification (the Einstein condition at the Planck curvature, and so the radii); derived (the three generations, and the equality of their number with c_1).

M-IV. The Color Charge Problem

M-014 Exact Color Symmetry and Non-Interchangeability. That the three colors of the strong force are the labels of the three positions of comparison held inside the content: each quark carries one of three colors, and the word is only a name. That by Axiom III the three positions of comparison are distinct, inseparable and non-interchangeable. That the color unitaries, the group $SU(3)$ acting on the three colors, are exact symmetries of the metric of comparison: a color direction can be rotated into any other, the description of the strong force is unchanged, and no measurement has found a preference for one color over another. That the question this part answers is how an exact symmetry that rotates the colors stands with the non-interchangeability of the positions they label.

Sources: *The Story of the Particles*, Level 6, Section III; *The Kinds of Particle*, Chapter 4, Section VI; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section VIII; *The Concept Sequence*, Chapters 15, 22 and 23. Standing: an identification (the colors as the three positions of comparison held inside the content, which *Non-Commutative Geometry and the Immanent Metaphysics* carries as its postulate P-Rep).

M-015 Resolution: What Is Found of Color. That the law of symmetry says that what is found as content is found as a sameness of content across a difference of context. That two identifications give the law a form on the field theory: a content found is a ray of the space of states, and a difference of context that leaves the metric of comparison unchanged is the action of a unitary of the continuous subalgebra, $\mathbb{C} \oplus M_3(\mathbb{C})$: electromagnetism and color. That a content found is therefore carried into itself by every color unitary, up to a phase, and since $SU(3)$ is connected and equal to its own commutator subgroup (every element is a product of commutators), its only homomorphism into the circle is trivial: a content found carries no color, and it is a color singlet. That every finding there-

fore holds the three colors together, and no finding presents one position of comparison apart from the other two. That the exact color symmetry acts in the formalism, where the color states form a continuous space, while what is found of color is discontinuously symmetric: the formalism is continuous and the findings are discrete, as the Proposition of the Two Levels states for every physical theory. That the symmetry that rotates the color labels and the non-interchangeability of the positions are therefore statements at two levels, the formalism and the findings, and the findings show the inseparability of the positions: the three positions are found only together. That the step from the two identifications to the color singlet is a proof, and it shows validity; its result is a claim about what is found, and its soundness rests on the two identifications, which carry the law of symmetry to the field theory and are stated with it.

Sources: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section VIII (the confinement argument, P-Conf); *Incommensuration*, Chapter 10, Section VI (the Proposition of the Two Levels); *The Story of the Particles*, Level 6, Section III; *The Concept Sequence*, Chapters 13, 22 and 23. Standing: an identification (the two identifications of the confinement argument); a theorem (that SU(3), connected and equal to its own commutator subgroup, has only the trivial homomorphism into the circle).

> ***Why charge and color differ.***
>
> Electric charge and color are both exact symmetries of the metric of comparison, and both are differences of context across which a content found must remain the same ray. The circle of electromagnetism has characters, the homomorphisms $\theta \mapsto e^{ik\theta}$ for each integer k, so a content found may carry a charge, and it carries exactly one. SU(3) has only the trivial character, so a content found carries no color. That is why charge is superselected (every finding has one definite charge) and not confined, while color is confined. The unitaries of $\mathbb{H}$ outside the circle change the metric of comparison, so the law sets no condition through them: weak isospin is neither confined nor superselected.

M-016 Confinement as Inseparability. That confinement, the absence of any isolated quark, follows from the structure of comparison: each quark carries one of the three positions of comparison as its color, a single position taken alone is not a comparison, and only a state that holds all three positions, a complete comparison in color, can be observed. That a complete comparison in color holds one instance of each position: three quarks of three colors (a baryon), or a quark and its antiquark, whose conjugate color is the other two positions taken together as a unit (a meson). That the electron, which also fills one position, travels alone because its comparison is completed by things outside it, the photon and the proton, while the other two positions of a quark's comparison lie inside the same content, and only other quarks can fill them. That the leptons, the re-projection of the six intrinsics at the context level, carry no separated positions as internal labels, and go free. That on the finite spectral triple the same conclusion is reached from the law of symmetry (entry M-015). That the dynamics of quantum chromodynamics confine color, a result of physics that the collection reports; the argument here states the same fact on the kinematics, from the structure of comparison.

Sources: *The Story of the Particles*, Level 6, Section III; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section VIII (P-Conf); *The Kinds of Particle*, Chapter 4, Section VI; *The Concept Sequence*, Chapter 22 (confinement as inseparability). Standing: derived (from Axiom III, with the colors as the three positions of comparison); derived, given a stated identification (on the finite spectral triple, the two identifications of the confinement argument of entry M-015).

M-V. Anomaly Cancellation and the Charges

M-017 The Four Constraints on the Charges. That the electric charges of one generation are fixed by four constraints, stated with the frame of measurement, in which light is the comparing act and the elec-

tron's charge is the unit of the frame. The first constraint (whole charge): a complete comparison carries a whole number of units of charge, because a whole that light reads as one thing is read as a whole number of the unit. The second constraint (one unit within a doublet): the two sides of a doublet differ by exactly one unit of charge, as the lepton doublet shows, with the electron at -1 and the neutrino at 0; the quark doublet and the lepton doublet are the same two members, so the up-type and down-type quarks differ by the same unit. The third constraint (the leptons): the electron carries -1, the unit of the frame, and the neutrino carries 0. The fourth constraint (the sum rule): the charges of one complete generation, each quark counted once for each of its three colors, sum to zero, because the comparing act carries no charge, a generation holds one occupant for every position of knowing, and across the whole of what the act addresses there is nothing to set a total charge against except the act itself. That the first two constraints make every quark charge a whole number of thirds and leave a choice (an up-type charge of $+1$ with a down-type charge of 0 satisfies both), and the fourth decides it.

Sources: *The Story of the Particles*, Level 0, Level 3 and Level 6, Section IV; *The Kinds of Particle*, Chapter 5, Section I; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section IX; *The Concept Sequence*, Chapter 22. Standing: derived, given a stated identification (each of the four constraints, read from the structure of comparison in the frame of measurement).

M-018 The Determined Charges. That the four constraints of entry M-017 have exactly one solution. From the third constraint, $Q_\nu = 0$ and $Q_e = -1$. From the second, $Q_d = Q_u - 1$. From the fourth, $3(Q_u + Q_d) + Q_e + Q_\nu = 0$, so $3(2Q_u - 1) - 1 = 0$, and $Q_u = +2/3$, $Q_d = -1/3$. That every charge is a whole number of thirds of the unit, the proton (two up quarks and one down) has charge $+1$, and the neutron (one up and two down) has charge 0. That the hypercharges computed from the representation of the algebra and the unimodularity condition satisfy the same sum rule

over the particle states of one generation, so the algebraic route and the route of the four constraints reach the same charges. That the derivation uses no measured number beyond the electron's unit charge.

Sources: *The Story of the Particles*, Level 6, Section IV; *The Kinds of Particle*, Chapter 5, Section I and Summary; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 14, and Chapter 19, Section IX (Proposition 19.13); *The Concept Sequence*, Chapters 22 and 24. Standing: derived, given a stated identification (the four constraints of entry M-017); a measured input (the electron's unit charge).

> ***The charges, in summary.***
>
> Up-type quark: $+2/3$. Down-type quark: $-1/3$. Neutrino: 0. Charged lepton: -1. The electron's -1 is the unit of the frame; the neutrino's 0 is the neutral context member of the lepton doublet; the one unit between the two types is the lepton doublet's; and the sum rule decides between the two choices that whole charge and the doublet unit leave open. The fractional charges are never found isolated: confinement (entry M-016) means quarks are observed only as parts of composite states whose total charge is whole.

M-019 The Anomaly Conditions. That six anomaly conditions constrain the hypercharges of one generation, with left-handed fields counted with a plus sign and right-handed fields with a minus sign, and with the hypercharges quark doublet $1/6$, lepton doublet $-1/2$, right-handed up quark $2/3$, right-handed down quark $-1/3$, right-handed electron -1, right-handed neutrino 0 (convention $Q = T_3 + Y$). That two of the six anomaly conditions are satisfied by group theory: the $[SU(2)]^3$ condition and the $[SU(3)]^3$ condition. The $[SU(2)]^3$ condition is satisfied because $SU(2)$ has no independent cubic Casimir invariant (no symmetric invariant of degree three built from its generators). The $[SU(3)]^3$ condition is satisfied because the doublet structure places equal numbers of 3 and $\bar{3}$ in the left-handed spectrum, with the right-handed quarks written as left-handed antiquarks, each in $\bar{3}$. In the sign convention above, the two triplets of the quark doublet and the two right-handed quark triplets can-

cel. That the other four are non-trivial and are satisfied: gravitational, $\mathrm{Tr}[Y] = [6(1/6) + 2(-1/2)] - [3(2/3) + 3(-1/3) + (-1) + 0] = 0$; mixed $[SU(3)]^2\,U(1)$, $2(1/6) - (2/3) - (-1/3) = 0$; mixed $[SU(2)]^2\,U(1)$, $3(1/6) + (-1/2) = 0$; cubic $[U(1)]^3$, $\mathrm{Tr}[Y^3] = [6(1/6)^3 + 2(-1/2)^3] - [3(2/3)^3 + 3(-1/3)^3 + (-1)^3 + 0] = (-2/9) - (-2/9) = 0$. That in the physicists' account the sum rule of entry M-017 is one of the conditions for the cancellation of anomalies, the condition on two weak-force vertices and one hypercharge vertex (the hypercharges of the left-handed doublets sum to zero), and that with a right-handed neutrino present the anomaly conditions alone leave one charge assignment free, which the derivation closes with the neutral neutrino.

Sources: *The Kinds of Particle*, Chapter 5, Section I; *The Story of the Particles*, Level 6, Section IV, and note 17; *The Concept Sequence*, Chapter 23. Standing: a theorem (the six sums, evaluated on the hypercharges of the representation).

M-020 The Number of Colors. That the count of three colors is the count of the three positions of comparison held inside the content: three colors and three generations have one source, the number of positions, reprojected once as a label inside the content and once as an instance of it. That two conditions of physics converge independently on the same count. The first condition: with the hypercharges of entry M-019 held at their values, the cubic anomaly per generation is $\mathrm{Tr}[Y^3] = (3 - N_c)/4$, linear in the number of colors N_c, so it vanishes for $N_c = 3$ and for no other value. The second condition: the color count is odd, because the weak force is globally consistent over three generations only for an odd count, and because a proton of whole charge exists only for an odd count. The count must be odd for the weak force because an $SU(2)$ gauge theory needs an even number of left-handed doublets (Witten 1982), and three generations of $N_c + 1$ doublets each are even in number only when N_c is odd. The route through the weak force and the route through the whole charge of the proton share no premise, and each route excludes $N_c = 2$. That the first condition is a theorem with the hypercharges held at their

values, and it fixes the count of colors at the standing of that premise; the second is a theorem from the global consistency of the weak force and the whole charge of the proton.

Sources: *The Kinds of Particle*, Chapter 4, Section VI; *The Story of the Particles*, Level 6, Section III; Witten (1982); *The Concept Sequence*, Chapter 23. Standing: an identification (the colors as the three positions of comparison, which *Non-Commutative Geometry and the Immanent Metaphysics* carries as its postulate P-Rep); a theorem (the cubic anomaly $(3 - N_c)/4$ with the hypercharges held at their values, and the odd count).

> ***Written symbolically.***
> $\mathrm{Tr}[Y^3]$ per generation $= (3 - N_c)/4$, with the hypercharges held at their values in entry M-019. For $N_c = 3$: $\mathrm{Tr}[Y^3] = 0$. The color quadratic Casimir of the quark triplet (the number that measures the strength of the color charge a quark carries) is $C_F = (N_c^2 - 1)/(2N_c) = 4/3$. The count from comparison, the cubic anomaly, and the odd count are three routes to one integer, three.

M-021 Inseparability Carried by Anomaly Cancellation. That quarks alone are anomalous and leptons alone are anomalous: the mixed $[SU(2)]^2\,U(1)$ sum is $+1/2$ for the quarks and $-1/2$ for the leptons, and the cubic $[U(1)]^3$ sum is $-3/4$ for the quarks and $+3/4$ for the leptons; the anomalies cancel only when the colored and uncolored matter of one generation are taken together. That the three gauge factors $SU(3)$, $SU(2)$ and $U(1)$ realize Axiom III: they are distinct (each has its own coupling, generators and representations), inseparable (the matter content ties them together, and no factor can be removed without breaking the cancellation), and non-interchangeable (no automorphism of $SU(3) \times SU(2) \times U(1)$ permutes the three factors). That the chain to anomaly cancellation runs as follows: the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ from the structure of comparison (the Three-Carrier Theorem); the gauge group $U(1) \times SU(2) \times SU(3)$ from the unitaries of the algebra with unimodularity; the hypercharges of one generation as outputs of the representation and unimodularity,

reached again by the four constraints of entry M-017; and the six anomaly sums evaluated on them, all vanishing (entry M-019). That each step of the chain is a proof, and a proof shows validity: its result follows from its premises. The chain settles the claim about the particles of the world when the soundness of the chain is shown as well. The postulates give the clauses of the axioms the form of conditions on a spectral triple. The soundness of the chain is shown when the postulates truly render the clauses of the axioms, and when the representation of the algebra is the observed kinds. Both are identifications, stated with their reasons, and the chain's result about the world holds at their standing. That I identify the cancellation of the anomalies only across quarks and leptons together with the inseparability of the content and context projections within one generation.

Sources: *The Kinds of Particle*, Chapter 5, Sections I and II, and Summary; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapters 7 and 14, and Chapter 19, Section IX (Proposition 19.13); *The Connes Correspondence*, Chapter 6, Section II (2.1); *The Concept Sequence*, Chapters 22, 23 and 24. Standing: a theorem (each step of the chain, the Three-Carrier Theorem resting on the postulates of *Non-Commutative Geometry and the Immanent Metaphysics*); an identification (anomaly cancellation as carrying the inseparability of Axiom III).

Summary

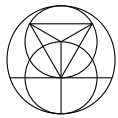
In this chapter I gather four structural results and show, for each result, the premises that the result rests on and the kind of step that reaches the result. Its first part fixes the kinds of step: a theorem shows that its result follows from its premises, an identification carries the structure of comparison to a structure of physics or geometry, in the first person and with its reasons, and a proved result settles a claim about the world when its statement renders the claim and its premises hold.

The second part counts the dimensions of context, as *Measurement* derives them. A dimension is an independent way of differing, and comparison supplies the numbers one, two, three, four and six. Time has one di-

mension, because from one stance any two findings stand in one order. Space has three dimensions, the least number that places every distinction twice. Possibility has six dimensions, one for each intrinsic, and the total is ten. A second road, from the three transitions and the one direction of the cycle of Axiom II, reaches the same counts, and the precedence of Axiom II gives the metric of space and time its Lorentzian signature, one sign for time against three for space. The three of space and the three of $\mathbb{C}P^2$ are one triad, carried by each factor in its own way.

The third part shows why the internal space is $\mathbb{C}P^2 \times S^2$. The algebra of comparison reaches it as the state spaces of its summands, and *The Gravitational Passage* reaches it without the algebra, by translating the grid of the six measures into geometry and applying the classification of compact homogeneous spaces: given the identifications of that translation, $\mathbb{C}P^2 \times S^2$ is the only space that meets them. The geometry it carries has positive curvature, radii fixed at the modal boundary on its context instance, and an isometry group one dimension smaller than the gauge group; the three generations come from the cycle of Axiom II, and their number equals $c_1 = 3$ as two derived integers are equal.

The fourth and fifth parts treat color and charge. The exact symmetry that rotates the colors acts in the formalism, and what is found of color is a singlet: the three positions of comparison are found only together, which is confinement. Four constraints read from comparison fix the charges of one generation, $+2/3$, $-1/3$, 0 and -1, the same charges the representation of the algebra gives; on them the six anomaly sums vanish, quarks and leptons cancel only together, and the cubic anomaly and the odd count converge with the count of the positions on three colors. The chain from the algebra to the cancellation is a sequence of proofs, and its result about the world stands on the identifications that each entry names.



Chapter N

The Discrete Symmetries and the Topological Classification

CHAPTER OVERVIEW

This chapter sets the account of the three discrete reversals C, P and T, as *The Branches of the Kinds* gives it, beside the tenfold classification of free-fermion quantum systems of Altland and Zirnbauer, one of the central organizing results of condensed-matter physics. Part N-I states the three reversals, the kind of number each marks, the two placements of each at the positions of comparison, and the three rules that govern which force keeps and which force breaks each reversal. Part N-II states the tenfold classification as standard physics, and the one place where it meets the account: its three Wigner-Dyson classes are Dyson's threefold way, by which the account pairs the reversals with the kinds of number. Part N-III states Bott periodicity, a theorem of mathematics, and the identification of *The Story of the Particles* that reads the eight-step sequence of Clifford algebras as a directed cycle through the three kinds of number.

The chapter rests on six chapters of *The Concept Sequence*: Chapter 13, The Incommensuration Theorem; Chapter 14, The Three Modalities; Chapter 15, The Three Axioms; Chapter 18, The Three Forces; Chapter 24, The Algebra of Comparison; and Chapter 36, Antiparticles.

N-I. The Three Reversals and Their Modal Character

N-001 Definition: Topological Insulator. That a topological insulator is a crystal that keeps time-reversal symmetry, whose interior is an insulator, and whose boundary conducts. That the boundary conduction is protected: the boundary conduction survives every change of the crystal that keeps time-reversal symmetry and keeps the energy gap of the interior open, and the boundary conduction can be removed only by a change that breaks time-reversal symmetry or closes that gap. That the protection rests on a two-valued ($\mathbb{Z}_2$) index of the crystal's bands, which exists because reversing time twice reverses the state of an electron, so that every state has a partner of the same energy and opposite spin (Kramers' theorem); a crystal that keeps time reversal with that index equal to one is a topological insulator.

Sources: *The Matter We Know*, Volume Two, Chapter 10, Section IX; Kramers (1930); Kane and Mele (2005). Standing: a definition.

> ***Why topology matters here.***
>
> Topology is the branch of mathematics concerned with properties that are preserved under continuous deformation: stretching and bending, with no tearing or gluing. A coffee mug and a doughnut are topologically equivalent (each has exactly one hole); a sphere and a doughnut are not. A property of a crystal is called "topologically protected" when it is fixed by an integer, or by a two-valued index, that keeps its value under every smooth change of the crystal that keeps its protecting symmetry and keeps its energy gap open. In a topological insulator the conduction at the boundary is a property of this kind. Which kinds of protection a crystal can have depends on which discrete symmetries it keeps, and the tenfold classification of Part N-II sorts every free-fermion system by those symmetries.

N-002 Definition: Symmetry Class. That a symmetry class, in the classification of Altland and Zirnbauer, is a class of single-particle (free-fermion) Hamiltonians H sorted by three discrete symmetries: time reversal, written T, an antiunitary operator that commutes with H; particle-hole symmetry, written C, an antiunitary operator that anticommutes with H, and so maps each state of energy E to a state of energy −E; and chiral symmetry, written S, a unitary operator that anticommutes with H. An antiunitary operator is one that preserves the size of the overlap of two states and turns every complex phase the other way. That S is the product of T and C (S = TC, up to a phase), so that S is present whenever both T and C are present, S is absent whenever exactly one of T and C is present, and S may be present or absent when neither T nor C is present. That each combination of these symmetries, present or absent, and, for T and C, with the value of its square when present, defines one symmetry class. That the particle-hole symmetry C of this classification is a property of a single-particle Hamiltonian (for example, the Bogoliubov-de Gennes Hamiltonian of a superconductor, which describes particles and holes together); it is a different operation from the charge conjugation C of quantum field theory in N-003, and the two share only a letter.

Sources: Altland and Zirnbauer (1997); Schnyder, Ryu, Furusaki and Ludwig (2008). Standing: a definition.

> ***What "discrete symmetry" means.***
>
> A symmetry of a physical system is a transformation that leaves the system's fundamental description unchanged. Some symmetries are continuous, such as rotation by any angle and translation by any distance, and these give the familiar conservation laws (conservation of angular momentum, conservation of linear momentum). Discrete symmetries are all-or-nothing transformations, like flipping a coin rather than turning a dial. Time reversal asks: is the description unchanged when the direction of time is reversed? Particle-hole symmetry, in the sense of the classification,

> asks: is the description unchanged when every occupied state is exchanged with an empty one and every energy changes sign? Each question has a yes-or-no answer for a given system.

N-003 The Three Reversals. That a measurement has three discrete labels that can be reversed, and that the three reversals are charge conjugation, parity and time reversal. That charge conjugation, written C, exchanges a particle with its antiparticle: every internal quantum number changes sign, while the spin and the momentum are unchanged. That parity, written P, exchanges the two senses of a turning with the direction of the order of time kept; in three spatial dimensions P reverses the whole of space, and no axis is chosen. That time reversal, written T, reverses the order of time: it exchanges initial and final states and turns every phase the other way. That P reverses three coordinates and has determinant $(-1)^3 = -1$ on space, T reverses one coordinate and has determinant -1, and PT reverses four coordinates and has determinant $+1$, so that P and T are improper transformations and PT is a proper one (a transformation is proper when its determinant is $+1$, and improper when it is -1).

Sources: *The Branches of the Kinds*, Chapter 3, Section I. Standing: a definition.

N-004 The Three Kinds of Number. That the three reversals are paired with the three kinds of number, the three finite-dimensional associative division algebras over the real numbers: $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{H}$. That C, the real structure, exchanges each state with its conjugate and squares to $+1$, and so marks the real numbers. That T is antilinear and squares to -1 on the states of content it acts on, and so marks the quaternions. That P leaves phases alone, and so marks the complex numbers. That the pairing is made by Dyson's threefold way: an antilinear symmetry squaring to $+1$ marks the real type, an antilinear symmetry squaring to -1 marks the quaternionic type, and the complex type is the type for which no such symmetry exists. That the real numbers and the quaternions are conjugate

($\mathbb{H} \otimes \mathbb{H}$ is the algebra of real four-by-four matrices), and that the complex numbers stand between them; P is C and T together, the composite that leaves the phases alone because each of the other two turns them round.

Sources: *The Branches of the Kinds*, Chapter 3, Section I; *The Story of the Particles*, Level 9, Section III, and note 25; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 4, Section VII; Dyson (1962). Standing: a theorem (Dyson's threefold way, and $\mathbb{H} \otimes \mathbb{H} \cong \mathrm{M}_4(\mathbb{R})$); an identification (the pairing of each reversal with its kind of number, by the type of its square).

N-005 The Two Placements of Each Reversal. That each reversal has two placements at the three positions of comparison, the act of comparing (the immanent), the thing compared (the omniscient) and the thing compared against (the transcendent), stated here in the labeling of *The Story of the Particles* and of *The Kinds of Particle*, in the focus of comparison as relation. That by the kind of number it marks, C (the real numbers) stands at the omniscient, T (the quaternions) at the transcendent, and P (the complex numbers) at the immanent. That by the measure of context it reverses, C (which reverses possibility) stands at the transcendent, T (which reverses time) at the immanent, and P (which reverses space) at the omniscient. That the two placements are one step apart round the directed cycle T → I → O → T of Axiom II, the axiom that orders the three positions: in each case the kind of number sits one position before the measure. That C stands at both members of the conjugate pair, the omniscient by kind and the transcendent by measure, which is what an exchange of the two terms of a state does; *Non-Commutative Geometry and the Immanent Metaphysics* names the real structure J, which on states is charge conjugation, "omniscient by type and transcendent by what it reverses" (Chapter 4, Section VII). That the three rules of N-007 use the placement by kind of number.

Sources: *The Branches of the Kinds*, Chapter 3, Section II; *The Story of the Particles*, note 25; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 4, Section VII; *The Concept Sequence*, Chapters 14 and 15. Standing: an identification.

N-006 The Three Reversals as a Triple of Axiom III. That the three reversals form a triple in the sense of Axiom III ("The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable"). That they are distinct: each is paired with one kind of number, and no two are paired with the same kind. That they are inseparable: each is the other two together; each alone, and each pair, is broken in nature, and only the three taken at once, CPT, is exact. That they are non-interchangeable: the real, complex and quaternionic types have different signatures under the squaring map. That CPT is an exact symmetry of every structure with the five properties of the CPT theorem (Lorentz invariance, locality, unitarity, a Hamiltonian bounded below, and the connection of spin and statistics), and that the Standard Model is such a structure; a particle and its antiparticle therefore have equal mass, equal lifetime, and equal and opposite magnetic moments. That measurement finds CPT exact at every precision reached: the neutral kaon mass difference is below 6×10^{-19} of the kaon mass (Particle Data Group 2026), the charge-to-mass ratios of the proton and the antiproton agree to about 16 parts per trillion (BASE Collaboration 2022), and the 1S-2S transition of antihydrogen agrees with that of hydrogen to a few parts in 10^{12} (ALPHA Collaboration 2018).

Sources: *The Branches of the Kinds*, Chapter 3, Sections I and III (Box 3.4); *The Story of the Particles*, Level 4, Section II; Lüders (1954); Pauli (1955); Jost (1957); Particle Data Group (2026); BASE Collaboration (2022); ALPHA Collaboration (2018); *An Immanent Metaphysics*, aphorism 1.3-1 (Axiom III). Standing: an identification (the triple of Axiom III); a theorem (the CPT theorem of Lüders, Pauli and Jost, for structures with the five properties); a measured agreement (CPT found exact at every precision reached).

N-007 The Three Rules. That three rules govern how the three reversals relate to the three ways of force: the strong way (SU(3), color), the weak way (SU(2), the doublet) and the phase way ($U(1)_Y$, hypercharge; electromagnetism after the Higgs condensate, the uniform value the Higgs field takes throughout space). That in the labeling of *The Story of the*

Particles, in the focus of comparison as relation, color stands at the omniscient, the weak way at the transcendent and the phase way at the immanent, and that each way is bound by kind of number to one reversal: color to C (real), the weak way to T (quaternionic), the phase way to P (complex). That Rule 1 states that each way keeps the reversal of its own kind of number: color keeps C, the weak way keeps T, and the phase way, as electromagnetism after the condensate, keeps P. That Rule 2 states that a way that breaks a reversal breaks exactly the two reversals not of its own kind, together; the count of two is the CPT theorem applied to each term, since a term even under CPT breaks none or exactly two of the three. That Rule 3 states that only the weak way breaks a reversal by its form (its coupling), because only its act carries a state across the partition between content and context; color and the phase way break none by their form. That the Incommensuration Theorem's requirement limits what is possible: a way whose act stays wholly content breaks no reversal by its form, and for the weak way, whose act carries a state across the partition, a difference between a process and its mirror-and-charge-conjugate is not excluded; the one phase of the quark mixing that three generations allow (the CKM phase, the irremovable phase of the quark mass matrices) is how that remaining possibility is realized, so that the violation of CP and T found in weak transitions is that phase, read through the weak transitions (*The Story of the Particles*, Level 7, Section III). That the computed signs of the standard terms agree with the three rules: the weak coupling is odd under C and P and even under T; the topological term of the color field is even under C and odd under P and T, and its own coefficient is zero; and the two terms that would break their way's own kind (the phase way's $F\tilde{F}$ and the weak way's topological term) are exactly the two terms with no physical effect.

Sources: *The Branches of the Kinds*, Chapter 3, Sections II to V (Boxes 3.1 to 3.7); *The Story of the Particles*, Level 7, Section III; Kobayashi and Maskawa (1973); Jarlskog (1985); *The Concept Sequence*, Chapter 18. Standing: derived (Rule 1, from the binding of each kind of number to its reversal and the computation of the commuting maps on each carrier); derived (Rule 2, from the CPT theor-

em); derived (Rule 3, from the law of symmetry and the Incommensuration Theorem); derived (the strong force's own CP angle, zero, from Rule 3); a measured agreement (the phase from the quark masses in the strong CP angle θ, carried as zero, with θ below about 10^{-10}).

N-II. The Tenfold Classification

N-008 Three Values for Each Antiunitary Symmetry. That each of the two antiunitary symmetries of the classification, T and C, has exactly three possible states for a given system: absent; present with square $+1$; present with square -1. That this is a theorem of mathematics: if an antiunitary operator A satisfies $A^2 = \lambda \cdot 1$ with $|\lambda| = 1$, then A commutes with A^2, and antilinearity gives $A\lambda = \bar{\lambda}A$, so $\lambda = \bar{\lambda}$ and $\lambda = +1$ or $\lambda = -1$. That the case of square -1 is the case of Kramers' theorem: time reversal on a particle of half-integer spin squares to -1, and every state then has a partner of the same energy.

Sources: Wigner (1932); Kramers (1930); Altland and Zirnbauer (1997). Standing: a theorem.

> ***Why three states.***
>
> Time reversal and particle-hole symmetry are antiunitary operators: they transform the quantum state and also turn every complex phase the other way. When such an operator is applied twice, the result is a multiple of the identity, and the antiunitary character forces that multiple to be $+1$ or -1. So there are two ways to be present and one way to be absent, three states where a plain present-or-absent count would give two.

N-009 The Tenfold Count. That the number of symmetry classes is exactly ten. That the two antiunitary symmetries T and C, each with three states (N-008), give three times three, nine, combinations. That in the one

combination where both T and C are absent, the chiral symmetry S may be present or absent, which gives two classes in place of one. That the total is therefore nine plus one, ten.

Sources: Altland and Zirnbauer (1997); Schnyder, Ryu, Furusaki and Ludwig (2008). Standing: a theorem (the count of Altland and Zirnbauer).

N-010 Definition: The Tenfold Way. That the tenfold way, the classification of Altland and Zirnbauer, is the classification of free-fermion quantum systems into the ten symmetry classes of N-009. That two of the ten are called the complex classes: the classes in which both T and C are absent (class A, with no symmetry, and class AIII, with chiral symmetry only). That the other eight are called the real classes: the classes in which at least one of T and C is present. That four of the real classes have exactly one antiunitary symmetry (AI, AII, D and C) and four have both (BDI, DIII, CII and CI). That this classification governs which topologically protected boundary states are possible in each class and in each spatial dimension.

Sources: Altland and Zirnbauer (1997); Schnyder, Ryu, Furusaki and Ludwig (2008); Kitaev (2009). Standing: a definition.

> ***The ten classes by name.***
>
> Each class is written with its values of (T^2, C^2, S^2), where 0 means the symmetry is absent. The two complex classes: A (0, 0, 0) and AIII (0, 0, 1). The four real classes with one antiunitary symmetry: AI (+1, 0, 0), AII (−1, 0, 0), D (0, +1, 0) and C (0, −1, 0). The four real classes with both: BDI (+1, +1, 1), DIII (−1, +1, 1), CII (−1, −1, 1) and CI (+1, −1, 1). The names are those of Élie Cartan's classification of the symmetric spaces, which Altland and Zirnbauer (1997) use.

N-011 Dyson's Threefold Way Within the Tenfold Way. That the three classes with no particle-hole symmetry and no chiral symmetry, AI ($T^2 = +1$), A (no T) and AII ($T^2 = -1$), are the three Wigner-Dyson classes, and that they are Dyson's threefold way: the real type, the complex type and the quaternionic type. That the tenfold way extends Dys-

on's three classes by adding the particle-hole and chiral symmetries. That Dyson's threefold way is the pairing by which *The Branches of the Kinds* and *The Story of the Particles* join the three reversals to the three kinds of number (N-004), and that by kind of number, in the labeling of *The Story of the Particles* and the focus of comparison as relation, those books place the real type at the omniscient, the complex type at the immanent and the quaternionic type at the transcendent (N-005).

Sources: Dyson (1962); Altland and Zirnbauer (1997); *The Branches of the Kinds*, Chapter 3, Sections I and II; *The Story of the Particles*, note 25. Standing: a theorem (the three Wigner-Dyson classes are Dyson's three types); an identification (the placement of the three types at the three positions, made in the books cited).

N-012 The Symmetries of the Classification and the Three Reversals. That the time reversal T of the classification is the time reversal of N-003 acting on single-particle states: it squares to -1 on particles of half-integer spin, as the T of *The Branches of the Kinds* does on states of content, and it squares to $+1$ on spinless particles. That the particle-hole symmetry C of the classification is a constraint on a single-particle Hamiltonian that describes particles and holes together; the charge conjugation C of N-003 is another operation, which exchanges a particle with its antiparticle in quantum field theory and squares to $+1$ on a single state. That the chiral symmetry S of the classification is unitary, and that the product of two antiunitary operators is always unitary; in the same way the P of *The Branches of the Kinds*, the composite of C and T, is linear and leaves the phases alone (N-004). That the chiral symmetry S of the classification acts on the internal (for example, sublattice) structure of a single-particle Hamiltonian and leaves space as it is, so it is not parity.

Sources: Altland and Zirnbauer (1997); Schnyder, Ryu, Furusaki and Ludwig (2008); *The Branches of the Kinds*, Chapter 3, Section I; *The Story of the Particles*, note 25. Standing: a definition; a theorem (a product of two antiunitary operators is unitary).

N-III. Bott Periodicity

N-013 Definition: Bott Periodicity. That Bott periodicity is the theorem that the stable homotopy groups of the classical groups repeat with period two for the unitary groups and with period eight for the orthogonal and symplectic groups (Bott 1959). That the same periodicity appears in the Clifford algebras, the algebras generated by elements whose squares are fixed numbers. The real Clifford algebras with n generators squaring to -1 are, for n = 0 to 7, matrix algebras over $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{H} \oplus \mathbb{H}$, $\mathbb{H}$, $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{R} \oplus \mathbb{R}$ in turn, and the sequence then repeats with period eight (Atiyah, Bott and Shapiro 1964). That the complex Clifford algebras repeat with period two. That in the tenfold way, the invariant that protects boundary states in a given class depends on the spatial dimension, and that this dependence repeats with period two in the dimension for the two complex classes and with period eight for the eight real classes (Schnyder, Ryu, Furusaki and Ludwig 2008; Kitaev 2009).

Sources: Bott (1959); Atiyah, Bott and Shapiro (1964); Schnyder, Ryu, Furusaki and Ludwig (2008); Kitaev (2009); *The Story of the Particles*, note 25. Standing: a theorem (Bott periodicity, and the Clifford sequence of Atiyah, Bott and Shapiro); a theorem (the periodic table of topological insulators and superconductors, derived for free-fermion band structures by Schnyder, Ryu, Furusaki and Ludwig and by Kitaev).

> ***Why periodicity matters.***
>
> The tenfold way says which symmetry classes exist. Bott periodicity says what protection each class can have in each spatial dimension. Class AII has no protecting invariant in one dimension and a two-valued invariant in two dimensions and in three; its two- and three-dimensional members are the quantum spin Hall insulator and the three-dimensional topological insulator. Class A has an integer invariant in two dimensions, the in-

> variant of the integer quantum Hall effect, and none in one or in three. The table of protections repeats when the dimension rises by eight for the real classes, and by two for the complex classes.

N-014 The Clifford Sequence as a Directed Cycle Through the Three Kinds. That *The Story of the Particles* identifies the eight-step sequence of real Clifford algebras of N-013 as "a directed cycle through the three kinds in which every passage between the real and the quaternionic goes through the complex, as Axiom I places the immanent between the two conjugate positions" (note 25). That the fact on which the identification rests is a theorem of mathematics: in the sequence $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{H} \oplus \mathbb{H}$, $\mathbb{H}$, $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R} \oplus \mathbb{R}$, every step from a real algebra to a quaternionic one, or from a quaternionic one to a real one, passes through a complex algebra. That in one period of the sequence the real kind holds three steps (0, 6 and 7) and the quaternionic kind three (2, 3 and 4), each run with its middle step doubled ($\mathbb{H} \oplus \mathbb{H}$ at step 3, and $\mathbb{R} \oplus \mathbb{R}$ at step 7, which lies between steps 6 and 0 when the sequence is read as a cycle), and the complex kind two steps (1 and 5), one in each passage between the real and the quaternionic: three, three and two, eight steps in all. That the KO-dimension 6 at which the finite algebra of the Standard Model is realized is a real position of that cycle, where the real structure squares to +1, which is the character of C (N-004); the KO-dimension is the number modulo 8 that the signs of the real structure J, the grading γ and the Dirac operator D of a spectral triple fix, and *Non-Commutative Geometry and the Immanent Metaphysics* proves that for a real even finite spectral triple the conditions of its postulate P-KO ($J^2 = +1$, $J\gamma = -\gamma J$, $JD = DJ$) hold if and only if the KO-dimension is 6 mod 8 (Proposition 4.2).

Sources: *The Story of the Particles*, note 25; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 4, Section VII (Postulate P-KO, Proposition 4.2); Atiyah, Bott and Shapiro (1964); Connes (2006); Barrett (2007). Standing: an identification (the sequence read as a directed cycle through the three kinds,

The Story of the Particles, note 25); a theorem (the Clifford sequence and the count of its steps by kind, and Proposition 4.2 of *Non-Commutative Geometry and the Immanent Metaphysics*); an identification (the postulate P-KO).

N-015 The Clifford Algebras of Spacetime. That a Clifford algebra of spacetime is generated by four elements, one for each direction of spacetime, which anticommute with one another and each square to $+1$ or to -1 by the sign the metric gives its direction. That the Clifford algebra of spacetime in which the three directions of space square to $+1$ and the direction of time to -1 is the algebra of real four-by-four matrices, $M_4(\mathbb{R})$, and that with the signs exchanged, the three directions of space squaring to -1 and the direction of time to $+1$, it is the algebra of quaternionic two-by-two matrices, $M_2(\mathbb{H})$. That both become the algebra of complex four-by-four matrices, $M_4(\mathbb{C})$, once complex numbers are allowed: $M_4(\mathbb{R}) \otimes \mathbb{C} \cong M_4(\mathbb{C})$, and $M_2(\mathbb{H}) \otimes \mathbb{C} \cong M_4(\mathbb{C})$, since $\mathbb{H} \otimes \mathbb{C} \cong M_2(\mathbb{C})$. That exchanging which of time and space carries the sign $+1$ therefore exchanges the real and the quaternionic, the two kinds that N-004 pairs as conjugate, while the complex is common to both, as the complex stands in every passage between the real and the quaternionic in the Clifford sequence of N-014.

Sources: Wernli (2019), Example 1.5.18, and Section 1.4.2 (the complexification of a Clifford algebra); Lawson and Michelsohn (1989), Chapter I, Section 4 (the classification tables of real Clifford algebras); N-004 and N-014 of this chapter. Standing: a theorem (the two isomorphisms and their complexification); the pairing of the real and the quaternionic as conjugate kinds is the identification of entry N-004.

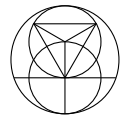
Summary

In this chapter I set two accounts of the discrete symmetries side by side and show the one place where the two accounts meet. The first of the two accounts is the account that *The Branches of the Kinds* gives. Charge conjugation, parity and time reversal reverse the three discrete labels of a meas-

urement, and each is paired with one of the three kinds of number by Dyson's threefold way: C, which squares to +1, with the real numbers; T, which squares to −1 on content, with the quaternions; and P, which leaves phases alone, with the complex numbers. The three reversals are a triple in the sense of Axiom III, and each has two placements at the positions of comparison, one by the kind of number it marks and one by the measure of context it reverses, a step apart round the cycle of Axiom II. Three rules, derived in that book, say which force keeps and which breaks each reversal: each way keeps the reversal of its own kind; a way that breaks, breaks exactly the other two, by the CPT theorem; and only the weak way breaks by its form. The CP violation found in weak transitions is, as *The Story of the Particles* states it, the one phase of quark mixing that three generations allow, realizing the one possibility the Incommensuration Theorem leaves open, and the strong force's own CP angle is zero.

The second account is standard physics: the tenfold classification of Altland and Zirnbauer, which sorts every free-fermion system by its time-reversal, particle-hole and chiral symmetries, and Bott periodicity, a theorem, which gives each class its protected boundary states dimension by dimension. The two accounts meet in the three Wigner-Dyson classes, which are Dyson's threefold way, the same pairing the first account uses for the reversals; the particle-hole and chiral symmetries of the classification are operations of their own, distinct from the charge conjugation and the parity of the reversals. On that meeting *The Story of the Particles* makes one identification: the eight-step sequence of real Clifford algebras is a directed cycle through the three kinds, in which every passage between the real and the quaternionic goes through the complex, and the KO-dimension 6 of the finite algebra of the Standard Model stands at a real position of that cycle, where the real structure has the character of C. The cycle holds three steps of the real kind, three of the quaternionic and two of the complex, and the Clifford algebras of spacetime show the same pair of

kinds: exchanging which of time and space carries the sign +1 exchanges the real algebra $M_4(\mathbb{R})$ with the quaternionic $M_2(\mathbb{H})$, and the complex algebra $M_4(\mathbb{C})$ is common to both.



Chapter O

Precision Phenomenology

CHAPTER OVERVIEW

This chapter sets the values of the account beside their measurements: the Higgs boson at the Large Hadron Collider, rare processes and the mixing of flavors, glueballs, the Dirac neutrino and the neutrino masses, and the predictions with the measurements that test them. Each value carries its source, its pull (its distance from measurement, the stated value less the measured value divided by the combined uncertainty, in standard deviations, written σ) and its standing, and each prediction carries the measurement that decides it and the step of the chain on which that measurement bears. In this chapter I derive nothing anew: each result is its owner's, chiefly *The Measures of the Kinds*, *The Story of the Particles*, *The Branches of the Kinds* (the Dirac neutrino and the absence of the seesaw) and *Non-Commutative Geometry and the Immanent Metaphysics* (the Higgs doublet the only scalar). Part O-IV gathers the predictions in two ledgers, one of the kinds and one of the values.

The chapter rests on these chapters of *The Concept Sequence*: Chapter 29, The Couplings: Rank, Running, Weights and Crossings; Chapter 30, The Spectral Action and the Form of the Standard Model (with the standings of the twenty-six numbers); Chapter 35, The Lepton Sector; Chapter 37, The Gauge Bosons; Chapter 38, The Higgs Boson; Chapter 40, The Mixing Matrices; Chapter 41, The Completeness of the Kinds; Chapter 42, Proton Stability; Chapter 45, Further Predictions; and Chapter 46, Predictions and the Measurements That Test Them. It also rests on two chapters of

this book: Chapter B, The Mass Spectrum (the top's coupling and the condensate's whole, the step formula and its constants, the nine charged masses, the Higgs self-coupling and the Higgs mass, quark and lepton mixing, the neutrino masses and the composites), and Chapter E, The Higgs Potential (its form, which the spectral action fixes, and its self-coupling).

The values are stated in two accounts of what measurement supplies, as *The Measures of the Kinds* sets them out. In the working account the measured inputs are the electron's mass (the unit), the strength of light α_{em}, the strength of color α_s and the vacuum value v (the value the Higgs field keeps throughout the vacuum), with the two offsets of the charged ladders. In the closure account the step of the mass ladder is written as a product, and the electron alone carries the chain to v, the top and the Higgs mass (Chapter B, entry B-014, which sets the two accounts side by side). Every value below names its account where the two differ.

O-I. The Higgs Boson at the Large Hadron Collider

O-001 Decay Width. That the decay width of an unstable particle is the rate at which it decays, measured in units of energy. That a wider particle decays faster: the decay width is inversely proportional to the particle's lifetime, with the reduced Planck constant as the factor of conversion (about 6.58×10^{-25} GeV s; CODATA 2022). That the total width is the sum of the widths of the channels, one width for each channel of decay open to the particle. That the width of each channel of the Higgs boson follows from its mass, $m_H = 125.17$ GeV (Chapter B, entry B-021, which gives the Higgs mass with its loop term), with the Standard Model's branching ratios, because the account's Higgs field has the Standard Model's form and couplings and the list of kinds adds no particle into which the Higgs boson could decay.

Sources: *The Measures of the Kinds*, Chapter 13, Section VI; *The Story of the Particles*, Level 9, Section IV (the closed list of kinds). Standing: a definition (the width); derived (the Higgs boson's widths are the Standard Model's at its mass).

> ***Decay width as a ball rolling off a hilltop.***
>
> Imagine a ball balanced on a narrow hilltop. The steeper the slopes on either side, the faster the ball rolls away. A large width belongs to a state that leaves quickly, like a ball on a steep hill; a small width belongs to a state that lingers, like a ball on a gentle one. The Higgs boson decays quickly, and far more slowly than the Z boson, whose measured width is 2.4955 ± 0.0023 GeV (Particle Data Group 2024).

O-002 Branching Ratio. That a branching ratio is the fraction of decays that go into one particular channel. That where a particle has several channels of decay, the branching ratios are the probabilities of the channels, and they sum to one. That the Higgs boson's largest branching ratio is to a bottom quark and its antiquark, and that its decays to a pair of W bosons and to a pair of Z bosons proceed with one of the two bosons off-shell (virtual, carrying less than its own mass), because the Higgs mass lies below twice the W mass and twice the Z mass. That in this account the branching ratios are the Standard Model's at the measured inputs: the account fixes the form of the Higgs sector (one doublet, the field on the crossed edge between the two sides of a doublet) and adds no channel of decay.

Sources: *The Measures of the Kinds*, Chapter 13, Section VI; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Proposition 19.10 (the Higgs field is the only scalar). Standing: a definition (the branching ratio); derived (the Higgs sector's one doublet, and no added channel).

Decays below threshold.

The Higgs boson is lighter than a pair of W bosons or a pair of Z bosons. It still decays into these channels, because one of the two bosons is produced off-shell: a virtual particle that at once decays into a pair of fermions. The computation integrates over the invariant mass of the off-shell boson.

O-003 Loop-Induced Decay. That a loop-induced decay is a decay that proceeds through virtual particles circulating in a closed loop, with no contribution at tree level (the simplest diagram, before any correction from virtual states). That the Higgs boson decays to two photons and to two gluons through loop-induced processes, because the photon and the gluons carry no mass and have no coupling to the Higgs condensate. That the decay to two gluons proceeds chiefly through a loop of top quarks, the top being the quark whose coupling to the condensate is one (Chapter B, entry B-001, which defines that coupling), and the decay to two photons through a loop of W bosons and a loop of top quarks that interfere destructively, which makes the two-photon channel a sensitive probe of those couplings. That the two-photon channel, though rare, gives a clean experimental signature, a pair of photons.

Sources: *The Measures of the Kinds*, Chapter 13, Section III; *The Story of the Particles*, Level 6, Section V (the top's coupling of one). Standing: a definition (the loop-induced decay); derived (the top's coupling of one).

O-004 The Masses of the W and the Z. That one Higgs doublet gives $m_W = m_Z \cos \theta_W$ at tree level, with θ_W the weak angle. That at the ideals of the account (the values at the structural shares) the W and the Z take one quarter of v^2, and the carriers' law states their two masses together as that quarter with one loop unit taken off: $m_W^2 + m_Z^2 = v^2(1/4 - 1/(16\pi^2))$. That with the measured v, 246.21967 GeV, and the measured Z mass, 91.1880 $\pm$ 0.0020 GeV (Particle Data Group 2024), the law gives m_W = 80.3547 $\pm$ 0.0023 GeV, at -0.97σ from the measured 80.3625 $\pm$ 0.0077

GeV (Particle Data Group 2026). That the carriers' law is selected by its agreement with the measured masses, so that agreement asks for its reason and is not an independent test of the law.

Sources: *The Measures of the Kinds*, Chapter 13, Section V (Box V-3); *The Strengths of the Kinds*, Chapter 4, Section II. Standing: derived ($m_W = m_Z \cos \theta_W$ at tree level); a measured agreement (the carriers' law).

O-005 Forward-Backward Asymmetry. That a forward-backward asymmetry measures how unevenly fermions emerge forward and backward of the incoming electron's direction when an electron and a positron annihilate at the Z pole, the peak of the Z resonance (about 91.19 GeV). That at tree level every pole asymmetry is a function of $\sin^2\theta_W$ alone, because it is a ratio of the Z's vector and axial couplings, and the overall normalization (the ρ parameter) cancels. That light's relation at the proton's mass (Chapter C, entry C-019, light's inverse strength at the proton's mass is 3π times the gap between the two electroweak inverse strengths at the end of the span) gives $\sin^2\theta_W(M_Z) = 0.23127$ to 0.23130, against the measured 0.23122 ± 0.00006 (Particle Data Group 2026), at $+0.83\sigma$ to $+1.33\sigma$. That the Standard Model's formulae evaluated at 0.23127 give the leptonic forward-backward asymmetry 0.01667 (measured 0.0171 ± 0.0010, -0.4σ), the bottom-quark asymmetry 0.10462 (measured 0.0992 ± 0.0016, $+3.4\sigma$), the charm-quark asymmetry 0.07474 (measured 0.0707 ± 0.0035, $+1.2\sigma$) and the leptonic asymmetry parameter 0.14908 (measured 0.1513 ± 0.0021, -1.1σ), and that five of the seven pole observables lie within 2σ. That the bottom-quark asymmetry at $+3.4\sigma$ is the excess that the Standard Model also has, and that R^0_b (the Z's width to bottom pairs over its hadronic width) at $+2.0\sigma$ comes from the incomplete correction at the vertex where the Z meets the bottom pair (the full vertex gives 0.2170, $+1.0\sigma$). That two extractions of the angle from single asymmetries, 0.23223 ± 0.00029 from the bottom asymmetry and 0.23098 ± 0.00026 from the lepton asymmetry measured at SLD (the SLAC Large Detector), differ from each other by 3.2σ.

Sources: *The Strengths of the Kinds*, Chapter 4, Sections IV and V (Box IV-1 and Table V-1); measured values: the combination of the LEP and SLD experiments (2006), LEP being the Large Electron-Positron Collider. Standing: a measured agreement (the weak angle at the Z mass, which restates the place of the first crossing of the couplings).

O-006 The One Mismatch and the Vacuum Value. That the one mismatch of the particle side, the residue left when a counted measure is read by a continuous one, is $\varepsilon = 11/(80\pi^2) = 0.013932$, with $11 = c_1^2 + n_C$ and $80 = N^2H$ in the kernel triangle (legs 3 and 4, hypotenuse 5, with $c_1 = 3$ and $n_C = 2$), a pure number of the content instance of the internal space, which has no size (Chapter B, entry B-009, the step factor and the one mismatch). That every small leftover between a counted measure and its continuous reading is ε times a whole-number ratio fixed by the kind of reading, a ratio of one at first order for readings counted in halvings (a halving being a factor of two in mass). That the vacuum value is measured from the Fermi constant, the strength of the weak interaction at low energy: $v = 246.21967 \pm 0.00006$ GeV, to 0.24 parts per million, and it is a measured input of the working account. That in the closure account the electron alone, with the step on the interval from the tau to $\sqrt{2}\,v$ written as a product, gives $v = 246.21966$ GeV, at -0.16σ, carrying 22.0 bits (binary digits of the measured value that the output gets right). That this product rests on the rule that orders multiply across two domains and add within one, which *The Measures of the Kinds* argues from the triple of addition, multiplication and exponentiation (Chapter 7, Section IX).

Sources: *The Measures of the Kinds*, Chapter 7, Section IX; Chapter 13, Sections I, VIII and X (Table X-1); *The Strengths of the Kinds*, Chapter 5. Standing: a measured input (v, in the working account); derived, given a stated identification (v in the closure account: the factor that each of the two domains supplies to the product step, with the domain rule itself argued); a measured agreement (the closure account's v against the measured v, -0.16σ).

> ***Four leftovers, one mismatch.***
>
> Four leftovers are measured at four places, in units of ε: 0.922ε on the step from one generation to the next, 0.916ε on the top quark's shortfall below $v/\sqrt{2}$, 0.85ε at the light quarks' floor, and 0.948ε on the span from the electron to the reduced Planck mass. They rest on different measured quantities (the rungs of the charged ladders; the top and v; the light quarks and the proton; the electron and Newton's constant), and all four lie within 0.15ε of the ε that the first order gives: independent routes meeting on one number.

O-007 The Higgs Self-Coupling. That the Higgs boson interacts with itself: two Higgs bosons can scatter into two Higgs bosons, and a single Higgs boson can produce a pair of Higgs bosons through the self-coupling λ, the coefficient of the quartic term $\lambda(\Phi^\dagger\Phi)^2$ of the Higgs potential. That the spectral action (the action computed from the spectrum of the geometry's Dirac operator) fixes the form of the potential and not its values. That the self-coupling at tree level is $\lambda = 1/8$, by the form route $\lambda = a^2/2$ with $a = 1/2$ the doublet's half (the weak isospin of each side of the doublet), at every number of colors, and by the share route at three colors; at that value the Higgs mass at tree level is $v/2 = 123.11$ GeV, -18.4σ from the measured mass. That the measured on-shell self-coupling, $\lambda = m_H^2/(2v^2) = 0.12914 \pm 0.00023$, exceeds 1/8 by $(0.980 \pm 0.054) \times 1/(24\pi^2)$, and that with $\lambda = 1/8 + 1/(24\pi^2) = 0.129222$ and the measured v the Higgs mass is 125.1715 GeV, at $+0.38\sigma$ from the measured 125.13 ± 0.11 GeV (Particle Data Group 2026). That the measurements *The Measures of the Kinds* names as deciding are of the Higgs mass: to ± 0.03 GeV it decides the relation reading of the top ($m_t^2 + m_H^2 = 3v^2/4$) and the midpoint ($m_H^2 = m_Z m_t$) against the mass formula's 125.17 GeV, and to about ± 0.005 GeV it separates the forms of the second order. That the production of Higgs pairs measures the self-coupling directly, and so tests the form of the potential that the spectral action fixes.

Sources: *The Measures of the Kinds*, Chapter 13, Sections IV, VI and IX (Boxes IV-1, IV-2 and VI-1); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13; Chapter E, Part E-I (the form of the Higgs potential); Chapter B, entry B-021 (the Higgs self-coupling and the Higgs mass). Standing: derived, given a stated identification ($\lambda = 1/8$ at tree level: $\lambda = a^2/2$, the self-coupling identified with half the square of the doublet's half); a measured agreement (the Higgs mass with its loop term).

O-008 The Higgs Sector: Standings. That the values of the Higgs sector stand as follows. The top's coupling $y_t = 1$: derived (the identity of the up-type sector that *The Story of the Particles* argues), and derived, given the stated identification $y_t = 2a$ (the top as the member at coupling one); its scheme is a setting, fixed by stating it. The self-coupling 1/8 at tree level: derived, given the stated identification $\lambda = a^2/2$. The Higgs mass 125.1715 GeV with its loop term: a measured agreement, $+0.38\sigma$. The vacuum value: a measured input in the working account; 246.21966 GeV in the closure account, -0.16σ. The masses of the W and the Z by the carriers' law: a measured agreement, the W at -0.97σ. The weak angle at the Z mass: a measured agreement; $\sin^2\theta_W = 3/8$ is the equality of the three traces, an identity of the definitions at the first crossing of the couplings, near 10^{13} GeV, and at no other scale. The pole observables: the Standard Model's formulae at the angle used, agreeing with measurement where the Standard Model agrees and disagreeing where it disagrees.

Sources: *The Measures of the Kinds*, Chapter 13, Sections II, III and IX, Tables X-1 and X-3; *The Strengths of the Kinds*, Chapter 3, Section IV, and Chapter 4, Section V. Standing: derived, derived given a stated identification, a measured input and a measured agreement, as stated for each value in the entry.

O-II. Rare Processes and Precision Flavor

O-009 Flavor-Changing Neutral Current. That a flavor-changing neutral current is a process in which a quark changes its flavor (from strange to down, or from bottom to strange, for example) and keeps its

electric charge. That the Standard Model gives such processes in loops only, with none at tree level, and that in the loops the contributions of the different quark generations cancel almost exactly (the Glashow-Iliopoulos-Maiani mechanism), leaving a small residual set by the differences of the quark masses. That in this account the list of kinds is closed (*The Concept Sequence*, Chapter 41, entry 41.10), so the loops of these processes contain only the kinds of the Standard Model, and their rates are Standard Model computations with the measured mixing elements as inputs. That the high-luminosity Large Hadron Collider, in its study of rare processes, will find heavier composites of the known kinds and no new kind; the measurement that decides is a confirmed departure from the Standard Model at short distance in these rates, which would bear on the completeness of the kinds and on the closure of the field content.

Sources: *The Measures of the Kinds*, Chapter 10, Section VIII; *The Story of the Particles*, Level 9, Section IV, and Coda, Section II; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Propositions 19.7 to 19.10. Standing: a definition (the process); derived (the closure of the kinds, and with it the rates as Standard Model computations).

O-010 B-Meson Oscillation and the Radiative B Decay. That the ratio of the oscillation frequencies of the two neutral B mesons, $\Delta m_s/\Delta m_d$, goes as $|V_{ts}V_{tb}|^2/|V_{td}V_{tb}|^2$ times hadronic factors (the V being elements of the quark mixing matrix), so it cancels the loop function of the top and most of the hadronic uncertainty; computed with the measured mixing elements it is 34.71, against the measured 35.05 (Heavy Flavor Averaging Group: $\Delta m_s = 17.766 \pm 0.006\ \text{ps}^{-1}$, $\Delta m_d = 0.5069 \pm 0.0019\ \text{ps}^{-1}$), 1.0 percent apart. That the branching ratio of $B \to X_s\gamma$ (a B meson decaying to a photon and strange hadrons), with the coefficient C_7 matched at the W mass and run to the bottom mass with the eight-operator anomalous dimension matrix (Buras, Misiak, Münz and Pokorski 1994), is 3.61×10^{-4}, at $+0.6\sigma$ of the measured $(3.49 \pm 0.19) \times 10^{-4}$ (Particle Data Group 2024); the Standard Model at next-to-next-to-leading order in the strong coupling, with the exact dependence on the charm quark's mass, gives $(3.54 \pm$

$0.14) \times 10^{-4}$ for photon energies above 1.6 GeV in the rest frame of the decaying meson (Czaja and others 2026). That the decay of the B_s meson to a muon pair is likewise a Standard Model rate, since the list of kinds adds nothing to the coefficients of the transition from a bottom quark to a strange quark and two leptons.

Sources: *The Measures of the Kinds*, Chapter 10, Section VIII, and Chapter 13, Table X-4 (the coefficients beyond the Standard Model are zero); *The Kinds of Particle*, Chapter 6, Section IV; Buras, Misiak, Münz and Pokorski (1994); Czaja and others (2026); Heavy Flavor Averaging Group (2026). Standing: derived, from measured inputs (Standard Model computations with the measured mixing elements).

O-011 The Golden Kaon Modes. That the decay of a charged kaon into a charged pion and a neutrino and antineutrino ($K^+ \to \pi^+ \nu\bar{\nu}$) is called the golden channel of kaon physics because its hadronic matrix element is fixed by isospin symmetry from the precisely measured rate of $K^+ \to \pi^0 e^+ \nu$, which makes the theoretical computation exceptionally clean. That scaled from the Standard Model with the inputs of the quarter-turn mass texture ($|V_{cb}| = 0.04212$, $\gamma = 63.7°$), a texture held as a hypothesis (Chapter B, entry B-024, the texture zero and the mass textures), its branching ratio is 8.33×10^{-11}, at -1.6σ of the NA62 measurement, $(13.0\ {}^{+3.3}_{-3.0}) \times 10^{-11}$ (2025); the Standard Model with the inputs $|V_{cb}| = 0.0407$ and $\gamma = 73.2°$ gives $(8.4 \pm 1.0) \times 10^{-11}$ (Buras, Buttazzo, Girrbach-Noe and Knegjens 2015). That the CP-violating mode $K_L \to \pi^0 \nu\bar{\nu}$ is governed by the imaginary part of $V_{td}V_{ts}^*$, and so by the Jarlskog invariant J, the one measure of CP violation in quark mixing that no choice of phases changes, which Chapter B, entry B-026, states with its standing.

Sources: *The Measures of the Kinds*, Chapter 10, Sections VII and VIII; Buras, Buttazzo, Girrbach-Noe and Knegjens (2015); NA62 Collaboration (2025). Standing: derived, from the inputs of the quarter-turn texture, which is held as a hypothesis (a scaling of the Standard Model's prediction).

O-012 B to K-Star Mu Mu. That the decay of a B meson into a K-star meson and a muon and antimuon is described by an effective Hamiltonian containing the Wilson coefficients C_9 (the coefficient of the vector current) and C_{10} (the coefficient of the axial current), which carry the physics of the loops at short distance. That with the list of kinds closed, no particle beyond the Standard Model's circulates in those loops, so the coefficients at short distance are the Standard Model's, and the contributions beyond it, ${C_9}^{NP}$ and ${C_{10}}^{NP}$, are zero. That the measurement that decides is a shift in a Wilson coefficient established at short distance, separated from the contributions of charm loops at long distance, which can mimic a shift in C_9; such a shift would bear on the completeness of the kinds.

Sources: *The Measures of the Kinds*, Chapter 13, Table X-4; *The Kinds of Particle*, Chapter 6, Section IV; *The Story of the Particles*, Level 9, Section IV. Standing: derived (from the completeness of the kinds).

O-013 Electric Dipole Moment and the Strong Angle. That an electric dipole moment is a separation of positive and negative charge within a particle; it violates both parity and time reversal, and so it is a sensitive probe of CP violation (the violation of the symmetry that exchanges matter with its mirror image). That the strong force's own CP angle is zero by structure: the strong force, the omniscient force in the frame of measurement of *The Story of the Particles*, preserves CP, and only the weak force breaks a reversal by its form. That the measured angle θ is the strong force's own angle plus the phase arg $\det(M_u M_d)$ from the quark masses, and that phase is carried as zero, a measured agreement. That the θ term gives the neutron an electric dipole moment of order $\theta \times 10^{-16}$ e cm, and the measured bound $|d_n| < 1.8 \times 10^{-26}$ e cm (Abel and others 2020) places θ below 1.8×10^{-10}. That with both parts of the angle at zero, a neutron electric dipole moment arises only from the phase of the quark mixing, whose leading contribution comes at three loops (Khriplovich and Zhitnitsky 1982); the measurement that decides is a neutron electric

dipole moment at a size the quark mixing cannot give, which would bear on the zero of the strong force's own angle and on the measured agreement of the quark-mass phase.

Sources: *The Branches of the Kinds*, Chapter 3, Section V (Boxes 3.6 and 3.7); *The Measures of the Kinds*, Chapter 13, Tables X-3 and X-4; *The Strengths of the Kinds*, Chapter 1, Section VII; Abel and others (2020); Khriplovich and Zhitnitsky (1982). Standing: derived (the strong force's own angle, zero); a measured agreement (the phase from the quark masses, carried as zero).

O-015 No Axion. That the conventional resolution of the strong CP problem is the Peccei-Quinn mechanism, which introduces a new symmetry and a particle, the axion, that relaxes θ to zero dynamically. That the kinds include no axion: an axion would be a scalar with no labels, a scalar field lives on a crossed edge, and a scalar with no labels would be a field on an edge with no sides, of which there is none. Since the kinds hold no scalar with no labels, there is no field for a Peccei-Quinn symmetry to act on and no pseudo-Goldstone axion. That the absence of the axion is a statement about the particle content and holds for any value of the strong angle, and that the structure fixes the strong force's own angle at zero without calling on an axion (entry O-013). That the account predicts no axion found as a particle; the detection of one would bear on the closure of the list of kinds.

Sources: *The Branches of the Kinds*, Chapter 3, Section V, and Chapter 4, Section III; *The Story of the Particles*, Level 9, Section IV. Standing: derived.

O-III. Glueballs, the Dirac Neutrino, and the Neutrino Masses

O-016 Glueball. That a glueball is a colorless whole of gluons alone, with no quark: each gluon carries a color and an anticolor, and the gluons attract one another, so gluons with no quark among them can combine into a color-neutral whole. That glueballs carry no electric charge, no baryon number and no flavor, and are found only mixed with neutral states of a quark and an antiquark with the same quantum numbers, never pure.

That lattice computations of quantum chromodynamics (the theory of the strong force, computed on a grid of points) with no quarks place the lightest glueball, of spin 0 and parity +1, at $1730 \pm 50 \pm 80$ MeV (Morningstar and Peardon 1999), and that the state X(2370), at about 2370 MeV with spin zero and odd parity, is the first shown to be made mostly of gluons, matched with the lightest glueball of that spin and parity. That the strength of color is a measured input, $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ (Particle Data Group 2024), so the glueball masses are computations of quantum chromodynamics with a measured input, and glueballs, like every composite, add no kind.

Sources: *The Matter We Know*, Volume One, Chapter 2, Section IX; *The Story of the Particles*, Level 9, Section I; Morningstar and Peardon (1999); BESIII Collaboration (2024). Standing: derived, from a measured input (the glueball masses); derived (that composites add no kind).

O-017 No Seesaw. That in the conventional seesaw mechanism a heavy right-handed Majorana mass M_R suppresses the physical neutrino mass to $m_\nu = m_D{}^2/M_R$, where m_D is a Dirac mass of ordinary electroweak size; a large M_R, say 10^{14} GeV, makes the neutrino mass small, and because M_R is a free parameter the mechanism accounts for how neutrinos can be light without fixing how light. That in this account every neutrino mass is a Dirac mass, because no state of content is its own reversal: the Majorana mass is zero, $M_R = 0$, so the seesaw does not operate, there is no seesaw scale, and no heavy right-handed neutrino exists. That the size of the neutrino masses is carried by the two routes of *The Measures of the Kinds* (Chapter B, entry B-028, the neutrino masses on two routes), and their lightness is the smallness of their coupling to the Higgs field, about 3×10^{-13} for the heaviest: "a question of value of the same kind as the value of every other coupling". That in the construction of Chamseddine and Connes the Majorana matrix of the right-handed neutrinos would produce light masses by the seesaw; the two accounts differ in what is put in an admitted entry.

Sources: *The Branches of the Kinds*, Chapter 1, Sections V and VI; *The Measures of the Kinds*, Chapter 12, Section I (quoted). Standing: derived.

O-018 The Boundary Placement. That beside the two routes of the neutrino masses stands a third placement of the neutrino's ladder, the boundary placement, which is a placement and no third branch of the fork. That its curvature is $d_\nu = -n_C/c_1 = -2/3$, and its offset adds to the lepton offset a boundary power $p = N^2 + 1/H = 16 + 1/5 = 81/5$, whose value equals c_1^4/H exactly, by the identity $N^2H + 1 = c_1^4$ ($16 \times 5 + 1 = 81 = 3^4$), which holds only at $c_1 = 3$. That the power is converted into halvings by p $c_1/(2 \ln 2)$ with no factor s (the step factor of the mass ladder), a shift of 35.057 halvings below the tau. That four numbers for the count of halvings from the tau to the heaviest neutrino stand side by side: the neutrino fit gives 34.979 ± 0.006; the count route's $\Delta_{lep}\Delta_{dn}s = 34.982$ stands at $+0.5\sigma$; the kernel integer $35 = H(c_1^2 - n_C)$ at $+3.5\sigma$; and the boundary shift 35.057 at $+13.1\sigma$. That the texture of the lepton angles uses the mass ratios of this placement, as a premise of that texture (*The Measures of the Kinds*, Chapter 11, Section V; Chapter B, entry B-030, the lepton mixing angles and phase).

Sources: *The Measures of the Kinds*, Chapter 12, Section VII (Box VII-1); Chapter 11, Section V (Box V-1); Chapter 7, Section VII (the identity). Standing: a theorem (the identity $N^2H + 1 = c_1^4$, which holds at $c_1 = 3$ only); a definition (the boundary placement, set beside the neutrino fit by its pulls).

O-019 The Dirac Question. That a neutrino is either a Dirac particle, distinct from its antiparticle, or a Majorana particle, its own antiparticle, and that the two have a decisive experimental signature, neutrinoless double beta decay, which changes lepton number by two units and occurs only for Majorana neutrinos. That the neutrino is a Dirac particle: every state of content is held toward the thing compared and the thing compared against, its two terms; a state that equaled its own reversal would make its two terms interchangeable, which Axiom III forbids ("The (classes/instances) of the immanent, omniscient, and transcendent are

distinct, inseparable, and non-interchangeable"); a Majorana mass joins a state to its own reversal, so no state of content has one, for either chirality. That the finite geometry, at KO-dimension 6 (the integer modulo 8 that fixes the signs of the real structure of a spectral triple), admits a Majorana entry for the right-handed neutrino alone and leaves its value open; the structure sets it to zero: "the geometry marks the one place where a Majorana mass could enter, and the structure closes it". That the right-handed neutrino is the sixteenth state of each generation, the only state that no force acts on, and joins the left-handed neutrino in one Dirac field.

Sources: *The Branches of the Kinds*, Chapter 1, Sections I to IV (Box 1.1; quoted); *The Story of the Particles*, Level 4, Section I, and Level 9, Section III; *An Immanent Metaphysics*, aphorism 1.3-1 (quoted); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 10, and Chapter 19, Proposition 19.9. Standing: derived.

O-020 The Effective Majorana Mass. That the effective Majorana mass $\langle m_{ee} \rangle$, the quantity that sets the rate of neutrinoless double beta decay, is exactly zero, to all orders, when there is no Majorana source; so $m_{\beta\beta} = 0$, and neutrinoless double beta decay does not occur at any rate, whatever the order of the masses. That the searches find no decay: the complete KamLAND-Zen 800 data set in ^{136}Xe gives $T_{1/2} > 3.8 \times 10^{26}$ years, and the germanium experiments LEGEND-200, GERDA and the MAJORANA DEMONSTRATOR combined give $T_{1/2} > 1.9 \times 10^{26}$ years in ^{76}Ge, each at 90 percent confidence. That LEGEND-1000, whose design reaches an effective Majorana mass of 9 to 21 meV, nEXO and CUPID will find no decay. That this is the plainest measurable contrast with the program of Chamseddine and Connes, which fills the admitted Majorana entry, so that the standard seesaw operates and the decay occurs. That the search for neutrinoless double beta decay is the measurement that decides between the two; an observed decay would bear on the closing of the Majorana entry, the step from the two terms of a state to a zero Majorana mass, and on the chain that rests on it.

Sources: *The Branches of the Kinds*, Chapter 1, Sections VI to VIII; *The Connes Correspondence*, Chapter 25, Section VI; *The Measures of the Kinds*, Chapter 12, Sections I and V; KamLAND-Zen Collaboration (2025); LEGEND Collaboration (2026); LEGEND Collaboration (2021, the design report of LEGEND-1000). Standing: derived ($m_{\beta\beta} = 0$); a fork that a named measurement decides (between this account and the program of Chamseddine and Connes: the search for neutrinoless double beta decay).

O-021 Neutrino Mass Ordering. That in the normal order the lightest neutrino is the first mass state, $m_{\nu 1} < m_{\nu 2} < m_{\nu 3}$, and in the inverted order the third mass state is the lightest. That the order of the masses is a value, which the kinds do not fix, and both value routes of *The Measures of the Kinds* give the normal order, on the premise they share (that the three mass states are read on the three rungs that the charged ladders give their generations): on the count route 15.149, 17.433 and 52.366 meV, and on the Koide route 0.370, 8.726 and 50.211 meV. That a measurement of the order therefore tests the value routes and not the kinds: an inverted order would exclude both routes together, and with them their shared premise, and it does not decide between the two routes. That global fits of the oscillation data (NuFIT 6.0) prefer the normal order at less than three standard deviations, and JUNO, which began taking data in August 2025, measures the order directly through the oscillations of antineutrinos from reactors.

Sources: *The Measures of the Kinds*, Chapter 12, Sections I, II, V and VI; *The Story of the Particles*, Level 5, Section IV; Esteban and others (2024); JUNO Collaboration (2026). Standing: derived, given a stated identification (the normal order: the three mass states read on the three rungs that the charged ladders give their generations).

O-022 The Mass-Squared Splittings. That the two splittings, the differences of the squared neutrino masses, are the quantities that oscillation experiments measure. That the count route, placed from the measured tau alone, predicts both: the solar splitting $\Delta m^2_{21} = 7.441 \times 10^{-5}$ eV², against $(7.60 \pm 0.17) \times 10^{-5}$ eV² (Particle Data Group 2026), at -0.94σ, and the atmospheric splitting $\Delta m^2_{31} = 2.5127 \times 10^{-3}$ eV², against $(2.521 \pm 0.023) \times$

10^{-3} eV² (Particle Data Group 2026, normal order), at -0.36σ. That JUNO reports $\Delta m^2_{21} = (7.50 \pm 0.12) \times 10^{-5}$ eV², and on either value, and on their combination, every route's pull on Δm^2_{21} stays within one standard deviation. That the Koide route takes both splittings as inputs; with the lightest mass of the Koide route fixed by the hydrogen relation $m_{\nu 1} = \alpha^4 m_e/4$, and with Δm^2_{31} as input, the Koide route predicts $\Delta m^2_{21} = (7.470 \pm 0.040) \times 10^{-5}$ eV². The predicted value lies at -0.74σ (Particle Data Group 2026) and at -0.24σ (JUNO).

Sources: *The Measures of the Kinds*, Chapter 12, Sections I, III, VI and VIII; Chapter 13, Table X-1; JUNO Collaboration (2026). Standing: a measured agreement (on the count route of the neutrino fork, and on the Koide route with the hydrogen relation).

O-023 The Neutrino Mass Sum. That the sum of the three neutrino masses decides the fork: 84.95 meV on the count route's full ladder from the tau, and 59.31 ± 0.25 meV on the Koide route. That the cosmological inference of the mass sum is a test under an analysis (a cosmological model together with a prior on the masses and a choice of data). That from the second data release of DESI with the cosmic microwave background (Elbers and others 2025), at 95 percent confidence, the bound is 64.2 meV in flat ΛCDM (the standard cosmological model with a constant cosmological constant and cold dark matter) with the prior $\Sigma m_\nu > 0$ and three equal masses, 101 meV with the prior of the normal order, and 163 meV in $w_0 w_a$CDM (the model whose dark energy evolves in time through two parameters); the bound of 64.2 meV lies between the two routes, and both sums lie below the other two. That the collection's own cosmology fixes $w = -1$ exactly, a constant cosmological constant, $\Omega_m = 0.311$ and $h = 0.678$, and a dark matter that does not decay, which closes three of the four freedoms that relax the bound in the published analyses; the fork is carried until the sum is measured at the Ω_m and h of *The Gravitational Passage*. That the beta decay mass, the measurement free of any

cosmological analysis, is about 17.6 meV on the count route and about 9 meV on the Koide route (8.93 meV); KATRIN bounds it below 0.45 eV, and Project 8 is designed to reach 40 meV.

Sources: *The Measures of the Kinds*, Chapter 12, Sections I, VI and VIII (Tables VI-1 and VI-2); *The Connes Correspondence*, Chapter 25, Section VI; Elbers and others (2025); KATRIN Collaboration (2025); Project 8 Collaboration (2022). Standing: a fork that a named measurement decides (the mass sum, read at the Ω_m and h of *The Gravitational Passage*, and the beta decay mass, free of any cosmological analysis).

O-024 The Neutrino Sector: Standings. That the values of the neutrino sector stand as follows. The neutrino is a Dirac particle, with no Majorana mass: derived. The effective Majorana mass is zero, $m_{\beta\beta} = 0$: derived. The lepton mixing matrix has three angles and one phase, and the Standard Model with the neutrinos has 26 numbers (28 would need Majorana neutrinos): derived. The three neutrino masses: a fork between the count route and the Koide route, decided by the mass sum, with the beta decay mass free of any cosmological analysis. The normal order: derived, given the identification the two routes share. The three lepton angles and the lepton CP phase: a fork between the carry family (δ_ℓ near 282°, fourth quadrant) and the segment family (near 204°, third quadrant), decided by the quadrant of δ_ℓ, which Hyper-Kamiokande and DUNE reach at the resolution of 25.8° that separates the families at 3σ. Charged-lepton flavor violation (the decay of a muon or a tau into a lighter charged lepton with no neutrino emitted), through loops of the three light Dirac neutrinos only: branching ratios of order 10^{-55} for $\mu \to e\gamma$, 10^{-54} for $\tau \to \mu\gamma$ and 10^{-55} for $\mu \to 3e$, and a rate of conversion of a muon to an electron in a nucleus, relative to the rate of muon capture, of order 10^{-52} to 10^{-51}: derived, with no signal expected at any foreseeable experiment.

Sources: *The Measures of the Kinds*, Chapter 11, Sections IV, VI and VII (Box VI-1); Chapter 12, Section VIII; Chapter 13, Tables X-1, X-3 and X-4; *The Branches of the Kinds*, Chapter 1, Section VII. Standing: derived, derived given a stated identification, and a fork that a named measurement decides, as stated for each value in the entry.

O-IV. Predictions and the Measurements That Test Them

O-025 The Values beside Measurement. That the values of the particle side stand beside their measurements as follows (sources of measurement: Particle Data Group 2026, or 2024 where named; CODATA 2022). The tau, 1776.969 MeV by Koide's relation, against 1776.93 ± 0.09 MeV: $+0.43\sigma$, derived, given a stated identification (each part of the amplitude vector measured by its share of the squared size). The muon, $m_\mu / m_e = 206.7682862$, against $206.7682827(46)$: $+0.75\sigma$, a measured agreement. The top, 172.5622 GeV from the electron alone, against 172.60 ± 0.27 GeV: -0.14σ, closure account; six readings of the top lie within 0.7σ, each a measured agreement. The bottom, charm, strange, down and up: every placement within two standard deviations at the floor $2m_p$, where the light quarks are read, bare and dressed (Chapter B, entry B-019, the nine charged masses against measurement), measured agreements. The Higgs mass, 125.1715 GeV, against 125.13 ± 0.11 GeV: $+0.38\sigma$, a measured agreement. The vacuum value, 246.21966 GeV from the electron alone, against 246.21967 ± 0.00006 GeV: -0.16σ, closure account. The W mass by the carriers' law, 80.3547 ± 0.0023 GeV, against 80.3625 ± 0.0077 GeV: -0.97σ, a measured agreement. The weak angle, 3/8 at the first crossing of the couplings; at the Z mass a measured agreement against 0.23122 ± 0.00006. The strengths $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ and $1/\alpha_{em} = 137.035999177(21)$: measured inputs. The quark CP phase, 67.15° ± 0.70°, against 66.12° ± 1.43°: $+0.65\sigma$, derived, given a stated identification (the right angle of the unitarity triangle). The Cabibbo element, 0.22495 or 0.22410 by its two laws: derived, given a stated identification (the angle between two copies

of a kind taken as the Cabibbo angle). The counted $|V_{cb}| = 0.04148$: -0.60σ, a measured agreement. $\sin^2\theta_{13} = 1/45$, against 0.0217 ± 0.0007: $+0.75\sigma$, a measured agreement. The lepton angles and phase on the carry family: $\sin^2\theta_{12} = 0.3185$ ($+0.96\sigma$), $\sin^2\theta_{23} = 0.5444$ ($+0.63\sigma$), $\delta_\ell = 281.37°$ ($+1.86\sigma$): a fork that a named measurement decides (the quadrant of δ_ℓ). The splittings on the count route: Δm^2_{21} at -0.94σ, Δm^2_{31} at -0.36σ: a measured agreement on the count branch of a fork that a named measurement decides (the mass sum). The neutrino mass sum, 84.95 meV or 59.31 $\pm$ 0.25 meV: a fork that a named measurement decides (the mass sum itself), so far set against bounds only. The span from the electron to the reduced Planck mass, $72 + \varepsilon(1 - k\varepsilon) = 72.0132104$, against 72.0132112 ± 0.0000162 (from Newton's constant and the electron): -0.05σ; the count is an identification tested by the measurement, and k is predicted. The number of generations, three: derived, as the three rotations of the content; their number equals $c_1 = \chi(\mathbb{C}P^2)$ as the equality of two derived integers (Chapter A, entry A-023; Chapter M, entry M-013; *Non-Commutative Geometry and the Immanent Metaphysics*, Corollary 19.11), and a mechanism of topology for that equality is a conjecture of that book (Conjecture 10.2). The effective Majorana mass, zero: derived. The strong force's own angle, zero by structure; the phase from the quark masses, carried as zero: a measured agreement.

Sources: *The Measures of the Kinds*, Chapter 9, Section VII; Chapter 13, Tables X-1, X-2 and X-3; *The Story of the Particles*, Level 5; *The Concept Sequence*, Chapter 25, entry 25.24. The cosmological values (the density of the cosmological constant, Ω_m, h and the ratio of dark to ordinary matter) are set out in Chapters F, G and I of this book. Standing: derived, derived given a stated identification, a measured input, a measured agreement, an identification and a fork that a named measurement decides, as stated for each value in the entry.

O-026 The Parameter Count. That the Standard Model as first written has nineteen adjustable parameters: nine masses of charged fermions, three quark mixing angles and one phase, three strengths of force, the Higgs mass and its vacuum value, and the strong force's angle θ; $9 + 3 + 1$

+ 3 + 2 + 1 = 19. That the masses and mixing of the neutrinos add seven (three masses, three angles, one phase), for 26; these are the numbers of a Dirac neutrino, and Majorana neutrinos would add two further phases, for 28. That the electron's mass is one of the nineteen, the unit; after it, eighteen remain. That the count is a count of parameters, not of values, so it holds on both routes of the neutrino masses and on both families of lepton mixing. That each of the twenty-six carries one standing in *The Measures of the Kinds* (Table X-3, set out in entry O-025 and in the Summary of Chapter B, which gives the standings of the values of that chapter): a measured input, derived, derived given a stated identification, a measured agreement, or a fork that a named measurement decides. That the measured inputs are the electron's mass on the side of content and Newton's constant on the side of context; for the particles, the working account adds α_{em}, α_s, v and the two offsets of the charged ladders, and the closure account carries the chain from the electron alone, with the step written as a product. That Newton's constant and the electron's mass are each measured, and each is derived from neither the other nor the Planck mass: the span of 72 halvings between them is a count whose identification with the span is assumed and tested by measurement.

Sources: *The Measures of the Kinds*, Chapter 13, Section X (Table X-3); *The Branches of the Kinds*, Chapter 1, Section VII; *The Strengths of the Kinds*, Chapter 1, Section VII, and Chapter 5, Section I. Standing: a definition (the count); derived (twenty-six rather than twenty-eight, from the Dirac neutrino).

O-027 The Ledger of the Kinds. That the predictions of the account about the kinds of particle are the items that follow, and each item gives the measurement that tests the prediction and the step of the chain on which that measurement bears. Neutrinoless double beta decay (LEGEND-1000, nEXO, CUPID): no decay, $m_{\beta\beta} = 0$; it bears on the closing of the Majorana entry, the step from the two terms of a state to a zero Majorana mass (entries O-019 and O-020). Proton decay (Hyper-Kamiokande): no decay; it bears on the reading of the content as a direct product and on the closure of the field content, since no force carrier joins color to the

weak charge, and the proton is stable if it is the lightest whole of baryon number one, which is measured (*The Concept Sequence*, Chapter 42, entries 42.12 and 42.15). New kinds at the high-luminosity Large Hadron Collider: heavier composites of the known kinds, and no new kind; it bears on the completeness of the kinds (a new kind would need a new label, an old label toward the wrong term, a fourth generation or a fourth kind of field, and there is none) and on the closure of the field content (no supersymmetric partner, no second Higgs doublet, no further gauge boson, no leptoquark). Charged-lepton flavor violation (MEG II, Mu3e, Mu2e, COMET): no signal, at rates of order 10^{-55} to 10^{-51}; it bears on the closure of the list of kinds and on the Dirac neutrinos as the only states in the loop. A neutron electric dipole moment at a size the quark mixing cannot give: it would bear on the zero of the strong force's own angle and on the measured agreement of the quark-mass phase (entry O-013). An axion detected as a particle: it would bear on the closure of the list of kinds (entry O-015).

Sources: *The Connes Correspondence*, Chapter 18, Section IV (Table 18.3); *The Branches of the Kinds*, Chapters 1 to 4; *The Story of the Particles*, Level 9, Section IV, and Coda, Section II; *The Measures of the Kinds*, Chapter 11, Section VI (Box VI-1); *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Propositions 19.7 to 19.10. Standing: derived (each prediction, from the chain named with it); a fork that a named measurement decides (each, by the measurement named with it).

O-028 How a Measurement Bears on the Chain. That the Incommensuration Theorem is a structural fact about the nature of knowing, a claim of logical form, settled by its proof, and every test of it is itself an act of comparison within its scope, so no measurement puts the theorem on trial: "Falsifiability is not a standard that the account of comparison fails". That the predictions of physics are claims about the world, each stated with the measurement that bears on it and scoped to its own chain of steps: each measurement bears on the identification and the chain of steps on which its prediction rests, and the steps that rest on other ground stand as they stand. That a result counts as confirmed when it is estab-

lished at five standard deviations by at least two independent kinds of measurement. That a measurement bears on a value according to the kind of value it is. That on a fork a measurement decides between the branches and leaves the steps outside the fork as they stand; on a measured agreement it bears on the agreement and on the reason for it; on a measured input it changes the input and the values computed from it; and on a derived value it bears on the premises named with it. That the discovery of new composite states (glueballs, tetraquarks, pentaquarks or other hadrons) bears on no step of the list of kinds, because composites are combinations of the kinds and not kinds.

Sources: *Physics Is Not Math*, Chapter 19, Section III (quoted); *The Gravitational Passage*, Chapter 20, Questions (the rule of confirmation: "confirmed means established at five standard deviations by at least two independent kinds of measurement"); *The Story of the Particles*, Level 9, Section I, and Coda, Section II. Standing: a definition (the scope of a measurement's bearing).

O-029 The Ledger of the Values. That the predictions of the account about the values are the items that follow, and each item gives the measurement that will decide the prediction and the step of the chain on which that measurement bears. The neutrino mass sum (the Simons Observatory with the Euclid space telescope and DESI, read at the Ω_m and h of *The Gravitational Passage*): 84.95 meV on the count route or 59.31 ± 0.25 meV on the Koide route; it decides the neutrino fork. The beta decay mass (KATRIN; Project 8): about 17.6 meV on the count route or about 9 meV on the Koide route; it decides the same fork free of any cosmological analysis. The order of the neutrino masses (JUNO): the normal order on both routes; it bears on the identification the two routes share, and not on the kinds. The lepton CP phase (Hyper-Kamiokande, DUNE): about 282° on the carry family or about 204° on the segment family; its quadrant decides the lepton-mixing fork. The solar angle (JUNO, whose first result is $\sin^2\theta_{12} = 0.3092 \pm 0.0087$): 0.3185 on the carry family with TM1 (the correction of the anchor that keeps the first column of tribimaximal mixing), at $+1.07\sigma$; JUNO tests TM1's solar angle directly. The Cabibbo element: a

measurement of $|V_{us}|$ to ± 0.00043 decides between its two laws. The up quark's mass: a measurement to ± 1 percent leaves the $\sqrt{2}$ law of the six masses and the constants' $2^{(11/20)s}$ 2.0 standard deviations apart, and three standard deviations need the other five masses measured to about half their present errors as well. $|V_{ub}|$ to about 1 percent, or the cross-row factor k to about ± 0.004: decides between the two readings of k. The tau's mass (Belle II, to ± 0.045 MeV): separates the ladder's tau with no head factor from Koide's at 4.4 standard deviations. The top's mass: to about ± 0.016 GeV it separates the span reading from the second-order tau ladder, and to ± 0.05 GeV the product step from the chain that places the top one bare ε (the one mismatch with no further factor) below $\sqrt{2}$ v. The Higgs mass: to ± 0.03 GeV it decides the relation reading of the top and the midpoint against the mass formula's 125.17 GeV, and to about ± 0.005 GeV it separates the forms of the second order.

Sources: *The Measures of the Kinds*, Chapter 8, Section VI; Chapter 9, Sections VI and VII; Chapter 10, Sections II, VI, VII and IX (Table VII-2); Chapter 11, Sections IV and VII; Chapter 12, Sections VI and VIII; Chapter 13, Sections VII, VIII and IX; *The Connes Correspondence*, Chapter 25, Section VI (the experiments of the mass sum and the beta decay mass); JUNO Collaboration (2026). Standing: a fork that a named measurement decides (the neutrino masses and the lepton mixing, each line decided by the measurement named with it); derived, derived given a stated identification, and a measured agreement (the other lines, each value with the standing that Chapter B states for it).

Summary

This chapter is the branch of the map that sets the values of the particle side beside their measurements, each with its source, its pull and its standing, and each prediction with the measurement that decides it. Its results belong to their owners: *The Measures of the Kinds* for the values and the forks, *The Story of the Particles* for the closed list of kinds, *The Branches of the Kinds* for the Dirac neutrino, the absence of the seesaw, the zero of

the strong force's own angle and the absence of the axion, and *Non-Commutative Geometry and the Immanent Metaphysics* for the Higgs doublet as the only scalar.

At the Large Hadron Collider the Higgs boson has the Standard Model's form and couplings and no added channel, so its widths and branching ratios are the Standard Model's at its mass. The self-coupling $1/8$ at tree level is derived, given the identification $\lambda = a^2/2$, and with the loop term $1/(24\pi^2)$ the Higgs mass is 125.1715 GeV, a measured agreement at $+0.38\sigma$. The carriers' law gives the W mass at -0.97σ, the weak angle at the Z mass is a measured agreement, and the closure account, with the step written as a product, carries the electron alone to the vacuum value at -0.16σ.

With the list of kinds closed, every rare process is a Standard Model computation with the measured mixing elements, the coefficients beyond the Standard Model are zero, the strong force's own angle is zero by structure, and no axion is called on. Glueballs are composites, and composites add no kind. The neutrino is a Dirac particle, so the Majorana mass is zero, the seesaw does not operate, and the effective mass of neutrinoless double beta decay is exactly zero. The neutrino masses stand as a fork of two routes, which share the normal order and part on the mass sum, 84.95 meV against 59.31 meV; the lepton mixing stands as a fork of two families, which part on the quadrant of the lepton phase.

The two ledgers gather the predictions: the ledger of the kinds names what the searches will find and which step each search bears on, and the ledger of the values names the precision at which each measurement decides between readings. A measurement bears on each value according to its standing, and the Incommensuration Theorem, a structural fact about the nature of knowing, is no outcome of any measurement. Together the parts make one account of the particle side as measurement meets it: the kinds are closed, the values are each carried at their standing, and every fork is carried with the measurement that will close it.

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