

The Unquantized Arena

A Structural Diagnosis of Quantum Gravity and the Measurement Problem

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What This Book Claims and How It Is Set Out

I. The Thesis

Quantum theory and general relativity are two descriptions of one world. Quantum theory describes the content of physics: what a measurement finds, in discrete classes with exact labels. General relativity describes the context of physics: the arena of space and time in which every content is found, smooth, and different from place to place. To quantize the arena is to apply to the context the treatment that belongs to the content. To quantize the arena is a category error; a category error is a question that applies a concept where the concept does not belong.

The diagnosis rests on one theorem about comparison. Every finding of physics is an act of comparison, and the Incommensuration Theorem states that symmetry and continuity cannot both be perfected in one comparison, and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric. I identify the findings of general relativity as continuously asymmetric and the findings of quantum theory as discontinuously symmetric. Quantization is a mode exchange: in one step the findings of a description pass from continuously asymmetric to discontinuously symmetric, and the character its formalism holds exact against those findings passes from symmetry to continuity. A content admits the mode exchange, since its perfected character is discontinuous symmetry and its two descriptions are laid out in one arena that stays as it is. The arena ad-

mits none: a quantized arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both.

The claim that the arena is not quantized is exact in two senses. The arena has no least step of length, area or duration, and nothing is a quantum of the arena itself, at any scale. Gravitation is not quantized. Small waves of geometry can be described by quanta, and the word "graviton" names at most a quantum of that description, never a quantum of the arena. The phonon is the nearest sense by which the word can be made intelligible: a phonon is a quantum of the description of the vibrations of a held structure, a solid whose bonds hold its atoms in a lattice. The arena is not a held structure, so even that sense does not make the graviton a quantum of the arena. A detector that registers a gravitational wave in single clicks would not count against this account, and it would not show gravitation quantized. The Planck scale is where the gravitational and the quantum descriptions of an object meet: an object's gravitational size equals its quantum size there, neither description dominates, and both are relevant. The Planck length is not a least length.

What remains when the error is removed is quantum matter on a classical geometry, and I identify the configuration in which each alternative of the matter carries its own classical geometry. The experiment of Page and Geilker (1981) supports that identification: one geometry sourced by the average of the alternatives fails it, and one geometry for each alternative passes it. A finding selects one outcome from among alternatives that already carry their geometries, so a finding changes no geometry. Two masses related only through gravity become entangled, and for the parameters that Bose and colleagues proposed in 2017 the entangling phase is 0.314 radians. Later designs use other masses, separations and times, and the same formula gives their phases.

The same diagnosis applies to five further questions. Physics uses three kinds of time, and the measurement problem is the residual where the time of evolution meets the time of findings. Each interpretation of quantum mechanics meets its difficulty where the time it drops returns,

and the theorems of Bell, of Kochen and Specker, and of Pusey, Barrett and Rudolph are instances of one rule. Each program of quantum gravity meets the kind of difficulty its founding commitment predicts. The correspondence between gravity in anti-de Sitter space and a conformal field theory on its boundary is a bridge between conjugate descriptions, with neither side more fundamental. Information is not destroyed in a black hole, and for a hole that evaporates while an outside world remains, the Page curve follows as a conditional result, turning at 0.538 of the evaporation time when the greybody factors of photons and of the small waves of the geometry are included, the small waves counted in the terms of their description by quanta.

II. The Premises and Their Kinds

The argument of this book is set out as numbered premises and propositions. Part I states premises P1 to P17, and later chapters add P18 to P23 where each is needed. Every premise names its kind, and there are three kinds.

A premise of IM ground is a statement of the Immanent Metaphysics, cited to *An Immanent Metaphysics* by aphorism number; the appendix "The Aphorisms Cited" prints each cited aphorism in full. Twelve premises are of this kind: the Root Tautology, that comparison is isomorphic with interaction and relation (P1); the six intrinsics of comparison and the inseparability of each of their three pairs (P3, P4); the inseparability principle of content and context, the two laws of symmetry and continuity, and the characterization of content and context (P5 to P7); the four compound concepts (P8); the Incommensuration Theorem (P9); the three modalities, and the dependence of their binding on the focus of the argument being made, with the rule that says what each focus pins (P10, P11); the three axioms, in their exact wording (P12); and the distinction between what exists and what is real (P14). The ground has no physical premise, so it applies to every domain in which comparison occurs.

A premise of established physics is a result of physics as it stands, cited to the literature. Five premises are of this kind: the six theories checked in Chapter 3, with general relativity and quantum field theory among them (P15); the singularity theorems (P17); the thermal radiation of quantum fields on the classical geometry of a black hole, and the evaporation of the hole (P20, P21); and the bound on the entanglement entropy of the radiation by its coarse-grained entropy (P22). Two further statements appear in Chapter 17 as conditions of the Page curve, (C) and (H), and the curve is stated as conditional on them. (C) is the purity of the whole formed by what fell in, the content inside the horizon and the radiation: the matter's state laid out on its geometries, with no geometry put into the state. (H) is the bound on the entanglement entropy of the content inside a hole by the hole's entropy.

An identification is a stated step from the structure of comparison to a physical quantity, made in the first person, with the observation that bears on it. Six premises are identifications: that every finding of physics is an act of comparison made from inside the world (P2); the roles of a comparison in the focus of measurement (P13); the character of the findings of each of six theories (P16); that gravity is the context of physics (P18); the three kinds of time as the three modalities, with the one-way order of findings as the direction of time (P19); and the equal weighting of the joint states of a black hole and its radiation that no comparison from outside tells apart, stated together with the established result of Page on random pure states (P23). P18 is the one identification in the chain from the ground to the category error that experiment tests directly. It is made by the match of four markers that Chapter 5 derives for any context before gravity is named, and it predicts that free fall stays universal for matter and antimatter at every sensitivity yet to be reached, and that no arrangement of matter shields gravity.

III. The Established Physics Taken as Given

General relativity, with Einstein's field equations, and quantum theory, with quantum field theory, are taken as established physics throughout. The Incommensuration Theorem states the character that an exact description can have, and the theorem states the specific content of neither theory. So I derive neither theory, and I take the content of both theories from physics. The structure of light cones is taken from relativity: news of a distant finding reaches a comparer only with a signal, at the earliest on that comparer's past light cone, and no comparison registers a surface of simultaneity, which this account does not need. Newton's constant is measured. The ground fixes its role, the strength of the coupling of every content to its context, and leaves its magnitude to measurement, because the axioms are structural and not dimensional.

Every result of physics the argument uses is cited to the literature by author and year, with the full entry in the bibliography. The results of the programs of quantum gravity are credited as results. With the criteria of Chapter 14 I assess what each program does to the arena at its first step, and I declare none of its computations void.

The cosmological constant, dark matter and dark energy are questions of cosmology, the science of the universe as a whole, and they lie outside this book. My argument needs none of the cosmological constant, dark matter and dark energy, and I make no claim about any of the three.

IV. How the Book Is Set Out

The book has six Parts. Part I, "The Ground as Premises" (Chapters 1 to 3), states the ground and establishes that every physical theory has two levels, its formalism and its findings, in conjugate modes. Part II, "The Diagnosis" (Chapters 4 to 6), reads quantization as the mode exchange, derives the four markers of any context and checks them against gravity,

and states the category error with its exact scope. Part III, "What Remains" (Chapters 7 to 9), sets out one geometry for each alternative, the entanglement test, what the word "graviton" can name, and the place of the Planck scale. Part IV, "Measurement" (Chapters 10 to 13), treats the three kinds of time, the measurement problem, the interpretations and the no-go theorems. Part V, "The Programs" (Chapters 14 to 16), judges ten programs of quantum gravity, reads AdS/CFT, and states the unity that the Incommensuration Theorem permits. Part VI, "Horizons, Objections and Tests" (Chapters 17 to 19), treats black-hole information, the objections at full strength, and five predictions with their evidence.

Propositions and equations are numbered by chapter. Each proposition is followed by its argument and the premises it rests on, and each equation is said in words beside it. The notes carry the ties to *An Immanent Metaphysics* and further detail of the literature, and the appendix, "The Numbers", gives every number the book prints with its calculation or source.

V. The Two Volumes

This book has a companion for the general reader, *Two Theories, One World; Gravity, the Quantum and the Mistake Behind the Search for One Theory*. That volume sets out the ground from nothing, with ordinary examples, and gives every step of the argument in prose, with no technical apparatus. This volume states the same ground as premises, names the companion's chapters where the reasons are set out at length, and makes the argument again in the form a specialist recognizes: premises, propositions, checks against six theories, the literature, the objections at full strength, and the predictions with their evidence. Each volume is complete for its reader. Each cites the other only beside a plain statement of what is cited, by title, chapter number and chapter title.

The ground that this book states as premises is set out at length, with its full arguments, in three volumes: *Comparison; The Necessary Basis* (ISBN 979-8-90710-062-6), cited here as Volume 1; *Incommensuration; The Theorem and the Three Axioms* (ISBN 979-8-90710-063-3), cited as Volume 2; and *Measurement; The Steps from Comparison to Physics* (ISBN 979-8-90710-064-0), cited as Volume 3. Volume 1 sets out comparison and its parts: the Root Tautology (Chapter 3, "One Act, Three Names"), the six intrinsics (Chapter 4, "The Six Intrinsics of Comparison"), the three roles and their binding by focus (Chapter 5, "Three Roles", and Chapter 6, "Which Pair Takes Which Role"), the order of finding (Chapter 7, "The Order of Finding") and the six measures (Chapter 8, "Three Six-Fold Systems"). Volume 2 states P1 to P14 with their sources (Chapter 5, "The Ground as Premises", and Chapter 10, "The Three Axioms and the Two Levels of a Theory"), proves the Incommensuration Theorem step by step (Chapter 6, "The Incommensuration Theorem: Statement and Proof") and derives the three axioms from it (Chapter 8, "From the Theorem to the Axioms"). Volume 3 continues the numbering with P15 to P19, which this book shares, and sets out the cross-modal principle, the mode assignment, the markers of any context and the three times (Chapter 8, "The Cross-Modal Principle", Chapter 9, "The Mode Assignment", Chapter 10, "Context and Its Markers", and Chapter 12, "Time"). Every premise this book uses is stated in it, with the shortest reason a specialist can check, and every step of its argument is made in it, so the book is complete without the volumes. The volumes give the longer reasons, and the appendix "Premises and Propositions" names, for each premise, where its longer reasons are set out.

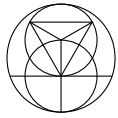
Table P.1 gives the chapters of each volume that answer to one another.

Two Theories, One World	***The Unquantized Arena***
Chapters 4 to 6, the ground	Chapters 1 and 2, the premises
Chapters 7 to 9, content, context and the four marks	Chapter 5, the content and context partition

Two Theories, One World	*The Unquantized Arena*
Chapter 10, how a theory sees	Chapter 3, the cross-modal principle
Chapter 11, what quantizing does	Chapters 4 and 6, quantization and the category error
Chapter 12, one geometry for each alternative	Chapter 7, one geometry for each alternative
Chapter 13, the experiment being built	Chapter 8, the entanglement test
Chapters 14 and 15, gravitons and the Planck scale	Chapter 9, linearized gravity, gravitons and the Planck scale
Chapters 16 to 20, the measurement problem	Chapters 10 to 13, measurement
Chapters 21 to 24, the programs, the correspondence, joined not merged	Chapters 14 to 16, the programs
Chapters 25 to 27, information, objections, tests	Chapters 17 to 19, horizons, objections and tests

PART I

THE GROUND AS PREMISES



Chapter 1

Comparison, the Intrinsics and the Incommensuration Theorem

What every finding of physics is, what every finding contains, and the one theorem that governs what a finding can be

This chapter states the first nine premises of the book. Every finding of physics is an act of comparison, and every comparison contains six intrinsic concepts, from which four compound concepts are formed. Of the ten ways of pairing the compounds, eight are excluded by the inseparability of sameness and difference, and two survive: continuity with asymmetry, and symmetry with discontinuity. That result is the Incommensuration Theorem, and it governs what any description exact at its foundation can find. At the close of this chapter, I separate my senses of "symmetry", "continuity" and "perfected" from the senses those words have in physics.

I. How the Premises Are Set Out

The ground of this book is stated in this Part as premises P1 to P17, and the numbering holds throughout the book. A premise that a later chapter needs takes the next number, from P18 on, and is stated where it is introduced. Each premise names its kind, and there are three kinds. A premise of IM ground, the ground of the Immanent Metaphysics, is a statement of that framework, cited to *An Immanent Metaphysics* by aphorism number

where the aphorism is given. A premise of established physics is a result of physics as it stands, cited to the literature. An identification is a stated step from the structure of comparison to a physical quantity, made in the first person, with the observation that bears on it. Propositions are numbered by chapter, and each is followed by its argument and the premises it rests on.

This Part states each premise exactly, with the shortest reason a specialist can check. The reasons are set out at length for the general reader in the companion volume, *Two Theories, One World*, Chapters 4 to 6, where comparison, the Incommensuration Theorem and the three roles of a comparison each have a chapter. The three volumes named in "What This Book Claims and How It Is Set Out", Section V, cited here as Volumes 1, 2 and 3, set out the full arguments for P1 to P19, among them the proof of the theorem and the derivation of the three axioms from it.

Three words recur with one meaning each. A comparer is whatever makes a comparison: an instrument, a detector, an interaction. A finding is the outcome of one act of comparison. An observer is a comparer: the word names a position inside the world, and it never names a mind.

II. Comparison, Interaction and Relation

Comparison is the act of finding whether two things are the same or different, in some respect.

> **P1 (IM ground; *An Immanent Metaphysics*, aphorism 1.42-4).** "The Root Tautology: Comparison is isomorphic with interaction and relation."

In plain words, comparison, interaction and relation are one act, approached three ways. Comparison names the act as a finding of sameness and difference. Interaction names the act as an event, something that hap-

pens. Relation names the act as a standing connection between the two things. P1 is a recognition that three names name one act. Its reason is reflexive: a denial of P1 compares the claim with something and finds the two different, so every attempt to reject the premise is itself an act of comparison.[1]

P2 (identification). I identify every finding of physics with an act of comparison made from inside the world.

The step from P1 is short. Physics reaches the world only through interactions, and by P1 each interaction is a comparison, so whatever constrains comparison constrains everything physics can find. The observation that bears on P2 is the form of every measured value: each is a count against a threshold, a reading against a standard, or a frequency against a reference, delivered by an instrument that is part of the world it measures. Every finding that physics records was obtained through an interaction with what is found, and so each one agrees with P2.

III. The Six Intrinsics

P3 (IM ground; *An Immanent Metaphysics*, aphorisms 1.42-5 and 1R1). The concept of comparison involves six intrinsic concepts: sameness, difference, content, context, the subjective and the objective. The six are necessary and sufficient for the formal consideration of comparison.

The six are what a comparison is, and they are not additions to it. A comparison gives a verdict, and the verdict is sameness or difference. The verdict is on something, and what the verdict is on is the content of the

comparison. The two things compared are two in some respect, and that respect is the context of the comparison. The content and the context cannot be one respect: a verdict on the respect that makes the two things two would be fixed before the comparison was made, and a verdict fixed in advance is no verdict. The subjective is the comparing: the side from which the comparison is made. The objective is what is compared: the side about which the comparison is made. For an instrument, the subjective is the instrument's own position and setting, and the objective is the system it registers. Every unit, scale and number a measurement uses is built from these six and comes after them.

P4 (IM ground; *An Immanent Metaphysics*, aphorisms 1R4 and 1R15). The six intrinsics form three pairs, and each pair is inseparable. Sameness and difference are inseparable: wherever one appears, the other is implicit. Content and context are inseparable: where there is content, there is context, and where there is context, there is content. The subjective and the objective are inseparable: there is no comparing without something compared, and nothing is compared without a comparing. Each pair is distinct from the other two, and no pair can stand in for another.

The inseparability of sameness and difference in P4 is co-presence: a verdict of sameness carries difference with it, and a verdict of difference carries sameness with it. Section VI uses P4 in exactly this form.

IV. Content and Context: Three Principles

Three further statements of the Immanent Metaphysics concern content and context, and Part II uses all three.

P5 (IM ground; *An Immanent Metaphysics*, aphorism 1R3). The inseparability principle: "Where there is content, there must also be context. Where there is context there must also be content."

P6 (IM ground; *An Immanent Metaphysics*, aphorism 1.83-5). The two laws: "Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context."

P7 (IM ground; *An Immanent Metaphysics*, aphorism 1.46-12). "All that is qualia is intercompositional, has the nature of non-local context, and is understood in terms of continuity. All that is quanta is interexclusionary, has the nature of local content, and is understood in terms of symmetry." I use P7 in this form: content is local and interexclusionary (two contents exclude one another from the same place), and content is understood in terms of symmetry. In the same form, context is non-local and intercompositional (contexts compose with one another), and context is understood in terms of continuity.

P6 has a reason, and it is given here at the standard that the theorem receives in Section VI. The two laws follow from the place of each side within a comparison (P3), from the inseparability of sameness and difference (P4), from the composition of contexts (P7), and from the definitions of the compounds (P8).

The law of symmetry. A content is what a verdict is on (P3). Two comparisons made in different contexts find one content only where that content is the same across the difference of context between them. A content that differed with every difference of context would be a new content in each comparison, and no two findings would be findings of one content. So what is found as content is found as a sameness of content across a difference of context, which is symmetry (P8).

The law of continuity. The reason has two steps. First, the context is connected. Contexts compose with one another (P7), and the context of two findings set side by side is the composition of their contexts. A context with a gap in it would leave two settings that no comparison could join, and the findings made in them could never be compared. Second, the context is continuous, and not only connected. A lattice and a causal set are connected, and each has a least step, so connection alone does not give continuity. By P4, sameness and difference are inseparable in context too, as in every respect of a comparison. So every difference of context holds a sameness within it, and so admits a finer comparison, one that finds the sameness within that difference. A least step would be a difference of context with no sameness within it, and P4 excludes it. So the context has no gap and no least step: between any two settings there is a third, and nothing in the context is sudden, which is continuity (P8).

Rests on: the six intrinsics (P3); the inseparability of sameness and difference (P4); the composition of contexts (P7); the definitions of symmetry and continuity (P8).

P4 states the inseparability of content and context within one act of comparison. P5 states the same inseparability as a principle of whatever is content or context, and P5 is the form in which later chapters apply that inseparability to physical content. P6 and P7 say which compound concept governs each side of the pair. Chapter 5, "The Content and Context Partition", derives from P5 to P7 four markers that any context in physics must have.

V. The Four Compounds

P8 (IM ground; *An Immanent Metaphysics*, aphorism 1R8). Where the subjective context is held the same (the stance of the comparing held the same), sameness and difference applied to the content and the context of what is compared give four compound concepts. Continuity is a sameness of content where there is a sameness of context. Discontinuity is a difference of content where there is a sameness of context. Symmetry is a sameness of content where there is a difference of context. Asymmetry is a difference of content where there is a difference of context.

Each compound concept is a pair of verdicts: one verdict, same or different, on the content, and one verdict, same or different, on the context. Continuity says that nothing is sudden; read at the limit, an infinitesimal change in context produces an infinitesimal change in content. Discontinuity says that something is sudden: the content changes while the context does not. Symmetry says that something is invariant: the content stays the same while the context changes. Asymmetry says that nothing is invariant: the content changes when the context changes.

Physics uses "discontinuity" in three senses, and P8 gives the third sense, from which the first two senses follow. The first sense of discontinuity is a gap in a spectrum. Two levels of one bound system are found in one and the same setting (the same system, preparation and apparatus), and the two levels differ, with no other level between the two levels. So the gap is a difference of content where the context is the same. The second sense of discontinuity is the suddenness of a finding. An outcome is registered while the setting of the measurement stays unchanged. So the passage from no record to a record is a difference of content where the

context is the same. The third sense of discontinuity is the literal sense of P8. The literal sense of P8 is the sense that fits identical preparations with different outcomes: a different content in the same context. In this book "discontinuity" always has the sense of P8, and the first two senses are cases of it.

Table 1.1 sets the four compounds out by their two verdicts.

	Same context	**Different context**
Same content	continuity	symmetry
Different content	discontinuity	asymmetry

The general reader's account of comparison and of the six intrinsics, with ordinary examples of each, is in *Two Theories, One World*, Chapter 4, "Everything Is Found by Comparison"; the four compounds open its Chapter 5, "Symmetry and Continuity Cannot Both Be Perfected".

VI. Ten Candidates and Three Criteria

The question the theorem answers is which conjunctions of two compound concepts can characterize one comparison, held exactly at its foundation. Four compounds can be paired in ten ways: each with itself, which gives four pairings, and each with each of the others, which gives six. A conjunction of two compounds carries two verdicts on the content and two verdicts on the context. I examine the ten candidates by three criteria.[2]

The first criterion is no self-application. A compound concept applied to itself is of no semantic value: it adds nothing to what the compound already says. The criterion removes continuity with continuity, symmetry with symmetry, discontinuity with discontinuity, and asymmetry with asymmetry.

The second criterion is no opposite pairing. Continuity with discontinuity asserts that the content is both the same and different in the same context. Symmetry with asymmetry asserts that the content is both the same and different under a change of context. Each violates the distinctness of sameness and difference, and the criterion removes both.

The third criterion is the balance of sameness and difference. By P4, sameness and difference are inseparable wherever a verdict is given: wherever one appears, the other is implicit. A conjunction of two compounds gives its verdicts in two respects, content and context, and within each respect verdicts are given. So the inseparability of sameness and difference holds within each respect: among the verdicts on the content, sameness must appear with difference; among the verdicts on the context, sameness must appear with difference. The criterion uses inseparability exactly as P4 states it, and it involves no counting.[3]

The candidates are conjunctions of exactly two compounds, and the criteria themselves give the reason. One compound alone gives a single verdict in each respect, so it cannot hold sameness with difference in either respect, which P4 requires. Any three of the four compounds contain an opposite pairing, continuity with discontinuity or symmetry with asymmetry, which the second criterion removes, and all four contain both. So a comparison perfected at its foundation is characterized by a conjunction of two compounds, and the ten candidates are all the conjunctions there are.

Table 1.2 applies the third criterion to all ten candidates and names the other criterion that also removes each excluded candidate.

Conjunction	**Verdicts on content**	**Verdicts on context**	**Balance of sameness and difference**	**Also removed by**
Continuity with asymmetry	same, different	same, different	passes	none: survives
Symmetry with discontinuity	same, different	different, same	passes	none: survives
Continuity with symmetry	same, same	same, different	fails: no difference of content	none
Discontinuity with asymmetry	different, different	same, different	fails: no sameness of content	none
Continuity with discontinuity	same, different	same, same	fails: no difference of context	no opposite pairing
Symmetry with asymmetry	same, different	different, different	fails: no sameness of context	no opposite pairing
Continuity with continuity	same, same	same, same	fails in both respects	no self-application

Conjunction	Verdicts on content	Verdicts on context	Balance of sameness and difference	Also removed by
Symmetry with symmetry	same, same	different, different	fails in both respects	no self-application
Discontinuity with discontinuity	different, different	same, same	fails in both respects	no self-application
Asymmetry with asymmetry	different, different	different, different	fails in both respects	no self-application

The balance criterion alone removes continuity with symmetry, where sameness of content appears twice and difference of content not at all, and discontinuity with asymmetry, where difference of content appears twice and sameness of content not at all. Neither of the other two criteria removes these two candidates. The balance criterion also fails the six candidates that the other two criteria remove. So the three criteria agree on the ten candidates, and inseparability alone, applied in each respect to a pair, gives the same two survivors. For conjunctions of three, the second criterion does work of its own: continuity with symmetry and asymmetry passes the balance criterion in both respects, and the second criterion removes it.

VII. The Incommensuration Theorem

P9 (IM ground; *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20). The Incommensuration Theorem: the concepts of symmetry and continuity cannot both be simultaneously and fundamentally applied to any event or comparison. The concepts of asymmetry and discontinuity cannot both be simultaneously and fundamentally applied to any event or comparison.

Any comparison, perfected at its foundation, must be either continuously asymmetric or discontinuously symmetric.

The reason for the theorem is the examination of Section VI: the ten candidates are all the conjunctions there are, the inseparability of sameness and difference in each respect excludes eight of them, and two remain.[4] The reason has three parts, and each part rests on a basis of its own kind. The count of ten candidates is settled as a count is settled: the four compounds pair in exactly ten ways, and every case is examined. The three criteria are argued from the inseparability of sameness and difference (P4). The match of the two survivors to general relativity and quantum theory is an identification (P16, Chapter 3), which observation tests. The appendix "The Ten Cases" lists the ten candidates as pairs of verdicts, each with the criterion that removes or keeps it. The theorem is a statement about what comparison requires, and it has no physical premise. It therefore applies to every domain in which comparison occurs, and by P2 physical measurement is such a domain.

The two survivors are continuity with asymmetry and symmetry with discontinuity. Continuity with asymmetry joins a sameness of content in a sameness of context with a difference of content in a difference of context. Symmetry with discontinuity joins a sameness of content in a difference of

context with a difference of content in a sameness of context. The two are exhaustive and exclusive: any act of comparison, perfected at its foundation, is characterized by one of them, and by no third. In each survivor the second compound is the exact complement of the first, since where one finds sameness, in content or in context, the other finds difference. So the theorem can be stated in one line: the only coherent conjunctions are a compound concept joined with its exact complement.

The two forbidden pairings are continuity with symmetry, and asymmetry with discontinuity. A description that held both continuity and symmetry exactly in what it finds would carry the first; a description that held both asymmetry and discontinuity exactly in what it finds would carry the second.

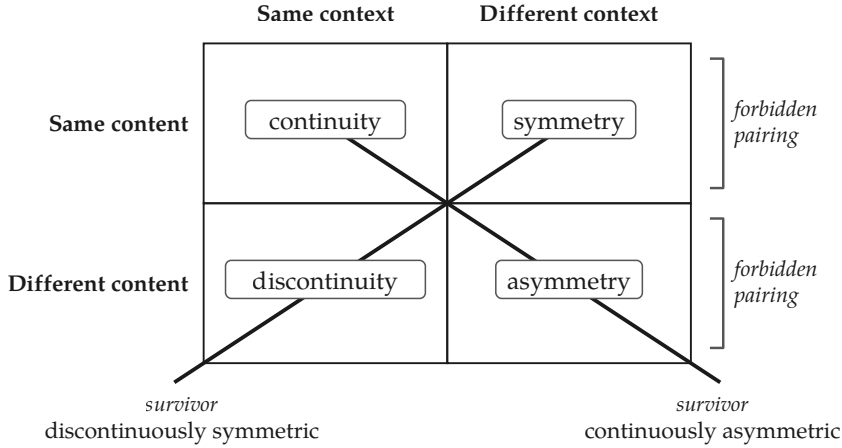


Figure 1.1. *The square of Table 1.1, with its rows and diagonals marked. The two forbidden pairings lie along the rows, where both compounds give the same verdict on the content; the two survivors lie on the diagonals, where each compound is the exact complement of the other.*

"Perfected" means holding exactly, at the foundation of a description. The theorem allows symmetry and continuity to appear together approximately, as they do in every real system. What it excludes is the two holding exactly at the foundation of one and the same description.

The theorem's verdict on any formalism follows. Any description of physics that is exact at its foundation finds either continuous asymmetry or discontinuous symmetry. No such description perfects continuity and symmetry together in what it finds, and none perfects asymmetry and discontinuity together. Chapter 2 shows that a physical theory has two levels, a formalism and its findings, and Chapter 3 shows that the verdict falls on what a theory finds, while its formalism carries the other character.[5] The theorem, its ten candidates and its two survivors are set out for the general reader in *Two Theories, One World*, Chapter 5, "Symmetry and Continuity Cannot Both Be Perfected".

VIII. Two Vocabularies

Physics uses the words "symmetry" and "continuity" of mathematical objects, while this book uses them of what a comparison finds. A continuous symmetry in physics is a group of transformations whose elements vary with continuous parameters. In this book a symmetry is a character of a finding, defined by P8. The two usages meet, and Chapters 2 and 3 show where: the invariants a theory finds are its symmetric findings, and the group that leaves them invariant acts in its formalism.

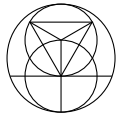
Table 1.3 sets the two vocabularies side by side.

Term	In this book	In physics	Where the two meet
Symmetry	A sameness of content across a difference of context: what is found does not change when its context changes.	Invariance of an action, an equation or a space under a group of transformations.	The invariants found are a theory's symmetric findings; the group acts in its formalism.

Term	In this book	In physics	Where the two meet
Continuity	A sameness of content across a sameness of context: nothing is sudden.	Smoothness of a function, a manifold, or the parameters of a group.	The book's continuity is a character of what is found; smooth parameters belong to a formalism.
Asymmetry	A difference of content across a difference of context: nothing is invariant.	Absence of an isometry; a generic spacetime; a broken symmetry.	A generic spacetime, with no isometry, is the physical case of what the book calls asymmetry.
Discontinuity	A difference of content across a sameness of context: something is sudden.	Discrete spectra, quantized levels, the outcome of a measurement.	The gaps of a spectrum and the suddenness of a finding are each a difference of content where the context is the same (Section V).
Continuous symmetry	A symmetry and a continuity at the two levels of one theory.	A Lie group of symmetries, with a conserved quantity for each continuous global symmetry.	The pairing that Proposition 2.2 requires of every theory, with Noether's theorem as the bridge between its levels.
Continuously symmetric	Both characters perfected in what one comparison finds: a forbidden pairing.	No single term.	The first of the two pairings the theorem forbids.
Perfected	Holding exactly, at the foundation of a description.	Exact, as against approximate.	An exact symmetry of a model holds of the model; the theorem concerns what a comparison finds.

From here on, “symmetry”, “continuity”, “asymmetry” and “discontinuity” carry the book’s sense. Where physics’ sense is meant, the text names the mathematical object: a symmetry group, the parameters of a group, a smooth manifold, a discrete spectrum.

Chapter 2 states the three axioms that order the roles within every comparison, and the two levels of every physical theory.



Chapter 2

The Three Axioms and the Two Levels of a Theory

The three roles within every comparison, the axioms that relate them, and why every physical theory has a formalism and a set of findings in conjugate modes

This chapter states premises P10 to P14 and establishes two propositions. Every comparison carries three roles, the omniscient, the immanent and the transcendent, and the three axioms of the Immanent Metaphysics, quoted here in their exact wording, relate the three. In the focus of measurement, I identify the roles with the record, the act and the stance of a measurement. Which concept stands in which role depends on the focus of the argument being made. I state the rule, and list every focus that I use (P11, Table 2.1). What exists stands as structure, and what is real is given in an actual act of comparison. From these premises the chapter establishes that every physical theory has two levels, its formalism and its findings, and that the two levels are always in conjugate modes.

I. The Three Roles

A modality is one of the three roles that the parts of a comparison play with respect to one another. Each of the three pairs of intrinsics of P4 plays a role with respect to the other two, and the same three roles appear wherever comparison occurs.

P10 (IM ground; *An Immanent Metaphysics*, aphorism 1.2-6). There are three modalities: the omniscient, the immanent and the transcendent. The omniscient modality is the role of that which is known, that which is structural, that which is pattern: what can be laid out and held in view as a whole. The immanent modality is the role of that which mediates, that which is processual, that which is happening: the act itself. The transcendent modality is the role of that which is potential, that which exceeds any particular instance: the stance and the possibility from which an act proceeds.

P11 (IM ground; *An Immanent Metaphysics*, aphorism 1.41-2). Axiom III alone does not fix which concept stands in which role: "Axiom III does not in itself identify which domain essential concept is associated with which modality, but only that such associations are always strongly definable." I take the rest of P11 as this book's own premise: which concept stands in which role depends on the focus, the context and frame of the argument being made. Arguments are not all aspects or variations of one argument, so no triple of concepts is expected to take the same binding in every focus. Three concepts form a triple where there is one identified focus in which all three axioms hold of them. In another focus an argument may assert only Axiom III, and then the three concepts are only distinct, inseparable and non-interchangeable. Or it may assert Axioms III and II and decline to pin the immanent: the three concepts then keep the cyclic order of Axiom II, and each of the three rotations of the binding is allowed.

The names of the modalities carry no theological meaning. They name roles, and P11 says that the concepts alone do not fix the roles. Aphorism 1.41-2 also says what each axiom contributes to a binding. Axiom I "has the effect of naming/identifying the term with an immanent modal associ-

ation", and Axiom II "distinguishes the omniscient modality from the transcendent modality". So a focus that asserts all three axioms pins the whole binding, and a focus that asserts fewer pins less. Table 2.1, in Section III, lists every focus this book uses, the binding in each, and the axioms each asserts.

II. The Three Axioms

P12 (IM ground; *An Immanent Metaphysics*, aphorism 1.3-1).

Axiom I. The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another.

Axiom II. A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent.

Axiom III. The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable.

Five terms of the axioms need their exact sense. A class is a kind, standing ready as a whole. An instance is one particular case of a kind, here and now. In Axiom III the bracket means that the statement holds both of the classes and of the instances; in Axiom I, "and/or" means of either one and of both together. Two things are conjugate when each can be perfected only at the expense of the other, and neither is prior to the other.

The three statements are three facets of a single structural fact, and they are not independent propositions. In plain words, Axiom I says that the act comes first, and that structure and stance are conjugate. Axiom II

says that the stance comes first, then the finding, then the finding laid out as a record, and the record shapes the next stance. Axiom III says that the three roles are distinct, that they are inseparable, and that none can do the work of another. The order of Axiom II is one of precedence, and it is not an order of clock time: the cycle can be entered at any point, and its direction cannot be reversed.

Each axiom is met in the structure of comparison already stated. Axiom III is met in P4: the three pairs of intrinsics are distinct, inseparable, and none can stand in for another. Axiom II is met in the order in which the pairs engage: a stance, then a finding, then the finding laid out as structure, which reshapes the stance from which the next comparison is made. Axiom I is met in the act of comparison being prior to the compounds formed from it, and in the conjugacy of symmetry and continuity, the characters that the focus of the two laws binds to the omniscient and the transcendent (Table 2.1). By P9 and the definition of conjugacy, symmetry and continuity are conjugate: each can be perfected in a comparison only where the other is not.

The three roles, the axioms and the cycle of Axiom II are set out for the general reader in *Two Theories, One World*, Chapter 6, "Three Roles and the Order of Finding".

III. The Roles in the Focus of Measurement

P13 (identification). In the focus of measurement, I identify sameness and difference as immanent, since the verdict of sameness or difference is given in the act. I identify content and context as omniscient, since they are the finding laid out as structure. I identify the subjective and the objective as transcendent, since they are the side from which the measurement is made, its stance, and the side about which it is made. In physics, I identify the omniscient modality with conservation laws, with structure, and with the content of measurement. I identify the immanent modality with interaction, with force, and with the process of measurement. I identify the transcendent modality with possibility, with the space of configurations, and with the boundary conditions within which events occur.

The observation that bears on P13 is the anatomy of every measurement. A measurement is an interaction, an event at a place and a time; it leaves a record, which can be laid out, copied and compared with other records; and it is made from a prepared setting, against a range of outcomes that could have occurred. Every measurement known leaves a record and is made against a range of possible outcomes, and so each one agrees with P13. P11 makes the binding of P13 a matter of focus. The focus of measurement is the identified focus in which all three axioms hold (Section II), so the three pairs of intrinsics form a triple. Other foci bind the three roles to other concepts, and in every focus that asserts Axiom II the direction of its cycle is the same.

Table 2.1 lists every focus this book uses, the concept that stands in each role, and the axioms the focus asserts.

Focus	Where it is used	Omniscient	Immanent	Transcendent	Axioms asserted
Measurement	P13; Chapters 2 and 11	content and context	sameness and difference	the subjective and the objective	I, II and III (Section II)
Physics at large	P13; Chapter 9	conservation laws; structure; the content of measurement	interaction; force; the process of measurement	possibility; the space of configurations; boundary conditions	I, II and III
The use of a theory	Proposition 2.1; Chapter 3	the formalism	each finding	the prepared setting, with its range of possible outcomes	I, II and III; Proposition 2.1 uses Axiom III
The two laws	P6; Proposition 5.7; Chapter 15, note 2	content, held to the law of symmetry	the comparing	context, held to the law of continuity	I, II and III (the binding follows *An Immanent Metaphysics*, aphorisms 1.83-2 and 1.83-3, and is set out in Volume 1, Chapter 6, Section V)

Focus	Where it is used	Omniscient	Immanent	Transcendent	Axioms asserted
The six measures	Table 5.1	pattern; space	force; time	probability; possibility	I, II and III, by an identification on which no argument of the book rests
Time	P19; Chapters 10 to 13	geometric time	event time	evolution time	I, II and III (Chapter 10)
A program of quantum gravity	Chapter 14, note 1	not used	each calculation of the program	its founding commitment	II and III; the note's conclusion holds in each of the three rotations
Counting the information of a region	Chapter 17, Section IV	the count made cell by cell, from a vantage that holds the whole region	the count made from a place, through interaction across the boundary	not used	I and III

Two pairs of rows may look opposed, and each binding belongs to its own focus. In the focus of the use of a theory, the quantum state is part of the formalism laid out as structure, and it stands in the omniscient role. In the focus of time, the state is what evolution time carries, the range of what could be found, and it stands in the transcendent role (Chapter 10, Definition 10.3). In the focus of the two laws, symmetry, the character that

the law of content requires, belongs to the omniscient role, while in the focus of time the omniscient role is taken by geometric time, whose findings are continuous and asymmetric. That symmetry belongs to the omniscient role is a binding of a law's character to a role. That the findings of geometric time are continuous and asymmetric is a statement of the character of what one bound concept finds. The binding and the statement are made in two foci, the focus of the two laws and the focus of time. The note to Chapter 14 shows the case that P11 allows where only Axioms III and II are asserted: a founding commitment precedes every calculation made under it in each of the three rotations of the binding, so the note's conclusion needs only the order of Axiom II.

IV. What Exists and What Is Real

P14 (IM ground; *An Immanent Metaphysics*, aphorisms 1.76-6 and 1.76-8). To say that something exists is to say that it stands as structure: timeless, the same in every instance, like the pattern of a law, or the state of a system at an instant. To say that something is real is to say that it is given in an actual act of comparison: temporal, occurring now, with the future still open. A law exists; a measurement is real.

Two kinds of reasoning follow from P14, one for what is said to exist and one for what is real. Reasoning about what exists is held to validity: within its own structure a deduction is certain. Reasoning about what is real is held to soundness: it must also ask whether its premises hold in the world, and the passage from a structure to the world carries risk, because the structure and the world are two domains.

Every deduction fixes its subject from outside. In physics that fixing is a property of the model and not of the world. The physical world is the whole of what comparison can reach from within it, and every measurement of it is made from inside, by a comparer that is part of it. History is structure in the sense of what exists. The past is known only through the impressions it leaves in the present, and from those impressions physics reconstructs one consistent history, which exists; the act of measuring, here and now, is real.

V. A Physical Theory Has Two Levels

The formalism of a physical theory is the mathematical structure in which the theory is written: its space of states, its equations, and the symmetry principles that constrain them. A finding is the outcome of one actual comparison (Chapter 1, Section I). The outcomes that a theory says such comparisons can have form its range of findings, and the findings of a physical theory are the outcomes of the actual comparisons against which its predictions are checked, each one member of that range.

Proposition 2.1. Every physical theory has two levels: its formalism, which exists, and its findings, which are real. The two levels are distinct, inseparable and non-interchangeable.

Argument. First, a physical theory is used only by comparing what its formalism predicts with what is found, and by P1 and P2 that use is an act of comparison. Second, the formalism stands as structure, the same in every application, so by P14 the formalism exists; each finding is given in an actual act of comparison, so by P14 each finding is real. Third, the use of a theory is a measurement, and it takes the focus of the use of a theory (Table 2.1), the focus of measurement applied to a theory, in which all three axioms hold: by P10 and P13 the formalism, laid out as structure,

stands in the omniscient role, each finding, given in the act of comparison, stands in the immanent role, and the setting prepared for the comparison, with its range of possible outcomes, stands in the transcendent role. Fourth, by Axiom III (P12) the two levels, in their two roles, are distinct, inseparable and non-interchangeable. A formalism with no findings is a piece of mathematics and not a physical theory; a set of findings with no formalism is a record and not a theory; and neither level can stand in for the other.

Rests on: the Root Tautology (P1); the identification of every finding with an act of comparison (P2); the three modalities (P10); the three axioms (P12); the roles in the focus of measurement (P13); the distinction between what exists and what is real (P14).

A mode is the compound character that governs one level of a theory. I call the mode of a theory's findings its constitutive mode: it is the character of the theory's range of findings, and each finding is one member of that range, given in an act. I call the mode of its formalism its instrumental mode.

Proposition 2.2. The constitutive mode and the instrumental mode of a physical theory are conjugate. Where the findings are governed by continuity, the formalism is governed by symmetry; where the findings are governed by symmetry, the formalism is governed by continuity.

Argument. First, by Proposition 2.1 the use of a theory is a comparison of its formalism with its findings. Second, if the formalism were governed by the same character as the findings, the comparison of the two would apply that character to itself. The criterion of no self-application (Chapter 1, Section VI) excludes that conjunction: the formalism would find in its findings only its own character, and the comparison would yield no verdict. Third, by P9 the findings, exact at their foundation, are continuously

asymmetric or discontinuously symmetric, so they are governed by continuity or by symmetry; a formalism exists (P14) and so is the same in every instance, and of the four compounds of P8 only continuity and symmetry find a sameness of content, so a formalism is governed by one of these two (Section VI, first question, sets the reason out at length). P9 makes symmetry and continuity conjugate, so the formalism is governed by the conjugate of the character that governs the findings. Fourth, the proposition places symmetry and continuity at two levels, and so it does not join them in one mode. The Incommensuration Theorem excludes the two perfected together in what one comparison finds. Proposition 2.1 places the formalism and the findings at two levels, which Axiom III keeps distinct, and a symmetry at one level with a continuity at the other is the pairing the theorem permits.

Rests on: the four compounds (P8); the Incommensuration Theorem (P9); the three axioms (P12); the distinction between what exists and what is real (P14); Proposition 2.1.

A continuous symmetry of physics is this pairing of a symmetry at one level with a continuity at the other. In quantum mechanics the rotation group acts on the states with continuous parameters, in the formalism, and what a measurement finds are the discrete values of total angular momentum, which label what the rotations leave unchanged and so are symmetric. In general relativity the symmetry, invariance under smooth changes of coordinates, belongs to the formalism, and the continuity is that of the geometry found. Noether's theorem joins the two levels: to each continuous symmetry of a formalism it assigns a conserved quantity that a comparison can find (Noether 1918).[1]

VI. Questions

Why is a formalism governed by symmetry or by continuity, and never by asymmetry or discontinuity? A formalism exists (P14), and what exists is the same in every instance. Of the four compounds of P8, two find a

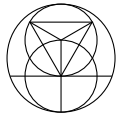
sameness of content: continuity, in a sameness of context, and symmetry, across a difference of context. A level that is the same in every application is therefore governed by one of these two, and Proposition 2.2 says which one, given the character of the findings.

A quantum formalism also carries symmetry groups. Is its mode then mixed? A formalism uses structures of several kinds, and its mode is the character on which its predictions depend. Quantum mechanics computes its predictions in a space of states with continuous amplitudes, evolving in continuous time, and its symmetry groups act on that space as transformations with continuous parameters. The symmetry those groups express appears in the findings, as invariants with discrete values. So the formalism is governed by continuity, and the symmetry is found.

A formalism is learned, checked and computed by acts of comparison. Is it not also real? The acts by which a formalism is learned, checked or computed are real, and what those acts establish exists. Proposition 2.1 concerns the formalism as the structure from which a theory's predictions are drawn. The computation of a prediction is an act whose result is then compared with a finding, and that comparison is the use of the theory that Proposition 2.1 describes.

The axioms are found in comparison, and comparison is described by the axioms. Is the ground circular? A circle needs an order: something assumed first and derived later. The three pairs of intrinsics, their roles and the pattern of their relations are present together in every comparison, and no one of them comes first. The axioms state that pattern, and they are found in every comparison because every comparison has that pattern.

Chapter 3 states the principle of Proposition 2.2 in its general form, for any framework and the content it investigates, and checks it against six established theories.



Chapter 3

The Cross-Modal Principle, Checked Against Six Theories

Why every investigation probes its content in the conjugate mode, how the principle reads in six established theories, and the residual each theory leaves

This chapter states the cross-modal principle in its general form and checks it against six theories. Any investigation is a comparison of a framework with a content, and the two are always in conjugate modes: a symmetric framework probes continuous content, and a continuous framework probes symmetric content. The reach of the framework into its content is dense and never exhaustive, so every theory leaves a residual. Newtonian mechanics, classical electrodynamics, general relativity, quantum mechanics, quantum field theory and thermodynamics all have conjugate modes, and the residuals of the two theories that describe the world at its foundation are the measurement problem and the singularity. Premises P15 to P17, which I add here, complete the ground.

I. The Puzzle the Principle Answers

Each of the two fundamental theories of physics seems to contain both characters. Quantum mechanics has continuous wavefunctions and discrete spectra. General relativity has invariance under smooth changes of coordinates and a smooth metric. A reading that assigned one character to each theory would have to say which of its features carries the theory's

character, and Chapter 2 supplies that distinction: a theory has two levels (Proposition 2.1), and the two levels are in conjugate modes (Proposition 2.2). Here I state the principle in its general form, apply the principle to physical theories, and check the principle.

II. The General Form

A framework is the structure by which an investigation is carried out. The content of the investigation, in the sense of P3, is what its verdicts are on. The reach of a framework into its content is dense when the framework approaches every finding of the content as closely as wished. A residual is what the framework approaches as closely as wished and does not contain.

Proposition 3.1 (the cross-modal principle). Framework and content are always in conjugate modes. A symmetric framework probes continuous content; a continuous framework probes symmetric content. The reach of the framework into its content is dense but never exhaustive.

Argument. For the first part, the argument is that of Proposition 2.2, made for any investigation. By P1 an investigation is a comparison of the framework with the content. A framework in the same mode as its content would apply that mode to itself, which the criterion of no self-application excludes (Chapter 1, Section VI). A framework exists (P14), so it is governed by symmetry or by continuity (Proposition 2.2, third step). So the framework takes the mode conjugate to the content's, and by P9 symmetry and continuity are the conjugate pair.

For the second part, the argument has two halves. The reach is dense because a framework in the conjugate mode can be refined without limit: a continuous framework assigns values to each discrete outcome to any

precision required, and a symmetric framework carries a smooth content along any path to any precision required. The reach is never exhaustive because a framework that contained its content exactly would carry the content's character perfected together with its own conjugate character, in one mode, which the Incommensuration Theorem (P9) excludes; and by Axiom III (P12) the two levels cannot be interchanged. So every framework leaves a residual in its content.

Rests on: the Root Tautology (P1); the Incommensuration Theorem (P9); the three axioms (P12); the distinction between what exists and what is real (P14); Proposition 2.2.

Proposition 3.1 is a statement about what comparison requires, and it holds of every description exact at its foundation. Its empirical consequence is a prediction about the findings of physics: they are reproduced at the foundation only by a theory of the conjugate kind, and a theory whose formalism and findings carry one character, as a cellular automaton would, does not reproduce them at the foundation (Section VI, fourth question).

III. The Principle Applied to Physical Theories

For a physical theory the framework is the formalism and the content is the set of findings, so the instrumental mode is the mode of the framework and the constitutive mode is the mode of the content (Chapter 2, Section V). Each mode is read from features that a physicist can check.

Discrete findings mark a discontinuously symmetric constitutive mode: values separated by gaps, quantum numbers taking exact integer or half-integer values, and outcomes sharply distinguished with no intermediate case. The discreteness is the discontinuity; the exactness and classifiability of the labels is the symmetry. Continuous findings mark a continuously asymmetric constitutive mode: values that vary smoothly with

no gaps, with no structural preference for any particular value, and generic outcomes that each differ from all others. The smoothness is the continuity; the generic non-repetition is the asymmetry.

A formalism that depends on continuous structures has a continuous instrumental mode: a space of states of infinite dimension, evolution in continuous time, amplitudes that vary smoothly. A formalism that depends on symmetry structures has a symmetric instrumental mode: gauge invariance, laws of transformation of tensors, groups that constrain the equations.

P15 (established physics). The six theories of Table 3.1 are established in their domains, and their findings and formalisms have the features that Table 3.1 lists. Among them, general relativity, with Einstein's field equations, and quantum theory, with quantum field theory, are taken as established physics throughout this book.

P16 (identification). I identify the constitutive mode of each theory from the features of its findings: the findings of quantum mechanics and of quantum field theory are discontinuously symmetric, and the findings of Newtonian mechanics, classical electrodynamics, general relativity and thermodynamics are continuously asymmetric.

The observations that bear on P16 are the features themselves. Bound quantum systems show discrete spectra, and the quanta of a field are counted in whole numbers; the trajectories, fields, metrics and equations of state of the four classical theories vary smoothly, and no discreteness has been found in them within their domains. P16 needs one observable feature from each theory, and the cross-modal principle then fixes the theory's instrumental mode as the conjugate. The instrumental modes in Table 3.1 are found independently, from the formalisms as written, and so they check the principle.

IV. Six Theories

Table 3.1 reads both modes of each of the six theories.

Theory	**What it finds (constitutive mode)**	**Its formalism (instrumental mode)**	**Conjugate**
Newtonian mechanics	Continuous positions and momenta; smooth trajectories, generically all different; no quantized observable. Continuously asymmetric.	Galilean invariance; the symplectic structure of phase space, preserved by canonical transformations; the algebra of Poisson brackets. Symmetric.	yes
Classical electrodynamics	Continuous electric and magnetic fields of any strength; smooth propagation of waves. Continuously asymmetric.	Gauge invariance under the group $U(1)$; the Lorentz covariance of Maxwell's equations. Symmetric.	yes
General relativity	A smooth metric and continuous curvature; a generic spacetime has no isometry; no discrete observable. Continuously asymmetric.	Invariance under diffeomorphisms; local Lorentz invariance; the transformation laws of tensors. Symmetric.	yes
Quantum mechanics	Discrete eigenvalues of bound systems; exact quantum numbers; one outcome from among the alternatives at each measurement. Discontinuously symmetric.	A Hilbert space; unitary evolution in continuous time; complex amplitudes that vary smoothly. Continuous.	yes

Theory	What it finds (constitutive mode)	Its formalism (instrumental mode)	Conjugate
Quantum field theory	Quanta counted in whole numbers; electric charge quantized and exactly conserved; particle species labeled by exact spin and charge. Discontinuously symmetric.	Fock space built on continuous modes; the path integral over continuous field configurations; the smooth running of couplings with energy. Continuous.	yes
Thermodynamics	Continuous temperature, pressure and volume; smooth equations of state; entropy that increases in an isolated system. Continuously asymmetric.	The Legendre duality among the thermodynamic potentials; the Maxwell relations; homogeneity of degree one in the extensive variables. Symmetric.	yes

All six theories have conjugate modes. The rows rest on the standard forms of the theories. Quantum mechanics is read in its Hilbert-space form (von Neumann 1932). The classical theories are read through their symmetry principles and through Noether's theorems, which tie those principles to conserved quantities (Noether 1918). Thermodynamics is read in its formal structure, whose potentials are related by Legendre transformations and whose fundamental relation is homogeneous of degree one (Callen 1985).[1] The same reading holds of quantum chromodynamics, whose findings are a discrete spectrum of hadrons with integer baryon number, and whose formalism is a continuous gauge field.

Two rows describe one domain in opposite constitutive modes. Classical electrodynamics finds continuous fields and is written in a symmetric formalism; quantum electrodynamics, the quantum field theory of the same domain, finds discrete quanta and is written in a continuous formalism. Both modes change together between the two rows, and the pair

stays conjugate. That change of both modes is the subject of Chapter 4, "What Quantization Is", which reads quantization as a mode exchange of formalism and findings together.

V. The Residual

P17 (established physics). Under the conditions of the singularity theorems, a spacetime that satisfies Einstein's field equations contains geodesics that cannot be extended (Penrose 1965; Hawking and Penrose 1970).

Proposition 3.2. Each theory meets a residual where the character of its findings exceeds what the mode of its formalism can carry. For quantum mechanics the residual is the measurement problem: the discontinuity of a definite finding, which a continuous formalism does not contain. For general relativity the residual is the singularity: geodesics that cannot be extended, along which, in the cases worked out in full, the curvature grows without bound, and which a symmetric formalism does not carry past the edge of its domain.

Argument. For quantum mechanics, the continuous formalism probes the discrete findings with very great precision. The magnetic moment of the electron has been measured to 0.13 parts per trillion, and the prediction of quantum electrodynamics agrees with the measurement to about one part in 10^{12} (Fan, Myers, Sukra and Gabrielse 2023).[2] The formalism yields the probability of each discrete outcome, and it does not itself contain the finding: the selection, by the comparer that finds, of one outcome from among its possibilities, by which something possible becomes manifest for that comparer. The Born rule, the probability rule of quantum mechanics, is stated beside the unitary evolution, and it is the bridge between the two levels. A finding is made by the comparer that finds, and

it changes no geometry, here or anywhere. What a distant finding changes for another comparer is that comparer's record, when a signal carrying the finding arrives, at the earliest on the light cone. Chapter 11, "The Measurement Problem as a Cross-Modal Residual", sets out the problem in its standard forms and reads the Born rule as that bridge.

For general relativity, the symmetric formalism carries the smooth geometry with great power, along every geodesic that can be extended. By P17, under the conditions of the singularity theorems, some geodesics cannot be extended, and in the cases worked out in full, such as the interior of a black hole of definite mass, the curvature grows without bound along them. The Planck length, about 1.6×10^{-35} m, is the length at which an object's gravitational size equals its quantum size. An object's gravitational size is the length at which its own gravity becomes strong, and its quantum size is the length within which it cannot be confined without creating further particles (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", sets both out). The smooth description holds below the Planck curvature, the curvature whose radius is the Planck length, so the residual of general relativity is met where the curvature reaches that value. There the smooth description reaches the edge of its domain; asking for the geometry at the singularity applies the concept of a smooth geometry where it does not belong. Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", sets out the Planck scale as the place where the gravitational and the quantum descriptions of an object meet.

The two residuals are conjugate. The residual of quantum mechanics is a discontinuity, the definite finding, met by a continuous formalism. The residual of general relativity is an asymmetry without bound, the growth of curvature along an incomplete geodesic, met by a symmetric formalism. Each residual lies in the theory's own instrumental mode: it is where a formalism, refined without limit in that mode, reaches the limit of the mode.

Rests on: the Incommensuration Theorem (P9); the three axioms (P12); the constitutive modes of the theories (P16); the singularity theorems (P17); Proposition 3.1.

VI. Questions

Quantum mechanics also has continuous spectra, such as the momentum of a free particle. Are its findings then continuous? A continuous spectrum is a feature of the formalism: a range of values over which the formalism assigns probabilities. A measurement finds one outcome, registered by a detector as one count among discrete alternatives. The bound systems of quantum mechanics show discrete spectra with exact quantum numbers. The findings are discontinuously symmetric; the continuous range belongs to the formalism that assigns them their probabilities.

Masses, coupling constants and mixing angles are exact invariants whose values lie on a continuum. Are they continuous findings? An invariant is found as one value, the same in every context in which it is measured: the mass of the electron is one value in every laboratory and every frame, and a coupling measured at a stated energy is one value wherever it is measured. That sameness across every difference of context is symmetry, and each such constant is found as one exact value and not as a quantity that varies from one finding to the next. The continuum is the range of values that the formalism leaves open, and the theory finds one point of that range. The running of a coupling with energy is a smooth function in the formalism (Table 3.1), and at each energy the coupling is found as one value. The constants label a discrete list of species and interactions, each with its own exact value, so what is found is discontinuously symmetric, and the continuous range belongs to the formalism.

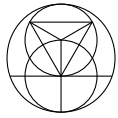
Newtonian mechanics conserves energy exactly. Is that not a symmetry found by a classical theory? The conservation law is the formalism's symmetry, invariance under translations in time, carried to the findings by Noether's first theorem. A measurement finds a value of the energy from a continuum, set by the initial conditions. The trajectories found from different initial conditions are generically all different. The findings are continuously asymmetric; the symmetry lies in the formalism and is the conjugate the principle requires.

Is the principle met by every conceivable theory, and so empty? The principle has content. It excludes, at the foundation of physics, a theory whose formalism and findings carry one character. A cellular automaton on a lattice, whose states are discrete and whose rule is the same at every site and every step, is the form such a theory would take. 't Hooft (2016) develops an account of quantum mechanics with a deterministic automaton of that kind beneath it. The principle predicts that the established findings of physics are not reproduced at the foundation by a theory of that form, and each of the six established theories of Table 3.1 is of the conjugate kind. A discrete rule can reproduce the statistics of quantum outcomes to any precision chosen in advance, and no discrete rule can be the continuous evolution itself, the continuous formalism that the cross-modal principle pairs with discrete findings. For any one fixed rule, beyond its smallest step, a sensitive enough measurement eventually tells the rule's statistics and the world's apart.

Part I is now complete: premises P1 to P17 state the ground, and Propositions 2.1 to 3.2 state the two levels of a theory, their conjugacy and their residuals. Part II applies them, and Chapter 4 begins with what quantization does to the two levels of a theory.

PART II

THE DIAGNOSIS



Chapter 4

What Quantization Is

Symmetry governance, continuity governance, and quantization as a mode exchange

This chapter establishes four propositions. What is governed by symmetry is found in discrete classes with exact labels (Proposition 4.1). What is governed by continuity is found as smooth values that do not repeat (Proposition 4.2). Every fundamental physical description therefore has one of two mode pairs, fixed by whether it describes content or context (Proposition 4.3). Quantization is the passage of a description from the one mode pair to the other, in which its findings and its formalism change character together (Proposition 4.4). At the close of this chapter, I check the four propositions against the classical and the quantum theories of matter.

I. The Ground This Chapter Uses

Four results of Part I carry this chapter. The first of the four results of Part I is the Incommensuration Theorem (P9), which I set out in Chapter 1, "Comparison, the Intrinsics and the Incommensuration Theorem". Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric. The two pairings that the Incommensuration Theorem forbids are continuity with symmetry and asymmetry with discontinuity. The second result of Part I is Proposition 2.1, that a physical theory has two levels: its formalism and its findings. The formalism of a theory is the mathematics in which the theory is written. Each finding of a

theory is the outcome of one actual comparison, and each finding is one member of the range of findings that comparisons can have according to the theory. Part I calls the mode of a theory's findings its constitutive mode and the mode of its formalism its instrumental mode, and it found the two modes conjugate (Proposition 2.2; Chapter 2, "The Three Axioms and the Two Levels of a Theory"). The third result of Part I is the cross-modal principle (Proposition 3.1), which I set out in Chapter 3, "The Cross-Modal Principle, Checked Against Six Theories". The cross-modal principle is that the formalism and the findings of every fundamental theory are in conjugate modes. A symmetric formalism serves continuous findings, and a continuous formalism serves symmetric findings. The fourth result of Part I is P6, the two laws of *An Immanent Metaphysics* (aphorism 1.83-5), whose reason I give in Chapter 1, Section IV. Every proposition of this chapter applies the two laws. The law of symmetry applies to all that is of content, and the law of continuity applies to all that is of context.[1]

I call the pair formed by a description's constitutive mode and its instrumental mode the mode pair of that description. A mode pair is written with the findings first: discontinuously symmetric findings with a continuous formalism is one mode pair, and continuously asymmetric findings with a symmetric formalism is the other.

Two terms name a phenomenon by the role it plays across all the comparisons of physics. A content-modal phenomenon is one that comparisons find as their content, within a context distinct from it: the thing judged the same or different. A context-modal phenomenon is one that is the context of every physical comparison, the comparisons whose content is a measurement of it included: the respect in which the things compared are two, within which every content is found.

The strongest objection to these terms comes from the ground itself. A gravitational-wave detector, a measurement of redshift and a measurement of the deflection of light each take the geometry as what their verdict is on, and P11 makes the binding of roles depend on the focus of the

argument being made. On that reading the geometry is the content of those comparisons, and gravity would be content-modal as well. The answer lies in what such an instrument finds. An interferometer finds a change in the phase of its light, and the light is content, compared against the arena as its context. What the measurement registers of the geometry is a relation within the context, the arena at one place and time against the arena at another, and that relation is found continuous and asymmetric, as Proposition 4.2 below requires of context. A small wave of the geometry about a smooth background can be described as if it were content, and that description can carry quanta; they are quanta of the description only, never quanta of the arena (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", sets that description out). The arena, as the background against which every comparison is made, stays context, and the two terms above are defined by that role. Proposition 5.6 states the general form: every measurement of the arena compares the arena with itself, so its verdict is a relation within the context.

The words "symmetry" and "continuity" keep the sense of P8, as Table 1.3 sets it beside the sense of physics: each names a character of what a comparison finds. The groups of transformations and the smooth functions that physics also calls symmetric and continuous belong to the level of the formalism (Chapter 2), and where this chapter means a group of physics, it names the group.

II. Symmetry Governance and Discrete Findings

Proposition 4.1. The findings of a content-modal phenomenon, perfected at their foundation, are discontinuously symmetric: they fall into classes with nothing between them, and the label of each class is the same across every change of context.

Argument. By the law of symmetry, what is found of a content is what stays the same where the context differs. Such a finding is an invariant. Invariants sort contents into classes, each class holding what the symmetry carries into itself. By the Incommensuration Theorem, a finding whose symmetry is perfected cannot also perfect continuity, since continuity with symmetry is a forbidden pairing, so the one surviving conjunction that contains symmetry contains discontinuity: the invariants that sort a content into classes take separated values. Two classes are found in one and the same setting of comparison, the same system, preparation and apparatus, and they differ with nothing between them. So the separation between two classes is a difference of content where there is a sameness of context: a discontinuity, in the sense of P8 that Chapter 1, Section V, shows to include the gaps of a spectrum.

Rests on: the law of symmetry (P6); the definitions of symmetry and discontinuity as compounds of the intrinsics (P8); the Incommensuration Theorem and its forbidden pairing of continuity with symmetry (P9).

The established physics of content agrees at every point. The forces of particle physics are gauge theories of the compact groups U(1), SU(2) and SU(3). By the theorem of Peter and Weyl (1927), every irreducible unitary representation of a compact group is finite-dimensional, and the inequivalent representations form a discrete set.[2] The labels that classify matter are accordingly discrete: spin comes in multiples of one half, electric charge in multiples of one third of the electron's charge, and color is the label of a representation of SU(3). Each label is invariant: the charge of an electron is the same in every frame and under every gauge transformation. The energy levels of bound systems, from atoms to nuclei to hadrons, are separated by gaps.

The discrete findings of content enter the argument above as a consequence, and not as a premise taken from observation. The law of symmetry and the Incommensuration Theorem together give the discrete findings of quantum theory, and the observed spectra serve as the check.

For SU(2) and SU(3) the check by the theorem of Peter and Weyl is direct. For electric charge, the compactness of the group U(1) and the steps of charge are one fact: the group is compact exactly where every charge is a whole multiple of one least charge, and physics takes the group to be compact because charge is observed in steps. The law of symmetry gives that fact its place, since a charge is a label of content. This account claims no explanation of the steps of charge beyond those that physics already has, Dirac's argument from a magnetic monopole (Dirac 1931) and the embedding of the charge in a simple group of grand unification (Georgi and Glashow 1974).

A specialist will ask about the quantities of matter that take continuous values, such as the momentum of a free particle, and will put the question in its sharpest form with the Poincaré group, the group of the flat arena. That group is not compact. Its irreducible unitary representations are classes with exact membership, labeled by the squared mass, which takes continuous values, and by the spin; the states within each class are labeled by a continuous momentum. So exact membership alone opens no gap between classes, and the argument above rests on the Incommensuration Theorem for that reason. The continuous labels of a representation of the Poincaré group describe a content's relation to the translations and rotations of the arena, and the species of particle found in nature carry a discrete list of rest masses, spins and charges. Momentum is the quantity a content carries with respect to the translations of the arena. The arena is space and time together, in which everything that is measured is measured. The continuous range of momentum belongs to that relation, and so to the context in which the content is found. The labels that classify the content itself are discrete. A particle confined to a bounded region has a discrete spectrum of momentum, and the continuous range returns only as the bound of the region is removed.

III. Continuity Governance and Smooth Findings

Proposition 4.2. The findings of a context-modal phenomenon, perfected at their foundation, are continuously asymmetric: they take smooth values with no gaps, and a generic finding differs from every other.

Argument. By the law of continuity, what is found of the context is a sameness of content where there is a sameness of context: nothing in it is sudden, and neighboring settings give neighboring values. A quantity of that kind does not sort into classes. Its values form a continuum, and no value is preferred over its neighbors. By the Incommensuration Theorem, a finding whose continuity is perfected cannot also perfect symmetry, and the one surviving conjunction that contains continuity contains asymmetry. Generic findings of the context therefore differ from place to place and from moment to moment.

Rests on: the law of continuity (P6); the definitions of continuity and asymmetry as compounds of the intrinsics (P8); the Incommensuration Theorem and its forbidden pairing of continuity with symmetry (P9).

General relativity agrees at every point. Space and time form a smooth four-dimensional manifold, the metric varies smoothly over it, and the curvature is a smooth tensor field. Every gravitational finding is a real number with no structural preference for any value: a redshift, a deflection of light, the delay of a signal, the decay of the orbit of a binary pulsar. A generic solution of the Einstein equations has no Killing vector field, so no two regions of a generic spacetime are related by an exact symmetry. The exactly symmetric solutions that physics uses, such as the spherically symmetric geometry of Schwarzschild, are the geometries of idealized sources: one body, alone, and perfectly round.

The objection from loop quantum gravity deserves its full strength. That program finds discrete spectra for the operators of area and volume (Rovelli and Smolin 1995; Ashtekar and Lewandowski 1997), and its proponents read those spectra as a discreteness of geometry itself. The spectra are spectra of operators defined after the arena has been quantized. They belong to the step whose coherence Chapter 6 examines, so they can count against Proposition 4.2 only on the assumption of that step. No observation has found a discrete area or volume. The arrival times of high-energy photons from distant sources, which many models of a granular arena would spread according to energy, show no such spread at the sensitivities reached (the evidence is set out in Chapter 19, "Predictions, Evidence and Decisive Experiments").

IV. Two Mode Pairs, and No Third

Proposition 4.3. (a) The findings of content, at their foundation, are discontinuously symmetric, and the formalism that describes them rests on continuous structures. (b) The findings of context, at their foundation, are continuously asymmetric, and the formalism that describes them rests on symmetry structures. (c) These are the only two mode pairs of a fundamental physical description.

Argument. For (a): by Proposition 4.1 the findings of content are discontinuously symmetric, and by the cross-modal principle the formalism takes the conjugate character, which is continuity. For (b): by Proposition 4.2 the findings of context are continuously asymmetric, and by the cross-modal principle the formalism takes the conjugate character, which is symmetry. For (c): content and context are distinct and inseparable intrinsics of every comparison (P3, P4), so every phenomenon that physics finds stands in its comparisons either as content or as context. The two

laws assign symmetry to content and continuity to context. The Incommensuration Theorem leaves exactly two surviving conjunctions to receive them.

Rests on: Propositions 4.1 and 4.2; the cross-modal principle (Proposition 3.1); the six intrinsics, with content and context distinct and inseparable (P3, P4); the two laws (P6); the Incommensuration Theorem (P9).

Part (a) is a statement about content as a complete description finds it. A description of content can also be made in the other mode pair, with smooth findings and a symmetric formalism, as Newtonian mechanics and classical electrodynamics are made. Section VI shows that such descriptions hold within known limits, and that each limit falls where the mode pair of the description differs from the mode pair of its content.

Table 4.1 lists each step of the assignment with its source and its kind.

Step	**Source**	**Kind**
Content is governed by symmetry; context is governed by continuity	P6 (*An Immanent Metaphysics*, aphorism 1.83-5); reason: Chapter 1, Section IV	Ground
A comparison perfected at its foundation is continuously asymmetric or discontinuously symmetric	P9, the Incommensuration Theorem (Chapter 1)	Ground
The findings of content are discontinuously symmetric	Proposition 4.1	Follows from the two rows above
The findings of context are continuously asymmetric	Proposition 4.2	Follows from the first two rows
The formalism of a theory is in the mode conjugate to its findings	Proposition 3.1, the cross-modal principle (Chapter 3)	Follows from P1, P9, P12, P14 and Proposition 2.2; checked against six theories
There are two mode pairs and no third	Proposition 4.3	Follows from the rows above
Gravity is the context-modal phenomenon of physics	P18 (Chapter 5)	Identification, checked by four markers

Each proposition of this chapter follows from the ground of Part I and the two laws. The step at which the argument passes from the structure of comparison to a named phenomenon of physics is the identification of gravity as context-modal (P18), which Chapter 5 makes in the first person, with the observations that bear on it. In Chapter 3 I identified the constitutive mode of each theory from one observable feature of that theory: the discrete spectra of quantum theory and the smooth metric of general relativity. Propositions 4.1 to 4.3 give the same modes from the two laws, so the observable feature of each theory is now a check as well as a ground. The two laws and the reason why content comes in steps are set out in plain words in *Two Theories, One World*, Chapter 7, "What Is Measured, and Where".

V. Quantization as a Mode Exchange

Classical electrodynamics and quantum electrodynamics describe one domain, the electromagnetic interaction, in the two mode pairs. The findings of the classical theory are the values of the electric and magnetic fields, which vary smoothly and take any amplitude at any frequency: they are continuously asymmetric. Its formalism rests on symmetry structures: invariance under the gauge group U(1), covariance under the Lorentz group, and Maxwell's equations as the constraints those groups impose. The findings of the quantum theory are discrete: whole numbers of photons in each mode, and a conserved charge that comes in fixed units. Its formalism rests on continuous structures: the Fock space of the field, the path integral over field configurations, and the smooth flow of its couplings under the renormalization group.

Proposition 4.4. Quantization is a mode exchange: in one step, the findings of a description pass from continuously asymmetric to discontinuously symmetric, and the character its formalism holds exact against those findings passes from symmetry to continuity. The findings and the formalism are conjugate before the step and after it.

I call this passage the mode exchange.

Argument. By the cross-modal principle, the classical description is conjugate at its two levels, and so is the quantum description. A change of the findings alone, with the formalism kept, would leave discontinuously symmetric findings in a symmetric formalism: the formalism would apply to the findings the very compound they perfect. A formalism that applied to the findings the very compound the findings perfect would apply a compound to itself. The criterion of no self-application, in the enumeration of the Incommensuration Theorem, excludes the application of a compound to itself. So the formalism changes with the findings, from symmetric to continuous.

Rests on: the cross-modal principle (Proposition 3.1); the criterion that excludes the application of a compound to itself (Chapter 1, Section VI, within P9); Proposition 4.3.

A specialist will object that quantization keeps the symmetries and the calculus of the classical theory: quantum electrodynamics keeps the gauge group $U(1)$ and the Lorentz group, and it is written in the calculus, as Maxwell's theory is. Both formalisms keep those structures, and the mode exchange concerns which character the formalism holds exact against the findings. In the classical theory the exact symmetries of the equations place findings that vary smoothly. In the quantum theory a continuous space of states gives the odds of findings that come in exact steps. The mode of each formalism is the character on which its predictions of the findings depend (Chapter 2, Section VI, second question).

The mode exchange describes, at the two levels of a theory, what the standard procedures of quantization do. In canonical quantization, the classical phase space and its Poisson brackets give way to a Hilbert space, on which position and momentum act as operators whose commutator is i times the reduced Planck constant:

$$[\hat{x}, \hat{p}] = i\hbar \tag{4.1}$$

The symmetry structure of the classical formalism, its canonical transformations, gives way to the continuous structure of the Hilbert space and its unitary evolution. The findings, which were smooth values, become the eigenvalues of operators, discrete wherever the system is bound. Deformation quantization reaches the same theory by deforming the product of classical observables in powers of ħ (Bayen, Flato, Fronsdal, Lichnerowicz and Sternheimer 1978; Kontsevich 2003). The mode exchange agrees with each procedure and adds no procedure of its own. What it adds is the reason why the formalism must change when the findings change.[3]

The mode exchange of a content is made within an arena that stays as it is. The classical field and the quantized field are laid out in one space and time. The two descriptions differ in what they find of the field and in the formalism with which they find it, and they share the arena in which both are compared. Quantizing the electromagnetic field is right because that field is content, whose perfected character is discontinuous symmetry: the quantum theory is its foundational form, and the classical theory is its limit. Chapter 6 asks whether the arena itself admits the same step.

VI. The Check on Classical Theories of Matter

Table 4.2 checks Proposition 4.3 against six theories, asking whether each has the mode pair that the proposition assigns to its phenomena.

Theory	Its phenomena	Findings	Formalism	Mode pair
Quantum mechanics	Content	Discrete eigenvalues; exact quantum numbers	Hilbert space; unitary evolution in continuous time	As assigned
Quantum electrodynamics	Content	Photon numbers; charge in fixed units	Fock space; path integral	As assigned
Quantum chromodynamics	Content	Discrete hadron spectrum; colorless bound states	Path integral over a continuous SU(3) gauge field	As assigned
General relativity	Context	Smooth metric; continuous curvature	Diffeomorphism invariance; tensor transformation laws	As assigned
Newtonian mechanics	Content	Continuous positions and momenta; smooth trajectories	Symplectic structure; canonical transformations	Conjugate mode pair, valid within known limits
Classical electrodynamics	Content	Continuous electric and magnetic fields	U(1) gauge invariance; Lorentz covariance	Conjugate mode pair, valid within known limits

Four of the six theories have the mode pair that Proposition 4.3 assigns to their phenomena. Newtonian mechanics and classical electrodynamics describe content in the mode pair that belongs to context. Each is conjugate at its two levels, as Chapter 3 found, and each is a valid description within limits that physics has long known.

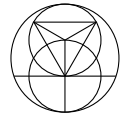
The classical theory of radiation in a cavity gives every mode of the field the same mean energy, so the energy density it predicts grows without bound as the frequency rises, the excess later named the ultraviolet catastrophe. Planck's law of 1900, published in full in Planck (1901), has no such excess, and it rests on energy exchanged in steps between the radiation and the oscillators of the walls. A classical electron in orbit about a nucleus radiates its energy continuously and spirals inward, so classical electrodynamics gives no stable atom; the stability of atoms, and their discrete spectra, come from the quantum description. The heat capacity of a solid falls toward zero at low temperature, against the constant value that classical equipartition gives, and the quantum description of the vibrations of its lattice gives the fall.

In each case the limit falls where the classical description meets the discreteness of its content, and the mode exchange removes the limit. I reason that this is the pattern Proposition 4.3 leads one to expect. A classical theory of matter describes content in the mode pair of context, which its content does not have at its foundation, so its reach into its content is dense and valid, and not exhaustive.

The same check shows why quantization is available to these theories. Their degrees of freedom are content. A content is found within a context that is not itself the content, so its description can take the mode exchange while the arena in which both descriptions are laid out stays as it is. By the mode exchange, made while the arena stays unchanged, Newtonian mechanics passed to quantum mechanics, and classical electrodynamics passed to quantum electrodynamics. Thermodynamics, the sixth theory

of Chapter 3, describes the bulk states of matter; its statistical foundation is quantum statistical mechanics, a theory of content, and the check above does not need it.

The plain form of this chapter, with the electromagnetic field as its example, is in *Two Theories, One World*, Chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It". Chapter 5 establishes which phenomenon of physics is context-modal.



Chapter 5

The Content and Context Partition

Four markers of any arena, derived before gravity is named, and checked against gravity

This chapter establishes that gravity is the context of physics. It states the ground on which the division of physics into content and context rests. From that ground it derives four properties that any context-modal phenomenon must have, before any phenomenon is named: universal coupling, no shield, one mass for coupling and inertia, and no discrete charge. It then checks each property against gravity with the experimental evidence. Gravity has all four, and each force of particle physics lacks universal coupling, one mass and a continuous charge. I identify gravity as the context-modal phenomenon of physics by that match. At the close of this chapter, I give what in that identification is empirical, and what every measurement of the arena compares. I also give there a reason, not used by the chain of the category error, for the arena to be uneven where there is content.

I. The Partition and Its Ground

Every derivation of this chapter applies the two principles of *An Immanent Metaphysics* that Chapter 1 states beside the two laws. P5, the inseparability principle, states that where there is content there must also be context, and where there is context there must also be content. P7, the characteriz-

ation of content and context, states that content is local and interexclusionary and is understood in terms of symmetry, and that context is nonlocal and intercompositional and is understood in terms of continuity.[1]

The content and context partition is the division of physical phenomena into content, governed by symmetry and found in discrete classes, and context, governed by continuity and found as smooth values. The arena, which Chapter 4 defines as space and time together, is the context of physics: everything that is measured is measured in it.

The matter and the forces of particle physics are content. Their findings are the discrete, exactly labeled findings of Proposition 4.1: charges in fixed units, spins in halves, color as a representation label. Each of these forces acts within space and time, couples to a discrete charge, and leaves objects without that charge untouched.

The ground also orders the six measures of physics in a grid of two rows and three columns (Table 5.1).

	Omniscient	**Immanent**	**Transcendent**
Content	pattern	force	probability
Context	space	time	possibility

The rows are content and context. The columns are the three modalities of Chapter 2. The laws belong to the rows: the law of symmetry governs every measure in the content row, and the law of continuity governs every measure in the context row. The columns place each measure in its role within the act of comparison, in the focus of the six measures (Table 2.1), and they carry no law of their own. The placing of each measure in its column is an identification, set out with its reasons in plain words in *Two Theories, One World*, Chapter 7, "What Is Measured, and Where". Nothing in the argument of this chapter rests on that placing.

The four markers below are derived for any context-modal phenomenon, in any domain where comparison occurs. Each derivation names no physical phenomenon. Each check then sets gravity beside the derived property.

II. Marker 1: Universal Coupling

Proposition 5.1. A context-modal phenomenon couples to every content, without exception.

Argument. By the inseparability principle, every content has a context. A context-modal phenomenon is the context of physical comparison itself, so every content that physics finds stands within it and is found through its relation to it. A content that did not couple to the context-modal phenomenon would be a content with no context, and the inseparability principle excludes it. To couple to a phenomenon, at the level of the ground, is to stand in a relation to it through which a comparison can find something: a content couples to an electric field when the field makes a difference to what is found of the content. A content that stands in the context-modal phenomenon is found through its relation to it, its place and its motion in it, so standing in it is already a coupling to it.

Rests on: the inseparability principle (P5); the definition of a context-modal phenomenon (Chapter 4).

Check. Gravity couples to the stress-energy tensor, the density and flow of energy and momentum, which every form of matter and radiation has. Light is deflected and delayed by the Sun as general relativity requires. In the post-Newtonian framework, the parameter γ measures how much curvature of space a unit of rest mass produces, and general relativity gives $\gamma = 1$. The Cassini measurement of the delay of radio signals passing the Sun found $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ (Bertotti, Iess and

Tortora 2003). Neutrons in an interferometer acquire the phase that quantum mechanics gives a mass in the Earth's gravitational potential (Colella, Overhauser and Werner 1975). Antihydrogen falls toward the Earth (Anderson et al. 2023). The share of nuclear binding energy in the mass differs between titanium and platinum, so the tests of free fall in Section IV also show that binding energy couples to gravity as rest mass does. The forces of particle physics couple only to their own charges: the neutron is neutral to electromagnetism, and the electron to the strong force. Gravity has no neutral object.

III. Marker 2: No Shield

Proposition 5.2. No arrangement of content shields a region from a context-modal phenomenon.

Argument. To shield a region from a phenomenon is to make a region, bounded by an arrangement of content, within which the phenomenon is absent while the region remains a region. A region is itself a structure of context. By the characterization, context is non-local and intercompositional, and a region is the composition of the contexts within it. A region from which the context were absent would have nothing to compose it. A region without context is not a region, so no shield of a context-modal phenomenon can be made.

Rests on: the characterization of context (P7); the inseparability principle (P5); the definition of shielding.

The marker of no shield is also the reason that a context-modal phenomenon is the arena itself and not a field laid out within an arena. A field within an arena can be absent from a region of that arena. The arena cannot be absent from any region, since the region is made of it.

The conclusion also has a physical mechanism, which agrees with the structural argument and draws on the next marker as well as this one. An electric shield works by covering. In a conductor, an applied field moves the free charges of opposite signs in opposite directions, and they settle where the field of their own arrangement cancels the applied field inside. Covering therefore needs two things: an opposite to the source, and parts that the field moves apart.

Gravity offers neither. It couples to energy, and the energy of every body yet weighed is positive. A particle and its antiparticle have the same mass, and antihydrogen falls toward the Earth with an acceleration of $a_g = (0.75 \pm 0.13 \pm 0.16)$ g, which excludes a repulsion of the size of g (Anderson et al. 2023). Every body falls alike, so gravity moves no arrangement of bodies apart into parts that could cancel it: titanium and platinum fall with accelerations equal to within a few parts in 10^{15} (Touboul et al. 2022). No arrangement of matter covers a source of gravity. The structural argument and the physical mechanism reach one conclusion by two routes.[2]

Check. Electric fields are screened routinely, and the strong force is confined within hadrons. No screening of gravity has been found. The hypothesis that matter absorbs gravity has been tested in the form proposed by Majorana, with a coefficient h giving the fraction of the attraction absorbed per unit of mass per unit of area traversed. Absorption within a large body would hide part of that body's own mass from bodies outside it. The Earth and the Moon, of very different sizes, would then fall toward the Sun with different accelerations. Lunar laser ranging finds no such difference, and its data give $h = (3 \pm 5) \times 10^{-23}$ cm^2 g^{-1}, a value smaller than its own uncertainty (Williams, Turyshev and Boggs 2009). There is no gravitational Faraday cage.

IV. Marker 3: One Mass for Coupling and Inertia

Proposition 5.3. For a context-modal phenomenon, the coupling of every content to it equals that content's inertia.

Argument. For a content-modal force the coupling is a charge, and a charge is a label of the content (Proposition 4.1). Inertia is a content's resistance to a change in its state of motion, which is a change in its relation to the arena. For such a force, the charge is a property of content and the inertia is a property of the content's relation to context. Content and context are distinct intrinsics of comparison, and neither can stand in for the other, so nothing in the ground ties the one property to the other, and the ratio is free to differ from one content to the next, as the ratio of charge to mass does. For a context-modal phenomenon, the coupling is the degree to which a content takes part in the context, and the inertia is the degree to which it resists a change in its relation to the context. I identify the two, for the context, as one relation of the content to the context, found twice. To take part in the context is to be found in it, at a place and in a state of motion there, and that is the only relation of a content to the context that the context offers. A charge is a separate label of the content, carried in addition to its relation to the context, so a charge and an inertia have two things to differ by; for the context there is only the one relation, so the coupling and the inertia have nothing to differ by. The coupling and the inertia are then one quantity, and their ratio is the same for every content.

Rests on: the distinctness of content and context as intrinsics (P3, P4); the inseparability principle (P5); Proposition 4.1; the definition of a context-modal phenomenon (Chapter 4); the identification stated in the argument.

Einstein's form of the equivalence principle follows by the same reasoning. A uniform acceleration of the frame of a comparison is a change in its context. Gravity is the context-modal phenomenon. A change in the context and the context-modal phenomenon are of one kind, so no comparison confined to a small region tells them apart, though a uniform acceleration and a gravitational field have different sources. A small region is one across which the separation of neighboring free paths does not change by enough for the comparison to register; across a larger region the two are told apart by that change, the tide of gravitation (Volume 3, Chapter 10, Section VI, sets out the three readings of inertia that this distinction separates). An electric field is not locally equivalent to an acceleration, because its source is a label of content and a change of frame is a change of context.

Check. Tests of the universality of free fall compare the accelerations a_A and a_B of two bodies of different composition in one field, through the Eötvös parameter, twice the difference of the accelerations divided by their sum:

$$\eta(A, B) = 2\,(a_A - a_B)\,/\,(a_A + a_B) \tag{5.1}$$

Rotating torsion balances in the line of Eötvös found $\eta(\text{Be, Ti}) = (0.3 \pm 1.8) \times 10^{-13}$ (Schlamminger et al. 2008). The MICROSCOPE mission, comparing test masses in orbit, found $\eta(\text{Ti, Pt}) = [-1.5 \pm 2.3\ (\text{stat}) \pm 1.5\ (\text{syst})] \times 10^{-15}$ (Touboul et al. 2022). Antihydrogen falls downward, with the acceleration given in Section III (Anderson et al. 2023). The ratio of electric charge to mass, by contrast, differs widely between the electron and the proton, and it is zero for the neutron.

In general relativity the equivalence principle is the founding principle of the theory. On the ground set out here it follows from the identification of gravity as context, and the same reasoning shows why no other interaction has a principle of its kind.

V. Marker 4: No Discrete Charge

Proposition 5.4. A context-modal phenomenon has no discrete charge: the quantity to which it couples takes continuous values.

Argument. By Proposition 5.3, the quantity to which a context-modal phenomenon couples is a content's relation to the context. By the law of continuity and Proposition 4.2, what is of the context is not sorted into classes with exact labels: its values are continuous. So the coupling quantity of a context-modal phenomenon is continuous, and no discrete charge carries it.

Rests on: the law of continuity (P6); Propositions 4.2 and 5.3.

Check. Electric charge comes in multiples of one third of the electron's charge, and every free particle yet observed carries a whole multiple of the electron's charge. Gravity couples to energy and momentum in total: rest mass, kinetic energy, binding energy and the energy of fields all count, and the total takes any positive value. The rest masses of the species of particle form a discrete list. The source of gravity is the total energy and momentum of a system. The energy of motion varies continuously, and no label quantizes the total; the steps in the energy of binding belong to the spectra of the content. An isolated bound system at rest has one of a discrete set of total energies, and still the quantity gravity couples to is continuous. What gravity couples to is the energy and momentum of a content in its relation to the context: as found at each place and in each state of motion, since the source of curvature is the density and flow of energy and momentum through the arena. That relation varies continuously with where the content is and how it moves, as Proposition 4.2 requires of

what is of the context, and the discreteness of the bound system's levels is a label of the content, in its rest frame, and not a step in the coupling. Gravity has no charge that comes in quanta.

VI. The Four Markers Together

Table 5.2 sets out each marker with the ground it rests on, its argument in one line, and its check against gravity.

Marker	Ground	Argument	Check against gravity
Universal coupling (Proposition 5.1)	Inseparability principle (P5)	Every content has a context, so every content couples to the context-modal phenomenon	Gravity couples to all stress-energy: light (Bertotti, Iess and Tortora 2003), neutrons (Colella, Overhauser and Werner 1975), antimatter (Anderson et al. 2023)
No shield (Proposition 5.2)	Characterization of context (P7); inseparability principle (P5)	A region without context is not a region; covering needs an opposite to the source and parts the field moves apart	No screening found; $h = (3 \pm 5) \times 10^{-23}\ \mathrm{cm^2\ g^{-1}}$ (Williams, Turyshev and Boggs 2009)
One mass (Proposition 5.3)	Distinctness of content and context (P3, P4); inseparability principle (P5); Proposition 4.1	Coupling and inertia are one relation of a content to the context	η(Ti, Pt) at the level of 10^{-15} (Touboul et al. 2022); η(Be, Ti) at the level of 10^{-13} (Schlamminger et al. 2008); antihydrogen falls
No discrete charge (Proposition 5.4)	Law of continuity (P6); Propositions 4.2 and 5.3	What is of the context is not sorted into labeled classes	Gravity couples to total energy and momentum, which take continuous values

No force of particle physics has the four markers. Electromagnetism, the weak force and the strong force each couple to a discrete charge and leave objects without that charge untouched. None of them has a ratio of charge to inertia that is the same for every body. Electric fields are screened by conductors, and the strong force is confined within hadrons.

VII. What Remains Empirical

Propositions 5.1 to 5.4 hold for any context-modal phenomenon, and they name none. The inseparability principle also requires that such a phenomenon exist wherever content exists.

Proposition 5.5. Physics contains a context-modal phenomenon, and that phenomenon has the four properties of Propositions 5.1 to 5.4.

Argument. The matter and the forces of particle physics are content (Section I and Table 4.2). By the inseparability principle, where there is content there must also be context. So physics has a context, and by Propositions 5.1 to 5.4 that context couples to every content, admits no shield, has one mass for coupling and inertia, and has no discrete charge.

Rests on: the inseparability principle (P5); Propositions 4.1 and 5.1 to 5.4; the established physics of particles and forces (P15).

The ground does not name this phenomenon gravity. It requires that something with the four markers that gravity has exist in any domain that has content.

P18 (identification). I identify gravity as the context-modal phenomenon of physics, by the match of all four markers.

P18 is the one identification in the chain from the ground to the category error that experiment tests directly. It predicts that free fall stays universal at every sensitivity yet to be reached, for matter and antimatter alike, and that no arrangement of matter will be found to shield gravity. Every measurement of free fall and every search for a shield of gravity tests P18, and P18 says that each will find gravitational and inertial mass equal and will find no shield. The experiments now planned are set out in Chapter 19, "Predictions, Evidence and Decisive Experiments".

With the identification, gravity is the arena itself. Its findings are the smooth and asymmetric geometry of Proposition 4.2, and general relativity is its description in the mode pair that Proposition 4.3 assigns to context. The four markers are set out for the general reader as four marks, each with its plain reason, in *Two Theories, One World*, Chapter 9, "Four Marks of the Arena".

The arena keeps its place as context in every comparison, the comparisons that measure it included, and the reason is general.

Proposition 5.6. Every measurement of the arena compares the arena with itself: it compares the arena at one place and time with the arena at another, so its verdict is a relation within the context, and the content it finds is always the content of some body or field.

Argument. The content and the context of a comparison cannot be one respect (P3; Chapter 1, Section III). By P18 the arena is the context of every physical comparison, the comparisons whose content is a measurement of it included (Chapter 4, Section I). A measurement of the arena therefore has the arena as its context, and what it registers of the arena is a relation between two settings of that context: the arena at one place and time against the arena at another. What its verdict is on, its content, is then

not the arena itself, and it is the state of some body or field that the arena carries: the phase of an interferometer's light, the rate of a clock, the path of a test mass.

Rests on: the six intrinsics, with content and context never one respect (P3); the identification of gravity as the context-modal phenomenon (P18); the definition of a context-modal phenomenon (Chapter 4).

A gravitational-wave detector is the sharpest case, and Chapter 4, Section I, applies Proposition 5.6 to it.

VIII. Why the Arena Is Uneven Where There Is Content

In the chain of the category error, the continuous asymmetry of the arena follows from the law of continuity (Proposition 4.2). A second argument gives a reason for the asymmetry to follow the content. Nothing in the chain of Chapter 6 rests on it.

Proposition 5.7 (not used by the chain of the category error). Where there is content, the context is uneven: the context carries, continuously, the asymmetry that neither the comparing nor the content can carry.

Argument. The argument takes the focus of the two laws (Table 2.1). A focus may assert fewer than the three axioms and then pins less of the binding (P11); the focus of the two laws asserts all three, so it pins the whole binding of its three terms, and the argument can use each axiom. It differs from the focus of measurement, which places content and context together in the omniscient role, because it considers a different thing: the focus of measurement considers the finding as recorded, where content and context are both written down, and the focus of the two laws considers the act that relates a content to its context under the two laws, where the content is what is laid out once found and the context is the set-

ting within which the act proceeds. First, the binding is that of the focus of the two laws: the comparing is immanent, the content, laid out once found, is omniscient, and the context, within which the act proceeds, is transcendent, following aphorisms 1.83-2 and 1.83-3 of *An Immanent Metaphysics*, which give the objective side of a comparison the omniscient modality with the lawfulness of content in symmetry, and the subjective side the transcendent modality with the lawfulness of context in continuity (Table 2.1; the binding is set out in Volume 1, Chapter 6, Section V, and Chapter 8, Section II, and in Volume 2, Chapter 1, Section III, and Chapter 10, Section II). Second, the Incommensuration Theorem is stated of a comparison, so it holds of the comparison as a whole (P9). The content is what is found and the context the framework in which it sits, so a symmetric content beside a continuous context is a conjunction split across two levels, the two modes holding of different things, which the theorem permits, as it permits the formalism and the findings of a theory to stand in conjugate modes (Proposition 2.2; Volume 2, Chapter 6, Section VIII); and continuity, wherever it is perfected in that comparison, travels with asymmetry, since continuity with asymmetry is the only surviving conjunction that contains continuity. The comparing connects without a break: to compare is to hold two things in one act, and a connection with a break in it holds nothing together. The content is symmetric: by Axiom II it is laid out after the act that finds it, and by the law of symmetry (P6) it is the same for every comparer that finds it. Neither demand admits an exception, since a comparing that did not connect would find nothing, and a content that differed between its comparers would be no one content. The comparing perfects no character of its own. A character is a pattern of findings across contexts, a sameness or a difference laid out over many settings (P8), and the comparing is the act of joining, here and now, not something laid out across settings; by Axiom III the act cannot do the work of what is laid out. In this focus the character of continuity belongs to the context, in the transcendent role, and the continuity that the comparing brings by connecting without a break is perfected in the context, where the comparing connects. By the theorem, held of the comparison as

a whole, that perfected continuity travels with an asymmetry in the comparison. The asymmetry cannot sit in the content, which is held symmetric. It cannot sit in the comparing, which is the act of joining and is not laid out across a difference of context, as an asymmetry is (P8). By Axiom III neither role can take on the work of another. So the context carries the asymmetry, together with the continuity perfected in it. The context is therefore continuously asymmetric, and its asymmetry is there because the content is there.

Rests on: the two laws (P6); the definitions of the compounds (P8); the Incommensuration Theorem (P9); the three axioms (P12); the binding of the focus of the two laws (Table 2.1); Proposition 2.2, for a conjunction split across two levels.

General relativity, as established physics (P15), says how the arena carries the asymmetry. The Ricci part of the curvature at a place is fixed by the stress-energy there, and the Weyl part, the free curvature, carries the curvature on into regions with no content, as around the Sun and across a passing gravitational wave. Proposition 5.7 gives the reason that a bending exists, that it is smooth, and that it goes with the content; the established theory gives its form and its size, with Newton's constant measured. The observation that bears on Proposition 5.7 is that the arena is found uneven wherever there is content: the flat arena of special relativity is the idealized case of an arena with no content anywhere and no wave crossing it. The argument is set out in plain words in *Two Theories, One World*, Chapter 8, "Why the Arena Bends", and Volume 3, Chapter 10, Section VIII, carries it on to where, and how much, the arena curves.

IX. Questions

Weinberg (1964) derived the equality of gravitational and inertial mass from the scattering theory of a massless particle of spin two. Does that not make the equivalence principle a property of quantized gravity? Weinberg's result shows that, in a description of small waves by quanta, a

massless spin-two quantum which couples at low energies must couple to all energy and momentum with one strength. This account allows that description of the small waves of geometry about a smooth background, as a description, and its quanta are quanta of the description, never quanta of the arena (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale"). Weinberg's derivation and Proposition 5.3 therefore agree: any description of the small waves of a context-modal phenomenon inherits its universal coupling. Neither gives the arena itself a quantum at any scale.

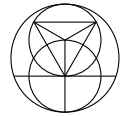
Is the absence of a gravitational shield anything more than the positivity of mass? The physical mechanism of Section III uses the positivity of energy and the universality of free fall, and so it draws on Marker 3 as well as Marker 2. The structural argument uses only the character of context. The physical mechanism is therefore the way the structural result shows itself in matter, and the two agree.

Gravity can be made to vanish locally by falling freely. Is that not a shield? Free fall removes the uniform part of the field in a small region by a change of frame, which is a change of context (Section IV). The tidal part, the curvature of the arena, remains in every frame, and no arrangement of content removes it from a region.

A gravitational-wave detector takes the metric as what its verdict is on. Is the geometry then content? The detector finds a change in the phase of its light, which is content, compared against the arena as its context. What it registers of the geometry is the arena at one place and time against the arena at another, a relation within the context, and that relation is found continuous and asymmetric, as Proposition 4.2 requires. The small wave that passes can be described as if it were content, and the quanta of that description are quanta of the description only, never quanta

of the arena; the arena, as the background of every comparison, stays context (Chapter 4, Section I, states the objection and this answer in full, and Proposition 5.6 gives the general form).

With gravity identified as the arena, Chapter 6 assembles the chain from the ground to the category error.



Chapter 6

The Category Error

The chain in full, its exact scope, and the relation the theorem permits

This chapter states the category error and its exact scope. It sets out what the program of quantum gravity asks of the arena, and assembles the chain from the ground to the diagnosis, step by step. It then states exactly what is excluded: the arena has no least step, and nothing is a quantum of the arena itself, at any scale. Small waves of geometry can still be described by quanta, and those quanta are quanta of the description. At the close of this chapter, I give the relation that the Incommensuration Theorem permits between the two theories, which is conjugation rather than unification. I also read canonical quantum gravity there as the arena put into the quantum state.

I. What the Program Asks of the Arena

The program of quantum gravity seeks a theory that quantizes the gravitational field as quantum electrodynamics quantizes the electromagnetic field. It makes two demands of that theory.

The first demand is quantization. The geometry is to be treated as a field whose findings are the eigenvalues of operators on a Hilbert space, discrete wherever the system is bound and classified by exact labels, and

whose formalism rests on continuous structures. By the first demand, the geometry is to have the mode pair of content: discontinuously symmetric findings with a continuous formalism.

The second demand is recovery. The theory is to reproduce general relativity in the appropriate limit, with a smooth metric, differentiable field equations and a continuous geometry. By the second demand, the theory is to recover general relativity, which has the mode pair of context: continuously asymmetric findings with a symmetric formalism.

Recovery is asked in a limit, as quantum electrodynamics recovers Maxwell's fields in a limit, and the two cases differ in which character is perfected at the foundation. Quantizing the electromagnetic field is right because that field is content, whose perfected character is discontinuous symmetry: the quantum theory is its foundational form, and the classical theory is its limit. The arena is the context of every finding, the findings of quantum electrodynamics included, and its perfected character is continuous asymmetry (Proposition 4.2). A program that makes the arena quantum at its foundation, and recovers smooth geometry only as a limit, inverts which character is perfected. The arena stays, at its foundation, the continuously asymmetric context in which the quantum findings themselves are laid out, and the program asks the same directions of the arena, at the same foundation, for the discontinuous symmetry of quantized findings. The two forbidden pairings are continuity joined with symmetry, a smooth geometry with exact quantum labels, and asymmetry joined with discontinuity, steps of geometry no one of which repeats another. A description of the arena that met both demands in one set of findings would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. The classical and the quantum descriptions of the electromagnetic field are compared within one arena that both keep; the classical and the quantum descriptions of the arena would need a further setting in which to be compared, which Proposition 6.1 examines.

II. The Chain in Full

Table 6.1 sets out the chain from the ground to the diagnosis, with what each step rests on.

Step	Statement	Rests on
1	Any comparison perfected at its foundation is continuously asymmetric or discontinuously symmetric; continuity with symmetry and asymmetry with discontinuity are forbidden	P9, the Incommensuration Theorem (Chapter 1)
2	The law of symmetry governs content; the law of continuity governs context	P6 (*An Immanent Metaphysics*, aphorism 1.83-5); reason: Chapter 1, Section IV
3	The findings of content are discontinuously symmetric; the findings of context are continuously asymmetric	Propositions 4.1 and 4.2, from steps 1 and 2
4	The formalism of each theory is in the mode conjugate to its findings	Proposition 3.1, the cross-modal principle (Chapter 3)
5	Every fundamental description has one of two mode pairs	Proposition 4.3, from steps 1 to 4
6	A description of content can be made in either mode pair, and quantization is the mode exchange between them	Proposition 4.4 and the check of Table 4.2
7	Gravity is the context-modal phenomenon of physics: the arena	Propositions 5.1 to 5.5 and P18, the identification of Chapter 5
8	The arena admits no mode exchange	Proposition 6.1, below
9	To quantize the arena is a category error	Proposition 6.2, below

Proposition 6.1. The arena admits no mode exchange.

Argument. The argument has two strands, one from the arena's findings and one from its role.

From its findings. A mode exchange would give the arena discontinuously symmetric findings. By the law of continuity and Proposition 4.2, the findings of the arena are continuous at their foundation, and the arena remains the context of every physical comparison after the step as before it. One description of the arena would then hold the continuity and the asymmetry of the context together with the discontinuous symmetry of a quantized finding: it would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both.

From its role. The mode exchange of a content is a passage between two descriptions of that content, and the two descriptions are compared within one arena that stays as it is (Chapter 4, Section V). The continuity of the arena is the condition under which any description of a content is laid out at all. A mode exchange of the arena would need a further setting in which its two descriptions could be compared, and that further setting would then be the context of physics in the arena's place, which contradicts step 7. The mode exchange is not difficult for the arena. It is incoherent.

Rests on: the law of continuity (P6); Propositions 4.2 and 4.4; the identification of gravity as the context-modal phenomenon (P18); the Incommensuration Theorem (P9).

Proposition 6.2 (the category error). To quantize the arena is to apply to context the treatment that belongs to content. To quantize the arena is a category error; a category error is a question that applies a concept where the concept does not belong.

Argument. The concept applied is quantization, the mode exchange, which belongs to descriptions of content (Proposition 4.4 and the check of Table 4.2). The domain to which it is applied is the arena, which is context (step 7). By Proposition 6.1, the mode exchange does not apply to the arena.

Rests on: Propositions 4.4 and 6.1; the identification of gravity as the context-modal phenomenon (P18).

The category error is meant in the philosophical sense. The category error is a claim about which concepts apply to which kind of thing, made on the ground of comparison. The category error is not a theorem of the mathematical theory of categories, and the category error borrows nothing from that theory.

The argument in seven sentences. (1) By the Incommensuration Theorem, any comparison perfected at its foundation is either continuously asymmetric or discontinuously symmetric. (2) Continuity with symmetry, and asymmetry with discontinuity, are forbidden. (3) General relativity describes the continuously asymmetric conjunction, and quantum theory describes the discontinuously symmetric conjunction. (4) Content is governed by symmetry and context by continuity; the matter and forces of particle physics are content, and gravity is the context of physics, its arena. (5) The program of quantum gravity asks that the arena, the context of every finding, the quantum findings included, be discontinuously symmetric at its foundation (to admit quantization) and recover its continuous asymmetry only as a limit (to reproduce general relativity). (6) That inverts which character the arena has perfected, and a description of the arena so quantized would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. (7) The program therefore makes a category error: it applies the treatment of content to the context of physics.

III. What Kind of Claim This Is

Each step of the chain follows from the ground of Part I, the two laws and established physics. The chain contains one identification, gravity as the context-modal phenomenon of physics (P18), and Chapter 5 names the observations that bear on it. The ground itself is stated as premises in Part I; its reasons are set out for the general reader in *Two Theories, One World*, Chapter 5, "Symmetry and Continuity Cannot Both Be Perfected", and Chapter 6, "Three Roles and the Order of Finding". The full chain in prose, without equations, is in *Two Theories, One World*, Chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It".

The long search for a quantum theory of gravity has produced results of lasting value, which Chapter 14, "Four Criteria and Ten Programs", credits one program at a time. The want of a complete quantum theory of gravity that holds across the Planck scale is what the diagnosis leads one to expect. That history agrees with the diagnosis, and the diagnosis does not rest on it.

IV. The Exact Scope

Proposition 6.3. That the arena is not quantized means exactly two things. (i) The arena has no least step: no least length, area or duration. (ii) Nothing is a quantum of the arena itself, at any scale.

Argument. A least step of the arena would be a discontinuity in its findings, and by the law of continuity and Proposition 4.2 the findings of the arena have none at their foundation. A quantum of the arena itself, at any scale, would be the arena in the mode pair of content, which is the mode exchange that Proposition 6.1 excludes. What a description with a limited domain quantizes is that description, and its quanta are quanta of the description, never quanta of the arena.

Rests on: the law of continuity (P6); Propositions 4.2 and 6.1.

Proposition 6.3 leaves four things allowed. The first and third are established physics, and the second follows from the first. The fourth joins established physics, the quantized matter, to this book's identification of one geometry for each alternative, which Chapter 7 sets out and which the experiment of Page and Geilker (1981) supports by excluding the averaged geometry.

First, small waves of geometry about a smooth background can be described by quanta. Those quanta are quanta of the description, steps in the energy of a mode of the description, and the lengths, areas and durations

of the four-dimensional geometry keep no least value. The phonon, a quantum of the description of the vibrations of a held structure such as a solid, is the nearest sense of such a quantum; the arena is not a held structure, so even that sense does not make a quantum of the description a quantum of the arena. The effective field theory of gravity computes the effects of the small waves in this description below the Planck curvature (Donoghue 1994), and Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", sets out that theory and its reading.

Second, a detector that registers a gravitational wave in single clicks would not count against this account, and it would not show gravitation quantized. A classical gravitational wave acting on a quantum detector produces the same discrete clicks (Carney, Domcke and Rodd 2024), as the photoelectric effect alone does not show that light is made of photons. [1]

Third, the integers that characterize the global form of a manifold, such as its Euler characteristic and its signature, are discrete, and they do not change under a smooth change of the metric. They are discontinuously symmetric already, and a sum over them, of the kind topological quantum field theories make, gives the arena no step.

Fourth, the matter in the arena is quantized in full, and each alternative of the matter carries its own classical geometry. A single geometry sourced by the average of the alternatives fails the experiment of Page and Geilker (1981). In that experiment the measured field followed the outcome of a quantum event, and not the average over outcomes (the argument is set out in Chapter 7, "One Geometry for Each Alternative").

The Planck length, about 1.6×10^{-35} m, is the length at which an object's gravitational size equals its quantum size. There neither description dominates, and both are relevant. The Planck length is the scale at which the gravitational and the quantum descriptions of an object meet, and it is not a least length of the arena.[2] The smooth description of the arena holds below the Planck curvature.

This account makes definite predictions below the Planck curvature. It predicts that two masses related only through gravity become entangled, with the phase that each alternative's own geometry gives (Chapter 8, "The Entanglement Test"). It predicts that free fall stays universal and that gravity stays unshielded (Chapter 5). It predicts that space and time show no granularity in the arrival times of high-energy photons (Chapter 19, "Predictions, Evidence and Decisive Experiments"). Below the Planck curvature, this account and the effective field theory of gravity give the same result for every experiment now proposed. The diagnosis is a claim about what the arena is. It holds that the arena has no step at any scale, and that at the Planck scale, where the gravitational and the quantum descriptions meet, no single description carries both characters.

V. Conjugation, Not Unification

Proposition 6.4. The Incommensuration Theorem permits a common source: two descriptions joined side by side, each keeping its own character. It forbids a merger: one formalism in which the directions of the arena themselves carry the character of content at the foundation, so that the same directions carry both characters.

Argument. The theorem forbids continuity with symmetry, and asymmetry with discontinuity, in one comparison perfected at its foundation. Two descriptions joined side by side each perfect one surviving conjunction, and no single set of findings carries a forbidden pairing. One formalism in which the same directions carried both characters would ask all four characters of those directions, and so it would carry both forbidden pairings in them. Quantum electrodynamics joins its field to the arena without a merger: the field is content laid on the arena, and the arena's

directions keep their own character. Content and context are distinct and inseparable intrinsics of every comparison (P3, P4): the side-by-side joining keeps both properties, and the merger removes the distinctness.

Rests on: the Incommensuration Theorem (P9); the distinctness and inseparability of content and context (P3, P4).

General relativity and quantum theory are conjugates in this sense. They are distinct, since each perfects a different conjunction. They are inseparable, since every finding has a content and a context. They are non-interchangeable, since neither can do the other's work.[3] They do not oppose one another, and they do not merge. They are two descriptions of one world, each capturing what the other leaves out. I read the relation as one of mutual amplification: a richer symmetric structure of content asks for a richer arena, and a more complex geometry hosts a richer content.

The Incommensuration Theorem forbids no interaction between quantum matter and geometry. The energy of the matter shapes the geometry, and the geometry guides the matter, in every alternative. What the theorem forbids is that both be subsumed into one formalism that perfects both characters. The configuration that results is quantum content on a classical arena, with each alternative of the matter carrying its own classical geometry. That configuration is not an approximation to a deeper theory of quantized geometry. The general form of the distinction between a common source and a merger, and what follows for the search, are set out in Chapter 16, "Common Source, Not Merger".

VI. Canonical Gravity: The Arena Put Into the State

Canonical quantum gravity applies the mode exchange to general relativity directly. It writes the theory in Hamiltonian form, with the metric of a spatial slice as its configuration variable, and promotes the constraints of

the theory to operators acting on a wave functional of spatial metrics (DeWitt 1967). The Hamiltonian constraint becomes the Wheeler-DeWitt equation:

$$\hat{H}\,\Psi[h_{ij}] = 0 \tag{6.1}$$

In words, the Hamiltonian operator annihilates the wave functional Ψ of the spatial metric h_{ij}. The equation contains no time parameter. The quantum state, which in every other quantum theory evolves in the time of an arena, has nothing in which to evolve. The problem of time (Isham 1993) is that the quantum state has nothing in which to evolve.

I read it as the category error in its plainest form. The context in which things happen has been put into the quantum state, instead of serving as the arena in which the state evolves. A continuous formalism needs a context in which its smooth evolution is laid out. Once the arena is itself the state, the formalism has no context left, and its evolution has no parameter. Many of the proposed resolutions recover a time by choosing some degree of freedom to serve as a clock (Isham 1993 reviews them). Each choice of a degree of freedom as a clock restores, within the description, a part of the context that the quantization removed.

Superspace gives the quantized geometry a structure and not a context. Superspace is Wheeler's name for the space of all the geometries that a slice of space can have, each of its points one whole geometry of the slice (Wheeler 1968), and the wave functional of equation (6.1) is laid out over it, with the metric on superspace that DeWitt gave (DeWitt 1967). Wheeler offered it as the arena that the equation keeps: "The arena must be a larger object: superspace" (Wheeler 1974, as quoted by Giulini 2009). Every geometry the slice could have is laid out in superspace at once, so superspace is a space of alternatives laid out, and what is laid out once found is content, while the context is the setting within which the act of comparison proceeds (Chapter 5, Section VIII). Were superspace the context of physics, it would be the further setting in which the two descriptions of the arena are compared, and the role strand of Proposition 6.1 ap-

plies to it: that setting would be the context of physics in the arena's place, and the geometry of spacetime would become one content within it, contrary to step 7 of Table 6.1. Superspace supplies no parameter either. Nothing in superspace orders one of its points before another, so a wave functional over it changes along no parameter of its own. DeWitt's metric on superspace has one direction of negative sign at each point, the direction in which the volume of space changes (DeWitt 1967), and the programs that recover a time from superspace choose that direction as a clock; that choice is one of the clocks named above, and it restores within the description a part of the context that the quantization removed. The conclusion stands: once the arena is itself the state, its evolution has no parameter. Wheeler and DeWitt gave the quantized geometry its space of states and its metric, and this account takes superspace as exactly that.

The canonical program gave gravity its Hamiltonian form and the algebra of constraints on which every later canonical approach builds. That achievement stands.

VII. Questions

Is the difficulty only that general relativity is not renormalizable? A quantum field theory is non-renormalizable when it needs infinitely many independent constants to absorb its divergences at high energy. Perturbative non-renormalizability carries less weight than the structural point. Effective field theory handles theories that are not renormalizable, and gravity is computed that way below the Planck curvature (Donoghue 1994). The diagnosis does not rest on renormalizability: it rests on Proposition 6.1. Chapter 9 sets out the effective theory, the place of its quanta, and the weight that non-renormalizability carries.

Quantum electrodynamics recovers the smooth classical fields of Maxwell's theory in a limit. Why may a quantum theory of the arena not recover the smooth metric in the same way, and does the argument not condemn quantum electrodynamics too? The answer is the role

strand of Proposition 6.1. The electromagnetic field is content, so its perfected character is discontinuous symmetry: the quantum theory is its foundational form, and Maxwell's theory is its limit. Both descriptions of the field are laid out and compared within one arena that both keep, and the arena's directions keep their own character throughout. The arena is the context of every finding, the findings of quantum electrodynamics included, and its perfected character is continuous asymmetry. A quantum theory of the arena puts discontinuous symmetry at the arena's foundation and leaves continuous asymmetry to a limit, which inverts the character that the arena perfects, and its two descriptions would need a further setting in which to be compared. So the chain condemns quantizing the arena, and it never condemns quantizing a field of content on the arena.

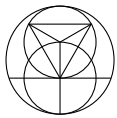
Does linearized quantum gravity not already quantize gravity? It quantizes a description of small waves about a classical background, and not gravitation. Its quanta are steps in the energy of modes of that description, which is the first of the four allowances of Section IV. It gives the arena no least step, and nothing in it is a quantum of the arena at any scale.

Does the diagnosis leave quantum matter unable to source gravity? No. The matter is quantized in full, and each of its alternatives sources its own classical geometry (Section IV). Chapter 7 shows why the single averaged geometry fails and the geometry of each alternative does not.

What takes the place of the search for a quantized arena? The joining of the two descriptions as conjugates, with each held exactly in its own character, and the account of where each reaches the limit of its domain. Part III sets out what remains when the error is removed, and Part V reads each program of quantum gravity in that light.

PART III

WHAT REMAINS



Chapter 7

One Geometry for Each Alternative

The semi-classical equation sourced by the expectation value, the experiment that excludes it, and the configuration that remains when the category error is removed

This chapter establishes three propositions. Each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by the stress-energy of that alternative alone, with the relative phases of the alternatives kept (Proposition 7.1). A finding changes no geometry, and the configuration sends no signal faster than light (Proposition 7.2). The single geometry sourced by the expectation value of the stress-energy, the semi-classical equation of Møller (1962) and Rosenfeld (1963), fails in both of its forms: without reduction of the state it fails the experiment of Page and Geilker (1981), and with reduction it signals faster than light (Proposition 7.3). I then place the configuration beside gravity-induced collapse (Diósi 1987; Penrose 1996) and beside the models of gravitational decoherence.

Below the Planck curvature, per-alternative geometry and a superposition of geometries assign the same geometry to each alternative of the matter. They predict the same for every experiment in which the geometry carries no degrees of freedom of its own beyond the small waves and those tied to the alternatives of the matter, the experiment of Chapter 8, "The Entanglement Test", among them. One formal difference separates them. Per-alternative geometry places no independent, non-perturbative degrees of freedom of the geometry in the quantum state: the label of each geometry is an alternative of the matter (equation 7.3). A superposition of

geometries allows such degrees of freedom. Chapter 19, "Predictions, Evidence and Decisive Experiments", names the observation that would separate the two (Prediction 19.2).

I. The Semi-Classical Equation

The first proposal for joining quantum matter to a classical geometry keeps one geometry for the whole quantum state. Møller (1962) wrote Einstein's equations with the classical stress-energy replaced by its quantum expectation value, and Rosenfeld (1963) examined whether the gravitational field must be quantized at all, holding that the question is one for experiment.[1] I call the resulting equation the semi-classical equation.

$$G_{\mu\nu}[g] = (8\pi G / c^4) \, \langle \psi | \, \hat{T}_{\mu\nu} \, | \psi \rangle \tag{7.1}$$

In words: the curvature of the one geometry g, on the left, equals Einstein's constant times the expectation value of the stress-energy operator of the matter in its quantum state ψ, on the right. The matter evolves on that geometry, and the geometry is sourced by the average of the matter's alternatives, each weighted by its probability.

In the Newtonian limit the semi-classical equation becomes the Schrödinger-Newton equation (Diósi 1984).

$$i\hbar \, \partial\psi/\partial t = [\, -(\hbar^2/2m) \, \nabla^2 + m\Phi \,] \, \psi, \qquad \nabla^2\Phi = 4\pi G \, m \, |\psi|^2 \tag{7.2}$$

In words: the wave function of a body of mass m evolves in a Newtonian potential Φ, and that potential is sourced by the body's own probability density, $|\psi|^2$, multiplied by its mass. The equation is nonlinear in ψ, because the potential depends on the state that it moves.

The semi-classical equation has two forms, and they must be judged separately. In the first form the quantum state is never reduced: the stress-energy in equation (7.1) is taken in a state that keeps every alternative, whatever is found. In the second form the state is reduced at each finding, so that the weights of the alternatives, and with them the source of the geometry, change when a finding is made. Section II tests the first form against experiment; Section IV tests the second against relativity.

Both forms are exact in one case. Where the matter is sharply one alternative, as for a star, a planet or a black hole of definite mass, the expectation value is the stress-energy of that one alternative, and equation (7.1) is Einstein's equation for it. The computations of astrophysics, and of quantum fields on a given background, are made in that case, and Proposition 7.1 leaves each of them as it stands.

Whether gravity must be quantized at all has a literature of its own, and it has found the standard arguments for quantization inconclusive. Huggett and Callender (2001) argue that a consistent treatment of quantum matter together with gravity is needed, that it need not quantize gravity, and that the argument of Eppley and Hannah (1977), taken up in Section VII, falls short of a no-go theorem. Wüthrich (2005) finds good physical reasons to quantize gravity, and he holds that a refusal to quantize it remains a tenable position. Carlip (2008) finds the theoretical arguments against coupling a classical gravity to quantum matter strong but not conclusive, and he leaves the question to experiment, as Rosenfeld did.

II. The Experiment of Page and Geilker

Page and Geilker (1981) tested the first form of the semi-classical equation directly, with a Cavendish torsion balance. A light bar carrying two small masses hangs from a fiber; two large masses placed near the small ones twist the bar, and the twist measures their gravitational pull. Before each

run, a radioactive decay registered by a counter selected one of two placements of the large masses, one placement twisting the bar one way and the other twisting it the opposite way.[2]

The two forms of the semi-classical equation predict different runs. Page and Geilker took the quantum state to be unreduced, as the first form requires, so that after each decay the large masses were in both placements, each with weight one half. On that form the expectation value sources the field of half the masses in each placement, the two pulls on the bar cancel, and the bar stays where it hangs on every run.

The bar turned on every run. It turned in each direction about equally often, and on each run it turned in the direction set by the actual placement of the large masses. The averaged prediction is right about the mean over many runs and wrong about every single run. So the first form of the semi-classical equation is excluded by experiment: the gravitational field followed the outcome of the quantum event, and not the average over its outcomes.

Page and Geilker titled their paper "Indirect Evidence for Quantum Gravity", and their abstract reads the result as support for quantizing the gravitational field. What the experiment establishes is narrower and exact: it excludes one geometry sourced by the expectation value in a state that is never reduced. Every account in which the field follows the outcome passes the experiment, and Section III sets out the account of this book, which is one of them.

Kuo and Ford (1993) reach a matching conclusion from within the semi-classical approach. They find that an equation sourced by the expectation value of the stress-energy is valid only where the fluctuations of the stress-energy about that value are small, and that the fluctuations are small for states near coherent states and large for others. A mass held in two placements is a state of the second kind, with large fluctuations, since its stress-energy at each placement differs between the alternatives by the whole mass.

III. One Geometry for Each Alternative

An alternative of the matter is one term of the decomposition of its quantum state into states of definite distribution of stress-energy, at the resolution at which the geometry is sourced: a mass in this placement, or the same mass in that placement. The relative phase of two alternatives is the phase of one term relative to the other in that decomposition.

Proposition 7.1. Each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by the stress-energy of that alternative alone, and the relative phases of the alternatives are kept.

$$|\Psi\rangle = \Sigma_i \, c_i \, |\psi_i\rangle, \qquad G_{\mu\nu}[g^{(i)}] = (8\pi G \,/\, c^4)\; T^{(i)}{}_{\mu\nu} \text{ for each } i \tag{7.3}$$

In words: the state of the matter is a sum of alternatives ψ_i with complex coefficients c_i, and each alternative has its own geometry $g^{(i)}$, whose curvature is sourced by the stress-energy $T^{(i)}$ of that alternative, while the coefficients, and so the relative phases, are kept. Each $g^{(i)}$ is a classical solution of Einstein's equations, and the label i belongs to the alternative of the matter, not to a state space of geometries. I call this configuration per-alternative geometry.

Argument. First, by the inseparability principle, where there is content there must also be context (P5; *An Immanent Metaphysics*, aphorism 1R3). Second, each alternative of the matter is a possible content: it is one of the outcomes a finding can select, and, selected, it is content. Third, the geometry is the context of the matter: the arena of space and time, whose shape is gravity, is the context of physics. Chapter 5, "The Content and Context Partition", identifies gravity as the arena by its four markers

(P18): it couples to every content, nothing shields it, its coupling to each content equals that content's inertia, and it has no discrete charge. Fourth, contexts compose with one another (P7), and I reason that they compose only where their contents are found together; alternatives exclude one another as contents and are never found together, so no context is the context of two alternatives. Fifth, Einstein's equations, taken as established physics (P15), fix the geometry from its source. So each alternative has its own context, which is its own geometry, sourced by its own stress-energy.

Rests on: the inseparability principle (P5); the composition of contexts (P7), with the step stated in the argument that contexts compose only where their contents are found together; Einstein's equations, as established physics (P15); the identification of gravity as the context of physics by its four markers (P18).

Proposition 7.1 is this book's identification, made on the ground of the argument above, and the experiment of Page and Geilker (1981) supports it by excluding the averaged geometry (Section II). Standard quantum field theory on curved space uses one fixed background, and per-alternative geometry extends that framework to matter whose alternatives differ in their distribution of stress-energy.

The same premises exclude one geometry shared by the alternatives. A geometry sourced by the alternatives together gives each alternative an Einstein tensor that does not vanish where that alternative has no stress-energy, so within that alternative Einstein's equations fail. The geometry then carries curvature sourced at a place by a mass that, in that alternative, is not at that place: a source that the alternative's content does not supply, which is context without its content; by the inseparability principle, no such context exists. The argument concerns the Ricci part of the curvature, the part that Einstein's equations fix at each place by the stress-energy there. The Weyl part, the free curvature that the arena carries and propagates where there is no content, lies outside it: in general relativity an empty region need not be flat, and the space around the Sun, a passing gravitational wave and the outside of a black hole are empty and curved.

Four checks follow, and each is met. First, per-alternative geometry quantizes nothing: within each alternative the geometry is continuous, classical and without a least step, and the discreteness of the alternatives is the discreteness of the content, which the matter carries. Chapter 6, "The Category Error", sets out the two senses in which the arena is not quantized: it has no least step, and nothing is a quantum of the arena itself, at any scale. Second, where the matter is sharply one alternative there is one term in equation (7.3), and equation (7.3) reduces to equation (7.1); every computation of Section I stands. Third, within each alternative the stress-energy is conserved and the Bianchi identity holds, so each geometry is a consistent solution.[3] Fourth, per-alternative geometry is not an approximation to a quantum theory of the geometry: the account contains no quantum theory of the geometry for it to approximate.

Equation (7.3) states the configuration at one time, and a law carries it forward. In the Newtonian regime, the regime of the experiment of Chapter 8, that law is the ordinary Hamiltonian of the bodies, with Newton's pair potential as an operator on their positions.

$$\hat{H} = \Sigma_k \, \hat{p}_k^{\,2} \,/\, (2m_k) - \Sigma_{j<k} \, G \, m_j \, m_k \,/\, |\hat{x}_j - \hat{x}_k| \tag{7.4}$$

In words: the energy of the bodies is the sum of their kinetic energies and of Newton's potential energy for each pair, with positions and momenta taken as operators. On an alternative in which each body has a definite placement, the potential takes the value that the Newtonian limit of that alternative's geometry gives, so the phase of each alternative advances at the rate set by its own energy. The kinetic terms spread the placements of each body, and equation (7.4) carries the alternatives forward with their relative phases kept. Chapter 8 derives the entangling phase from it. Beyond the Newtonian regime I take the law as given: the alternatives of the matter evolve by quantum theory, and Einstein's equations fix the geometry of each alternative from its stress-energy, as equation (7.3) states.

Per-alternative geometry passes the experiment of Page and Geilker. In each alternative the large masses sit in one placement and source the field of that placement, and the counter's registration of the decay has selected which alternative each run is. The bar therefore turns as the actual placement requires, on every run, as observed.

IV. No Signal Faster Than Light

A finding is the selection of one outcome from among its possibilities, by which something possible becomes manifest for the comparer that finds (Chapter 3, "The Cross-Modal Principle, Checked Against Six Theories").

Proposition 7.2. A finding changes no geometry, here or anywhere. News of a distant finding enters a comparer's record only at an event on or within the future light cone of the finding, in the geometry of the alternative that the finding selects, and no comparison registers a surface of simultaneity.

Argument. By Proposition 7.1 each alternative already carries its geometry before any finding is made. A finding selects among alternatives that carry their geometries, so it adds no geometry and alters none. What a distant finding changes for a comparer here is that comparer's record, and the record changes only when a signal carrying the finding arrives, which is no earlier than light from the finding could arrive. Each alternative carries its own geometry, and so its own light cones, and each finding is made on the light cone of the alternative it selects. Within one geometry the light cone is the same in every frame, so the account needs no preferred frame. With a geometry for each alternative, the order of two events may differ from one alternative to another, and I accept that consequence. Zych, Costa, Pikovski and Brukner (2019) show that a mass in a superposition of placements can entangle the temporal order of two timelike

events, and that the entangled order can violate a Bell-type inequality. I identify an event across alternatives by its content, by what happens and to which bodies, and not by its coordinates. Giacomini and Brukner (2022) extend Einstein's equivalence principle to superpositions of spacetimes, by changes of quantum reference frame, and find that such superpositions obey it. Until a signal carrying the finding arrives, every local statistic here follows quantum theory unchanged, whatever measurement was chosen far away (Eberhard 1978), so the observed results of local findings do not depend on which spacelike surface is used to describe them.[4]

Rests on: Proposition 7.1; relativity, as established physics (P15); the no-signaling theorem (Eberhard 1978).

Proposition 7.3. One geometry sourced by the expectation value fails in both of its forms. Without reduction of the state it fails the experiment of Page and Geilker; with reduction of the state it signals faster than light.

Argument. The first half is shown in Section II. The second half is a standard result about nonlinear quantum dynamics. Let a body here be entangled with a partner at a distance, and let the comparer at the partner choose between two measurements. One measurement leaves the body here, on the reduced form, sharply in one placement or the other; the other leaves it in an even sum of both placements. On the reduced form the weights here change with the distant choice, and a geometry sourced by those weights changes with them. A balance here would then register the distant choice before any light from it could arrive. Gisin (1990) showed that Weinberg's nonlinear quantum mechanics, combined with the standard reduction, signals in this way, and Bahrami, Großardt, Donadi and Bassi (2014) showed that the Schrödinger-Newton equation (7.2), togeth-

er with the standard reduction, permits superluminal signaling. Per-alternative geometry has no geometry sourced by the weights, and so it has nothing that could follow a distant choice.

Rests on: the experiment of Page and Geilker (1981); the signaling of nonlinear dynamics with reduction (Gisin 1990; Bahrami, Großardt, Donadi and Bassi 2014); relativity, as established physics (P15).

Table 7.1 sets the three configurations against the two tests of this chapter and the prediction of the next.

Configuration	**Page and Geilker (1981)**	**Signal faster than light**	**Entanglement of two masses through gravity**
One geometry, expectation value, state never reduced	excluded: predicts no deflection	none	none
One geometry, expectation value, state reduced at each finding	passes	yes (Gisin 1990; Bahrami et al. 2014)	none
Per-alternative geometry (Proposition 7.1)	passes	none (Proposition 7.2)	predicted (Chapter 8)

V. Gravity-Induced Collapse

Diósi (1987) and Penrose (1996) proposed that gravity itself ends a superposition of masses, and their proposals share the starting point of Proposition 7.1: two alternatives of a mass in different placements correspond to two different geometries. Penrose argued that a superposition of two dif-

ferent geometries has no well-defined common time translation, and that the superposition decays at a rate set by the gravitational self-energy of the difference of the two mass distributions.

$$\tau_{DP} \approx \hbar \,/\, E_G \tag{7.5}$$

In words: the lifetime τ_{DP} of the superposition (the subscript names Diósi and Penrose) is the reduced Planck constant divided by E_G, the gravitational self-energy of the difference between the mass distributions of the two alternatives. Diósi's master equation gives a decay of the same form, with the same dependence on that self-energy.

Gravity-induced collapse adds a new process to quantum dynamics: a reduction of the state made by gravity. Per-alternative geometry adds none. On this account a superposition ends only through a finding, and a finding is an act of comparison: an interaction that registers which alternative is the case, made by an instrument, an environment or any other comparer. Two alternatives with different geometries are both kept, each on its own geometry, for as long as nothing registers which one is the case, and their relative phase advances at the rate set by the difference of their energies.

So no mass, however large, no separation of the two placements, however wide, and no length of time ends a superposition of its own accord: only a comparison that registers which alternative is the case ends it. Gravitation alone ends none. What is found of the geometry is continuous and asymmetric, with no least step (Chapter 6, "The Category Error"), and what is found at a finding is discontinuous and symmetric (Definition 10.2), so the geometry of an alternative lays down no record of which alternative is the case. The gravitational decoherence of Section VI agrees: there the comparer that registers the difference is the internal motion of the body, a content, and not the geometry.

The two accounts therefore make different predictions, and experiment decides between them. Donadi, Piscicchia, Curceanu, Diósi, Laubenstein and Bassi (2021) searched underground for the faint radiation that the collapse noise would make charged particles emit. Their bound on the effective size of the mass density of nuclei rules out the natural version of the Diósi-Penrose model, the version with no free parameter. Versions with a larger effective size remain under test, and the entanglement experiment of Chapter 8 tests them in a new regime, because for its masses and placements equation (7.5) gives a lifetime far shorter than the time the experiment needs.[5]

VI. Gravitational Decoherence

Decoherence is the spreading of the relative phase of two alternatives into an environment that registers which alternative is the case, after which the alternatives no longer interfere in any measurement made on the system alone. Gravity produces decoherence of this ordinary kind, and per-alternative geometry describes it directly.

Pikovski, Zych, Costa and Brukner (2015) showed that time dilation decoheres the position of a composite body held in a superposition of heights. The proper time of the body's internal motion runs at a different rate in each alternative, so the internal state comes to record the height, and the interference of the two heights fades. That computation is per-alternative geometry at work. Each alternative of the height has its own geometry, and so its own proper time, and the internal degrees of freedom of the body are the comparer that registers the difference.

A second family of models makes decoherence a fundamental consequence of gravity being classical. Tilloy and Diósi (2016) source a Newtonian classical field from spontaneously localized matter, which avoids the signaling of Section IV and predicts an extra gravitational decoherence that depends on the localization model. Oppenheim (2023) couples a classical metric to quantum matter by dynamics that are linear, completely

positive and trace preserving, which forces both the metric and the matter to evolve stochastically, with decoherence arising from the coupling itself. Both models, that of Tilloy and Diósi and that of Oppenheim, keep one classical geometry for the matter's alternatives, and so neither lets two masses become entangled through gravity.

Per-alternative geometry predicts no decoherence beyond the ordinary kind. Every decoherence it allows is a comparison made by some environment: residual gas, thermal photons, the internal motion of the body, or a field that registers the placement. So the account and the fundamental models are separated by the same experiment, which also bounds any extra decoherence channel directly.

VII. Questions

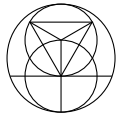
Eppley and Hannah (1977) argued that a classical gravitational field coupled to quantum matter must violate momentum conservation, the uncertainty principle or relativistic causality. Does their argument exclude per-alternative geometry? No. Their thought experiment measures a quantum body with a classical gravitational wave, and it presupposes one classical field for all the body's alternatives. In per-alternative geometry each alternative has its own field, so the measurement that the argument requires is itself a selection among alternatives. Mattingly (2006) showed separately that any device able to perform the proposed measurement would be massive enough to lie within its own Schwarzschild radius.

Is per-alternative geometry the many-worlds reading of quantum theory applied to gravity? No. Proposition 7.1 is a statement about what sources each geometry: the stress-energy of one alternative. It needs no claim about what becomes of the alternatives a finding does not select, and the finding is an act of comparison by which one outcome becomes manifest for the comparer that finds (Chapter 12, "The Interpretations as Modal Strategies", places the interpretations in that light).

Gravity sources itself, and a black hole can form from gravitational waves alone, with mass and with no matter. Proposition 7.1 keys each geometry to an alternative of the matter. What labels the alternatives where there is no matter? I reason that the free curvature of the arena, the Weyl part of Section III, belongs to the context of the content that produced it, and carries that content's alternatives. A gravitational wave is curvature that some content produced, and in each alternative of that content the wave is the one that alternative produced. Where no content is in superposition, there is one geometry. A hole formed from waves inherits the alternatives of whatever produced the waves. The reasoning rests on P5: the free curvature is context, and it is the context of the content whose motion produced it.

Which decomposition defines the alternatives? The decomposition into states of definite distribution of stress-energy, at the resolution at which the geometries differ. Two decompositions that differ only within that resolution give the same geometries to that resolution, and so give the same predictions for every gravitational measurement.

Per-alternative geometry keeps the relative phases of the alternatives and gives each alternative its own geometry, and so two masses related only by gravity should become entangled; Chapter 8 computes the phase for the design proposed in 2017, and for any other design.



Chapter 8

The Entanglement Test

Two masses, each in two places, related only by gravity: the phase, the budget, and what the outcome decides

This chapter establishes two propositions. Per-alternative geometry predicts that two masses, each held in two places and related only by gravity, become entangled, with an entangling phase fixed by the standard Newtonian interaction: 0.314 rad for the parameters that Bose and colleagues proposed in 2017 (Proposition 8.1). The outcome of the experiment separates the accounts with one geometry for all the alternatives, and gravity-induced collapse, from every account in which each alternative has its own geometry; it does not separate the account of this book from linearized quantum gravity (Proposition 8.2). I compute the phase explicitly, report the decoherence budgets as the proposals state the budgets, and read the argument from local operations and classical communication at full strength, with the premises of that argument.

I. The Proposals

The question that the proposals put to experiment was asked by Feynman at the Chapel Hill conference of 1957. He let a measurement of a spin send a ball one centimeter across into a superposition of two places, and reasoned that there must then be an amplitude for the ball's gravitational field, so that the field must be quantized (DeWitt and Rickles 2011,

Chapter 23). Per-alternative geometry gives his amplitude a reading of its own: one classical field for each place of the ball, with the relative phase of the two places kept.

Bose, Mazumdar, Morley, Ulbricht, Toroš, Paternostro, Geraci, Barker, Kim and Milburn (2017) proposed to entangle two microdiamonds through gravity alone, and Marletto and Vedral (2017) proposed the same arrangement with an argument for what its result would mean. In the scheme of Bose and colleagues, each diamond, of mass about 10^{-14} kg, carries one embedded electron spin. A magnetic field gradient splits each diamond into two paths according to its spin, so that its spin and its place become correlated. The two diamonds are released from traps 450 μm apart, each is held in a superposition of two places 250 μm apart for about 2.5 s, and the paths are then brought back together, so that the places are mapped back onto the spins.

The two diamonds interact only through gravity. The arrangement keeps their closest approach at about 200 μm, where Bose and colleagues estimate the Casimir-Polder interaction at a tenth of the gravitational one. At the end, correlations of the two spins are measured, and an entanglement witness certifies the result. An entanglement witness is a quantity whose value, beyond a stated threshold, is possible only for an entangled state. Their witness is $W = |\langle \sigma_x^{(1)} \otimes \sigma_z^{(2)} \rangle + \langle \sigma_y^{(1)} \otimes \sigma_y^{(2)} \rangle|$, built from the correlation of the spin of the first diamond along x with the spin of the second along z, and from the correlation of the two spins both along y. For every product state of the two spins, and for every mixture of product states, W is at most one, so a value greater than one certifies entanglement. Two further measures are used below. The concurrence of a state of two spins is zero for every unentangled state and at most one. The witness of Chevalier, Paige and Kim (2020), $W_1 = \langle 1 - \sigma_x \otimes \sigma_x - \sigma_z \otimes \sigma_y - \sigma_y \otimes \sigma_z \rangle$, is negative only for entangled states.

Label the two places of each diamond by their distance from the other diamond: n for the nearer place, f for the farther. There are four joint alternatives, (n, n), (n, f), (f, n) and (f, f), and the distance between the diamonds in alternative (a, b) is d_{ab}. For the parameters above, $d_{nn} = 200$ μm, $d_{ff} = 700$ μm, and $d_{nf} = d_{fn} = 450$ μm.

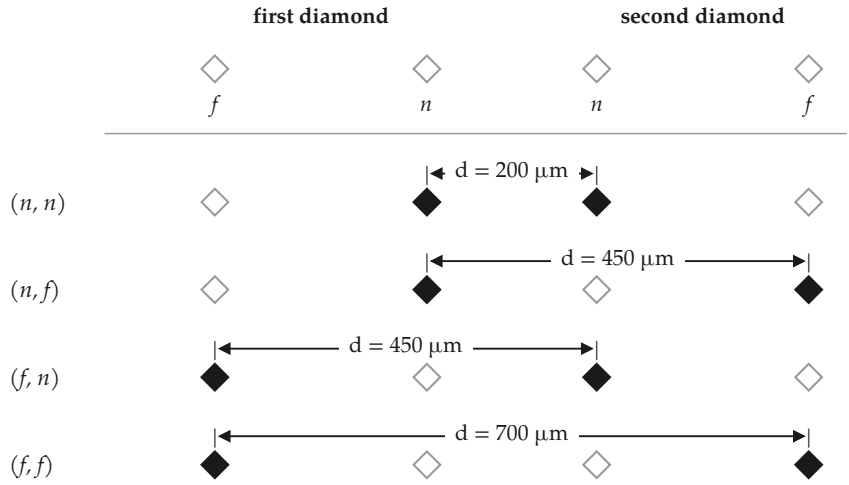


Figure 8.1. *The four joint alternatives of two diamonds, each held in two places 250 μm apart, with the traps 450 μm apart. The separations are $d_{nn} = 200$ μm, $d_{nf} = d_{fn} = 450$ μm and $d_{ff} = 700$ μm.*

Proposition 8.1. Two masses, each held in two places and related only by gravity, become entangled, with the entangling phase of equation (8.2); for the parameters of Bose and colleagues (2017), that phase is 0.314 rad.

Argument. First, by Proposition 7.1 each joint alternative carries its own classical geometry, sourced by the two diamonds in the places of that alternative, and the relative phases of the four alternatives are kept. Second, by the Hamiltonian of equation (7.4), the law that carries the alternatives forward in the Newtonian regime, the energy of the joint alternative (a, b) contains the interaction term $-Gm^2/d_{ab}$ once, the pair potential at that alternative's separation; while the places are held, the kinetic terms are the same in every joint alternative and add no relative phase. Third, by quantum theory taken as established physics (P15), the phase of each alternative advances at the rate set by its energy. Fourth, every other contribution to the energy, the rest energy and self-energy of each diamond among them, depends on one diamond alone and drops out of the combination of equation (8.2).

Rests on: Proposition 7.1; the Newtonian Hamiltonian, equation (7.4); quantum theory, as established physics (P15).

$$|\Psi(t)\rangle = \tfrac{1}{2}\, \Sigma_{a,b \in \{n,f\}}\, e^{i\varphi_{ab}}\, |a\rangle_1\, |b\rangle_2, \qquad \varphi_{ab} = G\, m^2\, t \,/\, (\hbar\, d_{ab}) \tag{8.1}$$

In words: after a time t the joint state is an equal sum of the four joint alternatives, and each alternative has acquired the phase φ_{ab}, which is Newton's constant times the square of the mass times the time, divided by the reduced Planck constant and by the distance between the diamonds in that alternative. Equation (8.1) is the evolution by equation (7.4) over the time t, applied to the four joint alternatives.

The state of equation (8.1) is a product of a state of the first diamond and a state of the second exactly when its matrix of amplitudes, $\tfrac{1}{2}e^{i\varphi_{ab}}$, has determinant zero. The determinant is zero when the phases of the two alternatives on one diagonal sum to the phases of the two on the other, modulo 2π. I call the difference of those two sums the entangling phase.

$$\Delta\varphi = \varphi_{nn} + \varphi_{ff} - \varphi_{nf} - \varphi_{fn} = (G\, m^2\, t \,/\, \hbar)\; (1/d_{nn} + 1/d_{ff} - 2/d_{nf}) \tag{8.2}$$

In words: the entangling phase is the phase of the two alternatives in which the diamonds are both near or both far, less the phase of the two alternatives in which one is near and the other far. It equals $Gm^2t/\hbar$ times the sum of the reciprocal nearest and farthest distances, less twice the reciprocal middle distance. A change of phase applied to the places of one diamond alone adds the same amount to one term of each sum and leaves $\Delta\varphi$ unchanged, so $\Delta\varphi$ is the one combination of the four phases that no local operation alters. The state is entangled whenever $\Delta\varphi$ is not a multiple of 2π, and its concurrence is $|\sin(\Delta\varphi/2)|$.

The numbers follow. With $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $m = 10^{-14}$ kg and $t = 2.5$ s, the prefactor is $G\,m^2\,t\,/\,\hbar = 1.582 \times 10^{-4}$ m. The three phases are $\varphi_{nn} = 0.791$ rad, $\varphi_{nf} = \varphi_{fn} = 0.352$ rad and $\varphi_{ff} = 0.226$ rad.

$$\Delta\varphi = 1.582 \times 10^{-4}\ m \times (1/200 + 1/700 - 2/450)\ \mu m^{-1} = 0.314\ rad \tag{8.3}$$

In words: for the parameters of Bose and colleagues, the entangling phase is 0.314 radians, about a twentieth of a full turn.[1] On the state of equation (8.1) with these phases, the concurrence is $|\sin(\Delta\varphi/2)| = 0.156$, and the witness of Chevalier, Paige and Kim takes the value $W_1 = -0.22$, which certifies the entanglement. The value belongs to the labeling of Bose and colleagues, in which each spin's first state is the left place of its diamond: the farther place for the first diamond and the nearer place for the second. In the labeling of equation (8.1), with the nearer place first for both spins, the same operator takes a positive value on the same state and certifies nothing, because W_1 changes when the two states of one spin are exchanged. The concurrence is the same in both labelings.

The phase of one alternative can also be written in Planck units.

$$\varphi_{ab} = (m\ /\ M_P)^2\ (c\ t\ /\ d_{ab}) \tag{8.4}$$

In words: the phase is the square of the ratio of the mass to the Planck mass M_P, times the distance light travels in the time t, divided by the distance between the diamonds. The square of the mass ratio is minute for a microdiamond, and the ratio of light's travel to the separation is enormous, and the experiment is designed so that their product is of order one. Christodoulou and Rovelli (2019) give the same phase a covariant form: the difference of proper time between the alternatives, read on a clock carried by one body, times that body's rest energy divided by the reduced Planck constant, so that the Newtonian energy of the interaction of the two bodies enters the phase once, as in equation (8.1); and they note its relation to the Planck mass.

III. Decoherence Budgets

The entangling phase is visible only if each diamond keeps the relative phase of its two places for the whole protocol. Bose and colleagues (2017) state the budget for their scheme. They require a pressure of 10^{-15} Pa and a temperature of 0.15 K. At that pressure and temperature, the time to the first collision with a residual gas molecule is of the order of the duration of the protocol, about 3.5 s including acceleration and recombination. They find the decoherence by scattering, emission and absorption of thermal photons negligible at that temperature, and they take an electron spin coherence time of about one second as achievable in microdiamonds below 77 K.

Later designs relax the budget. Van de Kamp, Marshman, Bose and Mazumdar (2020) place a thin conducting plate between the two interferometers, which screens the Casimir interaction between the diamonds and lets them approach more closely. Their design works with masses of 10^{-16} to 10^{-15} kg, a superposition of about 20 μm and a field gradient of 10^{4} T m^{-1}, and it adds the plate's own Casimir interaction and its vibrations to the budget. For any design, equation (8.2) with that design's masses, distances and time gives the phase that per-alternative geometry

predicts. The phase of 0.314 rad belongs to the design proposed in 2017. Later designs use other masses, separations and times (Bose and colleagues 2026), and the same formula gives their phases.

The budget fixes which experiment decides Proposition 8.1, whatever its result: one in which the coherence of each interferometer is verified to hold over the protocol, so that ordinary decoherence is excluded as a cause that could keep the entanglement from building. In such an experiment, per-alternative geometry (Proposition 7.1) says that the two diamonds become entangled with the phase of equation (8.2), and linearized quantum gravity says the same, so the experiment tests the two together.

IV. What the Outcome Separates

Table 8.1 sets the accounts against the prediction each makes for the experiment.

Account	The geometry of the alternatives	Prediction
Semi-classical equation (Møller 1962; Rosenfeld 1963) and the Schrödinger–Newton equation	one geometry, sourced by the expectation value	no entanglement
Classical stochastic gravity (Tilloy and Diósi 2016; Oppenheim 2023)	one classical geometry, with noise	no entanglement; added decoherence
Gravity-induced collapse (Diósi 1987; Penrose 1996)	the superposition ends within about 0.012 s	no entanglement builds
Per-alternative geometry (Proposition 7.1)	one classical geometry for each alternative, phases kept	entanglement, $\Delta\varphi = 0.314$ rad
Linearized quantum gravity	small waves described by quanta on a classical background	entanglement, $\Delta\varphi = 0.314$ rad
Superposition of geometries (Christodoulou and Rovelli 2019)	superposed proper times	entanglement, $\Delta\varphi = 0.314$ rad

Proposition 8.2. The outcome of the experiment separates every account with one geometry for all the alternatives, and gravity-induced collapse, from every account in which each alternative has its own geometry. It does not separate per-alternative geometry from linearized quantum gravity.

Argument. The first three rows predict no entanglement, each for its own reason. A single geometry sourced by the expectation value acts on each diamond as a classical field fixed by the other diamond's average, so it gives each diamond a phase that depends on that diamond's own place alone, and $\Delta\varphi$ is zero. Classical stochastic gravity keeps one classical field and adds noise, which can only remove coherence. Gravity-induced collapse ends each superposition before the phase can build: for a diamond of this mass held in two places 250 μm apart, equation (7.5) gives a lifetime of about 0.012 s, against the 2.5 s the phase needs (Chapter 7, Note 5, computes the self-energy). The last three rows predict entanglement with the same phase, because each gives every joint alternative its own interaction energy and keeps the relative phases.

Rests on: Proposition 8.1; Table 8.1 and the accounts it cites; the lifetime of gravity-induced collapse, equation (7.5).

I predict that the two diamonds become entangled with the phase of equation (8.3), and that a design with other masses, distances and times shows the phase that equation (8.2) gives for it. The experiment decides whether each alternative of the matter has its own geometry. The experiment works below the Planck scale, so it does not test the claim of Chapter 6, that the arena has no least step and that nothing is a quantum of the arena itself at any scale: below that scale this account and linearized quantum gravity predict the same for every experiment (Section V).

V. What the Outcome Leaves Together

Linearized quantum gravity predicts the same entanglement as per-alternative geometry, with the same leading phase. Bose and colleagues derive the phases by quantizing linearized general relativity, and they read the entanglement as showing nonzero off-diagonal terms in the coherent-state basis of the gravitational field.[2] Danielson, Satishchandran and Wald (2022) find no clear distinction between entanglement mediated by the Newtonian field of a body and entanglement with emitted gravitons, and

they conclude that Newtonian entanglement implies the existence of graviton entanglement. Belenchia, Wald, Giacomini, Castro-Ruiz, Brukner and Aspelmeyer (2018) find that a thought experiment with superposed masses is consistent with complementarity and causality when gravitational radiation is quantized and vacuum fluctuations limit the localization of a particle to no better than a Planck length.

Each of these results agrees with per-alternative geometry. The small waves of a geometry can be described by quanta, and this account allows that description below the Planck scale, as a description: its quanta are quanta of the description, never quanta of the arena (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale"). A bound on how sharply a body can be placed is a limit on localization; it is not a least length of the geometry. So the result of Danielson, Satishchandran and Wald, that Newtonian entanglement goes with what they call graviton entanglement, holds on this account as a statement within the description of the small waves by quanta, and it quantizes nothing of gravitation.

The two descriptions differ in what is superposed and in where each description ends. In per-alternative geometry the alternatives superposed are those of the matter, each carrying a classical geometry, and the smooth description holds below the Planck curvature, with no quantized arena carried beyond it. Linearized quantum gravity describes the small waves by a quantum field on a classical background, and it superposes states of that field. Read as the first term of a quantization of the metric itself, it belongs to a program that carries a quantized metric to the Planck scale and past it; that reading, and not the description of small waves, is what Chapter 6 excludes. Below the Planck scale the two predict the same for every experiment. The category error that Chapter 6 identifies is made by the reading of linearized quantum gravity as the first term of a quantization of the metric itself, a reading that carries a quantized metric to the Planck scale and past it; that reading parts from this account in what it predicts only at that scale and beyond, and the claim of Chapter 6, that nothing is a quantum of the arena itself, holds at every scale (Proposition 6.3).

VI. The Argument From Local Operations and Classical Communication

Local operations and classical communication are the operations that two parties perform, each on its own system, together with classical messages sent between them. Such operations cannot create entanglement between two systems that start in a product state. Bose and colleagues use that result to read their witness, and Marletto and Vedral build their argument on it: any system capable of mediating entanglement between two quantum systems must itself be quantum.

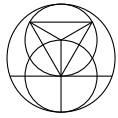
Read fairly, the argument has two premises. The first premise is locality: the diamonds do not interact directly, and every interaction passes through the mediator, here gravity. The second premise is a definition of a classical mediator: a system with a single observable, or with observables that all commute. From these premises, entanglement of the diamonds shows that the mediator has at least two observables that do not commute. Galley, Giacomini and Selby (2022) extend the result to generalized probabilistic theories: no theory describes gravity as generating entanglement, as acting as a local mediator, and as classical in that sense, all three together. The inference is valid, and the debate concerns the premises.

The premises have been examined in the literature. Hall and Reginatto (2018) argue that the claim that quantum systems cannot become entangled through a classical intermediary holds for mean-field models and fails for configuration-ensemble models, which supply explicit counterexamples. Anastopoulos and Hu (2018) argue that entanglement produced by Newtonian forces is agnostic about whether the true degrees of freedom of the gravitational field are quantum or classical. Martín-Martínez and Perche (2023) analyze the role of locality and find that the entanglement of the proposed experiments could be accounted for

without quantum degrees of freedom of the gravitational field, and they set out what must be assumed for the experiment to count as evidence of quantum gravity.

Per-alternative geometry meets the criterion of Marletto and Vedral for a non-classical mediator. The geometry takes a different state in each alternative of the matter, and the relative phases of the alternatives are kept, so the geometry is not a system with a single observable. A positive result would therefore agree with their conclusion as they state it.[3] What a positive result would not show is a quantum of the arena at any scale, or a least step of the arena. The property that the argument of Marletto and Vedral establishes is a mediator that differs from alternative to alternative with the relative phases of the alternatives kept. Proposition 7.1 assigns that same property to the geometry. The property excluded by Chapter 6 is a different one: a description of the arena itself in the character of content.

The experiment therefore separates the accounts as Proposition 8.2 states. Entanglement with the phase of equation (8.3) agrees with per-alternative geometry and with linearized quantum gravity, and it excludes the accounts with one geometry and gravity-induced collapse. Linearized quantum gravity speaks of gravitons, and in Chapter 9 I ask what the word "graviton" can name on this account, what its detection would show, and what the Planck scale is.



Chapter 9

Linearized Gravity, Gravitons and the Planck Scale

The quanta of a description of small waves, what a detection would show, the weight of non-renormalizability, and the scale at which the two descriptions meet

This chapter establishes four propositions. Gravitation is not quantized: the effective field theory of gravity quantizes a description, the description of the small waves of a classical geometry, and its quanta are quanta of that description at most, never quanta of the arena (Proposition 9.1). A detector that registers a gravitational wave in single clicks would not count against this account, and it would not show gravitation quantized (Proposition 9.2). The Planck scale is where an object's gravitational size equals its quantum size, so that neither description dominates and both are relevant; the Planck length is not a least length (Proposition 9.3). Newton's constant is measured: the ground fixes its role, and measurement fixes its magnitude, because the axioms are structural, not dimensional (Proposition 9.4). On the way, the chapter weighs the non-renormalizability of quantized general relativity, and finds that the structural argument does not need it.

I. Gravity as an Effective Field Theory

An effective field theory is a quantum field theory that describes a system below some energy, with every effect of higher energies summarized in local terms whose influence falls as a power of the energy. Donoghue (1994) showed that general relativity, quantized about a classical background, is an effective field theory of this kind, and that its leading quantum corrections at long distances are fixed by its low-energy content alone, with no free parameter.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu'} \qquad \kappa = (32\pi G)^{1/2} \,/\, c^2$$

(9.1)

In words: the metric g is written as a classical background $\bar{g}$ plus a small wave h, scaled by a constant κ built from Newton's constant, and the effective theory quantizes its description of the small wave h.

The best-known result of the effective theory is the correction to Newton's potential between two masses (Bjerrum-Bohr, Donoghue and Holstein 2003).

$$V(r) = -(G\, m_1\, m_2 \,/\, r)\, [\, 1 + 3G(m_1 + m_2)/(r\, c^2) + (41/10\pi)\, G\hbar/(r^2\, c^3)\,]$$

(9.2)

In words: the potential is Newton's, multiplied by one plus a classical relativistic correction of relative size $G(m_1 + m_2)/(rc^2)$, plus a quantum correction proportional to $G\hbar/(r^2c^3)$, which is the square of the Planck length divided by the square of the separation. At a separation of one meter the ratio $G\hbar/(r^2c^3)$ is 2.6×10^{-70}, and the quantum correction is $41/10\pi$, about 1.3, times that ratio.[1] The expansion of the effective theory is an expansion in that ratio, or equivalently in the curvature of the background in units of the Planck curvature, and its corrections are small below the Planck curvature and of order one at it.

Proposition 9.1. Gravitation is not quantized. The effective field theory of gravity quantizes a description: the description of the small waves of a classical geometry about its background, which holds below the Planck curvature. Its quanta are quanta of that description at most, and never quanta of the arena, of space-time or of the geometry.

Argument. First, by Proposition 7.1 the background of equation (9.1) is a classical geometry, one for each alternative of the matter. Second, a small wave of that geometry obeys linear equations about the background, and in its description each normal mode is a harmonic oscillator; quantum theory, taken as established physics (P15), applied to that description, gives each mode energy levels spaced by $\hbar\omega$, and a quantum of the description is one step in the energy of one mode. Third, the category error identified in Chapter 6, "The Category Error", is exact in two senses: the arena has no least step, and nothing is a quantum of the arena itself, at any scale (Proposition 6.3). A step in the energy of one mode of the description is a step in neither sense: it is not a step in the extent of the geometry, whose lengths, areas and durations keep no least value, and the description it belongs to has a domain, which ends at the Planck curvature. So what the effective theory quantizes is its description of a wave, and its quanta belong to that description. I call such a quantum a graviton, in that sense and in no other: the word names at most a quantum of a description, never a quantum of the arena, and this book never calls the graviton a particle, a carrier of gravitation or a kind of matter.

Rests on: Proposition 7.1; quantum theory, as established physics (P15); the exact scope of "not quantized" (Proposition 6.3).

The phonon is the nearest sense by which the word "graviton" can be made intelligible. A phonon is a quantum of the description of the vibrations of a held structure: a solid whose bonds hold its atoms in a lattice, so

that a vibration is a small motion of the atoms about the places the bonds hold. The arena is not a held structure. It has no parts held in place by bonds, and its curvature is set by the content in it (Chapter 5, "The Content and Context Partition"). So even the phonon's sense does not make the graviton a quantum of the arena.

II. Can a Single Graviton Be Detected?

No graviton has been detected. Dyson (2013) asked whether any conceivable experiment in the real universe could detect one. Rothman and Boughn (2006) worked through the candidate detectors and found that an idealized thought experiment can register a single graviton, but that detection becomes impossible once realistic physics is taken into account, which they read as making Dyson's conjecture very likely true.

Tobar, Manikandan, Beitel and Pikovski (2024) proposed a different route. A massive acoustic resonator cooled near its ground state, driven by a passing gravitational wave of the kind already observed, would absorb energy from the wave in single quanta, and continuous sensing of the resonator's quantum jumps would resolve each absorption. They conclude that signatures of single-graviton exchange are within reach of experiment. Carney, Domcke and Rodd (2024) agree that detectors able to register single gravitons can be built, and they show that the same signal can be produced by a classical gravitational wave acting on the quantum detector. They conclude that a demonstration that gravitational radiation is quantized would require measurements far harder than counting clicks, and that such measurements are likely impossible in practice.

III. What a Detection Would Show

Proposition 9.2. A detector that registers a gravitational wave in single clicks would not count against this account, and it would not show gravitation quantized. The clicks are steps of the detector, which is content; what no detection of this kind can find is a step of the arena itself, or a quantum of the arena at any scale.

Argument. First, by Proposition 9.1 gravitation is not quantized, and a count of quanta of a gravitational wave is a count made in the description of small waves, in that description's own terms. Second, by the result of Carney, Domcke and Rodd, the clicks of such a detector are produced equally by a classical wave acting on a quantum detector, as the photoelectric effect, taken alone, is produced equally by a classical light wave acting on quantized atoms. So the clicks do not decide whether the wave itself is quantized.[2] Third, this account reads the clicks as quantum jumps of the detector, which is content, driven by a classical wave of the arena, and the description of the wave by quanta counts the same clicks in its own terms; neither reading involves a step of the arena or a quantum of the arena at any scale. So a detection is accounted for on this account, and it separates this account from no rival.

Rests on: Proposition 9.1; the result of Carney, Domcke and Rodd (2024); the exact scope of "not quantized" (Proposition 6.3).

The consistency results that require quantized radiation also do not show gravitation quantized. Belenchia and colleagues (2018) find that a thought experiment with superposed masses respects complementarity and causality when gravitational radiation is quantized, and Danielson, Satishchandran and Wald (2022) find that the entanglement of Chapter 8 goes together with entanglement with emitted gravitons. Both are require-

ments on the description of the radiation, and the description of small waves by quanta meets them; the radiation itself is a small wave of a classical geometry, one for each alternative of the matter (Proposition 7.1), so meeting them quantizes nothing of gravitation.

IV. The Weight of Non-Renormalizability

Quantized general relativity is not renormalizable. 't Hooft and Veltman (1974) found that its one-loop divergences, once matter is coupled, cannot be absorbed into the parameters of the theory, and Goroff and Sagnotti (1986) found a divergence of pure Einstein gravity at two loops. A standard inference runs from that fact to the need for a quantum theory of gravity that completes the description at high energies.

The non-renormalizability of quantized general relativity carries less weight than the structural argument, and the structural argument does not need it. Effective field theory handles theories that are not renormalizable at low energies, and quantized general relativity is computed that way (Section I). The structural argument rests on what quantization is. Quantization is a mode exchange: it changes the character of a description's findings and of its formalism together, so that discontinuously symmetric findings, written in a continuous formalism, take the place of continuously asymmetric findings, written in a symmetric formalism (Chapter 4, "What Quantization Is"). The arena's findings are continuous and asymmetric, and a description that gave the arena itself findings of the other character would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both; the Incommensuration Theorem excludes each of them (P9; Chapter 6, "The Category Error"). That argument holds whatever the high-energy behavior of any perturbation series.[3]

Non-renormalizability marks the place where I expect, on my diagnosis, a domain to end. The Planck energy is the rest energy of the Planck mass of Section V, about 1.22×10^{19} GeV. The local terms of the effective

theory grow with powers of the energy in units of the Planck energy, so the expansion loses its order at the Planck scale, where the small wave h is no longer small beside the background. Where the perturbation and the background become comparable, the separation of a wave from the geometry it travels on no longer holds. I expect the description of small waves to reach the edge of its domain there, and I do not expect a quantized metric that continues past that edge.

V. Where the Two Descriptions Meet

Every object of mass m has two sizes. Its gravitational size is the length r_g at which its own gravity becomes strong, and its quantum size is the length λ within which it cannot be confined without creating further particles.

$$r_g = G\,m\,/\,c^2, \quad \lambda = \hbar\,/\,(m\,c), \quad r_g = \lambda \text{ when } m = M_P = (\hbar c/G)^{1/2} \tag{9.3}$$

In words: the gravitational size grows in proportion to the mass, the quantum size shrinks in inverse proportion to it, and the two are equal for one mass only, the Planck mass $M_P = 2.176 \times 10^{-8}$ kg. At that mass both sizes equal the Planck length, $l_P = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-35}$ m, and light crosses that length in the Planck time, $t_P = 5.391 \times 10^{-44}$ s. The Planck scale is marked by the full Planck mass, the mass at which the two sizes of equation (9.3) are equal.[4]

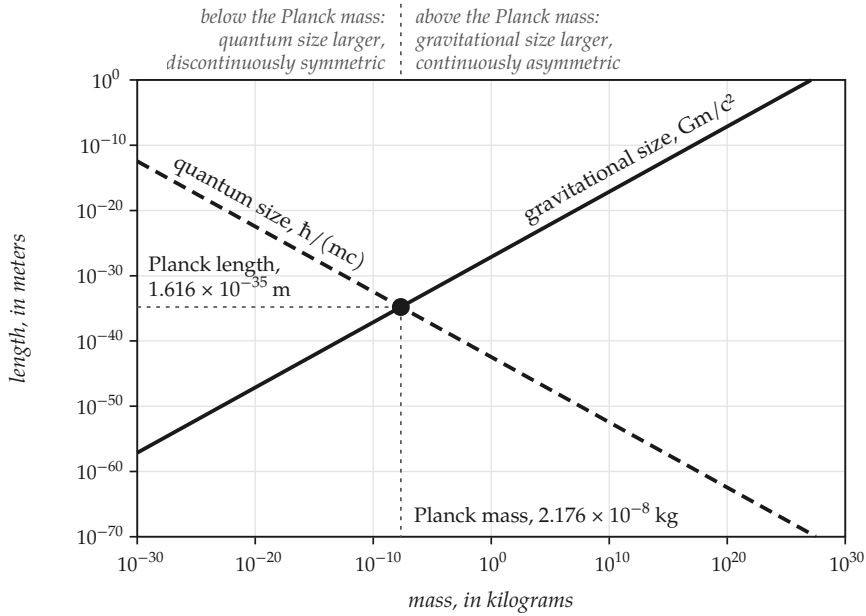


Figure 9.1. *The gravitational size Gm/c^2 and the quantum size $\hbar/(mc)$ against mass, on logarithmic axes. They cross at the Planck mass, 2.176×10^{-8} kg, where both equal the Planck length, 1.616×10^{-35} m.*

Proposition 9.3. The Planck scale is where the two descriptions meet: an object's gravitational size equals its quantum size, neither description dominates, and both are relevant. The Planck length is not a least length, area or duration of the arena. The smooth description holds below the Planck curvature.

Argument. First, for any mass far above the Planck mass the gravitational size exceeds the quantum size, so the quantum size is negligible beside the scale on which the object's gravity acts, and the continuously asymmetric description dominates. For any mass far below the Planck

mass the quantum size exceeds the gravitational size, and the discontinuously symmetric description of the content dominates. Second, at the Planck mass the two sizes coincide, so neither description dominates and both are relevant, and a single description that perfected both characters there would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both (P9). Third, nothing in either step gives the arena a step: the Planck length is the length at which two descriptions exchange dominance, and it is defined by a comparison of two sizes of an object, not by a grain of space. This account posits no third description beyond the Planck scale, and no new geometry there. Each existing description reaches the edge of its domain at that scale, and the corrections of equation (9.2) are of order one at the Planck curvature and small below it.

Rests on: the two sizes of a mass, equation (9.3); the Incommensuration Theorem (P9); the corrections of the effective field theory, equation (9.2).

The standard view holds that at the Planck scale space and time give way to quantum fluctuations, and that quantum theory and gravity can no longer be treated separately there. Proposition 9.3 reads the same scale differently. Both readings agree on where the scale lies; they differ on what lies there. On this account what lies there is the meeting of two descriptions, each perfect in its own character, and no single description can perfect both characters there.

A limit on localization is a different thing from a least length. Belenchia and colleagues (2018) find that vacuum fluctuations of the field limit the localization of a particle to no better than about a Planck length. That bound limits how sharply a comparison made from inside the world can place a body; it is a bound on placing, and it gives the geometry no least value. The arrival times of high-energy photons from distant sources test whether the arena itself has a grain at that scale, and Chapter 19, "Predictions, Evidence and Decisive Experiments", sets out that evidence and the observations that decide the prediction of smoothness (Prediction 19.4).

VI. Newton's Constant Is Measured

The measured value of Newton's constant is $G = 6.67430(15) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, the 2022 recommended value of the Committee on Data for Science and Technology (CODATA). The Planck mass, length and time of Section V follow from it, with the reduced Planck constant and the speed of light.

Proposition 9.4. The ground fixes the role of Newton's constant and not its magnitude. Its role is the strength of the coupling of every content to its context, universal, unshieldable and the same for every kind of matter; its magnitude is a fact about this domain of comparison, read off the world by measurement.

Argument. The first half follows from the four markers by which Chapter 5, "The Content and Context Partition", identifies gravity as the context of physics (P18): it couples to every content, nothing shields it, its coupling to each content equals that content's inertia, and it has no discrete charge. A constant that sets the strength of such a coupling takes the role those markers describe, whatever its value. The second half follows from the kind of premise the ground supplies, which Section VII sets out.

Rests on: the identification of gravity as the context of physics by its four markers (P18); the three axioms, which are structural and not dimensional (P12; Section VII).

A measurement of G is a comparison of a laboratory mass with the Planck mass. In units in which G, $\hbar$ and c are each one, every mass is a multiple of the Planck mass. So a measurement of G in kilograms, meters and seconds is a measurement of the Planck mass in kilograms, which is a

dimensionless ratio between a mass defined in the laboratory and the mass at which the two descriptions meet. This account makes no claim on the value of any such ratio, and it needs none.

VII. Structural, Not Dimensional

The axioms are structural, not dimensional. They determine relations: which compound characters can be perfected together, which roles a comparison has, and which concept takes which role in a given focus (P9, P12, P13). They do not determine an absolute magnitude, because a magnitude is a ratio to a unit, and the unit is a standard that some comparer brings. The axioms are about comparison as such, not about any particular domain of comparison, and the numerical value of G is one of the facts that selects the domain.

I identify the measured value of G as the point at which the structure of the ground meets this world. In the focus of physics at large, which asserts all three axioms (Table 2.1), structure is omniscient and measurement is immanent (P13); the bridge between them, the act of comparing a theory against the world, requires at least one datum read off the world, and for the scale at which gravity couples content to context that datum is G. A description with no measured input would fix its own relation to every comparer that could use it, and nothing in the axioms requires such a closure.[5] So the thesis of this book needs no value of G, and it gives none. It needs only the role of G, which the four markers fix, and the meeting of the two descriptions at the Planck scale, which equation (9.3) locates once G is measured.

VIII. Questions

If the small waves of the geometry can be described by quanta, why is the arena not quantized in the same way? A description of the small waves by quanta quantizes the description, and gravitation is not quant-

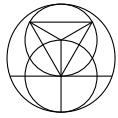
ized. A graviton, in the sense of Proposition 9.1, is a quantum of the description of a small wave; it does not give the geometry a least length, and its description fails where the wave is no longer small beside the background, at the Planck curvature. The phonon gives the word its nearest sense, a quantum of the description of the vibrations of a held structure, and the arena is not a held structure, so even that sense does not make the graviton a quantum of the arena. Quantizing the arena would require a quantum of the arena itself, and Chapter 6 excludes that at any scale.

Is the Planck length not the smallest length that can be measured? The bound of Belenchia and colleagues limits how sharply a body can be placed, at about the Planck length. The bound of Belenchia and colleagues is a property of comparison made from inside the world. Proposition 9.3 states the bound as a property of comparison made from inside the world. The bound is not a least length of the arena, and the lengths, areas and durations of the arena keep no least value.

Gravity is now placed as context, and gravitation is not quantized: a description of its small waves by quanta has quanta of that description only, and the Planck scale is located as the meeting of the two descriptions. Part IV turns from the arena to the finding, and asks what the three kinds of time in physics are and why a measurement has an outcome.

PART IV

MEASUREMENT



Chapter 10

Three Times

Geometric, event and evolution time as distinct mathematical objects, how each theory holds two, and the forbidden conjunction of a unified time

This chapter establishes that physics uses the word "time" for three mathematical objects of different kinds: the time of the geometry, the time of findings, and the time of evolution. I identify the three with the three modalities of comparison, and they meet the three conditions of Axiom III: they are distinct, inseparable and non-interchangeable. General relativity holds the first two, and quantum mechanics holds the last two. A single time that carried all three would need a pairing the Incommensuration Theorem forbids, and any formalism that holds two of the three holds the third only as external, as primitive, or not at all.

I. The Question

Physics speaks of time in three places. General relativity speaks of the time along a worldline and the order of events within light cones. Quantum mechanics speaks of the time in which a state evolves. Every experiment speaks of the time at which something was registered. The standard assumption is that these are three treatments of one time. The assumption meets its sharpest test in canonical quantum gravity. There the attempt to treat the geometry and the quantum state in one formalism produces an equation with no time in it at all, and the result is called the

problem of time (DeWitt 1967; Isham 1993).[1] I hold that there are three times, each a different kind of mathematical object, and that the problem of time is what follows from asking one formalism to make them one.

The words of this chapter keep the meanings of Chapter 1. A finding is the outcome of one act of comparison, and a comparer is whatever makes a comparison: an instrument, a detector, an interaction. Neither word refers to a mind.

Quantum theory already draws part of the distinction within its own domain. Busch (2008) separates three uses of time in quantum mechanics: external time, read from a laboratory clock and entering the equations as a parameter; intrinsic time, defined by the dynamical behavior of the system itself; and event time, the time at which something is found, such as the time of arrival of a particle at a detector.[2] I extend that division from quantum mechanics to physics as a whole, and I give the division a ground.

II. Three Mathematical Objects

Which concept stands in which role depends on the focus of the argument being made (P11). In the focus of time, which asserts all three axioms (Table 2.1), the three objects below take the three roles of P10. Each identification is stated beside its definition, and P19, after Table 10.1, collects them.

Definition 10.1 (geometric time). Geometric time is the structure of time that the metric of spacetime carries: the proper time along each timelike worldline and the causal order of events. A time function, a smooth real function on spacetime whose gradient is everywhere timelike, is a coordinate of it. Where the matter has several alternatives, each carries its own geometry (Proposition 7.1), and geometric time is the time of the geometry of one alternative.

The proper time along a worldline γ with parameter λ is

$$\tau[\gamma] = (1/c) \int \sqrt{(-g_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu})}\, d\lambda \tag{10.1}$$

In words, the proper time is the length that the metric assigns to the worldline, divided by the speed of light. What a comparison finds of geometric time is a set of such intervals and the order of events within light cones. The intervals vary smoothly from one worldline to its neighbors, so what is found is continuous. The intervals differ from region to region with the matter present, and a generic spacetime has no isometry, so what is found is asymmetric. Geometric time is therefore continuous and asymmetric in the sense of P8. The time-reversal invariance of Einstein's field equations and their invariance under diffeomorphisms are symmetries in physics' sense: they belong to the formalism, the instrumental mode of general relativity (Proposition 2.2). Geometric time carries no forbidden pairing, because its symmetries and its continuity lie at the two levels of the theory. I identify geometric time with the omniscient modality: it is time laid out as structure, the whole four-dimensional pattern held in view as one.

Definition 10.2 (event time). Event time, the time of findings, is the order of findings for a comparer: each finding is one outcome selected from a discrete set of possible outcomes and laid down as a record, and event time is the order in which those records are made.

As a mathematical object, event time is a discrete sequence of outcomes, each a label k of one element of a measurement (a projector P_k, or an effect E_k), ordered by precedence. What is found is discrete, since a finding is one outcome with no intermediate case. What is found is symmetric, since a record, once made, reads the same for every later comparison of it, and a repeated measurement of the same quantity returns the same outcome. Event time is therefore discontinuously symmetric in what is found, the character of every finding of quantum mechanics (P16). Its order is the order of Axiom II (P12): in a comparison the range of what is possible comes first, then the act, then the record, and the record shapes the next range. That order is a precedence and not a clock reading, and it cannot be reversed. I identify the one-way order of findings with the direction of time; every irreversible record in physics, a darkened grain of emulsion or a registered count, is an instance of that order.

Findings define the order of event time; they do not occur within a time given before them. I identify event time with the immanent modality: it is the time of the act.

Definition 10.3 (evolution time). Evolution time is the parameter t of the strongly continuous one-parameter group of unitary operators U(t) that carries the quantum state.

$$U(s+t) = U(s)\,U(t), \quad U(t) = exp(-iHt/\hbar), \quad i\hbar\, d\psi/dt = H\psi \tag{10.2}$$

In words, evolving for a time s and then for a time t is the same as evolving for s plus t, and the group is generated by the Hamiltonian H, so that the rate of change of the state is H acting on the state. By the theorem of Stone (1932), such groups and their self-adjoint generators correspond one to one.[3] Evolution time belongs to the formalism of quantum mechanics, its instrumental mode, which Table 3.1 reads as continuous. The state varies smoothly with t. The invariance of H under translations in t is a symmetry in physics' sense, a property of the law, carried to the findings by Noether's theorem as the conservation of energy. What evolution time carries is a state in a space of states, in which the eigenvectors of each observable label its possible outcomes and the amplitudes give those outcomes their probabilities. I identify evolution time with the transcendent modality: it is the time of possibility, of what could be found and with what probability. In the focus of time, which asserts all three axioms (Proposition 10.1, the order of Axiom II and the conjugacy of Axiom I), the state that evolution time carries stands in the transcendent role; in the focus of the use of a theory (Proposition 2.1) the formalism that contains the state is laid out as structure and stands in the omniscient role, and the two bindings belong to two foci (Chapter 2, Table 2.1).

Table 10.1 sets the three objects side by side.

Time	Mathematical object	Level of the theory	Character (sense of P8)	Modality
Geometric time	Proper time along worldlines and the causal order, carried by the metric; a time function as its coordinate	What general relativity finds	Continuous and asymmetric	Omniscient
Event time	A discrete sequence of outcomes, each the label of one element of a measurement, ordered by precedence	What every theory finds	Discontinuous and symmetric	Immanent
Evolution time	The parameter of a strongly continuous one-parameter unitary group on a Hilbert space	The formalism of quantum mechanics	Continuous (instrumental mode)	Transcendent

P19 (identification). In the focus of time, I identify geometric time with the omniscient modality, event time with the immanent modality, and evolution time with the transcendent modality. I identify the one-way order of findings, the order of Axiom II, with the direction of time.

The observations that bear on P19 are those stated with each definition above: the four-dimensional pattern that a metric lays out, the records that findings leave, the range of outcomes over which a state assigns probabilities, and the irreversibility of every record. The time of the grid of Chapter 5, Table 5.1, which stands in the immanent column of the context

row, is the order of happening in which force acts; geometric time is that order laid out as one structure, and in the focus of time it therefore stands in the omniscient role (P11).

III. Distinct, Inseparable, Non-Interchangeable

Proposition 10.1. Geometric time, event time and evolution time are distinct, inseparable and non-interchangeable.

Argument. The three conditions are taken in turn.

Distinct. The three are objects of different mathematical kinds. A time function is a smooth map from a manifold to the real numbers. Event time is a discrete ordered set of records. Evolution time is the argument of a one-parameter group acting on a Hilbert space. A coordinate is not a record, a record is not the parameter of a group, and the parameter of a group of states is not a direction of spacetime. The distinctness is a matter of mathematical category.

Inseparable. Each of the three requires the other two. A finding requires a state whose range of outcomes it selects among, and that state is given in evolution time; a finding also requires a place among the light cones where its record lies, and that place is given in geometric time. Geometric time is known only through findings, since every measured interval is a comparison of clocks and rods that leaves a record, and between those findings the state of each clock evolves. Evolution time requires findings, since a preparation is a finding that fixes the initial state and a later finding is what the evolution predicts; it also requires a clock in geometric time by which its parameter is read. Without geometric time, findings have no causal order and no separation in space.

Non-interchangeable. No one of the three does the work of another. A coordinate selects no outcome: assigning a coordinate time to an event decides nothing about what is found there. A finding cannot serve as the argument of a unitary group, because a finding is discrete and its record is not undone, while $U(t)$ is continuous and has an inverse. The space of states is not a direction of spacetime: it is a complex Hilbert space, generally of infinite dimension, and no signal travels through it from one place to another. The formalism of quantum mechanics keeps evolution time and event time apart in its own terms. Pauli's theorem, as Busch (2008) reports it, establishes that no self-adjoint operator is canonically conjugate to a Hamiltonian bounded below, so evolution time is not an observable of the theory. The time observables that quantum mechanics does use, such as a time of arrival, are positive operator-valued measures over the times at which a detector registers a count. They measure event time, and none of them is the parameter t.[4]

The three times meet the three conditions of Axiom III (P12), which the modalities meet.

Rests on: the three modalities and the dependence of their binding on the focus (P10, P11); Axiom III (P12); the mode of the findings of quantum mechanics (P16); the identifications of the focus of time (P19); Definitions 10.1 to 10.3; the theorems of Stone (1932) and of Pauli, as Busch (2008) states it.

IV. How Each Theory Holds Two

General relativity holds geometric time natively. It holds event time as the events of the manifold: every measurement the theory describes is a coincidence of worldlines at an event, laid down as a record, with a definite place in the causal order. It has no evolution time. It contains no state over

a range of alternatives, no probability, and no parameter that carries a range of possibilities forward. When canonical quantization puts the geometry itself into a quantum state, the Hamiltonian constraint becomes

$$\hat{H}\,\Psi[h] = 0 \tag{10.3}$$

In words, the wave functional of the spatial geometry h is annihilated by the constraint operator, and so it carries no time parameter along which it changes (DeWitt 1967). Evolution time disappears from the formalism at the step that puts the arena into the state. Chapter 6, "The Category Error", reads canonical gravity in exactly these terms: the context within which things happen is made part of the quantum state, and so it no longer provides the arena in which the state evolves.

Quantum mechanics holds event time and evolution time natively. Its findings are discrete outcomes with probabilities, and its states evolve unitarily in t. It holds geometric time only as an external parameter. The t of Schrödinger's equation is read from a clock outside the system and is not a dynamical variable of the theory. In quantum field theory the spacetime on which the fields live is fixed in advance, flat or curved, and is given to the theory rather than found by it.

Table 10.2 shows which times each theory holds natively.

	Geometric time (omniscient)	**Event time (immanent)**	**Evolution time (transcendent)**
General relativity	Held: what the theory finds	Held: findings as events of the manifold	Absent: no state over alternatives
Quantum mechanics	External: a background parameter	Held: findings, with probabilities from the Born rule	Held: the parameter of unitary evolution

Each theory holds two of the three times, and both hold event time. Both take event time as given: general relativity places each finding at an event and does not derive it, and quantum mechanics joins its findings to its formalism by a rule stated beside the unitary evolution. The time that each theory lacks is the one whose modality is conjugate to its strength. General relativity holds the omniscient time natively and lacks the transcendent; quantum mechanics holds the transcendent time natively and has the omniscient only as background. Axiom I states that the omniscient and the transcendent are in conjugate relationship to one another (P12), and Table 10.2 shows that conjugacy in the two theories.

V. The Forbidden Conjunction of a Unified Time

Definition 10.4 (unified time). A unified time is one time that is geometric time, event time and evolution time together, with all three perfected in one formalism, none of them external and none primitive.

Proposition 10.2. No unified time exists. A time found both as geometric time and as event time would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. A unified time asks all four of one time, and so it would carry both.

Argument. First, what is found of geometric time is continuous and asymmetric (Definition 10.1). Second, what is found of event time is discontinuous and symmetric (Definition 10.2). Third, a unified time is one time found as both, with each character perfected; it asks all four characters of the same direction of time, continuity and asymmetry from the geometry and discontinuity and symmetry from the record, and so it would carry the continuity of the geometry with the symmetry of the re-

cord, and the discontinuity of the record with the asymmetry of the geometry, in one comparison. Fourth, by the Incommensuration Theorem (P9) any comparison perfected at its foundation is either continuously asymmetric or discontinuously symmetric, and the two conjunctions of the third step are the two it forbids. Evolution time does not alter this conclusion, since its continuity lies on the same side as the geometry's.

Rests on: the four compounds (P8); the Incommensuration Theorem (P9); Definitions 10.1, 10.2 and 10.4.

The forbidden pairing lies in the demand for one time, and not inside any of the three. It appears only when one time must be perfected, in what is found, both as the smooth time of the geometry and as the invariant time of the record. A given solution of Einstein's equations generally has no time-reversal symmetry, and the invariance of the equations themselves belongs to the formalism, as Section II states.

Proposition 10.3. Any formalism that holds two of the three times natively holds the third only as external (a background parameter, not a dynamical variable), as primitive (a rule given without derivation within the formalism), or not at all.

Argument. The argument uses Axiom I (P12) in its two clauses. By its second clause, the omniscient and the transcendent are conjugate, so geometric time and evolution time can each be perfected only at the expense of the other. A formalism that perfects geometric time therefore does not hold evolution time natively; general relativity holds none, and the canonical quantization that demands one finds none (equation 10.3). A formalism that perfects evolution time holds geometric time as an external parameter, as quantum mechanics does. By its first clause, the immanent is more fundamental than the omniscient and the transcendent, so event

time is derived from neither of the other two. Every formalism that holds event time takes it as primitive: quantum mechanics takes it through the Born rule, and general relativity through the events of the manifold.

Rests on: Axiom I (P12); the identifications of the focus of time (P19); Proposition 10.1 and Section IV.

VI. Where the Two Theories Meet

The two theories together hold all three times, each holding two, and they share event time. They meet at the finding. Each alternative of the matter carries its own classical geometry, shaped by that alternative's stress-energy through Einstein's equations (the argument, with the experiment of Page and Geilker, is set out in Chapter 7, "One Geometry for Each Alternative"). A finding is one outcome selected from among its possibilities, among alternatives that already carry their geometries: something possible becomes manifest for that comparer, and no geometry changes, here or anywhere. A record of a distant finding reaches a comparer here only through a signal, which arrives at the earliest on that comparer's past light cone. No comparison registers a surface of simultaneity on which a finding would be settled for every comparer together, and this account needs none.

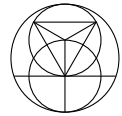
Quantum matter on a classical geometry for each of its alternatives is therefore the full account of the three times that two conjugate formalisms can give. Quantum theory holds event time and evolution time, general relativity holds geometric time and event time, and the finding joins the two theories. The three times require this arrangement, and nothing in it waits to be replaced by a single formalism. The plain form of this chapter is in *Two Theories, One World*, Chapter 16, "Three Kinds of Time", which shows the three times on a laboratory bench, with a clock, a counter and a radioactive source.

VII. Questions

Relativistic quantum field theory already joins spacetime and evolution: the Hamiltonian generates time translations within the Poincaré group. Is that not a unified time? It joins evolution time to the time coordinate of a fixed background geometry. The geometry is given in advance and is not a dynamical variable of the theory, so geometric time is held as external. Geometric time held as external is the first case of Proposition 10.3, and the success of quantum field theory is the success of holding geometric time as external.

Page and Wootters (1983) obtain evolution from a stationary state of a whole system, as correlations between a clock subsystem and the rest. Does that make the three times one? Their construction shows how evolution time can be read from a physical clock within a state that does not itself change.[5] The evolution appears when the rest of the system is conditioned on a reading of the clock, and a reading is a finding, so event time enters as the condition. The parameter of the conditional evolution is the label of the clock's readings, so evolution time is recovered as conditional on event time. The construction relates the three times to one another, and each keeps its own role in it.

Chapter 11 states the measurement problem in its standard forms and reads it as the residual of quantum mechanics, where evolution time meets event time.



Chapter 11

The Measurement Problem as a Cross-Modal Residual

The problem in its standard forms, decoherence and where it stops, and the probability rule as the bridge between the two levels of quantum mechanics

This chapter states the measurement problem in its standard forms and identifies it as the residual of quantum mechanics. Unitary evolution runs in evolution time and the finding belongs to event time, and the two are non-interchangeable (Proposition 10.1), so no evolution of the state yields a finding. The formalism gives the range of possible outcomes and their probabilities, and the finding is the act that selects one. Decoherence is the formalism's approach to the finding from its own side: it fixes the basis of the records and suppresses interference, and it selects no outcome. The Born rule is the bridge between the two levels, and its form is fixed once the Hilbert space is given and the bridge is granted its symmetry.

I. The Problem in Its Standard Forms

Von Neumann (1932) gave the problem its first exact form. His formalism has two processes. In one, a measurement takes the state of the system into a mixture of eigenstates of the measured observable, irreversibly. In the other, the state evolves by Schrödinger's equation, reversibly. He showed that the boundary between the measured system and the measuring chain can be placed at any link of the chain without changing any pre-

diction.[1] The formalism therefore says how to compute the outcome statistics, and it says nothing about where, or by what process, the first kind of change replaces the second.

Maudlin (1995) gives the standard modern statement. In Maudlin's statement, three claims are jointly inconsistent:

> 1.A. "The wave-function of a system is complete, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system."
>
> 1.B. "The wave-function always evolves in accord with a linear dynamical equation."
>
> 1.C. "Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes."

The inconsistency is shown by one line of the formalism. Let a system in the state $\Sigma_k \, c_k \, |s_k\rangle$ meet an apparatus in its ready state $|A_0\rangle$, with an interaction that takes each $|s_k\rangle|A_0\rangle$ to $|s_k\rangle|A_k\rangle$. By linearity,

$$(\Sigma_k \, c_k \, |s_k\rangle) \, |A_0\rangle \rightarrow \Sigma_k \, c_k \, |s_k\rangle \, |A_k\rangle \tag{11.1}$$

In words, the linear evolution takes a superposition of the system's states into a superposition of joint states of system and apparatus, each with its own pointer reading. If that state is complete (1.A), no pointer reading is determinate, and 1.C fails. Maudlin adds two further problems that any answer must meet. The problem of statistics: a complete state that evolves deterministically gives identical outcomes for identical states, against the observed statistics of the Born rule. The problem of effect: the outcome of a measurement has predictive power for later measurements, so an account must say how a finding bears on what follows it.[2]

Maudlin sorts the interpretations by which of 1.A, 1.B and 1.C each denies, and Chapter 12 uses that sorting where it applies. Here I give the reading that the ground of Part I yields.

II. The Residual

Proposition 11.1. The measurement problem is the residual of quantum mechanics. Unitary evolution, in evolution time, approaches the finding as closely as wished and does not contain it; the finding belongs to event time, and it is not a state of the formalism.

Argument. First, by Proposition 3.2 the residual of quantum mechanics is the discontinuity of the definite finding, which its continuous formalism does not contain. Second, by Proposition 10.1 evolution time and event time are non-interchangeable: no member of the group $U(t)$ is a finding, and no finding is a value of t. Third, in the focus of time, which asserts all three axioms (Table 2.1), the finding stands in the immanent role and the state in the transcendent role, and by Axiom I (P12) the immanent is more fundamental than the transcendent, so the finding, given in the act, is prior to the state, which describes the range of possibility from which the act selects. Fourth, a demand that the evolution produce the outcome is a demand that the formalism contain its residual, and by Proposition 3.1 the reach of a framework into its content is dense and never exhaustive.

Rests on: Axiom I (P12); the identifications of the focus of time (P19); the cross-modal principle (Proposition 3.1); the residual of quantum mechanics (Proposition 3.2); Proposition 10.1.

The formalism gives two things: the range of possible outcomes of a measurement, and the probability of each. A finding is one outcome selected from among its possibilities. When the apparatus registers, something possible becomes manifest for that comparer, and a record now exists that reads the same for every later comparison of it. The finding is made by the comparer that finds: the apparatus, as an interaction, and no mind is needed. A finding changes no geometry. Each alternative of the matter already carries its own classical geometry (Chapter 7, "One Geometry for Each Alternative"), and the finding selects among alternatives that carry them, so nothing in the arena changes, here or anywhere. For a comparer elsewhere, the record of the finding arrives only with a signal, at the earliest on that comparer's past light cone. No comparison registers a surface of simultaneity on which the selection would be made for every comparer together, and this account needs none.

The three claims of Maudlin read as follows. This account keeps 1.B: the state always evolves by a linear equation, and there is no dynamics of reduction. This account keeps 1.C: findings have determinate outcomes. This account reads the state by P14. The state exists: it is the complete description of the range of possible findings of the system and of their probabilities. The outcome is real: it is given in an actual act of comparison, and the state does not specify it in advance. The outcome is not a hidden property added to the state either, since nothing fixes it before the act. So 1.A, in Maudlin's wording, is the claim this account denies, and what the state leaves out is the act, not a further property.

The two further problems are met on the same ground. The problem of statistics is met because, on this account, two systems in one state can yield different outcomes: the Born rule gives the probabilities of findings, and no prior specification fixes which outcome is selected (the reason is given in Chapter 13, "Bell, Kochen and Specker, and PBR as Instances", Section VI). The problem of effect is met by Axiom II, whose third clause reads: "A class of the omniscient will precede an instance of the transcendent." In the focus of measurement, which asserts all three axioms (Table 2.1), the record is the finding laid out as structure, in the omni-

scient role, and the stance of the next comparison, with its range of outcomes, is in the transcendent role. The record of a finding precedes, and shapes, the range from which the next comparison is made. For the comparer that found, the state assigned after the finding is the range conditioned on its record, by the rule of Lüders. That conditioning is the entry of the record into the comparer's next stance. That conditioning is not a dynamical change of the system, and that conditioning is not a change at any other place. When two comparers later bring their records together, at an event in the common future of their findings, their records of one finding agree.

III. Decoherence and Where It Stops

Decoherence, the spreading of the relative phase of alternatives into an environment that registers which alternative is the case (Chapter 7, Section VI), is the formalism's account of what happens when a system and its apparatus interact with their surroundings (Zeh 1970; Zurek 1981; Joos and Zeh 1985; Zurek 2003).[3] The environment becomes correlated with the pointer states, and the joint state of system, apparatus and environment E is $\Sigma_k\ c_k\ |s_k\rangle|A_k\rangle|E_k\rangle$. Tracing out the environment gives the reduced density operator of system and apparatus,

$$\rho = \Sigma_k\ |c_k|^2\ |k\rangle\langle k| + \Sigma_{k\neq l}\ c_k\ c_l^*\ \langle E_l|E_k\rangle\ |k\rangle\langle l| \tag{11.2}$$

Here $|k\rangle$ stands for the joint state $|s_k\rangle|A_k\rangle$. In words, the reduced state has the Born weights on its diagonal, and each term off the diagonal is multiplied by the overlap of two states of the environment. For macroscopically distinct pointer states that overlap falls toward zero very quickly, and the interference between the alternatives becomes unobservable in practice.

In the formalism the interference is carried into the environment and is not destroyed there. The unitary evolution of system and environment together is never broken, so where no comparison has laid down a record of which alternative is the case, a measurement on the system and the environment together could in principle recover the interference. Quantum erasers recover interference in this way when the mark of which path was taken is held by one small system and has not spread further (Scully and Drühl 1982; Kim, Yu, Kulik, Shih and Scully 2000).

The achievements are large and established. Decoherence explains why interference between macroscopically distinct alternatives is not seen. Through environment-induced superselection, which Zurek calls einselection, it selects the pointer observable: the observable whose eigenstates remain stable while the environment monitors them. It answers the question of the preferred basis, in which the records of a measurement are laid down. It accounts for the quasi-classical behavior of open systems, whose einselected pointer states are stable while the rest of their state space is effectively closed to them.

I identify the pointer observable with the content of the finding: it is the observable that stays the same across the many interactions by which the environment records it, a sameness of content across a difference of context, which is symmetry in the sense of P8. The environment that carries the copies is the context of that content. Decoherence describes, in the formalism, where comparisons lay down durable records and in which basis.

Decoherence stops at the selection of one outcome. Schlosshauer (2005) states the point exactly: decoherence does not solve the problem of outcomes.[4] The joint state remains a superposition of all the alternatives, each with its pointer reading and its environment state. The reduced density operator of equation 11.2 becomes diagonal because the environment has been traced out, not because one of its entries has been selected, and so it is an improper mixture, in the standard term, and not a statement that one outcome has occurred.

Proposition 11.2. Decoherence is the dense reach of the formalism toward the finding. It fixes the basis of the records and makes interference among the alternatives unobservable, and it does not select an outcome.

Argument. Every product of decoherence, the diagonal reduced state included, is an operator of the formalism, given at a value of evolution time. A finding is not an operator; it is one outcome selected in event time. By Proposition 10.1 the two are non-interchangeable, so the most that a computation in evolution time can deliver is the range of outcomes in its stable basis, with their probabilities. Decoherence comes as close as a continuous formalism can come to the discrete finding, and the finding itself lies on the other side of the residual.

Rests on: Proposition 11.1; Proposition 10.1.

IV. The Probability Rule

The rule that joins the two levels was stated by Born (1926). For a state ρ and a measurement whose outcomes correspond to orthogonal projectors P_k, or more generally to effects E_k that sum to the identity,

$$p(k \mid \rho) = Tr(\rho\, P_k), \quad \textit{and in general} \quad p(k \mid \rho) = Tr(\rho\, E_k) \tag{11.3}$$

In words, the probability of each outcome is the trace of the state multiplied by the element of the measurement that corresponds to that outcome.

Gleason (1957) established that, on a Hilbert space, the rule is the only assignment of probabilities of its kind. In a Hilbert space of dimension at least three, every assignment of numbers in $[0, 1]$ to the projectors that sums to one over every complete set of orthogonal projectors has the form

$\mathrm{Tr}(\rho P)$ for some density operator ρ. Busch (2003) extended the domain from projectors to effects. Every assignment of probabilities to effects that is additive over effects summing to the identity has the form $\mathrm{Tr}(\rho E)$, in every dimension, dimension two included.[5] Both results take one condition as given: the probability of an element of a measurement depends on that element alone, and not on the other elements measured with it.

Proposition 11.3. Given the Hilbert-space formalism of quantum mechanics (P15), the Born rule is the only bridge from the formalism to the findings that assigns each element of a measurement the same probability in every measurement to which it belongs. I identify that condition as a symmetry of the bridge.

Argument. The first sentence is the content of the theorems of Gleason and Busch. The identification in the second sentence is made on the ground. A projector that belongs to two different measurements is one content found in two contexts, and a probability that is the same in both is a sameness of content across a difference of context, which is symmetry in the sense of P8. The bridge lands on the findings of quantum mechanics, whose mode is discontinuously symmetric (P16), and the frequencies of discrete outcomes that test the rule are findings of that mode, so the bridge carries the symmetry of the level it reaches. The observations that bear on the identification are the confirmations of the Born rule itself, in which the frequency of an outcome follows $\mathrm{Tr}(\rho P_k)$ whatever the other elements of the measurement are. The proposition does not derive the Hilbert space. It takes the space from established physics, and it says why, given the space, the rule has its form.

Rests on: the Hilbert-space formalism, as established physics (P15); the theorems of Gleason and Busch; the definition of symmetry (P8); the mode of the findings of quantum mechanics (P16).

The rule is stated beside the unitary evolution, and it is not derived from that evolution alone. By Proposition 11.1 the rule crosses the residual, and a derivation that stays within evolution time stays on one side of it. The derivations offered within the Everett interpretation bring in premises about rational choice among outcomes, and Chapter 12, Section III, reads those premises as event time entering the derivation.

V. What the Measurement Problem Is

The measurement problem is a feature of the structure of time. Unitary evolution does not produce a definite outcome, because evolution time and event time are distinct objects of different modalities, and neither can be turned into the other. The Born rule is the bridge between them: it gives the probabilities with which the findings occur, and, given the Hilbert space, its symmetry fixes its form. The usual demand is for a mechanism that carries the state into the outcome within one time. That demand rests on the premise of a single unified time, within which the mechanism would operate. Physics has three times, distinct, inseparable and non-interchangeable (Proposition 10.1), and a unified time is excluded (Proposition 10.2). Within quantum theory as it stands, the demand asks for a process that its equation does not contain. A process of that kind can be written only by changing the theory, as the collapse models do, and Chapter 12, Section V, sets out where their difficulty lies. This account therefore resolves the problem by showing that the demand for a mechanism rests on a false premise, and it adds no mechanism. The plain form of Sections II and III is in *Two Theories, One World*, Chapter 17, "Why a Measurement Has an Outcome", and Chapter 18, "How Close a Smooth Theory Comes".

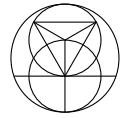
VI. Questions

Is this the Copenhagen interpretation under another name? It shares Copenhagen's priority of the finding, and it differs from Copenhagen in four respects. It needs no classical apparatus, since every interaction that lays down a record is a comparison. It places the boundary between the two levels of the theory, not between two classes of object. It reads the state as existing, as the description of possibility (P14), and not as a device of calculation only. It gives gravity a definite place, one classical geometry for each alternative of the matter. Chapter 12, Section II, sets out Copenhagen at its full strength.

What counts as a finding? Any comparison that lays down a record: an interaction after which some part of the world carries a record of the outcome. Decoherence locates such records in the formalism and fixes their basis. This account needs no further criterion, because von Neumann's result shows that the predicted statistics are the same wherever along the chain the finding is placed.

Wigner's friend measures a system inside a sealed laboratory. What has happened for Wigner, outside? For the friend, one outcome has become manifest, and the friend holds its record. For Wigner, who has made no comparison with the laboratory, the laboratory and its contents remain a range of possible findings with their probabilities. When Wigner compares with the friend's record, at an event in the common future of both, the two records of the friend's finding agree. Each finding is made by the comparer that finds, and no geometry changes at either finding.

Chapter 12 sets out the interpretations of quantum mechanics as strategies toward the three times, each with its achievement, what it drops, and what follows.



Chapter 12

The Interpretations as Modal Strategies

Copenhagen and the Heisenberg cut, Everett, Bohm, the collapse models, relational quantum mechanics, QBism and consistent histories: what each achieves, what it drops, and the difficulty that follows

This chapter reads seven interpretations of quantum mechanics as strategies toward the three times of Chapter 10. Each interpretation holds one or two of the times as primary and drops another, or absorbs it into another. Each has achieved something that stands, and each meets a characteristic difficulty that its own literature records. I identify the place of each difficulty: it lies where the time that the interpretation dropped or absorbed returns. That pattern is Axiom III at work, and it is the subject of Proposition 12.1.

I. The Rule

A modal strategy is the way an interpretation treats the three times: which it holds as primary, which it drops, and which it absorbs into another. Here I work in the focus of time, which asserts all three axioms (P19; Table 2.1). In the focus of time, event time stands in the immanent role, geometric time in the omniscient role, and evolution time in the transcendent role.

Proposition 12.1. An interpretation that drops one of the three times, or absorbs one into another, meets its characteristic difficulty where that time returns.

Argument. The three times are inseparable, so an interpretation that drops one still needs it, and the need appears as a question the interpretation cannot answer within its own terms. The three times are non-interchangeable, so an interpretation that absorbs one into another asks the second to do the work of the first, and the work is left undone at a definite place. The proposition is an identification of where each difficulty lies. Each difficulty is stated below in the terms of those who hold the interpretation or who have examined it, and none is introduced by this book.

Rests on: Proposition 10.1.

II. Copenhagen and the Heisenberg Cut

Copenhagen is the working practice of quantum physics from its founding. It takes the finding as primary, gives the state the role of computing the probabilities of findings, and never tries to derive the finding from the evolution. That order is the order of Axiom I (P12), in which the immanent is more fundamental than the transcendent, and of the interpretations it is the one that stands closest to the axioms. Its strategy holds event time as primitive, evolution time as a means of calculation, and geometric time as a fixed background.

The Heisenberg cut is the boundary between the system, described by a state, and the apparatus, described in classical terms. Von Neumann (1932) showed that the cut can be moved along the chain from the system to the final record without changing any prediction. I identify the cut as the boundary between the two levels of the theory, the formalism and the

findings, and not as a place among objects. Its mobility is the evidence: a boundary between two kinds of object would have a fixed location, and the cut has none.

What Copenhagen drops is a referent for evolution time. The state between findings is given no standing beyond calculation, so the transcendent is left without a place in the world. What follows is the familiar question of where the cut lies and what is happening on the quantum side of it. The question has no answer among objects, because the cut lies between modes. The account of Chapter 11 keeps Copenhagen's order and gives the state its standing: the state exists, as the description of possibility (P14).

III. Everett

Everett (1957) applied unitary evolution to everything, observers and apparatus included, with no second process. The program gives quantum mechanics as one linear dynamics, it applies without change to closed systems and to the universe as a whole, and with decoherence it gives a definite branching structure of quasi-classical worlds (Wallace 2012). Deutsch (1999) and Wallace (2012) recover the Born weights as the credences that a rational agent must assign to branches.[1]

The strategy holds evolution time as the whole of the account and absorbs event time into it: every outcome occurs, and a finding is read as a branching of the universal state. In Maudlin's terms it keeps 1.A and 1.B and denies 1.C for the whole, keeping it within each branch. By Axiom I the immanent is more fundamental than the transcendent, and this strategy makes the immanent derivative of the transcendent.

Two difficulties follow, and both lie where event time returns. The first difficulty is the preferred basis. The universal state can be written in many bases, and the state alone does not say which decomposition gives the branches. Decoherence supplies the basis by the stability of records, and the choice of that basis is the choice of which outcomes count as having

occurred, which is event time. The second difficulty is probability. When every outcome occurs, a probability needs an agent who faces outcomes and weighs them, and the decision-theoretic derivations take such an agent and its choices as premises. Choice among outcomes is event time again, entered as a premise of the derivation.

IV. Bohm

In the theory of Bohm (1952), particles have definite positions at all times, guided by the wave function through a deterministic equation. Given quantum equilibrium, the distribution $\rho = |\psi|^2$ of the configurations, it reproduces every prediction of non-relativistic quantum mechanics (Dürr, Goldstein and Zanghì 1992). Its non-locality is explicit, which agrees with Bell's theorem, and it explains a measurement with no special process. It keeps 1.B and 1.C and denies 1.A, since the positions belong to the complete description beside the wave function.

The strategy absorbs event time into geometric time. The outcome of a measurement is the position of the configuration at a time, and the finding becomes a reversible trajectory about which the comparer is ignorant. An irreversible finding is not the same as a reversible trajectory about which one happens to be ignorant, and the difference returns at a definite place.

For many particles the guidance equation needs the configuration on a spacelike hypersurface. The theory therefore needs a preferred foliation of spacetime, a structure that the geometry of general relativity does not supply (Dürr, Goldstein, Norsen, Struyve and Zanghì 2014).[2] Those authors propose to let the wave function fix the foliation covariantly, and the proposal is the program's own answer. Geometric time has been asked to do the work of event time, and it needs a surface of simultaneity to do it. Quantum equilibrium ensures that no finding registers the foliation. The account of Chapter 11 agrees with that result: no comparison registers a surface of simultaneity, and the account needs none.

V. The Collapse Models

The collapse models give one dynamics for microscopic and macroscopic systems. In the model of Ghirardi, Rimini and Weber (1986), each particle undergoes spontaneous localizations at a rate negligible for one particle and rapid for a macroscopic body. Continuous spontaneous localization (Pearle 1989; Ghirardi, Pearle and Rimini 1990) replaces the jumps by a continuous stochastic process. Diósi (1987; 1989) and Penrose (1996) tie the rate to gravity, through the gravitational self-energy of the difference between superposed mass distributions. The models make predictions that differ from those of quantum mechanics, and they are being tested (Bassi, Lochan, Satin, Singh and Ulbricht 2013). Donadi and colleagues (2021) searched for the radiation the gravity-related model predicts and ruled out its natural parameter-free version. Every test so far agrees with quantum mechanics. Matter-wave interference has been shown with molecules of more than 25,000 atomic mass units (Fein and colleagues 2019), and the rate of continuous spontaneous localization is bounded by the motion of cooled cantilevers (Vinante and colleagues 2016), by gravitational-wave detectors (Carlesso, Bassi, Falferi and Vinante 2016) and by the spontaneous emission of X-rays (Piscicchia and colleagues 2017). Tumulka (2006) gave a relativistic version of the model of Ghirardi, Rimini and Weber that needs no preferred foliation.[3]

The strategy absorbs event time into evolution time. The selection of an outcome becomes a stochastic, non-linear term of the evolution, so the models deny 1.B and keep 1.A and 1.C. Two difficulties follow, and both lie where event time returns. First, the evolution never reaches an exact eigenstate: the state after a localization keeps small amplitudes on the other alternatives, which is the problem of tails (McQueen 2015). The evolution approaches the discrete outcome without reaching it, and the residual of

Proposition 11.1 appears inside the dynamics itself. Second, the stochastic term heats matter, so energy is not conserved, and that heating is what the experiments look for.

An experiment decides between the gravity-related models and the account of this book. They tie the selection to a superposition of geometries. This account gives each alternative of the matter its own classical geometry and changes no geometry at a finding, so it predicts that two masses related only through gravity become entangled. The comparison of the entangling phase with the lifetime the Diósi-Penrose rule assigns to such superpositions is set out in Chapter 8, "The Entanglement Test", and the experiments being built decide between the two.

VI. Relational Quantum Mechanics

Rovelli (1996) dropped the observer-independent state. States and the values of quantities are relative to systems, every system can serve as an observer, and the theory describes the information that systems have about one another. The interpretation needs no special dynamics and no preferred frame. Its observer is any physical system, which is the sense of comparer that this book uses. It holds event time and evolution time relative to each system: a fact is a finding for the system that makes it, as in Chapter 11.

What the original form drops is the record laid out as structure, the same for every later comparison, which is the omniscient. What follows is the question of how the facts relative to different systems are related, so that two observers agree about one event. Adlam and Rovelli (2023) answer it by a postulate of cross-perspective links: the information an observer holds is stored in physical variables of that observer, which other observers can measure, so that observers can agree about past events.[4] The postulate restores the record as structure accessible to every comparer. The difficulty lay where the omniscient returned, and the repair restores it.

VII. QBism

QBism reads quantum probabilities as an agent's Bayesian degrees of belief (Caves, Fuchs and Schack 2002), and the Born rule as a rule that those beliefs must obey. Fuchs, Mermin and Schack (2014) set it out in full and show how it eliminates quantum non-locality from the theory's account of correlations. It makes the finding the agent's experience of the world's response to an action, and it gives the first person a place in the theory.

The strategy drops the transcendent as a feature of the world, since the state becomes the agent's belief, and it binds event time to an agent who uses the theory. In this book a comparer is any interaction, and an instrument with no user lays down a record. What follows is the question of why the probability structure that every coherent agent must adopt is one structure, the same for all agents and all systems. That shared structure is the transcendent, returning as the world's part in the agents' beliefs. The account of Chapter 11 keeps QBism's first person, as the comparer that finds, and reads the shared structure as possibility, which exists.

VIII. Consistent Histories

Griffiths (1984) gave quantum mechanics for closed systems with no measurement and no observer. A history is a sequence of projectors at chosen times. A consistent family is a set of histories that satisfies a consistency condition, which makes probabilities assigned to its histories additive, and every consistent family receives such probabilities. The approach reproduces the predictions of standard quantum mechanics, and it is used where no external apparatus exists, as in quantum cosmology.

The strategy takes the history, the sequence of events laid out as one structure, as primary, which is the omniscient, and it reduces event time to the choice of a consistent family: which projectors, at which times. What follows is the multiplicity of consistent families. Many exist, and they are

incompatible with one another. Dowker and Kent (1996) argue that the approach must therefore be supplemented by a principle that selects one family.[5] The selection of a family is the selection of the records that are actually made, which is event time, returning where it was reduced to a choice.

IX. The Pattern

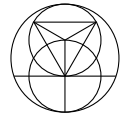
Table 12.1 sets out the seven strategies.

Interpretation	Holds as primary	Drops or absorbs	Where its difficulty lies
Copenhagen	Event time	Drops a referent for evolution time	The location of the cut, which lies between modes and not among objects
Everett	Evolution time	Absorbs event time into evolution	The preferred basis and the meaning of probability, where outcomes return
Bohm	Geometric time (configurations) with evolution	Absorbs event time into geometric time	The preferred foliation, a surface of simultaneity that no finding registers
Collapse models	Evolution time, made stochastic	Absorbs event time into evolution	Tails and the heating of matter, where the dynamics approaches the outcome and does not reach it
Relational	Event and evolution time, relative to each system	Drops the shared record	Agreement between observers, answered by restoring the record
QBism	Event time, for an agent	Drops the transcendent as a feature of the world	The one probability structure shared by every agent
Consistent histories	The history laid out as structure	Reduces event time to a choice of consistent family	The selection of one family among many

In every row the difficulty lies where the time that was dropped or absorbed returns, as Proposition 12.1 states. Each interpretation has the achievement credited to it above, and each achievement is the part of the strategy that keeps a time in its own role.

The account of Chapter 11 drops no time and absorbs none. It holds event time as primitive, by Axiom I; it holds evolution time as linear and as the complete description of possibility; and it holds geometric time as the classical geometry of each alternative of the matter. Its predictions agree with those of quantum mechanics wherever quantum mechanics has been tested, and where it parts from the gravity-related collapse models it is tested by the entanglement experiment of Chapter 8. The plain form of this chapter is in *Two Theories, One World*, Chapter 19, "The Interpretations as Choices About Time".

Chapter 13 reads the three best-known no-go theorems of quantum foundations as instances of one rule, and states the single-time result.



Chapter 13

Bell, Kochen and Specker, and PBR as Instances

The three theorems as cases of one structural rule, the single-time result, and causation without determinism

This chapter reads three theorems of quantum foundations as instances of one rule: by Axiom III no modality can do the work of another, and in the focus of time no one of the three times can stand in for another. The theorem of Bell shows that a finding cannot be reduced to a pattern laid out in geometric time with local variables. The theorem of Kochen and Specker shows that the range of possible findings cannot be replaced by one fixed table of values. The theorem of Pusey, Barrett and Rudolph, which the title of this chapter abbreviates as PBR, shows that the state cannot be read as ignorance of a pattern fixed beneath it. The single-time result follows from Chapter 10 with none of their assumptions: no interpretation that works within one unified time is complete. Causation is established by findings, and determinism is established by none.

I. One Rule

The rule is Axiom III (P12): the classes and instances of the immanent, the omniscient and the transcendent are distinct, inseparable and non-interchangeable. In the focus of time, which asserts all three axioms (Table 2.1), the three are event time, geometric time and evolution time (P19;

Chapter 10). A no-go theorem of quantum foundations assumes that one of these can be expressed in the terms of another, and derives from the assumption a conflict with the predictions of quantum mechanics. I identify each such assumption as a reduction of one modality to another, which the rule excludes.

The identification is one of kind. Each theorem is established within its own mathematical setting, and the reading given here adds nothing to its mathematics and changes none of it. The Incommensuration Theorem and Axiom III give the genus; the three theorems give species of it. What the genus adds is one reason for three results that arise in different settings, and a further result, Proposition 13.1, that needs none of their assumptions.

II. Bell

Bell (1964) considered two systems measured at separated places, with settings a and b and outcomes A and B, and a model in which the joint probabilities factorize over a variable λ whose distribution does not depend on the settings:

$$P(A, B \mid a, b) = \int d\lambda \, \rho(\lambda) \, P(A \mid a, \lambda) \, P(B \mid b, \lambda) \tag{13.1}$$

In words, the probability of the pair of outcomes is an average, over the variable, of the product of a probability that depends only on the near setting and one that depends only on the far setting. Every such model satisfies the inequality of Clauser, Horne, Shimony and Holt (1969),

$$| \, E(a, b) + E(a, b') + E(a', b) - E(a', b') \, | \leq 2 \tag{13.2}$$

where E is the mean of the product of the two outcomes, each ± 1, and the combination inside the bars is written S. In words, a combination of four correlations, each measured with one pair of settings, cannot exceed

two in size. Quantum mechanics predicts values up to $2\sqrt{2}$ (Tsirelson 1980). Loophole-free experiments have violated inequalities of this kind. Hensen and colleagues (2015) found $S = 2.42 \pm 0.20$ with electron spins 1.3 kilometers apart, and Giustina and colleagues (2015) and Shalm and colleagues (2015) found violations with entangled photons.[1] The theorem excludes local hidden variables in the factorizable sense of equation 13.1. It does not exclude hidden variables as such: the theory of Bohm is nonlocal, and it is not excluded.

The factorizable model demands that each outcome, or its probability, be fixed by the local setting and by variables laid down in geometric time before the finding is made. The factorizable model is a reduction of the immanent to the omniscient: the finding is replaced by a pattern laid out in spacetime in advance. Bell's theorem shows that the reduction fails.

Bell later gave the theorem a stronger form, built on a condition he called local causality (Bell 1987). It excludes every account in which each single result is produced by conditions local to its station, even when those conditions give only odds. This account answers that form in one way. A finding is a fact for the comparer that makes it. A distant finding becomes a fact for a comparer here only when its record arrives, at the earliest on the light cone. The joint pattern of the two results is a fact only where the records meet, and no comparer finds both results before they meet.

The account of Chapter 11 reads the correlations without any action at a distance. Each finding is made at its own wing by the comparer there, and it selects one outcome from among the possibilities of that wing. By the no-signaling theorem, the statistics at one wing do not depend on the setting chosen at the other (Eberhard 1978).[2] The correlation is found when the two records are compared, at an event in the common future of both findings, and the records reach that event through signals within its past light cone. The joint statistics are a property of the entangled state, which is the range of possibility of the two systems together. No finding changes anything at the other wing, and no geometry changes at either.

III. Kochen and Specker

Kochen and Specker (1967) considered an assignment of a value, 0 or 1, to every projector on a Hilbert space. The assignment must give the value 1 to exactly one projector in every complete set of orthogonal projectors, and the value of a projector must not depend on the set with which it is measured. They established that in a Hilbert space of dimension at least three no such assignment exists. Their construction uses 117 directions in three dimensions; Peres (1991) gave simpler constructions, and Cabello, Estebaranz and García-Alcaine (1996) gave one with 18 vectors in four dimensions. Bell (1966) reached the same conclusion from the theorem of Gleason.[3]

A non-contextual assignment of values demands that the range of possible findings be replaced by one fixed pattern of outcomes, the same for every measurement in which each projector appears. A non-contextual assignment of values is a reduction of the transcendent to the omniscient: possibility is replaced by a table laid out in advance. The theorem shows that the reduction fails.

The comparison with Chapter 11 locates the symmetry exactly. A non-contextual assignment of values asks for a sameness of the outcome across every context in which a projector appears, which is a symmetry in the sense of P8. Gleason's theorem and its extension by Busch admit that same symmetry for probabilities: the Born rule assigns each projector one probability in every measurement to which it belongs (Proposition 11.3). The symmetry is therefore carried by the range of possibility, the transcendent, and the Kochen-Specker theorem shows that it cannot be carried by a fixed table of outcomes, the omniscient.

IV. Pusey, Barrett and Rudolph

In an ontological model (Harrigan and Spekkens 2010), a system has at each preparation an ontic state λ, and each quantum state corresponds to a probability distribution over the ontic states. The model is ψ-epistemic when the distributions for two distinct pure states overlap, so that one ontic state is compatible with both, and ψ-ontic when the distributions for distinct pure states never overlap. Pusey, Barrett and Rudolph (2012) assumed preparation independence, that systems prepared independently have independent ontic states, and established that every such model that reproduces the predictions of quantum mechanics is ψ-ontic.[4]

The epistemic reading takes the state, which describes the range of possible findings, as a comparer's ignorance of a definite pattern beneath it. The epistemic reading is a reduction of the transcendent to ignorance of the omniscient. The theorem shows that the reduction fails: the transcendent is not reducible to ignorance. The reduction differs from the one excluded by Kochen and Specker. Their theorem concerns whether the outcomes are fixed in advance; the theorem of Pusey, Barrett and Rudolph concerns whether the state itself is knowledge of something fixed. The account of Chapter 11 reads the state as existing, as the description of possibility (P14), and not as any comparer's knowledge, and the theorem agrees with that reading.

Table 13.1 sets the three theorems side by side.

Theorem	What the excluded model demands	The reduction	Which time is kept in its role
Bell (1964)	Outcomes fixed by local variables at each wing	Immanent to omniscient	Event time is not reduced to geometric time
Kochen and Specker (1967)	One value for each projector, the same in every context	Transcendent to omniscient	The range of possible findings is not a fixed table
Pusey, Barrett and Rudolph (2012)	The state as knowledge of an ontic state beneath it	Transcendent to ignorance of the omniscient	The state of evolution time is not ignorance

V. The Single-Time Result

An interpretation of quantum mechanics is complete, in the sense of this section, when it accounts for all three times, each in its own role. An interpretation works within a single unified time when it holds the three times as one time, in the sense of Definition 10.4.

Proposition 13.1 (the single-time result). No interpretation of quantum mechanics that works within a single unified time is complete.

Argument. First, by Proposition 10.1 the three times are inseparable, so a complete interpretation must account for all three. Second, an interpretation that works within a single unified time holds the three as one time. Third, by Proposition 10.2 no unified time exists: one time found both as geometric time and as event time would carry a pairing the Incommensuration Theorem forbids. Fourth, such an interpretation therefore leaves one

of the three times unaccounted for, or makes one time do the work of another, and by Proposition 10.1 the times are non-interchangeable, so the second course is inconsistent. Either way the interpretation is not complete.

Rests on: Propositions 10.1 and 10.2.

The scope of the result is broader than that of the three theorems in one respect. It assumes nothing about hidden variables, value assignments or the ontic status of the state; it needs only the three times and Axiom III. Its scope is narrower in another respect. It gives a structural verdict, not an inequality with a numerical bound. What it predicts is where each interpretation meets its difficulty: at the time its strategy drops or absorbs (Proposition 12.1), and Table 12.1 is the check of that prediction against seven interpretations. The account of Chapter 11 does not work within a single time. It holds three times in two conjugate formalisms that meet at the finding, and so it is the account that the result does not exclude.

VI. Causation Established, Determinism Never

Causation, as this book uses the word, is the relation in which one finding, made by the comparer that finds, selects among alternatives and conditions the range of the findings that follow, in the one-way order of Axiom II. Determinism is the claim that a complete specification of the world at one time fixes every finding at every later time.

Proposition 13.2. Causation is established by findings. Determinism is established by no finding, and a complete specification of the kind it requires, perfected in what is found, would carry a forbidden pairing.

Argument. The argument has two parts, and the first is observational. Every finding is one outcome of one comparison made from inside the world, and no comparison delivers a complete specification of everything. So no set of findings establishes determinism. A preparation followed by a measurement, with the order of the two fixed, is causation, and every experiment finds it.

The second part is structural. By P8, a discontinuity is a difference of content where there is a sameness of context. By P9 and Proposition 4.1, the findings of content carry discontinuity perfected together with symmetry. So, among the findings of content, a sameness of context does not fix the content: identical preparations do not fix the outcome. No specification of the context, however complete, fixes every finding. A specification that did fix every finding would make every sameness of context a sameness of content, which is continuity, perfected together with the symmetry of the findings of content, and continuity with symmetry is a pairing that P9 forbids. So the specification that determinism requires is excluded from what is found, and no finding is fixed by a prior condition that is itself found. The Born rule has the form this proposition requires of a law: it is causal, since the state conditions the probabilities of the findings that follow, and it is not deterministic, since it fixes no single outcome.

Rests on: every finding is an act of comparison (P2); the definition of discontinuity (P8); the Incommensuration Theorem (P9); the order of Axiom II (P12); what is real is given in an actual act of comparison (P14); Proposition 4.1.

A deterministic formalism is permitted by the proposition, since the Incommensuration Theorem concerns what is found and a formalism is the other level of a theory (Proposition 2.2). The theory of Bohm is deterministic in its formalism, and by quantum equilibrium its determinism is registered by no finding (Dürr, Goldstein and Zanghì 1992). Its findings carry the Born statistics, as the proposition requires. The future is open in the sense of P14: what is real is given in an actual act of comparison, with

the future still open. The plain form of this chapter is in *Two Theories, One World*, Chapter 20, "The Walls Physics Has Already Found", and Chapter 18, "How Close a Smooth Theory Comes".

VII. Questions

Superdeterminism escapes Bell's theorem by giving up the independence of the settings from the hidden variables. Hossenfelder and Palmer (2020) argue that the objections to it rest on intuitions drawn from classical linear systems, and that it can be tested. Why is it set aside? Hossenfelder and Palmer identify the assumption of statistical independence exactly, and they propose tests of it.[5] A superdeterministic model fixes the settings and the outcomes together from a specification of the world that precedes both. That specification is the complete specification of Proposition 13.2, which no finding establishes and which the Incommensuration Theorem excludes from what is found. The tests they propose look for departures from the Born statistics, and this account predicts that the Born statistics hold.

The theorems are results within quantum theory. What does reading them as instances add? The reading adds nothing to their mathematics. It adds one reason for three results that arise in different settings; it adds the single-time result, which needs none of their assumptions; and it adds the location of each interpretation's difficulty, which Table 12.1 records and the literature of each interpretation confirms.

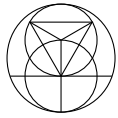
Does Bell's theorem not show that a finding at one wing acts on the other? It shows that no factorizable model reproduces the correlations. The no-signaling theorem shows that the statistics at each wing are independent of the setting at the other. The correlation is a sameness found

between two records when they are compared, and each record was made where its comparer stood. Nothing at one wing changes at the finding made at the other.

Part IV is complete: the three times, the measurement problem as the residual of quantum mechanics, the interpretations as modal strategies, and the no-go theorems as instances of one rule. Part V turns to the programs of quantum gravity, and Chapter 14 states the criteria by which each can be judged.

PART V

THE PROGRAMS



Chapter 14

Four Criteria and Ten Programs

How a program of quantum gravity is assessed, by the Incommensuration Theorem, from its founding commitment, and what ten programs have found

This chapter states four criteria by which any program of quantum gravity can be assessed before it is complete. It establishes that the commitment a program makes about the arena at its first step predicts the kind of difficulty the program meets, and that the kinds are three: multiplicity, dynamics and recovery. It then examines ten programs, each with its achievement first and then the difficulty its founding commitment predicts, checked against the primary literature. A summary table and a gradient close the survey, and the gradient shows that the programs nearest to the account of this book are the ones that do not quantize the arena.

I. The Question and the Method

A program of quantum gravity is a sustained effort to find one theory that holds both general relativity and quantum theory. Each program begins by deciding what is to be done to the arena, the space and time in which everything measured is measured. I call that decision the program's founding commitment.[1] A founding commitment is the operation a pro-

gram performs on the arena at its first step, before any calculation is made. It can be read from the program's own statement of its aims, so it can be assessed now, whatever the program later achieves.

The assessment rests on three results of this book. The Incommensuration Theorem (P9) excludes two pairings at the foundation of any comparison: continuity with symmetry, and asymmetry with discontinuity. Gravity is the arena, the context of physics, and not one more content within it (P18; Chapter 5, "The Content and Context Partition"). The arena admits no mode exchange, so it has no least step and no quantum of its own at any scale (Propositions 6.1 and 6.3; Chapter 6, "The Category Error"). From these three results it follows how a program built on an excluded commitment will be held up, and where. A difficulty of that kind recurs under every reformulation, because it belongs to the commitment and not to the technique.

II. The Four Criteria

Criterion 1 (the partition). Does the program treat gravity as the context of physics, or as one more content within it? The forces of particle physics are content: each acts on a charge that comes in discrete units, each can be screened, confined or cut off in range, and each has its own coupling. Gravity has the four markers of context: it couples to every form of energy, nothing shields it, its coupling to each body equals that body's inertia, and it has no discrete charge. A program fails the first criterion when it gives gravity a charge, a coupling that runs as a gauge coupling runs, and a field to be quantized as the gauge fields are quantized.

Criterion 2 (the excluded pairing). Does the program require one description whose findings, in the same respect, are both continuously asymmetric and discontinuously symmetric? Such a description would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both; P9 excludes each of them. The requirement is explicit when the extent of the geometry is given a discrete spectrum and the same geometry is asked to be smooth at large scales. The requirement is implicit when a discrete framework is asked to fix one smooth arena with nothing left over.

Criterion 3 (what is quantized). Which quanta does the program use? Quantum fields of matter on a classical geometry are admitted, each alternative of the matter carrying its own classical geometry (Chapter 7, "One Geometry for Each Alternative"). A description of the small waves of the geometry about a classical background by quanta is admitted, as a description: its quanta are quanta of the description only, never quanta of the arena (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale"). Sums over the discrete topological invariants of a manifold, such as its Euler characteristic, are admitted, since those integers are discontinuously symmetric already and give the arena no step. A quantum of the arena itself, at any scale, is excluded: a least step of length, area, volume or duration, or a quantum state of the geometry carried beyond the Planck scale as the foundation of the arena.

Criterion 4 (the predicted difficulty). Is the difficulty that the program meets in practice of the kind its founding commitment predicts? Section III states the three kinds. A program that meets the first three criteria is predicted to meet none of the three kinds, and its remaining difficulties are those of a construction not yet complete.

III. The Three Predicted Difficulties

Proposition 14.1 (the three difficulties). (a) A program that starts from a discontinuously symmetric framework and requires the arena to follow from it meets a difficulty of multiplicity: many arenas fit the framework, and no principle within the framework selects one. (b) A program that quantizes the arena directly meets a difficulty of dynamics: the quantized arena lacks a well-defined evolution. (c) A program that replaces one description entirely by a structure of the other character meets a difficulty of recovery: the replaced character does not follow from the surviving one alone, and the program must add a principle that selects the rare structures from which it does follow.

Argument. For part (a), a discontinuously symmetric framework fixes invariants, discrete labels that are the same in every context. A continuously asymmetric arena is specified by data that vary without repetition from place to place. For the framework to fix one arena uniquely, its findings would have to carry the arena's continuous asymmetry perfected together with their own discontinuous symmetry, in the same respect,

which P9 excludes. So the map from the framework to arenas is many-valued, and the selection of one arena must come from outside the framework.

For part (b), evolution is described along the arena: the time of a quantum system is a coordinate on its context. A program that quantizes the arena places that context inside the quantum state, and the state then has no parameter outside itself along which to evolve. In canonical gravity this lack of an outside parameter appears in the Wheeler-DeWitt equation (DeWitt 1967), equation (6.1) of Chapter 6, "The Category Error": the Hamiltonian constraint annihilates the wave functional of the spatial metric, and the equation contains no time.[2] The difficulty therefore appears in a constraint that is asked to generate the dynamics and resists being defined.

For part (c), by P9 the two surviving pairings are exhaustive and exclusive, so a structure of one character can approach the findings of the other only as a limit. A generic structure of the surviving character approaches no such limit, and the program must find a dynamics or a measure under which the rare structures that do approach it dominate. That added principle is where the difficulty appears.

Rests on: the two laws (P6); the Incommensuration Theorem (P9); the cross-modal principle (Proposition 3.1); for part (b), the category error (Proposition 6.2).

IV. The Ten Programs

Each entry states the program's achievement first, then its founding commitment, its reading on the criteria, and the difficulty that commitment predicts, with the literature. The achievements stand as results. The criteria assess the founding commitment, and by them none of the computations made under it is void.

1. String theory

String theory contains a massless state of spin two whose interactions at low energy are those of general relativity (Yoneya 1974; Scherk and Schwarz 1974). It also counts the microstates of a class of extremal black holes and recovers their Bekenstein-Hawking entropy (Strominger and Vafa 1996).[3]

Its founding commitment is to begin on the quantum side and to obtain the geometry of spacetime from the consistency conditions of the quantized string. It fails criterion 1, since what the program calls the graviton is one mode of the string beside the photon and the gluons, distinguished from them by quantum numbers and not by character. Its excluded pairing is implicit, in the requirement that the discrete framework fix one arena. On criterion 3 its description of small waves about a fixed background by quanta is admitted, as a description whose quanta are never quanta of the arena, and the reading of the full theory as a quantum theory of the geometry is excluded. The predicted difficulty is of multiplicity, and the record shows it. Estimates of the number of flux vacua reach about 10^{500} (Douglas 2004). No principle within the theory selects one of them, and the field has turned to studying the vacua statistically (Douglas 2003). The program's own reply to multiplicity is the swampland program, which seeks the conditions that separate the low-energy theories that can be completed into a consistent quantum gravity from those that cannot (Vafa 2005; review in Palti 2019). Its conditions are conjectures, each of which narrows the theories admitted at low energy, and none of them so far selects one vacuum, so the multiplicity that Proposition 14.1(a) predicts stands.

2. Loop quantum gravity

Loop quantum gravity constructs a background-independent quantization of the kinematics of general relativity, with a rigorously defined Hilbert space of spin-network states on which operators for area and volume have discrete spectra (Ashtekar 1986; Rovelli and Smolin 1995; Ashtekar and Lewandowski 1997).

Its founding commitment is to take the geometry of general relativity as the object and to quantize it. On criterion 1 its motivation is right and its operation is not: it treats gravity as geometry rather than as a force within the arena, and then applies to the geometry the operation that belongs to content. Its excluded pairing is explicit, since the extent of the geometry receives a discrete spectrum and the smooth arena is to be recovered from it in a limit. On criterion 3 the spectra of area and volume are steps of the arena itself, which are excluded in the account of this book. The predicted difficulty is of dynamics, and the record shows it. Thiemann (1998) constructed a Hamiltonian constraint operator, and the ambiguities of its construction and the connection of its solutions to classical gravity are the central questions of the program's dynamics (Nicolai, Peeters and Zamaklar 2005). The difficulty is the one that equation (6.1) exhibits.

3. Causal set theory

Causal set theory gives a discrete structure that preserves Lorentz invariance: a causal set obtained by Poisson sprinkling into Minkowski space picks out no frame (Bombelli, Henson and Sorkin 2009).

The program replaces spacetime by a locally finite partial order whose elements are counted to give volume (Bombelli, Lee, Meyer and Sorkin 1987). Its founding commitment is to remove the continuous arena and to obtain it as an approximation to a discrete order. It dissolves the partition of criterion 1, since the arena and its content are both to come from one discrete substrate. It asks a discontinuous structure to stand in for a con-

tinuously asymmetric arena, and its fixed number of elements per unit volume is a least step of the arena, which criterion 3 excludes. The predicted difficulty is of recovery, and the record shows it in the form the program itself describes. Almost all partial orders on many elements belong to a three-layered class (Kleitman and Rothschild 1975) that approximates no spacetime of four dimensions, so the dynamics must suppress the orders that are not like a manifold. One large class of them is strongly suppressed in a Lorentzian path integral with the causal-set form of the Einstein-Hilbert action (Loomis and Carlip 2018). Carlip, Carlip and Surya (2023) show that the Kleitman-Rothschild orders, the commonest orders that are not like a manifold, are strongly suppressed in the same path integral, and Carlip, Carlip and Surya (2024) extend the suppression to layered orders in every dimension, on the condition that the scale of the discreteness is at least a multiple of the Planck scale that the dimension fixes. Each of these results meets the difficulty of recovery in the form Proposition 14.1(c) predicts: the suppression comes from a chosen action and, in the last, from a condition on the scale of discreteness, and the order alone supplies neither. That action is built from operators that are non-local on the causal set and approximate local operators only in the continuum (Benincasa and Dowker 2010). Manifold-likeness and locality are the two faces of one recovery.[4]

4. Asymptotic safety

Asymptotic safety has found a non-Gaussian ultraviolet fixed point of the gravitational renormalization group in a long sequence of truncations of growing size (Weinberg 1979; Reuter 1998; review in Eichhorn 2019). With that fixed point it gave a Higgs mass of about 126 GeV before the particle was found (Shaposhnikov and Wetterich 2010).

Its founding commitment is to treat the metric as a quantum field like the gauge fields, with a path integral over metrics and a running Newton coupling. It fails criterion 1 for that reason. The scale invariance at its fixed point is a symmetry of the formalism, and it is not itself the excluded pair-

ing. On criterion 2 the excluded pairing is implicit, since the quantized metric is the foundation and the smooth arena is asked of it at low energies. On criterion 3 the program carries a quantum state of the geometry beyond the Planck scale as the foundation of the arena, which is excluded. Below the Planck scale no present observation separates this account from asymptotic safety. The predicted difficulty is of dynamics, and the record places it where the diagnosis places it. The fixed-point computations were for many years made almost entirely in Euclidean signature, and Donoghue (2020) identifies a well-behaved Lorentzian theory as the obstacle to be overcome. Computations in Lorentzian signature have since begun. Fehre, Litim, Pawlowski and Reichert (2023) compute the spectral function of what the program calls the graviton, a quantum of its quantized metric, directly in Lorentzian signature, and find a positive spectral function whose continuum scales as asymptotic safety requires at large spectral values; Assant, Litim and Reichert (2026) compute the spectral functions of the graviton modes and find that they pass from the behavior of general relativity at low spectral values to the scaling set by the fixed point at high ones. Each solves the flow of the functional renormalization group in an approximation that keeps a chosen set of terms, as the Euclidean computations do. In the full Lorentzian theory the time of evolution is itself among the variables integrated over. I expect each larger truncation, in Euclidean or in Lorentzian signature, to find the fixed point again, and the full Lorentzian theory, beyond truncation, to withhold it.

5. Causal dynamical triangulations

Causal dynamical triangulations produce, from a sum over causal geometries, an extended geometry of four dimensions at large scales (Ambjørn, Jurkiewicz and Loll 2004). The spectral dimension of that geometry runs from four at large scales to about two at short ones (Ambjørn, Jurkiewicz and Loll 2005).

The program's founding commitment is to define the gravitational path integral by gluing flat four-simplices under a fixed causal foliation, the discreteness serving as a regulator to be removed in a continuum limit. On criterion 1 it keeps the order of time as structure, and it sums over spatial geometry as a quantum variable. Its demand on criterion 2 is weakened, because the discreteness is not claimed as a step of the arena, and on criterion 3 its sum over geometries is a quantum state of the arena. The predicted difficulty is of recovery, here the recovery of a continuum limit. The program needs a second-order transition to define that limit. Evidence for one has been reported (Ambjørn, Jordan, Jurkiewicz and Loll 2011), and showing that the limit is general relativity is the task the program now addresses. The running of the spectral dimension is a property of the regulated sum, appearing near the Planck scale where the two descriptions meet (Chapter 9), and I do not read it as a property of the arena.

6. Noncommutative geometry

Noncommutative geometry obtains the Lagrangian of the Standard Model, coupled to gravity, from the spectral action of the product of a four-dimensional manifold with a finite noncommutative space (Chamseddine, Connes and Marcolli 2007).

Its founding commitment is to describe geometry by spectral data while keeping the four-dimensional geometry classical in the spectral action. It keeps the gauge content and the geometry of spacetime apart in its architecture, it requires no excluded pairing in its classical form, and it quantizes the matter on a classical geometry, which is admitted. None of the three predicted difficulties arises. Its difficulty is one of incompleteness: the finite algebra is chosen so that it reproduces the known content, and is not derived. The record holds one miss, and the survey records it. The spectral action in that form of 2007 gave a Higgs mass near 170 GeV; the particle was found in 2012 near 125 GeV (ATLAS Collaboration 2012; CMS Collaboration 2012), and Chamseddine and Connes (2012) restored

agreement with that mass by a neutral scalar singlet that the noncommutative geometry supplies, which changes the running of the couplings. The miss and its repair concern the content carried by the finite algebra, and the classical geometry of the spectral action stands through both.

7. Emergent gravity

Jacobson (1995) derived the Einstein equation as an equation of state, from the proportionality of entropy to horizon area and the Clausius relation applied to every local Rindler horizon.

The program's founding commitment is that gravity is the thermodynamics of underlying degrees of freedom and not a field to be quantized. Jacobson draws that conclusion himself: it may be no more appropriate to quantize the Einstein equation canonically than to quantize the wave equation for sound in air. Verlinde (2011) extended the idea to an entropic account of Newton's law, and Padmanabhan (2010) reviews the thermodynamic route to the field equations. The entropic account of Newton's law has met published objections. Kobakhidze (2011) argues that the quantum states of ultracold neutrons bound in the Earth's gravitational field are inconsistent with an entropic origin of the force, and Visser (2011) shows that the conservative character of gravity limits strictly the entropy and temperature an entropic account may use, limits that create serious problems for Verlinde's version and none for Jacobson's or Padmanabhan's. The objections bear on the entropic derivation of Newton's law, and the equation of state of Jacobson, on which the program's place in the survey rests, stands. The program keeps a partition in practice, and nothing of the arena is quantized: the horizons of Jacobson's derivation are horizons of a given smooth spacetime. None of the three predicted difficulties arises. Its difficulty is one of incompleteness: it does not say why gravity alone is thermodynamic. The partition of Chapter 5 supplies the reason, since gravity is the context and the gauge forces are content. Read instead as deriving the arena from microscopic quantum degrees of freedom, the program takes the commitment of Proposition 14.1(a).

8. ER=EPR

The eternal two-sided black hole in anti-de Sitter space has a dual description as two copies of the boundary theory in an entangled state (Maldacena 2003).

From that case Van Raamsdonk (2010) argued that connected spacetime is built from entanglement, and Maldacena and Susskind (2013) proposed that every entangled pair is joined by a bridge of geometry. The conjecture is called ER=EPR, after the bridges of Einstein and Rosen and the entangled pairs of Einstein, Podolsky and Rosen. The program's founding commitment is a correspondence between a continuously asymmetric description, the connecting geometry, and a discontinuously symmetric one, the entangled state. The correspondence agrees with the partition; the reading usually drawn from it, that gravity is quantum at bottom, does not. A correspondence joins two descriptions and requires no merger, while a derivation of geometry from entanglement requires the discrete side to fix the continuous one. Read as a correspondence, the program meets none of the three difficulties. Read as a derivation, it meets the difficulty of multiplicity, for the reason Chapter 15, "AdS/CFT as Cross-Modal Correspondence", gives: the entanglement data do not fix one geometry.

9. The amplituhedron

Arkani-Hamed and Trnka (2014) expressed the planar scattering amplitudes of maximally supersymmetric Yang-Mills theory through a positive geometry, the amplituhedron, from which locality and unitarity follow without being assumed.

The program's founding commitment is that amplitudes, and not fields in spacetime, are primary. The amplituhedron is a positive geometry in a continuous space, so the program is not a discretization. It applies so far to the content of a gauge theory, and it reformulates amplitudes already quantized without quantizing anything of the arena. None

of the three predicted difficulties arises in its present scope. Extended to gravity as a derivation of the arena from amplitude data, it would take the commitment of Proposition 14.1(a); extended to reproduce the description of small waves about a background by quanta, it would stay within what criterion 3 admits. Its place therefore stands between the two ends of the gradient, as the places of causal dynamical triangulations and of ER=EPR do: it depends on a further choice, here the form of its extension to gravity.

10. Twistor theory

Penrose's construction called the nonlinear graviton (1976) encodes every anti-self-dual solution of the vacuum Einstein equation in the complex structure of a deformed twistor space, and twistor methods made the computation of gauge-theory amplitudes far simpler (Witten 2004).

The program's founding commitment is to replace the points of spacetime by twistors, so that the light-cone structure is primary and the metric follows from it. It treats the conformal structure as prior to the full metric, a step toward reading the geometry as context that the program does not take, and it quantizes the gauge sector by twistor methods while treating gravity classically, which is admitted. None of the three predicted difficulties arises. Its difficulty is the full gravitational field: what the program calls the nonlinear graviton captures the anti-self-dual half of the vacuum field, and the general field has been the program's central task since. I read this as follows. The light-cone structure is the part of the geometry that stays the same under every change of the local scale of length, so it is symmetric in character and can be carried by a reformulation of that character. The full field adds data that vary from place to place without repeating, which is the continuous asymmetry of the arena, and that part resists the reformulation.

V. The Summary Table

Table 14.1 gives each program's founding commitment, its reading on the four criteria, and its place in the gradient of Section VI.

Program	Founding commitment	Gravity treated as	Excluded pairing required	What is quantized	Predicted difficulty	Place
String theory	Arena from the quantized string	Content (one mode among others)	Implicit (one arena fixed)	Waves on a background; geometry beyond the Planck scale	Multiplicity (number of vacua)	Low
Loop quantum gravity	Quantize the geometry	Geometry, then quantized	Explicit (spectra of area and volume)	The arena itself	Dynamics (Hamiltonian constraint)	Low
Causal sets	Replace the arena by a discrete order	Dissolved with content	Explicit (discrete for continuous)	Total replacement	Recovery (manifold-likeness, locality)	Lowest

Program	Founding commitment	Gravity treated as	Excluded pairing required	What is quantized	Predicted difficulty	Place
Asymptotic safety	Metric as a quantum field	Content (running coupling)	Implicit (smooth arena asked of a quantized metric)	Metric path integral	Dynamics (the full Lorentzian theory)	Low
Causal dynamical triangulations	Regulated sum over causal geometries	Partly context (time order kept)	Weakened (regulator)	Sum over geometries	Recovery (continuum limit)	Low to moderate
Noncommutative geometry	Spectral data, spacetime kept classical	Context (geometry kept apart)	None	Matter only	None of the three (incompleteness)	High
Emergent gravity	Gravity as thermodynamics	Not a field to quantize	None	Matter only	None of the three (incompleteness)	High
ER=EPR	Geometry and entanglement as one phenomenon	Depends on reading	Only if read as derivation	Inherited	Depends on reading	Moderate

Program	Founding commitment	Gravity treated as	Excluded pairing required	What is quantized	Predicted difficulty	Place
Amplituhedron	Amplitudes primary	Not yet addressed	None at present	Reformulation only	None at present; multiplicity if extended to derive the arena	Moderate (depends on its extension)
Twistor theory	Light cones primary	Partly context (conformal)	None	Gauge sector only	None of the three (full field)	Moderate to high

VI. The Gradient

Assessed on the four criteria, the ten programs fall along a gradient from those most in conflict with the account to those nearest to it. At the lower end are four programs. Causal set theory replaces the continuous arena entirely. String theory removes the partition. Loop quantum gravity quantizes the context directly. Asymptotic safety carries a quantum state of the geometry beyond the Planck scale. These four share one structural error. Each subjects the context of physics to the treatment that belongs to content, and they differ only in the direction and the method of the attempt.

In the middle are three programs whose standing depends on a further choice, each set out in Section IV: causal dynamical triangulations, ER=EPR and the amplituhedron, whose standing depends on the form of its extension to gravity. At the upper end are three programs that keep the arena classical: noncommutative geometry and emergent gravity, and, a step below them, twistor theory.

The programs closest to the account are those that do not attempt to quantize gravity. The programs furthest from it are those most committed to the proposition that gravity must be quantized as the other forces were. I identify that proposition as the structural error, and the question it hides is not "how do we quantize gravity?" but "why would we assume gravity is quantizable?" A program does not rise on the gradient by containing discrete and continuous elements together. It rises by keeping the arena out of the quantum state.

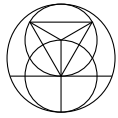
VII. What the Survey Shows

A program's standing with respect to the Incommensuration Theorem is fixed by its founding commitment, and later work cannot tune it. The number of string vacua, the Hamiltonian constraint of loop quantum gravity, the manifold-likeness of causal sets and the Lorentzian fixed point of asymptotic safety are not four independent computational challenges. They are different expressions of the same structural impossibility.

The survey makes a prediction that the record of the programs will go on testing. The difficulties of the lower end will persist under every reformulation that keeps the founding commitment, and the programs of the upper end will meet no difficulty of multiplicity, dynamics or recovery for as long as they keep the arena classical. The record decides the prediction at four places, as the observations of Chapter 19 decide the predictions for experiment, and at each the survey says what the record will show while each program keeps its founding commitment: no selection principle for string vacua, derived within string theory, will select one vacuum; no Hamiltonian constraint of loop quantum gravity will be shown to be unique, with general relativity as its classical limit; no dynamics of causal sets will make the orders like a manifold dominate with no added principle; and no fixed point of asymptotic safety will be established in the full Lorentzian theory, beyond truncation. Each of the four is a difficulty of the lower end that the founding commitment keeps in place, and the step of

the survey that the record tests is Proposition 14.1. The same survey is told for the general reader in *Two Theories, One World*, Chapter 22, "What Each Program Found", after its Chapter 21, "Four Questions for Any Theory", has set out the four criteria as four plain questions.

The strongest single case for the opposite view is the correspondence between gravity in anti-de Sitter space and a quantum theory on its boundary, and the next chapter examines it.



Chapter 15

AdS/CFT as Cross-Modal Correspondence

Two descriptions of one system in conjugate modes, a translation that loses information in both directions, and neither side more fundamental

This chapter answers the strongest objection to the account of this book: that the correspondence between gravity in anti-de Sitter space and a conformal field theory on its boundary shows gravity to be a quantum theory in another language. It establishes that the two sides of the correspondence are descriptions in conjugate modes, as the cross-modal principle requires of any systematic comparison between them. It then shows four results from the literature. Reconstruction of the bulk from the boundary reaches densely and not exhaustively. The formula of Ryu and Takayanagi and its corrections form a bridge that loses information. Holographic entanglement obeys a constraint that general quantum states do not. Neither side of the correspondence is more fundamental than the other.

I. The Objection at Full Strength

Maldacena (1998) conjectured that type IIB string theory on the product of five-dimensional anti-de Sitter space with a five-sphere is equivalent to four-dimensional maximally supersymmetric Yang-Mills theory with gauge group SU(N). Anti-de Sitter space is the maximally symmetric spacetime of constant negative curvature, and it has a timelike boundary at infinity. The correspondence between gravity in anti-de Sitter space

(AdS) and a conformal field theory (CFT) on its boundary is called AdS/CFT. The equivalence is made precise by an equality of generating functions (Gubser, Klebanov and Polyakov 1998; Witten 1998):

$$Z_{bulk}[\varphi_0] = \langle\, exp \int \varphi_0\, O \,\rangle_{CFT},$$

(15.1)

which says that the partition function of the bulk theory, with each bulk field set to the boundary value φ_0, equals the expectation in the boundary theory of the exponential of the corresponding operator O integrated against that value. The objection follows directly. If gravity in the bulk is exactly equivalent to a quantum theory on the boundary, then gravity is a quantum theory redescribed in geometric language, and the correspondence is the unification that the Incommensuration Theorem is said to exclude. The objection has a stronger form. At a finite number N the boundary theory is a complete quantum theory, and many physicists take it to define the bulk theory at every scale, the Planck scale included, which would make the boundary theory a full quantum theory of gravity.

I take the objection at that strength, and I answer it on the ground of Chapter 3. By Proposition 3.1, the cross-modal principle, any investigation compares a framework with a content in the conjugate mode, and the reach of the framework is dense and never exhaustive. Applied to two descriptions of one system, the principle predicts three features: a systematic relation between them; neither description reducible to the other; and a residual on each side that the other side does not contain. The correspondence shows all three, and the sections below set out the evidence for each. The dictionary between the two sides is a structural correspondence, not an ontological reduction.

II. The Two Sides in Conjugate Modes

Proposition 15.1. The findings of the boundary theory are discontinuously symmetric and its formalism is continuous. The findings of the bulk geometry are continuously asymmetric and its formalism is symmetric. The two sides of the correspondence are therefore descriptions in conjugate modes.

Argument. For the boundary, a conformal field theory is specified by its conformal data: the spectrum of scaling dimensions and spins of its primary operators, and the coefficients of their operator product expansion (Poland, Rychkov and Vichi 2019). The conformal group sorts every operator into discrete families, each labeled by exact quantum numbers, and the operator product of two operators is a discrete sum over primaries. These data form a discrete set, each item fixed exactly and the same in every context, so the findings of the boundary theory are discontinuously symmetric. Its formalism is continuous: the theory lives on a smooth manifold, and its correlation functions are smooth functions of the points at which operators are inserted.

For the bulk, the findings are the metric and its curvature, which vary smoothly from point to point and, in a generic solution of the Einstein equations, without repetition. They are continuously asymmetric. The formalism of the bulk is symmetric: its equations are organized by invariance under diffeomorphisms, by local Lorentz symmetry, and by the isometries of the anti-de Sitter background.

Rests on: the mode of the findings of quantum theory (P16); the cross-modal principle (Proposition 3.1).

The bulk description in this reading is a classical geometry with quantum fields on it. Heemskerk, Penedones, Polchinski and Sully (2009) conjectured that conformal field theories with a large number N of degrees of freedom per point and a large gap in their spectrum of dimensions have a local bulk dual of this kind. They supported the conjecture at the first nontrivial order in $1/N^2$. The quantum fields of matter in the bulk are content on the bulk geometry, and the small waves of the bulk geometry are described by quanta on the terms of Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", as quanta of a description and never quanta of the arena. A boundary state that is a superposition of states with different classical duals is a superposition of the content; in the account of this book no quantum state of the bulk geometry is read into that superposition.

III. Reconstruction and Its Reach

Reconstruction of the bulk from the boundary reaches far. Hamilton, Kabat, Lifschytz and Lowe (2006) wrote a local bulk field, in the semiclassical limit, as boundary operators smeared over a region of the boundary with a fixed kernel. They extended the construction to an operator inside the horizon of a three-dimensional black hole, written as a boundary operator on a region of complexified boundary coordinates (Hamilton, Kabat, Lifschytz and Lowe 2007). Jafferis, Lewkowycz, Maldacena and Suh (2016) found that boundary relative entropy equals bulk relative entropy at leading order. From that result Dong, Harlow and Wall (2016) established that every bulk operator in the entanglement wedge of a boundary region can be reconstructed on that region. The entanglement wedge is the bulk region between the boundary region and its extremal surface, and it is larger than the causal wedge that the first constructions reached.

Almheiri, Dong and Harlow (2015) described the reach of reconstruction as quantum error correction. The bulk effective field theory about a given geometry is a code subspace of the boundary Hilbert space. An op-

erator at the center of the bulk can be reconstructed on each of several boundary regions, so it survives the erasure of any one of them. The encoding is an isometry from the code subspace into the boundary Hilbert space. It is redundant, and it loses nothing within the code subspace.

The reach has three limits, and each is drawn from the literature. First, the code subspace about one geometry is small compared with the boundary Hilbert space, and the bulk effective field theory holds within it; states outside it have other geometries, or no geometry of this kind. Second, behind a horizon the reconstruction depends on the state. An interior operator is built from boundary operators chosen relative to the state of the black hole, and no single boundary operator represents it across all such states (Papadodimas and Raju 2013). Third, Hamilton, Kabat, Lifschytz and Lowe (2006) argue that at finite N the number of independent localized operators within a bulk volume reaches the holographic bound.

Proposition 15.2. The boundary description reaches the local geometric findings of the bulk densely and not exhaustively. It reaches them exactly within a code subspace about a given geometry, in a manner that depends on the state behind a horizon, and with a count of local operators bounded at finite N.

Argument. I read Proposition 15.2 as the reach and the residual of Proposition 3.1. A symmetric framework, the boundary theory, probes a continuous content, the bulk geometry, and its reach is dense and leaves a residual. The residual lies where the bulk is most continuous in character: the geometry inside horizons, and the fine structure of the geometry at the scale where the bulk description meets its own edge.[1]

Rests on: the cross-modal principle (Proposition 3.1); the results of reconstruction set out in Section III.

IV. The Ryu and Takayanagi Formula as a Bridge

Ryu and Takayanagi (2006) proposed that the entanglement entropy of a boundary region A is given by the area of the minimal bulk surface γ_A anchored on the edge of A and homologous to it,

$$S(A) = Area(\gamma_A) \,/\, 4G_N, \tag{15.2}$$

in units with $\hbar = c = 1$: the entropy of the region is the area of the surface divided by four times Newton's constant. Hubeny, Rangamani and Takayanagi (2007) gave the covariant form, with an extremal surface in place of the minimal one. Lewkowycz and Maldacena (2013) derived the formula by carrying the replica construction of the boundary entropy into the bulk gravitational path integral. Faulkner, Lewkowycz and Maldacena (2013) added the entanglement entropy of the bulk quantum fields across the surface, and Engelhardt and Wall (2015) extended the prescription beyond first order by extremizing the generalized entropy:

$$S(A) = ext_{\chi} \left[\, Area(\chi) \,/\, 4G_N + S_{bulk}(\Sigma_{\chi}) \, \right], \tag{15.3}$$

which says that the entropy of A is the extremum, over bulk surfaces χ anchored on the edge of A, of the area term plus the entropy of the bulk fields in the region Σ_{χ} between χ and A.

The two sides of (15.2) differ in character. The left side is symmetric in character: it depends only on the spectrum of the reduced density matrix of A, and it is the same under every unitary change of basis on A. In the continuum it is divergent, and the formula gives its universal part once the area is cut off near the boundary, as Ryu and Takayanagi found. The right side is continuous in character: the area of a surface in a smooth geometry, varying smoothly with the region and with the metric.

The Gauss-Bonnet theorem shows the same structure in elementary form. For a compact surface M with boundary,

$$\int_M K\, dA + \int_{\partial M} \kappa_g\, ds = 2\pi\, \chi(M), \tag{15.4}$$

which says that the integral of the Gaussian curvature over the surface, plus the integral of the geodesic curvature along its boundary, equals two pi times the Euler characteristic. The left side is continuous, and the right side is an integer that no smooth deformation changes. The relation is exact, it is mediated by a structural condition (compactness), and it is not invertible, since infinitely many distributions of curvature give the same Euler characteristic. The formula of Ryu and Takayanagi has the same three features, with extremality of the surface as its mediating condition.

Proposition 15.3. The formula of Ryu and Takayanagi, with its corrections (15.3), is a bridge: an exact, mediated and non-invertible relation between a symmetric quantity of the boundary description and a continuous quantity of the bulk description. It does not derive geometry from entanglement, or entanglement from geometry.

Argument. Three results of the literature support the bridge reading. First, the derivation of Lewkowycz and Maldacena assumes the dictionary (15.1), so the formula is obtained within the correspondence and not from the boundary theory alone. Second, the relation is not invertible. The entropy data are one real number for each boundary region, and they fix the geometry at most where extremal surfaces reach. By (15.3) the bulk fields enter only through their entropy, so different bulk states with equal entropies give equal data. A reading that derives the bulk from the entropy data therefore meets the difficulty of multiplicity of Proposition 14.1(a): many bulk descriptions fit the data, and the data select none. Third, the corrected formula (15.3) combines an area, which is geometric, with the

entropy of quantum fields, which is content, and Harlow (2017) found that the corrected formula holds in any quantum error-correcting code with complementary recovery. The formula is thus a property of the translation between the two descriptions. The geometric side reaches the further entropies of the spectrum, the Rényi entropies, only by a different geometry for each index, the area of a cosmic brane that deforms the bulk (Dong 2016).

Rests on: the derivation of Lewkowycz and Maldacena (2013); the difficulty of multiplicity (Proposition 14.1(a)); the result of Harlow (2017).

The "geometry from entanglement" reading is a one-directional interpretation of what is structurally a correspondence in both directions. The formula is a bridge of the same type as the Born rule, which takes the continuous state of the formalism to the probabilities of discrete findings (Chapter 11, "The Measurement Problem as a Cross-Modal Residual").

V. Holographic Monogamy

Hayden, Headrick and Maloney (2013) established that entropies given by (15.2) satisfy the monogamy of mutual information. For three boundary regions A, B and C,

$$I(A{:}B) + I(A{:}C) - I(A{:}BC) \leq 0, \tag{15.5}$$

which says that the information A shares with B and with C separately does not exceed what it shares with the two together. In general quantum systems the left side of (15.5) can take either sign, and in free field theories it can be positive (Hayden, Headrick and Maloney 2013). The holographic entropy cone is the set of values that the entropies of a fixed number of regions can take when each is given by (15.2). For up to four regions, strong subadditivity and monogamy are the complete set of inequalities that bound the cone, and from five regions on further families appear (Bao, Nezami, Ooguri, Stoica, Sully and Walter 2015).

I read these results as the residual on the other side. The bulk geometry reaches the entanglement of the boundary through areas, and areas carry only the patterns of entanglement that lie inside the holographic entropy cone. Patterns outside the cone belong to the boundary description, and no smooth geometry carries them. The constraint of the holographic entropy cone is a filter set by the continuous description. The constraint passes the entanglement that admits a geometric expression and stops the entanglement that admits no geometric expression. The residuals of the two sides are complements. The boundary does not fully reach the continuity of the bulk, and the bulk does not fully reach the symmetric structure of the boundary. Each side's blind spot is the other side's native domain.

VI. Neither Side More Fundamental

The reading most widely held in the field takes the boundary theory as fundamental and the bulk geometry as emergent from it. Its motivations are serious. The boundary theory is a well-defined quantum theory with a precise Hilbert space; it is defined on a fixed background, while the bulk geometry varies with the state; and reconstruction reaches far into the bulk, as Section III records. The conclusion drawn is that gravity is a quantum theory in disguise.

Proposition 15.4. Neither side of the correspondence is more fundamental than the other. Each is a description in its own mode, each reaches the other densely, and each leaves a residual that the other contains.[2]

Argument. A duality that is exact in both directions does not privilege one side. Emergence is an asymmetric relation, since what emerges depends on what it emerges from and not the reverse, and an exact equality

such as (15.1), between the bulk theory and the boundary theory, holds in both directions. By Propositions 15.2 and 15.3 and Section V, each description has a residual, and each description's residual is the other's strength: the boundary description handles the fine-grained entanglement natively, and the bulk's geometric description handles the smooth geometry natively. The correspondence shows that one system admits two irreducibly different descriptions, one in each mode, with a systematic translation between the two descriptions. The relation between the two descriptions is correspondence, not emergence.

Rests on: the exactness of the duality (15.1) in both directions; Propositions 15.2 and 15.3, and Section V; the conjugacy that Axiom I states (P12).

Philosophers of dualities have reached the same verdict on their own ground. Teh (2013) finds few convincing grounds for holding that gravity emerges from the boundary gauge theory, and so for treating the boundary theory as more fundamental than the bulk. Dieks, van Dongen and de Haro (2015) compare three holographic schemes and conclude that only Verlinde's entropic scheme straightforwardly instantiates emergence, so that the duality of AdS/CFT, read in the standard way, does not.

The stronger form of the objection, at finite N, is met on the same ground. What the boundary theory defines there is the content description, in its own character, together with the bulk theory dual to it. Where the bulk's curvature reaches the Planck curvature, the bulk's geometric description reaches the edge of its domain, as Chapter 9 places that edge, and the boundary supplies no geometry with steps in its place. The exact duality relates two theories, and it gives the arena no quantum of its own.

Tensor networks give a concrete model of the translation. Swingle (2012) read the network of entanglement renormalization as a discrete geometry of anti-de Sitter type, and Pastawski, Yoshida, Harlow and Preskill (2015) built tensor networks that are quantum error-correcting codes and reproduce the formula (15.2) for their own regions. These networks approximate the geometry from boundary data, and they do not re-

place a smooth bulk geometry with a discrete one. The conjugate reading accounts together for the exactness of the duality, the limits of reconstruction and the non-invertibility of the bridge.

VII. What the Reading Predicts

Proposition 15.5. Any systematic correspondence between a description whose findings are discontinuously symmetric and a description whose findings are continuously asymmetric is a bridge between conjugate descriptions, and not a unification.

Argument. A unification would be one description carrying both characters perfected in the same findings, which P9 excludes; a correspondence between two descriptions, each perfected in its own character, is what Proposition 3.1 predicts. AdS/CFT is such a correspondence. So it is not the unification the theorem excludes, and it shows no quantum of the arena. The claim of this book concerns the actual world, which has no timelike boundary at infinity, and the correspondence serves here as the strongest test case of the claim, not as its ground.

Rests on: the cross-modal principle (Proposition 3.1); the Incommensuration Theorem (P9).

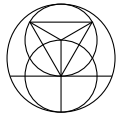
Table 15.1 lists what the reading predicts of the correspondence and the evidence that bears on each prediction.

Prediction	Evidence
Reconstruction of the bulk from the boundary is dense and not exhaustive	Code subspace about a given geometry (Almheiri, Dong and Harlow 2015); state dependence behind horizons (Papadodimas and Raju 2013); bound on localized operators at finite N (Hamilton, Kabat, Lifschytz and Lowe 2006)

Prediction	Evidence
The formula of Ryu and Takayanagi is a bridge, not a derivation	Derivation within the dictionary (Lewkowycz and Maldacena 2013); bulk entropy term (Faulkner, Lewkowycz and Maldacena 2013; Engelhardt and Wall 2015); the formula as a property of codes (Harlow 2017)
Neither side is more fundamental	Exactness of (15.1) in both directions; complementary residuals (Sections III to V)
The bulk acquires no discrete structure that replaces its smooth geometry	Tensor networks approximate the geometry and do not replace it (Swingle 2012; Pastawski, Yoshida, Harlow and Preskill 2015)
The boundary acquires no smooth geometric character of its own	The boundary theory is specified by discrete conformal data (Poland, Rychkov and Vichi 2019)
The translation loses information in both directions	Limits of reconstruction (Section III); non-invertibility of the bridge (Section IV); the holographic entropy cone (Hayden, Headrick and Maloney 2013; Bao and colleagues 2015)

Chapter 19, "Predictions, Evidence and Decisive Experiments", carries the first prediction forward with the others of this book. Chapter 17, "Black-Hole Information and the Page Curve", compares its own result in shape with the island formula, which applies the generalized entropy of (15.3) to radiation. The same chapter is told for the general reader in *Two Theories, One World*, Chapter 23, "Two Descriptions That Translate".

The deepest correspondence physics has found is a bridge and not a merger, so the question that remains is what unity the theorem permits, and the next chapter answers it.



Chapter 16

Common Source, Not Merger

What the Incommensuration Theorem permits and forbids of any unification, and what follows for the search

This chapter states, in general form, what the Incommensuration Theorem permits and what it forbids of any unification of the two descriptions of physics. It forbids a merger: one formalism in which the directions of the arena themselves carry the discontinuous symmetry of content, so that the same directions carry both characters. It permits a common source: two descriptions, each perfected in its own character, joined side by side from one origin. I answer the objection that the right theory must be a hybrid of discrete and continuous elements, and I state what follows for the search for one theory of quantum gravity.

I. Two Senses of Unification

The word "unification" is used in physics in two senses, and by the theorem one is forbidden and the other permitted. Chapter 6, "The Category Error", states the distinction as Proposition 6.4 and names the two senses a merger and a common source. Here I give each sense in general form and draw the consequences of each sense.

In general form, a merger is one formalism in which the directions of the arena themselves carry the discontinuous symmetry of content at the foundation, while the same directions are the continuously asymmetric

arena: both characters perfected in the same directions. By "the directions of the arena" I mean the respects in which every finding is placed: the coordinates, lengths, areas and durations of the geometry, and its degrees of freedom. Quantum electrodynamics joins its quantized field to the arena without a merger: the field is content laid on the arena, and the arena's directions keep their own character. In general form, a common source is two descriptions, each perfected in its own character and in its own directions, joined side by side and related systematically, with one origin for both.

Proposition 16.1 (what is forbidden). No merger is possible. No single formalism can give the directions of the arena the discontinuous symmetry of content at the foundation, so that continuous asymmetry and discontinuous symmetry are both perfected in the same directions.

Proposition 16.2 (what is permitted). A common source is possible. Two descriptions, one continuously asymmetric and one discontinuously symmetric, each perfected in its own directions, can be joined side by side and related by a systematic translation. The inseparability principle requires that both be present wherever either is.

II. What the Theorem Forbids

Argument for Proposition 16.1. A merger takes continuity and asymmetry from the one description and discontinuity and symmetry from the other, and it asks all four of the same directions of the arena. A description that gives the arena's own directions the character of content would carry at least one of the two forbidden pairings, and, where all four characters are

asked of the same directions, both. A merger asks all four, so it carries both: continuity with symmetry, and asymmetry with discontinuity, perfected together in the same directions, the two pairings that P9 excludes.[1] A merger also removes the distinctness of content and context (P4), which the joined form keeps. The proposition holds for any formalism, whatever its mathematics. The exclusion is in principle, and therefore in all practice.

Rests on: the Incommensuration Theorem (P9); the distinctness of content and context (P4).

Two familiar aims of quantum gravity are mergers in the general sense of Section I. A quantized arena is one: it asks the arena's own directions to carry the discrete symmetry of content (Chapter 6, "The Category Error"). A derivation of one arena, uniquely, from a discontinuously symmetric framework is another: it asks the framework's findings to fix the arena's continuous asymmetry in the same respects (Proposition 14.1(a)). Each of the four programs at the lower end of the gradient of Chapter 14, "Four Criteria and Ten Programs", aims at a merger of one of these two kinds.

The proposition forbids mergers and nothing more. Joining descriptions of one character with each other, and joining two descriptions of opposite characters side by side, are both left to physics.

III. What the Theorem Permits

Argument for Proposition 16.2. First, by P9 the two surviving pairings, continuity with asymmetry and symmetry with discontinuity, are exhaustive and exclusive, and each may be perfected in a description of its own. Two descriptions that keep their directions apart carry no excluded pairing, since no direction of either carries the character of the other. Second, by P5, the inseparability principle, where there is content there must also be context, and where there is context there must also be content. So the two descriptions are permitted together, and each requires the other wherever it applies.

Rests on: the inseparability principle (P5); the Incommensuration Theorem (P9).

The origin the two descriptions share is comparison itself. Every finding is an act of comparison (P2), and every act of comparison has a content and a context (P4). With the identifications P16 and P18, the discontinuously symmetric description is the description of the content, and the continuously asymmetric description is the description of the context. I reason that the two describe the two sides of every act of comparison, and so have one source in that act.[2]

Physics as it stands already has the permitted form. Quantum fields of matter live on a classical geometry, and standard quantum field theory on curved space uses one fixed background. The account of this book extends that framework: each alternative of the matter carries its own classical geometry, shaped by that alternative's energy through the Einstein equations (Chapter 7, "One Geometry for Each Alternative"). The small waves of that geometry can be described by quanta below the Planck scale; those quanta are quanta of the description, and the description quantizes nothing of the arena (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale"). The correspondence of Chapter 15, "AdS/CFT as Cross-Modal Correspondence", is the same form seen as a translation: two descriptions of one system, in conjugate modes, joined by a dictionary.

Unification within one character is also permitted, and the theorem is silent on it. Minkowski's union of space and time joined two descriptions of the context into one, and the electroweak theory joined two interactions of the content into one gauge description. In each case every direction of the joined description carries one character, so P9 neither forbids the union nor requires it. I claim no unification of the forces, and the thesis needs none.

Table 16.1 sets out the kinds of unity and the theorem's verdict on each.

Kind of unity	What is joined	Verdict
Merger	Both characters, perfected, in the directions of the arena itself, in one formalism	Forbidden (Proposition 16.1)
Common source, side by side	Two descriptions of opposite characters, each in its own directions	Permitted, and required by the inseparability principle (Proposition 16.2)
Common source, by correspondence	Two descriptions of one system, in conjugate modes, with a dictionary	Permitted (Proposition 15.5)
Union within one character	Descriptions of the same character	Neither forbidden nor required

IV. The Objection From Hybrids

A careful physicist will put the strongest objection in these terms. The right theory of quantum gravity, on this objection, is neither purely discrete nor purely continuous. It is a hybrid, in which discrete and continuous elements are both present and both irreducible, meeting at the Planck scale. Loop quantum gravity, with its smooth classical limit, and causal set theory, with its Lorentz invariance, already have this form, so by the theorem they should count as compatible, and they should not be placed at the lower end of the gradient.

The answer turns on where each element sits. Physics already holds discrete and continuous elements together, both present and both irreducible, in the two joined descriptions of Section III. In loop quantum gravity the discreteness sits in the arena's own findings, the spectra of area and volume, and the continuity is asked of the same arena in a limit. In causal set theory the discreteness sits in the arena's own count of elements, and the continuous arena is asked of the same order in a limit. In both, the arena itself, the context in which every description is laid out, is given discrete findings at its foundation and asked to recover its continuity in a limit, with no further arena in which the two descriptions could be compared, and that is a merger. A content's classical limit, as in electrodynamics, is a second description compared within an arena that both keep, and Proposition 16.2 permits it. A hybrid in the sense of the objection is therefore a merger, and Proposition 16.1 excludes it. A hybrid in the sense of two joined descriptions is the permitted form, and it needs no quantum theory of the arena. The ranking of Chapter 14 stands.

The objection also places the meeting of the two elements at the Planck scale, and there I agree on the place and differ on what lies there. At the Planck scale an object's gravitational size equals its quantum size, neither description dominates, and both are relevant (Chapter 9). A single description that carried both characters there would be a merger. I posit no third description at that scale, and the joined form requires none.

V. What Follows for the Search

The search for one theory of quantum gravity, in the sense of a merger, is excluded in principle, and therefore in all practice. The relation between quantum theory and general relativity is richer than a merger would allow: two descriptions, each perfect in its own character, each requiring the other. What the search has found stands. The achievements of the ten programs of Chapter 14 are results, and the correspondence of Chapter 15 is among the deepest results of theoretical physics. The effective field theory

of gravity computes the leading quantum corrections to the gravitational interaction below the Planck scale (Donoghue 1994), and its quanta are quanta of its description of the small waves of the geometry, never quanta of the arena. None of these depends on a merger, and each finds its place in the joined description.

The work the theorem leaves to physics is the work of joining. It includes the dictionaries between the two descriptions, of which AdS/CFT is the most developed. It includes the computations of quantum fields on classical geometries, one geometry for each alternative of the matter. It includes the experiments that test the joined form where it makes a definite prediction, such as the entanglement of two masses through gravity (Chapter 8, "The Entanglement Test").

The joined description, quantum content with a classical geometry for each of its alternatives, is not waiting to be replaced by something deeper. The joined description is what depth looks like when the structure of comparison itself is taken as the foundation. The same conclusion is set out for the general reader in *Two Theories, One World*, Chapter 24, "Joined, Not Merged".

VI. Questions

Is the joined description the semi-classical gravity that experiment has already excluded? The joined description is the form that agrees with that experiment. The theory of Møller (1962) and Rosenfeld (1963) keeps one geometry for all the alternatives of the matter, sourced by the expectation value of their energy. Page and Geilker (1981) found that the gravitational field followed the outcome of a quantum random event and not the average over outcomes. The joined description gives each alternative of the matter its own classical geometry, so the field follows the outcome, as Page and Geilker found (Chapter 7, "One Geometry for Each Alternat-

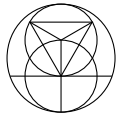
ive"). Where the matter is sharply one alternative, as for a star or a planet, the two forms coincide, and every computation made with the expectation value stands.

Does the theorem bear on the unification of the gauge forces with one another? The theorem is silent on it. The gauge forces are content, and a union of descriptions of content has one character in every direction, so P9 neither forbids it nor requires it. The theorem bears on a program of unification only where the program asks the arena's directions to carry the character of content. In string theory that is the reading of the full theory as a quantum theory of the geometry, which Chapter 14 assesses; whether the gauge forces share one origin is a question within the content, on which the theorem is silent.

The two descriptions meet most sharply at a black hole, where quantum fields radiate from the horizon of a classical geometry, and Part VI begins there.

PART VI

HORIZONS, OBJECTIONS AND TESTS



Chapter 17

Black-Hole Information and the Page Curve

The premises of the paradox, the premise removed, and the turning point of the radiation's entropy as a conditional result

This chapter states the premises of the black-hole information paradox exactly. Hawking radiation is real, and it is a result for quantum fields on a classical geometry. One premise of the paradox, that geometry and matter form one quantum state, is the category error of Chapter 6, "The Category Error", applied to a horizon. A second premise, that the classical geometry describes the whole history through its singularity, places the loss that Hawking found at the edge of that description's domain. With both premises set aside, the evolution of the matter on each geometry loses nothing, and information is not destroyed. For a hole small enough to evaporate while an outside world remains, the Page curve follows as a conditional result from the smaller of two capacities, and its turning point comes before the hole has lost half its entropy. For a hole formed from a star, the account requires only that the information is not destroyed and is not accessible from outside while the hole stands.

I. The Paradox and Its Premises

The information paradox is a conflict between a computation and a demand, both made about a black hole that forms from matter in a pure state and then evaporates. A pure state is a quantum state whose von Neumann entropy is zero; a mixed state is one whose von Neumann en-

tropy is positive. Hawking (1975) computed, for quantum fields on the classical geometry of a collapsing star, that the hole emits radiation with a thermal spectrum at the temperature

$$T_H = \hbar c^3 \,/\, (8\pi G M k_B) \tag{17.1}$$

In words, the temperature is inversely proportional to the mass of the hole; for one solar mass it is 6.2×10^{-8} kelvin.[1] Hawking (1976) then showed that the state of the radiation, computed on the same geometry, is mixed, and concluded that a hole which evaporates completely turns a pure state into a mixed one. He called the result a breakdown of predictability.

The premises of the paradox.

P20 (established physics; Hawking 1975). Quantum fields on the classical geometry of a black hole radiate with a thermal spectrum, and to leading order on that geometry the state of the radiation outside the horizon is mixed.

P21 (established physics). The hole loses mass to that radiation, and a hole that receives nothing loses its mass, down to the Planck mass, in a finite time.

(W) Hawking's further premise, of the whole history (Hawking 1976). The classical geometry describes the whole history, its singularity included, so that what the field carries into the singularity is gone, and after the evaporation the outside world is all that remains.

(Q) The premise of a quantum theory of gravity, of one quantum state. Geometry and matter together form one quantum state, whose evolution from before the collapse to after the evaporation is one unitary map.

P20 and P21 are premises of this book, taken as established physics. (W) and (Q) are the further premises of the paradox, and Section III sets both aside.

Hawking's conclusion, that information is lost, follows from P20, P21 and (W). The paradox is the conflict of that conclusion with (Q), which demands that the whole of the information present at the start be found, at the end, in the state of the radiation outside. The standard responses keep (Q) and revise P20, or the geometry near the horizon, at late times: correlations in the late radiation (Page 1993b), a firewall at the horizon, a layer of high-energy quanta that a body falling in would meet (Almheiri, Marolf, Polchinski and Sully 2013), a horizonless fuzzball (Mathur 2005), or the islands of Section VI. The paradox rests on (W) and (Q), and each of (W) and (Q) applies a concept where the concept does not belong.

II. Hawking Radiation on a Classical Geometry

Hawking radiation is a result for quantum content on a classical context. The computation expands the field in modes defined before the collapse and in modes defined far outside at late times, and relates the two sets by a Bogoliubov transformation fixed by the classical geometry. The vacuum of the early modes, restricted to the late modes outside, is thermal at the temperature (17.1), filtered by the greybody factors of the geometry (Page 1976). A greybody factor is the probability that a mode of given frequency and angular momentum, leaving the neighborhood of the horizon, passes the potential barrier of the geometry and reaches infinity. The metric enters every step as a classical background.

Unruh (1976) found the same structure in flat spacetime. A uniformly accelerated detector in the vacuum registers a thermal distribution at the temperature $\hbar a/(2\pi c k_B)$, where a is its acceleration. Flat spacetime is as classical as a geometry can be, and the thermal response follows from restricting the vacuum to the region that the accelerated detector can reach.

The derivation traces each outgoing mode back toward the horizon, to frequencies above the Planck frequency. Unruh (1995) changed the dispersion of the field at short distances in a sonic analogue of a horizon, and the evaporation was not altered. Corley and Jacobson (1996) found that, with such a change, the radiation produced at the horizon stays very close to a thermal spectrum. So the thermal radiation of the horizon does not rest on what happens beyond the Planck scale.

A familiar heuristic speaks of pairs created at the horizon, one member falling in and one escaping. The account of the radiation is the computation, and the heuristic is not used in this book. The computation gives the state of one field. In that state, the partners of the outgoing quanta are modes of the same field behind the horizon.

I identify the mixedness of the outside state as a fact relative to the outside as context. The state of the field on a surface that crosses the horizon is pure if it began pure. The part of that state outside the horizon is mixed, because the comparisons that reach the outside do not reach the partner modes behind the horizon. A comparer, in this book, is anything that makes a comparison: an instrument or an interaction, never a mind. The observation that bears on this identification is the spectrum of the radiation, which the identification leaves exactly as Hawking computed it.

The geometry follows the mass of the hole through the Einstein equations. A hole of definite mass is matter that is sharply one alternative, and for such matter the expectation value of the stress-energy is the source of the one geometry it has (Chapter 7, "One Geometry for Each Alternative", sets out this limit). For one solar mass, emitting photons only, the evaporation takes 2.1×10^{67} years.

III. The Premise Removed

Proposition 17.1. Premise (Q) applies the concept of a quantum state to the arena. It is the category error of Chapter 6.

Argument. The arena of space and time is the context of physics (P18; Chapter 5, "The Content and Context Partition"). To quantize a description is to exchange the characters of both of its levels together, its formalism and its findings (Chapter 4, "What Quantization Is"). The arena admits no mode exchange (Proposition 6.1): quantized, it would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. Each alternative of the matter therefore carries its own classical geometry (Proposition 7.1). Premise (Q) puts the geometry into the quantum state, so that its evolution and that of the matter form one unitary map.

Rests on: the identification of gravity as the context of physics (P18); quantization as a mode exchange (Proposition 4.4); the arena admits no mode exchange (Proposition 6.1); one geometry for each alternative (Proposition 7.1).[2]

Proposition 17.2. The loss in Hawking's conclusion is located at the singularity, where the classical description reaches the edge of its domain; it is not a result of either description.

Argument. On a given classical geometry, quantum field theory evolves the state of the field unitarily from one surface to any later surface that captures the whole field. While the hole stands, a surface through the outside and the inside carries the whole state. The one place where Hawk-

ing's computation registers a loss is where the field meets the singularity, under premise (W). Before the field reaches the singularity, the curvature passes the Planck curvature, and the smooth description holds only below that curvature (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale"). A loss placed at the edge of a description's domain is not a step within that description.

Rests on: quantum field theory on a classical geometry as established physics (P15); Hawking's premise of the whole history (W); the meeting of the two descriptions at the Planck scale (Proposition 9.3).

Proposition 17.3. Information is not destroyed in a black hole.

Argument. Information has two senses in this chapter. In the first sense, information is the form of the content: the specification of what the matter is and of which alternatives it holds, carried with the matter wherever the matter goes. In the second sense, information is a count of states relative to a context: the number of states that the comparisons available from that context cannot tell apart. Proposition 17.2 shows that no step of either description destroys information in the first sense. The Bekenstein-Hawking entropy is a count in the second sense of information:

$$S_{BH} = k_B \, c^3 \, A \, / \, (4G\hbar) \tag{17.2}$$

In words, the entropy of the hole is one quarter of the area A of its horizon, measured in units of the Planck area, times Boltzmann's constant (Bekenstein 1973; Hawking 1975). The Planck area is the square of the Planck length. The count is real. I read it as the number of states of the hole that no comparison from outside tells apart, a count relative to the outside as context.[3]

Rests on: the premise of one quantum state removed (Proposition 17.1); the loss placed at the edge of the classical description's domain (Proposition 17.2).

With (Q) removed, the paradox divides into two questions of different kinds. The first asks whether anything destroys the information, and Proposition 17.3 answers it. The second asks whether, and when, the information is accessible from outside, and the answer depends on the hole. Section V takes a hole that evaporates while an outside world remains, and Section VII takes a hole formed from a star. (The plain account is in *Two Theories, One World*, Chapter 25, "Where Does the Information Go?".)

IV. Why Area, Not Volume

The entropy of equation (17.2) grows with the area of the horizon, and it is established physics (Bekenstein 1973; Hawking 1975). Here I read that dependence on area on the ground of comparison. Section V then uses the count it reads as the capacity of the hole.

A bounded region has two natural counts of its information. The first count takes the interior cell by cell, one degree of freedom for each Planck volume, and it grows with the volume of the region. The second count takes what crosses the boundary of the region through interaction, and it grows with the area of the boundary, in units of the Planck area. The Planck volume is the cube of the Planck length. Each serves its count as a unit, and neither is a step of the arena.

I identify the first count as omniscient and the second as immanent (P10; P13), in the focus of counting the information of a region, which asserts Axioms I and III (Chapter 2, Table 2.1). To make the first count, a comparer would need every cell of the interior in view together and independently, from a vantage that holds the whole region as one structure laid out. That vantage is the omniscient role: what can be laid out and held in view as a whole. The second count is made from a place. A comparer outside the region registers the region only through the interactions

that reach it across the boundary: what falls in, the field that the region's mass, charge and spin set up outside, and the quanta the region emits. That count is the immanent role, interaction as the process of measurement. By P2 every finding of physics is a comparison made from inside the world, so every finding is made from a place.

P12 (IM ground; *An Immanent Metaphysics*, aphorism 1.3-1).

Axiom I. The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another.

I reason from Axiom I to the count of a horizon. By Axiom I the immanent is more fundamental than the omniscient, so the count made from a place through interaction is the more fundamental of the two natural counts. That count grows with the area of the boundary, and the boundary of a black hole, for every comparer outside it, is its horizon. I therefore identify the dependence of equation (17.2) on area as the immanent count: the number of states of the interior that the comparisons available from outside cannot tell apart, relative to the outside as context (Section III). The observation that bears on the identification is the area law itself. The Bekenstein-Hawking entropy grows with the area of the horizon, and the entanglement entropy of quantum fields across a surface grows with the area of the surface (Bombelli, Koul, Lee and Sorkin 1986; Srednicki 1993). The identification makes no claim on the coefficient one quarter, which equation (17.2) takes from Hawking (1975).

The count made from outside is real. It is the entropy that the generalized second law adds to the entropy outside the hole (Bekenstein 1974).

The objection, at full strength, runs as follows. A local quantum field theory has independent modes at every point of the interior, so the number of its states grows with the volume, and that number is the physical count. The answer is that the mode count of a local field theory is a count

of the structure of its formalism, laid out as a whole and held in view from no place. It is the omniscient count, and it describes that structure correctly. The entropy of a hole is a count relative to the outside as context, and every comparison made from that context is made from a place outside the horizon. So the physical count of a horizon's information is the count made from a place, and it goes by area.

Condition (H) of Section V takes this count as the capacity of the hole, the bound on the entanglement entropy of the content inside the horizon. The identification reads the established area law on the ground, and the bound that (H) states remains a condition of the curve.

V. The Page Curve for a Hole That Evaporates

For a hole small enough to evaporate while an outside world remains to receive its radiation, the question of access from outside can be asked over the whole life of the hole. The Page curve is the entanglement entropy of the radiation outside, plotted against time (Page 1993b). The Page time is the time at which that curve turns from rising to falling. Two conditions and two premises give the curve.

The conditions and the premises of the curve.

(C) Condition. The whole formed by what fell in, the content inside the horizon and the radiation began pure and stays pure, as described from one outside world. The whole is the matter's state laid out on its geometries, and the condition puts no geometry into the state. For a pure whole of two parts, the entanglement entropy of the radiation equals that of the content inside the horizon.

(H) Condition. The states of the hole that no comparison from outside tells apart number e^{S_{BH}/k_B}, so the entanglement entropy of the content inside the horizon cannot exceed S_{BH}, which falls as the hole evaporates. Almheiri, Hartman, Maldacena, Shaghoulian and Tajdini (2021) call this statement the central dogma; in this book it is a condition of the curve.

P22 (established physics). The entanglement entropy of the radiation cannot exceed its coarse-grained entropy S_{emit}.

P23 (established physics with an identification; Page 1993a). The joint state is one of very many that no comparison from outside tells apart, and each counts alike. For a random pure state of a system of dimension mn with $m \leq n$, the average entropy of the part of dimension m is about $\ln m - m/(2n)$, which lies within half a nat of its largest possible value.

The coarse-grained entropy of the radiation is the thermal entropy computed from its spectrum alone, the entropy it would have if its quanta were uncorrelated. The capacity of a part is the largest entanglement entropy that part can have. I identify the equal weighting of P23 with a symmetry: states that no comparison from outside tells apart count alike, as the states of a closed system that no measurement distinguishes count alike in statistical mechanics.[4]

Proposition 17.4. Under (C), (H), P22 and P23, the entanglement entropy of the radiation is the smaller of the two capacities, the capacity of the radiation and the capacity of the hole, to within half a nat:

$$S_{rad}(t) = min\ [\ S_{emit}(t), S_{BH}(t)\ ] \tag{17.3}$$

In words, the entropy of the radiation rises with the entropy it has carried away, until the entropy carried away equals the entropy the hole can hold; it then falls with what the hole can hold.

Argument. By (C) the two parts share one entanglement entropy, by P22 and (H) it is bounded by each capacity, and by P23 a typical state fills the smaller bound.

Rests on: the purity of the whole (condition C); the capacity of the hole (condition H); the bound by the coarse-grained entropy (P22); the typical state (P23).

The evaporation fixes both capacities as functions of time. In Planck units the mass falls as $dM/dt = -\alpha/M^2$, where α is set by the species emitted and their greybody factors. So the mass is $M_0(1-\tau)^{1/3}$ and the entropy of the hole is

$$S_{BH}(\tau) = S_0\ (1-\tau)^{2/3} \tag{17.4}$$

where τ is the fraction of the evaporation time that has passed and S_0 is the initial entropy. Let r be the ratio of the entropy emitted to the Bekenstein-Hawking entropy lost. For a hole that emits the same species throughout, r is constant, and $S_{emit} = r(S_0 - S_{BH})$. The two capacities are equal where $r(S_0 - S_{BH}) = S_{BH}$, which gives

$$S_{BH}(\tau_P) \,/\, S_0 = r \,/\, (1+r), \quad \tau_P = 1 - [r \,/\, (1+r)]^{3/2} \tag{17.5}$$

In words, the fraction of its entropy the hole keeps at the turning point is r over one plus r, and the Page time is one less the three-halves power of that fraction.

Proposition 17.5. Under (C), (H), P22 and P23, the turning point comes before the hole has lost half its entropy, and no later than 0.646 of the evaporation time.

Argument. The generalized second law states that the entropy of the hole plus the entropy outside it never decreases (Bekenstein 1974). The entropy emitted is therefore at least the entropy the hole loses, and $r \geq 1$. Zurek (1982) showed that emission into empty space is irreversible, so that r exceeds one. With $r \geq 1$, equation (17.5) gives $S_{BH}(\tau_P) \geq S_0/2$ and $\tau_P \leq 1 - 2^{-3/2} = 0.646$.

Rests on: the smaller of the two capacities (Proposition 17.4); the law of evaporation (equation 17.4); the generalized second law (Bekenstein 1974).

Table 17.1 gives the turning point for the bound and for the two values of r that the literature computes.

Case	r	S_{BH}/S_0 at the turning point	τ_P
Bound from the generalized second law	1	0.500	0.646
Black-body emission into empty space	4/3	0.571	0.568
Photons and the small waves of the geometry, with greybody factors (Page 2013)	1.4847	0.598	0.538

The value 0.568 is the black-body idealization. It takes the ratio of entropy to energy of black-body radiation, $s = 4u/(3T)$, and leaves out the greybody factors. Page (2013) computed $r = 1.4847$ for a hole emitting photons and the small waves of its geometry, with their greybody factors. He found that the entropy of the radiation peaks after 53.81 percent of the evaporation time, when the hole has lost 40.25 percent of its entropy. Equation (17.5) with $r = 1.4847$ gives 0.5975 and 0.538, which are Page's values.[5] Page counts the small waves in the terms of their description by quanta, and the count is a count of that description (Chapter 9); it quantizes nothing of the arena.

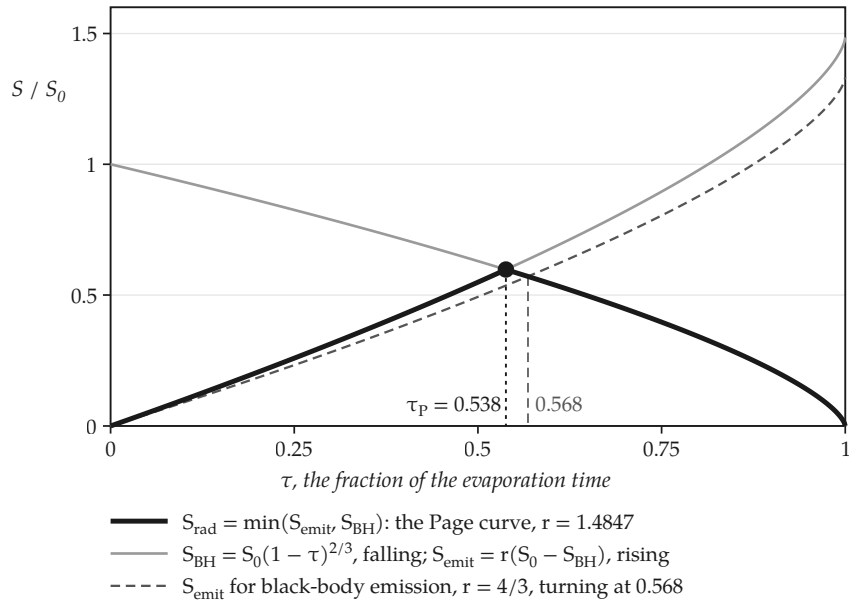


Figure 17.1. *The Page curve as the smaller of the two capacities, for $r = 1.4847$ (Page 2013), turning at $\tau_P = 0.538$. The dashed curve is the coarse-grained entropy for black-body emission, $r = 4/3$, whose turning point at 0.568 comes later.*

Proposition 17.4 uses no quantized geometry and no single geometry shaped by averaged alternatives. The leading-order computation on a given geometry gives Hawking's rise, S_{emit}; the turning point comes from the capacity of the hole. The shrinking horizon exposes nothing, since the computation on a given geometry leaves the partner modes behind it. Past the Page time, (C) and (H) require correlations in the late radiation that the leading-order computation on one geometry does not track; the island computation finds them within semiclassical gravity by including further classical saddles (Section VI). The last stage of the curve, as the mass approaches the Planck mass, lies where the two descriptions meet, and there the result rests on condition (C) alone.

VI. The Island Formula, Compared in Shape

The island formula computes the entropy of the radiation R as

$$S(R) = min_I \, ext_I \, [\, A(\partial I) \,/\, 4G + S_{matter}(R \cup I) \,] \tag{17.6}$$

The formula is written in units with $\hbar = c = k_B = 1$. In words, one adds to the radiation a region I behind the horizon, called the island. One then counts the area of the island's boundary in units of four Planck areas, adds the entropy of the matter in the radiation and the island, extremizes over the place of the boundary, and takes the smallest extremum. Before the Page time the smallest extremum has no island, and S(R) is Hawking's rise. After it, an island whose boundary lies near the horizon dominates, and S(R) follows the entropy of the hole (Penington 2020; Almheiri, Engelhardt, Marolf and Maxfield 2019; Almheiri, Mahajan, Maldacena and Zhao 2020). The formula comes from replica wormholes, new saddles of the gravitational path integral (Almheiri, Hartman, Maldacena, Shaghoulian and Tajdini 2020; Penington, Shenker, Stanford and Yang 2022), and the same five authors review it (Almheiri, Hartman, Maldacena, Shaghoulian and Tajdini 2021).

The island formula and Proposition 17.4 give one shape: a rise with the coarse-grained entropy, a turning point where it meets the capacity of the hole, and a fall with that capacity. Both express one fact of quantum information: the entanglement entropy of a part is bounded by the logarithm of the dimension of the smaller part.

I compare the two as shapes, and I decline the mechanism. Each saddle of the path integral is a classical solution of the gravitational equations. A replica wormhole is a saddle of the calculation of a power of the density operator of the radiation, a geometry that joins several copies of the black hole (Almheiri, Hartman, Maldacena, Shaghoulian and Tajdini 2020). It belongs to the calculation, and it is not the geometry that any one

alternative of the matter carries. What I decline is the reading of the sum over saddles as a quantum superposition of geometries, which would put the arena into the state and so repeat (Q). On the account, the curve is evidence that the whole of the content is pure relative to one outside world, and it is not evidence of a quantized geometry.[6]

VII. Holes Formed from Stars

A hole formed from a star has a mass of several solar masses or more. Its temperature is therefore below 6.2×10^{-8} kelvin, and its evaporation time is longer than 10^{67} years. For such a hole the account requires two things only. The information is not destroyed (Proposition 17.3). The information is not accessible from outside while the hole stands.

The second requirement, that the information is not accessible from outside while the hole stands, holds by established general relativity. The interior of the hole lies outside the past light cone of every event outside the horizon, so no comparison made outside registers what is inside. General relativity carries the matter that falls in to the end of the interior, where the classical description reaches the edge of its domain (Chapter 18, Section IV, sets out that edge). I assert neither the return of this information to the outside world nor its loss to it. The thesis of this book needs nothing more for these holes than Proposition 17.3 and the causal structure of the horizon.

VIII. Questions

Is condition (C) the unitarity that the paradox put in question, assumed under another name? Condition (C) is stated as a condition, and Propositions 17.4 and 17.5 are stated as conditional on it. What I add is that no step of either description destroys what (C) keeps: the content evolves without loss on each geometry, and the geometry is not a part of the state whose purity (C) asserts. Hawking's leading-order computation, carried

past the Page time, conflicts with (C) and (H) together; the conflict is the demand that the leading-order state be the whole state, and the turning point shows that it is not. Premise (W) adds the further conclusion that what reaches the singularity is gone, and Proposition 17.2 places that loss at the edge of the classical description's domain.

Does the account reproduce Hawking's own result? Yes. To leading order on a given geometry the entropy of the radiation is S_{emit}, rising for the whole life of the hole, and that is Hawking's result. The turning point is a statement about the whole under (C) and (H), and it leaves the leading-order spectrum on the geometry as Hawking computed it.

Does the turning point of the curve require a firewall at the horizon? No. I keep the smooth horizon and ordinary physics outside the hole, and I give up the independence of the partners behind the horizon from the early radiation. Almheiri, Marolf, Polchinski and Sully (2013) argue that three statements cannot all hold for a hole that evaporates: that the radiation is in a pure state, that low-energy effective field theory holds outside the horizon beyond a microscopic distance from it, and that a body falling in encounters nothing unusual at the horizon. Their argument runs as follows. Past the Page time, under (C), a late outgoing mode must be maximally entangled with the early radiation. A smooth horizon requires the same mode to be maximally entangled with its partner behind the horizon. The monogamy of entanglement is the rule that a system maximally entangled with one partner is entangled with no other system; so if the partner is a system independent of the early radiation, monogamy forbids both, and the horizon must carry a firewall instead. The step the argument needs, and I do not grant, is that independence. Past the Page time the island of equation (17.6), which contains the partner modes, belongs with the radiation (Penington 2020; Almheiri, Engelhardt, Marolf and Maxfield 2019), so the partner is encoded in the early radiation, and the entanglement of the late mode with its partner is a part of its entanglement with the early radiation, not a second entanglement that monogamy forbids. The island calculation expresses that the partner of the late mode is en-

coded in the early radiation. The answer keeps the equivalence principle, which a firewall would break: a body falling freely through the horizon meets there nothing that a body falling freely anywhere else would not meet, and Chapter 19, "Predictions, Evidence and Decisive Experiments", states that principle as a prediction without exception (Prediction 19.3). The question arises under (C) and (H), for a hole that evaporates while an outside world remains; for a hole formed from a star, Section VII states all that the account requires.

The next chapter states the objections a physicist brings against the whole account, each at full strength, with its answer.



Chapter 18

Objections at Full Strength

Nine objections a competent physicist brings, each in the words of those who make it, with its literature and its answer

This chapter states the nine strongest objections to the account and answers each. Every objection names a real result that seems to put quantum effects into gravity, or a real feature of physics that seems to demand a quantum theory of geometry. Each answer shows what the result is on the account: quantum fields on a classical geometry, a count of the degrees of freedom of content, a correspondence between two descriptions, or the edge of a description's domain. None of the nine exhibits a quantum of the arena at any scale.

I. Hawking Radiation

The objection. As it is often put: Hawking radiation is a quantum gravitational phenomenon. A black hole, an object made of geometry alone, radiates quanta. So quantum effects act within gravity, and a classical gravity could not make a black hole radiate.

The answer. Hawking's computation (Hawking 1975) is quantum field theory on a fixed classical geometry. The quantum object is the field, and the geometry enters as a classical background that fixes the relation between early and late modes. The thermal spectrum follows from restricting the field's state to the region outside the horizon, as the thermal

response of an accelerated detector in flat spacetime follows from restricting the vacuum to the region that detector can reach (Unruh 1976). The thermal radiation of the horizon survives changes to the field's behavior at short distances (Unruh 1995; Corley and Jacobson 1996). The small waves of its geometry that a hole emits are computed as a field on the same background, in their description by quanta (Page 1976); the quanta of that computation are quanta of the description, never quanta of the arena. The loss of mass is computed with the expectation value of the stress-energy as its source, which is exact for a hole of definite mass, since such matter is sharply one alternative (Chapter 7, "One Geometry for Each Alternative"). Hawking radiation is therefore what the account describes: quantum content on a classical context (Chapter 17, "Black-Hole Information and the Page Curve", Section II, sets out the computation).

II. Microstate Counting

The objection. The Bekenstein-Hawking entropy $S = A/4G$ implies that a black hole has $\exp(A/4G)$ microstates. Strominger and Vafa (1996) derive this number for a class of extremal black holes by counting states in string theory. Ashtekar, Baez, Corichi and Krasnov (1998) obtain an entropy proportional to the area by counting states of quantum geometry on the horizon. If gravity has microstates, it has a discrete state space, and gravity is quantizable.

The answer. The Bekenstein-Hawking entropy is real. The question is what is being counted. On the account it is a count relative to the outside as context: the number of states of the hole that no comparison from outside tells apart (Chapter 17, Section III). The entanglement entropy of quantum fields across a horizon grows with its area (Bombelli, Koul, Lee and Sorkin 1986; Srednicki 1993), so a count of the degrees of freedom of content across the surface already has the form of the area law.

Strominger and Vafa's count is one of the most exact matches in theoretical physics. A D-brane is an object of string theory on which strings can end, and it carries discrete charges. The count is of bound states of D-branes, made in a regime of weak coupling where the system is not a black hole. Supersymmetry protects the number of those states as the coupling grows, and at strong coupling the same charges form a five-dimensional extremal black hole whose area gives the same number. The count is of the states of a system with discrete charges, which is content, and the match across couplings is a correspondence between two descriptions of one system. The count does not exhibit a step of the arena.

The count of Ashtekar, Baez, Corichi and Krasnov rests on the discrete spectrum of the area operator of loop quantum gravity. The Barbero-Immirzi parameter is a free constant of loop quantum gravity that sets the scale of its area spectrum. The count of Ashtekar and colleagues is proportional to the area, and it equals A/4G for a chosen value of that parameter. In the account of this book a least step of area is excluded (Chapter 6, "The Category Error"), and Chapter 19, "Predictions, Evidence and Decisive Experiments", states that exclusion as a prediction with its tests. A count that matches A/4G through the choice of a parameter does not by its match show that the area spectrum is physical.

III. The AdS/CFT Correspondence

The objection. The AdS/CFT correspondence (Maldacena 1998) establishes an exact duality between a gravitational theory in anti-de Sitter space and a conformal field theory on its boundary. If gravity is exactly dual to a quantum theory, then gravity is a quantum theory, which only looks classical from the bulk. At a finite number N the boundary theory is complete, so it defines the bulk at every scale, the Planck scale included. The AdS/CFT correspondence is the strongest evidence that gravity can be fully quantized.

The answer. The correspondence is a map between two descriptions, and I read it as a map between conjugate ones (Chapter 15, "AdS/CFT as Cross-Modal Correspondence", sets out the reading in full). The boundary theory finds discrete and symmetric things: a discrete spectrum of scaling dimensions, gauge-invariant operators, exact quantum numbers. The bulk description finds a smooth geometry that differs from place to place, governed at large N by the Einstein equations. The dictionary between them works within limits. Local bulk operators are reconstructed from boundary data at large N (Hamilton, Kabat, Lifschytz and Lowe 2006). They act as logical operators of a quantum error-correcting code on a subspace of states (Almheiri, Dong and Harlow 2015), and each is reconstructible within the entanglement wedge of a boundary region, the bulk region between that boundary region and its extremal surface (Dong, Harlow and Wall 2016; Chapter 15, Section III). The Ryu and Takayanagi (2006) formula relates the area of a bulk surface to the entropy of a boundary region, and it needs quantum corrections beyond leading order (Faulkner, Lewkowycz and Maldacena 2013). The mutual information of states with geometric duals is monogamous (Hayden, Headrick and Maloney 2013), so the states that have a smooth bulk geometry form a restricted class of the states of the boundary theory.

An equality of partition functions shows that two descriptions describe one system. It does not show that the bulk geometry is a quantum state of the arena, since the geometry is the description that applies to the restricted class of states just named. What the correspondence calls bulk gravitons are quanta of the description of the small waves of the bulk geometry, never quanta of the arena. Neither side is more fundamental: each captures what the other leaves out. At finite N, what the boundary theory defines is the content description in its own character, together with the bulk theory dual to it; where the bulk's curvature reaches the Planck curvature, the bulk's geometric description reaches the edge of its domain (Chapter 15, Section VI).

IV. The Singularity

The objection. The singularity theorems (Penrose 1965; Hawking and Penrose 1970) show that, under the energy conditions, classical general relativity predicts incomplete geodesics inside every black hole, without any assumption of symmetry. Only quantum effects in gravity can resolve the singularity. If gravity cannot be quantized, the singularity stands unresolved, and general relativity has no complete account of its own interiors.

The answer. The incompleteness the theorems establish marks the edge of the domain of the classical description. In the cases worked out in full, the curvature grows without bound along a path that falls into a black hole. The corrections to Einstein's equations grow with the curvature, negligible far below the Planck curvature and of order one at it, so the smooth description holds below the Planck curvature, and the singularity lies beyond the curvature at which it holds. At the singularity a description reaches the edge of its domain, and the edge is a property of the description, not an event in the world. To ask for the geometry at the singularity is to apply the concept of a geometry where the concept does not belong, which is a category error. I place no quantized geometry and no new geometry at that edge or beyond it. For a hole of the Schwarzschild kind, the singularity lies in the future of every path that crosses the horizon; it is not a place in the outside world at any outside time.

The ultraviolet catastrophe of Chapter 4, Section VI, has the same form. Classical equipartition gave every mode of thermal radiation the same mean energy, and the sum over infinitely many modes diverged.[1] Planck's law of 1900 (Planck 1901), which has no such divergence, rests on energy exchanged in discrete steps by the radiating oscillators, and quantum electrodynamics later gave the field itself its quanta. The resolution was a quantization of content, and not a quantization of space. The divergence marked a description extended beyond its domain, and the

remedy quantized the description of the content. The singularity likewise marks the classical description of the arena extended beyond its domain, and I answer it by stating that domain, not by quantizing the arena.

V. Observable Gravitons

The objection. Gravitational waves have been detected (Abbott et al. 2016). Every other wave phenomenon has been quantized: light comes in photons, sound comes in phonons. Gravitational waves must come in gravitons, and Tobar, Manikandan, Beitel and Pikovski (2024) propose that single gravitons can be registered by a massive quantum resonator. If gravitons exist, gravity is quantized.

The answer. Gravitation is not quantized. The step from light and sound to gravitational waves takes the arena for one more content, and the arena is the context of every content (Chapter 5, "The Content and Context Partition"). The phonon, the objection's own example, is the nearest sense by which the word "graviton" can be made intelligible: a phonon is a quantum of the description of the vibrations of a held structure, a lattice whose bonds hold its atoms in place, and it is not an atom of the lattice. The arena is not a held structure: it has no parts held in place by bonds, and its curvature is set by the content in it. So even the phonon's sense does not make a graviton a quantum of the arena. The effective field theory of gravity describes small waves on a classical background by quanta and computes their interactions in that description (Donoghue 1994), and a graviton names at most a quantum of that description. A detector that registers a gravitational wave in single clicks would not count against the account, and it would not show gravitation quantized: what the detector registers is content, its own quantum jumps, against the arena as its context. Carney, Domcke and Rodd (2024) show that such a signal "could just as well be explained via classical gravitational waves" acting on a quantum detector, as clicks in a photodetector do not by themselves show the field quantized. Rothman and Boughn (2006) and Dyson (2013)

argued that detecting one graviton is out of reach for detectors of the kinds they examined, and the account does not depend on the outcome. What is excluded in the account of this book is a step of the arena itself, and a quantum of the arena at any scale. The description of small waves on a background applies while the waves are small against the background and the curvature lies below the Planck curvature; at the Planck curvature the two descriptions meet (Section VIII).

VI. Inflation and the Primordial Waves

The objection. Krauss and Wilczek (2014) state it in these words: the polarization of the cosmic microwave background produced by a background of gravitational waves from inflation "would firmly establish the quantization of gravity". The cosmic microwave background is the thermal radiation left from the early universe. Inflation is a proposed early epoch in which the universe expanded at a nearly exponential rate. Their argument is dimensional. Such an epoch is characterized by one rate of expansion, H, and the energy density of the gravitational radiation it leaves, measured invariantly, must be built from G, H, ħ and c. With G to the first power, as for the leading gravitational effect, the one combination of the right dimensions carries ħ to the second power. In their words, "there is no classical gravitational radiation from a classical de Sitter background", de Sitter space being the geometry of uniform exponential expansion. A positive power of ħ marks the effect as quantum, and no field but gravity enters it. So an imprint of these waves in the sky would show the metric quantized.

The answer. The dimensional argument is sound, and it shows exactly what its power of ħ carries: if such a background exists, its amplitude is computed from the vacuum fluctuations of the small waves of the geometry in their description by quanta. In the account of this book that description is allowed, as a description: its quanta are quanta of the description, never quanta of the arena (Section V; Chapter 9, "Linearized Gravity,

Gravitons and the Planck Scale"). In the standard computation the tensor perturbations are such small waves: the expanding geometry enters every step as a classical background, and only the description of the small waves about it is quantized. The vacuum fluctuation that fixes their amplitude belongs to that description. The power of $\hbar$ marks the amplitude as computed in a quantum description, and the quantum description in the standard computation is the description of the small waves, with the background classical at every step. A small wave about the background can be described as if it were content, and the background, the arena of every comparison, stays context. An imprint of the waves would therefore show that the description of small waves by quanta gives their amplitude. It would not show gravitation quantized: it would exhibit neither a least step of the arena nor a quantum of the arena at any scale. The answer holds whether or not such a background is found, and the account rests on neither outcome.

VII. Non-Renormalizability

The objection. The quantized metric is not renormalizable. 't Hooft and Veltman (1974) found divergences at one loop for gravity coupled to matter, and Goroff and Sagnotti (1986) found a divergence at two loops for pure gravity. New degrees of freedom must therefore enter at the Planck scale as a completion of the quantized metric, and an account that offers no such completion leaves gravity without a description at high energy.

The answer. Non-renormalizability carries less weight than the objection gives it. Effective field theory handles theories that are not renormalizable at low energy, and quantum general relativity is computed that way (Donoghue 1994; Burgess 2004). Its quantum correction to the Newtonian potential is $41/10\pi$, about 1.3, times $(l_P/r)^2$, and $(l_P/r)^2$ is 2.6×10^{-70} at a separation of one meter.[2] The structural argument does not need renormalizability. It rests on quantization as a mode exchange of both levels of a description, which the arena cannot take (Chapter 4, "What Quantiza-

tion Is"; Chapter 6). Non-renormalizability marks the scale at which the corrections of the effective theory become of order one, which is the Planck scale of Section VIII. It does not mark a place where the arena needs a quantum completion.

VIII. The Planck Scale

The objection. At the Planck scale, quantum effects and gravitational effects are both of order one. Some new physics must appear there. The claim that gravity is not quantized is inconsistent with the existence of the Planck scale.

The answer. The Planck scale is where the two descriptions meet. An object of mass M has a gravitational size GM/c^2 and a quantum size $\hbar/(Mc)$, and the two are equal when $M^2 = \hbar c/G$, at the Planck mass of 2.18×10^{-8} kilograms. The gravitational size of that mass is the Planck length, 1.62×10^{-35} meters. There neither description dominates, and both are relevant. A single description that carried both characters there would need a pairing that the Incommensuration Theorem forbids. At the Planck scale the description by quantum theory and the description by general relativity meet; both descriptions are relevant there, and no single description perfects both characters.

The Planck length is not a least length. The account has no least length, area or duration. The bound that I set out in Chapter 9, that no body can be placed more sharply than about the Planck length, limits the placing of a body. The bound is not a step of the geometry. The size of the Planck scale follows from Newton's constant, which is measured. The axioms fix the role of gravity (universal, unshieldable, with one mass for inertia and weight) and leave its magnitude to measurement, because they are structural, not dimensional. A measurement of Newton's constant is a measurement of the ratio of a laboratory mass to the Planck mass, a fact about this domain (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", sets this out).

IX. "Only Philosophy"

The objection. The Incommensuration Theorem is a philosophical argument, not a physical one. Physics is decided by experiment, not by metaphysical reasoning.

The answer. A structural argument belongs to physics when it forbids things that can be observed. Bell's theorem (Bell 1964) is a structural argument about every local hidden-variable account, and experiment decided it (Hensen et al. 2015). The account has the same form. Its premises are of three kinds, each marked as such: the ground of comparison, which has no physical premise (cited to *An Immanent Metaphysics*); established physics, cited to its literature; and identifications, each made in the first person with the observation that tests it. The category error is a philosophical diagnosis in its own sense, a question that applies a concept where the concept does not belong, and its consequences are physical. They are entanglement through gravity with the Newtonian phase, no effect in which the geometry carries quantum degrees of freedom of its own, an equivalence principle without exception, a speed of light without dependence on energy from any step of spacetime, and a dictionary of AdS/CFT that holds within limits. Chapter 19 states each with its present evidence and the observation that decides it. The account's consequences are predictions with outcomes that measurement settles, and each of the five is stated with the observation that would go against it (Chapter 19); the ground they rest on is not a prediction, and no test reaches it (*Physics Is Not Math*, Chapter 19).[3]

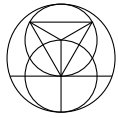
X. The Pattern

Table 18.1 sets each objection beside what its phenomenon is on the account.

Objection	What the phenomenon is	Does it exhibit a quantum of the arena?
Hawking radiation	Quantum fields on a classical geometry	No
Microstate counting	A count of states of content, or a count resting on a chosen area spectrum	No
AdS/CFT	A correspondence between two descriptions of one system	No
The singularity	The edge of the classical description's domain	No
Gravitons	Quanta of a description of the small waves of a classical geometry	No
Inflation and the primordial waves	Small waves of a classical geometry, their amplitude computed in a description by quanta, should such waves be found	No
Non-renormalizability	The scale where the corrections of the effective theory become of order one	No
The Planck scale	The meeting of the two descriptions	No
"Only philosophy"	A structural argument with observable consequences	Not applicable

Every phenomenon offered as evidence for quantum gravity is, on the account, one of four things: a result for content on a classical context, a count of the degrees of freedom of content, a correspondence between two descriptions, or the edge of a description's domain. None requires the arena to be quantized, and none carries continuity and symmetry together in what one description finds. The objections fix where the boundary between the two descriptions lies, and the account states that boundary. (The plain forms of these objections, all but the one from non-renormalizability, are in *Two Theories, One World*, Chapter 26, "What a Physicist Will Say".)

The next chapter states the account's predictions, each with its present evidence, the experiments under way, and the observation that decides it.



Chapter 19

Predictions, Evidence and Decisive Experiments

Five predictions, each with its present evidence, the experiments under way, and the observation that decides it

This chapter states the predictions of the account. Two masses related only through gravity become entangled with the Newtonian phase. Gravitation is not quantized: the geometry carries no quantum degrees of freedom of its own beyond those tied to the alternatives of the matter, and a description of its small waves by quanta has quanta of that description only. The equivalence principle holds without exception, and nothing shields gravity. Space and time have no least length, area or duration, so the speed of light does not depend on energy through any step of spacetime. The dictionary of AdS/CFT holds within limits that no construction removes. Each prediction is stated first, then its evidence, then the experiments under way, then the observation that decides it and the step of the account that observation tests.

I. The Form of the Predictions

A prediction in this chapter is a statement about what observation will find, derived from the premises of Chapters 1 to 16. The rule of decision is the same for each: a result counts when it is established at five standard deviations by at least two independent kinds of measurement. A null res-

ult where the account predicts one is consistent with the account, and other accounts may predict it too. Where the account predicts that something will not be found, the observations that look for it test the step of the account that the prediction names, and by the account none of them will find it. Where the account predicts a positive result, as for entanglement through gravity, the result established by the rule of decision, with ordinary decoherence excluded, tests the step of the account that the prediction names, and the prediction states what that result will be. Each prediction names the step of the account that its observations test, what it separates the account from, and what it leaves together with the account. (The plain forms are in *Two Theories, One World*, Chapter 27, "What the World Will Show".)

II. Entanglement Through Gravity

Prediction 19.1. Two masses, each held in a superposition of two places and related only through gravity, become entangled, with the Newtonian phase. For the parameters of Bose and colleagues (2017), the entangling phase is 0.314 radians.

Each alternative of the matter carries its own classical geometry (Chapter 7, "One Geometry for Each Alternative"). So each joint alternative of the two masses has its own Newtonian energy, and its phase advances at the rate that energy sets. Let the masses be m, let d_{ij} be the separation of their centers when the first is in place i and the second in place

j, and let t be the time they are held. The phase of the joint alternative ij is G m^2 t/(ħ d_{ij}), and the entanglement is fixed by the one combination of the four phases that no change to the phases of one body alone can alter:

$$\Delta\varphi = (G\, m^2\, t \,/\, \hbar)\ (1/d_{nn} + 1/d_{ff} - 1/d_{nf} - 1/d_{fn}) \tag{19.1}$$

In words, the entangling phase is the sum of the Newtonian phases of the two alternatives in which both bodies are near or both are far, less the sum of the phases of the two mixed alternatives. For m = 10^{-14} kilograms, places 250 micrometers apart, centers 450 micrometers apart and t = 2.5 seconds, the separations are 200, 700 and 450 micrometers, and $\Delta\varphi$ = 0.314 radians.[1] The computation is the standard one (Bose et al. 2017; Marletto and Vedral 2017; Christodoulou and Rovelli 2019; Danielson, Satishchandran and Wald 2022), and Chapter 8, "The Entanglement Test", sets it out with its budget of decoherence.

Present evidence. Page and Geilker (1981) set the masses of a Cavendish balance by a quantum random event, and the measured field followed the outcome, not the average over outcomes. That result excludes one geometry sourced by the average over the alternatives of a state in which no outcome is ever selected, and it agrees with one geometry for each alternative. Westphal and colleagues (2021) measured the gravitational coupling between gold spheres of about 90 milligrams, which extends gravitational measurement toward small sources. Fuchs and colleagues (2024) measured the gravitational pull of kilogram masses about half a meter away on a magnetically levitated particle of 0.43 milligrams, a signal of attonewtons, which carries the measurement of gravity to test masses below a milligram.

Experiments under way. The masses of the proposals are far smaller still, and the designs work to keep every other coupling below gravity. Van de Kamp, Marshman, Bose and Mazumdar (2020) place a conducting plate between the two interferometers to screen the Casimir force, which allows masses of 10^{-16} to 10^{-15} kilograms in superpositions about 20 mi-

crometers wide. A white paper of 2026 sets out the environmental noise in the band from 1 to 10 hertz that the same test must suppress, chiefly relative acceleration noise and gravity-gradient noise (Bose et al. 2026).

What the outcome separates. In the theory of Møller and Rosenfeld, which I set out in Chapter 7 together with the literature on that theory, a single geometry is sourced by the expectation value of the matter. Such a single geometry is a classical channel and creates no entanglement. Gravity-induced collapse, in Penrose's estimate (Penrose 1996; see also Diósi 1987), makes the superpositions of such masses decay before the entanglement can build. Classical gravity coupled stochastically to quantum matter (Oppenheim 2023) creates no entanglement. Oppenheim, Sparaciari, Šoda and Weller-Davies (2023) show that every consistent coupling of this kind decoheres the quantum system and makes the classical one diffuse, with a trade-off between the two: the longer the coherence of the matter, the stronger the diffusion of the metric. Classical gravity of that kind is therefore bounded from two sides, by the coherence that interferometers keep and by the precision of measurements of mass, and present bounds of both kinds already exclude part of its parameters. The outcome does not separate the account from linearized quantum gravity, which gives the same leading phase (Danielson, Satishchandran and Wald 2022). Christodoulou and Rovelli (2019) read the result as a superposition of geometries; their reading and one geometry for each alternative assign the same geometry to each alternative, and differ in whether that geometry is a quantum state of the arena. Marletto and Vedral (2017) read entanglement as showing that the mediator is not classical in their sense, and one geometry for each alternative meets their condition, since the geometry takes a different state in each alternative and the alternatives keep their relative phase. The result would agree with their conclusion as they state it. It would not show a quantum of the arena at any scale.

What decides it. An experiment with two such masses, isolated so that decoherence stays well below the entangling phase, decides the prediction by the rule of decision, and by the account the masses become entangled

with the phase of equation (19.1). The step of the account that this experiment tests is one geometry for each alternative of the matter (Proposition 7.1).

III. No Quantum of the Arena Itself

Prediction 19.2. Gravitation is not quantized. No effect will show the geometry carrying quantum degrees of freedom of its own, beyond those tied to the alternatives of the matter. A description of the small waves of the geometry by quanta holds below the Planck curvature, and its quanta are quanta of that description, never quanta of the arena.

Present evidence. The first gravitational wave detected matched the classical waveform of general relativity for the merger of two black holes (Abbott et al. 2016), and the effective field theory of gravity computes the low-energy effects of the small waves, in their description by quanta, without any quantum of the arena (Donoghue 1994). No graviton has been detected.

Experiments under way. Tobar, Manikandan, Beitel and Pikovski (2024) propose to register single gravitons from a passing gravitational wave by stimulated absorption in a massive quantum resonator. A detection agrees with the account. The same clicks would follow from a classical wave acting on a quantum detector (Carney, Domcke and Rodd 2024), and a count of such clicks as quanta of the wave is a count in the description's own terms.

What decides it. Two kinds of observation decide Prediction 19.2. The first kind of observation is the cross-section for the scattering of small waves of the geometry. The cross-section is computed in the description of the small waves by quanta, at energies where the corrections of the effect-

ive theory are large. By my account, no measured cross-section of the scattering that departs from the effective theory will agree with a calculation in which the metric is quantized beyond the effective theory. The second kind of observation is every search for an effect in which the geometry carries quantum degrees of freedom that belong to the geometry itself. One example of such an effect is a superposition of macroscopically different geometries that is not sourced by any superposition of the matter, and that is not a state of free small waves in the description of small waves by quanta. By my account, no such effect will be found. The entanglement of Prediction 19.1 is not such an effect, since its geometries are tied to the alternatives of the matter. The step of the account that these observations test is the category error of Chapter 6, "The Category Error": that the arena admits no quantum of its own (Proposition 6.3).

IV. The Equivalence Principle

Prediction 19.3. The Eötvös parameter is zero for every pair of materials, at every precision. Antimatter falls as matter does. Nothing shields gravity.

The Eötvös parameter is the one of equation (5.1): twice the difference of the free-fall accelerations of two bodies divided by their sum. The prediction follows from the identification of gravity as the context of physics (P18; Chapter 5, "The Content and Context Partition"): a context couples to everything, nothing shields it, and the mass it acts on is the mass that resists acceleration. General relativity holds the equality exact, and I give the reason for it. The account agrees with general relativity and not with theories with couplings that depend on composition, such as the string dilaton of Damour and Polyakov (1994), which predicts small violations.

Table 19.1 gives the present evidence, by kind of measurement.

Measurement	Bodies	Result
Rotating torsion balance (Schlamminger et al. 2008)	Beryllium and titanium	$\eta = (0.3 \pm 1.8) \times 10^{-13}$
Atom interferometry (Asenbaum et al. 2020)	Rubidium-85 and rubidium-87	$\eta = [1.6 \pm 1.8\ \text{(stat)} \pm 3.4\ \text{(syst)}] \times 10^{-12}$
Satellite in orbit, MICROSCOPE (Touboul et al. 2022)	Titanium and platinum	$\eta = [-1.5 \pm 2.3\ \text{(stat)} \pm 1.5\ \text{(syst)}] \times 10^{-15}$
Antihydrogen released from a trap, ALPHA-g (Anderson et al. 2023)	Antihydrogen	Falls toward the Earth; repulsion excluded

Experiments under way. A proposed satellite test, STEP, aims at one part in 10^{18} (Overduin, Everitt, Worden and Mester 2012), three orders of magnitude beyond MICROSCOPE.

What decides it. Three kinds of observation decide Prediction 19.3: measurements of the Eötvös parameter for pairs of materials, each result established by the rule of decision; measurements of the free fall of antimatter; and searches for a gravitational shield. By the account, the Eötvös parameter will be found zero for every pair of materials, antimatter will fall with the acceleration of matter, and no shield will be found. The step of the account that these observations test is the identification of gravity as the context of physics (P18). Prediction 19.3 is the sharpest of the predictions, since I predict no violation at any precision.[2]

V. Smoothness and Lorentz Invariance

Prediction 19.4. The arena has no least length, area or duration. The speed of light in vacuum does not depend on the energy of the light through any step of spacetime. No discrete spectrum of area or volume will be observed. Gravitational waves travel at the speed of light with the two polarizations of general relativity.

A step of spacetime would generally change the relation between energy and momentum of light at high energy. In the usual parametrization the speed of a photon of energy E differs from c by a term of order $(E/E_{QG,n})^n$, for n = 1 or 2. Photons of different energies from one distant source then arrive at different times, with a delay that grows with the energy difference and with the distance traveled. $E_{QG,n}$, the quantum-gravity energy scale, is the energy at which that term would become of order one, and the bounds below are stated against the Planck energy, $E_{Pl} = 1.22 \times 10^{19}$ GeV.

Table 19.2 gives the present evidence.

Observation	Bound
Fermi, GRB 090510, a photon of 31 GeV (Abdo et al. 2009)	$E_{QG,1} > 1.2\ E_{Pl}$
Fermi, bursts including GRB 090510, subluminal case (Vasileiou et al. 2013)	$E_{QG,1} > 7.6\ E_{Pl}$; $E_{QG,2} > 1.3 \times 10^{11}$ GeV
LHAASO, GRB 221009A (Cao et al. 2024)	$E_{QG,1} > 10\ E_{Pl}$; $E_{QG,2} > 6 \times 10^{-8}$ E_{Pl}
LHAASO, photons of the highest energies, superluminal case (Cao et al. 2022)	first order above about $10^{5}\ E_{Pl}$; second order above about $10^{-3}\ E_{Pl}$
GW170817 and GRB 170817A (Abbott et al. 2017)	$(v_{gw} - c)/c$ between -3×10^{-15} and $+7 \times 10^{-16}$

The first-order bounds already lie above the Planck energy, where the simplest discrete pictures of spacetime place the effect. These timing bounds test the discrete pictures that predict a dispersion. A discreteness that keeps Lorentz invariance, as in causal sets (Bombelli, Henson and Sorkin 2009), predicts no such dispersion and is tested by its own signatures. The account predicts that no observation will show a least length, area or duration, whatever its relation to Lorentz invariance.

Light of every energy follows the light cone of the one geometry, and the account's ground for that uses no step of spacetime. By Proposition 5.3, the coupling of every content to the arena equals its inertia, and the ratio is the same for every content; Einstein's form of the equivalence principle follows (Chapter 5, Section IV), so the free path of a content through a geometry is set by the geometry and not by the mass or the energy of the content. For light the free path is the light cone of the geometry, and within one geometry the light cone is the same in every frame (Chapter 7, Section IV). A speed of light that depended on the energy of the light would give light of each energy its own free path through one geometry,

set by a property of the content, and the equal ratio of Proposition 5.3 excludes that, whether the dependence came from a step of spacetime or from some other source. One such source has been proposed: a deformed symmetry of special relativity that keeps, beside the speed of light, a second quantity the same for every observer, an energy of the order of the Planck energy, set into the symmetry itself and not into a step of spacetime (Magueijo and Smolin 2002); in some versions of that symmetry the speed of light depends on its energy, and on the ground just given those versions are excluded. Others have argued against them on grounds of their own. Hossenfelder (2010) showed that a speed of light that depends on energy and is the same for every observer brings effects that are not local, and that their absence in observation rules out such a modification to first order in the energy over the Planck mass. Bodies made of many particles, whose total energy exceeds the Planck energy, raise a second problem for these symmetries: "The most pressing problem in these models is the treatment of multi-particle states, known as the 'soccer-ball problem'" (Hossenfelder 2014). The timing bounds of Table 19.2 test a deformed symmetry and a step of spacetime alike.

Experiments under way. LHAASO continues to record photons of the highest energies, and every new burst and every new source at greater distance tightens the bounds. Detectors of gravitational waves test the polarizations and the speed of each new signal.

What decides it. Four kinds of observation decide Prediction 19.4. The first kind of observation is the timing of photons of different energies from many sources. Each timing result is established by the rule of decision. A dependence of the speed of light in vacuum on energy is told apart from effects at the source by its growth with distance across many sources, and by the account the arrival times will show no such dependence. The second kind of observation is every observation of the spectrum of area or volume, and by my account no such observation will show a discrete spectrum. The third kind of observation is every experiment that looks for a least length, area or duration in the geometry, and by my account no such experiment will find a least length, area or duration. The fourth kind

of observation is the measurement of the polarizations and the speed of gravitational waves. By my account, every gravitational wave will carry the two polarizations of general relativity and travel at c. General relativity says the same of gravitational waves, so the fourth kind of observation tests general relativity as well. The step of the account that these observations test is the category error of Chapter 6: that the arena is not quantized and has no least step (Proposition 6.3).[3]

VI. The Limits of Reconstruction in AdS/CFT

Prediction 19.5. In the AdS/CFT correspondence, every reconstruction of the smooth bulk geometry and its local fields from boundary data holds within a restricted domain: a subspace of states, a wedge of the bulk, an expansion in 1/N. No construction will give an exact map, independent of the state, from every boundary state to a smooth bulk geometry and from every local observable of that geometry to a boundary operator.

The prediction follows from reading the correspondence as a map between conjugate descriptions, each of which keeps what the other leaves out (Chapter 15, "AdS/CFT as Cross-Modal Correspondence").

Present evidence. Local bulk operators are reconstructed from smeared boundary operators at large N (Hamilton, Kabat, Lifschytz and Lowe 2006). They are logical operators of a quantum error-correcting code on a subspace of states, and one bulk operator is represented by different boundary operators on different boundary regions (Almheiri, Dong and Harlow 2015). The reconstructible region of a boundary region is its entanglement wedge (Dong, Harlow and Wall 2016). Operators behind a black-hole horizon are reconstructed by maps that depend on the state (Papadodimas and Raju 2013). The area formula of Ryu and Takayanagi

(2006) needs corrections at each order beyond the leading one (Faulkner, Lewkowycz and Maldacena 2013; Engelhardt and Wall 2015). States with geometric duals satisfy a monogamy of mutual information that general quantum states violate (Hayden, Headrick and Maloney 2013).

Experiments under way. The test is theoretical, and its resolution depends on progress in mathematical physics.

What decides it. Theorems at finite N decide Prediction 19.5. By the account, no theorem at finite N will give an exact, state-independent map from every boundary state to a smooth bulk geometry and from every local bulk observable to a boundary operator, with no restriction on the observables. The step of the account that these theorems test is the reading of the correspondence as a map between conjugate descriptions (Propositions 15.4 and 15.5).

VII. What the Predictions Separate

Table 19.3 sets each prediction beside the accounts it separates from the account of this book and the accounts it leaves together with that account.

Prediction	Separates the account from	Leaves the account together with
19.1 Entanglement through gravity	One averaged geometry; gravity-induced collapse; classical gravity with stochastic coupling	Linearized quantum gravity
19.2 No quantum of the arena	Accounts in which the geometry carries quantum degrees of freedom of its own	The effective field theory of gravity below the Planck curvature
19.3 The equivalence principle	Theories with composition-dependent couplings	General relativity
19.4 Smoothness and Lorentz invariance	Discrete pictures with a dispersion of light; physical area spectra	General relativity
19.5 Limits of reconstruction	The reading of the boundary theory as the complete description of a quantized bulk geometry	The dictionary as it is used within its domain

Of the five, the equivalence principle gives the sharpest test now, since its bound can still be improved by orders of magnitude and the account predicts zero. Entanglement through gravity gives the test now under construction, and it decides between one geometry for each alternative and every account that keeps one geometry for all of them.

VIII. The Argument and Its Tests

The two established theories describe the two sides of every comparison: quantum theory the content, discrete and symmetric in what it finds; general relativity the context, continuous and asymmetric in what it finds. To quantize the arena is to ask the context to carry the character of the content: quantized, the arena would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both. What remains when the error is removed is quantum content with one classical geometry for each of its alternatives, and the two descriptions meet at the Planck scale, where neither dominates. That configuration is the joined description of Chapter 16, “Common Source, Not Merger”, and the five predictions of this chapter are where the world will decide it.

Appendix A

The Numbers

This appendix lists every number the book prints, with its value, the calculation or source that gives it, and the chapters that use it. The computed values follow from the stated inputs with the constants of CODATA 2022 (Mohr, Newell, Taylor and Tiesinga 2025), $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $\hbar = 1.054571817 \times 10^{-34}$ J s, $c = 2.99792458 \times 10^{8}$ m s^{-1} and $k_B = 1.380649 \times 10^{-23}$ J K^{-1}. The measured values are those of the works cited, as their authors report them. Each table is followed by its calculations in words.

I. Newton’s Constant and the Planck Scale

Table A.1 gives the measured constant and the Planck quantities that follow from it.

Quantity	Value	Source or calculation	Chapters
Newton’s constant G	$6.67430(15) \times 10^{-11}$ m^3 kg^{-1} s^{-2}	CODATA 2022 recommended value	8, 9, 19
Planck mass M_P	2.176×10^{-8} kg (also printed 2.18×10^{-8} kg)	$(\hbar c/G)^{1/2}$	6, 9, 18
Planck length l_P	1.616×10^{-35} m (also printed 1.6 and 1.62×10^{-35} m)	$(\hbar G/c^3)^{1/2}$	6, 9, 18
Planck time t_P	5.391×10^{-44} s	l_P/c	9
Planck energy E_{Pl}	1.22×10^{19} GeV	$M_P c^2$	9, 19
Ratio $G\hbar/(r^2c^3)$ at one meter, which the quantum correction multiplies by $41/10\pi$	2.6×10^{-70}	$(l_P/1\text{ m})^2$	9, 18
Coefficient of that correction	$41/10\pi$, about 1.3	Bjerrum-Bohr, Donoghue and Holstein (2003), equation (44)	9

The Planck mass is the one mass whose gravitational size, Gm/c^2, equals its quantum size, $\hbar/(mc)$. Setting the two equal gives $m^2 = \hbar c/G$, and the square root is 2.17643×10^{-8} kg. The common size at that mass is the Planck length, the square root of $\hbar G/c^3$, which is 1.61626×10^{-35} m.

Light crosses the Planck length in the Planck time, 5.39125×10^{-44} s. The Planck energy is the Planck mass times the square of the speed of light, 1.956×10^{9} J, which is 1.22×10^{19} GeV.

The quantum correction to the potential between two masses has the relative size $G\hbar/(r^2c^3)$, which is the square of the Planck length divided by the square of the separation. At a separation of one meter the square of the Planck length in meters is 2.61×10^{-70}. The full correction carries the further coefficient $41/10\pi$, which is 1.305.

II. The Equivalence Principle and the Absence of a Shield

Table A.2 gives the measured values that check the four markers of Chapter 5 and Prediction 19.3.

Quantity	Value	Source	Chapters
Post-Newtonian parameter, $\gamma - 1$	$(2.1 \pm 2.3) \times 10^{-5}$	Bertotti, Iess and Tortora (2003), Cassini	5
Free fall of antihydrogen, a_g	$(0.75 \pm 0.13 \pm 0.16)$ g	Anderson et al. (2023), ALPHA-g	5, 19
Eötvös parameter, beryllium and titanium	$(0.3 \pm 1.8) \times 10^{-13}$	Schlamminger et al. (2008)	5, 19
Eötvös parameter, rubidium-85 and rubidium-87	$[1.6 \pm 1.8\ (\text{stat}) \pm 3.4\ (\text{syst})] \times 10^{-12}$	Asenbaum et al. (2020)	19
Eötvös parameter, titanium and platinum	$[-1.5 \pm 2.3\ (\text{stat}) \pm 1.5\ (\text{syst})] \times 10^{-15}$, "a few parts in 10^{15}"	Touboul et al. (2022), MICROSCOPE	5, 19
Aim of the proposed STEP satellite	one part in 10^{18}	Overduin, Everitt, Worden and Mester (2012)	19
Majorana absorption coefficient h	$(3 \pm 5) \times 10^{-23}\ \text{cm}^2\ \text{g}^{-1} = (3 \pm 5) \times 10^{-24}\ \text{m}^2\ \text{kg}^{-1}$	Williams, Turyshev and Boggs (2009), lunar laser ranging	5
Unit of electric charge in the labels of matter; unit of spin	one third of the electron's charge; one half	Established physics of particles	4, 5

The Eötvös parameter of two bodies is twice the difference of their free-fall accelerations divided by their sum (equation 5.1). The STEP aim of one part in 10^{18} lies three orders of magnitude beyond the MICROSCOPE bound, which is at the level of 10^{-15}. The absorption coefficient is converted from the units of the paper by the factor 1 $cm^2\ g^{-1} = 0.1\ m^2\ kg^{-1}$.

III. One Geometry for Each Alternative, and the Entanglement Test

Table A.3 gives the parameters of the proposal of Bose and colleagues (2017) and the values that follow from them.

Quantity	Value	Source or calculation	Chapters
Mass of each diamond m; time held t	10^{-14} kg; 2.5 s	Bose et al. (2017)	7, 8, 19
Distance between traps; distance between the two places of one diamond	450 μm; 250 μm	Bose et al. (2017)	7, 8, 19
Separations d_{nn}, $d_{nf} = d_{fn}$, d_{ff}	200, 450, 700 μm	450 − 250, 450, 450 + 250	8, 19
Prefactor $Gm^2t/\hbar$	1.582×10^{-4} m	from the inputs	8, 19
Phases φ_{nn}, φ_{nf}, φ_{ff}	0.791, 0.352, 0.226 rad	prefactor divided by each separation	8
Sum of reciprocal separations, 1/200 + 1/700 − 2/450	1.984×10^{-3} μm^{-1}, which is 1984 m^{-1}	arithmetic	19

Quantity	Value	Source or calculation	Chapters
Entangling phase $\Delta\varphi$	0.314 rad, about a twentieth of a full turn	equation (8.2): prefactor times the sum above	8, 19
Relative phases of Bose and colleagues, $\Delta\varphi_{RL}$ and $\Delta\varphi_{LR}$	0.440 and −0.126 rad (quoted by them as ∼0.7 and ∼−0.2)	prefactor times (1/200 − 1/450) and (1/700 − 1/450) per micrometer	8
Radius of the diamond	about 0.88 μm	$(3m/(4\pi\rho))^{1/3}$, with ρ = 3510 kg m^{-3}	7
Self-energy of the difference of the two placements, E_G	9.1×10^{-33} J	$Gm^2(6/(5R) - 1/b)$, with R the radius and b = 250 μm	7
Lifetime of the superposition in the Diósi–Penrose estimate, τ_{DP}	about 0.012 s, about a hundredth of a second	$\hbar/E_G$ (Diósi 1987; Penrose 1996), with the whole sphere as the smearing length	7, 8
Concurrence of the predicted state	0.156 (zero for unentangled spins, one at most)	$\lvert\sin(\Delta\varphi/2)\rvert$, from the four phases above	8
Witness of Chevalier, Paige and Kim on the predicted state	−0.22 (negative only for entangled spins)	expectation of $1 - \sigma_x\otimes\sigma_x - \sigma_z\otimes\sigma_y - \sigma_y\otimes\sigma_z$ (Chevalier, Paige and Kim 2020)	8

Each joint alternative of the two diamonds carries its own Newtonian interaction energy, $-Gm^2/d$, and its phase advances by $Gm^2t/(\hbar d)$ over the time t. For m = 10^{-14} kg and t = 2.5 s the prefactor $Gm^2t/\hbar$ is 1.58223 × 10^{-4} m. Divided by 200, 450 and 700 micrometers it gives the three phases 0.79111, 0.35161 and 0.22603 radians. The entangling phase is the phase of the two alternatives in which both diamonds are near or both far,

less the phase of the two mixed alternatives: $0.79111 + 0.22603 - 2 \times 0.35161$, which is 0.31393 radians. The same value is the prefactor times 1984 per meter. It is 0.050 of a full turn of 2π radians.

Bose and colleagues measure their phases from the phase of the mixed alternatives. Their two differences are the prefactor times $(1/200 - 1/450)$ per micrometer, which is 0.43951 radians, and the prefactor times $(1/700 - 1/450)$ per micrometer, which is -0.12557 radians; the two sum to the entangling phase, 0.31393 radians.

The predicted state is one half the sum, over the four joint alternatives, of $e^{i\varphi_{ab}}$ times the alternative, with the phases above. Its concurrence is twice the modulus of the product of the amplitudes on one diagonal less the product on the other, which is $|\sin(\Delta\varphi/2)|$, and for $\Delta\varphi = 0.31393$ radians it is 0.156. The witness of Chevalier, Paige and Kim is the expectation of one less the three correlations $\sigma_x \otimes \sigma_x$, $\sigma_z \otimes \sigma_y$ and $\sigma_y \otimes \sigma_z$, taken with the spin states labeled by the left and right places of each diamond, as Bose and colleagues label them. On the predicted state it is -0.2225, and a negative value is possible only for an entangled state.

A diamond of mass 10^{-14} kg and density 3510 kg m^{-3} has a radius of 0.879 micrometers. For two placements of a uniform sphere separated by more than its diameter, the gravitational self-energy of the difference of the two mass distributions is Gm^2 times $(6/(5R) - 1/b)$. With $R = 0.879$ μm and $b = 250$ μm it is 9.08×10^{-33} J. The reduced Planck constant divided by that energy is 0.0116 s. Leaving out the term in $1/b$, which is about three parts in a thousand of the first, gives 0.0116 s as well. Either way the lifetime is about 0.012 s, some two hundred times shorter than the 2.5 s the phase needs. The radius of the whole sphere is the most lenient choice of the smearing length: a smaller length gives a larger self-energy and a shorter lifetime. Conventions for E_G in the literature differ by a factor of two, and with either factor the lifetime stays at least a hundred times shorter than 2.5 s. Both remarks strengthen the conclusion that the superpositions would decay before the entanglement could build.

Table A.4 gives the values that the proposals state for their budgets of decoherence, and the related measurements.

Quantity	Value	Source	Chapters
Pressure; temperature	10^{-15} Pa; 0.15 K	Bose et al. (2017)	8
Duration of the protocol with acceleration and recombination	about 3.5 s	Bose et al. (2017)	8
Spin coherence time; temperature below which it holds	about 1 s; 77 K	Bose et al. (2017)	8
Casimir–Polder interaction at closest approach	a tenth of the gravitational one, at about 200 μm	Bose et al. (2017)	8
Witness W as Bose and colleagues estimate it; threshold	about 1.16; 1	Bose et al. (2017), from their rounded phases of about 0.7 and −0.2	8
Masses; size of superposition; field gradient, with a screening plate	10^{-16} to 10^{-15} kg; about 20 μm; 10^{4} T m^{-1}	van de Kamp, Marshman, Bose and Mazumdar (2020)	8, 19
Frequency band of the environmental noise that the white paper addresses	1 to 10 Hz	Bose et al. (2026)	19
Mass of the gold spheres whose gravitational coupling was measured	about 90 mg	Westphal et al. (2021)	19
	0.43 mg; attonewtons	Fuchs et al. (2024)	19

Quantity	Value	Source	Chapters
Mass of the levitated particle on which the pull of kilogram masses was measured; size of the signal			

IV. Quantum Foundations

Table A.5 gives the numbers of Chapters 3 and 13.

Quantity	Value	Source	Chapters
Magnetic moment of the electron, g/2	1.001 159 652 180 59 (13), an uncertainty of 0.13 parts per trillion	Fan, Myers, Sukra and Gabrielse (2023)	3
Agreement of the prediction of quantum electrodynamics with that value	about one part in 10^{12}	Fan, Myers, Sukra and Gabrielse (2023)	3
Bound on the combination S for local models; quantum bound	2; $2\sqrt{2}$	Clauser, Horne, Shimony and Holt (1969); Tsirelson (1980)	13
Measured S, with electron spins 1.3 km apart	2.42 ± 0.20	Hensen et al. (2015)	13
Directions in the construction of Kochen and Specker; vectors in the construction of Cabello and colleagues	117 in three dimensions; 18 in four dimensions	Kochen and Specker (1967); Cabello, Estebaranz and García-Alcaine (1996)	13

V. The Programs

Table A.6 gives the numbers of Chapter 14.

Quantity	Value	Source	Chapters
Estimated number of flux vacua of string theory	about 10^{500}	Douglas (2004)	14
Higgs mass obtained with the fixed point of asymptotic safety	about 126 GeV	Shaposhnikov and Wetterich (2010)	14
Spectral dimension of causal dynamical triangulations	four at large scales, about two at short ones	Ambjørn, Jurkiewicz and Loll (2005)	14

VI. Black Holes and the Page Curve

Table A.7 gives the numbers of Chapter 17.

Quantity	Value	Source or calculation	Chapters
Hawking temperature of one solar mass	6.2×10^{-8} K (6.17×10^{-8} K in the notes)	$\hbar c^3/(8\pi G M k_B)$, M = 1.98892 × 10^{30} kg	17
Evaporation time of one solar mass, photons only	2.1×10^{67} years	$5120\pi G^2 M^3/(\hbar c^4)$	17
Holes formed from stars, several solar masses or more	below 6.2×10^{-8} K; longer than 10^{67} years	temperature falls as 1/M; time grows as M^3	17
Shortfall of the average entropy of the smaller part of a random pure state	within half a nat of ln m, the average being about ln m − m/(2n)	Page (1993a)	17
Bound from the generalized second law: r, S_{BH}/S_0 at the turning point, τ_P	1; 0.500; 0.646	equation (17.5) with r = 1	17
Black-body emission into empty space	4/3; 0.571; 0.568	equation (17.5) with r = 4/3 (Zurek 1982)	17
Photons and the small waves of the geometry, with grey-body factors	1.4847; 0.598 (0.5975); 0.538	equation (17.5) with r = 1.4847 (Page 2013)	17
		Page (2013)	17

Quantity	Value	Source or calculation	Chapters
Peak of the entropy of the radiation; entropy lost by then; entropy remaining	53.81 percent of the evaporation time; 40.25 percent; 59.75 percent		

The Hawking temperature of a hole of one solar mass, 1.98892×10^{30} kg, is $\hbar c^3$ divided by $8\pi G M k_B$, which is 6.17×10^{-8} K. Its evaporation time with the emission of photons only is $5120\pi G^2 M^3$ divided by $\hbar c^4$, which is 2.10×10^{67} years. The temperature falls in inverse proportion to the mass and the time grows as the cube of the mass, so a hole of several solar masses is colder than 6.2×10^{-8} K and lives longer than 10^{67} years.

The turning point of the Page curve comes where the entropy carried away, r times the entropy the hole has lost, equals the entropy the hole still holds. The hole then keeps the fraction $r/(1 + r)$ of its initial entropy. The entropy of the hole falls as the two-thirds power of the fraction of the evaporation time that remains, so the turning point comes at the fraction 1 less $[r/(1 + r)]^{3/2}$ of the evaporation time. For $r = 1$ the fraction kept is 0.5, and the turning point comes at $1 - 0.5^{3/2}$, which is 0.646. For $r = 4/3$ the fraction kept is 4/7, which is 0.571, and the turning point comes at 0.568. For $r = 1.4847$ the fraction kept is 0.5975, and the turning point comes at 0.538. Page (2013) reports a peak after 53.81 percent of the evaporation time and a loss of 40.25 percent of the initial entropy, so that 59.75 percent remains, and both figures agree with the formula at his value of r.

VII. Smoothness and Lorentz Invariance

Table A.8 gives the bounds of Prediction 19.4, each stated against the Planck energy of Table A.1.

Observation	Value	Source	Chapters
GRB 090510, photon energy; first-order bound	31 GeV; $E_{QG,1} > 1.2\ E_{Pl}$	Abdo et al. (2009)	19
Fermi bursts, subluminal case	$E_{QG,1} > 7.6\ E_{Pl}$; $E_{QG,2} > 1.3 \times 10^{11}$ GeV	Vasileiou et al. (2013)	19
GRB 221009A	$E_{QG,1} > 10\ E_{Pl}$; $E_{QG,2} > 6 \times 10^{-8}\ E_{Pl}$	Cao et al. (2024)	19
Photons of the highest energies, superluminal case	first order above about $10^{5}\ E_{Pl}$; second order above about $10^{-3}\ E_{Pl}$	Cao et al. (2022)	19
Speed of gravitational waves, $(v_{gw} - c)/c$	between -3×10^{-15} and $+7 \times 10^{-16}$	Abbott et al. (2017)	19
Rule of decision	five standard deviations, by at least two independent kinds of measurement	the standard of this book for every prediction	18, 19

The Ten Cases

This appendix lists the ten candidate conjunctions of Chapter 1, Section VI, with the verdict of each of the three criteria, so that each row can be checked and ticked.

The four compound concepts of P8, each a pair of verdicts, with the subjective context held the same:

Compound	Verdict on content	Verdict on context
continuity	same	same
discontinuity	different	same
symmetry	same	different
asymmetry	different	different

The first criterion is no self-application: a compound concept applied to itself adds nothing to what it already says. The second criterion is no opposite pairing: a compound joined with its opposite asserts that the content is both the same and different in the same context, or both the same and different under the same change of context. The third criterion

is the balance of sameness and difference: by P4, among the verdicts on the content sameness must appear with difference, and among the verdicts on the context sameness must appear with difference.

The ten candidate conjunctions are four self-pairings and six pairings of distinct compounds:

Conjunction	**Verdicts on content**	**Verdicts on context**	**No self-application**	**No opposite pairing**	**Balance of sameness and difference**	**Verdict**
1. Continuity with asymmetry	same, different	same, different	passes	passes	passes	survives
2. Symmetry with discontinuity	same, different	different, same	passes	passes	passes	survives
3. Continuity with symmetry	same, same	same, different	passes	passes	fails: no difference of content	forbidden
4. Discontinuity with asymmetry	different, different	same, different	passes	passes	fails: no sameness of content	forbidden
5. Continuity with discontinuity	same, different	same, same	passes	fails	fails: no difference of context	removed

Conjunction	Verdicts on content	Verdicts on context	No self-application	No opposite pairing	Balance of sameness and difference	Verdict
6. Symmetry with asymmetry	same, different	different, different	passes	fails	fails: no sameness of context	removed
7. Continuity with continuity	same, same	same, same	fails	passes	fails in both respects	removed
8. Discontinuity with discontinuity	different, different	same, same	fails	passes	fails in both respects	removed
9. Symmetry with symmetry	same, same	different, different	fails	passes	fails in both respects	removed
10. Asymmetry with asymmetry	different, different	different, different	fails	passes	fails in both respects	removed

The two survivors are continuity with asymmetry and symmetry with discontinuity. Each passes all three criteria, and in each the second compound is the exact complement of the first. By P9, any comparison perfected at its foundation is continuously asymmetric or discontinuously symmetric.

The two forbidden pairings are continuity with symmetry and discontinuity with asymmetry. Each passes the first two criteria and fails only the balance criterion, and P9 names both.

The other six are removed by the first or second criterion: the four self-pairings by no self-application, and the two opposite pairings by no opposite pairing. The balance criterion fails all six as well, so inseparability alone, applied in each respect, leaves the same two survivors. The table of tertiary conjunctions in *An Immanent Metaphysics*, aphorism 1R17, marks the two forbidden pairings [!z], the two opposite pairings [!y] and the four self-pairings [!x], in agreement with the rows above.

Appendix C

The Aphorisms Cited

Here I give verbatim the aphorisms of *An Immanent Metaphysics* that I cite in this book, in the order in which the aphorisms stand in *An Immanent Metaphysics*.

1.2-6. The three modalities are the three terms used to refer to each of the three primal roles that form the essential basis necessary to the consideration of each domain. To consider the modality of a concept is to consider the essential role that concept plays with respect to other primal concepts.
I cite aphorism 1.2-6 in Chapter 2, in P10.

1.3-1. Where the foundations of any domain can be subsumed by three necessary and sufficient concepts, known as the modalities, so too does the relationships between these three have a consistent pattern and form. This form/pattern, common to the foundations of all domains/worlds, is described by the three Axioms.
All of the descriptive power of this metaphysics ultimately descends from the Axioms.

> **Axiom I**: The immanent is more fundamental than the omniscient and/or the transcendent.
> The omniscient and the transcendent are in conjugate relationship to one another.
>
> **Axiom II**: A class of the transcendent
> will precede an instance of the immanent.
> A class of the immanent will precede
> an instance of the omniscient.
> A class of the omniscient will precede
> an instance of the transcendent.
>
> **Axiom III**: The (classes/instances) of the immanent, omniscient, and transcendent are distinct,
> inseparable, and non-interchangeable.

I cite the aphorism above, which states Axioms I, II and III, in Chapters 2, 6, 9, 10, 11, 13, 14, 15, 17 and 18.

1.42-5. The Intrinsics of Comparison: The six concepts of sameness, difference, content, context, objectivity, and subjectivity, when taken together as a group.
I cite aphorism 1.42-5 in Chapters 1 and 18.

1.42-4. The Root Tautology: Comparison is isomorphic with interaction and relation.
The concept of comparison is isomorphic with the concept of interaction and the concept of relation.
All relations are interactions and are comparisons. All comparisons are interactions and are relations.
I cite aphorism 1.42-4 in Chapters 1, 16 and 18.

1.83-2. *Parallel Aspect; 1st:* That which is external to the boundary of self, which is objective (has the nature of existence), also has the nature of the omniscient modality. As such, the basis of the lawfulness of that which is the content of perception will be defined in terms of symmetry. The deep lawfulness of the content of perceptions will be defined by the nature of symmetry laws.
I cite aphorism 1.83-2 in Chapters 2 and 5.

1.83-3. *Parallel Aspect; 2nd:* That which is internal to the boundary of self, which is subjective (having the nature of creativity/ creation), also has the nature of the transcendent modality. As such, the basis for the lawfulness of that which is the context of perception will be defined in terms of continuity. The deep lawfulness of the context of perceptions will be defined by the nature of continuity laws.
I cite aphorism 1.83-3 in Chapters 2 and 5.

1.83-5. Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context.
I cite aphorism 1.83-5 in Chapters 1, 4, 5, 6, 7, 17, 18 and 19.

1.41-2. The Theory of Practice:

Axiom III (as the being of Foundational Triplication) is an assertion that there *are* three modalities, or three terms/concepts, which together subsume the essence of a domain, and which fill (or have) the roles of the modalities of immanent, omniscient, and transcendent. Axiom III does not in itself identify which domain essential concept is associated with which modality, but only that such associations are always strongly definable.

Axiom I (as the essence of the Practice of Theory) distinguishes the immanent modality from the omniscient and transcendent modalities. The immanent modality is that term which (in a context which is both objective and non-temporal) is used as the necessary basis for the definition/description of both the omniscient and the transcendent terms, and/or that which can only be (completely/accurately) defined/described in terms of both the omniscient and the transcendent terms. Axiom I has the effect of naming/identifying the term with an immanent modal association, even though it does not identify which of the two remaining terms are to be associated with the omniscient and transcendent modalities.

Axiom II (as the essence of the Theory of Practice) distinguishes the omniscient modality from the transcendent modality. The omniscient modal term is identified by being that term which in practice (in a context which is both subjective and temporal) necessarily follows from the being of the term with an immanent modal association (that term already identified by Axiom I). The transcendent modal term is identified by being that term which in practice necessarily precedes the being of the term with an immanent modal association.

I cite the aphorism above, on Axiom II as the essence of the Theory of Practice, in Chapter 2.

1R1. Where discussing the concept of comparison in the common usage terms (i.e., where holding the terms subjective and objective as implicit), it is apparent that exactly four other concepts are both necessary and sufficient for the formal consideration of the concept of comparison. These concepts are 'sameness', 'difference', 'content', and 'context'. Any process of comparison involves all four of these terms as intrinsics. Where there is comparison there must be sameness, difference, content, and context. No comparison can be defined without implicitly making reference to all six of these concepts. As such, the collection of these six abstract concepts will be hereafter referred to as 'the intrinsics of comparison'.
I cite aphorism 1R1 in Chapter 1.

1R2. In consideration of the meanings of the intrinsics of comparison, certain statements of inseparability are apparent. For example, it is clear that the concepts of sameness and difference are inseparable. Whenever one appears, the other is implicit. Where there is sameness, there must also be difference. Where there is difference, there must also be sameness.

1R3. Further, the concepts of content and context are inseparable. Where there is content, there must also be context. Where there is context there must also be content.
I cite aphorism 1R3 in Chapters 1, 4, 5, 7, 17 and 19.

1R4. This inseparability of the concepts of content and context, and the concepts of sameness and difference, is assumed as being fundamental to, and irrevocably inherent in, all discussions of the concept of comparison, or of anything that is defined in terms of comparison.
I cite aphorism 1R4 in Chapter 1, in P4.

1R5. Where the above concepts of comparison, sameness, difference, content, and context, are inseparable, it is also clear that they are distinct concepts with distinct meanings. For example, the meanings of these terms are not interchangeable in any formal statement, without changing or altering the fundamental meaning of that statement.

1R6. It is understood that similar aspects of distinctness and non-interchangeability will apply similarly to any terms that are defined in terms of these.

1R7. It is clear that some of the intrinsic concepts of comparison are applicable to one another. In particular, the concepts of sameness and difference can be applied to the concepts of content and context. Also, (the concepts of) content and context can be applied to (the concepts of) sameness or difference. As it turns out, there are four primary concepts which are formed from these specific applications (i.e., as second order compounds and conjugations).

1R8. Where there is a sameness of subjective context, the following definitions hold about the objective.

> **continuity** is a reference to a sameness of content where there is a sameness of context,
>
> **discontinuity** is a reference to a difference of content where there is a sameness of context,
>
> **symmetry** is a reference to a sameness of content where there is a difference of context,
>
> **asymmetry** is a reference to a difference of content where there is a difference of context,

I cite aphorism 1R8 in Chapter 1.

1R9. Clearly the concepts of symmetry and continuity are not the same, for 1) there are discrete and non-continuous things which are symmetric, and 2) twisted continuous things that are non symmetric. For this reason, it is valid to consider the formation and inter-applicability of these compound concepts of the intrinsics of comparison, and their opposites, to one another. However, in considering the mutual compatibility and applicability of the secondary compounds to form tertiary conceptual compounds, certain conflicts become immediately apparent.

1R10. For example, some of these tertiary compound candidates can be rejected since the duplicate application of the same concept to itself is of no semantic value. In this way four potential candidates are discarded from the list of potentially valid mutual applications.

1R11. Another of these conflicts concerns the mutual applicability of the types of comparison to one another in the terms of 'opposite meanings'. Some of the pairs of these concepts are mutually incompatible as they are clearly opposites of one another. For example, the concept of continuity with the concept of discontinuity asserts that for the same context, something is both identically the same as and identically different than itself. Clearly something (in this case content) cannot both be the same and be different in the same way at the same time.

1R12. In the same way that the formation of a single compound concept out of 'sameness of content' and 'difference of content' would be invalid as it violates concept distinctness (as defined earlier). A compound that assumes that the context is both the same and different violates distinctness. For example, it would be problematic to assert that for the same content, its context is both the same and different. Thus, there is the limitation (in the formation of compound concepts of the four types of comparison) that something cannot be both the same and different in the same way at the same time.

1R13. Insofar as there are two pairs (of the four concepts which are available) which are opposites to one another, then clearly they will not be mutually compatible or applicable. In this way two additional potential third compound candidates can be dropped from the acceptable list of mutual applications. Other potential candidates will also be dropped where they violate the requirement of distinctiveness.

1R14. In addition, due to the inseparability and distinctiveness of the intrinsic concepts of comparison themselves, the formation of the compounds of using these are subject to certain limits. In particular, one of these limits is 1) where the concept of content appears, the concept of context must appear also, and 2) where the concept of context appears, the concept of content must also appear. This, however, is not a problem as the definitions of the terms always ensures that there will be an even matching of a content with a context and a context with a content, wherever these appear.

1R15. However, in a very similar manner (as in the immediately above paragraph), two more candidates for tertiary compound concepts must be rejected. This is a more subtle limitation. In the same manner that the concepts of content and context are inseparable, conceptually distinct, and non-interchangeable, the concepts of sameness and difference are also inseparable, distinct, and non-interchangeable. In particular, where the concept of sameness appears, the concept of difference must appear also, and vise versa. The 'number of appearances' of each concept must match the number of appearances of its opposite.
I cite aphorism 1R15 in Chapter 1, in P4.

1R16. Fortunately, all of these limitations can be accounted for fairly easily by noting that when any one of the concepts in each of the concept axis domains appears, its mate must appear also. All that is needed to determine whether the third order compounds of the intrinsics of comparisons are valid is to check that all of the concepts 'add up' or 'cancel out' under the inseparability relationships. In this way, the four compounds formed by the four types of comparison can be checked rather readily.

1R17. In summary form, the above limitations could be organized into three basic sets: where [!x] indicates an invalid application to self, [!y] indicates an invalid mutual application of opposites, and where [!z] indicates an invalid assumption of the separability of sameness and difference. As such, the results of the consideration of all candidates for the formation of the third compounds of the intrinsics of comparison may be listed as follows.

Table of Tertiary Conjunctions:			
[!x]	continuity	and continuity	(ss & ss)
[!z]	continuity	and symmetry	(ss & sd)
[!y]	continuity	and discontinuity	(ss & ds)
	continuity	and asymmetry	(ss & dd)
[!x]	symmetry	and symmetry	(sd & sd)
	symmetry	and discontinuity	(sd & ds)
[!y]	symmetry	and asymmetry	(sd & dd)
[!x]	discontinuity	and discontinuity	(ds & ds)
[!z]	discontinuity	and asymmetry	(ds & dd)
[!x]	asymmetry	and asymmetry	(dd & dd)

I cite the aphorism above in Chapter 1.

1R18. From the above list, it is clear that there are only two remaining combinations which are mutually compatible; 1) continuity with asymmetry and 2) symmetry with discontinuity. All other combinations are, for various reasons, incompatible with one another.

1R19. Concept inseparability and the associations as defined, require that an eventity/comparison is either 'continuously asymmetric' **OR** it is 'discontinuously symmetric'. An eventity/comparison cannot be 'discontinuously asymmetric' nor 'continuously symmetric'. This constitutes the fundamental result of this essay, and is called the 'Incommensuration Theorem':

> *The concepts of symmetry and continuity cannot both be simultaneously and fundamentally applied to any eventity/comparison.*
>
> *The concepts of asymmetry and discontinuity cannot both be simultaneously and fundamentally applied to any eventity/comparison.*

1R20. Any absolute application of the concept of comparison must ultimately be continuous and asymmetric, **or** symmetric and discontinuous.
I cite aphorisms 1R1 to 1R20 together, as the enumeration and statement of the Incommensuration Theorem, in Chapters 1, 17, 18 and 19.

1.46-12. All that is qualia is intercompositional, has the nature of nonlocal context, and is understood in terms of continuity.
All that is quanta is interexclusionary, has the nature of local content, and is understood in terms of symmetry.
I cite aphorism 1.46-12 in Chapters 1, 4, 5, 11, 17 and 19.

1.76-6. The term 'existence' has the nature of the omniscient modality. The term 'reality' has the nature of the immanent modality. The term 'objectivity' has the nature of the transcendent modality. As per their counterparts, 'that which is (objective) being' has the nature of the transcendent, 'that which is real' has the nature of the immanent, and 'that which exists' has the nature of the omniscient. Existence is characterized in the terms of independence. Reality is characterized in the terms of interdependence. Objectivity is characterized in the terms of dependence. The concepts of dependence, independence, and interdependence are triple. They are distinct, inseparable, and non-interchangeable. Typically, dependence has the nature of the transcendent, interdependence has the nature of the immanent, and independence has the nature of the omniscient.

1.76-8. All perceptions (comparisons in themselves) regardless of their 'type', are simply real. The stable patterns of comparisons of perception are reality. All Interactions are Real. Reality (an experience of a world) is constructed out of consistent sets of comparisons of those interactions (consensus, objectivity). Only the content of that set of comparisons 'exists'. The context of those comparisons (the self), does not exist, although it is an inseparable aspect of all that is real.
I cite the two aphorisms above in Chapter 2, in P14.

Appendix D

Premises and Propositions

This appendix lists every premise of the book with its kind and its statement, and every proposition, definition, criterion, condition and prediction in book order with what the chapter states it rests on.

The premises are numbered P1 to P23 in the order in which they are stated, and the numbering holds throughout the book. A premise of IM ground is a statement of the Immanent Metaphysics, cited to *An Immanent Metaphysics* by aphorism number where the chapter gives one. A premise of established physics is a result of physics as it stands, cited to the literature. An identification is a stated step from the structure of comparison to a physical quantity, made in the first person, with the observation that bears on it. In the second table, the column "Rests on" gives the list that follows each proposition in its chapter; where an item has no "Rests on" list, the column gives what its chapter derives it from, and for a prediction it also names the step of the account that the chapter says its observations test. A number standing alone in that column is a premise (P) or a proposition of the book; equations and works cited are named as the chapter names them.

I. The Premises

The twenty-three premises, with the chapter and section in which each is stated, and, for P1 to P19, the chapter of the three volumes named in "What This Book Claims and How It Is Set Out", Section V, where each is set out at length:

Premise	Kind, where stated, and where set out at length	Statement
P1	IM ground; aphorism 1.42-4; Chapter 1, Section II; at length in Volume 1, Chapter 3; Volume 2, Chapter 5	The Root Tautology: "Comparison is isomorphic with interaction and relation."
P2	Identification; Chapter 1, Section II; at length in Volume 1, Chapter 3, Section V; Volume 2, Chapter 5	Every finding of physics is an act of comparison made from inside the world.
P3	IM ground; aphorisms 1.42-5 and 1R1; Chapter 1, Section III; at length in Volume 1, Chapter 4; Volume 2, Chapter 5	The concept of comparison involves six intrinsic concepts: sameness, difference, content, context, the subjective and the objective. The six are necessary and sufficient for the formal consideration of comparison.

Premise	Kind, where stated, and where set out at length	Statement
P4	IM ground; aphorisms 1R4 and 1R15; Chapter 1, Section III; at length in Volume 1, Chapter 4; Volume 2, Chapter 5	The six intrinsics form three pairs (sameness and difference; content and context; the subjective and the objective), and each pair is inseparable. Each pair is distinct from the other two, and no pair can stand in for another.
P5	IM ground; aphorism 1R3; Chapter 1, Section IV; at length in Volume 2, Chapter 5	The inseparability principle: "Where there is content, there must also be context. Where there is context there must also be content."
P6	IM ground; aphorism 1.83-5; Chapter 1, Section IV; at length in Volume 2, Chapter 5	The two laws: "Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context." Chapter 1, Section IV, gives its reason, which rests on P3, P4, P7 and P8.
P7	IM ground; aphorism 1.46-12; Chapter 1, Section IV; at length in Volume 2, Chapter 5	In the form the book uses: content is local and interexclusionary (two contents exclude one another from the same place), and is understood in terms of symmetry; context is non-local and intercompositional (contexts compose with one another), and is understood in terms of continuity.

Premise	Kind, where stated, and where set out at length	Statement
P8	IM ground; aphorism 1R8; Chapter 1, Section V; at length in Volume 2, Chapter 5	With the subjective context held the same, four compound concepts: continuity is a sameness of content where there is a sameness of context; discontinuity is a difference of content where there is a sameness of context; symmetry is a sameness of content where there is a difference of context; asymmetry is a difference of content where there is a difference of context.
P9	IM ground; Technical Appendix, aphorisms 1R1 to 1R20; Chapter 1, Section VII; at length in Volume 2, Chapters 5 and 6	The Incommensuration Theorem: symmetry and continuity cannot both be simultaneously and fundamentally applied to any event or comparison, and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, must be either continuously asymmetric or discontinuously symmetric.
P10	IM ground; aphorism 1.2-6; Chapter 2, Section I; at length in Volume 1, Chapter 5; Volume 2, Chapter 10	There are three modalities. The omniscient is the role of what is known, structural, pattern; the immanent is the role of what mediates, processual, the act itself; the transcendent is the role of what is potential, the stance and the possibility from which an act proceeds.
P11	IM ground; aphorism 1.41-2; Chapter 2, Section I; at length in Volume 1, Chapter 6; Volume 2, Chapter 10	"Axiom III does not in itself identify which domain essential concept is associated with which modality, but only that such associations are always strongly definable." The rest of P11, which the book takes as its own premise: which concept stands in which role depends on the focus, the context and frame of the argument being made.

Premise	Kind, where stated, and where set out at length	Statement
P12	IM ground; aphorism 1.3-1; Chapter 2, Section II (Axiom I is quoted again in Chapter 17, Section IV); at length in Volume 2, Chapters 8 and 10	Axiom I: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another." Axiom II: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent." Axiom III: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable."
P13	Identification; Chapter 2, Section III; at length in Volume 1, Chapter 6, Section II; Volume 2, Chapter 10, Section IV	In the focus of measurement, sameness and difference are immanent, content and context are omniscient, and the subjective and the objective are transcendent. In physics, the omniscient is identified with conservation laws, structure and the content of measurement; the immanent with interaction, force and the process of measurement; the transcendent with possibility, the space of configurations and the boundary conditions within which events occur.
P14	IM ground; aphorisms 1.76-6 and 1.76-8; Chapter 2, Section IV; at length in Volume 2, Chapter 10	To say that something exists is to say that it stands as structure: timeless, the same in every instance. To say that something is real is to say that it is given in an actual act of comparison: temporal, occurring now, with the future still open. A law exists; a measurement is real.

Premise	Kind, where stated, and where set out at length	Statement
P15	Established physics; Chapter 3, Section III; at length in Volume 3, Chapter 8, Section IV	The six theories of Table 3.1 (Newtonian mechanics, classical electrodynamics, general relativity, quantum mechanics, quantum field theory and thermodynamics) are established in their domains, with the features Table 3.1 lists. General relativity, with Einstein's field equations, and quantum theory, with quantum field theory, are taken as established physics throughout the book.
P16	Identification; Chapter 3, Section III; at length in Volume 3, Chapter 8, Section IV	The findings of quantum mechanics and of quantum field theory are discontinuously symmetric; the findings of Newtonian mechanics, classical electrodynamics, general relativity and thermodynamics are continuously asymmetric.
P17	Established physics (Penrose 1965; Hawking and Penrose 1970); Chapter 3, Section V; at length in Volume 3, Chapter 8, Section VII	Under the conditions of the singularity theorems, a spacetime that satisfies Einstein's field equations contains geodesics that cannot be extended.
P18	Identification; Chapter 5, Section VII; at length in Volume 3, Chapter 10, Section VII	Gravity is the context-modal phenomenon of physics, by the match of all four markers of Propositions 5.1 to 5.4. It is the one identification in the chain from the ground to the category error that experiment tests directly.

Premise	Kind, where stated, and where set out at length	Statement
P19	Identification; Chapter 10, Section II; at length in Volume 1, Chapter 6, Section V; Volume 3, Chapter 12, Section III	In the focus of time, geometric time is omniscient, event time is immanent and evolution time is transcendent. The one-way order of findings, the order of Axiom II, is the direction of time.
P20	Established physics (Hawking 1975); Chapter 17, Section I	Quantum fields on the classical geometry of a black hole radiate with a thermal spectrum, and to leading order on that geometry the state of the radiation outside the horizon is mixed.
P21	Established physics; Chapter 17, Section I	The hole loses mass to that radiation, and a hole that receives nothing loses its mass, down to the Planck mass, in a finite time.
P22	Established physics; Chapter 17, Section V	The entanglement entropy of the radiation cannot exceed its coarse-grained entropy.
P23	Established physics with an identification (Page 1993a); Chapter 17, Section V	The joint state is one of very many that no comparison from outside tells apart, and each counts alike; the equal weighting is identified as a symmetry. For a random pure state of a system of dimension mn with $m \leq n$, the average entropy of the part of dimension m is about $\ln m - m/(2n)$, within half a nat of its largest possible value.

II. Part I: The Ground as Premises

The propositions of Chapters 1 to 3 (in Chapter 1 I state premises only):

Number	Chapter, section	Statement	Rests on
Proposition 2.1	2, V	Every physical theory has two levels, its formalism, which exists, and its findings, which are real; the two levels are distinct, inseparable and non-interchangeable.	P1; P2; P10; P12; P13; P14
Proposition 2.2	2, V	The constitutive mode (of the findings) and the instrumental mode (of the formalism) of a physical theory are conjugate: continuous findings go with a symmetric formalism, and symmetric findings with a continuous formalism.	P8; P9; P12; P14; Proposition 2.1
Proposition 3.1 (the cross-modal principle)	3, II	Framework and content are always in conjugate modes, and the reach of the framework into its content is dense but never exhaustive.	P1; P9; P12; P14; Proposition 2.2
Proposition 3.2	3, V	Each theory meets a residual where the character of its findings exceeds what the mode of its formalism can carry: for quantum mechanics the measurement problem, for general relativity the singularity.	P9; P12; P16; P17; Proposition 3.1

III. Part II: The Diagnosis

The propositions of Chapters 4 to 6:

Number	Chapter, section	Statement	Rests on
Proposition 4.1	4, II	The findings of a content-modal phenomenon, perfected at their foundation, are discontinuously symmetric: classes with nothing between them, each label the same across every change of context.	P6; P8; P9
Proposition 4.2	4, III	The findings of a context-modal phenomenon, perfected at their foundation, are continuously asymmetric: smooth values with no gaps, and a generic finding differs from every other.	P6; P8; P9
Proposition 4.3	4, IV	The findings of content are discontinuously symmetric with a formalism of continuous structures; the findings of context are continuously asymmetric with a formalism of symmetry structures; these are the only two mode pairs of a fundamental description.	Propositions 4.1 and 4.2; Proposition 3.1; P3; P4; P6; P9

Number	Chapter, section	Statement	Rests on
Proposition 4.4	4, V	Quantization is a mode exchange: in one step the findings pass from continuously asymmetric to discontinuously symmetric, and the character the formalism holds exact passes from symmetry to continuity; the two levels are conjugate before the step and after it.	Proposition 3.1; the criterion of no self-application (Chapter 1, Section VI, within P9); Proposition 4.3
Proposition 5.1	5, II	A context-modal phenomenon couples to every content, without exception.	P5; the definition of a context-modal phenomenon (Chapter 4)
Proposition 5.2	5, III	No arrangement of content shields a region from a context-modal phenomenon.	P7; P5; the definition of shielding
Proposition 5.3	5, IV	For a context-modal phenomenon, the coupling of every content to it equals that content's inertia.	P3; P4; P5; Proposition 4.1; the definition of a context-modal phenomenon (Chapter 4); the identification stated in its argument
Proposition 5.4	5, V	A context-modal phenomenon has no discrete charge: the quantity to which it couples takes continuous values.	P6; Propositions 4.2 and 5.3
Proposition 5.5	5, VII	Physics contains a context-modal phenomenon, and that phenomenon has the four properties of Propositions 5.1 to 5.4.	P5; Propositions 4.1 and 5.1 to 5.4; P15

Number	Chapter, section	Statement	Rests on
Proposition 5.6	5, VII	Every measurement of the arena compares the arena with itself, so its verdict is a relation within the context, and the content it finds is always the content of some body or field.	P3; P18; the definition of a context-modal phenomenon (Chapter 4)
Proposition 5.7 (not used by the chain of the category error)	5, VIII	Where there is content, the context is uneven: it carries, continuously, the asymmetry that neither the comparing nor the content can carry.	P6; P8; P9; P12; the binding of the focus of the two laws (Table 2.1, following aphorisms 1.83-2 and 1.83-3); Proposition 2.2, for a conjunction split across two levels
Proposition 6.1	6, II	The arena admits no mode exchange.	P6; Propositions 4.2 and 4.4; P18; P9
Proposition 6.2 (the category error)	6, II	To quantize the arena is to apply to context the treatment that belongs to content, which is a category error: a question that applies a concept where the concept does not belong.	Propositions 4.4 and 6.1; P18
Proposition 6.3	6, IV	That the arena is not quantized means exactly two things: the arena has no least length, area or duration; and nothing is a quantum of the arena itself, at any scale.	P6; Propositions 4.2 and 6.1

Number	Chapter, section	Statement	Rests on
Proposition 6.4	6, V	The Incommensuration Theorem permits a common source, two descriptions joined side by side, each keeping its own character; it forbids a merger, one formalism in which the same directions of the arena carry both characters.	P9; P3; P4

IV. Part III: What Remains

The propositions of Chapters 7 to 9:

Number	Chapter, section	Statement	Rests on
Proposition 7.1 (per-alternative geometry)	7, III	Each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by the stress-energy of that alternative alone, and the relative phases of the alternatives are kept. The chapter calls it the book's identification, supported by Page and Geilker (1981).	P5; P7; P15; P18; and the step of its argument that contexts compose only where their contents are found together
Proposition 7.2	7, IV	A finding changes no geometry, here or anywhere; news of a distant finding enters a comparer's record only on or within the future light cone of the finding, in the geometry of the alternative it selects; no comparison registers a surface of simultaneity.	Proposition 7.1; P15; the no-signaling theorem (Eberhard 1978)

Number	Chapter, section	Statement	Rests on
Proposition 7.3	7, IV	One geometry sourced by the expectation value fails in both of its forms: without reduction of the state it fails the experiment of Page and Geilker; with reduction it signals faster than light.	The experiment of Page and Geilker (1981); Gisin (1990); Bahrami, Großardt, Donadi and Bassi (2014); P15
Proposition 8.1	8, II	Two masses, each held in two places and related only by gravity, become entangled with the entangling phase of equation (8.2), which is 0.314 rad for the parameters of Bose and colleagues (2017).	Proposition 7.1; the Newtonian Hamiltonian, equation (7.4); P15
Proposition 8.2	8, IV	The outcome of the experiment separates every account with one geometry for all the alternatives, and gravity-induced collapse, from every account in which each alternative has its own geometry; it does not separate per-alternative geometry from linearized quantum gravity.	Proposition 8.1; Table 8.1 and the accounts it cites; the lifetime of gravity-induced collapse, equation (7.5)
Proposition 9.1	9, I	Gravitation is not quantized. The effective field theory of gravity quantizes a description, the description of the small waves of a classical geometry about its background, which holds below the Planck curvature; its quanta are quanta of that description at most, and never quanta of the arena, of space-time or of the geometry.	Proposition 7.1; P15; Proposition 6.3

Number	Chapter, section	Statement	Rests on
Proposition 9.2	9, III	A detector that registers a gravitational wave in single clicks would not count against the account, and it would not show gravitation quantized. The clicks are steps of the detector, which is content; no detection of this kind can find a step of the arena itself, or a quantum of the arena at any scale.	Proposition 9.1; the result of Carney, Domcke and Rodd (2024); Proposition 6.3
Proposition 9.3	9, V	The Planck scale is where the two descriptions meet: an object's gravitational size equals its quantum size, neither description dominates, and both are relevant. The Planck length is not a least length, area or duration of the arena, and the smooth description holds below the Planck curvature.	The two sizes of a mass, equation (9.3); P9; the corrections of the effective field theory, equation (9.2)
Proposition 9.4	9, VI	The ground fixes the role of Newton's constant, the strength of the universal and unshieldable coupling of every content to its context, and not its magnitude, which is read off the world by measurement.	P18; P12, with Section VII (the axioms are structural, not dimensional)

V. Part IV: Measurement

The definitions and propositions of Chapters 10 to 13:

Number	Chapter, section	Statement	Rests on
Definition 10.1 (geometric time)	10, II	The structure of time that the metric carries: the proper time along each timelike worldline and the causal order of events. What is found of it is continuous and asymmetric, and it is identified as omniscient.	A definition; the text cites Proposition 7.1, P8 and Proposition 2.2; its identification is collected in P19
Definition 10.2 (event time)	10, II	The order of findings for a comparer: a discrete sequence of outcomes, each laid down as a record, ordered by precedence. What is found of it is discontinuously symmetric, and it is identified as immanent.	A definition; the text cites P16 and the order of Axiom II (P12); its identification is collected in P19
Definition 10.3 (evolution time)	10, II	The parameter t of the strongly continuous one-parameter group of unitary operators that carries the quantum state; it belongs to the continuous formalism, and it is identified as transcendent.	A definition; the theorem of Stone (1932); its identification is collected in P19
Proposition 10.1	10, III	Geometric time, event time and evolution time are distinct, inseparable and non-interchangeable.	P10; P11; P12 (Axiom III); P16; P19; Definitions 10.1 to 10.3; the theorems of Stone (1932) and of Pauli, as Busch (2008) states it

Number	Chapter, section	Statement	Rests on
Definition 10.4 (unified time)	10, V	One time that is geometric time, event time and evolution time together, with all three perfected in one formalism, none of them external and none primitive.	A definition
Proposition 10.2	10, V	No unified time exists: a time found both as geometric time and as event time would carry at least one of the two forbidden pairings, and a unified time, which asks all four characters of one time, would carry both.	P8; P9; Definitions 10.1, 10.2 and 10.4
Proposition 10.3	10, V	Any formalism that holds two of the three times natively holds the third only as external, as primitive, or not at all.	P12 (Axiom I); P19; Proposition 10.1 and Section IV
Proposition 11.1	11, II	The measurement problem is the residual of quantum mechanics: unitary evolution, in evolution time, approaches the finding as closely as wished and does not contain it; the finding belongs to event time.	P12 (Axiom I); P19; Propositions 3.1, 3.2 and 10.1
Proposition 11.2	11, III	Decoherence is the dense reach of the formalism toward the finding: it fixes the basis of the records and makes interference unobservable, and it does not select an outcome.	Propositions 11.1 and 10.1

Number	Chapter, section	Statement	Rests on
Proposition 11.3	11, IV	Given the Hilbert-space formalism, the Born rule is the only bridge from the formalism to the findings that assigns each element of a measurement the same probability in every measurement to which it belongs; that condition is identified as a symmetry of the bridge.	P15; the theorems of Gleason (1957) and Busch (2003); P8; P16
Proposition 12.1	12, I	An interpretation that drops one of the three times, or absorbs one into another, meets its characteristic difficulty where that time returns.	Proposition 10.1
Proposition 13.1 (the single-time result)	13, V	No interpretation of quantum mechanics that works within a single unified time is complete.	Propositions 10.1 and 10.2
Proposition 13.2	13, VI	Causation is established by findings; determinism is established by no finding, and a complete specification of the kind it requires, perfected in what is found, would carry a forbidden pairing.	P2; P8; P9; P12 (Axiom II); P14; Proposition 4.1

VI. Part V: The Programs

The criteria and propositions of Chapters 14 to 16:

Number	Chapter, section	Statement	Rests on
Criterion 1 (the partition)	14, II	Does the program treat gravity as the context of physics, or as one more content within it?	P18 and the four markers of Chapter 5, which the criterion states
Criterion 2 (the excluded pairing)	14, II	Does the program require one description whose findings, in the same respect, are both continuously asymmetric and discontinuously symmetric?	P9
Criterion 3 (what is quantized)	14, II	Which quanta does the program use? Matter on a classical geometry, a description of small waves by quanta (whose quanta are quanta of the description only) and sums over topological invariants are admitted; a quantum of the arena itself, at any scale, is excluded.	Chapter 7 (Proposition 7.1) and Chapter 9 (Proposition 9.1), as the criterion cites them
Criterion 4 (the predicted difficulty)	14, II	Is the difficulty that the program meets in practice of the kind its founding commitment predicts?	Proposition 14.1

Number	Chapter, section	Statement	Rests on
Proposition 14.1 (the three difficulties)	14, III	(a) A program that requires the arena to follow from a discontinuously symmetric framework meets a difficulty of multiplicity; (b) one that quantizes the arena directly meets a difficulty of dynamics; (c) one that replaces one description by a structure of the other character meets a difficulty of recovery.	P6; P9; Proposition 3.1; for part (b), Proposition 6.2
Proposition 15.1	15, II	The boundary theory has discontinuously symmetric findings and a continuous formalism; the bulk geometry has continuously asymmetric findings and a symmetric formalism; the two sides of AdS/CFT are descriptions in conjugate modes.	P16; Proposition 3.1
Proposition 15.2	15, III	The boundary description reaches the local geometric findings of the bulk densely and not exhaustively: exactly within a code subspace about a given geometry, in a manner that depends on the state behind a horizon, and with a count of local operators bounded at finite N.	Proposition 3.1; the results of reconstruction set out in Section III

Number	Chapter, section	Statement	Rests on
Proposition 15.3	15, IV	The formula of Ryu and Takayanagi, with its corrections, is a bridge: an exact, mediated and non-invertible relation between a symmetric quantity of the boundary and a continuous quantity of the bulk; it derives neither geometry from entanglement nor entanglement from geometry.	The derivation of Lewkowycz and Maldacena (2013); Proposition 14.1(a); the result of Harlow (2017)
Proposition 15.4	15, VI	Neither side of the correspondence is more fundamental than the other: each is a description in its own mode, each reaches the other densely, and each leaves a residual that the other contains.	The exactness of the duality (15.1) in both directions; Propositions 15.2 and 15.3, and Section V; P12 (Axiom I)
Proposition 15.5	15, VII	Any systematic correspondence between a discontinuously symmetric description and a continuously asymmetric one is a bridge between conjugate descriptions, and not a unification.	Proposition 3.1; P9
Proposition 16.1 (what is forbidden)	16, I; argument in Section II	No merger is possible: no single formalism can give the directions of the arena the discontinuous symmetry of content at the foundation.	P9; P4

Number	Chapter, section	Statement	Rests on
Proposition 16.2 (what is permitted)	16, I; argument in Section III	A common source is possible: two descriptions of opposite characters, each perfected in its own directions, joined side by side and related by a systematic translation; the inseparability principle requires both wherever either is.	P5; P9

VII. Part VI: Horizons, Objections and Tests

The premises of the paradox, the conditions and propositions of Chapter 17, and the predictions of Chapter 19 (in Chapter 18 I state objections and answers, and no proposition):

Number	Chapter, section	Statement	Rests on
(W)	17, I	Hawking's further premise, of the whole history (Hawking 1976): the classical geometry describes the whole history, its singularity included, so that what the field carries into the singularity is gone.	A premise of the paradox, not of the book; Proposition 17.2 places the loss it yields at the edge of the classical description's domain
(Q)	17, I	The premise of a quantum theory of gravity: geometry and matter together form one quantum state, whose evolution from before the collapse to after the evaporation is one unitary map.	A premise of the paradox, not of the book; Proposition 17.1 sets it aside

Number	Chapter, section	Statement	Rests on
Proposition 17.1	17, III	Premise (Q) applies the concept of a quantum state to the arena: it is the category error of Chapter 6.	P18; Propositions 4.4, 6.1 and 7.1
Proposition 17.2	17, III	The loss in Hawking's conclusion is located at the singularity, where the classical description reaches the edge of its domain; it is not a result of either description.	P15; premise (W); Proposition 9.3
Proposition 17.3	17, III	Information is not destroyed in a black hole.	Propositions 17.1 and 17.2
(C)	17, V	Condition: the whole formed by what fell in, the content inside the horizon and the radiation began pure and stays pure, as described from one outside world; the condition puts no geometry into the state.	Stated as a condition of the curve, not derived
(H)	17, V	Condition: the states of the hole that no comparison from outside tells apart number e^{S_{BH}/k_B}, so the entanglement entropy of the content inside the horizon cannot exceed S_{BH}.	Stated as a condition of the curve; it takes as the capacity of the hole the count by area that Section IV identifies (P10; P13; Axiom I of P12)
Proposition 17.4	17, V	Under (C), (H), P22 and P23, the entanglement entropy of the radiation is the smaller of the two capacities, S_{emit} and S_{BH}, to within half a nat.	(C); (H); P22; P23

Number	Chapter, section	Statement	Rests on
Proposition 17.5	17, V	Under (C), (H), P22 and P23, the turning point of the Page curve comes before the hole has lost half its entropy, and no later than 0.646 of the evaporation time.	Proposition 17.4; the law of evaporation, equation (17.4); the generalized second law (Bekenstein 1974)
Prediction 19.1	19, II	Two masses, each in a superposition of two places and related only through gravity, become entangled with the Newtonian phase, 0.314 rad for the parameters of Bose and colleagues (2017).	Chapters 7 and 8; the step tested: Proposition 7.1
Prediction 19.2	19, III	Gravitation is not quantized. No effect will show the geometry carrying quantum degrees of freedom of its own beyond those tied to the alternatives of the matter; a description of the small waves by quanta holds below the Planck curvature, and its quanta are quanta of that description, never quanta of the arena.	The step tested: the category error of Chapter 6, that the arena admits no quantum of its own (Proposition 6.3)
Prediction 19.3	19, IV	The Eötvös parameter is zero for every pair of materials at every precision; antimatter falls as matter does; nothing shields gravity.	P18; the step tested: P18
Prediction 19.4	19, V	The arena has no least length, area or duration; the speed of light does not depend on its energy through any step of spacetime; no discrete spectrum of area or volume will be observed; gravitational waves travel at the speed of light with the two polarizations of general relativity.	The step tested: the category error of Chapter 6, that the arena is not quantized and has no least step (Proposition 6.3)

Number	Chapter, section	Statement	Rests on
Prediction 19.5	19, VI	Every reconstruction of the smooth bulk geometry and its local fields from boundary data holds within a restricted domain, and no construction will give an exact map, independent of the state, from every boundary state to a smooth bulk geometry.	Chapter 15; the step tested: Propositions 15.4 and 15.5

Notes

Chapter 1

1. P1 does not say that two particles compare in the way an instrument compares. It says that what every comparison requires, the six intrinsics of P3, is required of every interaction, since any description of an interaction is a comparison and nothing else is available to physics.

2. The enumeration of the ten candidates and the three criteria is given in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20, with the table of conjunctions at aphorism 1R17. There the three criteria are written [!x], "an invalid application to self"; [!y], "an invalid mutual application of opposites"; and [!z], "an invalid assumption of the separability of sameness and difference". The table marks continuity with symmetry and discontinuity with asymmetry [!z], the two opposite pairings [!y], and the four self-pairings [!x], as Table 1.2 does.

3. The balance criterion can also be stated as a count. Across the four verdicts of a conjunction, a candidate that passes in each respect carries sameness twice and difference twice. Counting is itself made of sameness and difference, since it treats items as units of one kind and tells them apart, so the count is a later description of the pairing in each respect and adds nothing to it.

4. The name follows the old sense of incommensurable magnitudes, which have no common measure. Symmetry and continuity have no common perfection.

5. The theorem fixes the character that an exact description can have. It does not give the specific content of any theory, and so it derives neither general relativity nor quantum theory; this book takes both as established physics.

Chapter 2

1. Noether (1918) gives two theorems. The first assigns a conserved current to each continuous global symmetry of an action: time-translation invariance gives the conservation of energy, and invariance under rotations the conservation of angular momentum. The second concerns symmetries that depend on arbitrary functions, such as the gauge invariance of electrodynamics and the coordinate invariance of general relativity, and it gives identities among the field equations, the contracted Bianchi identities in the case of general relativity. Both theorems relate a symmetry of the formalism to what the theory finds, which is the relation between the two levels that Proposition 2.2 states.

Chapter 3

1. Each row of Table 3.1 reads a theory in its standard form. The symplectic structure of Newtonian mechanics in Hamiltonian form is preserved by canonical transformations, and Noether's first theorem ties each continuous symmetry of the action to a conserved quantity (Noether 1918). Callen (1985) sets out thermodynamics from postulates, with the fundamental relation homogeneous of degree one in the extensive variables, the potentials related by Legendre transformations, and the Maxwell relations among their second derivatives; his closing chapter places symmetry at the conceptual foundation of the subject. Von Neumann (1932) gives quantum mechanics its form in Hilbert space, with unitary evolution in continuous time.

2. Fan, Myers, Sukra and Gabrielse (2023) report the electron's magnetic moment in Bohr magnetons as $g/2 = 1.001\,159\,652\,180\,59\ (13)$, an uncertainty of 0.13 parts per trillion. The prediction of the Standard Model, which is the prediction of quantum electrodynamics with small further terms, agrees with it to about one part in 10^{12}; the comparison is limited by the disagreement among measured values of the fine-structure constant, on which the prediction depends.

Volume 3, Chapter 8, "The Cross-Modal Principle", carries this chapter further in two sections: Section V reads how a continuous group is held in each theory, in its formalism or in what it finds, and Section VI sets out the place of Noether's two theorems.

Chapter 4

1. Chapter 1 states the two laws as P6 beside the two principles that accompany them in *An Immanent Metaphysics*: the inseparability principle (P5, aphorism 1R3) and the characterization of content as local and inter-exclusionary and of context as non-local and intercompositional (P7, aphorism 1.46-12). Chapter 5 applies all three. The same two laws, with everyday examples of each, are set out for the general reader in *Two Theories, One World*, Chapter 7, "What Is Measured, and Where".

2. F. Peter and H. Weyl, "Die Vollständigkeit der primitiven Darstellungen einer geschlossenen kontinuierlichen Gruppe", *Mathematische Annalen* 97 (1927), 737–755. The discreteness belongs to compact groups. Section II of the chapter treats the Poincaré group, which is not compact, and the continuous labels of its representations.

3. Canonical, deformation and geometric quantization differ in how they build the quantum description from the classical one, and each produces a description whose formalism rests on continuous structures and whose findings are the spectra of operators. The mode exchange is the shape that all of them share at the two levels of a theory; it does not choose among them. The reach of the continuous formalism into the dis-

crete findings that it describes is dense and not exhaustive, and Chapter 11, “The Measurement Problem as a Cross-Modal Residual”, sets out what that reach leaves over in quantum theory.

Volume 3, Chapter 9, “The Mode Assignment”, adds two sections to the argument of this chapter: Section V, on spin as the smallest intrinsic circulation, and Section VI, on why the continuous spectra of quantum mechanics, of position, momentum and the energies of scattering states, belong to the context.

Chapter 5

1. The inseparability principle is An Immanent Metaphysics, aphorism 1R3, and the characterization of content and context is aphorism 1.46-12. The inseparability principle is a principle of the ground and is distinct from the three axioms; Chapter 1 states it beside the inseparability of each pair of intrinsics, and later chapters apply it in this form. With the two laws of aphorism 1.83-5, the Incommensuration Theorem and the distinctness of content and context, these principles are the ground of the four derivations of this chapter.

2. The plain form of both routes, the structural argument and the physical mechanism, is in *Two Theories, One World*, Chapter 9, “Four Marks of the Arena”. The coefficient of absorption reported by Williams, Turyshev and Boggs (2009), $h = (3 \pm 5) \times 10^{-23}$ cm^2 g^{-1}, is $(3 \pm 5) \times 10^{-24}$ m^2 kg^{-1} in SI units.

Volume 3, Chapter 10, “Context and Its Markers”, adds Section VI, on the three readings of inertia, and Section IX, on why the two characters cannot be merged into one geometry; its Section VIII carries Proposition 5.7 on to where, and how much, the arena curves.

Chapter 6

1. D. Carney, V. Domcke and N. L. Rodd (2024) show that the signal of a detector registering one quantum of a gravitational wave is also produced by a classical gravitational wave acting on the quantum detector, so that, on their analysis, such a detection is consistent with quantized radiation without establishing it. In the terms of this book, the clicks of a detector are findings of the detector, which is content, and their discreteness is the discreteness of that content. The plain form, with the phonon as the nearest sense of the word "graviton" and the reason it does not apply to the arena, is in *Two Theories, One World*, Chapter 14, "Waves Without Particles of Space".

2. An object of mass m has a gravitational radius G m / c^2 and a quantum wavelength ħ / (m c). The two are equal when m^2 = ħ c / G, at the Planck mass, 2.18 × 10^{-8} kg, and the common length there is the Planck length, the square root of ħ G / c^3, which is 1.616 × 10^{-35} m. Newton's constant G is measured. The ground fixes the role of the Planck scale as the meeting of the two descriptions, and not its size, because the axioms are structural and not dimensional. Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", sets out both points; the plain form is in *Two Theories, One World*, Chapter 15, "Where the Two Descriptions Meet".

3. The three words are those of Axiom III, in the wording of An Immanent Metaphysics, aphorism 1.3-1: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable." I notice that the same three words hold of content and context as intrinsics of comparison: they are distinct, since the content and the context cannot be one respect; inseparable, by the inseparability principle; and non-interchangeable, since neither can do the other's work in a comparison. The reading of the two theories as conjugates, "each captur-

ing what the other leaves out", rests on that. The plain form of conjugation, and of the unity the theorem permits, is in *Two Theories, One World*, Chapter 24, "Joined, Not Merged".

Chapter 7

1. Møller's contribution appeared in the proceedings edited by Lichnerowicz and Tonnelat, *Les Théories Relativistes de la Gravitation* (Colloques Internationaux du CNRS 91, Paris, 1962). Rosenfeld (1963), "On quantization of fields", *Nuclear Physics* 40, 353–356. Bahrami, Großardt, Donadi and Bassi (2014) trace the semi-classical equation to these two works and read Rosenfeld as holding that whether the gravitational field must be quantized is for experiment, not for theory, to decide. The present chapter agrees with Rosenfeld on that point, and Chapter 8 sets out the experiment that now decides between the configurations of Table 7.1.

2. Page and Geilker (1981), *Physical Review Letters* 47, 979–982. Their analysis takes the quantum state to be unreduced, in the manner of Everett, so that the two placements of the large masses remain as two branches after each run; Huggett, Linnemann and Schneider (2022) set out the protocol and note that the experiment excludes the semi-classical equation on that assumption, and not on a reading in which the state is reduced at each finding. The reduced form is the second form of Section I, and Section IV shows that it signals faster than light. The plain account of the experiment, with the torsion balance described step by step, is in *Two Theories, One World*, Chapter 12, "One Geometry for Each Alternative".

3. In each alternative the stress-energy $T^{(i)}$ is conserved on its own geometry $g^{(i)}$, which is what the Bianchi identity of $g^{(i)}$ requires of its source. The reduced form of equation (7.1) meets a further difficulty here: at a finding its source jumps, while the Bianchi identity requires the divergence of the source to vanish at every event. Per-alternative geometry has no jump to reconcile, since no source changes at a finding (Proposition 7.2). The step from the inseparability principle (P5) to one geometry for

each alternative uses the two laws of *An Immanent Metaphysics*, aphorism 1.83-5 (P6), only through the identification of the geometry as context: the law of continuity governs the geometry within each alternative, and the law of symmetry governs the content whose alternatives are counted.

4. Proposition 7.2 needs only that no comparison registers a surface of simultaneity and that the account uses none. It does not need the stronger principle that a difference which no comparison can register is no difference.

5. For a diamond of mass 10^{-14} kg (density 3510 kg m^{-3}, radius about 0.88 μm) held in two placements 250 μm apart, the self-energy of the difference of the two mass distributions is $E_G = Gm^2(6/(5R) - 1/b) \approx 9.1 \times 10^{-33}$ J, with R the radius and b the separation of the placements, and equation (7.5) gives $\tau_{DP} \approx 0.012$ s, about a hundredth of a second, against the 2.5 s of the protocol of Chapter 8. The value spreads the mass of each diamond over its whole sphere, which is the most lenient choice of smearing length: any smaller length raises E_G and shortens the lifetime. The numerical constant in E_G is set differently in the literature, and the conventions differ by a factor of two; Diósi (2022) fixes it at one half from the master equation. Under either convention the lifetime stays two orders of magnitude below the 2.5 s of the protocol. Donadi, Piscicchia, Curceanu, Diósi, Laubenstein and Bassi (2021), *Nature Physics* 17, 74–78, set their lower bound on the effective size of the nuclear mass density about three orders of magnitude above earlier bounds.

Chapter 8

1. Bose and colleagues (2017) write the phases as $\varphi_{RL} \sim Gm^2\tau/(\hbar(d - \Delta x))$, $\varphi_{LR} \sim Gm^2\tau/(\hbar(d + \Delta x))$ and $\varphi \sim Gm^2\tau/(\hbar d)$, with $d = 450$ μm, $\Delta x = 250$ μm and $\tau = 2.5$ s, and they quote the differences $\Delta\varphi_{RL} = \varphi_{RL} - \varphi$ and $\Delta\varphi_{LR} = \varphi_{LR} - \varphi$ to order of magnitude, as ~ 0.7 and ~ -0.2. For the stated parameters the exact Newtonian values are 0.440 rad and

-0.126 rad, and their sum, $\Delta\varphi_{RL} + \Delta\varphi_{LR}$, is the entangling phase of equation (8.2) in the notation of this chapter: 0.314 rad. Their estimate of the witness, $W \sim 1.16$, was made from the phases as they quote them; this chapter gives instead the concurrence and the witness of Chevalier, Paige and Kim (2020), computed on the state with the exact phases. The plain account of the experiment, with the four joint alternatives pictured as four hands drifting out of step, is in *Two Theories, One World*, Chapter 13, "The Experiment Being Built", which gives the phase as about a third of a radian for the design proposed in 2017.

2. Huggett, Linnemann and Schneider (2022) set out how Bose and colleagues derive the phases from linearized general relativity quantized, and how the proposals read the result. The coherent-state reading and per-alternative geometry assign the same field to each alternative: the Newtonian field of the masses in the places of that alternative. They differ in what they take that field to be a state of: a quantum field of the geometry, or the classical geometry that the alternative sources.

3. Marletto and Vedral (2017) state their conclusion as a claim about the mediator: a system that mediates entanglement must have non-commuting observables. The experiment works far below the Planck scale, where per-alternative geometry and linearized quantum gravity agree, so no result of it bears on the claim of Chapter 6, that the arena itself is not quantized.

Chapter 9

1. Donoghue (1994), "General relativity as an effective field theory: The leading quantum corrections", *Physical Review D* 50, 3874. Equation (9.2) is equation (44) of Bjerrum-Bohr, Donoghue and Holstein (2003), *Physical Review D* 67, 084033, written there with $\hbar = c = 1$; the factors of c are restored here. The quantum term is $G\hbar/(r^2c^3) = (l_P/r)^2$, and with $l_P = 1.616 \times 10^{-35}$ m its value at one meter is 2.6×10^{-70}, multiplied by the coefficient $41/10\pi$, about 1.3. The plain account of the small waves and of

their description by quanta, with the phonon as the nearest sense of the word "graviton" and the reason it does not make the graviton a quantum of the arena, is in *Two Theories, One World*, Chapter 14, "Waves Without Particles of Space".

2. Tobar, Manikandan, Beitel and Pikovski (2024), *Nature Communications* 15, 7229, present single-graviton signatures as a possible first experimental clue to the quantization of gravity, by analogy with the photoelectric effect for light. Carney, Domcke and Rodd (2024), *Physical Review D* 109, 044009, show that the same signal can be explained by a classical gravitational wave. The photoelectric effect, taken alone, is accounted for by a classical light wave acting on atoms whose bound states are quantized; the evidence that light itself is quantized comes from other measurements, of the statistics of the field. Proposition 9.2 needs only the first point: a click registers a quantum of exchange between a wave and a detector, and the account admits every such quantum.

3. 't Hooft and Veltman (1974), "One-loop divergencies in the theory of gravitation", *Annales de l'Institut Henri Poincaré, Section A* 20, 69–94; Goroff and Sagnotti (1986), "The ultraviolet behavior of Einstein gravity", *Nuclear Physics B* 266, 709–736. The gauge groups of the Standard Model are compact, and the charges of content come in discrete labels; the diffeomorphism group of a manifold is not compact, and in general relativity it acts as a redundancy of the formalism, not as a symmetry whose labels are found (Chapter 4, "What Quantization Is", sets out why symmetry governance yields discrete findings). That contrast ranks above non-renormalizability among the reasons that the metric does not take the step that the gauge fields take.

4. The reduced Planck mass, $M_P/(8\pi)^{1/2}$, is the natural unit of the coupling in equation (9.1), but the equality of the two sizes in equation (9.3) is met at the full Planck mass. The values of the Planck mass, length and time are the CODATA values (2018 and 2022 agree to the digits given). Read on the ground, the Planck length is the bound of every comparison made from a place inside the world: the length below which one

comparison cannot resolve a placement without the two descriptions becoming equally relevant to it. The plain account is in *Two Theories, One World*, Chapter 15, "Where the Two Descriptions Meet".

5. The value of G is the CODATA 2022 recommended value, as published by the National Institute of Standards and Technology; it is unchanged from the CODATA 2018 value. The three axioms are quoted in the wording of *An Immanent Metaphysics*, aphorism 1.3-1 (P12). Each of them states a relation among the three roles of a comparison, and none of them contains a unit; so no magnitude expressed in a unit follows from them alone, and the one measured input of this book is G.

Chapter 10

1. DeWitt (1967) sets out the canonical quantization of general relativity, in which the Hamiltonian constraint, applied to a wave functional of the spatial metric, becomes the equation now called the Wheeler–DeWitt equation. Isham (1993) surveys the problem of time that follows from it and classifies the proposed responses: those that identify a time before quantizing, those that identify one after quantizing, and those in which time plays no fundamental role. The first two classes identify a time by choosing some physical variable to serve as a clock, and that choice is the step at which one of the three times of this chapter is brought into the formalism from outside it.

2. Busch (2008) distinguishes external time, "measured by an external, detached laboratory clock"; intrinsic time, defined through the dynamical behavior of the quantum objects themselves; and observable time, also called event time, such as a time of arrival or a time of flight. His external time is the parameter that this chapter calls evolution time, read from a clock that the theory does not describe; his event time is a case of the event time of Definition 10.2.

3. Stone (1932) establishes that every strongly continuous one-parameter group of unitary operators on a Hilbert space has a unique self-adjoint generator, and that every self-adjoint operator generates such a group. The parameter of the group is a real number with the additive structure of equation 10.2; it is not an element of the Hilbert space and not a point of spacetime, which is the distinctness of Proposition 10.1 in its mathematical form.

4. The theorem is Pauli's; Busch (2008) states it in the form used here: the semi-boundedness of a Hamiltonian precludes a self-adjoint operator T that generates translations of its spectrum, which is what a time operator canonically conjugate to the Hamiltonian would do. Quantities such as the time of arrival of a particle at a detector are represented instead by positive operator-valued measures over the possible times of registration. Each such measure assigns probabilities to the times at which a finding is made, so it is a measure on event time, and the theorem leaves it untouched.

5. Page and Wootters (1983) take the whole system to be in an eigenstate of its total Hamiltonian, so that the whole does not change, and show that the state of the rest, conditioned on the reading of a clock subsystem, changes with that reading as if under Schrödinger's equation. The reading on which the state is conditioned is the outcome of a comparison of the clock, which is a finding in the sense of Definition 10.2. The construction is a precise account of how evolution time is read from within; in the terms of Proposition 10.3 it holds event time as primitive and evolution time as conditional on it.

Volume 3, Chapter 12, "Time", states the same P19, sets out the three times for the reader of the foundations, and adds the two arrows of time.

Chapter 11

1. Von Neumann (1932) numbers the two processes. Process 1 is the change at a measurement, which is irreversible and not causal; process 2 is the evolution by Schrödinger's equation, which is causal and reversible. His argument that the boundary may be placed at any link of the chain from the system to the observer is the source of the movable cut that Chapter 12, Section II, reads as a boundary between the two levels of the theory and not a place among objects.

2. Maudlin (1995) states the three propositions of the first problem in the words quoted. His second problem, of statistics, and his third, of effect, are stated there in the same form. He notes that additional-variable theories meet the problem of statistics by denying completeness, and that non-linear stochastic theories meet it by making the evolution itself random. The account of this chapter meets it in neither way: the evolution is linear and deterministic, and the outcome is selected in the act of comparison.

3. Zeh (1970) first set out the role of the environment in measurement; Zurek (1981) introduced the pointer basis; Joos and Zeh (1985) computed the suppression of interference by scattering of the environment; Zurek (2003) reviews the program, with einselection and the stability of pointer states. The tie to the ground is P7 (*An Immanent Metaphysics*, aphorism 1.46-12): "All that is qualia is intercompositional, has the nature of nonlocal context, and is understood in terms of continuity." The environment that carries many copies of a pointer reading is the intercompositional context of that reading.

4. Schlosshauer (2005) separates three parts of the measurement problem: the preferred basis, the non-observability of interference, and the problem of outcomes. Decoherence answers the first two in large part, and

it leaves the third, which every interpretation then answers in its own way. The improper mixture is d'Espagnat's term for a reduced state obtained by tracing out part of an entangled whole.

5. Gleason's result is stated for separable Hilbert spaces of dimension at least three, with measures that are countably additive over orthogonal projectors. In dimension two it fails, since the projectors of a qubit fall into independent orthogonal pairs, and any assignment that gives each pair a sum of one is admitted. Busch (2003) replaces projectors by effects, the positive operators bounded by the identity that are the elements of a general measurement, and requires additivity over effects that sum to the identity; the larger domain fixes the form $\mathrm{Tr}(\rho E)$ in dimension two as well. Bell (1966) observed that Gleason's result already excludes states that assign a definite value to every projector in dimension three and above, which is the theorem of Kochen and Specker (Chapter 13, Section III). The tie to the ground is P8, the definition of symmetry as a sameness of content where there is a difference of context.

Chapter 12

1. Deutsch (1999) argues that the probabilistic predictions of quantum theory follow from its non-probabilistic axioms together with the non-probabilistic part of classical decision theory. Wallace (2012) develops the argument with axioms of rational preference among the outcomes of quantum games. In both, the premises concern an agent who chooses among acts whose consequences are outcomes, and the outcomes so chosen among are findings in the sense of Chapter 10, Definition 10.2.

2. Dürr, Goldstein, Norsen, Struyve and Zanghì (2014) state that a Bohmian theory on a relativistic spacetime requires a privileged foliation, and propose that the foliation be determined covariantly by the wave function, so that the theory may be Lorentz invariant at its foundation.

Dürr, Goldstein and Zanghì (1992) derive the quantum statistics from quantum equilibrium, and that derivation is the reason no measurement reveals the foliation.

3. Bassi, Lochan, Satin, Singh and Ulbricht (2013) review the collapse models and the experiments that bound their parameters. Donadi, Piscicchia, Curceanu, Diósi, Laubenstein and Bassi (2021) measured the rate of spontaneous radiation in an underground germanium detector and set a lower bound on the size of the nuclear mass density in the gravity-related model about three orders of magnitude above earlier bounds, which rules out its natural parameter-free version. Tumulka (2006) takes as the primitive ontology a random discrete set of spacetime points, the flashes, at which collapses occur, and his model reproduces the violation of Bell's inequality without a preferred foliation.

4. Adlam and Rovelli (2023) state the postulate as follows: all of the information possessed by a certain observer is stored in physical variables of that observer. They also propose an ontology in which quantum states remain relational while some quantum events are not strictly relational. The tie to the ground is P14, the distinction of what exists and what is real: a record, once made, stands as structure, and so it exists for every later comparison; the finding that made it was real for the comparer that found.

5. Dowker and Kent (1996) conclude that the consistent histories approach must be supplemented by some selection principle before it can serve as a fundamental theory that makes unconditional predictions.

Chapter 13

1. Bell (1964) derived the first inequality of this kind; Clauser, Horne, Shimony and Holt (1969) gave the form of equation 13.2, which experiments can test with imperfect detectors; Tsirelson (1980) showed that quantum mechanics bounds the combination by $2\sqrt{2}$. Hensen and colleagues (2015) used electron spins in diamond 1.3 kilometers apart and closed the locality and detection loopholes together; Giustina and col-

leagues (2015) and Shalm and colleagues (2015) closed them with entangled photons. The tie to the ground is Axiom III (*An Immanent Metaphysics*, aphorism 1.3-1): the immanent and the omniscient are non-interchangeable.

2. Eberhard (1978) showed that the predictions of quantum mechanics give no means of sending a signal by the choice of a setting at one wing, since the marginal statistics at the other wing do not depend on that choice. In the terms of Chapter 11, a finding reaches another comparer only through a signal, at the earliest on that comparer's past light cone, and the theorem shows that the entangled state provides no other route.

3. Kochen and Specker (1967) work in three dimensions with the squares of spin components of a spin-one system, whose three orthogonal squares sum to a constant, so that exactly one of each orthogonal triple of projectors takes the value 1. Bell (1966) derived the absence of such assignments in dimension three and above from Gleason's theorem, and so the theorem of Gleason and the theorem of Kochen and Specker are two faces of one fact: the Born rule is non-contextual in its probabilities, and no assignment of outcomes is non-contextual.

4. Harrigan and Spekkens (2010) set out the framework of ontological models and the division into ψ-ontic and ψ-epistemic models. Pusey, Barrett and Rudolph (2012) consider independently prepared systems in states chosen from a pair with overlapping distributions, and exhibit a joint measurement each of whose outcomes has probability zero for one of the joint preparations; an ontic state shared by the preparations would then have to give some outcome for which every preparation compatible with it gives probability zero. The tie to the ground is P14: the state exists as the description of possibility, and possibility is a modality of the world, not a lack in a comparer's knowledge.

5. Hossenfelder and Palmer (2020) argue that rejecting statistical independence, the independence of the hidden variables from the settings, may resolve the measurement problem and explain the correlations without non-locality, that the objections to it draw on intuitions from clas-

sical linear systems, and that the hypothesis can be tested in a nearly model-independent way. The causation of Proposition 13.2 is the causation of P14 and Axiom II: a finding is real, it occurs now, and the future is still open.

Chapter 14

1. In the terms of P10 and P12, a founding commitment is a stance: it belongs to the transcendent modality, the role of the possibility from which an act proceeds. Axiom II states that "A class of the transcendent will precede an instance of the immanent" (An Immanent Metaphysics, aphorism 1.3-1). A program's founding commitment therefore precedes every calculation the program makes and shapes each of them, and that order is the reason the commitment can be assessed before the program is complete. The conclusion needs only the order of Axiom II, and it holds in each of the three rotations of the binding (Chapter 2, Table 2.1).

2. Chapter 6, "The Category Error", reads canonical gravity as the arena put into the quantum state: the context within which things happen is incorporated into the state instead of serving as the arena in which the state evolves. Chapter 10, "Three Times", treats the absence of time from equation (6.1) through the three kinds of time and their irreducibility.

3. The count of Strominger and Vafa is a count of states, and the Bekenstein–Hawking entropy it recovers is a real count proportional to the horizon area. Chapter 18, "Objections at Full Strength", answers the objection that such counts show the geometry to be quantized: the states counted are states of the content, counted relative to the exterior as context.

4. The difficulty of causal set theory is the recovery of a manifold-like order and of locality. Lorentz invariance is preserved by sprinkling (Bombelli, Henson and Sorkin 2009), so it is not the quantity to be recovered. Lorentz symmetry belongs to the formalism of general relativity, its instrumental mode (Chapter 3, "The Cross-Modal Principle, Checked Against Six Theories"), and a continuous group of symmetries in a form-

alism is not the pairing of continuity with symmetry that P9 excludes, since P9 concerns what a description finds. What causal set theory must recover is the continuously asymmetric arena itself.

Chapter 15

1. In the bulk, Newton's constant measured in units of the anti-de Sitter radius is of order $1/N^2$, so the bulk Planck length is small compared with the radius when N is large. The bound on localized operators at finite N is reached as the scale of the bulk description approaches its Planck scale, which is where the two descriptions meet and neither dominates (Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale").

2. Proposition 15.4 applies conjugacy as An Immanent Metaphysics uses the word: two things each of which can be perfected only at the expense of the other, neither prior to the other, as symmetry and continuity are. In the focus of the two laws (Chapter 2, Table 2.1), symmetry, the character the law of content requires, has the character of the omniscient modality, and continuity, the character the law of context requires, has that of the transcendent, and Axiom I states that "The omniscient and the transcendent are in conjugate relationship to one another" (aphorism 1.3-1, P12). In the focus of time the omniscient role is taken by geometric time, whose findings are continuous and asymmetric; that is the character of what one bound concept finds, in another focus, and not a binding of continuity to the omniscient role. A description perfected in symmetry and a description perfected in continuity are conjugate in this sense, so neither is the source of the other.

Chapter 16

1. The third criterion of the enumeration, the balance of sameness and difference (Chapter 1, Section VI), shows the exclusion directly. Continuity with symmetry gives the verdicts "same" and "same" on the content,

and asymmetry with discontinuity gives "different" and "different". Either pairing loses the inseparability of sameness and difference (P4) in the very respect in which its verdict is given, and a merger asks both of the same directions.

2. The common source is the act of comparison named by the Root Tautology: "Comparison is isomorphic with interaction and relation" (An Immanent Metaphysics, aphorism 1.42-4, P1). Content and context are distinct and inseparable within that one act (P4), and no pair of intrinsics can stand in for another. The two descriptions of physics answer to the two members of that pair, so they are distinct, they are inseparable, and neither can do the other's work. That is the unity the theorem permits: one source, and two descriptions joined.

Chapter 17

1. The temperature is equation (17.1) for $M = 1.98892 \times 10^{30}$ kilograms, which gives 6.17×10^{-8} kelvin. The evaporation time for emission of photons only is $5120\pi G^2M^3/(\hbar c^4)$, which gives 2.1×10^{67} years for one solar mass; the time grows as the cube of the mass. Both values are listed with their inputs in "The Numbers".

2. The step from content and context to one geometry for each alternative rests on the inseparability principle (P5; *An Immanent Metaphysics*, aphorism 1R3): each alternative of a content is content, so each has its own context. By the two laws (P6; aphorism 1.83-5), symmetry governs the content and continuity governs the context. Premise (Q) puts the context under the law of the content, and the Incommensuration Theorem (P9; aphorisms 1R1 to 1R20) excludes a description whose findings carry continuity and symmetry together.

3. The reading of the entropy as a count relative to the outside follows the characterization of the two sides (P7; *An Immanent Metaphysics*, aphorism 1.46-12): content is local and interexclusionary, and context is non-local and intercompositional. A count of states that cannot be told apart is al-

ways a count made from some context. The entanglement entropy of quantum fields across a surface grows with the area of the surface (Bombelli, Koul, Lee and Sorkin 1986; Srednicki 1993), which agrees with the reading; the reading itself makes no claim on the coefficient one quarter, which equation (17.2) takes from Bekenstein (1973) and Hawking (1975).

4. Page (1993a) gives the average entropy of the smaller part of a random pure state; for $1 \ll m \leq n$ it is about $\ln m - m/(2n)$, so the shortfall from $\ln m$ is less than one half when $m \leq n$. The equal weighting is a symmetry in the sense of the law of symmetry for content (*An Immanent Metaphysics*, aphorism 1.83-5): states that no comparison distinguishes are the same in every respect that a comparison from outside can register, and so they count alike.

5. The arithmetic of the table. For $r = 1$, $r/(1 + r) = 0.5$, and $1 - 0.5^{3/2} = 0.646$. For $r = 4/3$, $r/(1 + r) = 4/7 = 0.571$, and $1 - (4/7)^{3/2} = 0.568$. For $r = 1.4847$, $r/(1 + r) = 0.5975$, and $1 - 0.5975^{3/2} = 0.538$. Page (2013) reports 53.81 percent of the evaporation time and a loss of 40.25 percent of the initial entropy, so that 59.75 percent remains; the general formula reproduces both figures from his value of r. The ratio exceeds one because emission into empty space is irreversible (Zurek 1982), and the greybody factors raise it above the black-body value $4/3$ (Page 1976; Page 2013). The values are listed in "The Numbers".

6. The boundary of the island is a quantum extremal surface, a surface that extremizes the area term plus the entropy of the matter outside it (Engelhardt and Wall 2015). Almheiri, Mahajan, Maldacena and Zhao (2020) obtain the curve from semiclassical geometry in a model whose matter has a higher-dimensional holographic dual, and Almheiri, Hartman, Maldacena, Shaghoulian and Tajdini (2021) review the whole development. The account takes from these works the shape of the curve and their computation on classical saddles, and it reads the purity of the whole as a fact about the content relative to one outside world.

Chapter 18

1. The analogy turns on where the quantization fell. Planck's steps are steps of energy at each frequency of the radiation, which is content; the space in which the modes are laid out kept its continuity. In the terms of the two laws (P6; *An Immanent Metaphysics*, aphorism 1.83-5), the remedy placed discreteness on the side governed by symmetry and left continuity with the context.

2. The Planck length is 1.61626×10^{-35} meters, so $(l_P/r)^2$ at $r = 1$ meter is 2.6×10^{-70}. Donoghue (1994) computes the leading corrections to the potential between two heavy masses; the quantum correction enters at relative order $G\hbar/(c^3 r^2) = (l_P/r)^2$, with a coefficient fixed by the long-distance part of the theory and free of unknown constants. The account reads these corrections as properties of the description of the small waves on a classical background, which the account admits below the Planck curvature.

3. The rule of decision used for every prediction of Chapter 19 is five standard deviations, established by at least two independent kinds of measurement. The premises of the ground cited to *An Immanent Metaphysics* are, among others, the Root Tautology (P1, aphorism 1.42-4), the six intrinsics (P3, aphorism 1.42-5), the three axioms (P12, aphorism 1.3-1) and the Incommensuration Theorem (P9, aphorisms 1R1 to 1R20).

Chapter 19

1. The arithmetic of equation (19.1). With $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $m = 10^{-14}$ kilograms and $t = 2.5$ seconds, $G\,m^2\,t/\hbar = 1.582 \times 10^{-4}$ meters. The separations give $1/200 + 1/700 - 2/450 = 1.984 \times 10^{-3}$ per micrometer, which is 1984 per meter. The product is 0.314 radians. A change to the phase of one place of one body alone enters two joint

phases, one counted with a plus sign in the combination and one with a minus sign, so the combination is fixed by the gravitational coupling alone. The value is listed with its inputs in "The Numbers". The ground of one geometry for each alternative is the inseparability principle (P5; *An Immanent Metaphysics*, aphorism 1R3): each alternative of a content has its own context.

2. The four markers of a context-modal phenomenon that make the prediction exact are derived in Chapter 5 from the ground before gravity is named. Universality follows from the inseparability principle (P5; *An Immanent Metaphysics*, aphorism 1R3): whatever has content has context, so the context acts on all of it. The absence of a shield follows from the characterization of context as non-local and intercompositional (P7; aphorism 1.46-12): a region is itself a structure of context, and no region can be without it. The physical picture agrees: a shield works by covering a source with an opposite, and gravity couples to energy, which has one sign.

3. A least step of the arena would give the description whose findings are continuous and asymmetric a discontinuity at its foundation, which is the pairing of asymmetry with discontinuity that the Incommensuration Theorem forbids (P9; *An Immanent Metaphysics*, aphorisms 1R1 to 1R20). The law of continuity applies to all that is of context (P6; aphorism 1.83-5). The bounds of the table therefore bear on the account directly: each extends the range over which the arena is found continuous.

Key Terms

AdS/CFT. The correspondence between gravity in anti-de Sitter space and a conformal field theory on its boundary.

Alternative. One term of the decomposition of the quantum state of the matter into states of definite distribution of stress-energy, at the resolution at which the geometry is sourced: a mass in this placement, or the same mass in that placement.

Anti-de Sitter space. The maximally symmetric spacetime of constant negative curvature, which has a timelike boundary at infinity.

Arena. Space and time together, in which everything that is measured is measured; the context of physics.

Asymmetry. A difference of content where there is a difference of context: nothing is invariant.

Barbero–Immirzi parameter. A free constant of loop quantum gravity that sets the scale of its area spectrum.

Born rule. The probability rule of quantum mechanics: the probability of each outcome of a measurement is the trace of the state multiplied by the element of the measurement that corresponds to that outcome.

Bridge. An exact relation between a quantity of one description and a quantity of the conjugate description, mediated by a structural condition and not invertible; it relates the two descriptions without deriving either from the other.

Capacity. The largest entanglement entropy that a part of a system can have.

Category error. A question that applies a concept where the concept does not belong.

Causation. The relation in which one finding, made by the comparer that finds, selects among alternatives and conditions the range of the findings that follow, in the one-way order of Axiom II.

Class. A kind, standing ready as a whole.

Coarse-grained entropy. Of the radiation of a black hole: the thermal entropy computed from its spectrum alone, the entropy the radiation would have if its quanta were uncorrelated.

Code subspace. The subspace of the boundary Hilbert space in which the bulk effective field theory about a given geometry is encoded.

Common source. Two descriptions, each perfected in its own character and in its own directions, joined side by side and related systematically, with one origin for both; the Incommensuration Theorem permits it.

Comparer. Whatever makes a comparison: an instrument, a detector, an interaction; never a mind.

Comparison. The act of finding whether two things are the same or different, in some respect.

Concurrence. A measure of the entanglement of a state of two spins: zero for every unentangled state and at most one; for the state of equation (8.1) it is $|\sin(\Delta\varphi/2)|$.

Conjugate. Said of two things each of which can be perfected only at the expense of the other, neither prior to the other. General relativity and quantum theory are conjugates in this sense: distinct, inseparable and non-interchangeable.

Consistent family. A set of histories, each a sequence of projectors at chosen times, that satisfies the consistency condition, so that the probabilities assigned to its histories are additive.

Constitutive mode. The mode of a theory's findings: the character of its range of findings.

Content. What the verdict of a comparison is on.

Content and context partition. The division of physical phenomena into content, governed by symmetry and found in discrete classes, and context, governed by continuity and found as smooth values.

Content-modal phenomenon. A phenomenon that comparisons find as their content, within a context distinct from it: the thing judged the same or different.

Context. The respect in which the two things compared are two.

Context-modal phenomenon. A phenomenon that is the context of every physical comparison, the comparisons whose content is a measurement of it included: the respect in which the things compared are two, within which every content is found.

Continuity. A sameness of content where there is a sameness of context: nothing is sudden.

Cosmic microwave background. The thermal radiation left from the early universe.

Cross-modal principle. Framework and content are always in conjugate modes: a symmetric framework probes continuous content, and a continuous framework probes symmetric content; the reach of the framework into its content is dense but never exhaustive.

D-brane. An object of string theory on which strings can end; it carries discrete charges.

De Sitter space. The geometry of uniform exponential expansion.

Decoherence. The spreading of the relative phase of alternatives into an environment that registers which alternative is the case, after which the alternatives no longer interfere in any measurement made on the system alone.

Deformed symmetry of special relativity. A change of the symmetry of special relativity that keeps a second quantity, an energy of the order of the Planck energy, the same for every observer beside the speed of light.

Dense. Said of the reach of a framework into its content when the framework approaches every finding of the content as closely as wished.

Determinism. The claim that a complete specification of the world at one time fixes every finding at every later time.

Difficulty of dynamics. The difficulty predicted for a program that quantizes the arena directly: the quantized arena lacks a well-defined evolution, because evolution is described along the arena.

Difficulty of multiplicity. The difficulty predicted for a program that requires the arena to follow from a discontinuously symmetric framework: many arenas fit the framework, and no principle within it selects one.

Difficulty of recovery. The difficulty predicted for a program that replaces one description entirely by a structure of the other character: the replaced character does not follow from the surviving one alone.

Discontinuity. A difference of content where there is a sameness of context: something is sudden.

Effective field theory. A quantum field theory that describes a system below some energy, with every effect of higher energies summarized in local terms whose influence falls as a power of the energy.

Einselection. Environment-induced superselection: the selection of the pointer observable by the interaction with the environment.

Entanglement wedge. The bulk region between a boundary region and its extremal surface; every bulk operator in it can be reconstructed on that boundary region.

Entanglement witness. A quantity whose value, beyond a stated threshold, is possible only for an entangled state.

Entangling phase. The sum of the phases of the two joint alternatives in which two bodies are both near or both far, less the sum of the phases of the two mixed alternatives; the one combination of the four phases that no local operation alters.

Eötvös parameter. Twice the difference of the free-fall accelerations of two bodies divided by their sum.

Euclidean signature. The form of a spacetime calculation in which time is turned into a fourth direction of space, so that the metric has the same sign in all four directions.

Event time. The time of findings: the order of findings for a comparer, each one outcome selected from a discrete set of possible outcomes and laid down as a record.

Evolution time. The parameter of the strongly continuous one-parameter group of unitary operators that carries the quantum state.

Exist. To stand as structure: timeless, the same in every instance, like the pattern of a law or the state of a system at an instant. A law exists; a measurement is real.

Factorizable model. A model in which the joint probabilities of outcomes at two separated places are an average, over a variable whose distribution does not depend on the settings, of products of local probabilities.

Finding. The outcome of one actual comparison: one outcome selected from among its possibilities, by which something possible becomes manifest for the comparer that finds. Each finding is one member of a theory's range of findings.

Firewall. A layer of high-energy quanta at the horizon that a body falling in would meet, proposed by Almheiri, Marolf, Polchinski and Sully (2013); in the account, the horizon is smooth.

Focus. The context and frame of the argument being made: what a comparison is taken to be about. Which concept stands in which of the three roles depends on the focus (P11; Table 2.1).

Forbidden pairings. Continuity with symmetry, and asymmetry with discontinuity: the two conjunctions that the Incommensuration Theorem excludes from any comparison perfected at its foundation.

Formalism. The mathematical structure in which a physical theory is written: its space of states, its equations, and the symmetry principles that constrain them.

Founding commitment. The operation a program of quantum gravity performs on the arena at its first step, before any calculation is made.

Framework. The structure by which an investigation is carried out.

Generalized second law. The entropy of a black hole plus the entropy outside it never decreases.

Geometric time. The structure of time that the metric of spacetime carries: the proper time along each timelike worldline and the causal order of events; continuous and asymmetric in what is found.

Gradient. The ordering of the programs of quantum gravity, assessed on the four criteria, from those most in conflict with the account to those nearest to it.

Graviton. In this book, at most a quantum of a description: a quantum of the description of a small wave of a classical geometry about its background by quanta. Never a quantum of the arena, of space-time or of the geometry, and never a particle, a carrier of gravitation or a kind of matter. The phonon, a quantum of the description of the vibrations of a held structure, gives the word its nearest sense; the arena is not a held structure, so even that sense does not make the graviton a quantum of the arena.

Gravity-induced collapse. The proposal of Diósi and Penrose that gravity itself ends a superposition of masses, at a rate set by the gravitational self-energy of the difference of their distributions.

Greybody factor. The probability that a mode of given frequency and angular momentum, leaving the neighborhood of the horizon, passes the potential barrier of the geometry and reaches infinity.

Heisenberg cut. The boundary between the system, described by a state, and the apparatus, described in classical terms; identified in this book as the boundary between the two levels of the theory.

Held structure. A structure whose parts are held in place by bonds, as the bonds of a solid hold its atoms in a lattice; its vibrations are small motions of the parts about the places the bonds hold, and their description by quanta has phonons as its quanta. The arena is not a held structure.

Holographic entropy cone. The set of values that the entropies of a fixed number of boundary regions can take when each is given by the formula of Ryu and Takayanagi.

Identification. A stated step from the structure of comparison to a physical quantity, made in the first person, with the observation that bears on it.

Immanent. The role of that which mediates, that which is processual, that which is happening: the act itself.

Improper mixture. A reduced density operator that is diagonal because part of an entangled whole has been traced out, and not because one alternative has been selected.

Incommensuration Theorem. The concepts of symmetry and continuity cannot both be simultaneously and fundamentally applied to any event or comparison, and neither can the concepts of asymmetry and discontinuity; any comparison, perfected at its foundation, must be either continuously asymmetric or discontinuously symmetric.

Inflation. A proposed early epoch in which the universe expanded at a nearly exponential rate.

Information. In the first sense, the form of the content: the specification of what the matter is and of which alternatives it holds, carried with the matter wherever the matter goes. In the second sense, a count of states relative to a context: the number of states that the comparisons available from that context cannot tell apart.

Inseparability principle. Where there is content, there must also be context, and where there is context, there must also be content.

Instance. One particular case of a kind, here and now.

Instrumental mode. The mode of a theory's formalism.

Island formula. The rule that computes the entropy of the radiation of a black hole by adding a region behind the horizon, the island, and taking the smallest extremum of the area of its boundary in units of four Planck areas plus the entropy of the matter in the radiation and the island.

Joint alternative. One assignment of a place to each of two bodies, each held in two places.

Kleitman–Rothschild orders. The three-layered partial orders into which almost all partial orders on many elements fall; they approximate no spacetime of four dimensions.

Local operations and classical communication. The operations that two parties perform, each on its own system, together with classical messages sent between them; they cannot create entanglement between two systems that start in a product state.

Lorentzian signature. The form of a spacetime calculation in which time keeps its own sign in the metric, opposite to the three directions of space, as in general relativity.

Marker. One of the four properties that any context-modal phenomenon has: universal coupling, no shield, one mass for coupling and inertia, and no discrete charge.

Merger. One formalism in which the directions of the arena themselves carry the discontinuous symmetry of content at the foundation, so that both characters are perfected in the same directions; the Incommensuration Theorem forbids it. Quantum electrodynamics is joined to the arena without a merger, since its field is content laid on the arena.

Mixed state. A quantum state whose von Neumann entropy is positive.

Modal strategy. The way an interpretation of quantum mechanics treats the three times: which it holds as primary, which it drops, and which it absorbs into another.

Modality. One of the three roles that the parts of a comparison play with respect to one another: the omniscient, the immanent and the transcendent.

Mode. The compound character that governs one level of a theory.

Mode exchange. Quantization as one step in which the findings of a description pass from continuously asymmetric to discontinuously symmetric and the character its formalism holds exact against them passes from symmetry to continuity.

Mode pair. The pair formed by a description's constitutive mode and its instrumental mode, written with the findings first.

Monogamy of entanglement. The rule that a system maximally entangled with one partner is entangled with no other system.

Non-contextual assignment. An assignment of a value to every projector that does not depend on the other projectors measured with it.

Non-renormalizable. Said of a quantum field theory that needs infinitely many independent constants to absorb its divergences at high energy.

Objective. What is compared: the side about which the comparison is made.

Observer. A comparer: the word names a position inside the world, and it never names a mind.

Omniscient. The role of that which is known, that which is structural, that which is pattern: what can be laid out and held in view as a whole.

Ontological model. A model in which a system has an ontic state at each preparation, and each quantum state corresponds to a probability distribution over the ontic states.

Page curve. The entanglement entropy of the radiation outside a black hole, plotted against time.

Page time. The time at which the Page curve turns from rising to falling.

Per-alternative geometry. The configuration in which each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by the stress-energy of that alternative alone, with the relative phases of the alternatives kept.

Perfected. Holding exactly, at the foundation of a description.

Planck area. The square of the Planck length; the unit in which the entropy of a horizon is counted, and not a least area.

Planck curvature. The curvature whose radius is the Planck length; the smooth description of the arena holds below it.

Planck energy. The rest energy of the Planck mass, the Planck mass times the square of the speed of light: about 1.22×10^{19} GeV.

Planck length. The length at which the gravitational size and the quantum size of the Planck mass coincide, the square root of $\hbar G/c^3$, about 1.616×10^{-35} m; the scale at which the two descriptions meet, and not a least length.

Planck mass. The one mass whose gravitational size, Gm/c^2, equals its quantum size, $\hbar/(mc)$: the square root of $\hbar c/G$, about 2.176×10^{-8} kg.

Planck scale. The scale where the two descriptions meet: an object's gravitational size equals its quantum size, neither description dominates, and both are relevant.

Planck volume. The cube of the Planck length; the unit of a count of the interior of a region cell by cell, and not a step of the arena.

Pointer observable. The observable of an apparatus whose eigenstates remain stable while the environment monitors them; it fixes the basis in which the records of a measurement are laid down.

Preferred foliation. A division of spacetime into spacelike hypersurfaces singled out as surfaces of simultaneity.

Problem of time. The absence of any time parameter from the Wheeler-DeWitt equation, so that the quantum state of the geometry has nothing in which to evolve.

ψ-epistemic. Said of an ontological model in which the distributions for two distinct pure states can overlap, so that one ontic state is compatible with both.

ψ-ontic. Said of an ontological model in which the distributions for distinct pure states never overlap.

Pure state. A quantum state whose von Neumann entropy is zero.

Quantum equilibrium. In the theory of Bohm, the distribution of configurations given by the squared modulus of the wave function.

Quantum-gravity energy scale. The energy $E_{QG,n}$ at which a dependence of the speed of light on energy, of order n in $E/E_{QG,n}$, would become of order one.

Range of findings. The outcomes that a theory says comparisons can have; the constitutive mode of a theory is the character of this range, and each finding is one member of it, given in an act.

Real. Given in an actual act of comparison: temporal, occurring now, with the future still open.

Relative phase. The phase of one alternative relative to another in the decomposition of the state of the matter into its alternatives.

Residual. What a framework approaches as closely as wished and does not contain.

Ricci part. The part of the curvature that Einstein's equations fix at each place by the stress-energy there.

Root Tautology. Comparison is isomorphic with interaction and relation: comparison, interaction and relation are one act, approached three ways.

Rule of decision. A result counts when it is established at five standard deviations by at least two independent kinds of measurement.

Semi-classical equation. Einstein's equations with the stress-energy replaced by its quantum expectation value, so that one geometry is sourced by the average of the alternatives of the matter, each weighted by its probability.

Single-time result. No interpretation of quantum mechanics that works within a single unified time is complete.

Stress-energy. The density and flow of energy and momentum, which every form of matter and radiation has; the quantity to which gravity couples.

Subjective. The comparing: the side from which the comparison is made.

Superspace. Wheeler's name for the space of all the geometries that a slice of space can have, over which canonical quantum gravity lays out the wave functional of the geometry.

Swampland. The low-energy theories that appear consistent but cannot be completed into a consistent quantum gravity; the swampland program of string theory seeks the conditions that separate them from those that can be completed.

Symmetry. A sameness of content where there is a difference of context: something is invariant.

Time function. A smooth real function on spacetime whose gradient is everywhere timelike; a coordinate of geometric time.

Transcendent. The role of that which is potential, that which exceeds any particular instance: the stance and the possibility from which an act proceeds.

Triple. Three concepts in the three roles, for which there is one identified focus in which all three axioms hold.

Truncation. A computation of the renormalization group of gravity that keeps a chosen, finite set of the terms of the theory.

Two laws. The law of symmetry applies to all that is of content; the law of continuity applies to all that is of context.

Ultraviolet catastrophe. The growth without bound, as the frequency rises, of the energy density that the classical theory of radiation in a cavity predicts.

Unified time. One time that is geometric time, event time and evolution time together, with all three perfected in one formalism, none of them external and none primitive.

Weyl part. The free part of the curvature, which the arena carries and propagates where there is no content, as around the Sun and in a passing gravitational wave.

Wheeler–DeWitt equation. The Hamiltonian constraint of canonical quantum gravity as an operator equation: the Hamiltonian operator annihilates the wave functional of the spatial metric, and the equation contains no time parameter.

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