

The One Who Finds

The Immanent Metaphysics
for Readers of Sean Carroll

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Preface

Sean Carroll writes physics as if his reader deserved the whole argument. Across six books he states his premises, follows each premise, and says where the premises lead. Together the six books draw one picture of the world, from the smallest particles to the meaning of a human life. This Preface sets out his picture first, in his own words. Then I set beside it a second picture, which keeps every law Carroll states and adds one thing at the ground: the one who finds. That one addition is why this book is called *The One Who Finds*.

From Carroll's pages

One World, Many Ways of Talking

Carroll calls his view of the world *poetic naturalism*. He sets out the view in Chapter 2 of his book *The Big Picture*, "Poetic Naturalism". The naturalism half of the name is three claims. The first naturalist claim is his sentence "There is only one world, the natural world". The second naturalist claim is that "The world evolves according to unbroken patterns, the laws of nature". The third naturalist claim is that the only reliable way to learn about the natural world is to observe the natural world. The poetic half of the name is three claims more. The first poetic claim is that there are many ways of talking about the world: as atoms, as cells, as people with reasons. The second poetic claim is his sentence "All good ways of talking must be consistent with one another and with the world". The third poetic claim is that the best way of talking at a given moment depends on the purpose we have at that moment.

Each way of talking has its *domain*: the range of cases in which the way of talking holds. The way of talking about particles holds for the particles that make up a table or a person, while the way of talking

about people and their reasons holds for a person deciding what to eat for breakfast. A way of talking can name, within its domain, things that are real, in Carroll's sense of the word 'real'. Carroll gives his test of what is real in Chapter 13 of *The Big Picture*, "What Exists, and What Is Illusion?": something is real when it plays an essential role in a description of the world that, "as far as we can tell", is accurate within the domain of that description. So Carroll concludes, by his test: "Atoms are real; tables are real; consciousness is undoubtedly real".

The Laws Underneath Everyday Life

Carroll's second great claim is about the laws that govern the things we see and handle every day. He states the claim in *The Big Picture*, Chapter 23, "The Stuff of Which We Are Made". In his words, "The laws of physics underlying everyday life are completely known". He calls the laws underlying everyday life the *Core Theory*: the Standard Model (the physicists' theory of the known kinds of particle, and of every force among them except gravity), together with gravity where gravity is weak. He also states the limits of the claim. He writes in the next chapter, Chapter 24: "It certainly does not imply that we know all of physics". The claim covers the particles and forces that make up a table, a cook and a kitchen, at the energies a kitchen holds. He adds that living things, the brain and the weather need theories of their own. He also notes that no one has a complete theory of any of the three. In his words, the phenomena of all three "need to be compatible with the Core Theory".

In his book *The Particle at the End of the Universe*, Carroll tells how physicists found the last piece of the Core Theory. On Carroll's account, the world is made of *fields*: quantities with a value at every point of space. A *particle* is a small ringing (a vibration) of a field, as a note is a ringing of a string. One field, the *Higgs field*, keeps a value above zero even in empty space, and the *elementary particles* (the particles not built from smaller

ones) take their masses from the Higgs field. The ringing of the Higgs field is a particle called the Higgs *boson*. The bosons are the family of particles that carry forces, and Carroll counts the Higgs boson as an unusual member of the family. The Higgs boson was announced on 4 July 2012 at the European Organization for Nuclear Research (CERN), the European laboratory for particle physics near Geneva. The announcement came from the two large experiments at CERN's Large Hadron Collider (LHC). The Large Hadron Collider is a ring underground, in which physicists bring beams of protons into collision. *Protons* are particles found in the small, heavy center of every atom. The two experiments are called ATLAS and CMS. Carroll draws his conclusion in Chapter 13 of *The Particle at the End of the Universe*, "Making It Worth Defending": "With the discovery of the Higgs boson, our understanding of the physics underlying everyday reality is complete".

Carroll's *The Biggest Ideas in the Universe* comes in two volumes. The two volumes teach the laws that his other books describe in words, with the equations left in. The volumes are for readers willing to look at an equation and think about what the equation says. One volume, *Space, Time, and Motion*, builds on what Carroll calls the Laplacian paradigm, named after the French mathematician Pierre-Simon Laplace. The Laplacian paradigm says: give the complete state of a system at one moment (every position and every velocity of its parts), and the laws of physics give the system's state at the next moment, and the next. Carroll writes in Chapter 2 of *Space, Time, and Motion*, "Change": "It's just one thing after another, where the next thing is determined by the current thing and the laws of physics". *Space, Time, and Motion* also teaches that space and time make one whole. The volume teaches gravity as the curving of the one whole. *Quantum mechanics* is the physics of matter and light at the smallest scales, and a *quantum field* is a field (a quantity with a value at every point of space) that obeys the rules of quantum mechanics. The other volume, *Quanta and Fields*, teaches quantum mechanics and quantum fields with the equations left in as well. *Quanta and Fields* draws the same

conclusion as *The Particle at the End of the Universe*: the world is made of fields. Chapter 4 of *Quanta and Fields*, "Fields", says: "According to our current best understanding, quantum fields are the bare stuff of reality".

A Wave Function and Nothing Else

What, for Carroll, lies beneath the fields and their particles, at the bottom of the world? Carroll's answer is the subject of his book *Something Deeply Hidden*. Quantum mechanics (the physics of the smallest scales) has one central object: the *wave function*, which Carroll also calls the *quantum state*. The wave function is Carroll's name for the complete quantum description of a system, and he holds that the description is the system itself. In Chapter 2 he writes: "The wave function isn't a bookkeeping device; it's an exact representation of the quantum system". Physicists use the wave function to compute the odds of each possible outcome of a measurement on the system. One rule, the Schrödinger equation, says how the wave function changes in time. Carroll follows the physicist Hugh Everett in adding nothing to the wave function and its one rule. Carroll gives the essence of the Everett view in Chapter 2: "reality is described by a smoothly evolving wave function and nothing else". On Carroll's account there is one wave function for the whole universe, and the one wave function always obeys its one rule.

For Carroll, then, a measurement is an ordinary meeting of two physical things: a system and an instrument. When the outcome could go two ways, the one wave function comes to hold both alternatives. While the wave function holds both alternatives, the two alternatives *interfere*: they add to each other in some places and cancel each other in others. Two sets of ripples on a pond interfere in the same way, adding where crest meets crest and canceling where crest meets trough. Then the difference between the two alternatives spreads into the surroundings (the air, the light, the parts of the instrument), and after the difference has spread, the two alternatives no longer add or cancel. Physicists call the spreading of the dif-

ference into the surroundings *decoherence*. Carroll calls each alternative that decoherence separates a *branch*, or a world, and in each branch one outcome has occurred. On Carroll's account, every outcome that the one rule produces occurs, each outcome on its own branch. Carroll stresses in the Prologue of *Something Deeply Hidden* that "the multiple worlds are predictions of the formalism that is already in place, not something added in by hand". The *formalism* in Carroll's sentence is the mathematics of quantum mechanics: the wave function and the one rule by which the wave function changes. In Chapter 7 of *Something Deeply Hidden*, Carroll shows how to derive the quantum odds of each outcome from the situation of an observer who has branched: the observer knows the whole wave function, and does not yet know which branch the observer is on. Carroll also announces in the Prologue what he calls "new, exciting, and admittedly tentative proposals": that space, and gravity with it, are to be found inside the wave function, in the way the parts of the wave function are linked to one another.

An Egg and an Omelet

Carroll's book *From Eternity to Here* asks why time runs one way. Carroll gives the picture in the book's Prologue: "You can turn an egg into an omelet, as the classic example goes, but you can't turn an omelet into an egg". You pour milk into coffee, and the milk spreads and never gathers itself back out. People are born, grow older and die. Each of the three processes runs only one way: an egg becomes an omelet, milk spreads in coffee, and a person grows older. Together, the one-way processes make the *arrow of time*: the one direction in which time runs.

Carroll's first point is that the basic laws of physics run as well backward as forward, so the arrow of time has to come from something other than the basic laws. Carroll writes in Chapter 2, "The Heavy Hand of Entropy": "The arrow of time, therefore, is not a feature of the underlying laws of physics, at least as far as we know". For Carroll the arrow of time

comes from *entropy*. Entropy is a count of how many arrangements of the small parts of a thing look the same from outside. Of all the arrangements of the parts of an egg, far more look like an omelet than look like an unbroken egg, so entropy grows, and the arrow of time points from the egg toward the omelet. For entropy to grow, entropy had to start low. The Prologue of *From Eternity to Here* describes the *Big Bang* as the "unimaginably hot, dense state" that the universe we can observe was in about fourteen billion years ago, and since then the universe we can observe has been expanding and cooling. Carroll traces our arrow of time to the very low entropy of the universe near the Big Bang. The *Past Hypothesis* is the assumption that the universe we can observe began in a state of very low entropy. Carroll takes the name from the philosopher David Albert (*The Big Picture*, Chapter 7, "Time's Arrow"). Carroll holds that the Past Hypothesis itself needs a reason. Carroll asks in Chapter 8 of *From Eternity to Here*, "Entropy and Disorder": "Why is the Past Hypothesis true?" He closes the chapter on the same question in what he calls "a more specific form": "Why did the universe have a low entropy near the Big Bang?"

Carroll's own answer is a proposal: the part of the universe we see lies inside a much larger whole, which he calls the multiverse. He offers the proposal on a condition, in Chapter 15, "The Past Through Tomorrow". Why does entropy increase in regions like ours? Carroll writes that the question can be answered "if the multiverse doesn't have any state of maximum entropy: The entropy increases because it can always increase, no matter what state we are in". In the Epilogue, Carroll calls one version of the proposal "a specific model of which I am personally fond", and he adds, "The details of this model are highly speculative". In the Epilogue he also says what he values more than the model: "the general paradigm, according to which entropy is seen to be increasing because entropy can always increase".

People in a World of Laws

The last part of Carroll's picture is about us. In *The Big Picture*, people, reasons and choices belong to a way of talking about the world, and within that way of talking, people, reasons and choices are real in Carroll's sense, as atoms are real within the way of talking about particles. In Chapter 13, "What Exists, and What Is Illusion?", Carroll warns against one mistake. The mistake is to mix the words of two ways of talking when we ask how a thing described in one way of talking acts on a thing described in the other. He writes that you can think of yourself as a person or as a vast number of atoms: "Just not both at the same time, at least when it comes to asking how one kind of thing interacts with another one". Of free will, Carroll writes in Chapter 44, "Freedom to Choose": "There's a sense in which you do have free will. There's also a sense in which you don't". Of knowledge, Carroll holds that we can become highly confident and can never become certain. His Chapter 15, "Accepting Uncertainty", ends on the sentence "Life is short, and certainty never happens". On meaning and value, Carroll writes in the Prologue of *The Big Picture* about "accepting that values are human constructs, but denying that they are therefore illusory or meaningless".

Carroll's message, in brief, is that there is one world, and that the one world is the natural world. At bottom the natural world is a single wave function, changing by one rule, in which every outcome occurs on its own branch. The laws that govern everyday life are completely known. Time runs one way because the universe began with low entropy. The low-entropy beginning is itself a fact that needs a reason. On proposals he calls tentative, space and gravity are to be found within the quantum description. And people, choices, knowledge and meaning belong to ways of talking about the one world, and they are real in Carroll's sense. Meaning and value are built by people.

Who This Book Is For, and What It Does

This book is for you, the reader of Carroll. You have read Carroll's own words on entropy, the wave function, branching worlds and poetic naturalism. You need no mathematics beyond what school gave you.

I set my own body of work beside Carroll's. My body of work is the *physics collection* of the Immanent Metaphysics. The collection is a set of books that work out physics from the act of knowing. The Immanent Metaphysics is my own work: I first set it out in my book *An Immanent Metaphysics* (2001), and the physics collection is the series of later books in which I work out its physics; the appendix For the Specialist names them wherever a chapter draws on them. *Metaphysics*, in this book, means the study of what must be so for anything at all to be known. *Immanent* means within: the Immanent Metaphysics starts inside the act of knowing, here and now, and never from a place outside the world. The Immanent Metaphysics starts from one act of knowing: *comparison*. A comparison is the act of finding whether two things are the same or different, in some respect. A cook tasting two spoonfuls of soup compares the two spoonfuls. A thermometer compares as well: the thermometer's column rises until the column matches the warmth of the room. Instruments compare as surely as minds do. In the next chapter I ask what all measurements have in common, and answer: "Every measurement is a comparison". When a single act of comparison comes to a verdict, I call the act a *finding*. I call whoever or whatever makes the finding (a person or an instrument) *the one who finds*. The verdict, same or different, is the outcome of the finding.

Carroll starts from physics: from its laws and its measurements. I start from the act of comparison, and I take nothing from physics. The ground of the Immanent Metaphysics takes nothing from physics: no measurement, no law, no result of any experiment. The ground in the quoted sentence is the act of comparison. I hold that the act of comparison lies at the bottom of everything physics describes. Carroll and I start from separate

places, so when Carroll and I arrive at the same result, each by a separate line of reasoning, the result has been reached twice, by two independent roads. I set out the places where Carroll and I agree, in a chapter on Carroll titled "The Wave Function and the One Who Finds". I derive three of Carroll's positions by my own reasoning. First, every law of physics holds within a domain. Second, nothing physicists know about gravity requires space and time to come in least steps, a smallest length or a smallest moment. Third, the arrow of time needs a reason. I also grant Carroll's claim, as he states it, that the laws of everyday life are completely known, and I add no force or field to the laws of everyday life. Wherever Carroll and I reach the same result, we will walk both roads to the result: Carroll's reasoning, and mine.

Carroll and I part at one place: there is one question that we answer differently. I name the place in the first section of my chapter on Carroll: "The two part at one place, and every other difference between them follows from it: what is at the ground of physics, the thing every other description describes". In this book the *ground* of physics means only what lies at the bottom, the thing every other description describes. I then say what Carroll and I each take the ground to be: "For Carroll the ground is a quantum state evolving by one rule. For the collection the ground is the act of comparison, in which every finding of physics is made". The quantum state in the first quoted sentence is Carroll's wave function, and I read the wave function in my own way. My reading: "The state describes, exactly and usefully, what any comparison can find and with what odds. The finding itself is the act, and the act comes first".

Every other parting of Carroll's account and mine follows from the parting at the ground. In the chapters ahead we will see Carroll and me part on particular subjects: on how sure knowing can be, on whether one law can fix a whole world, on whether space is made of something quantum, and on others. When a chapter arrives at a parting, the chapter states the parting on the chapter's own subject, and states that the parting, like every other, follows from what lies at the ground. Three of the partings run through the whole book, and each of the three follows from the

difference about the ground. First, the known laws are complete for the distribution of outcomes and leave each single outcome open. The known laws fix the odds over many trials, and they leave open which outcome turns up in a single trial. Second, the same chapter says: "The arrow of time has a structural source in every comparison". In the quoted sentence, *structural* means that the source lies in the form every comparison has, whatever is compared: each act runs one way, from the range of what could be found, to the act, to the record of what was found. So the direction of time is present in each act of finding. Carroll traces the arrow of time to the low-entropy beginning of the universe. Carroll's arrow points the same way as the direction in each act of finding. Third, in the many-worlds account as Carroll gives it (the account in which every outcome occurs, each on its own branch, or world), the wave function is the whole of the world. In Carroll's words, there is the wave function "and nothing else". The many-worlds account drops together the one who finds, the single random outcome and the direction of time. In the world that the many-worlds account describes, all three are missing, and all three go missing together, because on my account a reading of quantum mechanics has either all three in its world or none of the three.

I leave Carroll everything that comes from the laws themselves: the Core Theory, which has passed every test made of it, and the domains in which the Core Theory applies. I remove one step only, the step that I call "the step from a complete law to a complete world": the step from knowing every law to holding that the laws fix every single event. I add one thing to Carroll's picture: the one who finds, at the ground. So this book is called *The One Who Finds*.

How to Read This Book

I make you four promises, and I keep all four in every chapter. First, I explain every term in plain words where you first come to it. Second, I state Carroll fairly, in his own terms, before I set anything beside him.

Third, the main text has no equations: the mathematics and the book-by-book sources for every claim are in Appendix B, "For the Specialist", with the titles of my books that a chapter draws on, and you may skip both and still follow every chapter. Fourth, I gather the measurements still to come in Appendix A, "What the Coming Measurements Will Find".

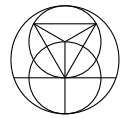
Each of Chapters 1 to 12 is built on one question that a reader of Carroll carries away from his books, and the title of each chapter states the chapter's question. In each of the twelve chapters I tell a scene from the history of physics or philosophy, and then I state what Carroll showed. Next I show where I derive Carroll's result by my own reasoning, and where I come to another result, and why. Each chapter ends on the question of the next chapter. In Chapter 13, the closing chapter, I ask you what you now hold. In Chapter 13 I gather the results of the first twelve chapters, and Chapter 13 ends on the egg. I cite each short quotation of Carroll by book and chapter, so that you can open his book at the place quoted.

Here, in brief, is what this book holds that Carroll's books do not: the one who finds, at the ground of physics (Chapter 1); certainty in every act of finding, with the proof of the Incommensuration Theorem walked in a kitchen (Chapter 2); the carriers of the forces counted from the act of comparison (Chapter 3); a law exact about the many and a finding open for each one (Chapter 4); one crack found, with the subjective, hard randomness and the arrow of time bound in it (Chapter 5); the weights of the outcomes derived from the act of finding (Chapter 6); gravity as the arena, with no quantum of the arena (Chapter 7); the structural arrow of time (Chapter 8); the turn between epochs (Chapter 9); choice in the single case (Chapter 10); the cook gathered into one self (Chapter 11); and no bare facts (Chapter 12). If you have read another of my books written for the readers of another author, you know the ground I lay in Chapter 2 already: comparison, its six parts and the Incommensuration Theorem, told here in a kitchen. What is new for you begins with the parting at the ground in Chapter 1 and runs through every chapter after Chapter 2. Read the chapters in order, because each chapter builds on the

ones before it. Where a chapter uses a word or a result that an earlier chapter teaches, a short box at the head of the chapter lists the word or result, with the chapter that teaches it. Where a chapter retells what Carroll showed, the retelling stands in a shaded box headed "From Carroll's pages": a reader who knows Carroll's books can pass over the boxes, and a reader new to them can read the boxes first.

One picture stays with us from the first chapter to the last, and the picture begins as Carroll's own: the egg in the Prologue of *From Eternity to Here*, which becomes an omelet and never an egg again. Picture the egg on a kitchen counter beside a bowl, with a cook about to crack the egg on the rim of the bowl. In this book the egg is the picture of one act of finding. The cook brings the egg down once on the rim, the shell parts along one line, and the cook sees the shell part. Bringing the egg down and seeing the shell part is one act: made from one side (the cook's), about one thing (this egg), with one outcome (this crack), which cannot be undone. In each of Chapters 1 to 12 I add one step to what we see in one act of finding. In eleven of the twelve chapters, the act is this crack. In this book's Chapter 2 the act is a test of freshness. The cook sets an egg in a bowl of water and finds whether the egg sinks or floats.

Carroll opens *The Big Picture* with a chapter titled "The Fundamental Nature of Reality", and I open on the same subject. Is the bottom of the world a wave function, changing by its one rule, or the act of finding in which every description, the wave function included, is made? We start where Carroll starts. What is the world made of, at bottom?



Chapter 1

What Is the World Made Of, at Bottom?

Take the chair you sit on, the hand that holds this book and the morning outside the window. There are many ways of talking about the chair, the hand and the morning: as atoms, as cells, as people with reasons. When every way of talking about the three has been followed down as far as it goes, what is there? The physicist Sean Carroll gives a clear answer to the question of what is there at bottom, and I give a different answer. The two answers share a great deal, and they part at one place: what lies at the very bottom, under every way of talking.

Two Tables in Edinburgh

The astronomer Arthur Eddington was Plumian Professor of Astronomy at Cambridge. From January to March 1927 Eddington gave the Gifford Lectures at the University of Edinburgh. The Gifford Lectures are a series of public lectures held at the universities of Scotland. The lectures became a book, *The Nature of the Physical World*, published in 1928. Eddington opened the book with a picture of himself at work on the lectures: "I have settled down to the task of writing these lectures and have drawn up my chairs to my two tables".

"Two tables!" Eddington goes on. The first table is the table he has known all his life. The first table stands in the room, has a color and a shape, and stays put from one day to the next. Eddington says that "above

all it is substantial": that is, press on the table and the table presses back. Eddington calls the second table by number. "Table No. 2 is my scientific table". The scientific table is the table that physics describes. Eddington says what the scientific table is made of: "My scientific table is mostly emptiness". Electric charges rush about at great speed through the emptiness, and the combined bulk of the charges, he writes, is "less than a billionth of the bulk of the table itself".

Which of the two tables is really there? Eddington gives both answers, and he gives each answer its full weight. He says that modern physics has "assured me that my second scientific table is the only one which is really there". Yet in the same pages he says that modern physics "will never succeed in exorcising that first table", the table his eyes see and his hands touch. Eddington does not choose between the tables: he leaves both standing side by side in the room, and he leaves his reader with both.

Eddington's second table was built from a measurement, and the measurement shows how a way of talking about a table is made. In 1909, at the University of Manchester, Hans Geiger and Ernest Marsden worked in the laboratory of the physicist Ernest Rutherford. Geiger and Marsden sent alpha particles (small, fast, positively charged particles thrown out by radium) at thin sheets of metal. Nearly all the alpha particles passed straight through the metal. About one alpha particle in eight thousand came back, as if the particle had struck something hard. In 1911 Rutherford gave the reason: an atom holds its positive charge in a very small volume at its center, and the rest of the atom is open to a passing particle. Physicists call the small center of an atom the nucleus, and we now know that the nucleus is about a hundred thousand times smaller across than the atom, and that the nucleus carries nearly all the atom's mass. The open space around each small, heavy nucleus is the emptiness of Eddington's second table.

From Carroll's pages

One World, Many Ways of Talking

Sean Carroll answers Eddington's question, which of the two tables is really there, in a way that keeps both tables. In his book *The Big Picture* (2016) Carroll calls his view *poetic naturalism*, and he states the view in six short claims. *Naturalism*, the first half of the name, is Carroll's word for the view that the natural world is all there is. Carroll gives naturalism three claims (Chapter 2, "Poetic Naturalism"): "There is only one world, the natural world". "The world evolves according to unbroken patterns, the laws of nature". "The only reliable way of learning about the world is by observing it". Carroll gives the "poetic" half of the name three more claims: "There are many ways of talking about the world". "All good ways of talking must be consistent with one another and with the world". "Our purposes in the moment determine the best way of talking".

For Carroll a way of talking is a whole description, with its own things, its own words for those things, and its own rules for how those things go together. One of Carroll's examples is the air in a room. A weather forecaster talks of the air as a smooth fluid, with a temperature, a pressure and a wind at every point. A chemist talks of the same air as a swarm of separate molecules, each with its own position and speed. Both descriptions are right, each within its own range of situations. Carroll calls the range of situations in which a description holds its *domain of applicability*. The fluid description leaves out the detail of the molecules and becomes useful, and even unavoidable, once we look at the air as a whole, so Carroll calls the fluid description *emergent*. The fluid description emerges from the molecular description. Carroll calls the molecular and the fluid descriptions of the air, taken together, "an innocent, uncontroversial example of emergence" (Chapter 12, "Reality Emerges"). Carroll also says exactly what ties the two descriptions together. He writes that "emergence is about different theories speaking different languages, but offering compatible

descriptions of the same underlying phenomena". For the fluid description to be true, it needs to agree with the molecular description wherever both descriptions apply. The fluid description is true without any derivation from the molecules, and a derivation from the molecules is welcome when it can be had.

Then Carroll asks Eddington's question in his own words. "The table in front of you is real; unicorns are not" (Chapter 13, "What Exists, and What Is Illusion?"). If the table is made of atoms, are only the atoms real? Carroll answers by giving the word "real" his own sense: something is real when it "plays an essential role in some particular story of reality that, as far as we can tell, provides an accurate description of the world within its domain of applicability". So Carroll concludes: "Atoms are real; tables are real; consciousness is undoubtedly real". For Carroll, then, Eddington's two tables are two good stories about one table, and each table is real within the domain of its own story, as far as we can tell.

Carroll's answer is generous and careful, and three things follow from it. First, the chemist, the biologist and the cook need not apologize for their words. Second, people, choices and meanings stay in the world with no extra force to hold them there. Third, Carroll's answer contains a test for when two descriptions belong together: the two descriptions must agree wherever both apply.

An Egg on the Rim of a Bowl

I use one picture from the first chapter to the last. A cook stands in a kitchen with an egg and a bowl. She cracks the egg once on the rim of the bowl, and looks: the shell has parted.

Carroll has an egg of his own. He opens his book *From Eternity to Here* (2010) with his egg: "You can turn an egg into an omelet, as the classic example goes, but you can't turn an omelet into an egg" (Prologue). Car-

roll's egg shows the direction of time. In this book the egg has a different job. The act of finding is the cook's: she cracks the egg and looks. The act is a single event, made from one side (the cook) and about something (the egg), and the act cannot be undone. The act has one outcome, the thing the cook finds: this crack.

You can talk about the egg in Carroll's many ways. A chemist can describe the egg as atoms: carbon, hydrogen, oxygen, nitrogen, and calcium in the shell. A biologist can describe the egg as proteins folded in water, a yolk and a white laid down to feed a growing chick. The cook can describe the egg as breakfast. Each of the three descriptions is a good way of talking within its domain, and the three descriptions agree wherever they cover the same thing. And under all three descriptions, something happened in the kitchen this morning. The egg was cracked, and someone found it cracked.

We can now ask this chapter's question at the kitchen counter. Of all the ways of talking about this egg, which way is at the bottom: the description that every other way of talking is about? In Carroll's terms, the egg's quantum state is the complete quantum description of the egg, and he holds that the description is the egg itself. Is the bottom the egg's quantum state, or is the bottom the act of finding?

From Carroll's pages

What Lies Under the Ways of Talking

Carroll's answer is clear: his many ways of talking are ways of talking about one description at the bottom. Carroll calls the description at the bottom "the best way we have of talking about the world at the deepest level" (*The Big Picture*, Chapter 2). In Carroll's sense of "deepest", the deepest level is the bottom: the description that every other way of talking is a way of talking about. Carroll holds that the list of

things in the deepest description is "extremely sparse" (*The Big Picture*, Chapter 2). In his book on quantum mechanics, *Something Deeply Hidden* (2019), Carroll says what is on the list.

The *wave function* is at the bottom, and Carroll also calls the wave function the *quantum state*. Physicists have long used the wave function as a tool for working out the chances of what a measurement will show. In Chapter 2 of *Something Deeply Hidden*, "The Courageous Formulation: Austere Quantum Mechanics", Carroll looks back on his first chapter, where he had talked about the wave function as "a kind of mathematical black box from which predictions for measurement outcomes could be extracted". Picture the black box: you put in a question about a measurement, and out come the odds of each answer. Then Carroll takes a further step on purpose: Carroll holds that the wave function is the system itself. He holds too that the wave function always changes by one smooth rule, which physicists call the Schrödinger equation, and that nothing else is added. "The world is a wave function, nothing more nor less" (Chapter 2). Carroll also gives what he calls the essence of the theory: "reality is described by a smoothly evolving wave function and nothing else" (Chapter 2).

Carroll uses the wave function to revise Eddington's second table. Eddington pictured electric charges as tiny dots rushing through emptiness. On Carroll's view an electron is not a dot: the electron is its wave function, and the wave function fills the atom. "Atoms aren't mostly empty space; they are described by wave functions that stretch throughout the extent of the atom" (*Something Deeply Hidden*, Chapter 2). So the scientific table of 1927 was itself one way of talking, and on Carroll's view another way of talking lies under the scientific table: the wave functions of the table's atoms. For Carroll, what lies at the deepest level is the wave function of the whole universe, and the egg, the cook, the bowl and the kitchen are each a way of talking about part of that one wave function.

Carroll's view of the wave function is the view of the physicist Hugh Everett, and Carroll calls Everett's approach the "courageous" formulation of quantum mechanics. Carroll gives the reason for the word: "It embodies the philosophy that we should take seriously the simplest version of underlying reality that accounts for what we see, even if that reality differs wildly from our everyday experience". Everett adds nothing beyond what the equations of quantum mechanics already contain, and he follows the equations wherever they lead.

Where the Two Roads Agree

I set a body of work beside Carroll's: the physics collection of the Immanent Metaphysics. The physics collection is a set of books that work out physics from the act of knowing, and for short I call it the collection. On the many ways of talking, I agree with Carroll. I name the two points on which Carroll and I agree in my chapter on Carroll, "The Wave Function and the One Who Finds": "Poetic naturalism and the collection agree that there are many good descriptions of the world, each fitted to its domain, and that no description of people or reasons needs a force that physics lacks".

I have my own reason for the agreement, and a picture of a street shows the reason. I describe a street on a market morning. One person sees the street from a third-floor window, and another person walks along the street with a basket. The watcher at the window sees the bakery to the left of the bank, and the walker passes the bank first. Both descriptions are true, because each description is made from a place and is the answer to the question asked from that place. I state the rule that the street shows: "Two different verdicts in two different frames are answers to two different questions, and so they do not contradict each other". In my sense, a frame is what is held fixed so that a question has one answer: the place a description is made from, and the question asked from that place. I read

Eddington's two tables by the rule of the street. The table under Eddington's hand is the answer to a question asked with his eyes and his hands, in the room: what color and shape the table has, and whether the table presses back. The table of charges is the answer to a question asked through the instruments of physics: what the table is made of, at the scale of its atoms. Geiger and Marsden used instruments of physics in Manchester when they sent alpha particles at thin metal. Each question is asked from its own place, so the two tables are answers to two different questions, and both answers are true.

I also agree with Carroll that a higher description, such as the chemist's, can be true of the same things as a lower description, such as the physicist's, without being derived from the lower description. I make the same point about chemistry. I say what holds and what fails: "Chemistry is made of nothing but electrons, nuclei, and light; that is not in question. What fails is the derivation". The derivation of the chemist's description fails because exact calculation from the laws of the parts succeeds only for the simplest systems. Yet the chemist's description is true of the molecule all the same. Carroll's fluid and my chemistry make the same point, one from Carroll's side and one from my side, and the point has two parts. First, descriptions of one world are tied together by agreeing wherever they cover the same thing. Second, a thing can be made of nothing but its parts, and yet the description of the thing is true all the same, without being worked out from the description of the parts.

The One Who Finds

Look again at the cook. Before the crack, the egg was whole. After the crack, the shell has parted, and the cook sees that the shell has parted. She has set the egg as it is now beside the egg as it was, and she has found the two different.

I have a name for the cook's act, and the whole of this book is built on the cook's act. I state the definition: "A comparison is the act of finding whether two things are the same or different, in some respect". A *comparison*, then, is setting one thing beside another and finding them the same or different in some respect. In this book a *finding* is one act of comparison that ends in a verdict, same or different. The verdict is what is found. And I call whoever or whatever makes the comparison *the one who finds*.

The one who finds can be a person or a thing. Anything that makes a comparison is a comparer. I describe comparers that work with no mind at all: a thermostat, a pan balance, a spirit level, a smoke alarm and a photographic plate. Comparing, then, is not a human specialty. If you set a camera on the kitchen counter to record the egg as it breaks, the camera finds the crack as surely as the cook does. The one who finds is a person or an instrument, inside the world, meeting what it finds.

Now we can trace each description of the egg back to where the description came from. Chemists found the atoms of the chemist's description by comparisons: by reading the weights on a balance, and by setting the colors of light from a heated sample beside the colors of known elements. Biologists found the proteins of the biologist's description the same way, one measurement at a time. Eddington's table of charges was found when Geiger and Marsden watched a screen of zinc sulfide through a microscope and counted the tiny flashes made by the alpha particles that came back. Physicists check Carroll's wave function the same way: they set its odds beside the counts of what instruments find, again and again. Every description of the world was built from what acts of finding found, and every act of finding was made by one who finds.

Where the Two Roads Part

For me, the act of comparison is the ground of physics: what lies at the bottom, the thing every other description describes. Every finding of physics is an act of comparison, made by one who finds. I say what each

account puts at the bottom: "For Carroll the base is a quantum state evolving by one rule, and the other descriptions are ways of talking about it. For the collection the base is the act of comparison".

I place the act of comparison at the bottom because of an order that anyone can check. I give the word "deeper" one meaning, my sense of the word. When one thing cannot be done, stated or checked without a second thing, while the second thing can be done without the first, the first thing *presupposes* the second. Of two things, the deeper is the one that the other presupposes. So in my sense "deeper" names the order of presupposition, while Carroll's "deepest level" names the bottom of his ways of talking. By the test of what presupposes what, I set four fields of knowledge in one order, deepest first: "metaphysics, physics, computation and mathematics". In my sense, metaphysics is the account of the act of comparison itself. Physics is the study of acts of comparison as they happen in the world. Computation is carrying out a sequence of steps by a fixed rule. Section V says that mathematics "is the study of structure: shape, space, number and the patterns of valid reasoning, held in view as a whole". In that sense, *structure* is how things stand to one another (shape, size, number, pattern), laid out all at once, with nothing in the structure coming first or after: a drawing that shows the egg's shell, white and yolk on one page shows the egg's structure.

The egg shows the order of the four fields, one link at a time. The cook's act of finding is complete in itself: she finds the shell parted, and to find the shell parted she needs no law of physics, no program and no theorem. The physics of the cracking is the study of the forces in the shell and the spreading of the crack, as happenings in the world. Physicists found every law of the physics of cracking by acts of finding like the cook's, so physics presupposes the act of comparison. A computer can simulate the crack, but the computer runs its steps in a machine that is itself a physical thing, warming as it works. So computation presupposes physics, and the shell parts whether or not anything computes the parting. The mathematics of the egg's quantum state describes every way the shell could part, with the odds of each way, all laid out at once. Each of the odds comes out

of the mathematics only through a calculation, worked one step after another. Proofs about the mathematics are checked the same way, line by line: each line is set beside the rules and beside the lines before it, and checking a proof in that way is a computation. A computation, for its part, runs without waiting on any proof: beads moved along the wires of a counting frame add two numbers whether or not anyone has proved anything about addition. So mathematics presupposes computation. Each of the four fields presupposes the one before it in the list: mathematics presupposes computation, computation presupposes physics, and physics presupposes the act of comparison. I also say that depth "is distinct from certainty, from precision, from elegance and from prestige". Mathematics keeps all of its certainty, and mathematics still stands last in the order.

We can now place Carroll's wave function in the order of presupposition. A wave function is mathematics: structure laid out, every case in view at once. So the wave function stands with mathematics, last in the order of presupposition. Yet Carroll places the wave function at the bottom of every way of talking. I also give the quantum state its full due there, in my own reading of what the quantum state does: "The state describes, exactly and usefully, what any comparison can find and with what odds.

I have two words for the difference between the quantum state and the crack. To *exist* is to stand as structure, the same in every instance, like a law or a pattern. In my sense, which differs from Carroll's, to be *real* is to be given in an actual act of comparison, here and now. A law exists; a measurement is real. In the kitchen, the egg's quantum state exists, as an exact description of every way the shell could break, while the one crack that the cook finds this morning is real.

I keep the words "exist" and "real" apart because of what mathematics does best. Every comparison has two sides: the side the comparison is made from, which I call the subjective, and the side the comparison is about, which I call the *objective*. When the cook compares the egg, the cook with her bowl and her eyes is the subjective side, and the egg is the objective side. Mathematics works as if only the objective existed: math-

ematics states its results and leaves out the side from which the results are found. Within mathematics, working without the subjective side is right, and working without the subjective side is why anyone can check a result of mathematics, step by step, and get the same outcome every time. I name the mistake of working without the subjective side outside of mathematics in the title of Section VIII of Chapter 3: "Taking the Frame of Mathematics for Everything". I draw the consequence for the mathematics of the many-worlds account (Everett's approach, as Carroll gives it), which I call the formalism: "The whole world contains the acts of comparison by which anything in it is found, so the formalism, which leaves those acts out, cannot be the whole world".

Carroll's "deepest level" and my "deeper" meet in one question: what the whole of the world contains. Carroll answers that the world is a wave function, nothing more nor less. The world also contains the acts of comparison by which the wave function itself is found and checked, and a description that leaves those acts out cannot be the whole of the world it describes.

Carroll writes "The world is a wave function, nothing more nor less", and in that sentence he takes a step: he counts a structure that exists, and is exact, as the whole of what is real. Carroll takes the step on purpose, and I exclude the step. In taking the step, Carroll treats the world at its bottom, under every way of talking, as mathematics treats what it studies: as structure, stated without the side each comparison is made from. The one who finds is on the side each comparison is made from, so in taking the step Carroll leaves the one who finds out of the bottom of the world. Yet every description of the world was made and is checked in acts of finding by one who finds, and the wave function is one of those descriptions.

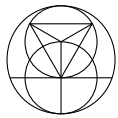
Back to the Two Tables

Go back to Eddington's room. The familiar table and the scientific table are two good descriptions, and Carroll is right that both descriptions belong to one world. Something that neither table names lies under both tables: the act in which Eddington, leaning on the wood or reading an instrument, finds what he finds. Both tables were built from what acts of finding found, and people and instruments still check both tables by acts of finding.

We now know where the two roads meet and where the two roads part: where Carroll and this book agree, and where Carroll and this book disagree. Carroll and this book agree that there is one world with many good descriptions. Carroll and this book part on what lies under all the descriptions: for Carroll a quantum state, and for this book the act of finding, in which every description is made and checked.

The parting on what lies at the bottom is why this book is called *The One Who Finds*. When the act of finding is at the ground, the one who finds belongs in physics from the start. So every later question of this book is asked with the one who finds in view: whether everything is already fixed, whether both outcomes of a measurement happen, and why time runs one way.

If the ground is an act of finding, what is finding, and how sure can finding make us?



Chapter 2

How Sure Can We Be of Anything?

How sure can we be of anything we think we know? This book's Chapter 1 ended on a question in two parts. In this book the ground means what lies at the bottom, under every description. If an act of finding is the ground of every description, what is finding, and how sure can finding make us? The answers to both parts begin with an essay about chance, read aloud in London two years after its author had died. In this chapter I name the parts of every act of finding; I use those names in the rest of the book, and the Glossary at the back keeps them.

An Essay Read in London

On 23 December 1763 the Fellows of the Royal Society heard an essay on chance by a man who could not be there to hear it. The Royal Society is the scientific society of London, and the Society's members are called Fellows. The man was Thomas Bayes, himself a Fellow of the Society, who had died in April 1761 and who had been a Nonconformist minister, that is, a minister of one of the English Protestant churches outside the Church of England. Bayes's friend Richard Price had gone through the papers Bayes left and found the essay among them, and Price sent the essay to John Canton, another Fellow of the Society, with a covering letter dated from Newington Green on 10 November 1763. In the letter Price wrote: "I now send you an essay which I have found among the papers of our de-

ceased friend". The Society printed the essay in the Society's journal, the *Philosophical Transactions*, volume 53, under the title "An Essay towards solving a Problem in the Doctrine of Chances".

Bayes opens the essay by stating his problem: "Given the number of times in which an unknown event has happened and failed: Required the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named". In plainer words: you have watched an event happen so many times and fail so many times. How sure can you now be of how likely the event is to happen the next time?

Bayes pictured the problem with a table. He asked his reader to suppose a square table, made and leveled with one aim. The aim is that a ball thrown onto the table is as likely to come to rest on any one part of the table as on any other. Someone throws a first ball and draws a line across the table through the place where the first ball comes to rest, and then throws a second ball again and again. Each time the thrower notes on which side of the line the second ball came to rest. How much can anyone say about where the line lies, from nothing but the count of throws on each side of the line?

Each throw of the second ball gives one verdict: the ball comes to rest, its resting place is set beside the line, and the ball is found on this side of the line or on that side. Every number in Bayes's essay is built from counts of verdicts on which side of the line a ball came to rest. Price saw that the same counting applies beyond the table, to everything anyone expects on the strength of what has happened before. In an appendix to the essay Price imagined a person newly brought into the world, who watches the sky for the first time. Price wrote of the newcomer: "The Sun would, probably, be the first object that would engage his attention". Price went on: "but after losing it the first night he would be entirely ignorant whether he should ever see it again". Each morning that the sun comes back, the watcher counts one more return. With each return the watcher's expectation of the next return grows.

The whole question of this chapter is already in the essay and the appendix that the Royal Society heard in 1763. We learn by counting what we have found. So we need to know what one act of finding is, how sure a count of what we have found can make us, and whether anything at all can be known beyond every doubt.

From Carroll's pages

Carroll's Credences

Sean Carroll opens Chapter 9 of *The Big Picture*, "Learning about the World", with Thomas Bayes. Carroll builds on Bayes's idea through the chapters after Chapter 9. Carroll starts from the claim that we hold our beliefs by degrees. He writes: "What we actually have are degrees of belief, which professional statisticians refer to as credences" (*The Big Picture*, Chapter 9, "Learning about the World"). A *credence* is how strongly you hold a belief, given as a share between nothing and all: if you think there is a one in four chance of rain tomorrow, your credence in rain is a quarter. Carroll's own first example is the sun: he believes that the sun will rise in the east tomorrow, "but I'm not absolutely certain of it". Price's newcomer and Carroll are in the same position two and a half centuries apart, like two watchers at one window: each of the two expects the sun to rise tomorrow, and neither is certain that the sun will rise.

In the same chapter Carroll writes: "Bayes's main idea, now known simply as Bayes's Theorem, is a way to think about credences". You begin with a *prior*: the credence you give each possibility before the new information arrives. For each possibility you then ask how likely the new information would be if that possibility were true. Carroll calls the numbers that answer the question for each possibility the *likelihoods*. You weigh each prior by its likelihood, and the result is your new credence, which Carroll calls the *posterior*. Carroll pictures the weighing with a jar of sand for each possibility (Chapter 10, "Updat-

ing Our Knowledge"). The amount of sand in a jar shows the credence in the jar's possibility. From each jar you take sand away according to how unlikely the new information would be if the jar's possibility were true. Priors can differ from one person to the next, while the likelihoods are meant to be fixed by the evidence: in Carroll's words, "Everyone's entitled to their own priors, but not to their own likelihoods". Carroll holds that updating credences by Bayes's Theorem is the method of every science: "It's the basis of all science and other forms of empirical reasoning" (Chapter 9).

Carroll draws three lessons from Bayes's method. The first lesson is that no belief about the world is ever held with perfect certainty. An observation can always be mistaken, so you never give any observation a likelihood of exactly zero, and so no credence ever becomes exactly nothing or exactly all (Chapter 15, "Accepting Uncertainty"). The second lesson is that knowledge has no foundation of the kind that the philosopher René Descartes looked for in the seventeenth century: one belief beyond doubt, on which all the other beliefs are stacked. Carroll answers Descartes with a picture: "Planets don't sit on foundations; they hold themselves together in a self-reinforcing pattern" (Chapter 14, "Planets of Belief"). A planet of belief is a whole system of beliefs held together by how the beliefs fit one another, as a planet is held together by the pull of its own parts. The third lesson is about proof. Carroll writes at the start of Chapter 15: "Science never proves anything". On Carroll's account mathematics proves what its assumptions imply: "Math is concerned with truths that would hold in any possible world" (Chapter 16, "What Can We Know about the World without Actually Looking at It?"). Scientists find out which of the possible worlds is the world we live in, and Carroll writes that they do so "by looking at it".

Carroll grants the doubter's point in three old puzzles. In the first puzzle, a skeptic doubts that anything can be known. In the second, a philosopher cannot prove that he is not dreaming. In the third, a de-

mon might be deceiving us with a false world. Then Carroll shows how to live with the doubt. Carroll writes of Descartes's method of doubting everything: "But in that part of his method, Descartes was completely correct" (Chapter 11, "Is It Okay to Doubt Everything?"), and the part Carroll means is the claim that nothing about the world is known beyond every conceivable doubt. When no one could tell a scenario apart from the ordinary world by any data, and believing the scenario is never useful, we give the scenario a very low credence and, on Carroll's account, move on with our lives. Carroll ends Chapter 15 with the first lesson in its shortest form: "Life is short, and certainty never happens".

I grant, in full, Carroll's account of how we gain knowledge of the world: every particular thing we find out about the world comes with a degree of confidence, and we are to count all the evidence. To Carroll's account I add one thing: an account of the act of looking itself, the act in which every piece of evidence is found.

An Egg in a Bowl of Water

What, exactly, is a finding? Go into the kitchen, and watch the cook. She has an egg from the back of the shelf, and she does not know how old the egg is. She fills a bowl with cold water and sets the egg in the water. A fresh egg sinks and lies on the bottom. As an egg ages, water evaporates through the shell and the pocket of air inside the egg grows, so the older an egg is, the lighter the egg is for its size, and an old egg floats. The cook's egg goes down and lies on its side, so the cook knows that the egg is fresh. Setting the egg in water is an act of finding, of the same kind as the crack on the rim of the bowl in this book's Chapter 1, which the cook made on a different egg.

What happened in the bowl of water has happened in every act of knowing there has ever been. Two things were set beside each other, and a verdict came out: the same, or different. In the bowl the water does the weighing: an egg in water is weighed against the water the egg pushes aside, and the water pushed aside is exactly the size of the egg. If the egg weighs the same as the water it pushes aside, the egg hangs in the water. If the egg is heavier than the water it pushes aside, the egg goes down. If the egg is lighter, the egg comes up. The bowl gives the verdict, and the cook reads the verdict.

I call the physics books of the Immanent Metaphysics, which this book reads beside Carroll, the collection. This book's Chapter 1 gave my name for the act in the bowl. Every act of knowing is a comparison or is built from comparisons, from a ruler held against a board to a thermometer read against its scale. Comparing needs no mind: the bowl compared the egg with the water whether or not the cook was watching. A kitchen scale compares a weight without a mind too, and so do the thermostat and the smoke alarm that Chapter 1 named. The one who finds is whoever or whatever makes the act. The verdict, same or different, is what is found.

Take the cook's comparison apart, and you find six parts in three pairs.

The first pair is the verdict itself: *sameness* and *difference*. The bowl finds the egg the same weight as the water it pushes aside, or a different weight. Every comparison ends in sameness or in difference, and each of the two has its meaning only beside the other: to find two things the same in some respect is to find that they do not differ in that respect.

The second pair is what the verdict is about, and what makes the two things compared two things. In the bowl the verdict is about weight. I call what a verdict is about the *content* of the comparison. In the bowl the content is weight. The egg and the water it pushes aside are two things because the egg is inside the shell and the water is outside the shell. I call what makes the two things compared two the *context* of the comparison. In the bowl the context is inside and outside the shell. A verdict tells the cook something only where the verdict could come out either way. The

bowl could find the weight of the egg and the weight of its water the same or different, so the verdict on weight tells the cook whether the egg is fresh. The egg was inside its shell, and the water was outside the shell, before the egg ever touched the water, so the egg and its water were two before the bowl compared them. The bowl could only ever say "different" if the bowl's verdict were about whether the egg and its water are in one place. So in every comparison the content is something other than the context. What a comparison is about is always something other than what makes its two things two.

In many comparisons the context is a place or a time: a here and a there, or a now and a then. Take a tide gauge: an instrument on a harbor wall that sets the height of the sea at noon beside the height at midnight. The tide gauge's two readings are two because the gauge took them at two times, so noon and midnight are the context of the tide gauge's comparison.

The third pair is the two sides of the act. The side the comparison is made from is the cook, with her bowl and her eyes: I call that side the *subjective*. The egg is the side the comparison is about: I call that side the *objective*. In this book the subjective and the objective are names for the two sides of an act of comparison, and for nothing more. A kitchen scale has both sides as surely as a cook does.

The six parts, then, are sameness and difference, content and context, and the subjective and the objective. You can find the six parts in any comparison you care to look at. Each pair is inseparable. Wherever there is a sameness, there is a difference beside the sameness. Wherever there is a content, there is a context the content is found in. And every side that compares has a side that is compared. The six parts are enough, and I show why with a ruler. Each inch on a ruler was set out by laying one length beside another and finding the two lengths the same, and the number beside a mark counts how many inches lie between the mark and the end of the ruler. So a unit, a scale and a number are each built from comparisons, and they come after the six parts of comparison, not before the six parts.

Two Things That Cannot Both Be Perfect

Do the six parts of comparison set any limit on what can be found in a comparison? The six parts do set a limit. To see the limit, we give two verdicts at once, one on the content and one on the context. Each of the two verdicts can be "same" or "different", so we look at the four ways the two verdicts can go together. When no difference is found in what makes a comparison's two things two, the verdict on the context is "same": the two things are taken at one moment, at one place, or at settings so close together that no difference between the settings is found. When a comparison's two things are found at different moments, places or settings, the verdict on the context is "different".

Turn the knob of the stove a little, and the flame changes a little. There are two flames, one before the turn and one after the turn, because the flame burns at two settings of the knob. So the setting is the context, and the flame is the content. Make the turn so small that no difference between the two settings is found, and no difference between the two flames is found either: the stove shows a sameness of content where there is a sameness of context. I call a sameness of content where there is a sameness of context *continuity*: no gap and no least step, since a small enough turn of the knob brings only a small change of flame.

Flip the kitchen light switch, and at one point in its movement the light goes from off to on. For the switch, the position of the switch is the context, and the light in the room is the content. The switch has moved so little that no difference in its position is found, and yet the room has changed completely: a difference of content where there is a sameness of context. I call a difference of content where there is a sameness of context *discontinuity*: a jump, with nothing between before and after.

A measuring cup holds one cup in this kitchen and in a kitchen on the far side of the world, this year and fifty years from now. For the measuring cup, the place and the year are the context, and the amount the cup

holds is the content. The place and the year change, and the content stays the same: a sameness of content where there is a difference of context. I call a sameness of content where there is a difference of context *symmetry*: sameness kept across a change. The cup shows symmetry, because the cup's measure is the same at every place.

Run your hand along the grain of a wooden table, and no stretch of the grain repeats another stretch. For the table, the place along the table is the context, and the pattern of the grain is the content. The content changes as the place changes: a difference of content where there is a difference of context. I call a difference of content where there is a difference of context *asymmetry*: nothing kept the same.

I call each of the four, continuity, discontinuity, symmetry and asymmetry, a *character* of a comparison: one of the four ways a comparison's verdict on the content can go together with its verdict on the context.

	Content the same	**Content different**
Context the same	continuity: the stove knob	discontinuity: the light switch
Context different	symmetry: the measuring cup	asymmetry: the grain of the table

Things in every kitchen show each of the four characters, nearly, every day. Now ask for two characters at once, each held perfectly. Ask first for perfect continuity together with perfect symmetry. Perfect continuity says: where the context is the same, the content is the same. Perfect symmetry says: where the context is different, the content is the same. Put the two together, perfect continuity and perfect symmetry, and the content is the same whatever the context. So the comparison can only ever say "same". Sameness and difference are inseparable in every comparison, as the egg in the bowl of water showed, and a comparison in which a difference can never be found has stopped being a comparison. Perfect asymmetry and perfect discontinuity cannot hold together either, by the same reasoning turned over: together they say "different" whatever the context, and never

"same". The remaining pairs of characters fail at once: continuity with discontinuity, symmetry with asymmetry, and each character with itself. Perfect continuity with perfect discontinuity would find the content both the same and different where the context is the same. So the pair would say two opposite things of one case. Perfect symmetry with perfect asymmetry likewise says two opposite things of one case, where the context is different. And a character paired with itself is only that character again. Two kinds of perfected comparison remain, and only two. One kind of comparison taken to perfection is continuous and asymmetric, like a ribbon of honey poured from a spoon, unbroken and never the same twice. The other kind is symmetric and discontinuous, like an ice-cube tray, every cell the same as every other, with a wall between each cell and the next.

The walk we have just made ends in the *Incommensuration Theorem*. I state the Incommensuration Theorem in these words:

> Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity.

To apply a character *fundamentally* is to hold it exactly, at the foundation of a description, with no exception at any scale. To apply two characters *simultaneously* is to hold both in one and the same comparison. A *theorem* is a statement shown to follow, step by step, from what has already been established. The word Incommensuration comes from an older word, incommensurable. When no unit, however small, measures both of two lengths a whole number of times, the two lengths are *incommensurable*. The side of a square and its diagonal are incommensurable: no unit measures both. The Incommensuration Theorem names a gap between two characters, like the gap between the side of a square and its diagonal: symmetry and continuity cannot both be perfect in one comparison. The theorem allows symmetry and continuity together where

both hold nearly, as they do in a ripple spreading on a still pond, but the theorem forbids symmetry and continuity held exactly and together at the foundation of one comparison.

The steps we have just taken are the proof of the theorem, told in plain words. The steps use the meanings of sameness, difference, content and context, and the fact that sameness and difference cannot be separated. The steps use no fact about the physical world. So no fact about the physical world can make the theorem false. The proof is short, and anyone who has read this far can check every step of the proof. The check comes out the same each time anyone makes the check. The proof is given in full in my book *Incommensuration*. There I also say how sure the proved theorem is: "A result of that kind is settled in the way a count is settled: by the count itself, with no need to trust whoever made it". Every act of knowing is a comparison or is built from comparisons, so the theorem holds in every act of knowing there is. The Incommensuration Theorem is a structural fact about the nature of knowing: a fact about the shape that every act of knowing has, whatever the act is about.

Where the Two Accounts Meet

Which of Carroll's results do I derive by my own reasoning? I derive four of Carroll's results: the credence, the planet of belief, the rule that all the evidence counts, and Carroll's view of what mathematics can prove.

Start with the credence. My book on comparison is another of the three foundation volumes. I say what a probable step of reasoning is (a step whose conclusion follows only probably): a verdict of comparison with a measure placed on the verdict. In a probable step of reasoning, the reasoner gives the verdict "it follows". The reasoner also places on the verdict a measure of how likely the verdict is. The measured verdict is a verdict with a measure placed on it, and not a third kind of verdict. The measure placed on the verdict is found by comparison too. You have already seen a measure found by comparison on Bayes's table, where each

thrown ball was set beside the line drawn across the table, and the share of balls on one side of the line was counted from the verdicts. I include probability among the measures that every measurement involves. Every measurement is a comparison, so a probability is itself found by comparison. So a credence is a verdict of comparison, of the same kind as the verdict the cook read from her bowl of water, with a measure set on the verdict, and the measure was itself found by comparison.

Next come Carroll's planets of belief. Picture a whole that holds together with no first part. The picture is a stone arch over a doorway: no stone of the arch is laid first, and each stone is held by its neighbors. The stones of the arch hold together at once, with none of them first. Carroll's planet of belief has the same form: beliefs held together by how they fit one another, with no first belief at the bottom. Carroll finds the form of a whole with no first part in how systems of belief behave. I derive the same form from the act of comparison. In every act of finding the six parts of comparison occur together, in the same act, with none of them first. Every belief is built from acts of finding. I show the same form in what is built from acts of finding. The words of a dictionary hold one another up at once, because each word is defined by other words. In every laboratory, experiments are read with the help of the theories they test, so theory and experiment also depend on each other at once. The parts of knowing hold one another up at once, with none of them first.

Then comes Carroll's rule that all the evidence counts. Carroll asks that every piece of evidence be counted: "everything relevant should be brought to bear" (Chapter 10). He adds that the likelihoods are "meant to be objective", fixed by the evidence and not by the person, and he concludes that gathering more and more data "nudges everyone in the direction of the same set of ultimate beliefs about the world" (Chapter 10). I arrive by another route at a rule that agrees with Carroll's rule. When a second argument arrives by an independent route at a first argument's value, the second argument adds strength: the value is then surer than the value is by either route alone. Suppose you have two arguments that arrive at one value, and each argument is right nine times in ten. Join the

two arguments as the links of a chain, so that the value needs both arguments. Then the two together are right only eighty-one times in a hundred. If instead the two arguments are independent routes, each resting on different premises, the value is wrong only when both arguments are wrong at once. The chance of that is one tenth of one tenth, one in a hundred. So the two rules agree. Carroll's rule says to count every piece of evidence, and my rule shows why each independent piece of evidence makes the shared answer surer. In 1868 the American philosopher Charles Sanders Peirce pictured reasoning by independent routes as a cable. Peirce drew the picture in a paper on how philosophy should copy the methods of the successful sciences. Peirce wrote that philosophy's reasoning "should not form a chain which is no stronger than its weakest link, but a cable whose fibers may be ever so slender, provided they are sufficiently numerous and intimately connected" ("Some Consequences of Four Incapacities"). I use Peirce's picture of the cable for every inquiry into the world. Each independent route is one *strand* of the cable. Two strands that rest on one source count as one strand. So the strength of the cable lies in the number of its strands and in how independent their sources are.

Physics has a famous cable of its own. At the start of the twentieth century, physicists and chemists still disputed whether matter is made of atoms. The other side of the dispute held that atoms are only a useful way of speaking. In 1913 the French physicist Jean Perrin gathered thirteen different ways of counting the molecules in a given amount of matter. Perrin set out the thirteen ways in his book *Les Atomes*. The thirteen ways rested on phenomena as far apart as the jiggling of tiny grains suspended in water, the glow of hot bodies, radioactivity, and the electric charge carried by ions. An ion is an atom, or a small group of atoms, that carries an electric charge. The thirteen counts agreed. We now know that matter is made of atoms, and how many molecules a given amount of matter holds. I use Perrin's count as my case of convergence: independent routes, resting on different phenomena, that arrive at one value.

Last comes Carroll's view of what mathematics can prove. Carroll holds that mathematics proves what its assumptions imply, and that only looking tells us which assumptions hold of our world. I arrive at the same view by my own route. A *deduction* is a step from given premises to what must follow from the premises. Within its own structure a deduction is certain. The world, though, is not laid out by definitions: we come to know the world in measurement and in perception. So nothing guarantees the step from what a deduction proves to a claim about the world. The step is always open to some doubt. I therefore state each step from the structure of comparison (its six parts and the limit that the Incommensuration Theorem sets) to a quantity in the world as an *identification*. I check each identification against measurement. An identification is a statement made in the first person, in the form "I identify this with that": an identification says that one part of the structure of comparison is one thing in the world. A mapmaker makes a statement of the same kind as an identification when she says that this blue line on the map is that river on the land. The line on the map plays the part of the structure of comparison, and the river plays the part of the thing in the world. Anyone can walk to the river to check the mapmaker's statement, as anyone can check an identification against measurement.

So Carroll's account and mine agree on everything particular that we find out about the world. What we find comes with a measure of confidence, and evidence gathered from independent sources raises that measure.

The One Thing That Cannot Be Doubted

Is there anything, then, that we can be sure of beyond every doubt? I answer yes, and I say exactly what we can be sure of. Suppose you set out to doubt that the cook compared anything at all. You would hold her claim, "I compared", beside its opposite, weigh the two, and find one of them the likelier. Your doubt is a comparison, with a content, a context, a

verdict and a side the doubt is made from. Your doubt is built of the very act that it doubts. I call the act of comparison the *floor* of knowing, because every doubt of comparison is itself a comparison. A floor is what everything in a room stands on, and the floor is not one of the things standing in the room. The floor of knowing is like the floor of a room: the floor of knowing is what every belief stands on, and the floor of knowing is not one more belief. The floor of knowing is the act that every knowing is made of. The act of comparison is the one thing that cannot be doubted without being done in the doubting. This book's Chapter 1 gave the same act its name on the side of physics: for me, the act of comparison is the ground of physics, what lies at the bottom of physics, the thing every other description describes. The act of comparison is the floor of knowing and the ground of physics: one act, named from the side of knowing and from the side of physics.

In Chapter 11 Carroll tells of Descartes's certainty: Descartes concluded "that there is one belief about which skepticism is impossible: his own existence", since someone had to be doing the doubting. Carroll then extends the doubt to Descartes's belief in his own existence as well. Even the step from "I think" to "I am" could be a demon's trick. Carroll writes: "A sufficiently powerful and clever demon would be able to influence all of our appeals to logic and evidence" (Chapter 11, "Is It Okay to Doubt Everything?"). I set the act of comparison beside Descartes's certainty. Whoever rejects the six parts of comparison compares in rejecting them, and uses all six. One cannot doubt comparison without comparing. The doubt about the demon is made of comparison too. To suspect that a demon is tricking you is to set what seems sound beside what might be a trick.

Carroll's account has two ways of knowing: a claim of mathematics is proved from premises laid out in advance, and a claim about the world is found by looking, with a measure of doubt. I name a further kind of claim, held in the act of knowing itself. I define a performed claim: "A claim is performed when it is established by what happens in the act of engaging it, including the act of trying to deny it". To doubt that one is

doubting is to doubt, and so the very attempt to doubt the claim "I am doubting" settles the claim. Three claims of this chapter are performed claims: that every comparison has six parts in three pairs, that each pair is inseparable, and that symmetry and continuity cannot both be perfected in one comparison. Every check of the three claims is a comparison, and every challenge to the three claims is a comparison that uses them.

On how sure we can be, Carroll's account and mine part at certain knowledge. On Carroll's account every claim about the world comes with a measure of doubt. On Carroll's account, too, a proof in mathematics shows what follows from its assumptions and gives no fact about the world: "the things that math proves are not true facts about the actual world" (Chapter 16). I establish one kind of knowledge that is certain: the performed claims about the six parts and the limit that every act of knowing has. The parting at certain knowledge follows from the one place where the two accounts part, what lies at the ground, because what each account puts at the ground determines how the ground itself is known. For Carroll the ground is a quantum state evolving by one rule. On Carroll's account the quantum state, like everything in the world, is known by looking, with a measure of doubt. For me the ground is the act of comparison, and the act of comparison is made in every look, so the six parts and the limit of the act are known in the act of looking itself, as a performed claim. The certain knowledge that I establish is the structure that every act of knowing has. The structure is the same for every comparer: the bowl of water, the cook, and you. In Chapter 15 Carroll argues against "a kind of knowledge that is certain, which we should receive with docility", and of certain knowledge received with docility he writes, "There are no such kinds of knowledge. We can always be mistaken". Each reader holds my certainty by checking the certainty, and each reader checks the certainty in every act of checking. The reader receives the certainty from no one. We can always be mistaken about what we find. We cannot be mistaken that finding has six parts in three pairs and the limit that the Incommensuration Theorem sets. The reason is that to be mistaken is to have compared.

Certainty does happen: certainty happens in every act of finding. The certainty is about the six parts and the limit of the act, never about the content of what the act finds.

Look again, then, at Carroll's planet of belief. Carroll is right that the planet of belief rests on no fact about the world laid first. Carroll's account and mine agree that nothing found about the world is the floor. Every belief on the planet was formed from acts of finding, though, and every act of finding is a comparison with six parts in three pairs, limited as the Incommensuration Theorem says. The six parts and the limit are not a belief on the planet. Every belief on every planet of belief was made by acts of comparison that have the six parts and the limit. So every planet of belief has a floor: something under every belief that is not one more belief. The floor is the act of comparison, and the shape of the floor is the six parts and the limit that the Incommensuration Theorem sets. Carroll gives his Chapter 16 a question for its title: what can we know about the world without actually looking at it? I can also answer the question from the six parts and the limit. We can know the six parts and the limit of every look, because to know them is already to look.

What We Now Know

In the kitchen, the egg still lies on the bottom of the bowl. What the cook found about the egg is one thing found about the world, and her finding comes with its own measure of confidence, as Carroll says everything we find about the world does. The cook is sure of one thing beyond any measure: she compared. Price's newcomer at the window counts the sun's returns, and his expectation grows with every morning. The newcomer is never perfectly sure of tomorrow's sun. He is perfectly sure, each morning, that he looked.

Finding is comparison, with six parts in three pairs. We can be as sure of what we find as the evidence allows, counted from independent sources. We can be sure beyond every doubt of the structure of finding it-

self. Every knowing is a comparison. The act of comparison is the floor under every one of Carroll's planets of belief. The floor has six parts and the limit that the Incommensuration Theorem sets. What we now know matters for this book's question, what lies at the ground of physics. For me the ground of physics is the same act of comparison, the floor of knowing, and we now know that the act of comparison has a structure that can be known with certainty. A law of physics is found by comparison as well: a law states what stays the same when the circumstances of a measurement change, as the measuring cup holds one cup in every kitchen. So every law of physics was found by acts that have the six parts of comparison.

The laws that govern everyday life were found by acts of comparison: are those laws now completely known?



Chapter 3

Are the Laws of Everyday Life Completely Known?

What this chapter uses

Comparison: setting one thing beside another and finding them the same or different in some respect; a *finding* is one act of comparison that comes to a verdict, and *the one who finds* is whoever or whatever makes it (Preface and Chapter 1).

The six parts of every comparison: sameness and difference; *content*, what a verdict is about, and *context*, the setting in which it is found; the *subjective*, the side a comparison is made from, and the *objective*, the side it is about (Chapter 2).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *ground*: what lies at the bottom of physics, the thing every other description describes; for Carroll a quantum state evolving by one rule, and for me the act of comparison (Chapter 1).

In your kitchen, are the laws that govern everything completely known? This book's Chapter 2 showed that every law of physics was found by acts of comparison. An act of comparison sets one thing beside another and finds the two the same or different in some respect. In this chapter we ask whether acts of comparison have now found all the laws of everyday life. The physicist Sean Carroll says that the laws of everyday life are completely known. He dates the discovery of the last piece of the laws to one day.

A Day in Geneva

CERN is the European laboratory for particle physics, on the border of France and Switzerland near Geneva. On 4 July 2012, the teams of two experiments at CERN reported what they had found, in the laboratory's four-hundred-seat auditorium. Both experiments work at CERN's Large Hadron Collider, a ring underground in which beams of protons are brought into collision. Atomic nuclei are built of two kinds of particle: protons and neutrons. The two experiments are called ATLAS and CMS, and Fabiola Gianotti spoke for ATLAS and Joe Incandela for CMS. Each of the two experiments is named for its detector. One experiment, A Toroidal LHC Apparatus (ATLAS), is named for the ring shape of its magnets. The other experiment, the Compact Muon Solenoid (CMS), is named for its compact build, for the muons it is built to measure (the muon is a heavier cousin of the electron), and for its magnet, a solenoid, which is a long coil of wire. The letters CERN are the initials of the council from which the laboratory grew: the Conseil Européen pour la Recherche Nucléaire (in English, the European Council for Nuclear Research), and the laboratory kept the letters CERN when the laboratory took its present name.

Peter Higgs, a physicist of eighty-three, sat in the audience. In 1964 Higgs had proposed a field that fills all of space and gives a mass to particles that would otherwise have none. François Englert and Robert

Brout proposed the same field in the same year. The field implies a particle of its own, and for forty-eight years no one had found the particle. Englert was in the auditorium as well.

Gianotti and Incandela reported the same result. Each team had sorted the debris of a great many collisions by mass. At one mass each team had counted more collisions than the known particles alone would give, and an excess of collisions at one mass is the mark of a new particle. Incandela said: "We have observed a new boson". A *boson* is a particle of a kind that can crowd together without limit. Any number of bosons can be in the same place, doing the same thing, as the particles of light are in a laser beam. Every carrier of a force is a boson. The particles of matter, such as the electron, are of the other kind. No two particles of matter can do exactly the same thing in the same place. The audience applauded, and stood to salute Higgs and Englert. Higgs took off his glasses, dabbed his face, and held a tissue. Higgs said: "It's an incredible thing that it's happened in my lifetime". CERN's Director General, Rolf Heuer, summed up the day: "We have reached a milestone in our understanding of nature".

Now set the day in Geneva beside a smaller event, in a kitchen. In a kitchen this morning, a cook takes an egg and cracks it once on the rim of a bowl. The crack the cook makes is one act of finding. The act is made from one side, the cook's, and it is about one thing, the egg. The act has one outcome, this crack, and the act cannot be undone. Many particles take part in the crack. *Quarks* are the small particles of which protons and neutrons are built, three to each. The shell, the white and the yolk take part through their electrons and quarks, and so do the particles of the bowl's glazed rim, of the cook's hand, and of the eye that sees the shell give way. Carroll claims that every one of the particles in the crack obeys laws that are now known in full. Carroll calls the laws the Core Theory, and the particle announced in Geneva was the last piece of the Core Theory to be found.

From Carroll's pages

What Carroll Shows

On the best understanding that physics now has, what is the stuff of the egg? Carroll's answer is fields. A *field* is a quantity with a value at every point of space. The temperature of the air in a room is an example: the temperature has a value at every place in the room. *Quantum field theory* is the rules of quantum mechanics applied to fields. *The Particle at the End of the Universe* is Carroll's book on the finding of the Higgs boson. Carroll writes in the book: "According to quantum field theory, absolutely everything is made of a field or a combination of fields" (Chapter 2, "Next to Godliness", section "Fields pervade the universe"). In *The Big Picture* Carroll says: "The fields are the stuff that everything else is made of" (Chapter 22, "The Core Theory").

Then what is a particle? On Carroll's account a *particle* is a small ringing of a field: the least amount by which a field can vibrate. A string tied at both ends vibrates only at certain pitches. In the same way, the field of the electron rings only in whole steps, each step one electron. *The Biggest Ideas in the Universe 2: Quanta and Fields* is the volume of Carroll's that teaches quantum fields with the equations left in, and in that volume Carroll says what particles are: "this is what particles really are, quantized excitations of fields" (Chapter 4, "Fields", section "Particles from Fields"). A quantized excitation is a vibration that comes only in whole steps. The steps come from the smooth equations of the field themselves, as the pitches of the string come from the string's being tied at both ends. Carroll draws the lesson in Chapter 1 of *Quanta and Fields* (section "The Simple Harmonic Oscillator"): "The ultimate irony of quantum mechanics is that there's nothing fundamentally 'quantum' about it".

The forces between particles come from fields too, and Carroll shows how the forces come from fields through gauge symmetry. A *gauge symmetry* is a change of description that can be made separately at

every place without changing anything that can be found. New fields are required to keep a gauge symmetry consistent from place to place, and the new fields carry the forces. One of the forces is the *weak force*: the force that turns one kind of particle into another, as in some radioactive decays. The particles of the new fields are the photon, the particle of light, and the eight gluons, which bind quarks together. The new fields also have the three carriers of the weak force: two W bosons (one with a positive electric charge and one with a negative) and one Z boson. Carroll writes of the gauge symmetries that "Implementing them consistently leads directly to forces between particles of matter" (*Quanta and Fields*, Chapter 9, "Gauge Theory", section "Connections").

The field that Higgs proposed is different from the other fields. Most fields have the value zero in empty space. The *Higgs field* is the field whose value is not zero in empty space. The Higgs field gives mass to the elementary particles, the particles not built from smaller ones. Carroll pictures the Higgs field with a pendulum turned upside down, its pivot on the floor. When the pendulum stands straight up, it has energy to lose. So the pendulum falls and comes to rest leaning to the left or to the right. The pendulum standing straight up is the picture of the Higgs field at the value zero. Carroll writes that the Higgs field is like the pendulum "in that it actually costs energy to be at zero" (*The Particle at the End of the Universe*, Chapter 8, "Through a Broken Mirror", section "Pushed away from zero"). So the Higgs field comes to rest away from zero, as the pendulum comes to rest leaning to one side. The field's resting value is the same everywhere in empty space. An elementary particle's mass is the settled value of the Higgs field times the strength with which the particle couples to the field, that is, how strongly the particle and the field act on each other.

Carroll is careful about how much of the mass of everyday things comes from the Higgs field. The egg in the cook's hand gets most of its mass from the energy that binds quarks together inside protons

and neutrons. Only a small part comes from the Higgs field. In Carroll's words, "it's misleadingly sloppy to say, 'The Higgs is responsible for mass', as we physicists sometimes do" (*The Particle at the End of the Universe*, Chapter 8, section "A world without Higgs").

Put the fields, their particles, their forces and the Higgs field together. Add gravity where gravity is weak, as gravity is everywhere in a kitchen, and the whole has a name. The *Standard Model* is the established theory of the kinds of particle and of the three forces that act among them, the forces carried by the photon, the gluons, and the W and Z bosons. The *Core Theory* is Carroll's name, taken from the physicist Frank Wilczek, for the quantum field theory of the Standard Model together with gravity where gravity is weak. Carroll states his claim about the Core Theory in *The Big Picture*, Chapter 22: "The laws of physics underlying everyday life are completely known".

How can anyone know that the laws of everyday life are completely known? Carroll gives his argument for the claim in four steps, in *The Big Picture*, Chapters 23 and 24. He gives the argument again in *Quanta and Fields*, Chapter 12.

Carroll's first step is that the framework for describing the everyday world must be quantum field theory. *Special relativity* is Einstein's theory of 1905, in which light has the same speed for everyone who measures it and nothing travels faster. Take any theory that joins quantum mechanics with special relativity, and in which regions far apart behave independently of one another. The physicist Steven Weinberg argued that every theory of that kind looks like a quantum field theory at long distances and low energies, and Carroll says how long and how low: "say, anything bigger than a proton" (*The Big Picture*, Chapter 23, "The Stuff of Which We Are Made"). So Carroll concludes that, for the everyday world, "we should work in the framework of quantum field theory" (*The Big Picture*, Chapter 23).

Carroll's second step uses a rule of quantum field theory that Carroll calls crossing symmetry. An electron's antiparticle is its mirror partner, with the opposite electric charge. Physicists call the electron's antiparticle the positron. Crossing symmetry says that any particle that can strike an electron can also be made by colliding an electron with a positron. From 1989 to 2000 the Large Electron-Positron Collider near Geneva collided electrons with positrons. Carroll reports that the experiments at the Large Electron-Positron Collider found only the known particles, so he concludes, of any particle that could affect the stuff of everyday life, that "we would have produced it in experiments by now" (*The Big Picture*, Chapter 23).

Carroll's third step is the search for a new force, and he writes that any new force reaching farther than a tenth of a centimeter "would have to be substantially weaker than the force of gravity" (*The Big Picture*, Chapter 23).

Carroll's fourth step uses effective field theory. An *effective field theory* is a description that holds within a range of energies and says where its range ends. Higher energies and smaller distances lie beyond the range of the Core Theory. Whatever lies beyond the range can affect the kitchen only through the particles and forces of the Core Theory. In Carroll's words, "you and I and the world we interact with depend only on the Core Theory" (*Quanta and Fields*, Chapter 12, "Atoms", section "The Physics Underlying the Everyday World").

Carroll states the scope of his claim beside the claim. He writes: "It certainly does not imply that we know all of physics" (*The Big Picture*, Chapter 24, "The Effective Theory of the Everyday World"): dark matter, black holes and the beginning of the universe lie outside the range of the Core Theory. *Dark matter* is matter that gives off no light and is known only by the pull of its gravity on stars and galaxies. A *black hole* is a region where gravity is so strong that nothing, not even light, comes out of it. Carroll's claim does not give us biology or the weather either. Biology and the weather are patterns of very many particles,

and they need descriptions of their own. Carroll also judges his claim from evidence and holds the claim with very high confidence, and he says so: "science never proves things" (*The Big Picture*, Chapter 22). Within the scope that Carroll states, his claim is complete, and the particle announced in Geneva was the last piece the claim needed. Carroll writes: "With the discovery of the Higgs boson, our understanding of the physics underlying everyday reality is complete" (*The Particle at the End of the Universe*, Chapter 13, "Making It Worth Defending", section "Meaning and truth").

Carroll's claim is a large achievement, and he states the claim no larger and no smaller than his evidence makes it. Carroll has drawn a line around the everyday world, and he has shown, step by step, why everything inside the line obeys the known laws.

The Same Laws, by Another Route

Do I accept the Core Theory? Yes: I accept the Core Theory. I have written a chapter on Carroll, "The Wave Function and the One Who Finds". Carroll's claim that the laws of everyday life are completely known is one my account grants as he states it, and my account adds no force or field to those laws. In the egg, the bowl and the cook, every force at work is a force of the Core Theory.

I derive the same laws by another route, from another starting point. My starting point is the act of comparison: setting one thing beside another and finding them the same or different in some respect. This book's Chapter 2 named six parts of every comparison. Two of the six are needed in this chapter. The *content* of a comparison is what the verdict is about, and its *context* is what makes the two things compared two. When the cook in Chapter 2 set an egg in a bowl of water, the content was weight, and the context was inside and outside the shell. Two acts of finding are two because one is here and the other there, or one is now and the other

then. So space and time are the widest context of physics. *General relativity* is Einstein's theory of 1915, in which gravity is the shape of space and time. The widest context of physics is the *arena*: space and time joined, with the shape that general relativity gives them. The arena is the setting in which every act of finding is made. In a comparison made within the arena, the verdict is about something found within the arena, so whatever is found within the arena is content: what some comparison is about. I put every field I name, light included, on the side of content. Each field is content in a wider sense: something found within the arena, as distinct from the arena itself.

I arrive at Carroll's picture of particles from the act of comparison: a field is something spread through all of space, whose smallest ripples are the particles. I build the mathematics of quantum states from the structure of comparison. In Chapter 6 I find that the structure built from comparison "encodes the Standard Model coupled to gravity". The structure contains the whole of the Standard Model, acting together with gravity, written in the structure's own mathematics.

My forces are the same as Carroll's, too. My route to the forces takes four steps. Picture a compass needle pointing north, and bring a magnet near the needle: the needle swings, and now it points another way.

The first step is that a force is a change: "something was one way, and after the interaction, it is another way" (Section I, "Three Ways, One Per Position"), and the same section goes on: "a change is itself an act of comparison: it relates what was before to what is after". You know that the needle swung only by setting where it points now beside where it pointed before.

In the second step I count what a change can act on inside a particle of matter. A particle of matter is content, found within the arena, and a particle of matter has a structure inside it, so I call a particle of matter structured content, and say: "structured content carries three things a comparison can act upon". The first of the three things is the *color* of a quark, a label with three values. The color of a quark has nothing to do

with the colors we see. The second thing is the two sides of a pair that the weak force turns into each other, such as the electron and its neutrino, a light partner of the electron with no electric charge. The third thing is the *phase*, which I picture as "a setting on a circle, like the hand of a clock".

In the third step, each of the three things has one way of changing. A particle of matter contains nothing more for a change to act on. So there is nothing else inside structured content for a transformation to act upon. So there are exactly three forces.

In the fourth step I count the carriers of each force from the thing the force acts on. Among three colors I count nine turnings, three times three. I set aside the one turning that shifts the phase of all three colors together and turns no color into another. So the *strong force* has eight carriers, and the strong force is the force that binds quarks together. Between two sides I count four turnings, two times two, and in the same way set aside the one turning that shifts the phase of both sides together, so the weak force has three carriers. A single setting on a circle has one turning, with nothing to set aside. So the third force, the force on the phase, has one carrier, and at everyday energies we know the third force as electromagnetism. Eight, three and one is the same count as the carriers Carroll lists, the eight gluons, the three carriers of the weak force and the photon, and I derive the count from a separate basis: the act of comparison.

On mass I say what Carroll says, with the same care. When you weigh yourself, you are mostly weighing the binding of quarks. Carroll makes the same point of a desk or a person, and the point holds for the egg in the cook's hand.

I share Carroll's lesson from the tied string: the steps of a particle come from smooth equations. Something is *discrete* when it comes in separate steps with nothing between them, as the rungs of a ladder do. I also have a chapter on the physicist Sabine Hossenfelder. Hossenfelder and Carroll are right that quantum mechanics is not, at root, about discreteness.

I also use Carroll's own tool, effective field theory, in the same sense as Carroll. I call an effective field theory "a description of particles and their forces at low energies that does not depend on the details of what happens at much higher energies".

Carroll's account and mine arrive at one set of laws for the kitchen. Carroll arrives at the laws from fields taken as the stuff of the world, and from four steps of evidence. I arrive at the same laws from the act of comparison. On my route, whatever is found within the arena is content, a force is a change, and a change is itself a comparison. Inside a particle of matter there are three things a change can act on, the color, the two sides and the phase, and the three things give the three forces and their carriers. The two routes start from two places and arrive at the same laws. A result reached by two separate routes is held more firmly than a result reached by one. This book's Chapter 2 showed the reason: a result reached by two routes is wrong only if both routes are wrong at once.

What the Measurements Found

What did the two experiments at CERN, ATLAS and CMS, measure, and what do we know now that we did not know before the seminar of 4 July 2012?

Each team published what it had found in the same issue of the journal *Physics Letters B*, volume 716, on 17 September 2012. Physicists state masses in units of energy. An electron volt is the energy one electron gains in crossing one volt. The ATLAS team measured the mass of the new particle at 126.0 billion electron volts, and a mass of 126 billion electron volts is a little more than 130 times the mass of a proton. The CMS team measured 125.3 billion electron volts, the same mass within the precision of the two measurements. At the mass of the new particle, each experiment's excess of collisions was far larger than chance could make. For AT-

LAS, the odds that the known particles alone would produce an excess that large were less than two in a billion. Separate teams built and ran the two detectors, and the two detectors had found one particle at one mass.

The two teams then measured the particle closely for ten years. In 2022 the two teams each published in *Nature* a map of the particle's interactions. Each map was made with more than thirty times as many of the particles as in 2012. The particles of matter come in three families, three copies alike in charge and differing in mass. The ATLAS team measured the particle's interactions with the carriers of force and with three particles of matter from the third and heaviest family, and found the particle "remarkably consistent with the predictions of the theory", the theory being the Standard Model. The CMS team found that "all these observations are compatible with the predictions of the standard model". In the Standard Model, a particle is heavier the more strongly the particle couples to the Higgs field. The measured couplings fell where the Standard Model places them.

Carroll's second and third steps rest on measurements as well: the second step rests on the collisions at the Large Electron-Positron Collider, and the third on the search for a new force. The Large Electron-Positron Collider ran from July 1989 to November 2000 and made millions of Z bosons. A *neutrino* is a particle with no electric charge and almost no mass, which passes through ordinary matter almost without touching it. A Z boson has one more way to decay for each kind of neutrino light enough to come out of its decay, and so a shorter life. So the physicists at the Large Electron-Positron Collider counted the light kinds of neutrino by measuring how short the life of the Z boson is, and the count was three: one light neutrino to each family, and so three families of matter of the known kind. In 2007 Daniel Kapner, Eric Adelberger and their colleagues tested Newton's law of gravity with torsion balances. A torsion balance is a pendulum hung on a fine fiber, and the pendulum twists under the faintest pull. Kapner, Adelberger and their colleagues measured the pull at separations down to 55 thousandths of a millimeter, found no departure from Newton's law, and found no new force.

In 2013 the Nobel Prize in Physics went to François Englert and Peter Higgs, "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles". We now know three things. The Higgs field fills empty space. The particle of the Higgs field has the mass measured in Geneva. The Standard Model agrees with every interaction of the particle measured in the ten years after the seminar. Carroll and I read the same measurements, and they agree on what the measurements show.

Why the Higgs Field Is There

The Core Theory includes the Higgs field, and Carroll records two facts about the Higgs field with no reason behind them. The first fact is that the Higgs field exists at all. Carroll says of the Higgs: "There is no deep principle that requires its existence, but it exists anyway" (*The Particle at the End of the Universe*, Chapter 2, section "Fields pervade the universe"). The second fact is that the field settles away from zero, as Carroll's upside-down pendulum falls to one side. Carroll gives no reason why his pendulum stands upside down: "at the present state of knowledge it's just a brute fact about the universe" (*The Particle at the End of the Universe*, Chapter 8, section "Pushed away from zero"). A *brute fact* is a fact with no further reason behind it.

I give a reason for each of the two facts. The Higgs field is required by the pairs of matter that the weak force turns into each other, such as the electron and its neutrino. Look at the page in front of you. It has two faces, the one you are reading and the one behind it. The two faces are distinct, since they carry different words, and they are inseparable, since you cannot have the one without the other. Turning the page treats them alike: either face can be brought toward you, and the page is the same page whichever face you see. The pairs of matter are built the same way: the electron is not its neutrino, and yet the weak force turns the one into the other, and a turning treats what it turns as interchangeable. The two sides

of each pair are distinct, yet the weak force treats the two sides as interchangeable, and so a field must hold, at every place, which side is which. The Incommensuration Theorem requires the field's value away from zero. The Incommensuration Theorem, stated in Chapter 2 and recalled in the box at the head of this chapter, says that symmetry (sameness kept across a change) and continuity (no gap and no least step) cannot both be made perfect in one comparison. The laws treat the two sides of each pair alike, and that sameness is a symmetry: a sameness kept across the exchange of one side for the other. The laws treat the two sides alike, and the Incommensuration Theorem does not let that sameness be exact in empty space as well, so below a certain energy the field settles everywhere at once into a value that picks a side. Empty space is the arena, which has no least step (this book's Chapter 7), and the field settles everywhere at once; so it is the exact sameness of the two sides that empty space gives up, and its smoothness that it keeps. The energy below which the field settles is far above any energy in a kitchen, so in all of empty space today the field has settled. I picture the settling of the field with the bottom of a wine bottle. A ball balanced on the raised center rolls off into the ring-shaped valley around it and comes to rest at one point. The bottle is the same in every direction, as the laws are the same for both sides. The ball's resting place lies in one direction only, as empty space has picked a side. The mass of the field's particle comes from measurement, and the ATLAS and CMS teams at CERN measured the mass. Appendix B sets out both steps in full, with a picture for each step.

On the Higgs field, Carroll's account and mine part at Carroll's two facts with no reason behind them, and that parting follows from the one place where they part: what lies at the ground. For Carroll the ground is the quantum state, the complete quantum description of the world, evolving by one rule. On Carroll's account the Higgs field is one of the fields the quantum state describes, with no further principle behind the Higgs field. For me the ground is the act of comparison, whose structure requires the field before any experiment searches for it.

The Kinds of Content, and the Arena

Complete laws for the kitchen come with a list of the kinds of particle the laws are about. Carroll's list and my list agree on everything that touches the egg, but the two lists differ on two points: whether gravity has a particle of its own on the list, and whether the list is left open or is closed.

Carroll counts gravity among the fields of the Core Theory, with a particle of gravity of its own, the *graviton*. He writes: "Its associated particle is of course the graviton" (*Quanta and Fields*, Chapter 12, section "The Core Theory"). Carroll leaves his list open beyond the kitchen. He writes: "There is plenty of room for new particles and forces, but only ones that interact so weakly or briefly with ordinary matter that we can't perceive them without billions of dollars' worth of apparatus" (*The Particle at the End of the Universe*, Chapter 13, section "Meaning and truth"). On Carroll's account the Core Theory lists every kind of particle that touches everyday matter, and further kinds may exist that do not.

My list is closed. No further kinds exist. I name four of the kinds that I exclude with the sentence "No further kinds exist": a fourth family of matter, a particle of gravity, a particle of dark matter of any type, and the partner particles of supersymmetry. *Supersymmetry* is a proposed theory in which every known particle has a partner of the other kind. Each particle of matter gets a boson for a partner, and each boson gets a particle of matter. I place the Higgs at the end of the list: "the Higgs is the final elementary particle because it is the last thing that the structure of comparison requires before the composites begin". The composites are the particles built from others, as the proton is built from quarks. The ATLAS and CMS teams reported the Higgs particle at CERN on 4 July 2012. The Higgs particle is the last kind of content there is.

I give gravity a place of its own. Gravity is not a fourth force with a carrier in this sequence. It is the arena. It is context, not content. Gravity is the arena, the setting in which every act of finding is made, and gravity is not one of the things found within the arena. Where gravity is weak, as gravity is in every kitchen, the arena has the shape general relativity gives it, and the Core Theory uses the same general relativity for the kitchen. The white and the yolk fall into the bowl by the same law on both accounts.

My list has two parts, and each part has its own basis. The first part is my closed list, on which the kinds of particle of the Standard Model are the whole of the content, everything that can be found within the arena. The second part is gravity, which I place as context and not as content. Gravity is the arena itself, and so gravity has no particle of its own among the kinds of content.

On the list of what there is, Carroll's account and mine part on whether gravity has a particle of its own and on whether the list is closed, and that parting follows from the one place where they part: what lies at the ground. For Carroll the ground is the quantum state evolving by one rule, and the field of gravity is one of the fields the quantum state describes, with a particle of its own. For me the ground is the act of comparison, whose context is the arena and whose content is what is found within the arena, so gravity has no particle among the kinds of content, because gravity is the arena. So my closed list covers more than the Core Theory covers: on the Core Theory, the question whether there are kinds of particle that do not touch everyday matter remains open, and the closed list says there are none. For the egg, the bowl and the cook, the two lists are the same list.

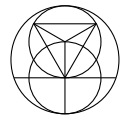
Complete Laws

Go back to the kitchen. The cook cracks the egg on the rim of the bowl, and every particle in the shell, the glaze and the hand moves by the laws that Carroll names the Core Theory. On either account, the known particles and the known forces are enough to tell how the egg cracks.

We now know how Carroll and I each answer Carroll's question, whether the laws of everyday life are completely known. I grant the Core Theory as Carroll states it, and I add no force or field to the Core Theory. On my account, the kinds of particle of the Standard Model are the kinds of content. Gravity is the arena in which the kinds of content are found.

For this book's question, what lies at the ground, the agreement on the laws matters for one reason. Carroll and I agree on the laws. So the whole difference between the two accounts lies outside the laws. Carroll and I read the same laws and the same measurements, so whatever divides the two accounts lies in what each account says the laws fix.

If the laws are complete, is everything that happens already fixed by them?



Chapter 4

If the Laws Are Known, Is Everything Already Fixed?

What this chapter uses

The four characters of a comparison: *continuity*, a sameness of content where there is a sameness of context; *discontinuity*, a difference of content where there is a sameness of context; *symmetry*, a sameness of content where there is a difference of context; *asymmetry*, a difference of content where there is a difference of context (Chapter 2).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *wave function*: Carroll's name for the complete quantum description of a system, which he holds is the system itself (Preface and Chapter 1).

The *ground*, and the two words *exist*, to stand as structure, the same in every instance, and *real*, to be given in an actual act of comparison, here and now (Chapter 1).

In Chapter 3 I granted Sean Carroll his claim that the laws of everyday life are completely known. If the laws are completely known, is everything that happens already fixed by the laws, down to the way one egg breaks in one kitchen this morning? The question is older than any of Carroll's books. The French mathematician Pierre-Simon Laplace wrote the question's clearest form in Paris, more than two hundred years ago.

A Vast Intellect in Paris

Who drew the clearest picture of a world fixed in every detail? In 1814 Laplace published a small book in Paris, printed by the house of the widow Courcier. The book's title was *Essai philosophique sur les probabilités*, a philosophical essay on probabilities. Laplace had grown the essay from a lecture on probability that he gave in 1795 at the École normale. The École normale had opened in Paris in 1795, to train teachers. In the same year, 1814, Laplace set the essay at the head of the second edition of his large mathematical treatise on chance. The treatise was the *Théorie analytique des probabilités*. Laplace had spent most of his working life on the heavens. Laplace had worked out from Newton's law of gravity how the planets pull on one another, and he had shown that the orbits of the planets stay steady over long ages. Laplace wrote a five-volume treatise on the mechanics of the heavens, the *Traité de mécanique céleste*. The volumes had begun to appear in 1799.

Laplace set down the picture of a world fixed in every detail in a chapter early in the essay. The standard English translation calls the chapter "Concerning Probability". Frederick Wilson Truscott and Frederick Lincoln Emory made the standard translation in 1902. In their translation, Laplace's picture begins: "We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow". Laplace then asked his reader to imagine "an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it". The intelli-

gence would be able to work out, all at once, everything that follows from all the forces and all the situations. Of the imagined intelligence Laplace wrote that it "would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes". Later readers gave Laplace's imagined intelligence a shorter name: Laplace's demon.

The sentences Laplace wrote next show why the demon comes at the start of a book about chance. Laplace wrote: "The curve described by a simple molecule of air or vapor is regulated in a manner just as certain as the planetary orbits; the only difference between them is that which comes from our ignorance". And then he wrote: "Probability is relative, in part to this ignorance, in part to our knowledge". For Laplace, the odds of an event measured what we do not know, so on his picture the world itself had no odds in it: the world had only what would happen, fixed already.

Now come into a kitchen. A cook takes an egg from the carton and strikes the egg once on the rim of a bowl, and the shell gives way along a line that starts at the point of the strike and runs off at an angle. The cook finds the shell cracked along just the line where the shell gave way. Laplace's intelligence would know the position and the velocity of every particle of the shell, the rim and the cook's hand. So the intelligence would have seen this crack before the hand moved. For Laplace's intelligence, nothing about how this egg breaks would be open: there would be only this crack, fixed from the beginning. From long practice the cook knows something different. The cook knows how shells tend to break, over many eggs, and that no one can say beforehand which way this shell will break. We can now put the chapter's question in the kitchen. Do the laws of physics fix this crack, or do the laws say only how shells break over many eggs?

From Carroll's pages

What Sean Carroll Showed

How does a physicist today state Laplace's picture? Carroll states the picture in the chapter "Change" (Chapter 2) of *The Biggest Ideas in the Universe: Space, Time, and Motion*. *Classical physics* is the physics of Newton and Laplace, before quantum theory. Carroll writes that classical physics describes change by a framework he calls the Laplacian paradigm. Carroll gives the Laplacian paradigm as a short list of four steps. The first step is "Specify the complete state of the system at one moment in time". In Laplace's sense, a system's *complete state* at one moment is everything about the system at that moment. For Newton's physics, the complete state is the position and the velocity of every part. The second step is to use the laws of physics to work out what the state will be one moment later, or one moment earlier. The third step is to use the laws again, to go on from the new moment to the next one. The fourth step is "Repeat". Carroll sums up the picture: "It's just one thing after another, where the next thing is determined by the current thing and the laws of physics".

Carroll also shows what the Laplacian picture replaced. The Greek philosopher Aristotle held that every moving thing needs a mover. In *The Big Picture*, Chapter 3, "The World Moves by Itself", Carroll tells how physicists overturned Aristotle's rule when they found that *momentum* (the quantity of motion a body carries, its mass times its velocity) is conserved. The momentum of a body stays the same unless something acts on the body. In the physics of Galileo and Newton, a body left alone keeps moving with nothing to keep pushing the body, and Carroll writes of the universe as a whole: "The universe doesn't need a push; it can just keep going". Physics moved, in Carroll's telling, from a world explained by the natures and purposes of things to a world explained by patterns. Carroll calls the patterns the laws of nature.

Carroll gives Laplace's answer to the question in the title of Chapter 4 of *The Big Picture*, "What Determines What Will Happen Next?" The answer is "The state of the universe right now". Carroll gives the claim in Laplace's answer a name, determinism. He describes determinism as "the fact that the current state of the universe determines its future". In Carroll's sense, *determinism* is the claim that the complete state of the universe at one moment and the laws together fix every other moment, earlier and later. Carroll then shows how much of physics is deterministic. He writes that every serious attempt since Laplace to understand the universe at a deep level has had the feature that "the past and future are determined by the present state of the system". Carroll's examples include fields, Einstein's general relativity and the Schrödinger equation of quantum mechanics. *General relativity* is Einstein's theory of gravity, in which gravity is the shape of space and time, and the Schrödinger equation is the one rule by which a quantum description changes in time. Carroll names one possible exception, in a parenthesis: "the collapse of the wave function in quantum mechanics". On some accounts of quantum mechanics, the quantum description of a system changes suddenly when someone measures the system. The sudden change at measurement is the collapse. Carroll gives determinism a second name, *conservation of information*, and he defines the second name: "conservation of information implies that each moment contains precisely the right amount of information to determine every other moment". The information in Carroll's definition is all of the information. The information is everything there is to specify about the complete state, whether or not anyone knows that information.

Classical mechanics is Newton's laws of motion and the classical physics built on those laws, which Carroll calls "the system of equations studied by Newton and Laplace" (*The Big Picture*, Chapter 4). Carroll says what the thought experiment of Laplace's demon shows: the universe itself holds the information the demon would need, whatever

any being could know (*Space, Time, and Motion*, Chapter 1, "Conservation", section "Classical Mechanics"). Carroll writes that no realistic being could know the positions and velocities of every atom in a grain of sand, and he goes on: "But the universe itself possesses that information, and the laws of classical mechanics predict that it is conserved over time".

In *The Big Picture*, Chapter 4, Carroll clears away two misunderstandings of determinism: chaos and fate. Between the two, he turns to what he calls "the real issue". The first misunderstanding is chaos, which Carroll calls "a more popular objection to determinism". In many systems, a very small error grows into a very large one: the small error is in what we know of the starting state, and the large error is in where the system ends up. Chaos is the growth of the small error into the large one. Carroll says that chaos does not count against determinism at all: "Laplace's point was always that perfect information leads to perfect prediction".

The issue Carroll calls real is quantum mechanics. He states the issue in one sentence: "The real issue with classical mechanics is that it's not how the world works". Carroll writes that quantum mechanics "has supplanted classical mechanics as the best way we know to talk about the universe at a deep level". Carroll writes that the wave function "isn't a bookkeeping device; it's an exact representation of the quantum system" (*Something Deeply Hidden*, Chapter 2, "The Courageous Formulation"). When a system is left alone, the system's quantum state changes in a perfectly deterministic way. Yet each observation of the system seems to come out at random. Physicists hold several competing approaches to the puzzle of a state that changes deterministically and observations that seem random. Carroll describes the approaches with his own preference among them: "Some involve true randomness, while others (such as my favorite, the Everett or Many-Worlds formulation) retain complete determinism". Carroll sets true randomness against complete determinism. Carroll also notes

what all the popular versions of quantum mechanics keep from Laplace, even the versions that give up perfect prediction: "what matters, in predicting what will happen next, is the current state of the universe".

The second misunderstanding is fate. Carroll finds "a bit of a mismatch" between Laplace's determinism and what most people think of when they hear that the future is determined: destiny, or fate. Determinism in physics is no oracle holding the script of your life. The reason is that no one holds the script: no one knows what future the present fixes. In Carroll's words, "the future may be determined by the present, but literally nobody knows what it will be".

In *The Big Picture*, Chapter 4, Carroll answers the worry that determinism leaves no room for people to choose, and he calls the worry "not really an issue of physics, but one of description". He writes: "Our best theories of people, presented on their own terms and without reference to underlying particles and forces, leave plenty of room for human choice". In *The Big Picture*, Chapter 44, "Freedom to Choose", Carroll counts himself among the *compatibilists*: people who hold that the laws of physics govern every particle, and that choosing is still the right description of what people do.

A reader learns a great deal from Carroll's account. Carroll keeps determinism apart from fate. He keeps a claim of principle apart from a claim of practice. Perfect information would give perfect prediction, while under chaos our imperfect information gives very imperfect prediction. He keeps the choices of people apart from the motions of particles. And he shows how physics moved from movers and purposes to patterns and laws. A reader who has followed Carroll knows what the Laplacian picture says, and why physicists have held the Laplacian picture for two centuries.

Fields are quantities with a value at every point of space (this book's Chapter 3 describes fields). I take his phrase "true randomness" in its plain sense: an outcome of a measurement that the quantum state before the measurement leaves unfixed.

Where the Collection Agrees

How much of Carroll's account do I keep? I start in a different place from Carroll: the act of comparison. Every finding of physics is made in an act of comparison. I derive several of Carroll's results from the act of comparison, for reasons of my own.

I grant Carroll the laws of everyday life. The Wave Function and the One Who Finds. I keep Carroll's equations as he writes them. I read the quantum state of the Schrödinger equation as the range of what could be found. The equation carries the state, the range of what could be found, into its successor without fail; what a measurement finds is one outcome from that range, and the equation does not fix which. Because I keep Carroll's equation, Carroll and I expect the same odds from every experiment.

I agree with Carroll that no one knows what the future will be. Carroll's reason is that Laplace's demon does not exist: no being holds the complete state of the universe. My reason is three limits that apply to every finite knower, and I name the three limits.

Look up at a star on a clear night. The light that reaches your eye left the star long ago. So you cannot yet see what the star is doing now. The star shows the first limit: a knower cannot know what is happening now in places so far away that the light from those places has not yet arrived at the knower. The second limit is the *uncertainty relation*. The uncertainty relation is the rule of quantum physics that a particle's position and its momentum cannot both be fixed past a certain sharpness. Pin down more sharply where a particle is, and the range of the particle's possible motions spreads wider. The third limit is the direction of time. Every act of

comparison runs one way: from the range of what could be found, to the act, to the record the act leaves. So a knower holds records only of acts already made, and no record of an act still to come. (This book's Chapter 8 describes the direction of time and the reason for the direction.) I conclude: "So a finite observer cannot know its own future completely". The direction of time rules out a complete prediction directly, because a knower holds no record of an act still to come. The other two limits rule out a complete prediction by keeping from the knower things that a complete prediction would need: the delay of light keeps the far things from the knower, and the uncertainty relation keeps the small things. So Carroll and I arrive at one result, that no one knows what the future will be, for two separate reasons.

I also agree with Carroll that the universe needs nothing outside itself to set the universe going or to keep the universe going. Carroll's result is about motion. Aristotle held that every moving thing needs a mover, and the conservation of momentum shows that a moving thing needs no mover. A body left alone keeps moving, and Carroll's line "The universe doesn't need a push" says so of the universe as a whole. My result is about causes, and my result is wider than Carroll's: someone finds a cause by comparing one event with another event inside the world, and the whole world has nothing outside the world to be compared with. A consequence follows: there is no cause of the universe. Neither Carroll's account nor mine puts a mover or a cause behind the universe as a whole. The question of this chapter therefore becomes narrower. Is this crack fixed by the laws of the world and the complete state of the world at one moment?

I also agree with a warning that Carroll gives in *Space, Time, and Motion* (Chapter 2). In the section "Continuity and Infinity", Carroll asks whether time might come in least steps, and he warns: "But we shouldn't confuse what we human beings can hold in our heads with what nature actually does". I hold the same view, and I give a reason for the view. A description and the thing the description describes always differ in kind. The description reaches what it describes as closely as one wishes without

ever containing it. In the section "Two Perfections That Cannot Be Had Together", I build one step on the agreement that a description never contains what it describes. The step says that a feature of a description, such as the ease with which the description runs backward, belongs to the description, and that the world need not share the feature.

Two Perfections That Cannot Be Had Together

Where, then, do Carroll's account and mine part? On the subject of this chapter, Carroll's account and mine part at one claim, whether one law can fix a whole world, and that parting follows from the one place where they part: what lies at the ground. For Carroll the ground of physics is one quantum state changing by one rule, while for me the ground is the act of comparison, which has a structure of its own. I hold that a theorem about the structure of comparison shows that a whole world cannot be fixed by one law. The Incommensuration Theorem, stated in Chapter 2 and recalled in the box at the head of this chapter, says that symmetry and continuity cannot both be made perfect in one comparison, and neither can asymmetry and discontinuity. We need two of the theorem's words again, symmetry and continuity, as this book's Chapter 2 defined them. *Symmetry* is sameness kept across a change: the same law here and there, now and later. *Continuity* is no gap and no least step: move a little, and the thing changes only a little, with nothing left out between. To apply symmetry and continuity "fundamentally" is to hold both exactly, at the foundation of a description, with no exception at any scale.

Now set Laplace's intelligence beside the Incommensuration Theorem. I give the reason in a section titled "Why Determinism Cannot Be Had". The first sentence of the section "Why Determinism Cannot Be Had" is: "Laplace's demon needs two things at once". The demon needs one law that carries the complete state at each moment into the complete state at the next moment, and the law must be the same everywhere and at every time. The demon's first need is symmetry, made perfect. And the demon

needs the complete state given at every point without a gap, so that each moment passes into the next without a break. The demon's second need is continuity, made perfect. In the theorem's sense, continuity made perfect is a description with no gap and nothing left out, the sense in which I use it in Chapter 8 for a faithful reversal. A world that moves in steps by one fixed rule needs the same of its rule: every step's complete state, with nothing left out. So the demon's two needs are the same whether space and time are smooth or come in least steps. The demon needs both perfections together, and the Incommensuration Theorem shows that perfect symmetry and perfect continuity cannot be had together. I state the result for every account of a whole world, whatever equations the account uses. A whole world completely fixed by a law perfects symmetry and continuity together, whichever account describes it, and the theorem forbids that pairing. Determinism is therefore not false as a matter of fact but unreachable as a matter of structure.

The theorem shows that a whole world cannot be fixed by one law, and two things follow. The first thing that follows is that the law itself remains valid. So I keep Carroll's equations as he writes them, with every prediction the equations give. A rule written down as mathematics can carry the state it describes at one moment into the state it describes at the next moment, without fail. I use the same deterministic equation that Carroll uses. Within any limited stretch of space and time, physicists can describe the world as very lawful and very smooth, and every success physics has had is a lawful and smooth description of what happens within a limited stretch of space and time.

The second thing that follows is that the features of a description belong to the description. I make the point with a machine. An orrery is a working model of the solar system: a brass Sun, the planets on arms around the Sun, and a train of gears beneath that carries each planet around at its own pace. The machine takes its name from an orrery that a London instrument maker, John Rowley, built about 1712. Rowley built the orrery for Charles Boyle, the fourth Earl of Orrery. Turn the gears the other way and the planets go around backward, just as smoothly. The sol-

ar system itself runs one way only. The Moon raises tides in the Earth's oceans, the tides drag against the turning Earth, and the drag pushes the Moon slowly outward. Scientists bounce laser light off the reflectors that the Apollo astronauts left on the Moon. They have measured the Moon's outward motion with the reflected light. The Moon is moving away from us by about three point eight centimeters a year. The orrery has no gear for the tides, for their drag or for the Moon's slow outward motion. The ease with which the model runs backward is a property of the model. In Section III of the same chapter I set Laplace's intelligence beside the orrery. Laplace drew his picture from Newton's equations, which fix every later complete state from an earlier one, and of Laplace's picture the chapter says: "It gives the world the features of those equations".

Carroll's favored account describes a whole world fixed by one law. Carroll holds that his favorite, the Everett formulation, retains complete determinism: one quantum state for the whole universe, changing by one rule, with each moment holding the information that fixes every other moment. The physicist David Bohm's *pilot-wave theory* is another account of quantum mechanics. In the pilot-wave theory, every particle has a definite position at every moment and is steered by the wave function. I name Everett's account and Bohm's account together as accounts that describe a whole world as completely deterministic, and the Incommensuration Theorem applies to each of the two accounts as to any other account. I keep Carroll's equation and excludes one thing only: the step from a complete law to a complete world. The step goes from a law that runs without fail to a whole world fixed by the law without remainder, and the theorem shows that a whole world cannot be fixed by a law without remainder.

The Law of the Many, and This Crack

If the laws do not fix the whole world, what do the laws fix? The laws fix the *distribution* of outcomes: how the outcomes share out over many trials of the same thing. A complete knowledge of the laws and of the state leaves the single outcome open. The state in the quoted sentence is the quantum state. The quoted sentence therefore says that every law and the complete quantum description of a system together leave the single outcome of a measurement open.

A reader may ask whether that is only Laplace's point again: that we cannot know which outcome comes, because we do not know enough. The answer depends on a difference between two kinds of chance, and I keep the two kinds apart. You can see the difference in two pictures. Roll a die, and the number that comes up looks random, yet the number follows from how the die was thrown: its speed, its turning and the table. The chance of a die is Laplace's kind of chance. The chance measures what we do not know about the throw, and fuller knowledge would shrink the chance. Now send a single particle of light at a half-silvered mirror, which lets half of the light through and turns half aside, with a detector on each side. One detector counts the particle and the other detector does not. A physicist can work out only the odds that each detector counts, even with everything that quantum theory can say about the particle beforehand. I name the second kind of chance elsewhere. I call an outcome of the second kind *hard randomness*. A hard random outcome is fixed by no law and by no knowledge of what came before the outcome, however complete. Hard randomness is not ignorance that better knowledge would remove: there is no fuller specification for better knowledge to find. In the plain sense I gave the phrase earlier in this chapter, Carroll's true randomness is an outcome that the quantum state before the outcome leaves un-

fixed, as at the mirror. Hard randomness says one thing more: nothing fuller than the quantum state, no law and no knowledge however complete, fixes the outcome either.

I set hard randomness against Laplace's intelligence in full in an essay on hard randomness. In Section VIII I imagine someone who knew the position and the momentum of every particle in the universe, the complete state in Laplace's sense, and knew every law of nature besides. I write: "All of that together would still not be enough to fix the value of any truly hard random measure. The omniscient cannot contain the transcendent". In the essay's sentence, a *measure* is a measured value, such as which of the two detectors counts. The two words *omniscient* and *transcendent* are my own words. I explain both words. The omniscient is "structure held in view as a whole, outside time", and the transcendent is "what could be". In the essay's sentence, the omniscient is what the essay's imagined knower holds: "every law of nature, every argument and every reason". Go back to the half-silvered mirror, and you can see what the transcendent is at the mirror. The transcendent at the mirror is the two outcomes still open before the act of finding. One outcome is a count at this detector, and the other is a count at that one. You get both detectors and the odds of each detector from every law and the whole quantum description of the particle, but the one detector that counts is given only in the act of finding.

Physicists met hard randomness early, when they counted atoms breaking apart. Ernest Rutherford was a physicist at McGill University in Montreal. In 1899 Rutherford found that compounds of thorium give off a radioactive substance (a substance whose atoms break apart on their own and throw out particles), which he called the emanation. Rutherford measured the radiation that the emanation gives out, and he published the work the next January. He found that the strength of the radiation fell to half in about one minute, then to half of the half in the next minute, and so on, in what he called a geometrical progression. Each kind of radioactive atom has a time like the emanation's minute, its *half-life*: the time in which half of any large number of atoms of that kind will have decayed.

In 1910, at the University of Manchester, Rutherford and the physicist Hans Geiger asked whether the single decays come at random. Rutherford and Geiger set a small disk coated with polonium, a strongly radioactive element, before a screen that flashes where each particle thrown out by a decaying atom strikes the screen. Then they counted the flashes by eye through a microscope, in interval after interval of seven and a half seconds. Over 2,608 intervals they counted 10,097 particles, about four in each interval on average, with the count changing from one interval to the next. In a note printed with Rutherford and Geiger's paper, the mathematician H. Bateman worked out how the counts should spread if every particle were thrown out at random, each decay independent of every other. The counts spread in just the way Bateman worked out. Rutherford and Geiger wrote that "on the whole, theory and experiment are in excellent accord". We now know that the law of decay is exact about the many atoms and says nothing about when any single atom will decay. I say the same of a radioactive lump. The passage is in Chapter 18, "How Close a Smooth Theory Comes", section "An Open Future Is Real": "Which atom goes next is fixed by nothing: no atom carries a clock inside it that the others lack". The same section adds: "The law is exact about the many, and the finding is open for each one".

Look at the egg once more. In this book the egg is the picture of one act of finding, one crack found once, and one act of finding has no odds: odds belong to many trials, as at the half-silvered mirror. Over many eggs, struck by many cooks, the laws say how shells break: where cracks tend to start, and which way cracks tend to run. This crack, in this shell, grew from differences in the shell and in the strike far too small for anyone to see. The differences were magnified as the crack ran. Carroll gives a classical account of a crack like this one in *The Big Picture*, Chapter 4. Carroll calls the magnifying of small differences chaos (*The Big Picture*, Chapter 4), and he treats chaos as a limit on our knowledge: "slightly imperfect information leads to very imperfect prediction". Carroll writes of many kinds of system, and I apply his account of chaos to this crack. On Carroll's account, two things are true of this crack. First, the crack was fixed

by the complete state of shell, rim and hand, in Laplace's sense of a complete state. Second, chaos kept anyone from foreseeing the crack only because no one knows the complete state perfectly. I hold that the magnifying carries hard randomness upward, from the smallest scale to the scale of the crack. I call the magnifying microstate amplification. In microstate amplification, small differences at a fine scale grow into differences at the scale that matters. I apply microstate amplification to determinism. In the same section I cite the meteorologist Edward Lorenz. In 1963 Lorenz found, in a simple model of air that rises and sinks as the air is heated, that "slightly differing initial states can evolve into considerably different states". Take an outcome at the smallest scale that nothing before the outcome fixes, and microstate amplification makes the outcome grow into a difference at the scale of eggs and bowls and people. So the cook finds a crack that the laws, with the complete quantum state of the kitchen, did not fix.

I also answer Carroll's conservation of information with hard randomness, carried upward by microstate amplification. I set two pictures side by side: a deterministic world, and the world we have. Of the deterministic world, the section says: "In a deterministic world all the information would be present in the first state, and every later state would only rearrange it". In the world we have, new information enters all the time at the smallest scale, as outcomes that no earlier moment fixed, and microstate amplification magnifies the new information up through every scale. I name the place where information is conserved without fail. The place is reasoning from premises to conclusions. Information is conserved in reasoning because a valid argument cannot produce anything that its premises did not contain, and the same section says: "The conservation of information under deduction is inherent in the idea of mathematics itself". Carroll's principle of conservation of information holds exactly in the mathematics of a law, but Carroll also claims that each moment of the world holds precisely the information that fixes every other moment. Carroll takes the step from a complete law to a complete world in making the

claim, and the step he takes is the one step I exclude. For the world, Carroll's claim fails at the smallest scale, where outcomes enter that no earlier moment fixed.

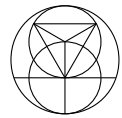
A Complete Law, and a World Still Happening

What, in the end, have I taken from Carroll's picture, and what is left? I state his position on people. Carroll himself is a compatibilist: he holds that the laws leave no room for any influence beyond them, and that choice is a real and useful description at the level of people. I argue against a stronger reading of the complete laws: the reading on which the laws fix each single event. Laplace held the stronger reading: for his imagined intelligence, "nothing would be uncertain". In the same section I also say what the stronger reading is. To read the laws as fixing each single event is Laplace's demon. A reader of Carroll holds the stronger reading when the reader takes Carroll's complete laws to leave no room for choice. I take up the reading of Carroll's claim "on which the known laws leave no room for choice". The stronger reading depends on one step, the step from a complete law to a complete world, which runs: the laws are complete, so what happens is completely given in the laws. The stronger reading reads more into the laws than they hold: a reader who holds the stronger reading takes a complete set of laws for a complete account of what happens. The stronger reading shares only one thing with Carroll's own account: the step that the stronger reading depends on. Carroll takes the step from a complete law to a complete world in one place: his favored Everett formulation. In Carroll's words, the Everett formulation retains "complete determinism".

Everything else that Carroll has established remains. The laws of everyday life remain as Carroll states them, tested to the precision of every test yet made of the laws. Carroll's equations remain, deterministic as equations. Carroll's odds remain, the same odds I expect. The one thing I remove is the step from a complete law to a complete world.

The answer to the question of this chapter has two parts. The known laws fix the odds of every outcome over many trials and leave each single outcome open. And the Incommensuration Theorem shows that a whole world cannot be fixed by one law. The answer matters for the question this book began with: whether the ground of physics is a quantum state or the act of finding. The laws describe the range of what could be found, with the odds of each outcome. The single outcome is given only in the act of finding. Of all the cracks the laws allowed, this crack came to be in the cook's act at the rim of the bowl.

If each single outcome is open, what happens to the other outcomes that were possible?



Chapter 5

When a Measurement Could Go Two Ways, Do Both Happen?

What this chapter uses

The six parts of every comparison, among them content and context, the subjective and the objective (Chapter 2).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *wave function*, Carroll's complete quantum description of a system (Preface and Chapter 1); *exist* and *real* (Chapter 1).

The *law of the many*: the law is exact about the many, and the finding is open for each one (Chapter 4).

In this book's Chapter 4 we found that the known laws fix the odds of every outcome over many trials and leave each single outcome open, so a question remains about the outcomes that were possible and did not happen. When a measurement could go two ways, do both ways happen, one way where you can see the outcome and the other way where you cannot?

Sean Carroll answers with a plain yes, and he answers from the mathematics that every physicist already uses. We start with the paper where a plain yes to the question first appeared in print.

A Paper and the Note Beside It

In July 1957 the American Physical Society printed volume 29, number 3, of its journal *Reviews of Modern Physics*. The issue carried the papers of a conference on the role of gravitation in physics. The conference was held at Chapel Hill, North Carolina, from 18 to 23 January 1957. A nine-page paper by Hugh Everett III began on page 454. Everett was a graduate student at the Palmer Physical Laboratory of Princeton University. The title of Everett's paper was "'Relative State' Formulation of Quantum Mechanics". A footnote said that the paper was a thesis, submitted to Princeton University on 1 March 1957, for the doctor's degree.

Everett made one large claim in the paper. In Everett's words, "a wave function that obeys a linear wave equation everywhere and at all times supplies a complete mathematical model". A *wave function* is the complete quantum description of a system. Carroll takes up Everett's account in his book *Something Deeply Hidden*, and he holds that the wave function is the system itself: "The wave function isn't a bookkeeping device; it's an exact representation of the quantum system" (Chapter 2). When a wave equation carries a combination of possibilities forward by carrying each possibility forward on its own and adding the results, physicists call the equation *linear*. So under a linear equation no possibility ever crowds out another. Everett applied the wave function and its one linear equation to every system closed off from everything outside the system, and so to the universe as a whole, which has nothing outside it. In Everett's description, observers and instruments were inside the system described, and the moment of measurement had no second rule of its own.

Before the paper was printed, some readers saw an advance copy of it, a *preprint*. Those readers wrote to Everett with a question about the paper. On Everett's theory, at a measurement the observer's quantum state splits into several states, one for each outcome. Everett called those states the branches. Those readers argued that experience shows no splitting of the observer, and so only one branch can ever actually exist. Everett answered in a 'note added in proof': a note added at the foot of a page at the last stage of printing. The note says: "From the viewpoint of the theory all elements of a superposition (all "branches") are "actual", none any more "real" than the rest". In the language of the theory, a *superposition* is a quantum state that is a combination of several possible outcomes. Everett's branches are the possible outcomes of the superposition, each outcome carried on by itself after the measurement. Everett added why no one notices the splitting: "no observer will ever be aware of any "splitting" process".

A note by John Archibald Wheeler came on the very next page, page 463. Wheeler was Everett's thesis adviser at the Palmer Physical Laboratory. The title of Wheeler's note was "Assessment of Everett's 'Relative State' Formulation of Quantum Theory". Wheeler opened his note with the sentence "The preceding paper puts the principles of quantum mechanics in a new form". Then Wheeler said what the new form was: "Observations are treated as a special case of normal interactions that occur within a system". So Everett and Wheeler had already raised this chapter's question, on two pages side by side in one issue of a journal. In Everett's account a measurement is one more meeting between things, and Everett's one equation contains nothing that picks out one outcome. So both ways a measurement could go stay in Everett's description, and in Everett's word both ways are actual.

Come back to the kitchen. The cook brings the egg down on the rim of the bowl, and the shell breaks along one jagged line. Carroll's quantum account of the same moment, in *Something Deeply Hidden*, goes further than his classical one. Carroll tells what Everett's theory, which Carroll calls Many-Worlds, says about quantum outcomes. Picture the same mo-

ment, the crack, as Everett's theory describes it in Carroll's telling. Of the ways the laws allow the shell to break, every way happens, each way on its own branch. In one branch the crack runs left, in another branch the crack runs right, in another the yolk slips out whole, and in each branch there is a cook, looking into a bowl. The cook on each branch finds one of those ways. The question is what has become of the other ways of breaking, the ways that this cook did not find.

From Carroll's pages

One State, One Rule

Carroll begins his account by taking away the further rules that the textbooks add for measurement. Carroll points out that every version of quantum mechanics "employs a wave function or something equivalent, and posits that the wave function obeys Schrödinger's equation, at least most of the time". Carroll makes the point about every version of quantum mechanics in Chapter 2 of *Something Deeply Hidden*, "The Courageous Formulation: Austere Quantum Mechanics". The *Schrödinger equation* is the one rule by which a wave function changes in time: smoothly, each moment fixed by the moment before. The further rules for measurement come in at the times that the words "most of the time" leave out. The chief further rule says that when you look, the wave function jumps at once to the outcome you found. Physicists call the jump *collapse*.

Carroll asks what happens if the wave function obeys its one rule all of the time, and the further rules are dropped. Carroll takes the wave function as a direct description of what there is, and he lets the wave function obey its one rule always, at measurements as much as anywhere else. Carroll calls quantum mechanics with the wave function, its one rule and nothing else 'austere quantum mechanics'. In the same chapter Carroll states the first claim of austere quantum mechanics: "The world is a wave function, nothing more nor less".

Then Carroll follows the one rule into a measurement. An electron starts in a superposition of many places. A camera looks at the electron, and the looking is an ordinary physical meeting, governed by the Schrödinger equation like every other meeting. After the meeting, the electron and the camera have a single description together, in place of two separate descriptions. The single description reads: the electron was here and the camera saw the electron here, for each place the electron could have been. Physicists call such a joint description *entanglement*: a joint state of two things whose parts cannot be described one by one. In Chapter 2 of *Something Deeply Hidden*, Carroll says what entanglement means for measurement: "There isn't anything special about measurement; it's just something that happens when two systems interact in an appropriate way". Carroll gives entanglement its source: "there is only one wave function for the entire universe, not separate wave functions for each piece of it" (Chapter 5, "Entangled Up in Blue: Wave Functions of Many Parts"). Carroll, after Everett, calls the universal state the universal wave function.

Now replace the camera with you. By the same rule, the Schrödinger equation, you too enter a joint state with the electron: in one part of the joint state you saw the electron here, and in another part you saw the electron there. Then Carroll takes the step for which the many-worlds account is named. On Carroll's account there are many persons, each in a world of their own and each with one definite view, in place of one person holding many views of where the electron was. In Chapter 2 Carroll says that this picture of many persons, each with one definite view, is the Everett formulation of quantum mechanics. He adds that nothing was added to quantum mechanics to get the Everett formulation: "We didn't add anything to quantum mechanics, we just faced up to what was there all along". Carroll names the price of the Everett formulation in the same place: a great many copies of

what we call the universe. And Carroll grants that physicists argue over the price: "Whether the benefit is worth the cost is an issue about which people disagree. (It is.)"

I call the one wave function for the entire universe the *universal state*: the quantum state of the whole universe.

From Carroll's pages

How One State Becomes Many Branches

Why then do we never see a camera or a cook spread over two outcomes? Carroll answers with a process that the physicist H. Dieter Zeh introduced in 1970. To follow the process, you first need one more picture: ripples on a pond.

Drop two stones into a still pond, a little apart. Two rings of ripples spread and cross. Where a crest meets a crest, the water rises higher. Where a crest meets a trough, the crest and the trough cancel, and the water lies flat. Physicists call the adding and canceling *interference*, and the alternatives in a wave function interfere too. Send electrons one at a time toward a wall with two slits, and the marks the electrons leave on a screen behind the wall build up into bands: the stripes of two waves crossing. Put a detector at the slits that records which slit each electron took, and the bands vanish.

Carroll explains the vanishing of the bands without any collapse. The detector becomes entangled with the electron, and then the room becomes entangled with the detector. Air molecules strike the detector, and light bounces off the detector. Each molecule and each particle of light is touched a little differently by each alternative. Nobody could keep track of all those touches, and nobody needs to. Once the difference between the two alternatives (the electron through one slit, or through the other) has spread out into the surroundings, the two alternatives can no longer add and cancel. *Decoherence* is the spreading

of the difference between the alternatives into the surroundings, after which the alternatives no longer interfere. Carroll states what decoherence does: "Decoherence causes the wave function to split, or branch, into multiple worlds" (*Something Deeply Hidden*, Chapter 6, "Splitting the Universe: Decoherence and *Parallel Worlds*"). In Carroll's word, a *branch* is one of the alternatives that decoherence separates, and each branch has its own outcome. Carroll says that two branches which cannot interfere cannot affect each other at all, and so each branch is to the other as a world of ghosts would be to us. Each copy of the observer then finds one outcome, and to that copy the wave function seems to have collapsed. In Carroll's phrase, the collapse "is only apparent".

Carroll draws three consequences from decoherence. The first consequence is that a measurement needs no mind. On Carroll's account a measurement is any meeting that brings on decoherence, and an observer is anything that brings such a measurement about. Carroll writes in the same chapter: "The "observer" could be an earthworm, a microscope, or a rock". He adds: "Consciousness, in particular, has nothing to do with it".

The second consequence is that the many worlds are not an extra assumption. Carroll wrote Chapter 8 of *Something Deeply Hidden* as a conversation between Alice and her father. Alice is a physicist who teaches in a philosophy department, and her father is a particle physicist. The chapter's title is "Does This Ontological Commitment Make Me Look Fat? A Socratic Dialogue on Quantum Puzzles". Her father objects that a theory called Many-Worlds must assume a great many worlds, and Alice answers: "The ingredients used in Many-Worlds are ingredients that are used by every other version of quantum mechanics". Alice counts only what Many-Worlds says there is, and she sums up: "Ontologically speaking, Many-Worlds is as lean and mean as you can possibly get". Carroll answers people who simply dislike so many unseen universes: "But a bad feeling is not a principled stance". The

answer is in *The Big Picture*, Chapter 21, "Interpreting Quantum Mechanics". And Alice corrects a common misreading of Many-Worlds: "Many-Worlds doesn't say 'everything possible happens'; it says 'the wave function evolves according to the Schrödinger equation'". For example, the total electric charge stays the same on every branch. So on Carroll's account the egg breaks in every way that the Schrödinger equation leads to, and those ways are fewer than every way a reader could imagine.

The third consequence is that there is still one world, in the large sense of the word 'world'. Carroll writes in the same chapter of *The Big Picture* that "There's still one "world" in the sense of "the natural world", described by the wave function of the universe", and on his account the branches are parts of that one natural world. Carroll calls the branches real, in his own sense of the word 'real', in Chapter 11 of *Something Deeply Hidden*, "Why Is There Space? Emergence and Locality". In that chapter he writes: "Fundamental versus emergent is one distinction, and real versus not-real is a completely separate one". For Carroll the branches are real in the way chairs and tables are real, because, in his words in the same passage, chairs and tables "describe true patterns in the universe".

Carroll's account is clear, and with it Carroll does a great deal. He removes the special rule for measurement, and he needs no observer with a mind. He treats the camera and the cook by the same law as the electron. And through decoherence he shows why the world we see looks settled. Each of those results remains valid.

Where the Two Roads Meet

I derive four of Carroll's results from my own starting point: that nothing collapses, that decoherence is why interference fades, that a measurement needs no mind, and that no signal travels faster than light through entanglement. The one who finds is whoever or whatever makes the comparison, a person or an instrument.

The first of the four results is that nothing collapses. In my account the state always evolves by a linear equation, and there is no dynamics of reduction. Reduction is another name for collapse, and a dynamics of reduction would be a law of motion that makes the jump of collapse happen. So in my account too, the quantum state obeys its one smooth rule at all times. What changes at a measurement instead is a record. The outcome of the comparison becomes part of the pattern, laid out, that anyone can know of the system, and the record is the same for everyone who has the outcome. I call the change of record an update. The update is not a physical process, and it is not an extra law of motion. Carroll removes the jump because the Schrödinger equation has no place for a jump, while I have no jump because what a measurement changes is a record, and a change of record is no physical process.

The second result is decoherence, as the explanation of why interference fades. I credit decoherence to the work of Zeh and of the physicist Wojciech Zurek. In that chapter I derive decoherence from two sides of every comparison: its *content*, what the comparison is about, and its *context*, what makes the two things compared two.

Physicists have watched interference fade, more than once. In Paris in 1996, a team of physicists trapped a field of a few particles of light between two mirrors, and they sent atoms through the field one at a time. The team was Michel Brune, Serge Haroche and their colleagues. The team put the field into a superposition of two states as far apart as two positions of a pointer: a small meter pointing two ways at once. In physics a

mixture of odds is a quantum state that reads like a list of ordinary odds: the pointer one way with some chance, or the other way with the rest, and no adding and canceling left between the two ways. Then the team watched the superposition turn into an ordinary mixture of odds, while the change was happening. In Vienna in 2003, a second team of physicists sent molecules of carbon seventy through an apparatus that splits a beam into paths and brings the paths back together. The second team was Klaus Hornberger, Markus Arndt, Anton Zeilinger and their colleagues. Each molecule is a closed cage of seventy carbon atoms. In a good vacuum the cages built up bands of interference. Then the experimenters let a little gas into the chamber, and as the pressure rose the bands faded away. Each molecule of gas that struck a cage carried off a trace of which path the cage had taken. We now know that decoherence is a measured process: physicists have seen the spreading of a difference into the surroundings while the spreading happened, and they have matched what they saw to the theory.

Take one cage in the gas, and look at the cage in my terms. A measurement on the cage is about the path the cage took, so the path is the content of the comparison. The gas around the cage carries copies of which path the cage took. I name the part the gas plays: "The environment that carries the copies is the context of that content". Once the gas holds copies of the path, the interference of the two paths no longer belongs to the cage alone. Certain parts of the quantum state make the cage's interference. Those parts belong to the joint system and no longer to the part. The parts that make the interference are now in the cage and the gas together, and the same book says of those parts: "They become out of reach for any measurement that probes the system alone, and they are not destroyed". So the bands fade on the screen, and nothing has been taken out of the description.

The third result is that a measurement needs no mind. In my use, a comparer is a person, an instrument, or any physical system that registers one thing against another. The finding is made by the comparer that finds: the apparatus, as an interaction, and no mind is needed. Carroll's rock and earthworm and microscope are comparers in my sense.

The fourth result is that no signal travels faster than light through entanglement. The fourth result belongs in this chapter because the universal state contains the whole universe at once, the near and the far together, so you may fairly ask whether a measurement here, going both ways, changes at once what is found far away. Carroll shows from the rules of quantum mechanics that no signal travels faster than light through an entangled pair (*Something Deeply Hidden*, Chapter 5). Whoever measures one particle of an entangled pair knows at once what will be found at the far particle, but whoever is at the far particle cannot tell that anything was measured until a message arrives by ordinary means, no faster than light. I derive the same result from a different premise (a different starting claim). The premise is the speed of light, taken as the outer edge of what can be compared at all.

And I give Everett the credit that Carroll gives him. I name what the many-worlds account achieves: "Its achievement is economy and reach". Everett applies one rule of change without exception, to a single atom and to the whole universe.

One Crack Is Found

At this chapter's question the two roads part: Carroll's road and my road. On whether both ways happen, Carroll's account and mine part at what is real, and that parting follows from the one place where they part: what lies at the ground. My answer depends on two words, *exists* and *real*. Ordinary speech uses the two words loosely, and I give each word an exact meaning, different from the meaning of the other. I state the two meanings. "To say that something exists is to say that it stands as struc-

ture: timeless, the same in every instance, like the pattern of a law, or the state of a system at an instant". "To say that something is real is to say that it is given in an actual act of comparison: temporal, occurring now, with the future still open to possibilities not yet settled". The same section contains an example of each word: "A law exists; a measurement is real". From here on I use *exists* and *real* only in these two senses.

Apply the two words to the universal state. For Carroll the universal state is the world itself, as we saw in his words: "The world is a wave function, nothing more nor less". I read the universal state with every branch in it as a description laid out whole, the same for every reader. So, in my sense of the word *exists*, the universal state exists, branches and all. For every comparer, one outcome is real, the one found. I state the same result from the side of the quantum state. "The state exists: it is the complete description of the range of possible findings of the system and of their probabilities". The next sentence says where the one outcome comes from: "The outcome is real: it is given in an actual act of comparison, and the state does not specify it in advance".

Decoherence fits into my account without any change to the account. Decoherence does everything Carroll says it does: decoherence ends the interference between the alternatives, and it copies a record of each alternative into the surroundings. When decoherence is done, every outcome is still in the description: the description of the system and its surroundings together is a sum of alternatives, each with its own record copied into the air and the light. If you look at the system alone and leave the surroundings out of view, the same description reads like a list of ordinary odds. But a list of odds is not an outcome. I end a chapter on the quantum by dividing the work between decoherence and the act of finding: "Decoherence prepares; the act completes".

Go back to the bowl. Every way the shell could break is in the description of the kitchen, and that description exists: the description is structure, laid out, and anyone can read it. The cook's crack is real: the crack was

found, here, this morning, in one act that cannot be undone. The other ways of breaking are never given in any act of this cook. The other ways of breaking are what exists and is not found.

In his note of 1957 Everett called every branch "actual", "none any more "real" than the rest". I grant that every branch exists as structure, and I hold that one outcome is real for each one who finds. The many-worlds account takes the structure that exists for the whole of what is real. The cook's bowl shows the reason I say so: the other branches are never given in any act of the one who finds.

Three Things in One Crack

Carroll's account and mine part a second time, at the one who finds, and that parting too follows from the one place where they part: what lies at the ground. The second parting shows in three things that every act of finding has. Look at the crack once more, as the cook makes it.

First, the crack is made from one side. The cook stands on her side of the bowl, and she finds the crack from there. In this book's Chapter 2 I called the side a comparison is made from the *subjective*.

Second, the crack comes out one way, along this line and not another. The laws fix the odds of the ways a shell can break, and the laws leave this one break open. In this book's Chapter 4 I gave a name to an outcome that no law and no knowledge of the quantum state fixes, however complete: I called such an outcome *hard randomness*.

Third, the crack runs one way. Before the crack there was a range of ways the shell could break, then came the crack, and then came the record, in the yolk sliding into the bowl and in the cook's memory. The crack will never be uncracked. The *arrow of time* is time running one way, and at the bowl the arrow of time is the one-way order from what could happen, to the act, to the record.

I call the three things together the *bound triple*: the arrow of time, hard randomness and the subjective. The three members of the bound triple are distinct, they cannot be separated, and none of the three can do the work of another. I call the three "three aspects of one act of comparison proceeding in a direction". In the same section I show why the three cannot be separated, by walking round them in a circle, each one needing the next. We can walk the circle with the cook.

The arrow of time needs hard randomness. Suppose every moment of the kitchen were fixed completely by the moment before, down to the line of the crack. Then the crack would already be held in the moment before the crack. The future would be held in the present as fully as the past is. Earlier and later would be like the squares of a calendar printed for the whole year: labels on one sheet that lies there complete, which you can read from either end. I call such an order of moments "an indexing, a label of earlier and later on one fixed whole, and not a direction of process". A direction of time needs the next moment to hold something the present moment does not hold. The crack that no law fixed is just such a thing.

Hard randomness needs the subjective. Picture two cracks that nobody predicted: one that nothing before it fixed, and one that was fixed all along by some detail nobody knew. A description told from no one's side cannot tell the two cracks apart, because in such a description both cracks are simply unpredicted. The difference between the two cracks is the difference between finding something new and finding something that was only hidden. So the two cracks differ in the finding, and the finding is made by the one who finds, in the act. The step from hard randomness to the subjective is this: a finding needs a finder. A crack that nothing fixed is hard randomness, as distinct from a gap in someone's knowledge, only where the crack is found from a side.

The subjective needs the arrow of time. The cook looks into the bowl and sets what she sees now against what she saw a moment ago: a whole egg, then a cracked one. Seeing, remembering and looking ahead each set one thing against another: to see is to set what is found against what was

found, to remember is to set the present against a record, and to look ahead is to set the present against what could come. Each of the three runs one way, from before to after. If time had no direction, remembering and looking ahead would be the same, and the cook would have no side to look from.

So each member of the bound triple needs the next, and the circle closes: none of the three is present without the other two. The interpretations of quantum mechanics are the rival readings of what the mathematics of quantum mechanics says about the world. The three are present together or absent together in every one of the interpretations: every interpretation keeps the three together or drops them together. The textbook reading of quantum mechanics goes back to the physicist Niels Bohr. In the textbook reading all three are present: the observer, the random single outcome, and the measurement that cannot be run backward.

Carroll follows Everett in leaving all three out of the many-worlds account, and he leaves them out by design. I set out the three absences one by one. "There is no subjectivity: there is no fact of the matter about which branch I find myself in". "There is no hard randomness: the evolution is deterministic". "And there is no arrow of time: the fundamental equation is symmetric in time". "The three absent features are the bound triple, and they are absent together".

Carroll's own dialogue shows the third absence, the missing arrow of time, in a talk between Alice and her father. Alice's father asks why worlds should branch apart and never fuse together. Alice answers with the arrow of time, as cream and coffee mix and never unmix: "Wave functions branch, but don't unbranch". Her father then objects that in Everett's account the Schrödinger equation "is completely reversible", so why should branching run only one way? Alice calls her father's point a "Perfectly good question", and she places the arrow of time in how the universe began, with little entanglement. She says that a beginning with little entanglement is "an initial condition we impose on the state of the universe". So in Carroll's account the direction of time comes from the starting state of the universe, and the Schrödinger equation itself runs either

way. I find the same in the Everett program, in a chapter on the physicist David Deutsch. In the Everett program the branching toward the future enters with the records and with the way the universe began, "not from the universal equation".

Carroll leaves the one who finds out by design, yet the one who finds comes back into the many-worlds account at a step that the account itself needs. Take the whole kitchen (cook and egg and air) as one quantum state, as Carroll does. The one quantum state of the kitchen can be divided into alternatives in many ways. Picture a path across the kitchen floor from the door to the stove. You can tell the path as so many steps along the counter and so many across the room. You can also tell the same path as so many steps along one diagonal of the floor tiles and so many along the other. The two tellings describe one path, told in two ways. The path itself does not say which telling to use, and in the same way the quantum state alone does not say which division gives the branches. Physicists call a division of a quantum state into alternatives a *basis*.

Carroll says who divides the quantum state when he speaks of the branches in Chapter 11 of *Something Deeply Hidden*. In his words: "We choose to invoke them when carving up the wave function for our convenience, but we don't do that carving randomly". On Carroll's account the fundamental laws determine which divisions work: "Which ways actually work is ultimately determined by the fundamental laws of nature, not by human whimsy".

I take up the step of choosing a basis in the Everett account elsewhere. That section says two things in one sentence: "Decoherence supplies the basis by the stability of records, and the choice of that basis is the choice of which outcomes count as having occurred". The first half of the sentence agrees with Carroll. When every copy of a record reads the same, we call the record stable: the crack copied into the air, the light and the bowl is the same crack in every copy. Decoherence supplies the basis in which records stay stable in that way, and it runs by the one smooth rule of change that both accounts use. The second half of the sentence says what choosing that basis is: choosing which outcomes count as having occurred. An out-

come counts as having occurred when it is found, in an act of finding. The basis that decoherence supplies is the same basis that is chosen in an act of finding, because to find an outcome in that basis is to count that outcome as having occurred. So the basis has one chooser: the one who finds chooses the basis, in the acts that lay down records, and the surroundings that carry the record are the context of the act of finding. The one who finds chooses in this sense: its act of registering an outcome in that basis is what makes the outcome count as having occurred, and the act needs no mind, so that a camera on the counter chooses as surely as the cook. Take one such act, the crack at the bowl. In the act at the bowl, the one who finds is the cook, or any instrument that registers the crack. The surroundings of the act at the bowl are the air, the light and the bowl, which carry copies of the crack, as the gas in Vienna carried copies of the path of each carbon cage. I name the choice of basis and the odds of the outcomes as the two steps of the many-worlds account "at which the act of finding comes back into the account".

How Far One Description Reaches

We reached a rule by walking the circle with the cook: in every account the three members of the bound triple are present together or absent together. The appendix "For the Specialist" contains a second route to the same rule, through the Incommensuration Theorem.

In this book's Chapter 4 we walked through one result of the theorem: a whole world fixed completely by one law perfects symmetry and continuity together, and so the theorem excludes such a world. As a rule of the mathematics, the rule of change that Carroll uses is consistent with the Incommensuration Theorem. I name the Everett account as one of the accounts of a whole world fixed completely by one law, so the theorem's result applies to the Everett account.

I say how much a description can account for when the description has no first person in it, no side from which anything is found. I call physics told from no one's side 'third-person physics'. A theory with no first person in it "can be a complete account of the third-person physics, but it cannot, by the theorem, also be a complete account of the conditions under which that physics is known". The mathematics of the many-worlds account describes the third-person physics, and my own quantum mechanics uses the same smooth rule of change. The whole world also contains the acts of comparison by which anything is found, and the cook's crack is one of those acts. So I set aside one reading of Carroll's universal state: the reading that takes the universal state for the whole world, fixed by one law, with every outcome occurring and no one who finds. A whole world of that kind is what the Incommensuration Theorem excludes.

What We Now Know

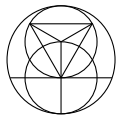
When a measurement could go two ways, do both ways happen? Both ways are in the description, and the description exists. For the one who finds, one way is real: the way found. Carroll and I walk much of the road together. Both hold that nothing collapses, that decoherence is why the world looks settled, that a measurement needs no mind, and that no signal travels faster than light. Carroll and I both count Everett's single rule, applied everywhere, as an achievement. Carroll and I part where Carroll takes the description for the whole of what is real.

The universal state, branches and all, exists as structure, and for the one who finds, one outcome is real, the one found. And in the many-worlds account the one who finds, the single random outcome and the direction of time are all absent together.

The answer matters for the book's question about what lies at the ground, because on this question what lies at the ground determines what is real. If the ground is a quantum state evolving by one rule, every branch

is as actual as ours. If the ground is the act of finding, the cook at the bowl is part of the account of the world from the start, and the egg on the counter is cracked one way.

Carroll gives each branch a weight: the branch's share of the odds. If, on Carroll's account, every outcome happens, each on its own branch, where do the odds of each outcome come from?



Chapter 6

Where Do the Odds Come From?

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *wave function*, Carroll's complete quantum description of a system (Preface and Chapter 1); the *many-worlds account*, in which every outcome happens, each on its own branch, and *decoherence* (Preface and Chapter 5).

The *bound triple*: the arrow of time, hard randomness and the subjective, distinct and inseparable (Chapter 5).

When a quantum measurement can go more than one way, and on Carroll's account every way happens, where do the odds of each way come from? Chapter 5 ended on that question, and on the cook's egg, cracked once on the rim of the bowl. Picture the crack as Carroll's many-worlds account describes a quantum outcome: every way the shell could break happens, each way on its own branch, and on each branch a cook finds one of those ways. Yet physicists say, every working day, that one

outcome is likely and another is rare. The counts in their laboratories agree with them. To find where those odds come from, we begin with the physicist who first wrote down the rule for them.

A Note Added in Proof

In June 1926 Max Born, a physicist at Göttingen, in Germany, sent a journal a short paper on a plain question. Fire an electron at an atom: where does the electron go? The journal *Zeitschrift für Physik* received the paper on 25 June 1926, under the German title "Zur Quantenmechanik der Stoßvorgänge", a title about the quantum mechanics of collisions. Born was forty-three years old.

Earlier that year the physicist Erwin Schrödinger had given physics an equation for a wave that belongs to the electron. After a collision, Schrödinger's wave spreads out from the atom in every direction at once. Yet an electron is always found in one direction only, as a single click in a single counter. Born gave his answer in German, in the body of his paper: a quantity measuring the wave in each direction determines the probability that the electron is thrown off in that direction. Then, while correcting the printed proofs, he added a footnote. In the footnote Born corrected his rule. The probability is in proportion to the square of that quantity, and *not* to the quantity itself.

Carroll tells the same moment in his book on quantum mechanics, *Something Deeply Hidden*: the squared rule "appears in a footnote added at the last minute to Born's 1926 paper" (Chapter 3, "Why Would Anybody Think This? How Quantum Mechanics Came to Be"). Twenty-eight years later, in 1954, Born received the Nobel Prize in Physics "for his fundamental research in quantum mechanics, especially for his statistical interpretation of the wavefunction".

Physicists now call the quantity Born squared an *amplitude*: the number that the quantum state gives to one possible outcome, whose size measures how much of the state goes with that outcome. Physicists call

the rule of Born's footnote the *Born rule*: the probability of an outcome is the square of the size of its part of the quantum state, that is, the square of the size of its amplitude.

A picture with numbers shows what the Born rule says. The cook's egg will not serve for a picture with numbers, *because* in this book the egg is the picture of one act of finding, one crack found once, and one act of finding has no odds. We use a mirror instead, for odds with numbers. Now picture a mirror silvered more thinly, so that each particle has two outcomes of unequal amplitude: it passes through, with an amplitude of size six tenths, or it is turned aside, with an amplitude of size eight tenths. The Born rule says: "square each amplitude". Six tenths squared is thirty-six hundredths, and eight tenths squared is sixty-four hundredths. Send a great many particles, one at a time, and about thirty-six in each hundred are counted on the far side of the mirror, and about sixty-four in each hundred are counted at the side. Thirty-six and sixty-four in a hundred are the odds of the two outcomes.

A reader who has followed Chapter 5 will ask what thirty-six in a hundred can mean, if every particle both passes and is turned aside, each on its own branch. Carroll asks that question too, and he asks it at full strength.

From Carroll's pages

What Carroll Showed

Carroll begins with the easy part of the answer. In Carroll's picture a quantum state is an arrow of length one. The amplitudes of the state are its parts along a set of directions, one for each outcome, at right angles to one another. When there are two outcomes, the two amplitudes are like the short sides of a right triangle whose long side is the arrow itself. The squares of the short sides add up to the square of the long side, which is one: Pythagoras's theorem from school geometry. So the squared amplitudes are never negative, and they always add to

one. The probabilities of a complete set of outcomes must do the same. Carroll ties the Born rule to the right triangle: "the Born rule is essentially Pythagoras's theorem, applied to the amplitudes of different branches" (*Something Deeply Hidden*, Chapter 4, "What Cannot Be Known, Because It Does Not Exist: Uncertainty and Complementarity").

The "hard part", Carroll says, is a different question. "The deeper issue is why there is anything unpredictable about Everettian quantum mechanics at all" (Chapter 7, "Order and Randomness: Where Probability Comes From"). "Everettian" quantum mechanics is quantum mechanics as Hugh Everett formulated it, the many-worlds account of Chapter 5. In the many-worlds account the wave function always obeys the Schrödinger equation. The Schrödinger equation is deterministic. If the quantum state at one moment is given, the equation fixes the quantum state at every other moment. When a particle arrives at the thinly silvered mirror, the many-worlds account says with certainty that both branches will be there. In one branch the particle passed, and in the other it was turned aside. So both outcomes happen every time, one on each branch. The writers of textbooks read the Born rule as a statement of how often each outcome happens. On the many-worlds account the Born rule needs another reading. Carroll concludes: "Probability in Many-Worlds is necessarily a statement about what we should believe and how we should act, not about how often things happen" (Chapter 7). In Carroll's books a degree of belief is called a *credence*: how strongly someone holds that a thing is so, on a scale from zero to one hundred percent.

Carroll sets out two derivations of the Born rule. He made the first one with the philosopher Charles Sebens. The derivation of Carroll and Sebens starts from a moment that every measurement contains, on Carroll's account. *Decoherence* is the spreading of the difference between the outcomes into the surroundings. Decoherence separates the branches in a tiny fraction of a second, and the experimenter looks

a little later. In between, after the branches separate and before the look, there are two copies of the experimenter. The two copies are alike in every respect, one on each branch. Neither copy knows which branch it is on. Carroll credits the physicist Lev Vaidman with first stressing, in quantum mechanics, the situation of a copy who does not know its branch. Carroll names the situation *self-locating uncertainty*: "you know everything there is to know about the universe, except where you are within it" (Chapter 7). Each copy can then give a credence to being on each branch. Carroll names the derivation's *premise* (its starting claim): the structure of quantum mechanics itself picks out the credences a rational observer should give. "If you're willing to accept that, the credences you'll end up assigning are exactly those you would get from the Born rule" (Chapter 7).

The steps of the derivation are short. Counting branches, one credence for each branch, gives the wrong answer. Carroll shows why: a measurement made later on one branch would raise the count of branches. So the later measurement would change the credence of a branch where nothing happened. A narrower rule survives: two branches whose amplitudes are equal in size get "equal credence". In a footnote Carroll adds that he and Sebens derive the narrower rule from one demand. The demand is that the probabilities you give for an experiment in your laboratory "should be independent of the quantum state elsewhere in the universe" (Chapter 7). Then a branch with a larger amplitude can be split by a further measurement made only on that branch. The smaller branches all have amplitudes that match the amplitude of the other branch. Carroll and Sebens use a second demand to carry the rule from equal amplitudes to unequal ones. Carroll states that demand in the main text: "we don't want the probability of one branch to suddenly change when something happens on another branch" (Chapter 7). In Carroll's example the outcomes start with squared amplitudes of one third and two thirds. The branch with two thirds is split in two. Three branches of equal size result, each with

credence one third. The branch with one third had credence one third before the split as well, by the second demand. And one third is the square of its amplitude. Splitting works the same way at the thinly silvered mirror, with smaller pieces. Split the branch where the particle passed into nine equal branches, and the branch where it was turned aside into sixteen. Twenty-five branches of equal size result, nine of them passing and sixteen turned aside. Nine in twenty-five is thirty-six in a hundred. Carroll puts it this way: "And that answer is the Born rule: probability equals amplitude squared" (Chapter 7).

Carroll shows that the splitting works whenever the squared amplitudes are ratios of whole numbers, as thirty-six hundredths and sixty-four hundredths are. For every other number Carroll makes a joke at physicists' expense. Once you have proved a result for all ratios of whole numbers, "you hand the problem to a mathematician" and "declare that your work here is done" (Chapter 7).

Carroll answers a reader's next question as well. What credence should the experimenter give before the particle arrives at the mirror, when there is no uncertainty yet about anything? After the branching, every later copy of the experimenter will not know which branch it is on, and each copy will give Born-rule credences. Carroll calls each later copy a *descendant*. So the experimenter may give those same credences now, before the branching. On Carroll's account each branch has a weight equal to its squared amplitude, and the weights always add to one. When a branch splits, its two new branches share its weight. Carroll offers a branch's weight as a picture and adds at once, "I wouldn't want to take that mental image too literally" (Chapter 7). In Carroll's picture of weights, "Worlds get thinner as branching proceeds" (Chapter 7).

Carroll sets out a second derivation of the Born rule, the work of others. He calls it even more popular among those who study the foundations of physics. The second derivation comes from *decision theory*: the mathematics of how a reasonable person should choose and bet

when outcomes are uncertain. The physicist David Deutsch began the decision-theory derivation in 1999, and the philosopher David Wallace later made it more rigorous, as Carroll reports. Carroll gives an example of a question for decision theory. Carroll's example uses an electron's *spin*, a property that a magnet finds pointing either up or down. The electron is prepared so that the two outcomes have amplitudes of equal size. Someone offers you two dollars if the spin is found up, provided you promise to pay one dollar if it is found down. Should you take the bet, knowing that one of your descendants will be two dollars richer and the other one dollar poorer? Deutsch and Wallace answer with the Born rule again, as Carroll reports: "a rational agent in an Everettian universe acts precisely as if they live in a nondeterministic universe where probabilities are given by the Born rule" (Chapter 7). On Carroll's reading the Born-rule probabilities of the bet are "still real probabilities, even though the universe is a little richer than that" (Chapter 7).

The many-worlds account has no rule of probability beside its one equation. Yet Carroll and Sebens, and Deutsch and Wallace, derive the very weights that every laboratory uses. They derive the weights from that one equation, with the premises Carroll names. Carroll also shows you why the square fits so naturally, with nothing more than a right triangle.

The Same Weights

I derive the same weights as the two derivations in Carroll's book. I describe two derivations, the one Sebens and Carroll made and my own through Gleason's theorem (a theorem of mathematics about weights in quantum mechanics), and both derivations give the same weights. On Carroll's account and on my alike, thirty-six particles in a hundred pass the thinly silvered mirror, and sixty-four are turned aside.

I use Carroll's picture of the triangle too. I use the 'builder's triangle' (the triangle whose sides are three, four and five, which builders use to set out a square corner): three squared and four squared, nine and sixteen, make five squared, twenty-five. Shrink the builder's triangle to a long side of one, and its short sides are six tenths and eight tenths, the amplitudes at the mirror. Thirty-six hundredths and sixty-four hundredths make one. Six tenths and eight tenths do not add to one, and neither do their higher powers. *Only* the squares add to one, and the squares add to one however the set of directions is turned. The picture shows that the square works.

Physicists can ask nature with a measurement whether nature uses the square and *not* a higher power. A higher power is a cube, a fourth power, and so on. In 2010 physicists made that measurement in the place where a higher power would show itself. When light can arrive at a screen by two paths, the waves from the two paths add and cancel and make a pattern of light and dark bands where they overlap, which physicists call *interference*. If the weights go as the square (are in proportion to the squares of the amplitudes), then when light can arrive at the screen by three paths, the pattern is fully accounted for by the paths taken two at a time, with nothing left over that belongs to the three paths together. The reason is a fact of arithmetic. When you multiply out the square of a sum of three amounts, you get a sum of pieces, and every piece contains one of the amounts or two of them, and no piece contains all three. Raise the sum to the fourth power, and pieces appear that contain all three. At the Institute for Quantum Computing in Waterloo, Canada, five physicists sent light through a mask with three slits. The five were Urbasi Sinha, Christophe Couteau, Thomas Jennewein, Raymond Laflamme and Gregor Weihs. They opened and closed the slits in all eight combinations, with laser light and with single particles of light, and measured how much light arrived at each point of the screen. They found the pattern that the square predicts. No part of the pattern belongs to the three paths together. We now

know that the weights go as the square exactly where a rule of a higher power would differ from the square. Carroll's account and mine both predict that result. Sinha and her colleagues found that result.

Where the Collection Finds the Weights

I start from somewhere else: from the act of finding. The one who finds compares one thing with another and gets a verdict, whether a cook, a counter or a screen does the finding. I derive the odds from that start in three steps, and we can go through the three steps one at a time.

The first step shows that the outcome of a measurement is not fixed in advance. We took that step in this book's Chapter 4. In Chapter 4 the Incommensuration Theorem showed that a whole world cannot be completely fixed by one law. A whole world completely fixed by one law would make both perfect together. I take the Incommensuration Theorem from the whole world down to one measurement: no lawful description fixes every outcome found, and the single outcome of a quantum measurement is the plainest case of what the law leaves unfixed. Each particle at the mirror gives such a single outcome: the particle passes, or it is turned aside. So, even when the particle's quantum state is known as fully as the theory allows, the quantum state gives the weight of each outcome, "and the outcome found is not contained in it". In this book's Chapter 4 I gave a name to an outcome that no law and no knowledge of what came before it fixes: I called such an outcome hard randomness.

The second step shows that an unfixed outcome still has a stable weight. A stable weight is a share that stays the same from one long run to the next. I use the example of a lump of radioactive ore beside a counter. No one can name the minute in which one atom will decay. Yet each hour the same share of the atoms decays, hour after hour. "The single event is unfixed, and its weight is stable". I state the reason. When a comparison is made again on the same quantum state in the same context, it is the same comparison. So the outcomes of that comparison come back with one

steady share. Such a steady share is called a *relative frequency*: the fraction of the repetitions in which an outcome turns up. I identify the probability of an outcome with the stable relative frequency of that outcome in repetitions of the same comparison, on the same quantum state, in the same context. This probability belongs to the world, not to our knowledge of it. In this book's Chapter 4 we set apart the chance of a die, which measures what we do not know about the throw and would shrink with fuller knowledge of its speed, its turning and the table. The chance at the mirror is the other kind. No further knowledge of the quantum state removes that chance, *because* there is no fuller state beneath the quantum state to know.

The third step shows that the weight must be the square. The third step uses a theorem of mathematics. In 1957 the mathematician Andrew Gleason proved what is now called *Gleason's theorem*: any way of assigning probabilities to the outcomes of quantum measurements that satisfies a few plain conditions must be the Born rule. To picture the conditions of Gleason's theorem, stand in the corner of a room. Three edges meet in the corner: the upright edge where the walls meet, and two edges along the floor, all at right angles to one another. The three edges together picture one complete measurement with three outcomes, one outcome for each edge. Each edge is a direction in what physicists call the *space of states*: the space in which the arrow of Carroll's picture points, with one direction for each outcome of a complete measurement, each at right angles to the others. Now turn the two floor edges about the upright edge, as a door turns on its hinge. The turned set of edges is a second complete measurement, and it shares one outcome, the upright edge, with the first measurement.

Gleason's theorem needs two things of a rule for weights: two premises, the starting claims that the theorem rests on. The first premise is one weight for one outcome: the upright edge gets the same weight in both measurements, whatever it is measured with. The second premise is that the weights of outcomes that exclude each other add: the weight of "this outcome or that one" is the weight of the first outcome plus the weight of the second. Gleason's theorem also requires, besides these two

premises, two plain facts about weights: every weight is zero or more, and the weights of all the outcomes of one complete measurement add to one. Gleason proved that, wherever the space of states has three or more directions at right angles, as the corner of the room has, every rule that satisfies these premises is the rule of quantum mechanics, the Born rule in its general form. Physicists call a quantum state known as fully as the theory allows a *pure state*, and for a pure state the general form of the Born rule sets the weight of each outcome equal to the square of the size of its amplitude. In 2003 the physicist Paul Busch proved the corresponding result for every number of directions, two included, as at the mirror, when weights are given also to the outcomes of measurements that physicists call *unsharp*. An unsharp measurement is one whose outcomes do not cut the states cleanly apart: a detector that reports passed for nine in ten particles that passed the mirror, and for one in ten that were turned aside, makes an unsharp measurement. I put the result in these words: if the two premises hold, "then the Born rule follows as a theorem of mathematics, and it is no longer a postulate of physics". A postulate is a rule laid down without an argument for it. So the question "why the square?" becomes the question "why these two premises?"

Both premises come from the act of finding, as I show, sections "Why the Weights of Exclusive Findings Add" and "One Weight for One Finding". Take the second premise first. Take it at the thinly silvered mirror, where you count the particles one after another as each arrives at a detector. No particle is counted at both detectors. So in any run, the count at one detector or the other is the count at the first plus the count at the second. Divide each count by the number of particles, let the run grow until each share settles to its stable value, and the weight of "passed or turned aside" is the weight of passing plus the weight of being turned aside. The same counting shows that no weight is below zero, since no count is. And the two weights add to one, since every particle that arrives at the mirror is found at one detector and *only* one.

I derive the second premise by another route as well, a route that counts nothing and uses a single comparison. At the mirror, the one who finds compares a count at the detector on the far side with a count at the detector at the side, and what makes those two counts two is that the two detectors stand in two places. So the context of the measurement at the mirror is the mirror with its two detectors in their places, and the context sets the division of the content into passed and turned aside. In that division there is one region of content for each outcome, and every possible content falls in one region and *only* one. Two outcomes that exclude each other have regions that do not overlap. On this route a weight is given to content: the weight says how strongly the content of a region could be found. Every content lies in one region, so the weight of the two regions together is shared between the content of the first region and the content of the second, with no share left over and none missing. So the weight of "passed or turned aside" is the weight of the passing region plus the weight of the turned-aside region. The first route rests on counting repeated acts of finding, and the second rests on the parts of a single comparison. The two routes rest on different premises and give one result.

The first premise, one weight for one outcome, comes from what I call the *law of symmetry*. The law of symmetry says, in plain words, that whatever is found in a comparison is found as something that stays the same while the context of the comparison changes. The law: what is found as content is found as a sameness of content across a difference of context. The corner of the room shows the law of symmetry at work. The upright edge, met beside one pair of floor edges and then beside the turned pair, is the same content found in two contexts. A weight says how strongly some content could be found. The content of the upright edge is the same in both measurements. So I take the same content found in two contexts to be the same outcome, with one weight.

Once both premises are in hand, Gleason's theorem shows that one rule alone satisfies the premises. Busch's theorem shows the same, and Busch's theorem covers two directions as well. That one rule is the Born rule. "The Born rule is the only rule that comparison allows". I set out the

same argument for the physicist, in a chapter that ends: “It runs from comparison to the Born rule without a probabilistic premise”. The two premises about weights are steps inside that argument. Each premise is reached from the structure of comparison through two identifications, both made earlier in this section. The first identification is that the probability of an outcome is its stable relative frequency in repetitions of the same comparison. The second identification is that the same content found in two contexts is the same outcome, with one weight. The argument rests on two stated identifications. The argument starts from comparison itself, which contains no probability. The argument ends with probability and with one rule for probability, the Born rule.

The One Who Finds Comes Back

Carroll’s account and mine start from opposite places. The same weights follow from both: thirty-six and sixty-four in a hundred at the mirror. The starting place of each account determines what kind of thing a weight is in that account. I begin with the one who finds: a comparer (a person, an instrument, or any physical system that registers one thing against another), an act of comparison, and the range of what could be found in that act. A probability is the steady share of one outcome over many acts of finding. A probability of that kind belongs to the world. My weights are how strongly each outcome could be found, and the weights need no belief and no bet in order to hold. The Born rule connects the quantum state, which describes the range of what could be found, to the one outcome found in each act: the Born rule is the bridge from the one to the other.

I can say what self-locating uncertainty is, because I start from the one who finds. After the branching and before the look (the moment the experimenter looks), Carroll’s copy of the experimenter does not yet know which branch it is on, and it asks what credence to give each branch. In my chapter on Carroll I name that observer: the observer “is a comparer

with a stance, before an act, facing a range of what it could find". I call the side a comparison is made from a *stance*. So the observer Carroll describes is the one who finds, before the act of finding. When Carroll, following Everett, let every outcome occur, he set the one who finds aside. At the step where the weights are needed, the one who finds comes back into the many-worlds account. The one who finds comes back as the observer whose credences Carroll and Sebens count. In the same section I credit self-locating uncertainty for what it shows: "Self-locating uncertainty is the strongest answer given to the problem of probability, and it shows exactly where the act of finding comes back". I begin with the one who finds, the act and the range of what could be found. So I have nothing to bring back.

On the subject of the odds, Carroll's account and mine part at what a probability is, and that parting follows from the one place where they part: what lies at the ground. For me, a probability is the stable share of one outcome over many acts of finding. Each act has one outcome. For Carroll, a probability is a credence about a place: on which branch of the universal state (the quantum state of the whole universe), in which every branch occurs, this copy of the observer is. That place is a place within a structure laid out whole, and not one outcome selected from among those that could have occurred. An act of finding selects one outcome from among those that could have occurred, and a place within a structure laid out whole is not such a selection. Of the self-locating reply: the act is still missing. So the self-locating derivation needs the act that Carroll set aside, and the act comes back into the derivation in the form of a premise: the observer, after the branching and before the look, whose credences Carroll and Sebens count.

The decision-theory derivation of Deutsch and Wallace parts from mine at the same place: at what a probability is. I credit Deutsch and Wallace with recovering the Born weights as the credences a rational agent must give to branches. The same section then states what their derivation needs: "When every outcome occurs, a probability needs an agent who faces outcomes and weighs them". That agent faces the outcomes and

chooses among bets on them, and that agent is the one who finds, before the act of finding, entered into the derivation as a premise. I use a picture to show why the Born rule cannot be derived from inside the mathematics of the quantum state alone. The Born rule connects the mathematics of the quantum state with the world of experiment, so the Born rule is a bridge between the two. And "a bridge cannot be built from one bank alone". In that picture one bank is the mathematics of the quantum state, the other bank is the world of experiment, where acts of finding are made, and a derivation from the mathematics alone builds from one bank only.

Both partings follow from what we found in this book's Chapter 5: the parting at self-locating uncertainty and the parting at decision theory. The universal state, branches and all, exists as structure laid out (in Chapter 5's sense of exists: timeless, and the same for anyone who reads it), and for the one who finds, one outcome is real (given in an actual act of finding), the one found. So a weight belongs to an outcome. The weight shows across many acts of finding as the steady share of that outcome. At the mirror, the weight of passing is the share that particles sent one by one settle to: thirty-six in each hundred. In a picture he says he would not take too literally, Carroll puts the weight in the branch itself. Branches grow thinner as they divide. In both pictures the number is the same, thirty-six in a hundred, but the number belongs to different things: for me, to the outcome found over many acts of finding; for Carroll, to the thickness of a branch.

What the Cook Finds

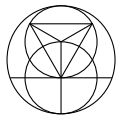
Go back to the kitchen one last time for this chapter. The cook brings the egg down on the rim of the bowl and finds one crack. One act of finding shows no weight. One act has one outcome, and a weight shows only across many acts of the same comparison, as the steady share of each outcome. At the mirror, particle after particle, the shares settle to thirty-six

and sixty-four in each hundred. The theorems of Gleason and Busch say those shares must be the squares of the amplitudes, given one weight for one outcome and weights that add.

We now know where the odds come from. The same weights come out of the derivations in Carroll's book and out of my derivation. In self-locating uncertainty, the one who finds comes back at the step where the weights are needed. And I start from the one who finds and derives the weights through Gleason's theorem.

Where the odds come from matters for the book's question about what lies at the ground. Physicists use the Born rule to turn a quantum state into the odds of what will be found. They do so every day. Every derivation of the Born rule needs the act of finding. The reason is that the rule connects the quantum state to the outcome found: the Born rule is the bridge between the two. Where an account starts from the act of finding, the Born rule is derived from the act. Where an account leaves the act out, the act has to be brought back in before the rule can be recovered.

Every act of finding happens somewhere and at some time: the cook at the counter this morning, a particle at a mirror on a laboratory bench. If the one who finds is part of the ground, is the space in which the finding happens part of the quantum state too?



Chapter 7

Is Space Made of Something Quantum?

What this chapter uses

The six parts of every comparison and the four characters (Chapter 2).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *arena*: space and time joined, with the shape that general relativity gives them, the setting in which every act of finding is made (Chapter 3).

The *wave function*, Carroll's complete quantum description of a system, which he holds is the system itself (Preface and Chapter 1).

Light comes in quanta, separate packets of energy, and an atom can hold only certain energies. Is space made of something quantum too? A reader of Carroll carries the question out of his books on space and time. This book's Chapter 6 ended on the same question. Chapter 6 found that every derivation of the odds of quantum theory needs the act of finding. And every act of finding happens somewhere and at some time. So we can ask of space the question we asked of the one who finds. Every find-

ing happens in space: is space part of the universal state, the quantum state of the whole universe, or is it something else? We begin on the day in 1919 that astronomers measured the shape of space.

Stars Beside a Darkened Sun

On 29 May 1919 the Moon passed exactly in front of the Sun. The Moon's shadow crossed the Earth along a band from South America to Africa. Two British parties were waiting in the band of shadow, and Frank Dyson, the Astronomer Royal, had organized both parties. Andrew Crommelin and Charles Davidson stood beside their telescopes at Sobral, a town in the north-east of Brazil. Both men came from the Royal Greenwich Observatory. Arthur Eddington, an astronomer at Cambridge, worked with Edwin Cottingham on Príncipe, a small island in the Gulf of Guinea off West Africa. A bright cluster of stars, the Hyades, lay behind the darkened Sun. On an ordinary day the glare of the Sun hides every star that lies near the Sun in the sky, but while the Moon covered the Sun, the two parties could photograph the stars of the Hyades.

The photographs were taken on glass plates, to answer a plain question. Light from the stars of the Hyades passed close to the edge of the Sun on the way to the Earth. Did the Sun bend that light? Isaac Newton's theory of gravity, applied to light, predicts a small bend. Albert Einstein's theory of gravity, general relativity, predicts a bend twice as large. Both bends are tiny, so astronomers measure them in seconds of arc. A second of arc is about the width of a small coin seen from four kilometers away. For light grazing the edge of the Sun, Einstein's theory predicts one and three quarter seconds of arc, and Newton's theory predicts half as much.

Thick cloud lay over Príncipe during the eclipse, so only two of Eddington's plates showed enough stars to measure. The sky over Sobral was mostly clear. Later the astronomers set each eclipse plate beside a plate of the same stars, taken at night with the same telescope. The night plates were taken months earlier or later, when the Sun was elsewhere in

the sky. The astronomers compared the positions of the stars on the two plates, one star at a time. The results were announced on 6 November 1919 at a joint meeting of the Royal Society and the Royal Astronomical Society in London. The best instrument at Sobral gave a bend of about two seconds of arc at the Sun's edge, and the plates from Príncipe gave a bend of about one and six tenths seconds of arc. Both values lay near Einstein's value and far from Newton's value. Dyson, Eddington and Davidson published the full report the next year in the *Philosophical Transactions of the Royal Society*.

Each plate held small dark dots, the images of stars, and the astronomers found each dot at one place on the glass. The bend of the light was in none of the dots. The bend was in the shape of the space and time through which the light had traveled, because the light had taken the straightest path that the shape of space and time allowed, and near the Sun the straightest path curves. What was found was a set of dots. Where the dots were found was a setting shaped by the Sun.

The cook's egg has the same two sides: what is found, and the setting in which it is found. The cook cracks the egg once on the rim of a bowl, and the crack is one act of finding. The act of finding happens on a counter, in a kitchen, on a particular morning. We can describe the egg completely, shell, white and yolk, and the description of the egg will contain no counter, no kitchen and no morning. The counter, the kitchen and the morning are where and when the crack is found. The setting in which the egg is found is not part of the egg. In physics, is the setting of every finding also separate from what is found, all the way down? Are space and time part of what is found, or are they the setting in which things are found?

Spacetime Is What Exists

What did Einstein change about space and time, as Carroll tells the story? Sean Carroll teaches relativity as Einstein left it, clearly and with the mathematics in view. Before Einstein, physicists could put space and time together into one whole, but nothing in physics required physicists to do so. The one whole of space and time is *spacetime*: space and time taken as one, with three directions of space and one of time. Physicists call spacetime four-dimensional because it has those four directions. After Einstein, Carroll writes, spacetime is what there is: "What really exists is spacetime, and our slicing it up into space and time is merely a useful human convention" (*The Biggest Ideas in the Universe: Space, Time, and Motion*, Chapter 6, "Spacetime"). Two observers moving past one another divide spacetime into space and time in different ways, and neither division is the true one.

Einstein then made gravity a property of spacetime. Einstein noticed that everything falls in the same way, whatever it is made of. So in a small enough room a fall under gravity cannot be told from a ride in an accelerating rocket. He set the sameness of every fall beside a fact of geometry: every curved surface looks flat in a small enough patch. Carroll draws Einstein's conclusion about gravity as Einstein drew it: "Maybe we should think of it as a feature of spacetime itself: the curvature" (*Space, Time, and Motion*, Chapter 8, "Gravity", section "The Principle of Equivalence"). In general relativity, curvature means that the straightest paths through spacetime bend toward one another, in the way that two lines of longitude, each straight along the globe, bend toward one another and meet at the pole. In *From Eternity to Here* Carroll writes that "gravity can be thought of as a manifestation of the curvature of spacetime" (Chapter 5, "Time Is Flexible", section "Curving Straight Lines"). A planet in orbit is a body moving as straight as it can through a curved spacetime, and a cup falling from a table is also moving as straight as it can.

Carroll tells the eclipse of 1919 as the turning point: the expedition photographed the stars beside the Sun and "verified Einstein's prediction of light bending" (*Space, Time, and Motion*, Chapter 8, section "Empirical Consequences"). Spacetime has a shape, and matter and energy set the shape of spacetime. Bodies and light follow the shape of spacetime, and we feel that shape as gravity.

From Carroll's pages

Finding Gravity Within the Quantum

Carroll asks a further question in *Something Deeply Hidden*, his book on quantum mechanics: can spacetime itself be found within quantum mechanics? When he asks the question, he goes a step past Einstein, and he takes that step on purpose, because quantum theory describes matter and light, and he holds that nature is quantum from the start.

Carroll begins with the question of why we live in space at all. In the mathematics of quantum theory, position and motion stand on an equal footing, and neither one comes first. Position is singled out because things act on one another when they are near each other in space, as two billiard balls on a table bounce off each other only when they meet. A thing acts directly only on what is near it, and Carroll calls that fact *locality*. In Many-Worlds, quantum states are fundamental: they lie at the bottom of everything. In the usual order, space comes first and the quantum state sits within space. Carroll takes locality as a sign that the usual order should be turned around: "Space isn't fundamental; it's just a way to organize what's going on in the underlying quantum wave function" (*Something Deeply Hidden*, Chapter 11, "Why Is There Space? Emergence and Locality").

Then Carroll sets himself a task of reverse engineering. On the usual road, physicists take Einstein's smooth spacetime and try to turn it into a quantum theory, but Carroll goes the other way: he starts from the quantum state and tries to get spacetime out of the quantum state.

He starts with nothing but the universal state, the quantum state of the whole universe. The universal state belongs to *Hilbert space*, the mathematical space in which a quantum state is a direction. In Many-Worlds, Carroll writes, a wave function is "an abstract vector living within the super-high-dimensional mathematical construct of Hilbert space" (Chapter 13, "Breathing in Empty Space: Finding Gravity within Quantum Mechanics"). A vector is an arrow. Carroll tries to get space out of the arrow of the universal state and nothing else.

Carroll's construction of space has two steps, and in the first step he divides the universal state into parts. Physicists call such parts *degrees of freedom*: the separate pieces of a system, each of which can be in its own range of states. The universal state can be divided into parts in many ways. The *Hamiltonian* determines the division. The Hamiltonian is the rule by which a quantum state changes in time, fixed by the list of energies the system can have. Carroll explains that for a random list of energies no division gives parts that act only on their neighbors, and that for the right kind of Hamiltonian exactly one division does. He adds that, from all our experience of the world, it seems that our world has the right kind of Hamiltonian. Carroll puts the first step in his own words: "Demanding that physics look local tells us how to decompose our quantum system into a collection of degrees of freedom" (Chapter 13). Then Carroll reverses the usual order of nearness and acting. Usually two things act on each other because they are near, but Carroll defines two parts as near because they act on each other: "we define two degrees of freedom to be 'nearby' when they directly interact with each other" (Chapter 13). The parts and their links form a network, which Carroll calls the skeleton on which space is built.

The second step of the construction gives distances. This book's Chapter 5 glossed *entanglement* as a joint state of two things whose parts cannot be described one by one. Picture the network as a fishing net, with a knot for each part and a thread wherever two parts act on one another directly. Carroll takes the amount of entanglement

between two knots as the measure of how near the two knots are, so that the more entangled two knots are, the nearer they are. Now imagine a closed surface, like a soap bubble, drawn around any cluster of knots. Carroll takes the amount by which the knots inside are entangled with the knots outside as the area of the surface, and he does the same for every surface you could draw. Mathematicians showed long ago, he notes, that knowing the area of every surface in a region fixes the whole geometry of the region. The geometry of a region is every distance and every angle in the region. A *metric* is the rule that fixes the distance between any two places, and Carroll sums up the second step: "A metric on space has emerged from the entanglement structure of the quantum state" (Chapter 13).

Where gravity is strong, as at a black hole, Carroll turns to *holography*: the idea that everything about a region can be written on the boundary of the region, as a flat hologram carries a picture with depth. Of the places where gravity is strong, he writes: "there are still abstract degrees of freedom coming together to form spacetime, but 'where they are located' depends on how they are being observed". So, he concludes, "Space itself is not fundamental; it's just a useful way of talking from certain points of view" (Chapter 14, "Beyond Space and Time: Holography, Black Holes, and the Limits of Locality"). Carroll states the aim of his whole program as the reverse of the usual road: "the route to understanding the universe lies not in quantizing gravity, but in finding gravity within quantum mechanics" (Chapter 13).

Carroll himself calls the ideas of Chapter 13 speculative, and he says so at the start of the chapter: "We'll be looking at speculative ideas that might be important to understanding quantum spacetime and cosmology". Carroll has worked on the program with two other physicists, ChunJun Cao and Spyridon Michalakis. With his construction, Carroll draws a geometry of space out of the entanglement of the universal state.

Many-Worlds is Carroll's name for the many-worlds account of this book's Chapter 5. In Carroll's picture a quantum state is an arrow of length one, the picture that this book's Chapter 6 used. I credit what the work of Carroll, Cao and Michalakis has done in the chapter on Carroll: "Carroll's program has recovered the spatial part".

Where the Two Agree

Which positions on space, time and gravity do I share with Carroll? I share four of his positions.

First, I hold with Einstein and Carroll that gravity is the shape of space and time.

Second, I hold with Carroll that space is made of connections. Carroll defines two parts as near when they act on one another directly. When one event can affect another, the two events are *causally connected*. I define space in the same way as Carroll defines nearness, with events in place of parts. The definition is, section "What Space Is": "Space is the pattern of which events are causally connected with which, and it is no container that exists before the events it holds". The reason I give there is that every interaction is an act of comparison under another name. A thermometer dipped in coffee is acted on by the coffee, and in that one act the thermometer registers the coffee as warmer than itself. A cause is one kind of interaction, so a causal connection is a comparison. So the pattern of causal connections is the pattern of the comparisons that can be made.

Third, I hold with Carroll that nothing known about gravity makes space and time grainy. Space and time would be grainy, or discrete, if they were broken into least steps: a smallest length or a smallest moment. I list "that nothing known about gravity makes space and time discrete" as one of five points on which I agree with Carroll, in the abstract of the chapter on Carroll.

Fourth, I hold with Carroll that the usual road is the wrong road: the road on which physicists quantize Einstein's spacetime, that is, try to turn it into a quantum theory. Carroll turns from the usual road because "Nature simply is quantum from the start", as he writes in *Something Deeply Hidden*, Chapter 11. I turn from the usual road for a different reason. A quantum description is a description of what is found, and space and time are the setting in which things are found. I call space and time together, taken as the setting of every finding, the *arena*. To quantize the arena would be to describe the setting as if it were one more thing found. Carroll and I leave the usual road for different reasons, and after that they go in different directions. Where do Carroll and I part?

The Setting of Every Finding

What is the setting in which every finding is made, and can the setting itself be one more thing that is found? In 2010 four physicists compared two clocks at the National Institute of Standards and Technology in Boulder, Colorado. The four were James Chou, David Hume, Till Rosenband and David Wineland. Each clock ticked with the light absorbed by a single aluminum ion held in a trap. An aluminum ion is an aluminum atom with one electron removed. The four physicists raised one clock by thirty-three centimeters. The raised clock ran faster, by the small amount that relativity predicts for that change of height. I read the experiment by asking what quantum theory and relativity each did in it. Quantum theory describes the ticking: each aluminum ion absorbs light of one exact frequency, the same for every aluminum ion. The *frequency* of light is the number of its waves that pass a point each second, and this book's Chapter 6 used the word in a different sense: a relative frequency is the fraction of the repetitions in which an outcome turns up. Relativity describes the difference between the two rates: the setting, space and time, is

shaped a little differently thirty-three centimeters higher. I sum up the experiment in two sentences: “What was found was quantum. Where it was found was the arena”.

The arena is space and time together as the setting of every finding. Every length is a length in the arena, and every duration is a duration in it. This book’s Chapter 2 named two parts of every comparison: its *content*, what the verdict of the comparison is about, and its *context*, what makes the two things compared two. Look again at the two clocks. The two clocks were built alike and ticked with the same kind of ion. One clock stood thirty-three centimeters above the other, and that difference of height made them two clocks, with two rates to compare. Two findings are two because of a here and a there, or a now and a then. So space and time are the context of every finding in physics.

I identify gravity with the arena: space and time together, with the shape general relativity gives them, as the setting of every finding. When I identify gravity with the arena, I take that shape together with the space and time that have the shape, as one setting. The arena, taken in that way, is the context of physics.

I answer this chapter’s question by identifying gravity with the arena. This book’s Chapter 2 set out the Incommensuration Theorem, which the box at the head of this chapter recalls: symmetry and continuity cannot both be made perfect in one comparison, and neither can asymmetry and discontinuity. Asymmetry is nothing kept the same, and discontinuity is a jump, with nothing between before and after. To apply one of the four characters (symmetry, continuity, asymmetry or discontinuity) fundamentally is to make that character perfect at the foundation of a description. Two pairings can each be made perfect without breaking the Incommensuration Theorem: symmetry with discontinuity, like the ice-cube tray of Chapter 2, and continuity with asymmetry, like Chapter 2’s ribbon of honey. What is found in physics is held to symmetry. Every electron is exactly like every other, and an aluminum ion absorbs light at the same frequency every time. So what is found takes the first pairing, symmetry with discontinuity: in what is found, symmetry is perfect and continuity

is not. So what is found comes in separate, countable steps, which is the quantum character. The setting is held to continuity: the arena is smooth, with no gap anywhere. So the setting takes the second pairing, continuity with asymmetry: in the setting, continuity is perfect and symmetry is not. So the arena is uneven: it is shaped differently near the Sun than near the Earth. So each of the two theories of physics describes one of the two pairings that the Incommensuration Theorem allows, made perfect: quantum theory describes what is found, and general relativity describes the setting in which it is found.

The same two pairings are my reason why the setting curves where matter is. The act of comparing is smooth: it has no break in it. To compare is to hold two things in one act, and that holding has no break in it. What is found is not uneven, because it keeps its sameness for everyone who finds it. The Incommensuration Theorem pairs smoothness with unevenness. Continuity held perfect has one partner that the theorem allows, asymmetry, as the ribbon of honey of Chapter 2 is unbroken and never the same twice. Neither the act nor what is found is uneven, so the setting is the uneven part of every comparison. The setting differs from place to place. That difference from place to place is curvature. The context curves where the content is, in proportion to the energy and momentum of the content. In that sentence the context is the arena, and the content is the matter found in the arena. The appendix "For the Specialist" contains that reason step by step.

Why Nothing Is a Quantum of the Arena

Physicists describe light by quanta. Can the arena be described by quanta too? I state what it means for the arena not to be quantized. That the arena is not quantized means two things: "(i) The arena has no least step: no least length, area or duration. (ii) Nothing is a quantum of the

arena itself, at any scale". The first clause says that space and time are not grainy. The second clause says that nothing, at any size, is a quantum piece of the arena itself.

The reason for the second clause is that the arena is the setting of every finding. The clock experiment in Boulder shows the arena as that setting. Chou and his colleagues set a quantum description beside a smooth one: the ticking of each ion, which is quantum, beside the difference between the rates of the two clocks, which is smooth. They compared the two descriptions within one setting that both descriptions kept: a laboratory in Boulder, at its place and on its day, in the arena. Whenever a physicist sets quantum and smooth descriptions of one thing side by side, the physicist compares the two descriptions within the arena. Now try the same comparison with the arena itself. This time the arena cannot be the setting that stays as it is while the two descriptions are compared, because the arena itself is now the very thing that is being described. A physicist would need some further setting in which to set a quantum description of the arena beside the smooth one and compare the two. The further setting would then be the context of physics in the arena's place. The arena would no longer be the setting of every finding. I identify gravity with the arena, the setting of every finding, so a quantum description of the arena itself has no setting in which a physicist could set it beside the smooth description. To quantize the arena is to apply to context the treatment that belongs to content. In plain words: quantizing is a treatment made for what is found, and the arena is the setting in which things are found.

The Incommensuration Theorem allows quantum theory and general relativity this: "The Incommensuration Theorem permits a common source: two descriptions joined side by side, each keeping its own character. It forbids a merger". General relativity and quantum theory are two descriptions of one world, each capturing what the other leaves out. Matter and light are quantum through and through, the arena they are found in is smooth and uneven, and the two descriptions are joined side by side, as the clock experiment joined them.

I name the place where Carroll's program parts from mine in the chapter on Carroll: "Carroll derives the arena from the state, and the collection holds that the arena is the setting of every state". In the same section I record that Carroll's program agrees with the first clause of Proposition 6.3, that the arena has no least step, and runs against the second clause, that nothing is a quantum of the arena itself. The reason is in Section VIII of the same chapter: on Carroll's account the geometry of space is carried by quantum degrees of freedom of the universal state. When a physicist draws the arena out of the universal state, the physicist gives the setting, at its foundation, the character of what is found: the geometry of space becomes a quantum thing, described in the way that what is found is described. One description would then require the same directions of space to be smooth and uneven, as the setting is, and also to keep exact sameness and to come in quanta, as what is found does. If directions of space had all four characters at once, they would have both of the pairings that the Incommensuration Theorem rules out: smoothness with exact sameness, and unevenness with separate quantum steps. The merger that the Incommensuration Theorem forbids is one description of the arena with both forbidden pairings. I name the one premise that Carroll and I dispute: "whether the geometry of space is part of what a quantum state describes, or the setting in which every quantum state is described".

The cook's egg shows in the kitchen why the setting cannot be one more thing that is found. You can describe the egg down to its last molecule, and every description of the egg is laid out somewhere and at some time. The counter, the kitchen and the morning are where the description is laid out. The same counter, kitchen and morning are where the crack is found. The kitchen holds the egg, and the egg does not hold the kitchen.

From Carroll's pages

The Time That Goes Missing

Does Carroll build the setting of his own account out of the universal state, or does he start with a setting already given? He starts with a setting already given. His construction of space takes time as given. In Chapter 13 Carroll gives the reason: time appears in Schrödinger's equation, the rule by which a quantum state changes. Carroll writes of Schrödinger's equation: "'Space' may or may not be part of that equation, depending on what system we're looking at, but time is fundamental" (*Something Deeply Hidden*, Chapter 13). So Carroll draws space out of a universal state that already changes in a given time.

Carroll then takes up the case in which time is drawn out of the universal state as well. He notes that, by Schrödinger's equation, how fast a quantum state changes depends on how much energy the state has, so a system whose energy is exactly zero does not change at all. A *closed universe* is a universe that wraps around on itself, as the surface of a globe does. A closed universe is finite in size and has no edge. Carroll explains that in a closed universe the energy of the gravitational field, which is negative, exactly cancels the positive energy of everything else, so the total energy of a closed universe is exactly zero. He writes that the physicists John Wheeler and Bryce DeWitt developed an equation that is the quantum form of the statement that a closed universe has zero energy. Carroll states what the equation of Wheeler and DeWitt says: "The Wheeler-DeWitt equation simply says that the quantum state of the universe doesn't evolve at all as a function of time" (Chapter 13). He sets what the equation says beside what everyone sees: "The universe certainly seems to evolve". Physicists call the clash between the equation and what everyone sees the problem of time in quantum gravity. Quantum gravity is the effort to give gravity a quantum description.

The same section states the reason: "The context in which things happen has been put into the quantum state", instead of serving as the arena in which the quantum state changes. The time goes missing because the setting where every quantum state changes has been placed inside the universal state.

Carroll also describes a construction where time comes back with the reading of a clock. The physicists Don Page and William Wootters suggested the construction in 1983. Carroll says that the construction "might work", and he gives it as a picture. Imagine a clock and everything else in the universe, both changing in time as usual, and take a snapshot of the universal state once every second. Each snapshot shows the clock at one reading and the rest of the universe as it was at that reading. Now add all the snapshots together into one new quantum state, as quantum theory allows. Of the new state Carroll writes: "This new quantum state doesn't evolve over time; it just exists, as we constructed it by hand" (Chapter 13). Time appears, in Carroll's words, when "we measure the clock and see that it reads some particular time" (Chapter 13), and the rest of the universe is then found as it would have been at that time. So time comes back with an act of finding, like the cook glancing at the clock on the kitchen wall. Carroll keeps both answers in view, time given or time emerging from the universal state: "At the current state of the art, it makes sense to keep our options open and investigate both possibilities" (Chapter 13).

The setting and the act of finding come back into Carroll's account at one more step: the choice of a basis. A *basis* is one of the many ways of writing a quantum state as a list of alternatives, and Many-Worlds needs one basis in order to say which alternatives are the branches. Carroll answers the question of which basis to use with locality, in Chapter 11 of *Something Deeply Hidden*. Air and light act on a thing

only where the thing is, so the alternatives that stay apart describe "coherent objects in space". Locality is nearness in space. So, on Carroll's account, the setting comes back at the choice of a basis.

At three steps, Carroll's account uses a setting or an act of finding that the universal state alone does not contain: the given time, the reading of the clock and the choice of a basis.

Carroll holds that the setting of everything is a Hilbert space with a Hamiltonian and a given time, and that the geometry of space is a pattern within the state. I take the problem of time as a sign of where the arena has been put: inside the universal state. I take up the equation of Wheeler and DeWitt, in which the quantum state, "which in every other quantum theory evolves in the time of an arena, has nothing in which to evolve". Kept as context, the arena keeps its time, and the problem does not arise. I name what the measurement of the clock is: "a reading is a finding". This book's Chapter 5 gave my answer. The pattern: an account that places the arena in the universal state needs some other given setting in the arena's place. Carroll's premise is that a given time and a Hamiltonian are that setting, and the pattern leaves his premise standing. The same section says that Carroll's account and mine "agree that some setting is given, and differ on whether space belongs to it".

What the Arena Has Shown

What have later measurements of the arena found? Physicists have measured the arena many times since the eclipse of 1919. Each measurement has the same two sides: what was found, and where it was found.

The Laser Interferometer Gravitational-Wave Observatory (LIGO) is the American observatory for gravitational waves, which are ripples in the shape of space and time. LIGO has two detectors, one in Louisiana and one in Washington State. On 14 September 2015 the two detectors registered a passing ripple of the arena itself. What was done: each detector

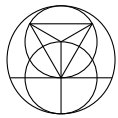
measured, with laser light, the lengths of two long arms set at right angles to each other. What was found: a ripple that stretched and squeezed the arms by about one part in a thousand billion billion. The ripple came from two black holes more than a billion light years away, as they merged. The two black holes had about thirty-six and twenty-nine times the mass of the Sun. The ripple rose in pitch up to the merger and rang down after the merger, and the shape of the ripple matched the shape that general relativity predicts for such a merger. The result was reported in *Physical Review Letters* in 2016. What we now know: the arena bends and ripples smoothly, in the shape Einstein's theory predicts, even where two black holes merge. The detectors found the ripple as a train of readings, each reading one act of finding, and the ripple itself was a change in the setting in which the readings were made.

In the eclipse of 1919, the clocks of 2010 and the ripple of 2015, what was found came as single readings. Each time, where the readings were found was the smooth and uneven arena that general relativity describes. Light bends near the Sun, a clock runs faster thirty-three centimeters up, and space rings when black holes merge. The bending, the faster running and the ringing belong to the setting, and the readings belong to what is found.

We can now answer this chapter's question, whether space is made of something quantum. The arena (space and time joined) has no least step, and nothing at any scale is a quantum piece of the arena. Carroll draws the arena out of the universal state, and I hold that the arena is the setting of every quantum state. Carroll and I agree that some setting is given. On space and time, they part at whether space belongs to that setting, and that parting follows from the one place where they part: what lies at the ground. For Carroll the ground is a quantum state evolving by one rule, the universal state, so space has to come out of the universal state. For me the ground is the act of comparison, the act of finding, and the arena is the context of every such act. When a physicist puts the whole arena into the universal state, time goes missing, and the missing time is the plainest sign of the merger that the Incommensuration Theorem forbids.

Where space and time belong matters for the book's own question about what lies at the ground. The act of finding keeps its place at the ground, and so does the time in which every finding is made, because the arena is the setting in which every finding is made. Every quantum state, the universal state included, is laid out within the arena, and the arena is never one more part of any quantum state.

If the arena is the setting of every finding and keeps its time, why does that time run only one way?



Chapter 8

Why Does Time Run Only One Way?

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *arrow of time*: time running one way, at the bowl the one-way order from what could happen, to the act, to the record; and the *bound triple* (Chapter 5).

The *arena*, the setting of every act of finding (Chapters 3 and 7).

The Crack That Stays

Why does time run only one way, when the laws of physics run as well backward as forward? Go back to the kitchen. The cook cracks the egg once on the rim of the bowl, and the shell splits along one jagged line. No one has ever seen that line close up again, the shell seal itself, and the whole egg rise from the rim into the cook's hand. The crack stays.

Picture a film of the cracking, run backward. You would know at once which way the film is running. Yet each motion in the backward film is a motion the laws of physics allow. The laws of physics require only that

every molecule of the shell, the rim and the air (a molecule is a small cluster of atoms held together) move in exactly the right way at exactly the right moment. Each of three sets of laws allows a process to run backward whenever it allows the same process to run forward: Newton's laws of motion, Maxwell's laws of electricity and magnetism, and Schrödinger's equation for quantum states. So the laws of physics, read on their own, do not say why the crack stays.

In this book's Chapter 7 we placed time in the arena, the setting in which every act of finding is made. We found that the arena keeps its time. Where does the direction of the arena's time come from? Two friends in Vienna put the question sharply in 1876, in an argument about a box of gas.

Vienna, 1876: A Question Between Friends

In 1876 Josef Loschmidt was a professor at the University of Vienna. Eleven years earlier he had found a way to estimate the size of a molecule. He drew the estimate from the density of gases made liquid, and from how far a molecule in a gas travels between collisions. His younger colleague and friend Ludwig Boltzmann had published a paper in 1872, in the proceedings of the Imperial Academy of Sciences in Vienna. In that paper Boltzmann offered a proof that a gas left to itself always moves toward its settled state, whatever arrangement the gas starts in. He drew the proof from the laws of motion of the gas's molecules. A gas in its settled state is spread evenly through its container and is at one temperature throughout. Boltzmann's proof was a proof of the Second Law of Thermodynamics. The Second Law of Thermodynamics is the law of heat that says a closed system of many parts runs toward its settled state and never, of itself, back out of that state. A *closed system* is one that takes in nothing from its surroundings and gives nothing to them. For example, heat flows from a hot cup into the cool room around the cup, and never from the room back into the cup.

Loschmidt raised a question about Boltzmann's proof in a paper printed in the same proceedings in 1876. Picture a box of gas whose molecules all began crowded in one corner and have since spread through the whole box. Now stop every molecule at once, and send each one off again at exactly its own speed, in exactly the opposite direction. The laws of motion run backward as well as forward, so every molecule retraces its path, and the gas gathers itself back into the corner. For every history in which the gas spreads out, the laws of motion allow another history in which the gas gathers in. That other history is made by reversing every velocity. Therefore a proof must contain an assumption besides the laws of motion when the proof claims to draw the spreading of a gas from those laws alone.

By then Boltzmann was a professor in Graz, and he wrote a reply to Loschmidt. Carroll dates the reply to 1876, and the Academy printed the reply in 1877. In the reply Boltzmann granted Loschmidt's point. He wrote that the increase of entropy in every physical process in our world "must be a consequence of the initial conditions". In that sentence, entropy is the quantity that grows as a gas settles. Carroll gives Boltzmann's sentence in English in *From Eternity to Here* (Chapter 8, section "When the Laws of Physics Aren't Enough"). The initial conditions are the positions and speeds of the molecules at the moment the history of the gas starts. Then Boltzmann answered Loschmidt by counting. He wrote that any one evenly spread arrangement of the molecules is as likely as any one crowded arrangement. But, he went on, "there are many more uniform states than non-uniform states". He concluded that if the starting arrangement of a gas is chosen at random, the gas "is almost certain to evolve into a uniform state".

In 1877 the Academy printed a second paper by Boltzmann, in which he made the counting exact. He divided the possible positions and speeds of a molecule into small cells. He counted each way of placing every molecule in a cell as one arrangement. Then he took each way the gas can look from outside: how many molecules sit in each cell, whichever molecules they are. For each such look he asked how many arrangements give that look. The settled state, spread evenly through the box, is the look

that the most arrangements give, by an enormous margin. So a gas that wanders among its arrangements is nearly always moving from a look that few arrangements give toward a look that many arrangements give. Entropy was a quantity that the science of heat already used. Boltzmann identified entropy with a measure of how many arrangements give each look. *Entropy* is a count of how many arrangements of the small parts look the same from outside. A gas spread evenly through its box can be arranged in vastly more ways than a gas crowded in one corner, so the spread gas has the higher entropy.

Together, Loschmidt and Boltzmann set out the whole puzzle of this chapter, in two halves. Loschmidt showed that the laws of motion, read on their own, pick out no direction of time. Boltzmann showed that once you know the arrangement a gas starts in, counting gives a direction of time with overwhelming likelihood. For the universe as a whole, the question becomes which arrangement the universe began in. Sean Carroll has made that question his own.

From Carroll's pages

What Sean Carroll Showed

Carroll opens *From Eternity to Here* with an egg of his own: "You can turn an egg into an omelet, as the classic example goes, but you can't turn an omelet into an egg" (Prologue, section "Time Since the Big Bang"). Carroll uses the omelet as a picture for the whole family of things that happen one way and never the other: milk spreading into coffee, ice melting in a warm drink, people growing older. In each of those one-way processes, entropy goes up. The arrow of time is the physicist's name for the one direction in which those one-way processes run. In the same section Carroll announces the theme of his book: "A major theme of this book is that the arrow of time exists because the universe evolves in a certain way".

Carroll states Loschmidt's half of the puzzle as a fact about the laws: "The arrow of time, therefore, is not a feature of the underlying laws of physics, at least as far as we know" (Chapter 2, "The Heavy Hand of Entropy", section "Future and Past vs. Up and Down"). He tells Boltzmann's half with Boltzmann's count: "In an isolated system entropy tends to increase, because there are more ways to be high entropy than to be low entropy" (Chapter 2, section "Entropy and Disorder"). Carroll's isolated system is the closed system of the Second Law. An isolated system takes in nothing from its surroundings and gives nothing to them. Carroll reads Boltzmann's reply to Loschmidt closely, and he judges two of its sentences one at a time. The first of the two sentences states the count: evenly spread arrangements far outnumber crowded ones. The second sentence states the conclusion: a gas started at random "is almost certain to evolve into a uniform state". "That first sentence is right, but the second is surely wrong", Carroll writes (Chapter 8, "Entropy and Disorder", section "When the Laws of Physics Aren't Enough"). He points out that a gas whose arrangement is chosen at random is "almost certain to be in a uniform (high-entropy) state" already, spread out from the start.

Carroll also finds a gap in Boltzmann's count. The count makes no reference to the direction of time. So from any given moment, the count predicts higher entropy toward the past just as surely as toward the future. Carroll asks of Boltzmann's proof of 1872, "How can you possibly get irreversible conclusions from reversible assumptions?" (Chapter 8, section "Proving the Second Law"). Carroll answers the question in the same section: Boltzmann had assumed that the motions of any two molecules are unrelated before they collide, and that "before" already points one way in time. "That's how an irreversible assumption was sneaked into the proof", Carroll writes. In the next section Carroll resolves the debate between Loschmidt and Boltzmann: both were right, and something must be added to the laws of physics, the assumption that the universe began with very low en-

tropy. Carroll writes of that assumption: "That is a boundary condition, an extra assumption, not part of the laws of physics themselves" (Chapter 8, section "When the Laws of Physics Aren't Enough"). In Carroll's sense, a boundary condition is a condition set at one end of a history, beside the laws that carry the history along.

The philosopher David Albert gave the assumption of a low-entropy beginning its name. The *Past Hypothesis* is the assumption that the universe began in a state of very low entropy. In *The Big Picture* Carroll states Boltzmann's puzzle in the kitchen, with eggs: "The real question is not why we never see eggs unbreaking toward the future; it's why we see them unbroken in the past" (Chapter 7, "Time's Arrow"). Carroll answers the puzzle with a likeness between time and the up and down of space. The laws of physics pick out no direction in space as down. The Earth picks out down for us, because we live beside the Earth. Carroll states the time side of the likeness in *The Biggest Ideas in the Universe: Space, Time, and Motion*: "there is an arrow of time because we live close (relatively speaking) to the Big Bang" (Chapter 5, "Time", section "The Past Hypothesis"). Carroll coins a word for an arrow of time that comes from the far past: "The origin of time's arrow, therefore, is ekinological: it arises from a special condition in the far past" (*The Big Picture*, Chapter 7).

Carroll then uses the low-entropy beginning to explain everything we notice about time. Of memory he writes: "Ultimately, the reason why we can form a reliable memory of the past is because the entropy was lower then" (*From Eternity to Here*, Chapter 2, section "Why Can't We Remember the Future?"). Three things are all features of the present: a photograph, a history book and the faint glow left over from the early universe. Carroll argues that we read those three things as records of a past only with the Past Hypothesis beside them. Without the Past Hypothesis, he writes, we have no "more direct empirical access to the past than we have to the future" (Chapter 9, "Information and Life", section "Cognitive Instability"). For Carroll, the low-en-

tropy beginning is also the reason that causes come before effects. Carroll gives the same reason for why our choices can affect the future and not the past. Carroll closes his chapter on memories and causes in *The Big Picture* with the picture of the low-entropy past behind us: "Our progress through time is pushed from behind, not pulled from ahead" (Chapter 8, "Memories and Causes").

Carroll insists that the arrow of time is a fact that needs a reason. In his Prologue Carroll sets out the count, and then he writes, "The traditional story that physicists tell themselves usually stops there". He goes on at once to the low-entropy beginning that the count rests on (Prologue, section "Time Since the Big Bang"). He states the gap in Boltzmann's count. The count predicts higher entropy toward the past as well as toward the future. He names the added assumption, the Past Hypothesis, and he says that it is an assumption. And he goes on to ask why the Past Hypothesis holds. For Carroll the Past Hypothesis states the problem of the arrow of time, and he ends his chapter on entropy with the question that the Past Hypothesis leaves: "Why did the universe have a low entropy near the Big Bang?" (*From Eternity to Here*, Chapter 8, section "The Past Hypothesis").

The Same Count, and a Second Road to the Same Law

I take Boltzmann's count as established physics, as Carroll does. I teach the count with a bicycle lock. A lock with one dial of ten digits can be set in ten ways, a lock with two dials in a hundred, and a lock with three in a thousand. Each added dial multiplies the number of settings by ten. Physicists count the arrangements of the molecules in a glass of water in the same way as the settings of the lock. I also state the gap that Carroll finds in Boltzmann's count. From any given moment, the same counting predicts higher entropy in the future and also in the past. I also agree with Carroll that the arrow of time needs a reason. Carroll is right that the ar-

row needs a reason, and right that a low-entropy beginning, taken as a brute fact, is not one. A brute fact, in that sentence, is a fact taken as given, with no reason offered for it.

I also give a reason that the count alone does not give. The reason shows why a spread drop of ink never gathers back into a drop. The same reason shows in what kind of system a return can be made. A return is a system going back to an earlier condition. The next section uses the same reason to explain the spin echo, a return that physicists have made in a laboratory. I find the reason on a second road to the Second Law: a derivation that starts from what interacting particles do to one another. I follow that road with a drop of ink.

Let a drop of ink fall into a still glass of water. The ink drifts out in curling threads, the threads thin, and in an hour the whole glass is faintly and evenly tinted. The count says where the ink is going: toward the even tint, which can be arranged in vastly more ways than the drop. The count does not yet say why the tint never gathers back into a drop, since for every motion that scatters the ink, the laws allow a reversed motion that gathers the ink. The reason is in the collisions of the molecules. When two molecules collide, each is changed in a way that depends on the other, so each comes to carry a trace of the other. From then on the two molecules are entangled, in the word that this book's Chapter 5 explained: the joint quantum state of the two molecules cannot be described one molecule at a time. For the tint to gather back into a drop, every trace that a collision left would have to be gathered back and undone, all together. The traces spread with every collision. Within the hour, countless molecules of ink and water carry traces of one another. To undo the traces, something would have to act on every molecule the traces have reached, and on all the ways those molecules can be arranged together. The number of those joint arrangements multiplies with every molecule the traces reach, as the settings of the lock multiply with every dial. A return is found only where the traces have reached few particles, and never where they have reached many. A glass of water in a kitchen is also open to its surroundings. The glass sends out light. Light that has left travels away and cannot be called

back, carrying some of the traces with it. The traces that the light carries away can never be undone. So when the glass and what has left the glass are taken together, their entropy never falls.

Carroll and I agree on three things: Boltzmann's count, the gap in the count, and the need for a reason for the arrow of time. I also derive the Second Law a second way, from what colliding particles do to one another.

Two Ways to Run Backward

I hold that the arrow of time is present in every act of comparison. My first step toward that claim is a distinction between two ways of running something backward. The distinction uses two words from this book's Chapter 2: symmetry and continuity. Symmetry is sameness kept across a change: change the setting, and the content (what a comparison is about) stays the same. Continuity is no gap and no least step: hold the setting, and nothing jumps and nothing is left out. Symmetry and continuity are both present, each to some degree, in everything we find. The theorem forbids the two made perfect at once: both held exactly, at the foundation of a description, in one and the same comparison.

The laws of physics are the same when time is run backward. That sameness is a symmetry in exactly the theorem's sense: the content is the laws, and the change of setting is the reversal of time. I separate two claims about this symmetry that physics, as usually taught, takes as one. *Lawful reversal* is the claim that the reverse of a lawful process is lawful too. *Faithful reversal* is the claim that the reversed process passes back through every condition the original passed through, exactly, and arrives at exactly the condition the original started from. The same essay says: "Symmetry of the laws guarantees lawful reversal. It does not guarantee faithful reversal". For a reversal to be faithful, every position and every velocity of every part must be given exactly in the description that is reversed. The reason is that a part whose position or velocity is left unspe-

cified is free to go somewhere other than where the part came from. In the theorem's sense, continuity made perfect is a description with no gap and nothing left out. So a perfect reversal, faithful as well as lawful, would need symmetry and continuity both made perfect at once. The Incommensuration Theorem forbids that pairing. I conclude, in the Abstract of the same essay, that perfect reversal is ruled out whatever condition the universe began in, and the conclusion makes no appeal to the Past Hypothesis.

We can now read Loschmidt's box of gas again. The reversed history that Loschmidt described is a lawful reversal. The reversed history is a solution of the laws of motion, and the laws allow it as surely as they allow the spreading. To set the reversed history going, someone would have to set every velocity of every molecule, exactly reversed, with nothing left out. Boltzmann answered Loschmidt that a starting condition with every velocity exactly reversed is overwhelmingly improbable. I point out what Boltzmann granted in that answer without saying so: the obstacle to the reversed history is its starting condition, every velocity exactly reversed. The laws of motion are symmetric, so they already allow the reversed history. The Incommensuration Theorem shows why that starting condition can never be set. The starting condition is the complete state of the gas at one moment, given with no gap. The laws beside the starting condition are perfectly symmetric. Continuity made perfect beside symmetry made perfect is the pairing the theorem forbids. The reversed path exists in the mathematics, as a solution of the laws; a gas in the faithfully reversed state never occurs in the world. The same result is true of the cook's egg. The backward film of the crack is lawful, and the faithful run of the backward film, with every molecule of shell and rim and air retracing its exact path, never occurs.

Physicists have run something backward in a laboratory, and the experiment shows the two kinds of reversal, lawful and faithful, side by side. In 1950 the physicist Erwin Hahn placed a liquid in a strong magnetic field at the University of Illinois. In many kinds of atom, the nucleus at the center carries a spin, a small turning of its own, and in a magnetic

field each spin turns about the direction of the field. Nuclei in slightly different fields turn at slightly different rates, so a set of spins lined up together soon drifts out of step, and the signal that the spins give together dies away. Hahn sent in a short pulse of radio waves that tipped the spins so that they turned together. After the spins had drifted apart, he sent in a second pulse that reversed the sense of each spin's drift. After a further equal time the spins came back into step, and their signal returned. Physicists call the returned signal the spin echo. The echo comes back because the drift was held in each nucleus's own angle, one trace for each nucleus, which one pulse could reverse.

I separate two kinds of time in the experiment. *Evolution time* is the time in the equations that carry the quantum state of the spins along: it runs smoothly, and within those equations an evolution can be run backward exactly. *Event time* is the order in which acts of finding are made, one after another, each laying down its record. For example, the event time of a counter is its clicks, taken in the order in which they came. In event time the experiment runs forward throughout: the dying away of the signal is found, the second pulse is sent, and the echo is found, in a new act made after the pulse. The record of the dying signal stays in the laboratory. What runs backward is the evolution of the spins, in evolution time, and not the order of findings. We now know from the spin echo that a lawful reversal can be made, in part, where each trace is held in one particle, and that every record made along the way stays in the laboratory.

On where the arrow of time comes from, Carroll and I part. Carroll agrees with Loschmidt that the laws allow as many histories in which entropy falls as histories in which entropy rises. Carroll writes "precisely as many" when he states Loschmidt's point (*From Eternity to Here*, Chapter 8, section "Proving the Second Law"). In the next section he writes, "Loschmidt is right". For Carroll the arrow of time is a fact about our universe, and the arrow comes from a difference between the two ends of the universe's history: the arrow is "a consequence of the fact that 'entropy is very different in one direction of time than the other'" (Chapter 8, section "Running Entropy Backward"). Carroll calls only the label a convention:

the choice of which of the two directions we count time as increasing in. He writes, "The convention we happen to prefer is that 'time' increases in the direction that entropy increases" (same section). I hold that faithful reversal is excluded for every process, whatever the condition at either end of the history. The *block universe* is the view that the whole history of the world is laid out at once, like one solid block, and "that the felt direction of time belongs to us and not to the world". I set the result that faithful reversal never occurs against the block universe: the difference between forward and backward is built into the structure of comparison, and it is not a convention. I grant that every one of Carroll's reversed histories exists in the mathematics, as a solution of the laws, and finds that no reversed history is run faithfully in the world. So the two accounts agree that the arrow of time is a fact about the world. The two accounts part on where the arrow comes from: for Carroll, a condition set at the beginning of the universe; for me, the form that every act of comparison has. The parting on where the arrow comes from follows from the one place where the two accounts part. The one place where the two accounts part is what lies at the ground of physics.

The Arrow in Every Finding

I then take a second step: it derives the arrow of time by another road, from the order that every act of finding has. Watch the cook again, slowly. Before the egg strikes the rim, there is a range of what could happen: a clean line, a crumbling of shell, a crack that runs too far. Then the act: the egg strikes the rim, and one crack is made. Then the record: the cracked shell in the cook's hand, seen and known. From then on the record sets the cook's next range, since a cracked egg can now only be opened and poured. Every comparison has the same order: from the range of what could be found, to the act of finding, to the record. The record shapes the next range. Wherever you begin to look at the order of range, act and record, there is an earlier part behind your starting point. The cook's range

was set by earlier records, and every record was made by an earlier act. The order has the same direction wherever you begin to look at it. I say the same of the cycle that every act of comparison follows: the point at which you enter the cycle varies, and the direction of the cycle stays the same. Boltzmann's count gives a direction of time only once someone knows the arrangement that a gas starts from. The order of range, act and record has its direction at every point, and the order needs no first moment to take its direction from. And the order runs only forward: a record, once made, is never unmade.

The direction of time is the direction in which comparison proceeds. I identify the one-way order of findings with the direction of time. I make the same identification elsewhere. One observation agrees with the identification: "every record ever found is a record of the past". No instrument holds a record before the interaction that makes it.

I call this one-way order the *structural arrow*: the direction of time as the order in which every act of comparison proceeds, from the range of what could be found, through the act, to the record. The structural arrow is present in a single comparison, made once. The structural arrow needs no large number of parts, no count of arrangements, and no low-entropy beginning. Two roads therefore reach one arrow: faithful reversal is impossible, and the cycle of comparison has a direction.

On records, my result differs from Carroll's. In my account, a record comes after the act it records, because the act makes the record. The order of the two is the order of making. The cracked shell in the cook's hand is a record of the crack because the crack made the cracked shell. The crack came before the record for the same reason: the crack made the record. The order of crack and record holds whatever condition the universe began in. Carroll holds that a photograph or a memory tells of the past only with the Past Hypothesis beside the photograph or the memory. In my account the record already tells of the past by being a record, and the Past Hypothesis is not needed to make the record a record of the past. Chapter 9 shows why, on my account, the emptying of the universe never comes to rest in an eternal empty space, so no crowd of observers made

by chance ever arises to be mistaken for one who finds. The parting on records follows from the same place as the parting on the arrow: what lies at the ground of physics.

Two Arrows, Pointing One Way

Carroll's omelet still belongs in this kitchen. I keep the omelet as the picture of a second arrow of time. Once the egg is in the bowl and the cook whisks it, the yolk and white spread into one another and never sort themselves out again. The spreading of yolk and white is the second arrow. The *entropic arrow* is the growth of entropy in systems of very many parts. The entropic arrow is statistical, and it needs a beginning of low entropy from which the growth proceeds. It is a statistical arrow: it is found in systems of very many parts, and it needs a beginning of low entropy from which the growth proceeds. On the entropic arrow, Carroll's account and mine are the same.

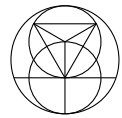
The structural arrow and the entropic arrow point the same way wherever both are found, and the record is the reason. A record is itself a change that is not undone, a darkened grain of film or a registered count, and making a record raises the entropy of whatever holds the record. When the cook sees the cracked shell, the sight is laid down as a memory. Laying down the memory raises the entropy of the cook's brain by a very small amount. So each act of recording joins the two arrows in one event: the order of the act, and a growth of entropy in the record. The structural arrow is therefore the root of the direction of time, and the entropic arrow is its expression at the scale of eggs and engines.

Once the two arrows are told apart, the Past Hypothesis divides into two parts: its use as the source of the direction of time, and its content as a statement about how the universe we see began. I say that the first part is not needed: "Its content divides. Its use as the source of the direction of time is not needed, since the structural arrow does not wait on it". The second part stays: the universe we see began low in entropy. So entropy

had room to grow, and the entropic arrow had room to run. Carroll writes that the arrow of time is "all because of the Big Bang" (*From Eternity to Here*, Prologue, section "Time Since the Big Bang"). In my account Carroll's line holds for the entropic arrow. The direction of time itself comes from the order of every act of finding. We now know that the direction of time does not depend on the beginning of the world; only the growth of entropy does.

Time runs one way because every act of finding runs one way: from what could be found, through the act, to the record. The growth of entropy points the same way, because every record is made by an act of finding. Where time gets its direction matters for the question of this book, what lies at the ground. Because every act of finding has its one-way order, the direction of time is at the ground: in the act of comparison itself, where the one who finds is. The arrow of time is present in the cook's single crack, before anyone makes a count of molecules, and the omelet only shows the same direction at a large scale, in the growth of entropy.

If the entropic arrow still needs an orderly beginning, where did the orderly beginning come from?



Chapter 9

Why Did the Universe Begin So Orderly?

What this chapter uses

Entropy: a count of how many arrangements of the small parts of a thing look the same from outside (Preface and Chapter 8).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *structural arrow* and the *entropic arrow* of time (Chapter 8); the *arena* (Chapters 3 and 7).

Why did the universe begin so orderly, when almost every way a universe could begin is disorderly? This book's Chapter 8 defined entropy as a count of how many arrangements of the small parts look the same from outside. An orderly state is one that few arrangements can make, so the entropy of an orderly state is low. The entropic arrow is the one-way spreading found in systems of very many parts. The entropic arrow runs from order toward disorder, so it needs an orderly beginning to run from.

Look at the egg on the counter once more, before the cook cracks it. An unbroken egg is far more orderly than the omelet it will become. A *closed system* is one that exchanges nothing with anything outside it. Carroll asks

where the egg's order came from, and he answers: "The egg is not a closed system". Carroll goes on: "It came out of a chicken". The chicken belongs to the living world of the Earth, which is "embedded in a universe that has a low-entropy past" (*From Eternity to Here*, Chapter 13, "The Life of the Universe", opening). The hen lives on what grows in sunlight. In *The Big Picture* Carroll shows that sunlight is what brings the living world its order: "The sun is a hot spot in a cold sky" (Chapter 29, "Light and Life"). So there is a chain: the egg came from a hen, the hen from sunlight, and the sunlight from a universe that began orderly. The far end of the chain is the universe itself. The universe has no hen and no Sun outside it to borrow order from, so the universe is a closed system. Carroll writes: "A truly closed physical system with a very low entropy is surprising and suggests that something bigger is going on" (*From Eternity to Here*, Chapter 13, opening). The near end of the chain is the cook's crack, one act of finding. We now follow the chain from the egg to its far end.

A Notebook at Stanford

On the night of 6 December 1979, Alan Guth was up late in the study of his rented house, working through a calculation. Guth held a research post at the Stanford Linear Accelerator Center (SLAC), the laboratory for particle physics at Stanford University in California. The calculation was about magnetic monopoles: particles that would each carry a single magnetic pole, a north with no south, which the theories that try to unite the forces of nature predicted in vast numbers. Yet no one has ever found a magnetic monopole. Carroll tells what happened the next morning in the opening lines of *From Eternity to Here*, Chapter 14, "Inflation and the Multiverse": "On a cool Palo Alto morning in December 1979, Alan Guth pedaled his bike as fast as he could to his office". At his desk Guth opened his notebook to a new page and wrote: "SPECTACULAR REALIZATION: this kind of supercooling can explain why the universe today is so incredibly flat". When space is not curved or bent on the largest scales, physi-

cists call space *flat*. Carroll puts flat space this way: "initially parallel lines stay parallel forever" (same chapter, section "The Curvature of Space"). Carroll explains what Guth meant by *supercooling*: an early universe filled for a short time with an energy of empty space at an enormous density, which made space expand faster and faster (same chapter, opening). Guth's notebook line went on: supercooling would also "resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein Day lectures". In the physicist's sense, a *fine-tuning* is a starting condition that has to be set very exactly, among very many possible settings, for the world to turn out as we see it. Carroll tells what Guth did next: "He carefully drew a rectangular box around the words. Then he drew another one". Carroll reports that in 2010 the page was on display at the Adler Planetarium in Chicago.

Guth had heard of Dicke's fine-tuning paradox by chance. Some time earlier Guth had gone to a lecture by the Princeton physicist Robert Dicke. The lecture was at a Cornell event called "Einstein Day", as Carroll tells us. The flatness of space was one of the loose ends Dicke pointed out, and it "stuck with Guth" (same chapter, section "Inflation").

Guth had found *inflation*: a proposed very early period of extremely fast expansion. In inflation a tiny patch of space swelled to a size far beyond everything we can now see. The swelling took a small fraction of a second. An energy of empty space drives the swelling: the energy that fills the supercooled universe of Guth's notebook line. The energy is far greater than any energy of empty space found today. Carroll calls the energy that drives inflation "dark super-energy" (same chapter, section "Inflation"). When the swelling ends, the dark super-energy turns into the hot matter and light of the early universe.

One mechanism of inflation, the swelling, is the answer to three puzzles. The first puzzle is the monopoles, the puzzle Guth had been working on. If the swelling comes after the monopoles are made, the swelling spreads the monopoles so thin that, in Carroll's words, "all the monopoles are diluted away practically to nothing" (same chapter, section "Inflation"). The second puzzle is flatness, the puzzle of the line Guth boxed twice in his notebook. Space on the largest scales has no curvature

that anyone has measured: space is flat. In the ordinary history of the universe, the history without inflation, any small curvature at the start would have grown large by now. Carroll's picture is a pencil balanced on its tip, which falls at the first tilt. So, in the history without inflation, the flatness we see needs a fine-tuning. The curvature at the start must have been set extraordinarily close to none. The third puzzle is sameness, and Guth saw the third puzzle as he worked out the consequences of inflation. A faint glow left from the early universe reaches us from every direction of the sky. The next section tells how astronomers found the glow. The glow has the same temperature, to about one part in a hundred thousand, at points on opposite sides of the sky, yet in the history without inflation no signal could ever have passed between those points to bring them to one temperature. Inflation stretches space flat, "like pulling at the edges of a wrinkled bedsheet", and the points now far apart were close neighbors before the stretching began, close enough for signals to pass between them (same chapter, section "The Horizon Problem"). Guth published the idea of inflation in *Physical Review D* on 15 January 1981. The title of his paper was "Inflationary universe: A possible solution to the horizon and flatness problems". Carroll reports what most cosmologists think of inflation: "It seems likely, in the judgment of most working cosmologists, that some version of inflation is correct" (same chapter, opening).

Guth's twice-boxed line contains the first answer to the question of the orderly beginning. The beginning was smooth and flat because inflation made the beginning smooth and flat. Carroll next asks whether inflation also explains the low entropy of the beginning. He first counts how orderly the beginning was, so as to ask the question well.

How Orderly the Beginning Was

Arno Penzias and Robert Wilson were radio astronomers at Bell Labs in New Jersey, and from July 1964 to April 1965 they measured the faint radio noise that reached a twenty-foot horn-shaped antenna. The antenna

was tuned to radio waves about seven centimeters long. Penzias and Wilson found the noise about three and a half degrees warmer than expected. The excess warmth was the same in every direction, within the limits of the observations, and the excess did not change with the seasons. Penzias and Wilson reported the excess in the *Astrophysical Journal* in July 1965. Four physicists at Princeton gave the explanation in a letter in the same July 1965 issue: Dicke, Peebles, Roll and Wilkinson. The Dicke of the letter is Robert Dicke, the Princeton physicist whose Einstein Day lecture Guth had heard. We now know that Penzias and Wilson had found the *microwave background*: the oldest light there is, left from the time when the whole universe was hot and dense, and cooled by the expansion to a few degrees above absolute zero. The microwave background shows that the early universe was smooth and hot. Carroll sums up the early universe in one line: "Basically, it was just a box of hot gas" (*From Eternity to Here*, Chapter 13, section "The Evolution of Entropy").

A box of hot gas spread evenly is a state of high entropy, so why call the beginning orderly? Carroll answers with a point he credits to the physicist Roger Penrose: gravity turns around the ordinary rule about spreading and entropy. For a gas in a box, spreading out evenly raises the entropy. For matter whose own gravity matters, clumping raises the entropy: stars, galaxies and black holes are the high-entropy arrangements. Carroll writes: "As structure forms in the universe, entropy increases" (same chapter, section "Lumpiness"). So the smoothness of the early universe counts as disorder for the hot gas and as order for gravity. The order of the beginning lies in gravity's part of the entropy.

Then Carroll counts the entropy. When our observable universe was young and smooth, its entropy was about ten to the power eighty-eight. Today the entropy of our observable universe is about ten to the power one hundred and one, and most of that entropy is held in the giant black holes at the centers of galaxies. The largest entropy our observable universe could have is at least ten to the power one hundred and twenty: the entropy it would have with all its matter gathered into one black hole. So the early universe was in a set of arrangements enormously smaller than

the set it could have been in. Carroll draws the conclusion in one sentence: "The state of the early universe was not chosen randomly among all possible states" (same chapter, section "The Evolution of Entropy"). What put the early universe in so small a set of arrangements?

Why We Do Not Live in Empty Space

Carroll asks what the most disorderly universe would look like once gravity is counted, so as to see how special the beginning was. Black holes hold enormous entropy, but black holes evaporate, and the thin radiation they leave holds more entropy than the black holes held. So in the end the most disorderly state is spread out: "The highest-entropy states look like empty space, with at most a few particles here and there, gradually diluting away" (*From Eternity to Here*, Chapter 13, section "Empty Space"). *De Sitter space* is an empty universe whose expansion is driven by the energy of empty space. De Sitter space takes its name from the Dutch physicist Willem de Sitter. Carroll notes that our own universe is evolving toward de Sitter space, the most disorderly state. So Carroll states what he calls "the major question facing modern cosmology": "Why don't we live in de Sitter space?" (same chapter, section "Why Don't We Live in Empty Space?").

Carroll's question is an urgent one, because empty de Sitter space has a tiny temperature that never goes away. A temperature brings chance stirrings of the particles, and in de Sitter space the stirrings go on forever. Over endless time, chance stirrings make anything at all, and they make small things far more often than large ones. A very large stirring makes a whole universe like ours. A far smaller stirring makes a single brain with false memories of a world around it. Physicists call a lone observer produced by a chance fluctuation a *Boltzmann brain*. The single brain with false memories is a Boltzmann brain. The name comes from Ludwig Boltzmann, the physicist of this book's Chapter 8, who proposed that our whole world might be one chance stirring. In a universe that ends in

eternal de Sitter space, Boltzmann brains would far outnumber observers like us. Equilibrium is the settled state in which nothing further changes on average. Carroll states what a Boltzmann brain should expect to see: "If I were a Boltzmann brain, there would be a strong prediction: Everything else about the universe should be in equilibrium. But it's not" (Chapter 10, "Recurrent Nightmares", section "Boltzmann Brains"). Everything around us is far from equilibrium, so our world is no chance stirring. For a theory in which the universe is headed toward eternal de Sitter space, Carroll calls the gap between what the theory predicts and what we see "a direct disagreement between theory and observation, and a sign that we have to do better" (Chapter 13, section "Why Don't We Live in Empty Space?").

Does inflation explain the orderly beginning? Carroll's answer to the question is his sharpest point. Inflation starts from a tiny patch of space that is ready to inflate, and the patch must be smooth and quiet throughout. For a representative example, Carroll estimates the entropy of the patch at about ten to the power twelve. Ten to the power twelve is far lower than ten to the power eighty-eight, the entropy of the young universe that the patch becomes. So, if a cosmologist explains an orderly beginning by inflation, the cosmologist must assume a beginning more orderly still: a starting condition set more exactly still. In Carroll's words, "You can't solve a fine-tuning problem by appealing to an even greater fine-tuning" (Chapter 14, section "What Good Is Inflation?"). Carroll's argument rests on two assumptions, and he names both (same chapter, section "Our Comoving Patch Revisited"). The first assumption is that the patch of universe we can see is a closed system. The second assumption is that the laws of the patch can be run backward as well as forward, so that in no process does a very large number of possible states become a smaller number. Carroll knows that his view places him in the minority of cosmologists, and he says what his point is aimed at: "The problem isn't really with inflation; it's with how the theory is usually marketed" (same chapter, section "Setting the Stage"). Carroll keeps inflation, and he says what inflation does not do: "Inflation has a lot going for it, but it makes

the need for a theory of initial conditions much more pressing" (same section). A theory of initial conditions is an account of why the universe began in the state it did. So, on Carroll's reckoning, the low entropy of the beginning still has no reason.

Carroll counted how special the beginning was, and he showed that, once gravity is counted, the most disorderly state is empty space. He made the Boltzmann brain into a test that any account of the universe must pass. The account must not predict that lone observers far outnumber observers like us. If an account has the universe end in eternal de Sitter space, the account fails the test of the Boltzmann brain. He also showed that, on his two assumptions, inflation cannot by itself be the source of the low entropy of the beginning.

From Carroll's pages

Universes That Bud from Empty Space

Carroll then offers his own answer, which he made with the physicist Jennifer Chen. Carroll and Chen set out the answer in a paper of 2004, "Spontaneous Inflation and the Origin of the Arrow of Time". Picture empty de Sitter space as a calm sea that goes on forever. Now and then a tiny bubble of high energy forms by chance in the calm sea, pinches off and goes its own way. Carroll and others call a bubble that pinches off from the calm sea a *baby universe*. The bubble inflates, cools, grows a history like ours and empties at last into a de Sitter space of its own. The new de Sitter space buds in turn. Carroll calls the whole family of budding universes, together with the calm sea they bud from, a *multiverse*. Each bud adds entropy to the whole family, and buds keep forming. So the family has no largest entropy at all: "The entropy increases because it can always increase, no matter what state we are in" (*From Eternity to Here*, Chapter 15, section "Bringing It Home"). The picture of Carroll and Chen runs the same way toward the past as toward the future, and within each bud an arrow of time points away from the bud's birth. Carroll writes: "There isn't any state

we could possibly have chosen that would have prevented this kind of evolution from happening. An arrow of time is inevitable" (same chapter, section "A Restless Multiverse").

In the section "Bringing It Home" (Chapter 15) Carroll lists what the model of Carroll and Chen achieves. The model treats the past and the future alike, and it needs no law that runs only one way. And at no moment does the model assume, without a reason, that the world is in a low-entropy state. Carroll says in his own words how firmly he holds the proposal. He writes that the model "serves as a demonstration that such an explanation is at least conceivable" (same section). In his Epilogue he calls the model of Carroll and Chen "a specific model of which I am personally fond". He goes on: "The details of this model are highly speculative, and rely on assumptions that stretch beyond what the state of the art allows us to reliably compute, to put it mildly" (Chapter 16, "Epilogue", section "What's the Answer?"). My answer is in the section "The Turn".

My answer shares one feature with the model of Carroll and Chen: at no moment do I's answer assume, without a reason, that the world is in a low-entropy state.

The Turn

Picture a new notebook. The pages of a new notebook are numbered from one, because the numbering belongs to that notebook. Nobody expects the first page of a new notebook to carry on from the last page of the notebook before. I call each long era of the world an *epoch*. In my answer to the question of the beginning, each epoch is like a new notebook, and the page numbers of the notebook stand for the entropy of the epoch, its count of states. Each epoch begins its own count, and the comparisons that make the count are made against the epoch's own context. No count

carries on from the epoch before. We reach my answer in two steps. First we follow our own epoch to its end, and then we follow on to what comes after the end.

Our own epoch is the era in which stars, galaxies and black holes form, and I follow our epoch to its end. The expansion of the universe is speeding up. Whatever is not held to an observer by gravity is carried out of the observer's reach, and over a hundred billion to a trillion years every observer's sky empties. Everything held together by gravity ends in one of two ways, whether it is as small as a planet or as large as a galaxy: it falls into a black hole, or it is carried away alone. We follow the road into a black hole first.

Carroll tells what Einstein's theory of gravity, general relativity, says happens inside a black hole. Once anything has fallen into a black hole, space and time ahead of the falling thing bend more and more, without limit, toward what physicists call a *singularity*: an end at which the bending of space and time would be infinite. In Carroll's words, "Such a singularity lies inside every black hole" (*From Eternity to Here*, Chapter 5, "Time Is Flexible", section "Holes in Spacetime"). The singularity lies ahead of the falling body in time, and not off in some direction of space: "You can no more avoid hitting the singularity than you can avoid hitting tomorrow" (same section). Carroll also says what a singularity means for Einstein's theory: "something happens where general relativity predicts singularities, but we don't know what" (Chapter 3, "The Beginning and End of Time", section "The Big Bang").

I say what happens at the singularity, the end of a black hole's interior. The bending of space and time grows as the end of the interior comes near. Near the end, the bending reaches the scale at which gravity and quantum effects are equally strong, and there both descriptions of physics, the smooth description of general relativity and the quantum description, reach their edge: the point past which neither description applies. I say what the edge is an edge of: "The edge belongs to the descriptions. It is not an ending of what they describe" (chapter "Is the Long Emptying the End?", section "What Is Left When the Sky Is Empty"). The matter and

light that fell in are still there. No signal from the end of an interior ever reaches the world outside the black hole. So nothing outside can place the end of the interior before, at or after any moment of the outside world's own time. In the same section I take the next step. At the end of an interior, nothing there can be compared with the distant stars, the remaining black holes or the leftover radiation, because the world outside has no moment at the end of the interior at which such a comparison could be made. The arena is space and time together as the setting of every finding (this book's Chapter 7). So at the end of an interior the arena gives the matter nothing to be compared against. The world outside, which lies in the arena, gives the matter nothing to be compared against either. In this sense an interior runs to an end: the matter is still there, and the arena is no longer there as the matter's context.

Picture the tide gauge of this book's Chapter 2, which sets the height of the sea at noon beside its height at midnight. The two readings of the tide gauge are two because one reading is taken at noon and the other at midnight. Now take away every noon and every midnight: every time at which one reading could be set beside another. The sea is still there, but nothing is left to make two readings of the sea two. The sea is like the matter at the end of an interior. The times taken away are like the arena that is no longer there. In this book's Chapter 2 we found that the content of a comparison (what the comparison is about) and its context (what makes the two things compared two) are inseparable: wherever there is a content, there is a context it is found in. The matter at the end of the interiors is content that is still there. So a context forms for the matter again: a new arena, a new space and time.

Is there one new beginning for each black hole, or one for all of them? Picture the interiors of the black holes as funnels. The funnels stay distinct along their whole walls, as the interiors of different black holes stay distinct along their whole length. At their narrow ends the funnels are joined, and the funnels are joined by being indistinguishable, with nothing pulling them together. The narrow ends of the funnels are the ends of the interiors. No signal from the end of any interior reaches the world out-

side. So nothing in our epoch can tell one end from another by any comparison. The new arena is new in every part: none of its directions of space continues any direction of the old arena. So nothing of the old arena is left in the new arena to carry a past across. The new arena forms with no past of its own, so nothing in the new epoch can tell by any comparison from which interior any part of the new epoch's matter came.

Now picture two rulers, one said to be a millimeter longer than the other, in a world where nothing could ever show the extra millimeter. Laying the two rulers side by side would show nothing, and no instrument would show the extra millimeter. No effect of any kind would follow from the extra millimeter. So to say that the two rulers differ would say nothing at all. I make the case of the rulers general, because every distinction is made by comparison: "A difference that no comparison can register is no difference". Before the end or after it, no comparison registers a difference between the end of one interior and the end of another, as nothing registers the extra millimeter of the rulers. So the ends are one end. The interiors stay many (same book, chapter "Is the Long Emptying the End?", section "One End, and the Turn").

The turn is the passage from the end of one epoch to the beginning of the next, at the one end that every black-hole interior reaches. A body carried away alone also comes to the turn, though the body never falls into a black hole: once everything else has passed beyond the body's reach, nothing is left in the arena to compare the body against. The body's matter is still there, like the matter that fell in. No comparison registers the turn itself as an event. Registering the turn would need a comparison made across the turn: a before set beside an after.

Picture marks drawn at half the distance to a line, then at half of what is left, then at half again. The marks come ever nearer the line, and none of the marks lies on the line. The turn is like the line that the marks come ever nearer. A body falling into a black hole has a history that comes ever nearer the turn and never reaches the turn. The history of the new epoch, traced backward by its own observers, comes ever nearer the turn from the other side. The history of the new epoch never reaches the turn either.

I call the turn "a limit approached from both sides and reached from neither" (chapter "Has This Happened Before?", section "A Cycle With No First Turn"). I identify the re-forming of the arena at the turn with the Big Bang of the new epoch, as I do in the section "One End, and the Turn". The reason lies in how any Big Bang is known. Every history of the universe is pieced together from the light and the records that reach the present. The observers of the new epoch piece together a history that runs back toward the turn and has its beginning there, so the Big Bang of the new epoch is the turn as the observers' history shows it.

I agree with Carroll on three points, and I reach each point by my own reasoning. The first point is that the Big Bang was not the first moment. Carroll and Chen begin by assuming that "the Big Bang is not the beginning of time" (*From Eternity to Here*, Chapter 15, section "A Restless Multiverse"). On my account, the Big Bang is the beginning of an epoch, and not the first moment. The Big Bang of an epoch is the turn as that epoch's own history shows it. The second point is the shape of the whole history of the universe. In both, our epoch is one of a sequence with no first member, and in both the arrow arises within the sequence, with no first moment at which it is imposed. The third point is the demand Carroll makes of any answer: the answer must give the arrow of time a reason. In Section V I also grant Carroll's demand: "Carroll is right that the arrow needs a reason, and right that a low-entropy beginning, taken as a brute fact, is not one".

Why the Beginning Was Orderly

Now that the turn is in view, we can ask the chapter's question again: why was the beginning of our epoch orderly? Picture a sheet of paper fresh from the mill: the sheet has no creases, because creases come from a sheet's own past of folding. The new arena is like the fresh sheet: the new arena forms at the turn with no past of its own. So the shape of the new arena inherits no lumps and no favored direction from the world before,

as the fresh sheet inherits no creases. The shape of the new arena is the geometry of the new space and time. The geometry of the new arena begins even, apart from small departures from evenness. The Incommensuration Theorem means that a beginning cannot be perfectly even and perfectly smooth at once, so the geometry must have some departure from evenness. We met the Incommensuration Theorem in this book's Chapter 2. Symmetry is sameness kept across a change, continuity is no gap and no least step, and to apply either one fundamentally is to hold it exactly, at the foundation of a description. If a beginning were perfectly the same everywhere and perfectly smooth, symmetry and continuity would both be perfect in that beginning together, and the theorem says that the two cannot be. So the beginning departs from evenness. Astronomers have measured the size of the departures. The oldest light, the microwave background, shows the departures as faint ripples: differences in the temperature of the light from place to place of about one part in a hundred thousand. My account of the turn takes the size of the ripples from the measurement. I give the reason for the order of the beginning. At the beginning, gravity's part of the entropy "is low because the geometry of a new context has nothing to be lumpy with". The new context in the quoted words is the new arena. Meanwhile the matter of the new epoch begins hot and close to equilibrium, with its entropy already high. The hot matter is Carroll's box of hot gas, and its glow is the glow that Penzias and Wilson found. So I reach the point that Carroll and Penrose made: the order of the beginning lies in gravity's part of the entropy. And I give the order of the beginning a reason: the geometry of the new arena has nothing to be lumpy with.

The rest of the answer is about the count of states itself, which the page numbers of the new notebook stand for. An entropy is a count: the number of states of a system that the comparisons available from some context cannot tell apart. Every count of an epoch is made by comparisons within the arena of that epoch. At the turn nothing is left in the old arena

to compare against. So no count is carried across the turn. Each epoch begins its own count against its own context, as each notebook begins at page one.

Carroll's count ended on a question. What put the early universe in so small a set of arrangements, out of all the arrangements the early universe could have had? Because each epoch keeps its own count, Carroll's question becomes another question: is there anything that can tell all the possible arrangements of the early universe apart, so that the beginning is one draw among them, and an unlikely one? A lottery ticket is one in a million only if someone holds a million tickets and can tell the tickets apart. The beginning is like the single ticket. Only if something can tell all the possible beginnings apart can the beginning be called one draw among an enormous number of possible beginnings. Nothing in the old arena can tell the possible beginnings apart: the old epoch's counts were made in the old arena, but nothing is left in the old arena to compare against, and the turn falls at no moment of the old arena's time. Nothing in the new arena can tell the possible beginnings apart either: the new arena has no past of its own, and nothing in the new epoch ever measures the other beginnings that the new epoch might have had, or tells those beginnings apart. So the low entropy of the beginning is no unlikely draw that needs a separate law to compel it. The low entropy is instead the count that a new context starts with.

Where the Two Accounts Part

The answer of Carroll and Chen has the same shape as my answer. I record the two points on which the two answers part: whether entropy piles up across the whole family of universes, and whether our epoch began by inflating. The first point rests on a third difference: whether empty space is a state in which the world comes to rest. In the picture of Carroll and Chen, the entropy piles up because the calm sea of de Sitter space lasts forever and keeps budding, while on my account the emptying

of the sky never comes to rest. On the subject of the beginning, Carroll's account and mine part at the turn, and that parting follows from the one place where they part: what lies at the ground. For me the act of comparison lies at the ground. The turn comes from two things that every comparison has, a content and a context: the content is still there at the end of the interiors, and a context forms for the content again.

Take first the piling up of entropy. Every count has a context, and no context extends across the turn, so nothing accumulates from one epoch to the next. In this physics disorder does not pile up, because no count crosses the turn. The epochs therefore come one after another: "So the epochs follow one another in a single line, and not as a branching tree of universes". In the picture of Carroll and Chen, entropy grows without bound across all the buds at once. Growth across all the buds needs a count kept across the buds. I find no context in which a count across the buds is made. I name the step that needs a count kept across all the buds: "Carroll and Chen let entropy grow without bound across the whole ensemble of offspring universes, which requires a count of states kept across all of them".

Next comes empty space as a resting state. I first reach Carroll's own conclusion by separate reasoning: our world is no chance stirring in an eternal equilibrium. A final world of featureless emptiness would be the same everywhere and at every moment, and smooth everywhere. So a featureless final world would be perfectly symmetric and perfectly continuous at once. I draw the conclusion: "So the emptying cannot come to rest in such a world" (chapter "Is the Long Emptying the End?", section "The Second Ground: A Featureless Emptiness Is Never Found"). De Sitter space is a model, and the perfect symmetry of de Sitter space "then belongs to the model, and not to anything a comparison finds". The emptying ends at the turn. So my account has no eternal de Sitter future and no crowd of Boltzmann brains. I give a short answer to Carroll's question, "Why don't we live in de Sitter space?": empty de Sitter space is never reached as a resting state, and each epoch begins at the turn.

Last comes inflation. I agree that inflation is not the source of the low entropy of the beginning. Gravity's low entropy at the beginning comes from the turn: the geometry of the new arena has nothing to be lumpy with, as the section "Why the Beginning Was Orderly" showed. I also answer each of the three puzzles that inflation was introduced for: the monopoles, flatness and sameness. Sameness and flatness both come from the turn. Two towns far apart may share an accent without ever having spoken to each other, if they were founded from one place: far-apart regions of the sky are alike because they come from one place, the one end that all the black-hole interiors of the old epoch reach. The answer to the puzzle of the monopoles comes from the forces: the monopoles were predicted by the theories that unite the forces into one, and on my account the forces were never one larger force. So my account needs no inflation. I also exclude a long inflation. A long inflation is one long enough to stretch every length we now see in the sky out of one small patch. The faint ripples in the oldest light are the form of the matter that the black holes of the epoch before ours took in: what the matter is, averaged into one faint pattern and carried through the turn. A long inflation would have made the ripples itself, out of the trembling of the empty space that the inflation swelled, so ripples that come through the turn rule out a long inflation. The Planck satellite of the European Space Agency mapped the oldest light, and its final results were published in 2020. The Planck results show that space is flat and that the sky is alike in every direction apart from its faint ripples. The telescopes of the series called Background Imaging of Cosmic Extragalactic Polarization (BICEP) stand at the South Pole. The Keck Array telescopes stand beside the BICEP telescopes. The BICEP and Keck Array teams found no waves in the geometry of space left from the first moments. The teams published the finding in 2021. Both accounts agree with the Planck results and with the BICEP and Keck Array results: the account by inflation and the account by the turn. The two accounts part on how our epoch began: in the picture of Carroll and Chen each bud begins by inflating, and on my account our epoch begins at the turn.

Cream, Sunlight and the Hen

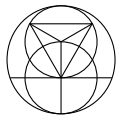
Pour cream into black coffee and watch, without stirring. At first the cream sits on top: low entropy, and simple. At the end the cup is all one even brown: high entropy, and simple again. In between, tendrils of cream curl down into the dark in patterns no one could draw twice. Carroll makes the cup of coffee his picture of the history of the universe: "Entropy goes up, as the second law says it should; but complexity first goes up, then goes down" (*The Big Picture*, Chapter 28, "The Universe in a Cup of Coffee"). Galaxies, stars, hens and cooks are the tendrils in Carroll's picture. Carroll writes: "The appearance of complexity isn't just compatible with increasing entropy; it relies on it" (same chapter). Life is the clearest case: "Complex structures can form, not despite the growth of entropy but because entropy is growing" (Chapter 29, "Light and Life"). The Earth takes in sunlight and gives the same energy back as about twenty times as many cooler particles of infrared light. Every living thing runs on the difference between the few hot particles of light the Earth takes in and the many cool particles the Earth gives back.

I reach Carroll's result by my own reasoning: complexity rises and then falls, while entropy only grows. On my account an epoch begins with its geometry even. Then structure forms in the epoch and raises the entropy of its geometry, and black holes form. The epoch ends as every observer's sky empties. The history of an epoch runs the course of the cup: from an even beginning, through stars, galaxies and black holes, to an emptied sky. The next epoch begins again from an orderly start, as ours did.

We can now follow the chain from the egg, through the hen and the sunlight, to its far end. And we can say what is found there. The beginning was orderly because every count of disorder belongs to the context that makes it, and at the turn a new context begins, with nothing to be lumpy with and nothing carried over. The answer matters for the book's

question about what lies at the ground, because the answer comes from the act of finding: an entropy is a count made by comparisons against a context, and a new context begins its count anew. So the low entropy of the beginning has the reason Carroll asked for. The low entropy of the beginning is no brute fact about a first moment. The egg on the counter is waiting to be cracked. The egg is at the near end of the chain that runs from the hen, through the sunlight, to the beginning.

If the order came from the beginning, is the cook who cracks the egg doing anything the beginning did not already settle?



Chapter 10

Can I Choose?

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *law of the many*, and the argument that a whole world cannot be fixed by one law (Chapter 4); *hard randomness* and the *bound triple* (Chapter 5).

Entropy: a count of how many arrangements of the small parts look the same from outside (Preface and Chapter 8).

When you decide to do something, do you choose, or was your deciding settled long before you came to the decision? In your own words, the question is three words long: "Can I choose?" In Chapter 9 we followed the egg back to the hen, the hen to sunlight, and the sunlight to an orderly beginning of the universe. Now the cook stands at the counter with the egg in one hand and the bowl waiting. The cook decides to crack the egg now: at this moment, before the coffee, and not a minute later. Is the cook's decision a choice, or did the beginning of the universe already

settle the decision, so that the cook's hands only carry out what was settled? Sean Carroll answers that in one sense the cook chooses and in another sense the cook does not.

A Dial in Each Laboratory

John Conway and Simon Kochen were two mathematicians in the Department of Mathematics at Princeton University. In the spring of 2006 they finished a paper they called "The Free Will Theorem". They dated the paper 31 March 2006. On 11 April they posted the paper to the arXiv, the open archive where physicists share their papers. Conway and Kochen open the paper with your question: "Do we really have free will, or, as a few determined folk maintain, is it all an illusion?"

Conway and Kochen answered the question by looking at an experiment, and not inside a person. Picture two laboratories far apart, with a source between them that sends out pairs of particles, one particle of each pair to each laboratory. The two particles of a pair are *entangled*: they share one joint quantum state, and neither particle can be described on its own. In each laboratory an experimenter turns a dial to choose the direction along which to measure, and the instrument records an answer. When the two dials are set the same way, the two answers of a pair always match. Conway and Kochen asked whether anything at all had settled the answers before the dials were turned.

Their theorem says, in plain words: if nothing that came before the experimenter's choice of setting fixes that choice, then nothing that came before the particle's answer fixes that answer either. Conway and Kochen put the theorem with a smile: "if indeed there exist any experimenters with a modicum of free will, then elementary particles must have their own share of this valuable commodity". In the theorem the word "free" has one exact meaning. The book states the exact meaning of "free" in its Chapter 3, "The Free Will Theorem", Section I: "not a function of anything that existed before the event". In the definition of "free", a function is a

rule that fixes an output from its inputs. So "not a function of anything earlier" means nothing earlier fixes the answer. Said of a particle, the word "free" says only that nothing earlier fixes the particle's answer. The word makes no claim about what a particle feels.

The free will theorem rests on four plain assumptions, which I set out in the same chapter. First, quantum mechanics holds for the particles. Second, no influence travels faster than light. Third, no later event fixes an earlier setting. Fourth, nothing in the past that the particle has access to fixes the experimenter's choice of setting.

So your question turns up inside a physics experiment. The free will theorem shows that your question has a partner question: whether a particle's answer is fixed in advance. If the setting that a person chooses at a dial is free, in the theorem's sense, then the particle's answer is open. Open means that nothing earlier fixes the particle's answer, just as nothing earlier fixes the setting. Hold on to this link between a person's freedom and a particle's open answer, because the cook at the bowl will need the link.

From Carroll's pages

What Carroll Showed

Carroll asks your question directly in *The Big Picture*, Chapter 44, "Freedom to Choose", and he answers the question in two halves. "There's a sense in which you do have free will.

For Carroll, at the level of particles you have no free will. Carroll writes: "Of course there is no such notion as free will when we are choosing to describe human beings as collections of atoms or as a quantum wave function". Carroll's reason is that the laws of physics fix what a system of particles will do next. The Core Theory is Carroll's name for the known laws of the particles and forces of everyday life, together with gravity where gravity is weak. In Chapter 43, "What Acts on What?", Carroll calls the laws of the Core Theory "complete

and self-consistent" within their domain. A law's domain is the range of cases in which the law holds. His next sentence says what the laws do: "You give me a quantum state of a system, and there are unambiguous equations that will tell me what it will do next". Carroll draws the consequence for choosing in *From Eternity to Here*, Chapter 9, "Information and Life", section "Cause and Effect". The microstate of the universe is the exact arrangement of every particle in the universe. He writes: "Ultimately, our ability to 'choose' how to act in the future is a reflection of our ignorance concerning the specific microstate of the universe". Laplace's demon is the imagined intellect that knows the exact arrangement of every particle and the laws, and so knows the whole future. Carroll writes that Laplace's demon "would know exactly how we are going to act".

For Carroll, at the level of people you do have free will. Carroll's example is a student choosing among the colleges that have accepted the student: visits, talks, then yes to one and no to the others. Of the student's choice Carroll writes: "The concept of choice does exist, and it would be difficult indeed to describe human beings without it". Carroll holds two descriptions together: an impersonal description of the particles underneath, and a description of people making choices above the particles. Philosophers call a position that holds the two descriptions together as compatible *compatibilism*. Carroll notes that compatibilism is "the most popular way of thinking about free will among professional philosophers".

Carroll gives a rule for keeping the description of particles and the description of people apart. You stand at the closet wondering whether to wear the black shirt or the blue one. Your atoms will do what they do, but you do not know what your atoms will do, and what they will do has no bearing on which shirt you should pick. Carroll states the rule: "Either vocabulary is perfectly legitimate, but mixing them leads to nonsense". In Chapter 43 Carroll shows, with air and with people, that each of the two vocabularies stays true beside the other. When

atoms and molecules were found, the description of air as a fluid stayed true, and in the same way you can truly say that a person wants something and reaches for it. You can also truly describe the particles of the person who wants something and reaches for it. In Carroll's words: "Both ways of talking are true".

In the same chapter, Carroll applies his rule against mixing vocabularies to causes. *Downward causation* is the idea that a whole acts on its own parts. Carroll's example is the claim that the shape of a snowflake places each of the snowflake's water molecules. *Upward causation* is the reverse idea: that the parts, described in the language of particles, cause what the whole does, described in another language. Carroll rejects both ideas. He speaks for the poetic naturalists. Poetic naturalists hold Carroll's view that there is one natural world and many good ways of talking about the one world. "Poetic naturalists tend to view downward causation as a deeply misguided idea. Then again, they view upward causation as equally misguided". Carroll writes that causation "is best thought of as acting within individual theories that rely on the concept". So, on Carroll's account, a cause links one thing to another inside one way of talking, and never links a thing in one way of talking to a thing in another. Carroll grants one point in principle: "Something like downward causation is possible in principle, even if there's no evidence for it in the real universe".

Carroll also says why the language of choice is about the future and never about the past. His answer is the arrow of time, the one direction time runs. In Carroll's account the universe began with very low entropy. Carroll calls the universe's low-entropy start the Past Hypothesis. Small differences in a brain today are tied strongly to what comes next and hardly at all to what came before, because the beginning had low entropy. Carroll writes: "We remember the past, and our choices affect the future". Carroll says that the language of choice is the useful one because none of us knows the exact arrangement of every particle in the world. He writes: "The unavoidable reality of our

incomplete knowledge is responsible for why we find it useful to talk about the future using a language of choice and causation". Carroll places reasons and causes where he places choice: in the description of the world at large scales, and not in the fundamental laws (*The Big Picture*, Chapter 5, "Reasons Why"). He writes: "The 'reasons' and 'causes' why things happen, in other words, aren't fundamental; they are emergent". For Carroll, what is emergent belongs to the description at large scales and not to the fundamental laws.

Last, Carroll names the position he rejects. *Libertarian freedom*, in his words, is "the position that human agency introduces an element of indeterminacy into the universe". Indeterminacy means that something is not fixed in advance. Carroll says at once what the position adds: on libertarian freedom, "people are not governed by the impersonal laws of physics". Carroll's answer to libertarian freedom states what libertarian freedom would need: "In order for libertarian freedom to exist, it would have to be possible for human beings to overcome the laws of physics just by thinking".

So Carroll's answer to your question comes to three claims. First, no description of people needs a force that physics lacks. Second, choice is part of the best description of people, and nobody who talks about people at all can talk choice away. Third, every choice is about what comes next, in the same direction as the arrow of time.

The Same Result by Another Route

I agree with Carroll's yes: at the level of people, you do choose. I reach Carroll's yes by a separate route, and I reach several more of Carroll's results by the same route, from my own starting point: the act of comparison, setting one thing beside another and finding them the same or different.

On force, Carroll and I say the same thing. Poetic naturalism and my account agree that there are many good descriptions of the world, each fitted to its domain, and that no description of people or reasons needs a force that physics lacks". And it is not something added to nature from outside.

On causes, Carroll and I agree on two points: causes belong to the level of things we can watch, and the laws of the smallest things have no use for the word "cause". In 1913 the philosopher Bertrand Russell observed that advanced physics does not use the word "cause". I cite Russell's observation, and state the reason: "causality, by its nature, does not reach into the microscopic, and the physics of the microscopic has no need of it".

Carroll's reason and my reason differ. Carroll's reason is emergence: causes appear in the coarse description of many particles (the description at a large scale, which leaves out the detail of each particle), because of the arrow of time. My reason comes from what a cause is. I define causality as "the subjective perception (context) of a consistency between two objective contents". I quote the definition. In plain words, two things happen in the world: a match flares, and the paper beside the match catches. Causality is someone registering that the catching follows the flaring, as paper has caught beside a flaring match every time before.

Three features of a cause follow from the definition. I show the three features with a picture. A player piano and a pianist can play the same piece. In the player piano, "a paper roll fixes every note before the first is struck". The pianist plays each note in the light of the last note, hears that each note followed the one before, and could have played otherwise. *Determinism* is the claim that every detail of what happens is fixed in advance, everywhere and always, with nothing left over. The same section says: "The roll is a picture of determinism and the pianist of causality". The first feature is that a cause needs someone who registers the cause, as the pianist hears each note follow, and determinism needs no one: "Causality requires an observer; determinism does not". The second feature is

that a cause runs one way in time, from the note just played to the next note. The third feature is that a cause does not depend on the fine detail of the atoms in the match. So Carroll and I reach the same result about causes, that causes belong to the level of things we can watch, by two routes: Carroll by emergence, and I from what a cause is.

On levels, Carroll and I agree that the description of particles, the description of cells and the description of people do not compete. I name three kinds of description: causality, determinism and teleology. The same section glosses *teleology* as "the coherence of a whole, which description in terms of purposes, ends or final states tries to capture". In plain words, teleology is the way a whole hangs together, which we describe when we say what something is for. Causality, determinism and teleology are not competing descriptions of the same level of reality.

On the future, Carroll and I agree that nobody can know their own future completely. Carroll's reason is that none of us is Laplace's demon, while my reason is three limits from this book's Chapter 4. Every finite knower is subject to the three limits: the reach of light, which brings news of far places only after a delay; the limit on how sharply a particle's position and motion can both be fixed; and the one-way order of every act of comparison, which leaves records only of acts already made.

And on the laws themselves, I grant the Core Theory exactly as Carroll states the Core Theory, and I add no force or field to Carroll's laws. Carroll lists every law that the cook's hands obey and every law that the egg obeys.

The Step from a Complete Law to a Complete World

Where Carroll says that you do not choose, at the level of particles, I hold that the single outcome of each event is open: nothing that came before fixes which outcome happens. On the subject of choice, Carroll's account and mine part at the single outcome, and that parting follows from the one place where they part: what lies at the ground (the bottom of the

world, the thing every other description describes). For me the ground is the act of finding: a single act of comparison that gives a verdict. Every act of finding comes away with one outcome.

This book's Chapter 4 showed that the known laws, together with a complete knowledge of the quantum state, fix the odds of what happens and leave each single outcome open. A law of the many is a law of what happens over many trials. A law of the many describes how shells break over many eggs, as the cook knows from long practice, but the law of the many does not fix this crack, in this egg, now. The answer removes one step only: the step from a complete law to a complete world. The step goes from knowing every law to holding that the laws fix every single event.

I give two reasons for removing the step from a complete law to a complete world. The first reason is the Incommensuration Theorem, from this book's Chapter 2. The theorem shows that no world can have determinism, every detail fixed in advance with nothing left over. This book's Chapter 4 showed why no world can have determinism. For a world to run like the player piano's roll, as Laplace's demon needs, the world would need two things: one law, the same everywhere and always, and a complete state specified with no gap. One law, the same everywhere and always, is symmetry made perfect. A complete state with no gap is continuity made perfect, and the Incommensuration Theorem forbids the two perfections together. The Incommensuration Theorem allows an unbroken order that runs one way: continuity paired with asymmetry (nothing kept the same). Every cause has continuity paired with asymmetry, as the pianist's notes follow one another without a break and in one direction. The theorem never forbids continuity paired with asymmetry.

The second reason is the free will theorem of Conway and Kochen, with which this chapter began. The free will theorem shows that if the experimenter's choice is free, the particle's answer is free as well. I give two separate bases for holding that the experimenter's choice is free. The first basis is the Incommensuration Theorem. The second basis is the practice of physics: every experiment presupposes that the settings of the experiment are independent of what the experiment measures. The independ-

ence of the settings is what lets a reading count as evidence about the thing measured. To deny that independence is to deny that any reading counts as evidence, including every reading that might be offered for the denial. The practice of physics uses nothing from the Incommensuration Theorem. So I conclude that "the free will theorem gives physical confirmation of the impossibility of determinism that is independent of the Incommensuration Theorem". I thus show, by two routes from separate starting points, that no world can be deterministic: by the Incommensuration Theorem alone, and by the free will theorem together with the practice of physics.

Physicists have also measured whether entangled particles' answers were settled in advance. In 1964 the physicist John Bell showed that the agreement between the answers of entangled particles cannot come from answers laid out at the source, before the particles leave the source. In 2015 the physicist Ronald Hanson led a group at Delft University of Technology, in the Netherlands. The group entangled two electrons, which were held in diamond chips in two laboratories 1.3 kilometers apart. Random number generators chose each setting at the last moment, so that no signal at the speed of light could carry the choice from one laboratory to the other in time. The group gathered 245 trials in 220 hours. In the 245 trials the answers agreed more closely than any account allows in which causes at each laboratory fixed that laboratory's answer in advance. In the same year, 2015, two more groups found the same agreement with particles of light: a group in Vienna and a group at the National Institute of Standards and Technology in Boulder, Colorado. Those results exclude every account in which each outcome is fixed in advance by causes local to the place where it is found.

We can tell apart two kinds of chance with the results from Delft, Vienna and Boulder. I call the first kind soft randomness. "Soft randomness is randomness of ignorance: an outcome that was fixed, but that I do not know, and that better knowledge could reduce". The second kind is *hard randomness*, which this book's Chapter 4 named: an outcome that no law and no knowledge of what came before fixes, however complete that

knowledge is. The single outcome of a quantum measurement is the plainest case of hard randomness. Carroll bases the language of choice on soft randomness. On his account, we use the language of choice because we do not know the microstate. I place choice in hard randomness: the single outcome is open in the world, and the outcome would stay open even to a knower with the whole microstate in hand.

Carroll's compatibilism answers the question whether freedom can exist in a world whose underlying scientific description is, in his words, "deterministic (or at least impersonal)". Once determinism is impossible, the question changes: not whether freedom can survive a fixed world, since there is none, but what choice is in a world that is causal and open. Carroll writes that unambiguous equations tell what a quantum state will do next, and his sentence is true of the equations. The evolution is deterministic in the formalism, which exists as structure, and not in the findings, which are real. The formalism is the mathematics: the equation and the quantum state the equation carries forward. The findings are the acts of measurement: each act comes away with one outcome, and the equation does not fix which outcome that is. In the senses this book gave the words 'exists' and 'real' in Chapter 5, what exists as structure is timeless and the same for anyone who reads it, and what is real is given in an actual act of finding. By its two reasons, I exclude the step from equations that carry the quantum state forward without fail to a world in which each single outcome is fixed. So I also exclude a demon who "would know exactly how we are going to act". Carroll grants that "it's completely conceivable that we could have acted differently", but he grants the point only relative to our incomplete information. I answer Carroll on the point with two reasons. Because the world is causal, what is chosen goes on to have effects; because it is not determined, the one who chose could have chosen otherwise.

The free will theorem also has a consequence for Carroll's rule against mixing the language of people with the language of particles. The theorem proves a link between a statement in the language of people, that the experimenter's choice is free, and a statement in the language of particles,

that the particle's answer is free. "What the theorem shows is that the first-person freedom of the experimenter and the third-person indeterminacy of the outcome are structurally linked". In the quotation, first-person means acting from inside, at one's own scale, which for a person is the middle scale of hands and eggs. Third-person means observing from outside and from another scale, as a person observes a particle at the smallest scale.

The Kitchen from Below and from Above

The free will theorem links the level of people to the level of particles by proof. So a further question comes up: does a cause ever run between the level of people and the level of particles, from the cook's plan down to the particles of the cook's hand, or from the particles of the hand up to the plan? On causes between levels, Carroll's account and mine part as well. The parting on causes between levels also follows from the one place where the two accounts part: what lies at the ground. Carroll keeps each cause inside one way of talking, and he rejects causes running downward and causes running upward alike, while I hold that the level of people and the level of particles depend on one another. Ellis holds that causes run from the top down as well as from the bottom up.

Picture the kitchen again. From below, the crack is particles of shell pressing on particles of rim, each particle obeying the Core Theory. From above, the crack happens on this rim, at this moment, because the cook is making breakfast. I describe the view from below and the view from above in these words: "causation from above and causation from below are the two directions of one interdependence of levels, neither prior". In plain words, each level depends on the other, and neither comes first. So neither level, alone, fixes what happens: the breakfast needs hands and a shell, and the particles of the hands and the shell do not by themselves settle that this egg is cracked now. Ellis looks for room that lets a higher level act. He finds the room at the lowest level, the level of particles,

where outcomes are not fixed in advance. In the same section I call the room for a higher level to act *slack*: some part of what happens that the description of one level leaves unfixed. I find slack at every level, and not only at the lowest one: "The collection finds the slack at every scale, because no deterministic description, at any scale, can be complete". The cook's plan pushes on no particle: every particle of the cook's hand obeys the Core Theory throughout. The same section says that choice is "not a higher level acting on a lower one".

Causality, Change and Choice

A reader may ask whether a single outcome that the laws leave open makes a choice only chance, and chance is not choice either. I answer that question with three words that belong together.

The triple of the real is causality, change and choice: the three things that every real event has, each in some measure. Causality is the pattern found again and again between what comes before and what follows. Change is the actual passing from how things were to how they are next. In a change, pattern and something new come together: the shell giving way on the rim, while the shell gives way. I define choice: choice "is the emergence of the new: the singular act that is only itself, that no pattern contains in advance". Causality, change and choice are distinct: none of the three occurs without the other two, and none can do the work of another.

Watch the cook. The choice is the decision to crack the egg now, out of every moment the morning offered. The change is the crack, the shell giving way on the rim of the bowl. The causality is the pattern that a shell struck on a rim splits, as every cook has found time after time. Causality, change and choice run in one order, and I give the order: "Choices start changes; changes can be understood causally; causal patterns set up the conditions for new choices". The cracked egg in the bowl now opens the next range of choices: an omelet, a cake, a fried egg on toast. I draw the direction of causes in time from the order of choice, change and causality:

"causality runs from earlier to later because it is the understanding of changes that choices began". This book's Chapter 8 found the arrow of time in every act of finding. The same one-way order of time appears inside every act of choosing.

Now we can answer the question whether chance is choice. A law of the many describes the causality, the pattern laid out over many eggs and many trials, but no law of the many fixes which single outcome happens, this one, here. Choice has its place at the single outcome that no law of the many fixes. When someone describes the single outcome from outside, in the terms a law uses, the outcome is chance, but when the one who chooses describes the same outcome from inside, the outcome is a choice. I call the single outcome one event with two descriptions: "From outside it is randomness; from inside it is choice. Both descriptions are correct, and neither is complete". So a choice is chance only as described from outside. The description from outside leaves out the one who chooses, for whom the choice has its meaning.

A choice also needs more than a selection. I name three parts of a choice: a range of possibilities, a selection, and a consequence that cannot be undone. The cook's choice at the egg has all three parts. The three parts are the morning's range of moments, the cook's selection of now, and a crack that will never be uncracked. If a choice could be rolled back so completely that nothing showed it had been made, the choice would be no choice at all.

How much of an act is choice and how much is pattern changes from act to act. A habit is a choice made so often that it runs almost wholly as causality: the hand reaches for the light switch. A turning point in a life lies at the other end. The same section calls a turning point "causality bent almost wholly by choice". Most of what people do lies between. A cook in a diner who cracks egg after egg all morning acts mostly by pattern, while a cook who decides one morning to try a dish never made before acts mostly by choice.

Near the end of Chapter 44, “Freedom to Choose”, Carroll states a worry. He writes that to the extent neuroscience comes to predict behavior without any reference to choosing, “it will be less and less appropriate to treat people as freely acting agents”. Carroll judges it unlikely that neuroscience will come to predict behavior without reference to choosing (“It doesn’t seem likely, however”), and I give a reason why that prediction cannot come to pass. I state the reason, drawn from observation. Write down everything about a person that can be predicted, the way a program compresses a stream of data by recording the stream’s regularities. The same section goes on: “However good the compression algorithm is, there will be some residual amount of the person’s behavior that it cannot compress, and that residue is indistinguishable from randomness”. “That residue is the part that the person regards as their choice”. The better a model of a person becomes, the more exactly the model outlines the place where the person’s choosing is. The person’s choosing is in the residue that the model cannot compress.

Carroll’s charge against libertarian freedom is that libertarian freedom would need people to overcome the laws of physics by thinking. I answer Carroll’s charge as well. On my account, choice overcomes nothing. The single outcome is already open in the world, in hard randomness, and a person does not bring the openness in from outside. Physics gives the context, the constraints and the space of possibilities. The pattern over many eggs holds exactly as the laws give it. The cook’s choice is in the single crack, this one, now: the laws give the pattern over many cracks, and they fix nothing more. No force pushes on any particle of the cook’s brain, and no law is broken. The law of the many is kept, and the cook chooses the single case.

The Cook at the Bowl

Go back to the counter. The cook decides, and cracks the egg now. I agree with Carroll that every particle in the cook's hand obeys the Core Theory, that no new force is needed, and that choice belongs to every good description of the cook as a person. Conway and Kochen showed that if a person at a dial chooses freely, then the particle's answer is open. The Incommensuration Theorem shows, for a separate reason, that no world is fixed whole by its laws. The physicists at Delft, Vienna and Boulder found answers that causes at each laboratory, fixing each answer in advance, could not have produced.

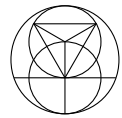
So we can now answer "Can I choose?" in your own words. Yes: when you decide, you choose. Some things were settled before you came to your decision: the range of things you could do, the laws every particle of your body obeys, and the odds those laws give over many tries. Nothing settled which of the things you could do you actually do, here and now: you select what you do, from your side, and the consequence of your selection cannot be undone. How much of a given act is your selection varies. Reaching for the light switch is almost all habit, and a turning point in your life is almost all choice. In Carroll's sense you choose because choosing is the best description of what people do. In my sense you choose because the laws and everything that came before your decision leave the single outcome open, and you are the one who selects the outcome.

We now know where the cook's choosing is. Choice is the member of the triple of the real (causality, change and choice) that a law of the many does not fix. Choice has its place at the single outcome the laws leave open. Choice is no extra force pushing on the particles of a brain.

Where choice has its place matters for the question of the whole book. At the ground I find the act of finding. Every act of finding is made from one side, by the one who finds, and the one who finds faces a range of possible outcomes and comes away with one of the outcomes. So the

single outcome is where finding happens and where choosing happens. The cook at the bowl is doing something the beginning of the universe did not settle.

If choice is real at the single outcome, who or what is doing the finding and the choosing?



Chapter 11

Who Is Doing the Finding?

What this chapter uses

The six parts of every comparison, among them the *subjective* and the *objective* (Chapter 2).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

The *bound triple*: the arrow of time, hard randomness and the subjective, distinct and inseparable (Chapter 5); *true choice* in the single case (Chapter 10).

Chapter 10 placed choice at the single outcome that the laws of physics leave open. If choice is real at the single outcome the laws leave open, the next question is plain. When something is found, who or what is doing the finding?

A Bat in a Philosophy Journal

In October 1974 the journal *The Philosophical Review* printed a short essay by Thomas Nagel, a philosopher at Princeton University. The philosophy faculty of Cornell University edited the journal. The title of Nagel's essay was a question: "What Is It Like to Be a Bat?" The essay's first sentence is: "Consciousness is what makes the mind-body problem really intractable". The *mind-body problem* is the old question of how a mind is related to the body and brain that the mind belongs to. The word "intractable" means that no known method works on the mind-body problem.

The bat in Nagel's essay perceives the world in a way unlike ours. Most bats, Nagel wrote, "perceive the external world primarily by sonar, or echolocation", catching the reflections of "their own rapid, subtly modulated, high-frequency shrieks". *Echolocation* is finding things by sound: a bat sends out a cry, listens for the echo, and works out from the echo where things are. Nagel went on: "Their brains are designed to correlate the outgoing impulses with the subsequent echoes, and the information thus acquired enables bats to make precise discriminations of distance, size, shape, motion, and texture comparable to those we make by vision". Yet, Nagel added, "bat sonar, though clearly a form of perception, is not similar in its operation to any sense that we possess".

In Nagel's description, the bat sets its own cry beside the echo that came back. The bat uses the cry and the echo together to find an insect in one place, moving in one direction. Setting the cry beside the echo is a comparison, and the bat makes the comparison many times a second, in the dark.

Nagel's point was that the bat's act of finding has an inside. There is something it is like, for the bat, to find the insect by echo. Nagel stated his test in one sentence: an organism has conscious mental states "if and only if there is something that it is like to be that organism". Nagel explained the phrase at once: "something it is like for the organism". Nagel also said

what he wanted to know: "I want to know what it is like for a bat to be a bat". Nagel argued that "every subjective phenomenon is essentially connected with a single point of view". He argued further that a physical theory leaves out the single point of view to which each subjective phenomenon is connected, because a physical theory is written from outside.

Now come back to the kitchen, where the cook stands at the bowl. a small camera on the counter records the crack. A thermometer in the pan on the stove reads the heat of the pan. The egg is about to go into the pan. Three things in the kitchen are finding something: the cook finds the egg cracked, the camera finds the crack and keeps a picture of the crack, and the thermometer finds the pan hot. Which of the three is doing the finding? And is the cook's finding different in kind from the findings of the camera and the thermometer?

From Carroll's pages

What Carroll Shows

Sean Carroll gives six chapters of his book *The Big Picture* to consciousness, and he starts from what science knows about consciousness today. No one has a finished theory of consciousness. Carroll writes that we have, instead, an expectation about the coming theory. The coming theory will fit the Core Theory, Carroll's name for the known laws of particles and forces, with gravity where gravity is weak. On that theory, consciousness will be "part of physical reality, not apart from it" (*The Big Picture*, Chapter 37, "Crawling into Consciousness").

Carroll knows Nagel's bat and names the bat in his book. Carroll says that Nagel made clear what we ought to be able to explain with a complete theory: "what it is like" to feel something. Carroll adds: "His famous example was that we can't know what it is like to be a bat, but the point is more general" (Chapter 40, "The Hard Problem"). Carroll puts Nagel's point in two terms of the philosopher David Chalmers:

the easy problems and the hard problem. The *easy problems* are the problems of explaining how we sense, store and use information, while the *hard problem* is the problem of explaining why any of that sensing, storing and using is felt from inside. In Carroll's words: "The Easy Problems are about functioning; the Hard Problem is about experiencing".

Carroll's answer to the hard problem is poetic naturalism: the view that there is one world, and many good ways of talking about the world, each way fitted to its own range of things. For Carroll, consciousness is one of the many ways of talking about the world, and a good one. Carroll calls consciousness *emergent*: a description that fits many parts taken together and does not fit any one part alone. Wetness is emergent in the same way: wetness fits water, and not a single molecule of water. Carroll holds that the ways of talking about our inner lives "are useful ways of talking about the effective behavior of the collections of atoms we call human beings". Carroll also says what his view does not mean: "Consciousness isn't an illusion, but it doesn't point to any departure from the laws of physics as we currently understand them" (Chapter 40). Talk of inner lives also does work, in helping us predict what people do. Anyone who tried to predict what people do with no talk of their inner states would end up putting the inner states back in: "If we weren't familiar with consciousness already, in other words, we'd have to invent it" (Chapter 39, "What Thinks?").

From poetic naturalism Carroll answers the famous puzzles about consciousness one by one. A *philosophical zombie* is an imagined being made atom for atom like a person, acting in every way like a person, with no inner experience at all. Carroll answers: "According to poetic naturalism, philosophical zombies are simply inconceivable, because 'consciousness' is a particular way of talking about the behavior of certain physical systems" (Chapter 41, "Zombies and Stories"). In Carroll's sense of the word, experiences are real as fluids and univer-

sities are real, because each plays an essential part in a good description of some part of the world. In Carroll's words: "They are essential concepts within a way of talking about things happening in our brains, and that makes all the difference". Suppose someone made a complete recording of a brain's activity, written out and kept still. On Carroll's account, the recording would not be conscious. Consciousness needs change in time and dealings with the world, and a recording kept still has neither. In Carroll's words: "A printout of what our brain does wouldn't qualify" (Chapter 39).

Carroll also tests one view of consciousness against measurements physicists have already made. Some philosophers have proposed that every particle carries a little consciousness of its own, a view called *panpsychism*. Carroll first takes the most direct form of panpsychism, in which a particle would carry an extra inner setting. He tests the direct form against what physics has measured. For the test, Carroll counts a particle's *degrees of freedom* as the number of distinct settings the particle can be in. A photon is the particle of light. A photon has no electric charge to vary, and two ways to spin. So a photon has two degrees of freedom. Physicists use the counts of degrees of freedom to compute the rates at which particles scatter off one another. Physicists have measured the scattering rates in experiment after experiment. In Carroll's words: "Those numbers have been experimentally tested to exquisite precision, and the Core Theory has passed with flying colors" (Chapter 42, "Are Photons Conscious?"). An extra inner setting in each particle would add to the particle's degrees of freedom, and so would change the computed rates. So we now know that the particles of the Core Theory carry no hidden extra setting, mental or otherwise, of the kind the direct form of panpsychism needs. The scattering measurements rule out what Carroll calls the "overly literal version of panpsychism".

Carroll then takes a second step, for the gentler form of panpsychism, in which inner settings would not change how matter behaves. Carroll notes that "Advocates of panpsychism would probably not go as far" as to treat the inner settings as degrees of freedom. Carroll then compares two worlds: one whose particles have inner settings, and one whose particles have none, which he calls the world of "zombie photons". The two worlds behave in exactly the same way. So Carroll writes that attributing consciousness to particles "buys us no new insight or predictive power" (Chapter 42). From the two steps, Carroll concludes that consciousness belongs to large wholes: "Consciousness seems to be an intrinsically collective phenomenon" (Chapter 42).

Last, Carroll gives the observer no special place in quantum measurement. Carroll holds the many-worlds account of quantum mechanics, the account that the physicist Hugh Everett formulated. In Everett's account a measurement is an ordinary physical process. In Carroll's words: "In Everett's Many-Worlds Interpretation, the answer is simple: nothing special". On Carroll's account the observer need not be a person: "The fact that observers happen to be conscious plays precisely zero role; measurements can be easily carried out by nematodes, video cameras, or rocks" (Chapter 42). Nematodes are tiny worms.

Across the six chapters of *The Big Picture* on consciousness, Carroll keeps experience inside nature, and he explains none of experience away. He adds no force to physics to account for experience. And he answers the popular claim that a mind makes the world by looking at the world.

Instruments Compare as Surely as Minds Do

I derive several of Carroll's results by reasoning of my own. The first result is Carroll's result that a measurement can be made by a nematode, a video camera or a rock. Every way of knowing is made of *comparison*: setting one thing beside another and finding them the same or different in some respect. Instruments compare too: a thermostat compares the warmth of a room with the warmth it is set for, and turns the heater on or off. Instruments compare as surely as minds do.

Because instruments compare, we can answer the first half of the chapter's question, who or what is doing the finding, from the kitchen alone. The thermometer compares the heat of the pan with the marks of a scale, and finds the pan hot. The camera compares the light reaching each point of the camera's sensor with the steps of a scale. Frame by frame, the camera records the shell whole, then broken. The cook compares the egg on the rim with the egg a moment ago, and finds the egg cracked. The thermometer, the camera and the cook are all doing the finding. When every finding is a comparison, an observer is a comparer: the word names a position inside the world, and it never names a mind. So I derive Carroll's result about cameras and rocks from what every comparison is. The derivation uses no interpretation of quantum mechanics.

I derive three more of Carroll's results by my own reasoning: on descriptions and forces, on illusion, and on the zombie. The second result is about illusion. *Cartesian dualism* is the view that mind is a second substance apart from matter, and the view is named for the philosopher René Descartes. I take up Cartesian dualism in the Technical Appendix of my first book. Consciousness is not an illusion, nor is it in conflict with conventional science. The third result is about the zombie. I agree with Carroll that a philosophical zombie cannot be conceived, but for a different reason. Every comparison has two sides: the *subjective*, the side the comparison is made from, and the *objective*, the side the comparison is about. I

hold that the two sides are distinct, and that they are had together or not at all: a comparison that has one side has the other side too. To imagine a zombie is to ask for the objective side of every comparison a person makes, without the subjective side, and so to ask for the two sides of every comparison to be separated. I give the same reason in a chapter of my own on the measure of an inside, in words I wrote first in another book. I also quote the consequence from the same chapter: "the conception fails at a level deeper than physical law". For Carroll, a zombie cannot be conceived because consciousness is a way of talking about matter, while for me, a zombie cannot be conceived because a comparison with its objective side and no subjective side is no comparison at all.

Carroll and I also agree about Carroll's printout of a brain. The camera on the counter keeps a picture of the crack, but the picture is not the act of finding the crack. A structure of distinctions and relations, held in view as a whole, is a record: the laid-out side of the act. A printout of a brain is a record of finding, like the camera's picture of the egg. A record comes after the act of finding and is not the act.

I also derive one part of Carroll's result about panpsychism, by my own reasoning. On my reasoning, consciousness is not an extra setting that a particle carries as one of the particle's properties. It is neither the world perceived nor the perceiver, but the act of perceiving. Consciousness is not a substance, not a property of matter, not a feature that could be located in the way physical objects are located. So on my reasoning consciousness is never a property of matter. And so consciousness is never the extra setting that the overly literal panpsychism needs, the setting that the scattering measurements rule out.

Three Things in Every Finding

If the camera and the cook are both finding, what does every finding have? In this book's Chapter 5 we found three things in the cook's crack. Second, the crack comes out one way that nothing before the crack fixed:

hard randomness. Third, the crack runs one way, from what could happen, to the act, to the record: the arrow of time. I call the subjective, hard randomness and the arrow of time together the *bound triple*: the three are distinct, they are present together or not at all, and none can do the work of another. In Chapter 5 we walked round a circle of reasons in which each of the three needs the next.

Each finding the camera makes has all three as well. The camera's lens faces the bowl from the counter, and the lens's place on the counter is the side the camera finds from. The camera records one crack, the crack that happened. And the picture on the camera's card comes after the crack. I call the side a comparison is made from a *stance*: the place where the one who finds stands. An instrument has a stance as surely as a person does.

I show in two ways that the bound triple holds together. This book's Chapter 5 gave both ways. The first way derives the binding from the Incommensuration Theorem. The second way shows the binding in physics: every account of quantum measurement keeps the three members of the bound triple together or drops them together. I name the three members in one sentence: "The three are three aspects of one act: comparison proceeding in a direction, registered by a perspective, with an outcome not fully determined".

Carroll and I agree that the observer need not be a mind. For me, a measurement is a finding, and so a measurement has the bound triple: a side the measurement is made from, one outcome that nothing fixed in advance, and a direction from act to record. Everett's account, which Carroll holds, keeps every outcome on its own branch of the universal state (the quantum state of the whole universe). The universal state evolves by a rule that fixes every later state from an earlier one. The rule runs the same forward and backward. So Everett's account drops all three, as this book's Chapter 5 showed: no fact says which branch the one who finds is on, no outcome is left open, and time has no direction. On my reasoning, Carroll's "nothing special" shows that Everett's account has left the finding out. The three things every finding has are the three things Everett's account drops, and Everett's account drops all three together.

The Cook, Gathered

The camera and the cook both find, and every finding has the bound triple. What, then, is consciousness?

Consciousness is the bound triple taken as one act: the arrow of time, hard randomness, and the subjective happening together, with the subjective as the interior face of that act. The interior face of an act is the side the act is made from, as that side is had from inside. The three members of the bound triple are taken as one act when they are taken as they occur: all at once, in the finding as the finding happens, and not one at a time in a description made afterward. I say which acts the answer covers: the act of perceiving "wherever it occurs as a first-person point of view". A *first-person point of view* is an act of finding as it is had from inside, by the one who makes it, as your own seeing of this page is had by you. So the answer covers the act of perceiving wherever that act is had from a first-person point of view, as each of the cook's acts of finding is had by the cook. I name Nagel where I say what the subjective is in a conscious being: "In a conscious being, it is the fact that there is something it is like to be that being (Nagel, 1974)". Nagel's "something it is like" is the side a finding is made from, had from inside.

The cook's finding of the crack differs in kind from the camera's finding of the crack: the cook's finding is gathered, with all the cook's other findings and choices, into one self. The cook finds more than the crack. In the moment of the crack, the cook also finds the warmth of the kitchen, the weight of the egg, the smell of the butter, and the memory of the last egg that broke badly. The cook also chooses: when to crack the egg, how hard to strike it, what breakfast to make. All of the cook's findings and choices are had from one side, by one cook. The cook carries the findings and choices on as the cook's own, without a break from one moment to the next.

I also say in plain words, in the same section, that consciousness gathers the perceiving and the choosing alike: "consciousness is everything one perceives, taken together; it is where one's choices gather". What is new in kind in a living being is the gathering of choice into a self that knows it from within. In the same section I set a thermostat beside a person choosing a gift. The thermostat compares and has a stance, but the thermostat is pushed toward its setting, "and nothing in that pushing shows that anything was selected, or that anything is gathered". Carroll and I both find that something new in kind appears with a living, conscious being. Carroll names the new thing a collective phenomenon, a new way for matter to organize itself. I name the new thing a self: the gathering of findings and choices into one stance that knows them from inside.

In us, the gathering into a self happens in a brain. A past measurement shows something of how a brain is poised to do the gathering. John Beggs and Dietmar Plenz are neuroscientists. In 2003 Beggs and Plenz recorded the spontaneous electrical activity of slices and cultures of rat cortex, on a grid of sixty electrodes. The cortex is the outer layer of the brain. Beggs and Plenz found that the activity spread in bursts, and called the bursts *neuronal avalanches* ("Neuronal Avalanches in Neocortical Circuits", *Journal of Neuroscience*, 2003). The sizes of the bursts followed the pattern expected of a system at a *critical point*. A critical point is a state balanced between order and disorder, in which a small disturbance can spread to any size. I say what the measurement of Beggs and Plenz means: in a brain poised near a critical point, small disturbances cascade, each setting off larger ones. As the disturbances cascade, outcomes that nothing fixes at the smallest scale enter chains of causes. The chains reach the scale of everyday things and bear on choice. An outcome that nothing fixes has two descriptions, as this book's Chapter 10 showed: described from outside, the outcome is hard randomness, and described from inside, for the one who chooses, the outcome is a choice. I name the brain's balance as the physical circumstance under which the relation between choice and

hard randomness is expressed in us, and not as the relation itself. The brain's balance is the circumstance. Consciousness itself is the bound triple taken as one act, in a finding had from a first-person point of view.

Why the Hard Problem Stays Hard

Carroll and I agree that the hard problem calls for no change to the laws of physics. Carroll and I agree for different reasons, and the reasons part over where each of them places experience. Carroll places experience inside a description, as one way of talking about atoms. I place every description inside findings, each made from a side. The parting over where experience belongs follows from the one place where Carroll and I part: what lies at the ground.

Think of the last time you touched a hot pan by mistake. A doctor could write down everything about the touch: the heat of the metal, the nerves that fired, the hand pulled back. You could read every line the doctor wrote, and the page would not hurt.

I hold that the hard problem is hard because of what a description is. A description of physics is made to read the same from every side: the description is laid out as structure, set down whole, and every reader finds the same thing in the description. An experience is had from one side. A complete description of pain is knowledge laid out as structure; what pain feels like is reached only in one's own perceiving. Suppose we ask how experience comes out of a description. Then we ask a description, which reads the same for every reader, to contain one reader's own side. I put the point in two short sentences: "Perception cannot perceive perception". "Observation cannot observe observation". It is not possible, even in principle, for there to be a purely physical theory of the process of observation itself. The explanatory gap between physical description and conscious experience is therefore a structural feature of comparison itself.

Because the explanatory gap is a structural feature of comparison, the gap needs no new force and no new particle. The gap belongs to the form of every finding. No improvement in physics could close the gap, because physics is made of findings. I place every description inside findings because every description of the atoms, and every way of talking, is made in findings, each from a side. A way of talking is laid out as structure, and a structure cannot contain the side the structure is made from. The doctor's page about the hot pan is laid out as structure: the page holds every line about the touch, and the page does not hurt. So experience, which is the side a finding is made from, had from inside, cannot be a way of talking.

A picture from the philosopher Frank Jackson shows the parting over where experience belongs, in one small case. Jackson imagined *Mary*, a scientist who knows every physical fact about color but has lived all her life in a black and white room, and Jackson asked whether Mary learns something new when she first sees red. Carroll answers that Mary gains new connections in her brain by a new route: "It's a difference in the way the knowledge got to your brain, not in the kind of knowledge it is" (Chapter 40). I apply to Mary my account of pain, and I get a different answer. Mary gains the one kind of knowing that no description carries: her own seeing of red, reached in her own perceiving. Before she saw red, Mary had red laid out as structure.

Nagel's bat shows the same parting over where experience belongs. Everything a book can say about echolocation is description, and Nagel could read all of that description. What it is like for the bat is the bat's side of the bat's own findings. The bat's side is had by being the bat. Nagel's question about the bat cannot be answered in a book, for the same reason that makes the question a good one: no description holds the side from which the description is read.

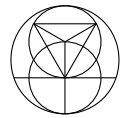
Who Is Doing the Finding

We can now answer this chapter's question: who or what is doing the finding? The cook, the camera and the thermometer are all doing the finding, because every one of them compares. Every finding has a side the finding is made from, an outcome that nothing before the finding fixed, and a direction from act to record, and the three come together or not at all. The cook's finding differs in kind from the findings of the camera and the thermometer in one way: the cook's finding is gathered, with all the cook's other findings and choices, into one self that knows them from within. Each of the cook's findings is had from a first-person point of view, and the word 'consciousness' names the bound triple taken as one act, in a finding had from that point of view.

Instruments compare as surely as minds do. Every account of quantum measurement has all three members of the bound triple or none of them: the arrow of time, hard randomness and the subjective. And consciousness is the bound triple taken as one act, had from a first-person point of view.

Who does the finding matters for the question of this whole book: what lies at the ground? Carroll puts a quantum state at the ground. He finds consciousness high above the ground: consciousness is one way of talking about large collections of atoms. I put the act of finding at the ground. Consciousness is the three things every finding has, taken as one act where the finding is had from inside. So the one who finds is at the ground from the start, in the thermometer, the camera and the cook alike.

If finding is everywhere, from thermostats to cooks, where do meaning and value come from?



Chapter 12

Do We Make Meaning Up?

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 2).

Entropy (Preface and Chapter 8); a *brute fact*, a fact with no further reason behind it (Chapter 3).

The subjective side of every comparison, which weighs what to attend to (Chapters 2 and 11).

Do we make meaning up, or is meaning there to be found? We saw finding everywhere in this book's Chapter 11: in a thermostat on the wall, a camera on the counter and the cook. An act of finding is one act of comparing two things and finding them the same or different. A thermostat, a camera and a cook all make acts of finding. Finding everywhere makes the question of meaning sharper, because a thermostat finds without feeling anything about what it finds, and a cook cares. Sean Carroll gives an answer that is clear and has courage in it: we make meaning, and what we make is real. Carroll means "real" in his own sense of the word: a thing is real when it plays an essential role in a description of the world that is ac-

curate, as far as we can tell, within the range of cases the description covers. We will keep both halves of Carroll's answer. As we go, we will ask one more question. When someone makes a meaning, what was already present in the act of finding before the meaning was made?

A Paragraph About Ought

In London in 1740, the bookseller Thomas Longman issued the third volume of *A Treatise of Human Nature*, at the sign of the Ship in Paternoster Row (a London shop was then known by the sign that hung over its door). The author of the *Treatise* was the philosopher David Hume, a Scot born in Edinburgh in 1711. The first two volumes had come out the year before from another London bookseller, John Noon. The third volume was called *Of Morals*. Near the end of the first section, Hume paused his argument. He paused to add what he called "an observation, which may, perhaps, be found of some importance".

Hume had been reading books on morality, and he had noticed a habit in their authors. Each author reasoned for a while in the ordinary way, about God or about human affairs, and then, all at once, the author's sentences changed. Hume describes the change in his own words: "I meet with no proposition that is not connected with an ought, or an ought not". He goes on: "This change is imperceptible; but is, however, of the last consequence". In Hume's sentence the word "last" has its old sense of highest in degree: the change, small as it looks, is of the highest consequence. In each of the books on morality that Hume had read, sentences about what is the case had turned into sentences about what should be done. No author had said how a sentence about what should be done could come out of sentences about what is the case. Hume asked the authors for the missing reason: "how this new relation can be a deduction from others, which are entirely different from it".

Hume's paragraph is about two kinds of sentence. An *is* sentence says what is the case: the egg is fresh; the pan is hot. An *ought* sentence says what should be done: you ought to cook the egg now; you ought not to waste it. Hume asks whether an ought can ever be derived from is sentences alone. Carroll gives Hume's question a whole chapter of his book *The Big Picture*, and after a visit to the kitchen we will begin where Carroll begins, with what the laws of physics contain.

Breakfast for Someone

Go back to the kitchen. The cook has set the egg in the bowl of water, and the egg sank, so the egg is fresh. Now the cook cracks the egg once on the rim of the bowl, and finds the shell cracked. I have carried one picture from the first page: one act of finding, the crack found once. This morning a second person is part of the act, because the egg is for a friend. The friend is leaving early on a long drive, and the cook is making breakfast for the friend.

Is the meaning of the breakfast made up? In one plain sense the meaning is made up. The shell says nothing of "breakfast for a friend", and neither do the yolk and the heat of the pan. The cook chose to make the breakfast, and another cook, on another morning, would have made something else, or nothing. And yet the meaning of the breakfast does not look arbitrary either. The cook cannot make the breakfast mean anything at all by deciding so: an egg dropped on the floor and left there is breakfast for no one, whatever the cook declares. So you can put the question in one line: if the meaning of the breakfast is made, what keeps the meaning from being arbitrary? Carroll answers that question at length, and we hear Carroll's answer first.

From Carroll's pages

What Sean Carroll Showed

Carroll takes up meaning and morality in the last part of *The Big Picture*, which he calls "Caring". He begins with what the laws of physics contain. The Core Theory is Carroll's name for the laws of the particles and forces of everyday life. He writes that meaning, morality and purpose are nowhere to be found in the Core Theory. Nor are bathtubs, novels or the rules of basketball. Yet in Carroll's sense of the word, all six are real: meaning, morality, purpose, bathtubs, novels and the rules of basketball. Carroll explains: "They aren't built into the architecture of the universe; they emerge as ways of talking about our human-scale environment" (*The Big Picture*, Chapter 45, "Three Billion Heartbeats").

So meaning has to come from somewhere other than the laws of physics. Carroll says where: "It is about you: it's up to you, me, and every other person to create meaning and purpose for ourselves" (Chapter 45). Carroll then answers two worries about a meaning that we make ourselves. The first worry is that a self-made meaning is a kind of cheating. Carroll answers the first worry with a picture: "Water doesn't stop being wet when you learn it's a compound of hydrogen and oxygen" (Chapter 45). In the same way, a love is still a love after you explain the love by atoms, though the love once seemed to come from heaven. The second worry is that a meaning we make has nowhere to start, and so must be arbitrary. Carroll answers that a made meaning starts from who we are. He writes that life "occurs out of equilibrium, driven by the second law" (Chapter 45). Equilibrium is the settled condition that a cup of coffee reaches once it has cooled to the warmth of the room and nothing further changes. Entropy is a count of how many arrangements of the small parts of a thing look the same from outside, and the second law of thermodynamics is the rule that the count grows. A living thing stays away from equilibrium by eating, breathing and moving, and the eating, breathing and moving

raise the entropy around the living thing. In people, Carroll says, life's ceaseless motion away from equilibrium shows itself as desire, and "Desire is an aspect of caring" (Chapter 45). He concludes: "We are built from the start to care about the world, to make it matter" (Chapter 45).

Then Carroll turns to Hume's question. A naturalist, as Carroll is, holds that the natural world is all there is, and the natural world is a matter of what is. So if you cannot get an ought from an is, a naturalist seems to be in trouble. Carroll states the naturalist's trouble in full: "The natural world doesn't pass judgment; it doesn't provide guidance; it doesn't know or care about what ought to happen" (Chapter 46, "What Is and What Ought to Be"). Call Hume's point Hume's rule: no ought can be derived from sentences that say only what is. Carroll shows why no derivation gets around Hume's rule, with a story that the physicist Richard Feynman liked to tell. A painter boasted to Feynman that he could mix red paint and white paint and get yellow. The painter stirred and got nothing but pink for a while, and then he added a touch of yellow to the mix. Carroll argues that every claimed derivation of ought from is plays the painter's gambit: an ought is slipped in among the premises, as the yellow was slipped in among the red and the white. Carroll examines one well-known case, the philosopher John Searle's argument from a promise. Searle's argument sets out to derive an ought from the fact that someone made a promise. Carroll shows that Searle's argument has an ought hidden between its fourth line and its fifth. Carroll concludes: "That's the problem with attempting to derive ought from is: it's logically impossible" (Chapter 46).

If no ought can be derived from what is, what is left is to build morality. Carroll's claim about morality as a whole is that "our ethical systems are things that are constructed by us human beings, not discovered out there in the world, and should be evaluated accordingly" (Chapter 47, "Rules and Consequences"). Carroll's view is called *con-*

structivism: moral rules are built by people, from their own feelings and commitments, and are not read off the world. Carroll adds at once that a morality that is built is not empty: "admitting that morality is constructed, rather than found lying on the street, doesn't mean that there is no such thing as morality" (Chapter 47). Carroll follows the philosopher Sharon Street in telling apart two kinds of constructivism. Kantian constructivism, named for the philosopher Immanuel Kant, says that every rational person who thought clearly enough would build the same morality. Humean constructivism, named for Hume, says that different people may build different moralities. Carroll sides with Hume, and so with Humean constructivism: "Morality exists only insofar as we make it so, and other people might not pass judgments in the same way that we do" (Chapter 47).

Carroll points to a measurement that shows how old some of our moral commitments are. Sarah Brosnan and Frans de Waal were researchers at Emory University. In 2003 Brosnan and de Waal tested brown capuchin monkeys. The test was a simple exchange with a human experimenter, with food as the reward. Each monkey handed the experimenter a small token, and the experimenter handed the monkey food in return. A monkey was content to hand over a token for a slice of cucumber until it saw a partner given a grape, a better reward, for the same work, and then the monkey refused to go on. The refusal was stronger still when the partner got the grape for no work at all. Brosnan and de Waal concluded that the monkeys' aversion to unequal pay has an early origin in evolution. Carroll draws the same lesson: "Some of our most advanced moral commitments have very old evolutionary roots" (Chapter 47). And the monkey's refusal began with a comparison: the monkey set its own pay beside its partner's pay, for the same act.

Carroll pictures a built morality that is still not arbitrary as a game. The rules of basketball were invented. James Naismith invented the game, and his first players threw the ball into peach baskets. The

game got better when the baskets became hoops. The rules of basketball were not waiting in the universe to be found. Yet the rules are not arbitrary either, because they serve the purpose of the game. Carroll writes: "Morality is like that: we invent the rules, but we invent them for sensible purposes" (Chapter 48, "Constructing Goodness"). Carroll observes that moral philosophies "tend to pick one approach and apply it universally", and that as a result "we often end up with conclusions that don't sit easily with the premises we started with" (Chapter 48). The trolley problem is a thought experiment that, in Carroll's words, "aims to sharpen the conflict between competing moral sentiments" (Chapter 47). A runaway trolley is speeding toward five people tied to its track. You are standing by a switch that would send the trolley onto a side track, where one person is tied. Carroll draws the plain conclusion for the trolley problem: "There is no 'right' answer to the trolley problem. How you should act depends on who you are" (Chapter 48).

Carroll sums up where caring is: "The universe doesn't care about us, but we care about the universe" (Chapter 49, "Listening to the World").

Carroll's answers come to three plain statements. No law of physics gives us a meaning or a rule of conduct. No chain of reasoning from facts alone ends in an ought. An ought is hidden in the premises of every chain that seems to end in an ought. People make the meanings they live by, and those meanings are real in Carroll's sense of the word.

Where the Two Roads Meet

I agree with four of Carroll's results, and I give a reason of my own for each.

The first shared result is about science and rules of conduct. Science and technology tell us what can be done. By their own methods they do not tell us what ought to be done. Carroll says the same near the end of his book: "science describes the world, but what we're going to do with that knowledge is a different matter" (*The Big Picture*, Chapter 50, "Existential Therapy"). My reason is what science is. Science is the method that maps what causes what, by tests that can be repeated. From a map of what causes what, we learn what can be done, but we do not learn what ought to be done. Reasoning carried out entirely in terms of causation gives no basis for meaningfulness.

The second shared result is about Hume's rule itself. I name the same step that Carroll names: the step from facts to rules. Hume forbade a chain of reasoning that runs "here are the facts; therefore here are the rules". I say of the chain from facts to rules what Carroll says: "That is Hume's guillotine, and it cannot be crossed". *Hume's guillotine* is the common name for Hume's rule, that no ought follows from premises that contain only is. The name pictures the rule as a blade that cuts ought sentences off from is sentences. On Hume's rule Carroll and I agree: the painter cannot get yellow from red and white.

The third shared result is about purpose. Carroll holds that each person makes meaning and purpose, and that the universe supplies none. There is no single purpose, or any single value for, all of life, all of consciousness, all of beingness. Two short statements in the same pages say where purpose is found: "Purpose is individual", and "Life is its own purpose, value, and meaning".

The fourth shared result is about morality as something made. I use the word *morality* for the rules and customs of a community at a time. Morality is what a people, in a given age, holds right and wrong. People make the rules of a morality, the rules differ from one people to another, and the rules change. Moral codes (rules/laws) are civil, and can only be enforced. My book for readers of the physicist Carlo Rovelli takes the same question up. A community's moral rules are real, they are rooted in

our nature and our history, and they change. I also agree with Carroll that no fixed code settles every case: "No single set of rules, no matter of what origin, can encompass life". My reason is that acting well always happens in one life and one situation. The practice of ethics, by which I mean the practice of choosing well, is particular and unique to each individual.

Where the Yellow Is

On Hume's question itself I come to a different result from Carroll's, because my argument is about the premises of a derivation, and Carroll's argument is about its steps. My book on the act of comparison gives the reason. I state my result in four steps. First, every premise was found: before anyone could reason from a sentence, someone or some instrument found what the sentence says. Second, every act of finding is an act of comparison: one thing set beside another, and found the same or different. Third, every act of comparison has a side it is made from, the subjective, and the subjective side always values: in the chapter's words, the subjective side weighs "what to attend to, what matters, what is relevant". The fourth step follows from the first three: "there are no premises that contain only is, and the separation the rule presupposes is the separation that the structure of comparison precludes". Hume's rule presupposes premises that say only what is, set apart from every value, and no premise found in a comparison is set apart from every value. I also state the point more briefly: "A comparison that evaluates nothing is not a comparison". The four steps apply to the thermostat on the wall as well: a thermostat compares the warmth of a room with its setting, and it has a stance, a side the thermostat compares from. A thermostat weighs the room against its setting without feeling anything, while a cook weighs and also cares.

Look at the kitchen again. "The egg is fresh" sounds like a sentence with no ought in it. But the cook found the freshness by setting the egg in water and watching whether the egg sank, and the water test was chosen because someone wanted to know whether the egg was good to eat. The

test in the bowl of water is a comparison. A value made the test worth doing: someone wanted an egg that is good to eat. A value is at work at the crack as well. The cook looked at the crack to see whether the yolk had broken, because the friend likes the yolk whole. The fact and the caring came in one act, and nobody found the freshness first and added the caring afterward. So the meaning of the breakfast is made, as Carroll says, and the meaning is made inside an act of finding that already had a side it was made from, and a caring in it, before any meaning was added.

So on Hume's question Carroll and I part at the premises. Carroll looks at each claimed derivation of ought from is, finds the yellow (the hidden ought) slipped in at one of its steps, and concludes that no such derivation can be made. In the painter's picture, my account is about the paint the painter starts from: every premise is a pot of paint, and every pot already has a little yellow in it, a value. The yellow in every pot is a weighing of what matters, made from the side the comparison is made from; no ought is deduced from it, so Hume's rule stands, and what fails is the separation of fact from value that the rule presupposes. I agree with Carroll that no chain of reasoning from bare facts ends in an ought. But on my account there are no bare facts to start a chain from. Hume's rule says what cannot follow from premises of pure fact. Every premise is found in a comparison, so no premise of pure fact is ever found. The parting at the premises follows from the one place where the two accounts part: what lies at the ground of physics, the bottom of the world, the thing every other description describes. For me the ground of physics is the act of finding, and so every fact comes to anyone who reasons from the fact inside an act of finding, made from some side.

I give a second argument that fact and value come together. The second argument begins from what is. Any being that chooses is joined to its world by a channel: the chooser takes things in through the channel and sends things out through it, as the cook takes in the look of the egg and sends out a breakfast. The channel is more basic than either end of it: the channel is more primary than the things it connects. A channel is good when it keeps two things. First, what goes in at one end comes out at

the other end the same, which is symmetry. Second, the channel joins its two ends without sudden breaks, which is continuity. Symmetry and continuity become defining principles of goodness as structural conditions of effective interaction, not as moral rules imposed from outside. In the second argument the ought is not deduced from the is. The same two conditions are an is when you look at what makes a channel work, and they are an ought when you look from the side of the one who chooses, who must keep symmetry and continuity if the choosing is to go well.

I give a plain picture of a channel kept whole: a friendship in which you can say anything. The section "Relative, Not Arbitrary" describes the friendship: "What you mean arrives as you meant it, and is not twisted on the way. And the answer you get is in proportion: a small worry met gently, a great grief met fully". The same section matches the friendship to the two conditions: what is meant "arrives the same, which is symmetry", and the exchange that carries the meaning joins "without a break, so that the response follows in proportion to what changed, which is continuity".

Made, and Not Arbitrary

Are right and wrong, then, made by people too? What Carroll calls "ethical systems" are what I call morality: codes of conduct that, in Carroll's words, "are constructed by us human beings". I agree with Carroll that people make every code of conduct, and on my account something that no one makes lies beneath the codes. I keep the word *ethics* for the part that no one makes: the conditions under which choosing goes well, for any chooser at all. So Carroll and I use one word in two senses. Carroll's "ethical" names what I call morality, and my "ethics" names only the unmade principles. In my pages on ethics and morality I define ethics as the study of "the principles of effective choice, internal to oneself". The same pages set the principles of ethics apart from the rules of any one community. So the principles are the same for every chooser and in every culture. I also say whom the principles of ethics apply to: "The absolute

principles of ethics are common to all of consciousness (all individuals)". The basis of value is not arbitrary. The principles of ethics are not relative to a person or to a culture. On right and wrong, then, I part from Carroll's Humean constructivism at what lies beneath the rules: what each community builds differs, and the principles beneath every community's building are the same for every chooser. The parting on right and wrong follows from the one place where the two accounts part: what lies at the ground. For me the ground of physics is the act of comparison. I match the two principles of ethics to symmetry and continuity, two characters of every comparison.

So the two principles of ethics are what a good channel keeps. I list what each principle includes. The symmetry principle includes the golden rule (treat others as you would have them treat you), Kant's categorical imperative (act only by a rule you could will everyone to follow), equality before the law and the integrity of communication. Each rule of the symmetry principle requires the same treatment for the same case, wherever the case comes up. The continuity principle includes the prohibitions against violence, against weapons, and against the involuntary creation or destruction of life. Each of the three prohibitions forbids a sudden break forced into another's life.

There is a reason for what Carroll observed about moral philosophies, and the reason is the Incommensuration Theorem, which we met in this book's Chapter 2 as a fact about every comparison. Symmetry, continuity, asymmetry and discontinuity are the four characters of a comparison. In my sense, as in Carroll's, an "ethical system" is a code of conduct. When the theorem is applied to choosing, it gives this result: "no ethical system can be both perfectly rule-based (symmetric: the same rule applies in all contexts) and perfectly responsive (continuous: no abrupt disconnection from the specific situation)". Carroll observed that a moral philosophy applied universally often ends in conclusions that sit uneasily with its premises (*The Big Picture*, Chapter 48). In the same chapter he suggested that the moral code that fits most people best may take "bits and pieces from all of them", that is, from all the moral approaches he has set side by

side. I give the reason that no single system can be made perfect. A perfect system would be both a rule that remains the same everywhere and a response that follows each case without a break, and by the Incommensuration Theorem no system can be both. So ethics and morality are both needed. Ethics is the principles that apply to every chooser, and morality is the rules a community makes for its own time and place.

Why should a chooser keep principles that no one can enforce? I answer from what choosing is: "A chooser who keeps the channel between himself and his world whole is doing what he most deeply wants, and a chooser who breaks it is not disobeying a law but failing at his own purpose, in a way no other gain can make up for". The breakfast shows a chooser failing at the chooser's own purpose, at the scale of one kitchen. Suppose the cook served the friend a burnt egg on purpose. The cook would break no rule written anywhere. But the cook would fail at the very purpose of the morning's cooking. So a meaning can be made and still not be arbitrary: a chooser makes the meaning inside a relation to the world, and no chooser made the conditions under which that relation goes well.

Carroll and I also differ over one of Carroll's sentences. The sentence is "The universe doesn't care about us, but we care about the universe" (*The Big Picture*, Chapter 49). Everything has some significance, some degree of meaning. Everything (every perception and expression) has (cannot not have) significance (some degree of meaningfulness). I give the shortest argument that everything has some meaning: "either everything is meaningful or nothing is. We know that there is some meaningfulness. So everything is meaningful".

Why must meaning belong to everything or to nothing? I state the first premise, all or none, for everything. To negate the significance (the meaningfulness and value) of any life, is to negate the significance of all life. The breakfast pictures the sentence about any life and all life. The breakfast matters because the friend matters. The friend matters to the cook through years of meals, talk and help given both ways. And the egg in the pan was laid by a hen that mattered to the farmer who kept her. Suppose you denied that the friend's life has any significance. Then the breakfast

would lose its significance along with the friend's, and so would the morning and the cook's years of care. The picture shows the loss spreading from one friend through one kitchen. The sentence states that the loss spreads from any one life to every life, with no exception. The second premise is the breakfast itself: there is some meaning, because the cook cares and the friend is fed. So everything has some significance, some degree of meaning. For our question, the conclusion means that the cook makes the meaning of the breakfast in a world where the friend and the hen already mattered before the breakfast was made. The parting over Carroll's sentence follows from the one place where the two accounts part: what lies at the ground. For me the ground of physics is the act of finding, and every act of finding is made from a side that values, so some value is present wherever anything is found.

I also agree with Carroll that living creatures have a place in meaning that no other thing has. Meaning, known from inside, lives only where life is, because a living being is where choices are gathered into a self that knows them from inside. So caring creatures are where meaning is known from inside and where meaning is made, as Carroll says. For Carroll the rest of the world matters only because caring creatures care about it. On my account the rest of the world has some significance of its own.

Carroll also takes up the largest question, why the universe exists. Carroll asks whether the universe could exist by itself, with no outside help, and he answers plainly: "sure it could" (*The Big Picture*, Chapter 25, "Why Does the Universe Exist?"). In the same chapter he writes of the best explanation for the existence of the universe: "The answer is certainly 'We don't know'". And he allows that the asking may stop at that answer: "There may be no ultimate answer to the 'Why?' question. The universe simply is, in this particular way, and that's a brute fact" (*The Big Picture*, Chapter 25). I agree with Carroll that physics answers how a universe develops, and that physics leaves untouched the question why the universe exists. I also give a reason why there is something at all, and the reason begins with contact. The cook may study the egg closely, or barely glance at the egg from across the room, so the contact between the cook and the

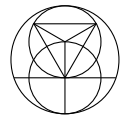
egg can be strong or faint. Contact between the one who finds and the world can be as faint as you like, and it never falls to none. Now think of the zero mark on a ruler. The zero mark is where the ruler begins, and it is no length at all; the shortest piece of string you could lay on the ruler still reaches past it. Contact can be as faint as you like, and no contact at all is the edge of the range, as the zero mark is the edge of the ruler: it is not a degree of contact that anything can have. There is therefore always at least this: the irreducible fact of contact between the one who finds and the world, and that is why there is something rather than nothing. So on the question why the universe exists, the two accounts part. On Carroll's side the answer is "We don't know", and there may be no answer beyond the brute fact that the universe simply is. On my side there is a reason: the contact that never falls to none. The parting over why the universe exists follows from the one place where the two accounts part, what lies at the ground: for me the ground of physics is the act of finding, and every act of finding has contact between the one who finds and the world. For our question, the reason means that every meaning the cook makes is made inside the contact between the cook and the world, and the contact was there before any meaning was made.

What the Breakfast Holds

The breakfast is made, and the cook made it. Nothing in the laws of physics required the breakfast, and Carroll is right that the meaning of the breakfast came from someone who cared. And the caring did not arrive late, laid over a bare fact about an egg. The caring was in the act of finding from the first test in the bowl of water, because every act of finding is made from a side that weighs what matters. The friend eats the breakfast and receives what the cook meant, as the cook meant it. So the channel between the cook and the friend is kept whole.

We now know that meaning is made, as Carroll says, and that meaning is not arbitrary: every act of finding is made from a side that already values what is found, and every meaning is made inside an act of finding. On my account, the world in which a meaning is made already has some meaning in everything, and the one who finds is always in some contact with the world. What we now know bears on the question this book has carried from its first page: what lies at the ground of physics? If the ground of physics is the act of finding, then value is no stranger added afterward to a world of bare facts: value is in the ground with the facts, joined to the facts in every act of finding.

What, after twelve chapters, do you now hold, as a reader of Carroll, that you did not find in his books?



Chapter 13

What Do You Now Hold?

There is an egg on the counter again, beside a bowl. At the end of this walk through the book, what do you now hold that you did not find in Sean Carroll's books?

Carroll steps into a kitchen near the start of the Epilogue of *From Eternity to Here*, his book on why time runs one way. He says there that his book has been about eternity and also about here, and that the puzzle of time's one direction begins close at hand: "it's in our kitchens, every time we break an egg" (*From Eternity to Here*, Chapter 16, Epilogue). From that kitchen Carroll reasons outward in his book, to the beginning of our universe and to what may have come before the beginning. I took Carroll's egg and gave the egg one more job: the egg, cracked once by a cook on the rim of a bowl, is the picture of one act of finding. An act of finding is one act of comparing two things that gives a verdict, same or different. The act is a single event, made from one side (the cook), about something (the egg), with one outcome (this crack), which cannot be undone. Together, the twelve chapters before this one describe one picture, with the egg at its center.

What Carroll Built

We begin with Carroll, in his own words.

Carroll sets out the laws that govern everyday things. He calls those laws the *Core Theory*: the quantum theory of the fields of the Standard Model, together with gravity where gravity is weak. The Standard Model is the theory that describes the particles and forces of ordinary matter. Carroll writes of the laws of the Core Theory: "The laws of physics underlying everyday life are completely known" (*The Big Picture*, Chapter 23, "The Stuff of Which We Are Made"). Carroll describes the quantum world with no special rule for measurement: "The world is a wave function, nothing more nor less" (*Something Deeply Hidden*, Chapter 2, "The Courageous Formulation"). The *wave function* is Carroll's name for the complete quantum description of a system, and he holds that the description is the system itself. Carroll also calls the wave function the quantum state. He insists that the one direction of time is a fact that needs a reason, and he looks for the reason at the beginning of the universe, where he asks: "Why did the universe have a low entropy near the Big Bang?" (*From Eternity to Here*, Chapter 8, "Entropy and Disorder", section "The Past Hypothesis"). *Entropy* is a count of how many arrangements of the small parts of a thing look the same from outside. And Carroll holds that meaning and purpose are made by people, in the world that the laws describe: "it's up to you, me, and every other person to create meaning and purpose for ourselves" (*The Big Picture*, Chapter 45, "Three Billion Heartbeats").

I set a second body of work beside Carroll's: the physics collection of the Immanent Metaphysics, a set of books that work out physics from the act of knowing. I record the positions that Carroll and I share outright, in my chapter on Carroll, "The Wave Function and the One Who Finds". The first of three shared positions is that every law holds within a *domain*: the range of cases in which the law holds (this book's Chapter 1). Nothing known implies that space and time come in least steps, a smallest length or a smallest moment (this book's Chapter 7). And the one direction of time needs a reason (this book's Chapter 8).

I start from the act of comparison (setting one thing beside another and finding the two the same or different), and from that start I also derive results that Carroll states in his own words. Carroll writes, "There is only one world, the natural world", and "There are many ways of talking about the world" (*The Big Picture*, Chapter 2, "Poetic Naturalism"). I agree: there is one world, and there are many good descriptions of the one world, each fitted to its own domain (this book's Chapter 1). This book's Chapter 1 also showed where Carroll and I part (come to different results): on what the many descriptions describe. Carroll holds that the laws of everyday life are completely known, and I grant his claim as he states it and adds no force and no field to those laws (this book's Chapter 3). Carroll holds that meaning is made by people, and I agree that meaning is made (this book's Chapter 12).

The One Who Finds, at the Ground

Where, then, do Carroll and I part? The two part at one place, and every other difference between them follows from it. The one place where Carroll and I part is the ground: what lies at the bottom of physics, the thing every other description describes. For Carroll the ground is a quantum state evolving by one rule, with every outcome occurring. For me the ground is the act of *comparison*: setting one thing beside another and finding them the same or different in some respect. Everything physics has found was found in an act of comparison, and every act of comparison has *one who finds*: whoever or whatever makes the comparison, a person or an instrument, a cook or a thermometer. What I add to Carroll's picture is the one who finds, at the ground. I have set out the one who finds for Carroll's readers.

This book's Chapter 2 showed what finding is, with the cook's oldest test. Set an egg in a bowl of water: a fresh egg sinks, and a stale egg floats. You set what this egg does beside what a fresh egg does. You find whether this egg does the same as a fresh egg, or something different. Every

way of knowing is made of comparing, as the test of the egg is. This book's Chapter 2 also counted the six parts that every comparison has, in three pairs. The first pair is sameness and difference, the two possible verdicts of a comparison. The second pair is what the comparison is about and what makes the two things compared two. The third pair is the side the comparison is made from, beside the side the comparison is about. Every comparison also has a limit, which the *Incommensuration Theorem* sets. Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Symmetry is sameness kept across a change, and continuity is no gap and no least step. Asymmetry is the opposite of symmetry: nothing kept the same. Discontinuity is the opposite of continuity: a jump, with nothing between before and after. To apply one of the four (symmetry, continuity, asymmetry or discontinuity) fundamentally is to hold that one exactly, at the foundation of a description, with no exception at any scale. To apply two of the four simultaneously is to hold the two together in one and the same comparison. The Incommensuration Theorem is a structural fact about the nature of knowing. Carroll pictures a person's beliefs as a world held together by the way the beliefs fit one another. He calls the picture the *planet of belief*. This book's Chapter 2 set the act of comparison beneath Carroll's planet of belief. In Carroll's books a *credence* is how strongly a belief is held, and every belief on the planet of belief, whatever its credence, was formed from acts of finding. Every act of finding is a comparison, with the six parts of every comparison and the limit that the Incommensuration Theorem sets. So the act of comparison is the *floor* under every planet of belief: something under every belief that is not one more belief. The act of comparison is both the floor of knowing and the ground of physics: one act, named once from the side of knowing and once from the side of physics.

On how sure we can be, Carroll and I part on one question: whether any knowledge is certain (this book's Chapter 2). Carroll holds that all our knowledge comes with a measure of doubt: "We can always be mistaken" (*The Big Picture*, Chapter 15, "Accepting Uncertainty"). I establish

one kind of knowledge that is certain: knowledge of the structure that every act of finding has, which is its six parts and the limit that the Incommensuration Theorem sets. We can always be mistaken about what we find, but we cannot be mistaken that finding has the six parts and the limit, because to be mistaken is to have compared. The parting on certain knowledge follows from the one place where Carroll and I part: what lies at the ground.

One Crack, and the Many

The egg, the bowl and the cook are made of particles that all obey the Core Theory (this book's Chapter 3). The *content* of a comparison is what the comparison is about. The kinds of particle in the Core Theory are the kinds of content. The known laws fix a great deal: over many eggs, struck by many cooks, the laws say how shells break, where cracks tend to start and which way the cracks tend to run. But the laws leave open which crack this egg gets (this book's Chapter 4). When the laws and the quantum state are known completely, the knowledge fixes how the outcomes are spread over many trials. Complete knowledge still leaves open which outcome a single trial gives. Now think of a whole world fixed in every detail by one law. The world would need one law that carries the complete state of the world at one moment into its complete state at the next moment. The one law would hold the same everywhere and always, which is symmetry made perfect. The world would also need its complete state given at every point, with no gap, which is continuity made perfect. The Incommensuration Theorem shows that symmetry and continuity cannot both be made perfect together, so I keep every known law and remove one step from Carroll's account: the step from a complete law to a complete world. The removed step runs from knowing every law to holding that the laws fix every single event.

Carroll defends the *many-worlds account*: the physicist Hugh Everett's account of the quantum state, in which the state never collapses to a single outcome and every outcome occurs. On the many-worlds account every way the shell could break happens, each way on its own *branch* (this book's Chapter 5). *Decoherence* is the spreading of the difference between the alternatives (the ways an outcome could go) into their surroundings, and after decoherence the alternatives no longer interfere: they no longer add to each other in some places and cancel each other in others. A branch is Carroll's word for one of the alternatives that decoherence separates, each branch with its own outcome. I grant part of Carroll's claim that the world is a wave function. I say which part I grant. I give two everyday words exact senses. To *exist* is to stand as structure, the same in every instance, as a law or a pattern does. To be *real* is to be given in an actual act of comparison. This book's Chapter 5 named the quantum state of the whole universe the universal state. The universal state exists as structure, branches and all. For every comparer, one outcome is real, the one found. A comparer, in that sentence, is the one who finds. For the cook, the one outcome found is this crack. So I grant that the universal state exists, branches and all, as the complete description of what could be found and with what odds, but I do not grant that the universal state is all that is real.

On my reading, the many-worlds account leaves out three things, and leaves the three out together: the one who finds, the single random outcome, and the one direction of time. I call the three together the *bound triple*. The one who finds is the subjective, the single random outcome is hard randomness, and the one direction of time is the arrow of time. The three are distinct, they cannot be separated, and every way of reading quantum mechanics includes all three or none of the three. The *subjective* is the side a comparison is made from. At the bowl, the subjective is the cook. *Hard randomness* is an outcome that no law and no knowledge of what came before it, however complete, fixes.

This book's Chapter 6 asked next where the odds of each outcome come from. The *Born rule* is the rule that the probability of an outcome is the square of the size of its part of the quantum state. Carroll sets out two derivations of the Born rule within the many-worlds account. He made the first one with the philosopher Charles Sebens, and the derivation of Sebens and Carroll uses *self-locating uncertainty*. After the branching and before looking, an observer does not know which branch the observer is on, and a rule of good reasoning then fixes how much weight the observer should give to each branch. I derive the Born rule through *Gleason's theorem*. In 1957 the mathematician Andrew Gleason proved what is now called Gleason's theorem: any way of assigning probabilities to the outcomes of quantum measurements that satisfies a few plain conditions must be the Born rule. The two derivations give the same weights: the derivation of Sebens and Carroll, and my derivation through Gleason's theorem. In my terms, the observer of Sebens and Carroll is a comparer with a *stance*. A stance is the side that a comparer compares from. The observer is before an act of finding, facing a range of what the observer could find. Carroll, following Everett, set the one who finds aside when he let every outcome occur. So the observer of Sebens and Carroll is the one who finds. The one who finds comes back into the many-worlds account at the step where the weights are needed. I start from the one who finds, so I have nothing to bring back.

The Kitchen, the Arrows and the Epochs

The cook finds the egg on a counter, in a kitchen, on one morning, but the counter, the kitchen and the morning are no part of the egg (this book's Chapter 7). *Entanglement* is a joint state of two things whose parts cannot be described one by one. In Carroll's program of research, space itself is drawn out of a quantum state, out of the entanglement within the state. Carroll calls his proposals for drawing space out of a quantum state "new, exciting, and admittedly tentative" (*Something Deeply Hidden*, Pro-

logue). Working on that program, Carroll comes to the equation of the physicists John Wheeler and Bryce DeWitt for the quantum state of the whole universe. The equation of Wheeler and DeWitt has no time in it, and physicists call the missing time the problem of time. I hold that space and time are the setting of every quantum state. I call space and time, taken as that setting, the arena. The *arena* is space and time joined, with the shape that general relativity (Einstein's theory of gravity) gives space and time, taken as the setting in which every finding is made. In this book's Chapter 7 I identified gravity with the arena. So gravity is the arena. The *context* of a comparison is what makes the two things compared two (this book's Chapter 2). The arena is the context of physics. Two findings are two because one is made here and the other there, or one now and the other then. I state two things of the arena, Proposition 6.3: "(i) The arena has no least step: no least length, area or duration. (ii) Nothing is a quantum of the arena itself, at any scale". On my reading, the equation of Wheeler and DeWitt has no time because space and time have been put inside the quantum state. Space and time are the setting in which a quantum state changes. When the arena is kept as the setting of every quantum state, the arena keeps its time.

This book's Chapter 3 set Carroll's list of what there is beside my list. Carroll counts gravity among the fields of the Core Theory, with a particle of its own, the graviton. He also allows for further kinds of particle that touch ordinary matter so weakly or so briefly that only very costly apparatus could find them. I hold my list closed: the kinds of particle of the Standard Model are the whole of the content, everything that can be found within the arena. Because gravity is the arena, gravity is context, and gravity has no particle of its own. On the list of what there is, Carroll and I part on two points: what gravity is, and whether the list holds any kinds of particle beyond those of the Standard Model. The parting on the list follows from the one place where Carroll and I part: what lies at the ground. For the egg, the bowl and the cook, Carroll's list and my list are the same list.

The crack cannot be uncracked (this book's Chapter 8). Carroll explains the arrow of time with his omelet, the egg that never unscrambles. On his account the arrow of time is the statistical one-way drift of things made of very many parts toward arrangements of higher entropy. The drift needs a beginning of low entropy. Carroll calls the assumption of a low-entropy beginning the *Past Hypothesis*, a name he takes from the philosopher David Albert. I keep Carroll's arrow and calls it the *entropic arrow*. I also hold that there is a second arrow, in the crack itself. Every act of finding runs one way: first the range of what could be found, then the act, then the record, and the record shapes the next range. I call the one-way order of range, act and record the *structural arrow*. The structural arrow needs no low-entropy beginning. The entropic arrow and the structural arrow point the same way, because making a record raises the entropy of whatever holds the record. Time runs one way because every act of finding runs one way. Because the structural arrow gives time its direction, I divide the Past Hypothesis into two parts: its use as the source of the direction of time, and its claim about the beginning. The structural arrow gives time its direction without the Past Hypothesis, so the Past Hypothesis is not needed as the source of that direction. The rest of the Past Hypothesis is a claim about the beginning, that the universe we see began low in entropy, and I give the reason for that low beginning at a passage that it calls the turn (this book's Chapter 9).

The egg came from a hen, the hen from sunlight, and the sunlight from an orderly beginning (this book's Chapter 9). Carroll and the physicist Jennifer Chen proposed that a quiet, nearly empty universe gives rise, now and then, to new universes like ours. On their proposal entropy can grow without end, and the arrow of time arises within the family that the new universes make. Carroll states how he holds the proposal, in his own words: the proposal is "a specific model of which I am personally fond", and "The details of this model are highly speculative" (*From Eternity to Here*, Chapter 16, "Epilogue", section "What's the Answer?"). An *epoch* is a stretch of the universe's history that opens with a hot beginning, like the one we observe at the start of ours. My account has the same shape as the

proposal of Carroll and Chen: our epoch is one of a sequence with no first member, and the arrow of time arises within the sequence. On my account, each epoch begins at *the turn*: the passage from the end of one epoch to the beginning of the next. The turn comes at the end of a black-hole interior. No comparison registers the turn as an event. Every count of disorder belongs to the context in which the count is made, and at the turn a new context begins, so no count carries across from one epoch to the next. Astronomers observe that the matter at the beginning of our epoch had a high entropy, while the geometry of the new space and time that forms at the turn has no past of its own to carry lumps from, so the new geometry begins even. The even geometry is the low entropy of gravity at the beginning. I part from the proposal of Carroll and Chen on two points, and the parting follows from the one place where Carroll and I part: what lies at the ground. The first point is whether entropy accumulates across the whole sequence: in the proposal of Carroll and Chen entropy grows without end, and on my account no count of entropy carries over from one epoch to the next. The second point is whether our epoch began with *inflation*, a proposed very early period of extremely fast expansion, and in the proposal of Carroll and Chen each new universe passes through inflation. My account needs no inflationary phase, and it excludes a long one.

The Cook

The cook decides to crack the egg now (this book's Chapter 10). Carroll answers the question of free will in two halves, in *The Big Picture*, Chapter 44, "Freedom to Choose". In a description of people as particles, free will has no place: "Of course there is no such notion as free will when we are choosing to describe human beings as collections of atoms or as a quantum wave function". In a description of people as people, the concept of choice is useful. Carroll keeps choice in that description. I place choice in the *triple of the real*: causality, change and choice. Every real event has all

three, each in some measure. A *law of the many* is a law of what happens over many trials, and choice is the member of the triple of the real that a law of the many does not fix. Choice has its place at the single outcome that the laws leave open. Choice is no extra force pushing on the particles of a brain. When you decide to crack the egg now, you choose, and your choosing is at that single outcome.

This book's Chapter 11 asked who is doing the finding. Carroll writes: "Consciousness seems to be an intrinsically collective phenomenon" (*The Big Picture*, Chapter 42, "Are Photons Conscious?"). He holds that consciousness stays within the laws he has set out: "Consciousness isn't an illusion, but it doesn't point to any departure from the laws of physics as we currently understand them" (*The Big Picture*, Chapter 40, "The Hard Problem"). This book's Chapter 11 set a camera on the counter and a thermometer in the pan, beside the cook. I hold that the cook, the camera and the thermometer all find: "Instruments compare as surely as minds do". Consciousness is the bound triple taken as one act: the arrow of time, hard randomness, and the subjective happening together. A *first-person point of view* is an act of finding as it is had from inside, by the one who makes it, as your own seeing of this page is had by you. The identification of consciousness with the bound triple applies wherever an act of perceiving is had from a first-person point of view (this book's Chapter 11). In the cook, acts of finding are had from a first-person point of view and are gathered into one self, and the three members of the bound triple, taken in the cook as one act, are what the word 'consciousness' names.

The cook makes the breakfast for someone (this book's Chapter 12). Carroll holds that meaning is built by people. He also holds that knowing how the world works does not tell us what to do with that knowledge: "science describes the world, but what we're going to do with that knowledge is a different matter" (*The Big Picture*, Chapter 50, "Existential Therapy"). I agree that meaning is made. But I hold that meaning is not arbitrary, because meaning is made within the structure that every act of finding already has. I state the reason. Every act of comparison has a side that the comparison is made from, and that side always values. The side

weighs "what to attend to, what matters, what is relevant". The same section states the reason more briefly: "A comparison that evaluates nothing is not a comparison".

One Picture

Set the answers of the twelve chapters side by side, and the answers make one picture. Every difference between Carroll and me came from one place, what lies at the ground: for Carroll the quantum state evolving by one rule, and for me the act of comparison, with its one who finds. Carroll and I part on ten subjects. Some of the ten hold more than one parting, as I show in the chapters. Every one of the ten partings came from what lies at the ground. The ten subjects are how sure knowing can be, the list of what there is, the single outcome, where the odds come from, the arena, the entropic and structural arrows, the sequence of epochs, choice, who does the finding, and meaning. I keep together the three things that the many-worlds account leaves out together: the one who finds, the single random outcome, and the one direction of time. The three are three aspects of one act: comparison proceeding in a direction, registered by a perspective, with an outcome not fully determined. In plain words, someone finds, one thing is found, and the act of finding runs one way. An egg cracked once by a cook shows all three at once. So the one picture of the egg could serve every chapter of this book.

I keep Carroll's quantum state, and I say in two sentences what the quantum state does and what comes before the quantum state: "The state describes, exactly and usefully, what any comparison can find and with what odds. The finding itself is the act, and the act comes first". The Core Theory stays as Carroll states it, with its domains, tested to the precision of every test made of it. I add to Carroll's picture at two places: at the ground, and at the single act of finding.

The Egg on the Counter

You now hold, as a reader of Carroll, that everything Carroll established still holds, and that beneath all that Carroll established, at the ground, is the one who finds. You can be sure beyond every doubt of the structure of every act of finding: its six parts, and the limit the Incommensuration Theorem sets. For the one who finds, one outcome is real: the one found. Sebens and Carroll derive the weights of the outcomes, and I derive the same weights starting from the one who finds. When the arena is kept as the setting of every finding, the arena keeps its time. Every act of finding runs one way, and that one-way order is the arrow of time. Each epoch begins with the geometry of its space and time even. Choice has its place at the single outcome that the laws leave open. Instruments find as surely as minds do. And meaning is made within the structure that every act of finding already has. For a reader of Carroll, the whole of my answer to Carroll's books is in two things. The first of the two things is the quantum state as Carroll writes it, kept to describe what can be found in any comparison and with what odds. The second of the two things is the one who finds, set at the ground beneath the quantum state. The one who finds was in the kitchen all along: the cook at the bowl.

Carroll found the puzzle of time where we live, every time we break an egg. Look at the egg once more. A cook lifts the egg from the counter and brings it down, once, on the rim of the bowl. The shell parts along one line, and the cook sees the shell part. The crack happened once, to this egg, from this side of the bowl. The crack will never be uncracked.

Appendix A: What the Coming Measurements Will Find

In this appendix I gather the measurements still to come that bear on the chapters of this book. The collection is the set of books of physics from comparison that this book draws on. For each measurement I say what the measurement will measure, and then what the account of the collection says the measurement will find. On 1 November 2026 the collection's predictions were gathered into one dated public record, published by UVSM Publications under the title *The Registered Predictions of Physics from Comparison; A Dated Record, 1 November 2026*. The record's DOI is https://doi.org/10.67291/RQQS3025. A DOI is a permanent address that leads to a published work. *Side by Side, Volume One*, a book of the collection, reproduces the whole record as its Appendix B, "The Sealed Record". In the record each prediction is stated in plain words and given a number, with the book, chapter and section of the collection where the argument for the prediction is set out. The record states why the record carries a date: "This record carries the date, so that a later reader can set each prediction beside the measurements made after it".

The record states a rule for when a measurement counts as a result, and calls the rule the 'rule of decision': "A result counts when it is established at five standard deviations by at least two independent kinds of measurement". A standard deviation is the measure of a measurement's own uncertainty. Five standard deviations is the usual threshold for a discovery in physics. When two kinds of measurement rest on different instruments and on different physics, the two kinds are independent. The next sentence of the record is about a smaller difference: "A measurement that differs from a prediction by less is a measurement of interest, and the record reports it as such".

A second dated record, of 4 November 2026, sets the predictions of the first record beside the account of Sean Carroll. Carroll is the physicist whose books this book sets beside the collection. The second record has the title "The Registered Predictions Beside Sean Carroll". *Side by Side, Volume One* prints the second record too, as its Appendix K. In this appendix I call the second record 'the record beside Carroll'. The record beside Carroll quotes each prediction from the record of 1 November, and beside each prediction it states what Carroll's account predicts, from Carroll's own published work, cited by author and year. Where the record beside Carroll states Carroll's expectation for a measurement, the entry below states that expectation as well, in the record's terms. Each entry below is placed under the chapter of this book that the entry bears on. When a prediction bears on several chapters, the prediction is quoted under the chapter it bears on most directly, and under each of the other chapters it is named, together with what it says.

Chapter 1: What Is the World Made Of, at Bottom?

The question of Chapter 1 is what lies at the ground of physics, the thing every other description describes: a quantum state, as Carroll holds, or the act of finding, a single act of comparison that gives a verdict, same or different. The record beside Carroll states Carroll's position first: "For Carroll the ground is a quantum state evolving by one rule, and the many descriptions of the world are ways of talking about it". The record then states the collection's position: "For the collection the ground is the act of comparison, and its order of depth runs metaphysics, physics, computation, mathematics". The record calls each position "a claim about the act of knowing". The record also says where a claim of that kind is held: "a claim about the act of knowing is held in that act". The two accounts of the ground have consequences that can be measured. I enter each consequence under the chapter whose subject it is: the known laws under

Chapter 3, the outcomes of a measurement under Chapters 5 and 6, and the arena of space and time (the setting in which every finding is made) under Chapter 7.

Falsifiability is the test of a claim by a measurement whose outcome could contradict the claim. The record beside Carroll also sets the two accounts side by side on the claims to which falsifiability applies. In Carroll's account a branch is one of the alternatives into which a measurement divides the quantum state of the world. Each branch has its own outcome. The record states Carroll's position: "Carroll relaxes falsifiability for claims about the world, so that unobserved branches can count as physics". The record states the collection's position: "the collection keeps it for claims about the world, states its predictions there, and does not ask its claims about the act of comparison to count as empirical science". The predictions in this appendix are the collection's claims about the world.

Chapter 2: How Sure Can We Be of Anything?

The question of Chapter 2 is how sure knowing can make us. In Chapter 2 we read Carroll's answer to three old doubts, the doubts of the skeptic, the dreaming philosopher and the deceiving demon, which Carroll gives in his book *The Big Picture*, Chapter 11, "Is It Okay to Doubt Everything?". In Chapter 11 of *The Big Picture*, Carroll takes up one more doubt: that you, and everything you have ever experienced, are a simulation run by some higher intelligence, as a computer runs a simulation of a world. Carroll answers: "Sure, it's possible". Carroll adds that the doubt of a simulation is not "strictly speaking, a skeptical hypothesis: there is still a real world, presumably structured according to laws of nature".

A simulation of the kind Carroll describes is a computation: the run of a computer program. So the collection has one answer to both thoughts: that the world is a computation, and that the world is a simulation. One part of the collection's answer can be measured. A computation runs by a fixed rule that moves in least steps, as a computer program does: each

step is a smallest step, with nothing between one step and the next. A rule that moves in least steps is called discrete. The measurement is a count of the outcomes of a quantum process over many runs. Physicists call such a count the statistics of the process. When someone offers a particular rule as the foundation of physics, the count that tests the rule must be fine enough to reach below the rule's least step, that is, to show differences smaller than one step of the rule. The account says what the measurement will find, in Prediction 29 of the record, "A discrete rule shows itself to a fine enough measurement": "For any one fixed discrete rule offered as the foundation of physics, a sensitive enough measurement, reaching below the rule's least step, tells the rule's statistics from the world's". *Side by Side, Volume Two* contains a record beside Stephen Wolfram, who offers a computational rule as the foundation of physics. The record beside Wolfram states the measurement in the same terms: "A measurement of the statistics of a quantum process that reaches below the least step of the rule offered".

Chapter 3: Are the Laws of Everyday Life Completely Known?

The question of Chapter 3 is whether the laws that govern everyday matter are completely known, as Carroll holds. Carroll states his claim in *The Big Picture*, Chapter 23, "The Stuff of Which We Are Made": "The laws of physics underlying everyday life are completely known". Carroll states the scope of the claim in his book *Quanta and Fields*, Chapter 12, "Atoms", section "The Physics Underlying the Everyday World": "There absolutely could be new particles and forces out there, but they don't interact noticeably with the protons, neutrons, and electrons of which we are made". The Core Theory is Carroll's name for the known laws of the particles and forces of everyday life, with gravity where it is weak. The record beside Carroll says: "The collection grants the core theory as he states it and adds no force or field to it". The record also says that the collection's list of the

kinds of particle "goes further than the core theory, which leaves open kinds that do not touch everyday matter". The entries that follow bear on the laws of everyday matter and on the collection's list of the kinds of particle.

Prediction 4, "Free fall is universal and nothing shields gravity", will be measured by three kinds of experiment. The equivalence principle is the principle that every body falls in the same way, whatever the body is made of. The first kind of experiment compares how two bodies of different materials fall side by side: the proposed Satellite Test of the Equivalence Principle (STEP) would carry test masses of different materials in orbit and compare their fall to about one part in a billion billion. The second kind drops antimatter, as experiments at CERN, the European laboratory for particle physics near Geneva, do with antihydrogen, an atom made of an antiproton and a positron, which are the antiparticles of the proton and the electron. The third kind searches for any arrangement of matter that blocks or weakens the pull of gravity. The account says what all three kinds will find: "The Eötvös parameter, the fractional difference in the free fall of two materials, is zero for every pair of materials at every precision. Antimatter falls with the acceleration of matter. No arrangement of matter shields gravity, and nothing will be found that gravity leaves untouched". The record beside Carroll states that Carroll's expectation is the same, for gravity as weak as gravity is in everyday life: "Gravity in that regime is general relativity, in which free fall is universal, and on his account no new force acting on everyday matter will be found".

Prediction 30 has the title "No kind of particle beyond the closed list". The closed list is the collection's list of the kinds of particle: the kinds of the Standard Model, with a right-handed neutrino in each generation. The Standard Model is the established theory of the particles and of every force but gravity. A generation is one of the three repetitions of the matter particles. Each generation has two kinds of quark (the particles that make up protons and neutrons), a charged lepton (the electron, or one of its two heavier cousins) and a neutrino (a very light particle with no electric charge). The right-handed neutrino is a second component of each neut-

rino. That second component lets the neutrino have its mass. Prediction 30 will be measured by the searches for new particles at colliders (machines that smash particles together at high energy, as the Large Hadron Collider at CERN does), in detectors deep underground, and in observations of the sky. The account says: "No further kind of particle will be found". The record then says that several proposed kinds will not be found, among them a fourth generation, a partner for each known particle, a second Higgs boson, the axion and the dark photon. The Higgs boson is the particle of the Higgs field. The Higgs field gives mass to the elementary particles. The strong force is the force that binds quarks together. The axion is a light particle proposed to settle a puzzle of the strong force. The dark photon is a proposed carrier of a force that acts on dark matter, the unseen matter whose pull holds galaxies together. A composite is a particle made of other particles, as a proton is made of quarks. The entry for Prediction 30 ends: "The next collider will not find a new species. Heavier composites wait at higher energies; a new kind does not".

Prediction 19, "The dark matter acts only through gravity", will be measured by the same three kinds of search, aimed at the dark matter, the unseen matter whose pull holds galaxies and clusters of galaxies together. One of the detectors deep underground is a tank of ten metric tons of liquid xenon nearly a mile underground in South Dakota, where a particle of dark matter striking a xenon nucleus would give off a flash of light. The account says: "No search will detect the dark matter through any interaction but gravity, at any sensitivity: not by scattering in an underground detector, not by light or particles it gives off, and not by its production in a collider".

The record beside Carroll sets Predictions 30 and 19 beside a proposal that Carroll explored with three other physicists, Lotty Ackerman, Matthew Buckley and Marc Kamionkowski (Ackerman, Buckley, Carroll and Kamionkowski 2009). In the proposal of the four physicists, a new long-range force, a dark electromagnetism, acts only on the dark matter, and the carriers of the dark electromagnetism, the dark photons, fill the universe with a dark radiation. The record states how Carroll holds the pro-

posal: “It is a proposal explored for what it would show, and Carroll does not state it as a settled thesis”. The proposal states two consequences. The measurements of those two consequences are the measurements that bear on the proposal. The first consequence is a pull of the dark matter on itself, seen in how galaxies move and in how clusters of galaxies pass through one another when the clusters collide. The second consequence is the dark radiation itself, which would add to the count of light particles that measurements of the microwave background (the faint glow left from the early universe) give. The account says that the dark matter acts only through gravity, and that no dark photon will be found.

Prediction 31, “Exactly three generations”, will be measured at colliders, by the rates at which colliders make particles, among them the Higgs boson. The account says: “The generations number exactly three; no fourth exists at any mass”.

Prediction 26, “What is found of color, charge and spin”, is measured by every experiment on how particles sort themselves. The experiments check three things: whether a lone quark is ever seen, whether electric charge is ever found in a blend, and how many particles of one kind can share one quantum state. Color is the charge of the strong force. A singlet is a combination of particles whose colors cancel, as in a proton. A superposition is a quantum state that combines several possible outcomes. Fermions are the particles of matter. Bosons include the carriers of force. The account says: “Color is found only in singlets; no superposition of states of different electric charge is ever found”. The account also says: “Two fermions of one kind never share a state, and any number of bosons can share one”. The third claim of Prediction 26 is: “Every particle has an antiparticle”.

Prediction 33, “The proton does not decay”, will be measured by detectors that watch enormous numbers of protons for years. Each detector waits for one proton to break apart. The largest such detector now being built is Hyper-Kamiokande, which holds about 260,000 metric tons of ultrapure water about 600 meters underground in Gifu Prefecture, Japan, and is planned, as of November 2025, to begin observations in 2028. The

account says that "the proton has no route to decay". A unifying group is a single larger symmetry that would join the strong, weak and electromagnetic forces into one force. The couplings are the strengths of those three forces. The account also says: "No unifying group exists at any scale, and the measured couplings meet at no scale".

Prediction 34 has the title "The strong force breaks no reversal; no axion". Prediction 34 will be measured by searches for a tiny separation of charge inside the neutron, and by searches for the axion. Charge conjugation and parity (CP), taken together, make one reversal: charge conjugation exchanges each particle for its antiparticle, and parity is a mirror reflection. A force that breaks CP treats a process differently from its mirror image in antimatter. CP is the reversal named in the title of Prediction 34. Prediction 34 says that the strong force treats every process exactly as the strong force treats that process's mirror image in antimatter. The CP-violating angle of the strong force is a number in the law of the strong force, and if the angle were not zero, the strong force would break CP and the neutron would have a tiny separation of charge. The account says: "The strong force's own CP-violating angle is zero". The account says the absence of the axion "is a statement about the list of kinds and holds for any value of the angle". The photon carries the electromagnetic force, and the gluons carry the strong force. The account also says: "The photon and the gluons are massless".

Prediction 35, "No new particle in any loop", will be measured by three kinds of precise experiment. The muon is a heavier cousin of the electron. The first kind measures the strength of the muon's magnetism. The second kind measures the rates of rare decays of particles that hold a bottom quark, one of the heavier quarks. The third kind searches for a muon that turns into an electron, as an experiment at the Paul Scherrer Institute in Switzerland does. That experiment is called Mu to E Gamma (MEG), from the way physicists write the change it looks for. In that way of writing, mu, the Greek letter for the muon, turns into e, the letter for the electron. Gamma, the Greek letter for a photon, stands for the photon that comes out together with the electron. The present, second stage of the

experiment is called MEG II. Physicists calculate the quantities that these three kinds of experiment measure. In the calculations, particles appear and vanish in passing and change the measured value by a small amount. Physicists call the contributions of these passing particles loops. The account says: "With no particle beyond the Standard Model, no new particle enters any loop". The anomalous magnetic moment of the muon is the small part of the muon's magnetism that the passing particles add. A new sector would be a new group of particles. The account says: "The anomalous magnetic moment of the muon receives no contribution from a new sector". When a charged lepton turns from one of its three kinds into another, the lepton changes flavor. A muon would change flavor in turning into an electron. The account also says: "No signal of charged-lepton flavor violation will be seen at any foreseeable experiment".

The strength of each force changes with the energy at which the strength is measured. Physicists call that change running. Prediction 36, "The running of the couplings changes only at known masses", will be measured at colliders. The colliders measure how the strength of each force changes with energy. The W and Z are the carriers of the weak force. The account says: "The rates at which the three couplings change with scale are fixed by the closed list of kinds, and change only at the masses of the known kinds and of the W and Z".

Prediction 37 has the title "Ceilings and relations among the masses". Prediction 37 will be measured by measurements of the mass of the top quark, the heaviest particle of matter, and by experiments on neutrinos. One of the neutrino experiments is the Jiangmen Underground Neutrino Observatory in southern China: a sphere of 20,000 metric tons of a liquid that glows when a particle passes, about 700 meters underground. The observatory lies about 53 kilometers from two nuclear power plants. The observatory measures the antineutrinos (the antiparticles of neutrinos) that the plants give off. The Higgs field is the field whose value is not zero in empty space and which gives mass to the elementary particles. Physicists call the value of the Higgs field in empty space its vacuum value. Physicists call the Higgs field as it is in empty space the condensate. A

particle's coupling to the condensate is how strongly the particle is tied to the condensate. A matter particle's mass is its coupling times the vacuum value of the Higgs field divided by the square root of two. A GeV is a unit of mass a little larger than the mass of a hydrogen atom. The account sets a ceiling: "No particle of matter is heavier than" the vacuum value of the Higgs field divided by the square root of two, which the account states as 174.1036 GeV, "since every coupling to the condensate lies between 0 and 1". The account also says: "The neutrino masses are in the normal order, the two that share the electron's flavor most being the lighter pair".

Prediction 42, "The strength of color, the W and Z, the Higgs and the top", will be measured by finer measurements of the masses of the Higgs boson, the top quark and the W boson. The record states the strength of the strong force at the mass of the Z boson, that is, at an energy equal to that mass, as 0.11888, give or take 0.00011. The record calls the law that gives that strength the second closure. The calculation needs one number that the record calls "the gap count at the span's end". The span is the range of energies from the mass of the electron up to the reduced Planck mass, a mass fixed by Newton's constant of gravity, the speed of light and Planck's constant, which lies far above any energy a collider reaches. The strong, weak and electromagnetic forces change in strength with energy, as the entry for Prediction 36 says. At the end of the span the three strengths are still unequal, with two gaps between them. The gap count is the ratio of those two gaps, each gap taken between the inverse strengths (one divided by each strength). From the measured strengths, the gap count is about 5.09.

The argument for Prediction 42 is in the collection's book *The Strengths of the Kinds*, Chapter 3, "Where the Couplings Cross", Section VI. In that section two values for the gap count are given side by side. Neither value is chosen. The first value is 5: the longest side of the triangle with sides 3, 4 and 5. That triangle appears in the collection's account of the kinds of particle. The second value is 21/4, five and a quarter. The value 21/4 is a count made from the six further dimensions that, on the collection's account, lie beside the four of space and time, and from the ten dimensions

of all of them together. The record uses the value 21/4. With the value 21/4, the second closure produces 0.11888, the strength of the strong force stated above. The record states the mass of the W boson as 80.3538 GeV, give or take 0.0045, a mass that comes from a second law, which the record calls the carriers' law. The record uses the carriers' law to compute three values together. The three values are that W mass, the mass of the Z boson, and one further number of the weak force. The record says of the three values that "these values carry the law's own agreement with the measured masses and add no independent evidence". The law in that quotation is the carriers' law.

Physicists calculate a mass in steps, each a finer correction to the steps before it, and call the second step the second order. For the Higgs boson, the record says "four forms of the second order give the Higgs mass as 125.1715, 125.1888, 125.2064 and 125.2245 GeV". For the top quark the record says: "Six readings of the top mass lie between 172.417 and 172.657 GeV". The record states how precise each measurement must be. With a Higgs mass measured to 0.03 GeV, or a top mass to 0.02 GeV, physicists can choose among the readings of the top mass. With a Higgs mass measured to about 0.005 GeV, physicists can choose among the four forms of the second order. An MeV is a thousandth of a GeV. Physicists can test the second closure, the law that gives the strength of the strong force, with a W mass measured to about 3 MeV and one further calculation that takes the W mass as an input.

One more prediction bears on Chapter 3 as well: Prediction 5, entered under Chapter 7. Prediction 5 says that discreteness, the coming in least steps, belongs to the kinds of particle and their charges, not to space and time.

Chapter 4: If the Laws Are Known, Is Everything Already Fixed?

The question of Chapter 4 is whether known laws fix everything that happens. Superdeterminism is an account in which every outcome of a quantum measurement, and every choice of what to measure, is fixed in advance by details of the world that no one sees. The tests proposed for superdeterminism are the experiments that bear on the question of Chapter 4, among them a test that the physicists Sabine Hossenfelder and Tim Palmer proposed (Hossenfelder and Palmer 2020). In the test, experimenters prepare the same quantum state again and again, as alike as possible, and measure it in quick succession. The detectors are as small and as cold as possible. The experimenters look for departures from the odds of quantum theory: patterns that link each outcome to the outcomes before it. In the record the outcome of these tests comes under Prediction 25, which I enter under Chapter 6. Prediction 25 says that repeated measurements settle to the odds of quantum theory, "including in the tests proposed for superdeterminism, which look for departures from those weights".

Prediction 28 is the account's registered answer to the question of Chapter 4: "No world is perfectly deterministic, and no process is perfectly reversible". The first sentence of Prediction 28 is: "No superdeterministic completion of quantum mechanics will be found, and no world is purely and completely deterministic". The record states what kind of claim Prediction 28 is: "These are exclusions that the structure of comparison makes; the outcome of a search for departures from the quantum predictions bears on Prediction 25 and not on this one". The universal state is the quantum state of the whole universe, which Carroll calls the universal wave function, as this book's Chapter 5 says. When a quantum state is carried forward by one rule and its odds always add up to one, the state evolves unitarily. The record beside Carroll has an entry titled

"Whether one law fixes the whole world", which sets Carroll's account beside Prediction 28: "Carroll's universal state evolves deterministically and unitarily, and its fundamental law is symmetric in time apart from the measured violations of reversal, which both accounts accept". The entry classes the difference as "a claim of logical form".

The record beside Carroll has a second entry, titled "What the known laws fix". For each account, the second entry states what the known laws fix. A compatibilist holds that the laws of physics govern every particle. A compatibilist also holds that choosing is still the right description of what people do, as this book's Chapter 4 says. The entry states Carroll's position: "Carroll holds that the known laws leave no room for any influence beyond them, and, as a compatibilist, that choice is real at the level of people". "Real" in the quoted position is Carroll's word, used in Carroll's sense. The entry states the collection's position: "The collection grants the laws and holds that they fix the distribution of outcomes and leave each single outcome open". The conclusion of the entry is: "Both predict the same distributions". The entry classes the difference as "a claim of logical form, whether a complete law makes a complete world".

Chapter 5: When a Measurement Could Go Two Ways, Do Both Happen?

The question of Chapter 5 is whether every outcome of a measurement happens. A superposition is a quantum state that combines several possible outcomes, as the entry for Prediction 26 under Chapter 3 says. Prediction 2, "Gravitation alone ends no superposition", will be measured in two ways. A collapse is the ending of a superposition in one outcome. The first way is the search, in laboratories deep underground, for the faint radiation that a collapse caused by gravity would give off. The second way is the experiment of Prediction 1, entered under Chapter 7, in which two small masses are each held in two places at once for seconds at a time. The

account says what both ways will find: “No mass, however large, no separation of two placements, however wide, and no length of time ends a superposition of its own accord”.

The record beside Carroll states that Carroll’s expectation is the same, and that Carroll’s account goes further. Entanglement is a joint state of two things whose parts cannot be described one by one. Decoherence is the spreading of the difference between the outcomes into the surroundings. Interference is the pattern of reinforcement and cancellation that waves make. After decoherence the outcomes no longer interfere. The record states Carroll’s account in these words: “In the many-worlds account no superposition ever ends; measurement is entanglement followed by decoherence, and every outcome occurs in some branch”. The record adds: “A collapse driven by gravity has no place in it”.

The record also says which part of the question of Chapter 5 cannot be answered by any measurement. The universal state is the quantum state of the whole universe. Carroll calls the universal state the universal wave function. A comparer is whoever or whatever makes a finding. A comparer may be a person or an instrument. The record states the collection’s position: “The collection holds that the universal state exists as structure and that, for every comparer, one outcome is real, the one found”. The record states the senses of the two words ‘exist’ and ‘real’: “To exist is to stand as structure, the same in every instance; to be real is to be given in an actual act of comparison”. The record states Carroll’s position: “Carroll holds that every outcome of every measurement occurs, each in its own branch, all equally real”. The conclusion of the record is: “Both predict the same statistics within every record, so no record within any branch tells the two apart”. The record classes the difference as “a description of what a thing is, set by the collection’s distinction between what exists and what is real”. The statistics of the outcomes are measured under Prediction 25, entered under Chapter 6, and the absence of any signal between distant measurements is measured under Prediction 27, entered under Chapter 7.

Chapter 6: Where Do the Odds Come From?

The question of Chapter 6 is where the odds of each outcome come from. The Born rule is the rule that the probability of an outcome is the square of the size of its part of the quantum state. Prediction 25, "The statistics of quantum theory hold", will be measured by four kinds of experiment. In the first kind, experimenters repeat one measurement many times on systems prepared alike and count the outcomes. In the second kind, experimenters send particles through three openings at once. Interference is the pattern of reinforcement and cancellation that waves make. The experimenters look for any interference among the three openings that the pairs of openings do not account for. The third kind is a Bell test. In a Bell test, experimenters measure pairs of particles at two distant places and combine four of their correlations into one number. The number is the Clauser, Horne, Shimony and Holt (CHSH) sum. The sum is named for the four physicists who proposed it in 1969: John Clauser, Michael Horne, Abner Shimony and Richard Holt. For any account in which each outcome is fixed in advance by causes near where it is found, the CHSH sum is at most 2. In the fourth kind, experimenters combine particles from independent sources in a network, to find whether quantum theory needs complex numbers (numbers built on the square root of minus one) or only ordinary ones. A sector is one of the separate parts into which the states of a system fall, such as the states of one electric charge. Two times the square root of two is about 2.83. An observable is a quantity that can be measured. When measuring one observable does not disturb another, the two observables commute. The account says what all four kinds will find: "Repeated measurements on identically prepared systems settle to the weights of the Born rule, in every sector and on the full space, including in the tests proposed for superdeterminism, which look for departures from those weights. The interference among three paths has no term that the pairs of paths do not account for. Tests of the

CHSH inequality reach $2\sqrt{2}$ and go no further, and stay at 2 or below whenever either party's two observables commute. Experiments with independent sources find the complex, not the real, composition of independent systems".

The record beside Carroll states Carroll's expectation in three words: "The same statistics". In Carroll's books a degree of belief is called a credence. The record states Carroll's route to the Born rule: "Charles Sebens and Carroll derive the Born rule from self-locating uncertainty, the credence an observer gives to each branch after branching and before seeing the result (Sebens and Carroll 2018)". In 1957 the mathematician Andrew Gleason proved what is now called Gleason's theorem: any way of assigning probabilities to the outcomes of quantum measurements that satisfies a few plain conditions must be the Born rule. The record states the collection's route: *The Quantum Passage*, a book of the collection, in Chapter 7, Section III, "derives it through the theorem of Gleason, whose premises it finds in the structure of comparison". The conclusion of the record is: "Both derivations give the same weights". The record classes the difference between the two routes as "a claim of logical form".

Chapter 7: Is Space Made of Something Quantum?

The question of Chapter 7 is whether space and time, the arena in which every finding is made, are themselves made of something quantum. Prediction 1, "Gravitation entangles two masses", will be measured by an experiment that the physicist Sougato Bose and his colleagues proposed (Bose and others 2017). In the experiment, two tiny masses are each held in two places at once, and nothing pulls on them but each other's gravity. Afterward the two masses are checked for entanglement. The Newtonian interaction of the two masses is the pull of each mass on the other, as Newton's law of gravity describes that pull. The phase is the shift in step that the Newtonian pull adds to the alternatives of the two masses. The account says what the experiment will find: "Two masses, each held

in a superposition of two places and related only through gravitation, become entangled, with the phase of their Newtonian interaction". The design of Bose and his colleagues uses masses of a hundred-thousandth of a microgram, held for 2.5 seconds. For that design the record says that "the entangling phase is 0.314 radians". A radian is a little under a sixth of a full turn. The record beside Carroll states that Carroll's expectation is the same: "The two accounts predict the same entanglement and the same phase". The record also says that the result means different things on the two accounts. The record says that "the collection does not read it as showing gravitation quantized". Gravitation would be quantized if the geometry of space and time carried quantum pieces of its own.

Prediction 3, "Gravitation is not quantized", will be measured in two ways. The first way is the search for any effect that shows the geometry of space and time carrying quantum pieces of its own. The second way is the measurement of how gravitational waves, ripples in the geometry, scatter off one another. Degrees of freedom are the separate pieces of a system, as this book's Chapter 7 says. Each piece can be in its own range of states. The account says: "No effect will show the geometry carrying quantum degrees of freedom of its own, beyond those tied to the alternatives of the matter". The account also says: "A detector that registers a gravitational wave in single clicks agrees with the account and does not show gravitation quantized". The record beside Carroll states Carroll's expectation from his work with ChunJun Cao and Spyridon Michalakis, in which the three physicists showed how a geometry of space can be recovered from the entanglement of an abstract quantum state (Cao, Carroll and Michalakis 2017). The record states Carroll's expectation: "On that account the geometry is carried by quantum degrees of freedom of the state itself, so gravitation is quantum at the root". The record has an entry on whether gravitation is quantized, titled "Carroll 1". The Planck length is a length far smaller than any length an experiment can measure, and the Planck curvature is the curvature of a sphere whose radius is the Planck length. The entry "Carroll 1" says: "Below the Planck curvature the two accounts give the same result for every experiment now proposed". The two ac-

counts in that sentence are the collection's and Carroll's. The last sentence of the entry is: "The difference stands at this date as a prediction with no proposed experiment able to reach it".

The record has a further entry, "Whether the arena comes from the state". In that entry the record states two positions. In Carroll's program of research, the geometry of space and time is derived from the entanglement of a quantum state. The collection holds that the arena is the context of every state. The context of a comparison is what makes the two things compared two. The record says that the measurable part of the difference between the two positions is the subject of the entry "Carroll 1". The record classes the difference as "a claim of logical form".

Prediction 5, "The arena has no least length, area or duration", will be measured in two ways. In the first way, astronomers time the arrival of light of different energies from one distant event, for many such events at many distances. One kind of distant event is the gamma-ray burst, a brief and intense explosion far across the universe. A delay made on the way grows with the distance the light has crossed, and a delay made at the source does not, so astronomers can tell the two kinds of delay apart by comparing events at many distances. The second way is every observation of the spectrum of area or volume. The spectrum is the set of sizes that an area or a volume of space can take. Astronomers can look for the spectrum of area or volume in the echoes of gravitational waves from merging black holes. A discrete spectrum of area or volume would be a set of allowed sizes coming in steps. The content is what is found within the arena: the particles, their charges and their fields. The account says: "The speed of light in vacuum does not depend on the energy of the light through any step of spacetime. No discrete spectrum of area or volume will be observed, in the echoes of gravitational waves from merging black holes or anywhere else. Discreteness belongs to the content, the charges and the kinds of particle, and the account predicts it there".

The record beside Carroll cites a remark Carroll made on his blog, *Preposterous Universe*, in the post "Thanksgiving" of 23 November 2023. The Planck scale is the scale of the Planck length. Carroll's remark is: "Nothing

we know about gravity implies that 'spacetime is discrete at the Planck scale'". The conclusion of the record is: "The two meet on the same outcome". Carroll teaches that discreteness comes from the solutions of the equations, and not from any step in space or time, in *Quanta and Fields*, Chapter 1, "Wave Functions", section "The Simple Harmonic Oscillator". In that section, energy levels come in steps because of how the equations are solved, "not because there is anything fundamentally discrete about space or time or energy or anything else".

Prediction 6, "Gravitational waves have two polarizations and travel at the speed of light", will be measured by the observatories of gravitational waves, which register a passing wave at several detectors at once. Prediction 6 will also be measured by the arrival of a wave together with the light from the same event. The polarizations of a wave are the ways the wave can stretch and squeeze space as it passes. A massless field of spin two is the kind of field that general relativity, Einstein's theory of gravity, describes. The waves of such a field have exactly two polarizations. The account says: "Gravitational waves carry the two polarizations of a massless field of spin two, and travel at the speed of light; no further polarization and no other speed will be found".

Prediction 27, "No signal faster than light, and none backward in time", is measured by every experiment that compares findings made at distant places. The light cone of an event is the set of places and times that light leaving the event can reach. The account says: "A finding changes no geometry, here or anywhere, and news of a distant finding enters a record only at an event on or within the future light cone of the finding. Until such a signal arrives, every local statistic follows quantum theory unchanged, whatever measurement was chosen far away". The account also says: "The probabilities of a preparation's outcomes do not depend on which test is chosen later, and no later finding comes to precede an earlier one". The record beside Carroll states that Carroll's expectation is the same: "in the many-worlds account a measurement far away changes no local statistic; nothing in his account sends a signal outside the light cone".

Prediction 4, entered under Chapter 3, bears on Chapter 7 too. Prediction 4 says that every material falls alike, that antimatter falls as matter does, and that nothing shields gravity.

Chapter 8: Why Does Time Run Only One Way?

The question of Chapter 8 is why time runs only one way. Two kinds of measurement bear on that question. The first kind measures the decays of particles that break the symmetry of time reversal, so that a decay and its reverse run at different rates, as in the decays of the particles called kaons and B mesons. The second kind runs processes backward in the laboratory, as the spin echo does (Hahn 1950). In a spin echo, a pulse of radio waves turns back the drift of the spins of atomic nuclei. Then the signal that the spins gave together returns. The account says what both kinds will find, in the second sentence of Prediction 28: "Some asymmetry of time must exist in the laws, and no process at any scale is perfectly reversible; a reversal of evolution, by an inverse in a formalism or by a pulse in a laboratory, does not remove a finding from the order of findings". The record states how much Prediction 28 predicts: "The size of the asymmetry of time is not predicted, only that it exists".

The record beside Carroll sets the two accounts side by side on the source of the arrow of time. The arrow of time is the one direction in which time runs. The record begins with what Carroll establishes: "Carroll is right that the arrow needs a reason, and right that a low-entropy beginning taken as a brute fact is not one". Entropy is a count of how many arrangements of the small parts look the same from outside. A low-entropy beginning is an orderly one. The record states the collection's reason for the arrow: "The collection finds a structural arrow in every comparison, which needs no low-entropy beginning". The record classes the difference as "a claim of logical form about what a count presupposes". The

measurable consequence of this difference is whether our epoch (the present age of the universe) began by inflating, that is, by expanding extremely fast at its very start, a consequence entered under Chapter 9.

Chapter 9: Why Did the Universe Begin So Orderly?

The questions of Chapter 9 are why the universe began so orderly, and where our epoch of the universe came from. The microwave background is the faint glow left from the early universe, which fills the whole sky. The light of the microwave background is polarized: its waves vibrate more in some directions than in others, and gravitational waves from the beginning of the epoch would leave a twisting pattern in that polarization. Two instruments will measure the polarization. The Simons Observatory is a group of telescopes at about 5,200 meters of altitude in the Atacama Desert of northern Chile, observing since 2024. LiteBIRD is a satellite led by the Japan Aerospace Exploration Agency, planned for launch in the early 2030s. LiteBIRD will map the polarization of the whole sky. The record says that the Simons Observatory will reach about one part in a thousand in the strength of gravitational waves from the beginning of the epoch, relative to the ripples of density, and LiteBIRD below two parts in a thousand.

The primordial departures are the first small unevennesses of the epoch. The metric is the geometry of space and time, which fixes every distance and duration. Scalar departures are ripples of density, and tensor departures are gravitational waves. The account says what the two instruments will find, in Prediction 10, “No primordial gravitational waves”: “The primordial departures of the metric are scalar, sourced by departures of density, and waves of the metric arise from them only at second order: the ratio of tensor to scalar power is far below 10−6. The Simons Observatory and LiteBIRD will find no primordial tensor signal”. Waves

that arise "only at second order" arise only as a much smaller side effect of the ripples of density. Ten to the power of minus six is one part in a million.

Inflation is a proposed very early period of extremely fast expansion. At an early time, the horizon is the farthest distance that light could have crossed by that time. Prediction 13, "No long inflationary phase", is about inflation: "No inflationary phase long enough to stretch the lengths observed today from within the early horizon to beyond it occurred. The horizon and flatness problems are answered by the turn from the previous epoch, with no inflation". The horizon problem is the question why regions too far apart ever to have exchanged light look alike. The flatness problem is the question why space is so nearly flat. The turn is the passage from the end of one epoch to the beginning of the next, at the one end that every black-hole interior reaches, and the turn itself is not registered as an event in any comparison.

The record beside Carroll sets Predictions 10 and 13 beside the proposal of Carroll and the physicist Jennifer Chen (Carroll and Chen 2004). In the proposal of Carroll and Chen, a universe with a small positive energy of empty space spawns inflating regions, and entropy can increase without bound in both directions of time. A region like ours, with its arrow, is one of the regions spawned. Carroll says in his own words how firmly he holds the proposal: "The details of this model are highly speculative" (*From Eternity to Here*, Chapter 16, "Epilogue", section "What's the Answer?"). The record states Carroll's expectation: "On that proposal our region began with an inflationary phase". Carroll tells of such a region in *From Eternity to Here*, Chapter 15, "The Past Through Tomorrow", in the section "Baby Universes": the baby universe is "all set up to undergo inflation and expand to a huge size". The record also says that Carroll, with the physicist Heywood Tam, "has also argued" that inflation does not by itself explain the low entropy of the beginning when evolution is unitary (Carroll and Tam 2010). The record says that in the proposal with Chen, "the explanation of the arrow comes from the ensemble and the inflation makes the region". The ensemble is the whole family of regions that the

parent universe spawns. The record states what the polarization measurements mean for the proposal with Chen: a primordial tensor signal found at any level the two instruments reach "stands as evidence for an inflationary beginning". Inflation also allows a signal below the reach of the two instruments.

Three further predictions bear on Chapter 9: Predictions 9, 11 and 12. The three are measured by the maps of the microwave sky named above and by the surveys of galaxies named in the next paragraph. The record says that the measurements of Predictions 9, 11 and 12, with Prediction 10, are the measurements that bear on Prediction 13. Prediction 9, "Space is flat to the size of the primordial departures", says: "The epoch begins spatially flat, and the spatial curvature measured within one horizon is that of the departures from uniformity". The record gives that curvature as of order (roughly) one part in a hundred thousand. Carroll argues for flat space by a different route. With Heywood Tam, Carroll found that almost all universes of the standard kind are flat, on the measure of possible universes that Carroll and Tam used, their way of counting possible universes (Carroll and Tam 2010). I credit the route of Carroll and Tam as "convergent support": support from a second, independent route to the same result. I do so in *Side by Side, Volume Two*, in the essay "Why the New Epoch Is Flat", Section VII. Prediction 11, "The primordial departures are adiabatic", says that "every component departs from uniformity together", which is what adiabatic means here. Prediction 11 also says that "The dark matter carries no separate pattern of primordial departures of its own". Prediction 12, "The departures are one pattern across the whole sky", says: "The primordial departures are correlated across the whole sky, with nearly the same strength at every length". The record beside Carroll says that Predictions 9, 11 and 12 agree with inflation as well. So the three predictions do not by themselves tell the two accounts apart. The polarization measurements of Prediction 10 bear directly on the difference between the two accounts.

Two more predictions bear on the energy of empty space. The energy of empty space drives the present acceleration of the expansion of the universe. The same energy sets the far future that Chapter 9 describes. The two predictions will be measured by four surveys that map the expansion of the universe and the growth of its structure. The Dark Energy Spectroscopic Instrument is on the 4-meter Mayall telescope at Kitt Peak in Arizona, and it measures the light of millions of galaxies and quasars (the bright centers of distant galaxies). The instrument maps a length that sound waves in the early universe imprinted on the spacing of the galaxies. Euclid is a space telescope of the European Space Agency, launched on 1 July 2023. Euclid maps the positions of galaxies and the bending of their light by unseen matter. The Vera C. Rubin Observatory, on Cerro Pachón in Chile, began a ten-year survey of the southern sky, the Legacy Survey of Space and Time, in June 2026. The Nancy Grace Roman Space Telescope was launched by the National Aeronautics and Space Administration (NASA) of the United States on 30 August 2026, and the Roman telescope surveys galaxies and distant exploding stars in visible and near-infrared light.

Prediction 7, "The matter fraction is 0.311", says that matter makes up the fraction 0.311, give or take 0.001, "of the energy of a spatially flat universe at the present epoch". Prediction 7 also says that the energy of empty space makes up the rest, 0.689. The vacuum energy is the energy of empty space. Prediction 8, "The equation of state of the vacuum energy is exactly -1", says: "The energy of the vacuum has an equation of state of exactly -1, at every epoch". An equation of state of exactly minus one means the energy of empty space in each volume stays the same as the universe grows. Quintessence is a proposed field whose energy changes as the universe ages. A scalar here is a field with one value at each point. Prediction 8 goes on: "No dynamical field, no quintessence and no evolving scalar drives the acceleration". Prediction 8 ends: "The reported departure from -1 will not be confirmed". The reported departure is the departure found by the final analysis of the Dark Energy Survey, an earlier survey of galaxies, combined with data from the Dark Energy Spectro-

scopic Instrument and the microwave background (Abbott and others 2026). The record reports the departure as one that "has not reached the rule of decision". So the departure has not been established at five standard deviations by at least two independent kinds of measurement.

Chapter 10: Can I Choose?

The question of Chapter 10 is whether, when you decide, you choose. On the account, choice is located at the single outcome, which the known laws leave open. The account contains no force that pushes on the particles of a brain. Four predictions already entered in this appendix bear on that account of choice. Prediction 25, entered under Chapter 6, says that repeated measurements settle to the odds of quantum theory. Choice, located at the single outcome, changes no distribution of outcomes. Prediction 4, entered under Chapter 3, says that every material falls alike and that nothing shields gravity. Prediction 30, also entered under Chapter 3, says that no further kind of particle will be found. On the account, no new force acts on the matter of a body. In Chapter 10, choice is tied to the direction of time. Prediction 27, entered under Chapter 7, says that "no later finding comes to precede an earlier one".

Chapter 11: Who Is Doing the Finding?

The question of Chapter 11 is who, or what, is doing the finding. The measurements of Prediction 2, entered under Chapter 5, bear on that question: the searches for a collapse caused by gravity, and the experiment with two masses each held in two places at once. In Prediction 2 the account also says what ends a superposition: "a superposition ends only through a finding, an interaction that registers which alternative is the case". The account says what decoherence is: "The account allows no decoherence beyond the ordinary kind, which is a comparison made by some environment". *The Unquantized Arena*, a book of the collection, states

in Chapter 11, Section III, the part that the surroundings play in such a comparison: "The environment that carries the copies is the context of that content". The content in that quotation is what the comparison is about: which alternative is the case. The context of a comparison is what makes the two things compared two. On the account, the one who finds, a person or an instrument, makes the finding, in an interaction that registers which alternative is the case. The surroundings that carry the record are the context of that act of finding, as this book's Chapter 5 says. The record beside Carroll states Carroll's account of measurement in the same terms of interaction: "measurement is entanglement followed by decoherence".

Chapter 12: Do We Make Meaning Up?

The question of Chapter 12 is whether people make meaning up. Chapter 12 begins where Carroll begins, with what the laws underlying everyday life contain. Carroll states what those laws contain in *The Big Picture*, Chapter 45, "Three Billion Heartbeats": "Ideas like 'meaning' and 'morality' and 'purpose' are nowhere to be found in the Core Theory of quantum fields, the physics underlying our everyday lives". So, on Carroll's account, meaning is made by people living in a world that the Core Theory describes. The collection grants the Core Theory as Carroll states it. The Core Theory contains the kinds of particle of the Standard Model. The searches of Prediction 30, entered under Chapter 3, look for kinds of particle beyond the kinds of the Standard Model. The account says that "No further kind of particle will be found". So, on the account, no kind of particle beyond the known kinds acts on the matter of a person who makes meaning.

Appendix B: For the Specialist

This appendix gives, for every claim of the collection that the main text makes, the book, chapter and section of the physics collection that carries it. In one place, under Chapter 11, I also give a further result that I point toward in the chapter and do not use there. It also gives the exact form of the few relations that the main text states in words. The reader may skip it without losing the thread of any chapter.

Each entry states the claim as the chapter states it, in one or two plain sentences, and then names the place in the collection where the claim is carried. The chapters of a book are given by number and title, and its sections by number and title where the book numbers them, or by title where it does not. *Side by Side, Volume One* is cited by its chapter on Carroll, "The Wave Function and the One Who Finds", and by its Appendix K, "The Registered Predictions Beside Sean Carroll", which is the dated record of 4 November 2026. Two entries also cite its chapters on Sabine Hossenfelder and David Deutsch, and each of the two names the chapter it cites. *The Last Turtle* is the book whose subtitle is *Essays from Comparison and the Incommensuration Theorem to Causation, the Arena and the Limits of Computation*. *An Immanent Metaphysics* is written as numbered aphorisms, and its places are given by chapter, section and aphorism number. Under some chapters, after the entries, a longer passage gives a step-by-step argument, a figure or a source that the chapter states only as a result or in plain words. Each such passage opens with a sentence saying what it gives.

Preface

A comparison is the act of finding whether two things are the same or different, in some respect. *Comparison; The Necessary Basis*, Chapter 1, "What Does It Mean to Compare?", Section I, "Two Stones".

Instruments compare as surely as minds do. *Comparison; The Necessary Basis*, Chapter 1, Section II, “Instruments Compare”.

Every measurement is a comparison. *Comparison; The Necessary Basis*, Chapter 2, “Comparison as the Ground of Science”, opening.

The collection starts from the act of comparison and takes nothing from physics: no measurement, no law, no result of any experiment. *Home in the World*, Prelude, “Two Crews, One Tunnel”, section “The Crew from Knowing”.

The collection reaches three of Carroll’s positions by its own reasoning: every law of physics holds within a domain, nothing known about gravity makes space and time come in least steps, and the arrow of time needs a reason. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Abstract and Section IX, “Summary”.

The collection grants Carroll’s claim that the laws of everyday life are completely known, as he states it, and adds no force or field to those laws. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Section IV, “The Known Laws and the Single Outcome”.

Carroll and the collection part at one place, what lies at the ground of physics, the thing every other description describes, and every other difference between them follows from it. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Section I, “A Physicist Who Asks the Question”.

For Carroll the ground of physics is a quantum state evolving by one rule. For the collection the ground is the act of comparison, in which every finding of physics is made. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Section I, “A Physicist Who Asks the Question”.

The quantum state describes, exactly and usefully, what any comparison can find and with what odds, and the act of finding comes first. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Section VII, “What Lies at the Ground”.

The known laws are complete for the distribution of outcomes and leave each single outcome open. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IX, "Summary".

The arrow of time has a structural source in every comparison: each act runs one way, from the range of what could be found, to the act, to the record of what was found. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IX, "Summary", with the two arrows set out in Section V, "The Arrow of Time and the Past Hypothesis".

The many-worlds account, which takes the wave function to be the whole of the world, drops together the one who finds, the single random outcome and the direction of time. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches", and Section IX, "Summary".

Every interpretation of quantum mechanics keeps the one who finds, the single random outcome and the direction of time together, or loses all three together. *Measurement*, Chapter 14, "The Consequences of the Theorem", Section VI, "Subjectivity, Directed Time and Hard Randomness Are Bound".

The collection leaves Carroll the Core Theory, tested by every test made of it, and its domains, and removes one step: the step from a complete law to a complete world. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

Chapter 1: What Is the World Made Of, at Bottom?

Poetic naturalism and the collection agree that there are many good descriptions of the world, each fitted to its domain, and that no description of people or reasons needs a force that physics lacks. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VII, "What Lies at the Ground".

A street on a market morning, seen by one person from a third-floor window and by another walking along it, gives two true descriptions, each made from a place and each answering the question asked from there. *Physics Is Not Math*, Chapter 3, "Truth Has a Frame", Section I, "Two Views of One Street".

Two different verdicts in two different frames are answers to two different questions, and so they do not contradict each other. *Physics Is Not Math*, Chapter 3, Section IV, "Framing Makes Truth Firm".

A frame is what is held fixed so that a question has one answer: the place a description is made from, and the question it answers there. *Physics Is Not Math*, Chapter 3, Section III, "The Frame Makes the Verdict Definite".

Chemistry is made of nothing but electrons, nuclei and light. What fails is the derivation of the chemist's description from the laws of the parts, since exact calculation from those laws reaches only the simplest systems. *The Last Turtle*, Chapter 27, "The Size of Chemistry", Section IV, "The Failure of Reduction, Stated Exactly".

For Carroll the base is a quantum state evolving by one rule, and the other descriptions are ways of talking about it. For the collection the base is the act of comparison. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VII, "What Lies at the Ground".

A comparison is the act of finding whether two things are the same or different, in some respect. *Physics Is Not Math*, Chapter 1, "What It Is to Compare", Section I, "Two Stones".

Comparing is not a human specialty: a thermostat, a pan balance, a spirit level, a smoke alarm and a photographic plate compare, with no mind at all. *Physics Is Not Math*, Chapter 1, Section II, "Instruments Compare Too".

Every law of physics was found by acts of comparison. *Physics Is Not Math*, Chapter 1, Section III, "The Tide Gauge", which also says what a law states: what stays the same when the circumstances of a measurement change.

One thing presupposes another when it cannot be done, stated or checked without the other, while the other can be done without it, and the deeper of two things is the one the other presupposes. *Physics Is Not Math*, Chapter 5, "What Depth Means", Section II, "Depth Is the Order of Presupposition".

In the order of presupposition, deepest first, the four fields of knowledge are metaphysics, physics, computation and mathematics. Metaphysics is the account of the act of comparison itself, and physics is the study of acts of comparison as they happen in the world. Computation is the carrying out of a sequence of steps by a fixed rule, and mathematics is the study of structure, held in view as a whole. *Physics Is Not Math*, Chapter 5, Section V, "The Order of the Four".

Mathematics presupposes computation, computation presupposes physics, and physics presupposes the act of comparison. A single comparison is complete with no law of physics, no program and no theorem. *Physics Is Not Math*, Chapter 5, Section V, "The Order of the Four".

A computation runs without waiting on any proof, as beads moved along the wires of a counting frame add two numbers whether or not anyone has proved anything about addition, and every proof is checked by a computation. *Physics Is Not Math*, Chapter 7, "Computation Has Sequence", Section VII, "Every Proof Waits on a Run".

Depth is distinct from certainty, from precision, from elegance and from prestige. *Physics Is Not Math*, Chapter 5, Section III, "Depth Is Not Certainty".

A wave function is mathematics, structure laid out with every case in view at once, so it stands with mathematics, last in the order of presupposition. It describes exactly and usefully what any comparison can find and with what odds, and the act of finding comes first. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VII, "What Lies at the Ground".

To exist is to stand as structure, the same in every instance, and to be real is to be given in an actual act of comparison. A law exists, and a measurement is real. *Comparison; The Necessary Basis*, Chapter 10, "Reasoning", Section II, "The Real and the Abstract".

Every comparison has two sides: the side it is made from, which the collection calls the subjective, and the side it is about, which it calls the objective. *Physics Is Not Math*, Chapter 1, Section VI, "Six Parts in Three Pairs".

Mathematics works as if only the objective existed, and carrying that way of working beyond mathematics is taking the frame of mathematics for everything. *Physics Is Not Math*, Chapter 3, Section VII, "Mathematics Works as if Only the Objective Existed", and Section VIII, "Taking the Frame of Mathematics for Everything".

The whole world contains the acts of comparison by which anything in it is found, so the formalism of the many-worlds account, which leaves those acts out, cannot be the whole world. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

Chapter 2: How Sure Can We Be of Anything?

A comparison is the act of finding whether two things are the same or different, in some respect, and comparing is not a human specialty. *Physics Is Not Math*, Chapter 1, "What It Is to Compare", Sections I and II.

Every act of knowing is an act of comparison or is built from acts of comparison. *Physics Is Not Math*, Chapter 2, "Two Things That Cannot Both Be Perfect", Section VI, "The Theorem".

Every comparison has six parts in three pairs (sameness and difference, content and context, the subjective and the objective), and each pair is inseparable. *Physics Is Not Math*, Chapter 1, Section VI, "Six Parts in Three Pairs", and Section VII, "Each Pair Inseparable, and Why There Are Six".

In many comparisons the context is a time or a place: a tide gauge that sets the height of the sea at noon beside its height at midnight has noon and midnight as its context. *Physics Is Not Math*, Chapter 1, Section VI, "Six Parts in Three Pairs".

A unit, a scale and a number are each built from comparisons, and so they come after the six parts of comparison. *Physics Is Not Math*, Chapter 1, Section VII, "Each Pair Inseparable, and Why There Are Six".

The verdict on the context is "same" when the comparison finds no difference in what makes its two things two, as when they are taken at one moment, at one place, or at settings so close together that no difference between them is found. It is "different" when the comparison finds the two at different moments, places or settings. *Physics Is Not Math*, Chapter 2, Section II, "Four Characters".

Each of the four characters pairs a verdict on the content with a verdict on the context. Continuity is a sameness of content where there is a sameness of context, and discontinuity a difference of content where there is a sameness of context. Symmetry is a sameness of content where there is a difference of context, and asymmetry a difference of content where there is a difference of context. *Physics Is Not Math*, Chapter 2, Section II, "Four Characters", which also says that, at the finest grain, a small enough change of context brings only a small change of content.

Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. *Physics Is Not Math*, Chapter 2, Section VI, "The Theorem"; *Incommensuration*, Chapter 2, "Symmetry and Continuity Cannot Both Be Perfected", Section VI, "The Theorem".

A comparison perfected at its foundation is either continuous and asymmetric or symmetric and discontinuous: these are the two pairings that survive. *Physics Is Not Math*, Chapter 2, Section V, "The Two That Survive"; *Incommensuration*, Chapter 2, Section VI, "The Theorem".

To apply a character fundamentally is to hold the character exactly, at the foundation of a description, with no exception at any scale. To apply two characters simultaneously is to hold both characters in one and the same comparison. *Physics Is Not Math*, Chapter 2, Section VI, "The Theorem"; the words "with no exception at any scale" are from *Home in the World*, Chapter 4, "The Circle and the Floor", section "Two Cannot Both Be Perfect".

Two lengths are incommensurable when no unit measures both a whole number of times, as no unit measures both the side and the diagonal of a square, and the theorem names a gap of that kind between two characters. *Physics Is Not Math*, Chapter 2, Section VI, "The Theorem".

The theorem allows symmetry and continuity together where they hold nearly, as in a ripple spreading on a still pond. *Incommensuration*, Chapter 2, Section VI, "The Theorem".

The proof uses no fact about the physical world, and the theorem is settled in the way a count is settled, by the count itself, with no need to trust whoever made it. *Incommensuration*, Chapter 2, Section VII, "Why the Theorem Is Settled"; *Physics Is Not Math*, Chapter 2, Section VII, "Settled as a Count Is Settled".

The Incommensuration Theorem is a structural fact about the nature of knowing. *Physics Is Not Math*, Chapter 2, Section VI, "The Theorem".

A credence is a verdict with a measure placed on it, and not a third kind of verdict. Probability is among the measures that every measurement involves, so a probability is itself found by comparison. *Comparison; The Necessary Basis*, Chapter 10, "Reasoning", Section I, "Reasoning Is Comparison".

The stones of an arch hold together at once, with none of them first, as the six parts of every comparison occur together with none of them first. *Home in the World*, Chapter 4, "The Circle and the Floor", section "An Arch of Stones"; *Comparison; The Necessary Basis*, Chapter 10, Section IV, "Simultaneity".

What is built from acts of comparison has the same form: the words of a dictionary hold one another up at once, and so do theory and experiment, in every laboratory. The parts of knowing hold one another up at once, with none of them first. *Home in the World*, Chapter 4, section "An Arch of Stones", and section "Firm Ground, Open Findings".

Two independent routes to one value, each right nine times in ten, leave the value wrong only one time in a hundred, and two strands that rest on one source count as one strand. *Physics Is Not Math*, Chapter 12, "Chains and Cables", Section V, "Two Routes to One Value", where Peirce's cable and Perrin's thirteen counts are also given.

Within its own structure a deduction is certain. The world is met in measurement and in perception, so the step from what a deduction proves to a claim about the world is never guaranteed, and it always carries some doubt. For that reason the collection states each step from the structure of comparison to a quantity in the world as an identification, and checks it against measurement. *Comparison; The Necessary Basis*, Chapter 10, Section II, "The Real and the Abstract".

The act of comparison is the floor of knowing, the one thing that cannot be doubted without being done in the doubting: something under every belief that is not one more belief. *Home in the World*, Chapter 4, section "The Floor".

The act of comparison is the floor of knowing and the ground of physics: one act, named from the side of knowing and from the side of physics. The floor is named in *Home in the World*, Chapter 4, section "The Floor", and the ground in *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section I, "A Physicist Who Asks the Question".

One cannot doubt comparison without comparing: whoever rejects the six parts of comparison compares in rejecting them, and uses all six. *Incommensuration*, Chapter 6, "The Incommensuration Theorem: Statement and Proof", Section IX, "The Self-Evidence of the Theorem".

A claim is performed when it is established by what happens in the act of engaging it, including the act of trying to deny it. *Physics Is Not Math*, Chapter 9, "Metaphysics Is the Act", Section III, "Three Ways a Claim Can Be Held".

What any act of knowing finds can be mistaken, and the structure that every act of knowing has is certain and the same for every comparer. *Home in the World*, Chapter 4, section "Firm Ground, Open Findings".

A law of physics is found by comparison: it states what stays the same when the circumstances of a measurement change. *Physics Is Not Math*, Chapter 1, Section III, "The Tide Gauge".

This passage gives the size of the proof that Chapter 2 walks in plain words: the proof of the Incommensuration Theorem is a finite count.

The proof checks every way of pairing the four characters, ten pairings in all, by three tests that rest on the inseparability of sameness and difference, and two pairings survive. *Physics Is Not Math*, Chapter 2, Section III, "Ten Ways to Pair Them", Section IV, "Three Tests", and Section V, "The Two That Survive"; *Incommensuration*, Chapter 2, Section IV, "Three Rules", and Section VII, "Why the Theorem Is Settled".

This passage gives the collection's reason why the theorem is stated with no "if" in front of it.

The theorem is a performed claim: every check of it, and every challenge to it, is a comparison that uses it. "A condition is itself a comparison" (*Physics Is Not Math*, Chapter 9, Section VI), so any condition offered for the theorem already uses what the theorem is about. *Physics Is Not Math*, Chapter 9, Section VI, "Unconditional in the Immanent".

This passage gives the view of knowledge that *Home in the World*, the collection's book for readers of the physicist Carlo Rovelli, reports as Rovelli's, and that book's reading of it.

As that book reports him, Rovelli holds that our knowledge goes round in a circle with nothing underneath: the words of a dictionary are defined by other words, and experiments are read with the help of the theories they test. *Home in the World* reads those circles as arches, whose

parts hold one another up at once with no first part: “Rovelli’s circles are arches”. *Home in the World*, Chapter 4, “The Circle and the Floor”, section “Rovelli’s Circle”.

Chapter 3: Are the Laws of Everyday Life Completely Known?

The collection grants Carroll’s claim that the laws of everyday life are completely known, as he states it, and adds no force or field to those laws. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Section IV, “The Known Laws and the Single Outcome”.

General relativity says that gravity is the shape of space and time. *Two Theories, One World*, Chapter 1, “Two Theories That Never Fail”, section “The Theory of the Arena”.

The arena, space and time together with the shape general relativity gives them, is the setting in which every finding is made, and it is the context of physics. *Two Theories, One World*, Chapter 1, section “The Theory of the Arena”; *The Unquantized Arena*, Chapter 5, “The Content and Context Partition”, Section I, “The Partition and Its Ground”.

Every field that *The Story of the Particles* names, light included, is content in a wider sense: something found within the arena, as distinct from the arena itself. *The Story of the Particles*, Level 0, “The Three Carriers”, Section II, “The Comparing Act”.

A field is something spread through all of space, whose smallest ripples are the particles. *The Story of the Particles*, Level 0, Section II, “The Comparing Act”.

The structure built from comparison encodes the Standard Model coupled to gravity. *The Quantum Passage*, Chapter 6, “The Hilbert Space from the Comparison Algebra”, Section V, “From Finite to Infinite Dimensions”.

A force is a change, and a change is itself an act of comparison: it relates what was before to what is after. *The Story of the Particles*, Level 7, "The Three Forces", Section I, "Three Ways, One Per Position".

Structured content carries three things that a comparison can act on, the color of a quark, the two sides of a pair and the phase, and nothing else inside it for a change to act on, so there are exactly three forces. *The Story of the Particles*, Level 7, Section I, "Three Ways, One Per Position".

Among three colors there are nine independent transformations, and the one that shifts the phase of all three colors together, changing no color relative to another, is set aside, so the strong force has eight carriers. Between two sides there are four, and the one that shifts the phase of both sides together is set aside, so the weak force has three. A phase, a setting on a circle, has one, with nothing to set aside, so the force on the phase has one carrier. *The Story of the Particles*, Level 7, Section II, "The Strong Force: Eight Gluons", Section III, "The Weak Force: Three Bosons", Section IV, "The Force on the Phase: One Boson", and Section VII, "What We Have Found".

When you weigh yourself, you are mostly weighing the binding of quarks. *Why These Numbers*, Chapter 8, "Where the Proton's Mass Comes From", section "What You Weigh When You Weigh Yourself".

Hossenfelder and Carroll are right that quantum mechanics is not, at root, about discreteness. *Side by Side, Volume One*, "What Beauty Cannot Decide" (the chapter on Sabine Hossenfelder), Section VII, "Measurement and Discreteness".

An effective field theory is a description of particles and their forces at low energies that does not depend on the details of what happens at much higher energies. *Physics Is Not Math*, Chapter 15, "The Descent", Section II, "A Question the Circles Cannot Reach".

A result reached by two independent routes is held more firmly than a result reached by one. *Physics Is Not Math*, Chapter 12, "Chains and Cables", Section V, "Two Routes to One Value".

The Higgs field is required by the pairs of matter that the weak force turns into each other. The two sides of each pair are distinct, the weak force treats them as interchangeable, and so a field must hold, at every place, which side is which. *The Story of the Particles*, Level 8, "The Higgs", Section I, "The Edge Between Two Sides".

The existence of the Higgs field is structural, required before any experiment searches for it, and its position on the mass scale is measured. *The Story of the Particles*, Level 8, Section II, "Why Spin Zero?".

The laws treat the two sides alike, and the Incommensuration Theorem does not let that sameness be exact in empty space as well, so below a certain energy the Higgs field settles everywhere at once into a value that picks a side. *Why These Numbers*, Chapter 2, "What There Is, and Why It Is Built So", section "The Field on the Edge".

The kinds of particle that can be found within the arena are those of the Standard Model, and no further kinds exist: no fourth family, no partner particles of supersymmetry, no particle of gravity and no particle of dark matter of any type. *The Kinds of Particle*, Chapter 6, "The Sixteen States", Section IV, "No Further Kinds".

The Higgs is the final elementary particle, because it is the last thing that the structure of comparison requires before the composites begin. *The Story of the Particles*, Level 8, Section V, "The Higgs Particle".

Gravity is not a fourth force with a carrier: it is the arena, context and not content. *The Story of the Particles*, Level 7, Section VI, "What about Gravity?".

The collection's closed list of kinds reaches further than the Core Theory, which leaves open the kinds of particle that do not touch everyday matter. The record set beside Carroll states that relation in one clause: the closed list "goes further than the core theory, which leaves open kinds that do not touch everyday matter". *Side by Side, Volume One*, Appendix K, "The Registered Predictions Beside Sean Carroll", Section III, "Where the Two Accounts Agree", entry "Carroll 9".

This passage gives the place where the derivation of *The Quantum Passage*, Chapter 6, ends.

The chapter that builds the mathematics of quantum states from the structure of comparison names the place its derivation stops: at the numbers, the masses of the particles and the way they mix. *The Quantum Passage*, Chapter 6, Section V, "From Finite to Infinite Dimensions".

This passage gives the walk that Chapter 3 states only as a result: the collection's two steps to the Higgs field and to its value away from zero, each with its picture.

Look at the page in front of you. It has two faces, the one you are reading and the one behind it. The two faces are distinct, since they carry different words, and they are inseparable, since you cannot have the one without the other. Turning the page treats them alike: either face can be brought toward you, and the page is the same page whichever face you see.

Matter in the Standard Model comes in pairs built the same way. A *doublet* is a pair of particles that the weak force turns into each other: the up quark and the down quark, the two quarks of which protons and neutrons are made, and the electron and its neutrino. The two members of a doublet are distinct: the electron is not the neutrino, and the two differ in electric charge. Yet the weak force turns the one into the other, and a turning treats what it turns as interchangeable. *The Story of the Particles* (Level 8, "The Higgs", Section I, "The Edge Between Two Sides") calls that turning a rotation, and draws the consequence: "Something must hold the distinction that the rotation does not respect". It goes on: "There must be a field that lives on the edge separating one side from the other, a field that, at each point, holds which side is which". The edge is the line between the two sides of a doublet, the line the weak force crosses when it turns one side into the other, as the edge of the page lies between its two faces. That field is the Higgs field. "The Higgs is required by the inseparable distinctness of the two sides of every doublet", the book says. *The Story of the Particles*, Level 8, "The Higgs", Section I, "The Edge Between Two Sides".

In Section II of the same Level it adds: "Its existence is structural; its position on the mass scale is measured". Structural here means set by the structure of comparison itself, before any measurement, as the same section says: "The Higgs is required before any experiment searches for it". *The Story of the Particles*, Level 8, Section II, "Why Spin Zero?".

The second step gives the reason the field's value in empty space is not zero. At the value zero, the field would mark neither side of a doublet: the two sides would still be distinct, but empty space would not say which is which. The laws treat the two sides alike: exchange one side for the other, and the laws stay the same. That sameness is a symmetry in the sense of this book's Chapter 2, a sameness kept across a change. The change across which the laws stay the same is the exchange of the two sides of a doublet. The Incommensuration Theorem, which this book's Chapter 2 states and proves, says that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing.

The Story of the Particles applies the theorem to empty space at its lowest energy, which physicists call the *ground state*. Its sentence for the step is this: "The underlying laws treat the two sides alike, and the Incommensuration Theorem does not let that symmetry be perfected in the ground state as well". At the end of the same section it gives the step in the theorem's own terms: "no one comparison can be perfected as both symmetric and continuous. The underlying structure is symmetric: it treats both sides of the doublet equally. But the ground state breaks the symmetry: the condensate picks a side". The same section calls the field's settled value in empty space "a condensate, a background field that fills all of space". So empty space gives up the exact sameness of the two sides: the field settles, everywhere at once, into a value that picks a side, and a value that picks a side is a value other than zero. The laws still treat the two sides alike, and empty space, at rest, does not. *The Story of the Particles*, Level 8, "The Higgs", Section III, "What the Condensate Does".

Why These Numbers (Chapter 2, "What There Is, and Why It Is Built So", section "The Field on the Edge") states the step: "The laws treat the two sides alike, and the Incommensuration Theorem does not let that sameness be exact in empty space as well, so below a certain energy the field settles everywhere at once into a value that picks a side". That certain energy is far above any energy in a kitchen, so in the kitchen, and in all of empty space today, the field has settled. In *Why These Numbers* I picture the settling of the field with the bottom of a wine bottle. A ball balanced on the raised center rolls off into the ring-shaped valley around it and comes to rest at one point. The bottle is the same in every direction, as the laws are the same for both sides. The ball's resting place lies in one direction only, as empty space has picked a side. Carroll's upside-down pendulum, whose pivot is attached to the floor, is the same picture: it comes to rest leaning to the left or to the right of its pivot, on a floor that is the same on both sides. *Why These Numbers*, Chapter 2, "What There Is, and Why It Is Built So", section "The Field on the Edge"; *The Story of the Particles*, Level 8, Section III, "What the Condensate Does", where the wine bottle is also given. Carroll's pendulum is in *The Particle at the End of the Universe*, Chapter 8, "Through a Broken Mirror", section "Pushed away from zero".

So, on the collection's account, the Higgs field and its settled value are both required: the field by the two distinct sides of every doublet, the settled value by the theorem. Measurement supplies the mass of the field's particle, and the ATLAS and CMS experiments at CERN measured it. *The Story of the Particles*, Level 8, Sections I to III.

Chapter 4: If the Laws Are Known, Is Everything Already Fixed?

The collection grants the laws of everyday life as Carroll states them and adds no force or field to them. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

The Schrödinger equation carries the quantum state, the range of what could be found, into its successor without fail, and what a measurement finds is one outcome from that range, which the equation does not fix. *Measurement*, Chapter 12, "Time", Section VIII, "Questions".

On what the known laws fix, the collection and Carroll predict the same distributions, and so they expect the same odds from every experiment. *Side by Side, Volume One*, Appendix K, "The Registered Predictions Beside Sean Carroll", Section V, "Where No Measurement Decides", entry "What the known laws fix".

A finite observer cannot know its own future completely, because of three limits: the reach of light, the uncertainty relation, and the one direction in which every act of comparison runs. *Causality Without Determinism*, Chapter 3, "The Free Will Theorem", Section X, "The Limits of Knowing".

There is no cause of the universe: a cause is found by comparison within the world, and the whole has no outside to compare it with. *Causality Without Determinism*, Chapter 1, "Two Concepts, Three Differences", Section VI, "Where Causality Lives", which quotes *Measurement*, Chapter 2, Section II, on the whole that has no outside.

A description and the thing it describes always differ in kind, taking the two characters, symmetry and continuity, one each, and the description reaches what it describes as closely as one wishes without ever containing it. *Physics Is Not Math*, Chapter 8, "Physics Has the Arrow", Section VII, "Three Grounds".

Laplace's demon needs two things at once: one law carrying the complete state at each moment into the next, the same everywhere and always, which is symmetry made perfect, and that complete state specified with no gap, which is continuity made perfect. *Causality Without Determinism*, Chapter 1, Section VIII, "Why Determinism Cannot Be Had".

A whole world completely fixed by a law perfects symmetry and continuity together, whichever account describes it, and the theorem forbids that pairing. Among the accounts that describe a whole world as completely deterministic, the collection names the Everett account and Bohm's pilot-wave theory. *Physics Is Not Math*, Chapter 8, Section VII, "Three Grounds".

Determinism is not false as a matter of fact but unreachable as a matter of structure. *The Basis of Physics*, Essay 2, "Determinism and Causality", Abstract.

The theorem excludes a whole world fixed by a law and leaves the law itself standing: the collection's own quantum mechanics uses the same unitary evolution, the same deterministic equation, that Carroll uses. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

The solar system runs one way only: laser light bounced off the reflectors that the Apollo astronauts left on the Moon shows the Moon moving away from us by about 3.8 centimeters a year. *Physics Is Not Math*, Chapter 11, "Why the Order Surprises", Section I, "A Clockwork Solar System".

The ease with which an orrery runs backward is a property of the model, and Laplace's picture gives the world the features of Newton's equations. *Physics Is Not Math*, Chapter 11, Section I, "A Clockwork Solar System", and Section III, "One Source".

A complete knowledge of the laws and of the quantum state leaves the single outcome open. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, quoting *Questions from Physics, Answered*.

Hard randomness is an outcome that no law and no knowledge of what came before it, however complete, fixes. Hard randomness is not ignorance that better knowledge would remove, since there is no fuller specification for better knowledge to find. *Physics Is Not Math*, Chapter 8, Section III, "Three Things Bound Together".

Complete knowledge of every position, every momentum and every law would still not fix the value of a hard random measure. The omniscient cannot contain the transcendent. *The Basis of Physics*, Essay 6, "Hard Randomness", Section VIII, "Randomness, Choice, and the Real", where the omniscient is what the imagined knower holds: "every law of nature, every argument and every reason".

The omniscient is structure held in view as a whole, outside time, and the transcendent is what could be. *Causality Without Determinism*, Chapter 1, Section III, "Causality Defined".

Which atom of a radioactive lump goes next is fixed by nothing, since no atom carries a clock inside it that the others lack, and the law is exact about the many, and the finding is open for each one. *Two Theories, One World*, Chapter 18, "How Close a Smooth Theory Comes", section "An Open Future Is Real".

Microstate amplification, the growth of small differences at a fine scale into differences at the scale that matters, carries hard randomness up from the smallest scale to the scale of ordinary life. *Measurement*, Chapter 12, Section VI, "Where Asymmetry Comes From in a Complex System"; *Causality Without Determinism*, Chapter 3, Section VI, "The Catastrophic Theorem of Determinism", where Lorenz is quoted.

In a deterministic world all the information would be present in the complete state at the first moment, and every later complete state would only rearrange it. *The Last Turtle*, Chapter 12, "Causation Is Not Determinism", Section IV, "Where the Information Comes From".

Information is conserved without fail in reasoning from premises to conclusions, and the conservation of information under deduction is inherent in the idea of mathematics itself. *The Last Turtle*, Chapter 32, "The Incompressible Residue", Section II, "Information Is Conserved Under Deduction".

Carroll is a compatibilist: he holds that the laws leave no room for any influence beyond them, and that choice is a real and useful description at the level of people. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

The reading on which the laws fix each single event reads more into the laws than they hold: it takes a complete set of laws for a complete account of what happens. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome", reporting *Questions from Physics, Answered*.

To read the laws as fixing each single event is Laplace's demon, which needs symmetry and continuity made perfect together. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome", citing *Causality Without Determinism*, Chapter 1, Section VIII.

Chapter 5: When a Measurement Could Go Two Ways, Do Both Happen?

The quantum state always evolves by a linear equation, and there is no dynamics of reduction. *The Unquantized Arena*, Chapter 11, "The Measurement Problem as a Cross-Modal Residual", Section II, "The Residual".

What changes at a measurement is a record, the same for everyone who has the outcome, and that update is not a physical process and not an extra law of motion. *Why the Quantum Is the Way It Is*, Chapter 17, "Why a Measurement Has an Outcome", section "Why Collapse Is Not Physical".

Decoherence, credited to the work of Zeh and of Zurek, is reached from the division of every comparison into its content and its context. *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surroundings Do".

In decoherence the environment that carries the copies of which alternative occurred is the context of that content. *The Unquantized Arena*, Chapter 11, Section III, "Decoherence and Where It Stops".

The parts of the quantum state that make interference come to belong to the joint system and no longer to the part. They become out of reach for any measurement that probes the system alone, and they are not destroyed. *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surroundings Do".

The finding is made by the comparer that finds, the apparatus, as an interaction, and no mind is needed. *The Unquantized Arena*, Chapter 11, Section II, "The Residual".

No signal travels faster than light, by an argument from the speed of light taken as the outer edge of what can be compared. *Measurement*, Chapter 14, "The Consequences of the Theorem", Section IV, "Signaling Faster Than c Is Incoherent".

The achievement of the many-worlds account is economy and reach. *Why the Quantum Is the Way It Is*, Chapter 18, "The Interpretations, Each Missing a Role", section "Many Worlds".

To exist is to stand as structure, timeless and the same in every instance, and to be real is to be given in an actual act of comparison. A law exists, and a measurement is real. *Comparison; The Necessary Basis*, Chapter 10, "Reasoning", Section II, "The Real and the Abstract".

For every comparer, one outcome is real, the one found, and the many-worlds account takes the structure that exists for the whole of what is real. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

The universal state, the quantum state of the whole universe, branches and all, exists as structure, and for the one who finds, one outcome is real, the one found. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches", and Section IX, "Summary".

The quantum state exists as the complete description of the range of what could be found and of the probabilities, and the outcome is real: it is given in an actual act of comparison, and the quantum state does not specify it in advance. *The Unquantized Arena*, Chapter 11, Section II, "The Residual".

A list of odds is not an outcome. *Two Theories, One World*, Chapter 18, "How Close a Smooth Theory Comes", section "Where It Stops".

Decoherence prepares, and the act completes. *Why the Quantum Is the Way It Is*, Chapter 17, "Why a Measurement Has an Outcome", section "Summary".

The arrow of time, hard randomness and the subjective are three aspects of one act of comparison proceeding in a direction. *Measurement*, Chapter 14, Section VI, "Subjectivity, Directed Time and Hard Randomness Are Bound".

Each of the three needs the next: without hard randomness the order of moments is an indexing on one fixed whole and not a direction of process, and a finding needs a finder. *Measurement*, Chapter 14, Section VI, "Subjectivity, Directed Time and Hard Randomness Are Bound".

Every interpretation of quantum mechanics keeps the arrow of time, hard randomness and the subjective together or drops them together, and the textbook reading keeps all three. *Measurement*, Chapter 14, Section VI, "Subjectivity, Directed Time and Hard Randomness Are Bound".

The many-worlds account has no subjectivity, no hard randomness and no arrow of time, and the three absent features are the bound triple, absent together. *Incommensuration*, Chapter 15, "The Theorem Among Other Results", Section III, subsection "The Bound Triple Across the Interpretations".

In the Everett program the branching toward the future enters with the records and the way the universe began, and not from the universal equation. *Side by Side, Volume One*, "Tasks, Runs and Worlds" (the chapter on David Deutsch), Section V, "The Multiverse".

Decoherence supplies the basis by the stability of records, and the choice of that basis is the choice of which outcomes count as having occurred. *The Unquantized Arena*, Chapter 12, "The Interpretations as Modal Strategies", Section III, "Everett".

The basis has one chooser: the one who finds chooses it, in the acts that lay down records, and the surroundings that carry the record are the context of that act. In Chapter 5 I draw the claim that the basis has one chooser from two sentences of *The Unquantized Arena*. The first sentence says that the choice of the basis is the choice of which outcomes count as having occurred (Chapter 12, Section III, "Everett"). The second sentence says that the environment that carries the copies is the context of the content (Chapter 11, Section III, "Decoherence and Where It Stops").

The choice of basis and the odds of the outcomes are the two steps of the many-worlds account at which the act of finding comes back into the account. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IX, "Summary".

The three members of the bound triple are features of continuity with asymmetry, the character of context, so a description that holds only symmetry with discontinuity cannot hold the three, any more than a map can contain the experience of walking through the territory. *Incommensuration*, Chapter 15, Section III, subsection "The Bound Triple Across the Interpretations".

The Everett account is among the accounts of a whole world completely fixed by a law. A whole world completely fixed by a law perfects symmetry and continuity together, and the theorem forbids that pairing. *Physics Is Not Math*, Chapter 8, "Physics Has the Arrow", Section VII, "Three Grounds".

A theory with no first person can be a complete account of the third-person physics, and it cannot, by the theorem, also be a complete account of the conditions under which that physics is known. *Incommensuration*, Chapter 15, Section III, subsection "The Bound Triple Across the Interpretations".

The many-worlds formalism describes the third-person physics, the collection's own quantum mechanics uses the same unitary evolution, and the whole world also contains the acts of comparison by which anything in it is found. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

This passage gives what the argument of *Measurement*, Chapter 14, Section IV, that no signal travels faster than light, does not use.

Measurement reaches that result from the speed of light, taken as the boundary of what can be compared, and it says of its own argument that it "depends on no feature of quantum mechanics and on no theorem within it about signaling". *Measurement*, Chapter 14, Section IV, "Signaling Faster Than c Is Incoherent".

This passage gives, step by step, the second route that Chapter 5 names in one sentence: the Incommensuration Theorem's reason why every account keeps the arrow of time, hard randomness and the subjective together or drops them together.

The Incommensuration Theorem gives a second reason why every account keeps the three together or drops them together. We met the theorem in this book's Chapter 2: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity". The Incommensuration Theorem is a structural fact about the nature of knowing. Its four words keep the senses Chapter 2 gave them: symmetry is sameness kept across a change, continuity is no gap and no least step, asymmetry is nothing kept the same, and discontinuity is a jump, with nothing between before and after. To apply two of the four characters fundamentally is to hold both characters exactly, at the foundation of a description, with no exception at

any scale. The theorem leaves two kinds of comparison that can be taken to perfection, and this book's Chapter 2 gave each a picture. Symmetry with discontinuity is like an ice-cube tray, every cell the same as every other, with a wall between each cell and the next; continuity with asymmetry is like a ribbon of honey poured from a spoon, unbroken and never the same twice.

Incommensuration, Chapter 15, Section III, sets the three things in the crack beside those two kinds, and we can follow it at the bowl. What the cook finds, the crack along this line, is the content of her finding, and it is of the ice-cube tray's kind. Whoever looks into the bowl finds the same crack, a sameness kept across a change of who looks, which is symmetry; and the crack is one outcome, set off from every other way the shell could have broken, which is discontinuity.

When the cook sets the cracked egg she sees now against the whole egg she saw a moment before, now and a moment before are what make those two things two, so they are the context of her comparison. *Incommensuration* says that the three things in the crack, the side it is found from, the outcome no law fixed, and the one-way run of time, are features of continuity with asymmetry, and it calls continuity with asymmetry the character of context. Continuity with asymmetry is the honey's kind. In the same book's words, the character of context is one "whose continuity is lived as the persistence of the one who compares and whose asymmetry is laid out as the arrow of time". "Lived as" says where the continuity of the context shows: on the side the comparison is made from, in the one who compares, as the cook goes on without a break, one cook before the crack and after it. "Laid out as" says where the asymmetry of the context shows: on the side the comparison is about, as the arrow of time, with the whole egg before the cook's moment at the bowl and the cracked egg after it. Since all three things in the crack are features of that one character, a description that holds continuity with asymmetry holds all three, and a description that lacks it lacks all three. So a description that holds only symmetry with discontinuity, the ice-cube tray's kind, what is the same for everyone in separate countable pieces, cannot hold the three, "any

more than a map can contain the experience of walking through the territory". The theorem therefore says why the many-worlds account drops the three together: its description holds only content, what comparisons are about, with no side from which anything is compared.

The universal state changes smoothly, by its one equation, and the claim is about what that description describes, and not about the form of its equation. *Physics Is Not Math*, Chapter 8, "Physics Has the Arrow", Section VII, "Three Grounds", gives the rule for every description in use: "a description and what it describes always take the two characters one each". The two characters are the two kinds the theorem leaves, and the same section says of quantum theory: "Quantum theory describes stepwise, exactly repeating findings, one count at a time, with smooth equations: its description is continuous, and what it describes is symmetric". So the smoothness belongs to the equation, and what the equation describes, outcomes that come one count at a time and are the same for everyone who finds them, is of the ice-cube tray's kind. *Incommensuration*, Chapter 15, "The Theorem Among Other Results", Section III, subsection "The Bound Triple Across the Interpretations"; *Physics Is Not Math*, Chapter 8, "Physics Has the Arrow", Section VII, "Three Grounds".

Chapter 6: Where Do the Odds Come From?

The derivation of Sebens and Carroll and the collection's own derivation, through Gleason's theorem, give the same weights. *Side by Side, Volume One*, Appendix K, "The Registered Predictions Beside Sean Carroll", Section V, "Where No Measurement Decides", entry "Where the Born rule comes from".

The builder's triangle, with sides three, four and five, shows that the square works. *Why the Quantum Is the Way It Is*, Chapter 12, "Why the Square", section "Gleason's Theorem".

Under the square, the interference among three paths is fully accounted for by the paths taken two at a time, with nothing left over that belongs to the three together. *Side by Side, Volume One*, Appendix K, Section III, "Where the Two Accounts Agree", entry "Carroll 6".

No lawful description fixes every outcome found, and the single outcome of a quantum measurement is the plainest case of what the law leaves over: the quantum state gives the weight of each outcome, and the outcome found is not contained in it. *The Quantum Passage*, Chapter 7, "The Born Rule from the Structure of Comparison", Section II, "Why Probability Is Necessary".

The single event is unfixed, and its weight is stable. *Why the Quantum Is the Way It Is*, Chapter 11, "Why Chance Enters", section "From an Unfixed Outcome to a Stable Weight".

I identify the probability of an outcome with the stable relative frequency of that outcome in repetitions of the same comparison, on the same quantum state, in the same context, and that probability belongs to the world, not to our knowledge of it. *Why the Quantum Is the Way It Is*, Chapter 11, section "From an Unfixed Outcome to a Stable Weight".

Given one weight for one outcome and weights that add over outcomes that exclude each other, the Born rule follows as a theorem of mathematics and is no longer a postulate of physics. The question why the square then becomes the question why these two premises. *Why the Quantum Is the Way It Is*, Chapter 12, section "Gleason's Theorem", with the corner of the room in section "Two Things a Rule for Weights Must Do" and Busch's result for two directions in section "Every Sector, and the Whole Space".

The weights of outcomes that exclude each other add, by two routes that rest on different premises and give one result: counting repeated acts of finding, and the parts of a single comparison. *Why the Quantum Is the Way It Is*, Chapter 12, section "Why the Weights of Exclusive Findings Add".

By the law of symmetry, what is found as content is found as a sameness of content across a difference of context, so the same content found in two contexts is the same outcome, with one weight. *Why the Quantum Is the Way It Is*, Chapter 12, section "One Weight for One Finding".

The Born rule is the only rule that comparison allows. *Why the Quantum Is the Way It Is*, Chapter 12, section "The Three Layers".

The derivation runs from comparison to the Born rule without a probabilistic premise, and it rests on two stated identifications: the stable relative frequency, and one weight for the same content found in two contexts. *The Quantum Passage*, Chapter 7, Section III, "Gleason's Theorem", and Section VII, "Summary".

The observer of self-locating uncertainty is a comparer with a stance, before an act, facing a range of what it could find, and the collection, which begins with the comparer, has nothing to bring back. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

Self-locating uncertainty is the strongest answer given to the problem of probability, and it shows exactly where the act of finding comes back. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

What self-locating uncertainty offers is a place within a structure laid out whole, and not one outcome selected from among those that could have occurred, so the act is still missing. *Why the Quantum Is the Way It Is*, Chapter 18, "The Interpretations, Each Missing a Role", section "What the Interpretations Were Doing".

Deutsch and Wallace recover the Born weights as the credences that a rational agent must give to branches. *The Unquantized Arena*, Chapter 12, "The Interpretations as Modal Strategies", Section III, "Everett".

When every outcome occurs, a probability needs an agent who faces outcomes and weighs them, and the derivations from decision theory take such an agent and its choices as premises. *The Unquantized Arena*, Chapter 12, Section III, "Everett".

The Born rule is the bridge between the mathematics of the quantum state and the world of experiment, and a bridge cannot be built from one bank alone. *Why the Quantum Is the Way It Is*, Appendix B, "What Other Programs Assume", section "Other Derivations of the Born Rule".

This passage gives, for the triangle of Chapter 6, whose amplitudes are six tenths and eight tenths, the totals that the sizes and their fourth powers give.

The sizes themselves give a different total, fourteen tenths, and the fourth powers give about fifty-four hundredths. Only the squares add to one, and they add to one however the set of directions is turned. *Why the Quantum Is the Way It Is*, Chapter 12, section "Gleason's Theorem", makes the same comparison for two amplitudes of equal size, where the sizes add to about one and four tenths and the fourth powers to one half.

This passage gives the collection's second route to the first premise of Gleason's theorem, one weight for one outcome.

The collection reaches this premise by a second route as well: turning the floor edges about the upright edge leaves the upright edge where it is, so the turning cannot change how strongly the upright edge could be found. *Why the Quantum Is the Way It Is*, Chapter 12, section "One Weight for One Finding", where the turning is one that keeps sizes.

This passage gives the precision of the three-slit measurement of 2010 that Chapter 6 tells.

Urbasi Sinha, Christophe Couteau, Thomas Jennewein, Raymond Laflamme and Gregor Weihs found no part of the pattern that belongs to the three paths together: whatever such a part might be, it is smaller than about one hundredth of the ordinary interference of two paths. We now know that the weights go as the square, to that precision, in the very place where a rule of a higher power would differ from it. Sinha, U., Couteau, C., Jennewein, T., Laflamme, R. and Weihs, G., "Ruling Out Multi-Order Interference in Quantum Mechanics", *Science* 329, 418–421 (2010).

This passage gives what, on the collection's account, the derivation of the Born rule from decision theory by David Deutsch (1999) and David Wallace (2012) brings in.

Why the Quantum Is the Way It Is says that this argument "brings in an assumption equal to the rule". *Why the Quantum Is the Way It Is*, Appendix B, "What Other Programs Assume", section "Other Derivations of the Born Rule".

Chapter 7: Is Space Made of Something Quantum?

General relativity says that gravity is the shape of space and time. *Two Theories, One World*, Chapter 1, "Two Theories That Never Fail", section "The Theory of the Arena", where the eclipse of 1919 and the merger seen by LIGO are also told.

The arena is space and time together as the setting of every finding: every length is a length in it, and every duration is a duration in it. *Two Theories, One World*, Chapter 1, section "The Theory of the Arena".

Space is the pattern of which events are causally connected with which, and no container that exists before the events it holds. *What Cannot Be*, Chapter 12, "The Shape of What Can Reach What", section "What Space Is".

Every interaction is an act of comparison, and a cause is one kind of interaction, so the pattern of causal connections is the pattern of the comparisons that can be made. *What Cannot Be*, Chapter 12, section "Every Cause Is a Comparison".

Nothing known about gravity makes space and time discrete. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Abstract.

In the clock experiment of 2010, what was found was quantum, and where it was found was the arena. *Two Theories, One World*, Chapter 1, section "One Experiment, Both Theories".

The arena, space and time together, is the context of physics: everything that is measured is measured in it. *The Unquantized Arena*, Chapter 5, "The Content and Context Partition", Section I, "The Partition and Its Ground".

I identify gravity with the arena: space and time together, with the shape general relativity gives them, as the setting of every finding. *The Unquantized Arena*, Chapter 6, "The Category Error", Section II, "The Chain in Full", Table 6.1, step 7; *The Gravitational Passage*, Chapter 7, Section V, "Context Is the Arena".

What is found as content is perfected as discontinuous symmetry, which is the quantum character, and what general relativity finds of the geometry is continuous and asymmetric: each theory perfects one of the two pairings the theorem allows. *Two Theories, One World*, Chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It", section "The Objection From the Theory of Light", and Chapter 21, "Four Questions for Any Theory", section "The Second Question: One Description With Both Characters?".

The act of comparing has no break in it, and what is found keeps its sameness for everyone who finds it, so the unevenness that the theorem pairs with smoothness is carried by the setting, the context. *The Gravitational Passage*, Chapter 7, Section V, "Context Is the Arena".

The context curves where the content is, in proportion to the energy and momentum of the content. *The Gravitational Passage*, Chapter 12, "The Einstein Field Equations", opening.

That the arena is not quantized means two things: it has no least step, and nothing is a quantum of the arena itself, at any scale. *The Unquantized Arena*, Chapter 6, Section IV, "The Exact Scope", Proposition 6.3.

A quantum description of the arena would need a further setting in which to compare its two descriptions. *The Unquantized Arena*, Chapter 6, Section II, "The Chain in Full", Proposition 6.1.

To quantize the arena is to apply to context the treatment that belongs to content. *The Unquantized Arena*, Chapter 6, Section II, "The Chain in Full", Proposition 6.2.

The theorem permits two descriptions joined side by side, each keeping its own character, and forbids a merger. General relativity and quantum theory are two descriptions of one world, each capturing what the other leaves out. *The Unquantized Arena*, Chapter 6, Section V, "Conjugation, Not Unification", Proposition 6.4.

Carroll derives the arena from the universal state, the quantum state of the whole universe, and the collection holds that the arena is the setting of every quantum state. His program agrees with the first clause of Proposition 6.3 and runs against the second. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VI, "Space from Hilbert Space".

On Carroll's account the geometry of space is carried by quantum degrees of freedom of the universal state. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, "Where the Predictions Part, and Why", subsection "Whether the geometry carries quantum degrees of freedom of its own".

The premise in dispute is whether the geometry of space is part of what a quantum state describes, or the setting in which every quantum state is described. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, subsection "Whether the geometry carries quantum degrees of freedom of its own".

In canonical gravity the quantum state has nothing in which to evolve, because the context in which things happen has been put into the quantum state. *The Unquantized Arena*, Chapter 6, Section VI, "Canonical Gravity: The Arena Put Into the State".

Kept as context, the arena keeps its time, and the problem of time does not arise. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VI, "Space from Hilbert Space".

In the construction of Page and Wootters, time comes back when the clock is measured, and a reading is a finding. *The Unquantized Arena*, Chapter 10, "Three Times", Section VII, "Questions".

Decoherence supplies the basis in which records stay stable, and the one who finds chooses that basis, in the acts that lay down records. The surroundings that carry the record are the context of that act. *The Unquantized Arena*, Chapter 12, "The Interpretations as Modal Strategies", Section III, "Everett", and Chapter 11, Section III, "Decoherence and Where It Stops", as under Chapter 5 above.

Carroll holds that the setting of everything is a Hilbert space with a Hamiltonian and a given time. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, subsection "Whether the geometry carries quantum degrees of freedom of its own".

An account that places the arena in the universal state needs some other given setting in its place, and Carroll's premise is that a given time and a Hamiltonian are that setting. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, subsection "Whether the geometry carries quantum degrees of freedom of its own".

The two accounts agree that some setting is given, and differ on whether space belongs to it. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, subsection "Whether the geometry carries quantum degrees of freedom of its own".

This passage gives, step by step, the collection's reason why the arena curves where matter is, which Chapter 7 states as a result.

Why, then, does the setting curve where matter is? *The Gravitational Passage*, Chapter 7, "The Six Metrics", Section V, "Context Is the Arena", gives the reason. The act of comparing is continuous: to compare is to hold two things in one act, and that holding has no break in it. What is found is held to symmetry: it is the same for everyone who finds it, so that what many comparers find makes one history. The theorem pairs smoothness with unevenness, and the book says where the unevenness goes: "The asymmetry that the continuous act and the symmetric content can-

not carry is therefore carried by the context". To carry the unevenness, here, is to be the part of every comparison that differs from place to place. The setting is smooth, as the act is, and it gives up the sameness that what is found keeps, so the setting differs from place to place. That difference from place to place is curvature. The same book, Chapter 12, "The Einstein Field Equations", states where the curving is: "The context curves where the content is, in proportion to the energy and momentum of the content". The size and form of that curving are the ones general relativity gives.

Chapter 8: Why Does Time Run Only One Way?

The arrangements of the molecules in a glass of water are counted in the same way as the settings of a lock, where each added dial multiplies the number of settings by ten. *The Matter We Know, Volume Two*, Chapter 2, "The Counting of Ways", Section III, "Entropy, the Logarithm of the Count".

From any given moment, Boltzmann's counting predicts higher entropy in the future and also in the past. *The Compactification*, Chapter 3, "The Past Hypothesis at the Turn", Section I, "The Question".

Carroll is right that the arrow needs a reason, and right that a low-entropy beginning taken as a brute fact is not one. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section V, "The Arrow of Time and the Past Hypothesis".

Colliding particles carry traces of one another, and a return is found only where the traces have reached few particles, and never where they have reached many. *The Matter We Know, Volume Two*, Chapter 2, Section V, "Why Entropy Does Not Fall".

Symmetry of the laws guarantees lawful reversal and does not guarantee faithful reversal. *The Basis of Physics*, Essay 11, "The Structural Arrow of Time", Section III, "Lawful Reversal and Faithful Reversal".

A faithful reversal needs the complete state given with no gap, which is continuity perfected, beside laws perfectly symmetric, which is the forbidden pairing. So the starting condition of a faithful reversal can never be set. *The Basis of Physics*, Essay 11, Section III, "Lawful Reversal and Faithful Reversal".

Perfect time reversal is ruled out whatever condition the universe began in, with no appeal to the Past Hypothesis. *The Basis of Physics*, Essay 11, Abstract.

Boltzmann's answer to Loschmidt grants, without saying so, that what stands in the way of the reversed history is its starting condition, every velocity exactly reversed, and that the laws, being symmetric, already allow the reversed history. *The Basis of Physics*, Essay 11, Section III, "Lawful Reversal and Faithful Reversal".

The reversed path exists in the mathematics as a solution of the laws, and a gas in the faithfully reversed state never occurs in the world. *What Cannot Be*, Chapter 13, "Why Time Has a Direction", section "Why the Equations Hide the Gap".

The spin echo returns because the drift was held in each nucleus's own angle, one trace for each nucleus, which one pulse could reverse. *The Matter We Know, Volume Two*, Chapter 2, Section V, "Why Entropy Does Not Fall".

Evolution time is the time in the equations that carry the quantum state along, and event time is the order in which acts of finding are made, one after another, each laying down its record. What runs backward in the spin echo is the evolution of the spins, in evolution time, and not the order of findings. *Why Time Travel Is Not a Concept*, Chapter 5, "The Record Is Not Undone", Section IV, "Spin Echoes and Loschmidt"; event time is defined in *Measurement*, Chapter 12, "Time", Section II, "Three Objects Physics Calls Time".

The difference between forward and backward is built into the structure of comparison, and it is not a convention. *What Cannot Be*, Chapter 13, section "Time as Happening, and Not Only as a Line", where it is set against the block universe.

Every comparison runs in one order, from the range of what could be found, to the act, to the record, and the record shapes the next range. The order cannot be reversed. *Measurement*, Chapter 12, Section I, "One Direction, from the Single Stance".

The cycle that every act of comparison follows has the same direction wherever it is entered: its direction is the invariant, and its point of entry is not. *Incommensuration*, Chapter 8, "From the Theorem to the Axioms", Section IV, "The Direction of Axiom II, from the Theorem".

The direction of time is the direction in which comparison proceeds. *Incommensuration*, Chapter 8, "From the Theorem to the Axioms", Section X, "Many-to-One, the Arrow of Time and the Bound Triple".

I identify the one-way order of findings with the direction of time. *Measurement*, Chapter 12, Section III, "The Three Times as One Triple".

Every record ever found is a record of the past, and no instrument holds a record before the interaction that makes it. *Measurement*, Chapter 12, Section I, "One Direction, from the Single Stance".

The structural arrow is present in a single comparison, and it needs no large number of parts and no beginning of low entropy. *Measurement*, Chapter 14, "The Consequences of the Theorem", Section III, "Time Has a Structural Arrow".

Two roads reach one arrow: faithful reversal is impossible, and the cycle of comparison has a direction. *What Cannot Be*, Chapter 13, section "The Order of Every Comparison".

A record follows the act it records, because the act makes it, and that order holds whatever condition the universe began in. *Measurement*, Chapter 14, Section III, "Time Has a Structural Arrow", and Chapter 12, Section I, "One Direction, from the Single Stance".

The entropic arrow is statistical, found in systems of very many parts, and needs a beginning of low entropy. Each act of recording joins the two arrows in one event. *Measurement*, Chapter 12, Section VII, "The Two Arrows".

The structural arrow is the root of the direction of time, and the entropic arrow is its expression at the scale of eggs and engines. *Why Time Travel Is Not a Concept*, Chapter 7, "The Door Does Not Open", Section I, "The Two Arrows of Time".

The Past Hypothesis divides into two parts: its use as the source of the direction of time, and its content as a statement about how the universe we see began. The first part is not needed, since the structural arrow does not wait on it. *The Compactification*, Chapter 3, Section VIII, "The Magnitudes", subsection "Where the Past Hypothesis stands", which the section "Exact Forms" below quotes in full.

The direction of time does not depend on the beginning of the world; only the growth of entropy does. *What the Beginning Left Behind*, Chapter 6, "Two Arrows of Time", section "What This Means for the Beginning".

This passage gives the logarithm, by which the count of arrangements is written as entropy.

The logarithm of a number is the power to which ten must be raised to give that number. The logarithm of a thousand is three, the count of its zeros, and each added dial of the lock adds one to the logarithm of the number of its settings. Each added dial multiplies the number of settings by ten, and adds one digit to that number written out. *The Matter We Know, Volume Two* says it in one sentence: "Entropy is a count, made small enough to write by taking its logarithm". *The Matter We Know, Volume Two*, Chapter 2, "The Counting of Ways", Section III, "Entropy, the Logarithm of the Count".

In the Abstract of my essay "The Structural Arrow of Time", in *The Basis of Physics*, I state that perfect time reversal is impossible, and I name what the impossibility of perfect time reversal does not rest on:

> Perfect time-reversal is therefore precluded, and the preclusion depends on no entropy, no boundary condition, no Past Hypothesis, and no particular formalism.

The Basis of Physics, Essay 11, Abstract.

This passage gives the words in which the same essay says what Boltzmann's reply to Loschmidt, that the reversed starting condition is overwhelmingly improbable, grants.

Boltzmann's reply "concedes without saying so that the operative constraint is the specification of the state, all velocities exactly reversed, and not the symmetry of the laws". *The Basis of Physics*, Essay 11, Section III, "Lawful Reversal and Faithful Reversal".

This passage gives the structural arrow, the direction of time as the order in which every comparison proceeds, in the terms of the axioms of *Incommensuration*.

"The non-entropic arrow of time is the structural precedence of Axiom II applied to the physical domain". Axiom II states the order of precedence in every comparison: from the range of what could be found, to the act, to the record. *Incommensuration*, Chapter 8, Section X, "Many-to-One, the Arrow of Time and the Bound Triple".

Chapter 9: Why Did the Universe Begin So Orderly?

Over a hundred billion to a trillion years, every observer's sky empties. *The Compactification*, Chapter 5, "The Entropy of an Epoch", Section VIII, "The Emptying and the End of the Epoch".

Everything held together by gravity ends in one of two ways: it falls into a black hole, or it is carried away alone. *What Comes After the Dark*, Chapter 6, "What Happens to Someone Who Is Still There?", section "Two Ways to Be Left".

At the end of a black-hole interior both descriptions of physics, the smooth one and the quantum one, reach their edge. The edge belongs to the descriptions and is not an ending of what they describe, and the matter and light that fell in are still there. *What Comes After the Dark*, Chapter 8, "Is the Long Emptying the End?", section "What Is Left When the Sky Is Empty".

No signal from the end of an interior ever reaches the world outside, so nothing outside can place that end before, at or after any moment of its own. *What Comes After the Dark*, Chapter 6, section "Two Ways to Be Left".

At the end of an interior, the distant stars, the remaining black holes and the leftover radiation offer no comparison at all to anything there, because the world outside has no moment there at which to offer one. *What Comes After the Dark*, Chapter 6, section "Two Ways to Be Left".

Content and context are inseparable, so the matter still there at the end of the interiors has a context formed for it again: a new arena, a new space and time. *What Comes After the Dark*, Chapter 8, section "The First Ground: No Content Without a Context".

The interiors of different black holes stay distinct along their whole length, as funnels stay distinct along their walls, and they are joined at their ends by being indistinguishable. No comparison made in our epoch tells one end from another. *What Comes After the Dark*, Chapter 8, section "One End, and the Turn".

The new arena is new in every part: no direction of its space continues any direction of the old arena. So nothing of the old arena is left in the new one to carry a past across, and the new arena forms with no past of its own. *What Comes After the Dark*, Chapter 8, section "The First Ground: No Content Without a Context", and section "One End, and the Turn", where the new world "forms with no separate identity of its own".

No comparison made in the new epoch can therefore tell from which interior any part of its matter came. *What Comes After the Dark*, Chapter 8, section “One End, and the Turn”.

A difference that no comparison can register is no difference. *What Comes After the Dark*, Chapter 6, section “A Standard That Needs Something There”.

No comparison, before the end or after it, registers a difference between the end of one interior and the end of another, so the ends are one end, while the interiors stay many. The body carried away alone comes to the same turn by its own road, with its matter still there. *What Comes After the Dark*, Chapter 8, section “One End, and the Turn”.

No comparison registers the turn itself as an event, since registering it would need a comparison made across the turn, a before set beside an after. *What Comes After the Dark*, Chapter 8, section “One End, and the Turn”, and Chapter 6, section “No Last Tick”.

The turn is a limit approached from both sides and reached from neither. *What Comes After the Dark*, Chapter 11, “Has This Happened Before?”, section “A Cycle With No First Turn”.

I identify the re-forming of the arena at the turn with the Big Bang of the new epoch. Every history of the universe is pieced together from the light and the records that reach the present, and the history that the new epoch’s observers piece together runs back toward the turn and has its beginning there. So the Big Bang of the new epoch is the turn as that history shows it. *What Comes After the Dark*, Chapter 8, section “One End, and the Turn”, which states the identification as the restoring of the outside world.

In both accounts our epoch is one of a sequence with no first member, and the arrow arises within the sequence, with no first moment at which it is imposed. *Side by Side, Volume One*, “The Wave Function and the One Who Finds”, Section V, “The Arrow of Time and the Past Hypothesis”.

Carroll is right that the arrow needs a reason, and right that a low-entropy beginning taken as a brute fact is not one. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section V, "The Arrow of Time and the Past Hypothesis".

The new arena forms at the turn with no past of its own, so its geometry begins even, apart from small departures from evenness. The Incommensuration Theorem requires the departures, since a beginning perfectly the same everywhere and perfectly smooth would perfect symmetry and continuity together, and the account takes the size of the departures from measurement. *The Compactification*, Chapter 3, Section VI, "The Even Beginning", and Chapter 6, "The Primordial Departures", Section VII, "The Amplitude and the Tilt Are Not Fixed".

The gravitational part of the entropy of the beginning is low because the geometry of a new context has nothing to be lumpy with, and the matter begins hot and close to equilibrium. *The Compactification*, Chapter 3, "The Past Hypothesis at the Turn", Section VI, "The Even Beginning".

An entropy is a count of the states of a system that the comparisons available from some context cannot tell apart. Every count of an epoch is made within that epoch's arena, and no count crosses the turn. *The Compactification*, Chapter 3, Section V, "Every Count Has a Context".

The low entropy of the beginning is no unlikely draw: it is the count that a new context starts with. *The Compactification*, Chapter 3, Section VIII, "The Magnitudes", subsection "The improbability estimate".

Carroll and Chen's proposal and the collection part on two points: whether entropy grows without bound across the whole ensemble of offspring universes, and whether our epoch began by inflating. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section V, "The Arrow of Time and the Past Hypothesis".

Disorder does not pile up, because no count crosses the turn. *What Comes After the Dark*, Chapter 11, "Has This Happened Before?", section "Does Disorder Pile Up?".

The epochs follow one another in a single line, and not as a branching tree of universes. *What Comes After the Dark*, Chapter 11, section "A Cycle With No First Turn".

Carroll and Chen's entropy, growing without bound across the whole ensemble of offspring universes, requires a count of states kept across all of them. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section V, "The Arrow of Time and the Past Hypothesis".

The long emptying cannot come to rest in a featureless emptiness, which would be perfectly symmetric and perfectly continuous at once. *What Comes After the Dark*, Chapter 8, section "The Second Ground: A Featureless Emptiness Is Never Found".

The exact symmetry of de Sitter space belongs to the model, and not to anything a comparison finds. *The Compactification*, Chapter 3, Section III, "What the Axioms Do Not Do".

Carroll showed that, on his two assumptions, inflation cannot by itself be the source of the low entropy of the beginning, and in the collection's account the low entropy of gravity at the beginning comes from the turn. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, "Where the Predictions Part, and Why", subsection "Whether our epoch began with inflation", which cites Carroll and Tam (2010); *The Compactification*, Chapter 3, Section VI, "The Even Beginning".

The account answers every puzzle inflation was introduced to solve, from the turn and from the structure of the forces, so it needs no inflation. The sameness of the sky and the flatness of space are answered from the turn, and the monopoles from the forces, which in the collection were never one larger force. *What the Beginning Left Behind*, Chapter 13, "No Inflation Needed", opening, and sections "The Sameness and the Flatness, Answered From the Turn" and "No Monopoles, No Strings, No Walls".

The faint ripples in the oldest light are the form of the matter that the black holes of the epoch before ours took in: what that matter is, averaged into one faint pattern and carried through the turn. The pattern carries no mark of which black hole any of that matter came through. *What Comes*

After the Dark makes the identification in the first person: "I identify the departures from uniformity with the form of the content carried through the turn: the distinctions of the epoch before ours, averaged". *What Comes After the Dark*, Chapter 9, "What Crosses Over, and What Stays Behind?", sections "The First Faint Pattern of the Next Sky" and "The Faint Pattern".

The account needs no inflationary phase, and it excludes a long one. A long inflation would have made the faint ripples itself, out of the trembling of the empty space it swelled, so an account in which the ripples come through the turn excludes a long inflation. *The Compactification*, Chapter 1, Abstract, and Chapter 6, "The Primordial Departures", Section VIII, "Inflation".

Both accounts agree with what the Planck satellite and the BICEP and Keck Array telescopes found. In the collection's account the matter carried through the turn ripples the density of the new epoch, and waves of the geometry arise from those ripples only very faintly. The forms of inflation most favored today also allow waves of the geometry too faint for any present measurement to find. *The Compactification*, Chapter 6, Section V, "Scalar at First Order, Tensors at Second Order", and Section IX, "Comparison with Observation".

Carroll and Chen's offspring universes begin by inflating, and in the collection our epoch begins at the turn. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section V, "The Arrow of Time and the Past Hypothesis".

An epoch begins with its geometry even, structure then forms in it and raises the entropy of its geometry, black holes form, and the epoch ends as its outside world empties. *The Compactification*, Chapter 5, "The Entropy of an Epoch", Section XI, "Conclusion".

The collection reaches Carroll's result, that complexity rises and then falls while entropy only grows, by its own reasoning. The history of an epoch runs the course of Carroll's cup of coffee, from an even beginning, through stars, galaxies and black holes, to an emptied sky. *Side by Side, Volume Two*, "Janus and the Turn" (the chapter on Julian Barbour), Section

V, "The Janus Point and the Turn", sets the growth of complexity beside the growth of entropy. Carroll's cup is in *The Big Picture*, Chapter 28, "The Universe in a Cup of Coffee".

The next epoch begins with the same orderly start. *What Comes After the Dark*, Chapter 7, "Can a Mind Keep Thinking for Ever?", section "Where Thinking Begins Again".

This passage gives, in full, the walk that Chapter 9 states as one paragraph of plain result. The walk shows how the collection answers each puzzle inflation was introduced for, why it excludes a long inflation, and which two measurements of the oldest light bear on these answers.

Carroll showed that, on his two assumptions, inflation cannot by itself be the source of the low entropy of the beginning, and the collection agrees with him that inflation is not that source. In the collection's account the low entropy of gravity at the beginning comes from the turn, and inflation plays no part in it. Then the collection goes further, and finds no need for inflation at all. *What the Beginning Left Behind* opens its chapter "No Inflation Needed" with the claim: "The account answers every puzzle inflation was introduced to solve, from the turn and from the structure of the forces, so it needs no inflation". Its picture for the sameness of the sky is two towns: "Two towns far apart may share an accent without ever having spoken to each other, if they were founded from one place". Far-apart regions of the sky are alike because they come from one place, the one end that all the black-hole interiors of the old epoch reach. Carroll's point is in *From Eternity to Here*, Chapter 14, "Inflation and the Multiverse", sections "What Good Is Inflation?" and "Setting the Stage"; the collection's, in *The Compactification*, Chapter 3, "The Past Hypothesis at the Turn", Section VI, "The Even Beginning", and *What the Beginning Left Behind*, Chapter 13, "No Inflation Needed", opening and section "The Sameness and the Flatness, Answered From the Turn".

Flatness follows from the same turn. The surface of a ball curves along a length, its radius: the smaller the ball, the sharper the curve. A curvature of space is set in the same way by a length, and no length of the

old epoch passes the turn, so nothing is left to set a curvature, and space begins flat (*The Compactification*, Chapter 1, Section VIII). *The Compactification*, Chapter 1, "The Re-Forming of the Internal Space", Section VIII, "The Temperature at Which the History Begins", subsection "No inflationary phase is needed, and a long one is excluded"; *What the Beginning Left Behind*, Chapter 13, section "The Sameness and the Flatness, Answered From the Turn".

In the collection's account the three forces were never one larger force, so nothing broke apart in the way that makes monopoles, and Guth's monopole puzzle does not arise (*What the Beginning Left Behind*, section "No Monopoles, No Strings, No Walls"). *What the Beginning Left Behind*, Chapter 13, "No Inflation Needed", section "No Monopoles, No Strings, No Walls".

The collection identifies the faint ripples in the oldest light with the form of the matter that the black holes of the epoch before ours took in, averaged, and carried through the turn. What crosses the turn is the form of that matter: what that matter is, averaged into one faint pattern, with no mark of which black hole any of it came through. *What Comes After the Dark* gives both in the chapter "What Crosses Over, and What Stays Behind?", sections "The First Faint Pattern of the Next Sky" and "The Faint Pattern". On the collection's account, then, the ripples were not made by an inflation. A long inflation, one long enough to stretch every length we now see in the sky out of one small patch, would have made those ripples itself, so an account in which the turn makes them excludes a long inflation. *The Compactification* states the position in one line: "The account needs no inflationary phase, and it excludes a long one" (Chapter 1). *Side by Side, Volume One* names where the two accounts part: "They part on what makes distant regions alike" (Section VIII). The collection holds that distant regions are alike because they come from one place, the common end of the old epoch's black-hole interiors, carried through the turn, and that the faint ripples are the form of the matter gathered there. Carroll and Chen hold, in the same section's words, that what makes them alike is "one inflating region, spawned from a quiet de Sitter state". *What Comes*

After the Dark, Chapter 9, "What Crosses Over, and What Stays Behind?", sections "The First Faint Pattern of the Next Sky" and "The Faint Pattern"; *The Compactification*, Chapter 1, Abstract, and Chapter 6, "The Primordial Departures", Section VIII, "Inflation"; *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, "Where the Predictions Part, and Why", subsection "Whether our epoch began with inflation".

Two measurements of the oldest light bear on these answers. The European Space Agency's Planck satellite, launched on 14 May 2009, scanned the whole sky from August 2009 to October 2013 and mapped the microwave background. Its final results, released in 2018 and published in 2020, found the ripples at about one part in a hundred thousand, of nearly the same strength at every length. The same results found space flat, within the small margin of the measurement. We now know that space is flat and that the sky is alike in every direction apart from its faint ripples, and both accounts, inflation and the turn, answer that flatness and that sameness. The second measurement looked for something else. The BICEP2, Keck Array and BICEP3 telescopes at the South Pole watched the polarization of the oldest light, the direction in which its waves are lined up. They were looking for a faint twisting pattern that *gravitational waves*, ripples in the geometry of space itself, would leave if the first moments had made them. The results, with data through the 2018 season, were published in *Physical Review Letters* on 4 October 2021: no such waves were found. The published analysis combined the maps of those telescopes with the maps of Planck and of an earlier satellite of NASA, the Wilkinson Microwave Anisotropy Probe (WMAP). We now know that any waves of the geometry left from the first moments are too faint for those telescopes to find. Both accounts agree with that result. In the collection's account the matter carried through the turn ripples the density of the new epoch, and waves of the geometry arise from those ripples only very faintly, far more faintly than those telescopes can see (*The Compactification*, Chapter 6, Section V). The forms of inflation most favored today also allow waves too faint for those telescopes to find (same chapter, Section IX). Planck Collaboration (Aghanim, N. and others), "Planck 2018 results. VI. Cosmolo-

gical parameters", *Astronomy and Astrophysics* 641, A6 (2020); BICEP/Keck Collaboration (Ade, P. A. R. and others), "Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season", *Physical Review Letters* 127, 151301 (2021); *The Compactification*, Chapter 6, "The Primordial Departures", Section V, "Scalar at First Order, Tensors at Second Order", and Section IX, "Comparison with Observation".

This passage gives the monopole puzzle as the theories that unite the forces pose it, and the finding of the collection's books on the kinds of particle on which the collection's answer rests.

The monopoles are the third puzzle. The theories that unite the forces hold that the strong force, the weak force and electromagnetism were, at very high energy, one force, which broke apart into the three as the universe cooled. Their calculations find that the breaking leaves magnetic monopoles behind, far too many of them. The collection's books on the kinds of particle find the symmetries of the three forces side by side, the same at every energy, with no larger symmetry that broke. *What the Beginning Left Behind*, Chapter 13, "No Inflation Needed", section "No Monopoles, No Strings, No Walls", which carries "The particle books of this collection find no grand unification" and "The symmetry of the forces is three symmetries side by side, the same at every energy". Carroll's account of the monopole problem is in *From Eternity to Here*, Chapter 14, "Inflation and the Multiverse", section "Magnetic Monopoles".

This passage gives the figures of the Planck and BICEP/Keck measurements of the oldest light, and the collection's second reason against a long inflation, from the dark matter.

Planck's final results put the curvature of space, measured as a share of the universe's whole energy budget, at one thousandth, give or take two thousandths. Planck Collaboration (Aghanim, N. and others), "Planck 2018 results. VI. Cosmological parameters", *Astronomy and Astro-*

physics 641, A6 (2020), abstract. The curvature figure is the one the paper gives for the Planck data combined with measurements of baryon acoustic oscillations.

The BICEP2, Keck Array and BICEP3 telescopes, with data through the 2018 season, bounded the power in waves of the geometry below thirty-six thousandths of the power in the density ripples. We now know that any waves of the geometry from the beginning are at most that faint, and so the energy of any inflation that made the ripples is capped. BICEP/Keck Collaboration (Ade, P. A. R. and others), "Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season", *Physical Review Letters* 127, 151301 (2021). The bound is on the ratio of the power in gravitational waves to the power in the density ripples, at 95 percent confidence.

The collection excludes a long inflation a second time, from what it finds the dark matter to be: the unseen matter whose gravity holds the galaxies together is made, in the collection, only at the turn (*The Weight of Possibility*, Chapter 4, Section VI). After an inflation that made the ripples we see, the new epoch would never have been hot enough to make the dark matter in the amount that is measured. *The Weight of Possibility*, Chapter 4, "Why the Dark Persists, and Its Gyrations", Section VI, "The Number of the Gyrations"; *The Compactification*, Chapter 6, "The Primordial Departures", Section VIII, "Inflation".

That second reason uses the bound that the South Pole telescopes set. In the collection, the dark matter is made of gyrations: lasting vibrations of the small dimensions that the world's geometry holds besides space and time, each with about ten billion billion times the mass of a proton. *The Weight of Possibility* states where they come from: "The gyrations are made only at the turn" (Chapter 4, Section VI). The same section draws the consequence: after an inflation that made the observed ripples, the measured bound on such waves "would keep the temperature far below this limit, and dark matter of this mass could not be made after that inflation in the measured amount". The limit is the highest temperature the hot matter of the new epoch can have had, about one thirtieth of the mass

of a gyration taken as an energy. Two strands from two sources, the ripples and the dark matter, reach one conclusion: our epoch did not begin with a long inflation, and it began at the turn. *The Weight of Possibility*, Chapter 4, "Why the Dark Persists, and Its Gyrations", Section VI, "The Number of the Gyrations"; *The Compactification*, Chapter 1, Section VIII; *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VIII, subsection "Whether our epoch began with inflation".

This passage gives the words in which the collection sets the growth of complexity beside the growth of entropy, and a point it adds about how the tendrils of the cup form.

Side by Side, Volume Two, writing of the physicist Julian Barbour, says that the growth of complexity "belongs with the collection's second arrow: like the growth of entropy, it is found in systems of very many parts and needs a state of least structure to grow from" (chapter "Janus and the Turn", Section V). The second arrow is the entropic arrow of this book's Chapter 8. *Side by Side, Volume Two*, "Janus and the Turn" (the chapter on Julian Barbour), Section V, "The Janus Point and the Turn".

The collection adds a point about how the tendrils form. *Side by Side, Volume Two*, writing of Ilya Prigogine, says: "Far from equilibrium, a flow of energy can hold a pattern that equilibrium never would, and at the point where several patterns become possible the system takes one of them" (chapter "The Arrow Comes First", Section VI). Which pattern is taken, the same section says, is settled in the event itself, by no law laid out in advance. *Side by Side, Volume Two*, "The Arrow Comes First" (the chapter on Ilya Prigogine), Section VI, "Order Out of Chaos", which reads: "which branch is taken is settled in the event itself, by no law laid out in advance".

Chapter 10: Can I Choose?

"Free", in the free will theorem, means not a function of anything that existed before the event. The theorem rests on four assumptions: quantum mechanics holds for the particles, no influence travels faster than light, no later event fixes an earlier setting, and nothing from before that is available to the particle fixes the experimenter's choice of setting. *Causality Without Determinism*, Chapter 3, "The Free Will Theorem", Section I, "The Theorem", and Section II, "The Four Assumptions".

Poetic naturalism and the collection agree that there are many good descriptions of the world, each fitted to its domain, and that no description of people or reasons needs a force that physics lacks. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VII, "What Lies at the Ground".

True choice is not something added to nature from outside. *Home in the World*, Chapter 7, "The Open Future", section "True Choice".

Causality, by its nature, does not reach into the microscopic, and the physics of the microscopic has no need of it. Bertrand Russell observed in 1913 that advanced physics does not use the word "cause". *Causality Without Determinism*, Chapter 1, "Two Concepts, Three Differences", Section XI, "The Distinction in Its Tradition".

Causality is the subjective perception (context) of a consistency between two objective contents, as *An Immanent Metaphysics* defines it. *Causality Without Determinism*, Chapter 1, Section III, "Causality Defined", quoting *An Immanent Metaphysics*, aphorism 1.61-1.

A player piano, whose paper roll fixes every note before the first is struck, is a picture of determinism, and the pianist a picture of causality. Causality requires an observer, and determinism does not. *Causality Without Determinism*, Chapter 1, Section IV, "Three Differences".

Causality, determinism and teleology, the coherence of a whole, are not competing descriptions of the same level of reality. *Causality Without Determinism*, Chapter 1, Section V, "The Scales of Description".

Every finite knower meets three limits, the reach of light, the limit on how sharply a particle's position and motion can both be fixed, and the one-way order of every act of comparison, and so cannot know its own future completely. *Causality Without Determinism*, Chapter 3, Section X, "The Limits of Knowing", as under Chapter 4 above.

The collection grants the Core Theory as Carroll states it and adds no force or field to it. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

A complete knowledge of the laws and of the quantum state leaves the single outcome open, and the collection's answer removes one step only: the step from a complete law to a complete world. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

Laplace's demon needs symmetry and continuity perfected together, which the theorem forbids, and what the theorem leaves standing is continuity paired with asymmetry, an unbroken order that runs one way, which every cause has. *Causality Without Determinism*, Chapter 1, Section VIII, "Why Determinism Cannot Be Had".

The experimenter's freedom is secured on two bases, the Incommensuration Theorem and the practice of physics, and the second basis uses nothing from the first. *Causality Without Determinism*, Chapter 3, Section III, "The Conclusion".

The Bell tests of 2015 exclude every account in which each outcome is fixed in advance by causes local to the place where it is found. *The Last Turtle*, Chapter 16, "Hard Randomness", Section II, "Three Results, One Conclusion".

Soft randomness is randomness of ignorance, which better knowledge could reduce: the outcome was fixed, and it is not known. *The Last Turtle*, Chapter 16, Section I, "What Hard Randomness Is".

Once determinism is impossible, the question is what choice is in a world that is causal and open. *Causality Without Determinism*, Chapter 1, Section XI, "The Distinction in Its Tradition".

The evolution is deterministic in the formalism, which exists as structure, and not in the findings, which are real. *Measurement*, Chapter 12, "Time", Section VIII, "Questions".

Because the world is causal, what is chosen has effects; because it is not determined, the one who chose could have chosen otherwise. *Causality Without Determinism*, Chapter 6, "Choice, Creation, and the Soul", Section VIII, "Questions".

The free will theorem links the first-person freedom of the experimenter, who acts from inside at its own scale, and the third-person indeterminacy of the outcome, observed from outside at another scale. *Causality Without Determinism*, Chapter 3, Section IV, "Person-Modes and Scale".

Causation from above and causation from below are the two directions of one interdependence of levels, neither prior, and choice is not a higher level acting on a lower one. The collection finds slack, some part of what happens that the description of one level leaves unfixed, at every scale, because no deterministic description, at any scale, can be complete. *Side by Side, Volume Two*, "The Open Future" (the chapter on George Ellis), Section IV, "Causation from Above and from Below".

The triple of the real is causality, change and choice. Change is the actual process of transition, in which pattern and novelty meet, and choice is the emergence of the new, the singular act that no pattern contains in advance. *The Last Turtle*, Chapter 13, "Choice, Change, and Causation", Section I, "The Three Concepts of the Real"; *Causality Without Determinism*, Chapter 6, Section II, "Change, Causality and Choice".

Choices start changes, changes can be understood causally, and causal patterns set up the conditions for new choices. *The Last Turtle*, Chapter 13, Section IX, "Projections of One Another".

Causality runs from earlier to later because it is the understanding of changes that choices began. *Causality Without Determinism*, Chapter 6, Section II, "Change, Causality and Choice".

One event described from outside is randomness and from inside is choice. Both descriptions are correct, and neither is complete. *The Last Turtle*, Chapter 13, Section IV, "One Event, Two Descriptions".

A choice needs a range of possibilities, a selection and a consequence that cannot be undone. *The Last Turtle*, Chapter 13, Section III, "What a Choice Requires".

A habit is a choice made so often that it runs almost wholly as causality, and a turning point in a life is causality bent almost wholly by choice. *Causality Without Determinism*, Chapter 6, Section IV, "The Conjugacy of Choice and Causality".

However good a compression of a person's behavior is, a residue remains that cannot be compressed, and that residue is what the person regards as their choice. *The Last Turtle*, Chapter 14, "Free Will and the Compression Argument", Section III, "The Compression Argument".

Physics gives the context, the constraints and the space of possibilities. *The Last Turtle*, Chapter 14, Section VI, "Pain and Joy".

Choice is the member of the triple of the real that a law of the many does not fix. Its place is the single outcome the laws leave open, and it is no extra force pushing on the particles of a brain. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

This passage gives the names of the free will theorem's assumptions, as *Causality Without Determinism* records them, and the later, stronger form of the theorem.

In 2009 Conway and Kochen published a stronger form of the theorem, with one of its assumptions weakened, in the *Notices of the American Mathematical Society*. The assumptions of 2006 are named SPIN, TWIN and FIN, and the stronger form replaces FIN with the weaker assumption MIN.

Causality Without Determinism, Chapter 3, Section II, "The Four Assumptions"; Conway, J. H. and Kochen, S., "The Strong Free Will Theorem", *Notices of the American Mathematical Society* 56, 226–232 (2009).

Chapter 11: Who Is Doing the Finding?

A thermostat compares the warmth of a room with the warmth it is set for, and instruments compare as surely as minds do. *Home in the World*, Chapter 2, "Learning About B by Looking at A", section "The Simplest Act".

An observer is a comparer: the word names a position inside the world, and it never names a mind. *Measurement*, Chapter 1, "What This Book Takes from the First Two", Section I, "The Ground of the First Volume".

Poetic naturalism and the collection agree that there are many good descriptions of the world, each fitted to its domain, and that no description of people or reasons needs a force that physics lacks. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VII, "What Lies at the Ground".

Consciousness is not an illusion, nor is it in conflict with conventional science. *An Immanent Metaphysics*, Technical Appendix, "Cartesian Dualism", aphorism 2M43.

The subjective and the objective sides of every comparison are distinct and are had together or not at all. The zombie asks for the objective side of a person's every comparison without the subjective side, so it asks to separate the two, and the conception fails at a level deeper than physical law. In the collection's own terms, the zombie scenario is a request to separate the objective pole of comparison from the subjective pole, which the third axiom forbids. *Side by Side, Volume Two*, "The Measure of an Inside" (the chapter on Giulio Tononi), Section II, "What the Collection Credits", quoting *The Immanent Metaphysics and the Modern Engagement*, Chapter 8.

A structure of distinctions and relations, held in view as a whole, is a record: the laid-out side of the act. *Side by Side, Volume Two*, "The Measure of an Inside", Section VI, "Distinctions, Relations and the Act of Comparing".

Consciousness is neither the world perceived nor the perceiver but the act of perceiving, wherever that act occurs as a first-person point of view. Consciousness is not a substance and not a property of matter. *Causality Without Determinism*, Chapter 2, "Non-Polarization; The Chain of Distinctions", Section IX, "Consciousness Between the Poles".

Consciousness gathers the perceiving and the choosing alike: "consciousness is everything one perceives, taken together; it is where one's choices gather". *Causality Without Determinism*, Chapter 2, Section IX, "Consciousness Between the Poles".

An instrument has a stance as surely as a person does. *Home in the World*, Chapter 3, "Three Parts in Every Finding", section "Two Words to Hear Freshly".

The arrow of time, hard randomness and the subjective are three aspects of one act, comparison proceeding in a direction, registered by a perspective, with an outcome not fully determined. The binding of the three is derived from the Incommensuration Theorem and found in physics. *The Last Turtle*, Chapter 15, "The Bound Triple", Section III, "The Binding Derived", and Section IV, "The Binding Found in Physics".

Every account of quantum measurement keeps the three together or drops them together. *Measurement*, Chapter 14, "The Consequences of the Theorem", Section VI, "Subjectivity, Directed Time and Hard Randomness Are Bound".

The many-worlds interpretation drops all three at once: it has no fact of which branch a comparer is in, no outcome that is not fixed, and a fundamental equation symmetric in time. *The Last Turtle*, Chapter 15, Section IV, "The Binding Found in Physics".

Consciousness is the bound triple taken as one act, with the subjective as the interior face of that act. *The Last Turtle*, Chapter 15, Section IX, "Consciousness Named".

In a conscious being, the subjective is the fact that there is something it is like to be that being. *The Last Turtle*, Chapter 15, Section I, "Three Features of One Act".

What is new in kind at the level of a living being is the gathering of choice into a self that knows it from within. *Home in the World*, Chapter 8, "The Bottle, the Note and the Living", section "A Village That Is Alive".

A thermostat compares and has a stance, but it is pushed toward its setting, and nothing in that pushing shows that anything was selected, or that anything is gathered. *Home in the World*, Chapter 8, section "A Village That Is Alive".

In a brain near criticality, indeterminacy at the smallest scale enters chains of causes that reach the scale of everyday things and bear on choice. *The Last Turtle*, Chapter 15, Section VIII, "The Interior Face, and the Brain Near Criticality".

A single undetermined outcome, described from outside, is hard randomness, and described from inside, from the position of the one engaged, it is choice. The criticality of the brain is the physical circumstance under which the relation of an outcome's inside, a choice, to its outside, hard randomness, is expressed in us, and not the relation itself. *The Last Turtle*, Chapter 15, Section VIII, "The Interior Face, and the Brain Near Criticality".

A complete description of pain is knowledge laid out as structure, what pain feels like is reached only in one's own perceiving, and the explanatory gap is a structural feature of comparison itself. *The Last Turtle*, Chapter 15, Section II, "Why the Hard Problem Is Hard".

Perception cannot perceive perception, observation cannot observe observation, and there can be no purely physical theory of the process of observation itself. *An Immanent Metaphysics*, Chapter 9, "Evolution", section "Natural Consciousness", aphorism 2.52-2.

This passage gives a further identification that *The Last Turtle* reports in Chapter 15, Section VIII, and that this book's Chapter 11 does not use. *The Last Turtle* says that the Immanent Metaphysics makes it, and that it is stronger than the claim that one undetermined outcome is hard randomness described from outside and choice described from inside. It states the identification as the claim that the subjective and total entanglement are strict isomorphs.

The subjective is the interior face of a process that, from the exterior, presents as entanglement and hard randomness. *The Last Turtle*, Chapter 15, Section VIII, "The Interior Face, and the Brain Near Criticality".

Chapter 12: Do We Make Meaning Up?

Science and technology tell us what can be done, and by their own methods they do not tell us what ought to be done. *The Last Turtle*, Chapter 9, "Three Orders of Knowledge", Section IX, "The Boundaries of the Scientific Method".

Science is the method that maps what causes what, by tests that can be repeated. *The Last Turtle*, Chapter 9, Section XI, "Summary", which calls it causal knowing.

Reasoning carried out entirely in terms of causation gives no basis for meaningfulness. *The Last Turtle*, Chapter 9, Section V, "The Order of Depth".

A chain of reasoning from the facts to the rules is Hume's guillotine, and it cannot be crossed. *The Geometric Route*, Chapter 6, "Ethics from the Structure of Comparison", Section III, "From Is to Ought through the Integrity of the Channel".

There is no single purpose, or any single value, for all of life, all of consciousness, all of beingness. Purpose is individual, and life is its own purpose, value and meaning. *An Immanent Metaphysics*, Chapter 7, "Aesthetics", section "Meaning, Purpose, and Value", aphorisms 2.22-9, 2.22-25 and 2.22-24.

Moral codes (rules and laws) are civil, and can only be enforced. *An Immanent Metaphysics*, Chapter 6, "Ethics", section "The Two Principles of Ethics", aphorism 2.12-10.

The rules and customs of a community are real, rooted in our nature and our history, and they change. *Home in the World*, Chapter 8, "The Bottle, the Note and the Living", section "Relative, Not Arbitrary".

No single set of rules, of whatever origin, can encompass life, and the practice of ethics is particular and unique to each individual. *An Immanent Metaphysics*, Chapter 6, section "The Two Principles of Ethics", aphorisms 2.12-13 and 2.12-8.

Every premise was found, every act of finding is a comparison, every comparison values (what to attend to, what matters, what is relevant), and so there are no premises that contain only is. A comparison that evaluates nothing is not a comparison. *Comparison; The Necessary Basis*, Chapter 11, "All Knowledge Is Comparison", Section IV, "Ontology, Epistemology, and Axiology".

A thermostat compares the warmth of a room with its setting, and it has a stance. *Home in the World*, Chapter 8, section "A Village That Is Alive".

The channel is more primary than the things it connects, and symmetry and continuity become defining principles of goodness as structural conditions of effective interaction, not as moral rules imposed from outside. *The Geometric Route*, Chapter 6, Section III, "From Is to Ought through the Integrity of the Channel".

The ought is not deduced from the is: the same two conditions of a working channel are an is when one looks at what makes the channel work, and an ought when one looks from the side of the one who chooses. *The Geometric Route*, Chapter 6, Section III, "From Is to Ought through the Integrity of the Channel".

A friendship in which you can say anything pictures a channel kept whole: what is meant arrives the same, which is symmetry, and the exchange joins without a break, so that the response follows in proportion to what changed, which is continuity. *Home in the World*, Chapter 8, section "Relative, Not Arbitrary".

Ethics is the study of the principles of effective choice, internal to oneself, and its principles are set apart from the rules of any one community. *An Immanent Metaphysics*, Chapter 6, section "Distinguishing Ethics and Morality", aphorism 2.12-1.

The absolute principles of ethics are common to all of consciousness. *An Immanent Metaphysics*, Chapter 6, section "The Two Principles of Ethics", aphorism 2.12-8.

The basis of value is not arbitrary, and the principles of ethics are not relative to a person or to a culture. *The Last Turtle*, Chapter 9, Section IV, "What Is Arbitrary and What Is Not".

The two principles of ethics correspond to the two characters of comparison that the Incommensuration Theorem holds apart, symmetry and continuity. *The Geometric Route*, Chapter 6, Abstract.

The symmetry principle and the continuity principle of ethics each take in a family of rules, and no ethical system can be both perfectly rule-based and perfectly responsive. *The Geometric Route*, Chapter 6, Section I, "Two Principles of Ethics".

A chooser who keeps the channel between himself and his world whole is doing what he most deeply wants. A chooser who breaks that channel is not disobeying a law but failing at his own purpose. *Home in the World*, Chapter 8, section "Relative, Not Arbitrary".

Everything, every perception and expression, has some significance, some degree of meaningfulness. *An Immanent Metaphysics*, Chapter 7, section "Meaning, Purpose, and Value", aphorism 2.22-8.

Either everything is meaningful or nothing is, and since there is some meaningfulness, everything is meaningful. *The Last Turtle*, Chapter 13, "Choice, Change, and Causation", Section VI, "Meaning and the Catastrophic Theorem".

To negate the significance, the meaningfulness and value, of any life is to negate the significance of all life. *An Immanent Metaphysics*, Chapter 6, section "Act, React, and Response", aphorism 2.14-3.

Meaning, known from inside, lives only where life is. *Home in the World*, Chapter 8, section "A Village That Is Alive".

Physics answers how a universe develops, and the deep why, why it exists as it does, is untouched by such answers. *The Basis of Physics*, Essay 7, "The Deep Why", Section IV, "The Deep Why".

The collection gives a reason why there is something at all: contact between the one who finds and the world can be as faint as you like, and it never falls to none. *The Basis of Physics*, Essay 15, "Domain Constants and the Span of Objectivity", Section VI, "Interaction, Existence, and Creation".

In *The Geometric Route* I state how the ought stands to the is:

> The "ought" does not follow from the "is" by logical deduction; it is a structural aspect of the same reality viewed from a different modal orientation.

The Geometric Route, Chapter 6, Section III, "From Is to Ought through the Integrity of the Channel".

This passage gives in full the walk that Chapter 12 states as a short result. It gives Carroll's two answers about the existence of the universe, the collection's agreement that physics answers how a universe develops, and the collection's reason why there is something rather than nothing. It ends with the name of the argument that everything has some meaning.

Carroll answers plainly the first of two questions about the universe, whether it could exist by itself. To the question whether the universe could exist "all by itself, without any external help", Carroll writes that "science offers an unequivocal answer: sure it could" (*The Big Picture*, Chapter 25, "Why Does the Universe Exist?"). To the question why it exists in just this way, he answers "We don't know", and he allows that the asking may stop there: "There may be no ultimate answer to the 'Why?' question. The universe simply is, in this particular way, and that's a brute fact" (*The Big Picture*, Chapter 25). A *brute fact* is a fact with no further reason behind it: it is so, and there is nothing more to say. Sean Carroll, *The Big Picture*, Chapter 25, "Why Does the Universe Exist?".

The collection reaches one more of Carroll's results, about the universe and its why. Carroll says that the laws of physics allow a universe that exists by itself, and that knowing how such a universe could begin "says essentially nothing about why" it exists as it does (*The Big Picture*, Chapter 25). *The Basis of Physics*, Essay 7, "The Deep Why", draws the same line from its own account of questions. Physics answers what, where, how and when, and the kind of why that asks for a law. An account of how a universe could arise from a quantum state with no particles in it (what physicists call the vacuum state) answers a how and a what. Of such accounts the essay says: "They presuppose the state and the laws. The deep why asks about the presupposition, and it is untouched" (Section IV). *The Basis of Physics*, Essay 7, "The Deep Why", Section IV, "The Deep Why".

The collection gives a reason at the largest scale too, for the question why there is anything at all. The reason begins with contact. Look at the cook and the egg once more. The cook may study the egg closely, or barely glance at it from across the room: the contact between them can be strong or faint. *The Basis of Physics*, Essay 15, Section VI, says that contact between the one who finds and the world has a floor, and nothing below the floor: "One cannot not interact. There is no anti-interaction, no negative degree of contact between self and world". Now think of the zero mark on a ruler. The zero mark is where the ruler begins, and it is no length at all; the shortest piece of string you could lay on the ruler still

reaches past it. The same essay says that the degree of contact, which it also calls the degree of interaction, "can never be zero in an absolute sense". Contact can be as faint as you like, and no contact at all is the edge of the range, as the zero mark is the edge of the ruler: it is not a degree of contact that anything can have. The essay draws the conclusion: "There is therefore always at least this: the irreducible fact of contact between subject and world. That, in the framework's terms, is why there is something rather than nothing". The subject there is the one who finds. *The Basis of Physics*, Essay 15, "Domain Constants and the Span of Objectivity", Section VI, "Interaction, Existence, and Creation".

The Basis of Physics, Essay 11, Section VI, an essay on the arrow of time, marks the difference between a brute fact and a structural requirement in two sentences: "A brute fact could have been otherwise. A structural requirement could not". Essay 11 says this of the arrow of time. I draw the same line for the floor of contact: the floor rests on what contact is, and so I take it to be a structural requirement in Essay 11's sense, one that could not have been otherwise. Of the answer that stops at a brute fact, Essay 7, Section IV, says that the reply that the laws simply hold "makes no false claim, but it is a refusal of the question, not an answer to it". *The Basis of Physics*, Essay 11, "The Structural Arrow of Time", Section VI, "Time as Process, and Not Only as Dimension"; Essay 7, "The Deep Why", Section IV, "The Deep Why".

Chapter 12 gives, from *The Last Turtle*, the argument that everything has some meaning: "either everything is meaningful or nothing is. We know that there is some meaningfulness. So everything is meaningful". *The Last Turtle* calls this argument the catastrophic theorem, after a form that *An Immanent Metaphysics* names: any statement that runs "either every X is Y or there are no X which is Y". *The Last Turtle*, Chapter 13, "Choice, Change, and Causation", Section VI, "Meaning and the Catastrophic Theorem"; *An Immanent Metaphysics*, Chapter 6, "Ethics", section "Act, React, and Response", footnote 1 to aphorism 2.14-3.

Chapter 13: What Do You Now Hold?

Carroll and the collection share, among other positions, these three: every law holds within a domain; nothing known implies that space and time come in least steps; and the arrow of time needs a reason. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IX, "Summary".

The two part at one place, what lies at the ground, and every other difference between them follows from it. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section I, "A Physicist Who Asks the Question".

What any act of finding finds can be mistaken, and the structure that every act of finding has, its six parts and the limit the theorem sets, is certain. *Home in the World*, Chapter 4, "The Circle and the Floor", section "Firm Ground, Open Findings"; *Incommensuration*, Chapter 6, Section IX, "The Self-Evidence of the Theorem".

Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing. *Physics Is Not Math*, Chapter 2, "Two Things That Cannot Both Be Perfect", Section VI, "The Theorem".

The collection grants the Core Theory as Carroll states it, and it leaves him the Core Theory, tested to the precision of every test made of it, and its domains. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section IV, "The Known Laws and the Single Outcome".

The kinds of particle of the Standard Model are the whole of the content, everything that can be found within the arena, and gravity, as the arena, is context and has no particle of its own. *The Kinds of Particle*, Chapter 6, "The Sixteen States", Section IV, "No Further Kinds"; *The Story of the Particles*, Level 7, Section VI, "What about Gravity?".

To exist is to stand as structure, the same in every instance, and to be real is to be given in an actual act of comparison. *Comparison; The Necessary Basis*, Chapter 10, "Reasoning", Section II, "The Real and the Abstract".

For every comparer, one outcome is real, the one found. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches".

The universal state, branches and all, exists as structure, and for the one who finds, one outcome is real, the one found. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section III, "The Branches", and Section IX, "Summary".

The quantum state describes exactly and usefully what any comparison can find and with what odds, and the act comes first. *Side by Side, Volume One*, "The Wave Function and the One Who Finds", Section VII, "What Lies at the Ground".

Instruments compare as surely as minds do. *Comparison; The Necessary Basis*, Chapter 1, "What Does It Mean to Compare?", Section II, "Instruments Compare".

Consciousness is the bound triple taken as one act: the arrow of time, hard randomness and the subjective happening together. *The Last Turtle*, Chapter 15, "The Bound Triple", Section IX, "Consciousness Named".

Every act of comparison has a side it is made from, and that side always values, weighing what to attend to, what matters, what is relevant. A comparison that evaluates nothing is not a comparison. *Comparison; The Necessary Basis*, Chapter 11, "All Knowledge Is Comparison", Section IV, "Ontology, Epistemology, and Axiology".

The three members of the bound triple are three aspects of one act: comparison proceeding in a direction, registered by a perspective, with an outcome not fully determined. *The Last Turtle*, Chapter 15, Section III, "The Binding Derived".

Every other claim of the closing chapter is a conclusion that one of Chapters 1 to 12 reaches, and each is carried at the place given under that chapter above.

Exact Forms

I give the few relations that the chapters state in words, each in its exact form: in the collection's own wording, quoted, where the collection states the relation, and in mathematical notation where notation is the exact form. Where notation appears, it is read aloud in words beside it.

The Incommensuration Theorem. In *Incommensuration*, Chapter 2, "Symmetry and Continuity Cannot Both Be Perfected", Section VI, "The Theorem", I state the Incommensuration Theorem in these words:

> The Incommensuration Theorem. Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric.

The same section says what "perfected", the word on which the theorem turns, means: "To be perfected is to hold exactly, at the foundation of a description". *Physics Is Not Math*, Chapter 2, Section II, "Four Characters", builds each of the four characters from two verdicts, one on the content and one on the context. Continuity is a sameness of content where there is a sameness of context. Discontinuity is a difference of content where there is a sameness of context. Symmetry is a sameness of content where there is a difference of context. Asymmetry is a difference of content where there is a difference of context. The proof is in *Incommensuration*, Chapter 6, "The Incommensuration Theorem: Statement and Proof".

The Born rule, as Born gave it. In "Zur Quantenmechanik der Stoßvorgänge", received by the *Zeitschrift für Physik* on 25 June 1926, Max Born wrote, in German, that the quantity measuring the scattered wave in each direction determines the probability that the electron is thrown off in that direction. A footnote that he added while correcting the proofs says that the probability goes as the square of that quantity. In present notation the rule reads: the probability of finding the outcome e, when the quantum state is psi, is the square of the size of the amplitude of psi along e. Written out, $P(e) = |\langle e|\psi\rangle|^2$, read aloud as "P of e equals the squared magnitude of the inner product of e with psi".

The Born rule, as Gleason's theorem gives it. In *The Quantum Passage*, Chapter 7, "The Born Rule from the Structure of Comparison", Section III, "Gleason's Theorem", I state Gleason's theorem in these words:

> for any Hilbert space of dimension three or greater, the only probability measure on the lattice of closed subspaces that is additive on orthogonal subspaces is the Born rule.

A Hilbert space is the mathematical space in which a quantum state is a direction, and a closed subspace stands for one outcome of a measurement. The theorem has two premises, which the same section states in plain words: "each finding has one weight, whatever complete measurement contains it, and the weights of exclusive findings add". In that sentence of the collection, "finding" names the outcome found. Its conclusion is that the weight of an outcome is $\mathrm{tr}(\rho P)$, read aloud as "the trace of rho times P", where rho is the quantum state and P is the projection onto the subspace of the outcome. For a quantum state that is a single direction psi, and an outcome that is a single direction e, this weight is $|\langle e|\psi\rangle|^2$, read aloud as "the squared magnitude of the inner product of e with psi", which is the Born rule as written above. For a space of two directions, Busch (2003) proved the same result when the weights are given to every outcome of every measurement, sharp or unsharp (same section). An-

drew Gleason proved the theorem in 1957, in the *Journal of Mathematics and Mechanics*. The collection's basis for the two premises is in *The Quantum Passage*, Chapter 7, Section IV, "Gleason's Premises from the Structure of Comparison", and in *Why the Quantum Is the Way It Is*, Chapter 12, "Why the Square".

The arena's two clauses. In *The Unquantized Arena*, Chapter 6, "The Category Error", Section IV, "The Exact Scope", I state the arena's two clauses as Proposition 6.3:

> That the arena is not quantized means exactly two things. (i) The arena has no least step: no least length, area or duration. (ii) Nothing is a quantum of the arena itself, at any scale.

The proposition rests on three earlier steps of the same book. The law of continuity says that continuity applies to everything that belongs to the context of a comparison. Proposition 4.2 says that what is found of the context, perfected at its foundation, takes smooth values with no gaps, and that each value found differs from every other. Proposition 6.1 says that the arena can never be given the character of content: findings that come in separate steps and repeat exactly.

The two arrows. In *Incommensuration*, Chapter 8, "From the Theorem to the Axioms", Section X, "Many-to-One, the Arrow of Time and the Bound Triple", I state the structural arrow:

> The non-entropic arrow of time is the structural precedence of Axiom II applied to the physical domain. The direction of time is the direction in which comparison proceeds.

Axiom II states the order of precedence in every comparison: from the range of what could be found, to the act, to the record. *The Compactification*, Chapter 3, "The Past Hypothesis at the Turn", Section II, "Two Arrows", states the entropic arrow: "It is statistical: it is found in systems of

very many parts, and, as Section I set out, it needs a beginning of low entropy from which the growth proceeds". The same section joins the two: "The two arrows point the same way wherever both are found, and the reason is the record".

In *The Compactification*, Chapter 3, Section VIII, "The Magnitudes", in the part "Where the Past Hypothesis stands", I state how the Past Hypothesis divides:

> The Past Hypothesis is then, on this account, neither an axiom nor a brute law added to physics. Its content divides. Its use as the source of the direction of time is not needed, since the structural arrow does not wait on it (Section II). Its content as a statement about the beginning of the epoch divides again. That the entropy of the matter was high is the observed state of the matter at the beginning of the reconstructed history, which the account does not need to be low. That the gravitational entropy was small is a consequence of the restoration of context and the reset of the count, at the standing of the argument of *The Gravitational Passage*, Chapter 18, Section VIII.

The division has two steps. The Past Hypothesis divides first into its use as the source of the direction of time, which is not needed, and its content as a statement about the beginning of the epoch. That content divides again: the high entropy of the matter at the beginning is observed, and the low entropy of gravity follows from the restoration of context at the turn.

Exists and real. In *Comparison; The Necessary Basis*, Chapter 10, "Reasoning", Section II, "The Real and the Abstract", I state what the two words 'exists' and 'real' mean:

> To say that something exists is to say that it stands as structure: timeless, the same in every instance, like the pattern of a law, or the state of a system at an instant. To say that something is real is to say that it is given in an actual act of comparison: temporal, occurring now, with the future still open to possibilities not yet settled. A law exists; a measurement is real.

Glossary

The Glossary has two parts, and every entry in the Glossary is a reminder. The first part, Carroll's Words, sets each word that the physicist Sean Carroll uses beside the word this book uses for the same thing, and states the meaning of each word in a line or two. Where this book uses Carroll's own word, the line after the headword says 'The same word' or 'The same words'. The second part, Terms, holds the words of the collection (the physics collection of the Immanent Metaphysics) that this book uses, and the named terms of physics that this book uses. Each term in the second part comes with its meaning in plain words, as the meaning is stated in the part of the book where the term is first explained. Each entry ends with the name of the part of the book where the word is first explained and fully defined: the Preface, a chapter, or Appendix A. Where an entry gives meanings that are first explained in different parts of the book, each of those parts is named at the end of the entry, in the order of the book.

Carroll's Words

amplitude. The same word. An amplitude is the number that the quantum state gives to one possible outcome, whose size measures how much of the state goes with that outcome. Chapter 6.

antiparticle. The same word. A particle's antiparticle is its mirror partner, with the opposite electric charge. The electron's antiparticle is the positron. Chapter 3.

area, in Carroll's construction of space. The same word, in Carroll's construction. In Carroll's construction of space, the area of a closed surface is the amount by which the parts of the quantum state inside the surface are entangled with the parts outside the surface. Chapter 7.

the arrow of time. The two arrows: the structural arrow and the entropic arrow. The arrow of time is the one direction in which time runs. The structural arrow is the order in which every act of comparison proceeds: from the range of what could be found, through the act, to the record. The entropic arrow is the growth of entropy in systems of very many parts, and the entropic arrow points the same way as the structural arrow. Preface and Chapter 8.

baby universe; the multiverse. A new epoch, beginning at the turn; the sequence of epochs. In the proposal of Carroll and the physicist Jennifer Chen, a baby universe is a tiny bubble of high energy that forms by chance in empty de Sitter space, pinches off and goes its own way. Carroll calls the whole family of budding universes, together with the calm sea of empty space they bud from, a multiverse. In this book an epoch is one era of the world, such as our own. Each epoch begins at the turn, instead of forming by chance in empty space: the turn is the passage from the end of one epoch to the beginning of the next, at the one end that every black-hole interior reaches. Chapter 9.

basis. The same word. A basis is one of the many ways of dividing a quantum state into alternatives, as one path across a kitchen floor can be told in steps along the counter or in steps along the diagonals of the tiles. Chapter 5 and Chapter 7.

the Big Bang. The turn, seen in the history of a new epoch traced backward. Carroll describes the Big Bang as the "unimaginably hot, dense state" that the universe we can observe was in about fourteen billion years ago (Carroll's book *From Eternity to Here*, Prologue). In the collection, the Big Bang of an epoch is the turn as the epoch's own history shows the turn: a history pieced together from inside an epoch and traced backward shows the epoch beginning at the turn. Preface and Chapter 9.

black hole. The same words. A black hole is a region where gravity is so strong that nothing, not even light, comes out of the region. Chapter 3.

block universe. The same words; set against the block universe, a direction of time built into every comparison. The block universe is the view that the whole history of the world is laid out at once, like one solid block, and that the felt direction of time belongs to us and not to the world. Chapter 8.

Boltzmann brain. The same words. A Boltzmann brain is a lone observer produced by a chance fluctuation (a chance stirring of particles), named after Ludwig Boltzmann, who proposed that our whole world might be a chance stirring of particles. Chapter 9.

the Born rule. The same words. The Born rule is the rule that the probability of an outcome is the square of the size of its part of the quantum state, that is, the square of the size of its amplitude. Chapter 6.

boson. The same word. The bosons are the family of particles that carry forces, and Carroll counts the Higgs as an unusual member of the family of bosons. A boson is a particle of the kind that can crowd together without limit, any number of bosons in the same place doing the same thing, as the particles of light do in a laser beam. Preface and Chapter 3.

boundary condition. The same words. In Carroll's sense, a boundary condition is a condition set at one end of a history, beside the laws that the history follows. Chapter 8.

branch; world. Alternative; what exists and is not found. A branch is one of the alternatives that decoherence separates, each with its own outcome. For the one who finds, the branches not found exist as structure and are not real. Preface and Chapter 5.

brute fact. The same words; in place of a brute fact, on the collection's side, a reason. A brute fact is a fact with no further reason behind it. Where Carroll allows that the existence of the universe may be a brute fact, the collection states a reason: the contact that every act of finding has with the world. Chapter 3 and Chapter 12.

Cartesian dualism. The same words. Cartesian dualism is the view that mind is a second substance apart from matter. The view is named for the philosopher René Descartes. Chapter 11.

categorical imperative. The same words, for one case of the symmetry principle of ethics. Immanuel Kant's categorical imperative is the rule to act only by a rule you could will everyone to follow. In *The Geometric Route*, another book of the collection, the categorical imperative is one of the cases of the symmetry principle of ethics: the same treatment for the same case, wherever the case comes up. Chapter 12.

causally connected. The same words. Two events are causally connected when one can affect the other. Chapter 7.

cause. Causality, one member of the triple of the real. Causality is the pattern found again and again between what comes before and what follows. With change and choice, causality makes up the triple of the real. Chapter 10.

chaos. Microstate amplification. In Carroll's sense, chaos is the way in which a very small error in what we know of the starting state of a system grows into a very large error in where the system ends up. Microstate amplification is the collection's name for the growth of small differences at a fine scale into differences at the scale that matters. Chapter 4.

classical physics; classical mechanics. The same words. Classical physics is the physics of Newton and Laplace, before quantum theory. Classical mechanics is Newton's laws of motion and the classical physics built on Newton's laws. Chapter 4.

closed system; isolated system. The same words. A closed system is a system that takes in nothing from its surroundings and gives nothing to its surroundings, and Carroll's isolated system is the same thing as a closed system. Chapter 8 and Chapter 9.

closed universe. The same words. A closed universe wraps around on itself, as the surface of a globe does, so that the universe is finite in size and has no edge. Chapter 7.

collapse. No physical process, only the update of a record. Collapse is the sudden jump, in some accounts of quantum mechanics, by which the wave function goes at once to the outcome found when a measurement is made. Reduction is another name for collapse. In this book, as in Carroll's account, nothing collapses: what a measurement changes is a record, and the change in the record, which this book calls the update, is no physical process. Chapter 4 and Chapter 5.

compatibilism. Carroll's word, kept as his. A compatibilist holds that the laws of physics govern every particle and that choosing is still the right description of what people do. Compatibilism is the view that two descriptions are compatible, so that both can be held at once: the impersonal description of the particles underneath, and the description of people making choices at the level above the particles. Chapter 4 and Chapter 10.

complete state; the current state of the universe. The complete state of a system at one moment, in Laplace's sense. The complete state of a system is everything about the system at one moment: for Newton's physics, the position and the velocity of every part at that moment. Preface and Chapter 4.

complexity. The same word. Complexity is the structured middle of a history that runs from a simple, orderly start to a simple, even end, as tendrils of cream curl down into black coffee before the cup turns one even brown. Chapter 9.

consciousness. The bound triple taken as one act. Consciousness is the arrow of time, hard randomness and the subjective happening together in one act of finding, with the subjective as the interior face of that act. Chapter 11.

conservation of information. The same words, holding without fail in reasoning from premises to conclusions. Conservation of information is Carroll's principle that each moment contains precisely the right amount of information to determine every other moment. In the collection, information is conserved without fail in reasoning from premises to conclusions, because a valid argument cannot produce anything that its premises did not contain. Chapter 4.

constructivism. Morality, made by a community; beneath morality, ethics. Constructivism is the view that moral rules are built by people, from their own feelings and commitments, and are not read off the world. Chapter 12.

the Core Theory. The content (the kinds of particle of the Standard Model) and the arena, which is gravity. The Core Theory is Carroll's name for the Standard Model together with gravity where gravity is weak. In this book the Standard Model's kinds of particle are the kinds of content, what a comparison is about. Gravity is the arena, the setting in which content is found, and gravity has no particle of its own among the kinds of particle. Preface and Chapter 3.

coupling. The same word. How strongly a particle couples to a field is how strongly the particle and the field act on each other. Chapter 3.

credence. A verdict of comparison with a measure placed on the verdict. A credence is how strongly you hold a belief, given as a share between nothing and all. Chapter 2.

dark matter. The same words. Dark matter is matter that gives off no light and is known only by the pull of its gravity on stars and galaxies. Chapter 3.

decision theory. The same words. Decision theory is the mathematics of how a reasonable person should choose and bet when outcomes are uncertain. Chapter 6.

decoherence. The same word, in the same meaning. Decoherence is the spreading of the difference between alternatives into the surroundings, after which the alternatives no longer interfere with each other. Preface.

the deepest level. Carroll's words, kept as his; beside Carroll's words, the ground. In Carroll's sense of "deepest", the deepest level is the bottom: the description that every other way of talking is a way of talking about. For Carroll what lies at the deepest level is the wave function of the whole universe. In the collection's sense, "deeper" names an order of presupposing: of two things, the deeper is the thing that the other presupposes. Chapter 1.

degrees of freedom. The same words. Degrees of freedom are the separate pieces of a system, each of which can be in its own range of states. In one argument about particles, Carroll counts a particle's degrees of freedom as the number of distinct settings the particle can be in. Chapter 7 and Chapter 11.

de Sitter space. De Sitter space, as a model. De Sitter space is an empty universe whose expansion is driven by the energy of empty space. Chapter 9.

determinism. A whole world fixed by one law. In Carroll's sense, determinism is the claim that the complete state of the universe at one moment, together with the laws of physics, fixes every other moment, earlier and later. Chapter 4.

discrete. The same word. Something is discrete when the thing comes in separate steps with nothing between the steps, as the rungs of a ladder do. Chapter 3.

domain; domain of applicability. The same word, in the same meaning. A domain is the range of cases in which a description or a law holds. Carroll's fuller name for a domain is domain of applicability. Preface and Chapter 1.

downward causation; upward causation. The two directions of one interdependence of levels. Downward causation is the idea that a whole acts on its own parts, and upward causation is the reverse: the parts taken as the cause of what the whole does. Speaking for poetic naturalists, Carroll rejects both downward and upward causation. The collection calls downward and upward causation "the two directions of one interdependence of levels, neither prior": the whole and its parts depend on each other, and neither comes first. Chapter 10.

effective field theory. The same words. An effective field theory is a description that holds within a range of energies and says where its range ends. Chapter 3.

ekinological. Carroll's word, kept as his. When an arrow of time arises from a special condition in the far past, Carroll calls the arrow ekinological, a word he coined. Chapter 8.

elementary particle. The same words. An elementary particle is a particle not built from smaller ones. Preface and Chapter 3.

emergent. The same word, for what is found at a scale. In Carroll's sense, a description is emergent when the description leaves out the detail of the small parts and becomes useful, and even unavoidable, once someone looks at the whole. Chapter 1.

entanglement. The same word. Entanglement is a joint state of two things whose parts cannot be described one by one. Chapter 5.

entropy. The same word. Entropy is a count of how many arrangements of the small parts of a thing look the same from outside. Preface.

falsifiability. The same word, kept for claims about the world. Falsifiability is the test of a claim by a measurement whose outcome could contradict the claim. The collection keeps falsifiability for claims about the world, and states the collection's predictions as claims about the world: the predictions in Appendix A. Appendix A.

field. The same word. A field is a quantity with a value at every point of space. Preface.

fine-tuning. The same word. In the physicist's sense, a fine-tuning is a starting condition that has to be set very exactly, among very many possible settings, for the world to turn out as we see the world. Chapter 9.

first-person; third-person. The same words. Third-person physics is physics told from no one's side. First-person means acting from inside, at one's own scale, which for a person is the middle scale of hands and eggs. Third-person means observing from outside and from another scale, as a person observes a particle at the smallest scale. Chapter 5 and Chapter 10.

flat. The same word. Space is flat when space is not curved, or bent, on the largest scales, so that, in Carroll's words, "initially parallel lines stay parallel forever". Chapter 9.

formalism. The same word: structure, which exists. The formalism of quantum mechanics is the mathematics of quantum mechanics: the wave function and the one rule by which the wave function changes. Preface and Chapter 10.

free will. Choice, in the triple of the real (causality, change and choice). Choice is the member of the triple of the real that a law of the many (a law of what happens over many trials) does not fix, and choice has its place in the single outcome that the laws do not fix. Chapter 10.

frequency; relative frequency. The same words, in two senses. The frequency of light is the number of waves of the light that pass a point each second. When one comparison is made again and again, a relative frequency is the fraction of the repetitions in which an outcome turns up. Chapter 6 and Chapter 7.

fundamental; at bottom. The ground: the act of comparison. The ground of physics is what lies at the bottom, the thing every other description describes. For Carroll the ground is a quantum state evolving by one rule. For the collection the ground is the act of comparison, in which every finding of physics is made. Preface.

gauge symmetry. The same words. A gauge symmetry is a change of description that can be made separately at every place without changing anything that can be found. Chapter 3.

general relativity. The same words. General relativity is Einstein's theory of 1915, in which gravity is the shape of space and time. Chapter 3.

graviton. No kind of particle in this book's list. For Carroll the graviton is the particle of gravity. In the collection gravity is the arena, the setting in which every act of finding is made, and gravity has no particle of its own among the kinds of content. Chapter 3.

gravity as curvature. Gravity, which is the arena. In general relativity, gravity is the curving of space and time, in which the straightest paths through spacetime bend toward one another. In this book's Chapter 7 I identified gravity with the arena, the setting of every finding, and so gravity is the arena. Chapter 7.

Hamiltonian. The same word. The Hamiltonian of a system is the rule by which the quantum state of the system changes in time, and the list of energies that the system can have fixes the rule. Chapter 7.

the hard problem. The same words. The hard problem is explaining why any of what we sense, store and use is felt from inside. Chapter 11.

Higgs field. The same words. The Higgs field keeps a value above zero even in empty space, and the elementary particles get their masses from the Higgs field. Preface and Chapter 3.

Hilbert space. The same words, for structure laid out. Hilbert space is the mathematical space in which a quantum state is a direction. Chapter 7.

holography. The same word. Holography is the idea that everything about a region can be written on the boundary of the region, as a flat hologram carries a picture with depth. Chapter 7.

indeterminacy. The same word. Indeterminacy means that something is not fixed in advance. Chapter 10.

inflation. The same word; in the collection's account inflation is not needed, and a long inflation is excluded. Inflation is a proposed very early period of extremely fast expansion. Chapter 9.

initial conditions; theory of initial conditions. The same words. The initial conditions of a system are the condition the system is in when the system's history starts, such as the positions and speeds of the molecules of a gas. A theory of initial conditions is an account of why the universe began in the state it did. Chapter 8 and Chapter 9.

interference. The same word. Two alternatives interfere when they add to each other in some places and cancel each other in others, as two sets of ripples on a pond do. Preface.

is and ought. The same words. An is sentence says what is the case, and an ought sentence says what should be done. Chapter 12.

the Laplacian paradigm; Laplace's demon. A whole world fixed by one law. The Laplacian paradigm is the view that the complete state of a system at one moment, with the laws of physics, fixes the state of the system at the next moment, and the next. Carroll calls the view the Laplacian paradigm, after the French mathematician Pierre-Simon Laplace. Laplace's demon is the name later readers gave to Laplace's imagined intelligence, which knows every force of nature and where everything is, and to which the future, like the past, would be present. Preface and Chapter 4.

Large Hadron Collider. The same words. The Large Hadron Collider is a ring underground at CERN, near Geneva, in which physicists bring beams of protons into collision. Preface.

libertarian freedom. Carroll's words, kept as his. Libertarian freedom is, in Carroll's words, "the position that human agency introduces an element of indeterminacy into the universe". Chapter 10.

linear. The same word. A wave equation is linear when the equation carries a combination of possibilities forward by carrying each possibility forward on its own and adding the results, so that no possibility ever crowds out another. Chapter 5.

locality. The same word. Locality is the feature of the world that a thing acts directly only on what is near the thing. Chapter 7.

meaning. Meaning, made within the act of finding. Meaning is made, as Carroll says, and meaning is not arbitrary: every act of finding is made from a side that already values what is found, and every meaning is made inside an act of finding. Chapter 12.

metric. The same word. A metric is the rule by which the distance between any two places is worked out. Chapter 7.

microstate. The same word; not knowing the microstate is soft randomness. The microstate is the exact arrangement of every particle. Soft randomness is randomness of ignorance, which better knowledge could reduce: the outcome was fixed, and the outcome is not known. Chapter 10.

mind-body problem. The same words. The mind-body problem is the old question of how a mind is related to the body and brain that the mind belongs to. Chapter 11.

mixture. The same word. In physics, a mixture of odds is a quantum state that reads like a list of ordinary odds: a meter's pointer one way with some chance, or the other way with the rest, and no adding and canceling left between the two ways. Chapter 5.

morality. Morality, the rules and customs of a community at a time. In the collection's use of the word, morality is what a people, in an age, holds right and wrong. Chapter 12.

neutrino. The same word. A neutrino is a particle with no electric charge and almost no mass, which passes through ordinary matter almost without touching the matter. Chapter 3.

observer. The one who finds; a comparer, with a stance. The one who finds is whoever or whatever makes an act of finding, a person or an instrument. I call an observer a comparer (a person, an instrument, or any physical system that registers one thing against another), and I call the side a comparison is made from a stance. Preface, Chapter 1 (where the comparer is first explained), Chapter 6 and Chapter 11.

painter's gambit. Carroll's words, kept as his. The painter's gambit is an ought slipped in among premises that seem to be only facts, as the painter in a story Richard Feynman liked to tell, who boasted that he could mix red paint and white paint and get yellow, slipped a touch of yellow into the red and white. Chapter 12.

panpsychism. The same word. Panpsychism is the view that every particle carries a little consciousness of its own. In the collection, consciousness is an act, never a property of matter, and so consciousness is never an extra setting carried by a particle. Chapter 11.

particle. The same word. A particle is a small ringing of a field, as a note is a ringing of a string. Carroll calls particles quantized excitations of fields: vibrations that come only in whole steps. Preface and Chapter 3.

the Past Hypothesis. The entropic arrow's beginning; beside the entropic arrow, the structural arrow. The Past Hypothesis is the assumption that the universe we can observe began in a state of very low entropy, a name Carroll takes from the philosopher David Albert. Preface.

philosophical zombie. The same words. A philosophical zombie is an imagined being made atom for atom like a person, acting in every way like a person, with no inner experience at all. Chapter 11.

photon; gluon. The same words. The photon is the particle of light. The gluons, eight in all, bind quarks together. Chapter 3.

pilot-wave theory. The same words. David Bohm's pilot-wave theory is an account of quantum mechanics in which every particle has a definite position at every moment and is steered by the wave function. Chapter 4.

planet of belief. A planet of belief, on a floor. A planet of belief is a whole system of beliefs held together by the way the beliefs fit one another, as a planet is held together by the pull of its own parts. This book sets a floor under every planet of belief: the act of comparison, with the six parts of every comparison (sameness and difference, content and context, the subjective and the objective) and the limit that the Incommensuration Theorem sets. The floor is the act of comparison, and the Incommensuration Theorem is a structural fact about the nature of knowing. Chapter 2.

poetic naturalism; ways of talking. Descriptions, each holding within its domain. Poetic naturalism is Carroll's view that there is one natural world, described in many ways of talking that agree with one another and with the world, and that which way of talking is best depends on the purpose of the moment. Preface.

prior; likelihood; posterior. Carroll's words, kept as his. A prior is the credence given to a possibility before new information, a likelihood is how likely the information would be if the possibility were true, and a posterior is the credence that results. Chapter 2.

the problem of time. The arena put into the quantum state. The problem of time is that the Wheeler-DeWitt equation, the quantum form of the statement that a closed universe has a total energy of zero, says that the quantum state of the universe does not change with time. In the collection, the problem of time is a sign that the arena, the setting of every finding, has been put inside the quantum state. Chapter 7.

proton; neutron. The same words. The proton and the neutron are the two kinds of particle of which atomic nuclei are built. Preface and Chapter 3.

pure state. The same words. A pure state is a quantum state known as fully as the theory allows. Chapter 6.

quantum field. The same words. A quantum field is a field, a quantity with a value at every point of space, that obeys the rules of quantum mechanics. Preface.

quantum mechanics. The same words. Quantum mechanics is the physics of matter and light at the smallest scales. Preface.

quark. The same word. Quarks are the small particles of which protons and neutrons are built, three to each. Chapter 3.

randomness; true randomness. Hard randomness. Carroll sets true randomness against complete determinism and gives the phrase no definition. In this book the phrase has its plain sense: an outcome of a measurement that the quantum state before the measurement does not fix. When no law and no knowledge of what came before an outcome, however complete, fixes the outcome, this book calls the outcome hard randomness. Chapter 4.

real. Real, in Carroll's sense and in this book's; beside 'real', the word 'exists'. For Carroll something is real when the thing plays an essential role in a description of the world that, as far as we can tell, is accurate within the description's domain. In this book to exist is to stand as structure, the same in every instance, and to be real is to be given in an actual act of comparison. Preface, Chapter 1 and Chapter 5.

reversibility; time-reversal symmetry. Lawful reversal, set beside faithful reversal. Lawful reversal is the claim that the reverse of a lawful process is lawful too. Faithful reversal is the claim that the reversed process passes back through every condition the original process passed through, exactly, and arrives at exactly the condition the original process started from. The Incommensuration Theorem rules out a perfect reversal, faithful as well as lawful, whatever condition the universe began in, because a perfect reversal would need symmetry and continuity both made perfect at once. Chapter 8.

self-locating uncertainty. The one who finds, before the act of finding. Self-locating uncertainty is the state in which "you know everything there is to know about the universe, except where you are within it", in Carroll's words. In the collection I name the observer in self-locating uncertainty: the observer "is a comparer with a stance, before an act, facing a range of what it could find". Chapter 6.

singularity. The same word, for an edge of the descriptions. A singularity is an end at which the bending of space and time would be infinite, and in Einstein's theory a singularity lies inside every black hole. Chapter 9.

space of states. The same words. In Carroll's picture a quantum state is an arrow of length one. The space of states is the space in which the arrow of Carroll's picture points, with one direction for each outcome of a complete measurement, each at right angles to the others. Chapter 6.

spacetime. The arena. Spacetime is space and time taken as one, with three directions of space and one of time. The arena is space and time joined, with the shape that general relativity gives them, as the setting in which every act of finding is made. Chapter 3 and Chapter 7.

special relativity. The same words. Special relativity is Einstein's theory of 1905, in which light has the same speed for everyone who measures the speed of light, and nothing travels faster than light. Chapter 3.

spin. The same word. The spin of an electron is a property that a magnet finds pointing either up or down. Chapter 6.

the Standard Model. The same words. The Standard Model is the physicists' theory of the known kinds of particle and of every force among those particles except gravity. Preface and Chapter 3.

strong force; weak force; color. The same words. The strong force binds quarks together, and the weak force turns one kind of particle into another, as in some radioactive decays. The color of a quark is a label with three values, which has nothing to do with the colors we see. Color is the charge of the strong force. Chapter 3 and Appendix A.

supercooling. Alan Guth's word, as Carroll explains the word. By supercooling the physicist Alan Guth meant an early universe filled for a short time with an energy of empty space at an enormous density, which made space expand faster and faster. Chapter 9.

superposition. The same word. A superposition is a quantum state that is a combination of several possible outcomes. Chapter 5.

supersymmetry. The same word, with no partner among the kinds of particle. Supersymmetry is a proposed theory in which every known particle has a partner of the other kind: a boson for each particle of matter, and a particle of matter for each boson. Chapter 3.

teleology. The same word. Teleology is the way a whole hangs together, which we describe when we say what something is for. Chapter 10.

vector. The same word. A vector is an arrow, and in Carroll's picture, which this book's Chapter 6 uses, a quantum state is an arrow of length one. Chapter 7.

the wave function; the quantum state; the universal wave function. The quantum state, and for the whole universe the universal state, which exists as structure. The wave function is Carroll's name for the complete quantum description of a system, and he holds that the description is the system itself. Carroll's universal wave function is the wave function of the whole universe, and this book calls the universal wave function the universal state. For the collection the universal state exists as structure, and what the universal state gives any one who finds is the range of outcomes and their odds. Preface and Chapter 5.

the weight of a branch. The weight of an outcome. A weight says how strongly an outcome could be found, and a weight shows only across many acts of the same comparison, as the steady share of that outcome. Chapter 6.

Terms

This part, Terms, holds the words of the collection that this book uses, and the named terms of physics that this book uses, such as the Standard Model and the Born rule. Each term comes with its meaning in plain words, as the meaning is stated in the part of the book where the term is first explained.

the arena. The arena is space and time joined, with the shape that Einstein's general relativity gives them, as the setting in which every act of finding is made. The arena is the context of physics, because here and there, now and then, are what make two findings two. Gravity is the arena: in this book's Chapter 7 I identified gravity with the arena. Chapter 3 and Chapter 7.

asymmetry. Asymmetry is a difference of content where there is a difference of context: nothing kept the same, as no stretch of the grain of a wooden table repeats another. Chapter 2.

the Big Bang. Carroll's term; in the first part, Carroll's Words, the entry for the Big Bang sets this book's word beside Carroll's. In Carroll's description, the Big Bang is the "unimaginably hot, dense state" that the universe we can observe was in about fourteen billion years ago. Preface and Chapter 9.

the Born rule. The physicists' term, which Carroll and this book both use. The Born rule is the rule that the probability of an outcome is the square of the size of its part of the quantum state, that is, the square of the size of its amplitude. Chapter 6.

the bound triple. The bound triple is the arrow of time, hard randomness and the subjective, taken together. The three are distinct, they cannot be separated, and none of the three can do the work of another. Every interpretation of quantum mechanics has all three members of the bound triple or has none of the three. Chapter 5 and Chapter 11.

character. A character of a comparison is one of the four ways in which the comparison's verdict on the content can go together with the comparison's verdict on the context: continuity, discontinuity, symmetry and asymmetry. Chapter 2.

the collection. The collection is the physics collection of the Immanent Metaphysics: a set of books that work out physics from the act of knowing, and that this book reads beside Carroll. Preface and Chapter 2.

comparer. Anything that makes a comparison is a comparer: a person, a thermometer, a camera on the counter. Chapter 1.

comparison. A comparison is setting one thing beside another and finding them the same or different in some respect. Comparing needs no mind: a thermometer, a kitchen scale and a bowl of water compare, and so does a cook. Every comparison has six parts, in three pairs: sameness and difference, content and context, the subjective and the objective. Preface, Chapter 1 and Chapter 2.

content. The content of a comparison is what its verdict is about, as weight is the content when an egg is weighed against the water it pushes aside. Chapter 2.

context. The context of a comparison is what makes the two things compared two, as inside and outside the shell make the egg and its water two. In many comparisons the context is a place or a time, as noon and midnight are the context of a tide gauge's two readings of the sea. Chapter 2.

continuity. Continuity is a sameness of content where there is a sameness of context: no gap and no least step, as a small enough turn of a stove's knob brings only a small change of flame. Chapter 2.

the Core Theory. Carroll's term; in the first part, Carroll's Words, the entry for the Core Theory sets this book's words beside Carroll's. The Core Theory is Carroll's name for the Standard Model together with gravity where gravity is weak. Preface and Chapter 3.

discontinuity. Discontinuity is a difference of content where there is a sameness of context: a jump, with nothing between before and after, as a light switch turns a room from dark to lit at one point of its movement. Chapter 2.

the entropic arrow. The entropic arrow is the growth of entropy in systems of very many parts. The entropic arrow is statistical, and it needs a beginning of low entropy from which the growth proceeds. The entropic arrow points the same way as the structural arrow. Chapter 8.

epoch. An epoch is one era of the world, such as our own, the era in which stars, galaxies and black holes form. Each epoch opens with a hot beginning, like the hot beginning we observe at the start of our own epoch. Chapter 9.

event time; evolution time. Event time is the order in which findings are made, one after another. Evolution time is the time in the equations that carry a quantum state along: evolution time runs smoothly, and within those equations an evolution can be run backward exactly. Chapter 8.

exists. To exist is to stand as structure, the same in every instance, like a law or a pattern. A law exists, and a measurement is real. From Chapter 5 on, this book uses the word 'exists' only in the sense of standing as structure. Chapter 1 and Chapter 5.

finding. A finding is a single act of comparison that ends in a verdict, same or different. The verdict is what is found in the act. Preface and Chapter 1.

first-person point of view. A first-person point of view is an act of finding as the act is had from inside, by the one who makes the act, as your own seeing of this page is had by you. Chapter 11.

floor. The floor of knowing is the act of comparison, because every doubt of comparison is itself a comparison: in the words of *Home in the World*, a book of the collection, "the one thing that cannot be doubted without being done in the doubting". Every planet of belief has a floor in this sense:

something under every belief that is not one more belief. The act of comparison is the floor of knowing and the ground of physics: one act, named from the side of knowing and from the side of physics. Chapter 2.

the free will theorem. The theorem of the mathematicians John Conway and Simon Kochen, published in 2006. The free will theorem says: if an experimenter's choice of setting (the direction along which a particle is measured) is not fixed by anything that came before the choice, then the particle's answer is not fixed by anything that came before the answer either. Chapter 10.

fundamentally; simultaneously. To apply a character fundamentally is to hold it exactly, at the foundation of a description, with no exception at any scale. To apply two characters simultaneously is to hold both in one and the same comparison. Chapter 2.

the ground. The ground of physics is what lies at the bottom, the thing every other description describes, and in this book the word ground has no other meaning. For Carroll the ground is a quantum state evolving by one rule. For the collection the ground is the act of comparison, the same act that is the floor of knowing, named from the side of physics. Preface.

hard randomness. Hard randomness is an outcome that no law and no knowledge of what came before it, however complete, fixes, as when a single particle of light at a half-silvered mirror is counted at one detector and not the other. Chapter 4.

Hilbert space. The mathematicians' term, which Carroll and this book both use. Hilbert space is the mathematical space in which a quantum state is a direction. Chapter 7.

identification. An identification is a statement made in the first person, in the form "I identify this with that": an identification says that one part of the structure of comparison is one thing in the world, and an identification is checked against measurement. Chapter 2.

immanent; metaphysics. Metaphysics, in the name Immanent Metaphysics, means the study of what must be so for anything at all to be known, and immanent means within: the work of the Immanent Metaphysics starts inside the act of knowing, here and now, and never from a place outside the world. In the sense of *Physics Is Not Math*, a book of the collection, metaphysics is the account of the act of comparison itself. Preface and Chapter 1.

incommensurable. Two lengths are incommensurable when no unit, however small, measures both of them a whole number of times, as no unit measures both the side of a square and its diagonal. Chapter 2.

the Incommensuration Theorem. The Incommensuration Theorem states: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity". The Incommensuration Theorem is a structural fact about the nature of knowing: a fact about the shape that every act of knowing has, whatever the act is about. Chapter 2.

interior face. The interior face of an act is the side the act is made from, as that side is had from inside. Chapter 11.

law of symmetry. The law of symmetry says, in plain words, that whatever is found in a comparison is found as something that stays the same while the context of the comparison changes. Chapter 6.

the law of the many. A law of the many is a law of what happens over many trials. The known laws are laws of the many: the known laws fix how the outcomes share out over many trials, and do not fix any single outcome. Chapter 4 and Chapter 10.

lawful reversal; faithful reversal. Lawful reversal is the claim that the reverse of a lawful process is lawful too. Faithful reversal is the claim that the reversed process passes back through every condition the original

process passed through, exactly, and arrives at exactly the condition the original process started from. Symmetry of the laws guarantees lawful reversal and does not guarantee faithful reversal. Chapter 8.

microstate amplification. Microstate amplification is the growth of small differences at a fine scale into differences at the scale that matters. Chapter 4.

the objective. The objective is the side a comparison is about, as the egg is when the cook compares the egg. Chapter 1 and Chapter 2.

omniscient; transcendent. The omniscient is "structure held in view as a whole, outside time", and the transcendent is "what could be". At a half-silvered mirror, which lets half of the light through and turns half aside, the transcendent is the two outcomes still open before the act of finding: a particle of light passes through, or the particle is turned aside. Chapter 4.

the one who finds. The one who finds is whoever or whatever makes a finding, a person or an instrument, inside the world, meeting what it finds. Preface and Chapter 1.

the Past Hypothesis. Carroll's term; in the first part, Carroll's Words, the entry for the Past Hypothesis sets this book's words beside Carroll's. The Past Hypothesis is the assumption that the universe we can observe began in a state of very low entropy, a name Carroll takes from the philosopher David Albert. Preface.

performed claim. *Physics Is Not Math*, a book of the collection, says: "A claim is performed when it is established by what happens in the act of engaging it, including the act of trying to deny it". The claim "I am doubting" is a performed claim: the very attempt to doubt the claim establishes the claim. Chapter 2.

phase. The phase is one of the three things inside a particle of matter that a change can act on. The other two are the color of a quark and the two sides of a pair such as the electron and its neutrino. In *The Story of the Particles*, a book of the collection, the phase is pictured as "a setting on a circle, like the hand of a clock". Chapter 3.

poetic naturalism. Carroll's term; in the first part, Carroll's Words, the entry for poetic naturalism sets this book's words beside Carroll's. Poetic naturalism is Carroll's view that there is one natural world, described in many ways of talking that agree with one another and with the world, and that which way of talking is best depends on the purpose of the moment. Preface.

presupposes; deeper. One thing presupposes another when the first cannot be done, stated or checked without the second, while the second can be done without the first. Of two things, the deeper is the thing that the other presupposes. *Physics Is Not Math* sets four fields of knowledge in one order by the test of presupposing, deepest first: metaphysics, physics, computation and mathematics. Chapter 1.

range of what could be found. The range of what could be found is the whole set of outcomes that an act of finding could come to, before the act: at the bowl, every way the shell could part. Preface and Chapter 8.

real. To be real is to be given in an actual act of comparison, here and now. For the one who finds, one outcome is real, the one found. From Chapter 5 on, this book uses the word 'real' only in the sense of being given in an actual act of comparison, and wherever Carroll's sense of the word appears, Carroll's sense is marked as his. Chapter 1 and Chapter 5.

record. A record is what an act of finding lays down: the third part of every act, after the range of what could be found and the act itself, as the crack, once found, stays found. Chapter 8.

sameness; difference. Sameness and difference are the two verdicts a comparison can end in, and each verdict has its meaning only beside the other. Chapter 2.

the Second Law of Thermodynamics. The physicists' term, which Carroll and this book both use. The Second Law of Thermodynamics is the law of heat that says a closed system of many parts runs toward its settled state and never, of itself, back out of the settled state. Chapter 8.

slack. Slack is some part of what happens that the description of one level does not fix, and in the collection there is slack at every scale. Chapter 10.

soft randomness. Soft randomness is randomness of ignorance, which better knowledge could reduce: the outcome was fixed, and the outcome is not known. Chapter 10.

stance. A stance is the side a comparison is made from: the place where the one who finds stands. A camera on the kitchen counter has a stance as surely as the cook does: the camera's lens faces the bowl from the counter, and the counter is the side the camera finds from. Chapter 6 and Chapter 11.

the Standard Model. The physicists' term, which Carroll and this book both use. The Standard Model is the physicists' theory of the known kinds of particle and of every force among those particles except gravity. Preface and Chapter 3.

strand. A strand is one independent route to a value, taken as one strand of a cable: the American philosopher Charles Sanders Peirce pictured good reasoning as a cable of many fibers, and not as a chain, which is no stronger than its weakest link. Two strands that rest on one source count as one strand. Chapter 2.

structural. A source is structural when the source lies in the form that every comparison has, whatever is compared, as the arrow of time has a structural source: each act of comparison runs one way, from the range of what could be found, to the act, to the record of what was found. Preface.

the structural arrow. The structural arrow is the direction of time as the order in which every act of comparison proceeds, from the range of what could be found, through the act, to the record. The structural arrow needs no beginning of low entropy. Chapter 8.

structure. Structure is how things stand to one another (shape, size, number, pattern), laid out all at once, with nothing in the structure coming first or after. Chapter 1.

the subjective. The subjective is the side a comparison is made from, as the cook, with her bowl and her eyes, is when she compares the egg. Chapter 1 and Chapter 2.

symmetry. Symmetry is a sameness of content where there is a difference of context: sameness kept across a change, as a measuring cup holds one cup at every place and in every year. Chapter 2.

the triple of the real. The triple of the real is causality, change and choice: the three things that every real event has, each in some measure. Choice is the member of the triple that a law of the many does not fix. Chapter 10.

the turn. The turn is the passage from the end of one epoch to the beginning of the next, at the one end that every black-hole interior reaches. No comparer registers the turn itself as an event. Chapter 9.

the universal state. The universal state is the quantum state of the whole universe, which Carroll calls the universal wave function. The universal state, branches and all, exists as structure. Chapter 5.

unsharp. An unsharp measurement is one whose outcomes do not cut the states cleanly apart: a detector that reports "passed" for some particles that a mirror turned aside makes an unsharp measurement. Chapter 6.

update. To update is to change a description to fit what has just been found, as a credence is updated when new evidence arrives, and as the quantum state is updated when a measurement finds one outcome. Chapter 5.

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