

The Gravitational Companion

Extensions, Comparisons and Open Questions
for The Gravitational Passage

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Preface

In *The Gravitational Passage*, which this book calls the Passage, I derive general relativity, Einstein's theory of gravitation, from the Immanent Metaphysics, the metaphysics that I set out in my book *An Immanent Metaphysics*. The Passage is one chain of argument. The chain runs from the structure of comparison to the Einstein equations, the field equations of general relativity. From the Einstein equations the chain runs to the black hole interior and the origin of an epoch (one age in the history of the universe), and last to the value of the cosmological constant. The cosmological constant, written Λ, is the constant term of the Einstein equations. The chain begins with the structure of comparison, and the Root Tautology of the Passage says that an interaction is a comparison (the Passage, Chapter 1, Section IV).

I set out what lies beside the Passage's chain of argument: comparisons, extensions and open questions. The comparisons are comparisons in the everyday sense, in which a step of the Passage is set beside the work of other researchers. The extensions add to what the Passage states, and the chain of argument does not use the extensions. The open questions are the steps of the Passage that are still open, taken up as questions for research.

The dependence between the two books runs one way. This book depends on the Passage, and nothing in the Passage depends on this book. So if anything in this book is shown to be wrong, no step of the Passage fails as a result.

How to read it. The opening of each chapter of this book contains the sentence of the Passage that the chapter extends, quoted in full, with the chapter and section of the Passage where the quoted sentence appears. Where each section of a chapter is about a particular step of the Passage, the opening of each section contains instead the sentence of the Passage that the section extends. Each section's quotation is also complete, with the chapter and section of the Passage where the quoted sentence appears. In this book, a chapter, section or appendix named by number is a chapter, section or appendix of *The Gravitational Passage*. I name the chapters of this

book by title, such as "Comparisons at Single Steps". I give the computation behind every number that this book computes, with the inputs of each computation, in the Passage's additional materials (*The Gravitational Passage; Additional Materials*, UVSM Publications, 2026, at uvsmpublications.com). The additional materials also contain the computations of this book's chapter "The Spectral Invariants of $\mathbb{C}P^2 \times S^2$".

How it argues. In this book, as in the Passage, I prove each theorem or cite a proof of the theorem, and I give each computation with the computation's inputs. When a step goes from the structure of comparison to a physical quantity, I call the step an identification, and I state each identification in the first person. When I describe a step of the Passage in the terms of the Immanent Metaphysics, I call the description a reading. Where a reading contains no new prediction, the chapter that contains the reading says so. Each chapter states its standing, that is, the grounds of the chapter's claims and what those claims add to the Passage.

> **On scope.** I make no claim about the Standard Model or about the particle spectrum, in this book as in the Passage. The Standard Model is the theory of the elementary particles and of every force except gravitation, and the particle spectrum is the list of the particles that exist, with their masses. I also leave the dark component of the universe out of this book: the dark component is the part of the universe's matter that gives off no light. The Passage states an identity in Chapter 13, Section III. The identity is an equation whose solution is the total matter fraction: the share of the density of the universe that is matter of every kind. The identity is indifferent to what the matter is made of, so what the dark component is made of makes no difference to the identity.

> **The map of this book.** The table below lists the chapters of this book. For each chapter, the table shows the step of the Passage that the chapter extends and the chapter's kind (comparison, computation, reading, collection, open questions or index). The last column of the table shows whether each chapter adds a test to the predictions that the Passage states. No

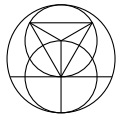
chapter adds a test to the predictions of the Passage: each chapter contains a step of the Passage set beside the work of other researchers, or a reading of a step of the Passage in the terms of the Immanent Metaphysics, or a statement of what is still open in the Passage.

Chapter	Step of the Passage it extends	Kind	Adds a test?
Part I			
Other Programs That Obtain General Relativity	the claim of derivation (Preface; Chapter 20)	comparison	No
What the Passage Obtains That Others Assume	what other derivations assume (Chapters 10 to 13)	comparison	No
Comparisons at Single Steps	single steps: the internal space (Chapter 9), the interior (Chapter 18), the entanglement (Chapter 12), Λ and the Hubble constant (Chapter 13), inflation (Chapters 11 and 18)	comparison; computation	No: the Hubble constant is the test of the matter fraction
Two Accounts of Causation	causation as a route to law (Chapter 5, Section III)	reading	No
Refutation Conditions Named in This Book	the steps named in the chapters above	collection	No
Part II			
Why the Balance Criterion Holds	the third rejection criterion (Chapter 3, Section II)	reading	No
The Categorical Form of the Incommensuration Theorem	the theorem (Chapter 3); the theorem on connections (Chapter 9, Section V)	comparison	No
The Cells of $\mathbb{C}P^2$ and the Six Metrics	the internal space (Chapter 9)	reading	No
	the shape of the internal space (Chapter 9, Section IX)	computation	No

The Spectral Invariants of $\mathbb{C}P^2 \times S^2$			
The Grid as the Space of Unentangled States	the internal space and its metric (Chapter 9)	reading; computation	No
The Modal Readings of the Inertial Forces and of Free Fall	the equivalence and the geodesic (Chapters 14 and 15); the prior direction of time (Chapter 10)	reading	No
One Mass, Three Elevators	the equality of the masses, inertia and spin (Chapters 12, 14 and 18; Appendix A, Sections II and IV)	reading	No
Two Formal Triples of a Domain	existence and reality; the turn (Chapters 5 and 18)	reading	No
Part III			
Open Questions as Research	the open steps of the Passage	open questions	No
Back matter			
Where This Book Meets the Passage	every chapter of the Passage	index	No

PART I

Comparisons



Other Programs That Obtain General Relativity

On What Others Have Achieved and Where They Stop

Several programs obtain general relativity, or parts of it, from principles other than its standard postulates. In this chapter I survey eight: the constructive axiomatics of Ehlers, Pirani and Schild (1972); the thermodynamic derivation of Jacobson (1995); the entropic gravity of Verlinde (2011) and the emergent gravity of Padmanabhan (2010; 2015); the spectral action of Chamseddine and Connes (1997); loop quantum gravity and causal set theory; the constructor theory of time of Deutsch and Marletto (2025); the derivation of the field equations from a self-coupled massless field of spin 2 (Gupta 1954; Kraichnan 1955; Deser 1970; Weinberg 1964a; 1965a); and string theory, in which the field equations are a condition of the consistency of the string (Callan, Friedan, Martinec and Perry 1985). Section IX notes three further approaches more briefly. For each I state what the program obtains, what it takes as input, and where it stops. Section X sets the results side by side, in four columns: the manifold, the signature, the field equations and the two constants G_N and Λ.

> *The Passage, Preface:* "Several programs obtain general relativity, or parts of it, from principles other than its standard postulates: the constructive axiomatics of Ehlers, Pirani and Schild (1972); the thermodynamic derivation of Jacobson (1995); the entropic gravity of Verlinde (2011) and the emergent gravity of Padmanabhan (2010; 2015); the spectral action of Chamseddine and Connes (1997); loop quantum gravity (Ashtekar 1986; Rovelli 2008) and causal set theory (Bombelli and colleagues 1987; Surya 2019); and the constructor theory of time of Deutsch and Marletto (2025)".

Rests on: the published programs as cited, and, for the row of the Passage in Section X, Chapters 8 to 12 and Chapter 13. *Used by:* "What the Passage Obtains That Others Assume".

I. Ehlers–Pirani–Schild (1972)

The EPS axiomatization builds the kinematic structure of GR from axioms about light rays and particle worldlines. From the empirical behavior of light signals and free fall, EPS constructs: a differentiable manifold, a conformal structure (light cones), a projective structure (free-fall trajectories), and a Weyl geometry compatible with both. The compatibility of the conformal and projective structures is itself an axiom of the construction. The final step, from Weyl geometry to Riemannian geometry, requires a postulate: that freely falling clocks measure metric arclength.

EPS derives the kinematic arena but not the dynamics. The Einstein field equations, G_N, and Λ are outside its scope.

II. Jacobson (1995)

Jacobson showed that the Einstein equations follow from two inputs: the proportionality of entropy to horizon area, and the Clausius relation $\delta Q = T\, dS$. Applied to every local Rindler horizon, with the Unruh temperature and boost-energy flux as heat, the two inputs yield the Einstein equations.

Jacobson derives the dynamics. He assumes the kinematic structure: the manifold, the metric, the causal structure, the Lorentzian signature, and the Unruh effect. G_N enters through the entropy-area relation; Λ appears as an undetermined constant.

III. Verlinde (2011) and Padmanabhan (2010; 2015)

Verlinde derives Newton's law as an entropic force on holographic screens. The relativistic extension recovers the Einstein equations by an argument parallel to Jacobson's. The program assumes the holographic principle, the Bekenstein bound, and the Unruh effect. Its use of relativistic inputs, the Unruh temperature among them, to obtain a non-relativistic law has been criticized as circular (Visser 2011), and the holographic principle it starts from is itself a conjecture.

Padmanabhan develops the thermodynamic interpretation further: the Einstein equations reduce to $T\,dS = dE + P\,dV$ on every horizon; the Einstein-Hilbert action is a free-energy functional; cosmological expansion is driven by the difference between bulk and surface degrees of freedom. The program also fixes the observed size of Λ by a count of information: the number of modes that cross the Hubble radius over the history of the universe, taken to be 4π, gives Λ in Planck units of the observed order (Padmanabhan and Padmanabhan 2013), and the same count fixes the amplitude of the primordial departures (Padmanabhan and Padmanabhan 2017). It gives an interpretation of the field equations; it does not identify the microscopic degrees of freedom.

IV. The Spectral Action of Chamseddine and Connes (1997)

Of the programs surveyed here, the Connes program is the closest to the Passage in form: it obtains gravity on a product of four-dimensional spacetime with an internal space. The spectral action principle, applied to the noncommutative geometry $C^\infty(M) \otimes A_F$ with $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, yields together the Einstein-Hilbert action and the Standard Model Lagrangian (Chamseddine and Connes 1997).

Connes *assumes* the algebra A_F: it is an input, not a derivation. He also assumes a four-dimensional Riemannian manifold, of Euclidean signature, as the commutative factor M. Barrett (2007) gives a formulation of

the program with a Lorentzian metric; there the Lorentzian signature is chosen at the outset. In both formulations the dimensionality, the signature and the product structure $M \times F$ are inputs. The next chapter, in its Section III, sets these inputs beside what the Passage obtains. The Passage makes no claim about the finite algebra A_F or about the Standard Model Lagrangian.

V. Loop Quantum Gravity and Causal Set Theory

LQG quantizes GR directly. It recasts GR in the connection variables of Ashtekar and quantizes them with methods from lattice gauge theory. Its states of geometry are spin networks, and its area and volume operators have discrete spectra, with area in quanta of the order of the Planck area. From these it counts the Bekenstein-Hawking entropy of black holes, and in loop quantum cosmology the big bang is replaced by a bounce. It does not derive GR from simpler principles; it takes GR as input and produces a quantum version. The discrete area and volume spectra and the bounce in place of the big bang are results downstream of GR, not derivations of it.

Causal set theory proposes that spacetime is a locally finite partial order. Its slogan, "order plus number equals geometry", states that causal ordering and element counting determine the metric. A causal set that embeds faithfully in a Lorentzian manifold is conjectured to determine that manifold essentially uniquely (the Hauptvermutung); the conjecture is not yet proved in general. The dimensionality of the emergent manifold is not derived, and the Einstein equations have not yet been recovered from causal set dynamics.

Causal set theory predicted the order of the cosmological constant before 1998, when the expansion was first observed to accelerate. In that prediction Λ fluctuates about zero, with a magnitude of order the inverse square root of the four-volume of the universe in Planck units, and that magnitude is today of the observed order (Sorkin 1997; Ahmed, Dodel-

son, Greene and Sorkin 2004). The prediction is a claim about every epoch: Λ changes from epoch to epoch, and at each epoch it is of the order of the critical density. The prediction of causal set theory is a stronger claim about time than the Passage makes. The Passage fixes Λ once, from the horizon of the measurement, and predicts the equation of state $w = -1$ (Chapter 13, Section V).

VI. The Constructor Theory of Time (Deutsch and Marletto 2025)

Constructor theory states physical laws as statements of which transformations are possible and which are impossible, rather than as equations of motion, and its laws make no reference to time. In the constructor theory of time, Deutsch and Marletto ask how duration and dynamics arise from such laws. Their answer is the timer: a physical system prepared with an attribute that changes, whose change to a fixed attribute marks the passage of a definite duration. Time is then defined by what such systems can do, not taken as a parameter.

The program derives a definition of time that needs no prior time parameter, and it avoids the problem of time in quantum gravity by never introducing time at the level of its laws. It assumes its own framework, the distinction between possible and impossible tasks, and the existence of substrates able to serve as timers. It does not yet derive the structure of spacetime, the field equations or the gravitational interaction. Its contact with gravity is the proposal, made within constructor theory by Marletto and Vedral (2017) and independently by Bose and colleagues (2017), to witness quantum effects of gravity by entanglement, a program of experiment rather than a derivation, and its later work (Marletto, Deutsch and Vedral 2026) extends its tests without producing the Einstein equations. The Passage predicts the outcome of that experiment: the two masses in that experiment become entangled, with the phase of their Newtonian interaction, because each alternative of the matter carries its own geometry

(Chapter 12, Section VI). The result would separate the Passage's account from a single geometry sourced by the weights of the alternatives. Marletto and Vedral (2017) prove that a mediator which creates entanglement between two systems cannot be classical in their sense, a system with a single observable whose states cannot be superposed. Per-alternative geometry meets their condition for a non-classical mediator, since the geometry takes a different state in each alternative of the matter and the alternatives keep their relative phase, so the result of the experiment would confirm their conclusion as they state it. What it would not show is a quantum of the arena: the Passage holds that the arena has none, at any scale (Chapter 12, Questions).

Constructor theory and the Passage both obtain time instead of taking it as a parameter. Deutsch and Marletto reach time operationally, through what timers can do. In the Passage I reach it structurally. I identify time with the immanent context of the context triad (Chapter 7): the single prior direction, the cut that divides before from after (Chapter 10, Section IV); Appendix C obtains the one dimension of time by a second route. Constructor theory takes the distinction between the possible and the impossible as its starting framework. In the Passage the central impossibility, the Incommensuration Theorem, is proved from the six intrinsics and four compounds of comparison by enumeration (Chapter 3).

VII. The Spin-2 Bootstrap (Gupta 1954; Kraichnan 1955; Deser 1970; Weinberg 1964 and 1965)

> *The Passage, Chapter 6, Section II:* "A quantization of the metric would carry spin 2, the only spin of a massless particle that couples universally to the stress-energy tensor (Weinberg 1964)".

The bootstrap obtains the Einstein equations from a field on flat spacetime. Its starting object is a symmetric tensor field $h_{\mu\nu}$ of zero mass and spin 2 on Minkowski spacetime, whose free field equation is the linearized Einstein equation (Fierz and Pauli 1939). The divergence of that lin-

ear equation vanishes identically, so the field can be coupled only to a conserved source. The stress-energy of matter coupled to the field is conserved only when the energy and momentum of the field itself are counted, so the field must couple to the total stress-energy, its own included. Gupta (1954) and Kraichnan (1955) imposed this requirement order by order in the coupling, and Feynman did so in his lectures at Caltech in 1962 and 1963 (Feynman, Morinigo and Wagner 1995). Deser (1970) completed the construction in one step: written in first-order form, the theory needs a single cubic term and no further iteration. The result is the Einstein-Hilbert action for the metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, in which the flat metric $\eta_{\mu\nu}$ no longer appears on its own. The construction is classical: it uses classical field equations and quantizes nothing.

Weinberg reached the universality by a second route, from the Lorentz invariance and the pole structure of the S-matrix (Weinberg 1964a; 1964b). A massless particle of spin 2, emitted with an energy that tends to zero, can be coupled consistently only if its coupling is the same to every particle; that equality is the equality of gravitational and inertial mass. The same argument for spin 1 gives the conservation of charge, and it explains why no macroscopic field corresponds to a massless particle of spin 3 or higher (Weinberg 1964a). Weinberg then treated the emission of soft gravitons to all orders (Weinberg 1965b) and obtained the Einstein equations from the perturbation theory of gravitons (Weinberg 1965a). In this book the word graviton names at most what it names in the Passage: a quantum of the description of the small waves of a classical geometry, never a quantum of the arena, a particle, a carrier of gravitation or one of the kinds (the Passage, Chapter 6, Section II); Weinberg's gravitons are quanta of that description. Weinberg's results are the ground of the sentence from Chapter 6, Section II quoted above.

The bootstrap takes four things as given: a flat four-dimensional spacetime with Lorentzian signature; a field of spin 2 and zero mass; field equations of second order; and coupling to the total stress-energy. From them it obtains the Einstein equations and the equivalence principle. G_N enters

as the strength κ of the coupling, and the bootstrap does not fix it. The flat metric solves the Einstein equations with a cosmological term only if $\Lambda = 0$, so a bootstrap on flat spacetime takes $\Lambda = 0$ as given with its background. Deser (1987) carried out the construction on an arbitrary background, and there a cosmological term of any value can be included. Barceló, Carballo-Rubio and Garay (2014) find that the self-coupling can also yield a theory with essentially the structure of unimodular gravity, in which Λ enters as a constant of integration.

The derivation has been contested. Padmanabhan (2008) argues four points: a Lorentz-invariant field theory has infinitely many conserved symmetric tensors of rank two, so the tensor to which the field couples is a choice; the choice that yields the Einstein-Hilbert action, and a total divergence that the construction must add, are known only from the answer; the second order of the final equations is put in by hand; and the surface term of the Einstein-Hilbert action, which is tied to the entropy of horizons, cannot be reached by expansion in the coupling. Butcher, Hobson and Lasenby (2009) give a version in which the stress tensor is fixed by a stated rule, and Deser (2010) replies that the derivation of 1970 stands. I therefore record where the derivation stands: the Einstein equations follow from the four inputs together with a stated choice of stress tensor, and whether that choice is forced is disputed.

On what holds below the Planck scale, the bootstrap and the Passage agree. Both arrive at the Einstein equations (Chapter 12, Section VII). In both, the gravitational field is in the end the metric of a geometry: in the bootstrap, $h_{\mu\nu}$ and $\eta_{\mu\nu}$ enter the result only through $g_{\mu\nu}$. The bootstrap begins from the small waves of the metric, and the Passage admits those waves and their description by quanta. The Passage, Chapter 12, Questions: "The small waves of the metric are described by quanta, and the quanta are quanta of the description of the wave, not things in the arena".

The bootstrap and the Passage differ in their starting object. The bootstrap starts from a field on a fixed flat spacetime and finds that its self-coupling sums to a geometry. The Passage takes the geometry as given,

and reads the field as a description of the geometry's small waves that ends at the Planck scale. The Passage, Chapter 11, Section I: "The reduction takes a metric as given: the context factor is a smooth manifold with a metric (Chapter 8, Section II), and the reduction starts from the ten-dimensional metric on the product". A field picture is not by itself a quantization: Deser's construction is classical throughout. The quantized versions, Feynman's gravitons and Weinberg's perturbation theory, concern scattering at energies far below the Planck energy, where the Passage admits the description by quanta as the effective field theory of gravity; their quanta are quanta of that description, not quanta of the arena. Neither version makes a claim at the Planck scale or beyond, and only there do the Passage's account and the description by quanta differ in what they predict. The Passage holds that the arena is not quantized at any scale. The Passage, Chapter 12, Questions: "The claim that quantum gravity is a category error holds at every scale; this account and linearized quantum gravity differ in what they predict only at the Planck scale and beyond". The bootstrap therefore neither supports nor refutes the Passage's Marker 3 (Chapter 6, Section II: "This book holds that the metric itself is not quantized (Marker 3)".), and no observation below the Planck scale separates the bootstrap's reading from the Passage's.

Weinberg and Witten (1980) prove that in every theory with a Lorentz-covariant energy-momentum tensor, massless particles of spin greater than 1, elementary or composite, are forbidden. General relativity escapes the theorem because the energy of the gravitational field has no Lorentz-covariant local tensor. This agrees with the Passage's Marker 2 (Chapter 6, Section II: "It is the arena, not a field within the arena".); the agreement is a reading and adds no prediction.

The bootstrap's reading and the Passage's both predict gravitational waves with the two polarizations of a massless field of spin 2, traveling at the speed of light. Observation agrees. The three-detector event GW170814 favors purely tensor polarization over purely vector and purely scalar polarization (Abbott and colleagues 2017b). The merger GW170817 and its gamma-ray burst bound the difference between the speed of grav-

itational waves and the speed of light to between -3×10^{-15} and $+7 \times 10^{-16}$ of the speed of light (Abbott and colleagues 2017a). At 90 percent credibility, the absence of dispersion in the waves bounds the mass that a quantum of their description could carry at no more than 2.42×10^{-23} eV/c^2 (Abbott and colleagues 2025, who state the bound as a mass of the graviton). If gravitational waves are found to carry a further polarization, or to travel at another speed, then the bootstrap's starting field is not the field of gravity, and the step of the Passage that fails is the uniqueness of Chapter 12, Section VII: that, with the moduli held, the four-dimensional equations have the Einstein form, whose waves have the two polarizations and the speed that the Passage states in Chapter 16, Section V.

VIII. String Theory (Callan, Friedan, Martinec and Perry 1985)

The Passage, Chapter 10, Section I: "The total is therefore $4 + 6 = 10$".

The Passage, Chapter 11, Section I: "On the ten-dimensional product manifold $M^4 \times \mathbb{C}P^2 \times S^2$, the gravitational action is built from the metric alone".

String theory obtains the field equations as a condition of consistency. A string moving through a spacetime with a metric $G_{\mu\nu}$, an antisymmetric tensor field $B_{\mu\nu}$ and a scalar field Φ, the dilaton, is described by a two-dimensional field theory on the surface that the string sweeps out, its worldsheet; the three spacetime fields are the couplings of that theory. The theory is consistent only if it is unchanged by local rescalings of the worldsheet metric, which is its conformal invariance, and that requires the beta functions of all its couplings to vanish. At leading order in α', the square of the string length, the condition on the metric is

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - \tfrac{1}{4} H_{\mu\lambda\sigma} H_\nu{}^{\lambda\sigma} = 0, \text{ with } H = dB$$

(Lovelace 1984; Fradkin and Tseytlin 1985; Callan, Friedan, Martinec and Perry 1985). With $B = 0$ and Φ constant it is $R_{\mu\nu} = 0$, the vacuum Einstein equation. The conditions are the field equations of an action that

contains the Einstein-Hilbert term, so with the other fields as sources they are the Einstein equations in the dimension of the target. A second route gives the same result: the closed string has a massless state of spin 2, and in the limit of zero slope its interactions are those of Einstein's theory (Yoneya 1974; Scherk and Schwarz 1974), as Section VII says they must be.

The same consistency fixes the dimension. Conformal invariance must survive quantization, and for that the central charges of the worldsheet fields must cancel. Each spacetime direction contributes 1 through its coordinate field and, in the superstring, a further 1/2 through its fermion; the ghost fields of the worldsheet symmetries contribute -26 and, in the superstring, a further $+11$. Cancellation gives $D = 26$ for the bosonic string (Polyakov 1981a) and $(3/2)D = 15$, so $D = 10$, for the superstring (Polyakov 1981b). For the bosonic string, Lorentz invariance in light-cone gauge gives the same number (Goddard, Goldstone, Rebbi and Thorn 1973). The anomaly cancellation of Green and Schwarz (1984) is a different result: it assumes ten dimensions and fixes the gauge group as $SO(32)$ or $E_8 \times E_8$. Ten is fixed by the anomaly of the worldsheet, not by the anomalies of spacetime.

String theory takes three things as given: the string with its worldsheet theory; a target manifold with a metric, which enters as a coupling; and a Lorentzian signature with one time direction, for which the absence of states of negative norm is proved (Brower 1972; Goddard and Thorn 1972). It obtains the total dimension, and the field equations at leading order in α', with corrections at higher orders. Three qualifications bound the dimension it obtains. A string in a background whose dilaton grows linearly is consistent in other dimensions (Myers 1987). At strong coupling, one of the superstrings has an eleventh dimension (Witten 1995). And the cancellation condition fixes a central charge, 15, rather than a number of geometric directions: with four extended dimensions, the rest must be a superconformal field theory of central charge 9, and that theory need not describe strings on a six-dimensional manifold (Gepner 1988). The split into four extended and six compact dimensions is chosen to match obser-

vation. In four dimensions G_N is fixed in terms of the string length, the string coupling and the volume of the internal space. The string coupling and the volume are values of fields that the theory does not fix by itself, and Λ depends on the choice among very many vacua (Bousso and Polchinski 2000; Kachru, Kallosh, Linde and Trivedi 2003; Douglas 2003). The constants are therefore related, and their values are open.

String theory and the Passage share a route: a ten-dimensional theory, reduced on six compact dimensions to four, with moduli that must be held. Both take a metric as given. In string theory the metric is a coupling of the worldsheet theory; in the Passage the reduction starts from it (Chapter 11, Section I). They obtain the four-dimensional field equations by different means: string theory from the consistency of the worldsheet theory, the Passage from Lovelock's theorem once the moduli are held (Chapter 12, Section VII).

Ten. The superstring and the Passage both have ten dimensions, and both divide them into four extended and six compact. The agreement does not support either account, for three reasons.

First, the string's count of ten and the Passage's share no step. The superstring's ten solves $(3/2)D = 15$, and its inputs are the central charges of the worldsheet fields and ghosts. The Passage's ten is $4 + 6$, and its inputs are the six metrics of comparison (Chapter 9, Section I) and the three transitions and one direction of the cycle of Axiom II (Chapter 10, Section IV). No quantity of either count enters the other. In string theory ten is a condition of consistency. In the Passage ten is a count: the Einstein-Hilbert action and Lovelock's theorem are consistent in every number of dimensions.

Second, the string's count does not register what the Passage derives. At leading order every six-dimensional internal space contributes the same central charge, 9. The count therefore neither favors nor excludes $\mathbb{C}P^2 \times S^2$, and it does not fix the split into 4 and 6.

Third, the conditions of string theory that do constrain the internal space are not met by the Passage's ten-dimensional geometry. (a) $\mathbb{C}P^2$ admits no spin structure (Hawking and Pope 1978), and so $M^4 \times \mathbb{C}P^2 \times S^2$ admits none. The type IIA, type I and heterotic superstrings need a spin structure, since their gravitino is a spinor field that couples to no gauge field. Type IIB can be defined on a manifold without a spin structure when a bundle of its duality group compensates (Debray, Dierigl, Heckman and Montero 2022). On $M^4 \times \mathbb{C}P^2 \times S^2$ that requires seven-branes and a string coupling that varies over the internal space, and the product $\mathbb{C}P^2 \times S^2$ occurs in that form as a base of compactifications of F-theory (Halverson and Taylor 2015). (b) With the metric, the dilaton and B alone, at leading order in α', and with a flat external factor, the conditions force the internal space to be Ricci-flat, with constant dilaton and $H = 0$. The Passage's internal metric has positive Ricci curvature. The Passage, Chapter 9, Section VIII: "$\mathbb{C}P^2$ with the Fubini–Study metric has Ricci tensor $R_{ij} = 6g_{ij}/{R_O}^2$, everywhere positive, where R_O is its radius in this normalization". (c) With a de Sitter external factor, supergravity with two derivatives and without singular sources has no compactification at all (Maldacena and Núñez 2001). The Passage's ten-dimensional action is built from the metric alone, with no dilaton, no form fields and no branes. Its ten-dimensional geometry is therefore not a vacuum of the superstring at leading order, and the two tens are counts of different things.

The agreement would carry weight if a derivation connected the two counts: for example, a consistent worldsheet theory of total central charge 15 whose target is $M^4 \times \mathbb{C}P^2 \times S^2$ with the Passage's metric, or a step from the six metrics of comparison to an internal superconformal field theory of central charge 9. Neither book has such a derivation. By (a) to (c), a worldsheet theory of the first kind would need fields that the Passage's ten-dimensional action does not contain.

IX. Three Further Approaches

> *The Passage, Preface:* "The book takes a metric geometry as given, as general relativity does; what it obtains from comparison is the number of dimensions, the signature and the product form of that geometry, and the form of the field equations then follows from Lovelock's theorem (Chapter 12, Section VII)".

Three further approaches obtain parts of general relativity; the table of Section X does not list them. Sakharov's induced gravity (Sakharov 1967; Visser 2002) takes a Lorentzian manifold with a classical metric and quantized matter fields on it. The effective action of the matter at one loop then contains the Einstein-Hilbert term, with G_N set by the highest energy to which the matter fields are counted. It also contains a cosmological term that is generically of the order of the fourth power of that energy, so the small observed Λ is put in by hand. Induced gravity shares with the Passage a metric that is not quantized; it differs in computing G_N, which the Passage takes from measurement.

Asymptotic safety (Weinberg 1979; Reuter 1998; Niedermaier and Reuter 2006) quantizes the metric and proposes that its dimensionless couplings reach a fixed point at high energy, so that the quantum theory holds at every scale. The evidence comes from truncations of the effective action, computed mostly in Euclidean signature, and the observed values of G_N and Λ are free parameters, fixed by experiment (Eichhorn 2019). The program continues a quantized metric past the Planck scale, which the Passage excludes (Chapter 6, Section II: "This book holds that the metric itself is not quantized (Marker 3)".). No present observation tells the two apart.

The approaches that put entanglement first read the connectedness of spacetime as an effect of the quantum entanglement of underlying degrees of freedom (Van Raamsdonk 2010). In holographic theories, the Einstein equations linearized about anti-de Sitter space follow from the first law of entanglement (Lashkari, McDermott and Van Raamsdonk 2014; Faulkner, Guica, Hartman, Myers and Van Raamsdonk 2014). Jacobson (2016) shows that, for conformal quantum fields and variations at

first order, the entanglement in small balls is stationary at fixed volume if and only if the Einstein equation holds, with Λ undetermined. These approaches take the manifold and its signature as given; the holographic ones also take the anti-de Sitter boundary as given. Like Jacobson (1995), they obtain the field equations from entropy.

X. The Comparative Table

The table records, for each program, whether it obtains or takes as given four things: the spacetime manifold with its dimension; the Lorentzian signature, one direction of time and three of space; the Einstein field equations; and the values of the two constants G_N and Λ. "Derived" means obtained from the program's own starting principles; "Assumed" and "Input" mean taken as given; "Open" and "Not yet" mean not yet obtained; "No" means outside the program's results.

Program	**Manifold and dimension**	**Lorentzian signature**	**Einstein field equations**	**Values of G_N and Λ**
EPS	Derived	In the light axioms	No	No
Jacobson	Assumed	Assumed	Derived	G_N assumed; Λ undetermined
Verlinde	Assumed	Assumed	Recovered, by an argument parallel to Jacobson's	No
Padmanabhan	Assumed	Assumed	As identity	Λ: the observed order, from a count of information
Connes	Assumed	Assumed (Euclidean; Lorentzian in Barrett 2007)	Derived	Relations only

LQG	Input	Input	Input	No
Causal sets	Emergent (conjectured); dimension open	From the order	Open	Λ: order predicted, fluctuating
Constructor theory	Not yet	Not yet	Not yet	Not yet
Spin-2 bootstrap	Assumed: flat, four-dimensional	Assumed	Derived, given second order and coupling to the total stress-energy; the choice of stress tensor disputed (Padmanabhan 2008)	G_N a free coupling; $\Lambda = 0$ assumed with the flat background, free on a curved one
String theory	Manifold assumed; total dimension derived (10 for the superstring, 26 for the bosonic string); split 4 + 6 chosen	Assumed	Derived at leading order in α', in ten dimensions, with dilaton and form fields; in four after compactification	Relations only; values open
The Passage	**Derived†, as a factor of the product (Chapters 8 and 10)**	**Derived† (Chapter 10)**	**Derived†, by reduction and Lovelock's theorem (Chapters 11 and 12)**	**Λ derived† (Chapter 13); G_N measured**

† With the identifications of the chapters named. In the row of the Passage, "derived" has the sense the Preface gives it: the result follows by theorem or standard technique from the axioms together with the identifications stated in the chapters named. Jacobson, Verlinde, Padmanabhan, Connes and loop quantum gravity take the metric as given, and so does the Passage (Preface; Chapter 11, Section I). String theory takes it as given too, as a coupling of the worldsheet theory. EPS and causal set theory seek to obtain it. The spin-2 bootstrap obtains a curved metric from a flat metric and a field of spin 2.

Questions

Is it fair to score each program against aims it did not set itself?

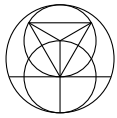
The table does not rank the programs. It records, for each, what the program takes as input and what it obtains, in the four respects that the Passage's aims require. Each program is described by its own aim in Sections I to VIII: EPS aims at the kinematic arena, Jacobson at the dynamics, Padmanabhan at an interpretation of the field equations, Connes at gravity and the gauge sector from one principle, loop quantum gravity at a quantum version of GR, constructor theory at time from possible and impossible tasks, the spin-2 bootstrap at the field equations from the consistency of a field of spin 2, and string theory at a consistent quantum theory of strings, which contains gravity. A "No" or "Input" in a column says that the item lies outside that program's results; it is not a judgment that the program has failed. The comparison shows where each program begins, and so which of its inputs a program that began earlier could supply.

The bootstrap obtains the Einstein equations from a field of spin 2. Does that not make the metric a quantized field, against Chapter 6, Section II?

No. Deser's bootstrap (1970) is classical and quantizes nothing. Its starting field is the small waves of the metric, which the Passage admits together with their description by quanta (Chapter 12, Questions). Its quantized versions concern energies far below the Planck energy, and their quanta are quanta of that description, not of the arena. The Passage holds that the arena is not quantized at any scale, and its predictions part from those of the description by quanta only at the Planck scale and beyond, where the bootstrap makes no claim (Section VII).

String theory fixes the total dimension at ten by consistency, and the Passage obtains ten as 4 + 6. Is the agreement significant?

No. The two counts share no step; the string's count does not register the split or the internal space; and the Passage's ten-dimensional geometry does not meet the conditions by which string theory constrains its internal spaces (Section VIII). The agreement would carry weight only through a derivation that connects the two counts, and neither book has one.



What the Passage Obtains That Others Assume

On the Depth of the Derivation

In the previous chapter, "Other Programs That Obtain General Relativity", I set out what the researchers of several other programs take as their starting inputs when those researchers obtain general relativity, Einstein's theory of gravitation. In this chapter I take those inputs one at a time, and for each input I state whether I obtain that input in the Passage, and how.

I obtain three things in the Passage from the axioms of the Passage, with the identifications of Chapters 8 to 10. In the Preface of this book I call a step from the structure of comparison to a physical quantity an identification. The first of the three things is the product structure: the geometry of the Passage is a product of a four-dimensional factor and a six-dimensional internal space. The second of the three things that I obtain is the number of dimensions, four external and six internal. The third of the three things that I obtain is the Lorentzian signature. The Lorentzian signature is the pattern of signs in a metric by which the direction of time counts with the opposite sign from the directions of space.

I take a metric as given, like Jacobson, and like Ehlers, Pirani and Schild (EPS for short) in the last step of their construction. A metric is the rule for the interval between any two nearby events. The reduction of Chapter 11 of the Passage starts from a ten-dimensional metric, and from that metric the reduction produces the four-dimensional metric. Like Jacobson, I also take Newton's constant G_N from measurement, and I take the entropy-area relation as given. The cosmological constant Λ is the constant term of the Einstein equations. The Passage fixes Λ once, from the information deficit at the observer's horizon, while causal set theory predicts a Λ that fluctuates from epoch to epoch.

The Passage, Chapter 11, Section I: "The reduction takes a metric as given: the context factor is a smooth manifold with a metric (Chapter 8, Section II), and the reduction starts from the ten-dimensional metric on the product".

Rests on: the previous chapter; the Incommensuration Theorem and its physical reading (Chapter 3, Section V); the product, the internal space, the dimensions and the signature (Chapters 8 to 10); the reduction (Chapter 11); the field equations (Chapter 12); Λ (Chapter 13); geodesic motion (Chapter 15). *Used by:* no step of the Passage; in this chapter I set the Passage beside the other programs.

I. What the Passage Obtains That EPS Assumes

Ehlers, Pirani and Schild build the geometry of general relativity from axioms about light rays and freely falling particles. The starting objects of the EPS construction are the light rays and the falling particles themselves, with their observed behavior. In the Passage neither light rays nor falling particles are starting objects.

The light cones of the Passage are the cones of the order of events. At each event the cone is bounded by the maximum speed of signals, and two premises of the Passage fix that maximum together (Chapter 10, Section III, and Questions; Chapter 17, Section I). The first premise is the directed precedence of Axiom II, the order of before and after among events, which runs in one direction only. The directed precedence excludes signals that take no time. The second premise is the macroscopic boundary of the domain, and the second premise makes the maximum speed finite. In the Passage I take the macroscopic boundary to be one ratio of spatial to temporal measure, the same for every comparer in uniform motion (a comparer is one who compares, such as an observer).

In the Passage, free fall is geodesic motion: motion along the straightest paths that the metric allows. Geodesic motion follows from the field equations through the Bianchi identity, an identity that the curvature of every metric satisfies (Chapter 15, Section I).

Ehlers, Pirani and Schild need a postulate for the last step of their construction: that clocks measure metric arclength. With the postulate, the Weyl geometry of the EPS construction becomes a Riemannian geometry. In a Weyl geometry the length of a measuring rod can change when the rod is carried around a closed path, and in a Riemannian geometry the metric fixes every length.

I do not obtain the step from a Weyl geometry to a Riemannian geometry in the Passage either. I take a metric as given. The context factor is the four-dimensional factor of the product. In the Passage the context factor is a smooth manifold with a metric (Chapter 8, Section II). The Kaluza-Klein reduction of Chapter 11 starts from the ten-dimensional metric on the product. A Kaluza-Klein reduction obtains a four-dimensional theory from a theory in more dimensions, by treating the extra dimensions as small and closed.

Because the reduction starts from a metric, the reduction does not produce the existence of a metric. The reduction produces the four-dimensional metric with its field equations (Chapter 12), and the internal metric with its shape fixed (Chapter 9, Section IX). I obtain three things from the axioms, with the identifications of Chapters 8 to 10: the product on which the metric lives, the dimensions of that product, and the Lorentzian signature of the metric.

Ehlers, Pirani and Schild also postulate that two structures of their construction are compatible: the conformal structure, which is the light cones, and the projective structure, which is the paths of free fall. In the Passage I read both structures from one metric. The light cones are the null directions of the metric, the directions of zero interval, and free fall follows the geodesics of the metric (Chapter 15). Because both structures come from one metric, the compatibility of the two structures follows from the one metric that I take as given, and the compatibility is not a further assumption.

II. What the Passage Obtains That Jacobson Assumes

Jacobson obtained the Einstein equations from thermodynamics (Jacobson 1995). He assumes the manifold, the Lorentzian metric, the Unruh effect, and the entropy-area proportionality $S = A/(4G_N)$. The Unruh effect is the warmth that an accelerated observer finds in empty space: an observer with constant acceleration finds the vacuum to be a bath at a temperature in proportion to the acceleration. The entropy-area proportionality says that the entropy S of a horizon is in proportion to the area A of the horizon, as the formula states.

In the Passage I obtain the four-dimensional context factor as one factor of the product (Chapters 8 and 10). I obtain the Lorentzian signature from the directed precedence of Axiom II, which is the asymmetry of the context factor (Chapter 10, Section III). Like Jacobson, I take a metric as given (Section I), and I take G_N from measurement. I also take the entropy-area relation as given, as Jacobson does: Chapter 13, Section II, uses the Bekenstein-Hawking count, the count of the states of a horizon that the entropy-area relation gives.

Jacobson and I take different routes to the field equations. Jacobson's route runs through the Clausius relation on local horizons, the relation by which heat is temperature times the change of entropy. The route of the Passage runs through the reduction and Lovelock's theorem (Chapter 12, Section VII). Lovelock's theorem fixes the form of the four-dimensional field equations: in four dimensions, the Einstein equations with a constant term are the only field equations of the usual kind built from the metric alone.

Jacobson concluded that the Einstein equation is an equation of state, and that it may be no more appropriate to quantize it canonically than to quantize the wave equation for sound in air (Jacobson 1995). An equation of state is a relation among bulk quantities, such as the relation among the pressure, volume and temperature of a gas. To quantize an equation ca-

nonically is to turn the equation into a quantum theory by the standard method. In the Passage I reach the same position as Jacobson by another route.

The Incommensuration Theorem states that symmetry and continuity are incommensurable: the two pairings that survive the theorem are symmetry with discontinuity and continuity with asymmetry. The arena is the geometry of spacetime itself. Quantizing the arena would need a pairing that the Incommensuration Theorem excludes (Chapter 3, Section V), because a quantized arena would be symmetric and the arena is continuous. The small waves of the metric are described by quanta, but those quanta are quanta of the description of the waves, and not quanta of the arena (Chapter 12, Questions).

The phonon is the nearest sense in which a quantum of a description can be made intelligible: a phonon is a quantum of the description of the vibrations of a solid. In the phonon picture, the small waves of the metric correspond to the vibrations of the solid, and the quanta of the small waves correspond to the phonons. The phonon picture fails at the solid itself: a solid is a held structure, and the arena is not a held structure. So even the sense of the phonon does not make the quantum of the small waves a quantum of the arena.

> *The Passage, Chapter 12, Section V:* "It curves where findings can be made, and in proportion to how many can be made there".

The Passage also states the reason the curvature goes with the content, the matter that spacetime holds (Chapter 12, Section V). In the Passage the context has the asymmetry that neither the content nor the act of comparison can have. The context curves where findings can be made, in proportion to how many findings can be made there. Energy and momentum measure that count of findings, because every finding is an interaction, and energy and momentum are what an interaction exchanges.

Jacobson's route also runs from a count to curvature: Jacobson counts the states behind every local horizon, and the count is in proportion to the area of the horizon. So the source of curvature is a count both in Jacob-

son's route and in the Passage, but the two counts are counts of different things. Jacobson counts the states that a horizon hides, and in the Passage the count is the count of the findings that can be made where the content is. Whether the two counts are one count is the question of Section XVII of "Open Questions as Research".

III. What the Passage Obtains That Connes Assumes

Chamseddine and Connes obtain gravity from the spectral action of a noncommutative geometry (Chamseddine and Connes 1997). Connes assumes a four-dimensional manifold M. In the Passage I obtain the four external dimensions from the cycle of Axiom II, with its three transitions and its one direction. I obtain the six internal dimensions from the grid of the six metrics, a grid of three columns and two rows with one cell for each metric. The internal space is $\mathbb{C}P^2 \times S^2$, the product of the complex projective plane $\mathbb{C}P^2$ and the sphere S^2. The internal space realizes the grid (Chapters 9 and 10). Both results depend on the identifications of Chapters 9 and 10, so both results are as firm as those identifications are.

Connes assumes the product structure $M \times F$, in which F is a finite internal space. In the Passage I obtain the product from the Incommensuration Theorem. Two conjunctions survive the Incommensuration Theorem, symmetry with discontinuity and continuity with asymmetry, and the two surviving conjunctions require separate geometric factors (Chapter 8, Section III).

Chamseddine and Connes formulated the spectral action in Euclidean signature, in which every direction counts with the same sign (Chamseddine and Connes 1997). Barrett (2007) gives a formulation of the spectral action with a Lorentzian metric. In both formulations the researchers choose the signature at the outset. In the Passage I obtain the Lorentzian signature from the directed precedence of Axiom II (Chapter 10, Section III).

Connes takes three geometric inputs as given: the dimension of the manifold, the signature of the manifold and the product. In the Passage I obtain all three of those inputs. Connes also assumes a finite algebra, the algebra of the internal space F, and the spectral action principle, by which the action of the theory depends only on the spectrum of an operator of the geometry. The Passage contains no claim about the finite algebra or about the spectral action principle.

Connes assumes the commutative factor $C^{\infty}(M)$, the algebra of the smooth functions on M, and in the Connes program the continuum enters through that factor. Connes does not obtain the four-dimensionality of M from the finite algebra. In the Passage the context factor is continuous from the start, because the law of continuity applies to all that is of context (Chapter 7). So the Passage contains no step from a finite structure to a continuum of points.

Chamseddine, Connes and Mukhanov (2014; 2015) obtain quanta of volume for four-dimensional geometry from a higher Heisenberg relation, a relation for geometry modeled on Heisenberg's relation between position and momentum. Chamseddine, Connes and Mukhanov write their construction in four dimensions from the start.

IV. The Cosmological Constant, Quantization and the Internal Space

Causal set theory treats spacetime as a set of events ordered by before and after, with only a finite number of events between any two. Causal set theory predicted the order of Λ before astronomers observed that the expansion of the universe accelerates. In the prediction, Λ fluctuates about zero, with a magnitude of order the inverse square root of the four-volume in Planck units. Today that magnitude is the observed order (Sorkin 1997; Ahmed, Dodelson, Greene and Sorkin 2004). In that prediction Λ changes from epoch to epoch.

The claim of the Passage is narrower than the claim of causal set theory. In the Passage I fix Λ once, from the information deficit at the observer's particle horizon (Chapter 13, Section II). The particle horizon is the greatest distance from which light has had time to reach the observer. The value of Λ rests on the identifications stated in Chapter 13, Section II, with G_N as its single measured constant. The identity of Chapter 13, Section III, does not use the measured horizon. The identity predicts the matter fraction $\Omega_m = 0.311$, the share of the density of the universe that is matter. The equation of state of the constant term is $w = -1$ (Chapter 13, Section V), where w is the ratio of the pressure of the term to its energy density.

The two claims differ in the history of Λ: in the claim of causal set theory Λ varies between epochs, and in the Passage Λ has one value. If astronomers confirm a variation of Λ between epochs, the step of the Passage that fails is the step of Chapter 13, Section V: that the deficit fixes Λ once, as a constant of the reconstructed history.

Three researchers each argue that gravity should not be quantized in the usual way: Jacobson, the mathematical physicist Roger Penrose, and the physicist Jonathan Oppenheim. Jacobson argues from thermodynamics (Jacobson 1995), Penrose from the collapse of the quantum state (Penrose 1996), and Oppenheim from a consistent classical coupling of gravity to quantum matter (Oppenheim 2023). My reason in the Passage rests on the Incommensuration Theorem: to quantize the arena would need a pairing that the Incommensuration Theorem excludes (Chapter 3, Section V). The reason also rests on an identification stated in Chapter 3, Section V: the identification of the characters of content and context.

Jacobson, Penrose, Oppenheim and I hold positions that predict different things. In the Passage each alternative of the matter carries its own geometry, so two masses held in superposition become entangled through gravitation (Chapter 12, Section VI). A mass held in superposition is a mass held, in the quantum sense, in two alternatives at once, such as two places. In Oppenheim's account the two masses do not become entangled.

In Penrose's account each superposition decays before entanglement can build, at the masses of the experiment proposed to test whether gravitation entangles two masses (Chapter 12, Questions).

String theory admits a vast set of compactifications, the ways of curling its six extra dimensions into a small closed space. The choice among the compactifications is open ("Comparisons at Single Steps", Section I). In the Connes program the researchers classify the finite geometries that satisfy the program's conditions. The researchers' choice among those geometries is guided by the particle content that the program is built to reproduce. The physicist Edward Witten (1981) took as internal spaces the circle bundles M^{pqr} over $\mathbb{C}P^2 \times S^2$, and Witten chose those spaces for their isometry group. A circle bundle over $\mathbb{C}P^2 \times S^2$ is a space in which a small circle sits over each point of $\mathbb{C}P^2 \times S^2$. The isometry group of a space is the group of the motions that leave every distance in the space unchanged.

Chapter 9 shows that $\mathbb{C}P^2 \times S^2$ is the only compact homogeneous product that satisfies three conditions (Chapter 9, Sections VI and VII). The three conditions rest on the identifications of Chapter 9, Section I. None of the three conditions refers to the particle spectrum, the list of the particles that exist with their masses, or to the isometries of the internal space.

Questions

Chamseddine and Connes obtain the Einstein-Hilbert action and its coupling relations from one principle, the spectral action. The Einstein-Hilbert action is the action whose field equations are Einstein's. In the spectral action a researcher can choose the signature either way. Does obtaining the signature add anything that matters?

Obtaining the signature matters for the aims of the Passage. The Lorentzian signature sets the order of events: the cone at each event, divided into a before and an after (Chapter 10, Section III). In the Passage I state the horizons, the collapse to a black hole and the interior passage (Chapter 18), and the particle horizon from which I fix Λ in Chapter 13, each of them within that order of events. In the Connes program the researchers choose the signature, in Barrett's formulation as in the Euclidean one. In the Passage the signature follows from the directed precedence of Axiom II. If a metric with no directions of zero interval can carry precedence, the step of the Passage that fails is the step of Chapter 10, Section III: that the metric's cone carries the limit of precedence.

Jacobson, Penrose and Oppenheim also conclude that gravity is not quantized in the usual way. What does a reason from the Incommensuration Theorem add to the reasons of Jacobson, Penrose and Oppenheim?

The reason from the Incommensuration Theorem adds a definite prediction on the question where the three accounts differ: whether two masses in superposition become entangled through gravitation. Jacobson's thermodynamic argument by itself contains no answer to that question. Oppenheim's postquantum classical gravity predicts that the masses do not become entangled. In postquantum classical gravity, gravity stays classical while matter is quantum. Gravity-driven collapse (Penrose 1996) predicts

that each superposition decays first. The Passage predicts that the masses become entangled, with the phase of their Newtonian interaction (Chapter 12, Questions).

The Passage also states what is not quantized. The four-dimensional geometry of each alternative is continuous, with no least step. The small waves of that geometry are described by quanta, which are quanta of the description and not of the arena (Chapter 12, Questions). If an experiment shows that the masses do not become entangled, the step of the Passage that fails is the step of Chapter 12, Section VI: that each alternative of the matter carries its own geometry.

The reduction gives a cosmological term of Planck size and negative sign, and Lovelock's theorem makes Λ the constant term of the four-dimensional equations. By what step do those equations carry $3fM_{Pl}^{\ 4}/S$ in place of $fM_{Pl}^{\ 4}$?

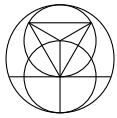
The four-dimensional equations carry $3fM_{Pl}^{\ 4}/S$ in place of $fM_{Pl}^{\ 4}$ by an identification, which Chapter 13, Section II, states, and not by a term in the action. The reduction fixes the classical part of the external value, the bare term $-3/(8\pi)\ M_{Pl}^{\ 4}$ (Chapter 11, Section III), where M_{Pl} is the Planck mass. In the Passage I take the ground-state part of the external value as $M_{Pl}^{\ 4}$. Together the two parts are $fM_{Pl}^{\ 4}$, the vacuum density at the modal boundary (Chapter 13, Section II). The modal boundary is the Planck scale, at which each existing description reaches the edge of its domain.

In the Passage I identify the constant that a measurement finds with the share of the external value that is accessible from inside, $3/S$ of the external value. The number S is the number of degrees of freedom in the count of the particle horizon. The field equations of the geometry that a comparer finds carry that share as their constant (Chapter 13, Section VI). The external value itself is the constant of no geometry that a comparer finds. A referee who rejects the identification rejects the value of Λ and keeps the form of the equations. The Passage sets out the four parts of the identification in Chapter 13, Section III.

The position of the Passage is akin to the positions of Jacobson and of the physicist Thanu Padmanabhan: the field equations are an equation of state, and the vacuum energy of the fields does not enter as a term of the action (Jacobson 1995; Padmanabhan 2010). In Padmanabhan's account Λ is likewise a constant whose value a count of information fixes (Padmanabhan and Padmanabhan 2013). Unimodular gravity is a third account in which the vacuum energy of the fields does not act through the field equations. Unimodular gravity is the theory named in "Other Programs That Obtain General Relativity", Section VII. In unimodular gravity Λ enters as a constant of integration, a constant that the law leaves free and each solution sets.

Einstein (1919) kept only the trace-free part of his field equations. The trace-free part is the part that remains when the trace of the equations, a kind of average over directions, is subtracted. The stress-energy is the quantity that describes the density and flow of energy and momentum. When a vacuum energy is the same at every place, the vacuum energy enters the stress-energy as a multiple of the metric, and so the vacuum energy drops out of the trace-free equations. The cosmological constant then returns through the conservation of energy and momentum, as a constant of integration and not as a constant of the law. Once the constant of integration returns, the equations become Einstein's, with a cosmological term of some value (Einstein 1919). Unimodular gravity differs from general relativity only in the status of the cosmological constant, and in every other test unimodular gravity agrees with general relativity (Weinberg 1989, on the cosmological constant problem, sets out the agreement).

The Passage agrees with unimodular gravity that the vacuum energy as a whole is not the constant of the field equations: the external value is the constant of no geometry that a comparer finds. The Passage and unimodular gravity differ in what fixes the constant. In the Passage the constant is the share of the vacuum's energy at the modal boundary that is accessible from inside, 3/S of that energy. The present measurement fixes that share (Chapter 13, Sections II and VI). In unimodular gravity nothing in the theory fixes the constant.



Comparisons at Single Steps

On the Work of Others Beside Particular Links

In this chapter I set single steps of the Passage beside the work of other researchers that bears on each step. In the first two sections I compare the internal space of the Passage with the internal spaces of string theory and with the spaces that Witten studied. In the third section I compare the Passage's account of a universe begun inside a black hole with earlier accounts of the same idea, and with the string cosmology in which three directions of space grow large. In the fourth section I compare the Passage with loop quantum gravity, in which a bounce replaces the singularities of general relativity, and in the fifth section I compare the Passage with the effective field theory of gravity. In the sixth section I name the programs that the entanglement experiment of the Passage can separate from the Passage. In the seventh section I compare the Passage with accounts that tie the energy of the vacuum to a horizon, and ask whether the Higgs field could be a dark energy that changes with time. In the eighth section I give the Hubble constant that the Passage's identity implies, and in the ninth section I show that the Passage does not need inflation. Each section opens with the sentence of the Passage that the section extends.

Rests on: the steps of the Passage named in each section, and the published work as cited. *Used by:* no step of the Passage.

I. String Theory

> *The Passage, Chapter 9, Section VIII:* "The first Chern class of $\mathbb{C}P^2 \times S^2$ does not vanish, so the internal space is not a Calabi–Yau space and admits no Ricci-flat Kähler metric".

The internal space of the Passage is $\mathbb{C}P^2 \times S^2$, the space that the Passage sets at each point of the four dimensions of space and time. The internal space has positive curvature, and the positive curvature is a fixed fact of the internal space's structure. Positive curvature separates the geometry of the Passage from the standard compactifications of string theory, which are Calabi-Yau spaces.

How does the internal space of the Passage differ from the internal spaces of string theory?

String theory posits a ten-dimensional (or eleven-dimensional) total space, and compactifies the total space to four dimensions: the extra dimensions are curled up too small to be seen. The consistency of string theory fixes the total dimension, through the conformal invariance of the worldsheet ("Other Programs That Obtain General Relativity", Section VIII). The worldsheet is the surface that a string sweeps out as the string moves. Once fluxes are counted, string theory admits a very large number of vacua, each vacuum a possible lowest-energy state with its own internal geometry (Douglas 2003; Ashok and Douglas 2004). The choice among the vacua is open. In the Passage I derive the product of space and time with an internal space from the Incommensuration Theorem (Chapter 8, Section III). I derive the internal geometry, $\mathbb{C}P^2 \times S^2$, from the structure of comparison (Chapter 9). The internal geometry has the standing of the identifications that Chapter 9 makes. An identification is a step from the structure of comparison to a physical quantity. Where string theory has many internal geometries, the Passage has one. The internal space of the Passage has positive curvature and is not a Calabi-Yau space (Chapter 9, Section VIII).

The internal space of the Passage also admits no spin structure, because $\mathbb{C}P^2$ admits none (Hawking and Pope 1978). A space needs a spin structure for spinor fields, the fields of particles such as the electron, to be defined on the space. A Calabi-Yau space always admits a spin structure, because the first Chern class of a Calabi-Yau space vanishes. A spinor field of ten dimensions therefore cannot be defined on $M^4 \times \mathbb{C}P^2 \times S^2$ without a

compensating connection, such as a $U(1)$ connection that carries a magnetic monopole on $\mathbb{C}P^2$ (Witten 1985). In the Passage I hold the spinor fields of the content, the matter in space and time, on M^4 alone (Chapter 12, Questions), and so the Passage needs no compensating connection.

II. Witten's Spaces

The Passage, Chapter 9, Section III: "The product $\mathbb{C}P^2 \times S^2$ is the base of the seven-dimensional spaces M pqr that Witten (1981) studied".

Has $\mathbb{C}P^2 \times S^2$ appeared in physics before?

Yes. Witten (1981) looked for internal spaces whose isometry group is $SU(3) \times SU(2) \times U(1)$, the gauge group of the Standard Model. The isometry group of a space is the group of motions that keep every distance in the space the same. Seven is the least dimension of an internal space whose isometry group is $SU(3) \times SU(2) \times U(1)$. Witten's seven-dimensional spaces, M^{pqr}, are circle bundles over $\mathbb{C}P^2 \times S^2$: each space attaches a circle to every point of $\mathbb{C}P^2 \times S^2$, which is called the base. The product $\mathbb{C}P^2 \times S^2 \times S^1$ is the trivial circle bundle over the same base, $\mathbb{C}P^2 \times S^2$. In the trivial circle bundle the circles are attached with no twist. The internal space of the Passage is the base $\mathbb{C}P^2 \times S^2$ itself, without a circle. The internal space of the Passage has the isometry group $SU(3) \times SU(2)$, locally, with no $U(1)$ factor. The Passage states the missing $U(1)$ factor as a difference from Witten's spaces (Chapter 8, Section IV). In the Passage I derive $\mathbb{C}P^2 \times S^2$ from the grid of comparison, not from a gauge group. The Passage contains no claim about the spectrum of the content, that is, about which particles exist and what their masses are.

III. Universes Inside Black Holes, and Strings Grown Large

The Passage, Chapter 18, Section XII: "A black hole interior as the origin of a universe has precedents (Pathria 1972; Frolov, Markov and Mukhanov 1989; 1990; Smolin 1992; Popławski 2010), and so has a succession of epochs, each joined to the next (Penrose 2010)".

The Passage, Chapter 18, Section VII: "The restoration re-forms the product geometry, and the number of large spatial directions is the number of spatial directions of the context factor (Chapter 10, Section IV)".

How does the Passage's account compare with earlier proposals that a universe begins inside a black hole?

Several researchers have proposed that the interior of a black hole is the origin of a universe. Pathria (1972) proposed that the universe may be the interior of a black hole, and noted that the universe is about the size of its own Schwarzschild radius. The Schwarzschild radius of a mass is the radius inside which the mass forms a black hole. Frolov, Markov and Mukhanov (1989; 1990) joined the interior of a black hole to a de Sitter core, a region that expands at an accelerating rate, through a layer of limiting curvature. Smolin (1992) proposed that each black hole gives rise to a new universe, and drew from the proposal a selection of the constants of nature. Popławski (2010) obtained a bounce, a contraction that turns into an expansion, from the coupling of spin to torsion in Einstein-Cartan theory. Einstein-Cartan theory is a version of general relativity in which space and time can twist as well as curve. Popławski proposed that the contraction before the bounce may be the matter inside a black hole. In his conformal cyclic cosmology, Penrose (2010) joins the remote future of one epoch to the origin of the next epoch, and no black hole lies between the two epochs. Chapter 18 of the Passage has one identification in common with the first four proposals, those of Pathria, of Frolov, Markov and Mukhanov, of Smolin and of Popławski. In Chapter 18 and in each of the four proposals, the interior of a black hole is identified with the origin of a universe.

In Chapter 18 I add three things to the four proposals. First, all black hole interiors are one interior, by a principle of comparison (Chapter 18, Section II), so there is one new epoch, and not one new epoch for each black hole. Second, the Passage calls the change from the interior of a black hole to the origin of the new epoch the turn. The turn is the restoration of the exterior factor of a product geometry (Chapter 18, Section VII). Third, I build the account of the turn without a model of the matter at the turn. The Passage divides what exists into the content (the matter) and the context (space and time), and calls the division the content/context partition. General relativity cannot continue its description of the interior to the turn, because the turn lies at the limit of the context-modal description. The Incommensuration Theorem describes the limit of the context-modal description from both sides, because the theorem is prior to the content/context partition. The Incommensuration Theorem contains no description that continues through the limit, since no comparison registers the turn as an event. So my account of the turn is structural, not dynamical: I identify what kind of transition the turn is, and I do not compute the detailed dynamics of the transition (Chapter 18, Section XI).

Brandenberger and Vafa (1989) built a cosmology on de-compactification, the growth of curled-up directions of space to a large size. In the account of Brandenberger and Vafa all nine spatial directions of string theory begin small, and three directions grow large. Winding strings are strings wrapped around a small direction, and winding strings keep the direction small. Three directions grow large because winding strings annihilate efficiently in at most three spatial directions. In the Passage the restoration re-forms the product geometry. The number of large spatial directions is the number of spatial directions of the context factor (Chapter 10, Section IV), the factor of the product that is space and time.

IV. Loop Quantum Gravity, and the Expectation of a Deeper Theory

> *The Passage, Chapter 19, Section IV:* "Loop quantum gravity, which constructs a model of that region, also reaches a finite curvature at the Planck scale (Ashtekar, Pawlowski and Singh 2006; Ashtekar and Bojowald 2006)".

In loop quantum gravity a bounce replaces the singularity, the place where the curvature of general relativity becomes infinite. Loop quantum cosmology is loop quantum gravity applied to the universe as a whole. In loop quantum cosmology the universe contracts to a minimum volume and expands again (Ashtekar, Pawlowski and Singh 2006). In the interior of a black hole the curvature has a maximum, and the discreteness of the quantum geometry sets the maximum (Ashtekar and Bojowald 2006). The program of loop quantum gravity is set out in the chapter "Other Programs That Obtain General Relativity", Section V.

I read the results of loop quantum gravity differently from the way the researchers who obtain the results read the results. Researchers in loop quantum gravity remove the singularity by quantizing the gravitational field, that is, by treating the arena (space and time itself) as made of quanta. A quantized arena needs one of the pairings that the Incommensuration Theorem forbids (Chapter 3, Section V).

The *result* of loop quantum gravity is a structure of finite curvature at the Planck scale in place of the singularity. The result agrees with the Passage's statement that the continuously asymmetric description (the description of space and time as continuous and asymmetric) has a finite domain, which ends at the modal boundary. In loop quantum gravity the Planck scale comes from a quantized geometry. In the Passage the Planck scale comes from the modal boundary, which quantizes nothing.

Researchers in loop quantum gravity obtain a finite curvature in place of the singularity by quantization, and in the Passage the reason for the finite curvature is the modal boundary. The modal boundary is the Planck

scale. At the Planck scale both descriptions, the description by quanta and the continuously asymmetric description, are relevant, and neither description can be extended alone. Researchers in loop quantum gravity construct a specific model of the region of transition. In the Passage I show that the transition is necessary, and I do not specify the detailed physics of the transition.

In Chapter 19, Section IV, I state the boundary of the domain of the continuously asymmetric description, and I construct no model of the region beyond the boundary. So in the Passage no model takes the place of the singularities.

V. The Effective Field Theory of Gravity

> *The Passage, Chapter 12, Questions:* "The description is the effective field theory of gravity (Donoghue 1994a; 1994b), whose corrections to the Newtonian potential are of relative size $(l\,Pl\,/r)^2$, of order 10^{-70} at a separation of one meter".

The effective field theory of gravity describes the metric by quanta. Does the effective field theory refute the Passage's claim that quantizing the arena is a category error?

No. A category error, in the philosophical sense, is the application of a concept to a domain where the concept does not belong. The Passage's claim of a category error concerns any quantum of the arena itself, at every scale. Below the Planck scale the small waves of the metric (the geometry of space and time) are described by quanta, as the effective field theory of gravity describes the small waves. In the Passage I admit the description by quanta (Chapter 12, Questions). An effective field theory is a description that holds below some scale and makes no claim beyond that scale. In the effective field theory of gravity, the quanta are quanta of the description, not quanta of the arena. The description by quanta ends at the

modal boundary (Chapter 17). The claim that quantizing the arena is a category error holds at every scale. The Passage and the description by quanta predict different things only at the Planck scale and beyond.

VI. The Programs the Entanglement Experiment Separates

> *The Passage, Chapter 12, Questions:* "For these numbers the entangling phase is 0.314 radians, and the account predicts that the entanglement appears with that phase".

The Passage predicts that masses in superposition become entangled through gravitation, with the entangling phase of 0.314 radians that Chapter 12 computes. The entanglement experiment can therefore separate the Passage from the programs that have one geometry for all the alternatives of a superposition, because every program with one geometry for all the alternatives predicts no entanglement. Sourcing by a continuous measurement record (Tilloy and Diósi 2016) is one program with one geometry for all the alternatives. In sourcing by a continuous measurement record, a continuous record of measurements of the matter is the source of gravitation. Sourcing by a continuous measurement record also predicts a specific gravitational decoherence, a loss of superposition caused by gravitation, which is a second channel of test. Postquantum classical gravity (Oppenheim 2023) is a second program with one geometry for all the alternatives. In postquantum classical gravity, gravitation stays classical while the matter is quantum. The experiment can also separate the Passage from collapse driven by gravity (Penrose 1996). In Penrose's account each superposition decays within about a hundredth of a second, before entanglement can build, for masses of 10^{-14} kilograms of the density of diamond, about two micrometers across.

The Passage's framework is per-alternative geometry: each alternative of a superposition has a geometry of its own. Per-alternative geometry is not the semi-classical theory of Møller (1962) and Rosenfeld (1963). In the semi-classical theory one geometry is sourced by the expectation value of

the stress-energy. The expectation value of the stress-energy is the average of the energy and momentum of the matter over all alternatives together. So the semi-classical theory has one geometry for all alternatives, and predicts no entanglement. Where the matter is sharply one alternative, the expectation value is the stress-energy of that one alternative, and only there does the equation of the semi-classical theory appear in the Passage (Chapter 12, Section VI).

VII. Horizons and the Vacuum Energy in Other Accounts

The Passage, Chapter 13, Questions: "The share is of a count, not a fluctuation in one".

In the Passage the cosmological constant Λ is a share of a count of degrees of freedom, and not a fluctuation in that count. Sorkin's everpresent Λ has the same order of magnitude as the Passage's Λ. Sorkin obtains the order of magnitude by a fluctuation, of order ${M_{Pl}}^4/\sqrt{N}$ with N the four-volume (the volume of space and time together) in Planck units (Sorkin 1997; Ahmed, Dodelson, Greene and Sorkin 2004). In Sorkin's account Λ fluctuates from epoch to epoch, while in the Passage Λ is a constant of the reconstructed history (Chapter 13, Section V).

Could the Higgs field supply a dark energy that changes with time?

No. The Higgs field could in principle contribute a vacuum energy that changes as the field takes its value. But the Higgs field took its value at the electroweak transition, long before the epoch of acceleration, the recent epoch in which the expansion of the universe speeds up. With the measured Higgs mass, 125.13 ± 0.11 GeV, the electroweak transition is a crossover (a smooth change with no sharp jump) centered at 159.5 ± 1.5 GeV (D'Onofrio and Rummukainen 2016; The Coming of the Kinds, Chapter 4, Section IV). Below the crossover the Higgs field settles toward its present value, and from then on the energy of the Higgs field is constant. In the Passage I give ρ_Λ, the energy density of Λ, as the accessible

share of the vacuum's energy at the scale of the modal boundary, the share accessible from inside (Chapter 13, Section II). I do not build Λ as a sum of the vacuum energies of separate fields.

Holographic dark energy ties the vacuum energy to a horizon, and holographic dark energy fails. How does the Passage's account differ from holographic dark energy?

Several researchers have written on the idea that the size or entropy of a horizon sets the vacuum energy. In cosmology a horizon is a boundary in the expanding universe. The particle horizon is the distance that light has traveled since the start of the epoch. The Hubble radius is the distance at which the expansion carries things away at the speed of light. The event horizon bounds the region from which light sent now will ever reach the observer. Fischler and Susskind (1998) wrote on holographic bounds in cosmology. Cohen, Kaplan and Nelson (1999) bounded the vacuum energy in a region by the size of the region. Hsu (2004) and Li (2004) wrote on holographic dark energy, in which an energy that tracks the Hubble radius or the particle horizon through time gives no acceleration. Padmanabhan and Padmanabhan (2013; 2017) fix Λ by a count of 4π units of information at the horizon, as the Passage fixes Λ by 3 degrees of freedom among the S degrees of freedom that the particle horizon counts. The count of Padmanabhan and Padmanabhan also fixes the amplitude of the primordial departures (the small departures from evenness in the early universe), which is open in the Passage (Chapter 18, Section IX). Banks (2001) and Bousso (2000) wrote on Λ as the inverse of the number of states.

The Passage's account differs from the holographic accounts of the vacuum energy in two respects. First, the coefficient of the Passage's formula, 3, comes from the three channels of comparison, one degree of freedom for each channel (Chapter 13, Section II). Second, Λ is fixed once, by the horizon of the measurement, and is not carried through time (Chapter 13, Section V). The ratio of the pressure of the dark energy to the density of the dark energy is written w. If an energy tracked the particle horizon

through one history, the energy would behave as $w = -0.125$ today, and astronomers' measurements exclude $w = -0.125$. In Chapter 13, Section V, Λ does not track the particle horizon through time.

The comparison with holographic dark energy can be stated exactly. If the formula of the Passage were allowed to track the particle horizon, the formula would be holographic dark energy on the particle horizon, with the coefficient of the formula fixed by c_1, the count of the channels of comparison. The number f is the number for which fM_{Pl}^4 is the vacuum density at the modal boundary (Chapter 13, Section II). In Li's form, $\rho = 3c^2M_p^2/L^2$ with the reduced Planck mass and L the size of the horizon, the formula has $c^2 = 8f$, so $c = 2.65$. On the particle horizon the formula would give $w = -1/3 + 2\sqrt{\Omega_\Lambda}/(3c) = -0.125$, with Ω_Λ the dark-energy fraction.

Holographic dark energy differs from the Passage's account in where the density enters the field equations. In the Passage I put the density in the constant term of one geometry. The field equations require the constant term to stay the same at every place and epoch (Chapter 13, Section VI): "If Λ varied from place to place or from epoch to epoch, the divergence of the left side would be the gradient of Λ, and the equations would set that gradient to zero". The argument of Chapter 13, Section VI uses no value of w. A density that followed the particle horizon would be a property of the content at each epoch, with a frame of rest of its own. For a density that followed the particle horizon, w exceeds $-1/3$ for every Ω_Λ, so the density would never make the expansion accelerate. For that reason Li (2004) set the particle horizon aside.

Suppose a version of the Passage's account fixed Λ once on the Hubble radius or on the event horizon, instead of on the particle horizon. The version would have no value to fix. On the Hubble radius and on the event horizon the formula's value exceeds $8f = 7.05$ times the constant at every epoch (Chapter 13, Section VI). If the formula were allowed to follow the

Hubble radius, the formula would give a dark-energy fraction of $c^2 = 7.05$, which no flat history allows. If the formula were allowed to follow the event horizon, the formula would give $w = -1/3 - 2\sqrt{\Omega_\Lambda}/(3c) = -0.54$.

The comparison that fixes Λ is not an open question: the comparison is the present measurement, by the definition of the real (Chapter 13, Section VI).

VIII. The Hubble Constant

The Passage, Chapter 13, Section IV: "This is the same test as Ω_m, not an independent one".

The identity of the Passage is an equation whose solution is the total matter fraction, the share of the density of the universe that is matter of every kind. The identity implies a value of the Hubble constant (the present rate of expansion of the universe), $h = 0.678 \pm 0.003$. The Passage states the implied Hubble constant as the same test as the matter fraction, seen through one more measurement (Chapter 13, Sections IV and VII; Chapter 20, Questions). I set the implied Hubble constant beside the two measured values of the Hubble constant, from the microwave background and from the distance ladder. I then solve the radiation fraction together with the Hubble constant, state the standing of the inference, and give the range of the Hubble constant that the identity allows.

With the radiation fraction $\Omega_r = 9 \times 10^{-5}$, the solution of the identity is the matter fraction $\Omega_m = 0.3108$ (Chapter 13, Section III). The microwave background is the oldest light in the sky. The Planck satellite measured $\Omega_m h^2 = 0.1430 \pm 0.0011$ from the microwave background (Planck 2018, temperature, polarization and lensing; Aghanim and colleagues 2020). So $h = \sqrt{(0.1430/0.3108)} = 0.678$, with an uncertainty of ± 0.003 from $\Omega_m h^2$: a Hubble constant of 67.8 ± 0.3 km s^{-1} Mpc^{-1}. The radiation fraction itself depends on h: $\Omega_r h^2 = 4.18 \times 10^{-5}$ for the photons and three light neutrinos. When the radiation fraction and h are solved to-

gether, $\Omega_r = 9.1 \times 10^{-5}$, and the value of h is unchanged at the precision of $h = 0.678 \pm 0.003$. Within the standard model of cosmology, Aghanim and colleagues fit the microwave background alone and obtain a Hubble constant of 67.4 ± 0.5 (Aghanim and colleagues 2020), which is 0.8 standard deviations from 67.8. Riess and colleagues measure 73.04 ± 1.04 by the distance ladder, in which distances are measured step by step from nearby stars out to distant galaxies (Riess and colleagues 2022). The value from the distance ladder is 4.9 standard deviations from 67.8, with the two uncertainties, 0.3 and 1.04, added in quadrature (as the square root of the sum of the squares of the two uncertainties).

The Passage's value of the Hubble constant is computed, and the computation uses the measured $\Omega_m h^2$ as well as the inputs of the identity. The measured $\Omega_m h^2$ is inferred from the microwave background in a spatially flat universe with $w = -1$. The Passage also states that the universe is spatially flat and that $w = -1$ (Chapter 13, Section V; Chapter 18, Section IX), so the inference and the identity are made in the same model. Suppose the Hubble constant were settled near 73 while the microwave background's $\Omega_m h^2$ remains as measured. Then Ω_m would be near 0.268, away from the identity's 0.311, and the test of the Passage would be whether a matter fraction away from 0.311, the value the Passage predicts, is confirmed (Chapter 13, Section III; Chapter 20, Questions).

The range of the Hubble constant that the identity allows also follows from the microwave background's $\Omega_m h^2$. With the microwave background's $\Omega_m h^2$, the matter fraction would lie more than three standard deviations of the measured matter fraction (0.007) from the identity's 0.311 whenever the Hubble constant lay outside 65.6 to 70.2. So the identity excludes a Hubble constant of 73, which lies outside the allowed range, and does not exclude a Hubble constant of 70, which lies inside the allowed range. Freedman and colleagues measured the Hubble constant from the tip of the red giant branch (the brightest red giant stars), calib-

rated with the Hubble and James Webb space telescopes. The value of Freedman and colleagues is 70.39 $\pm$ 1.22 (statistical) $\pm$ 1.33 (systematic) (Freedman and colleagues 2025), 1.4 standard deviations from 67.8.

IX. Inflation

The Passage, Chapter 18, Questions: "The account therefore needs no inflationary phase for the horizon and flatness problems".

Inflation is a proposed early phase in which the expansion of the universe sped up enormously. Physicists introduced inflation to solve three problems: the horizon problem, the flatness problem and the monopole problem. The horizon problem asks why regions of the sky too far apart ever to have exchanged light look the same, and the flatness problem asks why space is so nearly flat. In the Passage I answer the first two problems from the turn (Chapter 18, Section IX, and Questions). I answer the third problem, and a fourth problem for which inflation is also credited: the sameness of the expansion in every direction. Then I apply to inflation the diagnostic of method, form and function, and show that the held internal space, whose size and shape are held in place, supplies no field that could drive inflation.

Does the monopole problem arise in the Passage?

No. Grand unified theories join the forces of the Standard Model in one simple group, and predict magnetic monopoles (particles with a single magnetic pole) from the breaking of the simple group. The predicted monopoles come in numbers that would overclose the universe (give the universe more mass than its measured expansion allows), and inflation dilutes the monopoles. My books on the particles contain no grand unification. In my books on the particles the finite algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ has a gauge group that is the product of its factors at every scale, and there is no X or Y boson and no force carrier joining quark to lepton (The Branches of the Kinds, Chapter 2; The Grand Synthesis, Chapter 14, Section V). SU(3) and SU(2) act as the isometry groups of $\mathbb{C}P^2$ and S^2,

taken as spaces of states, and the U(1) of hypercharge acts as the phase of a line bundle over $\mathbb{C}P^2$ (The Kinds of Particle, Chapter 4, Section IV). The gauge group is not the remnant of a larger group. There is therefore no breaking of a simple group to leave monopoles behind, and the monopole problem does not arise. A magnetic monopole would explain the quantization of electric charge, the fact that every electric charge is a whole multiple of one smallest charge. In my books on the particles the quantization of electric charge follows instead from the charge rule of The Story of the Particles (The Alphabet of Particles, entry N11).

Does the sameness of the expansion in every direction need inflation?

No. In the Passage the sameness of the expansion in every direction comes from the restoration (Chapter 18, Section VIII). The metric of the new epoch has no prior identity, and a metric with no prior identity inherits no large-scale anisotropy (difference between directions) from the old exterior. So the new epoch begins conformally flat (flat up to an overall scale), up to the departures that the Incommensuration Theorem requires. Standard physics has a second route to the sameness of the expansion in every direction, from other premises. The *shear* of an expansion is the part of the expansion that changes the shape of a small region without changing the region's volume. In a homogeneous expansion (the same at every place) whose slices of space at one time are flat, with no stress that pulls differently in different directions, the shear falls as the inverse cube of the scale of distances (Ellis and MacCallum 1969; Tsagas, Challinor and Maartens 2008, Section 1.5.2). In a homogeneous expansion with flat slices of space and no stress that pulls differently in different directions, the square of the rate of expansion is a third of the sum of the density and the square of the shear, in units in which $8\pi G_N$ and the speed of light are one (Tsagas, Challinor and Maartens 2008, Section 1.5.2). The square of the shear falls as the inverse sixth power of the scale, faster than the density of radiation, which falls as the inverse fourth power. So, whatever the shear was at the start, the shear comes to add little to the rate of expansion.

Once the shear adds little, the rate of expansion falls as the inverse square of the scale while radiation carries most of the energy, and as the inverse three-halves power once matter does. The shear itself falls as the inverse cube of the scale, so the shear's share of the expansion (the shear divided by the rate of expansion) falls as the inverse of the scale while radiation carries most of the energy, and as the inverse three-halves power once matter does.

The second route needs the slices of space to be flat. Collins and Hawking (1973) showed that among homogeneous models the approach to isotropy, sameness in every direction, is not general. Collins and Hawking concluded that the models expanding at just the escape velocity, which are the flat models, would in general approach isotropy. A Robertson-Walker model is a model of a universe that is the same at every place and in every direction. A Robertson-Walker model expands at just the escape velocity when the energy of its expansion exactly balances the pull of its content, and the Robertson-Walker model that expands at just the escape velocity is the Robertson-Walker model whose slices of space are flat. In the Passage the homogeneous model of the new epoch is spatially flat, and the geometry found is curved only by its departures (Chapter 18, Section IX), so the new epoch satisfies the condition of the second route, flat slices of space.

For one stretch of the history of the universe, I can compute how far the shear's share falls. At first order, content close to equilibrium carries no stress that pulls differently in different directions (Chapter 18, Section IX). The hot gathering of the kinds is the hot mixture of every kind of particle in the early universe. The hot gathering of the kinds stays close to balance until the neutrinos go free, between about 3 and 1 MeV (The Coming of the Kinds, Chapter 9). In a region that grows with the expansion the entropy stays the same, and a kind that falls out of balance passes its share of the entropy to the rest (The Coming of the Kinds, Chapter 10, Section II). So while radiation carries most of the energy, the scale grows at least as fast as the temperature falls, and the density falls at most as fast as the inverse fourth power of the scale. From the electroweak crossover at

159.5 GeV (Section VII of this chapter) to 3 MeV, the temperature falls by the factor $3/159{,}500 = 1.9 \times 10^{-5}$, so the scale grows at least by the inverse of the factor, about 53,000. Suppose that at the crossover the shear is as large as the rate of expansion. Then the square of the shear makes up one third of the sum of the density and the square of the shear, and the density makes up the other two thirds. As the scale grows by some factor, the square of the shear falls by the sixth power of the factor, and the density falls by at most the fourth power of the factor. So the shear's share, which starts at one, falls below $\sqrt{(3/2)}$ divided by the factor. By 3 MeV the shear's share is therefore less than $\sqrt{(3/2)} \times 1.9 \times 10^{-5} = 2.3 \times 10^{-5}$: less than three parts in a hundred thousand of the rate of expansion at that time.

In a contraction the shear's share grows, by the law of the shear's share read the other way in time. Near the center of a black hole the geometry takes the Kasner form, in which each direction of space scales as a power of the time to the singularity, with exponents $-1/3$, $2/3$ and $2/3$ (Appendix B, Section II). So near the singularity the geometry is stretched along one direction and squeezed along the other two, and the collapse grows uneven, whereas the expansion grows even. There are two routes to the sameness of the expansion in every direction, and the two routes depend on different premises. The first route depends on the new context having no prior identity, and the second route depends on the law of a homogeneous expansion. The two routes have one result: the expansion of the new epoch is the same in every direction, up to its departures. Neither route needs inflation.

Does the Passage need inflation at all?

No. The diagnostic of method, form and function is a test of a proposed mechanism in three questions. The first question is by what means the mechanism works (its method). The second question is what the structure of the account supplies for the mechanism to work on (the form). The third question is what work the mechanism is wanted for (its

function). Two further questions follow: whether the method matches the form, and whether the function requires the method. For inflation, the questions of the diagnostic have these answers:

Method: inflation is a dynamical mechanism, a phase of accelerated expansion driven in its usual form by a scalar field with specific properties of its potential. A scalar field is a field with one number at each point, and the potential of a field is the energy of the field at each value of the field.

Form: the geometry $\mathbb{C}P^2 \times S^2$ provides no scalar field of the kind that inflation needs. The two moduli of $\mathbb{C}P^2 \times S^2$ are the overall size of the internal space and the ratio of its two radii. The two moduli are held in place with masses of about the Planck mass (Chapter 11, Section II). In the Passage I identify the primordial departures with the form of the content carried through the turn (Chapter 18, Section IX).

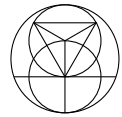
Function: in the Passage I answer in other ways the problems that inflation was designed to solve. I answer the sameness of the sky by the one domain from which the new exterior re-forms (Chapter 18, Questions), and the flatness by the lengths that remain at the turn (Chapter 18, Section IX). I answer the sameness of the expansion in every direction, for which inflation is also credited, by the restoration (Chapter 18, Section VIII) and by the fading of shear in a flat epoch, and the monopoles by the absence of grand unification.

So the method of inflation does not match the form that the Passage supplies, and the function does not require the method.

Can either modulus of the internal space drive inflation?

No. Slow-roll inflation needs a scalar field whose potential V is nearly flat. With $M_{Pl,red} = M_{Pl}/\sqrt{(8\pi)}$ the reduced Planck mass, and primes for derivatives along the field, one of the two conditions of slow-roll inflation is $|\eta| = {M_{Pl,red}}^2|V''/V| << 1$. At first order the masses of the two moduli of the internal space are 2.25 and 3.09 Planck masses (Chapter 11, Section II; Appendix D, Section III). For a potential dominated by its constant part, $V \approx 3H^2{M_{Pl,red}}^2$, the condition reads $|\eta| = m^2/(3H^2) << 1$ for a field of mass

m and an expansion rate H. For masses of about the Planck mass, the condition would need an expansion rate far above the Planck mass, beyond the modal boundary at which the description of the exterior ends (Chapter 17). Only the order of the masses enters the step from the masses to the expansion rate, as only the order of the masses enters the Passage's bound on the range of the force that the moduli carry (Chapter 11, Section II). Neither modulus can drive slow-roll inflation.



Two Accounts of Causation

On the Russell Objection, and on the Counterfactual and Interventionist Accounts

The Passage grounds science in comparison, and it takes causation as a second route from comparison to law: a persistent relation found in observation, with an observer, a direction in time, and no need of a complete specification (Chapter 5, Section III). Two bodies of philosophy are often set against that ground. Russell argued in 1913 that the notion of cause should be removed, because physics has only functional dependence. The counterfactual account of Lewis (1973a), and the interventionist account of Woodward (2003) after it, analyze causation in ways that critics charge are circular. In this chapter I answer both. Each step of the Russell objection reaches determinism, a mathematical description or an opinion, and none reaches causation. The charge of circularity mistakes the simultaneity of the intrinsics of comparison for a circle. What remains of the counterfactual and interventionist accounts is a correct description of causation as comparison.

> *The Passage, Chapter 5, Section III:* "Causation and determinism are distinct concepts, and they cannot be used interchangeably".

Rests on: causation and determinism (Chapter 5, Section III); exists and real, and validity and soundness (Chapter 5, Section II); simultaneity (Chapter 5, Section IV); Russell (1913); Lewis (1973a; 1973b; 1979); Woodward (2003). *Used by:* no step of the Passage.

I. The Russell Objection

Russell's case, in his order and in my words, has seven parts. First, the word "cause" carries misleading associations, and philosophy would be better without it; he called the law of causality "a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm". Second, "same cause, same effect" is of no use: a cause described narrowly enough to recur does not guarantee its effect, and one described fully enough to guarantee it never recurs. Third, cause and effect can be separated neither by an interval, in which something might intervene, nor by contiguity, since continuous time has no next instant. Fourth, the advanced sciences have no causes, only differential equations; "the reason why physics has ceased to look for causes is that, in fact, there are no such things". Fifth, what science relies on is the uniformity of nature, read as functional dependence among the states of a system, which Russell granted is trivial unless the functions are restricted to simple ones. Sixth, causation has no direction, since the equations run backward as readily as forward. Seventh, the notion that a cause compels its effect is an anthropomorphism.

Each part is placed by the distinctions of Chapter 5. The first part of Russell's case is a recommendation about vocabulary, an opinion, and the first part states the conclusion of the case in advance. The second requires a guarantee against every interference, which is the demand of determinism; causal law states a persistent relation with the confidence observation supports, and never claimed the guarantee. In the third, Russell treats cause and effect as static items that exist, located on a continuum of instants, and asks how two such items can be joined; causation is real, found in the act of observing, across the observer's interval, with the future open, and the continuum of point instants is a property of a mathematical model of time. In the fourth, Russell reads physics as its equations: an equation written to assume perfect information has the form of determinism, and the knowledge it records is causal. In the fifth, Russell places the uniformity in nature as it is in itself, where no observation reaches, which

is the move Kant objected to in dogmatic metaphysics; and the restriction to simple functions that makes Russell's replacement say anything comes from the scientist's preference for the law that keeps holding in what is observed, which is causation. In the sixth, Russell notes that a timeless description has no direction, which says nothing of a relation that is real. The seventh is true, and it is about a notion causation does not contain.

So every step in which Russell argues reaches determinism or a mathematical description, and the remaining steps are opinions. No step reaches causation as Chapter 5 defines it, and the conclusion is unsupported. Russell himself later relied on "causal lines", persistent chains of events, as a postulate of scientific inference (1948).

II. The Counterfactual Account

The Passage, Chapter 5, Section IV: "Where nothing comes first, nothing can return to it".

Hume gave two definitions of a cause in one sentence, the first by similarity and the second by what would have happened otherwise: "we may define a cause to be an object, followed by another, and where all the objects similar to the first are followed by objects similar to the second. Or in other words where, if the first object had not been, the second never had existed". Lewis took up the second. An event e depends causally on a distinct event c when, had c not occurred, e would not have occurred; such a counterfactual is true when some possible world in which its antecedent and consequent both hold is more similar to the actual world than any world in which the antecedent holds and the consequent does not; and c causes e when a chain of such dependences leads from c to e. Lewis intended an analysis: causation in terms of something that does not contain it.

The charge of circularity, at full strength, has two parts, and they need separate answers. Ordinary judgements of similarity give the wrong answers, so the ordering of worlds has to be weighted, and Lewis (1979) gave the weights. The general charge is that the analysis presupposes

sameness and the direction of time, which it was meant to explain. The specific charge is that the weights were chosen so the counterfactuals come out as particular causal facts require.

What the account presupposes is sameness, since the similarity of worlds is a degree of sameness, and time, since "had c not occurred" refers to a before and an after. Both are involved in every comparison, and the counterfactual is a comparison: the actual world set against a world that is not actual, in the respect of whether e occurs. They are involved together, with no lead of one over the other. Where sameness and time are involved together, with no lead of one over the other, the relation between sameness and time is simultaneity. A circle needs an order, from an assumption back to the same assumption, and simultaneity has no such order. The critic's charge is itself a comparison, of what the analysis assumes with what it concludes, and it presupposes the same two things in the same way. So the account is not circular in the general sense.

The specific charge is of the ordinary kind, and simultaneity is no answer to it (Questions). If the weights were fixed by checking them against causal verdicts already in hand, the analysis assumes particular conclusions and then derives them. I concede this. The counterfactual analysis is not reductive: its weights carry what is already known of which events depend on which, as Woodward concedes of interventions (Section III). What remains is its descriptive part, causation as a comparison of the actual with the possible, and that part needs no reduction.

Lewis's aim of reduction and his critics' charge share one assumption: an order of grounding, in which either the similarity of worlds comes first and causation is derived from it, or causation comes first and is smuggled into the similarity. Simultaneity removes that order in both directions. What stands is a description: causation as a comparison of the actual with the possible, made with what comparison already supplies. The description is a description of causation and not a foundation beneath causation, because nothing lies beneath the intrinsics. Lewis's account is written from the vantage of a survey of complete worlds, each of which exists as

timeless structure, of a relation that is found in an actual act; that is the register in which it is written, and the reply above does not depend on that register.

III. The Interventionist Account

Woodward (2003) replaces worlds with interventions: X causes Y when an intervention on X would change Y, with the other relevant variables held fixed, and an intervention is an idealized manipulation that changes X and affects Y, if at all, only by way of X. Woodward states that the account is not reductive, since an intervention is described in causal terms, and argues that the circle is not vicious.

The reply to the general charge of circularity (Section II) applies, and it shows why his defense is right. An intervention is an act: a difference introduced in the context of X, followed by a finding of sameness or difference in Y. It is a comparison performed, and it involves sameness and difference, content and context, and a before and an after, all together. What Woodward calls a harmless circle is simultaneity. His account places causation in an act of doing, where comparison is actually performed, and it adds nothing to the reply beyond what the counterfactual account raised.

IV. The Same Objection Turned on the Passage

The objection made to Russell in Section I, that he places the uniformity in nature as it is in itself, where no observation reaches, can be turned on the Passage. The Passage steps from the structure of comparison to the world (Chapter 1, Questions), and its Root Tautology says that an interaction is a comparison (Chapter 1, Section IV). For two particles that interact with no comparer present, a reader will ask whether the Passage has made Russell's move in the other direction. It has not. The Root Tautology does not say that two particles compare in the sense that a comparer does. It says that what every comparison requires, the six intrinsics, is required

of every interaction, because any description of an interaction is a comparison and nothing else is available. Stated so, it is a claim about every description of an interaction, and the Passage claims no more than that. The world as physics reaches it is reached only by interaction; every prediction of the Passage is a prediction about what a measurement finds; and the Passage defines "exists" and "real" from the act of comparison (Chapter 5, Section II). So the Passage stands on the side of Kant's objection, and the objection costs it nothing it claims.

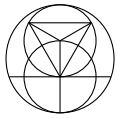
V. Standing

The Russell objection reaches determinism and mathematical description at every step in which Russell argues, and it does not touch a ground of science in causation as Chapter 5 defines it. The general charge of circularity against the counterfactual and interventionist accounts is answered by the simultaneity of the intrinsics, which every account, the critic's included, presupposes. The specific charge, that the weights are tuned to particular causal verdicts, is conceded, and with it the claim to reduction, which the description does not need. Both accounts remain as descriptions of causation as comparison. The Passage uses none of this: its argument from comparison to physics runs through measurement and law (Chapter 1, Section II).

Questions

Does the reply to the charge of circularity make every circular argument harmless?

It applies only to what every comparison presupposes all at once: the intrinsics and the before and after of the act. An argument that assumes a particular conclusion and then derives it has an order, from that assumption back to it, and it is circular in the ordinary sense. Simultaneity removes the order only among what co-occurs in every act.



Refutation Conditions Named in This Book

Collected, with the Step Each Would Fail

The chapters of this book name six observations that would bear on the Passage, each with the step of the Passage that would fail. In this chapter I collect them in one list. Each is a condition on a step of the Passage; none rests on a step of this book.

> *The Passage, Chapter 20, Questions:* "Here, as throughout the book, confirmed means established at five standard deviations by at least two independent kinds of measurement".

Rests on: the chapters named at each condition. *Used by:* no step of the Passage.

I. The Conditions

A further polarization of gravitational waves, or a speed other than the speed of light: the uniqueness of the Einstein form of the four-dimensional equations (Chapter 12, Section VII), whose waves have the two polarizations and the speed that the Passage states in Chapter 16, Section V ("Other Programs That Obtain General Relativity", Section VII).

A variation of Λ between epochs: the fixing of Λ once, as a constant of the reconstructed history (Chapter 13, Sections V and VI) ("What the Passage Obtains That Others Assume", Section IV).

Precedence carried by a metric with no directions of zero interval: the limit of precedence carried by the metric's cone (Chapter 10, Section III) ("What the Passage Obtains That Others Assume", Questions).

Two masses, each held in two places and related only through gravitation, that do not become entangled: one geometry for each alternative of the matter (Chapter 12, Section VI) ("What the Passage Obtains That Others Assume", Questions; "Comparisons at Single Steps", Section VI).

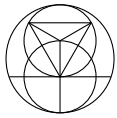
A matter fraction confirmed away from 0.311, or a Hubble constant outside 65.6 to 70.2 with the microwave background's $\Omega_m h^2$: the accessible share c_1/S of the particle horizon's count (Chapter 13, Section II) ("Comparisons at Single Steps", Section VIII).

Primordial departures with the large-scale spectrum of independent pieces: the identification of the ends of the interiors (Chapter 18, Section II) ("Open Questions as Research", Section XIII).

In each, confirmed has the sense the Passage gives it. The fourth and the fifth bear on two of the principal predictions the Passage sets out (Chapter 20, Questions); the others bear on steps argued in the Passage, in the chapters named with them.

PART II

Extensions



Why the Balance Criterion Holds

Four Independent Grounds for the Third Rejection Criterion

The Incommensuration Theorem removes eight of the ten pairings of the four compound concepts by three criteria, and the third, the balance of sameness and difference, is the one readers most often press. It is the best-defended step of the theorem. The Passage states it in its direct form and gives three of its grounds in Chapter 3, Questions. In this chapter I set out four grounds at full length, for readers who come to the theorem from logic, from mathematics or from category theory. Each ground is independent of the other three, and any one of them suffices.

> *The Passage, Chapter 3, Section II:* "Criterion !z: sameness/difference balance".

Rests on: the compound concepts as pairs of verdicts (Chapter 2, Section II); the inseparability of sameness and difference (Chapter 2, Section I); the criterion and its table (Chapter 3, Section II). *Used by:* no step of the Passage.

I. The Objection

The objection, at full strength, starts from inseparability, which says that where sameness appears, difference appears too. The criterion, stated as a count, requires that the appearances match one for one. The step from "each appears where the other does" to "they appear in equal number" is said to be a further commitment, not contained in inseparability, and so the weakest link of the theorem. I answer it on the four grounds below.

II. The First Ground: Inseparability in Each Respect

A comparison gives its verdicts in two respects, content and context, and content and context are themselves inseparable. So inseparability holds within each respect: among the verdicts on content, sameness must appear with difference; among the verdicts on context, sameness must appear with difference. This uses inseparability exactly as stated, and it uses no count.

Applied to the ten pairings, inseparability in each respect removes all eight that the theorem removes and keeps the two it keeps (the table of Chapter 3, Section II). In continuity with symmetry, sameness of content appears twice and difference of content not at all: sameness without difference, in the very respect where the verdict is given. The failure is an absence, not a shortfall, and no arithmetic enters. The other two criteria remain correct and independently grounded; inseparability in each respect reaches the same two survivors by itself.

III. The Second Ground: Counting Comes Later

To count is to treat several items as units of one kind, which is sameness, and to tell them apart so that each is counted once, which is difference. Counting is built out of sameness and difference, and it is not available beneath them.

The objection treats "equal in number" as an arithmetical requirement added to inseparability. At the level where the theorem works there is no arithmetic to add. What the equal totals record is pairing: every appearance of sameness matched with an appearance of difference, in its respect, and none left over. That pairing is inseparability applied to every appearance, and the equal count is its later description in the arithmetic that sameness and difference make possible. The two readings the objection separates are one reading, stated before and after arithmetic exists.

IV. The Third Ground: What Imbalance Costs

A comparison has two things to do: to find what is the same, and to find what is different. Both carry weight. A conjunction in which sameness has no answering difference, in some respect, has lost the capacity to differentiate there: everything in that respect is the same, and the comparison collapses into identity. A conjunction in which difference has no answering sameness has lost the capacity to relate there: nothing is the same as anything, and there is nothing for the difference to be a difference of. Either way, the conjunction describes no comparison, rather than a weaker comparison. Imbalance fails as a kind, not as a degree.

V. The Fourth Ground: The Form of an Inseparable Pair

Category theory has an exact form for an inseparable pair: the *adjunction*. An adjunction pairs two constructions, each given only in terms of the other, so that every component on one side has its answering component on the other. The two sides of an adjunction correspond exactly, because the binding between the two sides is constitutive. The exact correspondence is not the result of anyone counting the components of the two sides and finding the totals equal. An adjunction with one half and no other is not a defective adjunction. The one-sided pairing is not an adjunction at all. A conjunction with an unpaired sameness fails in the same way, for the same reason.

This ground concerns the form of inseparability, and it is independent of the categorical form of the whole theorem, which I set out in the chapter on that form.

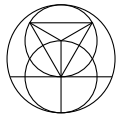
VI. Standing

The balance criterion stands on four independent grounds: inseparability in each respect, with no count; the count as a later description of a pairing inseparability already requires; the cost of imbalance, which ends the comparison; and the exact correspondence of an inseparable pair, which the adjunction expresses. The objection reads the criterion as an arithmetical rule added to inseparability. Under each of the four grounds it is no such rule, and the result it secures, that the only conjunctions available to any comparison are the continuously asymmetric and the discontinuously symmetric, stands on all four.

Questions

Why does the Passage state the criterion in each respect, and give the count only second?
Because the reading in each respect is inseparability itself, with nothing added, and a reader who checks the table can see each failure as an absence in one respect. The count is a correct summary of the same test across both respects, and a reader who meets it first is the reader the objection of Section I speaks for.

Is the criterion the same as complementarity, and is that a fifth ground?
The balance criterion is the same as complementarity, and complementarity is not a fifth ground for the balance criterion. In each respect one compound of a surviving pair gives sameness and the other difference; that is to say, the second compound is the exact complement of the first, which is how the Passage states the theorem in one line (Chapter 3, Section III, with its Figure 1). Complementarity is the first ground stated as a property of the pair rather than of each respect, and it stands or falls with that ground.



The Categorical Form of the Incommensuration Theorem

What the Topos Proves, and What It Leaves to Reading

The Passage proves the Incommensuration Theorem by enumeration (Chapter 3) and calls the quantization of the arena a category error in the philosophical sense: the application of a concept to a domain where it does not belong. *Category Theory and the Immanent Metaphysics* gives a rendering of the theorem in cohesive toposes, and proves that rendering there. This chapter states what that book proves and where it places a smooth spacetime. The category error is not among the theorems of that book, and I say why. In its last section I state the theorem of Ambrose and Singer on connections that the Passage cites, and read the Gribov ambiguity of gauge theory as an instance of it.

> *The Passage, Chapter 3, Section IV:* "The enumeration of Sections I to III is the proof this book gives".

Rests on: the Incommensuration Theorem (Chapter 3); the forbidden pairing (Chapter 3, Section V); *Category Theory and the Immanent Metaphysics*, Theorem 13.1 (its Main Theorem), Theorem 11.5a and Proposition F.3.3. *Used by:* no step of the Passage.

I. The Theorem in the Topos

Category Theory and the Immanent Metaphysics works in a non-degenerately cohesive topos over the category of sets, with Lawvere's Nullstellensatz. It identifies symmetry with the property of being discrete, and

continuity with the property of being codiscrete, in the topos's own sense. The two questions "is it discrete?" and "is it codiscrete?" form a two-by-two grid, whose four cells are the four ways of answering both.

Its Main Theorem (Theorem 13.1, with Theorem 11.5a) proves that the two cells with one yes and one no are exactly the discrete and the codiscrete objects with two or more points. It proves that the cell of both holds only the initial and the terminal object (Proposition F.3.3). Non-degeneracy is what keeps every object with two or more points out of that cell. In a degenerate topos the discrete and the codiscrete object on two points would be one object, and that object would be both.

The cell of neither is occupied. The cell of neither holds the genuinely cohesive objects, among them the real line of smooth sets (Proposition 8.3). In the category theory book I exclude the whole cell of neither as a fundamental mode, and not object by object. In my definition a pairing is a fundamental mode when its non-trivial objects, those other than the initial and the terminal object, are exactly the image of one of the two canonical embeddings, the discrete or the codiscrete, on sets with two or more elements (Definition 11.4). The cell of neither is the image of neither embedding, so it is not a fundamental mode (Proposition 11.9). In the category theory book I mark Definition 11.4 as a reading, and I prove the rest.

II. Where a Smooth Spacetime Falls

A smooth manifold of positive dimension is genuinely cohesive. The manifold is not discrete, because the manifold has a smooth curve that is not constant. The manifold is not codiscrete, because not every assignment of the points of the manifold to the points of a line is a smooth curve. So it lies in the cell of neither, which the topos allows and does not count as a fundamental mode. The context factor of the Passage, a smooth four-dimensional manifold, stands in that cell. In the category theory book (Section 19.6.2) I place the fields of continuous mathematics under

the continuously asymmetric mode by the continuous tools of those fields, not by the class of the objects of those fields. In the same section I mark the placement as a reading.

Read without care, the topos would place the context factor of the Passage in the cell that the Passage's own theorem forbids, since the cell of neither is the cell of discontinuous asymmetry. The topos's classification and the Passage's theorem concern different things. The topos classifies objects by their probes. The Passage's modes classify what a comparison finds, and every theory has two modes, the mode of what it finds and the mode of its formalism (Appendix A, Section III). The category theory book makes the same cut when it places continuous mathematics under the continuously asymmetric mode by its tools and not by the class of its objects (Section 19.6.2). So the manifold, as an object, is genuinely cohesive, and general relativity, as a theory, is continuously asymmetric in what general relativity finds. The statement about the manifold and the statement about general relativity do not contradict each other.

III. What the Topos Does Not State

The category error of the Passage is not a theorem of the topos. Under the book's identification, "the arena is continuous" would place spacetime among the codiscrete objects, and a smooth manifold of positive dimension is not codiscrete. In the category theory book I do not speak of the arena, of spacetime or of quantization, and I leave physical content unformalized (Section 13.12.3). What I prove in the category theory book on this point is that, under the identification of symmetry with being discrete and continuity with being codiscrete, no object with two or more points is both symmetric and continuous. That the arena is continuous, and that a quantized arena would be symmetric, are identifications the book does not make.

One other identification places smooth manifolds in the continuously asymmetric cell. It keeps symmetry as being discrete, and takes continuity as having two distinct points in one connected piece. Under it no object of any cohesive topos is both symmetric and continuous, and every manifold with a component of positive dimension is continuously asymmetric. That exclusion holds in degenerate toposes as well, so it does not use non-degeneracy. It excludes symmetry with continuity as a pairing of opposites, in the manner of criterion !y, and not by the balance of sameness and difference by which the Passage excludes it (Chapter 3, Section II). It is therefore a different theorem, and the book uses its own identification.

IV. The Theorem on Connections and Its Instance in Gauge Fixing

The Passage, Chapter 9, Section V: "the theorem of Ambrose and Singer exhibits the second of the two forbidden pairings as a fact about connections".

Rests on: the Ambrose-Singer theorem and the two connections (Chapter 9, Section V). *Used by:* no step of the Passage.

The statement about connections is a theorem. By the Ambrose-Singer theorem (Ambrose and Singer 1953), the Lie algebra of the restricted holonomy group (the holonomy around the loops that can be shrunk to a point) is spanned by the values of the curvature, so a connection with curvature anywhere has a holonomy group whose identity component is nontrivial, hence continuous; and a connection with discrete holonomy has zero curvature. The flat connections, which are the only ones with discrete holonomy, carry no field: their holonomy is a representation of the fundamental group alone, which is topological data, not dynamics. The reading of the theorem on connections as the second clause of the Incommensuration Theorem is an identification (curvature as asymmetry, dis-

crete holonomy as discontinuity): on that identification, the bundle admits no connection that carries the forbidden pairing of asymmetry with discontinuity.

I read that condition as having an instance in gauge theory: the Gribov ambiguity. To fix a gauge is to choose one smooth representative from each gauge orbit, the set of field configurations that gauge transformations carry into one another, and in a gauge theory with a non-abelian group the choice cannot be made globally: every smooth gauge-fixing condition fails at certain field configurations, the Gribov copies (Gribov 1978; Singer 1978), where the smooth structure of the space of orbits and the discrete structure of the gauge identification come into conflict. The reading places no condition on the theorem on connections, and it adds no prediction.

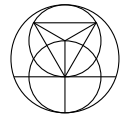
Questions

Does the Passage depend on the categorical form?

No. The Passage proves the Incommensuration Theorem by enumeration (Chapter 3, Sections I to IV) and argues the category error from the forbidden pairing (Chapter 3, Section V; Appendix A). The categorical form is a rendering of the theorem in another language, and no step of the Passage rests on it.

Is the category error of the Passage weakened because the topos does not state it?

No. The error is argued from the forbidden pairing: a quantized arena would join the two kinds, symmetric and continuous, that the Incommensuration Theorem keeps apart. The topos proves the prohibition for objects with two or more points. The step from that prohibition to the arena is the Passage's own reading of the arena, and it was never a step of the topos.



The Cells of $\mathbb{C}P^2$ and the Six Metrics

On Which Cell Carries Which Term

The Passage uses the counts of the cells of the internal space: three for $\mathbb{C}P^2$, two for S^2, six for their product, one for each metric (Chapter 9, Sections IV and VII). This chapter states which cell carries which term. In $\mathbb{C}P^2$ the point is immanent, the two-cell transcendent and the top cell omniscient; in S^2 the point is context and the two-cell content. Their products place the six metrics one to a cell, with time on the point cell. The assignment is argued from Axiom II, through the order of findings made in two steps. No result of the Passage depends on it.

> *The Passage, Chapter 9, Questions:* "The grid is a product of three columns and two rows, and the Euler characteristic of a product is the product of the Euler characteristics: $\chi(\mathbb{C}P^2 \times S^2) = 3 \times 2 = 6$, one cell for each metric (Section VII)".

Rests on: the cell decompositions of $\mathbb{C}P^2$ and S^2 (Chapter 9, Section IV); the grid of the six metrics (Chapter 7, Section III); $\mathbb{C}P^2$ as the space of states of three alternatives (Appendix C, Section VIII); Axiom II and its test, An Immanent Metaphysics, 1.41-2. *Used by:* no step of the Passage.

I. The Assignment

$\mathbb{C}P^2$ has three cells in its standard decomposition: a point, a two-cell (a copy of S^2), and the four-dimensional top cell. I identify them with the three modalities: the point with the immanent, the simplest; the two-cell with the transcendent, the middle; the top cell with the omniscient, the

most structured. S^2 has two cells, a point and a two-cell, and I identify them with the two rows: the point with context, the minimal, and the two-cell with content, the structured.

II. The Six Cells

The cells of the product are the products of the cells of the factors, and each pairs one modality with one row. Each such pair is one metric of the grid (Chapter 7, Section III).

Cell of $\mathbb{C}P^2 \times S^2$	Modality × row	Real dimension	Metric
point × point	immanent × context	0	time
point × two-cell	immanent × content	2	force
two-cell × point	transcendent × context	2	possibility
two-cell × two-cell	transcendent × content	4	probability
top cell × point	omniscient × context	4	space
top cell × two-cell	omniscient × content	6	pattern

The cells in each dimension number 1, 2, 2 and 1, which are the Betti numbers of $\mathbb{C}P^2 \times S^2$. Time lands on the point cell, of dimension zero. That agrees with the Passage's reading of time (Chapter 10, Section IV): "The one temporal dimension has the character of minimal dimensionality: it does not extend; it divides".

III. The Argument from Axiom II

The point cell is immanent in every reading, and the question is which of the other two cells, the two-cell and the top cell, is omniscient and which is transcendent. Axiom II determines the answer. *An Immanent Metaphysics*, 1.41-2: "The omniscient modal term is identified by being that term which in practice (in a context which is both subjective and temporal) necessarily follows from the being of the term with an immanent mod-

al association (that term already identified by Axiom I). The transcendent modal term is identified by being that term which in practice necessarily precedes the being of the term with an immanent modal association". The test requires an order in time, for a comparer. The order in which a cell complex is built is not such an order: it is a construction, objective and without time, and the dual decomposition reverses it.

$\mathbb{C}P^2$ is the space of the pure states of a system with three alternatives (Appendix C, Section VIII), and a comparer meets it through findings. A coarse finding that excludes one of the three alternatives leaves the state on a line, the states of the two alternatives that remain: the two-cell. A further finding leaves the state at a point. After the finding, the found state is carried on by its dynamics and is again generic, in the top cell. The order in time is: top cell, two-cell, point, top cell. With the point immanent, the top cell follows it and the two-cell precedes it. By Axiom II the top cell is omniscient and the two-cell transcendent.

A second order is read from *An Immanent Metaphysics*, 1.87-9: "An instance of observation (structured in no dimensions immanent) resolves an instance of an actuality (structured in one dimension; omniscient) from a class of potentialities (structured in two dimensions; transcendent)". Read on the cells in complex dimension, a point, a line and a plane, it takes the plane as the potentialities, the point as the observation, and as the actuality the line of alternatives that the finding excludes. The line of the alternatives that the finding excludes is a relation in the geometry of $\mathbb{C}P^2$, available to one who surveys the geometry as structure. In the frame of one who surveys the geometry as structure, the line is omniscient and the plane is transcendent. Both orders are valid, each in its own frame: the order of 1.87-9 is the binding when the geometry, held in view as structure, is the focus; the order of findings is the binding when the act of finding is the focus. Which term fills which role depends on the focus of the context (the Passage, Chapter 4, Section II). The Passage's focus is the act of meas-

urement, and the test of Axiom II quoted above applies to what is met in practice, in a context both subjective and temporal, so for the Passage the order of findings gives the binding.

The passage on dimensions in *An Immanent Metaphysics* (note 3 to 1.88-6) agrees. It gives the default association and its limit: "In this manner, one may regard fractals (and/or points) as immanent, lines as omniscient, and planes as transcendent. However, depending on context (as for example, when considering the duality between points and lines), these modality associations would be changed". In $\mathbb{C}P^2$ the duality of points and lines holds: through any two points passes one line, and any two lines meet in one point. So $\mathbb{C}P^2$ is the context the note names, and the assignment there is the one the order of findings gives.

IV. Standing

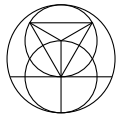
The assignment is argued, from Part I of the Passage and the exact topology of $\mathbb{C}P^2$. A finding that selects one of three alternatives at once goes from the top cell to a point without passing the two-cell, so "necessarily precedes" holds only for findings made in two steps. No result of the Passage depends on the assignment: $c_1 = \chi(\mathbb{C}P^2) = 3$ counts all three cells, and the classification of Chapter 9 uses only the counts. The assignment is interpretive: no observable distinguishes the binding by the order of findings from the binding by the order of 1.87-9.

Questions

Does the Passage need the assignment for any result?
No. The classification of Chapter 9, Section VI, uses the number of independent classes of each factor; $c_1 = 3$ and $\chi(\mathbb{C}P^2 \times S^2) = 6$ count all the cells together; and the Questions of Chapter 9 use only those counts. The assignment says which cell carries which term, and nothing downstream reads it.

Does the assignment give the count of channels of the Passage's Chapter 13?
Not by itself. The count of three needs only the counts of the cells, three for $\mathbb{C}P^2$ and two for S^2, and the Passage's ground that one row, the content, is what a measurement registers (the Passage, Chapter 13, Section II). Which cell of S^2 carries content, and which cell of $\mathbb{C}P^2$ carries which modality, does not enter. The assignment places the content row on the products with the two-cell of S^2, and the context row on the products with its point; each row has three cells, so the count is three under either binding.

What does the order of 1.87-9 give, and where does it apply?
The order of 1.87-9 gives the binding for the geometry surveyed as structure: the two-cell omniscient and the top cell transcendent. In the table, the rows of possibility and probability would then take the cells of space and pattern, and the reverse; time and force keep the cells of dimension zero and two. That binding is valid in its frame, and the order of findings is valid in the frame of measurement, which is the Passage's. No count differs between them, so the Passage reads the same under either.



The Spectral Invariants of $\mathbb{C}P^2 \times S^2$

On the Determinants of the Laplacians of the Einstein Product Metric

The Passage fixes the shape of the internal space by one identification: the internal metric is an Einstein metric, with radii $R_O = \sqrt{6}\, l_{Pl}$ for $\mathbb{C}P^2$ and $R_T = l_{Pl}$ for S^2 (Chapter 9, Section IX). This chapter computes four spectral invariants of that metric: the derivatives at $s = 0$ of the spectral zeta functions of the scalar Laplacian and of the Laplacian on 1-forms on each factor, which give the functional determinants of those operators. It states the values, derives them, states two identities among them and four checks, and gives the heat-kernel coefficients of the product. The invariants are results of mathematics about the shape of the internal space. They enter no coupling, and no step of the Passage uses them.

> *The Passage, Chapter 9, Section IX:* "The identification therefore makes the internal metric an Einstein metric, with Ricci tensor λg and $\lambda = 1/l_{Pl}^2$, one constant on both factors".

Rests on: the Einstein metric of the internal space and its radii (Chapter 9, Section IX); the spectra of the Laplacians on $\mathbb{C}P^2$ and S^2 (Ikeda and Taniguchi 1978); the heat-kernel coefficients (Gilkey 1975; Gilkey 1995; Vassilevich 2003). *Used by:* no step of the Passage.

I. The Spectra and the Unit

The invariants are built from the spectra of four operators. The unit of length is the radius R_O of $\mathbb{C}P^2$, in which both factors have the Einstein constant 6 ($\mathrm{Ric} = 6g$), so that S^2 has radius $1/\sqrt{6}$. In that unit: the scalar Laplacian on $\mathbb{C}P^2$ has eigenvalues $4k(k+2)$ with multiplicities $(k+1)^3$,

for $k = 0, 1, 2, \ldots$; the scalar Laplacian on S^2 has eigenvalues $6l(l+1)$ with multiplicities $2l+1$, for $l = 0, 1, 2, \ldots$; the Laplacian on 1-forms of $\mathbb{C}P^2$ has two series (Ikeda and Taniguchi 1978), the eigenvalues $4k(k+2)$ of the scalar Laplacian with doubled multiplicities $2(k+1)^3$, for $k \geq 1$, and the eigenvalues $4n(n+1)$ with multiplicities $(n-1)(2n+1)(n+2)$, for $n \geq 2$; and the Laplacian on 1-forms of S^2 has the eigenvalues $6l(l+1)$ of the scalar Laplacian with doubled multiplicities $2(2l+1)$, for $l \geq 1$. The factor $k(k+2)$ is the quadratic Casimir of the $SU(3)$ representation (k, k) in which the scalar harmonics of degree k lie. The doubled multiplicity on $\mathbb{C}P^2$ holds because on a Kähler manifold the exact form df and its image under the complex structure are both eigenforms with the eigenvalue of f; the doubled multiplicity on S^2 holds because df and its Hodge dual $*df$ are both eigenforms, and S^2 has no harmonic 1-forms. The second series on $\mathbb{C}P^2$ is the pair of conjugate representations $(n+1, n-2)$ and $(n-2, n+1)$, whose Casimir is $n(n+1)$ in the normalization in which the scalar Casimir is $k(k+2)$, so both series carry the same factor 4.

For an operator with eigenvalues λ_n and multiplicities d_n, the spectral zeta function is $\zeta(s) = \Sigma\, d_n\, \lambda_n^{-s}$, the sum excluding any zero eigenvalue. It converges for large $\mathrm{Re}(s)$, continues to a function regular at $s = 0$, and gives the functional determinant by $\log \det = -\zeta'(0)$. The four invariants are $S1 = \zeta'_{sc,\mathbb{C}P^2}(0)$ and $S2 = \zeta'_{sc,S^2}(0)$, for the scalar Laplacians, and $S3 = \zeta'_{vc,\mathbb{C}P^2}(0)$ and $S4 = \zeta'_{vc,S^2}(0)$, for the Laplacians on 1-forms.

The unit is part of each value. If every eigenvalue is multiplied by one constant factor, $\zeta(s)$ is multiplied by that factor to the power $-s$, so $\zeta(0)$ is unchanged and $\zeta'(0)$ shifts by $-\zeta(0)$ times the logarithm of the factor. None of the four values of $\zeta(0)$ is zero, so each invariant is a statement in a unit. In the internal space of the Passage $R_O = \sqrt{6}\, l_{Pl}$, so in the Planck unit every eigenvalue is divided by 6 and every invariant becomes $S + \zeta(0) \ln 6$. In that unit the S^2 factor is the sphere of radius 1.

II. The Four Values

Invariant	$\zeta(0)$	Value in the unit R_O	Value in the Planck unit
S1 = $\zeta'_{sc,\mathbb{C}P^2}(0)$	$-89/120$	$+0.0728791222367072120...$	$-1.2560091507741002469...$
S2 = $\zeta'_{sc,S^2}(0)$	$-2/3$	$+0.0328217380168996170...$	$-1.1616845748018037169...$
S3 = $\zeta'_{vc,\mathbb{C}P^2}(0)$	$-13/60$	$+0.4454932213553557534...$	$+0.0572786696892771699...$
S4 = $\zeta'_{vc,S^2}(0)$	$-4/3$	$+0.0656434760337992340...$	$-2.3233691496036074337...$

The value in the Planck unit is $S + \zeta(0) \ln 6$. The Planck-unit value of S2 is the value on the unit sphere, $4\zeta'_R(-1) - 1/2$, where ζ_R is the Riemann zeta function.

III. The Derivation

S1. On the scale $k(k+2)$, write $k(k+2) = m^2 - 1$ with $m = k + 1$, so that the sum runs over $m \geq 2$ with multiplicity m^3, and expand $(1 - 1/m^2)^{-s}$ in its binomial series. Interchanging the two sums gives $\zeta(s) = \Sigma_{j\geq 0}\,[s(s+1)...(s+j-1)/j!]\,[\zeta_R(2s+2j-3) - 1]$, the Riemann zeta values with the term $m = 1$ removed. At $s = 0$ two terms contribute to $\zeta(0)$: the term $j = 0$ gives $\zeta_R(-3) - 1 = 1/120 - 1$, and the term $j = 2$, whose coefficient $s(s+1)/2$ meets the pole of $\zeta_R(2s+1)$, gives $1/4$; so $\zeta(0) = 1/120 - 1 + 1/4 = -89/120$. Differentiating each term at $s = 0$ gives $\zeta'(0) = 2\zeta'_R(-3) + [\zeta_R(-1) - 1] + [\gamma_E/2 - 1/4] + \Sigma_{j\geq 3}\,(1/j)[\zeta_R(2j-3) - 1]$, with $\gamma_E = 0.5772...$ the Euler-Mascheroni constant. The four parts are $+0.0107571527155486...$, $-13/12$, $+0.0386078324507664...$ and $+0.0786791525731399...$; the terms of the tail fall as $2^{-(2j-3)}$, because $\zeta_R(m) - 1$ falls as 2^{-m}. Their sum is $-0.9552891955938783302....$ The factor 4 of the unit adds $-\zeta(0) \ln 4 = +1.0281683178305855420...$, and the

total is S1. A second method, the binomial expansion into Hurwitz zeta functions with $\zeta(0)$ and $\zeta'(0)$ read off by contour integration around $s = 0$, gives the same value.

S2. On the unit sphere, with eigenvalues $l(l+1)$, write $x = l + 1/2$, so that $l(l+1) = x^2 - 1/4$ and $2l + 1 = 2x$, and expand in powers of $1/(4x^2)$. The sum becomes $\zeta(s) = \Sigma_{j\geq 0}\, [s(s+1)...(s+j-1)/j!]\, 4^{-j} \cdot 2\zeta_H(2s + 2j - 1, 3/2)$, with ζ_H the Hurwitz zeta function, and its terms fall geometrically. At $s = 0$ the term $j = 0$ gives $2\zeta_H(-1, 3/2) = -11/12$, and the term $j = 1$ meets the pole and gives $1/4$, so $\zeta(0) = -2/3$. The derivative sums to the closed form $4\zeta'_R(-1) - 1/2 = -1.1616845748018037169...$. The Einstein constant 6 of S^2 in the product adds $-\zeta(0) \ln 6 = +1.1945063128187033...$, and the total is S2.

S3. The first series of the Laplacian on 1-forms of $\mathbb{C}P^2$ has the scalar eigenvalues with doubled multiplicities, so it contributes $2 \times$ S1, with $\zeta(0)$ twice $-89/120$. For the second series, on the scale $n(n+1)$, write $x = n + 1/2$; the multiplicity $(n-1)(2n+1)(n+2)$ is then $2x^3 - (9/2)x$, and the same expansion gives the sum as Hurwitz zeta values at $5/2$. Its $\zeta(0)$ is $19/15$, and its $\zeta'(0)$ is $+2.0557078343004694...$; the factor 4 of the unit adds $-(19/15) \ln 4$, so the second series contributes $+0.2997349768819413...$. So $\zeta_{vc,\mathbb{C}P^2}(0) = 2 \times (-89/120) + 19/15 = -13/60$, and S3 $= 2 \times$ S1 $+$ $0.2997349768819413...$.

S4. The multiplicities of the Laplacian on 1-forms of S^2 are exactly twice the scalar multiplicities, and the eigenvalues are the same, so its zeta function is twice the scalar zeta function at every s: S4 $= 2 \times$ S2, with $\zeta(0) = -4/3$.

IV. Two Identities and Four Checks

Two identities hold among the four invariants. S4 $= 2 \times$ S2, exactly, from the doubled multiplicities on S^2. S3 $= 2 \times$ S1 $+ \zeta'_2(0)$, where $\zeta'_2(0) = +0.2997349768819413...$ is the contribution of the second series in the unit

R_O, from the two series of the spectrum on 1-forms of $\mathbb{C}P^2$. The four invariants therefore rest on three independent computations: the scalar zeta derivative on each factor, and the second series on $\mathbb{C}P^2$.

Three independent checks verify the computation (Checks 1, 3 and 4), and one consistency relation fixes a further heat-kernel coefficient (Check 2).

Check 1. $\zeta_{sc,\mathbb{C}P^2}(0) = -89/120$ matches the heat-kernel formula of Gilkey and Vassilevich exactly. With the curvature invariants of $\mathbb{C}P^2$, $R = 24$, $|Ric|^2 = 144$, $|Riem|^2 = 192$, and its volume $\pi^2/2$, the coefficient $a_2 = (4\pi)^{-2} \int (5R^2 - 2|Ric|^2 + 2|Riem|^2)/360$ is $(2880 - 288 + 384)/(360 \times 16\pi^2) \times \pi^2/2 = 31/120$, and removing the one zero mode gives $\zeta(0) = 31/120 - 1 = -89/120$.

Check 2. $\zeta_{vc,\mathbb{C}P^2}(0) = -13/60$, which with the relation of the Euler characteristic, $2a_2(\Delta_0) - 2a_2(\Delta_1) + a_2(\Delta_2) = \chi(\mathbb{C}P^2) = 3$, gives $a_2(\Delta_2) = 41/20$ for the Laplacian on 2-forms.

Check 3. $\zeta_{sc,S^2}(0) = -2/3$ exactly, which matches the known Seeley-DeWitt coefficient on the round two-sphere.

Check 4. Direct numerical summation to 10,000 terms converges to the values of the expansions within the envelope of order $1/N$ expected of it, with N the number of terms.

V. The Heat Kernel of the Product

The Laplacian on $K = \mathbb{C}P^2 \times S^2$ is the sum of the Laplacians of the two factors, so its eigenvalues are sums and its heat trace is a product: $K_K(t) = \mathrm{Tr}\, e^{-t\Delta_K} = K_{\mathbb{C}P^2}(t) \times K_{S^2}(t)$, for every $t > 0$. Each heat trace has an expansion as $t \to 0$ in powers $t^{k - d/2}$, with d the dimension, whose coefficients a_k are the Seeley-DeWitt coefficients, and the coefficients of the product are the convolutions $a_k(K) = \Sigma_j\, a_j(\mathbb{C}P^2)\, a_{k-j}(S^2)$.

In the unit R_O, writing each heat trace as a sum over $m = k + 1$ or over $l + 1/2$ and taking its Mellin transform, the expansions follow from the Riemann and Hurwitz zeta values at negative odd integers:

$$K_{\mathbb{C}P^2}(t) \sim 1/(32t^2) + 1/(8t) + 31/120 + (241/630)\, t + \ldots,$$
$$K_{S^2}(t) \sim 1/(6t) + 1/3 + (2/5)\, t + (16/35)\, t^2 + \ldots$$

The leading terms agree with the volumes, $\mathrm{Vol}(\mathbb{C}P^2)/(4\pi)^2 = (\pi^2/2)/(16\pi^2) = 1/32$ and $\mathrm{Vol}(S^2)/(4\pi) = (4\pi/6)/(4\pi) = 1/6$, and the constant term $31/120$ of $\mathbb{C}P^2$ is the coefficient of Check 1. The convolution gives the coefficients of K:

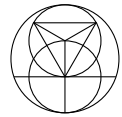
Coefficient of K	Power of t	Value
$a_0(K)$	t^{-3}	$1/192$
$a_1(K)$	t^{-2}	$1/32$
$a_2(K)$	t^{-1}	$7/72$
$a_3(K)$	t^0	$1619/7560$

Three of these agree with the local formulas on K, whose volume is $\mathrm{Vol}(K) = (\pi^2/2)(4\pi/6) = \pi^3/3$ and whose curvature invariants are $R = 36$, $|\mathrm{Ric}|^2 = 216$ and $|\mathrm{Riem}|^2 = 192 + 144 = 336$: $a_0(K) = \mathrm{Vol}(K)/(4\pi)^3 = 1/192$; $a_1(K) = (4\pi)^{-3} \int R/6 = 1/32$; and $a_2(K) = (4\pi)^{-3} \int (5R^2 - 2|\mathrm{Ric}|^2 + 2|\mathrm{Riem}|^2)/360 = (56/3)/192 = 7/72$. The coefficient $a_3(K)$ is the one at t^0 in six dimensions, and the constant functions are the one zero mode of Δ_K, so the zeta function of the scalar Laplacian on K has $\zeta_K(0) = a_3(K) - 1 = 1619/7560 - 1 = -5941/7560$. In six dimensions a_3 does not change with the unit of length, so $\zeta_K(0)$ is a property of the shape alone; it depends on the ratio R_O/R_T, and the value stated is the value at $\sqrt{6}$. Because $\zeta_K(0)$ is not zero, the determinant of Δ_K is a statement in a unit, as the determinants of the factors are (Section I).

VI. Where the Invariants Stand

The Passage, Appendix D, Section II: "where $\zeta(s) = \Sigma\, m^{-2s}$ runs over the masses".

The invariants S1 to S4 are invariants of the Einstein metric on $\mathbb{C}P^2 \times S^2$, the shape of the internal space of the Passage, stated in the unit R_O. They enter no computation of the gauge couplings, which run with the content of the particles and take no threshold from a tower of Kaluza-Klein modes (The Strengths of the Kinds, Chapter 2, Sections III to V; The Kinds of Particle, Chapter 1, Section XIII, and Chapter 3, Section VII). In the Passage the computation that does use a spectral zeta function is the zero-point energy of the small waves of the metric (Appendix D), which takes $\zeta(-2)$ and $\zeta'(-2)$ over their four-dimensional masses and not the invariants of this chapter. The invariants S1 to S4 stand in this book as results of spectral geometry about the shape of the internal space, with their checks (Section IV) and their unit (Section I).



The Grid as the Space of Unentangled States

On $\mathbb{C}P^2 \times S^2$ Inside $\mathbb{C}P^5$

In the Passage I read $\mathbb{C}P^2$ and S^2 as the spaces of the pure states of systems with three and with two alternatives (Appendix C, Section VIII). In this chapter I take one step further. The grid of the six metrics has the shape of a tensor product of a space of three states and a space of two, a six-dimensional space with one basis state for each metric. Its pure states form $\mathbb{C}P^5$, and the states that factor, a state of the columns times a state of the rows, form exactly $\mathbb{C}P^2 \times S^2$ inside it. The internal space is then the space of the unentangled states of the grid. The reading is a concordance, and no step of the Passage uses the reading.

> *The Passage, Appendix C, Section VIII:* "The complex projective space $\mathbb{C}P\ n$ is the space of the pure states of a quantum system with $n + 1$ alternatives, and its Fubini–Study metric measures how distinguishable one such possibility is from another (Wootters 1981)".

Rests on: the grid of the six metrics (Chapter 7, Section III); the product of Chapter 9, Section I; $\mathbb{C}P^n$ as a space of states (Appendix C, Section VIII); the Segre and Veronese embeddings of algebraic geometry (Griffiths and Harris 1978). *Used by:* no step of the Passage.

I. The Embedding

Let $\mathbb{C}^3$ have one basis state for each column of the grid, the three modalities, and $\mathbb{C}^2$ one for each row, content and context. Their tensor product $\mathbb{C}^3 \otimes \mathbb{C}^2 = \mathbb{C}^6$ has one basis state for each of the six metrics. The pure states of $\mathbb{C}^6$ form $\mathbb{C}P^5$. A state of the form $u \otimes v$, a column state times a

row state, is unentangled, and the map $(u, v) \mapsto u \otimes v$, taken on rays, is the Segre embedding of $\mathbb{C}P^2 \times \mathbb{C}P^1$ into $\mathbb{C}P^5$. Its image is exactly the set of unentangled states. Since $\mathbb{C}P^1$ is S^2, the image is $\mathbb{C}P^2 \times S^2$, a complex submanifold of complex dimension 3 and real dimension 6 inside $\mathbb{C}P^5$, of real dimension 10.

II. The Reading

I read $\mathbb{C}P^2 \times S^2$ as the possibilities of the grid in which the modal index and the row index are each fixed on their own. They are distinct, since each is a state of its own factor; inseparable, since every unentangled state has both; and non-interchangeable, since the factors have three states and two. An entangled state of $\mathbb{C}^6$ correlates a modality with a row, and so does not hold the two indices apart. This gives a second motivation for the product of Chapter 9, Section I, beside the counts.

The reading also makes the Euler characteristics read as counts of basis states. $\chi(\mathbb{C}P^{n-1}) = n$ is the number of basis states of $\mathbb{C}^n$: 3 for $\mathbb{C}P^2$, 2 for S^2, and their product, 6, the number of basis states of $\mathbb{C}^6$, one for each metric (Chapter 9, Questions).

III. What the Reading Does Not Fix

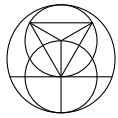
The embedding fixes the topology and the complex structure of the internal space. It does not fix the metric of Chapter 9, Section IX. The metric that $\mathbb{C}P^5$ induces on the image is the product of the Fubini-Study metrics with one normalization for both factors, and on it the Ricci constants of $\mathbb{C}P^2$ and of S^2 stand in the ratio 3 to 2. The Einstein condition of the Passage requires them equal, which fixes $R_O/R_T = \sqrt{6}$. So the induced metric is not the metric of the Passage, and the radii come from the Passage's own condition, not from the embedding.

In the normalization of the Passage (Chapter 9, Section VIII), the induced metric has $R_O/R_T = 2$. No other choice of inner product on $\mathbb{C}^3 \otimes \mathbb{C}^2$ changes this, since the inner product that the unitary groups of the two factors preserve is unique up to a constant. The metric of the Passage is induced by another embedding of the unentangled states: the map that sends $u \otimes v$ to $u \otimes u \otimes u \otimes v \otimes v$, three copies of the column state with two copies of the row state, into the projective space of the symmetric tensors, of complex dimension 29. With the inner product that the unitary groups preserve, the Fubini-Study metric of that space induces three times the Fubini-Study metric of $\mathbb{C}P^2$ plus twice that of S^2, which is the Einstein metric, with $R_O/R_T = \sqrt{6}$. The map that sends $u \otimes v$ to $u \otimes u \otimes u \otimes v \otimes v$ is the anticanonical embedding of $\mathbb{C}P^2 \times S^2$. The anticanonical embedding of $\mathbb{C}P^2 \times S^2$ is the embedding fixed by the first Chern class of $\mathbb{C}P^2 \times S^2$, which is 3 on $\mathbb{C}P^2$ and 2 on S^2. So distinguishability with one rule for both factors gives the ratio 2, and distinguishability of three copies of the column state with two copies of the row state gives $\sqrt{6}$.

Questions

Does the Segre reading derive the product?

No. It presupposes the grid as a tensor product of a space of three states and a space of two, which is a reading of the grid, and it then identifies the product with the unentangled states. In the Passage I reach the product from its own identifications (Chapter 9, Section I) and the classification (Chapter 9, Section VI). The reading agrees with that result and adds a second reason for it; it carries none of the Passage's weight.



The Modal Readings of the Inertial Forces and of Free Fall

On the Triple and the Geodesic in the Terms of the Immanent Metaphysics

The Passage gives the equivalence of the inertial forces and the law of free fall as physics (Chapters 14 and 15). In this chapter I give the readings of both in the terms of the Immanent Metaphysics: the modal assignment of centripetal force, linear acceleration and gravitation, which the Passage does not make, and the reading of the geodesic, which the Passage states in one sentence (Chapter 15, Section III) and I carry further here. I identify centripetal force, linear acceleration and gravitation with the omniscient, immanent and transcendent modalities, and their relation with Axiom III; and I identify the geodesic as the path that preserves the structure of the context. In Section II I read the split of the metric into lapse, shift and spatial metric by the columns of the grid and by Axiom I. None of the readings adds a prediction, and no step of the Passage rests on any of them.

> *The Passage, Chapter 14, opening:* "That equivalence is the weak equivalence principle, extended to the inertial forces of any non-inertial frame".
>
> *The Passage, Chapter 15, opening:* "The contracted Bianchi identity makes the stress-energy conserved, and conservation gives the geodesic equation: exactly for dust, and for any body small enough that its own field is negligible (Geroch and Jang 1975)".

Rests on: the modalities and Axiom III (Chapter 4, Sections II and V); the prior direction of time from Axiom I (Chapter 10, Sections III and IV); the equivalence of the three phenomena (Chapter 14); the geodesic law (Chapter 15, Section I); the content/context partition (Chapter 6); the senses of continuity and asymmetry (Chapter 3, Section V). *Used by:* no step of the Passage.

I. The Modal Assignments

I identify centripetal force, linear acceleration and gravitation with the omniscient, immanent and transcendent modalities.

I identify centripetal force as *omniscient-modal*. It is structural, geometric, determined by the knowable shape of the trajectory. Centripetal force is the pattern of the motion: the curvature of the path.

I identify linear acceleration as *immanent-modal*. It is processual, dynamic, the bare fact of interaction between the accelerating body and whatever provides the thrust. It is mediation: the engine pushes the floor, the floor pushes the person, the person feels weight.

I identify gravitation as *transcendent-modal*. It is potential, environmental, determined by the mass-energy distribution of the entire surrounding region. It is context: the geometric arena within which both trajectories and interactions occur. The gravitational field at a point depends on the distribution of mass-energy everywhere else, and so exceeds any local specification.

II. Axiom I and the Split of the Metric

> *The Passage, Chapter 10, Section IV:* "Axiom I makes time the single prior direction, and Axiom II supplies one direction for the cycle's order and three for its transitions".

Rests on: the prior direction of time from Axiom I (Chapter 10, Sections III and IV). *Used by:* no step of the Passage.

The standard split of the metric of general relativity can be read by the columns of the grid. A foliation of spacetime divides it into a family of spatial surfaces, its leaves, one for each value of a time. In the decomposition of Arnowitt, Deser and Misner (1962; the standard textbook form is in Misner, Thorne and Wheeler 1973, Chapter 21), a foliation splits the metric into a lapse function N, which gives the proper time that passes between neighboring leaves, a shift vector N^i (three components), which

gives how the spatial coordinates move from one leaf to the next, and a spatial metric h_{ij} (six components) on each leaf. I identify the lapse and the shift as the quantities of the immanent column, and the spatial metric as the quantity of the omniscient column. I read Axiom I as stating that the lapse and the shift are structurally prior: the spatial geometry h_{ij} is defined only relative to a choice of foliation, while the existence of foliations by Cauchy surfaces (surfaces that every causal path without end crosses exactly once) is a feature of the causal structure of a globally hyperbolic spacetime, which exists prior to any spatial metric on the leaves.

Stated formally, the condition of temporal priority is that the spacetime M is globally hyperbolic: M admits a Cauchy surface and a smooth time function whose gradient is everywhere timelike. The causal structure (the field of light cones) is more fundamental than the spatial metric: two Lorentzian metrics with the same causal structure are conformally equivalent (Hawking, King and McCarthy 1976; Malament 1977). Global hyperbolicity is the standard condition in general relativity for a well-posed initial-value problem. On this reading, Axiom I identifies it as a structural requirement of physics. The immanent modality, the temporal process, is prior because the causal structure is what makes physics possible: without a definite before and after, neither measurement nor interaction can occur. The reading adds no prediction.

III. Axiom III Applied

The equivalence principle, read as a triple, states that in each of gravitation, linear acceleration and centripetal force every body is affected alike, whatever its composition: the force each body feels is proportional to its inertial mass (Appendix A, Section II). At a point, gravitation and linear acceleration are locally indistinguishable (Chapter 14, Section III); rotation is registered at a point against the local inertial frame. I take the relation of the three phenomena to be Axiom III in the gravitational domain: the three modalities are *distinct, inseparable, and non-interchangeable*.

Distinct: the three phenomena have different physical origins (trajectory geometry, engine interaction, mass distribution).

Inseparable: the three are available together in every frame of description. Every trajectory is described with its curvature, every body with the interactions that act on it, and every region with its gravitational field. In a particular case the force felt may vanish, as in uniform straight motion, or in free fall, where no linear acceleration is felt; the three descriptions do not come apart, and in each the force felt is of one kind, borne alike by every body.

Non-interchangeable: they are globally distinct. Centripetal force requires a trajectory about a center; linear acceleration requires a thrust source; gravitation requires a mass distribution. Their causes are not interchangeable, and their global structures differ: an axis, uniformity, a center.

IV. What the Triple Reveals

The triadic structure shows the equivalence principle as a statement about the structure of force in the gravitational domain: three phenomena in each of which every body is affected alike.

The duality gravity = acceleration names two of the three. The triple of gravitation, linear acceleration and centripetal force contains all three modalities, in the relation that Axiom III states. The one quantity found in all three phenomena is the mass of the body, and "One Mass, Three Elevators" gives what that quantity measures.

V. The Geodesic as the Path That Preserves the Context

The Passage, Chapter 15, Section III: "On the freely falling observer no content-modal interaction acts".

The Passage already places the free body as the body on which no content-modal interaction acts, with its path the one the context alone governs (Chapter 15, Section III). The standard derivation is mathematically

complete: the Bianchi identity gives conservation, and conservation gives geodesic motion (Chapter 15, Section I). I identify a geodesic with the path along which the structure of the context is preserved: the parallel-transport condition states that the tangent vector does not change relative to the local context. What I add in this section is the character of that path, as a reading and not as a further derivation.

I identify the geodesic as the continuously asymmetric path, with both words in the sense of Chapter 3, Section V. It is continuous: along it nothing is sudden, with no abrupt change of direction and no jump. It is asymmetric in the sense of a difference of content across a difference of context. In a curved region the context differs from point to point, and two geodesics that start parallel do not stay parallel (Chapter 15, Section V): what neighboring free bodies find of each other, their separation and their relative velocity, differs as their place along the paths differs. The asymmetry is that of the context the path follows, not a property of the path taken alone.

In the reading of the Immanent Metaphysics, the Bianchi identity is the geometric expression of the requirement that the context be preserved under the dynamics, and the conservation of stress-energy is the mathematical translation of the principle that content does not create or destroy context.

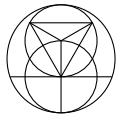
Questions

Are the modal assignments of Section I a result, or a reading?
A reading. I identify centripetal force, linear acceleration and gravitation with the omniscient, immanent and transcendent modalities, and their relation with Axiom III; none of the three identifications is a theorem. The field equations, the geodesic law and the Newtonian limit do not rest on the assignments (Chapters 12, 15 and 16). What Chapter 14 establishes

without them is its physics (Chapter 14, Sections II and III): that every body is affected alike in each phenomenon, and that the three are told apart across any room of nonzero size.

Is the reading circular: geodesics defined as the paths that preserve the context, and then found to be the paths of a phenomenon of context?
No. The geodesic equation is not obtained from the reading; it follows from the field equations by the Bianchi identity and conservation (Chapter 15, Section I). The content/context partition, which follows from the Incommensuration Theorem, read in physics (Chapters 3 and 6), assigns gravity to the context independently of any law of motion, and the mathematical expression of preserving the structure of a geometry along a path is parallel transport. The reading identifies the geodesic with the path that preserves the structure of the context; it is an identification, stated as one in Section V, and nothing in Chapter 15, Section I rests on it.

If the reading predicts nothing beyond general relativity, what does it add?
It adds no prediction. Geodesic motion in the Passage's account is geodesic motion in general relativity, and the reading does not modify the geodesic equation. What it adds to Chapter 15, Section III, which already places the free body as the body on which no content-modal interaction acts, is the character of its path: continuous and asymmetric in the senses of Chapter 3, Section V, with the Bianchi identity read as the preservation of the context under the dynamics.



One Mass, Three Elevators

On Pattern, Inertia and Weight as One Relation, and on Spin as the Twist of a Point Against the Whole

The Passage argues that gravitational mass equals inertial mass, and that pattern, inertia and weight are one relation of a content to its context, found three times (Appendix A, Section II). In this chapter I carry that argument further, in the terms of the Immanent Metaphysics: through three elevator rooms of the equivalence principle, through the principle of Mach, through the proton and light, and down to a point, where what is left of the relation of turning is spin. I identify spin as the permanent, quantized twist of a content of no extent against the frames that the comparable content fixes. No step of the Passage rests on this chapter, and I add no prediction.

> *The Passage, Appendix A, Section II:* "So pattern, inertia and weight (the gravitational mass) are one relation of a content to its context, found three times: as what there is of it to be found, as its resistance to a change of its place in the context, and as the hold the context takes on it".
>
> *The Passage, Chapter 18, Section IV:* "Inertia is a comparison".
>
> *The Passage, Appendix A, Section IV:* "Spin labels the representations of the rotations of the frames of M^4, a compact group whose double cover is the SU(2) within Spin(1,3), and a content keeps its spin across every change of context; so spin is a content, registered against the frames that the context supplies, and its half-integer values are why the content's fields of half-integer spin are spinors of M^4 (Chapter 12, Questions)".

Rests on: the Root Tautology (Chapter 1, Section IV); the six metrics (Chapter 7, Section I); the source of the curvature (Chapter 12, Section V); the equivalence as a triple (Chapter 14); inertia as a comparison and the frames that the comparable

content fixes (Chapter 18, Section IV); the context markers and the mode assignment (Appendix A, Sections II and IV); the modal readings of "The Modal Readings of the Inertial Forces and of Free Fall". *Used by:* no step of the Passage.

I. Three Rooms

> *The Passage, Chapter 14, Section III:* "Rotation keeps its axis at every point, and it is read against the local inertial frame, which the comparable content fixes (Chapter 18, Section IV): a gyroscope registers it, and so does the deflection of any moving body".

Begin with a small closed room, an elevator, and a person inside it who cannot see out. In the first case the room is far out in empty space, pulled steadily upward by a rope. In the second case it rests on the surface of a planet. In the third case it hangs at the end of a long cable and is swung in a wide circle. In each room the person is pressed to the floor (in the swung room the floor is the outer wall), and a ball let go falls to the floor.

In all three rooms one thing holds without exception: every body is affected alike, whatever it is made of. A hammer and a feather let go together reach the floor together. The push each body feels is in proportion to its inertial mass, and on the planet the pull of gravity on each body is in proportion to that same mass. The one thing that holds in all three rooms, that every body is affected alike, whatever the body is made of, is the equivalence principle read as a triple (Chapter 14).

Across any extent the three rooms are told apart. Two balls hung on strings from two points of the ceiling hang closer together than their points of hanging on the planet, parallel in the room pulled through empty space, and farther apart, along the direction of motion, in the swung room (Chapter 14, Section III). The rooms are distinguished by how the separation of two neighboring findings changes, and a separation can do only three things: shrink, stay or grow.

As the room shrinks, the difference between the strings shrinks faster than the push itself. At a point, the push felt in each room is of one kind, borne by every body in proportion to its mass, and gravitation and linear acceleration are not told apart at all. Rotation keeps its axis at every point (Chapter 14, Section III); Section VI says what that axis is at a point.

"The Modal Readings of the Inertial Forces and of Free Fall" identifies the three phenomena with the three modalities: rotation, structural and knowable from the shape of the motion, is omniscient; linear acceleration, the bare fact of being pushed, is immanent; gravitation, set by what lies beyond the room, is transcendent. The three are distinct, inseparable and not interchangeable, as Axiom III requires (Chapter 4, Section V), and each has its own source: an axis, an engine, a center. One quantity is found in all three rooms, the mass of the body, and the rest of this chapter is the account of what that quantity measures.

II. Inertia Is a Comparison

Newton hung a bucket of water from a twisted rope and let the rope unwind (Newton 1687). At first the bucket turned and the water stayed still, with a flat surface; then the water caught up, turned with the bucket, and its surface rose at the rim into a hollow. The hollow showed that the water was turning. The water was not turning relative to the bucket, since at the end the water and the bucket turned together and the surface was hollow, while at the start the water and the bucket turned relative to each other and the surface was flat. Newton concluded that the water turned relative to absolute space.

Mach gave the other answer (Mach 1883). The water turns relative to the rest of the universe: the earth, the sun, the distant stars, the whole of the matter there is. A gyroscope set spinning holds its direction against the distant stars, and a Foucault pendulum swings in a plane that stays fixed against them while the earth turns beneath it. What counts as not turning is fixed by everything else.

Picture an empty universe with a single spacecraft in it. Nothing in the ship can tell whether it is spinning, since to spin is to spin relative to something. In our universe a gyroscope, or light sent both ways round a ring (the Sagnac effect), registers rotation against the local inertial frame, and that frame is fixed by the content it can be compared with: the earth, the stars, all the matter within the horizon. With nothing else in the universe, no content fixes such a frame, and nothing is there against which the ship's turning could register. General relativity alone does not say this, since its equations allow empty solutions that still have inertial frames; it follows from comparison: a difference that no comparison can register is no difference. Now put two spacecraft in the empty universe, and nothing else. They turn only relative to each other: each ship's rest or turning is a relation of that ship to the other, the only content there is to compare it against. Only when something else is added can either ship's motion be stated relative to that something else. The relations make the motion what it is, and no frame of reference stands outside them, for time, for space or for anything else.

The Passage makes that answer a consequence of comparison (Chapter 18, Section IV). To be at rest, to move uniformly or to rotate is always to do so relative to something. No comparison registers a difference between one state of motion and another when there is nothing to compare them against, and a difference that no comparison can register is no difference. So a state of motion is a relation of a content to its context, and inertial frames are frames only through the comparable content that fixes them: the content within the horizon, since content beyond it is not comparable in principle.

Inertia, then, is the resistance of a content to a change in its relation to the context. To set a body moving is to change where it is found, and how it is found to be moving, against everything else, and the body resists that change in proportion to its mass. The first finding of the one quantity, mass, is inertia, which is mass found as resistance. Inertia is found in the act of changing the place of a content in the context of that content.

III. One Relation, Found Twice

The second finding is weight. For every force other than gravity, the strength with which the force takes hold of a body and the body's resistance to being moved differ. An electric field takes hold of a body by its charge, and the body resists by its inertia, and the ratio of charge to inertia differs widely from body to body: it is large for the electron, small for the proton and zero for the neutron.

The reason for that difference is the reason for the equality. A charge is a label of the content, a property it carries in addition to its place in the context. Inertia is a property of the content's relation to the context. Content and context are distinct, and neither stands in for the other, so nothing ties a charge to an inertia, and their ratio is free to be anything.

Gravity is the context-modal phenomenon (Chapter 6; Appendix A, Section II). It couples to everything, nothing shields it, it carries no charge in steps, and the coupling of every body to it equals that body's inertia. The last mark, that coupling equals inertia, rests on what it is for a content to take part in the context. To take part in the context is to be found in it, at a place and in a state of motion there, and that is the only relation of a content to the context that the context offers. A charge is a separate label of the content, carried in addition to that relation, so a charge and an inertia have two things to differ by. For the context there is only the one relation, so the coupling and the inertia have nothing to differ by. *Measurement; The Steps from Comparison to Physics* states the same ground as its Proposition 10.3 (*Measurement*, Chapter 10).

So weight and inertia are one relation of the content to the context, found twice: once as how strongly the context takes hold of the content, and once as how hard the content is to move within it. Because weight and inertia are one relation of the content to the context, the hammer and the feather fall together. For the same reason, the room on the planet and the room pulled through empty space are not told apart at a point. The

two rooms are not told apart because a uniform acceleration is a change of context, gravity is a phenomenon of context, and so a uniform acceleration and gravity are of one kind.

IV. The Third Finding: Pattern

The third finding is pattern, and it completes the identity (Appendix A, Section II). The coupling of a content to its context is the degree to which the content takes part in the context, and to take part is to be found. The degree to which a content can be found is how much of it there is to find: how many comparisons it can enter, how many findings can be made of it. How much of a content there is to find is the pattern of that content. A content with more arrangement, more relations and more structure offers more to be found, and a content with nothing to be found would take no part in the context at all. So the coupling of a content to its context is the measure of its pattern.

> *The Passage, Chapter 12, Section V:* "It curves where findings can be made, and in proportion to how many can be made there".

The Passage states the same result from the side of the context (Chapter 12, Section V). Where there is content, the context carries the asymmetry that the content and the act of comparison cannot carry, and it curves where findings can be made, in proportion to how many. Energy and momentum measure that count by the Root Tautology (Chapter 1, Section IV): to compare, to interact and to relate are one act, so every finding is an interaction, and energy and momentum are what an interaction exchanges. The capacity of a content to be found is its capacity to interact, which physics measures as its energy, and, for a body at rest, as its mass. *Measurement* argues the unevenness of the context where there is content as its Proposition 10.7 (*Measurement*, Chapter 10, Section VIII).

The three findings stand in the three modal roles. Pattern is how much of a content can be found: the content as laid out, the omniscient reading. Inertia is how hard it is to change where and how the content is found: the

content in the act of change, the immanent reading. Weight is how strongly the context is shaped around the content, and takes hold of it in turn: the content as the whole surrounding field answers to it, the transcendent reading. The rooms of Section I stand in the same roles, and each carries one finding in particular: the swung room, the structural reading, carries pattern; the pulled room, the reading of the act, carries inertia; and the room on the planet, the reading set from beyond the room, carries weight. Each finding is the one relation of the content to its context, the relation of being found there, and each is measured by one quantity, mass.

Pattern, inertia and weight share one cell of the six metrics (Chapter 7, Section I) because each of the three findings is the one relation of being found, and each finding is measured by the one quantity, mass. They are three concepts of one reality, distinct as concepts and inseparable in the thing, as Axiom III requires of any three that belong together. It is also why the identity is exact. For the context there is only the one relation, so the three findings have nothing to differ by, and measurement finds gravitational and inertial mass equal to about one part in 10^{15} (Touboul and colleagues 2022).

Einstein found the same identity from the side of energy (Einstein 1905). He showed that a body that gives off light loses inertia in proportion to the energy it gives off, and he asked, in the title of the paper, whether the inertia of a body depends on its energy content. It does. A body's inertia is its energy content divided by the square of the speed of light, and its energy content is the measure of what in it can interact, which is to say, what in it can be found. What Einstein showed for inertia, and what the equivalence principle shows for weight, are the two halves of the identity that pattern completes.

V. The Proton and Light

The proton shows that mass measures relations and not stuff. A proton is made of quarks, held together by the gluon field of the strong force. If mass measured an amount of stuff, the mass of the proton would be the sum of the masses of its quarks. The mass of the proton is not the sum of the masses of the proton's quarks: the rest masses of the quarks inside a proton add up to a small part of the proton's mass, and nearly all of the rest comes from the energy that binds the parts. The division of the proton's mass into the energy of the quarks, the energy of the gluon field and the small share of the quark masses was set out within the theory of the strong force by Ji (1995), and the masses of the proton and its relatives were computed from that theory, with only the quark masses and one overall scale taken from measurement, by Dürr and colleagues (2008).

What the proton weighs, and how hard it is to set moving, is then almost wholly the pattern of the relations among its parts: how the quarks move, how the field between them is arranged, how they are bound. Take the parts apart and that pattern is gone, and so is nearly all of the mass. The proton is the clearest case, and it is not a special one. A hot body weighs more than the same body cold, because its parts move more. A wound spring weighs more than the same spring unwound, because its parts are arranged under strain. A closed box of mirrors with light bouncing inside it weighs more than the empty box. In every case the mass follows the pattern, the arrangement and motion of what is there.

Light has pattern too: a frequency, a direction and a polarization, all of which can be found. Its pattern takes part in the context. Light passing the sun is bent by the sun's gravity, as the eclipse expeditions of 1919 found (Dyson, Eddington and Davidson 1920), and light shut in a box adds to the box's weight. What light lacks is a frame in which it is at rest. Rest mass is the measure of the pattern a content keeps in its own frame of rest. A photon has no such frame; its whole pattern is carried in its motion, and so it is measured whole by its energy and momentum, with nothing left over that is at rest.

So the identity, in its full scope, is that the pattern of any content, in any state, is measured by its energy and momentum, and these are what the context answers to (Chapter 12, Section V). The pattern a content keeps at rest is measured by its rest mass, and for every body that can be at rest, its inertia, its weight and its rest pattern are one. Light shows the general case; the brick, the planet and the proton show the case at rest.

VI. Rotation at a Point

The Passage holds that at a point gravitation and linear acceleration coincide, while rotation keeps its axis and is read against the local inertial frame (Chapter 14, Section III). *Measurement* observes that every instrument for rotation has extent: a gyroscope, a Foucault pendulum, the ring of a Sagnac interferometer, the deflection of a body moving across a room (*Measurement*, Chapter 10, Section VI). The claim of the Passage and the observation reported in Measurement both hold, and an elementary particle is a case in which the claim and the observation hold together.

An electron, measured as finely as it can be, shows no size and no interior structure (Dehmelt 1988). Its spin is nonetheless measured. In the experiment of Gerlach and Stern (1922) a beam of silver atoms, whose magnetism is that of the spin of one outer electron, passes through a magnetic field that changes in strength across the beam, and the beam divides in two, one part for each sense of turning. The extent in that measurement is the magnet's. The field that changes across the beam is the outside structure against which the spin is compared. The instrument has extent; what it registers need not.

The rotation of an extended body is found by comparing its inside with its outside: how the parts within it change their relation to everything beyond it. A point has no inside. What rotation keeps at a point is its relation to the frames, and a content of no extent that keeps such a relation is a spin. An ideal gyroscope of no extent is a spin. So the

push felt in each room is of one kind at a point, gravitation and linear acceleration are not told apart there, and the axis of rotation is kept there as a relation to the frames, which an instrument with extent registers.

VII. Spin as the Twist of a Point Against the Whole

The Passage, Chapter 18, Section IV: "Inertial frames are frames only through the comparable content that fixes them".

Spin cannot be a turning of a point about its own center: there is no extent in which such a turning could take place. The Passage states the half of the account that physics already holds: a content keeps its spin across every change of context, so spin is a content, registered against the frames that the context supplies (Appendix A, Section IV). The principle of Mach, as the Passage states it, supplies the other half. Those frames are the inertial frames, and inertial frames are frames only through the comparable content that fixes them (Chapter 18, Section IV). So spin is a relation of a content of no extent to the frames that the comparable content fixes, and through them to the whole of the comparable content. It is turning with an outside and no inside.

The relation is a twist, and it has the exact behavior of a twist. A state of spin one half changes sign when the frames are turned through one full turn, and returns to itself only after two full turns. The change of sign has been seen directly. A beam of neutrons is split, one half is turned through a full turn in a magnetic field, and when the halves are brought together their interference shows the turned half arriving with its sign reversed (Werner, Colella, Overhauser and Eagen 1975). That behavior is the behavior of anything whose orientation is tied to its surroundings. Hold a cup level on the palm of the hand and turn it through one full turn under the arm: the arm is left twisted. Continue through a second full turn, over the head, and the arm comes back untwisted. One turn leaves a twist that no smooth motion can remove; two turns leave none. Misner, Thorne and Wheeler (1973), in the chapter of their book on spinors, call this the ori-

entation-entanglement relation: the orientation of an object is entangled with the orientation of everything it is connected to, and the mathematics of spin one half is the mathematics of exactly that entanglement.

The twist is quantized. There are only two classes of twist against the surroundings, the untwisted and the once twisted, and no state lies between them, since a twist cannot be half removed. That there are two classes is a theorem of mathematics about the turnings of three-dimensional space. Picture a turning carried out gradually, from no turn at all through a full turn and back to where it began, as a closed path through all possible turnings. A path of one full turn cannot be unwound, by any gradual change, into no turning at all; a path of two full turns can. The space of turnings in three dimensions has exactly two kinds of closed path, those that can be unwound and those that cannot, and any two paths of the kind that cannot be unwound together make a path of the kind that can. That there are exactly two kinds is a fact about three dimensions. In two dimensions no number of full turns can be unwound, and the kinds of closed path of turning are unlimited in number. So there are exactly two kinds of spin: whole values, for contents whose state returns after one full turn, and half values, for contents whose state returns only after two. A content keeps its spin across every change of context, so spin is content, and content is governed by symmetry. The Incommensuration Theorem pairs symmetry with discontinuity, so a symmetric content is found in separated steps. The values of spin about any axis lie symmetrically on either side of zero, one step apart, so either zero is among them and every value is whole, or zero falls midway between two of them and the least value is one half, found as one of exactly two outcomes, one for each sense of turning (*Measurement*, Chapter 9, Section V).

The twist is permanent. No motion, no field and no change of place removes it, because it is a relation to the whole of the comparable content and not to any part of it. Nothing local can untie a twist that is tied to everything.

Spin is real angular momentum, the same quantity as the turning of an extended body, measured against the same frames. Einstein and de Haas (1915) hung an iron cylinder on a fine thread and changed its magnetization along its length, and the cylinder turned. The magnetization of iron comes almost wholly from the spins of its electrons, so the angular momentum the spins took up or gave off was given off or taken up by the cylinder as a whole: the spin of a point and the turning of a body exchange one quantity against one set of frames. The swung room and the spinning point are one relation. The swung room has an inside, which turns against its outside; the point has no inside, and only the relation to the outside remains.

I identify spin, then, as the permanent, quantized twist of a content of no extent against the frames that the comparable content fixes.

Questions

Is the identity of pattern, inertia and weight a result, or an identification?

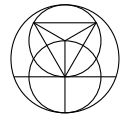
An identification, stated in the first person. The Passage takes the coupling of a context-modal phenomenon as the degree of participation in the context, and the inertial response as the degree of resistance to change of context, and identifies the two as one quantity (Appendix A, Section II); the step to pattern identifies participation with being found. The equality of gravitational and inertial mass is measured, and no step of the Passage rests on the step to pattern.

An electron has no extent, and every instrument for rotation has extent. How then is its spin found?

By an instrument with extent, against which the spin is compared (Section VI). In the experiment of Gerlach and Stern the magnet supplies the extent: its field changes across the beam, and the beam divides by the sense of turning of each spin. The comparison needs extent, and the outside supplies all of it.

Does the electron then turn?

No part of it turns, since it has no parts. Its spin is real angular momentum, exchanged with the turning of bodies against one set of frames (Einstein and de Haas 1915), and it is a relation of the point to those frames, not a motion inside it (Section VII).



Two Formal Triples of a Domain

On Interaction, Existence and Creation, and on the Real, the Existent and the Objective, Across the Turn

The Passage distinguishes what exists from what is real (Chapter 5, Section II), and it carries the physical domain through a turn into a new epoch (Chapters 18 and 19). Two formal triples of the Immanent Metaphysics say more about both. The first of the two formal triples, interaction, existence and creation, is the triple of which any domain is made. The second of the two formal triples, the real, the existent and the objective, is the triple of the three claims that can be made of anything in a domain. Read together across the turn, they say what carries from one epoch to the next and what does not. The reading is an extension: the reading adds no prediction, and no step of the Passage rests on the reading. I read the projection of Chapter 19 by the first triple in Note 12 of the Passage. I carry both triples across the turn.

> *The Passage, Chapter 5, Section II:* "To say that something exists is to say that it stands as structure: timeless, the same in every instance, like the pattern of a law or the state of a system at an instant".

Rests on: exists and real (Chapter 5, Section II); the physical domain from inside (Chapter 5, Section II; Chapter 13, Section I); the two limits of the domain (Chapter 17, Section I); the turn, and the form of the content carried through it (Chapter 18, Sections VIII and IX); the structure the same in every epoch (Chapter 18, Questions); records that agree (Chapter 12, Questions); Axiom III (Chapter 4).
Used by: no step of the Passage.

I. Interaction, Existence, Creation

A domain is a coherent region of comparison: the physical domain is the whole of what comparison can reach from within it. Three concepts together make a domain, and none alone does. *Interaction* is the act: one thing engaging another, a force exchanged, a measurement made. It is immanent, and its scale is the mesoscopic, where events occur and are registered. *Existence* is the persisting structure: the pattern that is the same across occasions and perspectives, the laws and the forms by which a domain is recognizable as that domain. It is omniscient, and its scale is the macroscopic, where patterns consolidate. *Creation* is the origination of what is new: what begins, and is not a rearrangement of what was. It is transcendent, and its scale is the microscopic, the lower boundary at which what is potential becomes actual.

The three are distinct: an act of measurement is not the persistence of what is measured, and neither is the beginning of something new. They are inseparable: no claim of existence has content without some conceivable interaction by which it could be established; every interaction brings something new, however slight; and what is created, once created, exists. They are non-interchangeable: more structure does not make up for less creation, nor more interaction for less structure. By Axiom III they are a formal triple. They stand in the directed cycle of Axiom II: interaction precedes existence, since a structure is the stabilization of a pattern of acts; existence precedes creation, since the new is new relative to what exists; and creation precedes interaction, since without new events no further act occurs.

II. The Real, the Existent, the Objective

Three claims can be made of anything in a domain, and ordinary language runs them together. To say that something is *real* is to say that it is given in an actual act, to a comparer, here and now: it involves exactly one

interaction. To say that something *exists* is to say that it stands as structure, the same in every instance, available in principle to any comparer: it involves no particular interaction at all. To say that something is *objective* is to say that many comparers, in many contexts, converge on it: it involves many interactions, and the convergence of their records.

The real is immanent, the existent omniscient, the objective transcendent. The three are distinct, since they rest on different numbers of acts; inseparable, since a claim of existence is drawn from real acts, and objectivity is the agreement of real acts across comparers; and non-interchangeable, since a single real finding is not thereby objective, and a structure that exists is not thereby found. The real is the most fundamental of the three, as Axiom I states of the immanent: existence is abstracted from many real acts by comparing them, and objectivity is their agreement.

The Passage already uses all three without the names. The cosmological constant is fixed once, by the present measurement, in the first person (Chapter 13, Section VI): that finding is real, and the constant term it fixes exists, as the law of the one geometry. The field equations and the internal geometry are the same in every epoch: that is existence. Two comparers reconcile their records by a further comparison, and their records of one finding agree (Chapter 12, Questions): that is objectivity.

> *The Passage, Chapter 12, Questions:* "Two comparers reconcile their records by a further comparison, at an event in the common future of their findings, and their records of one finding agree".

III. Across the Turn

> *The Passage, Chapter 18, Section VIII:* "Temporal language applies to the approach on each side and to the history reconstructed from inside the new epoch; it does not apply to the turn, which no comparison registers as instantaneous or as gradual".

Read together, the two triples say what carries across the turn from one epoch to the next.

Existence carries. The structure the Passage derives, the product geometry, its shape and its radii in Planck units, is the same in every epoch (Chapter 18, Questions). A claim of existence involves no particular interaction, so nothing in it requires an act that spans the turn.

Interaction does not carry. No comparison registers the turn as an event, so no act spans it, and no finding made in one epoch is real in the other. The real is confined to its epoch, as every act is.

Objectivity does not carry either. Objectivity is the agreement of the records of many comparers, and records are reconciled by further comparisons; no comparison reaches across the turn, so no record of one epoch is reconciled with a record of the other.

Creation is what the turn is. The new epoch begins at the microscopic boundary, where the internal factor gives up its compact form and the exterior is restored (Chapters 18 and 19). What begins there is the new context, a geometry with no prior identity (Chapter 18, Section VIII); it is not a rearrangement of the old exterior, and its origination is the transcendent member of the first triple. The content and its form are carried, not created (Chapter 18, Section XIII), and the next paragraph says in what sense.

The form of the content also carries. The Passage counts the state of a system at an instant as existing (Chapter 5, Section II), and the form of the content, the specification of what has been selected, passes through the turn and becomes the primordial departures of the new epoch (Chapter 18, Section IX). It is particular to the epoch before, not the same in every epoch; but it is existence in the Passage's sense, and it carries without any act that spans the turn.

The pattern of the microwave background carries the form of the content, and it can be read as a spectrum. The physical domain has two limits (Chapter 17, Section I): Planck's constant $\hbar$ is its microscopic limit, and the speed of light c is its macroscopic limit. The speed of light is not, at root, the speed of any one thing but the macroscopic boundary of the domain of comparison itself; light is only the perfected example of c. At each limit, what comparison finds carries the form of comparison. At the $\hbar$ lim-

it, comparison finds content as discrete kinds, and the particle books of this collection set out that spectrum: the six quarks, the six leptons, the gauge bosons and the Higgs. The particle spectrum is not a catalogue of independent things discovered by experiment; it is the structure of comparison, found as discrete content. Those books fix its kinds and charges by the structure of comparison (The Story of the Particles; The Kinds of Particle), and they contain its masses and couplings with the standing of each number, several of them measured inputs (The Strengths of the Kinds; The Measures of the Kinds).

At the macroscopic limit, by the same reasoning, I look for a macroscopic spectrum: a pattern of observable features in which what comparison finds at that limit is laid out. I identify the pattern of the microwave background as that spectrum. The microwave background meets the macroscopic limit in two ways. It is the oldest light that any comparison receives: its light went free when the first atoms formed (The Coming of the Kinds, Chapter 13), and nothing earlier reaches us as light. And its departures from uniformity are the form of the content carried through the turn, the limit our history takes for its beginning (Chapter 18, Section IX).

I trace the correspondence between the two spectra through three pairs. Each pair is an identification of this chapter, not a result of the books I draw on, and no number rests on the identification or on any of its three pairs.

At the ħ limit, content is found as discrete kinds: what quantum theory describes, the matter and the light, is discontinuously symmetric. At the macroscopic limit, the arena is found smooth and even: the context, which general relativity describes, is continuously asymmetric (Incommensuration; The Theorem and the Three Axioms, Chapter 2). In the focus of the two laws, symmetry carries the omniscient character and continuity the transcendent (Incommensuration, Chapter 1, Section III). Each theory applies to the domain of the opposite modality: general relativity, built on continuity, applies to the macroscopic, which is omniscient-modal, and quantum theory, built on symmetry, applies to the microscopic, which is transcendent-modal (Incommensuration, Chapter 8). The microwave

background stands where the two meet: its light is content, quanta whose blackbody spectrum is the spectrum of light in balance, while its sameness across the sky is that of the large-scale geometry of the arena, and its departures are the form of the content within that geometry.

At the ħ limit, confinement prevents the isolation of individual quarks. At the macroscopic limit, the microwave background is observed from within, and the horizon prevents access to the whole of its field: no comparison reaches beyond the cosmological event horizon (At the Edge of Reality, Preface). In both cases, the limit sets a structural bound on what can be isolated or reached.

At the ħ limit, the particle masses span a hierarchy across the generations (The Measures of the Kinds, Chapter 7). At the macroscopic limit, the departures span a range of angular scales, and the transfer functions of standard physics, which carry each departure forward to the time the light went free and depend on the densities of matter, shape their angular power spectrum. In both cases, the detailed structure is laid out across a range of one variable: the generation in the first case, the angular scale in the second.

The parallel is what makes the pattern a spectrum. The particle spectrum does not show what matter is apart from comparison; it shows what comparison finds when it finds content as discrete kinds at the ħ limit. The pattern of the microwave background does not show what the turn is; it shows what the new context registers of the content that passed the turn.

So the epochs are connected by existence alone: by the sameness of structure and by the form of the content, with no act and no shared record between them. They are not collected into one larger domain; a domain of all epochs would need both the common character of every member and an unbroken possibility of interaction among them, and the turn breaks the second. The Incommensuration Theorem gives a second ground, which does not use the turn: no domain contains every domain. If a domain contained every domain, then that domain would have to give all its

members one common character, the same sense of being a member. A common character shared by all the members is symmetry, sameness of content across a difference of context. A domain that contained every domain would also have to connect every member with every other member, none cut off from the rest. A connection of every member with every other member is continuity, what is unbroken and encompassing (Chapter 4, Section IV). Both demands are made of one comparison, the comparison in which the members are held together as one domain: the common character is a sameness found by comparing the members, and the connection is a continuity found by comparing them. Such a domain would perfect both at once in that comparison, and the two cannot both be perfected in one comparison (Chapter 3, Section IV). Each epoch is a whole physical domain, coherent from inside, and the series of epochs is a series of domains of the same structure, not one totality containing them. What physics calls the universe is a physical domain, the whole of what comparison can reach from within it (Section I), and not a completed totality; a model that treats it as a closed whole, given all at once, carries that wholeness as a property of the model, not of the domain.

IV. Standing

The two triples are formal triples of the Immanent Metaphysics, and the reading of the turn by them is an extension of the Passage. It agrees with what the Passage states: the structure is the same in every epoch, no comparison registers the turn, and every measurement is made from inside one epoch. It adds no prediction, and it is offered for the later discussion of what lies beyond a single epoch.

Questions

If nothing real crosses the turn, in what sense is the next epoch the same kind of world?

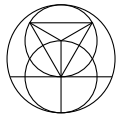
In the sense of existence: the same structure, the same laws, the same geometry, recognizable by any comparer inside either epoch, and in the new epoch the form of the content of the old one, as its primordial departures. Neither needs an act that spans the two. What no comparer can have is a record from both.

Does the reading imply that the epochs never end, or that they began?

It implies neither. The Passage describes a cycle of epochs, each beginning at a turn (Chapter 18, Section XII); the reading says only what carries across one turn. A first or a last epoch would be a claim about a totality of epochs, and the reading of Section III shows the epochs are not collected into one.

PART III

Open Questions



Open Questions as Research

On What the Passage Leaves Open, and What Would Settle It

Each chapter of the Passage states what it leaves open. This chapter takes those items, and five more that careful readers of the Passage raise (Sections XII to XVI), and one that "One Mass, Three Elevators" raises (Section XVII), as a program of research: for each, what is open and what kind of result would settle it. None of them is a step of the Passage's chain; each is a place where the chain stops, as the Passage says where it does.

Rests on: the statements of the Passage quoted or cited in each section. *Used by:* no step of the Passage.

I. Why This Epoch, Asked from Outside

> *The Passage, Chapter 13, Section VI:* "The comparison that fixes Λ is real, so it is the present measurement, by the definition of the real; no further identification is made".
>
> *The Passage, Chapter 13, Questions:* "Section VI answers the third part from Axiom I, and the step rests on Axiom I for that part alone".

The Passage states which comparison fixes Λ: the present measurement, by the definition of the real and by Axiom I (Chapter 13, Section VI). The values the formula gives at the particle horizons of other epochs are computed from the reconstructed history; they exist, as structure, and none of them is real. The two triples of "Two Formal Triples of a Domain" state the same answer: the present finding is real; the constant term it fixes exists; and its objectivity is the agreement of the records of many comparers, which are present impressions of the one history.

A reader who surveys the whole history asks why the present falls at the epoch where the identity holds. The Passage divides that question into three parts (Chapter 13, Questions): the ratio of the present age to the de Sitter time, 0.792, which the identity gives; the absolute epoch, about 10^{61} Planck times, which is a question about when comparers exist; and uniqueness, which rests on Axiom I. I give the numbers for the first two parts. Both stand outside the Passage's chain, and the second rests on facts about the content, which the Passage does not derive.

The absolute epoch. Three senses of time meet at the present. The past runs from the beginning of the reconstructed history to now, 13.8 billion years. The exterior route ends with the evaporation of the holes the exterior holds: a hole of mass M evaporates in about $5120\pi\, t_{Pl}(M/M_{Pl})^3$, about 2×10^{67} years for one solar mass. The interior route ends at the modal boundary: an observer falling from rest has $4GM/(3c^3)$ of proper time left at the horizon (Chapter 18, Section II), and the last interval a comparison registers at the end is the Planck time. On the logarithmic scale the present is the midpoint of the exterior route: $t_0^{\,2} = t_{Pl} t_{evap}$ holds for a hole of 17.5 solar masses. For the holes that stars leave, from 3 to 100 solar masses, the midpoint falls between 1.0 and 189 billion years, and the present lies between 0.491 and 0.510 of the logarithmic span from the Planck time to evaporation. The interior route balances only through its Planck end: with its full proper time in place of the Planck time, the balance would hold for a hole of 5×10^{21} kilograms, a mass that no process selects.

The balance is Dicke's relation in another form (Dicke 1961; Carter 1974). A star's mass is of order $M_{Pl}^{\,3}/m_p^{\,2}$, with m_p the mass of the proton, so the hole it leaves evaporates in a time of order $t_{Pl}(M_{Pl}/m_p)^6$; a star lives for a time of order $t_{Pl}(M_{Pl}/m_p)^3$, up to factors of the fine-structure constant; and the square of the second is the Planck time times the first. The era of star formation runs from about 10^6 to about 10^{14} years (Adams and Laughlin 1997), with its logarithmic midpoint at 10^{10} years, and the

rate of star formation has fallen for eleven billion years (Sobral and colleagues 2013). Comparers of the kind that measure Λ exist while stars shine, so these facts answer the second part, the absolute epoch, to an order of magnitude.

The ratio. The balance contains no cosmological constant, so an act at 0.5 or at 1.5 de Sitter times would meet it as well as the present does: a factor of three in time is a factor of two in the mass of the hole. Within general relativity with a cosmological constant, the only cosmological clock is the de Sitter time, and the natural balances of past against future fall at other epochs: past reach equal to future reach (the particle horizon equal to the event horizon) at 0.240 de Sitter times, the onset of acceleration at 0.439, equal densities of matter and vacuum at 0.587, an age equal to the Hubble time at 0.859, an age equal to the event-horizon distance over c at 0.973, and an age equal to the de Sitter time at 1.000. None of them falls at 0.792, so the ratio comes from the identity alone.

Uniqueness. A selection account stated from outside gives a distribution of epochs: Lineweaver and Egan (2007) find, from the ages of terrestrial planets, a 68 percent probability that observers find the densities of matter and vacuum at least as close as they are now. A distribution singles out no epoch as real. What remains for research is the statement of Chapter 13, Section VI, put so that a reader who takes the survey of all epochs as prior can check it step by step. The computations behind the numbers of this section are given in the Passage's additional materials.

II. The Coefficients of Order One

The Passage, Chapter 13, Questions: "Three coefficients of order one, none of them set by this comparison".

The ground-state part of the external value is taken as exactly M_{Pl}^4; the internal curvature as exactly the Planck curvature; and the classical energy of the higher Lovelock terms is open. Each enters f, and with it the

predicted Ω_m. What would settle them is a statement of the ground-state density at the modal boundary as a definite multiple of $M_{Pl}{}^4$, and of the coefficients of the higher Lovelock terms in the ten-dimensional action.

III. The Magnitude of G_N

The Passage, Chapter 17, Questions: "Whether the magnitude of G_N, that is, the size of a content relative to the boundary, can be derived from a deeper structural principle is not addressed in this book (Chapter 20)".

The Passage takes G_N as its one measured constant: the axioms fix that the domain has two limits and that content curves context through a universal coupling, and not the size of any content relative to the boundary (Chapter 17, Section I). What would settle the question is a principle that fixes such a ratio, a laboratory mass over the Planck mass, from the structure of comparison; the step of the Passage that would then change is the one of Chapter 17, Section I, where the Passage takes G_N as its one measured constant.

IV. The Dynamics of the Turn

The Passage, Chapter 18, Questions: "The quantitative dynamics of de-compactification: the rate at which the internal geometry expands, the temperature at which the reconstructed history of the new epoch begins, and the amplitude and tilt of the departures (Section XI)".

The Passage identifies the turn and its two sides and does not compute its dynamics (Chapter 18, Section XI). Among the open numbers are the amplitude and tilt of the primordial departures, which the measurements of the microwave background fix to high precision. What would settle them is a dynamical account of the restoration of the exterior factor, from which those two numbers follow.

V. The Count of States Across the Turn

The states across the turn are not yet counted in numbers.

The Passage gives the structural reason that the entropy of the new epoch begins low: the gravitational entropy of the interiors is counted relative to an exterior that ends (Chapter 18, Section VIII). A count, in numbers, of the states on each side of the turn would test that reset.

VI. From the Ordinary Interior to the Arc

What carries the ordinary interior from its first form to the upper end of the Kasner curve, and at what depth, is not yet derived.

The ordinary interior, with exponents $-1/3$, $2/3$ and $2/3$ and an internal space of fixed size, is a valid Kasner point off the curve (Appendix B, Section II). Near a generic singularity, empty-space gravity in ten dimensions also oscillates through an endless sequence of Kasner forms, and how the product geometry selects one is not yet derived; a single Kasner form with each group moving together is consistent within the product geometry, and it is not the generic behavior (Appendix B, Section VII). What would settle both is a dynamical treatment of the interior, in the product geometry, from the ordinary form to the arc.

VII. Three Accounts of De-Compactification

Whether the three accounts of de-compactification that the Passage gives (Appendix B, Section VII) are one process seen in three ways is not yet argued.

The Passage gives a global account (Chapter 18), a density account (Chapter 19) and a geometric account (Appendix B, Section IV). The argument that joins them will use the time of the infalling observer and the time of the exterior (Chapter 18, Sections I and II).

VIII. The Return to Compact Form

The dynamics by which the internal space, once restored to the ten-dimensional geometry, returns to its compact form in the new epoch are not yet given.

The Passage names the conditions under which the compactification holds (Chapter 18, Section VI). What would settle the question is an account of how the internal space, restored with the rest of the ten-dimensional geometry, comes to be held again at the modal boundary in the new exterior.

IX. The Floor Beyond First Order

> *The Passage, Appendix D, Questions:* "The energy beyond first order; the floor on a background that is on shell in its four-dimensional part; and the metric on the space of fields once the coefficients of the higher Lovelock terms are fixed".

The computation of Appendix D is the first order of one mechanism for the hold; no parameter makes the next order small (Appendix D, Section VI). The hold itself rests on the argument of Chapter 11, Section II. The three items would refine the stiffness of the hold; none of them bears on the radius, which Chapter 9, Section IX, fixes.

X. The Doubling Step of Appendix C

> *The Passage, Appendix C, Section VI:* "The placing rule: the distinction between two ways of a context is placed in that context when, and only when, the context has a way that is not composed from those two".

The doubling is the Passage's statement that every finding is placed twice, in a context and by a comparing (Appendix C, Section II). By that rule three meets the doubling: in a context of three ways, the distinction between any two of them is placed by the third. The rule is met by every

number of ways from three upward, so the count of three for space rests on the minimality principle of Appendix C, Section IV, as before. Two premises of the rule are argued and not derived. The first premise is that every way of a context is one of the independent ways of that context or is composed from the independent ways of that context, and that all the largest sets of independent ways of that context have the same size. The second premise is that, for any two ways of a context, a way of that same context that is not composed from the two ways is enough to place the distinction between the two ways. What would settle the question is a derivation of the second premise from Chapter 2, Section I. The count of three for space has a second route in the Passage, Chapter 10, Section IV, which does not use this step.

XI. Beyond the Boundary

> *The Passage, Chapter 19, Section IV:* "Beyond the Planck scale no description takes over (Chapter 17, Section IV): the full axiom structure holds there without the macroscopic partition into separate content and context geometries, and I construct no description of that region".

At the modal boundary each existing description reaches the edge of its domain, and no third description takes over (Chapter 17, Section IV). The Passage identifies the boundary and constructs no geometry or law beyond it, and the structure of comparison holds on both sides of it. No form of a further description is sought. What remains open at the boundary is the dynamics of the region that approaches it, which Section IV takes.

XII. The Grounds of the Accessible Share

> *The Passage, Chapter 13, Section II:* "I take the accessible share of the external value to be c_1/S, with $c_1 = 3$: one degree of freedom for each channel, among the S degrees of freedom the horizon counts".

The Passage argues the share in three steps (Chapter 13, Section II). The share is a ratio of counts because the S degrees of freedom hold the external value in equal parts; this is exact once the sum of parts and the absence of any distinction among the degrees of freedom are granted, whatever their correlations. The channels are three because a measurement registers one content in each modal role, and the context of each column is where that content is registered, not a second thing registered; in the internal space they are the three classes of $\mathbb{C}P^2$ taken with one class of S^2. The count is $\chi(\mathbb{C}P^2)$, which on every complex projective space equals the coefficient of the first Chern class. Each channel engages a different degree of freedom. The cell assignment of "The Cells of $\mathbb{C}P^2$ and the Six Metrics" supplies the vocabulary but not the ground: content and context are inseparable, and so are the three modal roles, so inseparability alone cannot make the count three rather than one or six. Three things remain open. The first thing that remains open is why the S degrees of freedom hold the external value as a sum of parts. The second thing that remains open is why the vacuum's energy reaches a measurement only through the degrees of freedom that the measurement engages. The third thing that remains open is why each channel engages exactly one degree of freedom, counted in nats. One bit for each channel, in place of one nat, would give $\Omega_m = 0.407$, and one qutrit for each channel, ln 3 nats, would give $\Omega_m = 0.288$. What would settle the three things that remain open is a model in which a comparer inside its domain receives, through each modal role, the part held by one degree of freedom of the horizon's count.

XIII. The Principle of Indiscernibility

The Passage, Chapter 18, Section II: "The principle is this: a difference that no comparison can register is no difference".

The Passage states the principle with its scope fixed and as an equivalence (Chapter 18, Section II). It counts every possible comparison, by any comparer in any domain, and it identifies two things when every possible

comparison gives each of them the same verdict with every thing whatever; so stated it is an equivalence, which the pairwise reading is not. Under that statement it identifies the ends of the interiors, not the interiors, which a comparer inside each registers apart; and it needs two identifications that the Passage states: that a comparison registers nothing it only predicts, and that the context of the new epoch registers no distinction that the old exterior drew. What remains open is a ground, in the structure of comparison, for those two identifications. If the primordial departures of an epoch were found to have the large-scale spectrum of independent pieces, the identification of the ends would fail (Chapter 18, Section II).

The principle is a verificationist principle: it counts as a difference only what some comparison can register. The Passage owns it as such, and it is of a piece with the Passage's definition of the real (Chapter 5, Section II). The principle of indiscernibility differs from the pairwise identity of indiscernibles. It compares the whole pattern of verdicts of each thing, and it counts comparisons, not properties. Black's two spheres (Black 1952) are two under it, since a comparison that takes one sphere as content and not the other registers a difference between them. The ends of the interiors are one because no comparison takes either of them as content. Hacking (1975) sets out where the classical principle stands.

XIV. The Phase Across Geometries

> *The Passage, Chapter 12, Questions:* "In each, the geometry is correlated with each alternative of the matter, and the alternatives keep their relative phase".

Each alternative of the matter carries its own classical geometry, and the alternatives keep their relative phase (Chapter 12, Section VI). The Passage compares the alternatives in the region far from the bodies, with the phase of each advancing at the rate of the whole energy its geometry carries; to first order in G_N the entangling phase is then the same

whichever single common term is taken, and it is the Newtonian value. Below the modal boundary the Passage adopts the construction of linearized quantum gravity, and it states the three assumptions of that adoption. What remains open is the relative phase where the geometries of the alternatives differ by more than small departures, or share no far region. Penrose (1996) raised the difficulty for every superposition of geometries, small departures included: two geometries supply no common measure of time, and he took the ill-definedness of the comparison, measured by the gravitational self-energy of the difference, as the rate of collapse. Where the departures are small and a far region exists, the Passage's answer to that difficulty is the far region as the common term; where either condition fails the Passage has no construction, and that is what is open. What would settle it is a construction of the phase in that regime, or a ground in the structure of comparison for the far region as the common term.

XV. The Grounds of the Einstein Condition

The Passage, Chapter 10, Section V: "The product structure $M^4 \times \mathbb{C}P^2 \times S^2$ admits a natural Einstein condition on the internal metric".

The ratio of the internal radii, $R_O/R_T = \sqrt{6}$, follows by theorem from the Einstein condition on the internal metric (Chapter 10, Section V). The reading of "The Grid as the Space of Unentangled States" makes the choice of the Einstein condition visible: the metric the embedding induces has Ricci constants in the ratio 3 to 2, not the equality the condition requires. Part of the question is settled. The curvature a single direction has is its Ricci curvature, and equal Ricci curvature in every direction is exactly the Einstein condition, so the ground is exact once the Passage takes the modal boundary to be one curvature for every direction. The Passage states that as its identification. Independence of the directions does not supply it, since independence holds at every ratio of the radii. Distinguishability with one rule for both factors gives the ratio 2; the metric that gives $\sqrt{6}$ weighs each factor once for each of its alternatives, which restates

the condition and does not ground it. What remains open is a ground, from the structure of comparison, for the modal boundary being one and the same curvature for every internal direction.

XVI. The Sign of the Tilt

> *The Passage, Chapter 18, Section IX:* "A spectrum that refers to no length is exactly scale invariant, and a tilt is a reference to a length".

The sentence holds: a dimensionless spectrum that depends on no length is constant, and a tilt needs a length. It does not fix the sign of the tilt. The departure of the metric on a length can depend on a length ℓ only through the ratio of the two lengths, and it may grow toward ℓ or fall toward it. The Passage names two lengths of the new epoch (Chapter 18, Section IX, under spatial flatness). One is the length of the turn, of the order of the Planck length, which lies below every observed length. The other is the de Sitter radius, which lies above them. Suppose the departures were largest on the length they refer to. Then the length of the turn would give an index above one, against the measured 0.965 ± 0.004 (Akrami and colleagues 2020) and 0.974 ± 0.003 (Louis and colleagues 2025), and the de Sitter radius would give an index below one. The Passage takes neither step (Chapter 18, Section IX).

Inflation shows that the length does not fix the direction. Inflation with a quadratic potential (Ade and colleagues 2016, Section 6.2) refers its departures to a short length, the horizon at the end of inflation. Its power grows away from that length, as the square of the logarithm of the ratio of the two lengths. With N that logarithm, about 60, its index is about $1 - 2/N$ and its running about $-2/N^2$. A reference to a short length thus gives a tilt below one there. A rule that the departures are largest on the length they refer to gives a tilt above one for the same length.

What would settle the sign is a statement, from the restoration of the exterior factor, of two things: which length the departures refer to, and whether the departure of the metric grows or falls toward it. The running

would then test it. Take a tilt below one and a reference that weakens as the logarithm of the ratio grows. Then the running is negative if the length lies below the observed lengths and positive if it lies above them. Its size is the departure of the index from one divided by the logarithm of the ratio of the two lengths: about 6×10^{-4} for the length of the turn and about 4×10^{-4} for the de Sitter radius (Chapter 18, Section IX). The measured running is -0.0045 ± 0.0067 (Akrami and colleagues 2020) and $+0.0062 \pm 0.0052$ (Calabrese and colleagues 2025).

XVII. The Count of Findings

The Passage, Chapter 12, Section V: "It curves where findings can be made, and in proportion to how many can be made there".

The Passage identifies the source of the curvature as the count of findings that can be made where the content is, and it gives the reason that energy and momentum measure that count: every finding is an interaction, and energy and momentum are what an interaction exchanges (Chapter 12, Section V). The identification is one of concepts. What remains open is a count of findings, made without geometry, that returns the energy of a content directly, so that the proportion of curvature to the count is shown as an identity of numbers as well as an identity of concepts. Jacobson's derivation of the Einstein equation from the proportion of entropy to the area of every local horizon (Jacobson 1995) is one route from a count to curvature ("What the Passage Obtains That Others Assume", Section II). What would settle the question is a construction in which the count of findings where content is, and the count of states behind a local horizon, are shown to be one count or two.

Where This Book Meets the Passage

For each chapter and appendix of the Passage, the places in this book that cite it, by chapter and section. The list shows the one-way dependence: every entry is a place where this book draws on the Passage.

Chapter 1, Comparison as the Ground of Science: "Two Accounts of Causation", Sections IV and V; "One Mass, Three Elevators", opening, Section IV.

Chapter 2, The Six Intrinsics and the Four Compounds: "Why the Balance Criterion Holds", opening; "Open Questions as Research", Section X.

Chapter 3, The Incommensuration Theorem: "Other Programs That Obtain General Relativity", Section VI; "What the Passage Obtains That Others Assume", opening, Sections II and IV; "Comparisons at Single Steps", Section IV; "Why the Balance Criterion Holds", opening, Section II, Questions; "The Categorical Form of the Incommensuration Theorem", opening, Section III, Questions; "The Modal Readings of the Inertial Forces and of Free Fall", opening, Section V, Questions; "Two Formal Triples of a Domain", Section III.

Chapter 4, The Modalities and the Three Axioms: "The Cells of $\mathbb{C}P^2$ and the Six Metrics", Section III; "The Modal Readings of the Inertial Forces and of Free Fall", opening; "One Mass, Three Elevators", Section I; "Two Formal Triples of a Domain", opening, Section III.

Chapter 5, Reason, the Real and the Abstract: "Two Accounts of Causation", opening, Sections I, II, IV and V; "Two Formal Triples of a Domain", opening, Section III; "Open Questions as Research", Section XIII.

Chapter 6, The Content/Context Partition: "Other Programs That Obtain General Relativity", Sections VII and IX, Questions; "The Modal Readings of the Inertial Forces and of Free Fall", opening, Questions; "One Mass, Three Elevators", Section III.

Chapter 7, The Six Metrics: “Other Programs That Obtain General Relativity”, Section VI; “What the Passage Obtains That Others Assume”, Section III; “The Cells of $\mathbb{C}P^2$ and the Six Metrics”, opening, Section II; “The Grid as the Space of Unentangled States”, opening; “One Mass, Three Elevators”, opening, Section IV.

Chapter 8, The Product Structure $M^4 \times \mathbb{C}P^2 \times S^2$: “Other Programs That Obtain General Relativity”, opening, Sections VII and X; “What the Passage Obtains That Others Assume”, opening, Sections I, II and III; “Comparisons at Single Steps”, Sections I and II.

Chapter 9, The Internal Space and Its Uniqueness: “Other Programs That Obtain General Relativity”, opening, Section VIII; “What the Passage Obtains That Others Assume”, opening, Sections I, III and IV; “Comparisons at Single Steps”, Sections I and II; “The Categorical Form of the Incommensuration Theorem”, Section IV; “The Cells of $\mathbb{C}P^2$ and the Six Metrics”, opening, Section IV, Questions; “The Spectral Invariants of $\mathbb{C}P^2 \times S^2$”, opening; “The Grid as the Space of Unentangled States”, opening, Sections II and III, Questions; “Open Questions as Research”, Section IX.

Chapter 10, Why Four Dimensions, Why Lorentzian: “Other Programs That Obtain General Relativity”, opening, Sections VI, VIII and X; “What the Passage Obtains That Others Assume”, opening, Sections I, II and III, Questions; “Comparisons at Single Steps”, Section III; “Refutation Conditions Named in This Book”, Section I; “The Cells of $\mathbb{C}P^2$ and the Six Metrics”, Section II; “The Modal Readings of the Inertial Forces and of Free Fall”, Section II; “Open Questions as Research”, Sections X and XV.

Chapter 11, The Kaluza–Klein Reduction: “Other Programs That Obtain General Relativity”, opening, Sections VII, VIII and X; “What the Passage Obtains That Others Assume”, opening, Section I, Questions; “Comparisons at Single Steps”, Section IX; “Open Questions as Research”, Section IX.

Chapter 12, The Einstein Field Equations: “Other Programs That Obtain General Relativity”, opening, Sections VI, VII, VIII, IX and X, Questions; “What the Passage Obtains That Others Assume”, opening, Sections I, II and IV, Questions; “Comparisons at Single Steps”, Sections I, V and VI; “Refutation Conditions Named in This Book”, Section I; “The Modal Readings of the Inertial Forces and of Free Fall”, Questions; “One Mass, Three Elevators”, opening, Sections IV and V; “Two Formal Triples of a Domain”, opening, Section II; “Open Questions as Research”, Sections XIV and XVII.

Chapter 13, Λ as a Prediction; Information Cost at the Horizon: “Other Programs That Obtain General Relativity”, opening, Sections V and X; “What the Passage Obtains That Others Assume”, opening, Sections II and IV, Questions; “Comparisons at Single Steps”, Sections VII and VIII; “Refutation Conditions Named in This Book”, Section I; “The Cells of $\mathbb{C}P^2$ and the Six Metrics”, Questions; “Two Formal Triples of a Domain”, opening, Section II; “Open Questions as Research”, Sections I, II and XII.

Chapter 14, The Equivalence Principle as a Triple: “The Modal Readings of the Inertial Forces and of Free Fall”, opening, Section III, Questions; “One Mass, Three Elevators”, opening, Sections I and VI.

Chapter 15, Geodesic Motion: “What the Passage Obtains That Others Assume”, opening, Section I; “The Modal Readings of the Inertial Forces and of Free Fall”, opening, Section V, Questions.

Chapter 16, Recovery of Classical Physics: “Other Programs That Obtain General Relativity”, Section VII; “Refutation Conditions Named in This Book”, Section I; “The Modal Readings of the Inertial Forces and of Free Fall”, Questions.

Chapter 17, The Planck Scale as Modal Boundary: “What the Passage Obtains That Others Assume”, Section I; “Comparisons at Single Steps”, Sections V and IX; “Two Formal Triples of a Domain”, Section III; “Open Questions as Research”, Sections III and XI.

Chapter 18, The Interior Passage: "What the Passage Obtains That Others Assume", Questions; "Comparisons at Single Steps", Sections III, VII, VIII and IX; "Refutation Conditions Named in This Book", Section I; "One Mass, Three Elevators", opening, Sections I, II and VII; "Two Formal Triples of a Domain", opening, Section III, Questions; "Open Questions as Research", Sections I, IV, V, VII, VIII, XIII and XVI.

Chapter 19, What Replaces Singularities: "Comparisons at Single Steps", Section IV; "Two Formal Triples of a Domain", opening, Section III; "Open Questions as Research", Sections VII and XI.

Chapter 20, The Complete Chain: "Comparisons at Single Steps", Section VIII; "Refutation Conditions Named in This Book", opening; "Open Questions as Research", Section III.

Appendix A, The Argument for the Content/Context Partition: "The Categorical Form of the Incommensuration Theorem", Section II, Questions; "The Modal Readings of the Inertial Forces and of Free Fall", Section III; "One Mass, Three Elevators", opening, Sections III, IV and VII, Questions.

Appendix B, The Dimensional Schema on Approach to the Singularity: "Comparisons at Single Steps", Section IX; "Open Questions as Research", Sections V, VI, VII and VIII.

Appendix C, A Parallel Derivation of the Dimensions of Context: "Other Programs That Obtain General Relativity", Section VI; "The Cells of $\mathbb{C}P^2$ and the Six Metrics", opening, Section III; "The Grid as the Space of Unentangled States", opening; "Open Questions as Research", Section X.

Appendix D, The Floor of the Metric at First Order: "Comparisons at Single Steps", Section IX; "The Spectral Invariants of $\mathbb{C}P^2 \times S^2$", Section VI; "Open Questions as Research", Section IX.

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