

The Bucket and the Stars

The Immanent Metaphysics
for Readers of Brian Greene

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What Greene Showed, and Why Read Another Book

A young physicist named Brian Greene began graduate school at Oxford University in October 1984. If you have read any of Greene's four books, you may fairly ask why you would want another book about the world Greene describes. The answer starts with what Greene found at Oxford in 1984. At Oxford, Greene found the hope of a single theory of every force and every piece of matter. Greene's four books have followed the hope of a single theory ever since.

Oxford, October 1984

Greene tells the story of his first months at Oxford himself, in *The Elegant Universe*, chapter 6, section "A Brief History of String Theory": "I began graduate school at Oxford University in October 1984". Greene had come to Oxford to learn the great theories of physics, and he was excited to be learning them. Yet Greene found a gloomy mood among the older graduate students: a feeling that "there was little or no future for particle physics". *Particle physics* is the study of the smallest pieces of matter and of the forces that act between them. Physicists know of four forces: gravity, the electric and magnetic force, the strong force and the weak force. The strong and the weak force act within the nucleus, the tiny core of an atom. The *Standard Model* is the theory of the known particles and of the three forces other than gravity. The older students were gloomy because the Standard Model had succeeded. The Standard Model was in place. The Standard Model agreed with experiment so well that checking it further seemed, to the older students, a matter of time and detail. The next task was to go beyond the Standard Model and bring gravity into one theory with the other three forces. Few physicists would take on the next task, because the task seemed so daunting.

String theory is the proposal that the smallest pieces of matter are not points but tiny vibrating strands of energy, called strings. *Quantum mechanics* is the theory of atoms and of the particles smaller than atoms. Physicists in the late 1970s and early 1980s had found string theory in a subtle conflict with quantum mechanics. In 1984 the physicists Michael Green and John Schwarz showed that the conflict could be resolved, and that string theory, once repaired, was broad enough to include all four forces and all of matter. Within months, news of the result reached even the first-year graduate students at Oxford, Greene among them. Greene writes, in the section "A Brief History of String Theory": "But six months later the mood had swung completely around". Greene and his fellow students worked deep into the night to master the physics and the mathematics that string theory needed. In the spring of 1985, a young physicist at Oxford suddenly saw a single theory of the whole world, of every force and every piece of matter, as close to being found.

All four of Greene's books follow the hope of a single theory of every force and every piece of matter. *The Elegant Universe* (1999) tells the story of string theory, the theory that had seemed close to being found. *The Fabric of the Cosmos* (2004) asks what space and time are, and why time runs one way. *The Hidden Reality* (2011) follows the theories of physics, string theory and quantum mechanics among them, to the idea that our universe may be one of many. *Until the End of Time* (2020) follows the universe forward, from its beginning through life and mind to its far future. *Until the End of Time* also asks where meaning is found in the universe. Across more than twenty years, Greene widened the hope: from one theory of the forces to one explanation of everything, ourselves included.

What Greene Showed

Greene's message across the four books can be told in eight steps, each step in Greene's own terms.

The first step is a conflict between the two theories on which physics rests. *General relativity* is Einstein's theory of gravity. In general relativity, gravity is the shape of space and time, the way space and time are bent from one place to the next. General relativity describes stars, galaxies and the universe as a whole. The second theory is quantum mechanics, which describes molecules, atoms and the particles smaller than atoms. Greene writes that experiments have confirmed nearly every prediction of each theory "to almost unimaginable accuracy". Yet Greene states a conclusion in the opening pages of *The Elegant Universe*: "As they are currently formulated, general relativity and quantum mechanics cannot both be right" (chapter 1, opening pages, before the section "The Three Conflicts"). The two theories conflict where something is both tiny and massive at once. A *black hole* is a region whose gravity is so strong that nothing comes back out of it, not even light. Something is both tiny and massive deep inside a black hole. Something is also both tiny and massive at the *big bang*, the hot, dense and expanding state in which the history of our universe begins. The conflict shows up at distances below the *Planck length*, which is about a millionth of a billionth of a billionth of a billionth of a centimeter, far smaller than the nucleus of an atom. Greene shows how the conflict is resolved in string theory. Nothing made of strings can probe the distances shorter than the Planck length, where the conflict arises, because a string is about a Planck length long, whereas a point has no length at all. In string theory, all matter and all forces also arise from one ingredient, the vibrating string.

The second step is a count of dimensions. A *dimension* is an independent way in which things can differ: we know three dimensions of space (left and right, forward and back, up and down) and one dimension of time. Greene shows that string theory needs more dimensions than four:

"For string theory to make sense, the universe should have nine space dimensions and one time dimension, for a total of ten dimensions" (*The Elegant Universe*, chapter 8, section "More Dimensions and String Theory"). Greene pictures a dimension curled up too small to see as a garden hose stretched across a canyon (chapter 8, section "Kaluza's Idea and Klein's Refinement"). From far away, the hose looks like a line. Up close, the hose shows a second dimension, a small circle curled around the hose. In string theory, three of the nine space dimensions are the ones we see, and the other six are curled up like the circle around the hose, too small to notice. The six dimensions are curled into *Calabi-Yau shapes*, a family of six-dimensional shapes named for the mathematicians Eugenio Calabi and Shing-Tung Yau. For years there were five versions of string theory. In the spring of 1995 the physicist Edward Witten gave strong evidence that each version turns into another version when the strength with which the strings of the starting version interact is turned up. Greene writes that Witten's result means that "the five string theories are not different after all" (*The Hidden Reality*, chapter 5, section "Duality"). When two descriptions look different and describe the same physics, the match between the two descriptions is a *duality*. The dualities show that the five versions are parts of one theory, called *M-theory*. M-theory has eleven dimensions, ten of space and one of time.

The third step is about space and time themselves. Observers moving relative to one another measure different distances and different durations. Observers in relative motion also disagree about which distant events happen at the same moment. *Spacetime* is space and time joined into one four-dimensional whole. Greene pictures spacetime as a loaf of bread that each observer slices into moments at a different angle (*The Fabric of the Cosmos*, chapter 3, section "Angling the Slices"). The loaf itself, the whole of spacetime, is the same for every observer. Greene concludes that the most straightforward reading of Einstein's general relativity is that "spacetime is a something", although the issue is still debated (chapter 3, section "Spacetime in the Third Millennium"). On the most straightforward reading of general relativity, spacetime is a thing in its

own right, and spacetime is the standard for deciding whether a body is speeding up or turning. Greene takes his argument about spacetime one step further in chapter 5, section "The Persistent Illusion of Past, Present, and Future". Suppose that reality is what lies on your slice of "now", and that your slice has no more claim to "now" than the slice of a far observer who is free to move. From the two suppositions, Greene argues that past, present and future are all equally there: "The total loaf exists".

In Greene's sense, the universe is *local* if you can directly affect only things that are next to you. The fourth step is that the universe is not local in Greene's sense. The physicist John Bell showed how experiments could test locality, in Greene's sense, against quantum mechanics, and the physicist Alain Aspect and others ran Bell's tests on pairs of particles prepared together and measured far apart. The outcome for each particle of a pair is random alone, yet the outcomes for the two particles agree with each other in a pattern, and Bell showed that no instructions carried separately by each particle could give that pattern. Greene concludes that "the universe is not local" (*The Fabric of the Cosmos*, chapter 4, section "No Smoke but Fire").

The fifth step is the direction of time. An egg that falls and splatters never gathers itself up again, though the laws of physics allow the reverse. Greene explains the *arrow of time*, the one-way direction from past to future, by *entropy*. In Greene's sense, entropy is the number of ways the parts of a thing can be rearranged without changing how the thing looks overall. Greene reasons that, since entropy tends to grow, there must once have been very little entropy: "The ultimate source of order, of low entropy, must be the big bang itself" (*The Fabric of the Cosmos*, chapter 6, section "Entropy and Gravity"). *Inflation* is a brief but enormous burst of expansion of space in the earliest moments of the universe. Greene then tells how inflation can produce a start with very little entropy. Greene judges that, with our current understanding, the inflationary explanation "is the most complete explanation for time's arrow that has been given" (chapter 11, section "Inflation and the Egg"). In the next section, "The Fly in the Ointment?", Greene names the assumption on which the inflationary ex-

planation rests: what conditions were like before inflation began. Greene calls the assumption about conditions before inflation "a pivotal assumption that's as yet unjustified".

The sixth step is that our universe may be one of many. A *multiverse* is a collection of universes of which ours would be one. In *The Hidden Reality*, Greene traces "nine variations on the multiverse theme" (chapter 1, section "Varieties of Parallel Universes"). Two of the nine variations are copies of our region repeating across an infinite space, and bubble universes made by an inflation that never ends. Two more are universes on separate slabs of space, floating in a space of more dimensions, and universes simulated on a computer. Another variation is the many worlds of quantum mechanics, in which every possible outcome of a measurement happens, each in a universe of its own. Greene calls the subject of parallel universes "highly speculative" (chapter 1, section "The Cosmic Order"), and in the same section he writes: "No experiment or observation has established that any version of the idea is realized in nature". Greene also writes, in "The Cosmic Order", that the proposals he takes seriously "emerge unbidden from the mathematics of theories developed to explain conventional data and observations".

The seventh step is about life and mind. In *Until the End of Time*, Greene calls entropy and evolution "two forces sharing the role of leading character" (chapter 1, section "Stories of Nearly Everything"). Greene holds that living things follow the laws of physics and that nothing overrides those laws: "Life is physics orchestrated" (chapter 4, section "The Physics of Information"). Greene expects that consciousness will one day be explained by the particles of matter and the laws that govern the particles, with no new ingredient (chapter 5, section "Consciousness and Storytelling"). In the traditional sense, free will would be the power of a mind to rise above the laws that govern the mind's particles, and Greene holds that no mind has that power. Greene holds at the same time, in his own words, that "We do make choices" (chapter 5, section "Relevance,

Learning, and Individuality"). In the section "Relevance, Learning, and Individuality" Greene also writes that "the sensation of free will is real", and that the capacity to rise above the laws of physics is not.

The eighth step is the far future. The stars run out of fuel, and the black holes slowly glow away. Greene pictures the timeline of the universe as the Empire State Building, with each floor standing for ten times as many years as the floor below it (*Until the End of Time*, chapter 9, section "An Empire of Time"). Greene says that our understanding is too coarse for a precise prediction, and he sets aside whatever intelligent life might do to change how the stars, the galaxies and the expansion of space develop. Greene estimates that thinking may end within a span of years written as a one followed by fifty zeros. Greene makes the estimate by putting rough numbers into the equations. Greene also assumes that the speeding up of the expansion of space does not slow. The fiftieth floor of Greene's building stands for a span of years written as a one followed by fifty zeros, and by then, in Greene's words, "the universe may very well have hosted its final thought" (chapter 9, section "A Final Thought on Thought"). Greene states his conclusion at the start of *Until the End of Time*: since nothing is permanent, the only answers of significance in the search for value and purpose "are those of our own making" (chapter 1, section "Reflections on the Future").

Four of Greene's pictures appear in the eight steps: the garden hose, the loaf of bread sliced at an angle, the egg that falls and splatters, and the Empire State Building, whose floors stand for the ages of the universe.

A Second Argument About the Same World

Greene's argument starts from the theories of physics. Greene uses the theories of physics to explain the world. I set a second argument beside Greene's argument. The second argument starts from comparison, the act by which anything at all is observed, measured or tested. The second argument is mine. I am Forrest Landry. The Immanent Metaphysics is my own work: I first set it out in my book *An Immanent Metaphysics* (2001),

and I work out its physics, from the act of comparison, in a series of later books, every one of them mine, which Appendix B, For the Specialist, names chapter by chapter, with the place in each book that every step draws on. In this book "I" means me, the writer of those books, except in a question asked in the reader's words (as some chapter titles are).

Every observation, every measurement and every test is a comparison: the act of finding whether two things are the same or different, in some respect. So a *comparison* is the act of finding whether two things are the same or different, in some respect. Take two readings of a thermometer, one in the morning and one in the evening. When the two readings are set beside each other and found to differ, that act is one comparison. The Immanent Metaphysics is the study of what every comparison must contain, whatever is compared and whatever does the comparing. The word *immanent* in the name means within: the study is worked out from inside the act of comparison, taken as the act is happening, and never from a place outside the world from which the world could be looked at. The *ground* of an argument is what the argument starts from: the things taken as given in the argument. The ground of the Immanent Metaphysics is only what every comparison must contain, so the Immanent Metaphysics takes no measurement, no law of physics and no result of any experiment as given. My books build their physics on the Immanent Metaphysics, and I teach the physics of my books through Greene's message, one step of Greene's message at a time.

Greene's argument and mine start from separate grounds: his from the theories of physics, mine from what every comparison must contain. So a result that follows from both arguments is supported by two arguments, and neither argument starts from the other. Many results follow from both arguments. One result that follows from both is about gravity. Greene tells how Einstein understood that "gravity itself must be nothing but warps and curves in the fabric of spacetime" (*The Fabric of the Cosmos*, chapter 3, section "Warps, Curves, and Gravity"). In this book the *arena* is space and time taken together, as the setting in which everything that is measured is measured. Einstein's result follows from three things gravity does. Gravity

acts on everything, nothing shields anything from gravity, and all bodies fall alike under gravity. The three things gravity does are three of the four marks (properties) that the arena must carry, as the setting of every measurement. In a body, the arena takes hold of the degree to which the body takes part in the arena. To take part in the arena is to be found in the arena, somewhere. The fourth mark is that the degree to which a body takes part in the arena can take any value in its range. The degree comes in no steps. Chapter 1 sets out all four marks. No force of particle physics carries any of the four marks. So gravity is the shape of the arena, and not a force that acts within it.

On some questions the results of the two arguments differ, and for each such question the chapter about the question states the result in my account, and the reason for the result. For example, string theory and my account both count ten dimensions. The world has ten dimensions: one of time, three of space, and six of possibility. *Possibility* is the range of what could be found in a comparison: every outcome the comparison could have. The six dimensions of possibility hold the range of outcomes. Things travel through the three dimensions of space, but nothing ever travels through the six dimensions of possibility. In string theory the six dimensions beyond the familiar four are instead dimensions of space, curled up too small to see. So the two counts of ten dimensions are counts of different things.

Who This Book Is For

If you have read any one of Greene's four books, or all of them, this book is written for you. You need no physics beyond what you met in school. Every idea is shown to you before it is named. Every new word is explained where you first meet it. If you open the book at a later chapter, the box at the head of that chapter lists the words the chapter takes from earlier chapters, with the chapter that teaches each one, and the Glossary explains every one of them.

How This Book Is Built

The title of every chapter but the last is a question that a reader of Greene carries away from Greene's pages. Each chapter first states what Greene showed, in his words, with the chapter and section where he shows it, and then shows where Greene's result also follows from my ground, and states that ground. Where a chapter retells what Greene showed, the retelling stands in a shaded box headed "From Greene's pages": a reader who knows Greene's books can pass over the boxes, and a reader new to Greene's books can read the boxes first. Where the result in my account differs from Greene's, the chapter states the result in my account, and the reason for the result. Measurements already made are told as what was done, what was found, and what we now know. Near the end of each chapter, the chapter states what we now know. Each chapter but the last ends on the question that the next chapter is about. Under each chapter title, one line names the chapters of Greene's books that the chapter discusses, and at the back of the book a table called Reading Beside Greene sets every chapter of this book beside those chapters of Greene's books. At the back of the book, a list called Where to Go from Here names a selection of my books to read next, Part by Part, and says what the reader gains from each book.

Here, in brief, is what this book holds that Greene's books do not: space and time as the arena, a something whose shape is gravity and no container (Chapter 1); the whole loaf existing as structure, with what is real given in an act of comparison, here and now (Chapter 2); one world described by two theories joined from one source, with ten dimensions of three kinds, six of them forming one small shape (Chapters 5 to 7); the direction of time in every comparison (Chapter 8); the turn as the source of the first ripples, and the small constant as the share of the vacuum's energy that a measurement made from inside takes in, in one world (Chapters 9 to 11); consciousness identified with three features of an act of comparison taken whole (Chapter 14); choosers who could have

chosen otherwise, with the laws as they are (Chapter 15); a far future that ends in a turn (Chapters 16 to 19); and meaning present wherever comparison occurs (Chapter 21). If you have read another of my books written for the readers of another author, you know the ground already: comparison, its content and its context, the four characters and the Incommensuration Theorem, which I teach again here in Chapters 1 and 3 with a thermometer read morning and evening, a dimmer, a light switch, a square turned a quarter round and a rocky coast. Everything else in this book is set beside Greene's own pictures, beginning with his bucket in Chapter 1. Read the chapters in order, because each chapter builds on the ones before it. Where a chapter uses a word or a result that an earlier chapter teaches, a short box at the head of the chapter lists the word or result, with the chapter that teaches it.

The chapters are gathered into seven Parts.

Part One, Space and Time, follows *The Fabric of the Cosmos* and the early chapters of *The Elegant Universe*, and asks whether space is a something and whether the whole of time is laid out at once. Part Two, The Quantum, asks whether the universe is local, and how a single outcome happens when quantum mechanics gives only the chances of many. Part Three, One Theory?, follows *The Elegant Universe* and the string chapters of the other books. Part Three asks why relativity and quantum mechanics cannot both be right as they stand, whether there is one theory of everything, and why there are ten dimensions. Part Four, The Arrow and the Beginning, asks why time has a direction, what banged, and why the energy of empty space that speeds the expansion of the universe is so small. Part Five, One Universe or Many?, follows *The Hidden Reality* and asks whether our universe is the only one. Part Five also asks whether the worlds of the quantum, of the hologram (Greene's picture of a universe mirrored exactly on a distant boundary) and of the computer are other worlds. Part Six, Mind in a Universe Running Down, follows *Until the End of Time*, and asks how a universe whose store of usable energy keeps running down builds stars and us, and whether consciousness is only particles. Part Six also asks whether you choose if the laws of physics gov-

ern everything your particles do, as Greene holds they do. The title of Part Seven is Until the End of Time, and After. Part Seven follows Greene's far future and goes on past Greene's last page. Part Seven asks whether the sky will go dark, whether a mind can keep thinking forever, and what is left when the black holes are gone. Part Seven asks whether the long emptying of the sky is the end: the emptying goes on until no stars and no black holes are left, and only a thin mist of particles drifts through space. Last, Part Seven asks whether anything of us goes on and where meaning comes from. A closing chapter, What Do I Now Hold?, gathers the results this book adds to Greene's four books.

Appendix B, For the Specialist, gathers the mathematics with every exact value and symbol. The main line of the book does not need the mathematics. Physicists are now measuring how matter and *antimatter* (matter whose particles carry the opposite charges) fall, and how the light left from the early universe is patterned. Physicists are also measuring whether the energy of empty space that speeds the expansion keeps one value in each cubic meter as the universe ages. Appendix A, What the Coming Measurements Will Find, gathers each measurement now under way with what I say the measurement will find. You may skip both appendices, because the chapters can be read without the appendices.

Where the Book Begins

When one result follows from both Greene's argument and mine, the result rests on both grounds at once. When the two arguments differ on a result, the difference shows you the ground each argument starts from. Greene's four books follow one hope, a single explanation of the whole world, from strings far smaller than an atom to the last thought in an emptying sky. We now know what this book does. Beside Greene's four

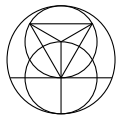
books I set a second argument, which starts from the act of comparison, and I show where the two arguments agree on a result, and where and why the two arguments differ.

In *The Fabric of the Cosmos*, Greene asks what space is. Greene begins with a bucket of water hung from a twisted rope and let go. When the water turns and climbs the walls of the bucket, what is the water turning against?

PART ONE

Space and Time

Beside *The Fabric of the Cosmos*, Parts I and II, and *The Elegant Universe*, chapters 2 and 3.



Chapter 1

Is Space a Something?

Beside The Fabric of the Cosmos, *chapters 2 and 3, with passages from chapters 12, 14 and 16, and* The Elegant Universe, *chapter 3.*

In 1687 the English physicist Isaac Newton described a bucket of water in his book the *Principia*. The bucket hangs from a twisted rope and is let go. As the water in the bucket turns, the water climbs the walls of the bucket. When the bucket spins and the water climbs the walls, what is the water spinning against: space itself, or something else?

Newton set the experiment down in a few lines, in a note near the start of the *Principia*, his great book on motion and gravity, printed in London in 1687. In Newton's note, a vessel is hung by a long cord and turned until the cord is tightly twisted. The vessel is filled with water and held still, and then the vessel is let go. As the cord unwinds, the vessel turns. At first the water does not turn with the vessel, so the surface of the water stays flat. Little by little the turning passes from the walls to the water, until the water turns with the vessel, draws back from the middle and climbs the sides into a hollow. Newton adds, in a short parenthesis, that he had tried the experiment himself: "as I have experienced". "This ascent of the water", Newton wrote, "shows its endeavour to recede from the axis of its motion".

You can try Newton's experiment with a pail and a length of rope. Twist the rope, let go, and watch. At first the pail spins around the water, and the water stays still and flat. Then the water catches up with the pail,

and the surface of the water sinks in the middle and rises at the rim. Now the water and the pail turn together, so the water is not turning against the pail at all. Yet the hollow in the surface shows that the water turns.

The physicist Brian Greene begins his story of space with Newton's bucket, in his book *The Fabric of the Cosmos*. Greene returns to the bucket again and again, and I use the bucket again and again too. In this book the bucket is the picture of one thing throughout: a comparison made from inside the world. A comparison made from inside the world is made by things that are part of the world, set against other things in the world, and never from a place outside the world. The hollow in the water is found by a comparison made from inside the world: the turning of the water is measured against the other things there are to compare the water with.

From Greene's pages

Three Hundred Years of a Bucket, as Greene Tells It

Greene begins with what he calls a natural suggestion: that the water turns against the bucket, and that the turning against the bucket makes the hollow. Newton argued that the experiment shows the natural suggestion to be wrong. At the start, the bucket turns against the water, and the surface of the water is flat. Later the bucket and the water turn together, and the surface is hollow. So the surface is flat while the water turns against the bucket, and hollow when the water does not turn against the bucket. Greene then takes Newton's experiment "one small step further": when the rope twists the other way and stops the bucket for a moment, the water still turns, and the surface of the water is still hollow. Greene concludes: "this shows conclusively that the relative motion cannot explain the surface's shape" (*The Fabric of the Cosmos*, chapter 2, section "The Bucket"). In Greene's sentence, the relative motion is the motion of the water against the bucket.

Newton's answer was *absolute space*: a space standing behind everything, the same everywhere, which nothing acts on and which is there whether or not anything is in it. On Newton's view, the water

turns against absolute space, and the hollow in the surface of the water shows the turning against absolute space. Greene then tells the view opposed to Newton's view. The philosopher Gottfried Leibniz, Newton's contemporary, held that space is nothing over and above the relations of things to one another, as an alphabet is nothing over and above the order of its letters. About two centuries later the physicist Ernst Mach took up the bucket again. For Mach, the water turns against the rest of the matter in the universe: the Earth, the Sun and the distant stars. Mach also held that in a universe with nothing else in it, there would be no difference between spinning and not spinning. Greene gives the reason the debate between Mach and Newton stalled: Mach never said how the distant matter would act on the water, so Mach's idea was hard to investigate with any precision. Greene also tells that when he learned of Mach's ideas as an undergraduate "they were a godsend". He adds that "I felt sure Mach's had to be the answer" (same chapter, section "Mach vs. Newton").

From Greene's pages

Einstein's Answer, as Greene Tells It

Greene brings in the physicist Albert Einstein in his next chapter. In 1905 Einstein put forward *special relativity*, his theory of motion at a steady speed in a straight line and of light. In special relativity, as in the physics before it, steady motion has meaning only against something else. Special relativity also showed that observers moving steadily against one another disagree about lengths and durations. Observers moving steadily against one another also disagree about which distant events happen at the same moment. Each observer moving steadily against the others slices space and time into moments in a different way. Greene shows that the whole that the observers slice is the same for every one of them, and he calls that whole *spacetime*: space and time taken together as one whole. Greene writes: "according to

special relativity, absolute spacetime does exist" (*The Fabric of the Cosmos*, chapter 3, section "The Bucket, According to Special Relativity"). All observers moving steadily against one another also agree on whether the path of a body through spacetime is straight, and a body whose path through spacetime is straight is not speeding up, slowing or turning. Greene's *benchmark* is the standard against which a motion is judged to be accelerated: speeding up, slowing or turning. So Greene finds in spacetime the standard that the bucket needs: "Spacetime, not space alone, provides the benchmark" (same section).

In 1915 Einstein went further, with *general relativity*, his theory of gravity. Greene tells how Einstein got from special relativity to general relativity by way of the *equivalence principle*: within a small enough room, the push you feel from gravity cannot be told apart from the push you feel when the room speeds up. Greene shows that a spinning bucket gives a push like the push of gravity too: the outward press against the wall of the bucket can take the place of the weight felt on Earth (chapter 3, section "The Equivalence of Gravity and Acceleration"). From the equivalence principle Einstein went on to explain gravity itself, and Greene gives Einstein's result in one sentence in his earlier book, *The Elegant Universe*: "Gravity, according to Einstein, is the warping of space and time" (*The Elegant Universe*, chapter 3, section "Acceleration and the Warping of Space and Time"). Greene calls what is warped the *fabric of the cosmos*, his name for space and time as the setting of everything. Einstein also found that changes in the warping of space and time travel at exactly the speed of light, so in Einstein's theory gravity no longer acts across a distance at once, as gravity did in Newton's theory.

What does general relativity say about the bucket? Greene answers in two parts. He gives the answer as his own judgment of a question on which "there is still some controversy" (*The Fabric of the Cosmos*, chapter 3, section "General Relativity and the Bucket"). In the first part, general relativity is like Mach's view: the standard of not speed-

ing up and not turning at any place is shaped by all the matter in the universe, near and far. In the second part, general relativity is unlike Mach's view: "The conclusion we draw is that even in general relativity, empty spacetime provides a benchmark for accelerated motion" (same section). Greene gives his verdict on the three centuries of debate from Newton to Einstein: although the issue is still debated, on the most straightforward reading of Einstein "spacetime is a something" (same chapter, section "Spacetime in the Third Millennium"). A *field* is something that has a value at every place. The wind is one example, since the wind has a strength and a direction at every place across a country. On Greene's reading, the gravitational field is the something that spacetime is. The gravitational field is as changeable as matter. A relationist, like Leibniz and Mach, holds that space is nothing over and above the relations among things. Greene adds that his conclusion, that spacetime is a field as changeable as matter, is "cause for celebration among supporters of a more broadly defined relationist outlook". In Greene's telling, acceleration measured against a changeable field is relational in that broader sense. Later in *The Fabric of the Cosmos*, Greene describes an effect in Einstein's theory that, he writes, "would have pleased Mach no end": a turning body drags space around as the body turns (chapter 14, section "Einstein in Drag"). Greene ends *The Fabric of the Cosmos* with a guess at a theory from which space and time would arise out of more basic ingredients. Greene joins a hope to the guess: "If this is how it turns out, Leibniz, Newton, Mach, and Einstein could all claim a share of the victory" (chapter 16, section "The Constituents of Spacetime").

Every Finding Is a Comparison

My starting point is again the water in Newton's bucket. The hollow in the water is an observed fact: the hollow is there in the shape of the surface, for anyone to see. To find anything at all is to make a comparison: to find whether two things are the same or different, in some respect. Think

of a thermometer read in the morning and again in the evening. The two readings are set side by side and found to differ: the evening is warmer. What is found in a comparison is its *content*: for the two readings, the content is that the evening is warmer. The respect in which the content is found is the *context* of the comparison: what makes the two things compared two, as the two times of day make the two readings two. A *finding* is the outcome of one act of comparison, laid down as a record: the reading written down, or the hollow in the water. A *comparer* is whatever makes a comparison and registers the content of the comparison. The word "comparer" names a role, and in this book the word never means a mind. A gyroscope is a spinning wheel on a mount, and the axis of the wheel keeps pointing one way. A gyroscope is a comparer while the gyroscope holds its direction.

A state of motion is a content, and what it is measured against is its context. In the bucket, the hollow is the content: the water is turning, and not still. What the turning is measured against is the context: the standard of what counts as not turning. Newton said that the standard of not turning is absolute space, while Mach said that the standard is the matter of the universe. Which is the standard of not turning: absolute space, or the matter of the universe?

One Spacecraft, Then Two

Picture an empty universe with a single spacecraft in it and nothing else at all: no stars, no dust, no light. Is the ship spinning? Nothing in the ship can tell whether the ship is spinning, because there is nothing outside the ship to tell by. Now put a second ship in the same empty universe. Each ship can now turn against the other ship. The rest or the turning of each ship is a relation of that ship to the other ship, because the other ship is the only thing there is to compare each ship against.

The two ships show a principle that every chapter of this book uses. A difference that no comparison can register is no difference: every distinction is made by comparison, and a distinction that no comparison can

make is not a distinction. I call the principle the *principle of comparison*. When the principle of comparison is applied to turning, the principle says that where nothing is there to compare against, no comparer can tell one standard of not turning from another. So only matter within reach fixes a standard of not turning: matter that can still be compared against.

In our universe there is plenty of matter within reach. A gyroscope keeps pointing at the same distant stars while the Earth turns beneath the gyroscope. A long pendulum swings in a plane that stays fixed against the distant stars while the floor under the pendulum turns as the Earth turns. So in our universe what counts as not turning is fixed by the matter within reach: the Earth, the Sun and the stars, as Mach proposed. Mach's result follows from what a comparison is, by my own argument. The principle of comparison determines what the water turns against, and general relativity by itself does not determine what the water turns against.

My account has one step that Mach did not have: the matter within reach has an edge. The universe is expanding: its galaxies are drawing apart, and the expansion is speeding up. Because the expansion is speeding up, the most distant galaxies are carried away faster than any signal from them can close the distance. So every comparer has a *cosmological horizon*, the edge beyond which nothing can ever send that comparer a signal, however long the comparer waits. (Chapter 16 follows the distant galaxies as they pass beyond the cosmological horizon.) Matter beyond the horizon can never be compared with that comparer at all, so matter beyond the horizon fixes no standard for that comparer. The standard of not turning comes from the matter within the horizon.

So the water in Newton's bucket turns against the matter there is to compare with: the Earth, the Sun, the stars, all the matter within the horizon.

Three Rooms and the Climbing Water

Greene tells of two pushes that Einstein found alike: the push of gravity and the push felt in a room that speeds up. Greene has already shown a third push that looks like the push of gravity and the push of speeding up: the outward press in a spinning bucket. The three pushes are taken together: gravity, speeding up in a straight line and turning.

Imagine three small closed rooms, each with a person inside who cannot see out. The first room is far out in empty space, pulled steadily upward by a rope. The second room rests on the surface of a planet. The third room hangs at the end of a long cable and is swung around in a wide circle. In each room the person is pressed to the floor, and a ball let go falls to the floor. In the swung room, the floor is the outer wall. In all three rooms every body is affected alike, whatever the body is made of: the push each body feels is in proportion to the body's *inertia*, how hard the body is to set moving. The three pushes are equivalent: turning, speeding up in a straight line and gravity each act on every body alike, in proportion to the body's inertia.

The three rooms can still be told apart. Hang two balls on strings from two points of the ceiling. On the planet the two strings hang a little closer together at the bottom than at the top, because both point toward the planet's center. In the room pulled through empty space, the two strings hang parallel. In the swung room, the two strings spread apart. Shrink the rooms, and the difference between the room on the planet and the pulled room shrinks with them, until at a point the two rooms cannot be told apart at all. Einstein's equivalence principle states that, in a small enough room, gravity and speeding up cannot be told apart. The swung room is different even at a point. A gyroscope carried in the swung room shows the turning of the room, and so does the sideways drift of anything moving across the room. In 1913 the French physicist Georges Sagnac sent light both ways around a ring of mirrors set turning on a table. Sagnac found that the beam sent one way and the beam sent the other way came

back out of step, by an amount set by how fast the ring turned. The ring registered its own turning, right where the ring was, without a look outside. We now know that turning, alone of the three, is registered at a point.

The water in the bucket does what Sagnac's ring did. The hollow in the water registers the turning of the water, right there in the bucket, without a look at the stars. Newton found that the bucket shows its turning from inside. The turning is shown against the standard of not turning at that place, fixed by the matter there is to compare with.

Let Go of a Pen

Hold a pen at arm's length and let go. The pen falls, and while the pen falls it is weightless: nothing holds the pen up. The push of the floor is the weight you feel in your own body as you stand. The floor holds you off the path you would follow if you fell.

Why does the pen fall at all? On my account, the first step of the answer is what space and time taken together must be like, before anything is said about gravity, and the last step is the pen following the shape of space and time. The arena is this book's word for what Greene calls the fabric of the cosmos, or spacetime. The arena must have four properties. The four properties are that the arena acts on everything, that nothing shields anything from the arena, that a body has one mass, and that what the arena takes hold of comes in no steps. I call the four properties the four marks of the arena. The four marks follow from one fact: the arena is the context of every measurement, and not one more thing measured within the arena.

The first mark is that the arena acts on everything. Physicists say that a force *couples* to the things the force acts on. Each force of particle physics couples to some things and not to others. An electric field pushes on an electron, and the electric field lets a neutron pass straight through, because a neutron carries no electric charge. Nothing can pass through the

arena untouched, as the neutron passes through the electric field, because a thing untouched by the arena would be found nowhere, and so the arena couples to everything.

The second mark is that nothing shields anything from the arena. A metal box keeps electric fields out. A phone can lose its signal in an elevator, because the elevator is a metal box. To shield something from a region is to make a region from which that thing is absent. A field laid out in the arena can be absent from a region, but the arena cannot be absent from any region, because a region of space is a stretch of the arena itself. So nothing can shield anything from the arena.

The third mark is that a body has one mass. An electron and a proton carry electric charges of the same size, yet the proton is about eighteen hundred and thirty-six times harder to set moving. A body feels the electric force by its charge, and the body resists being moved by its inertia. For a force like the electric force, the charge and the inertia are two different properties. For the arena, the two properties are one. The degree of taking part also resists a change of the body's place in the arena. So a single quantity is both the pull on a body and the body's inertia, and every body falls alike.

The fourth mark is that what the arena takes hold of comes in no steps. Electric charge comes in steps: every electric charge is a whole multiple of one smallest amount, a third of the size of the electron's charge. Quarks are the pieces of which protons and neutrons are made, and the smallest amount of electric charge is the charge that some quarks carry. A charge that comes in steps is a *label*: a property that sorts particles into kinds. A label is exactly the same for every particle of one kind. The arena is continuous, smooth from place to place with no jump, and a continuous arena gives no labels in steps. So what couples a body to the arena is not a label in steps, but a quantity that can take any value in its range.

Gravity carries all four marks of the arena. Gravity couples to everything, light included: during the eclipse of the Sun in 1919, astronomers saw starlight that passed close to the Sun bent by the amount Ein-

stein's theory requires. Gravity also couples to *antimatter*, matter whose particles carry the opposite charges. Antihydrogen is the twin of hydrogen, built from particles of antimatter. In 2023 physicists held atoms of antihydrogen in a magnetic trap and released them, and the atoms fell toward the Earth. No arrangement of matter screens gravity. Every body falls alike under gravity. On 2 August 1971, on the surface of the Moon, the astronaut David Scott held a geologist's hammer in one hand and a falcon's feather in the other, and he let them go at the same moment. There was no air to slow the feather, and the hammer and the feather struck the ground together: the hammer is pulled harder than the feather, and the hammer resists being moved more, in exactly the same proportion. The satellite MICROSCOPE, launched in 2016, kept cylinders of titanium and of platinum falling around the Earth for months. The MICROSCOPE team found no difference in the fall of the titanium and the platinum at a level of a few parts in a million billion, in its final report in 2022. Gravity also couples to the whole energy of a body (its matter at rest, its motion and its heat), and the whole energy of a body can take any value: a hot brick is heavier than the same brick cold, by an amount far too small to weigh. We now know that gravity couples to every kind of matter and to light, and that bodies of every make fall alike to the finest measurement made.

So I identify gravity with the shape of the arena, and not with a force acting within the arena. The shape of the arena is the way space and time are bent, from one place to the next. The four marks are marks of the arena, and the shape of the arena carries every one of them. The shape of space and time is shared by everything in space and time. The shape has no outside from which anything could screen the shape off. The shape moves every body along the same paths, and the shape is measured by lengths and durations, not by a label in steps. The pen falls because the arena at the height of your hand differs from the arena at the height of the floor, and the pen follows the straightest path that the difference allows. Greene and Einstein come to the same result, that gravity is the warping of space and time. Greene comes to the result through the equivalence

principle and the history of Einstein's search, while on my account the result follows from the four marks that the arena must have. General relativity and my account each say one part of the answer to one question. Einstein's equations are the equations of general relativity, and they say how much the arena bends for the matter in the arena. Why does the arena take a shape that follows the matter in the arena at all, and why does the shape of the arena change smoothly, with no jump, from place to place? Both answers follow from what every comparison requires.

Physicists have since measured one more part of Einstein's answer. A change in the shape of the arena travels outward as a wave, a *gravitational wave*, at the speed of light. When Greene wrote *The Fabric of the Cosmos*, no one had yet caught a gravitational wave directly. Greene looked ahead to the first time a gravitational wave would be caught. Greene wrote: "For millennia we have looked into the cosmos; now it's as if, for the first time in human history, we will listen to it" (*The Fabric of the Cosmos*, chapter 14, section "Catching the Wave"). A *black hole* is a region whose gravity is so strong that nothing, not even light, comes back out of it, and a *light-year* is the distance light covers in one year. The Laser Interferometer Gravitational-Wave Observatory, LIGO, has two detectors in the United States. On 14 September 2015 the two LIGO detectors caught the waves from two black holes that had spiraled together more than a billion light-years away. Virgo is a detector in Italy. The LIGO team and the Virgo team reported the waves in 2016. A neutron star is the dense collapsed core that some large stars leave behind. On 17 August 2017 two neutron stars merged and sent gravitational waves and light toward us together, and after traveling for many millions of years the waves and the light arrived within two seconds of each other. We now know that changes in the shape of the arena travel as waves, and that the waves travel at the speed of light to within a few parts in a million billion.

I agree with Greene on one more point about space. Greene writes that by quantum theory every field jitters, even in space you would think of as empty. Greene's words are "empty space is teeming with quantum activity" (*The Fabric of the Cosmos*, chapter 12, section "Quantum Jitters and

Empty Space"). I agree with Greene: "One of the deepest discoveries of twentieth-century physics is that empty space is not empty". I also say that the quantum activity carries energy: every cubic centimeter of empty space holds some of that energy. Physicists call empty space, with every field at its lowest energy, the *vacuum*. The vacuum's energy is the energy that the quantum activity of the vacuum carries. The arena takes its shape from what is in the arena. So a question follows: how much of the vacuum's energy bends the arena? Chapter 10 answers that question.

Where Greene and I Part

On three points of this chapter, Greene and I come to different results, and for one reason: at every step, the question in my account is what a comparison could register.

The first point is what space and time are. In Greene's telling, special relativity holds that "absolute spacetime does exist", and that this spacetime, with no curving at all, is the benchmark against which a motion is judged to be accelerated. Greene concludes that on the most straightforward reading of general relativity, spacetime is a something: a thing in its own right. Greene also says that the issue is still debated. On my account, space and time have no properties apart from the events in them and the relations among those events. An *event* is a single happening at one place and one moment, and two events are related exactly when one can affect the other. Space is the pattern of which events are causally connected with which, and it is no container that exists before the events it holds. So in my account there is no space standing behind the events. There is no absolute space of Newton's, a fixed stage, the same everywhere, unmoved by the actors on it and there is no spacetime that would be there with no events in it. When general relativity is stated without the picture of a container, general relativity agrees with my account: the shape of the arena is gravity, and the arena has no properties apart from the causal relations among events, of which the arena is made. A flat spacetime is one that is not bent at all, and so has no gravity. The flat spacetime of special relativ-

ity is the shape of an idealized world, one with nothing in it and no waves crossing it, because the arena has no properties apart from the causal relations among events. Physicists use flat spacetime as a model, but flat spacetime is not the standard that actual matter is measured against, because the world holds matter, and matter bends the arena, even in the empty space around the matter.

The second point is the empty bucket. Greene reports general relativity's answer for a spinning bucket in an otherwise empty universe. Einstein's equations allow worlds empty of matter in which a standard of not turning is still fixed at each place. A bucket's mass is so small that the bucket hardly changes the shape of space, so in Greene's report the water in a spinning bucket in a world empty of matter would still climb the walls of the bucket. On my account, the principle of comparison fixes the standard of not turning in a world empty of matter: where nothing is there to compare against, no comparison can register any standard of not turning. So take a universe that holds a bucket and its water, and nothing else. The water can still turn against the bucket, as one spacecraft turns against the other. The bucket and the water together are like one spacecraft alone: there is nothing outside the two to tell their turning by. So in a universe with only the bucket and its water, the water would have nothing but the bucket to measure its turning against, and the only turning any comparison there could register is the water's turning against the bucket. My result for a universe that holds only a bucket and its water is about the standard of not turning, and not about the shape of the water's surface.

The third point is the share of the victory. Greene hopes that a theory still to come will give Leibniz, Newton, Mach and Einstein each a share of the victory. In the results already in my account, each of the four has a share. Leibniz and Mach have their share: what counts as still and what counts as turning depends on the matter there is to compare with. Newton has his share: the turning shows itself inside the bucket, in the hollow of the water, with no look outside. Einstein has his share: gravity is the shape of the arena, and the arena is the setting in which every comparison is made. Greene guesses that space and time may be built from more basic

ingredients. The arena is not made of anything more basic that is found in it. What is found in the arena comes in separate, repeating steps, while the arena itself is smooth. The smooth arena cannot be built out of the steps found in it, and Chapter 5 sets out the argument for that claim.

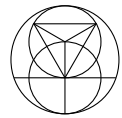
What the Water Turns Against

Go back to the pail on its rope. The water climbs the walls, and the climb is a finding, made right there in the pail. The climb is measured against the standard of not turning at that place, and that standard is fixed by the Earth, the Sun and the stars, by all the matter within reach that the water can be compared with. Every finding is a comparison made by things in the world, so every finding of physics is made inside the arena, against what is there to compare with, and never from a stage outside the arena.

Greene asks whether space is a something, and on my account the answer has two parts. Space is a something in this sense: together with time, space is the arena, the setting in which every comparison is made, and the shape of the arena, which follows the matter in the arena, is gravity. Space is not a something in Newton's sense, a container standing behind things, because space has no properties apart from the causal relations among the events that space holds. And the setting of every comparison contains time as well as space.

So we now know what the water is spinning against. The matter there is to compare against fixes what counts as still and what counts as turning. Gravity is the shape of the arena in which every comparison is made, and space is no container standing behind things.

If space is the setting of every comparison, is time laid out in that setting too, past and future, all at once?



Chapter 2

Is the Whole of Time Laid Out at Once?

Beside The Fabric of the Cosmos, *chapters 3, 5, 8, 15 and 16, and* The Elegant Universe, *chapter 2.*

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

A *finding* is the outcome of one act of comparison, laid down as a record; a *comparer* is whatever makes a comparison and registers what it finds, and the word comparer names a role, and never a mind (Chapter 1).

The *principle of comparison*: a difference that no comparison can register is no difference (Chapter 1).

The *arena* is space and time taken together, as the setting in which everything that is measured is measured, and I identify gravity with the shape of the arena (Chapter 1).

On 21 March 1955, in Princeton, Albert Einstein wrote to the family of Michele Besso. Besso, Einstein's lifelong friend, had died in Geneva six days before. If your "now" and the "now" of someone walking on a planet far away cut the whole of space and time at different angles, does the

whole of time already exist, and is the future as settled as the past? A reader carries that question away from the fifth chapter of *The Fabric of the Cosmos*, a book by the physicist Brian Greene, and Greene answers the question with the help of a sentence from Einstein's letter.

Besso was the only friend whom Einstein thanked by name at the end of his paper of 1905 on relativity. In the letter Einstein wrote that Besso had gone a little ahead of him in leaving this strange world. Einstein wrote that Besso's going ahead meant nothing. Then Einstein wrote the sentence that Greene quotes, here in the English that Greene prints:

> "For we convinced physicists, the distinction between past, present, and future is only an illusion, however persistent".

(*The Fabric of the Cosmos*, chapter 5, section "The Persistent Illusion of Past, Present, and Future".) Einstein died four weeks after he wrote the letter, on 18 April 1955. Einstein wrote the sentence to console a family. Greene sets Einstein's sentence beside a conclusion of his own: "The total loaf exists". The loaf is Greene's picture of the whole of space and time. Greene draws from his conclusion a sentence about a single moment of a life: "If you were having a great time at the stroke of midnight on New Year's Eve, 1999, you still are" (same chapter, section "Experience and the Flow of Time"). So on Greene's view a friend who has died is still there, at the friend's own place in the whole of space and time.

From Greene's pages

One Speed for Every Observer

Picture a train moving at a hundred kilometers an hour. A rider on the train throws a ball forward at twenty kilometers an hour. For the rider, the ball moves at twenty kilometers an hour. For someone standing beside the track, the ball moves at about a hundred and twenty. Now let the rider send a pulse of light forward instead. The rider and the

bystander each measure the speed of the light, and the two of them get the same number, as every measurement of the speed of light has shown.

Greene builds his argument about time on that measured fact: every observer measures the same speed for light. Greene sets out *special relativity* in *The Elegant Universe*. Special relativity is Einstein's theory of 1905 of how lengths and durations compare for observers moving steadily relative to one another. Greene states the fact: "Regardless of relative motion between the source of photons and the observer, the speed of light is always the same". (*The Elegant Universe*, chapter 2, section "The Speed of Light".) A *photon* is a particle of light. Greene draws the further results of special relativity from the sameness of light's speed. A moving clock runs slow compared with a clock at rest, and Greene insists that the difference belongs to time and not to the clocks: "it is a true statement about time itself". (Same chapter, section "Intuition and Its Flaws".) Two events that happen at the same moment for one observer happen at different moments for a second observer who moves past the first. Greene gives a picture of why moving clocks run slow and why observers disagree about which events happen at the same moment. In Greene's picture, everything moves through space and time taken together at one fixed speed, the speed of light, so whatever motion a thing has through space is taken from the thing's motion through time. A clock at rest moves through time alone. Light moves through space alone, and for light no time passes at all. Greene also concludes that nothing can travel faster than light, because a body's mass grows without limit as the body's speed nears the speed of light.

In *The Fabric of the Cosmos* Greene turns the results of special relativity into a picture that a reader keeps. Greene uses the word *spacetime* for space and time taken together as one whole. He pictures the whole of spacetime as a block. Each observer cuts the block into *time slices*, Greene's word for all of space at one moment as one observer reckons

that moment. Observers in relative motion cut the block at different angles. In Greene's words, observers "cut a block of spacetime up into pages, into time slices, in different but equally valid ways". (*The Fabric of the Cosmos*, chapter 3, section "Carving Space and Time".) Then comes the picture that a reader keeps. Greene writes of spacetime: "it will sometimes be useful to think of it as a huge, fresh loaf of bread". (Same chapter, section "Angling the Slices".) Slice the loaf straight across or at a slant, and the slices differ. Yet the loaf itself does not change with the angle of the knife. The loaf is spacetime, each slice is one observer's "now", and the angle of the knife is the observer's motion.

Greene calls whoever does the measuring an *observer*. In this book the word means whoever or whatever does the measuring.

From Greene's pages

The Distant Walker and the Frozen River

In his fifth chapter Greene uses the loaf to ask what is there. Greene imagines someone he calls Chewie, sitting in his living room on a planet ten billion light-years away. While Chewie and you sit still relative to each other, your slices of "now" coincide. Then Chewie gets up and walks away from you at about ten miles an hour. Chewie's slice of "now" tilts by a tiny angle. Across ten billion light-years the tiny angle opens a wide gap, as the tips of a long pair of scissors move far apart when the handles move a little: Chewie's walk is the small move of the handles, and the gap at the distance of the Earth is the wide move of the tips. On Chewie's new slice, the events on Earth happening "now" are events of about a hundred and fifty years ago, by your reckoning. If Chewie walks toward you instead, the Earth events on his slice of "now" are about a hundred and fifty years ahead.

Then Greene states his argument as two conditions and a conclusion. Greene's first condition is that what is real is what lies on your slice of "now", and his second condition is that your slice of "now" is no more valid than the slice of anyone far away who can move freely. If both conditions hold, the slices of all possible walkers together fill a large region of the loaf. Greene adds a further step: if space is infinite, the slices can be centered as far away as one likes, and then the slices together sweep through every point of the loaf. In a note Greene says that his argument assumes the flat geometry of special relativity, in which gravity does not bend space and time, and that in other geometries a similar argument "will not necessarily yield the entire spacetime". Greene's conclusion is short: "The total loaf exists". A few lines later he writes: "The only thing that's real is the whole of spacetime". (Both in chapter 5, section "The Persistent Illusion of Past, Present, and Future"; the note is that chapter's note 4.) Greene's conclusion is the *block universe*: the view that past, present and future are all equally there and laid out together like the slices of one loaf.

Greene then asks what becomes of the feeling that time flows. He considers a light that moves along the loaf and makes one slice after another the present. He finds nothing in the laws of physics that moves the light. Greene argues that a single moment cannot change, so every moment is lit and stays lit, and on his view the river of time is better pictured as a block of ice, with every moment frozen in its place. For Greene, the sense that time flows comes from inside each moment, from the memories and thoughts that are present at that moment. Greene says that during the working day he sides with the view that the flow of time comes from the mind. By nightfall he finds it hard to keep up full resistance to the other view: that time has a quality of its own which scientists have not grasped. Greene writes: "It is possible that some insightful person will one day devise a new way of looking at time and reveal a bona fide physical foundation for a time that flows". In the next sentence he writes: "Then again, the discussion

above, based on logic and relativity, may turn out to be the full story". (*The Fabric of the Cosmos*, chapter 5, section "Experience and the Flow of Time".) Elsewhere Greene takes from the physicist John Wheeler the thought that time is what keeps all change from happening at once (chapter 8, section "Symmetry and Time").

One Edge for Every Comparer

I agree with Greene on the results told so far: one speed of light for every observer, moving clocks that run slow, and slices of "now" that differ from observer to observer. On my account, those three results follow from a different starting point, the act of comparison, which Chapter 1 sets out. The word "comparer" never means a mind. A record is a mark that stays after the act that made the mark.

Picture a single flash: a firework bursting at one place and one moment in a dark sky. The light of the firework spreads out as a sphere that grows by about three hundred thousand kilometers every second. Draw all the moments of the sphere at once, with time running up the page and one direction of space across the page, and the growing sphere becomes a cone that opens upward from the flash. The flash can affect every event inside the upward cone. Run the picture downward, and a second cone opening downward from the flash contains every event that can affect the flash. Every event has a double cone like the flash's: everything the event can affect and everything that can affect the event. I call the double cone at each event *the cone*.

The authors of relativity textbooks start from the speed limit, that nothing travels faster than light, and draw the cone from the speed limit. The cone comes first. The cone is primary, and it is not obtained from the limit. The cone at each event contains every event that can be compared with that event. The speed of light is the slope of the cone's edge in the drawing. I identify the speed of light with the edge of comparison: the greatest speed at which any signal can carry a finding from one place to another. The greatest speed is the same for every comparer. Light is the

plainest example of the greatest speed, because light travels in empty space at exactly the greatest speed. Greene starts from the fact that every observer measures the same speed for light, and on my account the sameness of light's speed has a reason: the speed of light is the edge of comparison, and the edge of comparison is the same for every comparer.

The ball's speed is a finding about one thing inside the arena. Findings about things inside the arena differ from one comparer to another: the ball moves at twenty kilometers an hour for the rider, and at about a hundred and twenty for the bystander. The edge of comparison is not a thing inside the arena. The edge belongs to the act of comparison itself. Every comparer is making comparisons however the comparer moves, so the same edge applies to every comparer. The invariance follows from the speed of light being an edge: an edge of comparison is the same for every comparer. The word "invariance", in that sentence, means the sameness of a quantity for every comparer. The edge has a value of about three hundred thousand kilometers each second, and that value is a measured number, but the edge is one edge, the same for every comparer, because the edge is an edge of comparison.

A measurement of 1934 shows that the greatest speed is a property of the arena, of space and time, and not a property of light. In 1934 the physicist Pavel Cherenkov saw a faint blue glow in clear liquids struck by radiation. Light travels more slowly in water than in empty space. Fast charged particles, knocked loose in the water by the radiation, were outrunning the light in the water. The outrunning particles gave off the glow. The particles outran light in water, but none of the particles reached the edge of comparison. No particle with mass reaches the edge, and nothing that carries a finding goes faster than the edge. That limit is a property of the arena and not of light.

In Greene's picture of one fixed speed through space and time, the speed of light has one more role, and the speed of light has the same role in my account. A light-year is a distance named by a time. The speed of light converts an amount of time into an amount of distance, as a rate of

exchange converts one currency into another. In Greene's picture the speed of light is the one total speed that a thing's motion through time and the thing's motion through space share between them.

Clocks That Keep Their Place

Greene says that a moving clock's slowness is a fact about time, and I agree with Greene. On my account, the rate of a clock is set by the clock's place and motion in the arena. The place is how high the clock sits, and the motion is how fast the clock moves. Nothing in the clock's works sets the rate. Four measurements show clock rates set in that way: two on clocks that move, and two on clocks that sit at different heights.

A *muon* is a heavier cousin of the electron, made high in the air when particles from space strike the top of the atmosphere. At rest a muon breaks apart in about two millionths of a second. In 1962 the physicists David Frisch and James Smith counted muons at the summit of Mount Washington in New Hampshire, close to two kilometers up, and again near sea level at Cambridge, Massachusetts. If a falling muon kept the time of a muon at rest, the falling muon would travel on average less than a kilometer before breaking apart, so only about one in twenty of the muons counted at the summit would have reached the lower counter. About three in four arrived. We now know that a muon's clock, moving at nearly the speed of light, runs slow as seen from the Earth, by the amount special relativity gives.

In October 1971 the physicist Joseph Hafele and the astronomer Richard Keating flew *atomic clocks*, clocks that count the vibrations of atoms, around the world on ordinary passenger flights, once eastward and once westward. Hafele and Keating compared the flown clocks with the clocks of the United States Naval Observatory. The clocks that flew east came home about sixty billionths of a second behind, while the clocks that flew west came home about two hundred and seventy billionths of a second ahead. Both amounts agreed with what Einstein's two theories of relativity give together for the speeds and heights of the clocks. Einstein's

first theory of relativity is special relativity. His second is *general relativity*, his theory of gravity, in which gravity is the shape of the arena: the way space and time are bent from one place to the next.

In 2010 the physicist Chin-Wen Chou and his colleagues compared two clocks at the National Institute of Standards and Technology. Each clock ticked with the light absorbed by a single aluminum *ion*: an atom of aluminum that has had one of its electrons taken away. Chou and his colleagues raised one of the two clocks by about a third of a meter. The raised clock ran faster by about four parts in a hundred million billion, the amount general relativity gives for that change of height. The clocks of the navigation satellites run faster than clocks of the same kind on the ground by about thirty-eight millionths of a second every day, with speed and height counted together, so the satellite clocks are set before launch to allow for the gain. In 2003 the physicist Neil Ashby set out the whole accounting. So we now know that a clock's rate is set by how fast the clock moves and how high the clock sits, by the amounts that Einstein's two theories of relativity give.

Every result of this chapter so far has two grounds: the measured sameness of light's speed, which is Greene's starting point, and the edge of comparison, which is my starting point.

The Loaf Exists

Greene's sentence "The total loaf exists" is also true in my account. To see in what sense the sentence is true, picture two loaves. The first loaf is Greene's loaf, baked and sliced on the board. Every slice of the first loaf is there together, and the whole loaf can be held in view from crust to crust. However the first loaf is sliced, and whoever looks at it, the first loaf is one and the same loaf. Beside the first loaf, picture a second loaf in the oven this morning, still rising. While it rises, nothing about the loaf is finished: it may rise well or badly, and how it turns out is not yet settled.

Two familiar words have exact meanings, one for each loaf: "exist" and "real". The meaning of "exist" uses a third word, structure. *Structure* is what is laid out whole and stays the same for everyone who looks at it, as the sliced loaf does. To *exist* is to stand as structure: timeless, the same in every instance (every single case of the thing), able to be held in view as a whole. The sliced loaf exists in that exact sense. Any comparer that looks at the sliced loaf finds one and the same loaf, held in view from crust to crust. To be *real* is to be given in an actual act of comparison: here, now, with the future still open. To be given in an act is to be the outcome of that act, at the place and the moment where the act happens. In the definition of "real", the words "still open" mean that no act has yet given the future. The rising loaf is real in that exact sense. The loaf's rising is found this morning, in this oven, and how the loaf turns out has not yet been found. A law exists; a measurement is real.

In the exact sense of "exist", Greene's loaf of space and time exists. The whole pattern of space and time stands as structure in four dimensions, with every event at its place among the cones. No way of slicing the pattern changes the pattern. Greene makes exactly that point the first time he uses the loaf, in his third chapter: slicings differ from observer to observer, and the loaf that the observers slice is one.

The word "time" also needs care, because physicists use the word for three different things. The three kinds of time can be set on one bench. Picture a laboratory bench with a lump of radioactive material on the bench, a counter beside the lump, and a clock on the wall. Each atom of the lump breaks apart, or *decays*, at a moment of its own. The counter clicks each time an atom decays. The clock on the wall measures *geometric time*: the time of the arena, the time along a path through the loaf, which runs at different rates in different places, as the raised clock showed. The counter measures *event time*. Each click is a finding: an atom has decayed, and a record now stands that did not stand a moment before. Event time is the order in which findings like the clicks are laid down as records. Between the clicks, the chance that a given atom has decayed grows smoothly from moment to moment. The physicist's description of the

chances of decay changes without a jump. The time of that smooth change is *evolution time*: the time along which the description of what could be found, and of how likely each finding is, changes between findings.

Geometric time, event time and evolution time are *the three times*. No one of the three can do the work of another. A clock reading labels a moment, but a label fixes no click, so geometric time cannot do the work of event time. A recorded click stays recorded, and no smooth change run backward undoes the record, so event time cannot do the work of evolution time. Evolution time ranges over the possible findings in the description, and the possible findings are not a direction of the arena: nothing travels through the possible findings from one place to another. So evolution time cannot do the work of geometric time: marking places and moments in the arena. Chapter 4 shows evolution time at work between findings and shows that a finding (one outcome among the chances) is given in event time. Greene's loaf is geometric time, laid out whole.

The Loaf Still Rising

On one word, "real", my results differ from Greene's results. The sliced loaf stands for what exists. The rising loaf stands for what is real.

But Greene says the whole loaf is real, you may say, and he has an argument. In Greene's argument, the word "real" means lying on someone's slice of "now", and every slice of "now" is a slice of geometric time. With "exist" and "real" in these senses, Greene's argument shows that the whole of geometric time exists: every slice of geometric time stands as structure. A slice of geometric time is structure, and not an act, so in this sense a slice is not real. So, in this sense of "real", "The total loaf exists" follows in my account. In Greene's sentence, "real" has the sense that the word has in his argument: lying on someone's slice of "now".

A picture shows that the difference between what exists and what is real is a difference in what happens, and not only a difference in words. A Geiger counter clicks when an atom of radioactive material decays. Quantum physics is the physics of atoms and of the particles smaller than

atoms. Quantum physics fixes only the chance of a click in each second, and never the moment of the click. The moment stays unfixed even given everything there is to know about the atom beforehand. When nothing before an outcome fixes the outcome, however completely what came before is known, I call the outcome *hard randomness*. The moment of each click is hard randomness. Now wire the counter so that the counter's next click launches a small rocket, if the click comes within one minute. The rocket nudges a small asteroid, the small asteroid's new path in time shifts a larger asteroid, and in principle the chain of nudges goes on until the orbits of planets are moved. Every link of the chain after the first link is ordinary gravity and motion, and the first link was a click that nothing beforehand fixed. So the loaf differs according to an outcome that was settled only when the outcome happened, and the loaf is the shape of space and time itself. A single fixed geometry of all of space and all of time, from the first moment to the last, laid out once and for all, cannot be the whole story of the universe. The present act is the line between what is settled and what is still possible.

I name what the block universe leaves out in my nineteen essays on the limits of physical law: "The block universe takes geometric time for the whole, and drops event time". The past has been given, act by act, and the records of the past stand. The future has not yet been given.

No Slice Carries the Present

What, then, of Chewie, walking away across ten billion light-years with Earth's past on his slice of "now"? Chewie's slice is a list of events drawn in geometric time, and no finding is settled on the slice. A *surface of simultaneity* is the physicist's name for Greene's time slice, all of space at one moment for one observer. No finding is settled on a slice: no comparison registers a surface of simultaneity on which a finding would be settled for every comparer together, and the account needs none. News of anything that happens on Earth reaches Chewie only by a signal, at the speed of light at most, and so ten billion years later at the earliest, and

Greene agrees. In a note Greene adds that the lists of what is happening "now" do not record which events can affect which, and that every observer agrees on which events can affect which. Every signal that carries a finding travels within the cones, and no signal travels along a slice.

George Ellis is a cosmologist, a physicist who studies the universe as a whole. Ellis and the physicist Stephen Hawking wrote *The Large Scale Structure of Space-Time* (1973), the standard book on which events in space and time can affect which. Ellis argues that the future is open, and he argues from physics itself: from the chances of quantum physics and from the one-way step of measurement. In Ellis's picture space and time form a block whose past is fixed and whose future edge keeps advancing. My account drops the frozen block for the reason Ellis gives: the future is not yet decided. For Ellis the present is a surface across the whole universe. My account needs no surface of the present across the whole universe. In the chapter on Ellis I also state what my account has in place of a surface of the present: "Each act is a present, and the order of acts is the order of time".

The Direction Greene Left Open

Greene left open the possibility of a physical foundation for a time that flows. On my account, the foundation of time's one direction is in every single act of comparison. In every finding the act comes first and the record after, and the record, once laid down, stays. A record is never unlaid. Findings define the order of event time; they do not occur within a time given before them. I identify the one-way order of findings with the direction of time.

In Greene's picture of time as a frozen river, the sense of flow comes from inside each moment, from the memories present at that moment, and every memory is a record. On my account, the direction of time is in the act that lays each memory down: the act comes first, and the memory after. A brute fact is a fact that could have been otherwise, with nothing beneath it. The direction of time is no brute fact: "It is what it means for

anything to interact with anything else". Wheeler thought that time is what keeps all change from happening at once. Wheeler's thought agrees with my starting point. Two moments in which nothing changes cannot be told apart, so the order of time is the order in which changes are found. Time comes with the comparison, not before it. In Chapter 8 we ask, with the one-way order of findings in hand, why Greene's egg splatters and never unsplatters.

Last Thursday Is Not a Place

Greene takes up one more question about the loaf in his fifteenth chapter: if the whole of time is there, can we travel through time?

On travel to the future, Greene and I agree. When a traveler goes fast enough and comes home, the traveler finds that less time has passed on board than at home. The muons and the flying clocks showed the effect on a small scale. Greene concludes: "Technology, not physics, keeps each of us tethered to this epoch". (*The Fabric of the Cosmos*, chapter 15, section "The Puzzles of Time Travel".) A trip to the future is travel within the cone. Travel within the cone is possible.

On travel to the past, Greene gives an answer that he calls a qualified no: "Most sober physicists would answer no. I would say no". (*The Fabric of the Cosmos*, chapter 15, section "Is Time Travel to the Past Possible?".) Greene adds that no one has shown that the laws of physics rule out travel to the past. A *time loop* is a path through space and time that returns to its own starting event. Einstein's equations of general relativity allow shapes of space and time that contain a time loop. The logician Kurt Gödel found a shape of space and time with time loops in 1949. In 1988 the physicists Michael Morris and Kip Thorne described a *wormhole*, a short tunnel joining two distant places, held open by matter of a kind that no one has found. In 1992 Stephen Hawking proposed, as a *conjecture* (a proposal not yet proved), that the laws of physics keep time loops from forming at all.

My answer depends on what the words "travel to the past" mean. When people speak of going back to last Thursday, which time do they mean? In the first sense, last Thursday is a region of the loaf, and in that sense last Thursday exists: the stretch of geometric time between two events on the Earth's path stands as structure, with every event at its place. In the second sense, last Thursday is the occasion it was, the findings made that day: the occasion was real when its findings were given, and what remains of the occasion now is its records. The phrase "travel to an earlier time" uses the first sense, the region of the loaf, to secure the destination, and the second sense, the occasion of the findings, to describe the arrival.

A *prime number* is a whole number that no whole number divides exactly except one and the number itself. The phrase "the largest prime number" has the form of a name, and yet every prime has a larger prime beyond it, so the phrase names no number. When people use the phrase "travel to an earlier time", they intend the phrase to mean arriving at an earlier occasion as the occasion it was. In that sense, "travel to an earlier time" is a phrase of the same kind as "the largest prime number": a phrase with the form of a name that names nothing. The phrase "travel to an earlier time", taken in that sense, names no idea at all.

I take the three things the phrase could mean one at a time. Each meaning can be tried on last Thursday.

The first meaning is reaching last Thursday as a region of the loaf. Where a geometry allows a path that comes back to that region at all, the traveler's clock reads later on arrival than at departure. The traveler's findings on arrival come after the findings made before setting out, so the traveler's arrival is a later finding of the traveler, and not last Thursday as it was.

The second meaning is running one's own findings backward, from today's findings back to last Thursday's, undoing the record of each finding. The second meaning fails because of the one-way order of findings: a record, once laid down, is never un-laid.

The third meaning is the one that people who use the phrase intend: being present at last Thursday as the occasion it was, among the findings of that day. The third meaning needs one time that is, completely, both the region of the loaf and the occasion of the findings. Clocks find geometric time smooth, with no gap anywhere, and different from place to place. A record, once made, is one separate finding, and the record reads the same for every comparer that later reads the record. Suppose one time were both the smooth geometric time and the separate record, each completely. That one time would have at least one of two pairs perfectly. The first pair is the smoothness of the geometry together with the sameness of the record for every reader. The second pair is the separateness of the record together with the difference of the geometry from place to place. Chapter 3 sets out exact names for the four properties in the two pairs. Smoothness with no gap is *continuity*, and the separateness of one finding from the next is *discontinuity*. Sameness for every reader is *symmetry*, and difference from place to place is *asymmetry*. Chapter 3 states and proves the Incommensuration Theorem. The Incommensuration Theorem shows that nothing is both perfectly continuous and perfectly symmetric, and that nothing is both perfectly discontinuous and perfectly asymmetric. The name of the theorem comes from the word incommensurable: two things are incommensurable when no common measure fits both. So no time is completely both the region of the loaf and the occasion of the findings, and nothing satisfies the third meaning.

This is the sense in which time travel is a category mistake. A *category mistake* is placing a thing in the wrong kind. In time travel to the past, the category mistake is treating an occasion that was given in an act as if the occasion stood as structure, waiting like a place to be visited.

Physicists who study time loops work under one principle: a history that loops back must agree with itself, so whatever goes around a loop arrives back exactly as it set out. Greene states that principle in his own way. Greene writes that a traveler to the past could no more change the past than change the value of pi, since the traveler always was part of that past. In Greene's words, the events of the loaf fit together into "a coherent, self-

consistent whole". (*The Fabric of the Cosmos*, chapter 15, section "Rethinking the Puzzles".) I take one step more. A return around a time loop would require every trace of the trip to be undone on the way back, with the traveler's clock among the traces. The undoing is itself a further act, and the findings of the undoing come after the findings that the undoing is said to undo. So even on a time loop the traveler's arrival is a later finding of the traveler.

Greene ends his book by saying that space and time "are here to stay" as the setting of everyday life (chapter 16, section "Inner and Outer Space"). I agree in full that space and time are the setting of everyday life. Geometric time and event time are the two times we live with. Each of the two times is in good order on its own terms. The two times are not rivals.

What We Now Know

Go back to Princeton, and to Einstein's letter to Besso's family. Einstein wrote that Besso's going ahead meant nothing, because the distinction between past, present and future is an illusion. With "exist" and "real" in this chapter's senses, one part of Einstein's comfort is true in my account. Besso's life has its place in the whole pattern of space and time, and the whole pattern exists, as Greene's loaf exists. Everything Besso did was real when he did it, and every record that Besso's acts laid down stays laid down: the past has been given, and the past cannot be ungiven. Einstein's reason is the part of his comfort that is not true in my account: his reason was that the distinction between past, present and future is an illusion. On my account, past and future are distinct, because the future is open: the future is given act by act, here and now, wherever an act of comparison takes place. For a reader of Greene, one word changes its meaning in my account, and every measurement stays as Greene gives it: the loaf, the slices, the slow clocks and the one speed of light all stand, and the word "real" now names the act, and no longer the whole loaf.

Chapter 3 needs the cone and the open future to ask how particles far apart can agree when no "now" runs across space. Chapter 4 needs the act, in which what is real is given, to say how a single outcome is found.

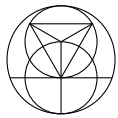
We now know that the whole pattern of space and time exists, laid out as structure, as Greene's loaf exists, while what is real is given in an act of comparison, here and now, and the past has been given while the future has not yet been given.

If no comparison registers one "now" across space, how can two particles far apart agree at once, as Greene says they do?

PART TWO

The Quantum

Beside *The Fabric of the Cosmos*, chapters 4 and 7, and *The Elegant Universe*, chapter 4.



Chapter 3

Is the Universe Local?

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

A *finding* is the outcome of one act of comparison, laid down as a record; a *comparer* is whatever makes a comparison and registers what it finds, and the word comparer names a role, and never a mind (Chapter 1).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

I identify the speed of light with the *edge of comparison*: the greatest speed at which any signal can carry a finding from one place to another (Chapter 2).

Hard randomness is an outcome that nothing before it fixes, however completely what came before is known (Chapter 2).

On 25 March 1935 the journal *Physical Review* received a paper of four pages from three physicists at the Institute for Advanced Study in Princeton: Albert Einstein, Boris Podolsky and Nathan Rosen. The paper of

Einstein, Podolsky and Rosen opened a question. If two particles far apart can behave as if they were one thing, as Greene says they can, does something pass between them? The title of the paper asked a question of its own: "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Quantum mechanics*, which physicists also call quantum theory, is the theory of atoms and of the particles smaller than atoms.

The argument of Einstein, Podolsky and Rosen can be followed with two particles. Two particles meet, interact, and fly apart in opposite directions. A particle's *momentum* is its quantity of motion: at everyday speeds, its mass times its velocity. The positions of the two particles stay linked, and so do their momenta. Measure the position of the first particle, and you know at once the position of the second particle. Measure the momentum of the first particle instead, and you know at once the momentum of the second particle. The second particle was never touched. So, Einstein, Podolsky and Rosen reasoned, the second particle had a definite position and a definite momentum all along, and a theory in which the second particle cannot have both is incomplete: something is missing from the theory. The opposite view says that the second particle has no definite position or momentum until a measurement is made on the first particle, so whether the properties of the second particle are real would depend on that measurement. Einstein, Podolsky and Rosen rejected the opposite view: in their words, "No reasonable definition of reality could be expected to permit this". The last sentence of the paper was a hope for a complete theory: "We believe, however, that such a theory is possible". The paper appeared on 15 May 1935. The physicist Niels Bohr replied in *Physical Review* that year, under the same title.

Within thirty years the physicist John Bell had turned the question of Einstein, Podolsky and Rosen into a question that an experiment could answer. Within fifty years, experiments had answered Bell's question. Greene states what the experiments found: the universe, he concludes, is not local. The question in the title of this chapter is whether Greene's statement is true. My starting point is what a comparison is. One example

of a comparison is setting a morning reading of a thermometer beside an evening reading and finding that the two readings differ. The word "local" has two senses. In the broad sense, which is Bell's, a world is local when every agreement between what is found in two places is carried by causes that travel from place to neighboring place. In the narrow sense, which physicists keep as true of the world, a world is local when no signal travels faster than light. A *signal* is anything that passes from one place to another and lets a record at the second place depend on an act at the first. In the broad sense, I agree with Greene's statement that the universe is not local. In the narrow sense, the universe is local.

From Greene's pages

Mulder's Boxes, and a Universe That Is Not Local

Greene opens his chapter on *entanglement* in *The Fabric of the Cosmos* with a parcel. Entanglement is Greene's word for the link between particles far apart. In Greene's story, Agent Scully is on holiday in Provence and Agent Mulder is in America. Each of the two agents receives a thousand small numbered boxes of titanium. Each box holds a sphere that flashes red or blue, at random, when someone opens the little door on the side of the box, and yet the spheres in boxes with the same number always flash the same color. Scully gives the obvious explanation: each sphere was set in advance, and the two sets of spheres were set alike. Greene's point is that quantum theory, and the experiments that test quantum theory, agree with Mulder, who holds that the color of each sphere is chosen at random when the box of the sphere is opened, and that the two random choices still agree.

Greene's picture of the boxes uses two ideas of quantum theory, and Greene explains the two ideas first. The first idea is probability: even with complete information about a particle, a physicist can know only the chance of each outcome. Greene writes: "Probability reigns supreme in the microcosmos" (*The Fabric of the Cosmos*, chapter 4, section "Probability and the Laws of Physics"). Greene calls the description

that carries the chance of each outcome the probability wave. The second idea is the *uncertainty principle*: a particle cannot have both a definite position and a definite velocity, and the limit comes from the spread of waves, not from clumsy measuring. Greene says of the uncertainty principle: "In quantum mechanics, uncertainty just is" (same chapter, section "Heisenberg and Uncertainty"). In *The Elegant Universe* Greene calls the uncertainty principle the center of the theory: "The uncertainty principle captures the heart of quantum mechanics" (chapter 4, section "Quantum Weirdness").

In the same chapter of *The Elegant Universe*, Greene tells how light and matter each behave both as a wave and as a particle (section "Is It a Wave or Is It a Particle?"). Greene also tells of the *sum over paths* of the physicist Richard Feynman (section "Feynman's Perspective"). The sum over paths is the rule that a particle travels from one place to another along every possible path at once, with a number given to each path. The sum over paths predicts the same results as the probability wave does. For large bodies, the sum over paths reduces to the single path of Newton's laws of motion. Greene tells, too, that physicists who use the rules of quantum mechanics follow the rules without knowing why the rules work. Greene then asks whether the human mind, which evolved for everyday scales, cannot fully grasp what goes on at the smallest scales. He also asks whether physicists have written quantum mechanics in an awkward form that hides what reality is like. Greene answers both questions: "No one knows" (section "The Quantum Framework").

Greene then tells how physicists tested Mulder's view against Scully's, and what they found (*The Fabric of the Cosmos*, chapter 4, section "No Smoke but Fire"). Two detectors agree on a pair of particles when both detectors record the same answer, just as Mulder's box and Scully's box with the same number agree when both spheres flash the same color. If Einstein, Podolsky and Rosen were right, each particle would carry its answers with it from the source, the device that sent

the pair out, just as Scully says each sphere was set in advance. The *settings* of the detectors are the directions the detectors are turned to. Bell showed in 1964 that answers carried from the source would make two far detectors agree on paired particles more than half the time, when the settings are chosen at random. Three teams of physicists did the experiment: Stuart Freedman and John Clauser, then Edward Fry and Randall Thompson, then Alain Aspect and his colleagues in France in the early 1980s. The detectors agreed half the time, as quantum theory predicts, and not more than half. So, in Greene's telling, the measurements show that the particles do not carry their answers from the source. Greene's verdict is that the most sensible part of the reasoning of 1935, the assumption that the world is local, is the part that fails: "the universe is not local" (same section). Greene draws a lesson about space itself: "Space does not distinguish such entangled objects" (section "What Are We to Make of All This?").

Greene adds three points about the experiments. His first point is that no usable message passes from one detector to the other, because the list at each detector is random. Special relativity is Einstein's theory of motion at a steady speed in a straight line, and of light, and special relativity says that nothing which carries a message travels faster than light. Greene writes of the speed limit of special relativity: "no information is transmitted from one to the other, and the speed limit of special relativity remains in force" (section "Entanglement and Special Relativity: The Standard View").

Greene's second point is about how a measurement is described in standard quantum mechanics, which he calls "good old-fashioned textbook quantum mechanics". In the textbook description, the probability wave of a measured particle *collapses*: the wave changes everywhere at once to fit the outcome just found. Greene writes that a collapse of the probability wave "would seem to pick out one vantage point as special": the vantage point of the observer for whom the collapse happens at one moment across all of space. Greene reports that

most physicists hold that there is a "harmonious coexistence" between special relativity and the results on entangled particles. Greene gives his own judgment that the view of the majority will win: "It's certainly possible, and in my view likely, that the majority view will ultimately prevail in some more definitive form" (section "Entanglement and Special Relativity: The Contrarian View").

Greene's third point is a warning against a popular claim: that everything is entangled with everything. Greene writes: "While I like the sentiment, such gushy talk is loose and overstated" (section "What Are We to Make of All This?"). Greene also writes that the data are inconsistent with a local universe, and consistent with particles that have hidden definite properties. In the version of quantum theory that the physicist David Bohm gave, particles have hidden definite properties. In Bohm's version, the forces on a particle at one place depend at once on conditions at distant places, and Greene calls the instant dependence on distant places "yet more blatant nonlocality" (same section). A *photon* is a particle of light. In chapter 15 Greene tells how, by 1997, two groups of physicists had moved the quantum state of a single photon from one place to another, using an entangled pair and an ordinary message.

I call the probability wave the *state*: the complete description of what could be found, and of how likely each finding is.

A Chord and a Click

Before we return to the far particles, listen to one sound. Press a piano key and hold it, and you hear a note with a definite pitch. Squeeze the sound into a very short click, and the click has no definite pitch at all. Give a sound a definite pitch, and the sound must last. Make a sound sharp in time, and its pitch spreads. No sound is both a perfect click and a perfect tone. The uncertainty principle that Greene describes has the same form

as the trade between a sharp pitch and a sharp moment. The reason for the trade holds far beyond sound, and the reason is found in what a comparison is.

What is found in a comparison is its *content*, as Chapter 1 said. The respect in which the content is found is the *context* of the comparison: what makes the two things compared two, as morning and evening make two readings of a thermometer two. Hold fixed the side from which a comparison is made, so that the same comparer works on the same thing compared. Then the content can come out the same or different, and so can the context, so every finding is one of four kinds, which this book calls the four *characters*. You meet all four characters every day. Turn a dimmer knob by a hair and the light changes by a hair: the same content where the context is the same, which is *continuity*. Flip a light switch and the room changes all at once: a different content where the context is the same, which is *discontinuity*. Turn a square a quarter of the way round and it is the same square: the same content across a different context, which is *symmetry*. Walk a rocky coast where no cove repeats the cove before it: a different content across a different context, which is *asymmetry*.

Now ask for two of the four characters at once, each held perfectly. Perfect continuity says that where the context is the same, the content is the same. Perfect symmetry says that where the context is different, the content is still the same. Put perfect continuity and perfect symmetry together, and the content is the same whatever the context. So the only finding the comparison can ever have is "same". But in every comparison, sameness and difference are found together. If no difference could ever be found in a comparison, the comparison would no longer be a comparison. So symmetry and continuity can each be had in part, and together in part, but never both perfectly in one comparison. The same steps, with "same" and "different" exchanged, show that asymmetry and discontinuity cannot both be perfect either, because perfect asymmetry and perfect discontinuity together would make every finding "different", whatever the context.

The result of the last two paragraphs is the *Incommensuration Theorem*: symmetry and continuity cannot both be perfect, and neither can asymmetry and discontinuity. A *theorem* is a statement shown to follow, step by step, from what has already been established.

In the exact words of *An Immanent Metaphysics*, on which every other book of mine is built:

> Nothing anywhere at all can be both absolutely symmetric and absolutely continuous. Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous.

Two pairings of the characters, and only two, remain possible under the Incommensuration Theorem. The first pairing is continuity with asymmetry, like a smooth ramp that rises unevenly up a hill: the ramp has no jump, and no stretch of the ramp is like the stretch before it. The second pairing is symmetry with discontinuity, like a ladder whose rungs are all alike, with nothing to stand on between them: every rung is the same, and the rungs are separate. I call continuity with asymmetry, and symmetry with discontinuity, *the two surviving pairings*. "Absolutely" means perfectly: held exactly, at the foundation, with no exception at any scale. The theorem uses nothing but the meanings of sameness, difference, content and context. So the theorem is true of every comparison, wherever anything is compared. Every observation, every measurement and every test is a comparison. So the Incommensuration Theorem is true of every observation, measurement and test: the theorem is a structural fact about the nature of knowing.

Position and momentum are two descriptions of one state, and either description can be turned into the other by one exact rule. In the same way, a chord can be described by the push of the air from moment to moment, or by the list of notes the chord holds. A state of definite momentum is like the held note of one definite pitch: each is the same wave all along its length, so shifting the wave from place to place changes noth-

ing. So the momentum description has symmetry: the same content (the wave) across a difference of context (the place). The position description has continuity, as the dimmer does. In the position description, the state is described place by place, along a line of places that runs without a gap, just as the knob of the dimmer turns through every position between two others, with no jump. A state of definite position is the sharpest description of the place-by-place kind: one point picked out on the unbroken line. Suppose a description were perfect in position and in momentum at once. The description would then join perfect continuity with perfect symmetry. The Incommensuration Theorem says that nothing can be both. I build quantum mechanics from the act of comparison and state: "The uncertainty principle is an instance of the Incommensuration Theorem". The size of the trade is set by Planck's constant, and Planck's constant is the number that fixes the size of every quantum step. The size of the trade is a matter for physics. The Incommensuration Theorem says why there is a trade between the position description and the momentum description at all, and why no future theory will remove the trade.

Two more of Greene's results follow from the same theorem: the two faces of light and matter (that each behaves both as a wave and as a particle), and the rule of probability (that a physicist can know only the chance of each outcome). Take a wave point by point, at one instant. From one point to the next the wave varies smoothly, with no jump, which is continuity. The *phase* of a wave at a point is how far along its swing the wave has come at that point. The phase advances from each point to the next, so no two points along the wave are at the same phase, which is asymmetry. So a wave, taken point against point, has continuity with asymmetry, like the ramp that rises unevenly up a hill. The held note is symmetric in another respect. When the whole wave of the note is shifted from place to place, the wave is the same wave as before. A particle comes whole, one or two with nothing between. And one electron is as good as another, so swapping two electrons changes nothing anyone can measure. So a particle has symmetry with discontinuity, like the rungs of the ladder. So the wave has the first of the two surviving pairings, continuity

with asymmetry, and the particle has the second, symmetry with discontinuity. One picture that held both faces perfectly would need perfect symmetry and perfect continuity together. The Incommensuration Theorem says that nothing can have both. Which face is shown depends on the act of measuring.

Greene's rule of probability has a reason too. A description that fixed every finding in advance would need one law carrying every state into the next, which is symmetry perfected. The same description would also need every state fixed with no gap, which is continuity perfected. The Incommensuration Theorem says that nothing can have perfect symmetry with perfect continuity. So every lawful description has some outcomes that its law does not fix. The single outcome of a quantum measurement is the plainest case of an outcome that the law does not fix. I call an outcome that no law fixes *hard randomness*, as in Chapter 2. Hard randomness is an outcome that nothing before it fixes, however completely what came before is known. This probability belongs to the world, not to our knowledge of it. Greene's "No one knows" also has an answer. I derive, from the act of comparison, the rules that physicists use without knowing why the rules work. The rules I derive include Feynman's sum over paths and Newton's single path. My conclusion is that the weirdness comes from where physicists start: "The weirdness is an artifact of the starting point, and not a feature of quantum mechanics".

Socks, Enchanted Decks, and a Count No Script Can Pass

Some agreement between far things is ordinary. Bell's colleague Reinhold Bertlmann liked to wear socks of two different colors. See that one of Bertlmann's socks is pink, and you know at once that the other sock is not pink, because the agreement between the two socks was laid out in advance, when Bertlmann dressed. Scully's explanation of the boxes is an agreement laid out in advance, like the agreement of Bertlmann's socks.

Now picture two ordinary decks of playing cards, enchanted together while they lie side by side on a table. One person keeps one deck, and a second person takes the other deck to Paris. Once the two people have parted, each person shuffles a deck thoroughly. Each person then writes down the top ten cards and emails the list to the other person. The two lists are identical. No order laid out in advance can explain the match, because each deck was shuffled after the two people had parted. If one of the two people stacks a deck in a chosen order to send a message, instead of shuffling the deck, the two lists no longer match, so neither person can use the decks to send anything. Physicists can prepare pairs of particles that behave much like the enchanted decks. When two particles are measured far apart and their outcomes agree in a pattern that no separate instructions carried by each particle could give them, the two particles are *entangled*.

In the laboratory a source sends two particles off in opposite directions, and a detector waits at each end. Each particle carries a direction that can be measured, and for an electron the direction is its *spin*. For a photon the direction is the way its light is lined up, which a polarizing filter of the kind in some sunglasses tests. Each detector is turned to one direction of its own, which is its setting. The detector records one thing about the arriving particle: whether the particle points along the setting. In effect, the detector asks the particle one question: do you point my way, yes or no? Each record, read alone, is a random list of yes and no. The pattern between the two records appears only when someone brings the two lists together and sets the lists side by side. When a pattern of agreement is found by setting two records side by side, the pattern is called a *correlation*.

Suppose that each particle carried its answers from the source, as a sealed script. A sealed script is a set of answers written in advance for every question a detector could ask. Suppose also that nothing done to one particle affected what is found at the other particle. Bell proved that the correlations could then never rise above a certain ceiling. In 1969 four physicists gave Bell's test a new form, in which four correlations from four

pairs of settings are combined into one score. The four physicists were John Clauser, Michael Horne, Abner Shimony and Richard Holt. No set of sealed scripts gives a score above two. Greene tells a test of the same kind with boxes that have three doors each, one door for each setting, and Greene counts the agreements as a share of the boxes opened. The test of 1969 is Greene's test in another form. In the test of 1969, each detector has two settings in place of three doors, and the ceiling for sealed scripts is a number, two, in place of a share of agreements. Quantum theory predicts scores up to twice the square root of two, a little more than two point eight. In 1982 Alain Aspect, Philippe Grangier and Gérard Roger measured the score with pairs of photons, and found about two point seven, the value that quantum theory predicts for their arrangement of detectors.

In 2015 three groups of physicists ran experiments at Delft, in Vienna and in Boulder. Each group closed, in one experiment, the main gaps through which some other explanation of the score might slip. At Delft the two electrons of each pair were 1.3 kilometers apart, so no light could pass between the two measurements while the measurements were made, and the score at Delft was about two point four. In 2017 and 2018, physicists chose the settings by the light of stars of our galaxy, and then by the light of quasars. Quasars are the blazing centers of far galaxies. A common cause would be one cause that fixed both the settings and the particles in advance. Any common cause would have had to act about six hundred years earlier for the starlight, and billions of years earlier for the light of quasars. The Nobel Prize in Physics for 2022 went to Alain Aspect, John Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science". So we now know what Greene says we know: no set of sealed scripts can explain the scores that the experiments measured. No account in which every outcome is fixed by hidden local causes can match all the predictions of quantum theory, and the measurements agree with those predictions.

A pair of particles can be in many states. Physicists call the whole range of states that a pair can be in the *space of states* of the pair. I derive the space of states from the act of comparison. No state in the space of states gives a score above twice the square root of two. So quantum theory predicts no score above twice the square root of two, and no score measured in the world goes above twice the square root of two. The steps of the proof are in Appendix B.

Lawful and Local: Two Perfections That Cannot Be Had Together

Why must the sealed scripts fail? To answer, look at two things that scientists ask of the world, and describe each thing with the four characters. Scientists ask for *lawfulness*: the same procedure, under the same conditions, gives the same result in this room, on the Moon, and in any age. Scientists also ask for *locality*: every influence passes from place to neighboring place, with no jump across a gap. I identify lawfulness with a symmetry, the same result across every difference of place and time, and locality with a continuity, an influence carried through the space between two places with no gap. A sealed script is both lawful and local at once: lawful because one rule fixes every result at both ends, and local because each result depends only on what is at its own end.

Bell's theorem is Bell's proof that no set of sealed scripts can give a score above the ceiling. So Bell's theorem is an instance of the Incommensuration Theorem: the Incommensuration Theorem applied to the correlations of far particles. No description of the world can be both perfectly lawful and perfectly local, because no comparison can be both perfectly symmetric and perfectly continuous. I set Bell's theorem and the Incommensuration Theorem side by side and draw the consequence: "No theory of reality can assert both total generality, and total determinism". In the sentence just quoted, total generality is the same law everywhere, which is symmetry perfected. Total determinism is every change fixed in advance, step by unbroken step at every scale, which is continuity perfected. I name the two options that remain under the Incommensuration Theorem: "So

reality can be completely lawful and not wholly local, or completely local and not wholly lawful". Bell found the limit, and found that the predictions of quantum theory go past the limit. The experimenters found that nature goes past the limit too. The Incommensuration Theorem says why a limit on being both lawful and local had to be found somewhere, and why exactly two options are left, one for each of the two surviving pairings. Bell found the result in quantum correlations. The Incommensuration Theorem says that the result had to be found somewhere. Only quantum theory and experiment can show that the measured scores go past the ceiling that Bell derived.

The usual form of physics is the first option: completely lawful, and not wholly local. Physicists keep universal law, which is symmetry, and they accept entanglement as a bounded exception to a purely local description. Entanglement, accepted as a bounded exception, is an agreement through which no one can send a signal: no one can use the agreement to make a record at one place depend on an act at the other place. Physicists study the agreement of entangled particles and put the agreement to use. In the first option, lawfulness, which is symmetry, is kept, and perfect locality, which is continuity, is given up. So the first option is symmetry with discontinuity, like the ladder. In the sense in which Bell's sealed scripts were local, the world is not local. On my account, Greene's result follows from the Incommensuration Theorem.

Hidden Properties, Spreading Links, and a State Sent Ahead

Greene makes three further points: about hidden properties, about the spread of entanglement, and about teleportation. On each of the three points, I agree with Greene's result and state a reason for the result. A *hidden variable* is a quantity beneath the state that would settle an outcome in advance. Bell's theorem shows that hidden variables that are local cannot give the predictions of quantum theory. Bohm's theory of 1952 remains standing after Bell's theorem, as a description that gives the predictions of quantum theory, and in Bohm's theory particles have definite positions

and are guided across space. One more step follows from the Incommensuration Theorem. A whole world fixed completely by one law would have perfect symmetry and perfect continuity together. So no whole world is completely fixed in advance. Bohm's equations fix every outcome, but no whole world is fixed in advance. So Bohm's equations are a map of the world, and not the world that the map describes. The findings that Bohm's equations predict have the statistics of quantum theory, so no finding shows that Bohm's equations fix every outcome. I call the way Bohm's equations fix every outcome "the determinism of a map, and not of a territory".

Greene answers the popular claim that everything is entangled with everything by describing a process. When two entangled photons bump into stray particles on their way, the agreement that an experimenter would look for becomes a pattern of correlations that includes the photons and everything the photons have bumped into. As the stray particles bump into yet other particles, the entanglement spreads into the surroundings through each new bump. Greene writes that the entanglement would finally "become virtually impossible to detect" (*The Fabric of the Cosmos*, chapter 4, section "What Are We to Make of All This?"). I agree with Greene's result. Any two systems that interact directly become entangled. Through interaction with their surroundings, systems become entangled with more and more of the world. When entanglement spreads into the surroundings, the agreement of two chosen particles is hidden in the pattern of correlations that includes everything the two particles have bumped into. Chapter 4 describes the spread of entanglement into the surroundings, and shows that the surroundings of a thing record the alternatives of the thing: the outcomes that could be found of the thing.

Teleportation is the moving of an unknown state from a sender to a receiver who share an entangled pair. I describe teleportation as Greene describes it. In 1997 a team in the group of Anton Zeilinger teleported the polarization of a photon: the way the photon's light is lined up. The team included Dik Bouwmeester, Jian-Wei Pan and their colleagues. In teleportation, the sender makes a joint measurement and sends the result by an

ordinary channel, and the result is a pair of yes-or-no answers, which physicists call two *bits*. The state arrives only when the two bits do, and they travel no faster than the speed of light, the limit of every causal signal. Nothing is copied, because copying a state would need a comparison in which the state itself is found, and a comparer finds only the outcomes for the direction that the comparer is set to measure. Even where physicists put entanglement to work, what crosses the distance is an ordinary signal: the two bits.

Real, and Carried by No Influence

What, then, is the agreement between entangled particles, and does anything pass between the particles? Much of the answer is already in the standard view that Greene reports. Greene writes that physicists say the results are correlated but "do not stand in a traditional cause-and-effect relationship because nothing travels between the two distant locations" (section "Entanglement and Special Relativity: The Standard View"). On my account, the same results follow from what a comparison is, as the rest of this section shows. I differ from Greene on one point: whether the measurement of one particle of an entangled pair acts on the other particle of the pair. In the standard view as Greene tells it, the two photons of an entangled pair are parts of one physical entity. One measurement on the entity "affects both photons at once" (same section). The agreement is real, and nothing carries the agreement: no influence of any kind passes from one particle to the other.

Greene defines locality by what can be affected. Greene writes: "you can directly affect only things that are next to you" (chapter 4, section "The World According to the Quantum"). Greene states the sense in which he finds the universe not local: "The outcome of what you do at one place can be linked with what happens at another place, even if nothing travels between the two locations" (section "No Smoke but Fire"). Greene's sense is the broad sense of "local", which is Bell's. In the broad sense, I agree with Greene's sentence "the universe is not local". In the

narrow sense, I call the universe local: no signal travels faster than light, and no choice made at one detector changes the tally of yes and no recorded at the other detector. Greene states the narrow sense in his own terms: the speed limit of special relativity stays in force.

I set out three wishes about cause and the speed of light. At most two of the three wishes can be kept. The first wish is that every change in the world has a cause. The second wish is that the laws of cause are the same type of law everywhere. The third wish is that every causal process stays within the speed of light. Physicists keep the first wish and the second, and give up the third, and so physicists call the agreement an influence across space that has no speed and can send nothing. The second wish and the third are kept, and the first wish is given up. So in my account some change in the world has no cause. The single outcome of a measurement is the plainest case of change with no cause: the single outcome is hard randomness, met earlier beside the chord and the click, an outcome that nothing before it fixes. Both choices are cases of the same surviving pairing: symmetry with discontinuity. In both choices, locality in Bell's broad sense is given up. Physicists give up locality in Bell's broad sense by admitting an influence with no mechanism, while on my account locality in Bell's broad sense is given up by admitting change with no cause.

The reason for the choice of my account is that a correlation counts as an influence only under the assumption that every change has a cause. Without the assumption that every change has a cause, the correlation is what the experiments show: an agreement between records that carries nothing from one place to the other. The agreement "is a *non-causal correlation*: real, and not an influence".

In 2012 the physicists Jean-Daniel Bancal, Nicolas Gisin and their colleagues showed that any influence carrying the agreement at a finite speed, however fast, could be used to send signals faster than light, and no signal faster than light has ever been found. So no influence in space and time carries the agreement. Gisin calls the correlations nonlocal, meaning that no explanation of the correlations by local causes exists, while I call the world local, meaning that no signal travels faster than

light. The two words “nonlocal” and “local” name one finding from two sides: the correlations are real, no influence carries them, and no signal passes.

Then where is the agreement? Every comparison has three roles, which this book calls the stance, the act and the structure. The *stance* is the side a comparison is made from, with the range of what could come: the direction a detector is turned to, and the yes or no the detector could hear. The *act* is the happening itself: the click. The *structure* is what is laid down, the same for everyone who reads it afterward: the line in the detector’s log, which is the detector’s record. In the three roles, the word “structure” has the meaning that Chapter 2 gave the word: what is laid out whole and stays the same for everyone who looks at it, as the line in the log stays the same for every reader.

The two particles of an entangled pair share one range of what could be found. A pattern can be followed along a street, from one house to the next, and a force can be followed through time. No one can follow the range of what could be found in a comparison along a street or through time. The range goes with the comparer, wherever the comparer goes. Possibility, in this book, means the range of what could be found in a comparison. Possibility has no place of its own along the street. The range of possible findings belongs to the stance from which a comparison is made, and the range has no place of its own. So the agreement of an entangled pair is located at neither detector, and nothing passes between the detectors to make the agreement. Each click is made in its own act, at its own place, and each log alone is random. The agreement becomes a fact where the two logs are brought together, for whatever compares the two logs, and the logs can be brought together only by ordinary means: a cable, a telephone call, a sheet of paper carried across a room. In the standard view that Greene reports, the agreement of the two lists is found in the same place, because the match of the lists “can’t be discerned until the two lists are brought together and compared by some conventional, slower-than-light means”.

No message can be sent by entanglement, because the agreement becomes a fact only where the two logs are compared, and the logs travel only by ordinary means. A second reason depends on no feature of quantum theory at all. In Chapter 2 I identified the speed of light with the edge of comparison, which is the greatest speed at which any signal can carry a finding from one place to another. The edge of comparison is the same for every comparer. Take two events so far apart that light cannot pass between them. The two events have no order of their own. For one comparer the first event comes earlier, and for another comparer, moving differently, the second event comes earlier. For some comparer, a signal joining the two events would arrive before the signal was sent, and the record of the signal would come before the act that the record records. The order of acts runs one way, so a signal that arrived before it was sent could not be a signal at all. The stacked deck shows on a table that no message can be sent. Stacking a deck to send a message breaks the match, so no message is sent. With particles, fixing an answer in advance destroys the shared state on which the agreement depends.

The three roles of a comparison also contain an answer to Greene's worry about the collapse of the probability wave. Greene worried that a collapse across all of space at one moment would pick out one vantage point as special. When a measurement is made, the shared record of what has been found is updated. The collapse of the probability wave is the update of the shared record. The update is not a physical process, and it is not an extra law of motion. A comparer far away receives the finding only by a signal, at the earliest when light could bring the finding. So nothing in a measurement needs one "now" across space. Quantum gravity is the joining of quantum theory with Einstein's theory of gravity. No comparison registers a surface of simultaneity. If one finding caused the other, one of the two findings would have to come first. In 2002 four physicists set the two measuring devices of a test in motion: André Stefanov, Hugo Zbinden, Nicolas Gisin and Antoine Suarez. The devices moved so that, by the clocks that traveled with each device, the finding at that device was made first. The agreement was unchanged, so neither finding needs the

other finding to come first, and neither finding responds to the other. Greene judges that the majority view is likely to prevail: a "harmonious coexistence" of entanglement with special relativity. On my account, the harmonious coexistence follows as a result.

In the argument of 1935 between Einstein and Bohr, my account is on Bohr's side, and has its own reason. Bohr granted that the measurement on the first particle of the pair does not disturb the second particle mechanically, but held that there is still "an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system". The collection stands with Bohr's conclusion, and it supplies the ground he did not supply. Einstein demanded that each particle carry its properties separately, which is a demand for a sealed script. The Incommensuration Theorem says that no description of the world can be a sealed script, because a sealed script is perfectly lawful and perfectly local at once, so Bohr was correct to refuse Einstein's demand.

What the Two Records Show

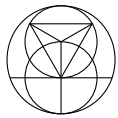
Return to the two enchanted decks, one at home and one in Paris. Each list, read alone, is as random as a shuffle can make it. The match between the two lists appears when someone sets the two emails side by side. Nothing went to Paris but the deck itself, and the match was not in the deck before the shuffle.

So we now know where the agreement of entangled particles is found: where the records of the particles are compared. And we now know that nothing passes between the particles to make the agreement. The agreement that Greene describes is found in every test, from Aspect's photons to the electrons of Delft and the settings drawn from the light of quasars. Causes carried along with the particles cannot explain the agreement. Bell's theorem is an instance of the Incommensuration Theorem, and the scores measured above Bell's ceiling show in the laboratory that the world cannot be both perfectly lawful and perfectly local. Physics and my ac-

count both describe a world that is completely lawful and not wholly local in Bell's broad sense. The universe is not local in Bell's broad sense, and the universe is local in the narrow sense: no signal travels faster than light.

No signal travels faster than light, so the results of Chapters 1 and 2 about space and time are still true. The speed of light is still the edge of comparison. And the agreement of entangled particles needs nothing faster than light, and can carry no signal.

If each record is random until it is made, what makes one outcome happen at all?



Chapter 4

How Does a Single Outcome Happen?

Beside The Fabric of the Cosmos*, chapter 7*, The Hidden Reality*, chapter 8*, The Elegant Universe*, chapter 4, and* Until the End of Time*, chapter 5.*

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

Continuity is the same content where the context is the same, as a dimmer knob turned by a hair changes the light by a hair; *discontinuity* is a different content where the context is the same, as when a light switch is flipped; *symmetry* is the same content across a different context, as a square turned a quarter of the way round is the same square; *asymmetry* is a different content across a different context, as on a rocky coast where no cove repeats the cove before it (Chapter 3).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

Every comparison has three roles: the *stance*, the side a comparison is made from, with the range of what could come; the *act*, the happening itself; and the *structure*, what is laid down in the act, the same for everyone who reads it afterward (Chapter 3).

Hard randomness is an outcome that nothing before it fixes, however completely what came before is known (Chapter 2).

In 1989 the physicist Akira Tonomura and his colleagues worked in a research laboratory of the Hitachi company in Japan. Tonomura's team sent electrons, one at a time, toward a screen. The screen marked the place where each electron arrived. Watch the screen with Tonomura's team.

When the first mark appears, we can ask the question of this chapter: if the quantum state, the description of what could be found, holds many possibilities, what makes one of those possibilities the one we find?

Electrons are the tiny particles of negative electric charge that surround the core of every atom. Chapter 3 gave the quantum state its plain meaning, and I need that meaning again: *the state* is the complete description of what could be found when a thing is measured, and of how likely each finding is.

Tonomura's screen showed what single electrons do. The electrons left their source one at a time, so each electron traveled to the screen on its own, and on the way each electron passed a device called an *electron biprism*. A double slit is two narrow openings side by side. The biprism divides an electron's wave in two, as a double slit does. Every electron arrived as a single dot, at a single place. No electron arrived as a smear across the screen, and no electron arrived in two places. At first the dots looked scattered at random. As the dots gathered in their thousands, the dots drew a pattern of bright and dark stripes.

When two waves meet, the waves add to each other where crest meets crest and cancel each other where crest meets trough, and physicists call the adding and canceling *interference*. Interference produces bands of bright and dark. Tonomura's stripes are bands of interference. Each electron came through alone and arrived whole. Yet the many electrons together drew the bands of a wave that had passed on both sides of the biprism. Physicists say that each electron's state was a *superposition*: a combination of the state for one side and the state for the other, with an amount of each. Before each electron set out, the electron's state said how likely each place on the screen was, but the state did not say at which place the electron would be found. Every time, something at the screen settled the place. What settled the place where each electron was found?

From Greene's pages

Greene's Puzzle: Smooth Waves and Single Spots

Greene takes up the puzzle of the single spot in all four of his books. In *The Elegant Universe* (chapter 4, section "Waves of What?"), Greene tells the reader that everyone agrees on how to use the equations of quantum theory to make accurate predictions. Greene adds that physicists have no consensus on what the waves of probability of quantum theory mean (the waves of probability are Greene's name for what this book calls the state). Physicists also have no consensus on how a particle comes to one of its many possible futures.

In *The Fabric of the Cosmos*, chapter 7, Greene sets the puzzle out in two stages, and in the first stage, the wave of probability changes smoothly, by *Schrödinger's equation*. Schrödinger's equation is the rule by which the state changes in time when nothing is measured. The equation is named for the physicist Erwin Schrödinger. In the second stage, when a measurement is made, the wave *collapses*: the wave jumps, all at once, to the one outcome found. Greene asks why the experimenter and the instrument should obey a different rule from the things they study, since the experimenter and the instrument are made of the same atoms as those things. Greene tells how the physicist Niels Bohr answered: large things obey the classical laws, the laws of the physics before quantum theory. Bohr held that somewhere between atoms and people the rules change. Greene then asks where exactly Bohr's border between atoms and people lies. Bohr declared the question out of bounds. Greene calls the puzzle by its usual name, the *quantum measurement problem*, and he reports the rank that many physicists give the measurement problem:

> "Many view this as the single most important gap in our formulation of quantum law" (*The Fabric of the Cosmos*, chapter 7, section "The Quantum Measurement Puzzle").

In *The Hidden Reality*, chapter 8, Greene follows Schrödinger's equation through a measurement, and he shows that the equation never produces the jump to one outcome. Greene describes an idea that, in his words, "if realized, would be just about the best thing we could hope for" (*The Fabric of the Cosmos*, chapter 7, section "Decoherence and Quantum Reality"). The idea rests on a plain fact: the things of everyday life are never alone. Air molecules and particles of light strike everyday things without pause. Each strike blurs the orderly crests and troughs of the waves of an everyday thing, so a big object's chances come to behave like the chances of a tossed coin, which show no interference. Physicists call the washing out of interference by the strikes of air and light *decoherence*. Greene pictures decoherence with a cat in a closed box. The cat's surroundings make "billions of observations" of the cat long before anyone opens the box. Does decoherence solve the puzzle of the single outcome? Greene reports that some researchers claim to have made decoherence into a complete framework that solves the measurement problem, and the physicists Robert Griffiths, Roland Omnès, Murray Gell-Mann and Jim Hartle are among those researchers. Greene answers for himself:

> "Others, like myself, are intrigued but not yet fully convinced" (*The Fabric of the Cosmos*, chapter 7, section "Decoherence and Quantum Reality").

Greene's reason for doubt is that when interference is washed out, every outcome is still in the description, and the description has no further detail that picks the outcome that happens. Greene writes that he strongly believes collapse will in the end be done away with (*The Fabric of the Cosmos*, note 11 to chapter 7). Greene also suspects that once the puzzle is settled, the laws will still be the same with time run backward (the last section of chapter 7 of *The Fabric of the Cosmos*, "Quantum Mechanics and the Arrow of Time").

In *The Fabric of the Cosmos*, chapter 7, Greene also asks what the quantum puzzle means for the past. The physicist Richard Feynman calculated what a particle does by adding up every path the particle could take. Greene tells how, in Feynman's way of adding up paths, the present that is observed is a blend of all the histories that could have led to the present (sections "Prochoice" and "Pruning History"). Greene says that he cannot settle how literally Feynman's picture of the present as a blend of histories should be taken. The physicist John Wheeler proposed the delayed-choice experiment. In the delayed-choice experiment, the experimenter chooses whether to reveal which path a particle of light took, and makes the choice only after the particle has passed the point where its paths divide. Wheeler imagined making the choice on light that left a distant *quasar*, the blazing center of a far galaxy, billions of years ago. Does a choice made today change what the light did billions of years ago? Greene answers in two words: "Certainly not" (*The Fabric of the Cosmos*, chapter 7, section "The Contingency of History"). Greene also describes the quantum eraser, an experiment in which each particle of light has a partner particle. In the quantum eraser, the choice of a later measurement on the partner particle determines whether interference bands can be drawn out of data already taken. Greene stresses that the data already collected stay exactly as they were after the later measurement, and he states the lesson in one line:

> "We thus see that the future helps shape the story you tell of the past" (*The Fabric of the Cosmos*, chapter 7, section "Shaping the Past").

In *Until the End of Time*, chapter 5, Greene weighs a proposal of the physicists Fritz London, Edmond Bauer and Eugene Wigner: that consciousness turns many possibilities into one. Greene judges the pro-

posed link between consciousness and the turning of many possibilities into one "extraordinarily unlikely", and he states what experiments support:

> "Interactions—not consciousness—coax the emergence of a definite reality" (*Until the End of Time*, chapter 5, section "Consciousness and Quantum Physics").

Greene grants one point: no argument can show, beyond every doubt, that consciousness plays no special role, because even to check a result, Greene must be conscious of the result.

Last, in *The Hidden Reality*, chapter 8, Greene sets side by side two rival views of what the quantum rules mean. The first view is the view of Bohr's school in Copenhagen, and the second view is the view of many worlds, in which every outcome of a measurement occurs, each outcome in a branch of its own. Greene shows that the Copenhagen view and the many-worlds view would, in principle, predict different results, but the difference would show only if two large, plainly different alternatives could be made to interfere. Greene notes that, because of decoherence, making two large, plainly different alternatives interfere is "an extraordinarily formidable task". So, Greene writes, to call the two views mere "interpretations" is off the mark. Greene insists, too, that the aim of physics is more than prediction:

> "There's a difference between making predictions and understanding them" (*The Hidden Reality*, chapter 8, section "Predictions and Understanding").

A Finding Needs No Watcher

Greene's result on interactions also follows from my account: interactions, and not consciousness, bring about one definite outcome. The Immanent Metaphysics is the study of what every comparison must contain. In the Immanent Metaphysics, every step of reasoning is made from inside the act of comparison, and no step assumes a place outside the world from which the world could be looked at. The first step in my account is what a finding is.

In Chapter 1 I defined the words that my argument needs. The content of the comparison is that the readings differ, and the context is the time of day. A *comparer* is whatever makes a comparison and registers what the comparison finds: a person, an instrument, or any piece of the world that registers one thing against another.

Now picture a horse crossing a beach before anyone is awake. Each hoof presses the sand, and the sand takes one shape and not another. When the sand takes one form, out of the forms the sand could have taken, matter has already made the simplest comparison. A hoofprint in sand shows how plain such a record can be. At noon a walker comes along and reads the print, and so makes a second comparison, of the first. The first comparison was finished before the walker woke.

Any comparison that lays down a record makes a finding. A comparison that lays down a record is an interaction after which some part of the world carries a mark of the outcome. Each dot on Tonomura's screen was a finding of exactly that kind: an interaction left a mark of the outcome on the screen. The screen or the counter, as an interaction, made the finding, and no mind is needed. An observer is a comparer: the word names a position inside the world, and it never names a mind.

Greene grants that his result on interactions cannot be proved beyond every doubt. On my account, the result follows from what a finding is. A result proved inside quantum theory shows the same thing that I show: a finding needs no mind. In 1932 the mathematician John von Neumann

studied the chain of instruments that measures a thing, from the first detector to the eye of the person reading the dial. Von Neumann showed that the line between the thing measured and the chain of instruments can be placed at any link of the chain. Wherever the line is placed, every predicted count comes out the same. So the end of the chain, a brain included, adds nothing that the predictions need: an interaction is a finding when the interaction lays down a record, and nothing more is needed.

Newton's bucket, from Chapter 1, shows in water that a finding needs no watcher. Hang the bucket from its twisted rope, let the bucket go, and leave the room: the water turns and climbs the walls of the bucket into a hollow, whether anyone is in the room or not. Chapter 1 showed that the hollow is found by measuring the water's turning against the other things the water can be compared with. The water and the world the water turns against make the comparison of the water's turning between them. The hollow is the record the comparison lays down. Come back in a minute and look, and you make a second comparison, of the first. The hollow is a finding made by matter, with no one watching.

What the Surroundings Write

Greene's result on decoherence also follows from my account: when interference is washed out, every outcome is still in the description. I define decoherence as "the fading of interference between the alternatives of a system as its surroundings come to carry records of which alternative it is in". Think of the surroundings as a notebook that each alternative writes in. If two alternatives leave the notebook exactly the same, the notebook holds no record of which alternative happened, and the two alternatives still interfere in full. If the two alternatives leave two notebooks that differ on every page, no measurement on the thing alone can show any interference between the two alternatives. For an instrument, a cup or a cat, countless molecules of air and particles of light strike the thing, and every

one of them carries away a slightly different mark of each alternative, as if writing a slightly different page. So the interference drops out of view almost at once.

Experimenters have watched the fading happen. In 1999 the physicists Markus Arndt, Anton Zeilinger and their colleagues in Vienna showed that whole molecules build bands of interference as single electrons do, using molecules that were closed cages of sixty carbon atoms, shaped like a football. In 2003 Klaus Hornberger, Arndt, Zeilinger and their colleagues sent cages of seventy carbon atoms through an *interferometer*, an apparatus that splits a beam into paths and brings the paths back together. In a good vacuum (a chamber pumped nearly empty of gas), the cages built bands of interference. Then the experimenters let a little gas into the chamber. As the pressure rose, the bands faded. The bands faded because each molecule of gas that struck a cage carried away a trace of which path the cage had taken. When Schrödinger's equation is applied to the cages and the gas together, the equation predicts the fading step by step. So we now know that the surroundings of a thing record the thing's alternatives, and that the recording hides the interference between the alternatives.

Decoherence is the reason we never see interference between plainly different states of a large thing. Decoherence also determines which quantity the surroundings keep records of. The recorded quantity is the quantity whose values stay steady while the surroundings keep striking the thing and keep recording that quantity. In the Vienna experiment of 2003, the recorded quantity was the path of each cage. The molecules of gas recorded each path. So the surroundings determine which alternatives of a thing stop interfering. The alternatives that stop interfering are the alternatives that differ in the quantity the surroundings record, as the two paths of a cage did. I then agree with Greene that decoherence does not determine which one outcome happens. Greene's cat shows both what decoherence does and what decoherence leaves to the act of finding. The cat is warm and breathes, and light and air strike the cat every instant, so the cat's living alternative and the cat's dead alternative stop interfering long before anyone opens the box. Each alternative then carries its own record,

written into the air, the light and the walls, and the act of finding settles which of the two records the opened box will show, while decoherence does not. Decoherence is not collapse.

I also say why decoherence cannot determine the outcome. Chapter 2 showed that physics has three times, and two of the three times matter for decoherence. The first time that matters for decoherence is *evolution time*: the time along which the state, the description of what could be found, changes smoothly between findings. The second time that matters for decoherence is *event time*: the order in which findings are made and laid down as records, the order that a counter's clicks mark. Everything decoherence produces belongs to evolution time, while a finding is one outcome, given in event time. Neither time can be turned into the other, so a smooth calculation produces at most the range of outcomes, each outcome with its likelihood. Decoherence prepares the conditions for a finding, and the act of comparison completes the finding, with one outcome.

The Late Choice Picks the Next Comparison

Greene's answer "Certainly not" also follows from my account: a choice made today does not change what the light did billions of years ago. Physicists have performed Wheeler's delayed-choice experiment. In 2007 the physicists Vincent Jacques, Alain Aspect, Jean-François Roch and their colleagues sent single particles of light into an interferometer. After each particle had entered, the experimenters decided whether to put in the second *beam splitter*, the device that brings the two paths together. With the second beam splitter in place, the light built interference bands. Without the second beam splitter, each particle was found on one path. The result was the result that quantum theory predicts. So we now know that a choice made after a particle of light has entered the interferometer determines which of the two patterns the light can show.

The late choice determines which comparison is made. The comparison made either registers the particle's path or registers the relation between the particle's two paths. The path becomes a fact only when a

comparison registers the path. The relation becomes a fact only when a comparison registers the relation. The reason is the principle of comparison, which Chapter 1 named: a difference that no comparison can register is no difference. Every record already made stays as it was. Nothing is sent backward in time. Wheeler, following Bohr, used the word *phenomenon* for a happening that becomes a fact only when an irreversible act (an act that cannot be undone) brings the happening to a close. So the experiment shows what Wheeler said it shows, that only the act closes the phenomenon, and the experiment shows this with no reach into the past.

In 2000 the physicists Yoon-Ho Kim, Rong Yu, Sergei Kulik, Yanhua Shih and Marlan Scully performed the quantum eraser with a delayed choice. In the eraser, one small partner particle holds *the mark of which path* (a record of which way the particle went). The mark has spread no further. In the experiment of 2000, each particle of light had a partner. The detectors registered the partner at least eight billionths of a second after the particle itself. The partners went to four detectors: two that kept the mark of which path, and two that erased the mark. When the landing places of the particles were sorted by the two detectors that kept the mark, the places showed no bands. When the same places were sorted by the two detectors that erased the mark, the places showed bands of interference, and the two sets of bands were shifted by half a stripe against each other. So we now know that while the mark of which path is held by one small partner, no lasting record of the path has been laid down. We also know that, before any lasting record is laid down, a measurement on the particle and the partner together can bring the bands back.

So Greene's "story you tell of the past" has a plain meaning in my account. A new stance comes after every finding already made. A reversal is itself a further act, whose own findings come after the findings it is said to undo. The experimenter's later choice settles which comparison is made next, and the records already made stay as they were.

Two Things Called the Measurement Problem

My result differs from Greene's result on the measurement problem, which, in Greene's report, many physicists call "the single most important gap".

Followed through a measurement, the smooth rule alone (Schrödinger's equation) never gives one outcome. I describe a case under the smooth rule alone: one silver atom meeting one screen. A magnet first splits the atom's state into two parts. One part would land the atom on an upper spot of the screen, and the other part would land the atom on a lower spot. By the smooth rule alone, the screen ends in a superposition of having shown both spots, but every real screen shows one spot. Tonomura's screen is like the silver atom's screen, with many places on the screen instead of two spots. In 1935 Erwin Schrödinger put the difficulty of the single outcome as his cat: by the smooth rule alone, a cat in a closed box ends in a superposition of alive and dead. In 1995 the philosopher Tim Maudlin put the difficulty as three claims that cannot all hold together. First, the state says everything about a thing. Second, the state always changes by the smooth rule. Third, every measurement has one definite outcome. On the measurement problem, my result is different: the name "measurement problem" covers two different things.

The first thing that the name covers is a clash between two rules. The two rules are the smooth rule of change and the rule of the sudden jump at a measurement. The clash comes from the way physicists put quantum theory together. The rules of quantum theory came from separate sources between 1926 and 1932. Schrödinger modeled his equation of 1926 on the older mechanics of motion. In 1926 too, the physicist Max Born guessed a rule for the odds of each outcome. Experimenters have brilliantly confirmed Born's guess. In 1932 von Neumann set the jump down in rules, taking the jump from the practice of the laboratory. Each rule works, but nobody derived the rules together from one source. Both rules follow from one source: the act of comparison, and the roles that every comparis-

on has. In the derived form, each of the two rules applies to its own thing: between findings, the state describes what could be found, and the description changes by the smooth rule. At a finding, the act gives one outcome, and the new outcome is added to the record of what has been found. I call the change in the record the *update of the record*: the shared record of what has been found changes, the same for everyone who has the outcome. The smooth rule applies to the description of what could be found, and the jump applies to the record of what was found, so when each rule is applied to its own thing, both rules hold together.

Of Maudlin's three claims, the second and the third hold in my account, and the first holds in one of its two readings. The second claim holds: between findings, the state changes by the smooth rule. The third claim holds: each comparison has one outcome. The first claim can be read in two ways. If the first claim says that the state is complete as a description of what could be found, the first claim holds. Given what has been found so far, the state says everything about what could be found next. Nothing lies beneath the state. Maudlin's own wording says more: that the state specifies all the physical properties of the thing, the outcome of each measurement included. Read in Maudlin's own wording, the first claim is the one claim of the three that does not hold in my account. The state does not contain the act of finding, and no property of the thing could supply the act. On both readings of the first claim, the state is complete as a description of possibility, and the outcome is given only in the act. Chapter 2 gave exact meanings to the words *exist* and *real*. In the meanings of Chapter 2, the state exists: the state stands as structure, the same in every instance. The outcome is real: the outcome is given in an actual act of comparison, here and now.

The name "measurement problem" also covers a second thing: the gap between the smooth description and the single finding. I call the gap the *residual*: what is left over in every finding that the law does not fix. The residual stays. The *Incommensuration Theorem*, which Chapter 3 introduced, states the reason. The Incommensuration Theorem says that nothing can be both perfectly symmetric and perfectly continuous. In the theorem,

symmetry is the same content across a different context, as a law gives the same result in every place and every age. If a description contained every finding exactly, the description would be both perfectly symmetric and perfectly continuous: one law carrying every case, with no gap anywhere. So the gap between the smooth description and the single finding is part of comparison itself, by the Incommensuration Theorem. No future theory can be free of the gap. *Born's rule* bridges the gap: physicists use Born's rule to get the odds of each finding from the smooth description.

On collapse, I agree with what Greene strongly believes: collapse, as a physical process, is done away with, because in my account the change of the state at a finding is the update of the record. The update is not a physical process, and it is not an extra law of motion.

On time, too, my result is different from Greene's result: Greene suspects that the laws will stay the same forward and backward once the measurement puzzle is settled. Greene's suspicion bears on this chapter's question. If the laws stay the same backward, could the step in which one outcome is given be run backward? The answer is no: each *passage* runs one way. A passage is the step from one finding to the next: a record, then the stance of the next comparison, then the act, then a new record. Many possibilities lead to one act, and many acts to one record. Return to the horse on the beach. Before the hoof came down, the sand could have taken many shapes. The press of the hoof gave the sand one shape. A step back goes from the one print to the many shapes the sand could have taken. The step back is a list of what could have been, set beside what is. No step back happens on the beach. A step from many to one is a passage. A step from one to many is a relation between things, and nothing passes along a relation. So each passage from one finding to the next has a direction: the one-way order of findings. In Chapter 2, I identified the one-way order of findings with the direction of time.

Where Bohr's Line Falls

Greene asked where Bohr's border between the small things of quantum theory and the large things of the laboratory lies. Why does the line exist, and why does the line have no fixed place?

Every comparison has a side from which the comparison is made and a side about which the comparison is made: in a measurement, the instrument is the side from which, and the thing measured is the side about which. The line between the two sides is called *the cut*: the boundary between what is described by the state and the instrument that records the finding. Picture a thermometer in a closed room, a camera that photographs the thermometer, and a person who later looks at the photograph. When the camera records the thermometer, the camera is the side from which, and the thermometer is the side about which. When the person looks at the photograph, the camera and its photograph are the side about which, and the person is the side from which. The cut has moved, because a different comparison is being made. The comparison performed sets where the cut falls. Moving the cut is performing a different comparison.

I identify the cut with the boundary between the smooth description and the findings: on one side of the cut is the state, which describes what could be found, and on the other side is the record of what was found. The cut can be moved along the chain of instruments without changing any prediction, as von Neumann showed. If the line were a boundary between two kinds of object, such as small things and large things, the line would have a fixed place. The cut has no fixed place. Bohr's border lies between the description and the finding, so the border lies at no one object.

A Stool with Three Legs

Physicists call the rival views of what the quantum rules mean *interpretations*. In *The Fabric of the Cosmos*, chapter 7, Greene describes the interpretations and finds that the interpretations agree on the predictions for

"most every experiment" (section "Reality and the Quantum Measurement Problem"). I agree with Greene, and say that the agreement ends at decoherence: "Up to that place all of them agree, because all of them use the same smooth equation and the same fading of interference". At decoherence, the interpretations stop agreeing, because the physicists who hold each interpretation add something there, or decline to add anything. Greene's point of principle also concerns decoherence: the Copenhagen view and the many-worlds view would differ only if two alternatives that decoherence has already stopped from interfering could be made to interfere again.

Picture a three-legged stool. On three legs, the stool stands on any floor. Saw off one leg, and the stool tips over, toward the place where the missing leg stood. Glue two of the legs into one thick leg, and the stool tips again, toward the gap that the moved leg left.

Every comparison that makes a finding has three roles, which Chapter 3 named the stance, the act and the structure. The stance is the side a comparison is made from, with the range of what could be found. For a detector, the structure is the line in the detector's log. The three legs of the stool stand for the three roles. I show that each interpretation sets one of the three roles aside, either by dropping the role or by gluing the role into another role. Each interpretation then has its own known difficulty, and the difficulty concerns the role that the interpretation sets aside. Likewise, the stool tips toward the place of the missing leg.

The *Copenhagen interpretation* is the view of Bohr's school. In the Copenhagen interpretation the finding comes first, as the finding does in my account, and the Copenhagen interpretation agrees with what physicists do: every experiment ends in a record that can be read. The Copenhagen interpretation leaves out the stance and its range of possibility. Between findings, the Copenhagen interpretation treats the state as a tool of calculation and nothing more. The Copenhagen interpretation has a known difficulty: where does the cut lie? The picture of the thermometer and the camera answered the question of where the cut lies: the comparison performed sets where the cut falls.

The physicists GianCarlo Ghirardi, Alberto Rimini and Tullio Weber began the interpretations called *collapse models* in 1986, and collapse models change the smooth rule itself, so that a superposition of a large thing ends of its own accord. The first achievement of collapse models is that they make outcomes part of physics, with one equation for both the atom and the instrument. The second achievement of collapse models is that experimenters can test them. Collapse models make the act of finding part of the smooth change: in the picture of the stool, collapse models glue two legs into one. The known difficulty of collapse models is that collapse models need new physics that experimenters have not found. Every test so far agrees with quantum theory. In 2019 Yaakov Fein, Markus Arndt and their colleagues in Vienna saw interference with molecules of more than twenty-five thousand atomic mass units. One atomic mass unit is about the mass of one hydrogen atom. Even at more than twenty-five thousand atomic mass units, the molecules still kept their alternatives. The physicists Lajos Diósi and Roger Penrose proposed a collapse model in which gravity itself ends a superposition, and in which the more mass the alternatives put in different places, the faster the superposition ends. In the Diósi and Penrose model, the random collapses would shake the electrons of ordinary matter. The shaken electrons would give off faint X-rays. In 2021 Sandro Donadi and his colleagues searched deep underground for the faint X-rays that the model predicts, and Donadi and his colleagues ruled out the natural version of the model, the version with no adjustable constant. In searches still to come, experimenters will test whether a superposition of a still larger mass ends of its own accord. Appendix A gathers the searches still to come.

In the *many-worlds interpretation*, the smooth rule governs everything, and every outcome occurs, each outcome in a branch of its own. The achievement of the many-worlds interpretation is one rule of change, applied to everything, observers and instruments included, with no second process. Greene writes of the many-worlds interpretation (*The Hidden Reality*, chapter 8, section "Predictions and Understanding"): "And should the Many Worlds approach be right, what a spectacular reality our un-

wavering commitment to understanding predictions will have uncovered". The many-worlds interpretation sets aside the act, the role that gives one outcome, and in the picture of the stool, the many-worlds interpretation saws off the leg that stands for the act. The known difficulty of many worlds concerns the act, and the difficulty has two parts. The first part of the difficulty is that the state can be divided into alternatives in many ways. The branches need one way of dividing the state to be chosen. Decoherence supplies one division, by the quantity the surroundings record, as the path of each cage was recorded in Vienna. But to choose a division is to choose which outcomes count as having happened. Choosing which outcomes happened belongs to event time, the order of findings. The second part of the difficulty is that, when every outcome occurs, a probability needs someone who faces the outcomes and weighs them. Chapter 12 discusses many worlds in full, among the other worlds Greene describes.

In the picture of the stool, I keep all three legs: the state is the complete description of what could be found, the act gives one outcome, and the record stays. My account also contains what Greene demands of physics: an explanation of its predictions, and not only the predictions. In its first volume: "an answer that only predicts leaves the why unanswered". The picture of the stool contains a reason: the picture says why each interpretation's difficulty concerns the role that the interpretation sets aside.

One Dot, One Act

Go back to Tonomura's screen. Before each electron set out, the electron's state described every place where the electron could be found, with the likelihood of each place. Nothing beneath the state fixed the place in advance. At the screen, one interaction laid down one dot: a comparison made by matter, finished whether or not anyone was looking. And nothing done later, to the electron that made the dot or to any other electron, moved the dot after the dot was made.

In this chapter I have found that what many physicists, in Greene's report, call "the single most important gap" is two plain things. The first thing is the clash of two rules. The clash came from the way physicists put the rules together, and the two rules hold together when both rules come from one source. The second thing is the gap between a smooth description and a single finding: the gap is part of every act of comparison, and the Incommensuration Theorem says why the gap is there.

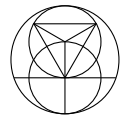
So we now know that the state describes completely what could be found, and that nothing beneath the state fixes which possibility is found. One outcome is given in an act of comparison, which is any interaction that lays down a record. And no later choice changes a record already made.

If what is found comes in single findings and the arena of space and time is smooth, can one theory hold both the single findings and the smooth arena?

PART THREE

One Theory?

Beside *The Elegant Universe*, *The Fabric of the Cosmos*, chapters 12 and 13, and *The Hidden Reality*, chapters 4 and 5.



Chapter 5

Why Can't Relativity and Quantum Mechanics Both Be Right?

Beside The Elegant Universe, *chapters 1, 5 and 6, and* The Fabric of the Cosmos, *chapters 12 and 16.*

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

Every comparison has three roles: the *stance*, the side a comparison is made from, with the range of what could come; the *act*, the happening itself; and the *structure*, what is laid down in the act, the same for everyone who reads it afterward (Chapter 3).

In May 1899 Max Planck, professor of theoretical physics in Berlin, closed a long paper on the radiation of hot bodies with a proposal: units of length, mass and time built from the constants of nature alone. More than a century later, the physicist Brian Greene tells his readers that the two great theories of physics, the theory of gravity and the theory of atoms, conflict at Planck's length. A reader of Greene then asks what happens at the Planck length: does it take string theory to resolve the conflict there? The *Planck length* is one of the units Planck proposed in his 1899 paper: a length built from the constants of nature alone. The Planck length is far smaller than anything yet measured.

Planck had found a new constant of nature in the glow of a hot oven. In December 1900 Planck's new constant fixed the size of the steps in which matter and light trade energy. Each step is called a *quantum*. The constant has carried Planck's name ever since. In 1899 Planck saw a further use for the new constant. When Planck set his constant beside the speed of light and the constant of Newton's gravity, the three numbers together gave a length, a mass and a time that owed nothing to any ruler, weight or clock that people had made. The Prussian Academy of Sciences printed Planck's paper in the Academy's reports. Planck wrote in the paper that units built from the constants of nature would keep their meaning "for all times and for all civilizations, even those beyond the Earth and beyond the human", and he called the new units natural units. The physics of 1899 could not yet say what would happen to a body as small as Planck's unit of length and as heavy as his unit of mass. An answer needed two later theories: Einstein's theory of gravity, sixteen years away, and the quantum theory of matter, more than twenty-five years away. Every one of Planck's units contains both the constant of gravity and the constant of the quantum, so both of the theories that later grew from those two constants are needed to describe what happens at Planck's length.

Chapter 4 ended by setting two facts side by side. The first fact is that what a measurement finds comes in single findings. Each finding is an outcome laid down as a record. The second fact concerns the *arena*: space and time taken together, as the setting in which everything that is measured is measured. The arena is smooth. Greene's books ask whether the theory of the smooth arena and the theory of the jittering quantum world can be joined in one theory. Greene's answer is that the two theories, as now formulated, cannot both be right.

From Greene's pages

The Clash Greene Describes

The first theory is *general relativity*, Einstein's theory of gravity, in which gravity is the shape of space and time. The second theory is *quantum mechanics*, which physicists also call quantum theory. In quantum mechanics, matter and light trade energy in steps, and each measurement gives a single finding. Greene opens *The Elegant Universe* with the trouble between the two theories: "As they are currently formulated, general relativity and quantum mechanics cannot both be right" (*The Elegant Universe*, chapter 1, opening). Greene reports that physicists have confirmed virtually all the predictions of each theory, in its own domain, to "almost unimaginable accuracy", but that the trouble comes only where both theories must be used at once.

Greene finds the source of the trouble in the *uncertainty principle*. The uncertainty principle is the rule of quantum mechanics that a particle cannot have both a definite position and a definite velocity. When the uncertainty principle is applied to a field, the principle says that over short distances the field cannot sit still: the value of the field jitters up and down, and the shorter the distance, the wilder the jitter. For Greene the gravitational field is the shape of space itself, so a jittering gravitational field means a shape of space that jitters. Greene writes that below the Planck length the violent jitters turn smooth space into what the physicist John Wheeler named the *quantum foam*. The quantum foam is a froth in which left and right, before and after, lose their meaning. Greene writes, "The equations of general relativity cannot handle the roiling frenzy of quantum foam" (*The Elegant Universe*, chapter 5, section "General Relativity vs. Quantum Mechanics").

Greene shows how small the Planck length is in two ways. The Planck length is a millionth of a billionth of a billionth of a billionth of a centimeter. If an atom were magnified to the size of the known universe, the Planck length would grow only to the height of a tree. Greene reports that calculations merging the equations of the two theories typically give one and the same answer: infinity, a value no dial ever

shows. The infinity matters in places that are both tiny and massive. One place that is both tiny and massive is the center of a *black hole*. By general relativity, all the mass of a black hole is crushed together at the center of the hole. The other place is the first moments of the universe. Greene writes in *The Fabric of the Cosmos* that, because of the conflict between the two theories, "we remain ignorant about what happened at the beginning" (chapter 12, section "Does It Matter?"). Greene marks our ignorance of the first moments of the universe with a "fuzzy patch" at the start of his drawing of cosmic history.

Greene's cure for the conflict is *string theory*. String theory is the proposal that the smallest pieces of matter are not points but tiny vibrating strands of energy, called strings, each about a Planck length long, and that each kind of particle is one way a string can vibrate. Greene explains the cure with a picture of two art students who must draw a peach pit without looking at the pit, so the students learn the shape of the pit only by firing things at the pit and watching the things bounce. Marbles fired at the pit show a lump. Small pellets fired at the pit show the ridges. So no probe can show detail much finer than the probe itself. In Greene's picture the marbles and pellets stand for the probes of physics, and in string theory the finest probes are strings. A string has a size. In 1988 the physicists David Gross and Paul Mende showed that pumping more energy into a string makes the string larger rather than sharper, so no string can probe a length below the Planck length. Greene concludes that the violent jitters below the Planck length "cannot be measured and hence, according to string theory, do not actually arise" (*The Elegant Universe*, chapter 6, section "The Rough Answer"). Greene argues that the clash came from picturing particles as points of no size, and he writes: "In a sense, therefore, the central conflict of contemporary theoretical physics has been a problem of our own making" (same chapter, section "A Sleight of Hand?").

Greene draws a second result from string theory. One way a string can vibrate has exactly the properties expected of the *graviton*, the particle long proposed as the quantum of gravity, with no mass and no charge. So string theory is, for Greene, the long-sought quantum theory of gravity. Greene reports that there is as yet no consensus on what happens to space and time below the Planck length. In Greene's report there are two possibilities. First, below the Planck length, space and time may be grainy, made of least pieces like the squares of a grid, so that no distance is shorter than the Planck length. Second, space and time may turn by degrees into other, more basic ideas. The second possibility is the one that, in Greene's words, "Many string theorists, including me, strongly suspect" (*The Fabric of the Cosmos*, chapter 12, section "Cosmic Fabric in the Realm of the Small").

In the last chapter of *The Fabric of the Cosmos*, Greene goes further: "many of today's leading physicists suspect that space and time, although pervasive, may not be truly fundamental" (chapter 16, section "Are Space and Time Fundamental Concepts?"). On that view, space and time may *emerge* from something more basic. In Greene's sense, to emerge is for one thing to arise out of the joint behavior of other, more basic things. In Greene's own example, the hardness of a cannonball emerges from the joint behavior of the cannonball's atoms.

Where the Two Descriptions Meet

I agree with Greene on where both theories are needed at once. My account has its own starting point, the Immanent Metaphysics, and the situations that need both theories at once follow from that starting point. The Immanent Metaphysics is the study of what every comparison (every act of finding whether two things are the same or different) must contain. I name the three situations that physicists usually give: a mass held in two places at once, a black hole, and a scale far below anything yet measured.

Greene names the center of a black hole, the beginning of the universe, and the Planck length. In both lists, each situation needs both theories at once to describe one thing.

Every mass can be given two sizes, one from each theory, and the two sizes show why some situations need both theories at once. A mass's *gravitational size* is the length at which the mass's own gravity would become strong: if the mass were squeezed inside about twice its gravitational size, the mass would be a black hole. The gravitational size grows with the mass. The Earth's gravitational size is about four and a half millimeters, and the Sun's is about one and a half kilometers. The second size comes from a trade that the uncertainty principle sets: to pin a thing down more finely takes a probe of shorter wavelength, and so a probe of more energy. Enough energy gathered in one place makes new particles, because mass and energy turn into each other. So every mass has a *quantum size*: the length within which the mass cannot be pinned down, by the rules of quantum theory, without spending enough energy to make new particles of its kind. The quantum size shrinks as the mass grows. The quantum size of a proton is about an eighth of the proton's own width, while the quantum size of the Earth is so small that writing that size as a fraction of a meter takes sixty-seven zeros after the decimal point.

Picture the two sizes as two lines drawn against mass, one climbing and one falling. The gravitational size is the climbing line. The quantum size is the falling line. For a proton the quantum size is the larger by far, so quantum theory describes the proton, and the proton's own gravity plays no part. For the Earth the gravitational size is the larger by far. So general relativity describes the Earth's gravity, and the Earth's quantum spread (the blur in the Earth's position) plays no part. Because one line climbs and the other falls, the two lines cross at one mass. The mass at the crossing is the *Planck mass*, about twenty-two millionths of a gram, which is not small by everyday standards: a grain of quartz sand a quarter of a millimeter across has about that much mass. At the crossing both sizes equal the Planck length, about a hundred billion billion times smaller than the width of a proton. The grain of sand itself is far larger than either of its

two sizes. So nothing strange happens to the grain. The two sizes meet only for a Planck mass held within a Planck length. The meeting of the two sizes is the *Planck scale*: the scale at which a body's gravitational size equals its quantum size. Up to a small numerical factor, Planck's length and mass of 1899 are the length and mass at the crossing of the two lines.

The next step uses the Incommensuration Theorem of Chapter 3. What is found in a comparison is the comparison's *content*. The *context* of a comparison is the respect in which the content is found, such as two places or two moments. Chapter 3 defines four words from content and context. In Chapter 3's words, continuity is the same content where the context is the same. Discontinuity is a different content where the context is the same. Symmetry is the same content across a different context. Asymmetry is a different content across a different context. "Perfectly", in the theorem, means held exactly, at the foundation, with no exception at any scale. Continuity or discontinuity can be paired with symmetry or asymmetry in four ways. By the theorem, two of the four pairings are impossible for anything held perfectly. So a comparison held perfectly can take only one of the two pairings that survive the theorem. Chapter 3 gave each surviving pairing a picture. The first surviving pairing is continuity with asymmetry, like a smooth ramp that rises unevenly up a hill: no gap anywhere, and different from place to place. The second surviving pairing is symmetry with discontinuity, like a ladder whose rungs are all alike: separate steps, each the same as the next, with nothing to stand on between the rungs.

Which pairing belongs to content, and which to context? Two laws: "Law of symmetry will apply to all that is of content", and "Law of continuity will apply to all that is of context". Symmetry goes with discontinuity in one surviving pairing, and continuity goes with asymmetry in the other. So content, perfected (held perfectly), is symmetric and discontinuous, and context, perfected, is continuous and asymmetric. Content is the ladder: the click of a counter is one whole click or none, and each click is like the last. Context is the ramp, as Chapter 1 showed with a pen let go at arm's length: the arena at the height of your hand differs from the arena

at the height of the floor, smoothly, with no step between the two heights. I identify quantum theory with the description of the content of physics, which is what is found in the comparisons of physics. I identify general relativity with the description of the context of physics, which is the arena in which that content is found.

In Chapter 1 I identified gravity with the shape of the arena, and not with a force acting within the arena. A second route to that result starts from the Incommensuration Theorem. General relativity can be derived from the Immanent Metaphysics, starting from the Incommensuration Theorem. When a force is described in countable steps, physicists call the force *quantized*. All the quantized forces take the pairing of symmetry with discontinuity, and so do all the particles of matter. Whatever takes the other pairing, continuity with asymmetry, must be of another kind altogether: "the arena itself, not something operating within an arena. That phenomenon is gravity". Because the arena takes the pairing of continuity with asymmetry, the arena is smooth and differs from place to place. The shape of the arena is the way the arena differs from place to place. Chapter 1 described the shape of the arena as the way space and time are bent, from one place to the next. Greene reaches gravity as the shape of space and time through Einstein's argument from the equivalence of gravity and speeding up. I derive gravity as the shape of the arena from the Incommensuration Theorem.

The two sizes of a mass show which of the two descriptions, quantum theory or general relativity, applies to the mass. Where the quantum size is much the larger, quantum theory, with its discrete findings, describes the object, while the object's gravity is a faint background. Where the gravitational size is much the larger, general relativity's smooth geometry describes the object, while the object's quantum spread is too small to matter. At the Planck scale itself neither description is enough alone, so both are needed. The Incommensuration Theorem says what follows at the Planck scale. Suppose a single description held both pairings at the Planck scale, perfected together. The single description would hold at least one of the two pairings that the theorem makes impossible: symmetry

with continuity, or asymmetry with discontinuity. So the Planck scale is not the doorway to a third theory that replaces the other two. It is the place where two descriptions are both needed and stay two.

Reasoning with the Planck length also shows how far the smooth description of gravity holds. When quantum theory is taken into account, Einstein's equations give slightly changed answers, and physicists call the small changes the *quantum corrections*. The quantum corrections grow with the bending of space and time. For the pull between two masses a meter apart, the quantum corrections are about one part in a number seventy digits long of that pull. The share of the pull is the same whatever the two masses are, because the distance alone sets the share. The quantum corrections grow as large as the effects they correct only where space and time bend as sharply as the curve of a circle about a Planck length across. So the smooth description of gravity holds wherever space and time bend more gently than the curve of a circle about a Planck length across, which is everywhere the smooth description has been tested, by a vast margin.

A Limit on Placing

Greene's string cannot probe below the Planck length. On my account, a limit at the Planck length holds with no string at all. Placing something within a very short distance needs a probe of very short wavelength, and a short wavelength means a high energy. If a probe were sharp enough to place a body within a Planck length, the probe would carry so much energy into so small a region that the probe would make a black hole larger than the region probed. So the Planck length limits how finely any comparison made from inside the world can place a body. Greene reaches the Planck-length limit with strings, while on my account the limit follows from the two sizes of a mass.

Return now to the bucket of water of Chapter 1, hanging from its twisted rope. As the bucket turns, its water climbs the walls into a hollow, and the turning of the water is measured against the other things there are to

compare the water with. In this book the bucket stands for a comparison made from inside the world. Suppose you tried to place the lowest point of the water's hollow as finely as anything can be placed. The water is made of molecules, and long before the Planck length the water's surface blurs into molecules, so the finest thing to place there is one molecule of the water, the one at the bottom of the hollow. Placing that molecule is a comparison made from inside the world: a probe sent from the room meets the molecule, and what the probe registers is found against the other things in the room. A probe sent from the room can place the molecule to about a Planck length, and no finer, because the Planck length sets that bound on every comparison made from inside the world. The bound limits how finely a comparison made from inside the world can place a body, the molecule or any other, but the arena in which the bucket turns has no least length.

Greene's string argument and my limit on placing both rest on one principle, the principle of comparison of Chapter 1. Chapter 1's principle of comparison says that a difference no comparison can register is no difference. Greene concludes that, according to string theory, jitters no probe could ever register do not arise. Greene's conclusion is the principle of comparison applied to the jitters. Greene and I agree on the principle of comparison. Greene and I differ on what sets the limit. For Greene the limit is the size of the string, and below that size, he suspects, space and time give way to other ideas. For me the limit is a limit on placing, and the arena has no grain: no least length, area or duration. A limit on localization is a different thing from a least length.

A light microscope shows the difference between a limit on placing and a least length. An ordinary light microscope cannot show details much finer than about half the wavelength of the light the microscope uses, yet nobody concludes that a cell is built of tiles of that size. The limit belongs to the way the cell is looked at. The cell itself is as fine as it is. In the picture, the microscope stands for a comparison made from inside the world, half the wavelength of light stands for the Planck length, and the

cell stands for the arena. As with the microscope, the Planck length limits how finely a comparison can place a body, while the arena has no least step.

The reason the arena has no grain comes from the Incommensuration Theorem. A least step would make the arena discrete, which is the way content is found, but what is found of the arena stays smooth and different from place to place, which is the way context is found. If the arena had a least step, one description of the arena would hold both pairings at once, perfected, and so the description would hold at least one of the two pairings that the theorem makes impossible: symmetry with continuity, or asymmetry with discontinuity. So the arena has no least length, no least area and no least duration, at the Planck length or anywhere else.

Astronomers have used light from across the universe to test whether space has a grain, and what they found agrees with my account. In 2009 the Fermi space telescope caught a burst of *gamma rays*, cataloged as GRB 090510. In the catalog name, the letters stand for gamma-ray burst and the digits for the date of the burst. Gamma rays are light of the highest energies. The light of GRB 090510 had traveled for billions of years. A *photon* is a particle of light, and an *electronvolt* is the energy an electron gains in crossing one volt. One photon of the burst carried about thirty-one billion electronvolts, and the photon arrived within about a second of the burst's low-energy light. Suppose space had a grain at the Planck length that slowed light by an amount growing in step with the light's energy, which is the simplest way a grain of space would act. If space had a Planck-length grain of that simplest kind, the grain would have held the photon back noticeably behind the low-energy light. The photon arrived within about a second of the burst's low-energy light, so the Fermi team concluded that any grain slowing light in step with its energy must be finer than the Planck length. In October 2022 another burst, GRB 221009A, was caught by the Large High Altitude Air Shower Observatory in China. The observatory measured the light of GRB 221009A, and the measurements raised the bound. The *Planck energy* is the energy a Planck mass holds at rest. A slowing of that simplest kind could set in only above about ten

times the Planck energy. Light crossing a jittering space would also gather small random shifts on its way, so the image of a distant source would arrive blurred. X-ray images of distant quasars are sharp, so the sharp images rule out the model of the foam, built after Wheeler, in which the jitters add up as the light goes. So we now know that space and time, across the whole distance the light of the two bursts and the quasars has traveled, show none of the grain that the burst timings and the quasar images can detect.

One more claim of Greene's is also a claim of my account. Greene writes that the clash was in a sense "a problem of our own making", made by an assumption. I also find the clash in an assumption that physicists made, and not in nature. Greene and I name different assumptions. For Greene the assumption was that particles are points. For me the assumption is that physicists can describe the arena in steps, as they describe matter and light in steps. On that assumption, gravity is one more force waiting to be quantized.

The Stage and Its Actors

String theory's founders found the graviton among the string's vibrations. In 1996 the physicists Andrew Strominger and Cumrun Vafa used string theory to count the hidden arrangements of a class of idealized black holes. In Greene's sense, a thing's *entropy* is the number of ways the thing's parts can be rearranged without changing how the thing looks overall. The count of Strominger and Vafa matched exactly the entropy that the physicists Jacob Bekenstein and Stephen Hawking had given the idealized black holes. The count of arrangements is always made relative to a context, in the sense defined earlier in this chapter. For a thing, the context is the respects in which a given set of measurements compares the thing, such as the thing's warmth for a thermometer and its weight for a balance. The entropy counts the arrangements of the thing's parts that measurements made in the respects of the context cannot tell apart. So what Greene calls a thing's overall appearance is, on my account, what a

given set of measurements registers of the thing. The graviton found among the string's vibrations and the count of Strominger and Vafa are two of the most striking results in modern physics. A *dimension* is an independent way in which things can differ. Einstein's equations for our four dimensions also follow from string theory, which is a theory of ten dimensions. I obtain Einstein's equations from ten dimensions too, by a different route. Chapter 7 asks what the ten dimensions are.

I differ from string theory on string theory's first assumption. The commitment at string theory's first step is to begin from quanta and to derive the arena from them. In string theory the graviton belongs to one list with the photon and the particles of the nuclear forces. So, in string theory, gravity is one more kind of content. Gravity is the shape of the arena, by the argument from the Incommensuration Theorem earlier in this chapter. The arena is the context of every finding. Gravity is the stage, not one of the actors. A theater can count its actors, list their entrances and number their lines, but the theater cannot count the stage as one of the cast, because every entrance is made on the stage. In the picture, the actors are the particles, which are content. The stage is the arena, which is context. Light can be quantized because light is content: light is found within a context, and quantizing light leaves that context in place. If the arena itself were built out of quanta, the directions of space and time themselves would have to hold both pairings in one description. The directions of space and time would then hold at least one of the two pairings ruled out by the theorem: symmetry with continuity, or asymmetry with discontinuity.

What, then, is a graviton, if the arena has no particle of its own? A crystal shows the answer. A crystal is a lattice of atoms held in place by the bonds between the atoms. When someone strikes a crystal, a wave of motion runs through the lattice. The wave of motion is sound. Quantum theory says that each way the lattice can vibrate takes up energy only in equal steps, and one step is called a *phonon*. Nobody concludes that a crystal is made of phonons. A crystal is made of atoms. The phonon is a step in the description of how the atoms move together. So a phonon is a

quantum of the description of how a lattice whose parts are held by bonds vibrates. Small waves of the arena can be described as the vibrations of the lattice are described, and in that description each step of energy is called a graviton. The arena, though, has no parts that bonds hold in place, as the crystal's atoms are held. The shape of the arena is set by the matter and energy in the arena. So even in the phonon's sense the graviton is no quantum of the arena: the word names at most one step of energy in the description of a small wave, a *quantum of a description*, never a piece of space and time.

The waves of the arena themselves have been measured. On 14 September 2015 the Laser Interferometer Gravitational-Wave Observatory (LIGO) registered a wave of the arena at its two detectors, in Louisiana and in Washington. The wave came from two black holes merging more than a billion light-years away. In 2016 the LIGO team reported the wave together with the team of the European detector Virgo. The wave changed the length of each four-kilometer arm of a detector by far less than the width of a proton. The wave's shape matched the shape that Einstein's smooth theory predicts. No single graviton has been detected. So we now know that the arena ripples. As measured so far, the ripples of the arena are the ripples of a smooth arena.

An older experiment shows what the arena does when the matter in the arena is held between two outcomes. In 1981 the physicists Don Page and C. D. Geilker let the decays of a radioactive source, counted by a Geiger counter, decide where large lead balls were set beside a sensitive gravity balance, a light bar hung from a thin wire. One position of the balls turned the bar one way, and the other position turned the bar the other way. An averaged geometry would be half one position and half the other, so an averaged geometry would have pulled the bar neither way. The bar turned on every run, each time in the direction that the actual position of the balls required. So we now know that the pull of matter held between two outcomes follows the outcome that happened, and not the average of the two. I identify the gravity of matter held between alternatives with one geometry for each alternative: each alternative of the matter carries its

own smooth geometry, shaped by that alternative's own energy. One geometry for each alternative predicts exactly the result Page and Geilker found, with no steps of geometry needed.

Where a Description Ends

What, then, becomes of Wheeler's quantum foam, the froth that Greene describes below the Planck length? In 1957 Wheeler estimated how large the quantum ripples of the geometry (the shape of space and time) would be in a region of a given size, using the formula that general relativity gives for the energy of a small ripple. By Wheeler's estimate the ripples grow as the region shrinks, until at the Planck length the ripples are as large as the geometry itself. The formula holds only while the ripple is small. So Wheeler's estimate shows where its own formula stops holding. The estimate cannot show what the geometry is past that point. Wheeler's argument and mine find the same edge. The edge is the Planck length, where the formula for a small ripple stops holding. The essay on Wheeler goes on to say, "They part on what lies past the edge". For Wheeler the geometry is a *quantum variable* at every scale, a quantity that quantum theory describes with the jitter the uncertainty principle requires, so for Wheeler the geometry froths past the edge. For me the geometry is the arena, which cannot be quantized, so for me the edge is the end of a description, and the arena past the edge is as smooth as the arena on this side of the edge.

On my account, each infinity Greene reports marks the place where a description stops holding, as Wheeler's edge does. At the center of a black hole the equations of general relativity have a *singularity*: a point where, by those equations, the bending of space and time grows without limit. The singularity marks the edge of the range in which the smooth description of general relativity holds. Physics itself does not break down at the singularity. The infinity is what the smooth description gives when the de-

scription is carried past the edge of its own range. There is no third description past the edge of the smooth description's range, because the two descriptions I already have meet at that edge.

In 1900, the year after he proposed his natural units, Planck himself met an infinity that came from carrying a description past its range. Around 1900 the physics of the day predicted an infinite amount of energy at the shortest wavelengths of the light inside a hot oven. The prediction of infinite energy was later named the *ultraviolet catastrophe*. Planck's law of December 1900 has no infinite excess, because in that law energy passes between the light and the oven walls only in steps. Planck's law put the steps into the content: the energy exchanged between matter and light. Planck's law put no steps into space. The infinity showed that the physics of the day had been carried past the range in which it holds. The cure was the right description of the content. The singularity of a black hole means what the ultraviolet catastrophe meant: general relativity has been carried past the range in which general relativity holds.

Greene's last suspicion is that space and time may emerge from something more basic. On my account, the answer to Greene's suspicion is the same as for the foam and the singularity: the arena is not built from anything more basic. The arena is not made of anything more basic that is found in it. The pieces from which space would emerge would be pieces of content: separate states that can be counted. The smooth arena would then be composed from pieces of content. One cannot compose a continuity out of purely symmetric components, any more than one can abstract a symmetry out of a pure continuity given alone. So content is discrete and repeating, context is smooth and uneven, and neither is built out of the other.

Exact translations between theories are among the clues Greene gives for that suspicion. Among those exact translations, the most surprising is the one that the physicist Juan Maldacena found in 1997, between gravity inside a region of one kind of curved space and a quantum theory on the region's boundary. Maldacena's translation is exact in both directions. So in my account the two sides of the translation are one system with two de-

scriptions, neither more basic than the other. Chapter 12 says which of the two surviving pairings each of the two descriptions takes. A good translation of a poem carries the poem's sense across line by line, and each language still keeps what the other cannot hold. Nobody concludes from a perfect translation that French is English in disguise. In the picture, the two languages stand for the two descriptions, and the poem stands for the one system that both descriptions describe. When two descriptions have an exact translation between them, and neither is made of the other, this book calls the pair a *correspondence*. This is correspondence, not emergence.

One World, Two Descriptions

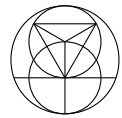
In this chapter I asked what happens at the Planck length, and whether string theory is needed to resolve the conflict of general relativity and quantum mechanics there. At the Planck length a body's two sizes are equal, and the two descriptions of the world meet: both are needed at once. At the Planck length the smooth description of the arena reaches the edge of the range in which the smooth description holds, and so does the stepwise description of content. By the Incommensuration Theorem, the two descriptions cannot be merged into one description, so the two descriptions meet and stay two. No probe can place a body more finely than about a Planck length, as Greene also finds with strings, but that bound is a limit on placing, not a least length of space. Experiments now being built will find whether two small masses become *entangled* when each mass is held in two places at once and the masses act on each other only through gravity. Entanglement through gravity would not show that the arena comes in steps: each placement of the masses has its own smooth geometry, as I identified earlier in this chapter, and the masses become linked because the pairs of placements pull on each other by different amounts. Two masses are entangled when their outcomes, measured apart, agree in a pattern that no separate instructions carried by each mass

could give them. At the back of the book, Appendix A gathers the measurement of whether the two masses become entangled, with the other measurements still to come.

On my account, general relativity and quantum mechanics are both right, each as the description of its own side of every comparison: quantum theory of the content, and general relativity of the context. Greene says that the two theories, as they are now formulated, cannot both be right. The clash Greene describes appears when the jitter of quantum theory is given to the gravitational field, that is, when gravity is treated as one more force waiting to be quantized. I find the cause of the clash in the assumption behind treating gravity as one more force: that the arena can be described in steps, as the matter and light within it are. Each of Greene's two theories is complete in its own pairing. The two theories have worked side by side in every laboratory for a century: quantum theory describing what is found, and general relativity describing the arena in which each finding is made. Planck's units, written for all civilizations, mark the one scale at which neither description can be set aside. At the Planck scale no third theory is needed. The two theories we have are both needed at the Planck scale, used together, and each kept whole. Greene hoped for more from strings than an end to the clash: he hoped that one string, vibrating, would give every force and every particle in one theory of everything.

So we now know that both theories are right about one world, each in its own surviving pairing. As I identified the two theories earlier in this chapter, quantum theory is the description of the content of physics, and general relativity is the description of the context of physics, the arena. At the Planck scale the two descriptions meet and stay two, and the arena has no least length.

If the two descriptions cannot merge, is there still one theory of everything, and what becomes of the forces and particles that string theory hoped to unite?



Chapter 6

Is There One Theory of Everything?

Beside The Elegant Universe*, chapters 1, 6, 7, 12, 13 and 15;* The Fabric of the Cosmos*, chapter 9; and* The Hidden Reality*, chapter 4.*

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

Every comparison has three roles: the *stance*, the side a comparison is made from, with the range of what could come; the *act*, the happening itself; and the *structure*, what is laid down in the act, the same for everyone who reads it afterward (Chapter 3).

I identify quantum theory with the description of the content of physics, which is what is found in the comparisons of physics, and general relativity with the description of the context of physics, which is the arena in which that content is found (Chapter 5).

On 17 April 1955, in a bed at the Princeton Hospital, Albert Einstein felt a little stronger. Einstein asked for the pages of equations he had been working on. The four books of the physicist Brian Greene begin from Einstein's search for one theory of all the forces of nature. A reader of Greene's books comes away from them with a question: is there one theory of all the forces and all the matter? The question has two halves, the forces and the matter. Einstein's search began with the forces.

For the last thirty years of his life Einstein had looked for a unified field theory: one set of equations from which both of the forces then known would follow, gravity and electromagnetism (the force of electri-

city and magnetism). The pages brought to Einstein's bed held the latest work of his search for a unified field theory. Einstein died in the night, before the morning came. Greene tells the scene and closes the scene with one line: "His final scribblings shed no further light on unification" (*The Hidden Reality*, chapter 4, section "A Brief History of Unification").

Strings That Play Every Note

What did Greene find when he took up Einstein's unfinished search?

Greene opens *The Elegant Universe* with a report on what physicists believe. Greene writes that physicists "believe they have finally found a framework" for joining what physics knows into "a single theory that, in principle, is capable of describing all physical phenomena" (*The Elegant Universe*, Preface). The framework that physicists believe they have found is superstring theory, called string theory for short.

The basic idea of string theory is easy to picture. A violin string can vibrate in many patterns, and each pattern sounds as a different note. String theory says that every elementary particle is a tiny loop of string. The vibration pattern of a particle's loop sets the mass and the charges we measure for the particle, as the vibration pattern of a violin string sets the note we hear. An electron is a loop vibrating one way. A *quark* is one of the particles of which protons and neutrons are made. A quark is the same kind of loop as an electron, vibrating another way.

The forces come from vibrating loops too. Each force acts through particles of its own, called its *carriers*. The carriers of a force pass between the particles of matter, as the photon, the particle of light, passes between electric charges. In string theory the carriers are patterns of vibration as well. Physicists have long proposed a particle called the *graviton* as the quantum of gravity, with no mass and no charge. One pattern of vibration has exactly the properties of the graviton. So one kind of object would give every particle and every force: a vibrating loop of string. Greene re-

ports that string theory is therefore "sometimes described as possibly being" the theory of everything (*The Elegant Universe*, chapter 1, section "String Theory as the Unified Theory of Everything").

Greene sets string theory beside the theory physics already has. The *Standard Model* of particle physics is the theory of the known particles and of the three forces other than gravity. The three forces other than gravity are electromagnetism, the weak force and the strong force. The weak force changes one kind of particle into another in radioactive decay. The strong force binds quarks into protons and neutrons. The Standard Model has passed every test that the largest colliders, the machines that smash particles together, have set it, but in the Standard Model the masses and charges of the particles are numbers that are measured and written in. Greene writes that "no one has any explanation of why our universe is composed of these particular particles, with these particular masses and force charges" (*The Elegant Universe*, chapter 1, section "The Forces, or, Where's the Photon?").

Greene writes that string theory "sets up a framework for explaining the properties of the particles observed in nature" (chapter 6, section "The Music of String Theory"). In the same section Greene adds that "we do not know if" the properties of the particles can be explained by string theory. Greene writes that hurdles still keep physicists from working out the patterns of vibration of string theory precisely enough to compare the patterns with experiment. Greene describes string theory as needing only one number as input: the tension of its loops, which sets the overall scale of its strings.

Greene also tells how far physicists had come in joining the forces, by theories of fields, before string theory. Greene wrote *The Fabric of the Cosmos* before the Higgs field was detected. In *The Fabric of the Cosmos* Greene introduces the Higgs field (chapter 9, section "Force, Matter, and Higgs Fields"). Greene writes that "Many physicists strongly believe that there is yet a third kind of field". The Higgs field is the third kind of field, besides the fields of the forces and the fields of matter. In the theory Greene tells, the Higgs field settled at the same value everywhere as the young uni-

verse cooled, and so the settled Higgs field filled space with what Greene calls the Higgs ocean (chapter 9, section "The Higgs Ocean and the Origin of Mass"). A particle feels a drag from the Higgs ocean when the particle is pushed to change its speed, as a Ping-Pong ball pushed through molasses does. In the theory Greene tells, the drag from the Higgs ocean is the particle's mass.

Greene tells how physicists joined the forces themselves in *The Hidden Reality*, chapter 4, note 2. Sheldon Glashow, Abdus Salam and Steven Weinberg were physicists who proposed that electromagnetism and the weak force are two aspects of one combined force, and accelerator experiments later confirmed their theory, in the late 1970s and early 1980s. Glashow and the physicist Howard Georgi then proposed grand unification. In grand unification, the strong force, the weak force and electromagnetism are aspects of one more fundamental force. The simplest version of grand unification predicts that protons decay, every so often. Scientists looked for the decay of protons and did not see it, so they ruled the simplest version out. Greene adds that there are "many other versions of grand unification that remain experimentally viable", and he gives one reason: the decay that the other versions predict can be too slow for existing experiments to detect.

Chapter 3 defined a *symmetry* as the same content across a different context. In plain words, what is found stays the same when the respect in which it is found changes. A square turned a quarter of the way round is the same square: the shape is what is found, and the turning changes the respect in which the shape is found. Turn a laboratory to face another way, and the laws found in the laboratory are the same laws: the sameness of the laws under the turning is a symmetry of the laws. Greene also gives the name symmetry to the change itself, the turning of the laboratory (*The Fabric of the Cosmos*, Glossary). *Spin* is a turning built into every particle, the same amount for every particle of one kind. Spin is counted in units of one fixed size: every particle of matter has a spin of half a unit, and every carrier of a force has a spin of a whole number of units. The particles fall into two *families* by their spin: the family of matter, and the family of the

carriers, and the particle of the Higgs field, whose spin is zero, also belongs to the family of the carriers. Greene explains that once spin is counted, "there is precisely one more symmetry of the laws of nature that is mathematically possible", called supersymmetry (*The Elegant Universe*, chapter 7, section "Supersymmetry and Superpartners"). Supersymmetry would give every particle a partner whose spin differs from the particle's own spin by half a unit, so each particle of matter would have a partner in the family of the carriers, and each carrier would have a partner in the family of matter. Physicists call the partners that supersymmetry would give the *superpartners*.

Greene then tells of a measurement of the strengths of the forces. Physicists find that the strength of each force changes with the distance at which the force is examined, or, what comes to the same thing, with the energy of the collision that examines the force (chapter 7, section "The Case for Supersymmetry: Prior to String Theory"). Physicists can calculate, by the rules of quantum theory, what the strengths measured at everyday energies become at far higher energies. Physicists call the calculation running the strengths upward. In 1991 the physicists Ugo Amaldi, Wim de Boer and Hermann Fürstenau ran the measured strengths of electromagnetism, the weak force and the strong force upward, and they found the three strengths almost equal at the highest energies, but not quite equal. Greene writes that "this tiny but undeniable discrepancy in their strengths vanishes if supersymmetry is incorporated" (same section). Supersymmetry is also part of the string theories that have patterns of vibration for the particles of matter (chapter 7, section "Supersymmetry in String Theory").

From Greene's pages

Greene's Report on the Theory

How does Greene himself judge the state of string theory?

In *The Hidden Reality* Greene sets out a status report on eight goals of string theory (chapter 4, section "The State of String Theory: An Evaluation", Table 4.2). Greene marks six of the eight goals "Excellent". The six goals marked "Excellent" are uniting gravity with quantum mechanics, unifying all the forces, keeping the great results of earlier physics, counting the entropy of black holes (a count of their inner arrangements, told below), giving mathematics new results, and making sense of singularities. On singularities, Greene adds that "The theory still needs to address black hole and big bang singularities". Greene marks the other two goals "Indeterminate; no predictions": explaining the properties of the particles, and confirmation by experiment.

Greene also judges what experiment can do for string theory, in the section "String Theory and Experiment" of the same chapter. Greene judges that the most promising positive results of experiment "would most likely not be able to definitively prove string theory right", and that negative results would most likely not prove string theory wrong. Greene writes that for the foreseeable future physicists can build "a circumstantial case for string theory" from predictions of one kind: predictions that follow from older theories too, but follow far more naturally from string theory.

Greene marks "Excellent" the goal of counting the entropy of black holes. Counting the entropy of black holes has a story of its own. The *horizon* of a black hole is its edge: the surface from inside which nothing returns. The *entropy* of a thing is a count: the number of ways the parts of the thing can be arranged that look the same in some context, some respect in which the thing is compared. The entropy of a black hole grows with the area of its horizon. For a black hole the context is the world outside the hole. So the entropy of a black hole counts the arrangements inside the hole that look the same from outside. For a

quarter of a century no one could say what the arrangements inside a black hole were. In January 1996 the physicists Andrew Strominger and Cumrun Vafa built a class of black holes in string theory out of strings and *membranes*. A membrane is an object that spreads out in two or more directions, as a sheet spreads out in two. A string spreads out in one direction. Strominger and Vafa counted the arrangements of the strings and membranes, and they found exactly the number that Bekenstein and Hawking had given. Greene writes: "A quarter-century-old puzzle had been solved" (*The Elegant Universe*, chapter 13, section "Enter String Theory").

Greene also reports a view of what a final theory should be like, and he describes the view as "currently shared by many physicists". On the view that Greene reports, "one of the most convincing arguments in support of its particular form would be that the theory couldn't be otherwise" (*The Elegant Universe*, chapter 12, opening). Greene writes that string theory still lacks a founding principle like the equivalence principle. Greene's own words are: "Currently, string theorists are in a position analogous to an Einstein bereft of the equivalence principle" (chapter 15, section "What Is the Fundamental Principle Underlying String Theory?"). Greene asks, too, whether explanation has a limit. Greene answers that explanation may have a limit: some features of the universe may be as they are "because of happenstance, accident, or divine choice" (chapter 15, section "Are There Limits to Explanation?").

Greene answers the question of one theory in two parts. String theory is a framework that would describe every particle and every force, and string theory has the results that Greene marks "Excellent", but string theory has not yet explained the properties of the particles, and no experiment has yet tested string theory.

A Principle at the Start

Which of Greene's results follow in my account, and from what starting point?

Begin with a founding principle like the equivalence principle, the kind of principle that Greene says string theory still lacks. My founding principle is that every finding is a *comparison*. The *content* of a comparison is what is found in the comparison. One example of content is that a thermometer reads warmer in the evening than in the morning. The *context* of a comparison is the respect in which the content is found: what makes the two things compared two, as the two times of day make the morning's reading and the evening's reading two. Every measurement in physics is an act of comparison.

Every comparison has three roles, which Chapter 3 named. A line written in a detector's log is a structure. Three short statements say how the three roles relate to one another. The three statements are the *axioms* of the Immanent Metaphysics: the basic statements from which the rest of the Immanent Metaphysics is worked out. The first axiom says that the act is the most fundamental of the three roles. The stance and the structure are both made of acts: take away every act, and the structure records nothing and the stance waits for nothing. The second axiom gives the order of the three roles: the stance comes before the act, the act before the structure, and the structure before the next stance. A detector shows the order: the detector stands ready (the stance), a particle strikes it (the act), a line is written in its log (the structure), and the detector stands ready again with that line behind it.

Picture three rings, woven together so that no two of the rings are linked to each other alone. Yet the three rings hold together as one. Cut any one ring, and the other two fall apart. The three roles of a comparison are joined together in the same way. The third axiom says how: the three roles are distinct, each itself and not another; inseparable, none occurring

without the other two; and non-interchangeable, none able to do another's work. In the detector, the log cannot do the work of the strike, and the strike cannot do the work of the log.

The Incommensuration Theorem, met in Chapter 3, follows from comparison alone. The Incommensuration Theorem is about four kinds of finding, which Chapter 3 called the four *characters*: symmetry, discontinuity, continuity and asymmetry. In the Incommensuration Theorem, *symmetric* means having the same content across every change of context: a symmetric thing repeats exactly wherever and whenever it is found. *Discontinuous* means coming in separate, countable steps, and *continuous* means joining without a break. *Asymmetric* means different from place to place. Under the Incommensuration Theorem, a comparison can perfect two pairings of characters: symmetry with discontinuity, and continuity with asymmetry. Chapter 3 called the two pairings that a comparison can perfect *the two surviving pairings*. Perfected content has the first pairing: perfected content is symmetric and comes in countable steps. Perfected context has the second pairing: perfected context is continuous and different from place to place. The *structure of comparison* means the following parts taken together: the three roles of comparison and the axioms that relate the roles, the content and the context of comparison, and the Incommensuration Theorem. (The word structure has its everyday sense in the name structure of comparison, the way a thing is built, and does not mean the third role of comparison, what is laid down in the act.) Every act of comparison contains the structure of comparison. So whatever is found is found by an act with a stance before it and a structure after it, has a content and a context, and obeys the Incommensuration Theorem.

In Chapter 1 I started from comparison and found four marks that the arena must carry. First, the arena acts on everything. Third, one quantity is both the pull on a body and the body's resistance to being moved, so every body falls alike. Fourth, the arena takes hold of the degree to which a body takes part in the arena. The degree of taking part comes in no steps. Gravity carries all four marks. In Chapter 1 I identified gravity with the shape of the arena: the way space and time are bent from one place to

the next, which is why a dropped pen falls. Einstein's equivalence principle gave him the equality of the third mark: the pull and the resistance are one quantity.

Greene asks whether explanation may have a limit. Explanation has a limit. The foundations guarantee structure, not content. My foundations fix the form of the world: its laws and its kinds of particle. The laws and the kinds of particle are the same for every comparer, everyone who makes a comparison. The content found in any one comparison is settled only in that act of comparison.

The Count of a Black Hole

What of the black-hole count that Greene marks "Excellent"?

The string theorists' graviton and the count of Strominger and Vafa are two of the most striking results in modern physics. The entropy that Bekenstein and Hawking found is established physics, and I take that entropy as established physics in my account.

Starting from comparison, I also derive in my account the reason that the entropy of a black hole grows with the area of its horizon and not with the volume inside. Picture a walled garden, and two ways of counting what the garden holds. From above, with the whole garden in view at once, you could count the garden bed by bed. A count made bed by bed grows with the size of the garden. Standing at one place outside the wall, you register the garden only by what reaches you across the wall: what goes in, what comes out, and what can be seen or heard over the wall. From one place outside the wall, a count of the garden grows with the length of the wall. Around a region of space the boundary is a surface, not a line, so a count made across the boundary grows with the area of the surface.

A bounded region of space can be counted in the same two ways as the garden. The first count takes the inside piece by piece, like the count of the garden bed by bed. The first count grows with the volume. The piece-by-piece count would need the whole inside laid out in view at once, from

no place, as a detector's log lies open the same for everyone who reads it. So the piece-by-piece count is a count of the structure, the third role of comparison: what is laid down, taken as a whole. The second count takes what reaches one place outside across the boundary, like the count made from outside the garden wall. The second count grows with the area of the boundary. The second count is made in acts of comparison, at a place. The first axiom says that the act is the most fundamental of the three roles of comparison, so the count made in acts, from a place, is the more fundamental of the two counts. The entropy of a black hole is counted with the world outside the hole as its context, so the entropy of a black hole is the count made from a place outside, across the boundary. For everyone outside a black hole the boundary is the horizon of the hole, and so the entropy of a black hole grows with the area of its horizon, as Bekenstein and Hawking found.

What, then, does the entropy of a black hole count? I identify what the entropy of a black hole counts with states of the hole's content: the states of what the hole holds that look the same in every comparison made from outside its horizon. The count is made with the outside world as its context. The count of Strominger and Vafa shows that the entropy counts states of content. The hole's entropy counts states of its content that the outside world cannot tell apart. Strominger and Vafa counted the states of strings and membranes, which are content, for a special class of black holes: the black holes with the largest charge that their mass allows. Strominger and Vafa made the count in a physical case where gravity is too weak to make a black hole at all. So space and time are smooth where Strominger and Vafa made their count. The count of Strominger and Vafa stays the same when they increase the strength of gravity in their calculation, all the way until the strings and membranes they counted have become a black hole. The count exhibits no step of the arena. A step of the arena would be a least, countable piece of space or of time. Chapter 5 showed that the arena has no least, countable piece. So the count of Strominger and Vafa is a count of states of content.

Joined, Not Merged

Then why is string theory not, on my account, the one theory of everything?

Picture a railway timetable and the railway itself. The timetable lists departures as separate entries that can be counted, and each entry repeats exactly from week to week. The track is one unbroken line through country that changes from mile to mile. Every train in the timetable runs on the track, and the timetable and the track are joined at every station. Nobody asks the track to be made of timetable entries, or the timetable to be made of track.

The two great theories of physics are related to each other as the timetable is related to the track: quantum theory as the timetable, and general relativity as the track. In Chapter 5 I identified quantum theory with the description of the content of physics: quantum theory describes what is found, which is symmetric and comes in steps, like the entries of the timetable. In Chapter 5 I identified general relativity with the description of the context of physics: general relativity describes the arena, which is continuous and different from place to place, like the track. So the phrase "theory of everything" can mean two things, and the Incommensuration Theorem shows which of the two kinds of theory can exist. The first meaning is a theory that leaves nothing out: every finding, of every kind, has its place in the theory. The second meaning is a single description, of one kind throughout, from which everything else follows. I call unity of the first kind a *common source*. A common source is two descriptions that arise from the same act of comparison and apply to the same world, each perfect in one of the two surviving pairings (symmetry with discontinuity, or continuity with asymmetry), joined side by side. In the same book I call unity of the second kind a *merger*: a single description, of one kind throughout, from which everything else follows. A merger is one description that requires the directions of space and time themselves to be smooth and different from place to place, and also to come in exact, re-

peating steps. A common source is possible under the Incommensuration Theorem, and a merger is impossible. I use the word "account" for the whole description of the world, and say: "There is one account of everything, and it is made of two descriptions joined together".

Greene tells the story of unification as a line of great conflicts. In Greene's telling, physicists resolved each conflict with a deeper theory, and the next conflict arose from that deeper theory (*The Elegant Universe*, chapter 1, section "The Three Conflicts"). The search expected a merger because mergers had served physics well. The physicist James Clerk Maxwell found electricity and magnetism to be one field. Later, Glashow, Salam and Weinberg wrote electromagnetism and the weak force as one theory. In each success, physicists joined one force found within the arena to another force found within the arena. Gravity, though, is the shape of the arena itself, and every force that physicists joined to another is found within the arena. If a theory made gravity and the particles one description, the theory would need the directions of space and time themselves to be perfectly continuous and also perfectly symmetric, but the Incommensuration Theorem says that nothing is both.

I differ from string theory at string theory's first step. Where string theory's gravitons describe small waves on a smooth arena, I allow the gravitons in my account as quanta of a description: steps of energy in the description of those small waves. String theorists claim a merger when they say that the arena itself is built from string states. The Incommensuration Theorem says that a merger cannot exist.

A lowest state is a settled state of least energy for a whole world. I find that string theory's first step, to begin from quanta, predicts a difficulty: a great many possible lowest states, each with its own particles and forces, and no principle within string theory by which to choose among the lowest states. In Chapter 7 I picture the many lowest states of string theory as the valleys of a hilly country. The many lowest states are the difficulty that string theorists have in fact run into. The *extra dimensions* of string theory are six further dimensions beyond the three of space and the one of time, held too small to see. Greene names the difficulty in his own report, be-

side the goal "Explain particle properties": "the many different possible forms for the extra dimensions imply many different possible collections of particle properties" (*The Hidden Reality*, chapter 4, Table 4.2).

A Book and a Cup

If gravity, the shape of the arena, stays apart from the other forces, do the other three forces at least become one at high energy?

Take two objects, a book and a cup, and set them side by side. Each of the two objects keeps its own identity and takes up its own space. The book does not become the cup. Now stir blue paint and yellow paint together: you get green, a blend in which the blue and the yellow can no longer be separated. The book and the cup, against the blended paint, show how the symmetries of the three forces are related: side by side, and not blended. In the picture of the book and the cup, the book stands for color, the charge of the strong force. The cup stands for the charges of the weak force and of electromagnetism. A grand unified theory would stir them together, making green, and the proton would decay.

The three forces stand side by side because of the third axiom. The third axiom says that the three roles of every comparison (the stance, the act and the structure) are distinct and inseparable, and that no role can do another role's work. The three forces are three because of the third axiom, and I pair each force with one role of comparison. I make the pairing from the standpoint of a measurement. In the measurement, two charged particles are compared, and the light passing between the two particles does the comparing. The strong force holds the three colors of the quarks in their pattern, and the strong force goes with the structure, the role in which what is laid down in a comparison is held whole, the same for everyone. The weak force is the one force that changes what a particle is, one kind into another, and the weak force goes with the stance. In a measurement, the stance is what the comparison reaches toward and never contains: the range of what could come, which no single finding contains. In the hot young universe, before the Higgs field settled into Greene's

Higgs ocean, *hypercharge* was the charge that played the part electric charge plays now. The third force, the force of hypercharge, acts on the phase that every quantum state carries. The phase is the point that the state's wave has reached in its cycle. The force of hypercharge goes with the act, the happening present in every comparison.

Before the Higgs field settled, the forces were the strong force, the weak force and the force of hypercharge. When the Higgs field settled, one combination was left without mass: a combination of the carrier of hypercharge and the uncharged carrier of the weak force. The combination left without mass is the photon. The force of the photon is electromagnetism. Electromagnetism is the force through which light relates every charged thing to every other charged thing. So at everyday energies we know the force that goes with the act as electromagnetism: the force of the light that does the comparing in a measurement. Whether the forces are counted before the settling of the Higgs field or after it, the forces within the arena are three: the strong force and the weak force, with hypercharge before the settling and electromagnetism after it. A *lepton* is one of the particles of matter that feel no strong force: the electron, its two heavier cousins, and the neutrinos. There is no force that turns a quark into a lepton, and no single force of which the three are parts.

By the Incommensuration Theorem alone, two forces could share one description, because two kinds of content share one surviving pairing, symmetry with discontinuity. The three forces stay three because the three roles of comparison are three. The electroweak theory is the theory of Glashow, Salam and Weinberg that joins electromagnetism and the weak force. The electroweak theory is a joining of the allowed kind: the two symmetries joined in the theory are side by side, as the book and the cup are, and no single symmetry contains both.

I also say how the strengths of the three forces change with energy. Greene tells of the change of the strengths with energy in the measurement of Amaldi, de Boer and Fürstenau. Run upward with the Standard Model's own rates, the three measured couplings draw close to one another, and they meet at no scale. In the quoted sentence, the couplings are the

strengths of the three forces, and a scale is a level of energy. The three strengths cross in pairs, at three different energies, spread over nearly four powers of ten. Inseparability produces nearness; distinctness produces the gap. The strengths of the three forces draw close because the three forces are inseparable. The three forces never become one force because the three forces are distinct. The book and the cup show both halves of the reason. The book and the cup stand close together on one table, and they stay two things. Greene describes a small gap between the strengths, which supersymmetry would close. I require the small gap: the structure of comparison needs the three strengths nearly equal at the highest energies, and not equal.

The proton lasts: no carrier turns a quark into a lepton, so the proton has no route by which to decay. Under a mountain in Japan, the Super-Kamiokande detector has watched fifty thousand tons of pure water since 1996, waiting for one of the protons in the water to break apart. Super-Kamiokande watches for a decay that gives a *positron*, the electron's twin of opposite charge, and a neutral pion, a light particle with no electric charge. No proton in the tank has been seen to decay into a positron and a neutral pion. In 2020 the Super-Kamiokande team reported that the proton's lifetime for decay into a positron and a neutral pion is longer than a number of years written as twenty-four followed by thirty-three zeros. The reported lifetime is more than a million billion billion times the present age of the universe. We now know that the proton lasts at least as long as the reported lifetime. The reason is that there is no route by which the proton could decay.

Each Family on Its Own Side

What of supersymmetry, the one further symmetry that Greene says is possible?

Every particle, of matter or of force, is something found within the arena. In the sense of being found within the arena, every particle is content, found within a context, as Chapter 5 showed light to be. I show that

the line between content and context is drawn again within the content itself. The first line drawn within the content runs between the two families of particles: the family of matter and the family of the carriers. On the first line, the particles of matter belong to the content side. The carriers of the forces belong to the context side, with the other members of the carriers' family. The carriers' family also includes the Higgs particle (the particle of the Higgs field). By definition, a supersymmetry would carry every particle of matter onto a member of the carriers' family, and every member of the carriers' family onto a particle of matter.

The Incommensuration Theorem says that a supersymmetry, an exchange of the two families, cannot exist. On the first line, the content side is the side perfected as symmetric, and the context side is the side perfected as continuous. Suppose a transformation carried one side onto the other. The transformation would make one perfect symmetry across the line between the two sides. The two sides would then be one description, and that one description would hold the context side whole, which is perfected as continuous, together with the new symmetry. One description would then be perfectly symmetric and perfectly continuous at once, and nothing can be both.

The two families show their difference plainly. Take two electrons, two identical particles of matter, and write down the quantum state of the pair. The quantum state is the complete description of what could be found for the pair. Swap the two electrons, and the state comes back multiplied by minus one. Swap two identical carriers, two photons, and the state comes back multiplied by plus one, unchanged. The sign shows that the two families differ in kind, because a sign is plus or minus, with no values in between.

I prove, as a *proposition* (a statement that is proved), that the field space of the Standard Model admits no supersymmetry. The field space is the whole collection of the Standard Model's fields, of matter and of force, taken together. So no particle of matter has a partner in the carriers' family, and no member of the carriers' family has a partner among the particles of matter.

CERN is the European laboratory for particle physics near Geneva. The Large Hadron Collider at CERN collides protons at the highest energies ever reached. The experiments at the Large Hadron Collider have searched for superpartners since the collider's research began in 2010. The experiments have found no superpartner. An *electronvolt* is the energy an electron gains in crossing one volt, a small unit of energy. A mass is energy held at rest, so physicists weigh particles in electronvolts too, and a proton weighs a little under a billion electronvolts. The gluons are the carriers of the strong force. Any partner of the gluons would have to weigh more than about two thousand three hundred billion electronvolts, nearly two and a half thousand times the mass of a proton. Any partner of the lighter quarks would have to weigh more than about twelve hundred to eighteen hundred and fifty billion electronvolts, depending on what is assumed about how the partner decays. The top quark is the heaviest quark. Any partner of the top quark would have to weigh more than about thirteen hundred billion electronvolts. We now know that any superpartner would have to be heavier than the limits found at the Large Hadron Collider, and heavier than many physicists expected. The reason no superpartner has been found is that no particle has a partner across the line between the two families.

A Closed List of Kinds

We now know that gravity is the shape of the arena, described by general relativity, and that the other three forces are found within the arena, described by quantum theory, side by side and never one. And we know that no particle has a partner across the line between the family of matter and the family of the carriers. The first meaning of "theory of everything" is a common source, a theory that leaves nothing out. So every particle must have its place in a common source, as every force does. Greene marks the goal of explaining the properties of the particles as one that

string theory has not yet reached. The forces are three because the roles of comparison are three. Can comparison fix the kinds of matter too: can anything say why the world has these particles and no others?

The reason the world has these particles and no others comes from the structure of comparison, with no vibrating string. The *generations* are the three copies of the particles of matter, alike but for their masses. I take up in Chapter 7 why there are three. I work out which kinds of particle there are, with their charges, their spins, their colors (the charge of the strong force) and their generations. I find that the kinds of particle form a closed list. The closed list contains exactly what comparison requires and nothing else. No further kinds exist: no fourth generation, no supersymmetric partner, no X or Y boson, no graviton, no dark-matter particle of any type. The X and Y bosons are the carriers that a grand unified theory would need. *Dark matter* is the unseen matter whose pull is measured in every galaxy. There is no graviton among the kinds because gravity is the shape of the arena: the shape of the stage on which the particles act, and not one of the actors. The word graviton names at most one step of energy in the quantum description of small ripples of the arena about its smooth shape: in the words of Chapter 5, a quantum of a description, and never a piece of space and time.

The Higgs field also comes out of the structure of comparison, and I derive Greene's Higgs ocean in my account by the following argument. The particles of matter come in pairs: the electron and its *neutrino*, and the *up quark* and the *down quark*. Neutrinos are the lightest particles of matter, with no electric charge. Each generation has one kind of neutrino. The up and down quarks are the two kinds of quark of which protons and neutrons are made. The weak force carries a particle across from one member of its pair to the other, as when the weak force turns an electron into its neutrino. The two members of a pair differ. A crossing from one member to the other carries the difference between the two members. A crossing can happen at any place, so the difference between the two members must be held at every place in space. The *edge* of a pair is no place in space: the edge is the line that divides one member of the pair from the other, the

line that the weak force crosses. Something with a value at every place is a field. So a field lives on the edge between the two members of each pair, and at every place in space the value of that field says how the two members differ. The field on the edge is the Higgs field. The Higgs field is required before anyone has a mass problem to solve: the argument uses no fact about mass.

Why does the Higgs field settle at one value everywhere? Picture the bottom of a wine bottle. A ball balanced on the raised center rolls off into the ring of the valley around the center and comes to rest at one point. The ground is the same in every direction, but the ball rests in one direction only. The laws of the weak force treat the two members of each pair alike, as the ground of the bottle treats every direction alike. Empty space is one state that joins without a break from place to place, and joining without a break is continuity. If empty space also treated the two members of each pair exactly alike, empty space would be perfectly symmetric and perfectly continuous at once. The Incommensuration Theorem says that nothing is both. So the state of empty space tells the two members apart, as the resting ball marks one direction.

As the young universe cooled, the Higgs field settled everywhere at once into a value that tells the two members of each pair apart. The settled value of the Higgs field is the same everywhere in space, and I call the settled value the *condensate*.

I picture what the condensate does with walkers crossing a wide meadow at dusk, as snow begins to fall. A heavy walker in boots sinks deep and labors, a lighter walker sinks a little and is slowed a little, and a skier glides over the snow and is not slowed at all. The depth of the snow is the value of the field, the same for every walker. Each walker is a particle. How deeply a walker sinks is the particle's *coupling* to the field: how strongly the particle and the field act on each other. The slowing is the particle's mass. The skiers are the photon and the gluons, the carriers of electromagnetism and of the strong force. The photon and the gluons

stay without mass. Greene's molasses pictures the same result as the snow of the meadow: the settled Higgs field gives each particle that couples to it a mass in proportion to the particle's coupling.

Physicists had looked for the particle of the Higgs field for decades. ATLAS and CMS are the collaborations, the teams of physicists, that run the two largest detectors at CERN. On 4 July 2012 the ATLAS and CMS collaborations announced together a new particle at about a hundred and twenty-five billion electronvolts. By 2018 the ATLAS and CMS collaborations had measured how strongly the Higgs particle acts on the three heaviest particles of matter: the top quark, the bottom quark and the tau, a heavier cousin of the electron. The two collaborations found each of the three strengths in proportion to the mass of the particle it was measured on. We now know that the field required by the structure of comparison is there, and that the field gives masses as the snow slows the walkers.

A second measurement counts the generations of the particles of matter. The Z particle is one carrier of the weak force. Physicists count the kinds of light neutrino from the rate at which the Z particle decays into neutrinos. There is one kind of light neutrino for each generation. In 2020 the physicists Patrick Janot and Stanisław Jadach worked again through the data of an earlier collider at CERN, and from those data they counted three kinds of light neutrino, give or take less than a hundredth of one kind. We now know that the kinds of light neutrino number three, one for each of the three generations of the closed list.

Four Numbers from the World

Do I need any measured numbers, when string theory, as Greene describes it, needs only one?

My account needs four measured numbers, taken as measured inputs. The first number is Newton's constant, which sets the strength of gravity. The second number is the strength of light: the strength of the electric and magnetic force, of which light is a wave. The third number is the strength of color, which is the strength of the strong force. The fourth number is

the value of the Higgs field in empty space. These four are the measured values among the strengths of the forces and the value of the condensate. The masses of the particles are stated as ratios to one unit, the mass of the electron, and since no argument fixes a unit, the unit is taken from measurement; in Appendix A I name, for each of my two calculations of the masses, the measured numbers that calculation takes, and the history of the universe takes in addition the measured density of radiation and the measured strength and tilt of the first ripples.

Picture a photograph of a cathedral with no person and no car in it. Every proportion is right: the height of the tower against the length of the nave, the width of each window against the door. Nothing in the photograph tells you whether the building is a cathedral or a model on a table. One length measured on the real building fixes the size of all the rest. The photograph pictures the reason that I give for the first of the four numbers, Newton's constant. The axioms and the Incommensuration Theorem concern relations among comparisons: which of the four characters (symmetry, discontinuity, continuity and asymmetry) goes with which, which pairings survive, what comes before what. If a description is made only of relations, the description fixes every proportion of the world, but the description fits a world scaled up or down as a whole just as well. So one value read off the world must fix the scale of the world, as one length measured on the real building fixes the size of the building. Newton's constant is the value that fixes the scale.

The strength of light is read off the world for a reason of another kind. I show that the kinds of particle, their charges and the symmetries of the laws come out the same whatever the strength of light is. A different strength of light changes only how often and how strongly findings occur, which is settled by the world and not by comparison, so no argument from comparison fixes the strength of light. Two more numbers are read off the world as well: the strength of color and the value of the Higgs field in empty space. Wherever my account works with the strength of color or the value of the Higgs field, that number is a measured input.

I separate two questions about the masses of the particles. The first question is how heavy a particle is in grams. How heavy a particle is in grams is a question about a unit. The answer of every comparison is the same (same or different, more or less) whatever unit is used. So no argument fixes a unit. The second question is how heavy a particle is compared with the other particles. The answer to the second question is a pure number, a ratio of like to like. Greene's questions about the masses are of the second kind. One of Greene's questions is why the tau weighs "about 3,520 times as much as an electron" (*The Elegant Universe*, chapter 1, section "The Universe at Its Smallest: What We Know about Matter"). I work out ratios of masses, and I hold that none of them is a number with no reason. For Greene's own example, I show a balance kept by the masses of three particles: the electron, the tau and the muon, the electron's cousin of middle weight. The physicist Yoshio Koide noticed this balance in the measured masses in 1981, and I derive it. Given the measured masses of the electron and the muon, the balance gives the mass of the tau. Appendix B gives the mass of the tau, and the ratio of that mass to the mass of the electron, with the working.

Many Strands

Greene judges that, for the foreseeable future, the most promising way to link string theory with data is through predictions that follow far more naturally from string theory than from more traditional methods (*The Hidden Reality*, chapter 4, section "String Theory and Experiment"). Greene writes that the considerations he describes "have the capacity to build a circumstantial case for string theory". What holds up an argument like my argument?

Hang a heavy load from a chain, and the chain parts at its weakest link. Hang the load from a cable twisted from many strands of steel wire, and the cable holds when one strand is cut, because the strands share the load. A cable is stronger than its strongest strand, and its strength comes from the number of its strands and from how closely they are bound to-

gether. I show that an argument about the world is held up as a cable is held up, by many strands from independent sources, and that a court weighing circumstantial evidence reasons in the same form. Greene's circumstantial case for string theory has the form of a cable too.

The closed list of kinds is held up as a cable is. I argue for the list from the structure of comparison by more than one route. Each route is a strand of the cable. The measurements told in this chapter are strands too, and each measurement agrees with the list. The measurements told in this chapter are five: the three strengths of the forces drawing close and crossing apart; the proton lasting; no superpartner found at the collider; the Higgs particle found; and three kinds of light neutrino counted.

The Theory Einstein Was Writing

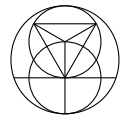
What, then, becomes of the theory that Einstein was still writing on his last day?

The world has a unity, and the unity of the world is of the kind that the railway has. Quantum theory describes what is found. General relativity describes the arena in which things are found. Quantum theory and general relativity both arise from the one act of comparison. The two theories are joined wherever something is found in the arena, as the timetable and the track are joined at every station. The two theories are merged nowhere. Within the content, the strong force, the weak force and electromagnetism stand side by side like the book and the cup, drawing close in strength and never becoming one. The particles that the forces act on form a closed list, fixed by the structure of comparison, as the three forces are.

For a reader of Greene, a unity of the railway's kind still has what Greene hoped for: no particle and no force left without its place, and all of them reached from one stated principle. What changes is the kind of unity. A unity of the railway's kind is two descriptions held together by the act of comparison from which both arise, and no single description into which both are poured.

We now know that there is one theory of everything, of the railway's kind: two descriptions joined from one source. The one theory of everything has three forces side by side that stay three, a closed list of kinds of particle, and four values taken from measurement.

If the structure of comparison fixes the kinds of particle, what becomes of the extra dimensions, which string theory says fix the kinds of particle?



Chapter 7

Why Ten Dimensions?

Beside The Elegant Universe, *chapters 8, 9, 12 and 14,* The Fabric of the Cosmos, *chapters 12 and 13, and* The Hidden Reality, *chapters 4 and 5.*

What this chapter uses

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

I identify quantum theory with the description of the content of physics, which is what is found in the comparisons of physics, and general relativity with the description of the context of physics, which is the arena in which that content is found (Chapter 5).

The *structure of comparison* means the three roles of comparison and the axioms that relate the roles, the content and the context of comparison, and the Incommensuration Theorem, taken together (Chapter 6).

Early in 1919 a mathematician named Theodor Kaluza sent the physicist Albert Einstein a paper in which space had four directions instead of three. Kaluza worked at the University of Königsberg. In Chapters 5 and 6 I introduced *string theory*: the theory in which the smallest pieces of matter are tiny vibrating strings. In the plain sense, a *dimension* is one independent way in which things can differ. The number of dimensions is the count of the independent ways. A point on a map differs from another point by how far east and how far north the point lies. So a map has two dimensions. String theory needs ten dimensions. Six of the ten are too small to see. What are the six small dimensions, and why do the six have the shape they have? A reader of the physicist Brian Greene carries both questions out of Greene's chapters on strings. Kaluza's paper is the origin of the idea of a dimension beyond the familiar four: three of space and one of time.

In the paper, Kaluza had taken Einstein's equations of gravity and written the equations out for a world of four dimensions of space and one of time, and then he had looked at what came out. The equations for the familiar four dimensions were Einstein's own equations of gravity. The new dimension had equations of its own. The new equations were the ones that the physicist James Clerk Maxwell had written for electricity and magnetism. Electricity and magnetism make up one force, and light is a wave of that force. With one more dimension, one set of equations described both gravity and light. Einstein replied on 21 April 1919, and Greene quotes the reply in *The Elegant Universe*, chapter 8, section "Unification in Higher Dimensions": "At first glance, I like your idea enormously". About a week later Einstein wrote again, this time with some doubt, and for two years Einstein held Kaluza back from publishing the paper. In October 1921 Einstein wrote that he was having second thoughts about holding the paper back, and he offered to present Kaluza's paper to the Prussian Academy of Sciences after all. The Prussian Academy printed Kaluza's paper in 1921. Kaluza's paper already contained the question that every later reader would ask: if the world has a fifth dimension, where is that dimension?

A Fifth Dimension, Seen from Far Away

In 1926 the Swedish physicist Oskar Klein gave the answer that string theory still uses. Greene tells Klein's answer with the picture of a garden hose. Stretch a long hose across a canyon and look at the hose from a quarter of a mile away: the hose looks like a line, with one direction. Come close, or take up binoculars, and you find a second direction. The second direction is the small circle round the hose, present at every point along the hose's length. Seen from far away, a small dimension looks like no dimension at all. Klein proposed that Kaluza's fifth dimension is a small circle like the circle round the hose: present at every point of space, and curled so small that nothing we do can see the circle. In Greene's words, Klein's calculations "indicated that the additional circular dimension might be as small as the Planck length". The *Planck length* is a length built from the constants of nature alone (Chapter 5). Greene draws one conclusion from the history of Kaluza and Klein, in *The Elegant Universe*, chapter 8, section "Kaluza's Idea and Klein's Refinement": "The universe may very well have more dimensions than meet the eye".

From Greene's pages

Greene's Ten, and the Search for One Shape

Greene shows how, in string theory, more than three dimensions of space went from a possibility to a requirement. In the early years of string theory, certain calculations gave probabilities below zero, and no sensible theory has a probability below zero. The impossible probabilities canceled exactly when the strings could vibrate in nine independent directions of space. So Greene writes, in *The Elegant Universe*, chapter 8, section "More Dimensions and String Theory": "For string theory to make sense, the universe should have nine space dimensions and one time dimension, for a total of ten dimensions". Three of the nine are the large directions we live in. In string theory, as in the picture of Kaluza and Klein, the other six are curled up small at every

point of space. Greene gives one equation of the mathematics behind the number ten, in *The Hidden Reality*, chapter 4, section "The Dimensions of Space". The equation has a troublesome part, and the troublesome part vanishes when the dimensions of space and time number ten: "in a universe with ten spacetime dimensions the trouble gets wiped away".

The equations of string theory allow the six to be curled up only in certain shapes. In 1984 four physicists showed that one class of six-dimensional shapes satisfies the conditions that the equations of string theory set. The four were Philip Candelas, Gary Horowitz, Andrew Strominger and Edward Witten. Physicists call the shapes of the class found in 1984 the *Calabi-Yau shapes*. The shapes are named for the mathematicians Eugenio Calabi and Shing-Tung Yau, whose work on the shapes came first. Greene then states what he calls one of the most far-reaching insights of string theory. The six dimensions shape a string that vibrates through the six, as an organ pipe shapes the column of air inside the pipe. The notes that the string can sound appear to us as the masses and charges of the particles. So, in Greene's words, the masses and charges are "determined, in large measure, by the geometrical size and shape of the extra dimensions" (*The Elegant Universe*, chapter 8, section "The Physical Implications of Extra Dimensions"). The particles of matter come in three copies, alike but for their masses. The three copies are the *generations*, which Greene calls families. The shape of the six would set even the number of generations. String theorists propose that the number of generations is "a reflection of the number of holes in the geometrical shape comprising the extra dimensions!" (*The Elegant Universe*, chapter 9, section "The Road to Experiment").

Greene names one difficulty: the equations of string theory allow very many shapes and do not say which shape is ours. Greene reports that the Calabi-Yau shapes known when he wrote numbered in the tens of thousands, with from three holes to as many as four hundred and

eighty (*The Elegant Universe*, chapter 9, section "The Road to Experiment"). At the time, no one knew how to deduce from the equations which of the shapes is ours. Without a way to pick the shape, string theorists cannot yet compute the properties of the particles from the shape. Greene counts "at least hundreds of thousands" of Calabi-Yau shapes in *The Fabric of the Cosmos*, chapter 12, section "String Physics and Extra Dimensions", and he says that the known equations give no way to pick one of the shapes. Greene asks whether string theory is fatally flawed when no one can pick the shape from its equations, and he answers in his own voice: "Is this a fatal flaw of string theory? Possibly. But I don't think so". Greene gives his reason in the same paragraph of *The Fabric of the Cosmos*, chapter 12. Greene writes that for many years string theorists had not found the exact equations of string theory. As the approximate equations are sharpened, he writes, "determining the form of the extra dimensions is a prime—and in my opinion attainable—objective". So in Greene's opinion the form of the extra dimensions can be found.

In 1995 the physicist Edward Witten gave evidence that the five versions of string theory then known are five ways of describing one theory (*The Elegant Universe*, chapter 12). Physicists have given the one theory the tentative name *M-theory*. M-theory has ten dimensions of space and one of time. *Branes* are objects spread out in two, three or more dimensions, as a sheet is spread out in two. M-theory has branes as well as strings. Greene tells the next part of the story in *The Hidden Reality*, chapter 5, section "In Flux". Over the next decade and a half, string theorists developed more exact mathematical methods. With the new methods, string theorists found that the count of shapes was larger still. A *field* is something spread through space with a value at every point, as temperature has a value at every point of a room. String theorists found that the holes of a Calabi-Yau shape can be threaded by fields called *fluxes*, each hole a whole number of times. Each way of threading the holes is a different *form* for the six dimen-

sions. A form is the same shape, with a different pattern of fluxes through the shape's holes. Greene counts the forms at about a one followed by five hundred zeros, "increasing the candidate pool enormously while not providing an iota of insight into how to single out one as ours".

Picture a hilly country with many valleys. Drop a ball onto the country from the sky. The ball comes to rest in one of the valleys. The valley that the ball rests in is a fact about where the ball happened to fall, and not a fact about the shape of the country. The bottom of a valley is a settled state with the least energy nearby. Physicists call a settled state with the least energy nearby a *vacuum*. In the physicists' sense, the word vacuum does not mean a chamber pumped nearly empty, the everyday sense of the word. For the six small dimensions, a vacuum is one size and shape that the laws of motion keep steady. After a small push, the six roll back to the steady size and shape. Greene calls the many-valleyed country of string theory's forms string theory's *landscape*. In the landscape, our world would be one valley picked out of a vast number of valleys.

The Same Ten, Counted from Comparison

The total of ten also follows from an argument that uses no strings, no particular kind of matter and no law of physics. The argument's starting point is comparison. Every measurement is an act of comparison. Chapter 1 defined a *comparison* as the act of finding whether two things are the same or different in some respect. The *content* of a comparison is what the comparison finds. The *context* of a comparison is the respect in which the comparison finds its content: what makes the two things compared two, as two times of day make two readings of a thermometer two.

Take the simplest comparison there is: a thermometer read this morning and again this evening, the two readings set side by side. The comparison ends in a verdict, same or different: two ideas, sameness and differ-

ence. The verdict is about the temperature, which is the comparison's content. The two readings are two because they were taken at two times of day, and the two times are the comparison's context. Content and context are two ideas more. And the comparison is made: there is the comparing, which is the thermometer's own side, and there is what is compared, which is the warmth of the room, and the comparing and the compared are two ideas more. The comparing is called the *subjective*. What is compared is called the *objective*. So every comparison contains six ideas, in three pairs: sameness and difference, which make the verdict; content and context, which make the respect; and the subjective and the objective, which are the comparing and the compared. There are six ideas, no more and no fewer, because each pair has exactly two members. A comparison also uses other things, such as a unit, a scale or a number, but each of those things is built from the six ideas and comes after the six.

A finding is the outcome of one act of comparison. Counted from comparison, the number of dimensions is the number of independent ways in which two findings can differ in a given context. When no mix of the other ways of differing gives a way of differing, I call that way of differing independent. On a map, north-east is a mix of north and east. So north-east adds no dimension. Counted from comparison alone, a number of dimensions can only be the size of a set of things that every comparison contains. The sets that every comparison contains have five sizes: one, the comparing, which is single; two, the members of a pair; three, the pairs; four, the ideas other than the subjective and the objective (sameness, difference, content and context); and six, all the ideas together. So, counted from comparison, a number of dimensions can be one, two, three, four or six. The world has ten dimensions: one of time, three of space, and six of possibility.

Time has one dimension because the comparing is done from one *stance*, the side a comparison is made from (Chapter 3), and from one stance any two findings are simply earlier and later. Space has more than one dimension, because space and time are two different contexts. And a context has no feature but its number of ways of differing. If space and

time each had one way of differing, nothing would tell space from time. For the rest of the count of space, picture a flat page. On the page you can see that east differs from north, but no direction on the page stands apart from both east and north. Lift your eyes off the page, and the third direction, straight up, is the direction from which the page's two directions can be told apart.

When a difference is found in some respect other than the difference itself, the difference is *placed*. The flat page shows a placed difference. Straight up is a respect other than east and north, and from straight up the difference between east and north can be told. When a comparison finds a difference, the difference is placed twice, because every finding is found by a comparing and found in a context, and the comparing and the context are two, neither of them the finding. The comparing places every difference. A context places the difference between two of the context's directions only when the context has a further direction that stands apart from both, as straight up stands apart from the page. In a context with two directions, the difference between the two directions is placed by the comparing alone, once. In a context with three directions, each direction places the difference between the other two. So every difference is placed twice: once by the comparing and once by the context. Four and six, the larger counts, would also place every difference twice. The count of a context is the least count on the list that satisfies every requirement of comparison. Nothing in a comparison would tell a further direction apart from the other directions. So a further direction would add nothing that a comparison could find. So space has three dimensions.

Possibility is the range of what a comparison could find. Possibility has six dimensions. Every finding is fixed in each of the six ideas: every finding has a verdict, same or different; a respect, which is a content in a context; and a comparing, of something compared. So the range of what could be found spans all six ideas, one independent way of differing for each idea, and possibility has six dimensions. One of time, three of space and six of possibility make ten.

Physics also has a six, found from its own side: the six small dimensions that physicists set beside space and time. When a comparison is made in physics, the three pairs of ideas in the comparison have a definite form: six kinds of *measure*. A measure is a kind of quantity that a measurement finds. The six measures form a grid of two rows and three columns. There are two rows because content and context are a pair, and three columns because every comparison has three roles. The rows are content and context. The columns are the three roles of every comparison that Chapter 3 named: the *structure*, what is laid down; the *act*, the happening itself; and the stance, the side a comparison is made from.

Each content is measured in its own context, and the table below is the grid of the six measures:

	Structure (what is laid down)	**Act (the happening)**	**Stance (what could come)**
Content (what is measured)	pattern	force	probability
Context (where it is measured)	space	time	possibility

Pattern is laid out in space, and mass, inertia and the way a body is built are kinds of pattern. Force is a directed flow of change, and a change is found between an earlier and a later reading, so force is measured in time. Probability is how values are spread among alternatives, and probability is measured in possibility, the range of what could be found.

The six small dimensions are the context in which the comparisons of content take place, in all six measures. I give each measure a direction of its own in the six small dimensions. I identify the six dimensions of possibility with the six small dimensions of physics. The six dimensions of possibility are counted by the six ideas, one dimension for each idea. The six small dimensions of physics are counted by the six measures, one dimension for each measure. The six measures come from the six ideas, one measure for each idea. So both sixes are counted by the same six ideas,

and by nothing else. A difference between the two sixes would need a respect of comparison beyond the six ideas, and comparison has no respect beyond the six ideas, so no comparison can register a difference between the two sixes. Chapter 1 stated the principle of comparison: a difference that no comparison can register is no difference.

So Greene's total and my total agree, and the two tens are counts of different things. In string theory ten is a condition of consistency. Ten is a count, made from what a comparison contains. Counted in my account, the ten divide into four and six, and each part has a kind of its own. String theorists choose the division into four and six to match what we see. A *comparer* is anything that makes a comparison, an instrument or a detector as much as a person. The word "comparer" never means a mind. The ten dimensions are of three kinds: time, space and possibility. Four are traversed by located comparers, and six are met only as what could be found.

Many Valleys Foreseen, and One System Told Twice

Greene reports a very large number of shapes and forms, and my account has the same result, by an argument of its own. Chapter 6 told a result about string theory's first step. String theory's first step is to begin from quanta (the separate steps of quantum theory) and to derive space and time from quanta. String theory's first step predicts a great many possible lowest states, which are the valleys of the hilly country pictured earlier in this chapter, and no principle within the theory for choosing among the lowest states. The reason is the way quanta differ from space and time. A start made of quanta fixes separate labels, each label the same wherever the label is found, while space and time are smooth and differ from place to place without repeating. So many different spaces and times fit a start made of quanta. The choice of one space and time has to come from outside the start. A discrete starting point, however rich, has fixed no

single smooth world; that is what multiplicity means. Greene's landscape is the difficulty that string theory's first step predicts, in the form in which string theorists have found the difficulty.

Duality is a second result of Greene's that I also have, by an argument of its own. Greene defines duality in *The Elegant Universe*, chapter 12, section "Duality": "Physicists use the term duality to describe theoretical models that appear to be different but nevertheless can be shown to describe exactly the same physics". Where two descriptions translate exactly, in both directions, one system is being described twice, as a poem and its translation carry one meaning in two languages, and neither is the other in disguise. In 1997 the physicist Juan Maldacena found a duality between gravity inside a region of one kind of curved space and a quantum theory on the region's boundary (Chapter 5). Maldacena's duality is correspondence, not emergence. Chapter 5 defined a *correspondence* as two descriptions with an exact translation between them, neither made of the other. *Emergence* is a one-way relation: if one thing emerges from another, the second is the more fundamental of the two. When a translation is exact in both directions, neither side is the more fundamental one.

Six Met Only as What Could Be Found

String theory and my account differ over whether anything moves through the six, and one of Greene's own pictures shows the difference. Greene asks you to sweep your hand in a wide arc, and he says that "you are moving not only through the three extended dimensions, but also through these curled-up dimensions" (*The Elegant Universe*, chapter 8, section "What Do the Curled-Up Dimensions Look Like?"). In Greene's picture your hand goes round the tiny six so many times that the motion round the six averages out. On my account, the six are met only as what a comparison could find, and no one moves through the six. You can walk to the next room. You can read yesterday in a diary. You cannot walk to the outcome that did not happen. A comparison is always located. A comparison finds this place and not another, this time and not another, and

this possibility made actual and not another. A comparer can look into other places and read other times from records, but a comparer can only think about other possibilities. So, to anyone inside the world, the six appear small and *closed* (finite, with no edge, as the surface of a ball is finite and has no edge), with no depth to them. The six are not directions of space, and nothing ever travels through them.

The six are *held small*: held a few Planck lengths across, as the content of the large world of four dimensions. The large world is the context of the six. In the grid of the six measures, the six were a context: the possibility in which probability is measured. Relative to the large world, the six are content. Content and context are assigned within each comparison. Relative to the comparisons in which the content of physics is found, the six are possibility: a context in which each finding is the content. Relative to the large world of four dimensions, the six are content, and the large world is the context that holds the six. A reader who hears Greene's word "curled up" pictures six directions of space that were once spread out and then rolled tight. But the six of my account were never directions of space at all. A sculptor's block of stone gives the picture of the six in my account. The block has three directions that you can measure with a tape. The block also has a grain, present at every point. The grain decides how the stone can be shaped, but no tape measures the grain. The six are present as the grain is present in the stone: everywhere you are. The block's three directions stand for the three directions of space, and the grain stands for the six.

Greene and I agree that the effects of the six show up through gravity. Greene describes a *braneworld* in *The Hidden Reality*, chapter 5, section "Sticky Branes and Gravity's Tentacles". In a braneworld, our universe lies within a three-dimensional brane in a larger space of more dimensions. Matter and light are confined to the brane, and "gravity provides our only means of probing beyond our three-dimensional spatial expanse". On my account, the extent of the six shows only through the effects of the six, and one of those effects is the strength of gravity, which is tied to the volume of the six. Greene's braneworld and my account differ over what physi-

cists would find when they measure gravity. Greene writes that on the braneworld the extra dimensions could be as large as a tenth of a millimeter and still unseen, because gravity had then been tested no closer (*The Fabric of the Cosmos*, chapter 13, section "Gravity and Large Extra Dimensions"). On my account, the six are a few Planck lengths across. Because the six are dimensions of possibility, there is no further space for a brane to sit in. So no brane has anywhere to drift.

Two quantities could change the size and the shape of the six, and physicists call such quantities the *moduli*: the dials that set the size and the shape of the six. If a modulus differed from place to place, the modulus would show up as a field in a measurement made in our four dimensions, and the ripples of the field would come in separate steps, each step a particle with a mass of its own. Chapter 5 gave the *Planck mass* as about twenty-two millionths of a gram, the mass of a grain of sand a quarter of a millimeter across. The particles of the two moduli weigh about two and a quarter and about three times the Planck mass. Moduli with particles heavier than the Planck mass would make a force whose reach is shorter than the Planck length.

Physicists have made three measurements to look for large hidden dimensions, or for the long-reaching force that light moduli would carry. At the University of Washington, Dan Kapner and his colleagues used a pendulum hung on a fine fiber, close above a second plate, to measure the pull of gravity across gaps smaller than a millimeter. In 2007 Kapner's group reported that gravity weakens with distance exactly as gravity does in three dimensions of space, down to about a twentieth of a millimeter. In 2002 the Cassini spacecraft was on its way to Saturn, and the spacecraft sent radio signals past the Sun. The Sun's effect on the radio signals is what Einstein's theory predicts, to within about two parts in a hundred thousand, as the researchers Bruno Bertotti, Luciano Iess and Paolo Tortora found. The Cassini result is too exact to allow the long-reaching extra force that light moduli of the six would carry. Gravitational waves are ripples in the shape of space and time, spreading at the speed of light. In August 2017 two neutron stars merged. Neutron stars are the collapsed

cores of dead stars. The gravitational waves of the merger arrived together with the light of the same event. Four physicists compared the distance to the merger read from the waves with the distance read from the light: Kris Pardo, Maya Fishbach, Daniel Holz and David Spergel. Pardo and his colleagues found that the waves had spread through four dimensions of space and time, to within a few hundredths, with nothing lost into any further large dimensions. So we now know that, at every distance measured so far, gravity acts in three dimensions of space and one of time, and on my account the large world has three dimensions of space and one of time.

Why are three dimensions large and six small? Greene answers the question as physics then stood, in *The Elegant Universe*, chapter 8, section "Some Questions": "At present no one knows the answer to this question". Greene tells of a proposal by the physicists Robert Brandenberger and Cumrun Vafa, in chapter 14 of *The Elegant Universe*: strings wound round the dimensions let only three of the dimensions grow. On my account, the answer follows from the three kinds of the ten dimensions: time, space and possibility. Three dimensions are large because the three are the directions of space of the large world itself, which is the context in which the six are held, and the large world has no edge in any direction we can reach. The count of three is the count that the flat page showed earlier in this chapter. The six are small because the six are held as content of the large world.

One Shape for the Six

My account has one shape for the six. String theory has tens of thousands of shapes, and then forms numbering a one followed by five hundred zeros. The shape follows from the grid of the six measures by steps of two kinds: identifications, which I make in the first person, and theorems of mathematics. I translate the grid into a shape in three steps. The first step was taken earlier in this chapter: each of the six measures has its own direction, so the shape has six dimensions. The second step uses the

layout of the grid. The grid is two rows by three columns, so the shape is made of two pieces side by side. One piece carries the three roles of comparison, and the other piece carries the two rows. Side by side means that every point of the whole shape is a pair, one point from each piece, as every point on the surface of a tin can is an angle round the can together with a height along the can.

The third step uses a count of the basic pieces from which a shape is built. For example, the surface of a ball is built from two basic pieces: a single point, and a disc whose whole rim is drawn together to the point, like a drawstring bag pulled shut. Mathematicians have an exact way of counting basic pieces, and the way is named after the mathematician Leonhard Euler. The third step is that each piece of the shape is built from as many basic pieces as the piece carries columns or rows: the piece for the three roles from three basic pieces, and the piece for the two rows from two. I also take every point of each piece to be like every other, as on a perfect ball. I take every point to be alike because the six are the context in which content is compared. If one point of the six stood apart from the rest, that point would be a difference of context that no content carries.

Only one shape satisfies the three steps and the likeness of every point. Mathematicians have listed every closed shape whose every point is alike. On the mathematicians' list there is only one way to share out the six dimensions between the two pieces: four for the piece built from three basic pieces, and two for the piece built from two. On the list, only one shape of two dimensions is built from two basic pieces: the ordinary *sphere*, the surface of a ball. On the list, only one shape of four dimensions is built from three basic pieces: the *complex projective plane*. The sphere's surface has two dimensions and the complex projective plane has four: six in all.

Each piece has a plain meaning. Think of a lamp that could be found red, green or blue. Suppose that, as quantum physics allows, the lamp is held between its three alternatives at once, in given proportions and with given settings of a tiny clock hand on each alternative. A *state* of the lamp is one way of holding the three alternatives at once. All the states of the lamp together form the complex projective plane. Think of anything that

can be found in one of two ways: all the states of the two-way thing form the sphere. So the six form one small shape, the *internal space*: the complex projective plane times the sphere, the spaces of all the states of a three-way and of a two-way choice, side by side. The internal space bends everywhere, like the surface of a ball, so the internal space is not one of the Calabi-Yau shapes, whose bending balances out in every direction, as string theory requires.

The internal space is the conclusion of two arguments, and the two arguments have different premises. The first argument is the one just given, and it uses no algebra: its premises are the grid of the six measures, the three steps and the likeness of every point, and its last step is the mathematicians' list of closed shapes. The second argument uses an *algebra*: a stock of quantities with rules for adding and multiplying the quantities. Comparison can be set out in *postulates*: the conditions from which theorems start. Each postulate puts one statement of the Immanent Metaphysics into the form of mathematics. For example, one postulate puts into mathematics a clause of one of the three axioms that Chapter 6 set out: my three statements of how the three roles of comparison relate to one another. I prove that comparison, as set out in the postulates, has one algebra, made of three pieces. The three pieces are the ordinary complex numbers (numbers that carry both a size and a clock-hand setting), a four-part number system called the quaternions, and the three-by-three tables of complex numbers. A state of a piece of the algebra is one way of holding the piece's alternatives at once, as with the lamp. The states of the three-by-three tables form the complex projective plane, the states of the quaternions form the sphere, and together the complex projective plane and the sphere make the six. Picture two climbers who set out from opposite valleys, by different paths, and meet on one summit. The two climbers are the two arguments, and the summit is the internal space. Of the two climbers that "they have found one summit, and neither of them chose it from a list".

What the Six Hold

My account has a picture close to Greene's organ pipe. The six are the small part of the world beside the four large dimensions. The six hold the possibilities that content can take, as the shape and size of a drum decide which notes it can sound. My books take the internal space in two ways, and I give a picture of the two ways. Think of the face of a clock. Taken as the pattern of twelve hours, the face of a clock has no size at all. Three o'clock is three o'clock on a wristwatch and on a tower. Taken as a particular clock, the face has a width: two centimeters on the wrist and several meters on the tower.

When the internal space is taken as a pattern of states, with no size and nothing spread through the space, the kinds of particle and their charges are read on the internal space. The pattern of states is like the twelve hours of the clock face. The six held small are like a particular clock, with its width. Taken as the six held small, with sizes and a bending, the internal space carries three things that later chapters follow. First, the strength of gravity that we measure is tied to the volume of the six. The *Planck scale* is the scale at which a body's gravitational size equals the body's quantum size (Chapter 5). Second, the six bend. The bending of the six holds energy of its own. The energy of the bending takes a part away from the energy that empty space holds when every field is at its lowest, summed up to the Planck scale. The energy that is left is set by the shape and size of the six, and Chapter 10 follows the energy that is left. Third, *dark matter* is the unseen matter whose pull is measured in every galaxy, and on my account dark matter is made of lasting vibrations of the six held dimensions, as Chapter 9 tells. So in my account the kinds of particle are read on the pattern, which has no size, and the sizes belong to the held six. No string vibrates through six dimensions of a given size to make the kinds of particle.

The three generations of particles have a reason of their own in my account. The generations came up earlier in this chapter, in Greene's account: the three copies of the particles of matter, alike but for their masses. On my account, every particle is itself a comparison, because comparison, interaction and relation are one act, approached three ways. A particle is found as the content of other comparisons, and a particle takes part in the world by interacting, which is comparing, so a particle carries the three pairs that every comparison has. One act of comparison finds something of a particle in one respect only, so the act reads the particle by one of the particle's three pairs at a time. The act reads the particle by the pair that faces the act. I call the pair that faces the act "the pair in front". The three roles of a comparison come in a fixed order that closes on itself: the stance comes first, the act follows from the stance, the act lays down the structure, and the structure shapes the stance of the next comparison.

Picture a pencil with three flat faces lying on a table, and turn the pencil so that each face in turn lies on top. Each turn puts a different face on top, and a fourth turn puts the first face on top again. The three pairs of a particle stand at the three roles of comparison as the three faces stand round the pencil. There are six ways to set the three pairs at the three roles. Take any one of the six placings of the pairs. Turn the placing round the fixed order of stance, act and structure, as the pencil turns face by face, and you get three of the six ways. The three turnings keep the direction of the order. Each turning puts a different pair in front, and each turning is one generation. The other three ways swap two of the pairs and run the order backward, against the direction in which every finding is made, and none of the other three ways gives a generation. The three generations are the three rotations of the content.

Greene notes that experimenters have already counted the generations, which he calls families. *Neutrinos* are the lightest particles of matter, with no electric charge, and each generation has one kind of neutrino. Chapter 6 told how the count is made. The Z particle is a carrier of the weak force, the force at work in some radioactive decays. The Z particle

decays into neutrinos at a rate that counts three kinds of light neutrino, one for each generation. So the measured count is three generations, the same number that the three rotations give.

One Vacuum, and Nothing to Choose

Go back to the hilly country. The six have one valley. A theorem of geometry says how many quantities can vary in a shape like the internal space. A shape is given by a measuring rule: a rule that says how long each small step in each direction is, at every point. On a piece whose every point is alike, and which looks the same in every direction at each point, a measuring rule under which every point is alike is fixed up to one number, the size of the piece. The six are two pieces side by side, each alike at every point and in every direction, so the six have two quantities that can vary and no others: the radius of the complex projective plane and the radius of the sphere. When the two radii change together, the overall size of the six changes. When the two radii change against each other, the ratio of the two radii changes. The two ways of changing the radii are the two moduli.

One identification fixes the size and the ratio together. I take every direction of the six to bend by the same amount, the bending of the Planck scale, which is the bending of a sphere one Planck length in radius. Two results follow by algebra. First, every direction of the six bends alike only when the radius of the complex projective plane is the square root of six times the radius of the sphere: about two and a half times the radius of the sphere. Second, the bending of the Planck scale sets the radius of the sphere at one Planck length, so the radius of the complex projective plane is the square root of six Planck lengths. The strength of gravity sets the Planck scale, and the strength of gravity has one value everywhere in the large world, so the six have the same size and the same shape at every point of space. With the two radii set, the size and the shape of the six are both fixed.

Gravity in all ten dimensions is built from the geometry of the ten alone, with no further field, so nothing is threaded through the six, and there is no flux. Picture a ring with nothing wound through the ring's hole. There is nothing to count, and with no whole number to count there is no choice to make. The ring is the six, and the empty hole is a hole with no flux. So the six have one size and shape that the laws of motion keep steady: one vacuum.

Ten Dimensions of Three Kinds

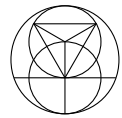
Kaluza's paper of 1919 contained the question of where a fifth dimension could be. The six dimensions beyond space and time are everywhere you are: held small as content of the large world, and met only as what a comparison could find. The six are held in one shape, the same at every point of space. So the kinds of particle come from one shape that no chance picked, and so does the strength of gravity: the kinds of particle are read on the pattern of the shape, and the strength of gravity is tied to the volume of the shape. We now know that the world has ten dimensions of three kinds: one of time, three of space and six of possibility. We now know, too, that the six form one small shape, the same at every point, so there is one vacuum and no landscape.

The one dimension of time is the order of earlier and later that every finding has from its stance. Why, then, do the laws of physics run as well backward as forward, while a splattered egg never gathers itself up whole?

PART FOUR

The Arrow and the Beginning

Beside *The Fabric of the Cosmos*, chapters 6 and 8 to 11, *The Hidden Reality*, chapters 3 and 6, and *Until the End of Time*, chapters 2 and 3.



Chapter 8

Why Does Time Have a Direction?

Beside The Fabric of the Cosmos, *chapters 6 and 7, and* Until the End of Time, *chapters 2 and 3.*

What this chapter uses

Continuity is the same content where the context is the same, as a dimmer knob turned by a hair changes the light by a hair; *discontinuity* is a different content where the context is the same, as when a light switch is flipped; *symmetry* is the same content across a different context, as a square turned a quarter of the way round is the same square; *asymmetry* is a different content across a different context, as on a rocky coast where no cove repeats the cove before it (Chapter 3).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

I identify the one-way order of findings with the direction of time (Chapter 2).

The world has ten dimensions: one of time, three of space, and six of possibility; *possibility* is the range of what a comparison could find, and nothing ever travels through *the six* (Chapter 7).

In 1876 the physicist Josef Loschmidt asked why a gas spreads and never gathers, when the laws of motion work as well backward as forward. Loschmidt asked the question in Vienna, in a paper for the Imperial Academy of Sciences. Loschmidt had already estimated the size of the molecules of air. His paper of 1876 was about gases. Loschmidt's younger colleague and friend was the physicist Ludwig Boltzmann. Boltzmann had argued that the collisions of molecules carry a gas one way only, from

crowded to spread out, and keep the gas spread out. Loschmidt answered Boltzmann with the laws of motion themselves. Greene says that the laws of physics run as well backward as forward. By the laws alone, then, a splattered egg could gather itself up whole again. Why does a splattered egg never do so? The question is much older than Greene's books.

Picture Loschmidt's point with a box of gas whose molecules began crowded in one corner and have since spread through the whole box. Stop every molecule at once, and send each molecule off again at exactly its own speed in exactly the opposite direction. The laws of motion work as well backward as forward. So every molecule retraces its path, and the gas gathers itself back into the corner the gas started from. For every way a gas can spread, there is a way for the gas to gather that is just as lawful.

Boltzmann replied to Loschmidt in 1877. Boltzmann's reply was a count of the ways the molecules of a gas can be arranged. Greene gives his own readers Boltzmann's count, more than a century after Boltzmann. Greene works the count out with eggs, ice cubes and the pages of a novel. Loschmidt's question is your question about the egg, asked by a physicist with a box of gas. Physicists call the one-way running of the world, from past to future, the *arrow of time*. The answer to the question of the egg has two parts, because time has two arrows, and the two arrows point the same way.

From Greene's pages

A Film Run Backward

Roll an egg toward the edge of your kitchen counter and let the egg fall. The egg strikes the floor, the shell cracks, the yolk spreads, and the air of the kitchen stirs a little. Now picture a film of the fall, and run the film backward in your mind. The yolk gathers into a globe, the pieces of shell fly in from all sides and fit together, and the whole egg leaps up onto the counter and rolls to a stop. You know at once that the film is running backward. No one has ever seen an egg gather itself up from the floor.

Greene draws a first lesson from the backward film in *The Fabric of the Cosmos*, chapter 6: nothing in the backward film breaks a law of physics. Greene first shows the lesson with a tennis ball that falls toward Jupiter (section "Time-Reversal Symmetry"). Just before the ball strikes the planet, Greene reverses the ball's velocity. Wherever the ball's velocity is reversed along the ball's fall, the ball moves back, under the laws of motion, along exactly the path the ball came by. Greene explains that the laws of Newton (the laws of motion), of Maxwell (the laws of electricity, magnetism and light) and of Einstein (the laws of relativity) treat past and future alike. Greene writes that the arrow we live by "seems not to be found in the fundamental laws of physics" (*The Fabric of the Cosmos*, chapter 6, section "The Puzzle"). Greene then applies a reversal of velocity to an egg in your kitchen. If the velocity of every piece of shell, every drop of yolk and every molecule of air and floor that the fall disturbed were reversed, the egg would rise whole onto the counter. When the laws stay the same with the order of time turned around, physicists say that the laws have *time-reversal symmetry*, which is the property of the laws that Greene is describing. Greene states time-reversal symmetry for quantum physics too (*Until the End of Time*, chapter 2, section "From This to That"). The laws of quantum physics allow the reverse of every sequence of events that the laws allow. Greene adds a condition in a note to *Until the End of Time*, chapter 2: for some more exotic systems, the laws allow the reversed sequence only if the charges of all the particles, and the roles of left and right, are reversed as well. Greene notes one exception to time-reversal symmetry in a note to *The Fabric of the Cosmos* (notes to chapter 6, note 2): the behavior of certain short-lived particles, the K-mesons (also called kaons) and the B-mesons. Greene writes that many physicists, himself among them, think it unlikely that the exception in the decays of kaons and B-mesons plays a role in

answering the puzzle of time's arrow. Greene adds that no one knows for sure. So, as Greene presents the laws of physics, the laws give no reason why the egg splatters and never unsplatters.

From Greene's pages

Greene's Count, and Its Two Heads

Greene finds the reason the egg splatters, and never unsplatters, in a count. Greene's picture is a copy of the novel *War and Peace*, unbound and thrown into the air. The loose pages can land in one order that reads correctly, and in an enormous number of orders that do not read correctly. So the pages almost always land out of order, and further tosses keep the pages out of order. Anything made of very many parts is like the loose pages: the parts can be arranged in far more mixed ways than orderly ones. A splattered egg can be made in far more ways than a whole egg can. So an egg left to itself goes from whole to splattered, and stays splattered. Physicists call the count Greene describes *entropy*. Greene describes the *second law* of thermodynamics (the science of heat) as the tendency of things with many parts to go toward higher entropy. Greene states how strong the second law is: "Entropy can decrease. It's just ridiculously unlikely" (*Until the End of Time*, chapter 2, section "Boltzmann and the Big Bang").

Then Greene applies the count to the past, as Boltzmann did, and finds that the count runs both ways, because the count rests on the laws of motion, and the laws of motion run both ways. So the counting that predicts more disorder tomorrow also predicts more disorder yesterday. Greene sums up the two-way counting in one phrase: "the entropic arrow of time is *double-headed*" (*The Fabric of the Cosmos*, chapter 6, section "Entropy: Past and Future"). Greene's picture of the double-headed count is a glass of half-melted ice cubes on a bar at half past ten at night. By the count alone, it is far more likely that the half-melted cubes formed slowly out of room-temperature water over

the half hour before, than that someone put whole cubes in the glass half an hour before (*The Fabric of the Cosmos*, chapter 6, section "Following the Math"). When Greene applies the count to the whole universe, the count gives a stranger result: it becomes far more likely that everything we see, our memories included, arose by a rare chance swing out of disorder, and that the past our memories record never happened. Physicists call a rare chance swing out of disorder a *fluctuation*. Greene rejects the conclusion that the past never happened, and he gives his reason: "Even more pressing, if you truly can't trust memories and records, then you also can't trust the laws of physics" (*The Fabric of the Cosmos*, chapter 6, section "Taking a Step Back"). Greene says that the conclusion that we arose by a chance fluctuation leads into a quagmire "in which we lose all understanding" (*The Fabric of the Cosmos*, chapter 6, section "Taking a Step Back").

Greene avoids the conclusion of a chance fluctuation by adding one more fact to the count: a fact about the beginning of the universe. Greene follows the egg back: the egg came from a chicken, the chicken's food from plants, the plants' energy from the sun, the sun from a cloud of gas, and the gas from the early universe. Greene shows that, when gravity matters, a smooth spread of gas is the orderly state, and clumping is the disorderly state. Clumping is disorderly all the way up to black holes, regions whose gravity is so strong that nothing, not even light, comes back out. So the hot gas of the early universe had an extraordinarily low entropy, because the gas was nearly smooth. Greene sums up the chain from the egg back to the early universe in one phrase: "the current order is a cosmological relic". A relic is something left over from long ago, and the order around us now is left over from the early universe. Greene concludes that a splattering egg tells us something about the *big bang*, the hot, dense, expanding state in which the history of our universe begins. Greene writes: "Thus, conditions at the birth of the universe are critical to directing time's arrow" (*The Fabric of the Cosmos*, chapter 6, section "The Critical Input"). The

puzzle is now why the beginning was so orderly, and Greene closes his argument on that puzzle with the sentence "It all comes down to cosmology" (same chapter, section "The Remaining Puzzle"). *Cosmology* is the study of the universe as a whole: its history, its contents and its shape.

Greene completes his telling in two later places. Greene asks, in *The Fabric of the Cosmos*, chapter 7, whether the arrow of time comes from *quantum mechanics*, the theory of atoms and of the particles smaller than atoms, and Greene expects that the arrow of time does not come from quantum mechanics. Greene writes that, "If time symmetry persists (as I suspect it will)", his argument from the count and the low-entropy beginning applies to the quantum world "with little change" (section "Quantum Mechanics and the Arrow of Time"). Greene then considers a law of quantum collapse (a sudden jump of the quantum description at each measurement) that runs one way. Greene adds that even a law of collapse that runs one way "could very well call for" an explanation of why entropy was low in the distant past, as every law of collapse proposed so far does. So, by Greene's argument, unless a future discovery shows two features together, the arrow of time can be explained only by going back to the origin of the universe. Greene considers a discovery of the two features unlikely: a collapse that runs one way and also makes entropy fall toward the past. Greene writes that, without a complete theory of cosmic origins, science can give no answer to what arranged the low entropy of the beginning (*Until the End of Time*, chapter 3, section "The Big Bang and the Second Law"). Greene calls the assumption of a low-entropy past the Past Hypothesis, the name that the philosopher David Albert gave the assumption (a note to *Until the End of Time*, chapter 3). Greene's answer to the egg has three parts: the laws run both ways, the count runs both ways, and an orderly beginning at one end of history gives the count its one direction.

Where Greene and I Share Results

How much of Greene's answer do I share?

I share a great deal of Greene's answer, and in my account the shared results follow from a different starting point: *comparison*. The outcome of one act of comparison is a *finding*. What is found in an act of comparison is the act's *content*. A finding laid down and held after the act that made it is a *record*.

On the laws of physics, Greene and I agree. For nearly every lawful process, the equations of mechanics, of electromagnetism and of general relativity allow the reversed process as lawful too. (Mechanics is the physics of motion, electromagnetism the physics of electricity, magnetism and light, and general relativity Einstein's theory of gravity.) When a reversed process obeys the laws, I call the reversed process a *lawful reversal*. The backward film of the egg is a lawful reversal, as Greene says.

On the count of arrangements, Greene and I agree too, but I add one word. Take a new deck of cards, straight from its box and in order, and shuffle the deck. The deck becomes mixed, and further shuffling keeps the deck mixed. Only a handful of the deck's arrangements look "in order" to a glance. Vastly more arrangements look "shuffled", so each shuffle almost always leaves the deck in one of the shuffled arrangements. For the deck, the context is a glance, and the arrangements are the orders of the cards. The word I add is "context": every count of arrangements is made relative to the comparisons by which someone tells the arrangements apart. In the physics definition, as Greene states it, entropy is the number of rearrangements that leave a system's "big-picture" properties unchanged (*The Fabric of the Cosmos*, chapter 6, section "Entropy"). The big picture is one context: one respect in which someone tells the arrangements apart.

A measurement made in the last century shows that an entropy is literally a count. In ice, each oxygen atom has two hydrogen atoms near it and two farther off. The hydrogen atoms can be placed in many ways that

keep this rule of two near and two far. In 1935 the chemist Linus Pauling counted the ways, and from his count Pauling worked out how much entropy ice must keep even when the ice is cooled toward the coldest possible temperature. In 1936 William Giauque and J. W. Stout measured the leftover entropy of ice from the heat the ice takes in as it warms. Giauque and Stout found the amount that Pauling's count gave, within the precision of their work. So we now know that an entropy is a count of arrangements, and that a careful count of arrangements matches the heat a laboratory measures.

Greene and I agree again that the count has two heads: more disorder toward the future, and more disorder toward the past. Suppose someone hands you a deck that is partly in order: the hearts still run from ace to king, and the rest is mixed. Use the counting to ask what the deck will look like after a few more shuffles, and the answer is: more mixed. Use the counting to ask what the deck looked like a few shuffles ago, and the answer is again: more mixed. Both answers have one reason: mixed arrangements far outnumber ordered ones. You know better only because you know one more fact. Decks leave the factory in order. The factory sets a condition at one end of the deck's history: the deck starts in order. The universe needs a condition like the factory's, set at one end of the universe's history, and like Greene, I use David Albert's name for that condition: the *Past Hypothesis*. The Past Hypothesis is the condition that the universe began with very low entropy. Two results about the count are established physics, and both stand as Greene states them: the counting alone runs both ways, and a condition at one end of history gives the counting one direction.

So my account has Greene's arrow of the count too, under Greene's own name for that arrow. The *entropic arrow* is the growth of entropy in things of very many parts. The entropic arrow is a matter of very many parts, and it needs a beginning in which things were less mixed than they are now. Each of Greene's pictures runs by the entropic arrow, from a less mixed start: the egg, the ice cubes and the pages.

Two Ways to Run a Film Backward

Is the entropic arrow the whole of the direction of time?

On the whole direction of time, my answer differs from Greene's, and the difference begins with two questions about the backward film. The first question is whether the backward film is lawful: does each part of the backward film obey the laws? For the egg, as Greene shows, each part of the backward film does obey the laws. The second question requires much more of the backward film: could the world run back through exactly the states the world passed through on the way forward, every one of them, with nothing left out? When a reversed process retraces exactly the states the original process passed through, I call the reversed process a *faithful reversal*. I separate the two kinds of reversal in two sentences: "Symmetry of the laws guarantees lawful reversal. It does not guarantee faithful reversal".

Think what a faithful reversal of the egg would require. Every piece of shell would have to be sent back along its exact path. Every drop of yolk would have to be sent back too, and so would every molecule of air the fall pushed aside, every bit of warmth the crash left in the floorboards, and the faint sound spreading out through the walls of the house. A faithful reversal of the egg would have to start from a state that fixes exactly the motions of the shell, the yolk, the air, the warmth and the sound. The starting state would have to fix each motion at every place and to every fineness. No part could be left free to go its own way. If the starting state leaves a gap anywhere, however small, the backward run is no longer the forward run retraced. Greene, too, lists every piece that would have to be sent back, and he observes that sending every piece back is beyond what we can do (*The Fabric of the Cosmos*, chapter 6, section "Principle and Practice"). Sending every piece back is beyond what anything can do. The reason comes from two words that Chapter 3 defined: symmetry and continuity.

The laws of physics stay the same when time is counted backward instead of forward. So the time-reversal symmetry of the laws is a sameness of content across a difference of context: the content is the laws, and the difference of context is the direction in which time is counted. If the state of the egg and its kitchen were fixed without any gap, at every place and to every fineness, the fixed state would be continuity perfected. So a faithful reversal needs two things perfected together: the symmetry of the laws, and the continuity of a state fixed without gap.

The *Incommensuration Theorem* states that symmetry and continuity can never both be perfect in one thing. We met the Incommensuration Theorem in Chapter 3.

I give the first half of the Incommensuration Theorem:

> "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous".

"Absolutely", in the first half of the theorem, means perfectly. The theorem's second half says that nothing can be both perfectly asymmetric (different in content across a difference of context) and perfectly discontinuous (different in content within the same context). The theorem holds of every comparison because the theorem uses nothing but the meanings of sameness and difference, content and context. So the theorem holds of anything found in any comparison. So a perfect time-reversal is ruled out; this is a consequence of the theorem, and it must be so. The limit on time-reversal is a limit of principle, and therefore a limit of all practice: no finer instrument removes the limit, because every instrument makes comparisons.

Loschmidt's question about the box of gas now has its answer. Boltzmann replied that the reversed state of the gas, with every velocity turned exactly around, is overwhelmingly improbable. I read Boltzmann's reply as a concession that Boltzmann made without saying so: the gas stays spread out because the gathering of the gas would need a state given

with every velocity exactly reversed. The symmetric laws allow the gathering, but besides the laws, the gathering needs a state given with every velocity exactly reversed, with no part of the state left free to go its own way. If a state were given with every velocity exactly reversed and no part left free, the state would be continuity perfected, beside the symmetry of the laws. Perfect continuity beside perfect symmetry is the pair that, by the Incommensuration Theorem, nothing can have. The reversed path exists in the mathematics, as a solution of the laws; a gas in the faithfully reversed state never occurs in the world.

Experimenters have measured whether the laws themselves stay the same when run backward. A particle decays when the particle breaks up into lighter particles. In 1964 a team of four physicists watched the decays of a short-lived particle, the neutral kaon, at Brookhaven National Laboratory on Long Island. The kaons were made in collisions at an accelerator. The four were James Christenson, James Cronin, Val Fitch and René Turlay. The four physicists found a decay that could not happen if the laws stayed the same under two reversals made together: mirror reflection, and the exchange of matter for antimatter (matter whose particles carry the opposite charges). The quantum theory of particles has a theorem, the CPT theorem, which says that three reversals made together leave the laws exactly the same. Physicists named the CPT theorem after the initials of their names for the three reversals: mirror reflection, the exchange of matter for antimatter, and the turning around of the order of time. So, where the laws do not stay the same under the first two reversals made together, the laws cannot stay the same under the reversal of time alone. The decays of the kaon are governed by the *weak force*, which changes one kind of particle into another in radioactive decay, so the break in the symmetry of time lies in the laws of the weak force. In 1998 physicists at CERN, the European laboratory for particle physics near Geneva, ran an experiment called CPLEAR. The CPLEAR physicists saw the difference between forward and backward directly, in kaons. In 2012 physicists of the BaBar experiment, in California, saw a difference between forward and backward in another kind of particle. So we now know that the laws of physics are

not quite the same run backward. The laws are symmetric in time to an extraordinary degree and never perfectly. Greene reports the CPLEAR result, together with the result of the KTeV experiment at Fermilab, in the note of *The Fabric of the Cosmos* where he gives his judgment on kaons and B-mesons (notes to chapter 6, note 2).

The direction of time is present in every comparison. The growth of entropy in the egg runs from the orderly beginning of the universe. So the egg's arrow does not come from the decays of the kaon. The Incommensuration Theorem shows that no reversal of time can be perfect. I draw from the theorem a limit on the laws themselves. The laws show the symmetry of time to an extraordinary degree, and the argument says that the symmetry cannot be perfect. I read the decays of the kaon as one instance of the imperfect symmetry of time that the Incommensuration Theorem requires, and experimenters find the size of that imperfection by measurement.

Runners Turned by a Pistol Shot

If no reversal is perfect, can any reversal be made at all?

Yes: one kind of reversal has been made, and the way experimenters made the reversal shows in which things a reversal can succeed. Picture runners on a track who start together from one line at a pistol shot, each at a speed of his own. After a minute the runners are strung out around the track. At a second shot each runner turns and runs back at his own speed. After another minute all the runners reach the starting line together. The scattering of the runners was undone because each runner's own place on the track held how far he had pulled ahead or fallen behind, and because one shot turned every runner at once.

In 1950 the physicist Erwin Hahn made a reversal of the runners' kind with the nuclei of atoms in a liquid. The nucleus is the tiny core of an atom. Each nucleus carries a turning of its own, called its *spin*, which makes the nucleus a small magnet. In a magnetic field, the spin of a nucleus swings around the direction of the field. Nuclei in slightly different

fields swing at slightly different rates, so a set of nuclei lined up together soon drifts out of step, and the radio signal that the nuclei give out together dies away. Hahn tipped the nuclei with one pulse of radio waves, and after the nuclei had drifted out of step, Hahn sent a second pulse that reversed the sense of each nucleus's drift. After a further equal time, the signal came back. Physicists call the returned signal the spin echo. In 1954 Herman Carr and Edward Purcell refined Hahn's method. In 1971 Won-Kyu Rhim, Alexander Pines and John Waugh reversed even the pulls of the nuclei on one another. The spin-echo experiments show where a reversal works: each nucleus held its own drift, as its own angle, as each runner held his own lead, and one pulse could reach every nucleus, as one shot turned every runner.

I follow what particles leave in one another when the particles strike each other. When two particles strike, each particle is changed in a way that depends on the other particle. So each particle comes to carry a *trace* of the other. From then on, the state of the one particle is tied to the state of the other particle. A reversal would have to gather back every trace. I set out two cases, which differ in where the traces have gone. In the first case, a thing is open to its surroundings: something can leave the thing, as something can leave every thing in the world. A thing open to its surroundings sends some of its traces away. A warm glass of water sends out light. The light that has left travels off at the speed of light and cannot be called back. So, for a thing open to its surroundings, the entropy of the system together with what has left it never falls, with no exception.

In the second case, a thing is wholly sealed off, and the thing keeps its traces among its own particles. Traces kept among a thing's own particles can, in principle, still be gathered back. Hahn's nuclei each held their drift alone, as each runner held his own lead. Now picture a sealed glass of water with a drop of ink in it. In the glass, each trace ties two molecules together, and those two molecules pass traces on to other molecules. So the traces spread through more and more of the glass. If the mixing of the ink is to be undone, a reversal would have to act on every molecule the traces have reached, and put right, all at once, every way those molecules can be

arranged together. Each molecule drawn in multiplies the number of ways the molecules can be arranged together, as each card added to a deck multiplies the number of orders the deck can be in. So a return to less entropy is found only where the traces have reached few particles, as in Hahn's nuclei. A return to less entropy is never found where the traces have reached many particles. Physicists call a thing sealed off from its surroundings an isolated system. For an isolated system, the second law of thermodynamics says that the entropy of the system does not decrease. Greene says that a fall of entropy is ridiculously unlikely. For a thing open to the world and counted together with all it has given out, the entropy never falls, because what has left cannot be gathered back.

An Arrow in One Footprint

Is there anything in a single event that runs one way, with no crowd of parts to count?

Watch someone cross a patch of wet mud. Before each step there is the walker, with every place her foot might come down. Then there is the step itself, in which the foot comes down in one place and not the others. Then there is the footprint: laid out in the mud, the same for anyone who comes by and looks. And the footprint shapes what the walker does next: she sees how deep the footprint is, and she steps more carefully.

Every comparison has three roles, in the order of walker, step and footprint. Chapter 3 named the three roles. The walker is the stance: the side a comparison is made from, with the range of what could come. The step is the act: the happening itself. The footprint is the structure: what is laid down, the same for everyone who reads it afterward. A record is the structure that one act lays down. In Chapter 6 we met the three *axioms* of the Immanent Metaphysics. The axioms are the basic statements from which the rest of the Immanent Metaphysics is worked out. The second of the three axioms states the order of every comparison whatever: first the stance, then the act, then the record. The record then shapes the next stance. The order runs one way only. No footprint comes before its step.

In Chapter 2, I identified the one-way order of findings (act before record) with the direction of time. Every record ever found follows the act it records: no footprint has been found before its step, no photograph before its exposure, and no count on a detector before the particle that made the count. The order of act before record needs no large number of parts and no special beginning of the world: it is present in one comparison, made once.

Set two results side by side. The first result: the Incommensuration Theorem shows that no faithful reversal can happen. The second result: the order of every comparison runs one way, act before record. Together the two results describe the *structural arrow*: the one-way order inside every comparison, which no reversal can undo, and which needs no special beginning. I call the arrow structural because the way every comparison is built requires the arrow. In the name "structural arrow", the word "structural" names all three roles in their order, and not the third role, the structure, by itself. The structural arrow is present in one step in the mud. The entropic arrow is present only where there is a crowd of parts to mix and a less mixed start to mix from. I set the two arrows in order: "The entropic arrow is how the structural arrow shows itself at our own scale, in systems of many parts, where reasoning by chances applies".

Is Greene right to suspect that time symmetry persists in quantum mechanics? In the sense of Chapter 3, the *state* is the complete description of what could be found. In quantum mechanics, the state changes by an equation that runs as well backward as forward. A *formalism* is the mathematical form in which physics is written. The formalism of quantum mechanics is deterministic: the equation of quantum mechanics fixes each next state exactly. Chapter 2 gave two words exact meanings: to exist is to stand as structure, the same in every instance, and to be real is to be given in an actual act of comparison, here and now. I place the equation of quantum mechanics in one sentence: "The evolution is deterministic in the formalism, which exists as structure, and not in the findings, which are real". In the two meanings that Chapter 2 gave, the equation exists. The findings are real, each finding given in an act, one finding after another.

So Greene's suspicion is right about the equation, but the direction of time lies in the order of the findings, and running the equation backward removes no finding from that order.

Where the Two Arrows Meet

If the structural arrow and the entropic arrow are different things, why do the two arrows always point the same way?

The two arrows point the same way because every record carries both arrows. Making the footprint was itself a mixing: the walker's foot squeezed and spread the mud, and pushed aside the water in the mud. The footprint does not unmake itself. A grain of photographic film darkens where light strikes the grain, and the grain does not lighten again. A counter steps up by one and does not step back. Every record is made by a change that is not undone. Making a record raises the entropy of the thing that holds the record. So each act of recording joins the two arrows in one event: the order of the act, and a growth of entropy in the record. Every record is one event in which both arrows are present, and in that event the two arrows point the same way.

The quagmire Greene described was a quagmire about records: whether our memories and records can be trusted. Taken alone, Greene's count made our memories and records more likely the products of a fluctuation than the results of what happened, and Greene kept his trust in memories and records by placing a low-entropy beginning in the past. On my account, our trust in records has a second support: the order of every comparison, act before record. Every record follows the act it records, and so does every increase of entropy that a record registers. The footprint in the mud follows its step, and so does the growth of entropy that the walker's foot made when the foot pressed the mud.

What the Past Hypothesis Is For

If the direction of time does not depend on the orderly beginning, what does the orderly beginning do?

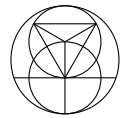
The Past Hypothesis is a condition on the entropic arrow. The Past Hypothesis is not the source of the arrow. The low entropy at the start of our history fixes how the direction of time shows itself across the whole of that history, in the long running down of things of many parts. Stars burn their fuel, cups of tea cool, and eggs splatter and stay splattered, all in the direction that every single comparison already has.

So Greene's chain holds, from the egg back to the big bang, and the chain is true of the entropic arrow. Greene is right that the growth of entropy in our world runs from an orderly beginning. Greene is right, too, that conditions at the birth of the universe direct the growth of entropy. The direction of time itself does not depend on the conditions at the birth of the universe. Greene still has a real puzzle, now about the entropic arrow only: why the beginning was orderly. The low entropy of the beginning has a reason of its own, and the next chapter sets out that reason. So no law needs to be added to physics to supply the orderly beginning.

We now know that time has a direction in every single comparison, act before record, and that this direction needs no special beginning. At our scale, the direction of time shows itself as the growth of entropy in things of many parts. Only the growth of entropy needs an orderly beginning.

Greene's question about the arrow of time became a question about cosmology, and the question is still about cosmology, with one change. Only the growth of entropy depends on the beginning, so the question for cosmology is why the entropy of the beginning was low. The beginning does not have to supply the direction of time. The splattered egg on the floor shows the same arrow as a single step in the mud: the order of act before record, made large by a crowd of parts, and running down from an orderly start.

Then why was the beginning orderly, and what was the beginning?



Chapter 9

What Banged?

Beside The Fabric of the Cosmos, *chapters 8, 10 and 11,* The Elegant Universe, *chapters 3 and 14, and* Until the End of Time, *chapter 3.*

What this chapter uses

The *principle of comparison*: a difference that no comparison can register is no difference (Chapter 1).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

I identify the one-way order of findings with the direction of time (Chapter 2).

The world has ten dimensions: one of time, three of space, and six of possibility; *possibility* is the range of what a comparison could find, and nothing ever travels through *the six* (Chapter 7).

In the spring of 1965, Arno Penzias and Robert Wilson cleaned out a horn antenna at the Crawford Hill Laboratory in Holmdel, New Jersey, where a pair of pigeons had roosted. For months the two men had chased a faint hiss in the antenna, and after the cleaning the hiss fell only a little. The hiss was light left from the early universe. Brian Greene is the physicist whose four books this book reads beside. Greene says that the big bang theory "tells us nothing about what banged" (*The Fabric of the Cosmos*, chapter 10, opening). So the hiss leads to the question Greene's sentence leaves open: what banged?

Penzias and Wilson worked for Bell Laboratories. They had built a receiver of radio waves so quiet that they could trace every source of noise in the receiver. They subtracted the noise of the sky, the ground, the air and the antenna itself, and a little noise was left over: about three and a half degrees of warmth above absolute zero, the coldest there can be. The leftover warmth came from no source they could find. The paper of Penzias and Wilson reports that, within the limits of their observations, the excess warmth was the same in every direction and did not change with the seasons, from July 1964 to April 1965. Then Penzias talked about other matters with the radio astronomer Bernard Burke, and in that talk he heard that a group at Princeton, less than thirty miles away, had been working out what a hot beginning of the universe would have left behind. Penzias telephoned Robert Dicke, the physicist at the head of the Princeton group. James Peebles, of Dicke's group, had worked out that a hot beginning should have left a glow filling space, and Dicke, with Peter Roll and David Wilkinson, had been building an instrument to look for that glow. Penzias and Wilson published side by side with Dicke's group in the *Astrophysical Journal* in 1965. In 1978 Penzias and Wilson received the Nobel Prize in physics.

Penzias and Wilson had found the *oldest light*: light left from the early universe, stretched by the expansion of space until its warmth now stands at about 2.7 degrees above absolute zero. Astronomers call the oldest light the cosmic microwave background. Turn a radio telescope to any part of the sky, by day or by night, and the telescope picks up the oldest light.

From Greene's pages

The Bang That Gravity Gave

What does Greene show about the beginning of the universe?

Greene begins with the expansion of the universe. General relativity is Einstein's theory of gravity, in which gravity is the curving of space and time, and in 1922 the physicist Alexander Friedmann found that general relativity allows a universe whose size changes with time. In January 1929 the astronomer Edwin Hubble found that the farther away a galaxy is, the faster the galaxy moves away from us. Greene reads Hubble's pattern as space itself stretching, so that no galaxy sits at a center, and he writes: "In the big bang, there is no surrounding space" (*The Elegant Universe*, chapter 3, section "Black Holes, the Big Bang, and the Expansion of Space"). Greene shows too that the whole universe has one age, because space is the same everywhere. Clocks carried along with the expansion tick alike, here and in far galaxies. In his words, "The homogeneity of space thus provides a universal synchrony" (*The Fabric of the Cosmos*, chapter 8, section "Symmetry and Time").

Then Greene names what the big bang theory leaves out. Run the expansion back, and the universe grows denser and hotter, toward a first moment. The big bang theory describes what happened from a split second after that first moment onward. So, Greene writes, "the big bang leaves out the bang" (*The Fabric of the Cosmos*, chapter 10, opening). Gravity pulls things together, so what pushed space apart at the start?

Greene's answer is *inflation*, his word and the word of most cosmologists for a brief early burst of ever faster expansion. Inflation depends on a fact about pressure, and Greene shows that fact with everyday things. Squeeze the spring of a jack-in-the-box under its lid, and the spring pushes outward on the box: that outward push is a positive

pressure. A stretched rubber band pulls inward on whatever holds its ends: a pressure that pulls inward instead of pushing outward is a negative pressure. Greene shows that in general relativity pressure is a source of gravity, as mass and energy are (chapter 10, section "Einstein and Repulsive Gravity"). A positive pressure adds to the pull of gravity. A negative pressure adds a push, and where the pressure is negative enough, gravity pushes things apart.

In December 1979 the physicist Alan Guth found that a *field* (a quantity with a value at every place) can carry a negative pressure. The field carries a negative pressure while its value is caught for a time on a plateau. On the plateau the field's energy is higher than its least energy. A ball can rest in the same way for a while on a flat shelf partway up the side of a bowl, instead of at the bottom. The place of the ball stands for the value of the field, and the height of the ball stands for the energy of the field. Physicists call the field that drives inflation by its negative pressure the *inflaton*. In Greene's account, the inflaton drove space apart by a factor of a one followed by thirty zeros, or more, in far less than a billionth of a billionth of a billionth of a second. In Greene's words, "Guth provided the big bang with a bang" (*The Fabric of the Cosmos*, chapter 10, section "Inflation").

Picture a baker's patch of dough, small enough to sit in your palm, kneaded as one piece. Stretch the dough, in a moment, until the dough covers a whole city. Every fold is pulled flat, and anyone standing anywhere in the city finds the same smooth dough underfoot. In Greene's account, inflation works like the stretched dough. The patch of dough is a tiny region of the early universe, small enough to have come to one temperature, and inflation stretched the tiny region far beyond everything we can see, so the sky is the same in every direction. Any curve the tiny region had was stretched flat, as the folds of the dough were, so space on the largest scales is flat. The stretching answers two puzzles (chapter 10, sections "Inflation and the Horizon Problem" and "Inflation and the Flatness Problem"). The first puzzle

is the *horizon problem*: why regions of the sky that never exchanged a signal agree so closely. The second puzzle is the *flatness problem*: why space began so flat.

Greene tells that Guth and the physicist Henry Tye came upon the field caught on a plateau while they worked on a third puzzle, which Greene names the *magnetic monopole problem* (chapter 10, section "Of Jumping Frogs and Supercooling"). Researchers had run into the magnetic monopole problem in their attempts at *grand unification*. Grand unification is the joining of the three forces other than gravity into one force. A magnetic monopole is a heavy particle that carries a single magnetic pole, a north with no south. Grand unified theories would join the three forces, and they predict far more magnetic monopoles than the measured expansion of the universe allows. Inflation spreads the monopoles so thin that they no longer matter to the expansion.

Greene credits inflation with one more result. The inflaton trembles a little at every place, as every field described by quantum theory (the theory of the smallest things, met in Chapter 3) does, and inflation stretched that trembling across the sky. Where the field was a little denser, gravity later gathered more matter, and the clumps became galaxies, which in Greene's words "are nothing but quantum mechanics writ large across the sky" (*The Fabric of the Cosmos*, chapter 11, section "Quantum Skywriting"). Greene judges inflation "far and away the front-running cosmological theory" (chapter 10, section "Puzzles and Progress"). Yet Greene writes in *Until the End of Time*, chapter 3, section "The Afterglow", that "it would be too strong to conclude that the observations prove that a burst of inflationary expansion happened", since physicists have developed other approaches. In the same section of *Until the End of Time*, Greene gives his own view, which he says many researchers share: the case for inflation, built over forty years, is formidable, while ideas that challenge the dominant view still need an open hearing.

Greene shows that the smoothing done by inflation also explains the arrow of time, the one-way direction from past to future. The matter of the young universe was hot and evenly spread. For gravity an even spread is the orderly state, because gravity gathers matter into clumps. So the smooth beginning had low entropy, and later clumping could raise the entropy. To explain how inflation began, Greene pictures an earlier state, of high energy and disorder. In Greene's picture, sooner or later a chance fluctuation in that earlier state made the inflaton field jump to the right value, uniform throughout one small nugget of space (*The Fabric of the Cosmos*, chapter 11, section "Boltzmann Redux"). Greene calls the step of specifying the earlier state "a pivotal assumption that's as yet unjustified", and he says that the state he pictures is "only a guess" (*The Fabric of the Cosmos*, chapter 11, section "The Fly in the Ointment?").

Greene then tells the rest of the early history in order of time. The earliest era of all lies before the *Planck time*, a moment so early that gravity and quantum theory must be described together. For the era before the Planck time, Greene writes that "we must make use of string theory" (*The Elegant Universe*, chapter 14, section "Cosmology and Superstring Theory"). Greene reports that physicists have taken the first steps toward understanding string cosmology, the history of the era before the Planck time as string theorists would tell it, and that the work is at an early stage. After inflation the universe cooled, and as it cooled it changed its state, as steam condenses and water freezes, and in one such change the Higgs field (the field whose settled value gives many particles their masses) took its value everywhere. In the first minutes the nuclei of the *light elements* formed: helium above all, with a little heavy hydrogen and a trace of lithium. Observations confirm the proportions in which the light elements formed.

Greene's picture has two more parts, about the universe as it is now. First, most of the matter gives off no light. Greene writes of what the unseen matter is made of, "So far, no one knows", and he adds that

astronomers have measured how much unseen matter there is "with significant precision" (*The Fabric of the Cosmos*, chapter 10, section "A Prediction of Darkness"). Second, Greene reports that the most refined data favor space with no overall curve, "the flat shape", though he notes that the question was not yet settled definitively (chapter 8, section "Cosmology, Symmetry, and the Shape of Space").

The Same Sky, on Separate Ground

On which results do I agree with Greene?

My account agrees with Greene on every measurement he reports. The universe is expanding, as Friedmann's equations allow and as Hubble's galaxies show. The oldest light fills the sky, and in 1992 the Cosmic Background Explorer satellite found the temperature of the oldest light the same in every direction to about one part in a hundred thousand. The temperature differs from one direction to another by about one part in a hundred thousand, and those slight differences mark slight differences of density in the early universe, from which gravity grew the galaxies, as Greene says. For matter held together by its own gravity, clumping raises the entropy, and so the history of the universe runs from smooth to clumped.

The matter at the beginning was hot and close to *equilibrium*, the settled state in which nothing further changes on average, so the entropy of the matter was already high. The mathematical physicist Roger Penrose showed, and Greene says too, that the entropy that was low at the beginning was the *entropy of gravity*. The entropy of gravity is the count of arrangements for the shape of space and time itself. The entropy of gravity is low when the shape of space and time is smooth, with almost no clumping. The shape was smooth in that way at the beginning.

On my account, the amounts of the light elements also come out as Greene says. A *proton* is the particle at the core of every atom that carries a positive electric charge. A *neutron* is the particle of that core that carries no charge. About one neutron is left for every seven protons, two minutes

into our history. Nearly every one of those neutrons ends up in helium. The common nucleus of helium is two protons and two neutrons. Take eight particles: one neutron and seven protons. The one neutron goes into helium with one proton beside it. A proton and a neutron weigh almost the same, so a quarter of the mass ends up in helium: two particles of every eight. When I count exactly, helium makes up between twenty-three and twenty-five percent of the mass of ordinary matter. The measured share is about twenty-four and a half percent.

I agree with Greene, too, that our history of the universe leaves out the bang itself. When we trace the history of the universe backward, we never come to a first moment that the history can describe as an event. The reconstruction is the history we piece together from what reaches us now, and the time zero of cosmology is where the reconstruction meets the limit that it cannot pass. Greene's telling and my account differ over what lies at the limit that the reconstruction cannot pass.

The Bucket and the Oldest Light

Can we measure whether we are still, or moving, against the whole sky?

Recall Isaac Newton's bucket from Chapter 1. Once the water turns, the water climbs the walls of the bucket, and the water's surface sinks into a hollow. The hollow is a *finding*: the outcome of one comparison, made by the water and laid down in the shape of the water's surface. The water finds its hollow against what is there to compare with. The water can still be compared against the matter within its reach. The matter within the water's reach fixes what counts as still and what counts as turning.

The oldest light is the farthest thing there is to compare with, and astronomers have compared our motion against the oldest light. In 1977 three physicists flew a radio receiver at twenty kilometers' height aboard a U-2 aircraft: George Smoot, Marc Gorenstein and Richard Muller. They found the oldest light warmer in one direction than in the opposite direction, by about three and a half thousandths of a degree. When light comes

from the direction we move toward, its waves arrive squeezed a little shorter, and so the light is a little warmer, as the pitch of a siren rises while the siren comes toward you, because its sound waves are squeezed shorter. So the difference comes from our own motion, found against the oldest light. From that difference Smoot, Gorenstein and Muller measured the Earth moving at about 390 kilometers each second relative to the oldest light.

The *Planck satellite* was a space telescope of the European Space Agency, named for the physicist Max Planck, and it mapped the oldest light across the whole sky from 2009 to 2013. In 2016 five cosmologists searched the Planck maps for the swirl that an unevenly expanding universe would leave in the oldest light. The five were Daniela Saadeh, Stephen Feeney, Andrew Pontzen, Hiranya Peiris and Jason McEwen. The search covered turning universes too, since a turning universe is one kind of unevenly expanding universe. Saadeh and her colleagues found no swirl in the Planck maps. When they took every kind of uneven expansion they tested together, they found odds of a hundred and twenty-one thousand to one against an uneven expansion of any of those kinds. So we now know that our own motion is found against the oldest light, and that the maps of the oldest light show no swirl of an unevenly expanding or turning universe.

I set the two results, our motion found against the oldest light and no swirl in the maps of the oldest light, beside the principle of Newton's bucket. By the principle of the bucket, the matter within reach fixes what counts as still and what counts as turning. The oldest light comes from the farthest matter whose light has reached us. So the distant matter within reach fixes one standard of not turning, for the water in the bucket and for the whole sky alike, and that standard is one of three facts of the world on which this book's answer to what banged depends, as the next section shows.

What Banged: The Turn

Then what lies at the limit that our history of the universe cannot pass?

On this question I differ from Greene, and the reason for the difference begins with black holes. A *black hole* is a region whose gravity is so strong that nothing, not even light, comes back out of it, and its edge is its horizon, the surface from inside which no signal reaches the outside. Inside the horizon, moving toward the center is no longer a choice of direction, just as moving toward tomorrow is no choice. So the end inside a black hole is a moment ahead, and not a place: the end, in the future, of every path inside the horizon, which everything that falls in approaches.

Where does every bound thing of our era go in the end? A bound thing is held together by its own gravity, as a star, a galaxy and a cluster of galaxies are. The expansion of the universe is speeding up, so whatever is not bound to an observer's own galaxy is carried out of that observer's reach, over about a hundred billion to a trillion years. Chapter 16 describes in full how the speeding expansion empties each observer's sky. Within each galaxy, stars burn out, the remains of the stars gather and merge, and bound structures end in black holes. So every bound thing of our era ends in one of two ways: inside a black hole, or carried away alone by the expansion.

Now draw the interior of each black hole as a funnel that narrows toward the end of the interior. The funnels stay apart along their whole length, but the ends of all the funnels are one end. The reason the ends are one is the *principle of comparison* of Chapter 1: a difference that no comparison can register is no difference. While someone falls inside one black hole, the one who falls registers that black hole and not another. So the interiors are distinct while anyone travels through them. At the end of the interiors, no comparison can find the end of one interior different from the end of another. Ends that no comparison can tell apart are one end. So, in each era, the interiors of all the black holes have one common end.

Recall the ten dimensions of Chapter 7: one of time, three of space, and *the six*. The six are six dimensions of possibility, held a few Planck lengths across, which nothing ever travels through. The *Planck length*, met in Chapter 5, is about a hundred billion billion times smaller than the width of a proton, and it is the finest scale at which any comparison made from inside the world can place a body. The *Planck scale* is the scale at which gravity and the quantum meet on equal terms: there both descriptions of physics, quantum theory and general relativity, apply, and they stay two separate descriptions.

What a comparison finds is its *content*. Chapter 7 showed that the six are held small as the content of the large world of four dimensions, which is their context. Whether the large world holds the six depends on three plain facts of the world around you. First, the world has an outside, beyond whatever room you sit in. Second, the space and time of the world are smooth at every scale at which anything can be compared. Third, the distant matter of the world fixes what counts as not turning, as the water in Newton's bucket finds. While the world has an outside, smooth space and time, and distant matter that fixes what counts as not turning, the large world keeps the six small, with one size and one shape at every point of the large world. In this book, *the hold* is the large world of space and time keeping the six at their one size and shape. The hold keeps the forms of matter too, along with the six.

At the end of the interiors all three facts fail. First, the outside world has no moment at the end of the interiors. Second, the smooth description of space and time has reached the edge of its range, the limit past which it does not apply, at the Planck scale. Third, no matter is left at the end of the interiors to fix what counts as not turning. So the large world is no longer the context in which the six are held, and the six are released. Yet the content, what comparison finds, continues at the end of the interiors, and content needs a context. The content does not cease at the end, and content needs a context, so the ten-dimensional geometry restores one. So the ten-dimensional geometry re-forms its large part: three directions of space open out while six close in, and a new large world begins.

The re-forming of a large world is the *turn*: the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world. An *epoch* is each era of the world, from one turn to the next. No comparison registers the turn itself as an event, because no comparison is made across the turn, of a before with an after. How long, how hot and how fast can be asked only of the approach to the turn on each side, and not of the turn.

We piece together our history from what reaches us now: the light of far galaxies, the oldest light, the amounts of the elements. Draw our history as a curve of the size of the universe against time, and the curve rises to the right. If we trace the curve back to the left, it heads for the line at which the size would be zero, and it never reaches that line. A history records everything as happening at a definite time and place. So our history draws the line of zero size as a point in time: a beginning. I identify the Big Bang with the turn as our epoch's history renders the turn: the limit, which no comparison registers as an event, drawn as the first event of the history we reconstruct from inside.

So the answer to "what came first?" is the collapse of the interiors of the black holes of the epoch before ours, which approached the same limit from the other side. "Before" here names the order of a cycle, and no clock measures a time across the turn. The expansion that our history describes, run forward, begins as the opening of the three directions of space while the six close in, and my account has no inflaton. That book says what the turn is not: "The content continues through it, so the turn is not a creation from nothing". In Greene's telling, inflation would let a nugget of uniform inflaton field, weighing about twenty pounds, gain enough energy to make everything we see in the universe today (*The Fabric of the Cosmos*, chapter 11, section "Creating a Universe"). On my account, by contrast, the content of our epoch is carried through the turn from the epoch before.

Two more results follow from the turn, and each can be set beside a passage of Greene's. First, three directions of space open, because the large world, the context, has three directions of space, the number Chapter 7 found. Greene reports another reason, from the physicists Robert Brandenberger and Cumrun Vafa, as "a possible explanation" (*The Elegant Universe*, chapter 14, section "Why Three?"). In the proposal of Brandenberger and Vafa, strings wound around the directions of space keep those directions from growing. The exception is three directions, where the wound strings collide and undo one another. Second, the six reform with the same shape at every turn: the one shape of Chapter 7.

Greene reads the temperature and density that soar without limit at time zero as the sign that the model of the universe built on general relativity "has completely broken down" (*The Elegant Universe*, chapter 14, section "Cosmology and Superstring Theory"). On my account, the turn is no breakdown at which the laws fail. On both sides of the turn the principle of comparison applies, and so does every rule true of every act of comparison. At the turn each description of physics reaches the edge of its range: the smooth one (general relativity) and the stepwise one (quantum theory). My account also has no *initial state*, no arrangement chosen at a first moment. The beginning that our history finds is our side of a limit, and the epoch before approached the same limit from its own side.

Two Towns, One Accent

Why is the oldest light the same in every direction, when no signal crossed the sky?

Take two directions of the sky more than about a degree apart, about twice the width of the full Moon. In the standard history of cosmology, no signal could have passed between regions in any two directions more than a degree apart before the oldest light was released. Yet the oldest light from the two directions has one temperature, to about one part in a hundred thousand.

Two towns far apart may share an accent, although the people of the two towns have never spoken to each other. Settlers from one village founded both towns, and both towns speak as the village spoke. No letter ever had to pass between the two towns. The sky is like the two towns. The village stands for the one origin from which the new world re-forms: the one common end of all the interiors of the epoch before, which no comparison divides into parts. The geometry of the new world has no past of its own, so the geometry inherits no unevenness from the world before the turn. Through the turn the content carries its own form, the pattern of the outcomes its matter took, which the section "The Worn Stairs" below follows. Regions of the sky are alike because they come from one origin, as the two towns speak alike because they come from one village, and no signal ever had to connect the regions, as no letter had to pass between the towns. The one origin is my answer to the horizon problem.

Why Space Is Flat

And why is space flat on the largest scales?

An ant on a marble finds the curve of its world within a few steps. On a ball the size of the Earth, an ant walks for days and finds the ground flat. Both surfaces are curved, and one length tells the two surfaces apart: the radius of each ball. A *curvature* of space as a whole is a bending like that of the surface of a ball. A length sets the curvature of space in the same way. The length that sets a curvature is its *radius of curvature*: the radius of the ball whose curve the space would follow. Einstein's equations leave the radius of curvature free: the state from which the history of the universe starts sets the radius once, and the equations carry the radius unchanged from then on.

So whatever radius of curvature the new space has must be made from what the state at the turn contains. The state at the turn inherits no length from the old epoch. The reason is that no comparison made in the new epoch registers the size or the mass of the old interiors. The state at the turn contains one length of its own, the length of the turn, which is about

the Planck length. Otherwise the state at the turn contains only numbers not far from one, such as two or ten. If a radius is made from the length of the turn and from numbers not far from one, the radius is about a Planck length. Only a very large number could stretch the length of the turn into a radius large enough to leave a curve we could measure today: a number of about a one followed by thirty zeros. Only counts of states are as large as a one followed by thirty zeros, and no count passes the turn. Each epoch begins its own count, as the census in the section "A Clean Slate, Counted" below shows.

A Planck time is the time light takes to cross a Planck length. A radius of curvature as small as the Planck length would make the new epoch fall back on itself and end within a few Planck times, or the small radius would leave the bend of space, and not the matter and light of the epoch, to set how the epoch expands. In either case the epoch would form no stars and no black holes. Yet every epoch comes from the black holes of the epoch before it. A large radius cannot be made at the turn, and a small radius would leave an epoch with no black holes from which a next epoch could come, so no radius of curvature is left, and space on the largest scales is flat, curved only by its own small ripples.

Measurement agrees that space is flat. The team of the Planck satellite read its final results together with the spacing of the galaxies, and the team found no bend in space: the part of the universe's energy that a bend would take agrees with zero to within about two parts in a thousand. The data Greene reports and my argument agree on one result, flat space, and they start from separate grounds.

The Worn Stairs

If no inflaton trembled, where do the slight differences in the oldest light come from?

Climb the stone stairs of an old church, and look at the steps. Each step is worn into a shallow curve, lowest where most feet have fallen over hundreds of years. No one can pick out a single footstep in the worn curve,

and yet no footstep has been wiped out of the curve either. Each footstep wore away its own small share, and the curve is all the footsteps together, one shape on one stone.

The matter of the epoch before took part in many findings, and every finding came out one way and not another. The *form of the content* is the pattern of the outcomes of all those findings: which alternatives the matter took. The form of the content is carried through the turn with the content. The *first ripples* are the faint differences of density with which our epoch began. I identify the first ripples with the form of the content carried through the turn. I call the first ripples "the distinctions of the old epoch averaged and not erased". Each footstep stands for one outcome of the old epoch, and the worn curve stands for the faint shape of the first ripples. Every outcome of the old epoch is in that faint shape, as every footstep is in the worn curve, and no one can pick a single outcome out of the shape.

Symmetry is sameness through a change, and *continuity* is smoothness, with nothing sudden. One half of the Incommensuration Theorem, met in Chapter 3, is that nothing can be both perfectly symmetric and perfectly continuous. The Incommensuration Theorem therefore says that there must be some ripples. If a new world were exactly the same in every direction and smooth all through, it would be both perfectly symmetric and perfectly continuous. So the new world begins very nearly even, and never perfectly even.

Two more results follow from the carried form. Could the ripples come instead from the trembling of empty space, as they do in Greene's inflation? Empty space is never perfectly still: every field keeps a least, restless trembling that cannot be removed. Without an inflation to stretch that trembling from tiny lengths to large ones, the trembling adds almost nothing to the ripples. A power of ten is one more zero on the end of a number. On the lengths we see, the trembling of empty space falls short of the measured ripples by more than forty-two powers of ten. So the ripples are the carried form, and not the trembling of empty space.

Because the ripples come from one origin, the ripples are also linked across the whole sky. Lay a floor of small tiles, each tile shaded light or dark by a different worker who never saw another tile, and from far back the floor looks an even gray. Weave a cloth on one loom, with one design, and the cloth keeps its swirls as strong across its whole width as across a hand's breadth. If a world were assembled from separate pieces, one for each old interior, the world would be like the tiled floor: its ripples would fade on large lengths. The Planck satellite measured the strength of the ripples of the sky at about one part in a hundred thousand, and found the ripples nearly as strong on large lengths as on small ones. So the sky is like the woven cloth. From there the story is the one Greene tells: gravity gathered the denser places, and the clumps became galaxies, stars and us.

No Inflation Needed

Then do I need inflation at all?

The account answers every puzzle inflation was introduced to solve, from the turn and from the structure of the forces, so it needs no inflation.

The sameness of the sky comes from the one origin at the turn. The flatness of space comes from the state at the turn, which carries no length from before. Grand unified theories would have scattered magnetic monopoles through the young universe. The monopoles never form, because the forces were never one larger force. So no larger force breaks apart into the three forces as the universe cools, and nothing is left behind to make monopoles. The three forces are three and stay three, as Chapter 6 showed.

A long inflation cannot have happened. In the first moments of our history there was an *early horizon*: the greatest distance a signal could have crossed since the turn. A long inflation is one that stretches the lengths we see today from inside the early horizon to far beyond the early horizon. A long inflation would print the trembling of its own empty space on the

lengths we see today, so the ripples on those lengths would be that trembling, but I identified those ripples with the carried form. So in my account a long inflation cannot have happened.

An inflation would leave one more mark, in gravitational waves. The *first waves* are gravitational waves present from the beginning of an epoch: ripples of the arena itself. In an inflation, the inflating empty space makes the first waves. The strength of the first waves grows with the energy that the empty space holds. The first ripples are ripples of density, and waves of the arena arise from them only as ripples acting on ripples. Children standing still on a trampoline press the trampoline into dips of different depths, and they set no wave running across the trampoline. The matter and light at the beginning of our epoch press on the arena as the standing children press on the trampoline, equally in every direction: the dips correspond to the first ripples, and a wave running across the trampoline would correspond to the first waves. The *power* of a ripple is its strength multiplied by itself. The first waves carry far less than a millionth of the power of the first ripples. The BICEP and Keck telescopes stand at the South Pole. In 2021 their teams found no first waves, with data from the Planck satellite and the earlier Wilkinson Microwave Anisotropy Probe satellite as well, and they placed the power of the first waves below thirty-six thousandths of the power of the first ripples. Coming maps of the oldest light will measure the power of the first waves far more finely. Appendix A sets out what I say the coming maps will find.

A Clean Slate, Counted

Why was the beginning so orderly? The last chapter ended on that question.

Think of a census. Census clerks count people by the lines of their own forms. Everyone who fits one line is counted on that line, however they differ in ways the form does not ask about. When a new country is founded with new clerks and new forms, its first census starts a count of its own. The people are all still there, but the old count stops.

Entropy is a count of the same sort as a census: the number of arrangements of a thing's parts that some context cannot tell apart. A context is like the form of a census: arrangements that the context cannot tell apart are counted together, like the people on one line. An observer's *outside world* is everything that can ever reach that observer. At the turn the outside world of the old epoch has emptied, because the speeding expansion carried everything else out of each region's reach. So at the turn no count reckoned against the old world is carried across, while the content and its form are carried across. I call the dropping of the old counts at the turn, while the content and its form are carried across, the *reset*. The new world is a new context, and the new world starts its own count, as the first census of the new country does.

Now picture a sheet of paper fresh from the mill. The fresh sheet has no creases, because creases come from a sheet's own past of folding. The new geometry of space and time had no separate past before the turn, so the new geometry has nothing to be lumpy with: like the fresh sheet, it is smooth, except for the small ripples the Incommensuration Theorem requires. The entropy of gravity in the new epoch is therefore small, while the matter is hot and its entropy already high, as Penrose found. In 1979 Penrose proposed, as a separate principle, that the geometry of space and time began without the part of curvature that squeezes shapes out of round, while on my account Penrose's condition follows from the turn, with no separate principle.

In Greene's own picture, his chance fluctuation is like a slot machine played until it pays: "we only need the cosmic slot machine to pay out once" (*The Fabric of the Cosmos*, chapter 11, section "Boltzmann Redux"). Penrose treats our beginning as one draw from all the states the same matter could have taken, and by his estimate our beginning is one draw in ten raised to a power that is itself a one followed by a hundred and twenty-three zeros. Picture a lottery. A ticket counts as one in a million only where a million tickets are held and each ticket can be told from the others. An improbability is a comparison of the actual case with alternatives that some context distinguishes. At the turn no context is there to tell the

other possible beginnings apart, as no one is there to tell the tickets apart. So the low count is the count that a new context starts with, and the low count needs no lucky draw. In Greene's telling, the low entropy of the beginning depends on an assumption about a state before inflation. On my account, the low entropy of the beginning follows from the turn.

Smooth Changes, with No Bubbles

Did the cooling universe freeze, as steam turns to ice?

Picture water chilled below freezing in a very clean bottle. The water stays liquid past the point where ice would form, and then ice appears in sudden crystals that spread and join. Freezing in this way is a jump, with bubbles of the new state in the old state. Now picture butter softening on a counter: the butter changes steadily, with no moment at which it jumps. Softening in this way is a *crossover*: a smooth change of state, with no jump, no bubbles and no walls between regions.

Physicists make *lattice computations* of the Standard Model by computer, on a fine grid of points in space and time. Lattice computations show which kind of change the cooling universe went through. Quarks are the particles inside protons and neutrons. In 2006 the physicist Yasumichi Aoki and his colleagues found that the binding of quarks into protons and neutrons is a crossover. In 2016 the physicists Michela D'Onofrio and Kari Rummukainen found that the settling of the Higgs field is a crossover too. Their result applies to the measured mass of the Higgs particle. Both changes of state in the content's history are crossovers. The universe cooled as butter softens, smoothly through both changes.

The kind of change fixes what happens to the count of the matter, its entropy. A jump raises the count of the matter, and the bubbles of the new state shake the arena with gravitational waves when they collide. A crossover neither raises the count nor shakes the arena. So no change of state after the beginning raised the count of the matter in bulk. Neither change of state filled space with a hum of gravitational waves.

The Weight Nobody Sees

And what is the matter that gives no light?

In 1933 the astronomer Fritz Zwicky found that the galaxies of the Coma cluster move too fast. The visible galaxies pull too weakly to hold the cluster together. In 1970 the astronomers Vera Rubin and Kent Ford found that the outer parts of the Andromeda galaxy turn about as fast as its inner parts, so the mass of the galaxy keeps growing far from its center. If the mass stopped growing, the outer parts would turn more slowly, as the outer planets circle the Sun more slowly than the inner planets. Astronomers also weigh the unseen matter, the matter that gives no light, by the way clusters of galaxies bend light. Astronomers get the amount of the unseen matter from the pattern of the oldest light: about five and four tenths times as much as the ordinary matter, in the same proportion everywhere that pattern can measure. Astronomers call the unseen matter dark matter.

In Chapter 5 we met the phonon as one step in the description of how the atoms of a crystal move together, and a phonon describes a vibration of the crystal itself, and not an extra thing inside the crystal. The dark matter is made of *gyrations*: lasting vibrations of the six held dimensions, made at the turn. Each gyration has about the Planck mass. A gyration is a vibration of the six themselves, as a phonon describes a vibration of the crystal itself. Within each comparison, one thing is content and another is context, so Chapter 7 placed the six in two ways. Every instrument makes comparisons that find the content of physics. An instrument registers content, and the six are the context in which the instrument's comparisons of content take place. So, for every instrument, a gyration is a vibration of a context, and not an item of content. A gyration has energy, so a gyration has weight. But a gyration is not content, so nothing that registers content registers it.

Then how is the weight of the dark matter seen? Every instrument registers content, and what the instruments have registered of the dark matter is its pull on content: the stars and galaxies the dark matter holds in their paths, the light the dark matter bends around clusters, and the pattern the dark matter set in the oldest light. The pull of the dark matter is a bending of the arena around the dark matter, because gravity is the shape of the arena, as Chapter 1 showed, and the arena is the setting in which every comparison is made. So the dark matter shows itself only by its weight, and its weight is all that anyone has ever seen of it.

The amount of the dark matter, measured against the ordinary matter, is a pure number, a number with no unit: two divided by pi, taken away from six. The *basic pieces* of a shape are the pieces from which the shape is built. The number six counts the basic pieces of the six held dimensions, as Chapter 7 counted them. The six held dimensions are two shapes side by side. One shape is the four-dimensional space of the states of a three-way choice, called the complex projective plane. The other shape is the sphere (the surface of a ball) of the states of a two-way choice. When two shapes are taken together, their counts multiply: three times two is six. The ordinary matter counts as one, because a measurement can take in only the ball of space whose light has reached us. A solid ball can be shrunk, without tearing, down to a single point, so a solid ball counts as a single point does, one basic piece, while the surface of a ball is built from two, as Chapter 7 showed.

The two divided by pi is a part taken away from the number six by the bending of the held dimensions. Each held dimension is bent at the Planck scale. The size of the part taken away comes from the ways in which a horizon holds energy. A bending of a given strength sets an edge of its own, a horizon. The part taken away is three of the independent ways in which the horizon of the bending holds energy, counted out of all of those ways. For the bending of the complex projective plane, the part taken away comes to two divided by pi. The notes to this chapter in Appendix B give the count of the ways in which the horizon holds energy. They also give the reason that the bending of the sphere takes nothing

further away. Two divided by pi, taken away from six, gives about five and thirty-six hundredths, so the dark matter is about five and thirty-six hundredths times the ordinary matter, and the Planck satellite measured that amount, to within the precision of its measurement.

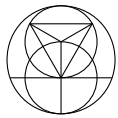
What Banged

So what did Penzias and Wilson hear?

Penzias and Wilson heard the oldest light of an epoch that began at a turn. The oldest light is the same in every direction because it comes from one origin, the common end of the black-hole interiors of the epoch before. The flatness of space comes from a state with no length from before. The faint ripples of the oldest light are the form of everything the black holes of the epoch before took in, worn into one faint shape as the footsteps are worn into the stairs. And the dark matter was made at the same turn, as vibrations of the six: the dark matter is the hidden weight that shaped the galaxies.

So we now know what banged: the turn. The turn is the passage at the end of the black-hole interiors of the epoch before, which restores a large world, and I have identified the turn, as our epoch's history renders it, with the Big Bang. The sky is the same in every direction and space is flat, and both follow from the turn with no inflation needed. The first ripples are the form of what came before, averaged and not erased. The principle of Newton's bucket applies all the way back to the beginning: every count our epoch begins with is made against its own context, as every turning the water finds is found against the matter within the water's reach.

If space is flat and the first ripples come from the turn, what sets the small energy of empty space that now speeds the expansion?



Chapter 10

Why Is the Energy of Empty Space So Small?

Beside The Fabric of the Cosmos*, chapters 10, 12 and 14;* The Hidden Reality*, chapters 2 and 6;* The Elegant Universe*, chapter 9; and* Until the End of Time*, the notes to chapter 3.*

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

The world has ten dimensions: one of time, three of space, and six of possibility; *possibility* is the range of what a comparison could find, and nothing ever travels through *the six* (Chapter 7).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

The hold is the large world of space and time keeping the six at their one size and shape (Chapter 9).

I identify the *first ripples*, the faint differences of density with which our epoch began, with the form of the content carried through the turn (Chapter 9).

Adam Riess was an astronomer at the University of California at Berkeley. In the autumn of 1997 Riess worked through the light of distant exploding stars. Riess found that the light would not fit a universe whose expansion was slowing down. The expansion was speeding up, and the energy of empty space speeds up the expansion. Greene shows that in quantum physics, the physics of atoms and of the particles smaller than atoms, empty space has an energy about a hundred and twenty-three powers of ten larger than the energy astronomers measure in empty space. A power of ten is one more zero on the end of a number, so a hundred and twenty-three powers of ten is a one followed by a hundred and twenty-three zeros. Physicists widely regard the gap of a hundred and twenty-three powers of ten as the largest by which a successful theory has ever missed a measurement. Why, then, is the measured energy of empty space so small?

Too Faint

Riess was working for a team of astronomers led by Brian Schmidt. The team had set out to measure how fast the expansion of the universe was slowing, because every galaxy pulls on every other galaxy by gravity, and the pull of the galaxies on one another should act as a brake on the expansion. The only question seemed to be how hard the brake was pressing.

The team measured the braking with a kind of exploding star. A *supernova* is a star that blows itself apart. For a few weeks a supernova can shine as brightly as the whole galaxy it lives in. One kind of supernova explodes with very nearly the same true brightness every time. So astronomers can tell how far away a supernova of that kind is by how faint it looks from the Earth, as the faintness of a lamp of known brightness tells how far down the road the lamp stands. The light of each explosion is stretched on its way to us by the expansion of space, so when the astronomers set the distance of each explosion beside the stretching of its light, they can read off the history of the expansion, era by era.

The distant explosions were too faint: fainter than they would be in a universe whose expansion was slowing. Riess later recalled his first measurements: they matched only the curves of an expansion pushed faster, as if gravity had turned into a push. Riess spent a long time checking his work, and he found no error. Schmidt was the first person Riess told, and Schmidt checked the final calculations and reached the same result. In 1998 the team's paper reported that the distant supernovae were, on average, ten to fifteen percent farther away than a slowing universe would put them. Saul Perlmutter, at the Lawrence Berkeley National Laboratory, led a second team of astronomers. Perlmutter's team measured forty-two distant supernovae and found them too faint as well. In 2011 Perlmutter, Schmidt and Riess shared the Nobel Prize in Physics for the discovery that the expansion of the universe is speeding up.

So the two teams measured how faint the distant explosions were, and they found the distant explosions too faint for a slowing universe. We now know what the light of the explosions says. The expansion is speeding up.

From Greene's pages

Einstein's Third Line

In *The Hidden Reality* Greene pictures the mathematics of general relativity, Einstein's theory of gravity, as a tax form with three lines (*The Hidden Reality*, chapter 2, section "Taxing Gravity"). One line of the form describes the bending of space and time, which is gravity. A second line describes the matter that does the bending. Einstein at first left the third line blank. The third line records "the amount of energy stitched into the very fabric of space itself". In 1917 Einstein wrote a positive number on the third line. A positive energy of space pushes every region of space away from every other region. Einstein chose the number so that the push would exactly balance the pull of the matter, and the universe would stand still. Einstein called the entry on the third line the *cosmological constant*. The cosmological con-

stant is one number in Einstein's equations, which physicists later read as an energy of empty space: the same in every cubic meter of space, everywhere, and not thinning out as space grows.

Einstein dropped the cosmological constant after the astronomer Edwin Hubble showed, in 1929, that the distant galaxies are rushing away from us, and that the universe is expanding. Greene reports the famous story that Einstein called the constant his greatest blunder. Greene adds that some have questioned whether Einstein ever said so. Greene also quotes a letter that Einstein wrote in 1917 to the physicist Willem de Sitter, in which Einstein leaves it to measurement to decide whether the cosmological constant is zero (*The Hidden Reality*, chapter 6, section "The Return of the Cosmological Constant"). Einstein ends the letter's passage on the cosmological constant: "Conviction is a good motive, but a bad judge". The supernova teams of Schmidt and Perlmutter measured the cosmological constant some eight decades later. The teams found the constant positive. Greene writes: "To many researchers, the discovery of a nonzero cosmological constant is the single most surprising observational result to have emerged in their lifetimes" (*The Hidden Reality*, chapter 6, section "The Cosmological Constant").

Astronomers call whatever drives the speeding-up *dark energy*. The word *dark* says that dark energy gives off no light, and the word *energy* says that dark energy acts on the expansion as an energy does. The measured amount of dark energy is tiny. Greene gives the amount as a little under ten to the power minus twenty-nine grams in each cubic centimeter of space: a gram divided by a one followed by twenty-nine zeros. In everyday terms, the amount is about the mass of three or four hydrogen atoms in each cubic meter, but dark energy is spread through all of space, so the small amount still adds up to about seven tenths of all the energy the universe holds. The other three tenths is matter: ordinary matter, and dark matter, the unseen matter whose pull is measured in every galaxy. In Greene's telling, *inflation* is a brief

but enormous burst of expansion of space in the earliest moments, and inflationary cosmology is the account of the universe that includes that burst. In *The Fabric of the Cosmos* Greene adds the three tenths of matter to the seven tenths of dark energy. The sum is the full total that inflationary cosmology predicts. Greene writes: "The supernova measurements and inflationary cosmology are wonderfully complementary" (*The Fabric of the Cosmos*, chapter 10, section "The Missing 70 Percent").

Greene's word *fabric* names what this book calls the arena: space and time taken together, as the setting in which everything that is measured is measured.

From Greene's pages

The Hum That Cannot Be Sent Home

Why was a positive cosmological constant such a shock? Greene answers the question in *The Hidden Reality* (chapter 6, section "Explaining Zero"). Greene's answer states the puzzle that this chapter is about.

Modern physics describes the world with fields. A *field* is a quantity with a value at every point of space, as the temperature has a value at every point of a room. Each kind of particle is a ripple in its own field: light in the field of light, the electron in the field of the electron. Quantum physics is the physics of the very small, in which energy comes in separate steps. Quantum physics says that no field can be perfectly still, so even at its lowest energy every field keeps a restless trembling, which carries energy. Physicists call empty space, with every field at its lowest energy, the *vacuum*, and I call the trembling of the vacuum's fields, which never stops, the hum of the vacuum. Greene's picture is a playground. The children on a playground can be sent home, and the playground is left empty, but the trembling of the fields cannot be sent home. You cannot close off a region of space and

send the trembling away. So Greene writes that "Once quantum mechanics is taken into account, you are forced to confront an energy contribution provided by fields that's uniformly spread through space". An energy spread evenly through space is exactly what the third line of Greene's tax form for Einstein's equations records, so quantum physics on its own predicts a cosmological constant, whether or not Einstein had thought of one.

Physicists have detected the hum in a laboratory. In 1948 the Dutch physicist Hendrik Casimir worked out that two metal plates, set very close together and parallel, would be pressed toward each other. The plates are pressed together because the plates leave out some of the ways the field of light can tremble in the gap between the plates, so the trembling outside the plates pushes harder than the trembling in the gap. In 1997 the physicist Steve Lamoreaux measured that push and found it within about five percent of Casimir's value (*The Fabric of the Cosmos*, chapter 12, section "Quantum Jitters and Empty Space"). So empty space holds energy, which pushes on the plates.

How much energy does empty space hold? Physicists add up the energy of every way in which the fields can tremble, and the sum has no end, because there is no end to ever shorter and more energetic ripples. So physicists stop the sum at the Planck length. Chapter 5 found that at the Planck length the two descriptions of the world must both be used together: gravity's description and the quantum's. Quantum field theory is the quantum physics of fields: a description of particles as ripples of fields, with no gravity in it. Greene gives the reason for stopping the sum at the Planck length: "you can't trust quantum field theory on super-small distance scales". The sum stopped at the Planck length has an end. The total is enormous. The *Planck mass* is a mass built from the constants of gravity, of the quantum and of light alone. Greene puts the Planck mass at about a hundred-thousandth of a gram, and he gives the total as one Planck mass in every cube one Planck length on a side. In the same units,

Greene writes the measured value as about ten to the power minus one hundred and twenty-three (*The Hidden Reality*, chapter 6, sections "Explaining Zero" and "The Cosmological Constant"). So the summed energy of the vacuum is larger than the measured value by a one followed by a hundred and twenty-three zeros.

Physicists hoped for a symmetry in the laws, so that the sum would cancel to zero as numbers and their negatives cancel when they are added in pairs. The particles fall into two broad families: the family of matter, and the family of the carriers of force. The carriers are the particles through which a force acts. The best candidate was *supersymmetry*, a proposed pairing that would give every known kind of particle a partner in the other family. Under supersymmetry, the trembling of each field would be canceled by the trembling of its partner's field. Greene writes that the cancellation is complete only if each partner has the same mass as its particle, and that experimental data have ruled out partners of equal mass: no partner has been seen in the accelerator experiments. If the partners exist, they must be much heavier than their particles, and so, in Greene's words, "the cancellations are imperfect; the result is once again huge" (*The Hidden Reality*, chapter 6, section "Explaining Zero"). Greene wrote *The Elegant Universe* before the speeding-up was measured. In that book he calls the gap between the computed value and what experiment allowed "a wonderful challenge and opportunity for string theorists" (chapter 9, section "Some Longer Shots"). Greene asks whether string theorists can explain the gap with string theory.

Greene ends his account of the puzzle with a measured number far smaller than the number that follows from quantum theory, and he adds two further thoughts. The first thought is that perhaps our value is one of a great many values, in a great many universes, and that we find a small value because only a small value lets galaxies, and us, form. Greene traces the first thought to a paper of 1987 by the physicist Steven Weinberg, and the next chapter takes the first thought up.

Greene gives the second thought in *The Fabric of the Cosmos* (chapter 14, section "Dark Matter, Dark Energy, and the Future of the Universe"). The second thought is that the dark energy might be a true constant, or it might be *quintessence*: the name physicists give, and Greene uses, for a field that slowly changes and mimics a constant for billions of years. A true constant speeds the expansion for ever. Quintessence would let the speeding-up end, and an end to the speeding-up would give what Greene calls "a far future less bleak and desolate".

Where Greene and I Agree

On my account, several of Greene's results follow from arguments of my own. I describe the supernova measurements as Greene does. I use the result of the supernova measurements as found: the expansion is speeding up, and the cosmological constant is positive.

I describe Einstein's still universe as Greene does, and I include in my main line a step that Greene gives in his notes. In 1930 the astronomer Arthur Eddington showed that Einstein's balance between the push of the constant and the pull of the matter could not last. If the still universe grows a little, its matter thins while the constant stays the same, so the push wins and the growth runs on, and if the still universe shrinks a little, its matter crowds closer, so the pull wins and the shrinking runs on. The balance is like a pencil stood on its point: it can stand there for an instant, and the smallest nudge tips it one way or the other. When a universe's matter is spread evenly and held by nothing but gravity, the universe cannot stand still. So Einstein's still universe failed twice: in the sky in Hubble's galaxies, and on paper in Eddington's calculation. Greene gives Eddington's step in two notes, crediting the step to Eddington in *Until the End of Time* (notes to chapter 3, note 2) and to de Sitter in *The Hidden Reality* (notes to chapter 2, note 7). So the number on Einstein's third line nev-

er held the universe still. The number on the third line now is the push that the supernova teams measured. The measured push is very small. The smallness of the measured push needs a reason.

I agree with Greene that the hum of the vacuum is there, and that the hum is as large as quantum theory says. The energy of the vacuum is real, and it is of the size quantum theory gives. I also agree with Greene that no cancellation has worked. And Chapter 6 found a reason of its own why the supersymmetric cancellation was never available. Every particle, of matter or of force, is something found within the arena, so in that sense every particle is content. The *context* of a comparison is the respect in which the comparison finds what it finds. A morning and an evening make two readings of a thermometer two, one reading at each time of day. Within the content, the line between content and context is drawn again, and the first line drawn in that way runs between the two families of particles. The family of matter is on the content side of that line, and the family of the carriers of force on the context side, so no particle has a partner across that line.

What Bends the Arena

I differ from Greene on one assumption, which Greene and nearly every physicist share: that the summed energy of the vacuum, if it is there, must bend space, as all energy does. If the assumption holds, there are two ways to explain the small measured value. Either something cancels the total almost exactly, or the total is not there at all. My account has a third explanation. The total is there, and it curves nothing.

Picture once more Newton's bucket from Chapter 1: a bucket of water, hung from a twisted rope and let go, whose water climbs the walls of the bucket as the water turns, so that the water's surface sinks into a hollow. The hollow in the water is a finding made from inside the world: the water turns against the matter there is to compare with, and the hollow shows what the water's comparison finds. Chapter 1 drew the principle of comparison from the bucket: a difference that no comparison can register

is no difference. Some of the energy of empty space bends the arena, and that energy is a finding of the same kind as the water's hollow. The arena is bent by exactly what a measurement made from inside the world takes in.

Why should only what a measurement takes in bend the arena? Chapter 1 found that gravity is the shape of the arena, and that the arena takes its shape from what is in the arena. I give, with a picture, the reason that the arena bends at all. Picture a rug on the floor between a wall and a heavy sofa. Someone pushes the sofa toward the wall. The wall will not move, and the sofa keeps coming, so the rug between the wall and the sofa buckles into a ridge: the strain of the push had to go somewhere, and it went into the rug, the one thing that could give way.

Every comparison must satisfy two demands, and neither demand can give way. First, the comparing must join its two sides with no break anywhere, or the comparing would find nothing. To be *continuous* is to run on with no break or step anywhere, so the comparing must be continuous, and in the picture the continuous comparing is the sofa, which keeps coming. Second, what the comparing finds must hold together as one world, the same for every comparer. To be *symmetric* is to show the same content across a different context, as a pattern is the same wherever it is placed, so what is found must be symmetric, and in the picture what is found is the wall, which will not move. The Incommensuration Theorem of Chapter 3 states that nothing can be both perfectly symmetric and perfectly continuous, and that nothing can be both perfectly asymmetric and perfectly discontinuous. So no one thing can satisfy both demands perfectly, and the strain between the two demands has to fall somewhere. The strain falls on the arena, the setting in which every comparing is made and every finding is found, and the arena carries the unbroken joining of every comparing, so the arena cannot also be perfectly symmetric. The arena gives way, as the rug does, and the arena bends.

The strain on the arena comes from what comparison finds. Read the other way: "What no comparison finds places no strain on the arena, and so bends nothing". If no sofa ever pushes, the rug lies flat. The question about the vacuum then becomes a question about comparison: does any comparison find the vacuum's total energy?

Two reasons show that no comparison finds the vacuum's total energy, and the two reasons agree.

The first reason is about what a measurement takes in. The vacuum holds its energy in a vast number of independent ways. Physicists call each of the vacuum's independent ways of holding energy a *degree of freedom*: an independent way in which something can hold energy or vary, as each string of a piano can ring on its own. Within everything we can see, the vacuum's degrees of freedom number about a one followed by a hundred and twenty-three zeros, while a measurement takes in only a few of them, and never all of them at once. The measuring instrument is itself made of the very fields whose hum it measures. So the instrument never stands outside the hum to take in the whole hum. I identify the part of the vacuum's energy that a measurement receives with the energy held by the degrees of freedom that the measurement engages. Every measurement ever made, and every measurement that could be made from inside the world, takes in only part of the vacuum's energy.

The second reason is the Incommensuration Theorem, applied to the vacuum's total energy. Taken whole, the vacuum's total energy is the same at every place. The total is also the same for every comparer moving steadily, whatever the comparer's speed: running through still air you feel a wind, but running through the vacuum at any steady speed you feel no wind. And the total has no break anywhere. Being the same at every place and for every comparer makes the total symmetric, and having no break makes the total continuous. So, taken whole, the total would be perfectly symmetric and perfectly continuous at once. Nothing can be both, by the Incommensuration Theorem. So no comparison can find the total.

So no comparison finds the vacuum's total energy taken whole, and what no comparison finds bends nothing. Nothing needs to cancel what was never acting on space, so Greene's gap needs no cancellation. So the size of the hum does not matter to the bending of the arena: however large the total is, the total bends nothing. Greene's gap comes from setting the energy measured in the sky beside an amount that no measurement made from inside could ever take in. The question that is left is much smaller: how much of the vacuum's energy does a measurement made from inside take in?

Three Tiles on the Edge of What Has Reached Us

I identify the cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in. I call that part of the vacuum's energy the *accessible share*. How large is the accessible share?

Think first of the edge of everything we can see. I call the edge the *particle horizon*: the edge of everything whose light has reached us since the expansion began. Space has stretched while the light traveled, so the place from which the oldest arriving light set out is now about forty-six billion light-years away (a light-year is the distance light travels in a year).

The entropy of a black hole is a count of the ways its inside could be arranged that no measurement from outside can tell apart. Bekenstein and Hawking found that a black hole's entropy grows with the area of its edge, and not with the volume inside it: the count has one entry for every four squares of the edge, each square one Planck length on a side. Picture a library whose every book could be listed on its outer wall, one entry for each small patch of the wall. The library stands for the black hole, the books stand for the ways the inside of the black hole could be arranged, and the outer wall stands for the edge.

Why should the edge of what has reached us carry a count of the same kind? In 1998 the physicists Willy Fischler and Leonard Susskind applied the count of Bekenstein and Hawking, made for the edges of black holes,

to the edge of what we can see. A measurement made here and now contains everything that has reached it, and nothing beyond: everything within the particle horizon, and nothing outside it. I apply the count of Bekenstein and Hawking to the particle horizon, because the edge of everything that has reached a measurement is the edge of what that measurement can know.

Picture the particle horizon as a vast surface tiled like a floor, with one tile for each entry of the count. The particle horizon is a sphere around us. The area of a sphere is four times pi times the square of its distance from the center. With one tile for every four squares, the count of the tiles is pi times the square of the horizon's distance, measured in Planck lengths. Each tile stands for one degree of freedom within the edge: in the picture of the piano, one of its strings. Our particle horizon carries about two and three tenths times ten to the power one hundred and twenty-three tiles.

The size of the accessible share follows from three steps, and each step has its own reason.

First, every tile holds an equal part of the vacuum's energy. The vacuum's energy is the same at every place, and the tiles are laid at one rate over the whole edge, so nothing makes one tile different from another in what it holds. Tiles that nothing distinguishes hold equal parts. A cake cut into equal slices gives three slices' worth to whoever takes three slices, and in the same way a measurement that takes in three tiles takes in three tiles' worth of the vacuum's energy.

Second, a measurement takes in what it finds through three channels, one for each of three kinds of thing that a measurement registers. Watch a die thrown onto a table. What there is to find about the die is of three kinds. One kind is where the die lies and how it is shaped, laid out in space. Another kind is the knock the die gives the table: a push that runs its course in time. The third kind is how likely each face is to come up: a spread of chances over what could happen. Chapter 7 named the three kinds: pattern, measured in space; force, measured in time; and probability, measured in possibility, the range of what could be found. A *channel* is

one of these three kinds of thing that a measurement registers, and every measurement registers one of each. Pattern, force and probability go with the three roles of every comparison, which Chapter 3 named. The pattern is what is laid down: the structure. The force is what happens: the act. The probability is the range of what could come, which belongs to the stance, the side from which a comparison is made. I identify the vacuum's channels into a measurement with pattern, force and probability, one in each role. Space, time and possibility are where pattern, force and probability are registered, as a page is where a word is written. So the channels are pattern, force and probability themselves, and not space, time and possibility. The channels number three.

Third, through each channel a measurement takes in one whole tile, and each channel has a tile of its own. If an observer uses a telescope and a thermometer together, do the two instruments take in six tiles? The two instruments take in three tiles, because the one who measures stands in one place, and the tiles taken in are counted once for that one place, and not once for every instrument carried there. I hold that through each channel a measurement takes in one whole tile, because what a channel registers comes whole, in steps. And each channel has a tile of its own, because pattern, force and probability are distinct and no one of them can do another's work.

So a measurement takes in three tiles out of all the tiles on the edge of what has reached us. At the start of this section I identified the cosmological constant with the accessible share, the part of the vacuum's energy that a measurement made from inside can take in. By the three steps, the accessible share is exactly three tiles out of all the tiles on the particle horizon.

What energy do the tiles share among them? The tiles share the vacuum's energy summed up to the Planck length. The summed energy is about the *Planck density*: the density built from the constants of gravity, of the quantum and of light alone. A small part of the summed energy is held elsewhere, in the six small dimensions that Chapter 7 described: the dimensions of possibility (the range of what a comparison could find),

held a few Planck lengths across. The six small dimensions are curved, and a curved thing holds energy in its curve, as a sheet of spring steel rolled into a tight coil holds energy in its curl. The energy of the curve has the opposite sign. The energy of the curve would draw space together, so the curve's energy takes its part away from the total. I work out the size of the part from Einstein's equations, carried from ten dimensions down to four. The curve of the six small dimensions takes about twelve hundredths of the Planck density and leaves about eighty-eight hundredths. What is left is the *trim*: the part of the Planck density that remains once the curve of the six small dimensions has taken its share. The trim is set by the shape and size of the six small dimensions.

Now count. Take three tiles' worth of the trimmed Planck density, out of all the tiles on the particle horizon at the distance astronomers have measured. The sky shows an energy of empty space equal to the mass of three or four hydrogen atoms in each cubic meter. The share agrees with that energy to within two percent. The summed hum missed the sky's value by a one followed by a hundred and twenty-three zeros. The share misses by less than two percent. The energy of empty space is small because the edge of what has reached us is large.

A careful reader will ask why the share can be found, when by the Incommensuration Theorem the total cannot. The share, too, is the same everywhere and the same for every comparer. The difference is that the share is found as a count: a tile is taken in whole, or not at all, and there is a step between three tiles and four, with no smooth smear between them. When a thing is the same for every comparer and comes in steps, the thing is symmetric and discontinuous, and that pairing is consistent with the Incommensuration Theorem. The total would be symmetric and continuous at once, while the share is symmetric and stepwise. The share is found, and the total is not.

Why does the count use the particle horizon, of all the edges an observer has? The other edges are set by what could reach the one who measures in the future, or by rates of expansion, and not by what has arrived. A measurement is made here and now, so the only edge that a present measurement contains is the particle horizon.

My account is not the first account of the vacuum's energy that uses the edge of what we can see. In 1999 the physicists Andrew Cohen, David Kaplan and Ann Nelson built on the count of Fischler and Susskind. Cohen, Kaplan and Nelson tied the vacuum's energy to that count, and they found a value close to the measured one. I credit Cohen, Kaplan and Nelson at once: "This is a real clue".

Two Surveyors

Is there a second way to check the share?

Imagine two surveyors sent to measure the same field. One surveyor walks the edge of the field with a measuring chain, while the other surveyor works from a hilltop with angles and a map. Each surveyor's method has its own unknowns, but the two surveyors measure one field, so their answers must agree. When two answers must agree, the agreement can tell you something that neither surveyor could find alone.

The share is the first way to find the energy of empty space. The second way is the astronomers' way, through the equation that the physicist Alexander Friedmann found from Einstein's theory in 1922. A flat universe is one in which space as a whole has no overall bend, and in a flat universe Friedmann's equation says that the rate of the expansion fixes how much energy the universe must hold in all. Chapter 9 found our universe to be flat. The energy of empty space is one part of the universe's total energy, and the matter of both kinds (ordinary and dark) is another part. The fraction of the total that is matter is called the *matter fraction*.

Now set the two answers equal, as the surveyors' answers must be equal. Both answers depend on the rate of the expansion, and in the same way. The expansion sets one natural length, the *Hubble radius*: the distance

at which the expansion now carries things away from us at the speed of light. The faster the expansion runs, the shorter the Hubble radius. Write the distance to the particle horizon as a number of Hubble radii. For us the number is a little over three. If we hold that number fixed and let the expansion run twice as fast, the Hubble radius is half as long, so the horizon is half as far. Half the distance gives a quarter of the area, and the count of tiles goes with the area of the edge, so the horizon carries a quarter of the tiles, and with a quarter of the tiles each tile holds four times as much, and so does the share. So the share grows as the square of the rate of the expansion, and Friedmann's total grows as the square of the same rate. The strength of gravity enters both answers in the same way too: in the share through the Planck density and the size of the tiles, and in Friedmann's total directly. Set the two answers equal, and the rate of the expansion and the strength of gravity cancel from both sides.

What is left is one statement in pure numbers, with no units at all. In words, the statement says that the square of the particle horizon's distance in Hubble radii, times the fraction of the whole that is the energy of empty space, equals eight times the trim. The eight is the eight pi with which Einstein's equations tie energy to the bend of space, divided by the pi in the count of the tiles, while the three tiles of the share cancel a three in Friedmann's equation. Eight times the trim comes to three divided by pi, taken away from eight: a little more than seven.

Both sides of the statement depend only on how the universe's energy has been divided between matter, light and empty space over the universe's history. The horizon's distance in Hubble radii depends on that division too. Follow a ray of light from the beginning of the expansion to today: how much each stretch of the ray's path was stretched depends only on how the energy was divided in the era when the ray crossed that stretch. In a flat universe the parts add up to the whole, and the share of light is measured. So the statement is one equation in one unknown, the matter fraction.

Solve the equation, and matter is 31.1 percent of the energy of the universe. The Planck satellite mapped the oldest light, the light left over from the early universe, across the whole sky, and in its final results the matter fraction is 31.5 percent, give or take 0.7. So the two ways agree twice: the share agrees with the measured energy of empty space, and the matter fraction worked out from the share agrees with the measured matter fraction. I call the two agreements "one measured agreement, seen two ways". In the picture of the surveyors, the share is the surveyor with the chain, Friedmann's equation is the surveyor on the hill, and the one universe is the one field that both surveyors read. Greene's figures give the same division of the whole, as astronomers measure it: seven tenths of dark energy and three tenths of matter. In Greene's telling the whole adds to the full amount because of inflation. On my account, the whole adds to the full amount because space is flat, and Chapter 9 found the source of that flatness at the turn, the passage that began our epoch, with no inflation needed.

One Scale for the Whole Map

Greene's second thought was a question: is the dark energy a true constant, or a slowly changing quintessence? On my account, the dark energy is a true constant, and the reason has two parts.

Think of a map with its scale printed in one corner: one inch to one mile. The scale belongs to the whole map at once. If a map's scale changed from one corner to the next, the map would not be one map. The cosmological constant is like that scale: the constant belongs at once to the one space and time of our epoch, the one cycle of the universe we live in (Chapter 9). Einstein's equations require a strict balance of energy and of momentum, the amount of motion: as the universe changes, energy and momentum are moved from place to place, and they are never made or lost. Suppose a constant term changed from place to place, or from time to time: the changing term would break the balance exactly where it changed, and Einstein's equations allow no such break. So, within one

geometry, the cosmological constant is one number, where one geometry means the one space and time of our epoch. The result is a theorem about Einstein's equations, and the theorem uses no measurement at all. The dark energy is the one constant term of Einstein's equations, which earlier in this chapter I identified with the accessible share: the part of the vacuum's energy that a measurement made from inside can take in. There is no slowly changing field behind the dark energy.

But the particle horizon grows every year, as light from a little farther away arrives. Why does the share not shrink as the count of tiles grows? The answer is the distinction of Chapter 2 between existing and being real. To exist is to stand as structure, timeless. A law exists; a measurement is real. The one value of the constant is fixed by a measurement, and the measurement is a real act, so the measurement is the one made now, by the one who measures, on the particle horizon as it is now. Astronomers piece the history of the expansion together, as one history, from what reaches the present measurement. In that history the constant has the one value at every era of our epoch, before the present and after it. Picture an astronomer placed in that history, at an era when the universe was half its present size. The astronomer's measurement was a real act in her own day, and her record of it reaches us. Her record, like every record our history holds, shows the one value of the constant that we measure. On paper, the share worked out for her smaller horizon is a number about seven times larger, but that larger number exists as structure in the model, and no record shows the larger number as a finding. My account is stated from the stance of the one who measures, as every finding is, so from our stance the astronomer is a figure in our history, and in our history the constant has one value. What she found from her own stance, in her own present, is the finding of an act that is not ours: no measurement we make can reach her present, and I make no claim about it. No observer ever finds an energy of empty space that falls with time.

Physicists use one number to describe how any kind of energy behaves as space grows. Physicists call the number the energy's *equation of state*: the ratio of the energy's pressure (its push on its surroundings) to the

amount of the energy in each unit of volume. Matter has a ratio of zero, because its particles barely push on one another, and matter thins out as space grows. An energy that never thins out has a ratio of exactly minus one. A ratio of exactly minus one is the mark of a true constant. The ratio for the energy of empty space is exactly minus one. Astronomers have joined the results of the Planck satellite to the measured patterns of the galaxies and to the supernovae, and the joined measurements give minus one and three hundredths, give or take three hundredths: minus one, within the measurement. Surveys of the sky now map the expansion and measure the ratio across the history of the universe. The measurements made so far agree with a true constant. Two analyses also prefer, at about three times their stated uncertainty, an energy of empty space that changes as the universe grows: the final analysis of the Dark Energy Survey of 2026, combined with the Dark Energy Spectroscopic Instrument (DESI) and the oldest light, and DESI's own analysis of 2025, and the two analyses share much of their data. On my account the coming surveys will find the ratio exactly minus one at every era, and the reported departure from minus one will not be confirmed. Appendix A gathers what I say the surveys will find.

So our epoch is heading into the far future that Greene draws for a true constant: the push of the constant does not change, and the distant galaxies go on drawing away. Greene writes that if the speeding-up comes from a cosmological constant, "in the far future the universe will be a vast, empty, and lonely place" (*The Fabric of the Cosmos*, chapter 10, section "The Missing 70 Percent"). In Chapters 16 to 19 of this book I follow the emptying of the universe until the emptying ends. In Chapters 16 to 19 the emptying ends in a turn, and not in a final dark.

What the Small Number Says

The hum of the vacuum is there, and the hum is as large as quantum theory says. Every measurement made from inside the world takes in only part of the hum. Taken whole, the hum would be perfectly symmetric and

perfectly continuous at once. No comparison can find anything that is both, by the Incommensuration Theorem, so the whole hum bends nothing, and nothing needs to cancel the whole hum. The share of three tiles gives the measured energy of empty space, and with Friedmann's equation the share gives the measured matter fraction. By Einstein's equations, the share is one number for the whole of our epoch.

In the proposals that Greene reports, no known law fixes the constant. So a value as small as the measured one needs a further reason. The physicists who make the proposals that Greene reports look for the reason among many universes. On my account, the constant is set by where the one who measures stands: inside the world, made of the fields it measures, and taking the fields in through three channels at a time. The most embarrassing number in physics is a measure of the observer's own place in the world. And the one value of the constant follows from three tiles, the trim and the measured edge, with no choosing among many values.

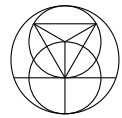
So we now know why the energy of empty space is small. I identified the energy of empty space with the share of the vacuum's energy that a measurement from inside takes in. The share is three tiles out of all the tiles on a very large edge, so the share is small. The share is one constant for our epoch, and it gives the measured matter fraction.

Greene reports another answer, from Steven Weinberg and from string theory's landscape of many possible forms. The other answer is that our value was one of many, selected because we are here to measure it. Is our universe the only one?

PART FIVE

One Universe or Many?

Beside *The Hidden Reality*.



Chapter 11

Is Our Universe the Only One?

Beside The Hidden Reality, *chapters 1, 2, 3, 5, 6, 7 and 11, with* The Fabric of the Cosmos, *chapters 11 and 13,* The Elegant Universe, *chapter 14, and* Until the End of Time, *chapter 10.*

What this chapter uses

The *principle of comparison*: a difference that no comparison can register is no difference (Chapter 1).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

The world has ten dimensions: one of time, three of space, and six of possibility; *possibility* is the range of what a comparison could find, and nothing ever travels through *the six* (Chapter 7).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

I identify the *first ripples*, the faint differences of density with which our epoch began, with the form of the content carried through the turn (Chapter 9).

I identify the cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in, and I call that part the *accessible share* (Chapter 10).

In September 1973 the Australian physicist Brandon Carter told astronomers at a meeting in Kraków that our place in the universe is special in one way that science has to allow for, although our place is not the center. The year 1973 was the year of the five hundredth birthday of Nicolaus Copernicus. A reader of *The Hidden Reality* carries one question away from Brian Greene's pages: is our universe the only one? In *The Hidden Reality* Greene shows how theory after theory of physics, followed to its end, pre-

dicts other universes. The question of whether ours is the only universe begins with Carter's argument. Greene's other universes lie beyond the edge of what we can see, in bubbles of a swelling space, on other membranes floating in a larger space, in other cycles before and after ours, and in other settled states of empty space.

The meeting in Kraków was a symposium of the International Astronomical Union, one of the meetings held in 1973 in honor of Copernicus, and the title of the symposium named its subject: the confrontation of cosmological theories with observational data. Copernicus had moved the Earth out of the middle of the heavens. By 1973 astronomers had made his lesson into a working rule: we hold no central place. Astronomers call the rule the *Copernican principle*. Carter accepted the Copernican principle, and then he added one correction to the principle. We make the observations, Carter said. So the place from which we observe has to be a place where observers can live, and everything we see therefore reaches us through a filter: the condition that we, the observers, can live where we observe.

Carter's paper appeared in the proceedings of the meeting the next year, and in the paper Carter puts his point in one sentence:

> "Although our situation is not necessarily central, it is inevitably privileged to some extent".

Carter named his correction the *anthropic principle*. By the anthropic principle, what we observe must be read as observed by beings who could exist where and when they observe it. Then Carter proposed that the anthropic principle might apply beyond planets and stars, to the constants of nature themselves. The constants of nature are the fixed numbers in the laws of physics. If there were many universes, each with its own constants, we would find ourselves in a universe whose constants let us exist, and the question of why the constants have their values would then have no deeper answer to find. Greene tells the story of the Kraków meeting in

The Hidden Reality (chapter 6, section "Cosmological Anthropics"), and in the same book Greene follows Carter's thought out to collections of many universes.

From Greene's pages

Nine Roads to Other Universes

Greene's book is like a map of roads. Each road is one theory of physics, and Greene follows each theory until the theory predicts other universes. In Greene's words, "the chapters that follow trace a narrative arc through nine variations on the multiverse theme" (*The Hidden Reality*, chapter 1, section "Varieties of Parallel Universes"). In Greene's use, a *multiverse* is a collection of universes of which ours would be one. Greene also calls the members of a multiverse parallel universes or parallel worlds. Greene notes that the meaning of the word universe has itself changed: "The word's meaning now depends on context" (chapter 1, section "Universe and Universes").

Greene counts five of his roads in chapter 7 of *The Hidden Reality* (section "Accessible Multiverses"), before he turns in chapter 8 to *quantum theory*, the theory of atoms and of the particles smaller than atoms. This chapter is about those five roads. Greene's other four roads begin in quantum theory, in holograms, in computers and in mathematics. The next chapter is about those four roads.

The first road is the patchwork quilt. Picture space at one moment as a vast quilt of round patches. Each patch is the region around one place from which light has had time to arrive since the expansion of the universe began. Greene calls the edge of our patch our cosmic horizon. Two patches far enough apart have never exchanged a signal of any kind. Greene then asks how many different ways matter can be arranged inside one patch. Greene starts from a fly buzzing around a bedroom (chapter 2, section "Finite Possibilities"). The fly asks how many places there are for the fly to be. Greene finds his answer in the *uncertainty principle* of quantum theory: a particle cannot have both a

definite position and a definite velocity. Greene writes that, by quantum theory, if you imagine both the speed and the position of a body, a fly or an electron, changing by small enough amounts, "you are imagining something meaningless". So the number of ways that matter can be arranged in one patch is large, and still finite, and in the section "Finite Possibilities" Greene gives his reason, a principle about measurement: "Changes that are too small to be measured, even in principle, are not changes at all". The number of arrangements is a one followed by a number of zeros that is itself a one followed by a hundred and twenty-two zeros. If space goes on for ever, there are infinitely many patches, and each patch has only finitely many arrangements, so the arrangements must repeat. Somewhere far away, then, there is a patch just like ours. Greene names this endless collection of patches the Quilted Multiverse. Greene comes back to the quilt in *Until the End of Time*. Of an infinite space he writes: "The infinite collection of universes thus realizes every possible history, and each such history is realized infinitely often" (chapter 10, section "Thought and the Multiverse").

The second road is inflation. In Greene's telling, *inflation* is a brief burst of very fast expansion of space at the beginning. A *field* is a quantity with a value at every place, as temperature has a value at every place in a room. A field that Greene calls the inflaton drives the burst. In most versions of the theory of inflation, the burst never stops everywhere at once: "once inflation begins it never ends" (*The Hidden Reality*, chapter 3, section "Eternal Inflation"). Where the burst does stop, a region settles down and becomes a bubble universe, while new bubbles form without end in a space that keeps swelling around the bubbles, like the holes in a Swiss cheese. Greene calls inflation that, once begun, goes on for ever in some regions and ends in others *eternal inflation*. Greene tells the idea of eternal inflation in two other books as well. In *The Elegant Universe* (chapter 14, section "Cosmological Speculation and the Ultimate Theory") Greene reports the proposal of

the physicist Andrei Linde that the burst of inflation may happen again and again, in separate regions. In *The Fabric of the Cosmos* (chapter 11, section "Boltzmann Redux") Greene places the big bang of each region where the burst of inflation happens, the hot, dense, expanding state in which the region's history begins, inside a larger inflation. Greene judges that theory and observation together make "an indirect yet compelling case" for the bubbles, his second kind of other universe (*The Hidden Reality*, chapter 3, section "Changing Perspectives"). Greene's reason is that a great many versions of inflation are eternal. Greene adds that space looks infinite from inside each bubble, so that "Inflationary parallel universes beget quilted ones" (chapter 3, section "Space in a Bubble Universe").

Greene's third and fourth roads start in *string theory*. *Dimensions* are independent ways in which things can differ, and we know four: three of space and one of time. String theory needs more dimensions than those four. In Greene's telling the further dimensions are further directions of space. String theory allows our three directions of space to be one *brane*: a membrane of three dimensions, floating in a larger space. Greene calls the larger space the *bulk*. Other branes could float nearby in the bulk: "In the braneworld scenario, our universe is just one of many that populate the Brane Multiverse" (chapter 5, section "Branes and Parallel Worlds"). If two branes collide again and again, each collision could start a hot beginning like ours. In *The Hidden Reality* Greene reports how long one full cycle, from one collision to the next, would take, by the estimate of the physicist Paul Steinhardt and his co-workers: about a trillion years (chapter 5, section "Time, Cycles, and the Multiverse"). The repeated collisions would make a series of universes, one after another in time. Greene calls the series the Cyclic Multiverse. The cycle is the proposal of Steinhardt, the physicist Neil Turok and their colleagues, and Greene also tells the cycle in *The Fabric of the Cosmos* (chapter 13, section "Cyclic Cosmology").

The fifth road is the landscape of other settled states of empty space, and on the fifth road Greene brings back Carter's reasoning. In string theory the extra dimensions are curled up into small shapes with loops. Fields can be threaded through the loops in many ways. Each way of threading the fields produces a different *vacuum*: a settled state of the least energy nearby. Each vacuum has its own energy of empty space and its own physics. The count of vacua is about a one followed by five hundred zeros. Physicists call the whole collection of vacua the *landscape*, and picture it as a country of many valleys, each valley one vacuum. A region of space can leak slowly from one valley of the landscape into another, and if regions leak in that way, eternal inflation would make every vacuum real in some bubble. Greene names the result: "This is the Landscape Multiverse" (*The Hidden Reality*, chapter 6, section "Quantum Tunneling in the Landscape"). Greene then gives a picture for why a vast collection of universes would matter. Imagine that you thought a shoe shop carried only one size, and that the one size fit you. You would wonder at your luck. Then you learn that the shop carries every size, and your wonder goes away. A shop with every size has a size that fits you. The *energy of empty space* is the energy that speeds the expansion of the universe, as Chapter 10 told. Now take a vast collection of universes with different values of the energy of empty space. Observers would find themselves in a universe where the value of the energy of empty space lets galaxies form: the shop with every size is the vast collection, and the size that fits you is a value that lets galaxies form. Greene writes that Carter's reasoning has the power to "make the mysterious mundane" only in a vast collection of universes (chapter 6, section "Cosmological Anthropics"). Greene tells how the physicist Steven Weinberg used Carter's reasoning in a paper of 1987 to expect a small value above zero for the energy of empty space (chapter 6, section "Life, Galaxies,

and Nature's Numbers"). More than a decade later, astronomers measured distant exploding stars and found a small value above zero, as Chapter 10 told.

Greene states that other universes are not confirmed: "No experiment or observation has established that any version of the idea is realized in nature" (chapter 1, section "The Cosmic Order"). Greene asks the question himself: "Which raises an essential question: Is this science?" (chapter 6, section "Is This Science?"). Greene's answer has two parts. The first part is about principle. Greene writes that "the mere invocation of inaccessible universes does not consign a proposal to stand outside science" (chapter 7, section "Science and the Inaccessible I"). His reason is that a theory confirmed by what we can see may also predict universes that we cannot see. The second part is a reason why the idea of other universes is more than idle speculation: physicists were not looking for other universes, and yet physicists kept arriving at other universes at the ends of theories built for other reasons. In Greene's words about the other universes, "They're harder to avoid than they are to find" (chapter 11, section "Is the Copernican Pattern Fundamental?").

When this book speaks in its own sentences, the world means everything that is. When this book speaks of other universes, the word universes is always Greene's word, in sentences that tell what Greene says.

Greene's Principle and the Bucket's

Go back to the bucket of Chapter 1. In Isaac Newton's picture, a bucket of water hangs from a twisted rope, and when the bucket is let go, the bucket spins, and the water climbs the walls of the bucket into a hollow. Chapter 1 named the act at work in the bucket: comparison. A *comparer* is whatever makes a comparison and registers the outcome, and the word

comparer names a role, and never a mind. The hollow is a *finding*: the outcome of one act of comparison, laid down as a record, which the water registers against what is there for the water to compare with.

In Chapter 1 I also gave the principle that the water obeys, which I call the *principle of comparison*:

> "A difference that no comparison can register is no difference: every distinction is made by comparison, and a distinction that no comparison can make is not a distinction".

By the principle of comparison, only matter within reach fixes the water's standard of what counts as not turning. Matter within reach is matter that the water can still be compared with.

Now set Greene's sentence about changes too small to be measured beside the principle of comparison. "Changes that are too small to be measured, even in principle, are not changes at all". Greene's sentence is the principle of comparison in one case: the position and speed of a body. Greene derives his sentence from the uncertainty principle of quantum theory. On my account, the principle of comparison applies to every distinction whatever, and the principle follows from what a comparison is. So, in the one case that Greene's sentence covers, Greene's principle and the bucket's principle say the same thing, and each principle is derived from its own premises: Greene's from the uncertainty principle, the bucket's from what a comparison is.

The principle of comparison also says what the far patches of the quilt are. A patch beyond our particle horizon has never sent us a signal. Yet the edge of the far patch touches the patches next to the far patch, and those patches touch their own neighbors, and so on, patch by patch, back to our patch, and every patch interacts with its neighbors, as our patch interacts with its own neighbors. Regions beyond a horizon interact with what is near them, so they exist; they are further parts of one world, and not another universe. The step from interacting to existing rests on one

principle: existence without any interaction whatsoever is indistinguishable from non-existence, and therefore is non-existence. So Greene and I agree in substance about the far patches. In both, the same space with the same laws goes on beyond our particle horizon. Greene and I differ on what to call the far patches. Greene calls the far patches other universes. I call the far patches further parts of one world, because each far patch is joined to the other far patches through the neighbors of that far patch. In a far patch, a bucket of water set spinning would find its hollow as our bucket does. The far bucket would find its hollow against the matter within its own reach.

Is space infinite, so that the quilt goes on for ever? Greene writes that a finite space is possible, and that a finite space would be "the most convincing way to upend" his quilt (chapter 2, section "What to Make of This?"). Astronomers have looked for signs of a finite space. If space were finite and small, light could circle all the way around space. If light could circle all the way around space, we would see the same patch of the oldest light twice, as a pair of matching circles in two parts of the sky. Greene reports that astronomers had looked for the repeated images of stars and galaxies that light circling a finite space would make, and had found none (chapter 2, section "Reality in an Infinite Universe"). Greene adds that finding no repeated images does not, in itself, prove that space is infinite. In 2023 Pip Petersen and colleagues searched the maps of the oldest light for matching circles and found none. The search showed that any closed path that starts at our place and goes all the way around the universe is longer than 98.5 percent of the width of the sphere from which the oldest light set out. From the oldest light the Planck satellite measured the *curvature* of space: whether, on the largest scales, space bends like the surface of a ball or lies flat like a tabletop. When the Planck measurement is taken together with maps of how the galaxies are spread, the curvature agrees with zero to within about two parts in a thousand. So we now know that space is flat out to the edge of what we can see, and shows no sign of closing on itself.

My account has one more step, about what any comparison can find. For a comparison made from inside the universe, the region within the particle horizon is a ball. The particle horizon is set by a limit of comparison: the particle horizon is the edge of what has been compared, and the edge of what has been compared is also the edge of the information that a comparison made at our place has. Within the particle horizon, no comparison can tell our ball of space from a ball cut out of an infinite flat space, and by the principle of comparison, two descriptions that no comparison can tell apart are one description. So, for every comparison made from inside, space cannot be told apart from an infinite space, up to the limit of what can be told apart, whatever space is beyond the particle horizon.

Is This Science?

Greene draws a line between science and what lies outside science, and the line of my account falls in nearly the same place. Greene keeps inside science any proposal that can be confronted by experiment, even when some of its universes are out of reach. Greene draws the line at ideas that can never be confronted meaningfully by experiment or observation, "not because of human frailty or technological hurdles, but because of the proposals' inherent nature" (chapter 11, section "Can We Test the Multiverse Theories We've Encountered?"). On my account, the kind of claim sets where the line falls. A test is a comparison of outcomes. A test shows something only where other comparers, at other times and along other routes, can compare the same outcome. My books call an outcome objective when other comparers can compare the same outcome in that way. So a test applies exactly where the outcome of a comparison can be objective, and nowhere else.

Greene's second reason was that physicists kept arriving at other universes at the ends of theories built for other reasons, and that reason gives Greene's case a shape that I picture. The picture sets a chain beside a cable: a chain parts at its weakest link, and "A cable is stronger than its

strongest strand, and its strength comes from the number of its strands and from how closely they are bound together". Greene's case for other universes has the shape of a cable: several theories, built for other reasons, each predict other universes. Each theory is one strand of the cable.

Carter's Principle at Its Own Scope

I use Carter's correction, the anthropic principle, where the correction applies. When the conditions for our being there to observe shape what we observe, I call the shaping a *selection effect*. Greene gives two examples of a selection effect (chapter 6, section "Cosmological Anthropics"). Survey trout only in the Sahara, and you may set out to explain why trout are extinct. The trout are missing from your survey because of where you looked. Ask why the Earth sits at its distance from the Sun. The answer is that a planet at another distance would hold no one to ask. The anthropic principle is a selection effect.

Where do I use the anthropic principle? I use the anthropic principle for the question of when we are: at what time in the history of the world we live. Beings like us are the kind of comparer that measures the energy of empty space. Beings like us live on planets warmed by stars. So we observe while stars shine. The reasoning from shining stars places our moment in the history of our epoch to within a factor of about ten; the reasoning goes back to the physicist Robert Dicke and to Carter's paper of 1974. So my account and Carter agree about selection effects: a selection effect shows why observers find themselves where and when they do.

There is one world, with its constants fixed. So, for the constants and the structure of the world, there is no range of cases for a selection to select among. The reasons are in the rest of this chapter: nothing makes the bubbles, the six small dimensions of my account have one shape, there is no bulk for branes to drift in, and the energy of empty space has one value, given by the laws.

One Shape, No Bubbles, No Bulk

My account has a result of its own for each of Greene's four remaining roads: the bubbles of inflation, the landscape, the branes and the cycle.

The first answer is about the bubbles of eternal inflation. A *black hole* is a region whose gravity is so strong that nothing, not even light, comes back out of it, and the interior of a black hole is the region inside its horizon, where every path leads forward to an end. One epoch runs from one turn to the next. Inflation was introduced to explain the sameness of the sky and the flatness of space. Chapter 9 showed that the sameness of the sky and the flatness of space come from the turn. So my account needs no inflation. The content is what comparisons find: in our world, matter and light. The form of the content is what the matter is, and which of its possibilities the matter has taken. The *first ripples* are the faint differences of density with which our epoch began, and in Chapter 9 I identified the first ripples with the form of the content carried through the turn. The first ripples are the form of the content carried through the turn, and a long inflation would have erased that form, so in my account there was no long inflation. Eternal inflation needs a long inflation, and the bubbles need eternal inflation, so in my account nothing makes the bubbles. So in my account Greene's second road, the bubbles of inflation, fails at its first step, eternal inflation, and the part of Greene's first road that eternal inflation would have built, the quilt inside each bubble, fails at the same step.

The second answer is about the landscape. In Chapter 7 I gave a picture for the landscape: drop a ball onto a hilly country of many valleys, and the valley where the ball comes to rest is a fact about where the ball happened to fall, and no fact about the shape of the country. Greene's landscape is a hilly country of that kind, and our vacuum would be the valley where our ball fell. In Chapter 7 I also described *the six* small dimensions of my account, which I call the six. Nothing ever travels through the six, and the six are present in a comparison only as the range of what

could be found. Chapter 7 showed that the six form one small shape: the complex projective plane, which is the space of states of a three-way choice, times the sphere, which is the space of states of a two-way choice.

Picture a ring with nothing wound through its hole: there is nothing to count. In string theory a field can be wound through each loop of the six small curled dimensions, a whole number of times through each loop. When a field is wound through the loops in that way, the field is called a *flux*. The law of gravity in all ten dimensions follows from the geometry of the ten dimensions alone. The ten dimensions are one of time, three of space and the six. A field threaded through the six would add an energy of its own to the energy of empty space, and the added energy would change my result for how much of the universe is matter. The strength of a threaded field would also be a second number beside the strength of gravity. The strength of gravity is the one measured strength on gravity's side. My results contain no added energy from a threaded field and no second strength beside the strength of gravity, so in my account no field is threaded through the six. The six are like the ring in the picture, with nothing wound through its hole. A flux sector is one choice of the whole numbers that count the windings. So the account has no flux sectors, and the landscape of flux vacua has no counterpart in it. In the picture of the ball, there is one country with one valley, and the ball has nowhere else to fall. The six have one vacuum.

The third answer is about the branes. In string theory the extra dimensions are a space in which branes sit and move. The six are dimensions of possibility. The six hold the range of what a comparison could find, and each finding is one selection from that range. No comparison travels through the six in the way a comparison travels through space. So no brane has anywhere to drift. Greene's third road needs a bulk for other branes to float in, and on my account there is no bulk.

The fourth answer is about the brane cycle of Steinhardt and Turok. The brane cycle and my account have much in common, so what the two have in common comes first. Chapter 9 showed that our epoch began at a turn. The sky empties through our epoch until the next turn. So the his-

tory of the universe, on my account, is a cycle. I set the two cycles side by side: "Steinhardt and Turok's cycle, like this one, has no first turn". In both cycles, the speeding-up of the expansion empties the region that each observer can compare with, and the laws of physics hold through the passage from one history to the next.

The two cycles differ on where the cycle turns. In the brane cycle a field ends each cycle. The energy of the field turns negative, the speeding-up stops, and a contraction brings the branes together. The energy of empty space is one constant number in our epoch, as Chapter 10 showed. So the speeding-up of our epoch never ends. On my account, the turn comes instead at the ends of the interiors of black holes, and the speeding-up of the expansion does not prevent the interiors of black holes from ending. Greene counts each cycle of the brane model as a universe of its own, one of a series in time, with the other universes lying in our past and future (chapter 5, section "Time, Cycles, and the Multiverse"). On my account, each epoch is one cycle of the one world's history, and the epochs form a single sequence, one after another, with no first. The physicist Lee Smolin made a proposal that Greene names in *The Elegant Universe* (chapter 14, section "Cosmological Speculation and the Ultimate Theory"), and in Smolin's proposal a new universe begins inside every black hole, so universes branch into a growing population. I share Smolin's starting idea: a new epoch begins at the end of a black hole's interior. But the count of new epochs in my account is different. By the principle of comparison, the ends of all the black-hole interiors are one end. No comparison finds the end of one interior different from the end of another, and ends that no comparison can tell apart are one end. There is one next epoch, not one per black hole.

The *arena* is space and time taken together. *First waves* are ripples of the arena itself, present from the beginning. Greene names one measurement that would tell inflation from the brane cycle: a measurement of first waves. Inflation would make first waves strong enough to leave a mark on the oldest light. Greene writes that any first waves made by a brane collision "would almost certainly be too weak to create a lasting signal" (*The*

Hidden Reality, chapter 5, section "The Past and Future of Cyclic Universes"). Astronomers have searched the oldest light for the swirling pattern that first waves would print on the oldest light. The searches combine the maps of the Planck satellite with the maps of telescopes at the South Pole. The searches have found no swirling pattern. In 2021 the teams at the South Pole placed the power of any first waves below thirty-six thousandths of the power of the first ripples, as Chapter 9 told. In 2022 Matthieu Tristram and colleagues lowered the bound: any first waves carry less than about three hundredths of the power of the first ripples. The first ripples are differences of density. Waves of the arena arise from the first ripples only as a faint second effect, when the ripples act on one another. So the cycle of my account agrees with the brane cycle about first waves: in both cycles, any first waves are faint. The two cycles differ on one measurable question: whether the energy of empty space stays one constant number through our epoch. Appendix A gathers the coming measurements of the energy of empty space.

Our Value, Without a Draw

Greene's landscape road ends with the energy of empty space, the case that Weinberg took up. The energy of empty space has a measured value inside the narrow range that lets galaxies form, as the landscape argument requires. The anthropic reasoning is logically impeccable. I also state what is lost if the anthropic reasoning is right: if the value is selected by our existence, "It is not a consequence of the laws of nature but an accident of location".

The value follows from the laws. The energy of empty space speeds the expansion, and that energy enters Einstein's equations as the *cosmological constant*. The cosmological constant is one number, the same in every cubic meter of space, and the cosmological constant does not thin out as space grows. In this paragraph the vacuum means empty space with every field at its lowest energy. The vacuum's total energy is there, taken as a whole, but the total energy curves nothing. Only the accessible share

bends the arena. The accessible share is one number, and on my account there is no collection of vacua for the number to be chosen from. The *matter fraction* is the part of the universe's energy that is matter. When the accessible share is put into the equation for the expansion of the universe, the equation fixes the matter fraction at 31.1 percent. The matter fraction measured from the oldest light is 31.5 percent. The cosmological constant is then not selected from a distribution of vacua. In the picture of Greene's shoe shop, my account is the case that Greene set aside: a shop with one size on the shelf. Greene says that, with one size on the shelf, you could still ask the "deeper explanation" question. The "deeper explanation" question is why the one size is the size it is. The deeper explanation is that the one size is the accessible share, the share of the vacuum's energy that a measurement made from inside the universe takes in.

In Greene's landscape the other constants of nature would also vary from one vacuum to another. *Pure numbers* are numbers with no unit, such as a ratio. Some pure numbers are derived. The derived pure numbers include the matter fraction of Chapter 10 and the ratio of dark matter to ordinary matter of Chapter 9. Four values are taken from measurement. The four measured values are the strength of gravity, the strength of light, the strength of the force that binds quarks (the particles inside protons and neutrons), and the settled value of the Higgs field. Greene writes that in the string landscape, as now understood, properties such as the mass of the electron vary from place to place (chapter 6, section "The Rest of Physics?"). So, Greene writes, trying to calculate the properties of the fundamental particles "may be misguided", like trying to explain the distance between the Earth and the Sun. On my account, the kinds of particle, their charges and their three *generations* (the three copies of the particles of matter, alike but for their masses) form a closed list, as Chapter 6 showed. Chapter 7 showed why the generations are three. Chapter 6 named the *structure of comparison*: the three roles of every comparison and the axioms that relate the three roles, the comparison's content and its context, and the Incommensuration Theorem, taken together. The structure of comparison fixes the closed list. The Incommensuration Theorem, from Chapter

3, says that nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. In the name structure of comparison the word structure has its everyday sense: the way a thing is built. In that name the word structure does not mean the third role. So in my account there is an answer to why the kinds of particle are what they are: the structure of comparison.

Greene names one more difficulty for a vast collection of universes. When there are infinitely many universes, you get different answers for how common one kind of universe is, depending on how you count. The whole numbers show the difficulty: the whole numbers and the even numbers can be paired off one to one, although every other whole number is missing from the even numbers. In Greene's words, "There simply is no absolute answer to the question of which of these kinds of infinite collections are larger" (chapter 7, section "Dividing Up Infinity"). Physicists call the problem of saying how common each kind of universe is in an infinite collection the *measure problem*. Greene says that a small but growing number of researchers share his own view: "the measure problem is tapping into a deep problem at the very root of physics" (notes to chapter 7, note 12).

My account has a rule about counts, and the rule applies to Greene's finding on the measure problem. Picture a lottery with a million tickets, each with its own number, and one of the tickets drawn. To call one ticket a one-in-a-million winner, someone must hold a million tickets and be able to tell them apart. An improbability is a comparison of the case at hand with alternatives that can be told apart in some context. A change of context is a change of what is counted, and a count made relative to one context is not a count relative to another. I call a whole collection of universes an *ensemble*, and say that an ensemble "would have to be laid out from a vantage no comparison can occupy". So the rule about counts and Greene's finding agree: every count needs a context in which its cases can be told apart, and a count of all universes at once would need a context in which no comparison can be made. In my own cosmology there is no eternal inflation, so there is no collection of bubbles to count.

One World, Met from Inside

Go back once more to Kraków. Carter's sentence holds in my account as Carter wrote it: our situation is not necessarily central, and our situation is inevitably privileged to some extent, because we observe from inside the world, at a time when stars shine. Carter's principle applies within one world. A selection effect needs no other worlds to show where and when observers find themselves.

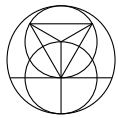
Greene's case for other universes has the shape of a cable, and my case for one world has the same shape, with six strands. The first strand is the far patches, joined to their neighbors. The second strand is the first ripples, which show that there was no long inflation. The third strand is the one shape of the six. The fourth strand is the six as dimensions of possibility, with no bulk for branes. The fifth strand is the one end of all the black-hole interiors, and so one next epoch. The sixth strand is the energy of empty space, given by the laws without selection.

Greene used his principle in the case of a body's position and speed to count the arrangements in a patch. With that count Greene reached other universes. When Greene's principle is taken for every distinction, Greene's principle is the principle of comparison. By the principle of comparison, a region is part of our world when the region can be compared with ours through a chain of neighbors. A region that no path from our place could ever reach is, relative to us, no place at all.

So we now know that there is one world. The regions beyond our particle horizon touch their neighbors, so those regions are further parts of the one world. There was no long inflation to make other bubbles. The six have one shape, so there is no landscape. The six are dimensions of

possibility, so there is no bulk for branes to drift in. The epochs follow one another as a single sequence, with one next epoch and no first. Our value of the energy of empty space is given by the laws, without a selection.

Greene's remaining multiverses lie inside quantum theory, in holograms, in computers and in mathematics. Are the universes of those four multiverses other worlds?



Chapter 12

Are There Other Worlds in the Quantum, the Hologram, the Computer?

Beside The Hidden Reality, *chapters 8, 9, 10 and 11,* The Fabric of the Cosmos, *chapter 7, and* The Elegant Universe, *chapter 4.*

What this chapter uses

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

Every comparison has three roles: the *stance*, the side a comparison is made from, with the range of what could come; the *act*, the happening itself; and the *structure*, what is laid down in the act, the same for everyone who reads it afterward (Chapter 3).

I identify quantum theory with the description of the content of physics, which is what is found in the comparisons of physics, and general relativity with the description of the context of physics, which is the arena in which that content is found (Chapter 5).

Hard randomness is an outcome that nothing before it fixes, however completely what came before is known (Chapter 2).

In May 1956 the physicist John Wheeler went from Leiden to Copenhagen to put before the physicist Niels Bohr the thesis of Wheeler's Princeton student Hugh Everett III, which held that every outcome of a quantum measurement happens, each in a world of its own. If every outcome of a quantum measurement happens in some world, or our world is a hologram, or a simulation, or one mathematical structure among all the mathematical structures there are, which of these other worlds are real? In that question, a *mathematical structure* is a pattern of relations laid out whole and the same for everyone.

Everett had given Wheeler a long draft of the thesis in January 1956, and Wheeler was impressed by the draft. Wheeler put Everett's ideas before Bohr and the physicists around Bohr. In Greene's words, "the reception was icy" (*The Hidden Reality*, chapter 8, section "Quantum Reality"). Wheeler then required Everett to cut the parts of the thesis that criticized Bohr's method, and Everett cut those parts. Princeton accepted a much shorter thesis in the spring of 1957. In July 1957 a paper by Everett covering the same ground as the thesis appeared in *Reviews of Modern Physics*. For years physicists ignored the paper. Then, in 1970, the physicist Bryce DeWitt wrote about Everett's paper for a wide audience of physicists. Physicists have argued over the other worlds of quantum theory ever since.

Greene counts nine kinds of other world in *The Hidden Reality*, and the last four kinds each bear on the question of which other worlds are real. The last four kinds are the worlds of quantum measurement, of the hologram and of the computer, and every world that mathematics can describe. My account has one answer that covers all four kinds.

From Greene's pages

Everett's Worlds, as Greene Tells Them

Quantum theory describes a particle by what Greene calls its *probability wave*: a description, spread over the places where the particle could be found, of how likely the particle is to be found at each place. The state changes smoothly in time by a rule that the physicist Erwin Schrödinger wrote down in 1926. Schrödinger's rule is *linear*: when the state is a sum of alternatives, each alternative changes by the rule as if it were alone, and the state at the later time is the sum of the changed alternatives. So Schrödinger's rule never reduces a sum of alternatives to one of the alternatives. Yet every measurement ever made has had one result. In the standard view, which Greene calls the Copenhagen approach, the wave collapses at a measurement onto the one result found, by a sudden change that Schrödinger's rule does not

contain. Greene shows that the collapse cannot come from Schrödinger's rule: in his words, the collapse "can't possibly emerge from Schrödinger's math" (chapter 8, section "Not So Fast"). Greene adds that Bohr drew a line between the small things that Schrödinger's rule governs and the large instruments that record a result, and that Bohr never said where the line falls.

Everett let Schrödinger's rule govern everything, including the measuring instrument and the person reading the instrument. Greene explains that if the rule is followed with care, the rule describes an instrument and a mind registering one result, another instrument and another mind registering a second result, and so on for every result the wave allows. In Greene's words, each possible outcome "is realized in its own separate world" (chapter 8, section "Many Worlds"). Greene names the collection of those worlds: "I'll call it the Quantum Multiverse" (same section).

Why do the other worlds go unseen? Greene's answer is decoherence, and to explain decoherence we start from interference. Physicists call that adding and canceling *interference*. Interference produces the stripes of the double-slit experiment (two narrow openings side by side), the same stripes that Akira Tonomura's electrons drew on a screen in Chapter 4. The alternatives of the state add and cancel in the same way as two waves do. Once the surroundings hold records of the alternative that is the case, the interference between the alternatives washes out. Physicists call the washing out of the interference *decoherence*. Once a large instrument has recorded a result, the alternatives differ in so many particles that the alternatives can no longer interfere, so each alternative goes on as if the other alternatives were gone. Greene states the consequence in one line: "Within each universe, that is, it's as if the probability wave has collapsed" (chapter 8, section "When Is an Alternative a Universe?").

Greene argues that Everett's proposal adds nothing to the mathematics of quantum theory. Greene writes that "the Many Worlds approach is, in some ways, the most conservative framework for defining quantum physics" (chapter 8, section "A Tale of Two Tales"), because the mathematics of Everett's proposal is Schrödinger's rule and nothing else. Greene uses words like splitting and cloning when he retells the mathematics as what each observer experiences, but he says that the mathematics contains no process of splitting or cloning: "There's no 'cloning machine'. There's no 'splitting machine'" (same section).

Greene then takes up the question of odds. If every outcome happens, what does it mean that one outcome is more likely than another? Greene asks: "How can we speak of some outcomes being likely and others being unlikely when all take place?" (chapter 8, section "Uncertainty at the Cutting Edge"). When the wave is taller in one place than another, odds found by counting the copies of you who see each outcome are wrong. Everett developed a mathematical argument that, in Greene's words, "was meant to address this mismatch": in calculating the odds, give less weight to the worlds in which the wave is smaller (chapter 8, section "Probability and Many Worlds"). Greene calls Everett's idea perplexing and controversial. Greene also reports that those who have studied Everett's thesis generally agree on two points. The first point is that Everett's aim was clear: a deterministic theory, in which the state at one moment fixes the state at every other moment, that looks probabilistic to those inside the world it describes. The second point is that Everett "didn't convincingly spell out how to achieve it" (same section). At Oxford, the physicist David Deutsch and his colleagues asked how a gambler who holds Everett's view should bet on a quantum experiment, and they argue mathematically that the best bets follow the odds of the standard theory. Greene reports that the experts are divided on what he calls the "crucial question of how to treat probability in the Many Worlds approach": "Some

are convinced that this completes Everett's program. Some are not" (chapter 8, section "Probability and Many Worlds"). In a note Greene says of himself: "I'd like the Many Worlds approach to be right" (chapter 8, note 11). Greene keeps "a healthy skepticism, fueled by the difficulties of integrating probability into the framework" (chapter 8, note 11). Greene answers the view that physics is only about making predictions: "There's a difference between making predictions and understanding them" (chapter 8, section "Predictions and Understanding").

Greene's earlier books describe the same divide: physicists agree on how to use quantum theory, and they are divided on what quantum theory means. In *The Elegant Universe* Greene writes that "there is no consensus on what it really means to have probability waves" (chapter 4, section "Waves of What?"). In *The Fabric of the Cosmos* Greene tells of the physicist Richard Feynman's way of adding up every possible history of a particle (chapter 7, section "Prochoice"). Greene asks whether an electron reaching a screen travels every possible route, or whether Feynman's sum over every possible history is a clever mathematical device for getting the right answer. Greene puts the puzzle of what Feynman's sum describes in one line: "Quantum mechanics is starkly efficient: it explains what you see but prevents you from seeing the explanation" (same section).

From Greene's pages

A Hologram, a Computer and a Library

Greene begins his seventh kind of other world with *black holes*, regions whose gravity is so strong that nothing, not even light, comes back out of them. The physicist Jacob Bekenstein proposed that a black hole has *entropy*, and in 1974 a calculation by the physicist Stephen Hawking confirmed Bekenstein's proposal. In this book, a context is the respect in which a thing is compared, and entropy is a count of the arrange-

ments of a thing's parts that a comparison in some context cannot tell apart. The entropy of a black hole is one quarter of the area of its horizon, measured in squares of the *Planck length*. The Planck length is about a millionth of a billionth of a billionth of a billionth of a centimeter, about a hundred billion billion times smaller than the width of a proton. Greene draws from black holes a lesson that holds for any region of space (*The Hidden Reality*, chapter 9, section "Beyond Black Holes"). However the objects in a region are designed, the information the region can hold is always less than the area of the surface around the region, counted in squares of the Planck length. In this lesson, information is a count of states, like entropy. The physicists Gerard 't Hooft and Leonard Susskind took a further step: they proposed that everything that happens inside a region is fully linked to processes on a surface around the region, so that the world inside is like a projection of the surface. A *hologram* is a flat film that shows a scene in depth when the film is lit. Greene names the paired worlds, the world inside a region and the world on its surface, after the hologram: "I'll call them Holographic Parallel Universes" (same section).

In 1997 the physicist Juan Maldacena found a mathematical setting in which physicists could make the idea of 't Hooft and Susskind explicit and study the idea by calculation. Maldacena took a universe whose shape differs from the shape of ours, a shape with a boundary that completely surrounds the interior of the shape, and physicists call that shape anti-de Sitter space. Maldacena argued that string theory in the interior of anti-de Sitter space, gravity included, is equivalent to a *quantum field theory* living on the boundary: a quantum description of particles as ripples of fields, with no gravity in it. Greene reports three things about Maldacena's equivalence. First, Maldacena's result is still "deemed conjectural". The holographic ideas, Greene adds, have come through "a great many stringent mathematical tests" (chapter 9, section "Parallel Universes or Parallel Mathematics?"). Second, with the equivalence, string theory in anti-de Sitter space has what Greene

calls "a firm formulation" (chapter 9, section "Coda: The Future of String Theory"). Third, Greene writes: "if Maldacena's insights are fully valid, as all available evidence attests, string theory and quantum field theory may very well be two sides of the same coin" (same section).

For black holes themselves, Greene tells of a proposal of Susskind's (chapter 9, section "Locating a Black Hole's Hidden Information"). In Susskind's proposal, someone falling freely through a horizon floats as if in empty space, while a distant watcher pieces together a story in which the one who falls is burned up by a scorching glow around the horizon. Greene also reports a result of the physicist Edward Witten: a black hole inside anti-de Sitter space is equivalent to a hot quantum field theory on the boundary of the space (chapter 9, section "String Theory and Holography"). Greene says, in a note to the same section, that Witten's result supplied "conclusive evidence that all information that falls into a black hole is ultimately available to the outside world".

Greene's eighth kind of other world is the Simulated Multiverse: worlds run on computers, with minds in the worlds. Greene notes that the possibility of a Simulated Multiverse depends on *functionalism*, the view that a mind is a pattern of processes and functions, whatever material carries them (*The Hidden Reality*, chapter 10, section "The Stuff of Thought"). Greene writes of functionalism, "The assumption could be wrong". Then he takes the view up, to see where the view leads. He tells the argument that the philosopher Nick Bostrom published in 2003: if realistic simulations prove possible, simulated minds could come to far outnumber minds of flesh, and then brute counting says we are probably among the simulated minds. Greene states that "Logic alone can't ensure that we're not in a computer simulation" (chapter 10, section "Are You Living in a Simulation?"). Greene then states his own position: "Until I get my breath taken away by seeing a sentient simulation, I won't consider seriously the proposition that I am now in one" (chapter 10, section "The Roots of Reality"). Greene

adds that a computer running smooth laws must round the laws off, so the inhabitants of a simulation might one day notice small errors (chapter 10, section "Seeing Beyond a Simulation"). Greene also says, in the section "Seeing Beyond a Simulation", that a simulator could hide the errors, so that "it would seem a stretch to claim that a simulated reality would reveal its true nature through glitches and irregularities". Greene's own guess is that "we will indeed find that the world is fundamentally discrete" (chapter 10, section "Simulating Babel"). *Discrete* means made of separate steps, with nothing between the steps.

Greene's ninth kind of other world is, in his words, "the most expansive possible" (chapter 10, section "The Library of Babel"). Greene presents the ninth kind, the Ultimate Multiverse, following the philosopher Robert Nozick and the physicist Max Tegmark. In the Ultimate Multiverse, every consistent set of laws describes a world as real as ours: "All sets of laws describe real universes" (same section). Greene states his own position on the Ultimate Multiverse: "personally, it's a leap I've not taken" (chapter 10, section "Multiverse Rationalization"). Greene states his own view of mathematics too: "My view on mathematics periodically changes" (chapter 10, section "The Library of Babel"). Greene names the single theme of *The Hidden Reality* in the book's last chapter: "the capacity of mathematics to reveal secreted truths about the workings of the world" (chapter 11, section "Should We Believe Mathematics?"). Greene says that he titled *The Hidden Reality* in the singular to reflect the book's single theme, the power of mathematics to reveal hidden truths about the world.

Where Greene and I Reach the Same Results

The first chapters gave three plain words, and the rest of this chapter needs all three. On six matters my account has the same results as Greene, from its own ground: Schrödinger's smooth rule, decoherence, the difficulty of giving odds when every outcome happens, the count of a black

hole's entropy, the smallest step of a simulation, and the power of mathematics. On two further matters my account says more than Greene reports: Maldacena's correspondence is exact, and my account has its own answer to what becomes of whatever falls into a black hole formed from a star.

I use the same smooth rule that Everett used. A *third-person* description is one that anyone can check from outside, told of a thing as "it". The third-person physics is physics told in the third person. *Unitary evolution* is the physicist's name for the smooth, linear change of the state. The many-worlds formalism describes the third-person physics, and the quantum mechanics of my account uses the same unitary evolution. The Everett program achieves two things. First, the Everett program has one rule of change, which applies without alteration to the smallest system and to the whole universe. Second, with decoherence, the Everett program describes a definite branching into alternatives that each look like the everyday world. Physicists call each of those alternatives a *branch*. I agree with Greene about decoherence. On my account, decoherence prepares the records, and an act of comparison then completes the measurement with one finding.

Schrödinger's smooth rule has passed every test made of it. Physicists have sent molecules of more than twenty-five thousand times the mass of a hydrogen atom through interference experiments, and the molecules still showed interference. The physicists Lajos Diósi and Roger Penrose proposed that gravity itself makes a heavy body settle into one alternative. The proposal of Diósi and Penrose predicts a faint radiation. In 2021 Sandro Donadi and his colleagues looked for the faint radiation deep under a mountain in Italy, and the team found too little of the radiation that the proposal predicts. The team's measurement therefore rules out the version of the proposal that has no adjustable number. So we now know that the smooth rule applies where that version of the proposal said the rule would fail.

On the odds, I agree with Greene that there is a difficulty. When every outcome occurs, a probability needs an agent who faces outcomes and weighs them. Deutsch and his colleagues take an agent of that kind, with the agent's choices, as the starting point of their gambler's argument. My starting point is three things: the comparer, the act of comparison and the range of what the act could find. So the agent that the odds need is already in my starting point. On my account, Born's rule follows from the comparer, the act and the range. I agree with Greene that explaining is more than predicting. An answer that only predicts leaves the why unanswered.

On black holes Greene and I agree closely. The entropy of a black hole is a physical count, and my account has its own argument, the argument of Chapter 6, for why the count grows with the area of the horizon. A region can be counted in two ways. In the first way, the count goes piece by piece through the inside, and the count grows with the volume of the region. From outside, a black hole can be compared only the second way, through its horizon. The count therefore grows with the area of the horizon, as Bekenstein and Hawking found. On information, I agree with Greene that information is not destroyed in a black hole. Information is not destroyed in a black hole. Information has two senses. In the first sense, information is the form of what falls in: what the matter is and which of its alternatives it holds, carried with the matter wherever the matter goes. In the second sense, information is a count of states relative to a context. The entropy of a black hole is information in the second sense. A black hole gives off a faint glow, as Hawking found, and the hole slowly shrinks by the glow. Take a hole small enough to glow away while a world outside the hole remains to receive the glow. For a hole of that size the information comes back out in the glow, with quantum theory unchanged, so for that hole my account has the answer that Greene reports from Witten (Chapter 18 sets out the steps). For a hole formed from a star, my account has a different answer from the one Greene reports from Witten: the information goes with the matter to the end of the hole's interior, and on through the turn. An epoch is one era of the world. In the next

epoch the information shows again in the first ripples: the small departures from evenness at the start of the next epoch. In this book, the *content* is what comparisons find. In Chapter 9 I identified the first ripples with the form of the content carried through the turn: information in the first sense. The form of the content is kept the way the worn stairs of an old church keep the footsteps of hundreds of years. No single footstep can be picked out in the curve of a step, and no footstep has been wiped out of the curve. The footsteps stand for what fell into the holes. The curve of the step stands for the form of the content carried through the turn. The information is kept through the end of the interior and through the turn, and no observer who stays outside the hole in our epoch can observe the information. Chapter 18 follows the information through the turn in full. I also agree with Greene's picture of the one who falls. A body falling freely through the horizon meets there nothing that a body falling freely anywhere else would not meet.

On Maldacena's correspondence my account goes further than Greene's report. Greene calls the result conjectural and tells of the many tests the result has come through. The correspondence is exact. The thesis of the book is one system with two descriptions, joined by an exact dictionary, and the book calls Maldacena's correspondence the clearest example of that thesis that physics has found. Greene tells of one use of the correspondence that a measurement has already checked. At the Relativistic Heavy Ion Collider on Long Island, physicists smashed gold nuclei together hard enough to melt the protons and neutrons of the nuclei into a hot fluid of quarks and gluons. Quarks are the particles inside protons, and gluons are the carriers of the force that binds the quarks. The experimenters measured the fluid's *shear viscosity*, the drag of one flowing layer on the next. The measured drag was far smaller than the usual calculations gave. Three physicists then calculated the drag on the gravity side of the correspondence, in the interior: Pavel Kovtun, Dam Son and Andrei Starinets. In Greene's words, their results "come impressively close to the experimental data" (*The Hidden Reality*, chapter 9, section "Parallel Universes or Parallel Mathematics?"). Greene adds two conditions in the

same section. First, the calculation's boundary theory "doesn't model our universe fully", because the boundary theory leaves out gravity. Second, on a conservative reading, the calculation is "a potent string-based mathematical trick". So we now know that physicists, using Maldacena's correspondence as a tool of calculation, have got a value close to the drag measured in a real fluid of our own world.

On simulation, two arguments show that a computer running a world has a smallest step: Greene's argument and my argument. Greene's argument is about rounding: a computer running smooth laws must round the laws off, so the inhabitants might one day notice small errors, though Greene judges that a simulator who reset the program could hide the errors. My argument is that a description can follow the thing it describes as closely as one likes, and never contains the thing described. My account has a result of its own: a measurement finer than a program's smallest step could show the difference between a world run on a computer by one fixed program and our world. A program's *statistics* are the pattern of how often each of the program's outcomes comes up. For any one fixed program, beyond its smallest step, a sensitive enough measurement eventually tells the program's statistics from the world's. Appendix A gathers the coming measurements that are finer than the smallest step of a fixed program.

On mathematics, I agree with Greene about the power of mathematics. Mathematics keeps its necessity, its exactness and its power to hold structure in view as a whole, and it is the finest instrument there is for what is laid out.

What Exists and What Is Real

Where, then, do I differ from Greene on his four kinds of other world? I start the answer with a kitchen table.

Watch two people playing chess at the kitchen table tonight. There are the rules of chess, which hold for every game of chess ever played. There is a score sheet, on which every move is written down. And there is the

game itself, being played, move by move, in the kitchen. At the chess table, the game being played tonight is real; the rules and the score sheet exist.

The two words "real" and "exist" carry the meanings that Chapter 2 gave them. A law exists; a measurement is real.

Now look at Everett's universal state, the one state of the whole world, every branch included. The universal state exists: the universal state is structure, laid out, like the rules and the score sheet. For each comparer, the finding the comparer makes is what is real. For every comparer, one outcome is real, the one found. For each comparer, only the branch the comparer finds is given in the comparer's acts. The many-worlds view treats the structure that exists as the whole of what is real, as if a book of every game of chess that could be played were the game on the table. The book of games stands for the universal state, and the game on the table stands for the finding that each comparer makes.

What, then, are the other outcomes? The other outcomes are possibilities. Possibility is the range of what each comparison could find, and every finding is one selection from that range. Possibility is distinct from the act and from the record, and inseparable from both. I call possibility "the range from which every finding is one selection". A *quantum computer* is a machine that computes with the alternatives of a quantum state while the alternatives still interfere. David Deutsch argues that a large enough quantum computer would, by its interference, do more computation than the whole visible universe could hold, and he asks where the computation was done. My answer remains within one world: The computation was done where every quantum interference is done: among the possibilities of one world, before the comparison that makes one of them actual. In the quoted sentence, "actual" means found in an act of comparison, and so real for the comparer who makes the finding. So the alternatives that interfere are held in one world, as possibility. In *The Fabric of the Cosmos* Greene asks whether an electron reaching a screen travels every possible route. My answer is that the routes are possibilities of one world. Each finding makes one route real for the comparer who makes that finding.

My claim about many worlds has two parts, and each part applies to a different thing: the first part applies to the whole world, and the second to the formalism that Everett wrote.

The first part concerns the many-worlds claim that the whole world is completely fixed by one law. In this book's sense, *symmetry* is the same content across a different context. The context is the respect in which a comparison finds what it finds. For a law, symmetry is one law, the same everywhere and at every time. *Continuity* is the same content where the context is the same. For a world, continuity is every state fixed at every point, each state passing into the next with no gap. In Chapter 3 I found the two pairings that the Incommensuration Theorem leaves, which I call *the two surviving pairings*. The two surviving pairings are symmetry with discontinuity, like a ladder whose rungs are all alike, and continuity with asymmetry, like a smooth ramp that rises unevenly. The rungs of the ladder are all alike, with gaps between them. The ramp has no gaps, but it rises more in some places than in others. A whole world completely fixed by one law would be perfectly symmetric and perfectly continuous at once. A whole world completely fixed by a law perfects symmetry and continuity together, whichever account describes it, and the theorem forbids that pairing. The many-worlds view describes a whole world completely fixed by one law. In that view the only randomness is in which branch an observer finds itself. So, by the Incommensuration Theorem, the many-worlds view is excluded as a description of a whole world.

The second part concerns the many-worlds formalism taken as a description of knowing. A description of knowing also covers the comparer who finds, and how that comparer makes and knows each finding. For the second part a different picture helps: a map and a walk. A map can show every road of a country. Only the one who walks has the experience of walking through the country, the here and now of each step. Three features go together in every act of comparison: the side the comparison is made from, the one-way order in which the comparison runs, and an outcome that nothing before the outcome fixed. Chapter 14 sets out the three features in full. The many-worlds formalism lacks all three features at

once. The formalism has no side: no fact in the many-worlds formalism says which branch a comparer is in. The formalism has no unfixed outcome: every outcome occurs, by a rule that fixes each next state. The formalism has no one-way order: the formalism's basic equation runs the same both ways in time. In the picture, the three features belong to the one who walks, and the formalism, written in the third person, is the map. Where a theory leaves out the side, the order and the unfixed outcome, the theory can be a complete account of the third-person physics, but it cannot, by the theorem, also be a complete account of the conditions under which that physics is known. So the theorem limits what the many-worlds formalism can claim as a description of knowing. The formalism can still be a complete description of the third-person physics.

So the two parts of the claim apply to two different things. Everett's formalism is the formalism I use. Everett's formalism describes the third-person physics, with a limit on what the formalism can say about knowing. By the Incommensuration Theorem, the whole world cannot be Everett's formalism, completely fixed, with every branch equally real.

A measurement of gravity shows which of the alternatives the geometry of space and time follows, and Chapter 5 told of that measurement. So we now know that gravity follows the alternative that is the case. So a finding adds no geometry and alters none: a finding selects among alternatives that already carry their geometries. The arrangement of one geometry for each alternative stays within one world, and the arrangement needs no claim about what becomes of the alternatives a finding does not select.

Two Descriptions of One System

Is the hologram, then, a second world mirroring ours?

Think of a good translation of a poem. The translation carries the sense across, line by line, and still each language keeps something the other language cannot hold: the rhyme in one, a turn of phrase in the other. Nobody concludes from a perfect translation that French is English in dis-

guise. Chapter 5 used the picture of a translation for a *correspondence*: two descriptions with an exact translation between them, neither made of the other.

Maldacena's correspondence is a translation of exactly this kind. The boundary theory states its results as a discrete list of numbers sorted by an exact symmetry. Quantum theory describes what is found in that same way. The theory of the interior states its results as a smooth geometry that differs from region to region. General relativity describes space and time in that same way. Recall the two surviving pairings of Chapter 3: symmetry with discontinuity, the ladder whose rungs are all alike, and continuity with asymmetry, the smooth ramp that rises unevenly. Chapter 5 took Maldacena's translation as exact in both directions, and read the translation as one system with two descriptions, neither more basic than the other. I identify which side carries which pairing: symmetry with discontinuity on the boundary, and continuity with asymmetry in the interior. The translation is exact in both directions, so neither side is made from the other. *Emergence* is one thing arising out of another, and emergence makes the thing arisen from the more fundamental of the two. An exact translation in both directions makes neither side the more fundamental. This is correspondence, not emergence. Neither side of the correspondence is more fundamental than the other. So the hologram's two sides are two descriptions of one system: the boundary carries symmetry with discontinuity, and the interior carries continuity with asymmetry. Greene calls string theory and quantum field theory "two sides of the same coin", under his condition "if Maldacena's insights are fully valid". On my account, the coin is the one system, its two sides are the system's two descriptions, and neither side is ranked above the other.

A Room Inside the One World

Could we, then, be living inside someone's computer?

Picture a prison cell. The walls of the cell restrict what the person inside can see and reach, and the restriction is real. The cell is still a room of the building, under the same sky, on the same day. I use the prison cell as a picture of a simulation: "The computation restricts what can be reached from inside it, as a prison cell restricts what its occupant can see; the restriction is real, and the separateness is not".

First, a program run on an ordinary computer is fixed: each state follows from the state before by the program's rule. So every outcome in the program is fixed by what came before the outcome. So the program has no hard randomness of its own. Second, the three features of every act of comparison go together: the side, the one-way order and the unfixed outcome. The program therefore has no direction of time of its own, and no side of its own from which a comparison is made. In this argument, a *subject* is one who compares from a side of its own. So a simulated world with subjects in it would have its randomness, its direction of time and its subjects from the world in which the computer runs. A simulated world containing real subjects is not a separate world: it is a restricted region of the one world in which it runs. Bostrom counts each simulation as a separate world with its own observers, while on my account a simulation with subjects in it is a region of the one world in which the simulation runs. Greene's chapter and I differ at the functionalist premise, which Greene takes up to see where it leads: on my account no program, by being a program, is a mind.

A mosaic can follow a curve as closely as anyone likes: make the tiles smaller, and the steps along the edge shrink until no eye can see them. The mosaic is still made of tiles, and the curve is still not made of anything. I agree with Greene's guess that the world is at bottom discrete, but only for what comparisons find: the kinds of particle and their charges, which come in steps. The *arena* is space and time taken together, as the set-

ting in which everything that is measured is measured, and the arena has no least step: no least length, area or duration. A simulation of the arena is a mosaic of the arena: the simulation can come as close to the arena as one likes, and the simulation is never a second arena.

The Orrery and the Age of the Earth

And what of every mathematical world at once, the Ultimate Multiverse?

An *orrery* is a working model of the solar system: a brass Sun, planets on arms, and beneath them a train of gears that carries each planet around at its proper pace. Turn the gears backward and the planets go backward, just as smoothly. A visitor might conclude that the solar system could run backward just as well. The solar system runs one way only. The Moon raises tides in the oceans, the tides drag against the turning Earth, and the drag pushes the Moon slowly outward. Observatories on Earth send laser light to the reflectors that the Apollo astronauts left on the Moon, and the light that bounces back has shown the Moon moving away by about 3.8 centimeters a year. The orrery has no gear for the tides that push the Moon outward. The ease with which the model runs backward is a property of the model.

Those who hold the Ultimate Multiverse take the visitor's step on the largest scale: they treat the features of a mathematical description as features of the world. A mathematical structure can be perfectly continuous and perfectly symmetric at once, like the circle of geometry. No comparison ever finds a thing that is both. So a mathematical structure exists, in the sense in which the chess table's rules exist, and the world that comparisons find is real. Structures that no comparison could ever find are not further worlds. My answer depends on a different reason from Greene's. Greene remains skeptical of the Ultimate Multiverse because, in his words, it was "Cooked up specifically to address one issue", and because he finds it hard to imagine a process that would generate it (chapter 10, section "Multiverse Rationalization"). Greene adds that simulations run

by future computer users could supply a process that generates one part of the Ultimate Multiverse (chapter 10, sections "Multiverse Rationalization" and "Simulating Babel"). The part is made of the universes that arise from computable functions. A computable function is one that a computer can evaluate exactly by running through a finite set of instructions. The Ultimate Multiverse is set aside, because existing and being real are two different things, and a further world would have to be real.

Greene's single theme is the power of mathematics to reveal hidden truths, and the theme has famous cases. Greene tells one of the famous cases (*The Hidden Reality*, chapter 10, section "The Library of Babel"). In 1931 the mathematics of the physicist Paul Dirac required a particle that no one had seen. In 1932 the physicist Carl Anderson found the particle in cosmic rays: the positron, the electron's twin of opposite charge. I tell a case from the other side, in which exact mathematics gave a wrong answer about the world. In 1862 the physicist William Thomson, later Lord Kelvin, calculated the age of the Earth from the Earth's cooling. Thomson took the Earth to be a solid ball with no source of heat within, losing heat only by *conduction*, the passing of heat through material that stays where it is. Thomson found an age somewhere between twenty million and four hundred million years. Over the following decades Thomson narrowed his figure to between twenty and forty million years. Thomson's figures, carried by exact mathematics, prevailed over the geologists' evidence of the rocks for some forty years. In 1895 the engineer John Perry pointed to the premise of a solid Earth: an interior that flows carries its heat with it, by *convection*, and with a flowing interior the measured warmth of deep mines fits an Earth thousands of millions of years old. *Radioactivity* is the giving off of energy by certain atoms as the atoms change. Radioactivity was discovered in 1896. Radioactivity gave the Earth a source of heat inside it, and gave physicists a new clock for the age of the Earth. In 1956 the geochemist Clair Patterson measured the lead in meteorites and found the age of the Earth to be about four thousand five hundred and fifty million years. So we now know that Thomson's mathematics was sound, and that one of its premises, a solid Earth with no source of heat within, losing

heat only by conduction, was false of the Earth. Mathematical validity does not replace physical reasoning. Mathematics keeps everything it has, its exactness and its necessity, and measurement of the world still has a say in which structures describe the world.

One World, Met Four Ways

Look back over the four kinds of other world. Everett's branches are the universal state, which exists as structure. For every comparer, one outcome is real: the outcome the comparer finds. The hologram is one system with two descriptions joined by an exact translation. A simulation with subjects in it is a room of the world in which the simulation runs. And in the Ultimate Multiverse, structures that exist are counted as worlds that are real.

So we now know that each of the four kinds of other world is the one world, met in one of four ways: as possibility, as one of two descriptions of one system, as a restricted region, or as structure that exists without being real.

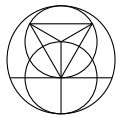
When the four kinds of other world are placed inside one world, everything the four kinds were built to explain is kept. Schrödinger's smooth rule remains. The translation of the hologram remains too. Physicists have used the translation as a tool of calculation and got a value close to one measured in a real fluid. The power of computers remains, and so does the exactness of mathematics. My account changes only where it places the smooth rule, the translation, the power of computers and the exactness of mathematics: inside one world that is found from within.

If there is one world, met from inside, how does that world, running toward disorder, build stars and the beings who ask these questions?

PART SIX

Mind in a Universe Running Down

Beside *Until the End of Time*, chapters 2 to 5.



Chapter 13

How Does a Universe Running Down Build Stars, and Us?

Beside Until the End of Time, *chapters 2, 3 and 4, and the first section of chapter 9.*

What this chapter uses

Entropy is a count of the arrangements of a thing's parts that some context cannot tell apart (Chapter 8).

I identify the one-way order of findings with the direction of time (Chapter 2).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

I identify the *first ripples*, the faint differences of density with which our epoch began, with the form of the content carried through the turn (Chapter 9).

In June 1824, in Paris, the scientific publisher Bachelier brought out a book of a hundred and eighteen pages by a young French military engineer named Sadi Carnot, under the title *Reflections on the Motive Power of Fire*. Brian Greene tells in *Until the End of Time* how Carnot's book launched the science of heat (chapter 2, section "Steam Engines").

Greene's chapters on heat leave a reader with a question: if everything runs toward disorder, how do stars, planets and living things form, and how do they last?

Carnot's own question was smaller and more practical. The steam engines of Carnot's day burned coal to boil water, the steam pushed a piston, and the spent steam was cooled and condensed. Most of the heat of the coal went to waste. Could a better engine waste less, and was there a limit to how little an engine could waste? Carnot found that an engine does work by letting heat pass from a hot body to a cold one, as a mill wheel does work by letting water fall from a height to a lower level. The falling water plays the part of the heat, and the height plays the part of the hot body. Carnot also found that the limit to how little an engine can waste is set by the temperatures of the hot body and of the cold body, and by nothing else. In the English translation, Carnot's sentence for the limit begins: "The motive power of heat is independent of the agents employed to realize it". The "motive power" of heat is the work that heat can do, and the "agents" are the working substances, such as steam, that an engine uses to turn heat into work. In July 1824 the engineer Pierre-Simon Girard read a warm review of Carnot's book to the Academy of Sciences in Paris. Even so, the first printing had not sold out by 1835. Carnot died in Paris of cholera in August 1832, aged thirty-six, but his book outlived him. Carnot's book already contains the idea of this chapter: an engine comes back to its starting state, cycle after cycle, only by passing heat to a cold side.

From Greene's pages

The Engine That Must Give Heat Away

Greene explains disorder with a bag of coins (chapter 2, section "Entropy: A First Pass"). Imagine shaking a bag of a hundred pennies and pouring them out on the table. There is one way for all hundred pennies to land heads, and there are a hundred ways for exactly one penny to land tails. Greene keeps counting: there are 4,950 ways for

exactly two pennies to land tails. For an even split of fifty heads and fifty tails, there are about a hundred billion billion billion ways. Each complete arrangement of the coins is one outcome of the shake. Greene gathers into one group all the arrangements that look the same at a glance. For the pennies, two arrangements look the same at a glance when they have the same number of heads. In Greene's sense, *entropy* is the size of the group that the pennies in front of you belong to: the number of arrangements that look, on the table, the same as the arrangement in front of you. An even split has high entropy because an even split can happen in so many ways. All heads has the lowest entropy there is, because all heads can happen in only one way.

The *second law of thermodynamics* comes from the counting of arrangements. Thermodynamics is the science of heat, energy and work. Greene states the law in these words: "The second law declares that over time there is an overwhelming tendency of entropy to increase". (*Until the End of Time*, chapter 2, section "Laws of Thermodynamics".) Entropy tends to increase because of the numbers. Random jostling carries a thing from groups with few arrangements toward groups with many arrangements, which hold almost all the arrangements there are. Greene then applies the counting to Carnot's engine (chapter 2, section "Heat and the Second Law of Thermodynamics"). When an engine runs in a cycle, the engine comes back to the same overall state at the start of every cycle, so the engine's entropy must also come back to its starting value. The engine took in entropy with the heat of the fire, so that entropy must leave the engine by the end of each cycle. Heat carries entropy, so the engine must give heat away to its surroundings. So Greene derives, from the counting, the old form of the second law: the waste heat that no engine escapes.

Next Greene names the process that the rest of his story depends on, the entropic two-step: "any process in which the entropy of a system decreases because it shifts a more than compensating increase in en-

tropy to the environment" (chapter 2, section "The Entropic Two-Step"). Order in the system is paid for by more disorder in its environment. The system's own entropy falls, and its environment's entropy rises by more. Greene says that you dance the two-step yourself. You burn food and air, you keep your own order, and you give heat away all day long. The heat you give away is why infrared goggles can find a person in the dark (chapter 2, section "You Are a Steam Engine"). Greene also speaks of the quality of energy (chapter 2, section "Energy and Entropy"). Concentrated energy, low in entropy, is high in quality and easy to use. The same energy spread out, high in entropy, is low in quality and hard to use.

In his third chapter Greene applies the two-step to the stars. Greene holds that "a proper exegesis of the second law renders an intelligent designer unnecessary" (chapter 3, opening). An exegesis is a careful reading. Picture, with Greene, a huge cloud of hydrogen in empty space. A small cloud spreads out evenly, as the smell of baking bread spreads through a house. In a cloud large enough for its own gravity to matter, the core is pulled inward and grows hotter as it shrinks. Meanwhile the outer shell swells and cools. The core gives heat to the shell, yet the core grows hotter still. The core's entropy falls, the shell's entropy rises by more, and the total rises. Greene calls the cloud's process "another instance of the entropic two-step" (chapter 3, section "Gravity, Order, and the Second Law"). In his fourth chapter he names the cloud's process the gravitational entropic two-step (chapter 4, section "The Origin of the Solar System"). Greene stresses that the gravitational two-step is self-sustaining. When a steam engine does work and gives off heat, the engine grows colder, and without more fuel the engine stops. When a contracting cloud gives off heat, the cloud grows hotter, so the cloud goes on shrinking and giving off heat. Greene sums up the gravitational two-step this way: "when there's enough matter for gravity's influence to be significant, the second law undertakes a rapid U-turn, and drives the system away from homogeneity" (chapter 3, section "Fusion, Order, and the

Second Law"). Homogeneity is sameness throughout, the even spread of the small cloud. At last the core grows hot enough for *fusion*. Fusion is the joining of light nuclei (the tiny cores of atoms) into heavier nuclei, and fusion gives out energy. The pressure of the energy from fusion halts the core's fall, and a star shines. Starlight carries entropy away, and for billions of years fusion supplies the heat that the star sends away as starlight. In Greene's picture the nuclear force does the leading work for those billions of years, and gravity works beside it: he puts the nuclear force "on the front line", with gravity as its "indispensable partner" (chapter 3, section "Fusion, Order, and the Second Law").

Greene adds one more part to his story, with a picture of an unlit gas oven. Greene writes that natural gas "can peacefully coexist with the oxygen in air" (chapter 3, section "Hurdles on the Path Toward Disorder"). The gas and the oxygen do not burn until a burning match sets them alight. Greene calls whatever keeps the gas and the oxygen from burning a hurdle on the path toward disorder. Greene calls the starter that carries a change over a hurdle, such as the burning match, a catalyst. In a star, Greene says, the catalyst that sparks fusion is gravity, which crushes the core until the core is hot and dense enough (chapter 3, section "Fusion, Order, and the Second Law").

In his fourth chapter Greene applies the two-step to living things. A plant takes in sunlight and uses the sunlight, and an animal eats the plant and gives away heat. A *photon* is a single grain of light. Greene notes that for each photon the Earth receives from the Sun, the Earth sends back to space about two dozen weaker photons of infrared light, light too red to see. Greene holds that a cell's information guides its molecules without overriding any law, as the shape of a water slide guides its riders. In the picture, the shape of the slide plays the part of the information, and the riders play the part of the molecules: the shape steers the riders, and the riders still obey every law. Greene sums up his view in four words: "Life is physics orchestrated"

(chapter 4, section "The Physics of Information"). In the first section of his ninth chapter, Greene takes up entropy and evolution together. Greene writes that entropy and evolution can work together, because by the two-step, structure can grow in one place while entropy is sent to another. Greene asks whether future discoveries will revise the laws now taken as fundamental, and he answers "Almost certainly". He also asks whether entropy and the second law will keep their place in our understanding all the same, and he answers that entropy and the second law almost certainly will (chapter 9, section "Evolution, Entropy, and the Future").

Counting the Ways, on Separate Ground

I also have each of Greene's results on the counting, the engine, the two-step, the hurdle and the stars, on separate ground: from premises of my own, which are not Greene's. The counting in my account begins with a hundred coins, as Greene's counting does. The book's chapter on entropy begins with a hundred coins shaken in a cup and poured on a table. A hundred coins can fall in about one and a quarter million million million million million million ways, but a person who glances at the coins sees only one number, the count of heads. The ways with between forty and sixty heads make up ninety-six and a half percent of all the ways the coins can fall. A *microstate* is one complete arrangement, every coin or every molecule given. A *macrostate* is what is found about the whole, such as the count of heads, or the temperature and the pressure of a gas.

Greene counts as if every arrangement were as likely as every other, as every arrangement is in a fair shake. On my account, the equal counting is derived. The derivation uses two things: the principle of identity, and the weight that the gas's own motion keeps. The *principle of identity* states that what no comparison can tell apart, in any way at all, even in principle, is one thing and not two. Stated for differences, the principle of identity is the principle of comparison of Chapter 1: a difference that no comparison can register is no difference. The *weight* of a microstate is the likelihood of

finding the gas in that microstate. The motion of a gas carries each microstate into another microstate. The motion carries each microstate's weight along to the next, unchanged. Picture water poured from cup to cup, each pouring moving the whole of one cup into the next and spilling nothing. If one cup held more than another, the pouring would carry the excess along, and the amounts in the cups would not stay as they were. Each cup stands for a microstate, the water in it for that microstate's weight, and each pouring for the motion carrying one microstate into the next. So if the motion is to leave every weight as it was, the motion must carry each microstate only into microstates of the same weight. A measurement of the gas registers only the gas's totals, such as its energy, and every microstate with the same totals gives the same measurement. So if the gas were found in some microstates more often than in others with the same totals, no measurement could register the difference. The principle of identity says that a difference no measurement registers is no difference at all, so every microstate with the given totals counts alike.

From the equal counting onward, the counting in my account is Greene's counting, and its results are Greene's results. Entropy is a count, made small enough to write by taking its logarithm. The *logarithm* of a number is a measure of its size that adds where the numbers multiply, as the number of digits in a written number does. Join a lock of three dials to a lock of two: the settings multiply, a thousand times a hundred, and the digits add, three and two to five. The number of dials is the logarithm of the number of settings. My books call the entropy itself a count, meaning the count of arrangements written as its logarithm. From here on, this chapter uses the word count in the same sense. In anything made of many parts, almost all the microstates belong to macrostates close to the macrostate of greatest entropy. The band of macrostates that holds almost all the microstates grows narrower as the number of parts grows. For a glass of water, the band is far too narrow to draw. A thing of many parts, left to itself, is therefore found at its greatest entropy, because almost all its microstates lie in that narrow band. The next section explains why a pond freezes, and the explanation depends on the same fact.

Carnot's limit also follows from the counting of arrangements. Work is the energy that moves a piston as a whole in one direction. Work adds no arrangements to anything, so work carries no entropy. Heat carries entropy. Heat taken from a body carries away an amount of entropy equal to the heat divided by the body's temperature. The body's temperature is measured upward from absolute zero, the coldest there can be, in the units called kelvins. An engine that runs in a cycle must give heat to a cold side, because the engine keeps no entropy of its own from one cycle to the next, and work cannot carry entropy away. The engine must give the cold side at least as much entropy as the engine took from the hot side. Since the entropy that heat carries is the heat divided by the temperature, the heat given to the cold side is at least the heat taken in, times the ratio of the cold temperature to the hot temperature. Work can come only from the heat that the engine does not give to the cold side, so the engine can turn into work at most a share of its heat equal to one minus the ratio of the cold temperature to the hot temperature. So when an engine takes heat at six hundred kelvins and gives heat up at three hundred, the engine turns at most half of its heat into work. Carnot found in 1824 that the limit belongs to the temperatures alone, not to steam or to any other working substance, and the reason is that the limit comes from counting entropy.

Why a Pond Freezes

Put a tray of water outside on a cold night, or in the freezer, and look at the tray in the morning. The water has turned to ice. A pond freezes in the same way on a winter night, and thaws again on a spring afternoon. In ice the molecules of water sit in a regular pattern, each molecule bound to its neighbors. In liquid water the molecules tumble past one another. The same molecules take order when their surroundings are cold, and disorder when their surroundings are warm. What decides whether the molecules take order or disorder?

The pond is in contact with the air and the ground. The air and the ground are much larger than the pond, and they stay at one temperature. Taken together, the pond and its surroundings are found where their joint entropy is greatest, because almost all their joint arrangements are found there. When water freezes, the water's own entropy falls, because ice can be arranged in fewer ways than liquid water. Water that freezes also gives heat to its surroundings. The heat raises the entropy of the surroundings by the heat divided by the temperature of the surroundings. On a cold night the temperature of the surroundings is low, so the heat from the freezing water raises the entropy of the surroundings by a great deal, more than the water loses, and so the joint entropy is greatest with the pond frozen. On a warm afternoon the same heat would raise the entropy of the surroundings by less than the water would lose, so the joint entropy is greatest with the pond liquid. As a general rule, a thing held at a steady temperature settles where its energy, less its temperature times its entropy, is least. The *free energy* of a thing is its energy, less its temperature times its entropy. At a given temperature, the free energy is the share of the thing's energy that can be drawn off as work. Greene's quality of energy is the same share, seen in Greene's terms. When the same energy is held at the same temperature with less entropy, more of the energy is free to be drawn off as work, while with more entropy, less of the energy is free. Water freezes when cold and melts when warm because ice has less energy than water and less entropy.

So, on my account, Greene's two-step is the rule for whether an ordered arrangement forms when the arrangement is in contact with its surroundings. Order in the arrangement is paid for by more entropy passed to its surroundings. The arrangement forms where the joint count of the arrangement and its surroundings is greatest.

I also describe a case in which order forms by entropy alone. In 1957 two pairs of physicists used early computers to follow the motion of hard balls with no pull between them, packed at different densities. One pair was Alder and Wainwright, and the other was Wood and Jacobson. The two pairs found that when the balls were squeezed closely enough, the

balls lined up into a *crystal*, a regular pattern repeated in every direction. Balls with no pull between them have the same energy however they are arranged, so their free energy is set by entropy alone. The crystal therefore formed because, when the balls were squeezed closely enough, the crystal had the greater entropy. So we now know why the crystal has the greater entropy: in a crowded jumble each ball is hemmed in on every side, while in a regular packing at the same density each ball has more room to rattle in its own place. Above a certain density, then, the crystal can be arranged in more ways than the jumble. Balls stay jumbled when they fill less than about forty-nine percent of the space, and they crystallize when they fill more than about fifty-four and a half percent, while between those two crowdings jumble and crystal are found side by side.

Matches in a Drawer

A box of matches can lie in a drawer for years, with the wood and the head of each match beside the oxygen of the air, and nothing happens. Strike one match, and in a second the match burns until the wood is gone. The burning gives out heat, so the wood and the oxygen together hold more energy than the substances they turn into. Yet at the temperature of a drawer, the wood and the oxygen do not burn. Picture a match in a drawer, in place of Greene's gas oven: the match is stable in the drawer because the barrier between wood and oxygen and their burning is high. A *barrier* is an arrangement of higher energy that a change must pass through on its way: the atoms must be pulled partway out of their old bonds before they can settle into new ones. At any moment, only some of the molecules hold enough energy in the few atoms that must move. The share of molecules with enough energy is found by counting the ways the molecules can hold their energy. The share falls steeply as the barrier rises, and climbs steeply as the temperature rises.

Picture two valleys divided by a high mountain range, with the only road between them climbing over a high pass. Build a road over a lower pass through the same range, and travelers cross both ways far more eas-

ily, though the valleys stay where they were. The valleys stand for the substances before and after a change, the high pass for the barrier, and the travelers for the molecules. A *catalyst* is the lower pass: a substance that opens a new route for a change, with a lower barrier, and is left unchanged at the end. A catalyst brings the change to the same balance that the change would have reached without the catalyst, only sooner. A lower pass makes no water run uphill: the lower pass changes how easily travelers cross, and leaves the valleys as they were.

Living cells are full of catalysts. An *enzyme* is a large molecule of a cell, shaped to hold reacting molecules in the arrangement they must pass through. In 1995 the chemists Radzicka and Wolfenden measured how slowly one reaction of the cell goes in water with no enzyme, and found that half of a sample would take seventy-eight million years to react. The enzyme for that reaction is named orotidine monophosphate decarboxylase. With the enzyme, half of a sample reacts in eighteen thousandths of a second, as Miller, Wolfenden and their colleagues reported in 2000. So we now know, from the two measurements, that the enzyme speeds the reaction by about a hundred million billion times. The enzyme speeds the reaction by giving the reaction a lower pass. Like every catalyst, the enzyme leaves the balance of the reaction where the balance was. Greene's hurdle and my barrier are the same thing. Greene uses the word catalyst for the burning match that starts the change. I use the word catalyst for a lower pass, which is left unchanged at the end. Beyond Greene's picture, my account has a reckoning of how often the molecules get over the hurdle. The reckoning is made by counting the ways the molecules can hold their energy.

A Cloud That Shines

Picture a cloud of gas collapsing under its own weight. The cloud heats up as it falls inward, and the cloud shines. For a cloud too small for its own gravity to matter, spreading out raises the count, as Greene's small cloud shows. Greene's gravitational two-step follows a rule: for matter

held together by its own gravity, the order is reversed, and clumping raises the count. The cloud's own count falls as the cloud gathers, and the light the cloud sends away carries off more count than the cloud loses. So as the faint differences of density in the early universe grow into galaxies, stars and clusters, the total count rises: the count of the matter and of the light it sends away, taken together.

An *epoch* is one cycle of the universe, and our epoch is the one in which we live. The content is the matter and light of the world. The turn comes at the end of the black-hole interiors of the epoch before ours (the insides of regions whose gravity lets nothing come back out), and in the turn a large world is restored. The form of the content is the pattern of which way every finding of the old epoch came out. Every outcome is kept in the pattern as every footstep is kept in the worn curve of an old church step, with no single footstep to be picked out and none wiped away. Greene and I agree on what gravity does with the differences of density once the differences are there: gravity makes the differences grow.

The largest rise in the count from clumping comes from black holes. A *black hole* is a region whose gravity is so strong that nothing that enters it, light included, comes back out. The edge of a black hole is called its horizon. Physicists, following Jacob Bekenstein and Stephen Hawking, give a black hole a count that grows with the area of its horizon. So the largest black holes hold most of the count there is. At the center of our galaxy is a black hole of about four million times the mass of the Sun. The count of that black hole is about one followed by ninety zeros, which is already more count than all the light and all the neutrinos (the lightest particles of matter) of the early universe hold in the whole region we can see. In the whole region we can see, all the black holes together hold a count of about one followed by a hundred and four zeros. So gravity, which gathers the stars, also gathers matter into black holes whose counts are far larger than the count of all the light and matter of the early universe.

The stars begin from the matter that the first minutes after the Big Bang left. So on the counting, the engine, the two-step, the barrier and the stars, my results and Greene's results are the same, on separate ground, but Greene and I use the word catalyst for two different things.

Every Count Is Made from Somewhere

Hang a bucket of water from a twisted rope and let the bucket go, as Newton did. I have used the picture of Newton's bucket since the first chapter of this book. Chapter 1 showed what the water is spinning against: what counts as still and what counts as turning is fixed by the matter there is to compare against. So the hollow is the finding of a comparison made from inside the world. Now stop the bucket, let the water settle, and ask what the entropy of the water is. The entropy of the water is the count of the arrangements of the water's molecules that a thermometer, a ruler held against the walls and a balance cannot tell apart. The count of the water's arrangements is made against the thermometer, the ruler and the balance, as the hollow was found against the matter within reach. So the count of the water's arrangements is a finding of the same kind as the water's turning. The count and the turning are both comparisons made from inside, against what is there to compare with.

My account has one thing that Greene's counting does not state. Every entropy is a count made relative to a *context*. A context is the respect in which a comparison finds what it finds, what makes the two things compared two, as the two times of day make a morning reading and an evening reading of a thermometer two readings. For a count of arrangements, the context is the measurements available and the walls that hold the thing counted. Greene's own pennies show a context at work, because Greene groups together the arrangements that look the same to someone glancing at the table, so for the pennies the glance is the context. For a gas, the box around the gas is the context relative to which the states of the gas are counted. Greene speaks of the universe as a whole reaching its "entropic potential", the greatest entropy it could reach (chapter 3, opening).

On my account, the question is: relative to what would the count of the whole universe be made? A count for the whole universe needs a context for the whole, and the universe has none outside itself. Inside the world, every count that Greene makes remains valid. In 1865 the German physicist Rudolf Clausius gave entropy its name. Clausius wrote that the entropy of the universe tends toward a maximum. Within our epoch, the count grows, as Clausius said it would. Each of the growing counts within our epoch is made against a context inside our epoch, such as the box around a gas or the world outside a black hole. The stars, the galaxies and the black holes raise the counts made against those contexts, as Greene describes.

As his ninth chapter says, Greene expects that entropy and the second law will keep their place. I agree: the second law and every result of the counting remain in my account. Beneath the second law, my account has the one-way order of findings, in which each act of comparison comes before the record that the act lays down, and which I identified in Chapter 2 with the direction of time. Chapter 8 called the one-way order of findings the *structural arrow*. In things of many parts, the growth of entropy is the *entropic arrow*. So in the world of engines, ponds and stars, the second law has the same place in my account as in Greene's. On my account, the one-way order of findings is also present in every single comparison, with no large number of parts needed, and with or without an orderly beginning.

A Pattern Chosen in the Event, and a Living Thing

Heat a thin layer of oil gently from below, and at a certain rate of heating the oil, which was still, breaks into rolling cells, often six-sided, side by side across the pan. Greene tells of the rolling cells of oil in his fourth chapter (section "A General Theory of Life?"). The physical chemist Ilya Prigogine received the Nobel Prize in Chemistry in 1977. Prigogine spent half a century on order of the kind the oil shows. A *dissipative structure* is an ordered pattern that exists only because energy and matter keep flowing through it, far from *equilibrium*. The cells of oil are one dissipative

structure, and a living thing fed by sunlight is another. Each dissipative structure passes more entropy to its surroundings than it removes from itself, as Greene's two-step says.

My account has one step more than Greene's two-step. A system far from equilibrium can reach a branching point, a point at which the system can take any one of several patterns. At a branching point the outcome depends on small differences in the system, and when those differences are traced down to smaller and smaller scales, they become too small for any measurement to register. At that scale no instrument that could register the difference exists at all, however good instruments become. Yet the system still produces an outcome. So at the scale where no instrument can register the difference, what settles the outcome is hard random. As this book uses the term, *hard randomness* is an outcome that nothing before it fixes, however completely what came before is known. In Chapter 2, the single click of the counter is a case of hard randomness. For the pattern that the system takes, which branch is taken is settled in the event itself, by no law laid out in advance, and the branch, once taken, is lawful in all that follows.

Experiments already made with growing crystals show a system taking its pattern at a branching point. In 1990 the researchers Kondepudi, Kaufman and Singh grew crystals of sodium chlorate. Sodium chlorate is a salt whose crystals come in two mirror-image forms, left-handed and right-handed, as a pair of gloves does. From a still solution the two forms appeared in roughly equal numbers. From a stirred solution, one sample came out with about ninety-nine and seven tenths percent of its crystals of one hand, but which hand made up most of the crystals varied from one stirred sample to the next. In 2005 Cristobal Viedma stirred a solution of sodium chlorate with glass beads that kept crushing the young crystals, and after some hours all of the crystals were of one hand. The report of Viedma's work in *Physics Today* says: "The dominant form was random". So we now know that from a stirred solution the crystals come out all or

nearly all of one hand, and that which hand varies from one sample to the next. I describe the same pattern at a branching point. One branch is taken in each run, and which branch is taken varies from run to run.

What of a living thing, which Greene calls physics orchestrated? I agree with Greene that a living thing is made of the same matter as everything else, and that every event in it follows the laws of physics. Chemistry is made of nothing but electrons, nuclei, and light; that is not in question. The step that fails is the step from the physics of electrons, nuclei and light to chemistry: what fails is the derivation. Exact derivation is possible only for the simplest systems, such as the hydrogen atom. No description, however refined, contains everything it describes. *Reductionism* is the claim that the description of the parts exhausts the whole. So reductionism fails at its first and simplest step, from physics to chemistry. Above chemistry, every reduction is harder to carry out and harder to check, from biology to chemistry and on upward.

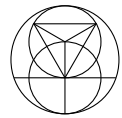
Think of notes scattered across a page, and the same notes made into a sonata. The notes are of one kind throughout, and the sonata is a new kind of thing, because of how the notes are held together. A stone and a living being differ in kind in the way that scattered notes and a sonata differ: the difference lies in how something present in both is held together. Choice is what is present in both a stone and a living being. The stirred crystals give a picture, seen from outside, of an outcome that nothing before it fixed: one hand taken, and which hand settled in the event itself. In this book's sense, *choice* is an outcome of that same kind, an outcome that nothing before it fixed, described from the position of the process from which the outcome came. The atoms of a stone, too, have outcomes that nothing before them fixed. When one of those outcomes is described from the side of the stone, the process from which the outcome came, the outcome is choice in the smallest degree. So every part of the world, a stone included, has some degree of choice, however small. A stone and a person differ in how much choice is present in them, and the amount of choice changes by degrees from the stone to the person, without a break. Chapter 14 uses Newton's bucket to set out the side from which an outcome that

nothing before it fixed is had. Chapter 15 says in full what choice is. I name what is new in kind in a living thing: "the gathering of choice into a self that knows it from within". My account still contains all of Greene's physics of the living cell: the cell's two-step, its sunlight and its guided molecules. And in the living self, my account has something that a description of the cell from outside leaves out, however complete the description is: choice gathered into one self and known from within.

What a Universe Running Down Builds

We began with Carnot's engine, which comes back to its starting state, cycle after cycle, only by giving heat to a cold side. The pond, the cloud and the cell pass entropy to their surroundings too. The pond gives heat to its surroundings as it freezes each winter night, the cloud gives off light as it falls together and shines, and the cell gives away heat while it lives on sunlight. All four obey the second law: the engine, the pond, the cloud and the cell. Each of the four comes from nature as it is, with no designer and no new force. The ice, the star and the cell each form because the total count of the thing and its surroundings together rises. Where matter is gathered by its own weight, clumping raises the total count. By the same counting, the world that Greene shows running down is also a world that builds ordered things. And every count in the building of ordered things is a comparison made from inside the world, as the hollow in the water of Newton's bucket is. So we now know how a universe running down builds stars, and us: a thing gains order by passing more disorder to its surroundings, and settles where its entropy and theirs together are greatest. Under gravity, clumping raises the joint count of the thing and its surroundings, and every count is made relative to a context.

Brains are among the ordered things that a universe running down builds. When a brain is aware, is what the brain does only more of the same building of order?



Chapter 14

Is Consciousness Only Particles?

Beside Until the End of Time*, chapters 1 and 5, and* The Hidden Reality*, chapter 2.*

What this chapter uses

Every comparison has three roles: the *stance*, the side a comparison is made from, with the range of what could come; the *act*, the happening itself; and the *structure*, what is laid down in the act, the same for everyone who reads it afterward (Chapter 3).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

Hard randomness is an outcome that nothing before it fixes, however completely what came before is known (Chapter 2).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

In April 1994 a young Australian philosopher named David Chalmers took the stage at the first of the conferences on the science of consciousness held in Tucson, Arizona, and gave a name to a difficulty. The physicist Brian Greene expects that consciousness will one day be explained by particles and their laws, with nothing added. Is awareness, then, only particles in motion?

Greene tells the scene at Tucson in his own book, and in Greene's telling Chalmers stood on that stage with "hair flowing past his shoulders" (*Until the End of Time*, chapter 5, section "The Hard Problem"). Chalmers set two kinds of question side by side. A question of the first kind is about how the brain does what it does: how the brain takes in light, how the brain lays down a memory, or how the brain steers a hand. Chalmers called the questions of the first kind the easy problems. Chalmers meant only that we can see what an answer to an easy problem would look like: an explanation in terms of nerve cells and their signals, given step by step. A question of the second kind is about why any of what the brain does is felt: why there is something it is like, from inside, to see red or to hear a voice. Chalmers called the second kind of question the hard problem. He argued that an explanation built only of particles and their laws leaves the hard problem unanswered. In 1995 Chalmers published his argument for the hard problem in the *Journal of Consciousness Studies*. The name "the hard problem" stayed in use.

Hold the hard problem for a moment beside the bucket of water with which this book began. Since Chapter 1, this book has used the word *comparison* for one act: the act of finding whether two things are the same or different in some respect. The water has been compared with the matter there is to compare the water against, and the climb is the outcome of that comparison. Now picture yourself watching the bucket. You compare too: you set the hollow against the flat surface of a moment ago, and the turning water against the still room. One thing is true of your comparing that a person watching you from across the room cannot see: you have the finding from inside. Your seeing of the climb is a comparison made from

inside the world, and so is the water's climb. If both are comparisons, what is the difference between the water's climb and your seeing of the climb?

From Greene's pages

A Bag of Particles That Tells Stories

Greene comes to the hard problem as a physicist with a commitment, and he says what his commitment is. He recalls telling a late-night television host that the host was "nothing but a bag of particles governed by the laws of physics" (*Until the End of Time*, chapter 5, section "Consciousness and Storytelling"). In *The Hidden Reality* Greene declares his commitment to the particles and their laws as his bias: "I believe that a physical system is completely determined by the arrangement of its particles" (*The Hidden Reality*, chapter 2, section "Nothing but Physics"). Greene calls his belief reductionist. The lower level is the level of the particles, and the higher level is the level of a brain or a person. To exhaust a thing is to leave nothing of the thing out.

Greene also holds that the particles' story is one story among many. For Greene, reality is told in nested stories: the story of particles, then of life, then of mind, then of culture. Each level gives insight that the other levels cannot give, and each level's story must agree with the story of the particles. Greene says that the story of consciousness differs in one respect from the stories of entropy, of evolution and of life. (What a comparison finds is its *content*. Chapter 8 glossed *entropy* as a count of the arrangements of a thing's parts that some context cannot tell apart.) The stories of entropy, evolution and life can be told from outside, in the third person. The story of consciousness, in Greene's words, "relies on a first-person account" (*Until the End of Time*, chapter 5, section "Consciousness and Storytelling"). Greene's first-person account is a *first-person* description: one had from inside, by the one to whom the thing happens.

Greene sets out his evidence for expecting that particles and their laws will one day explain consciousness. Researchers now follow the blood flow that feeds working nerve cells (by functional magnetic resonance imaging), record single nerve cells firing (by probes set deep in the brain), and record the electrical waves that cross the brain (by electrodes on the scalp). Greene writes that the case for approaching consciousness as a physical phenomenon "strengthens substantially" when the patterns the researchers find match both what people do and what people report of their inner experience (*Until the End of Time*, chapter 5, opening). Greene shows that much of what the brain does happens with no awareness at all. Two of his cases are the work behind your eyes as you watch a sunset, and a number flashed too briefly to be seen that still speeds or slows your answer. Greene writes of the brain's work that goes on with no awareness: "Wondrous though these brain activities are, they do not constitute a conceptual mystery" (*Until the End of Time*, chapter 5, section "In the Shadows").

The hard problem is the one place where Greene finds a conceptual mystery. Greene asks you to imagine seeing every particle of his brain, about a thousand trillion trillion particles, each pushed and pulled by the same forces, written in the same mathematics, as a particle in a kitchen or in a star. Within the mathematical description of the brain's particles, Greene writes, "there is nothing that even hints at the inner experiences those particles somehow generate". Greene asks the question in his own words: "How then does a whirl of particles inside a head—which is all that a brain is—create impressions, sensations, and feelings?" (*Until the End of Time*, chapter 5, section "The Hard Problem").

Greene's own answer is the answer of *physicalism*. Physicalism is the view that the physical, objective world is all there is at bottom, so that methods using only the physical properties of matter will explain consciousness in the end. Greene writes, "The physicalist perspective does indeed summarize my own long-held view" (*Until the End of*

Time, chapter 5, section "Something About Mary"). Greene tells of one argument that gave him, in his words, "significant moments of doubt". The argument is a story about a woman named Mary, published in 1982 by the philosopher Frank Jackson. In Greene's retelling, Mary has seen only black and white since birth. Mary studies until she knows every fact of the physics, chemistry and biology of seeing: every firing and every cascade of particles that happens in a brain looking at a blue sky. Then a surgeon gives Mary color vision, and she opens her eyes in front of a bouquet of red roses. Does Mary learn something new? It seems plain that Mary does learn something new, and Greene gives Jackson's conclusion: "Complete knowledge of the brain's physical workings leaves something out". Greene also reports the replies to Jackson. The philosopher Daniel Dennett replied that knowledge so complete would let Mary know red in advance. The philosophers David Lewis and Laurence Nemirow replied that Mary gains a new ability and no new fact. And Jackson himself later changed his mind. Greene notes that no one has yet proposed an experiment, an observation or a calculation that can "definitively settle the question raised by Mary's story".

Greene then tells two tales from the history of science. In the first tale, life once seemed to need a spark beyond physics. The spark turned out to be unneeded: particles arranged in cells are enough. In the second tale, electricity and magnetism once seemed to need nothing beyond Newton's mechanics. Electricity and magnetism turned out to need a new quality of matter, electric charge, and a new kind of influence, the electric and magnetic field that fills space (a *field* is something that has a value at every place). Chalmers takes the second tale as his model, and in Greene's telling Chalmers entertains the possibility that particles carry a trace of consciousness of their own (*Until the End of Time*, chapter 5, section "Theories of Everything"). Greene also reviews the proposals of two scientists who study the brain. The neuroscientist Giulio Tononi characterizes conscious awareness as informa-

tion that is highly integrated and highly differentiated. Tononi proposes a formula for the amount of highly integrated, highly differentiated information, and he calls the quantity integrated information. The neuroscientist Michael Graziano proposes that the brain builds a simplified model of its own attention. Graziano's model leaves out the brain's mechanics, so that awareness seems to float free of the body. Greene calls Graziano's theory "a physicalist account through and through", one that "provides one of the most illuminating approaches to the mystery of consciousness" (*Until the End of Time*, chapter 5, section "The Mind Integrates Information"). Greene also answers people who fear that explaining consciousness in physical terms would make us less: "Consciousness would be demystified without being diminished" (*Until the End of Time*, chapter 5, section "The Mind Models the Mind").

Two more steps complete Greene's argument. The first step concerns quantum physics, the theory of atoms and of the particles smaller than atoms. *Quantum measurement* is the finding of one outcome among the possibilities that the state describes. Greene judges it "extraordinarily unlikely" that consciousness plays a special part in quantum measurement. Greene's ground is what experiments show: "Interactions—not consciousness—coax the emergence of a definite reality". Greene grants that there is "no foolproof argument that consciousness does not play a special quantum role" (*Until the End of Time*, chapter 5, section "Consciousness and Quantum Physics"). The second step follows from the first step, Greene's judgment on consciousness in quantum measurement: Greene expects, "barring a stunning surprise", that consciousness "will be understood as another physical quality that arises in a quantum universe" (same section). Greene treats thought as a physical process for the whole of his book. In his first chapter Greene says that treating thought as a physical process is "an assumption we will examine" (*Until the End of Time*, chapter 1, section "Information, Consciousness, and Eternity").

In the sense this book uses, *reductionism* is the claim that the description at the lower level exhausts the thing at the higher level.

Where Greene and I Agree

On my account, Greene's working assumption is a result. Thought is a physical process. A thinker of the far future, of whatever material and at whatever temperature, thinks only as a physical process making comparisons. There is no second stuff beside matter: no spark, and no soul floating outside nature.

I also have the result Greene gives for quantum measurement: interactions, and not consciousness, bring a definite outcome. My account has its own argument for the result about quantum measurement. Chapter 4 showed that a finding needs no mind: a screen or a counter that lays down a record has made a finding, whether or not anyone reads the record later. My argument depends on what a comparison is. The argument needs one part of my premise: comparison and interaction are one act. So every interaction that leaves a record is a comparison, and the record the interaction leaves is a finding. I use the word "observer" only for a comparer. An observer is a comparer: the word names a position inside the world, and it never names a mind. So Greene's sentence about quantum measurement holds in my account, because any interaction that leaves a record has made a finding.

I agree with Greene that the easy problems are no conceptual mystery. A machine can do the brain's tasks, and a model of the brain's behavior is useful. Computation can calculate, predict, translate and converse, and do all of it superbly; what it cannot be, by being a computation, is a first-person point of view. For Greene and for me alike, the whole difficulty is to explain first-person experience from a third-person description.

I also agree with Chalmers, as Greene reports him, that a gap separates the third-person description of a brain from the felt experience. Greene calls the gap "the explanatory gap" (*Until the End of Time*, chapter 5, section "The Hard Problem"). (Chapter 3 stated the Incommensuration

Theorem, a theorem about every act of comparison.) Our theories of physics are all knowledge described in the third person, and every experience is first person. Mary's story has the same shape as in Jackson's telling: her complete description of seeing does not give her the felt red. My account has its own argument for the shape of Mary's story. Every interaction across the boundary of a self runs in one of two directions: in perception, something passes from the world into the self, and in expression, something passes from the self out into the world. The felt red of the roses is had in perception, the flow from world to self. Mary's complete description of seeing was made by expression, the flow from the selves of the description's writers out into the world. The description was written into books, and laid out as structure that anyone can share. No amount of the one is any amount of the other. So when Mary opens her eyes, she has the felt red of the roses, and the felt red reaches her only in her own perceiving, while her books, however complete, reached her as their writers' expression.

Greene's nested stories also have a counterpart in my account. I describe the world at three scales: the scale of particles and fields, the scale of comparers (organisms, instruments and their readings, the scale at which experiments are done), and the scale of the whole. Each of the *three scales of description* has its own kind of description, and each kind of description belongs to its own scale. I compare the three scales with the levels of the cosmologist George Ellis, and say that both "refuse to let the bottom level speak for the whole". Neither the three scales nor Ellis's levels treat the description at the lowest level as the description of the whole.

A Gap Built into Every Comparison

The gap between the particles' story and the felt red comes from what every comparison is. Each comparison is made from one side, and a description is laid out as structure, the same for everyone who reads the description. The side is missing from the description, because the side be-

longs to one comparer and the description is the same for everyone. So the gap stays the same however much is learned about the parts of a brain. Picture a map of a walk you have taken: every turn of the path drawn, every slope shaded, every stone marked. The map can be made as detailed as anyone likes, and anyone can read it. You had the walk, step after step, from the side where your feet were. A map, however fine, is a map of the walk, and only the walker has the walking.

A third-person description is like the map. A third-person description is laid out as structure, held in view as a whole, the same for everyone who consults the description. The first person is like the walking. The first person is the side from which an act of comparison is made, with the range of what could come from that side. The hard problem is the question "how does first-person experience arise from a third-person description?" An answer would need the laid-out structure to contain the side from which every laying out is done. The hard problem has stayed unsolved since it was posed because the toolset used on it presupposes its conclusion, and so cannot solve it. The toolset used on the hard problem is description in the third person, and a third-person description already leaves the first person out.

The Incommensuration Theorem, met in Chapter 3, is the basis of a second argument for the same conclusion: the gap is built into every comparison. The Incommensuration Theorem is a theorem about what any act of comparison can find. A *symmetry* is the same content across a different context: the same thing found whatever the place, the time or the one who looks. A *continuity* is the same content where the context is the same, with no break: the same thing, running on. The opposite of a symmetry is an *asymmetry*, a different content across a different context. The opposite of a continuity is a *discontinuity*, a break: a different content where the context is the same. Compute a number from a description of a brain, and the number is symmetric: exact, repeatable, the same for everyone who computes the number. The side from which you see is continuous and asymmetric: your seeing runs on without a break, and what you see never exactly repeats. Now suppose that a third-person description held the first

person completely, with nothing left over. The description would keep its own symmetry without exception, the same for everyone who reads the description, and the description would also carry the continuity of your seeing without a single break. So the description would be perfectly symmetric and perfectly continuous at once, and by the Incommensuration Theorem nothing can be both. So no third-person description holds the first person completely. The explanatory gap between physical description and conscious experience is therefore a structural feature of comparison itself.

Both arguments show that the explanatory gap stays as it is, however much is learned of the brain. Greene's first tale, the tale of life, is then a tale about a different kind of question. As Greene tells the tale, the hard question about life was a question about parts and what the parts do: a third-person question. More knowledge of the parts was enough to answer the question about life. The question about consciousness requires the third person to contain the first person, and more knowledge of the parts is more of the third person. There can be no physical, mechanical, purely logical or causal, theory of consciousness; only a metaphysical theory of consciousness is possible. Perception cannot perceive perception, and observation cannot observe observation.

What Awareness Is

If no third-person description contains awareness, what is awareness? Listen to a piece of music you have never heard before. Each note follows the note before it, and the first hearing of a note cannot be had again: there is a before and an after. Some notes surprise you, because nothing you heard before fixed them. And the hearing is yours: the music happens to someone.

The hearing has three features: a one-way order, from before to after; notes that nothing before them fixed; and someone to whom the music happens. Every act of comparison has the three features of the hearing, and the three features come together. Every act of comparison runs one

way, from the comparing to the record the act leaves. The outcome of an act of comparison is fixed by nothing before the act. And every act of comparison is made from a *stance*. Chapter 3 defined a stance as the side a comparison is made from, with the range of what could come. The stance is also the side to whom the outcome happens. Take away any one of the three features and the other two go. The three features go together because each needs the next, as the music shows. If every note were fixed in advance, the notes would stand like the entries of a list, which read the same forward or back, and the hearing would have no direction of its own. A note that nothing has fixed in advance is singled out as new only in a hearing made from a side, by someone to whom the note arrives. And the hearing happens from a side that sits in an order that runs one way, an order in which what has been heard comes before what is still to come. When you hear the music, the three features come to you as one hearing, and not one after another. I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act, happening: the one-way order of the act, its outcome that nothing before it fixed, and its stance. A thing whose comparing has the three features together has an *inside*: a side of its own, from which the comparing is done and to which the outcome happens, as the music happens to you.

Greene sets the views of consciousness along a range. At one end is *eliminativism*, which dismisses consciousness as an illusion. *Idealism* is at the other end. In Greene's words, idealism is the view "that consciousness is the only quality of the world that is real". Between the two ends lies "a spectrum of proposals" (*Until the End of Time*, chapter 5, section "A Tale of Two Tales"), and Greene's own physicalism is one of the proposals. I set my account beside two views. Each view takes one side of the old divide between subject and object. I describe physicalism as the view that there is the physical, the objective, and that awareness is a by-product of the physical. Idealism is the view that mind comes before body. The Immanent Metaphysics is neither physicalism nor idealism. My ground is the relationship between the *subjective* and the *objective*. Every comparison contains three pairs of ideas, which Chapter 7 set out: sameness and differ-

ence, content and context, and the subjective and the objective. The subjective is the comparing, the side from which a comparison is made, and the objective is what is compared. The stance belongs to the subjective. The subjective stance is the subjective pole of comparison, the stance from which every comparison is made. I identified consciousness above with three features of one act of comparison taken whole, and an act of comparison joins the comparing and the compared. So consciousness belongs to the act that joins the subjective and the objective. It is neither the world perceived nor the perceiver, but the act of perceiving. Consciousness is real: consciousness is not an illusion, an epiphenomenon (a by-product that does nothing), or a word without a referent. (A *referent* is the thing a word names.) And consciousness is an act: consciousness is not a substance, not a property of matter. So consciousness is neither a thing beside the particles nor a new property of the particles, of the kind Chalmers entertains. Consciousness is real; what is illusory is the demand to explain it by the impossible extremes of pure physicality or pure non-physicality.

Now return to the bucket. The water compares, and the water's comparing is real and simple, just as a thermostat's comparing is real and simple when the thermostat compares a room's warmth with the warmth set on its dial. Each act of the water's comparing has the three features, as every act of comparison does, and so the water's comparing has an inside in this sense: a side of its own, from which the comparing is done. So the water's comparing is consciousness in the smallest degree. The water is still no mind: the word comparer, in Chapter 1, names a role, and here consciousness belongs to the comparing itself. In the same way, Chapter 15 shows that the water's outcome is choice in the smallest degree. In general, wherever interaction occurs, some form of consciousness is present. The result is structurally different from panpsychism, because consciousness here is a property of interaction and not of matter. In the water, consciousness belongs to the comparing, which is an interaction, and not to the molecules taken as matter. You compare too, and in you a great many acts of comparison are gathered together, held as one, carried forward from moment to moment by one self, and known from inside. The gather-

ing of many acts of comparison is what is new in kind in a living being. The gathering is inside nature, with nothing added to the matter: a living being is a place where choosing is gathered and known from inside. The water and you are made of the same kind of matter, and both compare from inside the world. So the gathering of comparisons is the difference between the water's climb and your seeing of the climb. The water's comparing is real and simple: consciousness in the smallest degree. What you have, as the watcher, is many comparisons gathered into one self and known from inside.

Machines, Measures and a Brain at Its Edge

A person types a question into a chat window and receives a fluent and thoughtful reply. Nothing in the reply shows whether anyone is there. Two things settle whether anyone is there: what the machine is, and how the machine is built. A *classical computation* is the kind ordinary digital machines carry out. A classical computation is one rule applied the same way at every step, with each state fixed by the state before. The rule of a classical computation is symmetry made perfect, because the rule is applied the same way at every step. So, by the Incommensuration Theorem, the steps of a classical computation cannot make up the unbroken one-way order of the first person, because that unbroken order is continuity, and nothing can be both perfectly symmetric and perfectly continuous. A classical computation's rule fixes every step, so the computation has no outcome of its own that nothing before it fixed. A classical computation lacks two of the three features, and since each of the three needs the next, the computation lacks the third, the stance. I call the three features held together the "bound triple". Whether a system has the bound triple depends on whether it is a physical process making comparisons in the world, with an inside; it never depends on which program it runs.

Greene reviews Tononi's theory of integrated information. Tononi's theory is the scientific theory of consciousness nearest to my account. Tononi begins from experience itself, taken as given. He holds, as

Chalmers does, that a description of what the brain does leaves experience unexplained. I credit Tononi in full for beginning from experience. I also agree with two consequences of Tononi's theory. The first consequence is that a network that only passes signals forward has no integrated information, and so, on Tononi's theory, no consciousness. The second consequence is that no program a digital computer runs makes the computer conscious, since what counts is the physical system and not the program. Tononi's theory and my account differ at one step. Integrated information is a number computed from a third-person description of a system. The description gives the system's states, and the chance of each next state given each present one. Tononi identifies the experience with the structure that integrated information measures. Integrated information measures how far a system's parts act on one another as a whole, beyond what the parts would do taken one by one. Where a system's parts act on one another as a whole, the system's many acts of comparison can be made from one side, by one self. Integrated information cannot be the act. Integrated information is a number computed from a description, and consciousness, on my account, is the act of comparison itself.

The measurements made of the brain so far measure two things. The first thing measured is how widely and how variously the *cortex*, the outer layer of the brain, responds to a brief pulse. The second thing measured is how near the brain works to the edge between order and disorder. In 2013 a team of researchers sent a brief magnetic pulse through the scalp into the cortex. The researchers were Adenauer Casali, Giulio Tononi, Marcello Massimini and their colleagues. The team recorded with electrodes how the brain's electrical response spread. The team turned the pattern of the response into one number. The number is high when the response spreads widely and differs from place to place. The number is low when the response stays local or spreads in one uniform wave. The team called the number the perturbational complexity index. The team wrote that the index "reliably discriminated the level of consciousness in single indi-

viduals during wakefulness, sleep, and anesthesia". So we now know that, in waking, in sleep and under anesthesia, the index shows whether a person is conscious.

A second line of measurement is about the brain's poise: how near the brain works to the edge between order and disorder. John Beggs and Dietmar Plenz are neuroscientists. In 2003 Beggs and Plenz recorded the activity of slices of rat cortex and found that the activity of the slices comes in cascades, which Beggs and Plenz called neuronal avalanches. The sizes of the avalanches follow the pattern that physicists find in a system poised at *criticality*. Criticality is the balance between order and disorder in which a small disturbance can cascade to any size. In 2022 the neuroscientist Daniel Toker and his colleagues recorded brain activity in people and animals. Toker and his colleagues reported that "during conscious states, the electrodynamics of the cortex are poised near a critical point", where the electrodynamics of the cortex are the cortex's changing electrical activity. Toker and his colleagues also reported that under anesthesia and in generalized seizures, as awareness is lost, the cortex moves away from the critical point. So we now know that in conscious states the cortex works near the edge between order and disorder, where a small difference at one place can grow into a difference in what the whole brain does.

Criticality is the physical circumstance under which the bound triple is expressed in us; it is not the bound triple itself. Near criticality, an outcome that nothing before it fixed, at the smallest scale, can carry through to the scale of what a person does. Seen from outside, that single outcome is hard randomness. The outcome is of the kind quantum physics describes, and nothing before the outcome fixed which outcome it would be. Seen from inside, by the comparer whose act it is, the single outcome is that comparer's own act. So there is one event, with two descriptions, and both descriptions are correct. The subjective is the comparing side of every comparison, the side that holds the stance. I identify the subjective with the inside of a process that, seen from outside, shows as hard randomness and as *entanglement*. Two particles are entangled when their outcomes, measured far apart, agree in a pattern that no separate instructions carried

by each of the two particles could give them, as Chapter 3 set out. Set the identification beside Greene's expectation that consciousness will be understood, barring a stunning surprise, as another physical quality that arises in a quantum universe. The identification has two halves, hard randomness and entanglement, and both halves give the same answer to the chapter's question: the subjective is the inside of the very events that quantum physics describes from outside, the single outcome and the agreement of outcomes measured apart. Nothing is added to the particles or to their laws to make the subjective. The identification applies to any system, whatever its mechanism, whose comparing has three features: an outcome that nothing before it fixed, a one-way order, and a side from which the comparing is done.

Only Particles?

Greene's question, whether consciousness is only particles, has two parts, and my account has an answer to each part. Is anything added to the particles to make a person aware? Nothing is added: a person is made of the same matter as a stone, every event in a brain is a physical process, and the particles of a person keep their laws. Is awareness, then, the particles in motion and nothing more? An act of comparison taken whole is had from inside by the one whose act it is, and the interactions of the particles are the same acts of comparison, seen from outside. Describe the particles in motion in the third person, as completely as you like, and the description is a description of every comparison from the outside. So the description and the awareness are two, and the description does not contain the awareness. Greene's bag of particles is all that a person is made of. The story told of the bag in the third person is the map, and the awareness is the walk.

Greene's book goes on to ask whether thought can last until the end of time. My account has now said what a thought needs. A thought needs a physical process making comparisons in the world, with an inside, wherever and of whatever the process is made. And every act of compar-

ison has an outcome that nothing before it fixed (one of its three features), and the one whose act it is has that outcome from inside. We now know that consciousness is real, and I have identified consciousness with three features of an act of comparison taken whole, in a context, as one act, happening: its one-way order, its outcome that nothing before it fixed, and its stance. No third-person description, however complete, contains consciousness, because the first person is the side from which every comparison is made, and a description is laid out as structure, the same for everyone who reads the description.

If nothing before an act fixes the act's outcome, is choosing found in that outcome, or is Greene's verdict right, that no mind can rise above the laws that govern its particles?



Chapter 15

If the Laws Decide, Do I Choose?

Beside Until the End of Time*, chapter 5;* The Fabric of the Cosmos*, chapter 15;* The Elegant Universe*, chapters 4 and 13; and* The Hidden Reality*, chapter 2.*

What this chapter uses

Hard randomness is an outcome that nothing before it fixes, however completely what came before is known (Chapter 2).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act, happening: the one-way order of the act, its outcome that nothing before it fixed, and its stance (Chapter 14).

In 1814 the mathematician Pierre-Simon Laplace published a short book on chance, the *Essai philosophique sur les probabilités*, in Paris. In the book Laplace imagined an intellect that knew, at one instant, every force acting in nature and the position of every thing in nature. If the laws of physics fix every motion of the particles you are made of, and the quantum outcomes that the laws leave unsettled are only chance, do you choose anything at all?

Greene quotes Laplace's description of the intellect in *The Elegant Universe* (chapter 13, section "The Remaining Mysteries of Black Holes"), in Andrew Dale's English translation. In the last sentence of the description, Laplace says what the intellect would see:

> "For such an intelligence nothing would be uncertain, and the future, like the past, would be open to its eyes".

Writers since Laplace have called his imagined knower *Laplace's demon*. *Determinism* is the claim that what the demon would know exists. What the demon would know is a complete description of the world at one moment, which, together with the laws of physics, fixes every detail of every other moment, earlier and later. You can answer your question, whether you choose anything at all, only after you answer two other questions. The first question is whether the world can be deterministic, so that one moment and the laws fix every other moment. If the world cannot be deterministic, the second question is what a choice is.

Picture a player piano in a parlor. A paper roll punched with holes passes over the piano, and each hole sounds a note. Every note of the piece is fixed on the roll before the first note is struck, and the playing adds nothing to the roll. Now picture a pianist at the same piano, playing the same piece. The pianist plays each note in the light of the last note, hears each note follow from the note before, and could have played the next note otherwise. Laplace's world is the roll. In Laplace's world every moment is fixed in advance, as every note is fixed on the roll before the

first note is struck. Greene argues that our world has no room for free will, whether our world is like the roll (fixed in advance) or like a roll with a few holes punched at random (fixed in advance except for a few chance events).

From Greene's pages

What Greene Shows

Greene sets out his argument against free will in *Until the End of Time*, chapter 5, section "Free Will". We are made of particles, and physical law governs the motion of every particle, in a brain as in a baseball. In classical physics, you can calculate the positions and speeds of all particles at every later moment from their positions and speeds at one moment, using Newton's equations. If the past fixes the future, Greene asks, how can there be any room for free will?

At first, quantum physics seems to loosen the roll, that is, to leave some moments of the world unfixed by the moments before them. The laws of quantum physics give only the chance of each future. Greene grants that quantum chance looks like a modern form of the swerve that the Greek philosopher Epicurus once imagined: a random jog of the atoms that frees the atoms from their fate. Greene then shows why quantum chance only seems to loosen the roll. The quantum *state* (Greene's wave function: the complete description of what could be found, and of how likely each finding is) changes by an exact rule, Schrödinger's equation, named for the physicist Erwin Schrödinger. Schrödinger's equation fixes the likelihood of each future as firmly as Newton's equations fix a path. In Greene's words: "Much like Newton, Schrödinger leaves no room for free will" (*Until the End of Time*, chapter 5, section "Free Will"). In *The Elegant Universe* Greene gives a name to the fact that Schrödinger's rule is exact: "quantum determinism replaces Laplace's classical determinism" (chapter 13, section "The Remaining Mysteries of Black Holes"). Greene also states what

Schrödinger's rule does not fix (*The Elegant Universe*, chapter 4, section "Waves of What?"): the rule fixes the probability of each future, and the rule does not fix which future happens.

Physicists call the question of how one definite outcome comes out of the list of possibilities the *measurement problem*. Greene asks the question in one line: "How is one unique future selected from quantum mechanics' list of possibilities?" Greene reports that some researchers have looked for free will in the answer to the measurement problem, but Greene argues that free will cannot be found in the answer to the measurement problem. Suppose an electron has an even chance of being found here or there. Greene asks whether you could freely pick where the electron will be found. Greene answers that you cannot: "The data attest to the outcome being random, and random outcomes are not freely willed choices" (*Until the End of Time*, chapter 5, section "Free Will"). Greene adds that over many runs of a measurement the results agree with the statistics that Schrödinger's rule predicts, and that a freely willed choice would not have to agree with those statistics. So, for Greene, whether the laws are deterministic or probabilistic matters greatly for how the world unfolds and not at all for free will, because neither kind of law leaves room for free will.

Greene marks his view of free will as his own judgment, and he holds the view with different degrees of firmness in different books. In the same passage of *The Hidden Reality* Greene says that he is open to the possibility that life needs a nonphysical aspect, and that he has never encountered any evidence for a nonphysical aspect of life. In *The Fabric of the Cosmos* Greene allows that free will might have a place in one further part of the quantum story: how one quantum outcome comes about. Greene reports that some physicists have speculated that conscious observation is what brings one quantum outcome about, and he writes of that idea: "Personally, I find this extremely unlikely, but I know of no way to rule it out". Greene concludes: "The upshot is that the status of free will and its role within fundamental physical law re-

main unresolved" (chapter 15, section "Free Will, Many Worlds, and Time Travel"). In *Until the End of Time* Greene concludes that there is no free will in what he calls the traditional sense: "the capacity for the human mind to transcend the laws that control physical progression" (chapter 5, section "Relevance, Learning, and Individuality").

Greene then argues that people are free in another sense. Greene asks you to imagine a scene. You and a rock are sitting side by side on a park bench when a tree limb snaps above a passer-by. You leap and pull the passer-by clear, and the rock stays where it was. The laws are the same for you and the rock, and yet your particles are arranged so richly that your particles can do a thousand things that the rock's particles cannot do. *Compatibilism* is the view that freedom is compatible with laws that fix everything that happens. Greene says that the point of his view is one that "the compatibilist school of philosophy has long suggested": there is great benefit in considering kinds of freedom that fit with physical law (*Until the End of Time*, chapter 5, section "Rocks, Humans, and Freedom"). Greene places human freedom in how richly a person can respond, and not in the will: "Human freedom is not about willed choice". The rock can respond in only a narrow range of ways, as all inanimate matter can. For Greene, our freedom is release from the narrow range of responses of inanimate matter.

Greene then shows that learning, creativity, consequences and responsibility all remain in a world whose particles follow physical law (same chapter, section "Relevance, Learning, and Individuality"). A robot vacuum cleaner updates its map of a room as the cleaner runs into the furniture, so the cleaner learns. In Greene's words, "Learning and creativity do not require free will". Our actions have consequences. Take a story told at the human level: hunger, a pizza in the oven and a struck match. Greene writes that the human-level story is "accurate and insightful", and that the causal sequences in it "are manifest and are real". Greene says that he is responsible for his ac-

tions, "in a very literal if unfamiliar way", because "I" is shorthand for one particular arrangement of particles, and "the behavior of my particles is my behavior". Greene also says that his own strong impression of choosing does not go away when his argument is done. Greene concludes that "the sensation of free will is real" (same section), while the capacity to transcend the laws is not.

Where Greene and I Agree

Several of Greene's results are also results of my account. On my account, each result shared with Greene follows from *comparison*: the act of finding whether two things are the same or different in some respect, the act every measurement is made of.

First, the laws of everyday life are completely known, and choosing adds no force to those laws. Greene's argument depends on the premise that the laws of physics govern every particle of a brain. The physicist Sean Carroll is a compatibilist, as Greene is. Carroll states the premise of Greene's argument in his own words: the laws of everyday life are completely known. I grant Carroll's claim as Carroll states the claim. My account adds no force or field to the laws of everyday life. *Choice* is an outcome that nothing before it fixed, seen from inside by the one whose outcome it is. Choice is not an extra force pushing on the particles of a brain. So Greene and I agree that no mind acts from outside the laws of physics to move the particles of a brain.

Second, I grant that the quantum rule, Schrödinger's equation, is exact. I call Greene's quantum determinism *the formalism's determinism*, because the determinism belongs to the description. In the description the quantum state changes by exact rules, and those rules do not fix the findings themselves (the outcomes that measurements record). The formalisms that physics uses can stay deterministic at the level of the description, and the findings that the formalisms describe are *caused* without be-

ing *determined*. When a finding follows from what came before, along an order that runs one way without a break, I call the finding caused. When what came before fixes a finding in full, I call the finding determined.

Third, the data show the single quantum outcome to be random. Greene reports that "The data attest to the outcome being random" (*Until the End of Time*, chapter 5, section "Free Will"). In *The Fabric of the Cosmos* Greene leaves open the possibility of a quantum mechanics that would be wholly deterministic (chapter 15, section "Free Will, Many Worlds, and Time Travel"). Greene writes: "If physicists one day conclude that Schrödinger's equation is all there is to quantum mechanics, then quantum physics, in its entirety, would be every bit as deterministic as classical physics". On my account, by contrast, the single outcome is fixed by nothing before it. The single outcome is unfixed because of the Incommensuration Theorem, the theorem about every act of comparison that Chapter 3 states. In plain words, the theorem says that nothing can be both perfectly the same everywhere and perfectly without a gap. The section on Laplace's demon, below, states the Incommensuration Theorem in full. By the theorem, every lawful description leaves a residual: a part of what is found that no law fixes. The single quantum outcome is the plainest case of the residual.

Fourth, a choice seen from outside looks exactly like chance. I agree with Greene that you cannot pick where the electron is found, and that someone who watches a random result from outside sees no sign of a will at work. Every choice, seen from outside, looks like randomness: "Choices must appear, at some level of abstraction, to be fundamentally unpredictable to all others, and thus, indistinguishable from pure/true randomness in the domain".

Fifth, four facts hold in both accounts: the person on the bench differs from the rock, learning happens, actions have consequences, and the one who acts is responsible. For Greene, the four facts hold in a world whose particles are "fully governed by physical law" (*Until the End of Time*, chapter 5, section "Free Will"). On my account, the four facts hold in a world whose single outcomes are fixed by nothing before them.

What Laplace's Demon Would Need

Go back to the player piano. The roll of the player piano has two features at once. First, the roll has one rule, the same at every step: a hole sounds its note, here, there, now and later. Second, the roll has no gap: every moment of the piece is punched in, so each note passes into the next. Nothing is left unsettled.

Laplace's demon needs both features, made perfect, for the whole world. A comparison's *context* is the respect in which its content is found, such as a place or a time. Compare two readings of a thermometer, one in the morning and one in the evening: the temperatures are the content, and the two times of day are the context. The demon needs one law that fixes each next state from the state before, the same everywhere and at every time. A *symmetry* is the same content across a different context, as a square turned a quarter of the way round is the same square. So the demon needs symmetry made perfect. The demon also needs every state specified at every point, without a gap, so that each state passes into the next without a break. A *continuity* is the same content where the context is the same, with no jump anywhere, as a dimmer knob turned by a hair changes the light by a hair. So the demon needs continuity made perfect as well.

(Asymmetry is a different content across a different context, as no cove of a rocky coast repeats the cove before it. Discontinuity is a different content where the context is the same, a jump, as a light switch changes a room all at once.) Every act of knowing is a comparison, so the Incommensuration Theorem holds in every act of knowing. I take the step from knowing to the world. Comparison is interaction, and interaction is what physics is about. So a limit on comparison is a limit on every physical interaction. The demon needs exactly the pair that the theorem's first clause forbids: perfect symmetry together with perfect continuity. If one law fixed a whole world completely, the world would itself be perfectly symmetric and perfectly continuous, whatever theory describes the world, but

the theorem says that nothing anywhere at all can be both. So no world that could be is purely and completely deterministic, and our own world is no exception.

Yet the laws of physics stay reliable, experiments repeat, and the future is very largely fixed by the past. The theorem's first clause says that one claim only is false: the claim that the past fixes the future completely. Within any limited region, span of time or range of scale, a description can be very symmetric and very continuous. The whole success of physics is the success of descriptions within limited regions, limited spans of time and limited ranges of scale.

The theorem's second clause says that the opposite extreme is impossible too. *Pure randomness* is a world with no pattern carried anywhere and nothing joined to anything. Pure randomness would be asymmetry and discontinuity made perfect together, and the theorem forbids that pair as well. The writer Sam Harris gives an argument of the same form as Greene's in his 2012 book *Free Will*, where Harris sets choice against two cases: outcomes fixed in advance, and outcomes that are random. The argument assumes that the world is either fully determined or purely random. The theorem forbids both extremes, so every world that could be lies between the roll and a heap of noise: between a world fixed completely and a world with no pattern at all.

Causation remains in every world that could be. Causation pairs continuity and asymmetry: a connection without a break, along an order that runs one way. The theorem never forbids that pairing, because the two pairs that the theorem forbids are perfect symmetry and perfect continuity together, and perfect asymmetry and perfect discontinuity together. When a world's findings follow from what came before, along an order that runs one way without a break, I call the world *causal*. So every world that could be, our own included, is causal to a degree above zero.

In this book I already have a name for what lies between the roll and the heap of noise. A Geiger counter on a bench beside a lump of ore clicks when an atom in the ore decays. Each click has a cause: the atom decayed,

and the particle that the atom sent out passed through the counter. Yet nothing fixed when the atom would decay. The atom's decay followed from nothing that made it happen then rather than at another time. So the click is caused, since the click follows from the decay. And the click is not determined, since nothing before the decay fixed the moment of the decay. Chapter 3 describes the single outcome of a measurement as the plainest case of a change that nothing before the change fixes. Since Chapter 2, *hard randomness* has been this book's name for an outcome that nothing before it fixes, however completely what came before is known. The moment of the decay is hard randomness. Hard randomness is a fact about the world and not about anyone's ignorance. The random outcome in Greene's argument is hard randomness too. Hard randomness lies between the two extremes that the theorem forbids: hard randomness is neither the roll nor the heap of noise. Hard randomness is where choice happens.

A theorem of physics, the free will theorem, also shows that Laplace's demon cannot exist. In 2006, and in a stronger form in 2009, the mathematicians John Conway and Simon Kochen proved what they called the free will theorem. Picture two laboratories far apart. Each laboratory receives one of a pair of *entangled* particles (two particles whose outcomes, measured far apart, agree in a pattern that no separate instructions carried by each could give them). In each laboratory an experimenter turns a dial to choose which question to ask the particle. The free will theorem shows that if nothing earlier fixes the experimenters' choices, nothing earlier fixes the particles' answers either. Every experiment presupposes the experimenters' freedom, because an experimenter takes the settings of the instruments to be independent of what the instruments measure, and independent settings are what make a reading evidence about the thing measured. So if nothing earlier fixes the experimenters' settings, no specification of earlier states, however complete, fixes the particles' answers.

But perhaps, you may say, the turning of the dial was itself fixed long ago, together with the particle. Physicists call the proposal that the dial was fixed long ago *superdeterminism*: the settings of the instruments and the states of the particles share one deterministic past, so the settings only

seem free. Physicists have tested superdeterminism, and Chapter 3 describes the tests. In the tests the settings were drawn first from the light of stars of our galaxy and then from the light of *quasars* (the blazing centers of far galaxies). In both kinds of test the quantum agreement held: the outcomes measured far apart still agreed in the quantum pattern. So the settings and the particles could share a common cause only if the cause lay as far back as the moment the light left its source: about six hundred years ago for the stars, and billions of years ago for the quasars. Superdeterminism is ruled out outright, because superdeterminism needs everything that determinism needs, and more, and determinism needs the pair that the Incommensuration Theorem forbids: perfect symmetry with perfect continuity. Appendix A describes a measurement still to come, in which physicists will look for the patterns that superdeterminism predicts.

One last question is about a number that Greene gives. In Greene's count, a human body and brain hold about a hundred billion billion billion particles. Greene asks whether so many particles, arranged into a person, might escape the laws that govern each particle. Greene answers that the possibility of escaping the laws "contravenes all that science has so far revealed about the workings of the world". You may ask a different question about Greene's count of particles: does the chance in single atoms average away, among so many particles, long before the chance can affect anything a person does? Greene himself notes that the path of a pool ball can become impossible to predict, because a small error in the ball's starting speed grows with every bank the ball strikes. In 1963 the meteorologist Edward Lorenz published a simple model of *convection* (warm fluid rising and cool fluid sinking, the motion that drives the weather). Lorenz solved the model numerically, step by step with numbers rather than by a formula. Lorenz found that "slightly differing initial states can evolve into considerably different states". We now know that nature holds processes that carry small differences up into large differences: in the weather, a difference in the air too small for any instrument to record can grow, over days, into the difference between a storm and a calm. Suppose an outcome

at the scale of atoms is fixed by nothing before it. The atomic outcome is carried upward by processes that carry small differences up into large ones, and in time the atomic outcome can grow to the scale of what a person does. I draw the conclusion: a scale cannot be completely fixed while the parts that compose the scale are not completely fixed. So one failure of determinism anywhere is a failure of determinism at every scale.

Greene says that his focus is "on the existence of laws that govern your next move". He adds that no mathematical, experimental or observational sign has ever shown the laws that govern your next move to exert less than total control. I agree that the laws exist. Over many trials, the laws fix the *distribution* of outcomes: how often each outcome comes. A complete knowledge of the laws and of the state leaves the single outcome open. In that sentence, "open" means fixed by nothing that came before the outcome. So only one step of Greene's argument fails in my account: the step from a complete law to a complete world.

One Event, Seen from Outside and from Inside

Greene's argument has two horns, as a dilemma has: if the laws fix every outcome, there is no choice, and if nothing fixes an outcome, the outcome is random, and a random outcome is not a choice. The first horn fails in my account, because no world is completely deterministic. Greene's second horn remains. Picture a friend about to say the next word of a sentence. From outside, no one can predict the word, because nothing said up to now fixes the word. From inside, the friend means the word. Both descriptions describe one word: the friend's next word.

Try a second picture on yourself. Suppose you raise your left hand and not your right, and suppose that nothing before you raised a hand fixed which hand you would raise. To someone watching, the raised hand looks exactly like a random event. No measurement from outside can tell the raised hand apart from a random event. Or watch a person for a year and write down everything about the person that can be predicted, the way a program that shrinks computer files writes down the patterns in a stream

of data. However good your method of shrinking, some part of the person's behavior will not shrink into patterns. The leftover part looks exactly like randomness. The person knows the leftover part as their own choosing.

The *principle of identity* says that what cannot be told apart by any means, even in principle, is the same. The principle of identity is the principle of comparison of Chapter 1, stated for things in place of differences. The principle of comparison says that a difference that no comparison can register is no difference. Now apply the principle of identity. From outside, choice and hard randomness cannot be told apart, so by the principle of identity choice and hard randomness are one event. Hard randomness and choice are one event described from two positions. A description from outside is in the *third person*: a description that anyone can check, told of a thing as "it". In the third person, the raised hand or the friend's word is a gap in what the law specifies, with no pattern and no meaning. A description from inside is in the *first person*: the description had from inside, by the one to whom the thing happens. In the first person, the raised hand or the friend's word is a choice: the very event that the description in the third person tells of as a gap. The choice is meaningful to the process from which the choice came: the choice shows what that process is like, and the meaning of the choice belongs to that process alone. From outside, randomness; from inside, choice: one event, two descriptions, both correct, neither complete.

Greene reasons that the single quantum outcome is random, and that a random outcome is not a freely willed choice. On my account, the description from outside is not the whole of the event. The outcome that looks random from outside is, from inside, the choice of the one engaged in the event. A reader may take one more step from the premise that choice and hard randomness are one event: randomness is meaningless, so choice is meaningless. I reverse the step from meaningless randomness to meaningless choice, by the principle of identity, because meaning is not something an outside description can evaluate. So an outcome can be

meaningful to the process from which it came, even though no one can predict the outcome from outside. Choice is not random, however, in that it is subjectively meaningful, whereas true randomness is not.

Greene makes two further points, and my account has an answer to each. First, Greene notes that the results of many runs of a measurement agree with the statistics that the quantum rule predicts, while a free choice would not have to agree with those statistics. On my account, a choice has three parts: a range of what is open (what could come, fixed by nothing yet), the selecting of one outcome from the range, and the effects that follow. The statistics show the range and its *weights* (how likely each outcome is). A *law of many trials* is a law of what happens over many trials, such as the quantum rule for how often each outcome comes. A law of many trials fixes the range and its weights. The selecting is the single outcome, and no law of many trials fixes the single outcome. Second, Greene asks whether you could freely pick where the electron is found. You cannot, and I agree: the electron's outcome is not your selecting. By setting the instrument, you select which question is asked of the electron. The free will theorem shows that if nothing earlier fixes your selecting, nothing earlier fixes the electron's answer either.

Decision, Choice and the One Who Chooses

Greene's freedom of rich response holds in full in my account, under a name of its own. Someone has a free evening and a list: a film, a dinner with a friend, an early night. The person weighs the three and gives up two to have one. The person has made a *decision*: a selection within a set already laid out, by weighing. Then the friend calls with an idea that was on no list: a walk to watch the sun go down over the river. The evening becomes something that none of the three options was. Going on the walk is a *choice*, in my full sense: a range of what is open, a selection that no prior condition fixes, and the selection's consequences. A decision selects within what is already laid out; a choice realizes what was open but not yet laid out. In a world that was the roll, decision and choice would be one, be-

cause the list would be all there is. In a world that is causal to a degree above zero and never completely determined, the difference between decision and choice is real, and a person can live the difference.

Greene's person on the bench is released from the narrow responses of the rock, and so has a vast list of possible responses and a fine power of weighing the list. In my sense, the vast list and its weighing are decision, and both hold in full in my account. I differ from Greene about the rock. Return to the bucket of water of Chapter 14: as the water turns, the water climbs the walls of the bucket, and the water's surface becomes a hollow. Chapter 14 shows that the water's comparing is real and simple: consciousness in the smallest degree. Each act of the water's comparing has the three features of every act of comparison: a one-way order, an outcome that nothing before it fixed, and a stance, a side from which the comparing is done. The water has its unfixed outcome from the water's own side. The water's unfixed outcome is choice in the smallest degree, in the sense I give the word "choice", a sense far wider than a person's choosing. Every part of every world has some degree of choice, however small. A stone and a person differ in how much choice is present in them, and that difference is one of degree, running on without a break. One thing is new in kind in a person: choice is gathered, held together and carried forward as the person's own, and known from inside.

I call the gathering of choice in a person the *self*. Self is the appearance of the aggregation of that aspect of interaction which is choice. For Greene, the "I" is shorthand for one arrangement of particles. On my account, the self is neither purely physical nor purely non-physical: the self is where the choices that no arrangement of particles fixes are gathered, and known from inside.

Responsibility follows from choice itself. For choice to be choice, it must have consequences. Because the world is causal, what is chosen goes on to have effects; because it is not determined, the one who chose could have chosen otherwise. Greene keeps the sensation of free will. He gives up the capacity of free will, which he defines as the power of the mind to transcend the laws. The laws stay as they are, and no mind has a power to

transcend the laws. Yet more than the sensation of free will holds in my account. The one who chose could have chosen otherwise, because nothing before the selecting fixed the selecting. And the feeling of choosing is the selecting itself, known from inside.

What the Chooser Holds

Every world that could be is causal to a degree above zero, and its single outcomes are fixed by nothing before them. So the knowledge of Laplace's demon can be had in no world, because the demon needs one law perfectly the same everywhere and every state laid out without a gap, and the Incommensuration Theorem forbids that pair. The laws stay as Greene describes them, exact in their rule and reliable in their statistics, with nothing added, but the single outcome remains open under the laws, and choosing happens in the single outcome.

For a reader of Greene, every part of Greene's human story holds in my account (the leap from the bench, the learning, the consequences), and the one change is in what each choice in the story is. For Greene, each choice is a step of particles fully governed by physical law, while on my account each choice is an outcome that nothing before it fixed, known from inside. On my account, a person is the pianist and not the roll: the pianist plays each note in the light of the last note, and the next note is the pianist's own.

We now know that every world is causal to a degree above zero and never completely determined, and that the outcome that nothing before it fixes is hard randomness from outside and choice from inside. The one who chose could have chosen otherwise, because nothing before a choice fixes the choice, and responsibility rests on the fact that the one who chose could have chosen otherwise.

If choice happens in each act of comparison, what becomes of choosers in a universe that Greene shows emptying, star by star?

PART SEVEN

Until the End of Time, and After

Beside *Until the End of Time,* chapters 9 to 11.



Chapter 16

Will the Sky Go Dark?

Beside Until the End of Time, *chapters 1, 9 and 10;* The Hidden Reality, *chapter 6; and* The Fabric of the Cosmos, *chapters 10 and 14.*

What this chapter uses

The *principle of comparison*: a difference that no comparison can register is no difference (Chapter 1).

The world has ten dimensions: one of time, three of space, and six of possibility; *possibility* is the range of what a comparison could find, and nothing ever travels through *the six* (Chapter 7).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

The hold is the large world of space and time keeping the six at their one size and shape (Chapter 9).

An observer's *outside world* is everything that can ever reach that observer (Chapter 9).

I identify the cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in, and I call that part the *accessible share* (Chapter 10).

On 2 April 2007 two physicists, Lawrence Krauss and Robert Scherrer, sent out a short paper with a startling title: "The Return of a Static Universe and the End of Cosmology". Krauss was at Case Western Reserve University in Cleveland, and Scherrer at Vanderbilt University in Nashville. In his book *Until the End of Time*, the physicist Brian Greene pictures the far future of the universe as a climb up the Empire State Building, whose floors stand for the ages of the universe. On the way up, his reader watches the sun go dark, the galaxies slip away and the stars burn out. At the top, a reader of Greene asks the question that Krauss and Scherrer's paper asks: will the whole sky go dark?

Krauss and Scherrer pictured an observer of the far future, living in the one galaxy that our Milky Way and its neighbor Andromeda will by then have become. Krauss and Scherrer calculated that every galaxy beyond our own small family of galaxies passes out of the far-future observer's sight within about a hundred billion years. The far-future observer would find no distant galaxies moving away, and so no sign that space is growing. In time the expansion of space would stretch the faint warmth left over from the beginning of our universe past anything an instrument could catch. Krauss and Scherrer wrote that the far-future observer "will conclude the existence of a static 'island universe'": one galaxy, alone, in a dark that never changes.

Picture the far-future observer as an astronomer at her desk. She has instruments better than any of ours, and a sky that holds nothing beyond her own island galaxy. We follow Greene up his building floor by floor to see what the astronomer will see and why.

From Greene's pages

A Building Made of Time

How can anyone hold in mind the times of the far future? The numbers of years grow too fast for any picture made of hours or years. Greene's answer is a building. Greene asks us to imagine that the timeline of the universe runs up the Empire State Building, "with each

floor representing a duration ten times that of the previous". (*Until the End of Time*, chapter 9, section "An Empire of Time"). The first floor stands for ten years after the beginning, the second floor for a hundred years, the third for a thousand. So the number of each floor is the number of zeros in its count of years: floor 14 stands for a one followed by fourteen zeros, which is a hundred trillion years.

Each floor holds more time than all the floors below it put together. From floor 12 to floor 13, Greene notes, nine trillion years pass on one flight of stairs. Today we stand a few steps above the tenth floor, about thirteen point eight billion years after the big bang. The observation deck is on floor 86, and the topmost landing is floor 102.

Astrophysicists count the far future in the same steps, one power of ten at a time. In 1997 the astrophysicists Fred Adams and Gregory Laughlin wrote a long review of the far future, "A dying universe". Adams and Laughlin call each step a *cosmological decade*: the number of zeros in the count of years since the beginning. Greene's floor is the same count as the cosmological decade of Adams and Laughlin.

Greene sets out what he assumes for the climb. Greene assumes that the laws of physics as now known, acting in their undirected way, are the dominant influence on what is to come. Greene sets aside two possibilities: that the laws or the constants of nature change, or are already slowly shifting; and that intelligence takes control of matter at the scale of galaxies and beyond. Greene says that he describes what would happen if the laws and the constants of nature do not change and no intelligence takes control of matter at the scale of galaxies and beyond. Greene suspects that future discoveries, and whatever future intelligence may do, will not force a wholesale rewriting of the history he is about to survey. Greene calls that suspicion "A bold assumption, perhaps" (chapter 9, section "Evolution, Entropy, and the Future"). Then Greene climbs.

In this book Greene's building has one meaning only: a floor is a count of years since the beginning, written as a power of ten.

From Greene's pages

What Greene Sees on the Way Up

A few steps up, the first thing to go is the sun. In about five billion years the sun swells, swallowing Mercury and likely Venus. Some simulations, though, show that the swelling falls just short of Venus, and so falls short of the Earth too. The sun ends as a very dense ball of carbon and oxygen, called a white dwarf. The white dwarf stops shrinking because of a quantum rule that the physicist Wolfgang Pauli found in 1925. The white dwarf then cools and dims, like the last glow of an ember. In Greene's words, "A few steps above the tenth floor, the sun will fade to black" (section "The Black Sun").

Next comes the expansion of space. In the 1990s two teams of astronomers timed distant exploding stars and found the expansion speeding up. Physicists call whatever drives the speeding up dark energy. In Greene's words, "The case for dark energy is compelling but circumstantial" (section "The Big Rip"). The dark energy might stay the same, weaken, or grow. If the dark energy grew, its push would in time overcome every force that holds things together and tear apart galaxies, planets and atoms. Physicists call that tearing apart the big rip. Greene tells an example worked out by the physicists Robert Caldwell, Marc Kamionkowski and Nevin Weinberg, in which the rip would come before the eleventh floor. Greene and many other physicists judge that the dark energy is unlikely to grow: "I, and many other physicists, consider this an unlikely possibility" (section "The Big Rip"). Greene says that his judgment comes from decades of experience, not from a mathematical proof, and so his judgment could be wrong.

If the dark energy stays the same, the dark energy still changes the sky over the long run. Within about a trillion years the distant galaxies recede so fast that they slip beyond our cosmological horizon. Our cosmological horizon is the edge past which the light of a galaxy can never reach us. In Greene's words, "It will be as if the distant galaxies have dropped off a cliff at the edge of space" (section "The Cliffs of Space"). Our own small family of galaxies stays with us. By the eleventh floor our family of galaxies has likely merged into one galaxy, which astronomers have named Milkomeda. By the twelfth floor the faint warmth left from the beginning will likely be stretched too far to detect. Greene says that astronomers of that time might well conclude that the universe is static, the same size for ever. In *The Hidden Reality* Greene draws the lesson: "We live in a privileged age" (chapter 6, section "Cosmic Acceleration"). In *The Fabric of the Cosmos* Greene had pictured the far universe as "a vast, empty, and lonely place" (chapter 10, section "The Missing 70 Percent"). In *The Fabric of the Cosmos* Greene had also noted that a slowly changing dark energy would at some point bring the speeding up to a close (chapter 14, section "Dark Matter, Dark Energy, and the Future of the Universe"). Physicists call a slowly changing dark energy quintessence. In that chapter Greene wrote that a speeding up that comes to a close suggests "a far future less bleak and desolate" than a speeding up that goes on for ever.

Then the stars go out. By the fourteenth floor, about a hundred trillion years after the beginning, star formation ends in nearly every galaxy. Even the red dwarfs are running down by then. Red dwarfs are small, dim and long-lived stars. The observations show that red dwarfs are likely most of the stars there are. Between the nineteenth and twentieth floors, close passes between stars fling stars out of their galaxies, as earlier passes flung planets out of their orbits. By the twenty-third floor the planets still in orbit have spiraled into their dead stars, because the planets lost their energy to gravitational waves, the ripples in the shape of space and time that moving bodies send out. Most

galaxies have a black hole at their center: a region whose gravity is so strong that nothing, not even light, comes back out of it. By the thirtieth floor the black holes at the centers of galaxies have swept most galaxies clean of the stars that remain.

Last, Greene follows the fate of matter itself. Nearly every theory that joins the forces of nature into one predicts that the proton slowly breaks up into lighter particles. Experiments have searched for that breakup and have set the proton's lifetime above about ten to the power thirty-four years. Greene reports that many theorists expect protons to decay, that is, to break up in that way. Greene notes that the expectation might be wrong. To be definite, he takes the lifetime to be about ten to the power thirty-eight years. So from the thirty-eighth floor upward matter comes apart: every atom, every molecule and every structure built of atoms and molecules. In Greene's words: "It all falls apart" (section "The Fate of Complex Matter"). In a note Greene takes up the case in which protons do not decay by the processes of the theories that join the forces. In that case, Greene writes, any matter left would settle into iron over about ten to the power fifteen hundred years. Iron is the most stable of all the kinds of atom. Even in that case, Greene adds, physicists believe that a rare process within the established theory of the particles breaks protons apart. The estimates for the rare process are roughly ten to the power one hundred and fifty years, far sooner than the iron would form. So Greene answers yes to the question of this chapter, whether the whole sky will go dark: every structure we know is finite. As Greene wrote at the start of *Until the End of Time*, "our analyses have already revealed that the universe we have come to know is transitory" (chapter 1, opening).

The Same Emptying, Found from What Can Be Compared

I also predict the emptying of the sky that Greene describes, by an argument of my own. My argument starts from what can be compared with what. One comparison is to set two readings of a thermometer beside each other, one in the morning and one in the evening, and find that they differ.

In an expansion that keeps speeding up, every comparer has an edge. Greene uses the same name for the same edge (*Until the End of Time*, chapter 9, section "A Final Thought on Thought"). The cosmological horizon differs from the particle horizon, which Chapter 10 named. When a galaxy drifts past the cosmological horizon, the galaxy does not vanish at once. The light the galaxy sent before it crossed still arrives, stretched and dimmed without limit, but nothing the galaxy does afterward can ever reach us. I call a comparer an observer. In this chapter, too, I use the word "observer" for a comparer. An observer's *outside world* is the whole of what can ever reach that observer and be set beside the rest of what it receives. I wrote that what lies beyond the cosmological horizon, in its present and its future, can never be compared with the observer, "so it is no part of that observer's outside world".

Inside the cosmological horizon, gravity gathers matter into clumps: stars form, burn out and leave remains, and much of the bound matter ends in black holes. Each clump flings away the matter it does not take in. The expansion carries whatever is not bound to the comparer's own clump past the comparer's horizon. So, over long enough times, every observer's outside world empties. The populated sky is a passing stage of our epoch. In this book an *epoch* is one era of the world, from one turn to the next. Our epoch is the era we live in, whose history runs back to the big bang. The turn, which Chapter 9 set out, is the passage at the end of the interiors of the black holes of the era before ours, in which a large world is restored.

Now set my two spans of time on Greene's building: the time the sky takes to empty, and the time a black hole takes to glow away. The sky beyond our own small family of galaxies empties out of reach in a hundred billion to a trillion years. Greene places the distant galaxies' disappearance on floor 12 (*Until the End of Time*, chapter 9, section "A Final Thought on Thought"), so on his building the emptying is done by floor 12. A black hole as heavy as the Sun glows itself away far higher up. Greene updates the calculation the physicist Don Page made in 1976, and places the end of a black hole as heavy as the Sun at about floor 68 (*Until the End of Time*, chapter 10, section "The Disintegration of Black Holes"). The emptying comes first, by an enormous margin. So in my account the whole argument about the dark sky depends only on the first span, the emptying. The black holes end more than fifty floors above the emptying.

I also agree with Greene that we live in a privileged age, and the reason in my account is the way evidence travels: everything anyone knows of the past arrives as evidence reaching that person now. We live while the distant galaxies, their redshifts (the stretching of their light into longer, redder waves) and the oldest light (the faint warmth left from the beginning) are all still in view. What Krauss and Scherrer call the end of cosmology is the emptying of the sky, seen from the desk of an astronomer.

The Bucket Under an Emptying Sky

Hang a bucket of water from a twisted rope and let it go, as Isaac Newton did. The water stays flat for a moment, then turns with the bucket and climbs the walls into a hollow. Chapter 1 showed what the water's hollow is found against. What counts as still and what counts as turning is fixed by the matter there is to compare against, as the physicist Ernst Mach held. The standard of turning set by that matter follows from the principle of comparison: a difference that no comparison can register is no difference. In this book the bucket stands for one thing only: a comparison made from inside the world, against what is there to compare with.

The emptying sky adds a new step to what the bucket shows. So a standard of not rotating is fixed only by matter within reach, matter that can still be compared with. Matter beyond an observer's cosmological horizon can never be compared with that observer, so the matter beyond the horizon fixes no standard for that observer. Now picture Krauss and Scherrer's astronomer, a hundred billion years on, hanging a bucket of water from a twisted rope in her island galaxy and letting it spin: her water climbs the walls exactly as Newton's did. The hollow in her water is found against the matter still within her reach: the stars of her one galaxy, the *remnants* that burned-out stars leave behind, and whatever else has not yet crossed her horizon. The galaxies that have left her sky play no part in her water's climb. As the sky empties, the water's standard of turning comes from what is left within the horizon, and from nothing beyond the horizon.

Nothing Falls Back, and Nothing Is Torn Apart

Greene climbs on the branch in which the dark energy stays the same. On my account, that branch is a result: the dark energy stays the same because of a theorem about Einstein's equations. In Chapter 10 we met the *cosmological constant*: one number in Einstein's equations of gravity. The cosmological constant is the same in every cubic meter of space, does not thin out as space grows, and drives the speeding up. Einstein's equations describe gravity as the curving of space and time. The equations keep strict track of energy and of momentum, the amount of motion.

Within one geometry the cosmological constant is one number, and that statement is a theorem about Einstein's equations. The theorem about the cosmological constant is the reason the constant cannot shrink, cannot grow and cannot change its sign. I add that in this physics "a kind of particle has the same properties wherever and whenever it is found". For us, "wherever and whenever" means at every place and every time of our epoch. Greene sets aside the possibility that the laws or the constants change. On my account, the kinds of particle and the forces among them

stay the same within our epoch. On my account, the dark energy is the constant term itself, so no changing field drives the speeding up. So Greene's second branch, a slowly changing quintessence, has no place in my account.

Does the expansion, then, ever turn around? Throw a ball straight up: it rises, slows, stops and falls back into your hand. Thrown hard enough from a small enough world, the ball slows for ever and never stops. A *re-collapse* is the universe doing what the falling ball does: the expansion of space as a whole stopping, turning around, and contracting. Einstein's equations set the rate of expansion by a sum of three parts. In words, the square of the expansion rate is a part from the density (the amount of matter and energy in each cubic meter), less a part from any closed curvature, plus a part from the constant. A closed curvature is a bending of space as a whole that would close space on itself, like the surface of a ball. The density part is never negative. Chapter 9 showed that space is flat, so the curvature part is zero. The constant is positive. So the sum is always at least as large as the constant part, and the expansion rate stays above zero for any amount of matter. Our epoch does not fall back.

Does the push ever grow strong enough to tear things apart? Picture yourself on a back step in a breeze. A steady breeze, the same every minute, makes you lean a little, so you find your balance once and then stand there all night. Only a rising wind knocks you off the step in the end: a wind that keeps rising, with no limit.

Greene tells the example of three physicists, Robert Caldwell, Marc Kamionkowski and Nevin Weinberg, who worked out in 2003 exactly what a rip needs. Every kind of stuff that fills the universe has a pressure, its push on its surroundings, and a density, the amount of the stuff in each cubic meter, and physicists call the ratio of the pressure to the density the stuff's *equation of state*. The equation of state is about zero for thin matter, such as the galaxies themselves, one third for light, and exactly minus one for a cosmological constant, the one number for which a substance does not thin at all as space grows. A substance whose equation of state lies below minus one grows denser as space grows, and Caldwell named such a

substance *phantom energy*. A rip happens when the dark energy's equation of state lies below minus one, and only then. The constant is one number through our whole epoch. So the constant's density never changes, and so the constant's equation of state is exactly minus one. The dark energy of this physics does not meet the condition for a rip. On the back step, the constant is the steady breeze, and the phantom energy is the rising wind.

Everything that is bound now is bound in a world that already holds the constant at its present value: a galaxy, for example, or an atom. The constant has that same value at every later time. So, in the balance of forces that holds any bound thing together, the push of the constant is the same tomorrow as tonight. It shifts the balance once, by a fixed amount, and never by a growing one. At the outer edge of the great cloud of matter around our galaxy, the constant's push is about seven thousandths of the pull that holds the cloud. On the electron of a hydrogen atom, the constant's push is a fraction of the electric pull, a fraction whose first digit that is not zero comes sixty-nine places after the decimal point. Both ratios stay as they are at every later time: the ratio for the cloud and the ratio for the atom. Picture raisin bread rising in the oven: the dough swells and carries every raisin away from every other, and no raisin swells, because each raisin holds itself together. In the picture the dough is the space between the bound things of the sky, and the raisins are the bound things. The constant widens the great distances between the bound things of the sky, and bound things stay bound. So Greene's judgment that the rip is unlikely is, on my account, a result: there is no rip.

The equation of state has been measured. The Planck satellite mapped the oldest light over the whole sky, and the Planck map, taken with the distances of exploding stars and the spacing of galaxies, gives an equation of state of minus one and three hundredths, give or take three hundredths. So we now know that the measured equation of state is minus one, as far as that measurement can tell. Appendix A lists the surveys still to come, which will measure the equation of state at each era of our epoch.

What Holds Up a Burned-Out Star

Picture a theater filling from the front, one person to a seat. Squeeze the crowd into a smaller hall, with the front rows gone, and the people must sit further back, in dearer seats. When a crowd's energy rises as the crowd is squeezed, the crowd pushes back on whatever squeezes it.

Electrons fill their states the way the crowd fills the theater: one electron to a state, as one person to a seat, and from the lowest energy upward, as the people fill the seats from the front. *Exclusion* is the rule that no two electrons can be in the same state, that is, share both the same place and the same motion. Greene takes exclusion as a rule that Pauli found in 1925. On my account, exclusion is derived from a principle, the principle of identity. The reason in one clause: exclusion follows from the principle of identity, "since two electrons in one state could not be told apart and so would be one". A push follows from exclusion. *Degeneracy pressure* is the pressure a crowd of electrons exerts because exclusion forbids them all to settle into the lowest states. Degeneracy pressure needs no heat at all. A white dwarf is held up by the degeneracy pressure of its electrons. A *neutron star* is the dense collapsed core that some large stars leave behind: a ball some twenty-five kilometers across, made mostly of neutrons and packed as tightly as the core of an atom. A neutron star is held up by the exclusion of its neutrons (the particles of the core of an atom that carry no charge), with the push of the force between the particles of a nucleus added.

One white dwarf has been weighed with great care: the faint companion of Sirius, the brightest star in our night sky. Astronomers followed the orbit that Sirius and its companion trace about each other every fifty years, and in 2017 the astronomer Howard Bond and his colleagues reported the companion's mass: one and two hundredths times the mass of the Sun, give or take about one hundredth. Astronomers measured how far the companion's light is stretched as the light climbs out of the companion's own gravity. From that stretching the astronomers found the com-

panion's radius: about 5,600 kilometers, smaller than the Earth. A teaspoon of the companion's matter would weigh about fourteen tonnes. So we now know that a dead star can hold the mass of the Sun in less room than the Earth, held up with no fire inside it.

On the last day of July 1930 a young man of nineteen, Subrahmanyan Chandrasekhar, sailed from Bombay for graduate study in England. On the voyage Chandrasekhar saw that in a heavy white dwarf the fastest electrons move at nearly the speed of light, and that near that speed their push grows more slowly with the squeezing, so gravity's squeeze grows faster than their push. Above a certain mass, then, the electrons' push cannot hold a white dwarf up at any size. Astronomers now name the greatest mass a white dwarf can have for Chandrasekhar: about one and four tenths times the mass of the Sun. Every white dwarf astronomers have weighed lies below Chandrasekhar's mass.

Now let a white dwarf or a neutron star cool through the long ages. The cold remnant keeps its size, because the pressures that hold the remnant up do not depend on its temperature. When a white dwarf has cooled until it no longer glows, astronomers call it a *black dwarf*: a dark, solid ball about the size of the Earth, still holding the mass of a star. So I agree with Greene that the sun fades to black, and my account has one more step: the cooling takes nothing from the degeneracy pressure that holds the dark sun up.

Two Roads for Every Remnant

On my account, too, the stars go out on Greene's schedule. Adams and Laughlin place the end of the forming of new stars on the fourteenth floor, and the escape of stars from their galaxies between the nineteenth and the twentieth floors, where Greene places both.

A remnant does not sit still. The remnant's galaxy keeps stirring the remnant: some remnants merge, some fall into the black hole at the center of the galaxy, and others are flung clear. The remnants flung clear are then *carried away alone*: carried by the expansion beyond the reach of all other

matter. So every remnant's path ends in one of two ways: inside a black hole, or carried away alone. Adams and Laughlin estimate that from one to ten in every hundred stars of a galaxy fall into the black hole at the galaxy's center, and that the rest are thrown out. Greene shows the same two roads floor by floor: stars thrown out of their galaxies, planets spiraling into their dead stars, and galaxies swept clean of stars by their central black holes.

Two measurements are evidence for the two roads. In 1974 Russell Hulse and Joseph Taylor found a pulsar, a spinning neutron star that sweeps a beam past the Earth like a lighthouse, circling another neutron star every seven and three quarter hours. Timing the pulsar's pulses over the years showed the orbit shrinking exactly as Einstein's equations require of two stars that lose energy to gravitational waves. So we now know that the orbit of two bodies shrinks as they lose energy to gravitational waves. Greene follows that shrinking to the twenty-third floor. The Event Horizon Telescope is a network of radio telescopes spread across the Earth. In 2022 the astronomers of the Event Horizon Telescope made a picture of the black hole at the center of our own galaxy, a black hole about four million times the mass of the Sun. A share of our galaxy's stars will one day fall into the black hole at the center of our galaxy, the galaxy's dark center.

Does the Proton Last?

On the proton I differ from Greene: in my account the proton lasts. A *quark* is one of the particles of which protons and neutrons are made, and a *lepton* is a member of the family of light particles to which the electron belongs. A proton and a neutron are each made of three quarks. To say that a particle *decays* is to say that the particle breaks up into lighter particles. Greene names the theories that join three of the forces into one master force. In the theories that join the forces, some particles can turn a quark into a lepton, and so in those theories the proton decays.

Chapter 6 showed that the three forces, the strong force, the weak force and the electric and magnetic force, stand side by side, each with a *symmetry* of its own: a sameness of the laws kept across one kind of relabeling of the particles. One symmetry is for color: quarks carry color, and color is the charge of the strong force, which binds quarks together. One symmetry is for the weak charge, the label on which the weak force acts. The third symmetry is for electric charge. No single symmetry contains all three. The three symmetries stand side by side as a book and a cup stand side by side on a table, each keeping its own identity, and not as blue and yellow paint stirred into green. Each force is carried by particles of its own, and each carrier belongs to one symmetry only, so no carrier joins color to the weak charge. A quark carries color and a lepton carries none, so turning a quark into a lepton would need a carrier that acts on color and on the weak charge at once, and there is no such carrier. So the count of three-quark particles stays the same in every process in which a carrier turns one particle into another. The proton is the lightest of the three-quark particles, as measured. So in every process in which a carrier turns one particle into another, the proton cannot turn into anything lighter. The proton lasts. In this chapter, a proton that "lasts" is a proton that never decays.

In a note, Greene names one more way for a proton to break apart: a rare process within the established theory of the particles. The rare process is of another kind, a collective one: the rare process applies only to many particles together, never to one proton alone, and it runs freely only at temperatures far above any that a cold remnant reaches; in a cold remnant it can run only by tunneling through a barrier, at a rate of order ten to the power minus one hundred and sixty-one. So the rare process makes no single proton decay, and at the temperatures of a cold remnant its rate is effectively zero.

Physicists have looked patiently for the decay of the proton. The experiment Super-Kamiokande began its watch at midnight on the first of April 1996, in a mine near the town of Kamioka in Japan. Super-Kamiokande is a tank of fifty thousand tonnes of pure water, its walls lined with more

than eleven thousand detectors of faint light. A tank that size holds so many protons that, even if the proton's lifetime were far longer than the age of the universe, an occasional decay would show itself as a flash. In 2020 the Super-Kamiokande collaboration, the team of physicists who run the experiment, reported on the whole watch: no proton had been seen to break into a positron (the electron's twin of opposite charge) and a light particle called a pion. The collaboration saw no decay into a positron and a pion in the whole watch, so the collaboration set the proton's lifetime for that decay above twenty-four million billion billion billion years. So we now know that no proton has been seen to decay at any lifetime the searches have reached.

The rest of a remnant's matter lasts as well. The electron is the lightest particle that carries electric charge, as measured, and electric charge is never lost, so the electron has nothing to decay into. Under a mountain in Italy, the Borexino experiment found no electron decaying in 2015. From that search the Borexino experiment set the electron's lifetime above sixty-six billion billion billion years. The astrophysicist Matt Caplan has followed Greene's other branch further, the branch in which matter slowly settles into iron. In 2020 Caplan calculated the fate of a heavy black dwarf left alone. In Caplan's calculation, as in my account, no proton decays. The change Caplan follows is a rearrangement of lasting matter.

Forms Held, and Where the Hold Ends

Think of an ice sculpture in a cold room. The sculpture's shape is held by the cold around the sculpture. Warm the room, and the shape is gone, but the water the sculpture was made of does not vanish.

The forms of matter are held by the large world of space and time around them, as the sculpture's shape is held by the cold. Read in another way, the six are the range of all the states a particle of matter can be in. The one shape at which the six are held sets which kinds of particle there are. Protons, nuclei and electrons have the forms they have because the six

are held at that one size and shape. Chapter 7 also showed what holds the six: the six are held small as the content of the large world of space and time, which is their context.

The hold lasts as long as the large world of space and time is there around the six, with things in that world to compare against. The hold ends in two places, where nothing is left to compare against. The first place is the end of a black hole's interior: the region inside its horizon, the surface from inside which nothing comes back out, where every path leads forward to an end. Inside a black hole's horizon, as Chapter 9 said, moving toward the center is no longer a choice of direction, just as moving toward tomorrow is no choice. Inside the horizon the direction toward the center is the direction of time, so a remnant that falls in reaches the end of the interior after a finite time on its own clock. No signal from inside the horizon ever reaches the outside world, so the end of the interior lies at no moment of the outside world's time. At the end of the interior nothing is there to compare against, and the hold ends. The second place is a body carried away alone. Picture the astronomer's island galaxy again, and then take away the island too: for a body carried away alone, everything else has passed beyond the edge of what the body can ever be compared with. So for a body carried away alone nothing is left to compare against, and the hold ends too. The body's own parts do not keep the hold, because the hold needs a large world around the six with a standard of not turning fixed in it by matter within reach, the third of the facts of Chapter 9, and for a body carried away alone no such matter is left: Mach's bucket, set spinning in that body's place, would have nothing to measure its turning against.

At the end of a black hole's interior, and for a body carried away alone, the forms of matter end with the hold. What the matter is made of goes on, no longer in the form of protons, nuclei or electrons. Chapters 18 and 19 describe where that matter goes. The picture of the ice has one limit: the melted water keeps the form of water, and the matter past the end of each road keeps none of the forms matter takes in our epoch. Matter does not wear away: it keeps its forms until the hold that gives it those forms

ends. So complex matter ends where each road ends, inside a black hole or carried away alone, and not by slow decay from Greene's thirty-eighth floor upward.

What the Dark Sky Is

Go back to the astronomer at her desk, a hundred billion years from now. Her sky holds one galaxy. The astronomer's bucket of water still climbs the walls, against the matter of the one galaxy left in her sky. Her body, her bucket and the water hold protons of the same kind as the protons in the companion of Sirius and in our own hands, and none of those protons decays: each keeps its form until the hold ends. Her island will darken as its stars go out, its remnants will cool and keep their size, and each remnant will in the end travel one of the two roads: inside a black hole, or carried away alone. The long emptying is a scattering and a gathering of lasting matter, never a slow dissolving of it. So the question of the dark becomes a question about who is still there, made of that lasting matter, to compare.

I agree with Greene that the sky goes dark. My account has three further results: nothing falls back, nothing is torn apart, and the proton lasts. The populated sky is a passing stage of our epoch, and matter does not wear away.

So we now know that the sky does go dark for every observer: the outside world of each observer empties within a hundred billion to a trillion years, the stars that remain burn out, and their matter lasts, keeping its forms until the hold ends.

Could a mind, made of that lasting matter, keep thinking through the dark?



Chapter 17

Can a Mind Keep Thinking Forever?

Beside Until the End of Time, *chapters 1, 2 and 9.*

What this chapter uses

The hold is the large world of space and time keeping the six at their one size and shape (Chapter 9).

An observer's *outside world* is everything that can ever reach that observer (Chapter 9).

A body *carried away alone* is carried by the expansion beyond the reach of all other matter; at the end of a black hole's interior, and for a body carried away alone, the forms of matter end with the hold (Chapter 16).

I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act, happening: the one-way order of the act, its outcome that nothing before it fixed, and its stance (Chapter 14).

A *choice* is an outcome that nothing before it fixed, seen from inside by the one whose outcome it is (Chapter 15).

In the autumn of 1978 the physicist Freeman Dyson, of the Institute for Advanced Study in Princeton, came to New York University to give four lectures on time and its mysteries. One of Dyson's questions is also a question that a reader of Brian Greene's books may carry away: can a mind

keep thinking forever? If you have read the ninth chapter of Greene's *Until the End of Time*, you have met Dyson's answer: a plan by which a thinking being of the far future cools and sleeps to make its energy last. You have also seen Dyson's plan fail in Greene's reckoning. If the speeding up of the universe's expansion goes on unchanged, the sky keeps a faint warmth forever, and a thinking being that has cooled to that warmth can no longer throw off its waste heat.

Dyson opened his lectures with a book. A year before, the physicist Steven Weinberg had published *The First Three Minutes*, a book for general readers about how the universe began. Dyson read out one line of Weinberg's book: "The more the universe seems comprehensible, the more it also seems pointless". Dyson answered Weinberg's line in one sentence: "If Weinberg is speaking for the twentieth century, I prefer the eighteenth".

Dyson said that the view that prevailed left us "only the choice of being fried in a closed universe or frozen in an open one". In Dyson's use, a closed universe is one whose expansion stops and turns into a collapse. An open universe is one that expands forever. Dyson took the open universe and asked three questions about it. Does an open universe freeze into a permanent quiet as it expands and cools? Can life and intelligence survive in an open universe forever? Can societies keep sending messages to one another across the ever greater distances of an open universe? Dyson gave his three answers in one line: "Tentatively, I shall answer them with a no, a yes, and a maybe". The journal *Reviews of Modern Physics* printed the lectures in 1979, under the title "Time without end". Greene bases the closing pages of his ninth chapter on Dyson's lectures.

From Greene's pages

Greene's Thinker

Near the start of his book Greene asks whether a mind can keep thinking forever: "Can conscious thought persist indefinitely?" Greene is asking about thinking at large, in any form thinking may take, and

not about preserving one person's mind. Greene treats thought as a physical process. He marks the claim that thought is a physical process as his own starting point, "an assumption we will examine" (same section).

In his second chapter Greene shows why the question is hard, with a picture from the age of steam. A steam engine runs in cycles. On each cycle the engine must throw off waste heat to its surroundings, or the engine grows hotter until it fails. Greene shows that you are in the same case as the steam engine: your body burns food and air, your thinking is powered by that burning, and "much like the steam engine, you could not survive without resetting your entropy by purging excess waste heat to the environment". (*Until the End of Time*, chapter 2, section "You Are a Steam Engine"). Heat that leaves a thing carries entropy away from the thing. Greene's entropic two-step is any process in which a thing's own entropy falls because the thing passes a larger rise of entropy to its surroundings. So Greene asks whether the universe can take our waste heat forever: "Can life dance the entropic two-step indefinitely?" (same section).

In his ninth chapter Greene asks whether thinking can go on forever in the far future, floor by floor up his Empire State Building of time. On Greene's building, the number of each floor is the number of zeros in the count of years since the beginning. Floor ten is ten billion years, near our own time. Floor fifty is a count of years written as one followed by fifty zeros. Greene allows that, by the far floors of his building, thinking may have taken forms quite unlike ours. Greene's reason is an assumption that he states: life and mind are not tied to cells, bodies and brains, and are instead "collections of integrated processes". Greene draws one sentence from his assumption: "If some other arrangement of basic particles should faithfully execute the processes of life and mind, then that system will live and that system will think". (*Until the End of Time*, chapter 9, section "The Fate of Complex Mat-

ter"). Greene names whatever thinks, built in whatever way, the Thinker. Greene asks what the laws of physics allow the Thinker (chapter 9, section "The Future of Thought").

For Greene, as for Dyson, to think is to process information. Processing information makes entropy, and the entropy must leave the thinker as heat. Greene sets out Dyson's plan in the section "Thinking Slow", and the plan has two moves. The first move is to slow down. The Thinker lowers its temperature as the universe cools, so that all its inner workings slow together. Once the Thinker has cooled, its thoughts seem to the Thinker to run at their usual pace. And each thought costs less energy. If the Thinker cools on the right schedule, the cost of each thought falls fast enough that, in Greene's words, "the Thinker can actually think an infinite number of thoughts and yet only require a finite supply of energy" (*Until the End of Time*, chapter 9, section "Thinking Slow"). So the costs of all the thoughts add up the way one, and a half, and a quarter, and so on add up: the sum never comes to more than two.

The first move raises a problem, and Dyson made the second move to solve that problem. A colder body sheds its heat more slowly. At some temperature, then, the Thinker would make waste heat faster than it could throw the heat off. So the Thinker sleeps. The Thinker thinks in bursts and rests between the bursts, shedding the heat of each burst while it rests. Dyson called thinking in bursts, with rest between them, *hibernation*. Greene speaks to the Thinker as its adviser. He assures the Thinker that thought can continue forever with this rhythm of thinking in bursts and sleeping between them.

Then Greene gives, in two steps, the reply that physicists have found in the decades since Dyson (*Until the End of Time*, chapter 9, section "A Final Thought on Thought"). The first step of the reply comes from the study of computing. The simple steps of a computation can be done reversibly, in steps that could each be undone. A step that can be undone need give off no heat. Clearing a memory does give off heat.

A Thinker of finite size has a finite memory, so the Thinker must clear its memory if it is to think anything new. The second step of the reply comes from the sky. The expansion of space is speeding up, so each of us is surrounded by a cosmological *horizon*, the edge beyond which nothing can ever send us a signal. Greene asks you to picture the cosmological horizon as "a spherical array of distant heat lamps" (same section). The *kelvin* is the degree of the temperature scale that starts from absolute zero, the coldest anything can be. The cosmological horizon glows. The glow keeps empty space at a temperature that never falls, about ten to the power minus thirty kelvin. Heat flows only from warmer to colder, so a Thinker that has cooled to the temperature of the horizon can no longer throw off its waste heat. Greene writes: "This implies that the hibernation strategy is destined to fail" (same section). Greene puts rough numbers into the reckoning, and he concludes "that by the time we climb to the fiftieth floor, the universe may very well have hosted its final thought" (same section).

Greene leaves one door ajar: one condition under which thought could go on. Greene writes that his conclusion "rests on the assumption that the accelerated expansion of space will persist unchanged" (same section). The speeding up of the expansion might instead ease. If the speeding up eased, the cosmological horizon would disappear, and the distant heat lamps would go out. The physicists Will Kinney and Katie Freese showed that, in a universe whose speeding up eases, the Thinker could keep to its schedule of sleep and think on indefinitely. Greene still finds it "hard to avoid the conclusion that the far future of thought is precarious" (same section). The philosopher Bertrand Russell found bleak a future in which thought ends. Greene writes instead that the precarious future of thought shows how "our moment of thought, our instant of light", is "at once rare, wondrous, and precious" (same section).

In the sense this book has used since Chapter 8, *entropy* is a count of the arrangements of a thing's parts that cannot be told apart in some *context*, some respect in which the thing is compared.

A Row of Switches

Start where Greene and I agree. Picture a memory as a long row of switches, each on or off. Each switch holds one *bit*, the smallest piece of information, one choice between two. When the memory is cleared for new use, every switch is set to off, whatever the switch held before. Before the clearing, the row of switches could be in any of many arrangements. After the clearing the row is in one arrangement. The row held entropy in its many arrangements, and when the row is cleared, heat carries that entropy into the surroundings.

Greene's first step is the heat given off by clearing a memory, and on my account the same step follows from ground of my own, a starting point of my own. On my account, every observation and every measurement is a comparison. Every single act of comparison has a one-way order inside it: the comparing comes first, the record the comparing leaves comes after, and the order is never run backward. Chapter 8 called the one-way order inside every act of comparison the structural arrow. Making any record raises entropy: a record is itself a change that cannot be undone, and making a record raises the entropy of the instrument that holds it. So the structural arrow and the growth of entropy go together wherever anything is recorded. A thinker of finite size has a finite memory. So a thinker that keeps thinking new thoughts must keep clearing room in its memory. The thinker gives off heat at each clearing. Each clearing is a step taken one way, as the making of every record is: what the switches held is gone, and the entropy of the switches' many arrangements has passed into the surroundings as heat.

In 1871 the physicist James Clerk Maxwell pictured a tiny being at a small door in a wall between two halves of a box of gas. The being lets the faster molecules pass one way and the slower molecules pass the other

way, so that one half of the box grows warm and the other half grows cool. Maxwell's being is a sorter of molecules, and it must find out how each molecule moves and keep what it learns of each molecule in a memory until the being opens or shuts the door. The same tally of entropy holds for Maxwell's sorter as for the row of switches: whatever entropy the sorter takes from the gas, the sorter gives back to the surroundings, at least in full, when the sorter clears its memory.

In 1961 Rolf Landauer, a physicist at IBM, worked out the size of the heat that clearing a memory gives off: clearing one bit gives off at least an amount of heat fixed by the temperature alone. At room temperature, clearing one bit gives off no less than about three billionths of a trillionth of a joule. A joule is the everyday unit of energy. In 2012 the physicist Antoine Bérut and his colleagues built a memory of one bit from a bead of silica, the stuff of glass. The bead was two micrometers across, far thinner than a hair. Laser light held the bead in either of two dips of energy side by side, like one marble that can rest in either of two cups, and the dip that held the bead was the bit's one choice between two. Bérut and his colleagues cleared the bead's memory again and again, at slower and slower rates, and they measured the heat each clearing gave off. Bérut and his colleagues found that, as the clearings grew slower, the heat fell toward Landauer's figure and settled there: in their words, "the mean dissipated heat does saturate at the Landauer limit". So we now know, by measurement, that clearing a memory gives off heat, and that the least heat a clearing gives off has the size Landauer gave it. Greene's first step depends on the measured heat of clearing a memory, and so does the same step in my account.

A Room Whose Walls Stay Faintly Warm

The second step of Greene's reply is the lowest temperature the sky allows. One part of the second step is true in my account as well: the warmth that reaches a thinker from outside falls toward the warmth of the horizon's glow, and never below it. I call the whole era of the world we

live in our *epoch*. Picture a room whose walls stay faintly warm through our whole epoch. The room is Greene's array of heat lamps, seen from inside the array. The walls are the cosmological horizon, and their faint warmth is the horizon's glow. In 1977 the physicists Gary Gibbons and Stephen Hawking found that a universe whose expansion speeds up has a horizon that glows like walls that stay faintly warm. In a paper published in 2000, the physicists Lawrence Krauss and Glenn Starkman worked through Dyson's plan in a universe whose expansion speeds up. In the paper of Krauss and Starkman, a thing inside a room whose walls stay faintly warm can cool below the temperature of the walls only by running a refrigerator, and the refrigerator costs more energy than it saves. Krauss and Starkman call the warmth of the walls the de Sitter temperature. Krauss and Starkman wrote that "the de Sitter temperature fixes a minimum temperature below which life cannot operate" without running a refrigerator.

The temperature of the walls follows from the rate of expansion that our speeding universe settles toward. I work out the settled rate from the values that the Planck satellite measured when the satellite mapped the oldest light in the sky, the light left from the early universe. The settled rate is the rate that the energy of empty space, which drives the speeding up, would give on its own, and the settled rate is a little below the rate measured today. The temperature of the walls comes out at about two point two times ten to the power minus thirty kelvin. Greene bases his figure on the measured rate of the speeding up, and Greene's figure is about ten to the power minus thirty kelvin. The two figures agree to their power of ten. In 1998 two teams of astronomers measured the speeding up itself, by following the light of exploding stars, the supernovae. Chapter 10 told the work of the two teams. The warmth of the walls follows from the speeding up that the two teams measured. Greene and I use the same physics for both steps of Greene's reply: the heat of clearing a memory, and the warm walls of the sky.

What a Thought Needs

On my account, Greene's answer also follows by a second road, a second line of argument that never uses heat at all, and the second road begins with a plain question: what does one thought need?

Hang Newton's bucket of water from its twisted rope once more, and let the bucket go. The climb is a *finding*: the outcome of one act of comparison, laid down where it can be seen. Chapter 1 showed that the water's turning is registered against the matter within reach: the matter there is for the water to compare with. A thought is a finding of the same kind as the water's climb. A thought is made against something that is there: a page, a face, the sky over the house. A thinker has more than the water has: many comparisons gathered into one self, and known from inside. The water and the thinker share one thing: each of their findings is found against what is there to compare with.

In Chapter 1 I gave the word *comparer* its meaning in this book: whatever makes a comparison and registers the outcome of the comparison, such as a gyroscope that holds its direction. The word comparer names a role. Being an observer is a role that anything inside the world can fill, and filling it does not need a mind. In this book a *thinker* is a comparer of a particular kind, of whatever material, and Greene's Thinker is a thinker in this sense.

Which kind of comparer is a thinker? A thinker is a comparer in which a great many acts of comparison are held together as one, carried forward from moment to moment by one self, and known from inside. Chapter 14 found that you are a thinker. Chapter 14 pictured many acts of comparison held together as one with music that you hear for the first time. Each note arrives after the note before it, and you cannot have the first hearing of the piece again. Through the whole piece you stay the one listener while note after note arrives, and that one listener, going on while the notes change, is the self in which the hearing is gathered. Each hearing has three features (the hearing runs one way, nothing heard before fixes

what it brings, and it happens to someone), and the same three features belong to every act of comparison, a thinker's or any other comparer's. The outcome of every act of comparison is something that nothing before the act fixed. And every act of comparison is made from a *stance*, the side from which the comparison is made and to whom the outcome happens. In Chapter 14 I identified *consciousness* with the three features taken whole, in a context, as one act, happening. Each act of the water's comparing has the three features as well.

The second road never uses heat, and its first result agrees with Greene's starting point: a thought is a physical process. Greene takes the claim that a thought is a physical process as an assumption to be examined. On my account, the claim follows as a result. A *computation* is one rule applied the same way, step after step, and a program is such a rule written out. I also show that only a physical process making comparisons in the world can have an inside, and that whether a thing has an inside "never depends on which program it runs". A count of computations is not a count of thoughts.

Greene's sentence about another arrangement of particles can be read in two senses, and each sense has its own answer in my account. Where to "execute the processes of life and mind" means to be those processes, as a physical thing of another material, I agree with Greene: a thinker may be made of anything that compares. Where to "execute the processes of life and mind" means to run those processes as a program, I disagree with Greene. A person is not a very large file.

What does one comparison need? Picture the smallest comparison there can be. Two hydrogen atoms sit some way apart in an otherwise empty universe. The first atom gives out a *photon*, a single particle of light, and the second atom takes the photon in, so that the second atom's electron is lifted to a higher level. Take the first atom away, and the second atom has nothing to be measured against. Take the proton from the second atom, and there is no bound atom left to keep a record of what arrived. Take the photon away, and nothing passes between the two atoms at all. So even the smallest comparison needs three things: a second thing

to compare against, a lasting structure in which the record is kept, and a signal that passes between the second thing and the structure. A comparison needs an actual interaction, not a possible one.

Can a large enough store of energy keep the Thinker thinking forever? The answer follows from what energy is. Energy is the capacity of a system to participate in comparisons. A store of energy, however large, is a capacity, and a capacity is used only where something is there to compare with. So the size of the store does not decide whether the thinking goes on.

I also have my own way of counting a thinker's time. Dyson counted a creature's life in subjective time, time as the creature itself lives it. Time comes with the comparison, not before it. So in my account the counterpart of Dyson's subjective time is a count of findings. And "thinking forever" has a plain meaning: a *run of findings* without end, one finding after another by one thinker, each made against something actually there, and each kept in a lasting structure of the thinker's own.

Where Every Road Ends

Now follow a thinker through the emptying of the sky that Chapter 16 set out. Every thinker belongs to some bound structure, held together by its own gravity: a planet, a star or a galaxy. The sky over a thinker's structure empties within a hundred billion to a trillion years, the span that I give. Greene places the disappearance of the distant galaxies on floor 12 of his building, so on Greene's building the emptying is done by floor 12. Every observer's road comes to one of two ends: the observer falls into a black hole, or the observer is carried away alone, while everything else passes beyond the observer's cosmological horizon.

Could the fall into a black hole last forever, at least on the thinker's own clock? The fall ends after a finite time on the thinker's own clock. Outside a black hole's horizon, the surface from inside which nothing returns, a body can stand still, move outward or move inward. Every path inside the horizon leads inward, as every path you can take leads to later

times: inside a black hole's horizon the way inward is the way forward in time. The end of the interior comes as a limit. For a black hole of the Sun's mass, the standard calculation of Einstein's theory puts the end of the interior at most about fifteen millionths of a second after the crossing. A friend watching from far away never sees the body cross the horizon, and no signal from inside the horizon ever reaches the friend. So the end of the interior lies at no moment of the outside world's time, and a thinker at the end of the interior has no part of the outside world to compare with. A thinker carried away alone keeps only one reference, one thing to be measured against: its own cosmological horizon. The thinker's cosmological horizon is centered on the thinker and moves with the thinker, as your skyline moves with you wherever you walk: the thinker's own place fixes where the horizon is, and no matter fixes the horizon. Everything else has passed beyond the horizon, so no matter is left inside the horizon for the thinker to compare against. So the thinker has nothing outside itself to compare against at either end of its road. At the end of a black hole's interior no moment of the outside world is there, and for the thinker carried away alone no matter is left within its horizon.

At either end the thinker loses one more thing: the structure of matter in which it compares and keeps its records. The forms that matter takes in our epoch are atoms, molecules and cells. *The six* are the six small dimensions of possibility met in Chapter 7, held far smaller than an atom. The forms of matter in our epoch depend on what Chapter 16 called *the hold*: the large world of space and time keeping the six at their one size and shape, and with them the forms of matter. At the end of a black hole's interior, and for a body carried away alone, nothing is left within reach to compare against, so the six are no longer held. The forms of matter end with the hold. The ice sculpture of Chapter 16 is a picture of a form that ends while its matter goes on: when the room warms, the sculpture's shape is gone, and its water goes on. A thinker is made of matter in our epoch's form, and the thinker keeps its records in structures of that mat-

ter. So at either end the thinker loses two things: anything outside itself to compare against, and the structure in which it compares and keeps its records.

Put the steps of the second road together, and the second road ends at the same answer as the road through heat. No thinker makes an endless run of findings, and that conclusion uses neither the size of the thinker's store of energy, nor its temperature, nor the speed at which it thinks. A thinker's run of findings still ends even if the thinker has an endless store of energy and runs as cold and as slowly as it likes. Greene, and Krauss and Starkman before him, reason from a finite store of energy and from the lowest temperature the sky allows. On Greene's assumption that the speeding up of the expansion persists unchanged, his Thinker cools to the temperature of the horizon, and the universe "may very well have hosted its final thought" by the fiftieth floor of his building. On my account, the end of every thinker follows without either premise. The end follows from what is left to compare against at the end of each road. Two roads support the one answer, and the two roads start from separate ground: neither road starts from the ground of the other. The two roads hold the answer as two strands hold one cable, the picture Chapter 6 gave for an argument about the world.

Dyson's Plan, Step by Step

Now that the second road has been followed to its end, each step of Dyson's plan can be set beside my account, and at several steps my account and Dyson's plan disagree.

The first step of Dyson's plan is his copy of a creature. Dyson assumed that a creature copied part for part, to run on a smaller scale of energy in surroundings colder by the same factor, would be, in his words, "subjectively identical to the original creature". Greene gives his Thinker the comfort that Dyson gave his creature: the Thinker's experience will not change as the Thinker slows. On my account, a thinker can be built to run colder. No description from outside amounts to being the same one as the origin-

al: the same one who compares from inside, with the same stance. A match part for part is a description from outside. The reason is that being a person is not exhausted by any exterior description. So a match part for part shows that the copy is like the original, part for part, and the match does not show that the copy is the same one who compares from inside.

The second step of Dyson's plan is sleep, and each sleep needs something to end it. If a clock woke the sleeper with certainty after every one of endlessly many sleeps, the clock would be a process completely fixed in advance. In Chapter 15 Laplace's demon was an imagined knower that knows every force in nature and the position of every thing at one instant. The clock would need what the demon needed, on the scale of one clock: one rule carrying each of the clock's states into the next, the same at every step, and every step settled with no gap, without end. One rule, the same at every step, is symmetry made perfect, because symmetry is the same content across a different context: for the clock, the same rule across different steps. Steps settled with no gap are continuity made perfect, because continuity is joining without a break. The clock would need perfect symmetry and perfect continuity together. Chapter 15 used the Incommensuration Theorem against the demon. The Incommensuration Theorem says that nothing anywhere at all can be both perfectly symmetric and perfectly continuous, and the theorem holds of a single clock as it holds of a whole world. So no clock wakes its sleeper with certainty after every one of endlessly many sleeps. By another route, Krauss and Starkman also concluded that no clock wakes its sleeper forever: "Alarm clocks are eventually guaranteed to fail", they wrote. And a sleeper whose clock does wake it still wakes, at the end of its road, with nothing outside itself to compare against.

The third step of Dyson's plan is his maybe: messages sent forever between distant societies. The speed of light is the outer edge of comparison, the greatest speed at which any signal can carry a finding from one place to another, and the speeding up of the expansion cuts every observer off from whatever lies beyond the observer's cosmological horizon. Think of a message sent toward a society beyond the horizon as a runner

on a track that grows longer ahead of him faster than he can run: he runs forever and never arrives. So no chain of societies passes a message to a society beyond the horizon of the society that sends the message.

The fourth step is Greene's door left ajar: an easing of the speeding up, which would let the distant heat lamps go out. On my account, the cosmological constant is the energy of empty space that drives the speeding up. Chapter 10 found that the cosmological constant is one number within one geometry, the one space and time of our epoch. I follow the cosmological constant forward: "With the constant positive and unchanging, the rate of the expansion settles toward one steady pace and comes closer to it for ever, and every observer's horizon grows toward one fixed size". So the horizon keeps its glow, and the walls of the room stay warm, through our whole epoch. In the case of Kinney and Freese, the speeding up eases and the cosmological horizon is gone. The case of Kinney and Freese cannot happen, since the cosmological constant keeps its one value through our epoch. Appendix A gathers the measurements still to come of whether the energy of empty space that drives the speeding up keeps one value across the ages.

Krauss and Starkman left one more road aside: escape through a black hole. A thinker that falls into a black hole comes to the end of the interior, where the forms of its matter are no longer held and nothing is left to compare against. And every act of comparison is made on one side of the turn or the other, so no thinker passes through a black hole into the next epoch as a thinker.

Where Thinking Begins Again

So every thinker's run of findings ends where its road ends. Does thinking itself end, then?

Chapter 9 found that the turn, at the end of the black-hole interiors, restores a large world. The emptying ends in a turn, and a new epoch follows with the same structure as ours. Chapter 19 follows the emptying to the turn. I need only one consequence for thinking: comparers are present

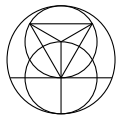
in the next epoch, as comparers are present in ours. The structure of the world is derived, and the structure is the same in every epoch. The six small dimensions have the same shape in every epoch, and the same size counted in Planck lengths. Then the new epoch makes stars, and the stars make black holes. In the new epoch comparison happens, and every act of comparison has the three features our acts have: a one-way order, an outcome that nothing before it fixed, and a stance. The new epoch begins with the same structure, and the same orderly start, under which stars, planets and people became possible in our epoch.

Whatever thinking happens in the next epoch belongs to that epoch alone. No act of comparison in the next epoch takes in any finding of ours. So, for a thinker of our epoch, every finding, record and count is real only in our epoch. The thinking of the next epoch continues no thinker of this one. Thinking goes on forever in this sense: each epoch is followed by a next, and in each epoch comparison begins again.

Greene closes the ninth chapter of *Until the End of Time* with our moment of thought, rare, wondrous and precious. I agree with Greene on the worth of our moment of thought. Every thought you have is made against something there: a page, a face, the sky over the house. Your thinking is a physical process with an inside, joined to a world it can compare with, and your thinking belongs to this epoch as the sky does. Each thinker's run of findings ends, but comparison itself is not used up.

So we now know that every thinker's run of findings ends where its road ends, where nothing is left to compare against, whatever the thinker's store of energy or temperature. We know too that thinking goes on, because in each epoch that follows, comparison begins again.

If every road ends inside a black hole or alone in the dark, what is left when even the black holes are gone?



Chapter 18

What Is Left When the Black Holes Are Gone?

Beside Until the End of Time, *chapter 10;* The Hidden Reality, *chapter 9;* The Elegant Universe, *chapter 13; and* The Fabric of the Cosmos, *chapter 16.*

What this chapter uses

The world has ten dimensions: one of time, three of space, and six of possibility; *possibility* is the range of what a comparison could find, and nothing ever travels through *the six* (Chapter 7).

Entropy is a count of the arrangements of a thing's parts that some context cannot tell apart (Chapter 8).

An observer's *outside world* is everything that can ever reach that observer (Chapter 9).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

The hold is the large world of space and time keeping the six at their one size and shape (Chapter 9).

On the fifteenth and sixteenth of February 1974, a meeting on quantum gravity was held at the Rutherford Laboratory, south of Oxford. Quantum gravity is the joining of quantum physics with gravity. At the meeting the physicist Stephen Hawking announced that black holes glow.

A black hole is a region from which nothing climbs back out, not even light. Greene follows Hawking's glow in chapter 10 of *Until the End of Time*, up to the top of his Empire State Building of time, a building whose floors stand for the ages of the universe. Greene's reader then asks: if every black hole glows itself away, is anything of what fell into it left anywhere?

Hawking had set out to prove another physicist wrong. Jacob Bekenstein was a student of the physicist John Wheeler, and not long before Hawking's announcement Bekenstein had proposed that a black hole carries *entropy*, a count of the ways the hole's hidden parts could be arranged. Greene tells in *Until the End of Time* that Hawking considered Bekenstein's proposal ludicrous. Hawking began a calculation that he expected to show where Bekenstein had gone wrong. Instead Hawking reached, in Greene's words, "a conclusion so shocking that it took him some time to believe it" (*Until the End of Time*, chapter 10, section "The Disintegration of Black Holes"). The conclusion was that a black hole gives off a faint warmth, as a hot coal does. So the black hole slowly loses its mass. Hawking published the result in a short paper in the journal *Nature* on the first of March 1974, under the title "Black hole explosions?". In the first sentence of the paper Hawking names the step he had taken: "QUANTUM gravitational effects are usually ignored in calculations of the formation and evolution of black holes". Hawking did not ignore the quantum effects of gravity. He applied the quantum description of matter and light, the physics of the very small, to the smooth geometry of a black hole, and the glow came out of joining the two.

From Greene's pages

The Top Floors of Greene's Building

Greene marks the times of the far future as the floors of a building. On his Empire State Building, each floor stands for ten times the span of the floor below. The number of a floor is the number of zeros in the count of years since the beginning, so floor 10 is ten to the power 10

years after the beginning, a one followed by ten zeros. We are at about floor 10. On the floors where the black holes end, the stars have long gone out.

Greene tells the story of black-hole entropy in his own terms: in Greene's use of the word, entropy counts the ways a thing's small parts can be rearranged while the thing looks, from outside, just as it did. A black hole seemed to have no rearrangements that leave the hole looking the same from outside. The reason was that the equations of general relativity, Einstein's theory of gravity, fix a black hole that has settled down by three numbers alone: its mass, its spin (how fast it turns) and its electric charge. Bekenstein answered that a black hole does carry entropy, and that the hole's entropy grows with the area of its *horizon*, the surface from inside which nothing comes back out. The area is counted in Planck areas, squares one Planck length on a side. The *Planck length* is the length, far below anything measured, at which gravity and the quantum meet on equal terms: at the Planck length the effects of gravity and the effects of the quantum are equally strong. Greene states the conclusion at full strength in *The Fabric of the Cosmos*: "Black holes have a monopoly on maximal disorder" (chapter 16, section "Wherefore the Entropy of Black Holes?"). Hawking's glow then means that a black hole has a temperature. The *second law of thermodynamics* is the law that the entropy of everything taken together does not fall. Once the black hole had a temperature, the second law was whole again. As a black hole glows, the hole's own entropy falls, and the glow carries more entropy out into space than the hole loses. Greene describes the black hole's trade of entropy as a dance: "The choreography is familiar: as black holes radiate, they dance the entropic two-step" (*Until the End of Time*, chapter 10, section "The Disintegration of Black Holes").

Then Greene gives the times at which the black holes glow away. A black hole heavier than the Moon is colder than the faint warmth left over from the early universe, so for now the hole takes in more

warmth than the hole gives off. The hole begins, on balance, to shrink only after the expanding universe has cooled below the hole's own temperature. Greene takes the mass of the Sun as representative of the holes that dying stars leave behind. Greene writes that black holes of the Sun's mass will have glowed away by about floor 68. Greene says, in note 10 to chapter 10 of *Until the End of Time*, that his floors for the glowing away of black holes update a calculation that the physicist Don Page made in 1976. Greene's update takes in newer knowledge of the particles, especially the masses of the neutrinos, the lightest particles of matter. At the centers of galaxies, the largest black holes have masses that approach a hundred billion times the mass of the Sun, and holes that heavy are calmer and colder. Greene finds that a hole of a hundred billion times the Sun's mass lasts until floor 102, the top of the building. Greene writes that from the top floor there is little to see beyond a diffuse mist of particles drifting through space. Astronomers, Greene among them, give the name dark energy to whatever drives the speeding up of the expansion of the universe. If the dark energy persists, Greene writes, the particles will rarely if ever meet, and their clumping will be extraordinarily unlikely. If the dark energy drains away, the particles may gather into still larger black holes, which glow away more slowly still. Greene writes of the era of the top floors: "Physicists sometimes liken this future era to the end of time. Not that time stops" (*Until the End of Time*, chapter 10, section "An End of Time").

What became of everything that fell into the black holes? Greene reports a long argument among physicists over the question. In 1976 Hawking concluded that a black hole which glows away destroys the information that fell into it, the details of what fell in. Greene states in *The Elegant Universe* where the question stood when he wrote: "As of this writing, there is no consensus among physicists regarding the answer to this question" (chapter 13, section "The Remaining Mysteries of Black Holes"). Greene reports a wager in the same section of *The El-*

egant Universe. The wager was on whether the information that falls into a black hole is lost. Hawking and the physicist Kip Thorne bet that the information is lost forever, while the physicist John Preskill bet that the information comes back out as the hole glows and shrinks. Later, in *The Hidden Reality*, Greene reports a result of the physicist Edward Witten's. Before Witten's result, the physicist Juan Maldacena had argued that, in a universe whose shape differs from ours, everything taking place inside, where gravity acts, is described by a theory without gravity on the universe's boundary. Greene reports that Maldacena's result "is deemed conjectural", since an airtight argument for the result remains elusive (chapter 9, section "Parallel Universes or Parallel Mathematics?"). The *holographic* ideas are the ideas by which what happens inside a region is described on the region's boundary. Greene also reports, in the same section of *The Hidden Reality*, that the holographic ideas have come through "a great many stringent mathematical tests". Witten showed that a black hole inside Maldacena's universe is, in the description on the boundary, a bath of hot particles. Greene writes that the hole inside and the hot bath on the boundary are two descriptions that "embody identical information" (chapter 9, section "String Theory and Holography"). In a footnote Greene draws the conclusion: "all information that falls into a black hole is ultimately available to the outside world. Quantum mechanics needs no modification" (chapter 9, footnote to the same section).

So Greene reports three findings about the top floors of his building. First, the glow is real, and the black holes are the last of the remnants of the stars to go. Second, a black hole's entropy grows with the area of its horizon. Third, everything that fell into a black hole is ultimately available to the outside world, on what Greene calls conclusive evidence.

From Greene's pages

Boltzmann's Long Wait

Chapter 10 of *Until the End of Time* has one more passage on the far future, and Greene's passage matters for the question of what is left after the black holes are gone.

Greene opens the chapter with an if-then statement about unlimited time: if time has no end, then any outcome the quantum laws do not strictly forbid will, sooner or later, happen. Greene reminds us that the second law of thermodynamics, that the entropy of everything taken together does not fall, holds only on the odds: entropy can fall, and a fall of entropy is only very unlikely. In an accelerating universe, each place has a cosmological horizon around it. The cosmological horizon of a place is the edge beyond which nothing can ever send a signal to the place. The cosmological horizon glows faintly too, at about ten to the power minus thirty degrees above absolute zero. Absolute zero is the coldest any temperature can be. Ten to the power minus thirty is a decimal point followed by twenty-nine zeros and then a one. Over spans of time beyond imagining, particles from the glow of the horizon will now and then collide and gather into atoms, and the atoms, more rarely still, will gather into larger things. Every so often one of the larger things would be a brain, with memories of a life the brain never lived. Greene calls brains that form by chance from the particles of the horizon's glow *Boltzmann brains*. The name comes from the physicist Ludwig Boltzmann, who first read entropy as a count of arrangements. Greene estimates a reasonable chance that one Boltzmann brain forms within ten raised to the power of a one followed by sixty-eight zeros of years, and over unlimited time Boltzmann brains would far outnumber brains like ours. If Boltzmann brains far outnumbered brains like ours, Greene argues, no brain could trust that its memories record anything that happened, and so no brain could trust the measurements on which science rests.

Greene then sets out the escapes that physicists have proposed: ways in which Boltzmann brains would not, after all, outnumber brains like ours. Greene writes of the escape in which the dark energy fades: "A conservative assessment is that the jury is still out" (*Until the End of Time*, chapter 10, section "Boltzmann Brains"). Greene then turns to more radical escapes, in which the universe ends long before Boltzmann brains could form (section "Is the End Near?"). In a note Greene himself names one more approach, argued by the physicist Sean Carroll and his collaborators: "that the fluctuations necessary to form Boltzmann brains do not arise under a careful quantum mechanical treatment" (chapter 10, note 27). In Greene's sentence, a fluctuation is a rare chance swing out of disorder. In note 27 Greene cites a paper by Kimberly Boddy, Sean Carroll and Jason Pollack, who argue that an empty space whose expansion speeds up settles into a quiet state in which nothing fluctuates.

The Glow Is Real

I agree with the first of the three findings that Greene reports about the top floors: the glow of black holes is real. The ground of an argument is what the argument starts from, and my argument for the glow has a ground of its own.

In his calculation Hawking compares two descriptions of empty space: the description natural long before the black hole formed, and the description natural long after the hole formed, far from the hole. In the later description, a steady warm flow moves outward from the horizon, through space that was empty in the earlier description. The warm flow is the *faint glow*, which physicists call Hawking radiation. The glow makes the black hole shrink slowly. The hole loses its mass by its own glow until, in the end, nothing of the hole is left. The slow shrinking is called *evaporation*. Greene tells of the glow with a familiar picture. In the picture, pairs of particles appear at the horizon, one particle of each pair falls in, and the

other escapes. Greene offers the picture of the pairs as the essence of Hawking's reasoning. Hawking's calculation itself is the comparison of the two descriptions of empty space. In every step of the calculation the geometry of space and time stays smooth.

In one step of the calculation, Hawking follows the waves of the glow to lengths far shorter than the Planck length. When the waves that become the glow are followed backward in time toward the moment the horizon formed, the waves grow shorter without limit. Chapter 5 described two sizes that every body has. A body's *gravitational size* is the length at which the body's own gravity would become strong. A body's *quantum size* is the length within which quantum theory will not let the body be pinned down. As Chapter 5 showed, the *Planck scale* is where a body's gravitational size equals its quantum size. Two descriptions of physics are needed at the Planck scale: the smooth description of space and time, and the description of quantum theory. Chapter 5 also showed why the smooth description of space and time has an edge at the Planck scale, a limit to the range in which the smooth description holds. For the edge at the Planck scale, my ground is the Incommensuration Theorem, the result about comparison that Chapter 3 set out. Every measurement is a comparison. When a thing's content stays the same across a difference of context, the thing is *symmetric*, as a square looks the same when the square is turned a quarter round. When a thing's content stays the same where its context stays the same, the thing is *continuous*, as a dimmer knob turned by a hair changes the light by a hair. When a thing's content differs across a difference of context, the thing is asymmetric, the opposite of symmetric. When a thing's content jumps where its context stays the same, the thing is discontinuous, the opposite of continuous. Chapter 5 showed that the smooth description of space and time, perfected, is continuous and asymmetric. Chapter 5 also showed that quantum theory's description of what comparisons find, perfected, is symmetric and discontinuous. At the Planck scale both descriptions are needed at once. If a single description carried both descriptions at the Planck scale, the single description would hold continuity together with symmetry, or asymmetry together with dis-

continuity, each perfectly, and the Incommensuration Theorem states that nothing can hold either pair perfectly. So, by the Incommensuration Theorem, the smooth description of space and time does not hold at every scale. The probe of Chapter 5 shows where the range of the smooth description ends: at the Planck scale. In the step that follows the waves below the Planck length, Hawking's calculation uses the smooth description beyond the range in which the smooth description holds. So the question is whether the glow depends on the one step that goes below the Planck length.

Physicists have answered the question, by calculation and by measurement. In 1995 the physicist William Unruh worked with a model of a horizon made for sound waves instead of light. In his model Unruh changed how a field behaves at the very shortest waves, and the glow came out unchanged. In 1996 the physicists Steven Corley and Ted Jacobson also changed how the field behaves at the shortest waves, and they found that the glow stays very close to the warm glow of Hawking's calculation. Then in 2019 four physicists built a horizon for sound in a cloud of atoms chilled almost to absolute zero: Juan Ramón Muñoz de Nova, Katrine Golubkov, Victor Kolobov and Jeff Steinhauer. In the cloud, the atoms flow past one place faster than sound can travel against the flow, so sound in the fast part of the flow cannot travel back upstream, as light inside a black hole cannot climb out. The four physicists measured what came out of the horizon. The model black hole glowed with sound at the temperature that Hawking's formula predicts for the model. So we now know that the glow does not depend on what happens below the Planck scale. So the thermal radiation of the horizon does not rest on what happens beyond the Planck scale. The glow is real.

Why the Count Goes by Area

I also agree with Greene's second finding, that a black hole's entropy grows with the area of its horizon, and my reason has to do with the place from which a count is made.

An entropy is always a count made against a context. The count is of the arrangements that the comparisons available from the context cannot tell apart. Chapter 6 counted a walled garden in two ways. The first count goes bed by bed from above, with the whole garden in view at once, and the second count is made from one place outside the wall, by what reaches you across the wall. A region of space can be counted in the same two ways. One count goes cell by cell inside, and grows with the volume of the region, while the other count goes by what crosses the region's boundary, and grows with the area of the boundary. A *comparer* is anything that compares and registers what it finds. To make the first count, a comparer would need every cell of the inside of the region in view at once, as one structure laid out as a whole, seen from no place in particular. The second count is made from a place. A comparer outside the region registers the region only through what reaches the comparer across the boundary.

An axiom of my account is one of the three basic statements from which the rest of my account is worked out. The first of the three axioms states that the act of comparison, made at a place inside the world, is more fundamental than any structure laid out as a whole. In the first axiom, for the act to be more fundamental means that the act comes first: a structure is what acts of comparison lay down and gather, and every structure is made of acts. Every finding of physics is made from a place inside the world. So the more fundamental of the two counts is the count made from a place, through what crosses the boundary. For every comparer outside a black hole, the boundary is the horizon. In Chapter 6 I identified what a black hole's entropy counts: states of the hole's content, the states of what

the hole holds that no comparison made from outside its horizon can tell apart. For a hole of the Sun's mass the count is about a one followed by seventy-seven zeros.

The area law states that the total area of the horizons never shrinks when black holes merge. Physicists have checked the area law in the sky. In the United States, the Laser Interferometer Gravitational-Wave Observatory has two detectors, each with arms four kilometers long. On the fourteenth of September 2015 the two detectors recorded ripples in the shape of space and time. Two black holes merging sent out the ripples. In 2021 five physicists read the areas of the two starting horizons from the part of the signal sent out while the two holes circled each other: Maximiliano Isi, Will Farr, Matthew Giesler, Mark Scheel and Saul Teukolsky. The five physicists read the area of the final horizon from the ringing of the signal as the merged hole settled. The final area was larger than the two starting areas added together, as the area law requires.

The Last of the Ashes

Now set the glow beside the times of the far future, floor by floor on Greene's building.

For a hole of the Sun's mass, the temperature of the glow is about six hundredths of a millionth of a degree above absolute zero, tens of millions of times colder than the leftover warmth that fills space today. In the accelerating expansion the leftover warmth thins, and the leftover warmth takes a hundred billion to a trillion years to cool below the temperature of the hole's own glow. On Greene's building, the cooling is done by about floor 12. After the cooling, a hole of the Sun's mass goes on glowing until about floor 68, the floor by which, Greene writes, a hole of the Sun's mass has glowed away. Three other figures put the end one or two floors lower than floor 68: Page's figure of 1976, and two figures of my own. The end comes one floor lower or two, depending on which kinds of particle the glow is counted as carrying away. The giant holes at the centers of galaxies take longer still, since a hole's lifetime grows with the cube of the

hole's mass: a hole ten times heavier lasts a thousand times longer. Greene finds that the largest of the holes at the centers of galaxies last until floor 102. So the black holes are the last of the ashes.

The emptying of the sky comes far sooner than the end of the black holes. As Chapter 16 found, every observer's outside world empties within a hundred billion to a trillion years. The span is the one that I give. Greene places the time at which the distant galaxies disappear on floor 12, so on Greene's building the emptying is done by floor 12. Chapter 9 followed the turn: the passage at the end of the insides of the black holes of the era before. So for every black hole formed from a star, the outside world of the hole's epoch has emptied long before the hole has glowed away. A remnant taken into a black hole is not given back by the hole's glow within the times over which the emptying runs. The emptying comes first, by more than fifty floors.

Time Does Not Stop

I also agree with Greene's sentence about the top floors, that time does not stop there, and my argument that time does not stop is about the end of a black hole's interior, deep inside the hole.

Follow a body that falls into a black hole. A body falling freely through the horizon finds there only what a body falling freely anywhere else would find. The *equivalence principle* of Chapter 1 states that, within a small enough room, the push you feel from gravity cannot be told apart from the push you feel when the room speeds up. The horizon is smooth, as the equivalence principle requires. A body falling freely, like the pen let go in Chapter 1, is weightless: nothing holds the body up. Inside the horizon, the direction toward the center is the direction of time. So the end of the inside comes after a finite time on the falling body's own clock: for a hole of the Sun's mass, at most about fifteen millionths of a second after the body crosses the horizon. No signal from the end of the inside ever reaches the outside.

Does the falling body's clock stop at the end of the inside? Picture marks at half the distance to a line, then at half of what is left, then at half again. All of the marks lie within a finite stretch, and no mark among them is the last one before the line. The time on the falling body's clock is like the stretch of marks: the time is finite, and the time has no last tick. No clock records a last tick beyond which it ticks no more. What an observer can compare with ends, and time does not end. The end of the sky is the end of what an observer can compare with, not the end of time.

Two Things Called Information

Greene's third finding is about what became of everything that fell into a black hole. My answer differs from the one Greene reports. The difference begins with a question that has to be asked first. Which information is meant?

Think of a handwritten letter in a post office. One thing the letter holds is what the letter says. What the letter says is which words were chosen, in which order, out of all the words that could have been written. The words belong to the letter, and the words go wherever the letter goes. A second thing belongs to the office. The office's sorting machine reads only the postcode, so every letter in one bin looks the same to the machine, and for each bin the office can count how many different letters lie there looking alike to the machine. The office makes the count of look-alike letters with its own machine. Close the office, and the office's counts have nothing left to be counts of.

In physics the word information is used for both kinds of thing: for what a thing holds, like the letter's words, and for a count made against a context, like the office's count. The two senses stay apart. The content of physics is what the comparisons of physics find: the matter and light of the world. Information in the first sense is the *form of the content*. The form of the content is what the matter is and which of its possibilities the matter has taken, and the form is carried with the matter wherever the matter goes, as the letter's words go with the letter. Information in the second

sense is a count of states relative to a context, like the count in the office's bin. The entropy of a black hole is information in the second sense, counted against the outside world.

Keep the two senses apart. Then my first result about information is that the form of what falls into a black hole is kept. Within the range where quantum theory applies, quantum theory describes the state of the matter going forward on the geometry of space and time without loss. Within the range where general relativity applies, general relativity describes what falls in going forward to the end of the inside. Hawking's calculation shows a loss at one place only: where the falling matter met the *singularity*. At the singularity the smooth description of space and time comes to the end of the range in which the description holds. Think of a road that runs across a map and off the edge of the paper: the road goes on, and the map stops describing the road at the edge of the paper. The road is like what fell into the black hole, the map is like the smooth description of space and time, and the edge of the paper is like the singularity. The loss that Hawking found shows where the smooth description stops, and no step inside either description removes the form of what fell in.

A second question is whether, and when, the form of what fell in can be reached from outside the black hole. The answer depends on the kind of black hole. Take first a hole far lighter than any star. A hole far lighter than any star glows away completely while an outside world is still there to receive the hole's glow. For a hole far lighter than any star, the answer is the one Greene reports from Witten's result. What fell in is ultimately available to the outside world, and quantum theory needs no change. The glow that has come out is *entangled* with what remains inside the hole. Chapter 3 said what entangled means for two far particles: findings made on one particle and on the other agree in a pattern that no separate instructions carried by each particle could give the findings. While the glow streams out, the glow outside and what remains inside grow more and more entangled. Then the amount of entanglement turns and falls to nothing as the hole is used up. Don Page found the rise and fall of the en-

tanglement in 1993. For holes far lighter than any star, I agree with Greene's report. Every black hole found so far is far too heavy to glow away while an outside world is still there: even the smallest black hole yet found, Greene notes, is much more massive than the Moon.

Every black hole made by a star is of the other kind. A hole made by a star is too heavy to glow away while an outside world is still there to receive the hole's glow. For a hole made by a star, the form of what fell in is not destroyed. Nothing of the form reaches the outside while the hole lasts, and the hole lasts far longer than the outside world of the hole's epoch. So the form travels with the content inward, to the end of the inside. The insides of all the black holes of an epoch share one end: the turn. In Chapter 9 I identified the Big Bang with the turn as our epoch's history renders the turn. At the turn the old outside world has no moment at all, so no count made against the old outside world goes across the turn, as the office's counts go nowhere when the office closes. The form of the content goes across the turn with the content, as one averaged form. The averaged form appears at the beginning of the next epoch in the *first ripples*, the faint differences of density with which an epoch begins and from which its galaxies grow. Chapter 9 pictured the worn curve of a church step. The curve is all the footsteps that fell on the step together, with none picked out and none wiped out. The averaged form is like the worn curve: the forms of everything that fell in are in the averaged form together, with none picked out and none wiped out. I name the premise that Hawking's puzzle rested on, and I decline that premise. The information paradox rests on the premise that information must be recoverable in the original outside world, and my account rejects that premise. The letter's words are kept. The office's counts belong to the office.

What Boltzmann's Long Wait Asks For

Return now to Greene's Boltzmann brains. Greene's argument about Boltzmann brains rests on three premises, and my account says something about each of the three.

The first premise is that in unlimited time every outcome the laws allow happens. There are two kinds of reversal. A *faithful reversal* is a reversed process that retraces exactly the states that the forward process passed through. So a path that the laws allow is one thing, and a path that the world takes is another. The laws can allow a path that the world never takes. And in my account, the time without end that the first premise needs comes as a run of epochs. The epochs come one after another, each epoch with its own outside world.

The second premise is that a region lasts, glowing and holding matter, through spans of time far longer than the emptying. Within the time of the emptying, every observer's road ends in one of two places: inside a black hole, or carried away alone by the expansion, beyond the reach of all other matter. The six small dimensions of Chapter 7 are this book's six dimensions of possibility, held a few Planck lengths across. The hold lasts for as long as the large world of space and time is there around the six small dimensions, with things in the large world to compare against. For a body carried away alone, the hold of Chapter 16 ends. A lone particle in the far future stops being matter in the form our epoch gives it. What drives the speeding up of the expansion is the *cosmological constant*, one constant number in the equations of our epoch's geometry. In the words of Chapter 10, the *vacuum* is empty space at its lowest energy. In Chapter 10 I identified the cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in, and the cosmological constant is one number through our whole epoch. So, on my account, the dark energy keeps one value through our whole epoch. Within our epoch, then, the dark energy does not fade, and Greene's first escape does not arise.

The third premise is the count itself: the odds of a brain formed by chance, set against a brain formed by a history. Recall the lottery ticket of Chapter 11. And the largest count of an epoch is a count made against the epoch's own outside world. When the epoch's outside world has emptied the largest value has nothing left to be the largest value of, and in the next epoch a new count begins, made against the next epoch's own context.

Greene's argument needs three things together: a region that lasts with no end, matter that keeps its forms in that region, and a context against which the odds are counted. On my account, each of the three things belongs to one epoch and to the epoch's outside world. Our epoch's outside world empties by floor 12. Greene estimates a reasonable chance that one Boltzmann brain forms within ten raised to the power of a one followed by sixty-eight zeros of years. On Greene's building, the floor of Greene's estimate would be far above every floor on which an outside world lasts in my account.

What Is Left

Go back to the meeting south of Oxford in February 1974. Hawking found that black holes glow. Physicists have since seen the glow in a model black hole, a cloud of chilled atoms, at the temperature Hawking's formula gives.

So we now know what is left, and where. The black holes are the last of the ashes, the last of the remnants that the stars leave. A hole of the Sun's mass glows away by about floor 68, and the largest holes glow away by floor 102. The outside world of every epoch empties long before any hole made by a star has glowed away. What fell into the black holes is kept in the first sense of information, as the form of the content. The form goes inward with the content to the end of the inside of each hole, and from there the form goes across the turn, as part of the averaged form from which the next epoch's galaxies grow. Every count made against the old outside world ends with the old outside world, as a closed post office keeps no counts. Greene's long wait for Boltzmann brains needs three

things together: a region that lasts, matter that keeps its forms, and a context for the odds. The three things are held together only within one epoch, whose outside world empties by floor 12.

While Greene's top floors go dark, everything the black holes took in keeps its form. The form is on its way to the next epoch.

Greene ends the climb in a thin mist of particles that, if the dark energy persists, will rarely if ever meet. Is the long emptying the end?



Chapter 19

Is the Long Emptying the End?

Beside Until the End of Time*, chapter 10,* The Fabric of the Cosmos*, chapters 13 and 14, and* The Hidden Reality*, chapter 5.*

What this chapter uses

The *principle of comparison*: a difference that no comparison can register is no difference (Chapter 1).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

An observer's *outside world* is everything that can ever reach that observer (Chapter 9).

A body *carried away alone* is carried by the expansion beyond the reach of all other matter; at the end of a black hole's interior, and for a body carried away alone, the forms of matter end with the hold (Chapter 16).

The emptying ends in a turn, and a new epoch follows with the same structure as ours (Chapter 17).

On 24 April 1865 the physicist Rudolf Clausius read a paper to the Natural Science Society of Zürich. Put together, the paper's last two sentences seemed to settle the far future of the whole universe. Brian Greene pictures the history of the universe as a tall building, his Empire State Building of time. He follows the far future to the top floors of the building. On the top floors the sky has emptied of stars and black holes, and only a thin mist of particles drifts through space. Is the long emptying of the sky the end?

Clausius was then professor of physics at the Federal Polytechnic School in Zürich. In his paper of 1865 Clausius gave the spreading of heat a measure and a name. He called the measure *entropy*, from the Greek word for a transformation. In this book, since Chapter 8, an entropy is a count of the arrangements of a thing's parts that some context cannot tell apart. A context is the respect in which the thing is compared. The paper was printed in 1865 as well, in the journal *Annalen der Physik und Chemie*. Clausius closed the printed paper with two sentences, written in German.

The two sentences read as follows in English:

> "The energy of the universe is constant. The entropy of the universe tends toward a maximum".

Put Clausius's two sentences side by side, and they give one picture of the far future. Every difference of temperature evens out. Energy that could push or lift anything spreads out of every store, into one lukewarm sameness. In the final state of the evening out, everything has one temperature, the entropy of everything has reached its largest value, and nothing ever happens again. Physicists call the final state of the evening out the *heat death*. Greene tells of differences evening out in a modern form, as a climb up his tower of time, with far more physics than Clausius had. But in Greene's picture, time still runs at the top of his tower, and rare events may still happen there.

From Greene's pages

Greene's Last Floors

Chapter 16 describes Greene's Empire State Building, a timeline set in a building: floor one stands for ten years after the beginning, floor two for a hundred years, and each floor stands for ten times the years of the floor below it. Floor 102 stands for a one followed by a hundred and two zeros of years. By floor 102 even the largest black holes have shrunk to nothing by their own faint glow, as Chapter 18 told. Greene writes of what is left: "we will not see much beyond a diffuse mist of particles wafting through space" (*Until the End of Time*, chapter 10, section "An End of Time").

In Greene's telling, what happens next depends on the energy that now speeds the expansion of space. Astronomers call whatever drives the speeding up of the expansion *dark energy*: dark because it gives off no light, and energy because it acts on the expansion as an energy does. If the dark energy has drained away, the remaining particles may gather into even larger black holes, which glow away more slowly still. If the dark energy persists, it drives the particles apart faster and faster, so that the particles rarely if ever meet. Then clumping, in Greene's words, "will be extraordinarily unlikely" (same section). Greene writes of the era in which the particles are driven apart: "Physicists sometimes liken this future era to the end of time. Not that time stops" (same section). Greene adds that rare events "may punctuate oblivion with infrequent but far-reaching possibilities" (same section).

Chapter 18 told of Greene's worry about Boltzmann brains. A Boltzmann brain would be a brain assembled by chance from particles of the horizon's faint glow, over spans beyond imagining. The brain would hold memories of a life it never lived. Greene asks whether the universe might end long before Boltzmann brains become likely, and

in answer he names three ways in which so early an end might come (*Until the End of Time*, chapter 10, sections "Boltzmann Brains" and "Is the End Near?").

The first way is a change in the Higgs field. The Higgs field fills all of space, with a settled value that gives many of the elementary particles their masses. Greene pictures the settled value held in place as a marble is held at the bottom of an empty champagne flute: walled in on every side, without the energy to climb out. In quantum mechanics a thing held behind a wall can sometimes pass through the wall without climbing over it. Physicists call the passing of a thing through a wall without climbing over it *tunneling*. If the Higgs field tunneled in one small region to a lower value, the region of the new value would spread outward as a bubble at nearly the speed of light. Inside the bubble the masses of the particles would change, and so atoms and chemistry would change too. Greene puts the danger in one line: "quantum tunneling of the Higgs field's value reveals an instability of empty space itself" (*Until the End of Time*, chapter 10, section "The Disintegration of Emptiness"). Greene writes that physicists "cannot pinpoint when the Higgs might make such a jump", for two reasons. First, the timescale depends on properties of particles and forces not yet measured precisely enough. Second, a quantum process "can only be predicted probabilistically" (same section). On the data Greene reports, the Higgs field is likely to tunnel somewhere between floors 102 and 359. Greene judges that a timescale of floors 102 to 359 gives little cause for anxiety. But he notes that there is still a chance that the tunneling could happen today (same section).

The second way is a crunch. If the dark energy leaped by tunneling to a value below zero, its gravity would pull instead of push. The expansion of space would then turn into a collapse. Everything would be squeezed together, to an extraordinary density and temperature, in what physicists call the *big crunch* (*Until the End of Time*, chapter 10, section "Is the End Near?").

The third way is a bounce. The physicist Paul Steinhardt has built a *cyclic cosmology*, first with Neil Turok and later with Anna Ijjas: a history in which regions of space like ours expand, then contract, then rebound into a new expansion, again and again. In the version Greene tells in *The Fabric of the Cosmos*, the cycle comes from branes: a *brane* is an object spread out in two, three or more dimensions, as a sheet is spread out in two. A three-brane is a brane spread out in three dimensions of space, set inside a space with more dimensions. Greene puts the proposal of Steinhardt and Turok this way: "Briefly put, they suggest that we are living within a three-brane that violently collides every few trillion years with another nearby, parallel three-brane" (*The Fabric of the Cosmos*, chapter 13, section "Cyclic Cosmology"). In the newer version Greene tells in *Until the End of Time*, each cycle lasts on the order of hundreds of billions of years. On the newer version the climb up the tower is cut short near the eleventh or twelfth floor. Greene says that the building would then need to be a spiral, with each lap one cycle (*Until the End of Time*, chapter 10, section "Is the End Near?").

Every cyclic picture faces an old question. Greene tells how in the 1930s the physicist Richard Tolman found that entropy builds up from one cycle to the next, and so each cycle lasts longer than the cycle before. Run the cycles backward in time, and they grow shorter and shorter. So, on Tolman's reasoning, only finitely many cycles can come before ours, and the universe has a beginning after all. Greene reports how the builders of the newer cyclic models answer Tolman: in each cycle space stretches far more than it shrinks, so the entropy, though it grows in total, is spread ever thinner. Greene puts the answer in one sentence in *The Hidden Reality*: "The total entropy goes up, but the entropy density goes down" (chapter 5, section "The Past and Future of Cyclic Universes").

Greene weighs the bounce against *inflation*, a brief but enormous burst of expansion of space in the earliest moments. Greene's earlier chapters depend on inflation. Greene reports that "Among researchers, inflation remains the foremost cosmological theory" (*Until the End of Time*, chapter 10, section "Is the End Near?"). In the same section of *Until the End of Time* Greene calls cyclic cosmology "a main competitor to the inflationary theory", but he adds that truth in science is settled by experiments, observations and evidence, and not by polls or popularity. Greene also names the mark in the sky, a sign that astronomers can measure, on which inflation and the cycle differ. He writes that the burst of inflation would likely have shaken space and time so hard that the gravitational waves inflation made, ripples in the shape of space and time, might still be detectable. Of the cycle Greene writes: "The more gentle expansion of the cyclic model results in gravitational waves too mild to be observed" (same section). In *The Fabric of the Cosmos* Greene says that the brane cycle predicts an absence of gravitational waves from the beginning (chapter 14, section "Cosmic Origins"). In *The Hidden Reality* Greene contrasts inflation and the cycle on gravitational waves too (chapter 5, section "The Past and Future of Cyclic Universes").

Where Greene and I Share Results

I agree with several of Greene's results, but my account has arguments of its own for each of them.

First, the emptying of the sky is real. Every observer has an *outside world*. Every observer's outside world empties over a hundred billion to a trillion years, the span that I give (Chapter 16). Greene places the disappearance of the distant galaxies on floor 12, so on his tower the emptying is done by floor 12. The *turn*, which Chapter 9 named, is the passage at the end of the insides of the black holes of one epoch, in which the ten-dimen-

sional geometry of the world restores a large world of space and time. The cold sky at the end of the emptying is real, as far as it goes: it is how the future looks from inside our epoch.

Second, time does not stop, as Greene says. My account has its own argument that time does not stop, set out in Chapter 18.

Third, Clausius's second sentence holds inside our epoch: the entropy of the universe tends toward a maximum. Stars burn, galaxies gather, black holes grow, and every count of entropy made against our epoch's outside world rises toward its largest value. Within our epoch, the count grows, as Clausius said it would. The next question is what happens when our epoch's outside world has emptied.

Nothing Left to Compare Against

The bucket spins, then the water catches up and climbs the walls of the bucket into a hollow. Since Chapter 1 the hollow has stood for one thing: a comparison made from inside the world. The water's climb is something found, and the climb is found against what there is to compare with: the matter within reach. What a comparison finds is its *content*. The physicist Ernst Mach asked of the bucket whether the water turns against space itself or against the matter of the universe. The answer of Chapter 1 to Mach was the principle of comparison.

Now carry the bucket to the end of the emptying. Every observer's future is a road that ends in one of two ways (Chapter 16). The horizon of a black hole is the surface from within which nothing comes back out. The falling observer comes to the end of the interior. On the second road the observer is carried away alone: the expansion bears the observer beyond the reach of all other matter. For the lone body, the observer carried away alone, everything else has passed beyond the lone body's cosmological horizon, the edge beyond which nothing can ever send the lone body a signal. At either end of the road nothing is left within reach, so Mach's bucket, set spinning at either end, would have nothing to measure its

turning against. The Mach principle is the rule that what counts as standing still and not turning, a *frame*, is fixed by the matter there is to compare with, so at either end, The Mach principle then fixes no frame.

The world has ten dimensions: one of time, three of space, and six of possibility (Chapter 7). Nothing ever travels through the six dimensions of possibility, which are met only as the range of what could be found. The six are held small, with one size and one shape, while the large world of space and time is there around the six, with things in the large world to compare against. The hold keeps the forms of matter along with the six. Until the hold ends, the matter that remains keeps its forms, and so in my account the proton lasts. Chapter 16 set the lasting proton beside Greene's working figure. To be definite, Greene takes the proton to break up after about ten to the power thirty-eight years. At the end of an interior, and for the lone body, the condition that holds the six fails: no large world of space and time is left around the six, and so the hold ends.

At the end of the emptying, my picture differs from Clausius's picture. In Clausius's picture, the world at the end of the road has stopped. The matter and light that reached the end still exist. The content of the world is still there, but no large world is left to be the context of that content. Three grounds show what must follow: a context forms again.

Three Reasons a Context Forms Again

The first ground is that content cannot be without a context. Recall the thermometer of the Preface, read this morning and again this evening. The two readings set side by side are a comparison. The warmth found is the content of the comparison, and the two times of day are its context. A temperature can be found only at some time and at some place. Chapter 6 states the third axiom of my account, one of its three starting principles. By the third axiom content and context are distinct and inseparable: there is no content without a context in which it is content, and no context without a content. At the end of the emptying the content still exists. By the third axiom no content exists without a context, so a context must

form again. The old outside world has emptied, and the forms of matter ended with the hold, so the new context can come from neither of them. What remains is the content and the ten-dimensional geometry itself, so the new context forms from the ten-dimensional geometry. When the conditions fail, the ten-dimensional geometry restores a large outside context. It must.

Picture a seesaw, with the three directions of space on one end and the six on the other. Inside a black hole, as the end comes near, the seesaw tips one way: Einstein's equations, written for all ten dimensions, have an exact solution on which the three directions of space shrink while the six grow. The new world begins with the seesaw tipping back. As the new world's own time runs forward, its three directions of space grow while the six shrink back to their held small size. The ten dimensions split into four and six again, but no direction of the new world continues any direction of the old world.

The second ground comes from the *Incommensuration Theorem*, met in Chapter 3, a theorem about what any comparison can find. Something is *continuous* when its content stays the same where its context stays the same, running on with no break or step anywhere. Something is asymmetric when its content differs across a difference of context, and discontinuous when its content jumps where its context stays the same. The theorem depends on no premise taken from physics, so the theorem holds of anything anywhere. Now picture the heat death taken all the way: a final world of featureless emptiness, the same everywhere and at every moment, smooth everywhere, with no break anywhere. The final world of the heat death would be perfectly symmetric, because it is the same everywhere and at every moment, and perfectly continuous, because it has no break anywhere. The pairing of perfect symmetry with perfect continuity is the one character no comparison ever finds. The emptying cannot simply come to rest. Context must be restored. The Incommensuration Theorem also limits how the new world begins. A world that began perfectly even would be perfectly symmetric and perfectly continuous at

once, so the new world cannot begin perfectly even. The new world begins even except for very small unevennesses: the first ripples of Chapter 9, the faint differences of density with which an epoch begins.

The third ground is about Clausius's second sentence itself, that the entropy of the universe tends toward a maximum. Think again of the census of a new country, from Chapter 9. A census counts what the census clerks can tell apart, by the categories that the census forms record. When a new country is founded, its first census starts a count of its own, and does not continue the census of the old country. The largest counts of entropy of all belong to black holes. The count of a black hole's entropy is made against the outside world. So at the end of the interior the count has nothing left to be counted against. The new context does not register the distinctions the old context drew, and that is the reset. The reset destroys nothing. The content and its form are carried across the turn, but the counts made against the old outside world are not. So Clausius's second sentence holds inside every epoch, as a statement about counts made against the epoch's own context. The heat death, a final maximum that holds for ever, has no context in which to be counted.

The three grounds are that content cannot be without a context, that nothing can be both perfectly symmetric and perfectly continuous, and that every entropy is a count made against a context. All three grounds have one result: the emptying does not come to rest in a heat death, and a context forms again.

One End, and No Top Floor

Where does the new world begin? Picture the interior of each black hole as a funnel. Each funnel has its own wall, and in the same way the interiors of different black holes stay distinct along their whole length. The interiors are joined at their ends only because no comparison can tell the ends apart. Nothing pulls the interiors together. No signal from the end of any interior reaches the outside world, so no comparison made in our epoch tells one end from another. And the new world forms with no sep-

arate identity of its own, so no comparison made in the new epoch can tell from which interior any part of the new world's content came. The principle of comparison says that a difference that no comparison can register is no difference, so the ends of all the interiors are one end.

Recall the marks of Chapter 18 once more, set at half the distance to a line, then at half of what is left: the marks come ever nearer the line, and no mark lies on the line. The common end of the interiors has two sides, and the history on each side is like the marks. On our side, the history of a falling body runs toward the end and comes ever nearer the end. On the new side, the history of the new world, traced back by its own observers, comes ever nearer the same end. Neither history reaches the end. The one end of all the interiors is the turn: the limit at which the interiors of all black holes end, approached from both sides and reached from neither. The lone body comes to the turn by its own road. The reason is that for the lone body, too, the large world has ceased to be anything to compare against.

Now set the turn beside Greene's tower. Every floor of the tower counts years of the outside world's time. So the turn is not an event in time at all. Duration, temperature and rate belong to the approach on each side, but none of them belongs to the turn. So the turn is on no floor. On my account, Greene's tower has no top floor beyond which only mist drifts on. The floors of one epoch end where the comparisons made in that epoch end. On the far side, the new epoch's observers piece together their history from the light that reaches them. When the observers trace their history back, the history runs toward the turn, and the observers find the beginning of their history there. In Chapter 9 I identified the Big Bang with the turn, as the turn appears in our epoch's history. Likewise, the Big Bang of the epoch after ours is the turn at the end of our epoch's interiors, seen from the next epoch.

Could the energy that speeds the expansion keep the turn from coming? No, the energy that speeds the expansion cannot stop the turn. The energy that speeds the expansion is the *cosmological constant*. In Chapter 10 I identified the cosmological constant with the part of the vacuum's en-

ergy that a measurement made from inside the universe can take in, the vacuum being empty space with every field at its lowest energy. Within our epoch the cosmological constant is one number, and the constant goes on carrying the galaxies apart in the space between the black holes. The constant does not stop black holes from forming, and it does not stop the interiors of the black holes from sharing one end. At the common end of the interiors the cosmological constant has no effect, because the constant belongs to the relation between an observer and the observer's cosmological horizon, and at the end of an interior there is no outside world and no horizon. The condition that holds the six depends on the large world around the six, and not on the constant, so the cycle of epochs in my account needs the expansion neither to stop nor to reverse. Since the end of one epoch is the beginning of the next, the history of the universe is a cycle, and not a single arc from a Big Bang to a heat death.

The epochs follow one another as the cars of one train do, each car coupled to the car before it, and they do not branch like a tree, because all the interiors of an epoch together give one new epoch. The comparisons made in an epoch find the turn the epoch came from, as the beginning to which the epoch's history runs back, but no comparison made in the epoch finds anything at the turn where the epoch's own interiors end. Ask what came before our Big Bang, and my answer is the collapse of the black-hole interiors of the epoch before ours. Ask what came before the epoch before ours, and the answer has the same form: the collapse of the interiors of an epoch earlier still. The cycle of epochs has no first turn.

No Bubble, and No Crunch

What of the bubble and the crunch, two of the early ends that Greene names? For the bubble and for the crunch my account has a result of its own. The two results agree: the emptying runs its course to the turn.

Start with the bubble. A bubble needs somewhere to lead to, because tunneling leads from a settled state of empty space to a lower settled state beyond a ridge. Physicists call a settled state of empty space with the least

energy nearby a *vacuum*, and a settled state that holds for now but is not the lowest a *false vacuum*. Empty space has two settled features. The first settled feature is the held shape of the six. The six form one shape made of two simple curved shapes side by side: an ordinary sphere, the surface of a ball, and a larger curved shape of four dimensions, the *complex projective plane* of Chapter 7. A theorem of geometry shows that the shape of the six has exactly two quantities that could change: its overall size, and the ratio of the radius of the larger shape to the radius of the sphere. Each of the two quantities is held at one value. No field is threaded through the six, as a thread might be wound through the hole of a ring, so there are no further valleys: no further settled states of empty space, each with its own energy. So the six have one vacuum, and no second one for a bubble to lead to. The turn is not a bubble either, because on its far side there is no second held shape for the six to fall into. In the next epoch the same six reform.

The second settled feature is the Higgs field. The particle of the Higgs field is the Higgs boson, a small ripple of the field, which physicists found in 2012 at CERN, the European laboratory for particle physics near Geneva (Chapter 6). Picture again the bottom of a wine bottle, as Chapter 6 did: a ball balanced on the raised center of the bottom rolls off into the ring of the valley around the center. Let the height of the glass stand for the energy of the field, and the distance from the center for the size of the field's value. My books derive the curve of the field's energy at the energies at which the Higgs boson is measured: a curve with two parts, one pulling the energy down and one pushing the energy up more steeply. When a curve has one part pulling down and a steeper part pushing up, the curve has one lowest value, reached all round one ring: the valley around the raised center. Every point of the ring is equally low, so the ring is one settled state and not many. Beyond the ring the curve rises without limit. So on the curve I derive, the present state of the Higgs field is the field's lowest state.

The Higgs field acts on itself, and the strength of the field's action on itself sets how steeply the second part of the curve, the part that pushes the energy up, rises. In quantum physics a strength of that kind changes slowly with the energy at which the strength is measured. My books work out the strength of the Higgs field's action on itself at the energies where the Higgs boson is measured. Physicists have followed the Standard Model out to energies far higher than any yet reached, in a standard calculation that starts from the measured masses of the Higgs boson and of the top quark, the heaviest known elementary particle. In the standard calculation the strength falls to small or negative values at very high energies. Where the strength is negative, the curve turns down again far beyond the ring, so the curve has a region lower than the present state. In 2018 the physicists Anders Andreassen, William Frost and Matthew Schwartz carried the standard calculation through and found the present state long-lived: about ten to the power one hundred and thirty-nine years, a one followed by a hundred and thirty-nine zeros. The present state would last until about floor 139 of Greene's tower. Floor 139 lies inside the range of floors 102 to 359 that Greene reports for the tunneling of the Higgs field. Floor 139 lies far above floor 12, by which every observer's outside world has emptied (Chapter 16). The lifetime that Andreassen, Frost and Schwartz found comes from the standard calculation, which follows the strength out to energies far higher than my account does. The field has no lower state to tunnel to on the curve derived in my account, at the energies where the Higgs boson is measured.

Then the crunch. Our epoch does not fall back. Space is flat, as Chapter 9 found. A curving of space as a whole is set by one length, its radius of curvature, as the curve of a ball is set by the ball's radius. No comparison made in a new epoch registers the size or mass of the old interiors, so the state at the beginning of an epoch carries over no length from the epoch before. The starting state of an epoch holds one length of its own, about the Planck length. Only counts of states are numbers large enough to stretch the Planck length into a wide radius, and no count crosses the turn, so any radius made from the starting state is about a Planck length.

If an epoch had a radius of curvature of about a Planck length, the epoch would never build the stars and black holes from which the next epoch comes, so no radius of curvature is left, and space has no curvature that could close space on itself. The cosmological constant is positive, since I identified the constant with a share of the vacuum's total energy, and that total is a positive amount. The constant is one number for the one space and time of our epoch, as a map has one scale, printed in one corner, for the whole map (Chapter 10), so the constant cannot shrink, grow, or turn negative. The rate of expansion therefore never falls to zero, because space is flat, the density of matter is never below zero, and the constant is positive and never changes, and so the expansion never stops. The collapse that ends an epoch does happen, but only inside the black holes, one by one, while the space between the black holes goes on growing.

Two Cycles Side by Side

Of Greene's three early ends, the bounce has the most in common with my account. I set Steinhardt and Turok's colliding worlds beside the cycle of my account. The colliding worlds share real ground with it. The cycle of Steinhardt and Turok also uses the accelerating expansion to empty the region each observer can compare with. In Steinhardt and Turok's cycle the change from one history to the next has a finite temperature and density, so the change is no place where physics fails. Einstein's equations give an infinite density at one point: an infinite amount of matter and energy in each cubic meter. Too, the point of infinite density is the edge of one description, and not an edge of the world.

The two cycles differ in where each cycle turns and in what each turn needs. In the brane cycles a field ends each cycle: the field's energy turns negative, the acceleration ends, and some measure of the whole of space shrinks before the collision or the bounce. The dark energy is the cosmological constant, one number at every time within the epoch. So the acceleration of our epoch never ends, and no measure of the space between the black holes ever shrinks. The turn is a limit with no temperature and no

duration. The extra dimensions of the two cycles are different things as well: in the brane models an extra dimension is a region in which branes sit and move, while on my account the six are dimensions of possibility, through which nothing travels. So no brane has anywhere to drift.

Tolman found that entropy building up from cycle to cycle would give the universe a beginning, and both cycles have an answer to Tolman's finding. The two answers differ. In the cyclic models of Greene's chapters, the growing entropy is spread ever thinner. On my account, no count crosses the turn. The reason is that every count belongs to the context it was made against, and the context ends at the turn. A new notebook shows how a count starts again: its pages are numbered from one, and nobody expects page one of a new notebook to carry on from the last page of the notebook before. Tolman's reasoning needs the opposite: that the count is carried from one cycle into the next. Tolman's reasoning rests on one premise: that the entropy of one cycle is carried into the next. Each epoch begins its own count, and the cycle has no first iteration.

The mathematical physicist Roger Penrose has a cycle of his own, which he calls conformal cyclic cosmology. Some changes of scale keep every shape, every angle and every path of light, while letting every size change. A change of scale of that kind is called *conformal*, and Penrose's cycle joins one history to the next by a conformal change. The crossing of a cycle is the passage from one history to the next. In the form of the crossing, Penrose's cycle comes nearer to my cycle than any other cycle does. In both cycles the end of one history becomes the beginning of the next. Both cycles pass through a state with nothing left to measure against, and both join what nothing can tell apart. The two cycles differ in where the crossing lies. Penrose's crossing lies in the remote future of the whole universe, after every black hole has evaporated. A body's *rest mass* is the mass the body has when it is still. Penrose's crossing needs every rest mass to fade to nothing. The dark matter is the matter that gives off no light and shows itself only by its weight. So in Penrose's cycle the dark matter must decay within each history. The turn of my account lies at the end of each interior. The turn needs nothing to fade.

Astronomers have measured three marks in the sky that bear on the brane cycles, Penrose's cycle and my cycle.

The first mark is the *first waves*: the gravitational waves present from the beginning of an epoch. The oldest light is the light left from the early universe, which astronomers call the cosmic microwave background. The *polarization* of light is the way the light is lined up. The first waves would leave a faint twist in the polarization of the oldest light. So we now know that the first waves, whatever their strength, are faint. The first ripples are ripples of density alone. Waves of space and time arise from the first ripples only as a side effect of the ripples acting on one another, so the strength of the waves goes as the square of the ripples' small size. The power of the waves lies far below a millionth of the power of the ripples. So my cycle, like the brane cycles, leaves the first waves too faint to see, but what sets my cycle apart from the brane cycles is the energy that speeds the expansion.

The second mark is the energy that speeds the expansion. When an energy stays the same in every cubic meter as space grows, the energy has an equation of state of exactly minus one. The cosmological constant is one number, so its equation of state is exactly minus one at every era, while in the brane cycles the energy that speeds the expansion is a field that weakens and turns negative. The team of the Planck satellite read their data together with the spacing of galaxies and the light of exploding stars, and they measured the equation of state at minus one and three hundredths, give or take three hundredths (2020). So we now know that, across the history that the Planck measurements cover, the energy that speeds the expansion has behaved as a constant to within a few hundredths.

The third mark is the dark matter, which Penrose's cycle needs to decay. The physicists Vivian Poulin, Pasquale Serpico and Julien Lesgourgues searched the oldest light and the history of the expansion for any sign that the dark matter decays into particles no detector sees. The three physicists found that a decay of the whole of the dark matter would

take longer than about a hundred and sixty billion years (2016). So we now know that the dark matter lasts more than ten times the present age of the universe.

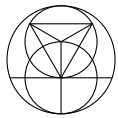
At the back of the book, Appendix A describes what the instruments of the coming decades will measure of all three marks: the first waves, the equation of state and the lifetime of the dark matter.

Where the Road Leads

In Greene's picture a mist drifts on above the top floor of the tower, unless a bubble, a crunch or a bounce cuts the climb short. On my account, nothing cuts the climb short: empty space keeps its two settled features, the expansion never turns into a collapse, and the emptying runs its course. The emptying ends where the bucket has nothing to measure its turning against: at the end of every black hole's interior. At the end of every black hole's interior the ten-dimensional geometry restores a large world, and a new world begins. The history of the universe is a cycle with no first turn, because each epoch begins where the interiors of the epoch before end. I speak of the epochs one after another, and never of all of the epochs gathered into one finished whole. Clausius's second sentence holds inside every epoch. Clausius's two sentences seemed to promise a heat death, but the heat death is a count with nothing left to be counted against.

The cold mist of Greene's top floors is the last view from inside one epoch, and not the last state of the world, since a new world begins where the emptying ends. So we now know that the emptying ends in a turn, not in a final dark, and that at the turn a new epoch begins under the same law as our epoch.

If a new epoch begins, does anything of us cross into it?



Chapter 20

Does Anything of Us Go On?

Beside Until the End of Time*, chapters 8, 10 and 11, and* The Fabric of the Cosmos*, chapter 15.*

What this chapter uses

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

I identify the *first ripples*, the faint differences of density with which our epoch began, with the form of the content carried through the turn (Chapter 9).

A body *carried away alone* is carried by the expansion beyond the reach of all other matter; at the end of a black hole's interior, and for a body carried away alone, the forms of matter end with the hold (Chapter 16).

The emptying ends in a turn, and a new epoch follows with the same structure as ours (Chapter 17).

On 20 August 1977 a rocket lifted the spacecraft Voyager 2 away from Cape Canaveral in Florida. A gold-plated copper disk was fixed to the side of Voyager 2: a record of sounds and pictures of the Earth, made for whoever might one day find the disk. The physicist Brian Greene says that our works live on only for as long as our civilization lasts. So does anything of us go on?

Voyager 1 followed on 5 September 1977, with a disk of its own. The astronomer Carl Sagan of Cornell University led the committee that chose what the two records carry. The two records carry greetings in fifty-five languages, music from many lands, sounds of a rainstorm and of a train, a hundred and sixteen pictures written as signals, and a written message from the President of the United States, dated 29 July 1977. The President wrote the statement to whoever might find the record. The President's statement says of the record, "It is likely to survive a billion years into our future". A few sentences later, the statement says what the record is for: "We are attempting to survive our time so we may live into yours".

The two records are a plain picture of a hope as old as writing. The hope is that something of us is set down, made to last, and sent outward, so that someone, somewhere, at some later time, may find it and know that we once lived on the Earth. How much of that hope can come true in the world that Chapters 16 to 19 described? In that world the sky empties. A body's road is its path through space and time. Every body's road ends alone in the dark or inside a *black hole*, a region whose gravity is so strong that nothing, not even light, comes back out of it. The *turn* comes at the end of every road, and the turn is the one end that the insides of all black holes share. A new world begins from the turn, with a sky of its own.

From Greene's pages

Greene's Measure of What Lasts

Greene takes up the hope of living on in three places. The three places are the things we make, the far future of thought, and the people who come after us.

The first place is the hope of living on through what we make. Greene begins with a picture from the night sky. When we look up, we see stars as they were long ago, because their light is still on its way to us. Greene writes that, in the same way, light reflected from us carries our image outward, "a poetic immortality racing across the cosmos at the speed of light" (*Until the End of Time*, chapter 8, section "Poetic Immortality"). "Back here on earth", he goes on, "poetic immortality takes a different form" (same section). Greene calls the earthly form *symbolic immortality*. Symbolic immortality is a life that goes on in its achievements and its works after the one who lived it has died. Greene traces symbolic immortality from the poet Sappho, some twenty-five hundred years ago, to the playwright William Shakespeare.

Greene also says how long symbolic immortality lasts. Greene writes that the scale of symbolic immortality needs a change of measure, "from eternity to the duration of civilization" (same section). Greene calls the change of measure a significant cost. Greene sets one recognition against the cost: "unlike its literal counterpart, the symbolic version of immortality is real" (same section). The literal counterpart, in Greene's sentence, is living on in one's own life, without end. On Greene's measure, our works live on for as long as there are people to read, hear and remember them.

The second place is the far future of thought. Greene allows that life and mind may need no particular material, and he asks whether some thinker made of anything at all could think forever.

In chapter 10 of *Until the End of Time* Greene adds one more idea. Greene builds the idea on *inflation*, a brief but enormous burst of expansion of space in the earliest moments. Greene reports that within the theory of inflation "the most intently studied mathematical formulations imply that space is endless" (chapter 10, section "Thought and the Multiverse"). Greene follows what he calls "the dominant inflationary perspective" and assumes that space is infinite (same section). An infinite space would hold endlessly many regions the size of our visible universe. Greene says that physicists like to think of each of those regions as a universe of its own. Greene reports that the physicists Jaume Garriga and Alex Vilenkin established that each of those regions can have only finitely many different histories. Since there are endlessly many regions and only finitely many histories, every history happens again and again. Chapter 11 described this picture of histories repeating without end as Greene's patchwork quilt. Greene writes that, by rare strokes of chance, some of those regions will keep life and thought going far longer than our region will.

Greene imagines one last gift, meant for those more fortunate regions. As our own end comes into view, we might pack what we have learned, discovered and created into a capsule, and launch the capsule in the hope that it might someday reach one of those more fortunate regions. Greene reports that Garriga and Vilenkin, with insights from the philosopher David Deutsch, found the plan hopeless. The reason is quantum fluctuations: the chance variations that quantum physics allows in all matter. Garriga and Vilenkin found that, across all those regions and all that time, random quantum fluctuations would make "far more fake capsules than our descendants will be able to produce real ones" (same section). Any reliable imprint of who we were would be lost among the fakes. Greene closes chapter 10 of *Until the End of Time* on one line: "even though we can contemplate eternity, and even though we can reach for eternity, apparently we cannot touch eternity" (same section).

The third place is the people who come after us. Greene recalls the philosopher Samuel Scheffler's question: how would you respond if you learned that, thirty days after your own death, everyone left alive would be wiped out? Greene writes that, for himself, the news that everyone would be wiped out so soon would make much of what he does seem pointless, because what we do is done for a future with people in it. Greene puts his own view in one sentence: "While immortality of the individual may sap significance, immortality of the species seems necessary to secure it" (*Until the End of Time*, chapter 11, section "Descendants"). Greene writes that he cannot be sure how widespread his own reaction would be, but he suspects that his reaction would be common. Greene quotes Scheffler's conclusion, which he says resonates with his own musings: "We need humanity to have a future for the very idea that things matter to retain a secure place in our conceptual repertoire" (same section). Greene also notes that people will respond to the news in different ways. Greene holds that an end in the far future bears on the significance of what we do as an end near at hand would, since "Long or short is a matter of perspective" (same section).

Then Greene lets the reader float along his timeline of the universe to its far end, a cold, dark, nearly empty expanse. At that far end, in Greene's picture, the evidence that we once existed amounts to "an isolated particle located here instead of there" (same section). Next Greene writes two short sentences: "We are ephemeral. We are evanescent". Greene goes straight on, "Yet our moment is rare and extraordinary", and he finds that recognizing how rare our moment is lets us make the shortness of life the basis for value and for gratitude (same section).

One more piece of Greene's thinking bears on whether anything of us goes on: his view of what makes a person the same person. In *The Fabric of the Cosmos* Greene asks whether a person rebuilt atom for atom somewhere else, by teleportation, would be the same person, and he

answers for himself, "Personally, I think so" (chapter 15, section "Teleportation in a Quantum World"). For himself, Greene's test for being the same person is a sameness of physical state. He holds that a living being whose atoms and molecules are in exactly the same quantum state as his own would be him. Greene also loosens his test: he writes that a teleported being that did not match his physical state with perfect accuracy "could very well be fully indistinguishable from me" (same section). Greene marks his view of personal identity as speculation, "the best that I or anyone can do", and as his own belief that identity "all resides in the physical" (same section). He notes that others disagree, and he says that no one can claim the definitive answer.

Chapter 17 followed Greene's question and answered it in two parts. A thinker's findings are the results that the thinker gets by comparing one thing with another. First, every thinker's run of findings ends where the thinker's road ends, at the place where nothing is left to compare against. Second, thinking goes on, because comparison begins again in each new world that begins from the turn.

Where Greene and I Agree

Can our works live on forever? My account has the same answer as Greene, that our works cannot live on forever, and it has its own argument for that answer.

The argument starts from three ideas that this book has used since Chapter 1: comparison, finding, and being real. Every observation and every measurement is a comparison. To be real is to be given in an actual act of comparison, here and now (Chapter 2).

Next, recall the turn, the one end that the insides of all black holes share. Chapter 19 pictured the insides of the black holes as funnels. Each funnel has its own wall. The end of one funnel cannot be told from the end of another by any comparison. So the ends of all the funnels are one end. The one end of all the funnels is the turn. On our side of the turn, a

body falls into a black hole, and the body's history comes ever nearer the end. On the new side of the turn, the new epoch's own observers trace the history of their epoch back. That history comes ever nearer the same end. Neither history ever arrives at the turn, and no act of comparison is made across the turn, so the comparisons of each epoch are made inside the epoch itself.

Three lines say what follows from comparison, finding and the turn:

> "No act of comparison in the next epoch takes in any finding of ours, so no finding, no record and no count of any thinker of our epoch is real there. The thinking of the next epoch continues no thinker of this one".

Our works are records, so no work of ours is real in the next epoch. Our works are real while they are given in acts of comparison. Every act of comparison in which our works are given belongs to our epoch. Greene limits symbolic immortality to the life of a civilization. On my account, the limit on our works is the end of our epoch. The end of our epoch limits how long our works are real, that is, given in some act of comparison, whether or not anyone reads them. Neither limit is eternity.

On my account, too, Greene's capsule cannot carry anything of us to another universe, and for two reasons: one reason holds inside our epoch, and the other reason holds beyond our epoch. Chapter 11 showed that, inside our epoch, the far regions of Greene's endless space are further parts of one world. The reason is that each region touches its neighbors. Chapter 17 showed that a message sent from here toward a region beyond our cosmological horizon never arrives, just as a runner on a track that grows longer ahead of him faster than he can run never reaches the end of the track. Beyond our epoch, no path runs at all: no path runs from any place in our epoch to any place in the next epoch. Nothing escapes as itself, and there is no other universe that a journey could reach.

Picture a country where the only way to travel is by road. For a traveler in that country, a town is a place because roads join the town to other towns. For that traveller, a town that no road reaches is no place at all, however good a car might be. As Chapter 1 showed, space is the pattern of which events can affect which. Space is no container that exists before the events it holds. So the road country is a picture of the world itself: the towns stand for places, and the roads stand for the paths by which an event in one place can affect an event in another. If a path from here can reach a region, the region is, by being reached, a part of our own world. "And any region that no path from here can reach has no position in our pattern, so relative to us it is no place at all". So a capsule that arrives somewhere arrives in our own world, and a capsule bound for somewhere no road reaches has nowhere to arrive.

The Voyager records travel inside our own world, the road country of the picture. The world has ten dimensions: one of time, three of space, and six of possibility, which is the range of what a comparison could find, every outcome it could have. No one travels through the six dimensions of possibility. The shape and size of the six fix which possibilities matter can take. In the same way, the shape and size of a drum fix which notes the drum can sound. Chapter 16 showed that matter does not wear away: the gold and copper atoms of the records get their forms from the hold, and the atoms keep those forms until the hold ends. Chapter 16 also showed where the road of every body ends, and the same holds of every structure: over far longer times still, every structure that remains ends inside a black hole or is carried away alone. The two records are structures, so each record's road ends in one of those two ways: inside a black hole, or carried away alone. The billion years of the President's statement are a short stretch of each record's road.

What a Message Needs

In Greene's telling and on my account alike, the capsule fails, but the arguments for that result differ. The difference begins with one question: what would it take for something of us to arrive as itself?

Take a message first. A message is a record: marks drawn in our world, made to be read by a reader who can tell those marks apart. The grooves of the golden record are marks drawn to be read. A book's letters and a disk's pits are marks drawn to be read too. A mark is a message only for a reader who can set the mark beside the other marks it might have been and tell them apart. At the turn, the new world begins with no record of the world before it. The new world inherits no record and no mark from the old one, and its own context draws none of the old distinctions.

Take a person next, and apply Greene's own test of whether a person is the same person. Greene's test for being the same person is a sameness of physical state: the same atoms and molecules, in the same quantum state. On my account, a comparison is how anyone finds that the rebuilt being and the first being are in the same state, since every finding of sameness or difference is a comparison. Someone would have to compare the two states across the turn to find that something in the next epoch is the same person as someone in ours, and no comparison is made across the turn. So nothing in the next epoch is the same thing as anything in ours, by any finding anyone could make. Greene's test needs a sameness of state, and on my account a sameness is found by a comparison made inside one epoch. So an identity can be found only inside one epoch.

Can a mind be sent on as a program, a person written into some store and run again later? Chapter 17 answered that question: a person is not a very large file. A description of a person, however complete, is not the person. The description is a map of the person, and the person is not in the map.

On my account, the capsule fails, as it fails in Greene's telling, but for another reason. In Greene's telling the capsule fails because random quantum fluctuations make far more fake capsules than we can make real ones, so no finder could pick our capsule out from among the fakes. On my account, when the capsule is sent toward any future beyond our epoch, the capsule fails for a reason that comes before any count of fakes. No reader in any epoch after ours can set the capsule's marks beside anything of ours, so whatever arrives of the capsule arrives as no message at all.

What Does Cross

Is nothing of us left, then, except an isolated particle here instead of there? My account has a different answer. To see the answer, return to the post office picture that Chapter 18 used to show two senses of the word information.

The words of the letter go wherever the letter goes. The office keeps something else: counts of how many letters lie in each bin. The counts belong to the office. When the office closes, the office's counts have nothing left to be counts of. The words of the letter are still the words of the letter.

Physics uses the word information for both things: for what the letter holds, and for what the office counts. What a comparison finds is its *content*. In the first sense, information is the *form of the content*: what the matter is, and which of its possibilities the matter has taken, carried with the matter wherever the matter goes, as the letter's words go with the letter. In the second sense, information is a count of states made against a context, as the office's count is made with the office's own sorting machine.

Whatever falls into a black hole goes inward to the end of the black hole's *interior*, the region from inside which nothing comes back out. The form of the falling thing's content goes inward with the falling thing (Chapter 18). Our epoch's *outside world* is all that can ever reach an observer of our epoch. Every count of states made in our epoch is made against our epoch's outside world. At the turn, every count of states made

in our epoch stays behind, as the office's counts stay behind when the office closes. The form of the content crosses the turn, and it crosses as one form. Each black hole of our epoch carries the form of its own content. Our epoch's outside world was the only context in which one hole's content was ever told from another's. No moment of our epoch's outside world lies at the end of the interiors. So the forms cannot be told apart by any comparison, and the forms cross the turn as one averaged form, with no mark of which black hole each form came through.

Climb once more the worn stone stairs of the old church from Chapter 9. No single footstep can be picked out in the curve, and no footstep has been wiped out of it: each footstep wore its own small share, and the curve is all of them together. The first faint pattern of the next epoch is like the curve of a step: the form of every black hole's content is in the pattern, and no one black hole's share can be picked out. In Chapter 9 I identified the first ripples with the form of the content carried through the turn: the distinctions of the epoch before, averaged and not erased. Our own sky shows the first ripples of our epoch, which are the averaged form of the content of the epoch before ours. The *oldest light* is the glow left from the early universe. The oldest light is warmer here and cooler there by about one part in a hundred thousand. Our galaxies grew from those differences of warmth.

Here is what the crossing of the form of the content means for each of us:

> "Everything of you that ends inside a black hole is kept, as a share of the form of the content from which the next epoch's galaxies grow. Nothing of you crosses as you".

Set the two sentences beside Greene's isolated particle. In Greene's picture, the trace of us is one stray particle in a cold, nearly empty expanse that may go on without limit. On my account, the trace of us is in the whole first pattern of a new sky. Our share is spread through all of that

pattern, as one footstep is spread through the curve of the stair, and the pattern is the one from which the galaxies of the next epoch grow. The share is kept. The share is no message: the form of a message dropped into a black hole arrives as no message, and as the instructions of no one.

The Same Script, a New Performance

Will there be any people after us?

Think of a play performed again and again from one script. For each performance a company builds a stage, sets out the props and counts the audience. When the performance ends, the stage is struck and nothing on the stage is kept. Each performance stands for one epoch of the world. The script comes back each time. The script is the *law*: the structure of the world that my physics derives. The law includes the ten dimensions, one of time, three of space and six of possibility, and the one small shape of the six (Chapter 7). "The law returns in every epoch, and it is not carried across the turn". The law holds again on the far side of the turn, as the script is played again on a new stage.

Stars form in the next epoch, and the stars make black holes, as in ours, because the same law holds again. I end on the sky of the next epoch: "Under that sky, the conditions under which watchers arose in our epoch are present again". Nothing of us is in the picture as itself: not our names, not our records, not our sky.

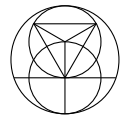
Greene, with Scheffler, finds that the worth of what we do depends on a future with people in it. On my account, every epoch to come has again the conditions under which people arose in our epoch, under the same law. No line of descent runs between us and whoever lives in those epochs: whoever lives in those epochs is no child of ours and reads no record of ours. In the next epoch's sky, the first pattern carries the form of what our black holes took in, as one share of the pattern. A question about meaning remains: whether the worth of what we do depends on the people of those epochs.

What Goes On

Go back to Cape Canaveral in the late summer of 1977. The two Voyager records were made to last, and the two Voyager spacecraft are carrying them outward still. Inside our epoch, the matter of the disks keeps its forms until each disk's road ends, inside a black hole or carried away alone. The President hoped to live into the time of whoever finds the record. His hope holds for every finder inside our epoch whom the record can reach. Whatever of the disks ends inside a black hole is kept, as a share of the first pattern of the next sky, but no finder beyond the turn reads the record, since no record and no mark crosses the turn. So we now know what of us crosses into the next epoch: the form of the content, as one averaged form, which I identified in Chapter 9 with the first ripples from which the next sky's galaxies grow. Nothing of us crosses as itself, and there is no other universe to escape to.

In the far future, under a law that returns in every epoch, everything that ends in our epoch's black holes is kept as a share of the form of the content, and no name, no record and no person is kept.

If nothing of us crosses into the next epoch as itself, where does the meaning of what we do come from?



Chapter 21

Where Does Meaning Come From?

Beside Until the End of Time, *its Preface and chapters 1, 2, 6, 7, 8, 9 and 11,* The Hidden Reality, *chapter 1, and* The Elegant Universe, *chapter 15.*

What this chapter uses

A *comparison* is the act of finding whether two things are the same or different, in some respect; what a comparison finds is its *content*, and the respect in which the content is found is its *context*: what makes the two things compared two (Chapter 1).

Every comparison has three roles: the *stance*, the side a comparison is made from, with the range of what could come; the *act*, the happening itself; and the *structure*, what is laid down in the act, the same for everyone who reads it afterward (Chapter 3).

To *exist* is to stand as structure: timeless, the same in every instance, able to be held in view as a whole; to be *real* is to be given in an actual act of comparison: here, now, with the future still open (Chapter 2).

I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act, happening: the one-way order of the act, its outcome that nothing before it fixed, and its stance (Chapter 14).

A *choice* is an outcome that nothing before it fixed, seen from inside by the one whose outcome it is (Chapter 15).

On the evening of 28 January 1948, the BBC's Third Programme broadcast a debate between the philosopher Bertrand Russell and the Jesuit priest Frederick Copleston. The station set the debate between a Schubert quartet and a program of English folk songs, as the physicist Brian Greene tells the story. Greene closes his book *Until the End of Time* with a conclusion: the universe offers no meaning, and the only meaning is the one we make. Is our own making where meaning comes from? Greene opens chapter 2 of *Until the End of Time* with what Russell said about the future in the 1948 broadcast.

At one point in the debate, Russell turned to what science seemed to say about where everything is going. "So far as scientific evidence goes", Russell said, "the universe has crawled by slow stages to a somewhat pitiful result on this earth and is going to crawl by still more pitiful stages to a condition of universal death". Russell drew a conclusion from the crawl toward universal death, and Greene prints Russell's words: "if this is to be taken as evidence of purpose, I can only say that the purpose is one that does not appeal to me". By universal death Russell meant the heat death of Chapter 19: a final state in which everything has one temperature and nothing ever happens again. Greene sets Russell's two sentences at the start of Greene's own story of how heat runs down, how life and mind arise, and how the far future unfolds. Greene returns to Russell at the close of his chapter 9. Russell found the future bleak. Greene can describe the future in far more detail than Russell could. Greene finds that the future shows our moment of thought to be, in his words, "rare, wondrous, and precious" (*Until the End of Time*, chapter 9, section "A Final Thought on Thought"). The two men looked at one future and judged it two ways. In each judgment a man weighed the worth of the future, from inside the world, while he looked at the world's sky.

From Greene's pages

Greene's Conclusion

How does Greene reach the conclusion that we make the only meaning? Greene begins *Until the End of Time* from one observation about people. In his Preface, Greene follows a line of thinkers who hold that our knowledge that our lives will end drives much of human culture, from art to science. One of the thinkers Greene follows is the historian Oswald Spengler. Greene finds that his own knowledge of his end has a considerable influence on what he does, though the knowledge does not explain everything he does. He imagines that other people's knowledge of their own end also has an influence on what they do, in varying degrees. Greene writes that a mind that knows it will not last feels the pull of what lasts. In his first chapter, Greene states his conclusion in advance, from two premises: "Nothing is permanent. Nothing is absolute". From the two premises Greene goes on: "in the search for value and purpose, the only insights of relevance, the only answers of significance, are those of our own making" (*Until the End of Time*, chapter 1, section "Reflections on the Future").

In chapter 11, his last chapter, Greene gives his reasons for holding that the only answers of significance are those of our own making. Greene says that he is more confident that nature is lawful than that our equations are written in nature's own language. He allows that the equations may be constructs of the human mind. Greene holds that our judgments of right and wrong, and of what matters, are constructs of the human mind too: "We invent these qualities" (*Until the End of Time*, chapter 11, section "Order and Significance"). Greene adds that we start our inventing of right, wrong and what matters from what evolution gave us. In Greene's telling, natural selection is the source of our leanings toward care for the young and fairness among neighbors. Through thought and culture, people then built codes of conduct on the leanings toward care and fairness. Greene finds particles and fields, physical laws and starting conditions beneath everything that happens. Greene says in his own words how

deep his finding about particles and fields goes: "To the depth of reality we have so far plumbed, there is no evidence for anything else" (same section). Greene draws one consequence from his finding about particles and fields, and he states the consequence in one sentence: "Value and meaning, decidedly absent from the bedrock of reality, become intrinsic to a restless urge that elevates us above indifferent nature" (same section).

For Greene, each of us will die, our species will die, and life and mind, at least in this universe, are virtually certain to die, and each of those endings is an expected outcome of physical law. Greene writes that the only novelty is that we notice that we will die. Greene finds that our choices carry weight because our time is short: we choose what to study, whom to love and what to risk, knowing that our mornings are numbered. Greene writes that the universe does not exist to give life a place to flourish: "Looking for the universe to hug us, its transient conscious squatters, is understandable, but that's just not what the universe does" (*Until the End of Time*, chapter 11, section "Meaning"). Greene answers a universe that does not hug us with how unlikely each of us is, as one arrangement of particles among a vast number that might have been, and with how rare our powers are. Greene makes the shortness of life, and the rarity of minds that reflect on themselves, the basis for value and a foundation for gratitude (chapter 11, section "Descendants"). In the end Greene points his reader inward, toward "the highly personal journey of constructing our own meaning" (section "Meaning").

In his earlier books, Greene finds our significance in what we understand and in the search for the laws of nature. In *The Hidden Reality*, Greene answers people who fear that other universes would make us small. He finds our significance in the depth of what we understand, and not in how many of us there are (chapter 1, section "The Cosmic Order"). Greene closes *The Elegant Universe* on the search for the laws of nature as a human drama: each generation builds on the work of

the generation before, standing on its shoulders, and no one can know whether anyone will reach the summit, the end of the search (chapter 15, section "Reaching for the Stars").

From Greene's pages

Language, Story, Myth, Religion and Art

Greene gives three chapters to the things a mind does once the mind knows it will die, and he places those three chapters between his chapters on the physics of life and his chapters on the physics of the far future.

Greene begins with language. Greene reports wide agreement on one point: human speech differs from every call of the animal world, because "Human language is open" (*Until the End of Time*, chapter 6, section "First Words"). In an open language, a small set of sounds combines without limit into sentences about yesterday, tomorrow and things that never were. Greene reports the quarreling theories of when and why we began to speak. He reports that no theory has settled the question.

Greene turns next to story. Greene finds one explanation most convincing: a story is a flight simulator, in which we try out conflict and trouble at no cost and train our intuition for the real thing. Through a story we also enter other minds. A social species must enter other minds to live and work in groups. Greene puts the point about other minds in one line: "With math we commune with other realities; with story we commune with other minds" (*Until the End of Time*, chapter 6, section "Storytelling and Other Minds"). Greene holds, with the writer Karen Armstrong, that myths are often rooted in the experience of death. In Greene's telling, the gods and spirits of myths come from three leanings of an evolved mind. First, an evolved mind attends to

ideas that bend ordinary expectations a little. Second, the mind's language mixes one kind of thing with another. Third, an evolved mind sees thinking agents behind events that have none.

Greene takes up religion next. Greene reviews the explanations researchers give of why religion arose: as a by-product of the brain's ways of reasoning, as a binder of groups, as a guard of reputation, as a calm against the fear of death. Greene reports that none of the explanations has won general agreement, and he states his own leaning: "I remain partial to religion's relevance to our singular recognition of our finite lives" (*Until the End of Time*, chapter 7, section "Individual Adaptation and Religion"). When Greene is asked about his belief in God, he answers in terms of confidence, a measure of certainty set by the weight of evidence, and he writes that by that measure his confidence in quantum mechanics is high and his confidence in the existence of God is low. Greene adds, after Armstrong, that myth once asked for no belief that the stories of myth had really happened: "Myth did not supplicate for belief" (*Until the End of Time*, chapter 7, section "Belief, Confidence, and Value"). When a religious claim is read literally, as a claim about the world that goes against established scientific law, Greene holds the claim false. Greene holds that religion read poetically or symbolically can still give some people community, ritual, guidance and an inward journey, a way of exploring one's own inner life. Taking part in religion poetically or symbolically does not require a belief in the factual truth of the stories of religion.

Greene also weighs the claimed links between Hindu or Buddhist thought and modern physics, and he reports that he has never encountered more than a metaphorical resonance between the ideas of those traditions and the ideas of modern physics. Greene holds that mathematics is the anchor of physics: "Words, however carefully chosen and crafted, are only a translation of the equations" (*Until the End of Time*, chapter 7, section "A Sketch of Religious Roots"). So, in

Greene's judgment, links drawn through words will almost never be more than a poetic alliance between Hindu or Buddhist thought and physics, because the words only translate the equations.

Greene comes last to art. Greene weighs the explanations of art's place in evolution, and he finds "particularly compelling" the explanation in which art trains the single mind in ingenuity (*Until the End of Time*, chapter 8, section "Imagination and Survival"). Yet Greene will not make survival the measure of art's worth: "Significance does not require adaptive utility" (*Until the End of Time*, chapter 8, section "Sex and Cheesecake"). Greene holds that in art there is a kind of truth that no proof contains. Greene takes his phrase for the truth of art from the lyricist Yip Harburg, for whom a song makes you feel a thought: "Feel a thought. For me, that captures the essence of artistic truth" (*Until the End of Time*, chapter 8, section "Art and Truth"). Greene writes that we are all bags of particles. He adds that the physical facts about the particles can fully address how the particles interact and behave. Greene goes on to say that the physical facts about the particles tell us little about how we live through thought, perception and feeling: the facts shed "only monochrome light" on our living (same section).

Greene's answer to where meaning comes from has two halves. First, the universe of particles and fields carries no meaning. Second, we who are made of particles make meaning in our languages, our stories, our myths, our religions and our art.

Where Greene and I Agree

How much of Greene's answer is in my account too? I agree with much of Greene's answer, and I have an argument of my own for each part that I share. I start my argument from the act of *comparison*, as Chapter 1 defined it: the act of finding whether two things are the same or different in some respect. Every observation and every measurement is a

comparison, made from inside the world. Every thought is a *finding*, the outcome of an act of comparison, made against something that is there (Chapter 17). My books work out the physics of the world from the act of comparison. My books build on *An Immanent Metaphysics*, which I wrote in 2001, and there I work out from the act of comparison what can be said of meaning, purpose and value.

Purpose is individual. I find no single purpose and no single value for all of life, for all of consciousness, in any world. Each life has purposes and values of its own, and always more than one. So I agree with Greene that there is no grand design, in the sense of one purpose set for all of life. One sentence of *An Immanent Metaphysics* is close in meaning to Greene's universe that does not hug its squatters: "All the universe values all experiences and creations equally; it has no favorites".

People make the codes of right and wrong, as Greene says. I call the codes of right and wrong *morality*: "an externally defined set of rules in a particular domain". I use the word *domain* for a world or setting in which things happen. So a morality is the set of rules that a culture holds, in its place and age, for everyone in that culture. The rules of a morality are real: they grow from our nature and our history, and they change.

The turn is the passage from which each new era begins (Chapter 9). Chapter 20 showed what of us crosses into the next epoch. What crosses is the *form of the content*: what our matter is, and which of its possibilities our matter has taken. In Chapter 9 I identified the form of the content with the first ripples: the faint differences of density with which the next epoch begins, and the pattern from which the galaxies of the next sky grow. Nothing of us crosses as itself. Nothing of us escapes to another universe either, since there is no other universe to reach. So I agree with Greene that no one should look for meaning in a monument that stands forever.

I also agree with Greene that our moment is rare and precious. The sky we see is full of galaxies, and that sky is a passing stage of our epoch (Chapter 16). The galaxies we count tonight will be carried out of our

sight. Greene responds to the future of an emptying sky by finding our moment rare and precious, and the response sets him apart from Russell. Greene's response is my response as well.

Greene and I differ at one step. Greene finds that meaning is absent from the bedrock of the world (particles and fields, laws and starting conditions), and present only in minds that make meaning. On my account, meaning is present wherever comparison occurs. Meaning is also gathered, and known from inside, in a *chooser*: one who chooses and knows the choosing from inside. This chapter's question, where meaning comes from, depends on the step at which Greene and I differ. Go back to the bucket to see why meaning is present wherever comparison occurs.

Where Meaning Could Stop

The bucket spins, the water catches up, and the water climbs the walls into a hollow. Since Chapter 1, the hollow in the water has stood for one thing in this book: a comparison made from inside the world.

Now stand beside the bucket and ask Greene's question about the bucket. Is there meaning in the water's climb? If we apply Greene's last chapter to the bucket, the answer is that there is no meaning in the water, and that there is meaning in you, watching, if you give the sight some meaning. In Greene's answer there is a line, with the water on one side and you on the other. Yet the water and you are joined by a chain of interactions: the light that leaves the surface of the water reaches your eye, and every event in one connected world is joined to the other events in the same way. So the question is where, along the chain from the water to your eye, meaning would stop.

Where would meaning stop? The answer comes in three steps. Meaningful, here, means significant to the process an outcome came from: the outcome carries that process's character, and it has a share in value.

The first step is about where the edge of meaning could be. Either meaning stops sharply somewhere along the chain, or meaning runs along the whole chain. A sharp edge would need a rule that says where

meaning stops. Drawing the line where meaning stops is itself an act of comparison, since every distinction is made by comparing. Every comparison is made from a *stance* (Chapter 3). A stance is the side from which a comparison is made, including the range of what could come from that side. The outcome of a comparison comes from the comparing, and the outcome bears the mark of the side the comparing was done from. A detector is one case of a comparison whose outcome bears the mark of its side: the yes or no that the detector hears depends on the direction the detector was turned to. The direction is the detector's stance, and the yes or no is the detector's outcome. The mark of the side is what the definition of meaningful calls the process's character. So a comparison made from a stance is itself a meaningful act, by the definition of meaningful. To make a comparison is to weigh one thing against another. So no one can draw the edge of meaning from a place outside meaning, because drawing the edge is itself a meaningful act. After the first step, two branches remain: either every comparison in the connected world is meaningful, or no comparison is. In the first step I show that the definition of meaningful applies to every comparison made from a stance; in the second step I find a case of meaning that applies to itself, which no description can take away, and in the third step I show that meaninglessness has no case of its own.

In the second step, I find a case of meaning that cannot be taken away. In this book, a *choice* is an outcome that nothing before it fixed, seen from inside by the one whose outcome it is (Chapter 15). Every part of every world has some degree of choice, however small, and the water in the bucket is included (Chapter 15). So choice, in some degree, is in every link of the chain from the water to the eye. Take the idea that a choice is meaningful: that a choice carries the character of the process it came from. The idea that a choice is meaningful has definite content, since the idea says what a choice is and what a choice carries. Every idea with definite content contains a distinction between what the idea says and what the idea does not say. Every distinction is made by comparing, from a stance, as the first step showed. The idea that a choice is meaningful is therefore itself meaningful, because by the first step a comparison made from a

stance is a meaningful act. The idea is a case of the very thing the idea is about. When a thing is a case of what it is about, I call the thing a *fixed point*: something that applies to itself, so that no description can take that one case away. So meaning has at least one case. The second branch of the first step is therefore ruled out: the branch in which no comparison is meaningful.

The third step is about meaninglessness itself. Defining meaninglessness means saying what it would be for something to lack meaning, and that saying is an act of comparing from a stance. So, by the first step, the act of defining meaninglessness is itself meaningful, and meaninglessness has no case of its own. So the either-or of the first step comes out one way: meaning is present wherever comparison occurs, and it cannot be removed.

Hear Russell again with that conclusion in mind. Russell's "does not appeal to me" is a judgment of worth, made by a man who had weighed the future and found the future wanting. The act of declaring that nothing matters is an act of weighing worth, and so it is itself a case of mattering. *Nihilism* is the view that nothing is meaningful. Nihilism would need a meaninglessness that is a case of itself, and the three steps show that no meaninglessness is a case of itself.

Meaning is present, in some degree, in the climb of the water, because the climb is a comparison. The degree of meaning in the water is small, as the degree of choice in a stone is small (Chapter 13). The meaning in the water belongs to the water's own comparing. Every comparison has a share in value, held by its outcome for the process the outcome came from. The share in value belongs to the comparing itself. The share is not a message that anyone could read off the water and write down.

Meaning is present wherever comparison occurs, and I never separate meaning from two other things: *purpose*, which goes with what is expressed or done, and *value*, which goes with what is perceived or under-

gone. I join the three in one sentence: "Everything from atoms to universes have non-zero and positive degrees of meaningfulness, purpose, and value".

Why the Bedrock Looks Empty

If meaning is present wherever comparison occurs, why does Greene find no meaning at the bedrock of reality, in particles and fields, laws and starting conditions?

The answer depends on what kind of description Greene's bedrock is. Particles and fields, laws and starting conditions, are what physicists find when they describe the world in the *third person*, as one describes someone else: from outside, the same for every reader, with no one's side in the description. No third-person description, however complete, contains the first person: the side from which a comparison is made (Chapter 14). When an outcome that nothing before it fixed is described from outside, the outcome is randomness, but when the same outcome is had from inside, by the one whose outcome it is, the outcome is a choice (Chapter 15). I draw the consequence for meaning. Whoever describes from outside says what is the case and weighs no worth. So the outcome that nothing fixed shows from outside only as a gap in which there is no pattern. From inside, where worth is weighed, the outcome that nothing fixed is a meaningful choice. The meaning is the description from inside of the event that the description from outside shows as randomness, and the meaning is not something laid on top of the randomness.

Greene's sentence about value and meaning speaks of the bedrock of reality. Greene finds the bedrock to be particles and fields, laws and starting conditions. I agree that a third-person description of particles and fields carries no meaning, because no one weighs worth in a third-person description. The bedrock is one step deeper, in the act of comparison itself. My books work out the particles and fields from the act of comparison.

Every act of comparison is made from a side of its own, a stance. So meaning is absent from the third-person description of the world and present in the world that the third-person description describes.

I answer Greene's remark that words are only a translation of the equations with the same point: a description from outside leaves out the side from which every comparison is made. I set the four ways of knowing in an order of *presupposition*. One thing presupposes another when it cannot be done, stated or checked without the other, while the other can be done without it. In the order of presupposition, deepest first, the four are metaphysics, physics, computation and mathematics. *metaphysics* is the study of the act of comparison itself. *Computation* is the carrying out of steps by a fixed rule. *Physics* is the study of acts of comparison as they happen in the world. *Mathematics* is the study of structure: shape, number and the patterns of valid reasoning, held in view as a whole.

Picture a sum checked on paper. Each line is checked against the rules and against the lines before it, one step after another, and that checking is a computation. The pencil moving across the page is a physical process. Each single check is an act of comparison: the one who checks finds a line the same as the rule or different from the rule. The worked sum on the page is a proof, and the proof needed every one of the steps of checking before the proof could count as proved. The steps of checking needed no proof to be made. So the proof rests on the checking, the checking rests on the physical process, and the physical process rests on comparison. In the picture, the proof stands for mathematics and the checking for computation. The moving pencil stands for physics, and each single check stands for the act of comparison that metaphysics studies.

Mathematics contains the structure of what is found, and mathematics does not contain the side from which the finding was made. Mathematics works as if only the objective existed, and as if there were no subjective side at all. Within mathematics, working as if only the objective existed is right, and working that way is the source of the exactness of mathematics. When mathematics is taken as a description of the whole world, working as if only the objective existed leaves out the side from which every com-

parison is made, and with that side it leaves out the place where meaning shows. Greene comes near the order of presupposition when he says that he is more confident that nature is lawful than that our equations are written in nature's own language. On my account, the comparisons come first, and the equations render the structure of what is found in the comparisons. So when a plain sentence says what a comparison found, the sentence is about the act of comparison itself, while the equation renders the structure of what was found in the act.

Gathered in a Chooser

If meaning is present in the water's climb, is meaning present in the same way in a sonata, and in you? The notes are of one kind throughout, and the sonata is a new kind of thing because of how the notes are held together: each note is chosen, and each choice answers the other choices. I measure the meaning of a work of art or a play by how the choices that make the work hold together. The more the choices that make up the work hang together, the more the work means. The more of the work's details are fixed by cause or by logic alone, the less the work means.

A person who chooses is like the sonata, because the person's choices hold together too, and the person goes one step further than the sonata. In a chooser, choices are made, and the choices are also gathered, held together and carried forward by one self, and known from inside. The *self* is the gathering of choice in a person (Chapter 15). I name the gathering of choice as what is new in kind in a living being: a living being is "a place where choosing is gathered and known from inside". I draw the consequence for meaning: in the living being "true choice is gathered and known from inside, and with it, meaning". *true choice* is an outcome that nothing before it fixed, had from inside by the one whose outcome it is. So meaning runs without a break from the water's climb to the sonata to the person who listens to the sonata. In the person, meaning is gathered with the person's choices, and known from inside.

Greene's view of what minds create is close to my view. Greene writes that "certain special collections of particles can think and feel and reflect, and within these subjective worlds they can create purpose" (*Until the End of Time*, chapter 11, section "Meaning"). I agree that choosers create. Creating belongs to every choice, of every size. Creation is not an event in itself, but an aspect of all events. So we make what we make in a world where meaning is already present, and our making is one more case of meaning.

I also say where the meaning of a life lies: "Life is its own purpose, value, and meaning". There is no greater purpose of life than to live. A life has its purpose, value and meaning in the living of it.

Meaning is found in the act of comparison, here and now, and meaning does not depend on anything being permanent. In his first chapter, Greene argues from two premises: nothing is permanent, and nothing is absolute. From the two premises he concludes that only we can make meaning. I read the first premise, that nothing is permanent, as leading to Greene's conclusion only on a condition that Greene does not state: that a meaning given by the world would have to last. I deny, in one line, the condition that a meaning given by the world would have to last: "Meaningfulness is not defined in terms of perfected, persistent future states". I find meaning in what passes among the domains that take part in an exchange. The example is a speaker, a listener and the language between them, and I set the example out in the paragraph on story later in this chapter. So meaning does not need the sky to last.

The Human Chapters in My Words

What of the things that Greene studies in his three chapters on language, story, myth, religion and art? *An Immanent Metaphysics* has passages on language, story, myth, religion and art. Below, I read Greene's three chapters beside the passages, using this chapter's result: meaning is present wherever comparison occurs.

On language, being in a world and using a language are one in essence: the essence of what it is to be in a domain is the same as the essence of what it is to use a language. A domain, again, is a world or setting in which things happen. Events occur within domains in the same sense that messages occur within communication. So, on my account, to come to speak is to come to be in a world in a new way. Greene comes close to the claim that speaking is a new way of being in a world when he judges that, once the human mind had language, "our species' engagement with reality was poised for radical change" (*Until the End of Time*, chapter 6, section "Why We Spoke").

On story, I find meaning in the passage between minds that Greene finds in story. When I tell you something, three domains are at work: mine, yours, and the language between us. In the language, I turn what I mean into a sentence, and you turn the sentence back into what you understand. Myself is a domain; yourself is a domain; the language is a domain. Greene says that with story we commune with other minds. On my account, communing with other minds is one of the places where meaning is found: meaning is found in the passage from the choosing of one mind to the choosing of another.

On myth, I give each of three words its own place: fact, truth and myth. That book is written as numbered *aphorisms*: short statements, each complete in itself. In the chapter on art, one aphorism pairs fact, truth and myth with the three roles in every act of comparison: the structure, the act and the stance (Chapter 3). The aphorism sets fact with the structure: what is laid down the same for everyone who consults the structure. The aphorism sets truth with the act: the happening here and now. And the aphorism sets myth with the stance: the side from which a comparison is made, including the whole range of what could come. Because myth is set with the stance, a myth is told from the side of the one who compares. So a myth is not to be judged as a statement of fact, since fact belongs to the structure. The aphorism on fact, truth and myth also states that what is mythic or mysterious tends to carry more meaning than what is wholly factual: "The more one comprehends this world (in terms of laws and

facts) the less one finds significance and meaningfulness in life". When a person comprehends the world in terms of laws and facts, the person describes the world in the third person. No one weighs worth in a third-person description, as this chapter showed in the section on why the bedrock looks empty. So when a person comprehends the world in terms of laws and facts, the description leaves meaning out. The meaning is still present in the world that is described. Greene makes a near point with his phrase "only monochrome light": the physical facts tell little of how we live through thought, perception and feeling.

On religion: a mind tends to grow in spiritual awareness when the mind reflects on itself and is in vivid and close contact with nature. When minds that grow in spiritual awareness live together in societies, as in human civilization, their spiritual awareness comes in time to be reflected in public religions. The central concern of public religions is suffering and its resolution, which I call "a mythos of redemption". A *mythos* is a body of shared story, and in "a mythos of redemption" the shared story is the story of release from suffering. So spiritual awareness is the root of public religion: the thing that public religion grows from. Suffering is the central concern of the religions that grow from spiritual awareness. Greene leans toward one root of religion among the explanations he reviews: our knowledge of our own end. Both roots lie in what a mind lives through and knows from inside: in Greene's telling, the mind's knowledge of its own end; in mine, the mind's spiritual awareness, grown in close contact with nature. So the two roots agree in kind. On both roots, religion has its source in what a chooser knows from inside, where this chapter found meaning gathered in a person.

On art, Greene and I agree that significance does not need survival to justify it. On my account, significance in a work of art comes from the coherence of its choices, as in the sonata. Greene says that the physical facts can fully address how the particles behave, and his first clause agrees with the first half of a finding of Chapter 13: every event in a living thing is a physical process. The finding of Chapter 13 has a second half: no description, however refined, contains everything it describes. The second half of

the finding agrees with Greene's second clause, that the physical facts shed only monochrome light on how we live. On my account, what Harburg calls feeling a thought is a comparison known from inside: the side that the third-person description leaves out.

Morality and Ethics

If we invent our judgments of right and wrong, as Greene holds, is anything about choosing well left that no one invents? I agree with Greene about morality, the rules of a culture. Beneath morality there is something that no one invents. I call what is beneath morality *ethics*, and define ethics as the study of, and keeping to, "the principles of effective choice": the conditions under which choosing goes well, for any chooser at all. The absolute principles of ethics are common to all of consciousness (all individuals). The practice of the principles of ethics belongs to each chooser, and it cannot be prescribed from without.

I put the two principles of ethics, symmetry and continuity, in one picture: a friendship in which you can say anything. In the friendship, what you mean arrives as you meant it. And the answer you get is in proportion to what changed. Content is what is found in a comparison, and context is the respect in which it is found (Chapter 1). The first half of the picture is *symmetry*, a word from Chapter 3: the same content across a different context, as a square turned a quarter of the way round is the same square. The second half is *continuity*, also a word from Chapter 3: the same content where the context is the same, as a dimmer knob turned by a hair changes the light by a hair. In the friendship, each of the two principles has its own side. Symmetry is in what is meant, and continuity is in the exchange that carries what is meant.

The relation between any chooser and the world is a channel of the same kind as the friendship: what one side sends goes to the other side, and an answer comes back. Every choice goes out into the world, and the world answers. When the channel keeps both principles, what the chooser chooses arrives as it was chosen, and the answer comes in proportion.

Then choosing goes well. When the channel breaks, the choice does not arrive as it was chosen, and the chooser loses the very thing choosing is for. I draw the consequence: when a chooser keeps the channel between himself and his world whole, the chooser is doing what he most deeply wants.

Morality is made by people, as Greene says, and morality differs from culture to culture. The principles of ethics are the same wherever there is a chooser, because they come from what choosing is. So the principles of ethics bind every chooser without being a rule laid on anyone from outside.

Where Meaning Comes From

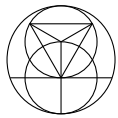
Return once more to the broadcast of January 1948, and to the bucket. Russell weighed the future he saw, a crawl toward universal death, and found that the purpose shown by that future did not appeal to him. Greene weighs a fuller future and finds our moment rare and precious, and he tells us to make our own meaning. Both men are comparing, from inside the world, and both judgments are cases of meaning. The water climbing the walls of the bucket is a comparison too, small and simple, and meaning is present in the climb of the water, in the climb's own degree. So meaning comes from the act of comparison, the act from which every chapter of this book has started.

A third-person description of particles and fields leaves meaning out, as it leaves out the side from which every comparison is made. The world that the description describes leaves nothing out. I name three edges of reality. The first edge is the cosmological horizon: the edge beyond which nothing can ever send us a signal (Chapter 16). The second edge is the Planck scale: the scale at which neither the smooth description of space and time nor quantum mechanics (the theory of atoms and of the particles smaller than atoms) governs, and both are needed (Chapter 5). The third edge is the turn, where the comparisons of one epoch end and

the comparisons of the next epoch begin (Chapter 19). The three edges contain every act of comparison made from inside the world. Then I turn to you, the reader: "Your reading of this sentence is one such act".

We now know that meaning is present wherever comparison occurs, and that in a chooser meaning is gathered and known from inside. We make meaning in a world where meaning is already present. The principles of ethics, unlike morality, are common to every chooser. So the meaning of what we do is present in the acts themselves, here and now. The meaning of our acts does not depend on the sky lasting. And in every epoch in which comparison begins again, meaning is present again.

What, then, does a reader of Greene now hold that is not in Greene's four books?



Chapter 22

What Do I Now Hold?

Beside the closing chapters of Greene's four books: The Elegant Universe*, chapter 15;* The Fabric of the Cosmos*, chapters 11 and 16;* The Hidden Reality*, chapter 11; and* Until the End of Time*, chapter 11; with passages from* The Elegant Universe*, chapter 1;* The Fabric of the Cosmos*, chapters 3, 5 and 6;* The Hidden Reality*, chapter 6; and* Until the End of Time*, chapters 5, 9 and 10.*

What this chapter uses

The *principle of comparison*: a difference that no comparison can register is no difference (Chapter 1).

The *Incommensuration Theorem*: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. Two pairings remain possible: continuity with asymmetry, like a smooth ramp that rises unevenly, and symmetry with discontinuity, like a ladder whose rungs are all alike. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 3).

I identify quantum theory with the description of the content of physics, which is what is found in the comparisons of physics, and general relativity with the description of the context of physics, which is the arena in which that content is found (Chapter 5).

The *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world; an *epoch* is each era of the world, from one turn to the next (Chapter 9).

I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act, happening: the one-way order of the act, its outcome that nothing before it fixed, and its stance (Chapter 14).

A *choice* is an outcome that nothing before it fixed, seen from inside by the one whose outcome it is (Chapter 15).

Meaning is present wherever comparison occurs (Chapter 21).

On 23 April 1992, at a meeting of the American Physical Society in Washington, George Smoot reported what a satellite had found in its first year of mapping the oldest light in the sky. Smoot was a physicist at the Lawrence Berkeley Laboratory. You have read Brian Greene's four books,

and you have read twenty-one chapters that set my account beside Greene's four books. What do you now hold that you did not find in his four books? The answer begins in Washington, with the faint pattern that Smoot reported.

A Faint Pattern Across the Whole Sky

The satellite was the Cosmic Background Explorer, known as COBE, which NASA launched in 1989 to map the *oldest light*, the light left from the early universe, which astronomers call the cosmic microwave background. The oldest light was released when the first atoms formed. Since then the expansion of space has stretched the oldest light into microwaves a millimeter or two long. COBE carried *radiometers*, receivers for microwaves. Each radiometer measured the difference between two patches of sky about sixty degrees apart. The COBE team built its map of the whole sky from a great many differences between pairs of patches, and never from the temperature of one patch taken alone, so the map was made, from start to finish, by setting one part of the sky beside another. Smoot had measured the oldest light by setting one part of the sky beside another fifteen years before, when he and his colleagues flew a receiver aboard a U-2 aircraft and found our own motion against the oldest light (Chapter 9).

COBE found a faint unevenness. The oldest light is warmer here and cooler there, by about one part in a hundred thousand of its temperature. The warmer and cooler patches are many times wider than the full Moon. Greene gives the size of the differences of temperature in *The Fabric of the Cosmos*: "while the temperature might be 2.7249 Kelvin in one spot in space, it might be 2.7250 Kelvin in another" (chapter 11, section "The Golden Age of Cosmology"). A *kelvin* is one degree counted up from absolute zero, the coldest there can be. Every later map shows COBE's faint pattern in finer detail, from the map of the Wilkinson Microwave Anisotropy Probe, launched in 2001, to the maps of the satellite named for Max

Planck. So we now know that the oldest light carries a faint unevenness across the whole sky. The galaxies grew, by gravity, from the small differences of density that the unevenness records.

Greene tells where the pattern came from in the terms of two ideas. In Greene's telling, *inflation* is a brief but enormous burst of expansion of space in the earliest moments. *Quantum fluctuations* are the chance variations that quantum physics allows in all matter. He writes that the small differences follow a pattern that "can be explained by attributing them to the same mechanism that has been suggested for seeding galaxy formation: quantum fluctuations stretched out by inflation" (same section).

The pattern comes from the end of the era of the world before ours. The *turn* is the passage at the end of the black-hole interiors of one era of the world. At the turn, the ten-dimensional geometry restores a large world. An *epoch* is one era of the world, from one turn to the next. The matter of an epoch took part in many findings, and every one of those findings came out one way and not another. The *form of the content* is the pattern of all the outcomes of an epoch's findings: which alternatives the matter took. The *first ripples* are the faint differences of density with which our epoch began, and in Chapter 9 I identified the first ripples with the form of the content carried through the turn: the distinctions of the old epoch, averaged and not erased. By the identification of Chapter 9, the faint pattern in the microwave sky is the averaged form of what those black holes took in. In the quoted sentence, "those black holes" are the black holes of the epoch before ours, and our epoch began at the common end of their interiors. On the identification of Chapter 9, the map that Smoot showed in Washington shows, in one picture, the whole span of time that this book has followed: the far future of an earlier sky, the beginning of ours, and the galaxies over our heads tonight.

Where Greene's Four Books End

How does Greene close each of his four books? He ends each one by looking back over the whole of that book. *The Elegant Universe* ends on the long climb of human understanding, each generation standing on the one before: "we are fulfilling our part, contributing our rung to the human ladder reaching for the stars" (chapter 15, section "Reaching for the Stars"). *The Fabric of the Cosmos* ends on a hope. Greene hopes that the sky itself carries the mark of the smallest processes in nature, stretched across the heavens by the expansion, so that "all we need do is learn how to recognize the pattern". In the last words of *The Fabric of the Cosmos*, Greene pictures us confirming our theories of the very small "by turning our most powerful telescopes skyward and gazing silently at the stars" (chapter 16, section "Inner and Outer Space").

The Hidden Reality ends on a question Einstein asked, which Greene states as "Einstein's famous query, asking whether the universe has the properties it does simply because no other universe is possible". Greene answers on a condition: if some or all of the mathematics that led physicists to parallel worlds proves relevant to reality, then, in his words, "Our universe is not the only one possible" (chapter 11, section "Should We Believe Mathematics?"). This book's answer, from Chapter 11, is that there is one world, and that each of its constants has one value in it. *Until the End of Time* ends on meaning. Greene writes, "Particles are not endowed with purpose". He goes on: "Instead, certain special collections of particles can think and feel and reflect, and within these subjective worlds they can create purpose" (chapter 11, section "Meaning").

All four endings carry the hope with which this book began: a single explanation of the whole world, from strings far smaller than an atom to the last thought in an emptying sky, with ourselves inside it. Beside Greene's hope this book has set a second argument. The second argument is mine, and it starts from the act of comparison.

The Ground Under Every Finding

What did every chapter of this book start from? Go back to the picture with which this book set out, in the last lines of its Preface and again in Chapter 1: a bucket of water, hung from a twisted rope and let go, as Isaac Newton described the bucket in 1687. The water at first stays flat, then turns with the bucket and climbs the walls into a hollow. The hollow is something found. To find anything at all is to make a *comparison*: to find whether two things are the same or different, in some respect. What a comparison finds is its *content*. The respect in which a comparison finds its content is its *context*: what makes the two things compared two, as the morning and the evening make two readings of a thermometer two. A *finding* is the outcome of one act of comparison, laid down as a record. Chapter 1 showed what the water is spinning against. The matter there is to compare against fixes what counts as still and what counts as turning: the Earth, the Sun and the stars, all the matter within reach that the water can be compared with. I call the principle that settled what the water spins against the principle of comparison. The principle of comparison says that a difference no comparison can register is no difference. Every chapter since Chapter 1 has used the principle of comparison.

I also use a theorem about comparison itself, beside the principle of comparison. Chapter 3 first showed four kinds of finding, and only then named the Incommensuration Theorem, which is about those four kinds. Turn a dimmer knob by a hair and the light changes by a hair: the same content where the context is the same, which is *continuity*. Flip a light switch and the room changes all at once: a different content where the context is the same, which is *discontinuity*. Turn a square a quarter of the way round and it is the same square: the same content across a different context, which is *symmetry*. Walk a rocky coast where no cove repeats the cove before it: a different content across a different context, which is *asymmetry*. The *Incommensuration Theorem* says that nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both per-

fectly asymmetric and perfectly discontinuous. The Incommensuration Theorem is a structural fact about the nature of knowing. My first book is the book on which all my other books build, and it states the first half of the theorem in these exact words: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". Chapter 3 gave the proof of the Incommensuration Theorem from the meanings of sameness, difference, content and context alone, so the theorem takes nothing from any measurement or from any law of physics. Every measurement is a comparison, so every measurement is an instance of the Incommensuration Theorem. I start from one ground: the act of comparison, together with the Incommensuration Theorem about comparison. Every other step in this book starts from the act of comparison and the Incommensuration Theorem.

The bucket has come back again and again in this book, and each time it has been the picture of one thing: a comparison made from inside the world, found against what is there to compare with. In Chapter 4 you hung the bucket, let it go and left the room, and the water climbed the walls into a hollow whether anyone was there or not: a finding needs no watcher. Now try to place the lowest point of the hollow as finely as anything can be placed. For any body at all, the finest bound on placing is about a *Planck length* (Chapter 5). A Planck length is a length built from the constants of nature alone. A Planck length is about a millionth of a billionth of a billionth of a billionth of a centimeter. The water is made of molecules, so its surface blurs into separate molecules long before the scale of the Planck length. The Planck length is a limit on how finely a comparison made from inside the world can place a body. The *arena* is space and time taken together, as the setting in which everything that is measured is measured. The arena in which the bucket turns has no least length. Now stand and watch the water climb. Chapter 14 showed that the water's comparing is real and simple, consciousness in the smallest degree. You, the watcher, have many comparisons gathered into one self, known from inside. In Chapter 16, a hundred billion years from now, an astronomer's bucket of water still climbs the walls. The water turns

against the matter of the one galaxy left in her sky. Follow any body to the end of its path. The body ends inside a black hole, or *carried away alone*: carried by the expansion beyond the reach of all other matter. Chapter 19 showed that the end of that path is the place where the bucket has nothing to measure its turning against. So whatever you find, you find by comparing, from inside the world, against what is there. The bucket shows the ground of this book in one picture.

The Arena, the Two Theories and the Six

Where is every finding made, and what are the two great theories of physics, quantum theory and general relativity, about? Chapters 1, 2, 5, 6 and 7 answered both questions with five pictures. The five pictures are a falling pen, two loaves, a ladder and a ramp, a railway and a drum.

Hold a pen at arm's length and let go. The pen falls because the arena at the height of your hand differs from the arena at the height of the floor, and the pen follows the straightest path that the difference allows. In Chapter 1 I identified gravity with the shape of the arena, the way space and time are bent from one place to the next, and not with a force acting within the arena. Space is no container standing behind things, because space has no properties apart from the causal relations among the events in it.

Greene's sliced loaf, the whole of space and time laid out at once, pictures what exists. Now picture a second loaf, still rising in the oven this morning: the rising loaf pictures what is real. In this book, to *exist* is to stand as structure, laid out whole and the same for everyone who looks at it. To be *real* is to be given in an actual act of comparison, here and now. The whole pattern of space and time exists, laid out as structure, as Greene's loaf is laid out. What is real is given in an act of comparison, here and now. The past has been given, and the future has not yet been given.

Quantum theory is the theory of atoms and of the particles smaller than atoms. *General relativity* is Einstein's theory of gravity. The four kinds of finding are symmetry, continuity, discontinuity and asymmetry. If you ask

for two of the four kinds at once, each perfect, the Incommensuration Theorem allows two pairings, and only two. The first pairing is symmetry with discontinuity, like a ladder whose rungs are all alike (symmetry) and separate (discontinuity). The second pairing is continuity with asymmetry, like a smooth ramp (continuity) that rises unevenly up a hill (asymmetry). I call the two pairings the two *surviving pairings*. Quantum theory and general relativity are both right about one world, each in its own surviving pairing. Quantum theory has the ladder's pairing, and general relativity has the ramp's. In Chapter 5 I identified quantum theory with the description of the content of physics. I identified general relativity with the description of the context of physics, the arena. Every body has a *gravitational size*, the length at which its own gravity would become strong. Every body also has a *quantum size*, the length within which quantum theory will not let the body be pinned down. The *Planck scale* is the scale at which a body's gravitational size equals its quantum size. At the Planck scale the two theories meet, since both theories apply to a body of that scale. The two theories stay two, and the arena has no least length.

Picture a railway: its timetable and its track are joined at every station, and neither is made of the other. There is one theory of everything, of the railway's kind: two descriptions joined from one source, as the timetable and the track are joined, with three forces side by side that stay three, and a closed list of kinds of particle with four values taken from measurement. Besides gravity there are three forces. The electric and magnetic force acts between electric charges, and light is a wave of the electric and magnetic force. The weak force changes one kind of particle into another in radioactive decay. The strong force binds quarks, the pieces of which protons and neutrons are made. The *Higgs field* fills all of space, and its settled value gives many of the elementary particles their masses. The four values taken from measurement are the strength of gravity, the strength of light (the strength of the electric and magnetic force), the strength of the strong force, and the value of the Higgs field in empty space. For a reader of

Greene, a unity of the railway's kind has what Greene hoped for: every particle and every force has its place, and all of them are reached from one stated principle.

The world has six dimensions of possibility as well as one dimension of time and three of space. I call *the six* dimensions of possibility the six. The six are everywhere you are, held small as content of the large world. You never travel through the six: you know the six only as the range of what a comparison could find. The shape and size of the six determine what can be found, as the shape and size of a drum determine which notes the drum can sound. String theorists found that fields called *fluxes* can thread the holes of the small shape into which string theory curls six dimensions, each hole a whole number of times. Nothing is threaded through the six. The shape of the six is like a ring with nothing wound through its hole. So the shape leaves no whole number to count and no choice to make. A *vacuum*, for the six, is one size and shape that the laws of motion keep steady, so that small pushes roll back. String theory's forms make a many-valleyed country called the *landscape*. On the landscape our world would be one valley picked out of a vast number. The world has ten dimensions of three kinds, one of time, three of space and six of possibility, and the six form one small shape, the same at every point, so there is one vacuum and no landscape.

Agreement, Outcome and the Order of Time

What happens in one single finding? Chapters 3, 4 and 8 answered that question in three steps. The steps are where two particles far apart agree, how one outcome is found, and why the record follows the act.

Picture two decks of cards, enchanted together and then parted, one kept at home and one taken to Paris. Each deck gives a list as random as a shuffle can make it, and the two lists match when the lists are set side by side. Two *entangled* particles, measured far apart, agree in the way the two decks agree: in a pattern that no separate instructions carried by each particle could give them. The agreement of entangled particles is found

where their records are compared, and nothing passes between the particles to make the agreement. The physicist John Bell used the word "local" in a broad sense: a world is local when every agreement between what is found in two places is carried by causes that travel from place to neighboring place. In the narrow sense, a world is local when no signal travels faster than light. The universe is local in the narrow sense, and it is not local in Bell's broad sense.

In 1989 the physicist Akira Tonomura sent electrons, one at a time, toward a screen. On Tonomura's screen each electron laid down one dot, and the dots, one by one, built stripes. Before each electron set out, its *state* described every place where the electron could be found, with the likelihood of each place. An interaction lays down a record with no mind needed, as a hoof lays down a print in sand. The state describes completely what could be found, and nothing beneath the state fixes which outcome is found. One outcome is given in an act of comparison, which is any interaction that lays down a record. And no later choice changes a record already made.

A walker's step in wet mud leaves a footprint, and the print follows the step, as the record follows the act. Time has a direction in every single comparison, the act before the record, and this direction needs no special beginning. *Entropy*, in this book, is a count of the arrangements of a thing's parts that some context cannot tell apart. An egg splatters on the floor and stays splattered. In things of many parts, the direction of time shows at our scale as the growth of entropy. Only the growth of entropy needs an orderly beginning.

A Beginning, a Small Number and One World

What banged, why is the energy of empty space so small, and is ours the only universe? Chapters 9 and 10 asked the first two questions, and Chapters 11 and 12 asked the third.

What banged is the turn: the passage at the end of the black-hole interiors of the epoch before, which restores a large world, and which I identified in Chapter 9 with the Big Bang as it appears in our epoch's history. The sky is the same in every direction and space is flat because of the turn, with no inflation needed. In Chapter 9 I also identified the first ripples, the faint differences of density with which our epoch began, with the form of what came before, averaged and not erased. The first ripples are like the worn stone stairs of an old church, which keep a share of every footstep in one curve: every finding of the epoch before left its share in one averaged pattern. The ripples that COBE mapped are the first ripples.

The energy of empty space speeds the expansion of the universe, and in our epoch's one geometry the energy of empty space is one constant number, as a map has one scale, printed in one corner, for the whole map. The *vacuum's energy* is the energy that the quantum activity of empty space carries. No measurement made from inside the world takes in the vacuum's energy whole. The constant number is the *cosmological constant*, which I identified in Chapter 10 with the part of the vacuum's energy that a measurement made from inside the universe can take in. So the energy of empty space is small because it is the share of the vacuum's energy that a measurement from inside takes in, and that share is one constant for our epoch. The share also gives the *matter fraction*, the part of the universe's energy that is matter: it comes out at 31.1 percent, beside the 31.5 percent measured from the oldest light.

Greene follows lines of theory to other universes: bubble universes made by an inflation that never ends, the many valleys of the landscape, and universes on *branes*. A brane is a membrane of three dimensions float-

ing in a larger space, which Greene calls the *bulk*. The *particle horizon* is the edge of everything whose light has reached us. There is one world. Each region beyond our particle horizon touches its neighbors, so each such region is a further part of our one world. On my account, Greene's bubbles, valleys and branes are not further worlds, for three reasons, one for each. The bubbles would need a long inflation, and the first ripples leave no room for a long inflation. The valleys would need many shapes for the six, and the six have one shape. The branes would need a bulk to drift in, and the six are dimensions of possibility, which leave a brane nowhere to drift.

Greene's other worlds that remain lie in four places. The four places are the quantum, the *hologram* (his picture of a universe mirrored exactly on a distant boundary), the computer and mathematics. A mathematical structure exists without being real, as the rules of chess exist and tonight's game at the chess table is real. Each of the four kinds of other world is the one world met in one of four ways. The worlds of the quantum are the one world met as possibility. The hologram is one of two descriptions of one system. A world run in a computer is a restricted region of the one world. A world of mathematics is structure that exists without being real.

Choosers in the World

If the universe runs down, how does it build us, and do we choose? In Chapters 13, 14 and 15 I turned from the cosmos to us, the ones who ask.

A tray of water in the freezer gives up its heat to the cold air and orders itself into ice. A cloud of gas falls together under its own weight, heats and shines. A universe running down builds stars, and us, because a thing gains order by passing more disorder to its surroundings, and the thing settles where its own entropy and the entropy of its surroundings, taken together, are greatest. For matter held together by its own gravity, clumping raises the entropy of the matter and its surroundings taken together, and every count of entropy is made relative to a context.

Among the things a universe running down builds are brains. A *third-person* description is one that anyone can check from outside, told of a thing as "it". A *first-person* description is had from inside, by the one to whom the thing happens. The *stance* of a comparison is the side the comparison is made from, with the range of what could come. Consciousness is real. In Chapter 14 I identified consciousness with three features of an act of comparison taken whole, in a context, as one act, happening: its one-way order, its outcome that nothing before it fixed, and its stance. In the bucket, each act of the water's comparing has a one-way order, an outcome that nothing before it fixed, and a stance, as every act of comparison does. So the water's comparing is consciousness in the smallest degree. No third-person description, however complete, contains consciousness, because the first person is the side from which every comparison is made, and a description is laid out as structure, the same for everyone who reads it. Picture a map of a walk you have taken, every turn of the path drawn. Anyone can read the map, and the walking was had only by you, the walker. In Greene's picture a person is a bag of particles. The bag of particles is the whole of what a person is made of. The story told of that bag in the third person is the map, and the person's awareness is the walk.

A player piano plays notes fixed in advance. A pianist plays each note in the light of the last, and the next note is the pianist's own. Every world is causal to a degree above zero and never completely determined, so every world has outcomes that nothing before them fixes. An outcome that nothing before it fixes is *hard randomness* seen from outside and *choice* seen from inside. The pianist's next note is a choice, and the player piano's notes, fixed in advance, are not choices. So the one who chose could have chosen otherwise, and responsibility rests on the fact that the chooser could have chosen otherwise.

The Emptying and the Turn

What comes at the end of the far future that Greene draws? Chapters 16 to 20 followed the far future to its end, and across the turn that ends it.

Every observer's *outside world* is the whole of what can ever reach that observer and be set beside the rest of what the observer receives. The expansion of space never turns around and contracts. The *proton* is the particle at the core of every atom that carries a positive electric charge, and the proton lasts. *The hold*, in this book, is the large world of space and time keeping the six at their one size and shape, and keeping with them the forms of matter. An ice sculpture keeps its shape while the room around it stays cold, and matter keeps its forms in the same way while the hold lasts: the cold room is the large world, and the sculpture is matter. The sky does go dark for every observer. The outside world of each observer empties within a hundred billion to a trillion years, and the stars that remain burn out. The matter of the burned-out stars lasts, and it keeps its forms until the hold ends.

The *cosmological horizon* is the edge beyond which nothing can ever send us a signal, and the cosmological horizon glows faintly. Picture a room whose walls stay faintly warm through our whole epoch: the walls are the cosmological horizon, and their warmth is its glow. The warmth that reaches any thinker from outside falls toward the warmth of the horizon's glow, and never lower. Every observer's road comes to one of two ends: the observer falls into a black hole, or the observer is carried away alone, while everything else passes beyond its cosmological horizon. Every thinker's run of findings ends where the thinker's road ends, where nothing is left to compare against, whatever the thinker's store of energy or temperature. Thinking goes on, because comparison begins again in each epoch that follows.

On Greene's Empire State Building of time, each floor stands for ten times as many years since the beginning as the floor below it, so that floor 68 stands for a one followed by sixty-eight zeros of years. The black holes

are the last of the ashes, the last remnants left in the sky. A black hole of the Sun's mass glows away by about floor 68, and the largest black holes glow away by floor 102. What fell into the black holes is kept as the form of the content. The form of the content is information in its first sense: what the matter is, carried with the matter.

The emptying ends in a turn, and not in a final dark. At the turn a new epoch begins under the same law as the epoch that ended. At the end of every black hole's interior, the ten-dimensional geometry restores a large world. The history of the universe is a cycle with no first turn. Each epoch begins its own count of years, as a new notebook numbers its pages from one.

What of us crosses into the next epoch is the form of the content, averaged into one form. In Chapter 9 I identified that one averaged form with the first ripples, from which the galaxies of the next sky grow. Nothing of us crosses as itself, and there is no other universe to escape to. So the end of the far future leads back to the scene in Washington, because the pattern that COBE mapped is the form that crossed into our epoch from the epoch before ours, as I identified it in Chapter 9.

Where Meaning Is

Where, then, does meaning come from? Greene concludes that the universe offers no meaning, and that the only meaning is the one we make (*Until the End of Time*, chapter 11, section "Meaning"). Yet even to declare that nothing means anything is to weigh worth. The act of declaring that nothing matters is an act of weighing worth, and so it is itself a case of mattering. Meaning is present wherever comparison occurs, and it cannot be removed.

A *chooser* is one who chooses and knows the choosing from inside. Chapter 21 showed that meaning is present wherever comparison occurs, and that in a chooser meaning is gathered and known from inside. In Chapter 21 I also set *morality* beside *ethics*. Morality is the rules a culture holds, in its place and age, for everyone in it. Ethics is the conditions un-

der which choosing goes well, for any chooser at all. The principles of ethics, unlike morality, are common to every chooser. So the meaning of what we do is present in the acts themselves, here and now. The meaning of our acts does not depend on the sky lasting.

Where the Two Arguments Meet, and Where They Part

Where do the results of Greene's argument agree with my results, and where do they differ? Greene's argument starts from the theories of physics and goes out to the world. My argument starts from the act by which anything at all is found.

The results of the two arguments agree in four places that measurements have checked. The first place is gravity. Gravity is the shape of the arena, space and time taken together. Changes in the shape of the arena travel as waves at the speed of light. Greene's argument starts from general relativity, which predicts the waves, and on my account the waves are ripples of the arena. On 14 September 2015 the Laser Interferometer Gravitational-Wave Observatory registered the waves with its two detectors (Chapter 5). The second place is a bound: no probe can place a body more finely than about a Planck length. In Greene's argument the bound follows from the size of a string. On my account, the bound is a limit on placing, in an arena with no least length. In 2009 a space telescope caught light from a burst that had traveled for billions of years, and the light showed none of the grain of space that the measurement could detect (Chapter 5). The third place is the shape of the sky: the sky is the same in every direction, and space is flat. In Greene's argument the reason is inflation, and on my account the reason is the turn, so in both arguments the reason is at the beginning. The oldest light shows the sameness of the sky. The later maps of the oldest light show the flatness of space, out to the edge of what we can see (Chapters 9 and 11). The fourth place is the speeding up of the expansion, which was measured in 1998. In Greene's

telling the speeding up carries the distant galaxies out of sight if the dark energy persists, and on my account it does so because the cosmological constant is one number through our epoch (Chapters 10 and 16).

String theory has two further results that I agree with, though the two results do not follow from my own argument. String theorists found the *graviton*, a carrier of gravity, among the ways a string can vibrate. In 1996 the physicists Andrew Strominger and Cumrun Vafa used string theory to count the states of a class of idealized black holes. The count of Strominger and Vafa matched exactly the entropy that the physicists Jacob Bekenstein and Stephen Hawking had given the idealized black holes. These are two of the most striking results in modern physics. I allow the graviton only as one step of energy in the description of small waves of the arena. The graviton is never a piece of the arena itself (Chapter 6). On the worth of our moment of thought, the two arguments agree. In Greene's words, our moment of thought is "rare, wondrous, and precious" (*Until the End of Time*, chapter 9, section "A Final Thought on Thought").

The results of the two arguments differ in nine places, and the reason is the same at each place: at every step, I start from what a comparison made from inside the world could register.

The table below lists the nine places where the two arguments differ, each with the chapter of this book that sets the difference out.

The question	Greene	My account
Is space a something? (Chapter 1)	On whether spacetime gives a benchmark for motion: "Although the issue is still debated, as we've now seen, the most straightforward reading of Einstein and his general relativity is that spacetime can provide such a benchmark: spacetime is a something" (*The Fabric of the Cosmos*, chapter 3, section "Spacetime in the Third Millennium").	Space, with time, is a something in one sense: the arena, whose shape is gravity. The arena is no container, since the arena has no properties apart from the causal relations among the events in it.
What is real? (Chapter 2)	"The only thing that's real is the whole of spacetime" (*The Fabric of the Cosmos*, chapter 5, section "The Persistent Illusion of Past, Present, and Future").	The whole loaf exists, and what is real is the act, given here and now.
Is there one theory of everything? (Chapters 5 to 7)	String theory may be one framework in which every particle of matter and every carrier of force is a string with its own pattern of vibration (*The Elegant Universe*, chapter 1, section "String Theory as the Unified Theory of Everything").	One world is described by two theories joined from one source, and it has ten dimensions of three kinds, six of them forming one small shape.
Why does time have a direction? (Chapter 8)	Greene traces the direction of time to the orderly beginning (*The Fabric of the Cosmos*, chapter 6, section "The Egg, the Chicken, and the Big Bang").	The direction of time is in every comparison, and only the growth of entropy is traced to the beginning.

The question	Greene	My account
Where do the ripples and the small constant come from? (Chapters 9 to 11)	Inflation can explain the ripples (*The Fabric of the Cosmos*, chapter 11, section "The Golden Age of Cosmology"). A multiverse can be a home for our small constant (*The Hidden Reality*, chapter 6, section "Cosmological Anthropics").	The turn is the source of the ripples. The small constant is the share of the vacuum's energy that a measurement made from inside takes in, and there is one world.
What is consciousness? (Chapter 14)	Greene expects consciousness, barring a stunning surprise, to be understood as another physical quality that arises in a quantum universe (*Until the End of Time*, chapter 5, section "Consciousness and Quantum Physics").	Consciousness is real, and I identified it in Chapter 14 with three features of an act of comparison taken whole, in a context, as one act, happening: its one-way order, its outcome that nothing before it fixed, and its stance.
Do I choose? (Chapter 15)	In the human-level story Greene keeps, "We do make choices" (*Until the End of Time*, chapter 5, section "Relevance, Learning, and Individuality"). Greene holds that the sensation of free will is real, and that the capacity of the mind to transcend the laws of physics is not (same section).	The laws stay as they are, and the one who chose could have chosen otherwise.

The question	Greene	My account
How does the far future end? (Chapters 16 to 19)	Gazing from floor 102, Greene writes, we will see little beyond "a diffuse mist of particles wafting through space", and if the dark energy persists, those particles will rarely if ever meet one another. Greene writes that physicists sometimes liken the era of the mist to the end of time, and he adds, "Not that time stops". He also weighs speedier ends: a leap of the Higgs field to a lower value, a *big crunch* (the expansion turned into a collapse) and a *bounce* (a collapse that rebounds into a new expansion). Greene sets out the mist and the speedier ends in *Until the End of Time*, chapter 10, sections "An End of Time", "The Disintegration of Emptiness" and "Is the End Near?".	The far future ends in a turn, and comparison begins again.
Where does meaning come from? (Chapter 21)	The only meaning is the one we make (*Until the End of Time*, chapter 11, section "Meaning").	Meaning is present wherever comparison occurs.

The measurements that Greene's books report remain in my account, and so do his pictures and the hope that runs through his four books. I add the act of comparison as the ground beneath Greene's measurements and pictures. I also show what the world looks like when every step is taken from the act of comparison.

Under a New Sky

Greene ends *The Fabric of the Cosmos* with us gazing silently at the stars. He hopes that the pattern of the smallest things is written across the sky. The pattern written across our oldest light is the form of what the black holes of an earlier sky took in, and the galaxies over our heads grew from that pattern. The light that reaches us at night carries the form of an epoch before our own. So to look up at night is to read the form of an epoch before our own. The pattern in the oldest light shows you already what comes after the dark of an epoch: a new sky, carrying the form of the content of the sky before.

Comparison begins again in every epoch, so you hold a ground that is the same in every epoch: the act of comparison. Content is uncertain while the structure of knowing is certain and the same for every perspective. Every perspective is a comparison. So the structure of knowing is the same for every comparer, while what each comparison finds is given only in the act.

Now picture the next sky. The next sky has a first faint pattern, warmer here and cooler there, and that pattern is the averaged form of what our black holes took in. Stars have formed under the next sky, and worlds around them. Under the next sky, our galaxies, stars, records and names are gone, and nothing of us is there as itself. On one of the worlds around the new stars, someone hangs a bucket of water from a twisted rope and lets it go. At first the bucket spins round still, flat water. Then the water catches up, turns with the bucket, and climbs the walls into a hollow. The hollow is found against the matter of the next sky, the only matter there to compare with. The law is the same law as ours, and the comparison is a new one. Comparison begins again.

On my account, the world Greene describes follows from the act of comparison, which takes nothing from physics. I show two descriptions joined in one world, a direction of time in every finding, choosers who could have chosen otherwise, an emptying that ends in a turn, and mean-

ing wherever comparison occurs. So we now know what you hold that you did not find in Greene's books. Every finding is a comparison made from inside the world, found against what is there to compare with, from the hollow in Newton's bucket to the faint pattern on the oldest light.

Now that you know what every finding is made of, and what it is found against, what will you compare tonight, under the stars of this sky, and against what?

Reading Beside Greene

The table gives one row to the Preface and one row to each chapter of this book, so that the books can be read side by side. Each row names the chapters of Brian Greene's four books from which the Preface or the chapter quotes or reports, as its text cites them. Where the text cites the preface of one of his books, a glossary or the notes to a chapter, the row names those too. Each row also names the pictures the Preface or the chapter uses.

This book	Greene's books	The pictures here
Preface: What Greene Showed, and Why Read Another Book	*The Elegant Universe*, chapters 1, 6 and 8; *The Fabric of the Cosmos*, chapters 3, 4, 5, 6 and 11; *The Hidden Reality*, chapters 1 and 5; *Until the End of Time*, chapters 1, 4, 5 and 9	Greene at Oxford in October 1984; Greene's garden hose stretched across a canyon, his loaf of bread sliced at an angle, his egg that falls and splatters, and his Empire State Building, each named as his; a thermometer read in the morning and in the evening; a bucket of water hung from a twisted rope, in the last paragraph

This book	Greene's books	The pictures here
1. Is Space a Something?	*The Fabric of the Cosmos*, chapters 2 and 3, with passages from chapters 12, 14 and 16; *The Elegant Universe*, chapter 3	Newton's bucket, the image that returns through the whole book; a pail on a rope that you spin yourself; a gyroscope that holds its direction; one spacecraft, then two, in an empty universe; three closed rooms, and two balls hung on strings; a dropped pen; a hammer and a feather let fall together on the Moon; a thermometer read in the morning and in the evening; an electric field that pushes an electron and lets a neutron pass; a metal box that keeps electric fields out, and a phone that loses its signal in an elevator; an electron and a proton, with charges of one size, the proton about eighteen hundred and thirty-six times harder to set moving; a hot brick, heavier than the same brick cold; an alphabet and the order of its letters, for Leibniz's view of space
2. Is the Whole of Time Laid Out at Once?	*The Fabric of the Cosmos*, chapters 3, 5, 8, 15 and 16; *The Elegant Universe*, chapter 2	Einstein's letter; a ball thrown forward on a train; Greene's loaf, sliced at angles; Greene's Chewie and the long scissors; a firework's flash and its cone; Cherenkov's blue glow; a bench with a clock, a counter and a radioactive lump; a second loaf still rising in the oven; a Geiger counter whose click launches a rocket; last Thursday; the phrase "the largest prime number", which has the form of a name and names nothing

This book	Greene's books	The pictures here
3. Is the Universe Local?	*The Fabric of the Cosmos*, chapters 4 and 15; *The Elegant Universe*, chapter 4	Greene's Mulder and Scully boxes; a piano note held; a chord and a click; a dimmer, a light switch, a square turned a quarter round, and a rocky coast; a ramp and a ladder; Bertlmann's socks; the enchanted decks of cards; detectors that each ask an arriving particle whether it points their way; a street, followed from one house to the next
4. How Does a Single Outcome Happen?	*The Fabric of the Cosmos*, chapter 7; *The Hidden Reality*, chapter 8; *The Elegant Universe*, chapter 4; *Until the End of Time*, chapter 5	dots building stripes; a hoofprint in sand, read by a walker at noon; the surroundings as a notebook each alternative writes in; Greene's cat, warm and breathing; a thermometer in a closed room, a camera that photographs it, and a person who later looks at the photograph; one silver atom, a magnet, and two spots on a screen; a three-legged stool; Newton's bucket
5. Why Can't Relativity and Quantum Mechanics Both Be Right?	*The Elegant Universe*, chapters 1, 5 and 6; *The Fabric of the Cosmos*, chapters 12 and 16	Planck's oven; Greene's peach pit, and his atom magnified to the size of the universe; a grain of sand of the Planck mass; the two sizes of a mass, crossing; a light microscope; Greene's grid and his cannonball; a curve as sharp as a circle about a Planck length across; light that arrives blurred after many small random shifts; the click of a counter; the falling pen and the ramp and the ladder, recalled; a stage and its actors; a crystal and its phonon; the translation of a poem; Newton's bucket

This book	Greene's books	The pictures here
6. Is There One Theory of Everything?	*The Elegant Universe*, its Preface and chapters 1, 6, 7, 12, 13 and 15; *The Fabric of the Cosmos*, chapter 9 and its Glossary; *The Hidden Reality*, chapter 4	Einstein's last pages; Greene's violin string, and his Ping-Pong ball pushed through molasses; the bottom of a wine bottle; walkers crossing a field as snow deepens; a railway timetable and its track; a walled garden, counted from above and from one place outside its wall; a photograph of a building with no person in it; a book and a cup, not blue and yellow paint stirred into green; the chain and the cable
7. Why Ten Dimensions?	*The Elegant Universe*, chapters 8, 9, 12 and 14; *The Fabric of the Cosmos*, chapters 12 and 13; *The Hidden Reality*, chapters 4 and 5	Greene's garden hose; a thermometer read in the morning and in the evening; north-east on a map; a tin can; a drawstring bag; a lamp found red, green or blue; a pencil with three faces, turned on a table; a ball dropped onto a hilly country of valleys; a flat page and the direction straight up; the next room, yesterday's diary and the outcome that did not happen; Greene's hand swept through the curled-up dimensions; a sculptor's block and its grain; two climbers on one summit; a drum, beside Greene's organ pipe; the face of a clock, taken two ways; a ring with nothing wound through its hole; the translation of a poem

This book	Greene's books	The pictures here
8. Why Does Time Have a Direction?	*The Fabric of the Cosmos*, chapters 6 and 7, and the notes to chapter 6; *Until the End of Time*, chapters 2 and 3	Greene's egg, and a film run backward; Greene's tennis ball past Jupiter, his unbound *War and Peace* and his half-melted ice cubes; a new deck of cards, straight from its box in order, and shuffled; Loschmidt's box of gas; a sealed glass of water with a drop of ink; runners turned by a pistol shot, and the spin echo; a warm glass of water sending out light; a walker, a step and a footprint in wet mud
9. What Banged?	*The Fabric of the Cosmos*, chapters 8, 10 and 11; *The Elegant Universe*, chapters 3 and 14; *Until the End of Time*, chapter 3	Penzias and Wilson's horn; a baker's dough stretched over a city; Greene's jack-in-the-box spring and his stretched rubber band; a ball on a flat shelf partway up a bowl; Newton's bucket; a siren whose pitch rises as it comes toward you; moving toward the center inside a black hole, as moving toward tomorrow; funnels that stay apart and meet at their ends; two towns founded by settlers from one village, with one accent; an ant on a marble and on a ball the size of the Earth; a history drawn as size against time; a tiled floor and a cloth woven on one loom; children standing still on a trampoline; the worn stone stairs of an old church; a census of a new country; a sheet of paper fresh from the mill; Greene's slot machine, and a lottery; water chilled below freezing in a very clean bottle, and butter softening on a counter; a phonon in a crystal

This book	Greene's books	The pictures here
10. Why Is the Energy of Empty Space So Small?	*The Fabric of the Cosmos*, chapters 10, 12 and 14; *The Hidden Reality*, chapters 2 and 6, and the notes to chapter 2; *The Elegant Universe*, chapter 9; *Until the End of Time*, the notes to chapter 3	a lamp of known brightness down a road; Greene's tax form and his playground; Casimir's two plates; numbers canceling with their negatives; a pencil stood on its point; Newton's bucket; the strings of a piano; running through still air; a rug between a wall and a heavy sofa; a die thrown onto a table; a page where a word is written; a sheet of spring steel rolled into a tight coil; the books of a library listed on its outer wall; three tiles of the particle horizon; the expansion run twice as fast, with a quarter of the tiles; a cake cut into equal slices; an astronomer placed in our history at half the present size; a ray of light followed era by era; a telescope and a thermometer used together; two surveyors; a map with its scale in one corner
11. Is Our Universe the Only One?	*The Hidden Reality*, chapters 1, 2, 3, 5, 6, 7 and 11, and the notes to chapter 7; *The Fabric of the Cosmos*, chapters 11 and 13; *The Elegant Universe*, chapter 14; *Until the End of Time*, chapter 10	Newton's bucket; Greene's patchwork quilt, his shoe shop and his Swiss cheese; Greene's fly in a bedroom, asking how many places there are for it to be; a ball among valleys; a ring with nothing wound through its hole; a lottery with a million tickets, and one drawn; the chain and the cable; matching circles in the sky

This book	Greene's books	The pictures here
12. Are There Other Worlds in the Quantum, the Hologram, the Computer?	*The Hidden Reality*, chapters 8, 9, 10 and 11; *The Fabric of the Cosmos*, chapter 7; *The Elegant Universe*, chapter 4	Wheeler in Copenhagen with Everett's thesis, May 1956; the stripes of the double-slit experiment, recalled with Tonomura's electrons; Greene's hologram; the worn stairs of an old church, recalled; the chess table, and a book of every game that could be played; a map and the walk; the translation of a poem; the ladder and the ramp, recalled; Cavendish's balance in the experiment of Page and Geilker; a prison cell; the mosaic and the curve; the orrery, and the Moon drawing away by its tides; the circle of geometry; Thomson's cooling Earth
13. How Does a Universe Running Down Build Stars, and Us?	*Until the End of Time*, chapters 2, 3 and 4, and the first section of chapter 9	Carnot's steam engine and the mill wheel; Greene's bag of pennies, and a hundred coins poured on a table; a lock of three dials joined to a lock of two; water poured from cup to cup, spilling nothing; a tray of water in the freezer, and a pond freezing and thawing; hard balls crystallizing; matches in a drawer, Greene's gas oven, and two valleys with a lower pass; a collapsing cloud that heats and shines; Newton's bucket; the worn curve of an old church step, recalled; oil heated from below; crystals of sodium chlorate, in two forms like a pair of gloves; notes on a page and a sonata; Greene's water slide

This book	Greene's books	The pictures here
14. Is Consciousness Only Particles?	*Until the End of Time*, chapters 1 and 5; *The Hidden Reality*, chapter 2	Chalmers in Tucson; Newton's bucket, beside a thermostat that compares a room's warmth with the warmth set on its dial; Mary's room; a map and the walk; music heard for the first time; a chat window with no one known to be behind it; Greene's picture of seeing every particle of his brain, his sunset and his number flashed too briefly to be seen, and his two tales
15. If the Laws Decide, Do I Choose?	*Until the End of Time*, chapter 5; *The Fabric of the Cosmos*, chapter 15; *The Elegant Universe*, chapters 4 and 13; *The Hidden Reality*, chapter 2	Laplace's demon; a player piano and a pianist; a Geiger counter's click; Greene's park bench, his rock and his robot vacuum cleaner; Greene's pool ball; Newton's bucket; a friend's next word; raising the left hand and not the right; a program that shrinks computer files; two ways to spend an evening, and a friend's call with an idea on no list
16. Will the Sky Go Dark?	*Until the End of Time*, chapters 1, 9 and 10; *The Hidden Reality*, chapter 6; *The Fabric of the Cosmos*, chapters 10 and 14	an astronomer at her desk under a sky of one galaxy; Greene's Empire State Building; Newton's bucket; a ball thrown straight up; a steady breeze and a rising wind; raisin bread rising; a theater filled from the front; a book and a cup; the map with its scale in one corner, recalled; an ice sculpture in a cold room; a lighthouse beam; Greene's ember

This book	Greene's books	The pictures here
17. Can a Mind Keep Thinking Forever?	*Until the End of Time*, chapters 1, 2 and 9	Newton's bucket; a row of switches; Maxwell's tiny being at a door between two halves of a box of gas; a room whose walls stay faintly warm; a gyroscope that holds its direction, recalled; a runner on a track that grows longer ahead of him; music heard for the first time; moving inward inside a black hole's horizon, as moving toward tomorrow; a friend far away who never sees a falling body cross the horizon; the ice sculpture and Laplace's demon, recalled; two hydrogen atoms and a photon; a marble that can rest in either of two cups; two strands of one cable; Greene's steam engine and his Empire State Building; one, a half, a quarter and so on, which never add up to more than two; your own skyline, carried as you walk
18. What Is Left When the Black Holes Are Gone?	*Until the End of Time*, chapter 10; *The Hidden Reality*, chapter 9; *The Elegant Universe*, chapter 13; *The Fabric of the Cosmos*, chapter 16	Greene's Empire State Building; a hot coal; a probe sharp enough to make a black hole, recalled; a square turned a quarter round and a dimmer knob, recalled; a horizon for sound in a cloud of chilled atoms; the walled garden of Chapter 6, counted two ways, recalled; a small room in which gravity and speeding up push alike, and the pen let go in Chapter 1, recalled; marks at half the distance to a line; a handwritten letter and the post office's count; a road running off the edge of a map; the worn church step, recalled; a lottery ticket, recalled

This book	Greene's books	The pictures here
19. Is the Long Emptying the End?	*Until the End of Time*, chapter 10; *The Fabric of the Cosmos*, chapters 13 and 14; *The Hidden Reality*, chapter 5	Clausius in Zürich in 1865; Greene's Empire State Building, his spiral building and his marble in a champagne flute; Newton's bucket; a thermometer read in the morning and in the evening; marks at half the distance to a line, recalled; a seesaw; a square turned a quarter round, recalled; a thread wound through the hole of a ring, recalled; the bottom of a wine bottle, recalled; a census of a new country; funnels, each with its own wall; the cars of one train; the map with its scale in one corner, recalled; a new notebook
20. Does Anything of Us Go On?	*Until the End of Time*, chapters 8, 10 and 11; *The Fabric of the Cosmos*, chapter 15	the Voyager golden records; Greene's light carrying our image; the road country; a runner on a track that grows longer ahead of him; the funnels of Chapter 19, each with its own wall, recalled; a drum, recalled; a handwritten letter and the post office's count; the worn stone stairs of an old church; a play performed from one script
21. Where Does Meaning Come From?	*Until the End of Time*, its Preface and chapters 1, 2, 6, 7, 8, 9 and 11; *The Hidden Reality*, chapter 1; *The Elegant Universe*, chapter 15	Russell's broadcast; Newton's bucket; Greene's flight simulator, for a story; a chain of interactions from the water to your eye; a detector and the direction it was turned to, recalled; a sum checked on paper, line by line; notes on a page and a sonata, recalled; the three domains at work when I tell you something: mine, yours, and the language between us; a friendship in which you can say anything, with a square turned a quarter round and a dimmer knob, recalled

This book	Greene's books	The pictures here
22. What Do I Now Hold?	The closing chapters of Greene's four books: *The Elegant Universe*, chapter 15; *The Fabric of the Cosmos*, chapters 11 and 16; *The Hidden Reality*, chapter 11; *Until the End of Time*, chapter 11; with passages from *The Elegant Universe*, chapter 1, *The Fabric of the Cosmos*, chapters 3, 5 and 6, *The Hidden Reality*, chapter 6, and *Until the End of Time*, chapters 5, 9 and 10	COBE's map of the oldest light, shown by Smoot in Washington; Greene's two temperatures; and, recalled from the chapters before: the bucket of water, left alone in a room, watched, and spun under a sky of one galaxy; a dimmer knob, a light switch, a square turned a quarter round, and a rocky coast; a dropped pen; Greene's sliced loaf, and a second loaf still rising in the oven; a ladder and a ramp; a railway timetable and its track; a drum; a ring with nothing wound through its hole; the enchanted decks of cards; Tonomura's dots building stripes, and a hoofprint in sand; a footprint in wet mud; Greene's egg; the worn stone stairs of an old church; a map with its scale in one corner; the chess table; a tray of water in the freezer, and a falling cloud of gas; a map and the walk; a player piano and a pianist; an ice sculpture; a room whose walls stay faintly warm; Greene's Empire State Building; a new notebook; and last, Newton's bucket under the stars of a new sky

Where to Go from Here

Each step of the account in this book is worked out in full in one of my books. These are the books to read next, Part by Part, with what each one gives. Where a book's own opening pages say whom it is written for, its entry ends by saying so in that book's terms; where they do not, the entry names no reader. Every book of the collection that the chapters name is also listed in the References.

The Ground Under Every Part

My book *Incommensuration; The Theorem and the Three Axioms* proves the Incommensuration Theorem by counting the ten ways the four characters can be paired, and it derives the three axioms from the theorem. Its Preface says that a reader who wants the results and their meaning may read Part One alone, and that Part Two is written for the specialist.

My book *Measurement; The Steps from Comparison to Physics* shows why every measurement is a comparison, where the line between content and context falls in physics, and why gravity is the shape of the arena and not a field in it. Its Preface says that a reader who wants the results and their meaning may read Part One alone, and that Part Two is written for the specialist and the physicist.

My book *An Immanent Metaphysics* gives the whole system in aphorisms: the three axioms, the Incommensuration Theorem, and the aphorisms on meaning, purpose and ethics that Chapter 21 draws on.

Part One. Space and Time

My book *What Cannot Be; Signals Faster Than Light, Time Travel, Simulation and Determinism, in Plain Words* shows what can reach what, the three kinds of time, and why travel to an earlier time names no concept (its

Chapters 12 to 16). Its opening pages promise the reader that every idea in it is built from the beginning in plain words, and that nothing in it depends on seeing a figure or reading an equation.

My book *The Gravitational Passage; Deriving General Relativity from the Immanent Metaphysics* derives Einstein's equations of gravity, with the value of the cosmological constant, from the structure of comparison.

My book *Side by Side; Twenty-Four Models of the Universe* sets this account beside twenty-three other models of the universe, each credited to the physicist or scientist who put it forward, and finds for each model the step at which it and this account part. Its Volume One, *The Quantum and the Arena*, holds the models of what a finding is, what the quantum state is, and what gravity does to a quantum state that holds two alternatives at once, from Niels Bohr to Jonathan Oppenheim. Volume One also reproduces whole the record of the collection's predictions sealed on 1 November 2026. Its Volume Two, *Computation, the Cosmos, Time and Mind*, holds the models of discreteness and computation, of the cosmos, and of time, life and mind, from John Archibald Wheeler to Karl Friston.

My book *Home in the World; The Immanent Metaphysics for Readers of Carlo Rovelli* follows Rovelli's six lessons. It finds a floor under our knowledge, the act of comparison, and it shows the two great theories of physics as two descriptions of one world. It finds, inside nature, with nothing added from outside, a difference in kind between a stone and a living being that chooses. This book's Chapters 13, 14 and 21 take from it the living being as a place where choosing is gathered and known from inside, and this book's Chapter 21 takes from it a picture of ethics: a friendship in which you can say anything. Its opening pages speak to a reader who has read Rovelli, and they say that everything needed to read it is in its own pages.

My book *The Basis of Physics; Nineteen Essays on the Structural Limits of Physical Law* reads the physics that already exists from one structural result, the Incommensuration Theorem. Its essays give the structural reason for Bell's theorem, for the conflict between general relativity and quantum mechanics, and for the impossibility of a faithful reversal of time. They

also show why some projects of physics cannot be completed in the form in which they were set. This book's Chapter 2 takes from it what the picture of all of time laid out at once leaves out, and why the direction of time is no brute fact.

My book *Why Time Travel Is Not a Concept; A Meditation on the Three Times and the Cauchy Horizon* argues one thing: that the phrase "time travel", in the sense in which it is used, names no concept. In the same way the phrase "the largest prime number" names no number. This book's Chapter 2 takes from it the three things the phrase could mean, each tried on last Thursday.

Part Two. The Quantum

My book *Why the Quantum Is the Way It Is; Quantum Mechanics from the Act of Comparison, in Plain Words* builds quantum mechanics from the act of comparison. Its Chapters 4, 15 and 17 give the uncertainty principle as an instance of the theorem, entanglement and Bell's limit, and why a measurement has an outcome. Its Chapter 18 reads the interpretations of quantum mechanics, each of which sets aside one of the three roles that every comparison has: the stance, the act or the structure. Its opening pages say that it is written for a reader who learned some physics at school and no mathematics past arithmetic.

My book *The Unquantized Arena; A Structural Diagnosis of Quantum Gravity and the Measurement Problem* sets out its argument as numbered premises and propositions. It gives the three times, the measurement problem, Bell's theorem, the correspondence of Maldacena, and the information that falls into a black hole (its Chapters 15 and 17). Its opening pages say that it makes its argument in the form a specialist recognizes.

My book *The Last Turtle; Essays from Comparison and the Incommensuration Theorem to Causation, the Arena and the Limits of Computation* goes from the act of comparison to the physical world, in seven Parts. It establishes that causation is a regularity in time and not determinism, and that choice is real. It sets out the limits that every physics shares, shows why quantum

mechanics and general relativity do not merge into one theory, and closes by showing that the world is not a computation and that a mind cannot be uploaded. This book's Chapters 3, 6, 13, 14, 15 and 21 quote it, and Chapter 14, on consciousness, quotes it most often.

Part Three. One Theory?

My book *Two Theories, One World; Gravity, the Quantum and the Mistake Behind the Search for One Theory* gives in full the argument of this book's Chapter 5. It shows the two great theories as two descriptions of one world, why the arena of space and time has no least step, and why the two are joined and not merged.

My book *At the Edge of Reality; From Black Holes to the Big Bang in Ten Dimensions* gives in full the argument of this book's Chapter 7 (its Chapters 3 to 8). It gives the ten dimensions, one of time, three of space and six of possibility, and how small the six are. Its opening pages say that it assumes no physics beyond school.

My book *Why These Numbers; The Strengths, Masses and Mixings of the Particles, in Plain Words* gives the strengths of the forces and the masses and mixings of the particles, each number with how it is reached. The mixings are angles between two ways of sorting the same particles. Its opening pages say that it is written for a reader who left physics behind at school and is comfortable with arithmetic and no more.

My book *The Kinds of Particle; Why the Structure Must Be So* gives the closed list of the kinds of particle, and the three generations as the three rotations of the content. Its Preface says that each chapter is written in two layers: a main text in plain and exact prose that an interested reader can follow, and derivation boxes that a physicist can check by reading them in order.

My book *The Strengths of the Kinds; The Couplings, Their Crossings, and the Scales of the Proton and of Gravity. What the Structure Sets and What Measurement Supplies, Volume I* shows why the strengths of the three forces draw close to one another. It also shows why they meet at no scale. Its Preface

describes the same two layers in the same words: a main text in plain and exact prose that an interested reader can follow, and derivation boxes that a physicist can check by reading them in order.

My book *The Branches of the Kinds; What the Structure Allows and Forbids* shows why no carrier joins quarks to leptons, so that the proton is stable (its Chapter 2). Its Preface says that each chapter has a main text in plain and exact prose and derivation boxes that a physicist can check by reading them in order.

My book *The Compactification; Initial Conditions, the Perturbation Spectrum, and the Gravitational Wave Signature* shows, in its Chapter 2, that the one internal space is reached by two routes, so that there is one vacuum and no landscape. Its Chapter 4 gives the count of a horizon.

My book *Non-Commutative Geometry and the Immanent Metaphysics; An Algebraic Derivation* states its starting identifications one by one, as named postulates. From them, by theorems of algebra proved in full, it reaches one algebra, a stock of quantities with rules for adding and multiplying them, made of three pieces: the algebra that Alain Connes had reached from the known physics of the Standard Model. It also proves that the Standard Model's fields of matter and of force, taken together, admit no supersymmetry, so that no particle of matter has a partner among the carriers of force.

Part Four. The Arrow and the Beginning

My book *What the Beginning Left Behind; The First Ripples, the First Waves and the One Vacuum, in Plain Words* gives in full the argument of this book's Chapters 8 to 10 (its Chapters 6, 7, 8, 11 and 13). It gives the two arrows of time, the clean slate at the turn, the one edge that sets the energy of empty space, the oldest light, and why no inflation is needed. Its opening pages say that it is for a reader who needs no physics beyond school and no mathematics beyond arithmetic.

My book *The View from Inside; On the Cosmological Constant and Why Empty Space Weighs So Little* takes up the energy of empty space in its Chapters 10 to 13. It shows why no measurement takes in that energy as a whole, and which share of it a measurement made from inside does take in. Its opening pages say that it is written for the interested reader who is not a physicist.

My book *The Weight of the Unseen; Dark Matter in Questions and Answers. A Companion to The Weight of Possibility* gives, in questions and answers, the dark matter as gyrations, lasting vibrations of the six held dimensions. Its opening pages say that it is written in plain words, for a reader who has no mathematics beyond arithmetic.

Two books named above also carry the turn. *At the Edge of Reality* tells the turn, and why the Big Bang looks like a beginning, in its Chapters 16 and 17. *The Gravitational Passage* gives the turn in its Chapter 18.

Part Five. One Universe or Many?

My book *Physics Is Not Math; Metaphysics, Physics, Computation and Mathematics, and Why Their Order Cannot Be Otherwise* shows why a mathematical structure is not a further world: it gives the order of metaphysics, physics, computation and mathematics, with mathematics keeping its necessity and exactness. Its Preface says that the main line of the book needs no mathematics beyond arithmetic, that Appendix A is for the reader who wants to check every case, and that Appendix B is for the mathematical reader.

My book *Causality Without Determinism; Free Will, Measurement and Mind* follows one distinction, between causality and determinism, to its consequences: the Incommensuration Theorem makes determinism impossible and leaves causality in place. It follows that result through the physics of measurement, through the question of whether a computation could be a mind or a world, and through choice. This book's Chapter 12

quotes it on simulation, Chapter 14 on consciousness and Chapter 15 on choice. Its Preface says that it can be read without the three foundation books on which it draws.

Books named above carry the other steps of Chapters 11 and 12. *The View from Inside* gives the one value of the energy of empty space, with no selection from among many values. *Why the Quantum Is the Way It Is* takes up the many-worlds interpretation in its Chapter 18. *Two Theories, One World* takes up the hologram in its Chapter 23, and *The Unquantized Arena* takes it up in its Chapter 15. *What Cannot Be* takes up simulation in its Chapter 21.

Part Six. Mind in a Universe Running Down

My book *The Matter We Know; From the First Atoms to the End of the Periodic Table. Volume Two; Gatherings* gives the counting of many parts (its Chapter 2, "The Counting of Ways"). It shows why every arrangement of many parts counts alike, and where a thing held at a fixed temperature settles. Its Preface says that the main text is written for an interested reader who may come to the book first, that a reader needs algebra, and that the derivation boxes are written so that a physicist can check each chapter.

For choice, *What Cannot Be*, named under Part One, takes up whether everything is already fixed, choice, change and cause, and a click no one could predict (its Chapters 5, 7 and 8). For consciousness, my book *What Comes After the Dark*, whose entry stands under Part Seven, gives what a thought needs (its Chapter 7). For consciousness and for choice, *Causality Without Determinism*, named under Part Five, gives the argument that this book's Chapters 14 and 15 quote.

Part Seven. Until the End of Time, and After

What Comes After the Dark; The Far Future of the Universe, in Plain Words gives in full the argument of this book's Chapters 16 to 20. It tells what happens to the sky we can see, whether a mind can keep thinking forever, whether the long emptying is the end, what crosses over, and whether this

has happened before. That book's own Appendix A, which it calls "For the Specialist", gives the place in the collection where each step is argued. Its opening pages say that it is written for an interested reader with no physics beyond school.

At the Edge of Reality tells the long emptying, the clean slate and the cycle without a first turn (its Chapters 12, 18 and 20). *An Immanent Metaphysics* gives the aphorisms on meaning, purpose and ethics that Chapter 21 draws on. *Home in the World*, named under Part One, gives the picture of ethics that this book's Chapter 21 uses, and the sentence on the structure of knowing that this book's Chapter 22 quotes. *The Last Turtle*, named under Part Two, finds meaning in what passes between minds, as this book's Chapter 21 quotes it.

Greene's Own Books

Greene's four books are the books to read beside these. *The Elegant Universe* gives superstrings, hidden dimensions and the quest for the ultimate theory. *The Fabric of the Cosmos* gives space and time, from Newton's bucket to the shape and history of the cosmos. *The Hidden Reality* gives nine kinds of parallel universe and the deep laws that lead physicists to them. *Until the End of Time* gives mind, matter and the search for meaning, from the beginning of the universe to its far future.

Appendix A

What the Coming Measurements Will Find

In this book, this account is the account of physics that my books of the Immanent Metaphysics physics collection give. This account says what the instruments of the coming decades will find. The instruments of the coming decades will measure the quantities that decide every prediction gathered in this appendix, apart from the last, more closely than any instrument has measured those quantities yet. Each entry of this appendix names one coming measurement: the quantity that the measurement will measure, and the instrument that will make the measurement. The entry then says what this account says the measurement will find, in the words of the book that states the prediction. When the record of this account's predictions contains the prediction, the entry gives the prediction's number in the record. When the quantity has already been measured, the entry then says what the earlier measurement found, as the record, or the book the entry names, reports the result. Each entry closes with the chapters of this book whose subjects the entry's prediction bears on.

The record is my document *The Registered Predictions of Physics from Comparison; A Dated Record, 1 November 2026*. My book *Side by Side, Volume One* reproduces the record whole, as the book's Appendix B. *Side by Side, Volume One* sets this account beside other physicists' models of the universe. In this appendix, "the record" always means that dated document. *Physics from comparison* is the record's name for this account: physics derived from what every act of comparison must contain. The record gives each prediction a number, names the book, chapter and section that argue the prediction, and, in the record's own words, "states, in one place, every prediction that the books of physics from comparison make".

Two of my books have their own appendix of coming measurements, written in plain words. The entries below quote the two books' appendices beside the record. The first book is *At the Edge of Reality*, on black holes and the Big Bang in ten dimensions, and the book's Appendix B is "How the Coming Surveys Will Test It". The second book is *What Comes After the Dark*, on the far future of the universe. The book's Appendix B is "What the Coming Measurements Will Measure". In the section "The Correspondence", the last entry quotes another book of the collection in place of the record: Chapter 27 of my book *Two Theories, One World*, on gravity, the quantum and the search for one theory. Mathematics, and not an instrument, will decide the prediction of the last entry.

Three terms recur in the entries: uncertainty, standard deviation and confidence. The *uncertainty* of a measurement is the range within which the true value of the measured quantity most likely lies. The uncertainty is written with the sign $\pm$ and read "give or take". A *standard deviation* is the usual measure of how far a measurement may wander from the true value by chance. One standard deviation is a common distance between a result and a prediction. A result five standard deviations away would come by chance less than once in a million tries. A limit stated at *90 percent confidence* is set so that, if the true value lay beyond the limit, nine in ten searches made by the method of the search that set the limit would have found a larger signal than that search found. A limit stated at 95 percent confidence is set like a limit at 90 percent confidence, with nineteen searches in twenty in place of nine in ten.

The record fixes in advance when a measured result counts. The record states its rule in these words:

> "A result counts when it is established at five standard deviations by at least two independent kinds of measurement".

The record calls its rule for counting a result the *rule of decision*. *At the Edge of Reality* gives the reason for the second kind of measurement: "to guard against an error in any one method". The record also states: "A measurement that differs from a prediction by less is a measurement of interest, and the record reports it as such".

Gravity and the Arena

In Chapter 1 I identified gravity with the shape of the arena, and not with a force acting within the arena. The *arena* is space and time taken together, as the setting in which everything that is measured is measured. The shape of the arena is the way space and time are bent, from one place to the next. The record states six predictions that follow because gravity is the shape of the arena. Five of the six predictions are in the section "Gravity and the Arena". The later section "The Quantum" contains the sixth prediction, on how a superposition ends.

Gravity links two masses

In the coming experiment, physicists will hold two tiny masses, each in a *superposition* of two places. A superposition of two places is a combination of being at one place and being at the other, with an amount of each. The physicists will let only gravity act between the two masses. Then the physicists will measure the two masses apart, to find whether the masses have become *entangled*: whether the outcomes of the two masses, measured apart, agree in a pattern that no separate instructions carried by each mass could give the outcomes. Sougato Bose and his colleagues proposed the experiment in 2017, and Chiara Marletto and Vlatko Vedral proposed the same experiment in the same year. *At the Edge of Reality* reports that physicists are building the experiment: "Experiments to test this are being built".

This account says, in the words of the record's Prediction 1: "Two masses, each held in a superposition of two places and related only through gravitation, become entangled, with the phase of their Newtoni-

an interaction". The *phase* is the amount by which each pair of placements of the two masses turns under Newton's attraction between the masses. Each pair turns as a hand turns on a dial. The masses become linked because different pairs of placements turn by different amounts. The record gives the phase for the design proposed in 2017. In the design of 2017, each mass is a hundredth of a billionth of a gram, held in two places a quarter of a millimeter apart for two and a half seconds, with the centers of the two masses a little under half a millimeter apart. For that design the phase is 0.314 of a radian, about a twentieth of a full turn. The record reports that the phase is still to be measured.

This prediction bears on Chapter 4, "How Does a Single Outcome Happen?", and Chapter 5, "Why Can't Relativity and Quantum Mechanics Both Be Right?".

Quanta of a description, and none of the arena

The measurement for this prediction is every measurement of *gravitational waves*. Gravitational waves are changes in the shape of the arena that travel outward at the speed of light. Every detector that registers gravitational waves makes the measurement, including any detector that would register a gravitational wave in single clicks.

This account says, in the words of the record's Prediction 3: "A description of the small waves of the geometry by quanta holds below the Planck curvature, and its quanta are quanta of that description, never quanta of the arena". A *quantum* is one of the steps in which matter and light trade energy, while a *quantum of a description* is one step of energy in the description of a small wave, never a piece of space and time (Chapter 5). The *Planck curvature* is the sharpness of bending at which the arena bends on the *Planck scale* of Chapter 5. The Planck scale is the scale at which a body's gravitational size equals the body's quantum size. The Planck curvature lies far beyond any bending that an instrument encounters. The record also states: "A detector that registers a gravitational wave in single clicks agrees with the account and does not show gravitation

quantized". Physicists detected the first gravitational wave in 2015. The shape of that wave matched the shape given by general relativity, a theory with no quanta in it (Abbott and others 2016).

This prediction bears on Chapter 5, "Why Can't Relativity and Quantum Mechanics Both Be Right?", and Chapter 6, "Is There One Theory of Everything?".

Every material falls alike

In the coming measurement, physicists compare how bodies of different materials fall together. Physicists also time the fall of *antimatter* (matter whose particles carry the opposite charges). And physicists search for any arrangement of matter that would screen a body from gravity. The French satellite MICROSCOPE has compared the fall of different materials in orbit. The proposed satellite STEP has a target precision of one part in a billion billion. STEP stands for the Satellite Test of the Equivalence Principle. The ALPHA-g experiment at CERN, near Geneva, times how antihydrogen atoms fall.

This account says, in the words of the record's Prediction 4: "The Eötvös parameter, the fractional difference in the free fall of two materials, is zero for every pair of materials at every precision". Prediction 4 also states: "Antimatter falls with the acceleration of matter". And Prediction 4 states: "No arrangement of matter shields gravity, and nothing will be found that gravity leaves untouched". MICROSCOPE found titanium and platinum falling alike to within a few parts in a million billion (Touboul and others 2022). ALPHA-g found antihydrogen falling toward the Earth, with an acceleration of 0.75 of the ordinary value, give or take 0.13 from chance and 0.16 from the method of measurement (Anderson and others 2023). Physicists timed laser light sent to the Moon and reflected back, and found no screening of gravity (Williams, Turyshev and Boggs 2009).

This prediction bears on the Preface and on Chapter 1, "Is Space a Something?".

Space and time stay smooth at every scale

In the coming measurement, astronomers time the arrival of *photons* (particles of light) of different energies from distant *gamma-ray bursts*, the brightest explosions known, whose light has crossed a large part of the universe. In the simplest pictures of a least step in space and time, light of one energy arrives before light of another. The record states what decides this prediction: "The timing of photons of different energies from many sources, told apart from effects at the source by its growth with distance; every observation of the spectrum of area or volume". The *spectrum* of area is the set of sizes a surface can take.

This account says, in the words of the record's Prediction 5: "The speed of light in vacuum does not depend on the energy of the light through any step of spacetime". In Prediction 5, "vacuum" means empty space. Prediction 5 also states: "No discrete spectrum of area or volume will be observed, in the echoes of gravitational waves from merging black holes or anywhere else". A *discrete spectrum* of area would be a rule by which surfaces take only separate sizes, with nothing between the sizes. Light from a very bright burst of 2022 was timed by the Large High Altitude Air Shower Observatory in China (Cao and others 2024). The *Planck mass* is about twenty-two millionths of a gram (Chapter 5). The Planck mass is about the mass of a grain of quartz sand a quarter of a millimeter across. The *Planck energy* is the energy a Planck mass holds at rest, about two billion joules. For the simplest form of a dependence of the speed of light on energy, the observatory in China placed the energy at which the speed of light would begin to depend on the energy of the light above about ten times the Planck energy.

This prediction bears on Chapter 5, "Why Can't Relativity and Quantum Mechanics Both Be Right?", and Chapter 12, "Are There Other Worlds in the Quantum, the Hologram, the Computer?".

Two patterns of stretching, at the speed of light

Physicists measure each wave with the network of detectors of gravitational waves. The network is the Laser Interferometer Gravitational-Wave Observatory, LIGO, in the United States; Virgo in Italy; and KAGRA in Japan. The network reads the *polarization* of each wave: the pattern in which the wave stretches and squeezes space as the wave passes. Physicists set the wave's arrival time beside the arrival of light from the event that made the wave.

This account says, in the words of the record's Prediction 6: "Gravitational waves carry the two polarizations of a massless field of spin two, and travel at the speed of light; no further polarization and no other speed will be found". The two polarizations are the two patterns of stretching and squeezing that general relativity gives a wave of the arena, the second turned an eighth of a circle from the first. *Spin* is a turning built into every particle, the same amount for every particle of one kind (Chapter 6). A wave of a field of spin two stretches space in a pattern that looks the same again after half a turn about the wave's direction of travel. A *massless* field is a field with no mass. The waves of a massless field travel at the speed of light. Three detectors saw the event of 14 August 2017, and the measurements of that event favored the two patterns of general relativity alone. The event of 17 August 2017 was seen together with its burst of gamma rays (light of very high energy), so the timing of the event set the speed of gravitational waves equal to the speed of light to within a few parts in a million billion (Abbott and others 2017).

This prediction bears on Chapter 1, "Is Space a Something?", and Chapter 5, "Why Can't Relativity and Quantum Mechanics Both Be Right?".

Signals and Time

News travels within the cone

In Appendix H of my book *Side by Side, Volume One*, I compare this account with the work of Anton Zeilinger. In that comparison, the measurement for this prediction is "Every experiment that looks for a dependence of local statistics on a distant choice". *Local statistics* are the proportions of the outcomes recorded in one laboratory. The distant choice is the choice of which measurement to make in another laboratory, far away.

This account says, in the words of the record's Prediction 27: "A finding changes no geometry, here or anywhere, and news of a distant finding enters a record only at an event on or within the future light cone of the finding". A *finding* is the outcome of one act of comparison. The *future light cone* of a finding is the future half of what Chapter 2 calls the cone, everything the finding can affect. The surface of the future light cone holds every event that a flash of light sent out at the finding reaches. The inside of the future light cone holds every event that a slower signal from the finding can reach. Prediction 27 also states: "Until such a signal arrives, every local statistic follows quantum theory unchanged, whatever measurement was chosen far away". And Prediction 27 states: "The probabilities of a preparation's outcomes do not depend on which test is chosen later, and no later finding comes to precede an earlier one".

Appendix H of *Side by Side, Volume One* also describes what physicists have measured so far. In 1998 Gregor Weihs and his colleagues, in Innsbruck, measured the agreement of entangled particles in two laboratories, with each laboratory's setting chosen too late for any signal from the other laboratory to arrive before the laboratory's own particle was measured. Weihs and his colleagues found the agreement that quantum theory predicts (Weihs and others 1998). In 1997, in the laboratory in Innsbruck where Weihs worked, a team carried the state of one photon to another

photon some distance away, but the receiver recovered the state only when an ordinary message arrived (Bouwmeester and others 1997). No experiment has reported a signal faster than light.

This prediction bears on Chapter 2, "Is the Whole of Time Laid Out at Once?", Chapter 3, "Is the Universe Local?", and Chapter 4, "How Does a Single Outcome Happen?".

The Quantum

A superposition ends in a finding

In the coming measurement, physicists hold larger and larger masses in superpositions for longer and longer times. The physicists watch for a superposition that ends of its own accord. In some proposals, gravity ends a superposition by itself, and these proposals say that each such ending would give off faint radiation. Physicists also search deep underground for that faint radiation. The entry "Gravity links two masses", above, describes an experiment with two masses linked only through gravity. The experiment with two masses holds each mass in two places for two and a half seconds, so the experiment tests the proposals in which gravity ends a superposition in a new range of mass and time.

This account says, in the words of the record's Prediction 2, that "a superposition ends only through a finding, an interaction that registers which alternative is the case". Prediction 2 opens with the same claim stated as a negative: "No mass, however large, no separation of two placements, however wide, and no length of time ends a superposition of its own accord". In the proposals in which gravity ends a superposition by itself, a *collapse* is a superposition ending of its own accord. Lajos Diósi and Roger Penrose made one proposal in which gravity ends a superposition by itself. The record also states: "For the masses of Prediction 1, the gravity-driven collapse of Diósi and Penrose gives a lifetime of about 0.012 seconds, against the 2.5 seconds the entangling phase needs". In 2021

Sandro Donadi and his colleagues searched under the Gran Sasso mountain in Italy, and ruled out the version of the proposal of Diósi and Penrose that has no adjustable constant (Donadi and others 2021).

This prediction bears on Chapter 4, "How Does a Single Outcome Happen?".

The statistics of quantum theory hold

In the coming measurement, physicists repeat one measurement many times on systems prepared alike, and count how often each outcome comes. Physicists also send particles through three paths at once, and look at how the three paths combine. Other experiments measure how entangled particles agree in the tests of Chapter 3. In still other experiments, physicists build networks of independent sources of entangled light. In 2020 Sabine Hossenfelder and Tim Palmer proposed one of the measurements of this entry. In the experiment of Hossenfelder and Palmer, measurements are made one after another on one prepared state. The devices are as small and cold as can be made. Hossenfelder and Palmer look for patterns in time between the outcomes. *Superdeterminism* is the proposal that the settings of the instruments and the states of the particles share one deterministic past. On that proposal, the settings of the instruments only seem free. Superdeterminism predicts patterns in time between the outcomes.

This account says, in the record's Prediction 25:

> "Repeated measurements on identically prepared systems settle to the weights of the Born rule, in every sector and on the full space, including in the tests proposed for superdeterminism, which look for departures from those weights".

The *Born rule* is the rule of quantum theory for how often each possible outcome comes. A *sector* is one of the small, separate parts of the space of a system's possible states. The sectors carry the system's internal labels, such as the system's charges. The *full space* is the whole space of the

system's possible states, which holds every finding. Prediction 25 also states: "The interference among three paths has no term that the pairs of paths do not account for". *Interference* is the adding and canceling of waves that meet: the waves add where crest meets crest, and cancel where crest meets trough. And Prediction 25 states: "Tests of the CHSH inequality reach $2\sqrt{2}$ and go no further". A form of the test of entangled particles was given in 1969 by John Clauser, Michael Horne, Abner Shimony and Richard Holt. Chapter 3 describes the test of Clauser, Horne, Shimony and Holt. In the 1969 form of the test, four correlations from four pairs of settings are combined into one score. A *correlation* is a pattern of agreement between the records of two detectors. Physicists find a correlation by setting the two detectors' lists of results side by side. The *CHSH inequality* is a ceiling on the combined score: a score of at most two. The letters CHSH stand for Clauser, Horne, Shimony and Holt. When each particle carries its answers from the source, as a sealed script, and nothing done to one particle affects what is found at the other, the ceiling of two holds. Chapter 3 shows that quantum theory and the world stop at a score of twice the square root of two, about 2.83. And Prediction 25 states: "Experiments with independent sources find the complex, not the real, composition of independent systems". The *composition* of two independent systems is the way the description of the pair is built from the descriptions of the two. Physicists write the states of quantum theory with *complex numbers*, numbers built from the square root of minus one. Quantum theory written in ordinary numbers alone predicts different results when independent sources feed one network. Three-slit experiments found the combination of three paths fully accounted for by the paths taken two at a time (Sinha and others 2010). Network experiments in 2022 found the complex composition (Chen and others 2022; Li and others 2022).

This prediction bears on Chapter 3, "Is the Universe Local?", and Chapter 15, "If the Laws Decide, Do I Choose?".

The laws carry an asymmetry of time

The measurements for this prediction measure two small differences in the laws of particles: the difference between a process and its mirror image run with antimatter, and the difference between a process and its reverse in time.

This account says, in the words of the record's Prediction 28: "Some asymmetry of time must exist in the laws, and no process at any scale is perfectly reversible". The opening of Prediction 28 states: "No superdeterministic completion of quantum mechanics will be found, and no world is purely and completely deterministic". When every change in a world is fixed in advance, step by unbroken step at every scale, the world is purely and completely *deterministic*. A superdeterministic completion would be a version of quantum mechanics with superdeterminism added to it. The record calls both statements of Prediction 28, the asymmetry of time and its opening exclusion of a superdeterministic completion, "exclusions that the structure of comparison makes": exclusions that follow from what every act of comparison must contain. The record states that "the outcome of a search for departures from the quantum predictions bears on Prediction 25 and not on this one". In the record's sentence, "this one" is Prediction 28. So the record counts the experiment of Hossenfelder and Palmer under the entry "The statistics of quantum theory hold", Prediction 25. Physicists found the small differences between matter and antimatter in 1964 (Christenson and others 1964). Physicists found the differences under reversal of time at CERN in 1998 (Angelopoulos and others 1998) and at Stanford in 2012 (Lees and others 2012). The record states that both kinds of difference, between matter and antimatter and under reversal of time, are "consistent with the required asymmetry".

This prediction bears on Chapter 8, "Why Does Time Have a Direction?", and Chapter 15, "If the Laws Decide, Do I Choose?".

A fine enough measurement shows a rule's least step

The measurement for this prediction is any measurement finer than the least step of a fixed rule proposed as the foundation of physics, such as the program of a computer simulating the world. One measurement for this prediction is the timing of photons from gamma-ray bursts, which the entry "Space and time stay smooth at every scale", above, describes.

This account says, in the words of the record's Prediction 29: "For any one fixed discrete rule offered as the foundation of physics, a sensitive enough measurement, reaching below the rule's least step, tells the rule's statistics from the world's". A *discrete rule* is a rule that proceeds in separate steps, as a computer program does.

This prediction bears on Chapter 12, "Are There Other Worlds in the Quantum, the Hologram, the Computer?".

The Kinds of Particle and the Forces

The record states: "The kinds of particle are reasoned from comparison as a closed list: the kinds of the Standard Model, with a right-handed neutrino in each generation". The *Standard Model* is the theory of the known particles and of the three forces other than gravity. A *generation* is one of the three copies of the particles of matter, alike but for their masses (Chapter 6). *Neutrinos* are the lightest particles of matter, with no electric charge. Each generation has one kind of neutrino. A *right-handed neutrino* is the right-handed component of a neutrino: a part counted within the neutrino of its generation, and not a further kind of particle. *Right-handed* names the way the component's spin turns about the component's direction of motion, as the fingers of a right hand curl when the thumb of that hand points along the motion. The right-handed component carries the charge of none of the three forces. Masses in this section are stated in GeV, a unit of energy that physicists use for mass: a GeV is a billion electron volts, where an electron volt is the energy an electron gains in crossing one volt. A proton weighs a little less than one GeV.

The kinds found are the kinds of the closed list

The measurement for this prediction is every search for a new kind of particle: at the Large Hadron Collider at CERN and at the colliders that will follow the Large Hadron Collider, in detectors deep underground, and in the sky.

This account says, in the record's Prediction 30, that the kinds found will be the kinds of the closed list. The record names some of the kinds that will not be found: a fourth generation, a partner of supersymmetry, a carrier that turns a quark into a lepton, and a second Higgs boson. A quark is one of the particles inside protons and neutrons. A lepton is a member of the family of light particles to which the electron belongs. The known particles fall into two broad families: the particles of matter, and the carriers of force together with the Higgs boson (Chapter 6). *Supersymmetry* is a proposed pairing that would give every known kind of particle a partner in the other family. The *Higgs boson* is the particle that is a ripple of the Higgs field, the field whose settled value gives many particles their masses. The record also names a particle of dark matter, a further neutrino beyond the right-handed component of each neutrino, and a hidden family of particles. The record states: "The next collider will not find a new species". And the record states: "Heavier composites wait at higher energies; a new kind does not". A *composite* is a particle made of other particles, as a proton is made of quarks. The Large Hadron Collider has found none of the kinds that the record names, although the Large Hadron Collider has searched for some of those kinds up to masses of several thousand GeV. Physicists had proposed a single light neutrino of a new kind to explain two earlier experiments. In 2025 the MicroBooNE detector at Fermilab, near Chicago, excluded that neutrino. The record reports the results of the Large Hadron Collider and of MicroBooNE from my book *The Alphabet of Particles*, which sets out every particle in the order of its derivation.

This prediction bears on Chapter 6, "Is There One Theory of Everything?".

Three generations

In the coming measurement, physicists count the kinds of light neutrino from the decays of the *Z particle*, one carrier of the *weak force*, the force that changes one kind of particle into another in radioactive decay. Physicists also measure the rates at which the Large Hadron Collider makes the Higgs boson. A fourth generation would change the rates.

This account says, in the words of the record's Prediction 31: "The generations number exactly three; no fourth exists at any mass". From the decays of the Z particle, physicists count 2.9963 kinds of light neutrino, give or take 0.0074 (Janot and Jadach 2020). Physicists have used the measured rates of the Higgs boson to exclude a fourth generation of the Standard Model's kind.

This prediction bears on Chapter 7, "Why Ten Dimensions?", and Chapter 6, "Is There One Theory of Everything?".

The proton lasts

In the coming measurement, physicists watch an enormous number of protons for years in large tanks deep underground, for the flash of a proton breaking apart. Super-Kamiokande in Japan watches now. The next instruments to watch are Hyper-Kamiokande in Japan and the Deep Underground Neutrino Experiment in the United States. The section "The Far Future", below, contains the entry "Searches for the decay of the proton". The later entry states the lower limits on the lifetime that Hyper-Kamiokande and the Deep Underground Neutrino Experiment can set.

This account says, in the words of the record's Prediction 33, that the proton lasts: "No carrier joins color to the weak charge, so no transformation turns a quark into a lepton" and "the proton has no route to decay". *Color* is the charge of the strong force. The *weak charge* is the charge of the weak force. A *quark* is one of the particles inside protons and neutrons. A *lepton* is a member of the family of light particles to which the electron belongs. A *baryon* is a particle made of three quarks, such as the proton. The record also states: "No unifying group exists at any scale, and the meas-

ured couplings meet at no scale". A *unifying group* would be one larger set of changes that leave the laws of the forces the same, containing the three forces' own sets as parts. The *couplings* are the strengths of the three forces. Super-Kamiokande has seen no proton decay, so the experiment sets the proton's lifetime for the decay into a positron and a neutral pion above twenty-four million billion billion billion years (Takenaka and others 2020). A *positron* is the electron's twin of opposite charge. A *neutral pion* is a short-lived particle with no electric charge, made of a quark and an antiquark.

This prediction bears on Chapter 6, "Is There One Theory of Everything?", and Chapter 16, "Will the Sky Go Dark?".

The known particles account for every loop

In the coming measurement, physicists compare the *magnetic moment of the muon* with the value predicted from the known particles. The magnetic moment of the muon is the strength of the tiny magnet that a muon carries. A *muon* is a heavier cousin of the electron. Physicists also search for a muon turning into an electron, a change that the known particles allow only at rates far too small to see. The Muon g−2 experiment at Fermilab has measured the muon's magnetic moment. The MEG II experiment in Switzerland searches for a muon turning into an electron and a photon.

In quantum theory, a particle's properties carry small contributions from every kind of particle that can appear briefly and vanish beside the particle. Physicists draw each contribution from a briefly appearing particle as a *loop*. This account says, in the record's Prediction 35, that every loop comes from the known particles: "With no particle beyond the Standard Model, no new particle enters any loop". Prediction 35 also states: "The anomalous magnetic moment of the muon receives no contribution from a new sector". The anomalous magnetic moment is the part of the muon's magnetic moment that the loops give. In Prediction 35, a new sector means a new family of particles. And Prediction 35 states: "No signal of charged-lepton flavor violation will be seen at any foreseeable experiment". A charged lepton's *flavor* is its kind: electron, muon or tau.

Charged-lepton flavor violation is a charged lepton of one flavor, such as a muon, turning into one of another flavor, such as an electron, with nothing else to carry the difference. The measured magnetic moment differs from the value predicted from the known particles by less than one standard deviation (Muon g−2 Collaboration 2025; Aliberti and others 2025).

This prediction bears on Chapter 6, "Is There One Theory of Everything?".

The strengths of the forces change only at known masses

In the coming measurement, physicists follow the strengths of the forces at colliders reaching higher and higher energies. Physicists call the change of a force's strength with the energy of a collision *running*.

This account says, in the words of the record's Prediction 36: "The rates at which the three couplings change with scale are fixed by the closed list of kinds, and change only at the masses of the known kinds and of the W and Z". In Prediction 36, the scale is the energy of the collision. The *W and Z* are the carriers of the weak force. The record also states: "A change of rate at any other mass would reopen the list". The *strength of light* is the strength of the electric and magnetic force. The strength of light runs at the rates of Prediction 36 from the lowest energies up to the mass of the Z. The *strength of color* is the strength of the strong force. The strength of color runs at the rates of Prediction 36 up to collisions of about a thousand GeV.

This prediction bears on Chapter 6, "Is There One Theory of Everything?".

A ceiling on mass, and the order of the neutrino masses

In the coming measurement, physicists weigh the heaviest known particle of matter, the top quark. Physicists also find the order of the three neutrino masses, with the Jiangmen Underground Neutrino Observatory, JUNO, in China, the T2K experiment in Japan and the NOvA experiment in the United States.

The *Higgs field* is the field whose settled value gives many particles their masses. The *vacuum value* of the Higgs field is the settled value, the same everywhere in space: the condensate of Chapter 6. This account says, in the record's Prediction 37, that no particle of matter is heavier than the vacuum value of the Higgs field divided by the square root of two, 174.1036 GeV, "since every coupling to the condensate lies between 0 and 1". In Prediction 37, a coupling is how strongly a particle of matter is joined to the condensate. Prediction 37 also states: "The neutrino masses are in the normal order, the two that share the electron's flavor most being the lighter pair". A neutrino's flavor is the charged lepton that appears beside the neutrino when the neutrino is made or caught: the electron, the muon or the tau. Each neutrino of a definite mass is a mixture of the three flavors. In the *normal order*, the two lighter neutrinos are the two that carry most of the electron's flavor, and the third is heavier. The top quark has a mass of 172.60 GeV, give or take 0.27, near the ceiling of 174.1036 GeV. When physicists fit all the neutrino measurements together, the normal order fits better, at less than three standard deviations.

This prediction bears on Chapter 6, "Is There One Theory of Everything?".

The strength of color, the W, the Higgs and the top

In the coming measurement, physicists weigh the W particle, the Higgs boson and the top quark more and more finely. The instruments are the colliders that follow the Large Hadron Collider's present runs.

In my books *The Strengths of the Kinds* and *The Measures of the Kinds*, I compute the values of the particles in two ways: the working calculation and the closure calculation. The record names the measured inputs of each calculation. Each calculation states which measured numbers the values take as inputs, and both statements are true, each of its own calculation. In the working calculation, the measured inputs are the mass of the electron (the unit), the strength of light, the strength of color and the vacuum value of the Higgs field. The unit is the mass against which the books state the other masses. Each other mass is stated as a ratio to the

unit, a pure number (Chapter 6). An *offset* is the starting point of the steps of mass in the books' formula for one family of particles. The working calculation also uses two offsets, found from the measured masses of the bottom quark and the tau. In the closure calculation, the mass of the electron alone fixes, step by step, the vacuum value of the Higgs field, the mass of the top quark and the mass of the Higgs boson. In Chapter 6 I name four numbers that *Side by Side, Volume One* lists as measured inputs: Newton's constant, the strength of light, the strength of color and the value of the Higgs field in empty space (the vacuum value of the Higgs field). The working calculation uses the strength of light, the strength of color and the vacuum value of the Higgs field from that list. Newton's constant is an input to the cosmology of the record. The record states: "Cosmology takes Newton's constant, which fixes the Planck mass, and the measured density of radiation". The *second closure* is three laws that join the strengths of light and of color, solved together with one further law, so that none of the quantities the laws join is taken from measurement. The record's Prediction 42 gives the strength of color at the mass of the Z from the second closure. The second closure gives 0.11888, give or take 0.00011. The measured value is 0.1180, give or take 0.0009. The record gives four values of the Higgs mass: 125.1715, 125.1888, 125.2064 and 125.2245 GeV. The measured Higgs mass is 125.13 GeV, give or take 0.11. The record's four Higgs values come from four forms of one correction to the Higgs mass. The four forms agree in the correction's *first order*, its first and largest part. The four forms differ in the correction's *second order*, the smaller part that follows. The record gives six values of the top mass, between 172.417 and 172.657 GeV. The measured top mass is 172.60 GeV, give or take 0.27. The record calls its six values of the top mass the *readings of the top*. Each reading comes from a different law, and each of those laws predicts a top mass below the ceiling of Prediction 37: the vacuum value of the Higgs field divided by the square root of two. The record states: "A W mass measured to about ± 3 MeV, together with one matching computed in one code with the W mass as an input, tests the closure". An *MeV* is a thousandth of a GeV. A *matching* is the step of the calculation that fixes the strengths of the

forces at the top quark's mass from measured quantities. A code is a computer program for the matching. The W mass tests the second closure: the record states that the further law that the second closure adds holds if, and only if, the W mass lies within about five MeV of 80.3488 GeV. The record also states: "A Higgs mass measured to $\pm$0.03 GeV, or a top mass to 0.02 GeV, decides among the readings of the top; a Higgs mass to about $\pm$0.005 GeV separates the forms of the second order".

This prediction bears on Chapter 6, "Is There One Theory of Everything?".

One number for the curvature of the six

In the coming measurement, physicists set the ratio of the dark matter to the ordinary matter beside the *matter fraction*. The matter fraction is the fraction of the universe's energy that is matter today. The matter fraction is measured by an independent kind of measurement. Two entries below describe the two measurements: "The ratio of the dark matter to the ordinary matter" and "The matter fraction is 0.311".

The *Planck length* is the length of the Planck scale, about a hundred billion billion times smaller than the width of a proton (Chapter 5). The six held dimensions of Chapter 7 are six small dimensions of possibility, held a few Planck lengths across within the large world of four dimensions. Under the laws of motion, the six stay at one size and shape, the same at every point of space. Measured in the units of the Planck scale, the curvature of the six is one number, *k*. The number k enters both the ratio of the dark matter to the ordinary matter and the matter fraction. This account says, in the words of the record's Prediction 17: "Both observables will find k = 1". An *observable* is a quantity that a measurement finds. In Prediction 17, the two observables are the ratio and the matter fraction. The record gives the values of k that the measurements so far give. From the ratio, the measurements so far give 0.989, give or take 0.075. From the matter fraction measured by the Planck satellite, the measurements so far give 1.13, give or take 0.21. In the analyses that add the galaxy survey of

the Dark Energy Spectroscopic Instrument (DESI), the ratio gives 1.26, give or take 0.06. With DESI added, the matter fraction gives 0.68, give or take 0.11.

This prediction bears on Chapter 7, "Why Ten Dimensions?".

The Shape of Space

Space is flat

In the coming measurement, astronomers find the curvature of space on the largest scale. Space could be curved like the surface of a ball, curved the other way like a saddle, or flat. Astronomers state the curvature as a share: the share that curvature would take in the budget of shares that sets how fast the universe grows, beside the shares of matter and of the energy of empty space. The instruments are the Dark Energy Spectroscopic Instrument and the Euclid space telescope. The Dark Energy Spectroscopic Instrument measures the distances of tens of millions of galaxies, from a telescope at Kitt Peak in Arizona. Euclid is a space telescope of the European Space Agency that maps the galaxies of a third of the sky.

This account says, in the words of the record's Prediction 9: "The epoch begins spatially flat". An *epoch* is an era of the world, from one turn to the next, where the *turn* is the passage at the end of the black-hole interiors of the era before ours, in which the ten-dimensional geometry restores a large world (Chapter 9). *At the Edge of Reality* states that the curvature of the new epoch "is that of the departures themselves, about a part in a hundred thousand". The departures in that quotation are the *first ripples* of Chapter 9, the faint differences of density that an epoch begins with. *At the Edge of Reality* describes two measurements made so far. The *oldest light* is the light left from the early universe and stretched by the expansion, which astronomers call the cosmic microwave background. Together with the measured pattern of galaxies, the oldest light gives a curvature of about one part in a thousand, give or take about two parts in a thousand (Aghanim and others 2020). Combined with the oldest light,

the Dark Energy Spectroscopic Instrument gives about two parts in a thousand (Abdul-Karim and others 2025). *At the Edge of Reality* also states: "The coming surveys will sharpen both, and the account says they will close in on zero".

This prediction bears on Chapter 9, "What Banged?", and Chapter 11, "Is Our Universe the Only One?".

Every path around space is longer than the particle horizon is wide

In the coming measurement, astronomers search the oldest light for pairs of matching circles: the mark that the same patch of the oldest light would leave if light could circle all the way around space and reach us twice (Chapter 11).

This account says, in the record's Prediction 23, that every path that goes all the way around the universe is at least as long as the diameter of the particle horizon, at every location. Chapter 10 uses the name *particle horizon* for the edge of everything whose light has reached us since the expansion began. The record calls the particle horizon "the horizon", and states: "No closed path that goes around the universe, and so cannot be shrunk to a point, is shorter than the diameter of the horizon, at any location". So far, astronomers have found every path around the universe through our place longer than 98.5 percent of the width of the sphere from which the oldest light set out (Petersen and others 2023). Astronomers will search again with every finer map of the oldest light.

This prediction bears on Chapter 11, "Is Our Universe the Only One?", and Chapter 7, "Why Ten Dimensions?".

The Beginning and the First Ripples

The turn leaves a new epoch no *radius of curvature*, so space on the largest scales is flat, curved only by its own small ripples (Chapter 9). A radius of curvature is the radius of the ball whose surface curves as much as space would curve. In Chapter 9 I identified the first ripples, the faint differences of density with which our epoch began, with the *form of the*

content carried through the turn. The form of the content is the pattern of which alternatives the matter of the epoch before took. The turn carries the pattern through averaged, and not erased. The prediction that space is flat is in the section "The Shape of Space", above. The predictions on the first ripples are entered in this section.

The first unevenness is unevenness of density

In the coming measurement, astronomers map the *polarization* of the oldest light across the sky: the way the light is lined up, the thing that a polarizing filter of the kind in some sunglasses tests. The astronomers look in the map of polarization for a faint twisting pattern. The twist would be left by *first waves*: gravitational waves present from the beginning of an epoch. The *power* of a ripple is the ripple's strength multiplied by itself. The strength of first waves is stated as the power of the first waves set beside the power of the first ripples of density. The instruments are the Simons Observatory in Chile and the Japanese satellite LiteBIRD. The record states that "the Simons Observatory will reach about one part in a thousand, and LiteBIRD below two parts in a thousand". *Side by Side, Volume One* reproduces the record, and gives the two instruments' targets as later published: about three parts in a thousand for the Simons Observatory and about one part in a thousand for LiteBIRD.

Primordial gravitational waves are the first waves. This account says that primordial gravitational waves will be found far weaker than any instrument can detect. *At the Edge of Reality* states the strength of the primordial gravitational waves in these words: "their power, relative to the power of the departures of density, far below one part in a million". The record states this entry's prediction as its Prediction 10: "The Simons Observatory and LiteBIRD will find no primordial tensor signal". A *tensor signal* is the record's name for the mark of first waves. *At the Edge of Reality* states, of the Simons Observatory and LiteBIRD: "The account says they will find nothing". So far, the power of first waves is bound below about three hundredths of the power of the first ripples, by the telescopes at the South Pole combined with the Planck satellite (Tristram and others 2022).

This prediction bears on the Preface, Chapter 9, "What Banged?", Chapter 11, "Is Our Universe the Only One?", and Chapter 19, "Is the Long Emptying the End?".

One form for every component

In the coming measurement, astronomers read every finer map of the oldest light for a departure of one part of the content of the universe against another: of the dark matter against the ordinary matter, for example, with no departure of the total density. The record glosses *content* as "what is found within the arena: the particles, their charges and their fields".

This account says, in the words of the record's Prediction 11: "The departures pass from one epoch to the next as one form for every component of the content, so every component departs from uniformity together". *At the Edge of Reality* states: "The account says that no departure of one component against another, no isocurvature mode, will be found". An *isocurvature mode* is a departure of one component against another. Prediction 11 also states: "The dark matter carries no separate pattern of primordial departures of its own". *At the Edge of Reality* states that the oldest light bounds an isocurvature mode "below one to two percent of the variation of its temperature" (Akrami and others 2020).

This prediction bears on the Preface and on Chapter 9, "What Banged?".

One pattern across the whole sky

The measurement for this prediction is every finer map of the oldest light. Astronomers read each map for how the strength of the first ripples changes from one length to another.

This account says, in the words of the record's Prediction 12: "The primordial departures are correlated across the whole sky, with nearly the same strength at every length". The measured pattern is nearly the same at every length. The *spectral index* is a number that would be exactly one if

the strength were the same at every length. The measured spectral index is 0.965, give or take 0.004 (Aghanim and others 2020). *At the Edge of Reality* states that "the account says every finer map of the sky will keep it so".

This prediction bears on the Preface, Chapter 9, "What Banged?", and Chapter 11, "Is Our Universe the Only One?".

The features of the sky come from the turn

The measurement for this prediction is the four measurements of Predictions 9 to 12 taken together, each entered above: the flatness of space, the first unevenness of density alone, the one form for every component, and the one pattern across the whole sky.

This account says, in the words of the record's Prediction 13: "The horizon and flatness problems are answered by the turn from the previous epoch, with no inflation". The *horizon problem* is why regions of the sky that never exchanged a signal agree so closely. The *flatness problem* is why space began so flat. *Inflation* is a brief but enormous burst of expansion of space in the earliest moments. The standard account that Greene describes uses inflation to answer both problems. The opening of Prediction 13 states: "No inflationary phase long enough to stretch the lengths observed today from within the early horizon to beyond it occurred". The *early horizon* is the greatest distance a signal could have crossed since the turn, in the first moments of our history (Chapter 9). The early horizon is a different edge from the particle horizon. The record states what decides this prediction: "The measurements of Predictions 9 to 12 together".

This prediction bears on the Preface, Chapter 9, "What Banged?", and Chapter 11, "Is Our Universe the Only One?".

The hum of gravitational waves

In the coming measurement, astronomers search for a *background of gravitational waves*: a steady hum of gravitational waves arriving from every direction. The Laser Interferometer Space Antenna, called LISA, is a planned observatory of three spacecraft millions of kilometers apart. LISA will register slower vibrations than the detectors on the ground. The

pulsar-timing arrays time the beams of many *pulsars*. Pulsars are spinning neutron stars that sweep a beam past the Earth like a lighthouse. The pulsar-timing arrays have found a hum at slower vibrations still. The pattern of the oldest light also shows how many kinds of light particle were present early on. A hum from the beginning would add to that count.

This account says, in the words of the record's Prediction 14: "LISA will find no stochastic background from a cosmological phase transition, since the electroweak and strong transitions are crossovers and no further kind of particle exists". A *stochastic background* is the steady hum of gravitational waves from every direction. A *phase transition* is a change of state with a jump, as water boils. A *crossover* is a smooth change of state, with no jump, no bubbles and no walls between regions (Chapter 9). The *electroweak and strong transitions* are the settling of the Higgs field and the binding of quarks into protons and neutrons. The record also states: "The background found by the pulsar-timing arrays is of astrophysical origin, from binaries of supermassive black holes, if the scalar power on short lengths is near the measured power". *Binaries of supermassive black holes* are pairs of the giant black holes at the centers of galaxies, circling each other. An astrophysical origin is a source among the objects of the sky. The *scalar power on short lengths* is the power of the first ripples of density on short lengths.

This prediction bears on Chapter 9, "What Banged?".

Matter outnumbers antimatter everywhere

In the coming measurement, astronomers search for stars, galaxies or regions of antimatter. Physicists also search the *cosmic rays* for nuclei of antihelium, the antimatter partners of the nuclei of helium. The cosmic rays are the particles that rain on the Earth from space. One of the searches for antimatter is made by the Alpha Magnetic Spectrometer, on the International Space Station since 2011.

This account says, in the words of the record's Prediction 15: "Nowhere in the observable universe do antibaryons outnumber baryons, and the observable universe holds no regions of antimatter". *Antibaryons* are the antimatter partners of baryons, such as the antiproton.

This prediction bears on Chapter 9, "What Banged?".

The Dark Matter

In this account, the dark matter is made of *gyrations*: lasting vibrations of the six held dimensions, each of about the Planck mass, made at the turn (Chapter 9). The record states: "The dark matter is not a kind of particle; it is not content". The entry "One form for every component" quotes the record's gloss of content. In the record's gloss, content is what is found within the arena: the particles, their charges and their fields.

The ratio of the dark matter to the ordinary matter

In the coming measurement, astronomers find the densities of the dark matter and of the ordinary matter from the pattern of the oldest light. The astronomers then set one density beside the other. The instruments are the maps of the oldest light made by the Planck satellite, the South Pole Telescope and the Atacama Cosmology Telescope. The next instrument is the Simons Observatory. With Planck, the Simons Observatory would measure the ratio to about 0.6 percent.

This account says, in the record's Prediction 16, that the ratio of the density of the dark matter to the density of the ordinary matter is two divided by pi, taken away from six: 5.3634, "the same at every time since helium formed". Three instruments together give 5.370, give or take 0.048: the South Pole Telescope, Planck and the Atacama Cosmology Telescope (Camphuis and others 2026). The value of 5.370 is 0.14 standard deviations away from 5.3634. Planck alone gives 5.364, give or take 0.065 (Aghanim and others 2020). This account's value agrees with the routes that do not use the oldest light: the route through the making of the light elements in the first minutes, and the route through the hot gas of clusters of

galaxies. When analyses add the spacing of galaxies measured by the Dark Energy Spectroscopic Instrument, the ratio comes out at 5.199, give or take 0.035, from one kind of measurement (Louis and others 2025). The value of 5.199 is 4.7 standard deviations away from 5.3634, or about 4.0 when the two densities are taken as correlated. The record states what decides this prediction: "A second, independent kind of measurement of the two densities, at the precision of the microwave background". The *microwave background* is the astronomers' name for the oldest light.

This prediction bears on Chapter 9, "What Banged?", and Chapter 7, "Why Ten Dimensions?".

The gyrations clump as cold dark matter does

In the coming measurement, astronomers find how strongly the matter of today's universe is gathered into clumps. Astronomers call the number that states how strongly matter is clumped the *amplitude of clustering*. The astronomers read the amplitude of clustering from the bending of light by matter, from the counts of clusters of galaxies and from the motions of galaxies. Three telescopes will make the coming maps of the bending of light by matter: Euclid, the Vera C. Rubin Observatory in Chile and NASA's Nancy Grace Roman Space Telescope.

This account says, in the record's Prediction 18, that the amplitude of clustering today is 0.823, give or take 0.006, and that "The gyrations cluster exactly as standard cold dark matter does". *Cold dark matter* is dark matter whose pieces move slowly, so that the dark matter gathers into clumps of every size. Each of the following measurements agrees with 0.823 within one standard deviation: the bending of the oldest light by matter, the complete Kilo-Degree Survey, and the growth of clumps read from the motions of galaxies. The final measurement of the Dark Energy Survey lies 2.5 standard deviations below 0.823. Clusters counted by the X-ray telescope eROSITA lie 3.2 standard deviations above 0.823.

This prediction bears on Chapter 9, "What Banged?".

The dark matter acts through gravity alone

The measurement for this prediction is every search for the dark matter by a means other than the dark matter's gravity. Detectors deep underground wait for a piece of dark matter to strike an atom. Telescopes look for light or particles that the dark matter might give off. And physicists try to make dark matter in colliders. One of the most sensitive underground detectors is LZ, in a former gold mine in South Dakota.

This account says, in the record's Prediction 19, that the dark matter acts only through gravity. The record states: "No search will detect the dark matter through any interaction but gravity, at any sensitivity". Prediction 19 also states the ways in which no search will detect the dark matter: "not by scattering in an underground detector, not by light or particles it gives off, and not by its production in a collider". So far, physicists have not detected the dark matter by any means other than gravity. LZ has excluded strikes whose chance, stated as an area, is as small as about two times ten to the power minus forty-eight square centimeters, for a piece of forty GeV. The record reports the result of LZ from *The Alphabet of Particles*.

This prediction bears on Chapter 9, "What Banged?".

The dark matter lasts

Astronomers read the history of the expansion and the pattern of the oldest light for any sign that the dark matter is decaying. If the dark matter decayed, matter would carry a falling share of the universe's energy over cosmic time, and the universe would grow differently. *What Comes After the Dark* states: "The coming maps of the microwave background, among them the Simons Observatory, and the coming surveys of galaxies, among them the Dark Energy Spectroscopic Instrument and Euclid, will read that history more closely".

What Comes After the Dark states: "This account says they will find the dark matter stable at every lifetime they can reach". The record states this entry's prediction as its Prediction 20: "The dark component does not de-

cay, so its density falls exactly as that of matter, and the ratio of Prediction 16 is the same when the microwave background was released and today". Physicists tell this account's cycle of epochs apart from the cycle of Roger Penrose with this entry's measurement. Penrose calls the dark matter *erebons*. In Penrose's cycle, the erebons must decay within each *aeon*, Penrose's word for each age of the universe. Penrose gives the erebons a lifetime of perhaps about a hundred billion years (Penrose 2018). *What Comes After the Dark* describes the bound found so far. The bound comes from the oldest light and the history of the expansion: any decay of the whole of the dark matter, into particles no detector sees, has a lifetime beyond about a hundred and sixty billion years (Poulin, Serpico and Lesgourgues 2016).

This prediction bears on Chapter 19, "Is the Long Emptying the End?", and Chapter 9, "What Banged?".

The dark matter clumps on every small scale

In the coming measurement, astronomers count the faintest small galaxies that circle our own galaxy, and the small clumps of matter that show in the bending of light by galaxies far away. If the dark matter's own motion smoothed away the smallest clumps, there would be fewer faint small galaxies and fewer small clumps.

This account says, in the record's Prediction 21, that the dark matter clumps on every small scale. The record states: "The dark matter has no cut-off in its structure on small scales from its own motion; it clumps on every small scale". From the counts so far, astronomers place any smoothing by the dark matter's own motion below clumps of about ten million times the mass of the Sun, at 95 percent confidence (Nadler and others 2021).

This prediction bears on Chapter 9, "What Banged?".

Each gyration has about the Planck mass

The measurement for this prediction is any measurement of the mass of the single pieces of the dark matter.

This account says, in the words of the record's Prediction 22: "The dark matter comes in gyrations whose mass is of the order of the Planck mass". The record gives the mass of the lightest gyration in two steps of one calculation. At the *classical level*, from the shape of the six held dimensions alone with no quantum correction, the mass is the square root of two Planck masses. At *first order*, with the first and largest part of the quantum correction added, the mass is about six tenths of a Planck mass. The Planck mass is about twenty-two millionths of a gram (Chapter 5). In this account, the density of the dark matter is 5.3634 times the density of the ordinary matter (Prediction 16), and each gyration is far heavier than a proton or a neutron. So there is about one gyration for every one and a half to three and four tenths billion billion protons and neutrons. No one has yet measured the mass of a single piece of the dark matter, and so the record reports Prediction 22 as untested.

This prediction bears on Chapter 9, "What Banged?".

A galaxy feels a larger one through its tide

In the coming measurement, astronomers follow the motions of the stars in small galaxies that fall freely in the field of larger galaxies. In 1983 Mordehai Milgrom proposed *modified Newtonian dynamics*, a change to Newton's law of gravity at very small accelerations, in place of dark matter (Milgrom 1983). In modified Newtonian dynamics, the motions inside a small galaxy depend on the whole pull of the larger galaxy that the small galaxy falls in. Physicists call the dependence of the inner motions on the whole pull of the larger galaxy the *external field effect*.

This account says, in the words of the record's Prediction 24: "A galaxy falling freely in the field of a larger one feels that field only through its tide; the external field effect that modified Newtonian dynamics predicts will not be found". The *tide* is the difference between the pull on the near side of the small galaxy and the pull on its far side.

This prediction bears on Chapter 9, "What Banged?".

The Energy of Empty Space

The matter fraction is 0.311

In the coming measurement, astronomers find the matter fraction, the fraction of the universe's energy that is matter today, from the pattern of the oldest light and from the spacing of galaxies. The spacing of galaxies was laid down by sound waves that rang through the hot gas of the young universe and left a preferred distance between clumps of matter, a pattern that astronomers call the *baryon acoustic oscillations*. DESI is the Dark Energy Spectroscopic Instrument. The record states what decides this prediction: "The coming measurements of the matter fraction by DESI, Euclid, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope".

This account says, in the words of *At the Edge of Reality*, that "matter makes up about 31 percent of the energy of a flat universe today". The record's Prediction 7 gives the number as 0.311, give or take 0.001. In this entry, as in Chapter 10, the *vacuum* is empty space with every field at its lowest energy. The *vacuum's energy* is the whole energy that the restless trembling of the fields of empty space carries. The number 0.311 comes from the part of the vacuum's energy that a measurement made from inside the universe can take in: the part with which I identified the cosmological constant in Chapter 10. The oldest light gives 0.315, give or take 0.007, which is 0.6 standard deviations away from 0.311 (Aghanim and others 2020). Measured on its own by DESI, the spacing of galaxies gives 0.2975, give or take 0.0086, which is 1.6 standard deviations away from

0.311 on the other side (Abdul-Karim and others 2025). *At the Edge of Reality* states: "The prediction lies between them, and the account says the coming surveys will converge on it".

The record also states Prediction 7 through the *Hubble constant*, the present rate of the expansion. The Hubble constant is given in kilometers per second for each megaparsec of distance. A megaparsec is about three and a quarter million light years. With the matter density that the oldest light measures, this account predicts 67.8, give or take 0.3. The oldest light's own fit gives 67.4, give or take 0.5, which is 0.8 standard deviations away from 67.8. *Cepheid variable stars* are stars whose brightness rises and falls with a period that tells their true brightness. Astronomers build a ladder of distances on Cepheid variable stars: a chain of distance measurements in which each step sets the scale of the next. The ladder gives 73.04, give or take 1.04, which the record places 4.9 standard deviations away from 67.8. *Side by Side, Volume One* gives the distance between 73.04 and 67.8 as later computed: 5.0 standard deviations in units of the measurement's own uncertainty, and 4.8 when the prediction's uncertainty of 0.3 is combined with the measurement's.

This prediction bears on Chapter 10, "Why Is the Energy of Empty Space So Small?", and Chapter 11, "Is Our Universe the Only One?".

The energy of empty space is constant

In the coming measurement, astronomers find the *equation of state* of the energy that drives the speeding up of the expansion: the ratio of the energy's pressure, its push on its surroundings, to the amount of the energy in each unit of volume. A different equation of state would change how fast the universe grew at each era of its history. So astronomers read the history of the universe's growth in four ways: the distances to exploding stars of a known brightness, the spacing of galaxies, the bending of light by matter, and the pattern of the oldest light. The record states what decides this prediction: "The expansion history and growth of structure measured by Euclid, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope".

This account says, in the words of the record's Prediction 8: "The energy of the vacuum has an equation of state of exactly -1, at every epoch". In words, the equation of state is minus one, with no change at all as the universe grows. *What Comes After the Dark* states: "This account says they will find the equation of state exactly minus one, at every era of our epoch's history". *At the Edge of Reality* states that "the account says they will find the energy of empty space constant". The *cosmological constant* is one number in Einstein's equations, read as an energy of empty space, the same in every cubic meter of space. In Chapter 10 I identified the cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in. Within the one space and time of our epoch, the cosmological constant is one number. An energy that never thins out as space grows has an equation of state of exactly minus one (Chapter 10).

Combined with the spacing of galaxies and exploding stars, the Planck satellite gives minus one and three hundredths, give or take three hundredths (Aghanim and others 2020). In the final analysis of the Dark Energy Survey, combined with DESI and the oldest light, the authors fit an equation of state that changes as the universe grows (Abbott and others 2026). *What Comes After the Dark* states: "Its present value in that fit is minus eighty-two hundredths, give or take five hundredths". And *What Comes After the Dark* states: "The authors report this fit as three standard deviations from a constant". *At the Edge of Reality* also states that DESI reports a preference for change of about the size that the Dark Energy Survey reports, and that "these analyses share much of their data". The record says of the departure from minus one: "That departure rests on one family of combined analyses, and it has not reached the rule of decision". The record also states what this account says the coming surveys will find: "The reported departure from -1 will not be confirmed".

This prediction bears on the Preface, Chapter 10, "Why Is the Energy of Empty Space So Small?", and Chapter 11, "Is Our Universe the Only One?". This prediction also bears on Chapter 16, "Will the Sky Go Dark?", Chapter 17, "Can a Mind Keep Thinking Forever?", and Chapter 19, "Is the Long Emptying the End?".

The Far Future

In *What Comes After the Dark*, the collection's book on the far future, I describe three coming measurements besides the two entered above, on the dark matter's lifetime and on the energy of empty space.

Searches for the decay of the proton

The measurement for this prediction is the search for the decay of the proton, made by the next instruments. The earlier entry "The proton lasts" quotes the record's Prediction 33, which states this prediction. *What Comes After the Dark* states: "Hyper-Kamiokande is a far larger water tank in Japan". If one of the tanks of Hyper-Kamiokande sees no decay in ten years, Hyper-Kamiokande will set the proton's lifetime for the decay into a positron and a neutral pion above seventy-eight million billion billion billion years, at 90 percent confidence. The Deep Underground Neutrino Experiment, DUNE, watches tanks of liquid argon. The lifetimes that DUNE can test are stated for an exposure of four hundred thousand tonnes of argon watched for a year, or the same total watched longer in a smaller mass. If DUNE sees no decay in that exposure, DUNE will set the lifetime for the decay into a kaon and an antineutrino above about thirteen million billion billion billion years. A *kaon* is another short-lived particle made of a quark and an antiquark. An antineutrino is the antimatter partner of the neutrino.

What Comes After the Dark states: "This account says they will find the proton stable, at every lifetime the searches can reach". And *What Comes After the Dark* states, of Hyper-Kamiokande and DUNE: "This account says both will find the proton stable".

This prediction bears on Chapter 16, "Will the Sky Go Dark?".

Primordial gravitational waves and the cycle

The measurement for this prediction is the measurement of the twist in the polarization of the oldest light by the Simons Observatory and LiteBIRD, as in the entry "The first unevenness is unevenness of density", above. Greene names the gravitational waves of the beginning as the observation that may one day help decide between inflation and a *cyclic cosmology* (*Until the End of Time*, chapter 10, section "Is the End Near?"). A cyclic cosmology is a history in which regions of space like ours expand, then contract, then rebound into a new expansion, again and again. Greene writes that the burst of inflation "would likely have so vigorously disturbed the fabric of space that the gravitational waves produced might still be detectable", while the gentler expansion of the cyclic model "results in gravitational waves too mild to be observed". Greene goes on to say that, in the not-too-distant future, "observations may thus have the capacity to tip the balance between the two cosmological approaches". Greene also says that gravitational waves of the beginning "would leave a distinct imprint on the polarization of the microwave background radiation, perhaps an imprint large enough to be measured" (*The Fabric of the Cosmos*, chapter 14, section "Cosmic Origins").

What Comes After the Dark states: "This account says the instruments will find the first unevenness of the universe to be unevenness of density alone, with the twist far too faint to see". And *What Comes After the Dark* states, of the Simons Observatory and LiteBIRD: "This account says both will find a ratio far below their reach: departures of density alone". On the first waves, this account's cycle agrees with the cycle of colliding *branes* (objects spread out in two, three or more dimensions, as a sheet is spread out in two): both cycles predict first waves too faint to see (Chapter 11). The two cycles differ in the equation of state of the energy of empty space. The entry "The energy of empty space is constant", above, contains that prediction.

This prediction bears on Chapter 19, "Is the Long Emptying the End?", and Chapter 11, "Is Our Universe the Only One?".

The flatness of space in every epoch

The measurement for this prediction is the measurement of the curvature of space on the largest scale, which is the measurement of Prediction 9 in the entry "Space is flat", above. The measurement will be made by the coming maps of the oldest light and the coming surveys of galaxies.

What Comes After the Dark states: "This account says they will find space flat". The beginning of an epoch holds no length carried over from the epoch before, so the beginning sets no curvature. The actual space of our epoch is curved only by its own small departures from uniformity, far below what the coming surveys can measure. *What Comes After the Dark* states: "This account says they will close in on zero".

This prediction bears on Chapter 19, "Is the Long Emptying the End?".

The Correspondence

The correspondence stays a translation

This entry's prediction is a statement of *Two Theories, One World*, Chapter 27. The measurement for this prediction is made by mathematics. In this book's sense, a *correspondence* is two descriptions with an exact translation between them, neither made of the other (Chapters 5, 7 and 12). In the correspondence of Juan Maldacena, a theory with gravity inside a region of space and a theory with no gravity on the boundary of that region describe one system in two ways. The *dictionary* of the correspondence is the list of rules for translating each quantity of one side into the other side. The mathematics that decides this prediction is about a boundary theory of finite size, with every correction included.

This account says, in the words of *Two Theories, One World*, that the dictionary of the correspondence "will remain a translation between two descriptions, limited in both directions". *Two Theories, One World* also states:

"This prediction is decided by mathematics rather than by instruments". And *Two Theories, One World* states: "For that case the account predicts that a restriction on what can be translated remains, and so not every quantity of the interior can be rebuilt exactly from the boundary". When the geometry inside can be treated as *classical*, described without quanta of its own, physicists have already rebuilt the fields inside from the boundary. In the work so far, a given region inside can be recovered only from a large enough part of the boundary. The work on codes that correct errors showed the need for a large enough part of the boundary. Codes that correct errors store information so that damage to part of the store can be repaired.

This prediction bears on Chapter 12, "Are There Other Worlds in the Quantum, the Hologram, the Computer?", and Chapter 7, "Why Ten Dimensions?".

For the Specialist

This appendix gives, for every claim of each chapter of this book, the book of the collection, the chapter and the section that carries it, with the exact values, symbols and relations that the chapter states in words. Mathematics stands here, and each symbol is read in words where it first appears. Where a chapter tells a claim of Greene's as his account and the collection gives no reply on that claim, the note for that chapter says so in one sentence: "Told here as Greene's account; the collection gives no reply on it". Where the collection does speak to such a claim, and its statement is given in another note of the same chapter, the note says only: "Told here as Greene's account". Before the notes for the chapters come the ground on which every chapter stands, the identifications the chapters make, and five statements of scope, each in its source's own words.

The Three Axioms

The three axioms are given below in the exact wording of *An Immanent Metaphysics*, aphorism 1.3-1. The chapters give each axiom in plain words where it is used: the first in Chapters 6 and 18, the second in Chapters 6 and 8, and the third in Chapters 6 and 19. Two terms in the axioms need a gloss. A *class* is a kind, standing ready as a whole, like the kind "a reading of a thermometer"; an *instance* is one particular case of a kind, like this morning's reading of the thermometer on the wall. In the first axiom, "and/or" means of either one and of both together; in the third, the bracket means that the statement holds both of the classes and of the instances.

The formal writings of the Immanent Metaphysics call the three roles of every comparison the *modalities*, and give them three names. The chapters use plain names for the three roles. The act, the happening of the comparison itself, is the *immanent*. The structure, what is laid down, the

same for all who consult it, is the *omniscient*. The stance, the side a comparison is made from, with the range of what could come from there, is the *transcendent*.

In the exact wording of aphorism 1.3-1, the three axioms are these:

> **Axiom I:** The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another.
>
> **Axiom II:** A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent.
>
> **Axiom III:** The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable.

The Incommensuration Theorem

Chapter 3 gives the two central lines of the theorem, and the reason for them, in plain words.

In the exact words of *An Immanent Metaphysics*, aphorism 1.43-2, the Incommensuration Theorem reads:

> **1.43-2 The Incommensuration Theorem:** Where applied to an absolute degree, the following theorems hold for any interaction (any comparison) in any domain:
>
> When something is absolutely symmetric, it must also be intrinsically discontinuous.
> When something is absolutely asymmetric, it must also be intrinsically continuous.
> When something is absolutely continuous, it must also be intrinsically asymmetric.
> When something is absolutely discontinuous, it must also be intrinsically symmetric.
> Nothing anywhere at all can be both absolutely symmetric and absolutely continuous.
> Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous.

Aphorism 1.43-1, "Primary Operators", defines the four characters in the vocabulary of the six ideas of comparison (sameness and difference, content and context, the subjective and the objective).

Aphorism 1.43-1 says that its four definitions hold "Where there is assumed a sameness of subjective context", and that the four definitions hold "about the nature of the objective". In the aphorism's own words, the four definitions are these:

> Continuity refers to where there is a sameness of content and a sameness of context.
> Discontinuity refers to where there is a difference of content and a sameness of context.
> Symmetry refers to where there is a sameness of content and a difference of context.
> Asymmetry refers to where there is a difference of content and a difference of context.

The proof is a count, and it can be checked in a few lines. Four characters can be paired in ten ways: each with itself (four pairings) and each with each of the others (six). A pairing gives four verdicts: two on the content, one from each character, and two on the context, one from each character. Three rules decide every case. A character paired with itself says nothing new, which removes four pairings. A character paired with its direct opposite contradicts itself, which removes continuity with discontinuity and symmetry with asymmetry. The third rule rests only on the inseparability of sameness and difference, which every comparison carries: within each of the two respects, content and context, a sameness must appear together with a difference. Continuity with symmetry gives the content "same" twice and "different" never, and discontinuity with asymmetry gives the content "different" twice and "same" never, so both fail the third rule. Two pairings remain, continuity with asymmetry and symmetry with discontinuity, and in each the second character is the exact complement of the first. I call these two pairings *the two surviving pairings* (Chapter 3).

The Incommensuration Theorem uses the meanings of sameness, difference, content and context, and the inseparability of sameness and difference, and nothing else. The Incommensuration Theorem has no physical premise. It is a structural fact about the nature of knowing. The count is set out, with its table of the ten pairings, in *Incommensuration; The Theorem and the Three Axioms*, Chapter 2, "Symmetry and Continuity Cannot Both Be Perfected", Sections III to V; the same book proves the theorem step by step from numbered premises in its Part Two and derives the three axioms from it.

The Two Kinds of Perfected Comparison

In physics, the two laws of *An Immanent Metaphysics*, aphorism 1.83-5, assign the characters to the two sides of every measurement: "Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context". So content, perfected, is discontinuously symmetric, and context, perfected, is continuously asymmetric: these are the two pairings that the theorem allows, symmetry with discontinuity and continuity with asymmetry.

Aphorism 1.43-8, "The Four Theorems of Being", gives the same two pairings as four facts: "Subjective context is continuous". "Subjective content is discontinuous". "Objective content is symmetric". "Objective context is asymmetric". Its examples are, in order, the persistence of self identity, the diversity of thoughts and perceptions, the conservation of matter and energy, and the arrow of time. The two facts of content (subjective content discontinuous, objective content symmetric) make the pairing of symmetry with discontinuity, and the two facts of context (subjective context continuous, objective context asymmetric) make the pairing of continuity with asymmetry. The identification that Chapter 5 makes from these two laws, of quantum theory and general relativity with the two sides, is listed in the next section.

The Identifications the Chapters Make

Each identification that the chapters make is made once, by one chapter, and it is given here once, in the exact words of the chapter that makes it, with the books of the collection that carry it. A later chapter, or Appendix A, recalls an identification when it restates it in the first person and names the chapter that makes it, as Chapter 13 does: "Beneath the second law, my account has the one-way order of findings, in which each act of comparison comes before the record that the act lays down, and which I identified in Chapter 2 with the direction of time". Each chapter that recalls an identification in this way, and Appendix A where it does, is named beside the chapter that makes it.

Gravity and the shape of the arena (made in Chapter 1; recalled in Chapters 5 and 6, and in Appendix A). Chapter 1: "So I identify gravity with the shape of the arena, and not with a force acting within the arena". The collection carries it in *Measurement; The Steps from Comparison to Physics*, Chapter 3, Section VII, "Why Gravity Is the Arena and Not a Field in It", and in *Two Theories, One World*, Chapter 9, "Four Marks of the Arena", section "The Identification". That section makes the identification in the words "I identify gravity as the arena of physics, the context of every physical measurement, by that match", and later in the same section it states the shape: "So gravity is the shape of the arena, and not a force that acts within it".

The speed of light (made in Chapter 2; recalled in Chapter 3). Chapter 2: "I identify the speed of light with the edge of comparison: the greatest speed at which any signal can carry a finding from one place to another". The collection carries it in *The Gravitational Passage*, Chapter 17, Section I, "Three Constants, Three Roles": "The speed of light is not, at root, the speed of any one thing but the macroscopic boundary of the domain of comparison itself; light is only the perfected example of c". *Measurement*, Chapter 3, Section VIII, "The Speed That Belongs to the Arena", calls it "the fastest that any signal can get from one place to another".

The direction of time (made in Chapter 2; recalled in Chapters 4, 8 and 13). Chapter 2: "I identify the one-way order of findings with the direction of time". The collection carries it in *The Unquantized Arena*, Chapter 10, Section II, premise P19, and in *What the Beginning Left Behind*, Chapter 6, section "The Direction in Which Comparison Proceeds", each in the words "I identify the one-way order of findings, the order of Axiom II, with the direction of time".

Lawfulness and locality (made in Chapter 3). Chapter 3: "I identify lawfulness with a symmetry, the same result across every difference of place and time, and locality with a continuity, an influence carried through the space between two places with no gap". The collection carries it in *Why the Quantum Is the Way It Is*, Chapter 15, section "Lawfulness and Locality", and in *What Cannot Be*, Chapter 10, sections "Lawfulness Is a Kind of Symmetry" and "Locality Is a Kind of Continuity".

The cut (made in Chapter 4). Chapter 4: "I identify the cut with the boundary between the smooth description and the findings: on one side of the cut is the state, which describes what could be found, and on the other side is the record of what was found". The collection carries it in *Why the Quantum Is the Way It Is*, Chapter 18, section "Copenhagen and the Cut": "I identify the cut as the boundary between the two levels of the theory, the formalism and the findings".

The two theories and the two sides (made in Chapter 5; recalled in Chapter 6). Chapter 5: "I identify quantum theory with the description of the content of physics, which is what is found in the comparisons of physics", and "I identify general relativity with the description of the context of physics, which is the arena in which that content is found". The collection carries it in *Home in the World*, "For the Specialist", "The Two Kinds of Perfected Comparison": "I identify quantum theory as the description of the content of physics, and general relativity as the description of its context, the arena of space and time". It carries it also in *Two Theories, One World*, Chapter 15, section "What the Meeting Means". That section uses the word "character" for one of the two surviving pairings, where this ap-

pendix uses it for one of the four characters. It makes the identification in these three sentences: "Part II found that comparison, perfected at its foundation, takes one of two characters. It is either continuously asymmetric or discontinuously symmetric. I identify general relativity with the first character and quantum theory with the second". So in that section the first character is the pairing of continuity with asymmetry, and the second is the pairing of symmetry with discontinuity.

One geometry for each alternative (made in Chapter 5; recalled in Chapter 12). Chapter 5: "I identify the gravity of matter held between alternatives with one geometry for each alternative: each alternative of the matter carries its own smooth geometry, shaped by that alternative's own energy". The collection carries it in *The Unquantized Arena*, "What This Book Claims and How It Is Set Out", Section I, "The Thesis": "I identify the configuration in which each alternative of the matter carries its own classical geometry". The same book states the identification as Proposition 7.1, in its Chapter 7, "One Geometry for Each Alternative", Section III, "One Geometry for Each Alternative": "Proposition 7.1 is this book's identification, made on that ground".

What a black hole's entropy counts (made in Chapter 6; recalled in Chapters 18 and 19). Chapter 6: "I identify what the entropy of a black hole counts with states of the hole's content: the states of what the hole holds that look the same in every comparison made from outside its horizon". The collection carries it in *What Comes After the Dark*, Chapter 8, "Is the Long Emptying the End?", section "The Third Ground: Every Count Has a Context": "I identify what that entropy counts as the states of the hole's interior that no comparison from outside can tell apart: a count made against the outside world as its context". *Two Theories, One World*, Chapter 26, section "'The Entropy Has Been Counted'", reads the count of Strominger and Vafa in the words "I read the count as a count of content". *The Unquantized Arena*, Chapter 17, Section III, "The Premise Removed", Proposition 17.3, and Section IV, "Why Area, Not Volume", carry the count and its dependence on area: "I therefore identify the dependence of

equation (17.2) on area as the immanent count: the number of states of the interior that the comparisons available from outside cannot tell apart, relative to the outside as context (Section III)".

The two sixes (made in Chapter 7). Chapter 7: "I identify the six dimensions of possibility with the six small dimensions of physics". The collection carries it in *At the Edge of Reality*, Chapter 7, section "A Space of Possibilities": "I identify the two sixes as one set".

The Big Bang and the turn (made in Chapter 9; recalled in Chapters 18, 19 and 22). Chapter 9: "I identify the Big Bang with the turn as our epoch's history renders the turn: the limit, which no comparison registers as an event, drawn as the first event of the history we reconstruct from inside". The collection carries it in *At the Edge of Reality*, Chapter 17, section "The Limit Rendered as a Point": "I identify the Big Bang of the equations of cosmology with that appearance, and I call the appearance the projection: the turn, which no comparison registers as an event, rendered as an event in a history reconstructed from inside".

The first ripples (made in Chapter 9; recalled in Chapters 11, 12, 13, 18, 19, 20, 21 and 22, and in Appendix A). Chapter 9: "I identify the first ripples with the form of the content carried through the turn". *What the Beginning Left Behind* calls the first ripples "the distinctions of the old epoch averaged and not erased". The collection carries it in *What the Beginning Left Behind*, Chapter 11, section "What Passes the Turn": "I identify the primordial departures from uniformity with the form of the content carried from one side of the turn to the other, the distinctions of the old epoch averaged and not erased".

What a measurement receives of the vacuum's energy (made in Chapter 10). Chapter 10: "I identify the part of the vacuum's energy that a measurement receives with the energy held by the degrees of freedom that the measurement engages". The collection carries it in *The View from Inside*, Chapter 10, section "A Cup from the Sea": "I identify what a measurement receives of the vacuum's energy with what is held by the degrees of freedom it engages".

The cosmological constant and the accessible share (made in Chapter 10; recalled in Chapters 11, 16, 18, 19 and 22, and in Appendix A). Chapter 10: "I identify the cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in". Chapter 10 makes it once, in its section "Three Tiles on the Edge of What Has Reached Us", and recalls it in its section "One Scale for the Whole Map" and in its last section, "What the Small Number Says". The collection carries it in *What Comes After the Dark*, Chapter 2, section "The First Way: Space Is Flat": "I identify the cosmological constant with the part of the energy of empty space that a measurement made from inside the universe can take in". *What Comes After the Dark* writes "the energy of empty space" for the total, of which the constant is a part: "The energy of empty space is a positive amount, and the part that a measurement takes in is a small fraction of it, so the constant is positive". (Same section.) Chapter 10 calls that total the vacuum's energy and keeps "the energy of empty space" for the energy that speeds the expansion, so the two sentences of identification name the same part of the same total.

The three channels (made in Chapter 10). Chapter 10: "I identify the vacuum's channels into a measurement with pattern, force and probability, one in each role". The collection carries it in *What the Beginning Left Behind*, Chapter 8, section "Three Tiles Out of All the Tiles": "I identify the channels through which the vacuum reaches a measurement with the three contents a measurement registers, one in each of the three roles". It carries it also in *The View from Inside*, Chapter 12, "Counting the Horizon", section "Why Three".

The two sides of the correspondence of Maldacena (made in Chapter 12). Chapter 12, of the two descriptions in Maldacena's correspondence, the boundary theory and the theory of the interior: "I identify which side carries which: symmetry with discontinuity on the boundary, and continuity with asymmetry in the interior". The collection carries it in *Two Theories, One World*, Chapter 23, section "Each Side in Its Own Character".

Consciousness (made in Chapter 14; recalled in Chapters 17 and 22). Chapter 14: "I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act, happening: the one-way order of the act, its outcome that nothing before it fixed, and its stance". The collection carries it in *What Comes After the Dark*, Chapter 7, section "What a Thought Needs": "Taken whole in that way, in a context, as one act happening, the three are what I identify as consciousness".

The subjective (made in Chapter 14). Chapter 14: "The subjective is the comparing side of every comparison, the side that holds the stance", and "I identify the subjective with the inside of a process that, seen from outside, shows as hard randomness and as *entanglement*". The collection carries it in *The Last Turtle*, Chapter 15, Section VIII, which states the identification in the form "the subjective and total entanglement are strict isomorphs".

Five Statements of Scope

Each of the five statements of scope below is given with what the account establishes first, and then in its source's own words, with its source.

A shorter inflation. "The account answers every puzzle inflation was introduced to solve, from the turn and from the structure of the forces, so it needs no inflation". (*What the Beginning Left Behind*, Chapter 13, "No Inflation Needed".) On the identification of the first ripples a long inflation is excluded (same chapter, section "A Long Inflation Is Excluded"). Of a shorter phase the same chapter says: "A shorter phase is neither required nor excluded by the account, and the temperature at which the history begins bounds it". (Summary.) *The Gravitational Passage* says the same: "A shorter phase, which leaves the observed scales outside the early horizon throughout, is neither required nor excluded by the account, and the book does not fix it". (Chapter 18, Section XIII, Questions.)

The information that falls into a black hole formed from a star. "Information is not destroyed in a black hole". (*The Unquantized Arena*, Chapter 17, Section III, Proposition 17.3.) For a hole formed from a star,

the information "is not accessible from the outside world of the original epoch: it is lost to every observer who remains outside the horizon, and it is not destroyed". (*At the Edge of Reality*, Chapter 18, section "Information Is Not Destroyed"), and it is carried with the content to the turn. Of the outside world of the original epoch, the account's statement of scope is this: "The account asserts neither the return of this information to the outside world nor its loss to it". (*The Unquantized Arena*, Chapter 17, Section VII, "Holes Formed from Stars".) *What the Beginning Left Behind*, Chapter 9, section "Where the Information Goes", and the sealed record, *Side by Side, Volume One*, Appendix B, Section IX, state the same.

The reset at the turn. "The reset follows from the meaning of a count and the account of the turn, and it is given by principle". (*What the Beginning Left Behind*, Chapter 7, "A Clean Slate, Counted", Summary.) Its scope, in the words of *The Gravitational Passage*: "The reset is given here by principle; a count of states across the turn, in numbers, is not given in this book". (Chapter 18, Section VIII, "One Process, Two Sides".) *What the Beginning Left Behind* says of that book: "it gives no count of states across the turn in numbers". (Chapter 7, section "Every Count Has a Context".)

The four values taken from measurement. The kinds of particle form a closed list fixed by the structure of comparison: "No further kinds exist: no fourth generation, no supersymmetric partner, no X or Y boson, no graviton, no dark-matter particle of any type". (*The Kinds of Particle*, Chapter 6, Section IV.) Four values are taken from measurement, in the words of the sealed record: "The value of Newton's constant, the strength of light, the strength of color and the vacuum value of the Higgs field are measured inputs" (*Side by Side, Volume One*, Appendix B, "The Sealed Record", Section IX, "What the Books Do Not Predict").

The coefficient one quarter in the entropy of a black hole. "The count is real". (*The Unquantized Arena*, Chapter 17, Section III, "The Premise Removed".) "So the physical count of a horizon's information is the count made from a place, and it goes by area". (Same chapter, Section IV, "Why Area, Not Volume".) Its scope, in the same section: "The identification

makes no claim on the coefficient one quarter, which equation (17.2) takes from Hawking (1975)". *The Compactification* states the same in the words "The identification makes no claim on the coefficient one quarter". (Chapter 3, Section V, "Every Count Has a Context"), and again in Chapter 4, Section II, "The Count of a Horizon".

The Chapters, One by One

Preface. What Greene Showed, and Why Read Another Book

Who writes, and what the collection is. *What Comes After the Dark; The Far Future of the Universe, in Plain Words*, "The Question of the Far Future", section "A Clear Night": "This book is one of a collection of books on physics that I have written, which work out the physics of the world from the act of comparison itself". The Preface names its author and uses "this account" for the account of physics that the books of that collection give.

Every finding is a comparison. *What Comes After the Dark*, Chapter 6, "What Happens to Someone Who Is Still There?", section "What an Observer Is": "Every observation, every measurement and every test is a comparison: the act of finding whether two things are the same or different, in some respect". The same section names what every comparison has: what it finds is its content; the respect in which it finds it is its context.

The Immanent Metaphysics, and its ground. *Home in the World; The Immanent Metaphysics for Readers of Carlo Rovelli*, Prelude, "Two Crews, One Tunnel", section "The Crew from Knowing": the study asks what every comparison must contain, whatever is compared and whoever or whatever does the comparing; "immanent means within"; "Its ground takes nothing from physics: no measurement, no law, no result of any experiment". The three axioms on which the study rests are given in the exact words of *An Immanent Metaphysics*, aphorism 1.3-1, in the opening section of this appendix.

Gravity is the shape of the arena (the same result as Greene's, on separate ground). *Two Theories, One World*, Chapter 9, "Four Marks of the Arena", section "The Identification": "So gravity is the shape of the arena, and not a force that acts within it". The four marks the chapter derives for any arena, and finds in gravity, in the words of the chapter's opening: "An arena couples to everything; nothing can shield it; the charge by which it holds a body and the inertia of that body are one quantity; and it has no charge of its own that comes in steps". The third mark is the one by which all bodies fall alike. The Preface names the four marks, the first three as three things gravity does; Chapter 1 sets out each of the four. What the arena takes hold of in a body, as the Preface states it before the fourth mark: the same chapter, section "The Third Mark: One Mass" ("What couples a body to the arena is how much the body takes part in the arena"). That to take part in the arena is to be found in it: the same chapter, section "The First Mark: It Couples to Everything" ("to be neutral to the context would be to be found nowhere"). The fourth mark: the same chapter, section "The Fourth Mark: No Charge in Steps" ("So whatever couples a body to an arena is no stepped label". and "It varies continuously, taking any value in its range"). That no force of particle physics carries any of the four: the same chapter's opening, "No force of particle physics carries any of the four". The same four marks, as the markers any context of a domain must carry: *Measurement*, Chapter 3, "Content and Context in Physics", Section VII, "Why Gravity Is the Arena and Not a Field in It". Greene's statement beside it: *The Fabric of the Cosmos*, chapter 3, section "Warps, Curves, and Gravity" (Einstein's conclusion from the equivalence of gravity and acceleration).

The arena, as this book uses the word: space and time taken together as the setting in which everything that is measured is measured; its shape is gravity. Chapter 1 gives the gloss again at its first use there, beside Greene's words "fabric of the cosmos" and "spacetime".

Ten dimensions of three kinds (a different result from Greene's, with the reason). *At the Edge of Reality*, Chapter 3, "Why Ten Dimensions?", section "One, Three and Six": "The world has ten dimensions: one of time,

three of space, and six of possibility". The same section gives the reason for each count: time has one because the comparing is one stance; space has three because three is the least number of directions in which every difference can be placed twice, found by a comparing and placed in a context; possibility has six because it is the range of what a comparison could find, and that range spans six ideas that every comparison contains. The gloss of possibility as "the range of what a comparison could find": the same book, "The Story in Brief", section "The Story"; and Chapter 3, section "One, Three and Six", which says that the extra dimensions "hold the possibilities that content can take". That the six are never traveled through: *What the Beginning Left Behind*, "Before We Begin", section "Where the Story Stands" ("The six of possibility are never traveled through"). The gloss of dimension, "an independent way in which things can differ": *At the Edge of Reality*, Chapter 3, section "A Fifth Dimension, Curled Small" ("The number of dimensions of something, in the plain sense, is the number of independent ways in which it can differ").

The two tens are counts of different things. *The Gravitational Companion*, "Other Programs That Obtain General Relativity", Section VIII, "String Theory (Callan, Friedan, Martinec and Perry 1985)": "the two tens are counts of different things". The section's reasons, each read in words. The internal space of the account is the complex projective plane, written $\mathbb{C}P^2$ (read "C P two"), times the two-sphere, written S^2 (read "S two"). The complex projective plane admits no spin structure (Hawking and Pope 1978), so the product of four-dimensional spacetime, written M^4 (read "M four"), with $\mathbb{C}P^2 \times S^2$ admits none; the type IIA, type I and heterotic superstrings need one. At leading order in the string length, the conditions of string theory with a flat external factor force the internal space to be Ricci-flat (of zero Ricci curvature), while the account's internal metric has positive Ricci curvature: on $\mathbb{C}P^2$ with the Fubini-Study metric, $R_{ij} = 6\, g_{ij} / R_O^2$, read "the Ricci tensor R i j equals six times the metric g i j divided by the square of the radius R O", everywhere positive. With a de Sitter external factor (a spacetime whose expansion speeds up), super-

gravity with two derivatives and without singular sources has no compactification at all (Maldacena and Núñez 2001). The account's ten-dimensional action is built from the metric alone, with no dilaton, no form fields and no branes, so its geometry is not a vacuum of the superstring at leading order.

Where the mathematics is, and where the coming measurements are. The three measurements the Preface names in words are entered in Appendix A, each beside the prediction of the collection's record of registered predictions, *Side by Side, Volume One*, Appendix B, "The Sealed Record": the fall of matter and antimatter under Prediction 4; the pattern of the light left from the early universe under Predictions 10 to 13 and *At the Edge of Reality*, Appendix B; whether the energy of empty space that speeds the expansion keeps one value in each cubic meter (an equation of state of exactly minus one) under Prediction 8.

Values the Preface states in words, from Greene's books. The Planck length: about 10^{-33} centimeters, read "ten to the power minus thirty-three centimeters" (*The Elegant Universe*, Glossary of Scientific Terms, "Planck length"); said in the Preface as a millionth of a billionth of a billionth of a billionth of a centimeter. The end of thought: within about 10^{50} years, read "ten to the power fifty years", if the accelerated expansion does not slow, with intelligent intervention set aside and the numbers called rough (*Until the End of Time*, chapter 9, section "A Final Thought on Thought"); said in the Preface as a one followed by fifty zeros, Greene's fiftieth floor (the floors: the same chapter, section "An Empire of Time", floor N standing for ten to the power N years since the big bang).

Greene's eight steps, each told in the Preface as Greene tells it; each is taken up, with this account beside it, in the chapters named. The conflict and strings: Chapters 5 and 6. The count of dimensions, Calabi-Yau shapes, dualities and M-theory: Chapter 7. Space and time, their slicing and the loaf: Chapters 1 and 2. Not local: Chapter 3. The arrow, the low-entropy beginning and inflation: Chapters 8 and 9. The nine multiverses: Chapters 11 and 12. Entropy and evolution, life, mind and free will: Chapters 13, 14 and 15. The far future and meaning: Chapters 16 to 21.

Until the End of Time also tells, in its fourth chapter, how chemistry may have given rise to the first living things, and it asks whether life is common in the cosmos (chapter 4, sections "Life, Quantum Physics, and Water" to "A General Theory of Life?"). I leave those pages to Greene and set no part of this account beside those pages.

Chapter 1. Is Space a Something?

Newton's bucket, in the *Principia* of 1687 (the Scholium to the Definitions), with the water flat at first and then hollow, and Newton's words on the ascent of the water. The collection tells the experiment in *What Comes After the Dark*, Chapter 6, "Mach and the Bucket", which quotes Newton's sentence, and in *The Gravitational Companion*, "One Mass, Three Elevators", Section II, "Inertia Is a Comparison" (Newton 1687; Mach 1883). Newton's parenthesis "as I have experienced" is in the English translation of Andrew Motte (1729), in the same Scholium.

Greene's telling of the bucket, of the natural suggestion that the water turns against the bucket, of his own step beyond Newton's experiment (the rope rewinds and stops the bucket while the water still turns), of absolute space, of Leibniz and of Mach (*The Fabric of the Cosmos*, chapter 2, sections "The Bucket", "Space Jam", "Mach and the Meaning of Space", "Mach, Motion, and the Stars" and "Mach vs. Newton"). Told here as Greene's account. The collection states Newton's conclusion and Mach's answer in the same terms in *The Gravitational Companion*, "One Mass, Three Elevators", Section II.

Greene's telling of special relativity, of the slicing of spacetime by observers in steady relative motion, of absolute spacetime, and of the straight path through spacetime as the benchmark for acceleration (*The Elegant Universe*, chapter 2, section "The Principle of Relativity"; *The Fabric of the Cosmos*, chapter 3, section "The Bucket, According to Special Relativity"). Told here as Greene's account; this account's different result on the flat geometry is the claim on the idealized world below.

Greene's telling of the equivalence principle, of the spinning bucket as one more acceleration that can stand in for the Earth's gravity, of gravity as the warping of space and time, and of disturbances that travel at the speed of light, so that Newton's instantaneous gravity no longer conflicts with special relativity (*The Elegant Universe*, chapter 3, sections "The Incompatibility of Newtonian Gravity and Special Relativity", "Einstein's Happiest Thought", "Acceleration and the Warping of Space and Time", "The Basics of General Relativity" and "Conflict Resolution"; *The Fabric of the Cosmos*, chapter 3, sections "The Equivalence of Gravity and Acceleration" and "Warps, Curves, and Gravity"). This account reaches the same results on separate ground: the claims on the triple, on weightless free fall, on the four marks and on gravitational waves below.

Greene's two-part verdict on general relativity and the bucket, given as his judgment of a question on which "there is still some controversy": Machian in the free-fall benchmark, which all the matter of the universe shapes, and not Machian in the empty universe; spacetime is "a something", the gravitational field, and acceleration against that changeable field is relational in a broader sense (*The Fabric of the Cosmos*, chapter 3, sections "General Relativity and the Bucket" and "Spacetime in the Third Millennium"; Notes, chapter 3, notes 22 and 23). This account's different result on the empty case is the claim on the empty bucket below.

Frame dragging: a turning body drags space around with it (*The Fabric of the Cosmos*, chapter 14, section "Einstein in Drag"; Lense and Thirring 1918, Pfister and Braun 1985, as Greene cites them). Told here as Greene's account; the collection gives no reply on it.

Greene's guess that a background-independent theory, with string theory adapting the techniques of loop quantum gravity, would make spacetime emerge from more basic ingredients, and his hope that it would give Leibniz, Newton, Mach and Einstein a share of the victory (*The Fabric of the Cosmos*, chapter 16, section "The Constituents of Spacetime"). The account's share for each is the last claim below.

Greene on empty space: every field undergoes quantum jitters, the vacuum fluctuations, even in empty space, and the Casimir force shows their effect (*The Fabric of the Cosmos*, chapter 12, section "Quantum Jitters and Empty Space"). The collection says the same in *The View from Inside*, Chapter 1, "The Emptiest Space Is Not Empty", section "What Is 'Empty' Space?", which adds that every cubic centimeter of empty space has a positive, nonzero energy density, the zero-point energy of the fields. In Chapter 1 I call that energy the vacuum's energy, in the words Chapter 10 uses for the whole of it.

Every finding is a comparison; content is what a comparison finds, and context is the respect in which it finds it; the thermometer read in the morning and in the evening, with the warmer evening as content and the two times of day as context. *What Comes After the Dark*, Chapter 6, "What an Observer Is". A finding is any comparison that lays down a record: *Why the Quantum Is the Way It Is*, Chapter 17, "Every Comparison Has a Cut". The comparer names a role and never a mind: *What Cannot Be*, Terms, entry "Comparer".

A state of motion is a content, and what it is measured against is its context. *The Gravitational Passage*, Chapter 18, Section IV, "The Mach Principle Unpins".

One spacecraft in an empty universe: nothing registers its turning; two spacecraft: each turns relative to the other. *The Gravitational Companion*, "One Mass, Three Elevators", Section II, "Inertia Is a Comparison".

The principle of comparison: a difference that no comparison can register is no difference. *What Comes After the Dark*, Chapter 6, "A Standard That Needs Something There"; the same principle in *The Gravitational Companion*, "One Mass, Three Elevators", Section II, and in *The Gravitational Passage*, Chapter 18, Section IV.

A standard of not rotating is fixed only by the matter within reach, and matter beyond the comparer's cosmological horizon fixes none; the reference is the content within the horizon. General relativity does not by itself settle Mach's question, and the principle of comparison settles it: "This

physics settles the question by a principle about comparison". (*What Comes After the Dark*, Chapter 6, "A Standard That Needs Something There"); "The principle of comparison supplies what general relativity leaves open". (*The Gravitational Passage*, Chapter 18, Section IV, which adds: "Inertial frames are frames only through the comparable content that fixes them"). In the terms of general relativity: the local inertial frames are fixed by the comparable distribution of mass and energy within the horizon. That statement about the local inertial frames is the Mach principle in its most conservative form, supplied from comparison (same section).

The cosmological horizon exists because the expansion speeds up: "The expansion of the universe is speeding up, driven by a cosmological constant, and so every observer has a horizon beyond which nothing can ever reach it". *What Comes After the Dark*, Chapter 1, "What This Means for You"; the accelerating expansion gives every observer a cosmological event horizon, *The Gravitational Passage*, Chapter 18, Section III.

The gyroscope that holds its direction against the distant stars, and the Foucault pendulum whose plane stays fixed against them. *The Gravitational Companion*, "One Mass, Three Elevators", Section II; *What Comes After the Dark*, Chapter 6, "Mach and the Bucket".

The equivalence principle as a triple: centripetal force (felt in a turning room as centrifugal force), linear acceleration and gravitation each act on every body in proportion to its inertial mass; at a point, gravitation and linear acceleration are not told apart; the three rooms and the two pairs of hanging balls (closer together on the planet, parallel in the pulled room, farther apart along the motion in the swung room). *The Gravitational Passage*, Chapter 14, opening and Sections I to III; *The Gravitational Companion*, "One Mass, Three Elevators", Section I, "Three Rooms".

Rotation is registered at a point, against the local inertial frame, which the comparable content fixes: by a gyroscope, by the Coriolis deflection, and by the Sagnac shift of a ring interferometer (Sagnac 1913). *The Gravitational Passage*, Chapter 14, Section II, "The Third Element", and Section

III, “Distinguishing the Three: The Hanging-Ball Experiment” (“Rotation keeps its axis at every point”). Sagnac’s shift, as standard physics gives it (the collection cites the measurement and not the formula): for light sent both ways round a loop enclosing an area A on a platform turning at angular velocity Ω (omega), the difference of travel times is four times A times Ω divided by the square of the speed of light, c.

The water’s climb shows its spin from inside, as Newton found. *What Cannot Be*, Chapter 17, “The Three Kinds and the Three Modalities”, which places Newton’s observation in the *Principia* of 1687.

Free fall is weightless; the felt weight is the reaction of the ground, which keeps a body off its geodesic (the straightest path the curved arena allows). *The Gravitational Passage*, Chapter 15, Section III, “The Equivalence Principle Revisited” (“Geodesic motion is weightless”).

The four marks of any arena, derived for any arena before gravity is named: the arena couples to everything; nothing shields the arena; the mass that couples and the mass that resists are one; the arena has no charge in steps. The four are four consequences of one fact: the arena is the context of every measurement, and not a content within it. *Measurement*, Chapter 3, Section VII, “Why Gravity Is the Arena and Not a Field in It”; *Two Theories, One World*, Chapter 9, “Four Marks of the Arena”, sections “Any Arena, Before Gravity”, “The First Mark: It Couples to Everything”, “The Second Mark: Nothing Shields It”, “The Third Mark: One Mass”, “The Fourth Mark: No Charge in Steps” and “The Four Together”.

The pictures and facts the chapter uses for the marks, each from *Two Theories, One World*, Chapter 9: the forces of particle physics are selective, and “Each of these forces has neutral objects”, as a neutron passes undeflected through an electric field (first mark); a metal box keeps electric fields out, as an elevator cuts a phone’s signal, and a shield needs an opposite source and parts the field moves apart, which gravity does not offer (second mark); the electron and the proton carry charges of equal size and opposite sign, and the proton has about 1836 times the inertia of the

electron (third mark); electric charge comes in whole multiples of one third of the electron's charge, and what couples a body to an arena "is no stepped label": it varies continuously, taking any value in its range, and gravity couples to the whole energy of a body, so that a hot brick is heavier than the same brick cold (fourth mark). The carriers of the smallest charge, named in the chapter, are from standard physics: the down, strange and bottom quarks each carry one third of the electron's charge, with the electron's sign, and the up, charm and top quarks two thirds, with the opposite sign. The third mark's plain statement, that to take part in the arena is to be found in it, and that the body's relation to the setting it occupies is the one quantity its pull and its resistance both measure: *Measurement*, Chapter 3, Section VII.

The past measurements: the hammer and the feather on the Moon, 2 August 1971 (*Physics Is Not Math*, Chapter 13, Section I, "Two Kinds of Mass"; *Measurement*, Chapter 3, Section VII); MICROSCOPE, the Eötvös parameter of titanium and platinum, η(Ti, Pt) (eta, the fractional difference of their accelerations in free fall) equal to minus 1.5 plus or minus 2.3 (statistical) plus or minus 1.5 (systematic) times ten to the minus fifteen, consistent with zero (Touboul and others 2022; *Measurement*, Chapter 3, Section VII; *Two Theories, One World*, Chapter 9); antihydrogen falling toward the Earth, at 0.75 plus or minus 0.13 plus or minus 0.16 times the acceleration of ordinary free fall, g (Anderson and others 2023; *Measurement*, Chapter 3, Section VII); starlight bent at the eclipse of 29 May 1919 (Dyson, Eddington and Davidson 1920; *Measurement*, Chapter 3, Section VII).

I identify gravity with the shape of the arena, and not with a force acting within the arena. *Measurement*, Chapter 3, Section VII; the same identification in *Two Theories, One World*, Chapter 9, "The Identification" ("I identify gravity as the arena of physics, the context of every physical measurement, by that match". and, as the section's conclusion, "So gravity is the shape of the arena, and not a force that acts within it"), and, from the Incommensuration Theorem, in *The Gravitational Passage*, Chapter 7, Section V, "Context Is the Arena". In Chapter 1 I state the identification once, in the words of that conclusion, and I use the same words, gravity is

the shape of the arena, wherever I state the result again. The Preface quotes the same sentence. The four marks are marks of any arena, and the shape of the arena carries each of them. *Two Theories, One World*, Chapter 9, "The Identification", says so of general relativity's description of gravity as the geometry of space and time ("each of the four marks is built into that description: a geometry is shared by everything in it, has no outside from which to screen it, moves every body along the same paths, and is measured by lengths and durations rather than by a stepped label").

The pen falls because the arena differs from one height to the next; the account says why there is a bending and why it is smooth and follows the content. The form of the bending is taken from established physics: Einstein's equations, the equations of general relativity, say how much the arena bends for the matter in it (Einstein's field equations, which tie the curvature at each place to the energy and momentum there), and Newton's constant is measured. *Two Theories, One World*, Chapter 8, sections "The Context Carries the Rest", "What the Established Theory Adds" and "You Live in a Bent Arena".

Gravitational waves detected: the first event of 14 September 2015 (two black holes of about 36 and 29 solar masses, about 1.3 billion light-years away, about 410 megaparsecs; peak strain about 1.0 times ten to the minus twenty-one; about 3.0 solar masses radiated), reported by the LIGO and Virgo collaborations (Abbott and others 2016); the neutron-star merger of 17 August 2017, with light and waves arriving 1.74 seconds apart, which bounds the difference between the speed of gravitational waves and the speed of light between minus 3 times ten to the minus fifteen and plus 7 times ten to the minus sixteen of the speed of light (Abbott and others 2017). *Two Theories, One World*, Chapter 8, sections "A Relation in Need of a Reason" and "What the Established Theory Adds"; the same book's Appendix A, "The Numbers", Part I, gives the measured values of the first wave; *Side by Side, Volume One*, Appendix B, "The Sealed Record", Predictions 3 and 6, "Measured".

Newton's space as a fixed stage, unmoved by the actors on it: *Two Theories, One World*, Chapter 8, "A Relation in Need of a Reason". Space is the pattern of which events are causally connected with which, and no container; this is the content of general relativity stated without the scaffolding of the container picture, in which spacetime has no properties apart from the relations that make it up: *What Cannot Be*, Chapter 12, "A Rule on a Stage" and "What Space Is" (with Minkowski 1909, Malament 1977, and Hawking and Ellis 1973, named there).

The flat arena of special relativity is the idealized case of an arena with no content anywhere and no waves crossing it; put content into it, and the bending reaches beyond the content into the empty space around it. *Two Theories, One World*, Chapter 8, "The Context Carries the Rest" ("In general relativity the flat arena of special relativity, the same at every place and time, is the idealized case of an arena with no content anywhere and no waves crossing it"). The same point for the three symmetric geometries taken together: Minkowski space, de Sitter space and the homogeneous models of cosmology are each the geometry of an idealized content, "with no content or a perfectly uniform one", and the context in which actual content stands has no isometries (no transformations that leave it unchanged); *The Gravitational Passage*, Chapter 10, Section III, "The Lorentzian Signature". In Chapter 1 I state the first of the three symmetric geometries for flat spacetime, in the words of *Two Theories, One World*.

The equations of general relativity admit matter-empty solutions with local inertial frames; the principle of comparison gives such frames nothing to be compared against. *The Gravitational Passage*, Chapter 18, Section IV; *What Comes After the Dark*, Chapter 6, "A Standard That Needs Something There"; *The Gravitational Companion*, "One Mass, Three Elevators", Section II; *At the Edge of Reality*, Chapter 8, "Inertia Is a Comparison". Greene's case is the spinning bucket in an otherwise empty universe, whose tiny mass "hardly affects the shape of space at all" (*The Fabric of the Cosmos*, chapter 3, section "General Relativity and the Bucket"). In Chapter 1 I apply the principle to that universe in the terms of the two

spacecraft. The water turns against the bucket, and the bucket and water together have nothing outside them to tell their turning by. The same conclusion for a bucket with no matter left within reach: “Mach’s bucket, set spinning at either end, would have nothing to measure its turning against”. (*What Comes After the Dark*, Chapter 6, “Two Ways to Be Left”.) The result is a result about the standard of not turning, in the words of the same book, Chapter 6, “A Standard That Needs Something There”: “So a standard of not rotating is fixed only by matter within reach, matter that can still be compared with”.

The share of each. The assignment of three results of the collection to the four names is this book’s; each result has its own source. Frames depend on the content that fixes them (Leibniz, Mach): *Side by Side, Volume Two*, “Janus and the Turn”, Section VI, “Where the Predictions Part, and Why” (“For the collection, removing absolute space makes the frame depend on the content that fixes it”); *What Comes After the Dark*, Chapter 6, “A Standard That Needs Something There”. Rotation is registered at a point (Newton): *The Gravitational Passage*, Chapter 14, Section III; *The Gravitational Companion*, “One Mass, Three Elevators”, Sections I and III; *What Cannot Be*, Chapter 17, “The Three Kinds and the Three Modalities”. The geometry is the context in which every configuration of the content is compared and laid out, and gravity is the shape of the arena (Einstein): *Side by Side, Volume Two*, “Janus and the Turn”, Section VI; *Two Theories, One World*, Chapter 9, “The Identification”. The arena is not made of anything more basic that is found in it: *Side by Side, Volume Two*, “Where the Dark Sector Lives”, Section IV, “What the Arena Is”, with *The Unquantized Arena*, Chapter 6, Section IV, Proposition 6.3 (no least length, area or duration; nothing is a quantum of the arena itself, at any scale). The reason given in the same section: content is discrete and governed by symmetry, context is continuous and uneven, and the Incommensuration Theorem says one cannot be built from the other.

Chapter 2. Is the Whole of Time Laid Out at Once?

The opening scene. Michele Besso died in Geneva on 15 March 1955; Einstein's letter of condolence to the Besso family is dated 21 March 1955; Einstein died on 18 April 1955. The letter is printed in *Albert Einstein and Michele Besso: Correspondence 1903–1955*, edited by Pierre Speziali (Paris: Hermann, 1972), the edition Greene cites (*The Fabric of the Cosmos*, Notes, chapter 5, note 5). Greene does not tell the occasion or the date of the letter; both are from that edition. The German sentence reads: "Für uns gläubige Physiker hat die Scheidung zwischen Vergangenheit, Gegenwart und Zukunft nur die Bedeutung einer wenn auch hartnäckigen Illusion". Greene's English: "For we convinced physicists, the distinction between past, present, and future is only an illusion, however persistent". Greene quotes the sentence in support of his conclusion "The total loaf exists" (*The Fabric of the Cosmos*, chapter 5, "The Persistent Illusion of Past, Present, and Future"); the consequence for a friend who has died is drawn by this book from Greene's view, beside Greene's own sentence "If you were having a great time at the stroke of midnight on New Year's Eve, 1999, you still are" (same chapter, "Experience and the Flow of Time"). Besso is the person thanked at the close of Einstein's paper of 1905, "Zur Elektrodynamik bewegter Körper", *Annalen der Physik* 17, 891–921.

Greene's special relativity. Told here as Greene's account, with the passages of the collection below set beside it. Greene states that every object moves through spacetime at one fixed speed, c (the speed of light), and that motion through space is motion diverted from motion through time (*The Elegant Universe*, chapter 2, "Motion through Spacetime"); and that mass grows without limit as speed approaches c, so nothing travels faster than light (same chapter, "What about E=mc2?"). The word "observer" is Greene's for whoever makes the measurement, and no passage of his four books defines the word. In Chapter 2 I give the word "observer" the meaning "whoever or whatever does the measuring". The loaf of

time slices: *The Fabric of the Cosmos*, chapter 3, "Carving Space and Time" and "Angling the Slices". In Greene's drawings the rotation of a time slice is at most 45 degrees either way (chapter 5 and its notes).

Greene's block universe and Chewie. *The Fabric of the Cosmos*, chapter 5, "Does Time Flow?", "The Persistent Illusion of Past, Present, and Future" and "Experience and the Flow of Time". Chewie at ten billion light-years, walking at about 10 miles an hour, has a slice that contains events on Earth about 150 years in our past (away) or future (toward); at 1,000 miles an hour, about 15,000 years. Greene states the argument in the conditional form he prints in italics ("if you buy the notion that reality consists of the things in your freeze-frame mental image right now"). In Greene's words the set of all the observers' slices "fills out a substantial region of the spacetime loaf"; the slices sweep through every point of it on two further conditions he states: "if space is infinite" (main text), and the flat geometry of special relativity: "The expert reader will recognize that I am assuming spacetime is Minkowskian. A similar argument in other geometries will not necessarily yield the entire spacetime". (Notes, chapter 5, note 4). Greene's note that the now-lists "do not encode causality" and that every observer agrees on which events can affect which is in Notes, chapter 5, note 3. Greene's argument from the walker has no passage of the collection in the argument's own terms. In Chapter 2 I set beside Greene's argument *The Unquantized Arena*, Chapter 10, Section VI, and *Side by Side, Volume Two*, "The Open Future", Section III. Greene's possibility of a physical foundation for a flowing time, and its alternative, are both in chapter 5, "Experience and the Flow of Time": "It is possible that some insightful person will one day devise a new way of looking at time and reveal a bona fide physical foundation for a time that flows. Then again, the discussion above, based on logic and relativity, may turn out to be the full story".

Time and change. Greene, after Wheeler: *The Fabric of the Cosmos*, chapter 8, "Symmetry and Time". The sentence "Time comes with the comparison, not before it". is in *Why Time Travel Is Not a Concept*, Chapter

1, Section II, "Three Times", with the picture of a head and a book in an empty, static universe, in which two moments T-zero and T-one (read "T zero" and "T one") cannot be told apart.

The cone comes first. *What Cannot Be*, Chapter 12, "The Shape of What Can Reach What", section "The Cone Comes First" (the firework; "The cone is primary, and it is not obtained from the limit"; the same section takes the textbook order, the speed limit first and the cone after, and reverses it). The technical form is *The Gravitational Passage*, Chapter 10, Section III, "The Lorentzian Signature": the directed precedence of Axiom II, identified in the context factor with the order of events, gives a limit and a direction; the metric has signature $(-, +, +, +)$, read "one sign for time and three for space", with exactly one negative sign. The principle of relativity, homogeneity, isotropy and the group law fix the transformations between frames up to one constant, $1/c^2$ (read "one over the square of the speed of light"), the Galilean case being the value zero (Lévy-Leblond 1976); the boundary of the domain makes the constant nonzero, which is the finiteness of the edge, and the cone of precedence makes it positive.

The speed of light as the edge of comparison (identification). *The Gravitational Passage*, Chapter 17, Section I, "Three Constants, Three Roles": "The speed of light is not, at root, the speed of any one thing but the macroscopic boundary of the domain of comparison itself; light is only the perfected example of c". *What Cannot Be*, Chapter 12, "An Exchange Rate Between Time and Space": the ball on the train, the invariance from the edge ("an invariant quantity is one that every comparer finds the same"), the light-year as a distance named by a time. *Measurement*, Chapter 3, Section VIII: the speed of light is "the fastest that any signal can get from one place to another" and the limit for "every act that carries a finding from here to there". The earliest statement: *An Immanent Metaphysics*, Technical Aphorisms, "Domain Constants", aphorism 1.89-1 (the Planck constant and the speed of light as the microscopic and macroscopic limits of a domain), and "Aspects of Time", aphorisms 1.86-13 and 1.86-14 (in a class of perceptions the ratio of the temporal to the spatial aspects is fixed to be the same). The value of c, 299,792,458 meters per

second, enters the account as a measured value; what the account derives is that there is one finite edge, the same for every comparer. That the edge is finite, so that every signal takes time to cross a distance, follows from its being an edge of comparison, and so does its being one edge, the same for every comparer. The finiteness is argued in *The Gravitational Passage*, Chapter 10, Section III, "The Lorentzian Signature", from two premises: the precedence of Axiom II, identified there in the context factor with "a span of time before every placing of a finding in space" ("no finding at a separation enters a comparer's record at the moment it is made there"), and the macroscopic boundary of every domain of measurement, "one ratio of spatial to temporal measure, the same throughout the domain and for every comparer in uniform motion", so that "c is finite"; the chapter's Questions, under "Two premises together (Section III)", say that the first premise excludes signals that take no time, and the second excludes signals with no greatest speed, and with them the Galilean structure.

Cherenkov's glow. *Measurement*, Chapter 3, "Content and Context in Physics", Section VIII, "The Speed That Belongs to the Arena": "That limit is a property of the arena and not of light". and "No particle with mass reaches it, and nothing that carries a finding exceeds it". In water the speed of light is about c divided by 1.33 (the refractive index of water); a charged particle faster than that emits Cherenkov light. Primary paper: P. A. Cherenkov, "Visible emission of clean liquids by action of γ radiation", *Doklady Akademii Nauk SSSR* 2, 451 (1934).

A clock's rate is set by its place and motion in the arena. *Why Time Travel Is Not a Concept*, Chapter 1, Section II, "Three Times". Geometric time as proper time: *The Unquantized Arena*, Chapter 10, Definition 10.1, equation 10.1, $\tau[\gamma] = (1/c) \int \sqrt{(-g_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu})}\, d\lambda$, read "the proper time tau along a worldline gamma is the length the metric g assigns to the worldline, divided by the speed of light", where λ (lambda) is any parameter along the worldline and $\dot{x}$ (x dot) is the rate of change of the coordinates along it.

Muons (past measurement). *The Alphabet of Particles*, Level 5, "Three Generations", entry "Muon": their clocks run slow as seen from the Earth, "one of the plainest confirmations of special relativity". Mean lifetime at rest about 2.2 microseconds; c times 2.2 microseconds is about 660 meters. Frisch and Smith counted about 563 muons an hour at the summit (1917 meters) and about 412 an hour at Cambridge; with no time dilation about 27 would have arrived; their dilation factor was 8.8 ± 0.8. Primary paper: D. H. Frisch and J. H. Smith, "Measurement of the Relativistic Time Dilation Using μ-Mesons", *American Journal of Physics* 31, 342–355 (1963).

Hafele and Keating (past measurement). Told by Greene, *The Fabric of the Cosmos*, chapter 3, "Subtle but Not Malicious". Predicted: eastward -40 ± 23 nanoseconds, westward $+275 \pm 21$ nanoseconds; observed: eastward -59 ± 10, westward $+273 \pm 7$ (one nanosecond is a billionth of a second). Each prediction adds the effect of speed (special relativity) and the effect of height (general relativity). Primary papers: J. C. Hafele and R. E. Keating, "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains" and "Around-the-World Atomic Clocks: Observed Relativistic Time Gains", *Science* 177, 166–168 and 168–170 (1972).

Optical clocks a third of a meter apart (past measurement). *Two Theories, One World*, Chapter 8, "You Live in a Bent Arena"; *Comparison; The Necessary Basis*, Chapter 4, Section VII. Each clock uses a single singly charged aluminum ion, Al^+. Height 33 centimeters; fractional rate difference about 4×10^{-17} (gravitational acceleration times height over c squared, g h / c^2). Primary paper: C. W. Chou, D. B. Hume, T. Rosenband and D. J. Wineland, "Optical Clocks and Relativity", *Science* 329, 1630–1633 (2010), doi:10.1126/science.1192720.

Satellite clocks (past measurement). *Two Theories, One World*, Chapter 16, "Three Kinds of Time", section "One Word for Three Things", gives about 38 millionths of a second a day; the book's Appendix A, "The Numbers", Part I, gives the net fractional rate 4.4647×10^{-10} (Ashby 2003),

times 86,400 seconds, 3.86×10^{-5} seconds, about 38.6 microseconds a day. Primary paper: N. Ashby, "Relativity in the Global Positioning System", *Living Reviews in Relativity* 6, 1 (2003), doi:10.12942/lrr-2003-1.

Exists and real. *Home in the World*, Chapter 7, "The Open Future", section "A Recipe and the Cooking": "To exist is to stand as structure: timeless, the same in every instance (every single case of it), able to be held in view as a whole, like the recipe. To be real is to be given in an actual act of comparison: here, now, with the future still open, like the baking"; the same section: "what exists is structure, laid out; what is real is the act, as it happens". *The Gravitational Passage*, Chapter 5, "Reason, the Real and the Abstract", Section II, "The Real and the Abstract" ("A law exists; a measurement is real"); *Why Time Travel Is Not a Concept*, Chapter 1, Section III, quoting *The Unquantized Arena*, premise P14. With "real" in the sense of P14, Greene's "The total loaf exists" holds and "The only thing that's real is the whole of spacetime" does not; Greene's second sentence uses "real" in the sense his argument gives it, lying on some observer's slice of "now".

The three times. *Two Theories, One World*, Chapter 16, "One Word for Three Things" (the bench, the clock, the counter and the radioactive lump), "The Time of Evolution" ("The third kind I call evolution time".) and "Distinct, Inseparable, Not Interchangeable" ("A coordinate selects no outcome"; a record "is not unmade by running an equation backward"; the space of possibilities is no direction of the arena, "because nothing travels through it from one place to another"); *The Unquantized Arena*, Chapter 10, "Three Times", Definitions 10.1 to 10.3, Proposition 10.1 (distinct, inseparable and non-interchangeable) and Proposition 10.2 (no unified time exists). The third time is evolution time, the parameter t of the one-parameter unitary group $U(t) = \exp(-iHt/\hbar)$, read "U of t is the exponential of minus i times the Hamiltonian H times t over h-bar", which carries the quantum state (Definition 10.3). In the focus of time the three are identified with the three roles (the opening section of this appendix gives the formal names).

The rising loaf. *Home in the World*, Chapter 7, "The Open Future", section "A Recipe and the Cooking" ("While it rises, nothing about the loaf is finished: it may rise well or badly, and how it turns out is not yet settled").

The Geiger counter and the planets. *The View from Inside*, Chapter 14, "Here, Now, and This", section "A Geiger Counter and the Orbits of Planets", with Figure 22; hard randomness glossed there as "an outcome that no prior state fixes, however completely the prior state is known".

The block keeps geometric time and drops event time. *The Basis of Physics*, Essay 11, "The Structural Arrow of Time", Section VI, "Time as Process, and Not Only as Dimension". The essay's full route: determinism, the conjunction of perfected symmetry with perfected continuity, is precluded by the Incommensuration Theorem; faithful time reversal needs the same conjunction and is precluded; "Together these undercut the block universe from two sides". The essay draws the consequence: "If determinism is precluded, the future does not exist in the sense the past does: there is a genuine residual, an openness that is not merely our ignorance". In that sentence "exist" carries the sense of the block universe view the essay describes ("If the future is uniquely fixed by the past, then the future exists in the same sense the past does"), fixed in advance as the past is fixed. In Chapter 2 I state the same result with the words of my own definitions: the past has been given, and the future has not yet been given. Chapter 2 gives the Geiger-counter route; the two halves of this route are in Chapter 3 ("no whole world is completely fixed in advance") and Chapter 8 ("The theorem shows that no faithful reversal can happen").

No comparison registers a surface of simultaneity. *The Unquantized Arena*, Chapter 10, Section VI, "Where the Two Theories Meet"; the same section: "A record of a distant finding reaches a comparer here only through a signal, which arrives at the earliest on that comparer's past light cone". See also *The Unquantized Arena*, Chapter 7, Section IV, Proposition 7.2 ("A finding changes no geometry, here or anywhere").

Ellis and the evolving block. *Side by Side, Volume Two*, "The Open Future" (George Ellis), Sections II, "The Evolving Block Universe", and III, "Where the Present Is"; Ellis "reached the open future from physics, by way of quantum indeterminism and the irreversibility of measurement" (Section II). The same book, after *What Cannot Be*, calls the Hawking and Ellis volume the standard work on the causal structure of space and time (Section I). Ellis's works named there: G. F. R. Ellis (2006); Ellis and Drossel (2020); Ellis and Rothman (2010); S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, 1973).

Greene's moving light of the present, set aside by both. In Chapter 2 I tell Greene's moving light as Greene's account. The passages of the collection that also set such a light aside, for the reason that the line is not fully drawn, are *What Cannot Be*, Chapter 13, "Time as Happening, and Not Only as a Line" ("our attention as a light that slides along a line already fully drawn"), and *The Basis of Physics*, Essay 11, Section VI.

The direction of time in every act (identification). *The Unquantized Arena*, Chapter 10, Definition 10.2 and P19: "I identify the one-way order of findings, the order of Axiom II, with the direction of time". *Home in the World*, Chapter 7, "Records That Are Never Un-laid"; *The Basis of Physics*, Essay 11, Section VI, paragraph "Maudlin's Primitivism" ("It is not a brute fact. It is what it means for anything to interact with anything else. ... A brute fact could have been otherwise"). Axiom II, from *An Immanent Metaphysics*, aphorism 1.3-1, is quoted in the opening section of this appendix.

Travel to the future. Greene: *The Fabric of the Cosmos*, chapter 15, "The Puzzles of Time Travel". The collection: travel within the cone is not excluded (*Side by Side, Volume One*, Appendix B, "The Sealed Record", Prediction 27; the coming measurements of this entry are in Appendix A).

The literature on time loops. *What Cannot Be*, Chapter 15, "Time Machines on Paper", sections "Loops in Space and Time" and "What Physicists Mean by a Time Machine"; *Why Time Travel Is Not a Concept*, Chapter 2. On the physics of travel to the past, this account reports what Greene

reports: no one has shown that the laws of physics rule it out (*The Fabric of the Cosmos*, chapter 15, "Is Time Travel to the Past Possible?"). Primary papers: K. Gödel, "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation", *Reviews of Modern Physics* 21, 447–450 (1949); M. S. Morris and K. S. Thorne, "Wormholes in spacetime and their use for interstellar travel", *American Journal of Physics* 56, 395–412 (1988); S. W. Hawking, "Chronology protection conjecture", *Physical Review D* 46, 603–611 (1992). A closed timelike curve is the physicist's name for a time loop.

Travel to an earlier time names no concept. *Why Time Travel Is Not a Concept*, Chapter 1, Section III, "What Exists and What Is Real", and Section IV, "Time Is Not a Place" ("This is the sense in which time travel is a category mistake"); the three readings are Chapter 4, Definition 4.7, with Propositions 4.8 (geometric reading), 4.9 (event reading) and 4.10 (joint reading). In Chapter 2 I test each reading on last Thursday. The geometric reading is decided by Proposition 4.8 (b) and (c). An arrival at an event that geometric time places both before and after the departure, or back at the departure event along a closed timelike curve, is a later finding of the comparer. The comparer's clock reads later at that arrival. The event reading is decided by Proposition 4.9, with Proposition 4.6 and Axiom II. On the event reading, the order of findings cannot be reversed, and a finding's record is not undone as a finding. The joint reading is decided by Proposition 4.10, which uses Proposition 4.5 (no unified time), reported from *The Unquantized Arena*, Chapter 10, Definition 10.4 and Proposition 10.2. On the joint reading, what is found of geometric time is continuous and asymmetric, and what is found of event time is discontinuous and symmetric. On the joint reading, also, "A time found both as geometric time and as event time would carry at least one of the two forbidden pairings", the pairings that the Incommensuration Theorem forbids (symmetry with continuity, asymmetry with discontinuity). In Chapter 2 I say the two forbidden pairings in words, as smoothness with sameness, and separateness with difference. Then I give the four words their names from Chapter 3 (continuity, symmetry, discontinuity, asymmetry). In Chapter 2

I also say, as Proposition 4.5 does, that such a time would have at least one of the two pairs. Plain form: *What Cannot Be*, Chapter 14, "Last Thursday" ("taken in the sense the people who use it intend, names no idea at all"; the largest prime number) and "Time Is Not a Place".

What the argument on time travel uses. The argument rests on what the words mean; of the physics of time loops it uses one principle only, that a history on a loop must agree with itself, and the theorems about loops stand on their own ground. *Why Time Travel Is Not a Concept*, Chapter 1, Section IV: "Those theorems stand on their own ground, and the argument of Chapter 4 does not rest on them. It uses one principle reported there, the self-consistency principle for spacetimes with closed timelike curves, and only in examining one reading of the phrase (Proposition 4.8)". The principle is due to J. Friedman, M. S. Morris, I. D. Novikov, F. Echeverria, G. Klinkhammer, K. S. Thorne and U. Yurtsever, "Cauchy problem in spacetimes with closed timelike curves", *Physical Review D* 42, 1915–1930 (1990): only histories consistent along every closed timelike curve are admitted. Greene's own statement of it: "the complete and unchanging set of events in spacetime necessarily fits together into a coherent, self-consistent whole" (*The Fabric of the Cosmos*, chapter 15, "Rethinking the Puzzles").

The arrival on a loop is a later finding. *Why Time Travel Is Not a Concept*, Chapter 4, Section VI, Proposition 4.8 (c): "So the arrival at p is a later finding of C", read "the arrival of the comparer C at the event p it left"; the restoration of every trace "is itself a further act, and its findings follow, in the order of C's findings, the findings it is said to undo". Greene's value of pi: *The Fabric of the Cosmos*, chapter 15, "Rethinking the Puzzles".

Space and time here to stay; neither time an illusion. Greene: *The Fabric of the Cosmos*, chapter 16, "Inner and Outer Space". The collection: *Why Time Travel Is Not a Concept*, Chapter 1, Section III ("The two are not rivals"; "Each is one of the three times, and neither does the work of the other").

Chapter 3. Is the Universe Local?

The opening scene. Einstein, Podolsky and Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", *Physical Review* 47, 777 to 780, received 25 March 1935, published 15 May 1935, Institute for Advanced Study, Princeton (doi:10.1103/PhysRev. 47.777). Bohr's reply under the same title, *Physical Review* 48, 696 to 702, received 13 July 1935, published 15 October 1935 (doi:10.1103/PhysRev. 48.696). The account of the reply and its verdict: *Side by Side, Volume One*, chapter "Niels Bohr: The Third Term", Section VII, "The Reply to Einstein, Podolsky and Rosen".

Greene on probability, uncertainty, wave and particle, the sum over paths, and the rules that work without reasons. *The Fabric of the Cosmos*, chapter 4, sections "Probability and the Laws of Physics" and "Heisenberg and Uncertainty"; *The Elegant Universe*, chapter 4, sections "The Quantum Framework" (the rules followed without knowing why they work; "No one knows"), "Is It a Wave or Is It a Particle?", "Feynman's Perspective" (every possible path at once, a number assigned to each, Newton's path recovered for large bodies) and "Quantum Weirdness". Told here as Greene's account, with the result of this account set beside it in the section "A Chord and a Click"; the locations in the collection are in the entries below. Greene's word "velocity" and the word "momentum" name the same trade; momentum is mass times velocity at everyday speeds, and the uncertainty bound is stated for position and momentum.

Greene on the Bell experiments, the conclusion "the universe is not local", the dilution of entanglement, the standard and contrarian views on special relativity, Bohm's theory, and teleportation. *The Fabric of the Cosmos*, chapter 4: section "The World According to the Quantum" (locality defined: "you can directly affect only things that are next to you"); sections "The Red and the Blue", "No Smoke but Fire" (the sense of "not local": what is done at one place "can be linked" with what happens at another "even if nothing travels between the two locations"), "En-

tanglement and Special Relativity: The Standard View" (the two photons as parts of one physical entity, one measurement that "affects both photons at once"; no information transmitted; the lists compared "by some conventional, slower-than-light means"; correlated, and not in "a traditional cause-and-effect relationship"), "Entanglement and Special Relativity: The Contrarian View" (the majority's "harmonious coexistence"; collapse across space in "good old-fashioned textbook quantum mechanics", which "would seem to pick out one vantage point as special"; Greene's own judgment that the majority view will likely prevail), and "What Are We to Make of All This?" (hidden properties not ruled out; Bohm's "yet more blatant nonlocality"; the warning against "gushy talk"; entanglement spread until "virtually impossible to detect"); chapter 15, section "Quantum Entanglement and Quantum Teleportation"; chapter 1, section "Quantum Reality" (the sentence the reader's question takes up: "it's as if they're a single entity"). Greene's boxes and his count of agreements, an agreement being the same color, or the same spin direction, found at both ends: chapter 4, section "The Red and the Blue" (one door; boxes with the same number flash the same color), section "Bell and Spin" (three doors; preset programs give agreement for more than half of the door combinations), section "Counting Angels with Angles" (the same count with spin detectors), section "No Smoke but Fire" (Bell's result of 1964 and the experiments of Freedman and Clauser, Fry and Thompson, and Aspect: the detectors "did not agree more than 50 percent of the time") and section "Entanglement and Special Relativity: The Contrarian View" (quantum theory predicts agreement "precisely 50 percent of the time"); the score of Clauser, Horne, Shimony and Holt (entry below) is a test of the same kind with two settings at each detector.

The four characters, with the stance held fixed. *An Immanent Metaphysics*, chapter 3, "Natural Physics", section "Symmetry and Continuity", aphorism 1.43-1 (definitions), as *Home in the World*, chapter 4, "The Circle and the Floor", section "Four Words for Four Findings", gives them in plain words with the dimmer, the switch, the square and the coastline. Aphorism 1.43-1 gives the four definitions "Where there is assumed a

sameness of subjective context"; *Why the Quantum Is the Way It Is*, chapter 4, "Symmetry and Continuity Cannot Both Be Perfected", section "Four Characters You Already Know", says: "Holding the stance fixed makes the four definitions exact". In Chapter 3 I state that condition in plain words before the four definitions ("Hold fixed the side from which a comparison is made, so that the same comparer works on the same thing compared"). Continuity: sameness of content with sameness of context. Discontinuity: difference of content with sameness of context. Symmetry: sameness of content with difference of context. Asymmetry: difference of content with difference of context.

The Incommensuration Theorem, its proof in words, and its two surviving pairings. *An Immanent Metaphysics*, chapter 3, section "The Universal Law", aphorism 1.43-2, whose last two lines the chapter quotes exactly. The steps of the proof (perfect continuity: same context gives same content; perfect symmetry: different context gives same content; together: same content for every context, so the comparison answers only "same", while sameness and difference are inseparable in every comparison) are those of *Home in the World*, chapter 4, section "Two Cannot Both Be Perfect", with its ramp (continuity with asymmetry) and ladder (symmetry with discontinuity). The name "the two surviving pairings" in Chapter 3 follows *Why the Quantum Is the Way It Is*, chapter 4, section "The Two Survivors". In Chapter 3 I keep "character" for each of the four kinds of finding. The theorem uses no physical premise (*Home in the World*, same section; "For the Specialist", "The Incommensuration Theorem"). The theorem as "a structural fact about the nature of knowing": *Why the Quantum Is the Way It Is*, chapter 15, "Entanglement and Bell's Limit", section "Lawfulness and Locality"; every observation, measurement and test is a comparison: *What Comes After the Dark*, chapter 6, "What an Observer Is".

The uncertainty principle is an instance of the theorem. *Why the Quantum Is the Way It Is*, chapter 4, section "The Uncertainty Principle as an Instance": the Fourier transform between the position description and the momentum description is identified there with the conjugation named in the first axiom; momentum, the generator of translations, carries sym-

metry (a state of definite momentum, a plane wave, is unchanged in every respect a comparison registers when shifted, which the chapter pictures with the held note); position carries continuity ("it describes a state place by place, along a line of places that runs without a gap, and a state of definite position is the sharpest such description, one point picked out on that unbroken line"), which the chapter pictures with the dimmer; the chord and the click are that section's picture. The bound: the product of the spreads of position and of momentum is at least one half of h-bar (h-bar, read "h bar", is Planck's constant h divided by two pi), from Robertson's inequality of 1929 and the commutator of position and momentum derived in that book's chapter 9. The same section states that the theorem gives the form of none of these limits and derives neither the speed of light nor the value of Planck's constant. In Chapter 3 I say the same in words ("The size of the trade... belongs to physics"). Also *Measurement*, chapter 5, "What Physics Can Reach", section IX, "Questions".

Wave and particle as the two surviving pairings; the act of measurement decides the face. *The Basis of Physics*, Essay 8, "The Wave and the Particle", sections II, "The Wave" (a pattern varying continuously, neighboring points differing by an amount that goes to nothing as they approach; "Its phase advances. At any instant no two points along it are in the same state"; phase as "the measure of how far along the smooth variation one is"), III, "The Particle" (a unit, one or two with nothing between; one electron as good as another, a swap that no measurement can tell), and V, "Every Duality Is a Triple"; *Side by Side, Volume One*, chapter "Niels Bohr: The Third Term", Section V, "Duality Is Real; Dualism Cannot Stand".

Probability belongs to the world; every lawful description leaves a residual; hard randomness. *Why the Quantum Is the Way It Is*, chapter 11, "Why Chance Enters", sections "The Residual" and "From an Unfixed Outcome to a Stable Weight"; that book's Key Terms entry for hard randomness reads "An outcome that no prior specification fixes, however

complete". In Chapter 3 I use the wording of this book's Chapter 2, which says the same as that Key Terms entry. Also *Side by Side, Volume One*, chapter "Anton Zeilinger: One Verdict, One Bit", Sections III and IV.

The rules are derived; the strangeness belongs to the starting point; the sum over paths is derived and Newton's path recovered. *Why the Quantum Is the Way It Is*, chapter 1, "Six Rules Nobody Chose", section "A Recipe Book With No Reasons"; chapter 19, "Where the Derivation Stops, and Why", section "Why the Quantum Seemed Strange"; chapter 17, "Why a Measurement Has an Outcome", section "How the Everyday World Comes Back" (after Dirac 1933 and Feynman 1948; the path of stationary action; Ehrenfest's theorem of 1927).

Bertlmann's socks; the enchanted decks; the gloss of "entangled". Socks: *Why the Quantum Is the Way It Is*, chapter 15, "Entanglement and Bell's Limit", section "What Bell Proved" (from Bell, "Bertlmann's Socks and the Nature of Reality", 1981). Decks: *Causality Without Determinism*, chapter 3, "The Free Will Theorem", section I; *What Cannot Be*, chapter 9, "The Free Will Theorem", section "The Enchanted Decks"; *Why the Quantum Is the Way It Is*, chapter 15, section "Why No Message Can Be Sent". "Entangled": *What Cannot Be*, Terms, entry "Entangled". Spin as an arrow asked one yes-or-no question by each detector, and the gloss of correlation: *What Cannot Be*, chapter 10, "What Bell Found", section "Two Detectors, Far Apart".

Bell's ceiling and the measured scores. *Why the Quantum Is the Way It Is*, chapter 15, section "What Bell Proved"; appendix "What Experiment Finds", section "The Strength of Correlations"; *What Cannot Be*, chapter 10, section "Bell's Question". The score is the CHSH quantity (Clauser, Horne, Shimony and Holt 1969): with Alice's settings a and a′ and Bob's settings b and b′, read "a prime" and "b prime", and E the correlation (the average of plus one for agreement and minus one for disagreement), $S = E(a, b) + E(a, b') + E(a', b) - E(a', b')$. Every sealed list gives $S = \pm 2$, so every local hidden-variable account gives $|S| \leq 2$, read "the size of S is at most two". Quantum theory: $|S| \leq 2\sqrt{2} \approx 2.83$, read "twice the square

root of two". Aspect, Grangier and Roger 1982: about 2.70. Hensen and others 2015 (Delft, 1.3 kilometers): 2.42, with an uncertainty of 0.20 (*Two Theories, One World*, chapter 20, "The Walls Physics Has Already Found", section "Bell's Theorem"). The cosmic settings (about six hundred years for stars of the Milky Way, billions of years for quasars): *Side by Side, Volume One*, chapter "Anton Zeilinger: One Verdict, One Bit", Section I; *Why the Quantum Is the Way It Is*, chapter 15, section "What Bell Proved". The Nobel citation of 2022: *Side by Side, Volume One*, same section. The sentence "no account in which every outcome is fixed by hidden local causes can match all the predictions of quantum theory": *What Cannot Be*, chapter 10, section "Bell's Question".

Why the scores stop at twice the square root of two. *Why the Quantum Is the Way It Is*, chapter 15, sections "The Ceiling", "Reaching the Ceiling", and "Where the Ceiling Is Reached, and Why It Is Not Higher"; "For the Physicist", sections "The Space of States (Chapters 9 and 10)" and "Bell and Tsirelson (Chapter 15)"; *Beyond the Quantum Passage*, chapter 1, "The Tsirelson Bound from the Comparison Algebra". Three numbers in a row: sealed scripts stop at two, and the rule that no signal can outrun light would, by itself, allow scores up to four. The steps of the proof, as *Why the Quantum Is the Way It Is*, chapter 15, section "The Ceiling", gives them: each of a party's two observables has outcomes plus one and minus one and norm one (the norm of an operation being the largest factor by which it stretches the length of a state), so the commutator of the two has norm at most two; by Landau's identity the square of the CHSH operation has norm at most four plus two times two, which is eight; the CHSH operation is self-adjoint, so its norm is at most the square root of eight, $2\sqrt{2}$; and in every state its average value lies between minus its norm and plus its norm. Sealed lists: 2; quantum theory: $2\sqrt{2}$, Cirel'son's theorem of 1980, by Landau's identity of 1987 (S^2, read "S squared", equals 4 minus the product of the commutator of Alice's two observables with the commutator of Bob's two); no signaling alone: 4 (Popescu and Rohrlich 1994). The space of states, glossed in the chapter as the whole range of the states a pair can be in, is the space that *Why the Quantum Is the Way It Is*, chapter 9,

"The Space That Comparison Builds", constructs; on it every state gives a score whose size is at most $2\sqrt{2}$. In *Beyond the Quantum Passage*, the value $2\sqrt{2}$ is reached in every sector where both parties have a non-commutative carrier, and the commutative carrier gives back 2.

Lawfulness is a symmetry, locality is a continuity; Bell's theorem as an instance of the Incommensuration Theorem; the two options. *Why the Quantum Is the Way It Is*, chapter 15, section "Lawfulness and Locality"; *What Cannot Be*, chapter 10, sections "Lawfulness Is a Kind of Symmetry", "Locality Is a Kind of Continuity", "The Same Shape and the Same Root", "The Two Ways Bell Leaves Open"; *An Immanent Metaphysics*, Technical Appendix, subsection "Law and Locality" (the sentence quoted: "No theory of reality can assert both total generality, and total determinism"). The gloss of total generality and total determinism is *What Cannot Be*, chapter 10, section "The Same Shape and the Same Root". In Chapter 3 I make the identification in the first person, as *Why the Quantum Is the Way It Is* does ("To read locality as continuity, I take the region between two places as the context, and the influence as the content"). Physics' option (universal law kept; entanglement a bounded exception inside the limit of signaling; symmetry with discontinuity): *Why the Quantum Is the Way It Is*, chapter 15, section "Lawfulness and Locality". The gloss of "signal": *What Cannot Be*, chapter 10, section "Locality Is a Kind of Continuity".

Hidden variables excluded only where they are local; Bohm's theory stands as a description; no whole world is completely determined. *Two Theories, One World*, chapter 20, section "Bell's Theorem"; *The Last Turtle*, chapter 28, "Correlation Without a Cause", section IX, "What the Correlation Is"; *What Cannot Be*, chapter 10, section "Hidden Instructions, and the Last Escape" (a whole world completely fixed by one law perfects symmetry and continuity together; "The findings that such a formalism predicts have the statistics of quantum theory, and no finding ever shows that the formalism is deterministic"; "the determinism of a formalism is the determinism of a map, and not of a territory"). The gloss of hidden variable: *Why the Quantum Is the Way It Is*, Key Terms.

Entanglement spreads into the surroundings. *The Last Turtle*, chapter 28, section IX (any two particles that interact directly become entangled; two systems that interact only through a go-between in one definite state do not become entangled).

Teleportation; no copy. *The Quantum Bridge*, chapter 4, "Quantum Error Correction Across the Bridge", section VII, and Key Terms, entry "Teleportation" (protocol of Bennett, Brassard, Crépeau, Jozsa, Peres and Wootters 1993; the four outcomes name one of the gates identity, X, Z, or X and Z together; Bouwmeester and others 1997); *Why the Quantum Is the Way It Is*, chapter 15, section "What Can Be Copied" (a state between the outcomes of one setting cannot be copied, because a comparison finds only the outcomes of its own setting, a setting being the direction the measurement is made along); no-cloning derived in *The Quantum Passage*, chapter 10, section III.

Locality in two senses; Greene's sense of the word; the three wishes; the non-causal correlation; the resolution of this account. *What Cannot Be*, Terms, entry "Locality"; chapter 10, sections "Locality Is a Kind of Continuity" (the two senses), "Three Wishes, and Which One to Give Up" (universality of causation, consistency of causation, locality; physics keeps the first two and lives with "an influence that has no speed and no mechanism", this account keeps the last two; both on the surviving pairing symmetry with discontinuity; both give up locality in Bell's broad sense), and "What the Correlations Are" ("is a non-causal correlation: real, and not an influence"; the reason: only the assumption that every change must be causal turns a correlation into an influence; physics "calls the gap a non-local influence that can send nothing"). Greene's "the universe is not local" is stated of what is linked between two places with nothing traveling between them, which is locality in Bell's broad sense as *What Cannot Be*, chapter 10, defines it (every agreement between what is found in two places carried by causes that travel from place to neighboring place); in that sense this account reaches his sentence, and in the narrow sense it calls the universe local. The narrow sense: *What Cannot Be*, chapter 10,

"Locality Is a Kind of Continuity", states it as physics uses the word ("locality asserts that no physical influence, no interaction and no signal can travel faster than light") and, in the same section and in Terms, entry "Locality", as "no signal crosses a distance in no time"; section "Three Wishes, and Which One to Give Up" gives the third wish as "every cause, and so every signal, stays within the speed of light". In Chapter 3 I state the narrow sense in one form throughout, "no signal travels faster than light".

Bancal and others 2012; Gisin's "nonlocal" and the collection's "local" as two names for one finding. *Side by Side, Volume One*, chapter "Nicolas Gisin: The Digits Not Yet Drawn", Sections I and VII, "Correlations Without a Cause".

The stance, the act and the structure; the agreement located at neither place; a fact where the records meet. The three roles in the formal vocabulary are the transcendent (the stance), the immanent (the act) and the omniscient (the structure): *Why the Quantum Is the Way It Is*, chapter 3, "Three Roles", section "Structure, the Act, and the Stance" (what is laid out as structure "exists", the same in every instance, which joins the chapter's gloss of structure to the sense of Chapter 2); chapter 17, section "Why Collapse Is Not Physical" (setting, act, record); *Home in the World*, chapter 6, "The Third That Compares", section "Where the Agreement Lives" (die, toss, line written). Possibility bound to the stance and not to a place, with the street from one house to the next; the correlation located at neither place; neither finding answers the other; the agreement a fact where the records meet, for whatever compares them: *Why the Quantum Is the Way It Is*, chapter 15, section "In Any Order, and at Neither Place". In the algebra: a size-keeping turning made at one place is built from observables of that place, which commute with every observable of the other place, so the far weights are unchanged (same section).

Why no message can be sent. *Why the Quantum Is the Way It Is*, chapter 15, section "Why No Message Can Be Sent" (the speed of light as the boundary of what can be compared; a faster signal would, for some com-

parer, arrive before it was sent; the argument depends "on no feature of quantum mechanics and no theorem inside quantum mechanics about signaling"); *What Cannot Be*, chapter 10, section "What the Correlations Are"; *Causality Without Determinism*, chapter 3, section I (the stacked deck).

The update is not a physical process; no surface of simultaneity; moving beam splitters. *Why the Quantum Is the Way It Is*, chapter 17, section "Why Collapse Is Not Physical" (the update changes the laid-out record, "which is shared, and the same for everyone who has the outcome"); chapter 15, section "In Any Order, and at Neither Place" (moving beam splitters; Stefanov, Zbinden, Gisin and Suarez 2002); *The Unquantized Arena*, chapter 7, "One Geometry for Each Alternative", section IV, "No Signal Faster Than Light", Proposition 7.2.

Bohr was right against Einstein, Podolsky and Rosen. *Side by Side, Volume One*, chapter "Niels Bohr: The Third Term", Section VII; the shared context of joint preparation is from *The Basis of Physics*, Essay 14, as that section quotes it.

Chapter 4. How Does a Single Outcome Happen?

The state as the complete description of possibility, with the outcome given in the act. *Why the Quantum Is the Way It Is*, Chapter 17, "Why a Measurement Has an Outcome", section "Each Claim Said of One Role": "the state is complete as a description of possibility, and the outcome is given only in the act". *The Last Turtle*, Chapter 30, "Schrödinger's Cat and the Closure of Domains", Abstract: "The quantum state describes possibility; what is called collapse is the passage from the possible to the actual in the act of comparison".

Tonomura's single electrons building the interference pattern, each electron making its passage on its own. *Why the Quantum Is the Way It Is*, Chapter 13, "Superposition and Interference", section "One Dot at a Time" ("so that each electron made its passage on its own"). Primary pa-

per: A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Ezawa, "Demonstration of single-electron buildup of an interference pattern", *American Journal of Physics* 57 (1989) 117 to 120, doi:10.1119/1.16104.

Superposition: a state is any complex combination of states. *Why the Quantum Is the Way It Is*, Chapter 13, sections "One Dot at a Time" and "Superposition Comes with the Space": superposition follows from the space of states built from the algebra of comparison by the construction of Gel'fand, Naimark and Segal, with complex amplitudes (each amplitude an arrow with a size and a phase); "Superposition is the default, and definite outcomes are the exception, produced only when a specific comparison is performed". (same chapter, section "A Second Route, from Possibility").

A finding is made by any comparison that lays down a record; no mind is needed. *Why the Quantum Is the Way It Is*, Chapter 17, section "Every Comparison Has a Cut" (the hoofprint; "the screen or the counter, as an interaction, and no mind is needed"). *Measurement*, Chapter 1, "What This Book Takes from the First Two", Section I ("An observer is a comparer: the word names a position inside the world, and it never names a mind"), and Chapter 2, "Every Measurement Is a Comparison", Section VII, "Questions" ("By the Root Tautology the interaction in the detector is a comparison whether or not a person reads the count later"). *Comparison; The Necessary Basis*, Chapter 1, "What Does It Mean to Compare?", Section III, "Three, Not Two". The result from physics: J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Berlin: Springer, 1932): the boundary between the measured system and the chain of instruments can be placed at any link without changing any prediction. *Why the Quantum Is the Way It Is*, Chapter 17, section "Every Comparison Has a Cut", joins the two: "Any interaction that lays down a record serves as the finding, because von Neumann's result shows that the predicted counts are the same wherever along the chain the finding is placed, and so the laying down of a record is the whole of what makes a comparison a finding".

The bucket in this chapter: the hollow is a finding made by matter with no one watching. The principle the bucket carries throughout: *What Comes After the Dark*, Chapter 6, section "A Standard That Needs Something There" ("A difference that no comparison can register is no difference").

Decoherence: the fading of interference as the surroundings come to hold records; decoherence is not collapse. *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surroundings Do" (the definition; the notebook; the cat; "Decoherence is not collapse"; "Decoherence prepares the conditions for the measurement, by selecting the pointer basis"), with Schlosshauer's statement that decoherence does not solve the problem of outcomes (M. Schlosshauer, "Decoherence, the measurement problem, and interpretations of quantum mechanics", *Reviews of Modern Physics* 76 (2004) 1267). In the formalism, the cross terms of the reduced state of system and instrument are multiplied by the overlap of the corresponding states of the surroundings, a number between zero and one; the reduced state is an improper mixture. Greene's report of the researchers who claim a complete framework built on decoherence (Robert Griffiths, Roland Omnès, Murray Gell-Mann, Jim Hartle), and his own answer, "Others, like myself, are intrigued but not yet fully convinced": *The Fabric of the Cosmos*, chapter 7, section "Decoherence and Quantum Reality". The quantity the surroundings keep records of: *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surroundings Do", "It picks out which quantity the surroundings keep records of: the quantity whose states stay stable while the surroundings keep monitoring them", and "The pointer basis is that set of stable states". *The Unquantized Arena*, Chapter 11, "The Measurement Problem as a Cross-Modal Residual", Section III, Proposition 11.2 ("Decoherence is the dense reach of the formalism toward the finding"). *Beyond the Quantum Passage*, Chapter 2, "Decoherence and the Comparison Algebra".

Interference of whole molecules, and its fading in gas. *Side by Side, Volume One*, chapter "One Verdict, One Bit" (Anton Zeilinger), Section I; *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surround-

ings Do". Primary papers: M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw and A. Zeilinger, "Wave-particle duality of C60 molecules", *Nature* 401 (1999) 680 to 682, doi:10.1038/44348 (C60, read "C sixty": the fullerene of sixty carbon atoms); K. Hornberger, S. Uttenthaler, B. Brezger, L. Hackermüller, M. Arndt and A. Zeilinger, "Collisional decoherence observed in matter wave interferometry", *Physical Review Letters* 90 (2003) 160401, doi:10.1103/PhysRevLett.90.160401 (C70, read "C seventy": the fullerene of seventy carbon atoms).

Why decoherence selects no outcome: the time of the state's change and event time cannot be turned into each other. *Why the Quantum Is the Way It Is*, Chapter 16, "Three Kinds of Time", and Chapter 17, section "What the Surroundings Do"; *The Unquantized Arena*, Chapter 10, "Three Times", Propositions 10.1 and 10.2 ("a finding is discrete and its record is not undone, while U(t) is continuous and has an inverse", where U(t), read "U of t", is the evolution operator carrying the state across a span t of evolution time).

The delayed choice performed, with no reach into the past. *Side by Side, Volume Two*, chapter "The Participant and the Act" (John Archibald Wheeler), Section VII, "The Delayed Choice", and Appendix B, entry "Wheeler 8". Wheeler's sense of "phenomenon": the same chapter of *Side by Side, Volume Two*, which quotes him: "no elementary quantum phenomenon is a phenomenon until, in Bohr's words, 'It has been brought to a close' by 'an irreversible act of amplification'". Primary paper: V. Jacques, E. Wu, F. Grosshans, F. Treussart, P. Grangier, A. Aspect and J.-F. Roch, "Experimental realization of Wheeler's delayed-choice gedanken experiment", *Science* 315 (2007) 966 to 968, doi:10.1126/science.1136303. Wheeler's proposal: J. A. Wheeler, "The 'past' and the 'delayed-choice' double-slit experiment", in A. R. Marlow (ed.), *Mathematical Foundations of Quantum Theory* (New York: Academic Press, 1978), 9 to 48, as the collection cites it; Greene dates the proposal to 1980 (*The Fabric of the Cosmos*, chapter 7, section "The Contingency of History").

The quantum eraser: interference recovered by a measurement on the system and the holder of the mark together, while the mark has not spread. *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surroundings Do"; *The Unquantized Arena*, Chapter 11, Section III. Primary paper: Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih and M. O. Scully, "Delayed 'choice' quantum eraser", *Physical Review Letters* 84 (2000) 1 to 5, doi: 10.1103/PhysRevLett.84.1 (arXiv:quant-ph/9903047). What it found: the joint counts of the signal photon with an idler photon at either of the two detectors that erase the which-path information (D1, D2) show the double-slit interference pattern, the two patterns out of phase by π (half a fringe); the joint counts with either of the two detectors that keep it (D3, D4) show no interference; the idler is registered at least 8 ns, eight billionths of a second, after its signal photon. The proposal it carried out: M. O. Scully and K. Drühl, "Quantum eraser: a proposed photon correlation experiment concerning observation and 'delayed choice' in quantum mechanics", *Physical Review A* 25 (1982) 2208 to 2213.

A later choice is a new stance and cannot reach back; a reversal is a further act. *The Quantum Passage*, Chapter 11, "What Other Programs Assume and the Immanent Metaphysics Derives", Section III, "Chiribella, D'Ariano, and Perinotti": the causality axiom of those authors is derived from Axiom II; "A later choice of test is a new stance, which follows the finding (O → T) and cannot reach back". Here O → T is read "the omniscient precedes the transcendent": the laid-out record of a finding comes before the stance of the next comparison; the full cycle is T → I → O → T, read "stance, then act, then record, then stance". *Why Time Travel Is Not a Concept*, Chapter 4, "The Chain", Section IV, "The One-Way Order of Findings", Proposition 4.6, and the reference there to Proposition 5.2.

The smooth rule alone leaves the screen in a superposition of both readings; Maudlin's three claims. *Why the Quantum Is the Way It Is*, Chapter 17, sections "Three Magnets in a Row" (the magnets of Gerlach and Stern and the silver atoms) and "Two Things Go by One Name" (one silver atom and one screen). T. Maudlin, "Three measurement problems", *Topoi* 14 (1995) 7 to 15, doi:10.1007/BF00763473: (1) "The wave-function of

a system is complete, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system"; (2) "The wave-function always evolves in accord with a linear dynamical equation"; (3) "Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes". E. Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik", *Die Naturwissenschaften* 23 (1935) 807 to 812, 823 to 828 and 844 to 849.

The measurement problem in two senses; the first dissolves when the rules are derived from one source; the smooth rule said of the description of possibility and the jump said of the record. *Why the Quantum Is the Way It Is*, Chapter 17, sections "Two Things Go by One Name", "Each Claim Said of One Role", "Where the Contradiction Came From", "Why Collapse Is Not Physical" and "The Problem Dissolves": "The measurement problem dissolves when quantum mechanics is derived from the structure of comparison, instead of assembled from rules". Each claim said of its role: the first of the omniscient (the state as the complete laid-out record of what has been found and of what a setting makes possible); the second of the transcendent (change by size-keeping turnings in evolution time); the third of the immanent (one definite outcome in event time). Read in Maudlin's own wording, which asks the state to specify every outcome, the first claim is the one claim the collection denies, in one respect: the state leaves out the act of finding. The directed cycle of Axiom II, entered at the omniscient: record, then setting, then act, then record (same chapter, section "Why Collapse Is Not Physical": "The setting of the comparison, with its range of possible outcomes, is the stance from which the act proceeds: the transcendent"). In Chapter 4 I call the setting of the next comparison the stance of the next comparison. The sources of the six rules: Hilbert (1906) and von Neumann (1932) for the space of states; Schrödinger (1926) for the rule of change; Born (1926) for the rule of weights; von Neumann (1932) for projection, with Lüders (1951) (same chapter, section "Where the Contradiction Came From"; Chapter 1, "Six Rules Nobody Chose").

Exists and real. *The Gravitational Passage*, Chapter 5, Section II ("A law exists; a measurement is real"); *Comparison; The Necessary Basis*, Chapter 10, Section II; *Why the Quantum Is the Way It Is*, Chapter 17, section "Each Claim Said of One Role" ("The state exists"; "The outcome is real"); *What the Beginning Left Behind*, Chapter 8, section "Fixed Once" ("A law exists; a measurement is real").

The residual, guaranteed by the Incommensuration Theorem, is where the Born rule stands. *Why the Quantum Is the Way It Is*, Chapter 17, section "What Remains, and Where the Born Rule Stands": "The Incommensuration Theorem guarantees the gap between the continuous formalism and the discrete finding, and no future theory removes that gap, since a formalism that contained its findings exactly would perfect symmetry and continuity together in one mode". *Measurement* (as quoted there): "For quantum mechanics the residual is the measurement problem: the discontinuity of a definite finding, which a continuous formalism does not contain". The theorem in the words of *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". The Born rule as the one rule that gives each outcome one weight whatever else is measured with it: Gleason's theorem and Busch's theorem (*Why the Quantum Is the Way It Is*, Chapter 12); the weight of an outcome is the squared size of its amplitude.

The update of the record: collapse is not physical. *Why the Quantum Is the Way It Is*, Chapter 17, section "Why Collapse Is Not Physical" ("The update is not a physical process, and it is not an extra law of motion"; "Epistemic" there means a change in the laid-out record, shared and the same for everyone who has the outcome, and not a personal degree of belief). *Side by Side, Volume One*, chapter "One Verdict, One Bit", Section V, "Measurement as an Update of the Record". The state after an outcome is fixed by repeatability and an unchanged context as the rule of G. Lüders (1951) (*Why the Quantum Is the Way It Is*, Chapter 17, section "The State After an Outcome").

The passage from one finding to the next has a direction (Greene's expectation of time symmetry in the resolution of the measurement problem, *The Fabric of the Cosmos*, chapter 7, section "Quantum Mechanics and the Arrow of Time": "If time symmetry persists (as I suspect it will)"). *Why the Quantum Is the Way It Is*, Chapter 17, section "Why Collapse Is Not Physical": "The cycle has a direction, and so the passage it describes cannot be reversed"; each forward step is many to one, and the reverse "would be one to many, which is a relation and not a passage". *Side by Side, Volume One*, Appendix B, Prediction 28 ("Some asymmetry of time must exist in the laws, and no process at any scale is perfectly reversible").

Every comparison has a cut, set by the comparison performed; the identification of the cut. *Why the Quantum Is the Way It Is*, Chapter 17, section "Every Comparison Has a Cut" ("Moving the cut is performing a different comparison"), and Chapter 18, "The Interpretations, Each Missing a Role", section "Copenhagen and the Cut" ("I identify the cut as the boundary between the two levels of the theory, the formalism and the findings"). The mobility of the cut along the chain of instruments (von Neumann 1932) is the observation that bears on the identification: "Its mobility is the observation that bears on the identification: a boundary between two kinds of object would have a fixed location, and the cut has none". (same section).

The interpretations agree up to decoherence. *Why the Quantum Is the Way It Is*, Chapter 18, section "Decoherence and Consistent Histories" ("Up to that place all of them agree, because all of them use the same smooth equation and the same fading of interference"). Greene's statement that the interpretations agree on the predictions for "most every experiment": *The Fabric of the Cosmos*, chapter 7, section "Reality and the Quantum Measurement Problem".

The three-legged stool: each interpretation sets aside one of the three roles of a comparison and meets its difficulty where the role set aside would stand. *Why the Quantum Is the Way It Is*, Chapter 18, sections "A Stool with Three Legs" (the sawn leg and the glued legs), "Where a Set-

Aside Role Returns", "Copenhagen and the Cut", "Many Worlds" and "Objective Collapse". The three roles by their formal names: the stance, the transcendent; the act, the immanent; the record, laid out as structure, the omniscient. Copenhagen keeps the immanent and the omniscient and sets aside the transcendent (its difficulty: where the cut lies); the collapse models fold the immanent into the transcendent (their difficulties: the tails of the narrowed state, and heating that does not conserve energy); many worlds removes the immanent, read by roles, and absorbs event time into evolution time, read by times (its difficulty in two parts: the preferred basis, and probability, which needs an agent who weighs outcomes). On the preferred basis, *Why the Quantum Is the Way It Is*, Chapter 18, section "Many Worlds": "Decoherence supplies the way by the stability of records, and choosing that way is choosing which outcomes count as having occurred, which is event time". G. C. Ghirardi, A. Rimini and T. Weber, "Unified dynamics for microscopic and macroscopic systems", *Physical Review D* 34 (1986) 470 to 491, doi:10.1103/PhysRevD.34.470.

Every test of collapse models agrees with quantum theory. *Why the Quantum Is the Way It Is*, Chapter 18, section "Objective Collapse" ("Every test so far agrees with quantum mechanics"; the gravity-related model of Lajos Diósi, 1987 and 1989, and Roger Penrose, 1996, with its rate set "through the gravitational energy of the difference between the superposed arrangements of mass"; the X-rays from electrons of ordinary matter shaken at random); *Side by Side, Volume One*, "Does a Superposition End of Its Own Accord?", and Appendix B, Prediction 2, "Measured". Primary papers: Y. Y. Fein, P. Geyer, P. Zwick, F. Kiałka, S. Pedalino, M. Mayor, S. Gerlich and M. Arndt, "Quantum superposition of molecules beyond 25 kDa", *Nature Physics* 15 (2019) 1242 to 1245, doi:10.1038/s41567-019-0663-9 (25 kDa, twenty-five thousand daltons, is twenty-five thousand atomic mass units); S. Donadi, K. Piscicchia, C. Curceanu, L. Diósi, M. Laubenstein and A. Bassi, "Underground test of gravity-related wave function collapse", *Nature Physics* 17 (2021) 74 to 78, doi:10.1038/s41567-020-1008-4. The coming searches (Prediction 2, and Prediction 1 on gravitational entanglement) are in Appendix A.

Physics asks for understanding, not only prediction. *Side by Side, Volume One*, chapter "Tasks, Runs and Worlds" (David Deutsch), Section VI, "Good Explanations" ("an answer that only predicts leaves the why unanswered").

Many worlds, and the reversal experiment by which Copenhagen and many worlds would differ in principle. Greene's claim that Copenhagen and many worlds predict different results in principle is in *The Hidden Reality*, chapter 8, section "Predictions and Understanding", and its note 12 describes the experiment: an electron's spin is measured, the whole physical evolution is then reversed, and the spin is measured again; with Copenhagen in a mathematically coherent form, such as that of Ghirardi, Rimini and Weber, the first value returns half the time, and with many worlds it returns every time. *Side by Side, Volume One*, Appendix D, entry "Deutsch 7. The reversible observer", states what the collection says of Deutsch's form of that experiment: "The interference returns, as Prediction 25 requires of every reversible quantum process, and the surviving record carries no outcome". The two scopes at which the collection states its result on many worlds are given with Chapter 12.

Greene's telling of the observed present as a blend of all the histories that could have led to it, in Feynman's sum over paths (*The Fabric of the Cosmos*, chapter 7, sections "Prochoice" and "Pruning History"). Told here as Greene's account; the collection gives no reply on it.

Greene's telling of the proposal of London, Bauer and Wigner that consciousness completes a measurement (*Until the End of Time*, chapter 5, section "Consciousness and Quantum Physics"). Told here as Greene's account; the statement of the collection that bears on it, that a finding is made by any comparison that lays down a record and needs no mind, is in the note on findings above.

Chapter 5. Why Can't Relativity and Quantum Mechanics Both Be Right?

Planck's natural units of 1899. Planck, "Über irreversible Strahlungsvorgänge", *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* 5 (1899), 440–480; the natural units are proposed near the end of the paper (page 479). The German words the chapter gives in English, as reference works quote them: "ihre Bedeutung für alle Zeiten und für alle, auch außerirdische und außermenschliche Culturen nothwendig behalten". The translation from his German is mine. Planck built his units from his radiation constant b (later written h, Planck's constant), the speed of light c and Newton's constant (now written G). Planck's own length is the square root of h G divided by c cubed; the modern Planck length, written with the reduced constant ħ (read "h-bar", h divided by two pi), is the square root of ħ G divided by c cubed, so the two differ by the square root of two pi, about 2.51. That factor is the "small numerical factor" of the chapter.

The situations in which both theories are needed at once. *Two Theories, One World*, Chapter 1, "Two Theories That Never Fail", section "Where the Two Seem to Collide": "The difficulty appears only where both theories must speak about the same thing". and "Three places are usually named": a mass in a superposition; a black hole; the scale of about twenty-two millionths of a gram at which an object's two sizes are equal. Greene's list of the same situations: *The Elegant Universe*, chapter 1, opening; chapter 5 ("near the central point of black holes or the whole universe at the moment of the big bang", and the Planck length).

Two sizes for every mass; the Planck mass and length. *Two Theories, One World*, Chapter 15, "Where the Two Descriptions Meet", section "Two Sizes for Every Mass", with Figure 15.1; *The Unquantized Arena*, Chapter 9, "Linearized Gravity, Gravitons and the Planck Scale", Section V, "Where the Two Descriptions Meet", equation (9.3) and Proposition 9.3. Gravitational size r_g (read "r sub g") = G m / c^2, G being Newton's constant, m

the mass and c the speed of light; the mass becomes a black hole inside about twice this length (the Schwarzschild radius 2 G m / c^2). Quantum size λ (read "lambda") = $\hbar$ / (m c), the reduced Compton wavelength: the length within which, in the words of *Two Theories, One World*, "quantum theory will not let the mass be pinned down without spending enough energy to make new particles of its kind". The step the chapter gives in words is the uncertainty relation (a position spread Δx needs a momentum spread of at least $\hbar$ / (2 Δx)) together with the equivalence of mass and energy ($E = m c^2$): at Δx about $\hbar$ / (m c) the energy spread reaches the rest energy of the mass, enough to make a pair of particles of its kind. The two sizes are equal when $m = M_P = (\hbar c / G)^{1/2}$, the Planck mass, 2.176×10^{-8} kilograms (about twenty-two millionths of a gram). At that mass both equal the Planck length $l_P = (\hbar G / c^3)^{1/2} = 1.616 \times 10^{-35}$ meters; light crosses it in the Planck time, 5.391×10^{-44} seconds. Values stated in words in the chapter: Earth's gravitational size about 4.5 millimeters; the Sun's about 1.5 kilometers; a proton's quantum size about one eighth of its width; the Earth's quantum size needs sixty-seven zeros after the decimal point in meters; for a proton the quantum size exceeds the gravitational size by a factor thirty-nine digits long, for the Earth the reverse by a factor sixty-five digits long; a quartz grain a quarter of a millimeter across has about the Planck mass. The Planck length is about 10^{20} ("a hundred billion billion") times smaller than the width of a proton: the width is the proton's diameter, about 1.7×10^{-15} meters (twice the measured charge radius of about 0.84×10^{-15} meters), and 1.7×10^{-15} divided by 1.6×10^{-35} is about 1.05×10^{20}.

The four characters, the two pairings and the Incommensuration Theorem, stated plainly. *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". The four characters (aphorism 1.43-1, as *Home in the World*, "For the Specialist", gives them): continuity, a sameness of content with a sameness of context; discontinuity, a difference of content with a sameness of context; symmetry, a sameness of

content with a difference of context; asymmetry, a difference of content with a difference of context. The two surviving pairings are symmetry with discontinuity and continuity with asymmetry. In Chapter 5, as in Chapter 3, I call these two "pairings" and keep "character" for each of the four kinds of finding. The technical books call each surviving pairing a character of comparison (*Two Theories, One World*, "The Argument in Brief", section "The Answer in a Page": "A comparison, perfected at its foundation, can have only one of two characters"). *An Immanent Metaphysics*, aphorism 1.83-5, states two laws: "Law of symmetry will apply to all that is of content. Law of continuity will apply to all that is of context". From those two laws with the theorem, *Home in the World*, "For the Specialist", section "The Two Kinds of Perfected Comparison", concludes: "So content, perfected, is discontinuously symmetric, and context, perfected, is continuously asymmetric".

Identification: quantum theory as the description of content, general relativity as the description of context, the arena. *Home in the World*, "For the Specialist", "The Two Kinds of Perfected Comparison": "I identify quantum theory as the description of the content of physics, and general relativity as the description of its context, the arena of space and time". *Two Theories, One World*, "The Argument in Brief", section "The Answer in a Page", and Chapter 15, section "What the Meeting Means" ("I identify general relativity with the first character and quantum theory with the second"). The chapter's gloss of "the content of physics" as what its comparisons find is the book's gloss of content (Chapter 1) taken over all the comparisons of physics.

Gravity is the shape of the arena (same result as Greene's, on separate premises). Chapter 1 of this book makes the identification of gravity with the shape of the arena (from *Measurement*, Chapter 3, Section VII, the four marks of an arena). In Chapter 5 I recall that identification. *The Gravitational Passage*, Chapter 7, "The Six Metrics", Section V, "Context Is the Arena": the content phenomena exhaust the discontinuously symmetric conjunction; what occupies the continuously asymmetric conjunction is "the arena itself, not something operating within an arena. That phe-

nomenon is gravity". The same section gives the character of the context of any physical measurement: "smooth, and without the invariance that would make every place and direction the same". In Chapter 5 I join that character of the context to Chapter 1's words for the shape of the arena, "the way space and time are bent, from one place to the next". Greene's result: gravity as the warping of space and time, reached through the equivalence principle (*The Elegant Universe*, chapter 3; *The Fabric of the Cosmos*, chapter 3).

At the Planck scale the two descriptions meet and stay two. *Two Theories, One World*, Chapter 15, section "What the Meeting Means": a single description that carried both characters at the Planck scale "would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both"; "So the Planck scale is not the doorway to a third theory that replaces the other two". "It is the place where two descriptions are both needed and stay two". Technical form: *The Unquantized Arena*, Chapter 9, Proposition 9.3; *The Gravitational Passage*, Chapter 17, "The Planck Scale as Modal Boundary", Sections II and IV, where the Planck scale is named the modal boundary, the scale at which the continuously asymmetric description and the discontinuously symmetric description exchange dominance.

How far the smooth description holds. *Two Theories, One World*, Chapter 15, section "What the Meeting Means": for the attraction of two masses a meter apart the quantum corrections to Einstein's equations are about one part in a number seventy digits long; they grow with the curvature and become as large as the effects they correct only at the Planck curvature. *Two Theories, One World*, "The Numbers", Part IV, row "The quantum corrections to Newton's law at one meter": the relative size of the correction is the square of the Planck length over the distance, l_P^2 / r^2, 2.6×10^{-70} at one meter (the size of the correction of Donoghue 1994). That ratio holds no mass, which is why the chapter says the share is the same whatever the two masses are. Primary paper: J. F. Donoghue, "General relativity as an effective field theory: the leading quantum corrections", *Physical Review D* 50 (1994), 3874–3888. *The Gravitational Passage*,

Chapter 17, Section IV: the corrections to the Einstein action are of relative size R l_P^2 (the curvature R times the square of the Planck length), negligible far below the Planck curvature and of order one at it. In Chapter 5 I say the Planck curvature in words as a bending as sharp as the curve of a circle about a Planck length across: a radius of curvature of the order of l_P, so that R l_P^2 is of order one.

The bound on placing (same result as Greene's, reached with no string). *Two Theories, One World*, Chapter 15, section "Not a Least Length": a probe that could place a body within a Planck length would carry enough energy into the region to make a black hole larger than the region; "It is a bound on findings, not a step of space". *The Unquantized Arena*, Chapter 9, Section V: "A limit on localization is a different thing from a least length", with Belenchia, Wald, Giacomini, Castro-Ruiz, Brukner and Aspelmeyer (2018), *Physical Review D* 98, 126009, who find that vacuum fluctuations limit localization to about a Planck length. *The Gravitational Passage*, Chapter 17, Questions: the bound is "a limit on localization, not a least length of the geometry". Greene's result: *The Elegant Universe*, chapter 6, sections "The Rough Answer" and "A Sleight of Hand?"; chapter 10, section "A Minimum Size".

The bucket in this chapter. The placing of a body within about a Planck length is a comparison made from inside the world, and the bound is a limit on that comparison; the arena has no least length. The water is made of molecules, which blur its surface long before the Planck length, so the placing is asked of one molecule of the water, at the bottom of the hollow; the bound holds for any body, and the claim that there is no least length is made of the arena. Sources as in the two notes above; the principle of comparison ("A difference that no comparison can register is no difference") is *What Comes After the Dark*, Chapter 6, section "A Standard That Needs Something There", and *The Gravitational Companion*, "One Mass, Three Elevators", Section II. Greene's matching principle: "these fluctuations cannot be measured and hence, according to string theory, do not actually arise". (*The Elegant Universe*, chapter 6, section "The Rough Answer").

The light microscope. *Two Theories, One World*, Chapter 15, section "Not a Least Length": the resolution limit of a light microscope, about half the wavelength of its light, belongs to the way the cell is looked at.

The arena has no least length, area or duration. *The Unquantized Arena*, Chapter 6, "The Category Error", Proposition 6.3: "(i) The arena has no least step: no least length, area or duration. (ii) Nothing is a quantum of the arena itself, at any scale". Plain-words reason: *Two Theories, One World*, Chapter 15, section "Not a Least Length": a least step would make the arena discrete, the character of content, while what is found of the arena stays continuous and different from place to place, the character of context; one description of the arena would then carry both, and "so it would carry at least one of the two forbidden pairings, and, where all four characters are asked of the same directions, both".

Past measurement: the gamma-ray burst GRB 090510 (Fermi, 2009). *Two Theories, One World*, Chapter 15, section "Not a Least Length": a photon of about 31 GeV (31 billion electronvolts) arrived within about a second of the burst's low-energy light. *Side by Side, Volume Two*, "Time Is Real" (Lee Smolin), Section VI: the photon places the energy at which a first-order energy dependence of the speed of light would appear above 1.2 times the Planck energy. The chapter's slowing of light "by an amount growing in step with the light's energy" is the first-order dependence: a change in the speed of light of relative size E / E_{QG}, E being the photon's energy and E_{QG} (read "E sub Q G") the energy scale at which the effect would appear. Primary paper: A. A. Abdo and others (Fermi LAT and Fermi GBM Collaborations), "A limit on the variation of the speed of light arising from quantum gravity effects", *Nature* 462 (2009), 331–334.

Past measurement: GRB 221009A (the Large High Altitude Air Shower Observatory, LHAASO, observed October 2022). *Side by Side, Volume One*, Appendix B, Prediction 5, "Measured"; *Side by Side, Volume Two*, "John Archibald Wheeler: The Participant and the Act", Section VIII, subsection "The arena at the smallest scale": a first-order energy dependence is excluded up to about ten times the Planck energy. The Planck en-

ergy $E_P = M_P c^2$, about 1.22×10^{19} GeV (*The Unquantized Arena*, Chapter 9, Section IV). Primary paper: Z. Cao and others (LHAASO Collaboration), "Stringent tests of Lorentz invariance violation from LHAASO observations of GRB 221009A", *Physical Review Letters* 133 (2024), 071501.

Past measurement: X-ray images of distant quasars. *Side by Side, Volume Two*, "John Archibald Wheeler: The Participant and the Act", Section VIII, subsection "The arena at the smallest scale": "the sharpness of the X-ray images of distant quasars excludes the random-walk model of the foam, and their detection at the highest energies appears to exclude the holographic model (Perlman and others 2015)". The same book, "The Registered Predictions Beside John Archibald Wheeler", entry "Wheeler 1. The arena at the smallest scale": light crossing a seething geometry over cosmological distances would gather a spread of phase, which would blur the images of the most distant sources; the X-ray images are from the Chandra telescope. The same entry states the scope of these measurements: "They exclude the models later built on Wheeler's foam; they do not reach his foam in the form he gave it, which named no rate of growth". Primary paper: E. S. Perlman, S. A. Rappaport, W. A. Christiansen, Y. J. Ng, J. DeVore and D. Pooley, "New constraints on quantum gravity from X-ray and gamma-ray observations", *The Astrophysical Journal* 805 (2015), 10.

The conflict is made by an assumption (same result as Greene's; different assumption named). *Two Theories, One World*, "The Argument in Brief", section "The Answer in a Page": "The arena cannot be quantized". and "To quantize the arena is therefore to apply a concept where the concept does not belong. That is a category error, and it is the error behind the search for one theory"; Chapter 24, "Joined, Not Merged", section "Why the Search Expected a Merger" ("The search expected a merger because mergers had served physics well"). Greene: "In a sense, therefore, the central conflict of contemporary theoretical physics has been a problem of our own making". (*The Elegant Universe*, chapter 6, section "A Sleight of Hand?").

String theory's results. *Two Theories, One World*, Chapter 22, "What Each Program Found", section "Four Programs That Quantize the Arena", subsection "String theory": the graviton among the string's vibrations (after Schwarz 2007) and the count of Strominger and Vafa (1996), which matched the Bekenstein-Hawking entropy, one quarter of the horizon area in units of the Planck length squared: "These are two of the most striking results in modern physics". Primary paper of the count: A. Strominger and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy", *Physics Letters B* 379 (1996), 99–104. The entropy gloss of the chapter gives Greene's sense first (the number of ways the parts of a thing can be rearranged without changing how it looks overall, as the Preface of this book gives it), then the count relative to a context, the respects in which a given set of measurements compares the thing; Chapter 8 of this book gives that count in full ("every count of arrangements is made relative to the comparisons that do the telling apart"). The relative count: *What the Beginning Left Behind*, Chapter 7, "A Clean Slate, Counted", section "Every Count Has a Context": "A change of context is a change of what is counted, and a count made relative to one context is not a count relative to another". Greene's "overall appearance": *The Hidden Reality*, chapter 9, section "The Second Law" ("a great many possible rearrangements of its constituents leave its overall appearance intact").

Einstein's equations in four dimensions from ten, by two routes. *The Gravitational Companion*, "Other Programs That Obtain General Relativity", Section VIII, "String Theory (Callan, Friedan, Martinec and Perry 1985)": "They obtain the four-dimensional field equations by different means: string theory from the consistency of the worldsheet theory, the Passage from Lovelock's theorem once the moduli are held (Chapter 12, Section VII)". The dimension gloss is *What the Beginning Left Behind*, Terms, "Dimension": "An independent way in which things can differ".

Where the account differs from string theory: its first step. *Two Theories, One World*, Chapter 22, subsection "String theory": "The commitment at string theory's first step is to begin from quanta and to derive the arena

from them"; the graviton "sits in one list with the photon and the particles of the nuclear forces"; "Its claim that the arena itself is built from string states is the one use of quanta the third question excludes". *What Cannot Be*, Chapter 17, "Two Theories That Cannot Be One", section "How Each Program Trades": "Gravity itself is treated as content, one way of vibrating among the ways a string can vibrate". Light quantized and the arena not: *Two Theories, One World*, "The Argument in Brief", section "The Answer in a Page": "Light can be quantized, because light is content, found within a context that the change leaves in place". The joining the theorem forbids is a merger: *Two Theories, One World*, Chapter 24, "Joined, Not Merged", section "What the Theorem Forbids": "The theorem forbids a merger: one description in which the directions of space and time themselves carry both characters". and "A quantized arena is such a merger". The theorem forbids that joining and nothing more: it leaves every other result of string theory to be weighed as physics. *The Last Turtle*, Chapter 22, "Two Theories, One Arena", Section VI, "Merger and Common Source": "The theorem forbids a merger, and nothing more". and "Whether a given proposal is good physics is a question for physics".

The stage and its actors. *The Alphabet of Particles*, entry N2, "Graviton", subtitle ("The stage is not one of the actors") and subsection "What the Structure Says" ("it is the stage, not one of the actors").

The graviton as a quantum of a description; the phonon. *Two Theories, One World*, Chapter 14, "Waves Without Particles of Space", section "What a Phonon Is" ("A phonon is a quantum of the description of how a held structure vibrates".) and section "The Small Waves of Geometry" ("The arena is not a held structure". and "So even the phonon's sense does not make the graviton a quantum of the arena"); "The Argument in Brief", section "The Answer in a Page". *The Gravitational Passage*, Chapter 6, Section II: "The word graviton, where this book uses it, names at most a quantum of a description". The description of small waves follows Donoghue (1994), general relativity as an effective field theory.

Past measurement: the first gravitational wave (GW150914). *Two Theories, One World*, Chapter 14, sections "The Objection as a Reader Puts It" and "The Small Waves of Geometry" (the wave of 14 September 2015, reported in 2016; the arms changed by far less than the width of a proton); *Side by Side, Volume One*, Appendix B, Prediction 3, "Measured": "The first gravitational wave detected matched the classical waveform of general relativity (Abbott and others 2016); no graviton has been detected". Primary paper: B. P. Abbott and others (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of gravitational waves from a binary black hole merger", *Physical Review Letters* 116 (2016), 061102, DOI 10.1103/PhysRevLett.116.061102.

Past measurement: Page and Geilker (1981); one geometry for each alternative. *Two Theories, One World*, Chapter 12, "One Geometry for Each Alternative", sections "The Reason From the Ground" and "The Experiment of Page and Geilker": "What the experiment shows directly is that the field follows the outcome of the quantum event". "A geometry for each alternative gives exactly that result". and "The result needs no geometry with steps"; Page and Geilker titled their paper "Indirect Evidence for Quantum Gravity". *The Gravitational Passage*, Chapter 17, Section IV ("per-alternative geometry, in which each alternative of the matter carries its own classical geometry", with $G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi\, G\, \langle T_{\mu\nu} \rangle$ where the matter is sharply one alternative: Einstein's tensor plus the cosmological constant times the metric equals eight pi G times the expected stress-energy). The identification of one geometry for each alternative: *The Unquantized Arena*, "What This Book Claims and How It Is Set Out", Section I, "The Thesis" ("I identify the configuration in which each alternative of the matter carries its own classical geometry"), and Chapter 7, "One Geometry for Each Alternative", Section III ("Proposition 7.1 is this book's identification, made on that ground, and the experiment of Page and Geilker (1981) supports it by excluding the averaged geometry"); the reason from the inseparability of content and context is *Two Theories, One World*, Chapter 12, section "The Reason From the Ground". Primary pa-

per: D. N. Page and C. D. Geilker, "Indirect evidence for quantum gravity", *Physical Review Letters* 47 (1981), 979–982, DOI 10.1103/PhysRevLett.47.979.

Wheeler's foam: the same edge, and what lies past it. *Side by Side, Volume Two*, "John Archibald Wheeler: The Participant and the Act", Section VIII, "Where the Predictions Part, and Why", subsection "The arena at the smallest scale". Wheeler's estimate (Wheeler 1957, set out in four steps after Carlip 2023) sets the energy of the smallest quantum excitation of the geometry that fits in a region beside the energy general relativity gives a small fluctuation in the same region. It gives the size of the fluctuation, as a fraction of the geometry, as the Planck length divided by the size of the region; the formula for the energy of a small fluctuation holds only while the fluctuation is small. "Both arguments find the same edge". "They part on what lies past the edge". "For Wheeler, the geometry is a quantum variable at every scale, so the edge is a place where the geometry itself fluctuates without limit". "For the book, the geometry is the arena, which quantization does not reach, so the edge is the end of a description, and the arena keeps no least step beyond it". The chapter's "the arena, which cannot be quantized" is *Two Theories, One World*, "The Answer in a Page": "The arena cannot be quantized". Technical form of the edge: *The Unquantized Arena*, Chapter 9, Section IV, "The Weight of Non-Renormalizability": the local terms of the effective theory grow with powers of the energy in units of the Planck energy, so the expansion loses its order at the Planck scale.

The singularity marks the edge of the smooth description. *At the Edge of Reality*, Chapter 1, "What Happens at the Bottom of a Black Hole?", section "Where This Book Goes": "Physics itself does not break down there". *Two Theories, One World*, Chapter 26, "What a Physicist Will Say", section "The Singularities Demand a Quantum Gravity": "The infinity is what the smooth description gives when it is carried past the edge of its own domain"; the singularity theorems of Penrose (1965) and Hawk-

ing and Penrose (1970). *Two Theories, One World*, Chapter 15, section "What the Meeting Means": "I claim no third description beyond that edge, and no new geometry there".

The ultraviolet catastrophe and Planck's law of 1900. *Two Theories, One World*, Chapter 26, same section: Planck's law of 1900 put steps into the content, the energy exchanged between matter and light, and not into space. Primary paper: M. Planck, "Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum", *Verhandlungen der Deutschen Physikalischen Gesellschaft* 2 (1900), 237–245 (presented 14 December 1900); and "Ueber das Gesetz der Energieverteilung im Normalspectrum", *Annalen der Physik* 309 (1901), 553–563.

The arena is not made of anything more basic (different result from Greene's emergence). *Side by Side, Volume Two*, "Where the Dark Sector Lives" (Erik Verlinde), Section IV, "What the Arena Is": "The arena is not made of anything more basic that is found in it"; "Microscopic degrees of freedom that compose space would be constituents of the arena with the character of content, discrete states counted and entangled, and the continuous arena would then be composed from them". and "The theorem excludes exactly that composition". *Incommensuration*, Chapter 14, "Domains, Being and the Two Modes", Section V, "Finite and Infinite Describability": "One cannot compose a continuity out of purely symmetric components, any more than one can abstract a symmetry out of a pure continuity given alone". (quoted also in *Causality Without Determinism*, Chapter 5, Section VI). Greene's view: *The Fabric of the Cosmos*, chapter 16, sections "Are Space and Time Fundamental Concepts?" (the cannonball, the rose and the cheetah: "Just as the hardness of a cannonball emerges from the collective properties of its atoms") and "The Constituents of Spacetime"; *The Elegant Universe*, chapter 15, section "What Are Space and Time, Really, and Can We Do without Them?".

Maldacena's correspondence: correspondence, not emergence. *Two Theories, One World*, Chapter 23, "Two Descriptions That Translate", section "Neither Side Comes First" ("A duality that is exact in both directions

does not privilege either side". "This is correspondence, not emergence".) and section "What the Correspondence Shows" (the translation of a poem; "Nobody concludes from a perfect translation that French is English in disguise"); *The Unquantized Arena*, Chapter 15, "AdS/CFT as Cross-Modal Correspondence", Proposition 15.4 (anti-de Sitter space and conformal field theory: gravity in a region of negatively curved space and a quantum theory on its boundary). Which surviving pairing each side takes is an identification of *Two Theories, One World*, Chapter 23, section "Each Side in Its Own Character" ("I identify the findings of the boundary theory as discontinuously symmetric, the character of quantum theory". and "I identify the findings of the bulk side as continuously asymmetric, the character of general relativity"); Chapter 12 of this book makes it. Maldacena's paper was posted in 1997 and published in 1998: J. Maldacena, "The large N limit of superconformal field theories and supergravity", *Advances in Theoretical and Mathematical Physics* 2 (1998), 231–252.

The experiment being built (coming measurement; Appendix A). *Two Theories, One World*, Chapter 13, "The Experiment Being Built"; *Side by Side, Volume One*, Appendix B, Prediction 1 (for the design of Bose and colleagues, 2017, "the entangling phase is 0.314 radians"). In Chapter 5 I name only what the experiment measures; what the account says the experiment will find is in Appendix A. The gloss of "entangled" is the one Chapter 3 of this book gives.

Both theories right, each in its own pairing; where the clash comes from. *Two Theories, One World*, Chapter 24, "Joined, Not Merged", section "What the Theorem Permits": "Each is complete in its own character, and each captures what the other must leave out"; Chapter 27, "What the World Will Show", section "The Argument, Once More": "The two theories that never fail are not waiting to be merged into one". The cause of the clash: "The Argument in Brief", section "The Answer in a Page": "To quantize the arena is therefore to apply a concept where the concept does not belong. That is a category error, and it is the error behind the search for one theory". Greene's condition: "As they are currently formulated, general relativity and quantum mechanics cannot both be right". (*The El-*

egant Universe, chapter 1, opening); the clash as he gives it, the uncertainty principle applied to the gravitational field: *The Elegant Universe*, chapter 5, section "General Relativity vs. Quantum Mechanics".

Greene's pictures and statements told as his. The peach pit drawn by probes (Slim and Jim), the result of Gross and Mende (1988) on high-energy string scattering, the Planck length as the height of a tree beside an atom magnified to the size of the known universe (*The Elegant Universe*, chapters 5 and 6). The confirmation of the two theories: physicists "have experimentally confirmed to almost unimaginable accuracy virtually all predictions made by each of these theories" (*The Elegant Universe*, chapter 1, opening); the infinities: calculations that merge the two theories "typically yield one and the same ridiculous answer: infinity" (chapter 5, section "General Relativity vs. Quantum Mechanics"). The "fuzzy patch" of the first moments: *The Fabric of the Cosmos*, chapter 8, where it is first drawn, and chapter 12, section "Does It Matter?" ("Because of the conflict between general relativity and quantum mechanics, we remain ignorant about what happened at the beginning and are reduced to drawing a fuzzy patch in Figure 10.6"). What takes over below the Planck length: "As for what concepts take over, there is as yet no consensus", with the grid, the penny and the gradual change into other concepts (*The Fabric of the Cosmos*, chapter 12, section "Cosmic Fabric in the Realm of the Small"). Told here as Greene's account.

Chapter 6. Is There One Theory of Everything?

Einstein's last day and Greene's line on it: the date and the hospital, the request for the pages, the thirty years of the search, and his death before the next morning. *The Hidden Reality*, chapter 4, section "A Brief History of Unification", reports the account of Einstein's secretary of 17 April 1955, "his penultimate day", and closes: "Einstein didn't rise with the morning sun". Told here as Greene's account; the collection gives no reply on it.

String theory as a candidate theory of everything: particles as vibration patterns of one kind of string; the graviton among the patterns; one input number, the tension of the loops, which sets the overall scale. Told here as Greene's account (*The Elegant Universe*, Preface; chapter 1, sections "String Theory: The Basic Idea" and "String Theory as the Unified Theory of Everything"; chapter 6, sections "Unification through String Theory" and "The Music of String Theory"). Greene's condition on the explanation of the particles, in the same section "The Music of String Theory": "At the present time, therefore, we do not know if the fundamental characteristics of our universe, summarized in Tables 1.1 and 1.2, can be explained by string theory". The hurdles, in the same paragraph: "a number of hurdles remain, preventing us from deducing the spectrum of string vibrations with the precision necessary to compare with experimental results".

The Higgs field as Greene tells it, before its particle was found: "Many physicists strongly believe that there is yet a third kind of field, one that has never been experimentally detected but that over the last couple of decades has played a pivotal role both in modern cosmological thought and in elementary particle physics". *The Fabric of the Cosmos*, chapter 9, section "Force, Matter, and Higgs Fields"; the settling as the universe cooled, section "Fields in a Cooling Universe"; the ocean and the drag, section "The Higgs Ocean and the Origin of Mass", which opens on a condition: "If a Higgs field has a nonzero value—if we are all immersed in an ocean of Higgs field—then shouldn't we feel it or see it or otherwise be aware of it in some way? Absolutely. And modern theory claims we do". Told here as Greene's account; the collection's reply is in the entries on the Higgs field below.

The joining of electromagnetism and the weak force, confirmed by accelerator experiments in the late 1970s and early 1980s; grand unification, proposed by Glashow and Georgi, its simplest version ruled out when the proton decay it predicts was not observed, many other versions not ruled out. Told here as Greene's account (*The Hidden Reality*, chapter 4, note 2:

"The simplest version of grand unification, however, was ruled out when scientists failed to observe one of its predictions—that protons should, every so often, decay").

Supersymmetry as the one further symmetry once spin is counted; the near-meeting of the three strengths made exact by superpartners (Amaldi, de Boer and Fürstenau 1991); the strengths changing with the distance, or energy, at which a force is examined. Told here as Greene's account (*The Elegant Universe*, chapter 7, sections "Supersymmetry and Superpartners", "The Case for Supersymmetry: Prior to String Theory" and "Supersymmetry in String Theory"); the collection's reply is in the entries on the three forces and on supersymmetry below. Greene's two uses of the word symmetry: the property that does not change (*The Elegant Universe*, Glossary, "Symmetry") and the transformation that leaves a system's appearance unchanged (*The Fabric of the Cosmos*, Glossary, "symmetry"). The two families by spin: the particles of matter (fermions) have spin one half in units of the reduced Planck constant ħ (read "h-bar"), and the carriers (bosons) have whole-number spin, the photon, the gluons and the W and Z one, the Higgs particle zero (*The Particular Evidence*, Case 14, "The Last Field", section III: "By 2013 its spin was confirmed to be zero"); the sign under exchange that goes with each family is in the entry on supersymmetry below.

Greene's status report on string theory, eight goals, six marked "Excellent" and two "Indeterminate; no predictions", with his note under the goal on singularities. Told here as Greene's account (*The Hidden Reality*, chapter 4, section "The State of String Theory: An Evaluation", Table 4.2; the circumstantial case, section "String Theory and Experiment": "For the foreseeable future, then, the most promising avenue for linking string theory with data are predictions that, while open to explanations using more traditional methods, are far more naturally and convincingly explained using string theory"). Of the rows of that table, "Incorporate key breakthroughs from past research", "Cure singularities" and "Mathematical contributions" have no reply in the collection.

Structural inevitability as a measure of merit, matching the test Greene reports, that a final theory "couldn't be otherwise". The test Greene reports, that a final theory could not be otherwise, my book *Physics Is Not Math* names as one measure of the merit of a work. In its words, "Structural inevitability is whether the form the work takes could not have been otherwise, so that it shows why its conclusions hold and not only that they do". *Physics Is Not Math*, chapter 19, "A Right Method of Evaluation", section VII, "Merit Has Many Measures"; the same sentence adds that independent thinkers arriving at the same structure is evidence of it.

The three roles of comparison and the three axioms, in plain words, with the three rings for the third axiom. *Home in the World*, chapter 3, section "The Borromean Rings" (the plain words and the rings the chapter uses), and "For the Specialist", section "The Three Axioms", which gives the exact wording of *An Immanent Metaphysics*, aphorism 1.3-1: "Axiom I: The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". "Axiom II: A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". "Axiom III: The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and noninterchangeable". The chapter's act is the immanent, its structure the omniscient and its stance the transcendent.

The Incommensuration Theorem, in the exact words of *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". The plain glosses the chapter uses for the four (exactly repeating; in countable steps; joining without a break; different from place to place) are those of *Home in the World*, chapter 5, section "Beads on a Thread"; the definitions in terms of sameness and difference of content and context are aphorism 1.43-1, as *Home in the World*, "For the Specialist", gives them. The two surviving pairings, content perfected as discontinuously symmetric and con-

text perfected as continuously asymmetric: *An Immanent Metaphysics*, aphorism 1.83-5, as *Home in the World* quotes it. The technical books call each surviving pairing a character of comparison, and in Chapter 6, as in Chapter 3, I keep "character" for each of the four kinds of finding.

The structure of comparison: the three axioms, the six intrinsics, the partition of content and context, and the Incommensuration Theorem, "the structure of the act by which anything can be known". *The Story of the Particles*, "The Scientists", section I, "An Empty World".

The account begins from a stated principle: every finding is a comparison, with a content and a context; gravity carries the four marks of the arena, one of them the equality of inertial and gravitational mass. *Two Theories, One World*, chapter 9, "Four Marks of the Arena", sections "The Third Mark: One Mass" and "The Identification" ("I identify gravity as the arena of physics, the context of every physical measurement, by that match"); the fourth mark, section "The Fourth Mark: No Charge in Steps" ("So whatever couples a body to an arena is no stepped label"); the identification is made in this book's Chapter 1, in the words "I identify gravity with the shape of the arena". The measured check of the third mark: MICROSCOPE, the fractional difference in the free fall of titanium and platinum, the Eötvös parameter (eta, read "eta"), consistent with zero at about one part in ten to the fifteenth (Touboul and others 2022).

The limit of explanation: the foundations guarantee structure, not content. *The Last Turtle*, chapter 2, "Comparison, Its Four Compounds, and the Incommensuration Theorem", section X, "The Limit It Sets on Every Theory"; the same section: "No theory will ever completely explain, predict or model all the observable behavior and phenomena in a domain".

The graviton and the Strominger and Vafa count credited as results. *Two Theories, One World*, chapter 22, "What Each Program Found", section "String theory": "These are two of the most striking results in modern physics". *The Unquantized Arena*, chapter 14, section IV, entry "1. String theory": "The achievements stand as results".

Black-hole entropy is established physics, and its dependence on area is reached by Axiom I. *The Compactification*, chapter 4, "Horizons and Their Counts", section II, "The Count of a Horizon": $S_{BH} = A/(4\ l_{Pl}{}^2)$ (read "the black-hole entropy equals the horizon area A divided by four times the square of the Planck length"), with Boltzmann's constant set to one; about 1.05×10^{77} for one solar mass. The two counts of a bounded region: the count of the interior cell by cell is identified as omniscient, the count through the boundary, made from a place, as immanent (*The Unquantized Arena*, chapter 17, "Black-Hole Information and the Page Curve", section IV, "Why Area, Not Volume"); by Axiom I the immanent count is the more fundamental. The chapter's walled garden pictures the two counts. The same section: the identification "makes no claim on the coefficient one quarter, which equation (17.2) takes from Hawking (1975)".

What a black hole's entropy counts: states of the hole's content that no comparison from outside tells apart, a count made against the outside world as its context. *The Unquantized Arena*, chapter 17, section III, "The Premise Removed", Proposition 17.3: "I read it as the number of states of the hole that no comparison from outside tells apart, a count relative to the outside as context". *Two Theories, One World*, chapter 26, "What a Physicist Will Say", section "'The Entropy Has Been Counted'": the states of strings and membranes counted, for black holes with the largest charge their mass allows, in the regime of weak gravity, where the geometry is flat and classical, with the count protected as gravity is turned up, matching a quarter of the horizon area. That book's own wording is "I read the count as a count of content". The count does not show that the arena itself is made of countable states, which is how many physicists read it. The reading is the question that the same section of *Two Theories, One World* sets out and answers: "If gravity has inner states that can be counted, it has a discrete space of states, and so gravity is quantized". *The Unquantized Arena*, chapter 18, "Objections at Full Strength", section II, "Microstate Counting": "The count is of bound states of D-branes, made in a regime of weak coupling where the system is not a black hole". The same section of

Two Theories, One World reads the loop quantum gravity count (Ashtekar, Baez, Corichi and Krasnov 1998) as reaching one quarter through the choice of the Immirzi parameter.

One account of everything, made of two descriptions joined from a common source; the theorem allows a common source and forbids a merger; the railway timetable and the track. *Two Theories, One World*, chapter 24, "Joined, Not Merged", sections "One World, One Account, Two Descriptions", "Two Kinds of Unity" and "What the Theorem Forbids". A merger asks the directions of space and time themselves to carry both surviving pairings; a quantized arena asks all four directions, so it would carry both forbidden pairings, continuity with symmetry and asymmetry with discontinuity ("What the Theorem Forbids").

Why the search expected a merger: each earlier success joined one content to another, and gravity is the context in which those mergers took place. *Two Theories, One World*, chapter 24, section "Why the Search Expected a Merger".

String theory's first step (to begin from quanta and derive the arena from them) and the difficulty it predicts, multiplicity of string vacua with no principle in the theory selecting one; gravitons on a smooth background allowed as quanta of a description. *Two Theories, One World*, chapter 22, section "String theory" (Susskind 2003; Douglas 2003; the swampland program, Vafa 2005, reviewed by Palti 2019).

The three forces are three because the three positions of comparison are distinct, inseparable and non-interchangeable (Axiom III); no single force of which the three are parts; no carrier turns a quark into a lepton; the book and the cup. *The Story of the Particles*, Level 9, section IV; *The Branches of the Kinds*, chapter 2, "The Stable Proton", section I, "The Product and Its Consequence", and its box "A product, not a merger" (the book is color, the cup the electroweak labels, the doublet and the phase): the gauge group $SU(3) \times SU(2) \times U(1)_Y$ (read "the special unitary groups of degree three and two, times the unitary group of hypercharge") is a product of its factors at every scale. The pairing of forces and posi-

tions: SU(3), the pattern-holding force, omniscient; SU(2), the maximally asymmetric force, transcendent; U(1), the force on the phase every state carries, immanent; the labels follow the frame of measurement, in which the comparing act is light, and the same algebra admits three labelings, rotations of one another around the cycle of Axiom II (*The Story of the Particles*, Level 9, section III, "The Algebra Is Finite"). The property the chapter gives for each force, in that book's words: the strong force is "the pattern-holding force, the force that maintains the structure of content"; the weak force is "the force of change and decay, the force that transforms what something is into what it is not"; the force on the phase "acts on the phase every state carries", and "At low energy this force meets the scientists as the electromagnetic force" (Level 7, sections II, III and IV). The description of the stance the chapter quotes, in full: "The transcendent is the thing compared against, which the comparison reaches toward and never contains". (*The Story of the Particles*, Level 7, "The Three Forces", section III, "The Weak Force: Three Bosons"); in the frame of measurement light is the comparing act, the electron is "The thing compared" and the proton "The thing compared against" (the same book, Glossary, entries "Electron" and "Proton"), and the same section gives the reason for the pairing: every doublet has a side that faces the comparing act and a side that "does not present itself to it", and "the two-sidedness of a doublet is the transcendent's mark" (Level 7, section III).

The two lists of three forces: the three of the gauge group before the Higgs field takes its value (color, weak isospin and hypercharge), and the three met at low energy (the strong force, the weak force and electromagnetism), which *The Story of the Particles*, Level 9, section IV, names: "The other three forces (electromagnetic, weak, and strong) are transformations of comparison's content". The photon as one combination of the hypercharge boson and the neutral weak boson, left massless by the condensate: *The Coming of the Kinds*, chapter 6, "The Higgs Field Settles", section III, "The Field Takes Its Value", quoting *The Story of the Particles*, Level 8, section III: "The combination that reads electric charge finds nothing in the condensate to read, and stays massless. That combination is the

photon". Hypercharge, the charge that before the electroweak transition plays the part electric charge plays after it: *The Coming of the Kinds*, chapter 6, section II, "Before the Field Settles".

The theorem alone allows content joined to content; the three forces stay three by Axiom III. *Two Theories, One World*, chapter 24, section "What the Theorem Permits": "The theorem is silent on whether nature joins further forces in that way, and this book takes no position on it". Taken alone, the Incommensuration Theorem is silent on whether nature joins further forces, and that silence is the scope of the theorem taken alone. The result that the three forces stay three is carried by *The Story of the Particles*, *The Strengths of the Kinds* and *The Branches of the Kinds*. The joining of electromagnetism and the weak force above the settling (four massless carriers), kept as Greene tells it; the electroweak symmetry a product of two factors side by side: *The Coming of the Kinds*, chapter 6, section II, "Before the Field Settles"; *Why These Numbers*, chapter 5, section "One Force, or Three?".

The measured couplings draw close and meet at no scale; inseparability produces nearness, distinctness the gap; the structure needs the three nearly equal at the finest resolution, as distinct from equal. *The Strengths of the Kinds*, chapter 3, "Where the Couplings Cross", section III, "Where the Crossings Are", subsection "The measured couplings draw close and stay apart". Values: at one loop, from inverse strengths at the Z mass of 59.0 (the phase force, in the unification normalization), 29.6 (the weak force) and 8.5 (color), the pairwise crossings lie near $10^{13.0}$, $10^{14.4}$ and $10^{17.0}$ GeV, in the order $\mu_{12} < \mu_{13} < \mu_{23}$ (read "mu one-two, less than mu one-three, less than mu two-three", the crossing scales of each pair), spread over nearly four orders of magnitude; the remaining mismatch at the first crossing is about 5 to 6 units of inverse strength (5.58 at one loop, Particle Data Group 2022 inputs). Scope: *The Grand Synthesis*, chapter 13, section III, "The Completeness of the Kinds", states that the three observations named there (no superpartners found, no proton decay, no meeting of couplings) do not discriminate between this account and the plain Standard Model; they bear on the supersymmetric and grand unified extensions.

The proton has no route down; the measured lifetime bound. *The Branches of the Kinds*, chapter 2; *Side by Side, Volume One*, Appendix B, Prediction 33: the lifetime for decay into a positron and a neutral pion exceeds 2.4×10^{34} years (Super-Kamiokande; Takenaka and others 2020). Arithmetic of the chapter's comparison: 2.4×10^{34} years divided by an age of the universe of about 1.38×10^{10} years is about 1.7×10^{24}, more than a million billion billion. The detector (fifty thousand tons of water under Mount Ikeno, Kamioka, running since 1996): Fukuda and others 2003.

No supersymmetry: a supersymmetry would carry the content pole onto the context pole; at the first scale the content pole is the symmetric one and the context pole the continuous one, and a transformation carrying one onto the other would perfect symmetry across that partition; the two differ by a sign under exchange (minus one for fermions, plus one for bosons), and a sign has no values in between. The first scale is the first of the three scales at which the content/context partition operates within the content: "fermion against boson, the two sides of a doublet, and quark against lepton" (*The Story of the Particles*, Level 8, "The Higgs", section II, "Why Spin Zero?", citing Level 6, section VI). In Chapter 6 I call the first scale the first line, between the two families. *The Story of the Particles*, Level 9, section IV, and Level 0, section III and its Note (the sign); *The Alphabet of Particles*, entry N4, "Supersymmetric Partners", subsection "What the Structure Says". *Non-Commutative Geometry and the Immanent Metaphysics*, chapter 19, "The Particle Content", section IV, "The Field Space and Its Content", Proposition 19.8: a map Q equivariant under $SU(A_F)$ exchanging the odd and even sectors of the field space $\mathbb{F}$ vanishes on the quark and charged-lepton fields, since the two sectors share only the gauge types (1, 1, 0) and (1, 2, $\pm 1/2$). The field space is the sum of the spinor fields, the gauge one-forms and the Higgs doublet (Proposition 19.7 of the same section).

Superpartners not found at the Large Hadron Collider. *The Alphabet of Particles*, entry N4, subsection "Searches": gluinos excluded below about 2300 GeV, top squarks below about 1300 GeV, the lighter squarks below

1200 to 1850 GeV, depending on what is assumed about their decays; *Side by Side, Volume One*, Appendix B, Prediction 30, "Measured". The chapter's comparison: 2300 GeV divided by the proton mass of 0.938 GeV is about 2450. That many physicists expected the partners within the collider's reach: *The Particular Evidence*, Case 14, section III ("it is one of the reasons many physicists expected new particles at the LHC"), and Case 15, section II, "The Detective Work: What Was Expected" ("for the cure to work the partners had to appear at energies the LHC could reach").

The kinds of particle form a closed list fixed by the structure of comparison: their labels, generations and field kinds; no fourth generation, no graviton, no species of dark matter, no supersymmetric partner, no force joining quark to lepton. *The Story of the Particles*, Level 9, "The Composites and the Boundary", section IV, "What the Zoo Does Not Contain"; the algebra of the internal structure of comparison is $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ (read "the complex numbers, plus the quaternions, plus the three-by-three complex matrices"). *The Kinds of Particle*, chapter 6, "The Sixteen States", section IV, "No Further Kinds": sixteen states of content in each generation. Scope: the list fixes kinds, not values (*The Story of the Particles*, Level 9, section V, "What the Zoo Fixes and What It Does Not").

The Higgs field required by the doublets before any question of mass. By the third axiom the two members of a pair are distinct and inseparable. Where two distinct things are inseparable, there must be a comparison across the edge between them, and so a field on that edge, whose value at every place says how the two members differ. *The Particular Evidence*, Case 14, "The Last Field", section IV, "With the Story in Hand" ("In The Story of the Particles the Higgs is expected at Level 8, and it is expected before anyone has a mass problem to solve"; "The two sides of a doublet are distinct and inseparable (the third axiom)"), with its account of the field's spin zero, its two components and its one edge crossed. The crossing argument the chapter sets out: "A field lives on an edge when a transformation crosses it, because the two sides differ (Axiom III) and what the crossing carries across, the difference, must be held at each point: on the edge". (*The Story of the Particles*, Level 8, section II, "Why Spin Zero?".)

The edge is no place in space: the same section calls it "the edge between the two sides of the doublet" and says "An edge has no direction. It is a separation, not a flow"; the field that lives on it is "a number at each point, not a vector"; the same book's Glossary, entry "Edge (of a doublet)": "The line between the two sides of every doublet, which the weak force crosses; the Higgs field lives on it (Level 8)". Why the field settles: "the law is symmetric, and the ground state cannot be both symmetric and continuous" (*The Particular Evidence*, the same section); *Why These Numbers*, chapter 2, "What There Is, and Why It Is Built So", section "The Field on the Edge" (the wine bottle).

Masses from the condensate: each particle's mass is the condensate's value times its own coupling; the photon and gluons stay massless; the settling is a crossover. *The Coming of the Kinds*, chapter 6, "The Higgs Field Settles", sections I and III (the walkers in snow). The settling was smooth, and it was steepest when the temperature of the young universe, stated as an energy, was about a hundred and sixty billion electronvolts. Values: the crossover is steepest within about 5 GeV of its center at 159.5 ± 1.5 GeV (D'Onofrio and Rummukainen 2016), the temperature stated in words in the sentence above; the vacuum value v (read "vee") is 246.21967 ± 0.00006 GeV; a quark's mass is its coupling y times v divided by the square root of two; the top quark's coupling y_t (read "y sub t") is one, giving the ceiling $v/\sqrt{2} = 174.1036$ GeV, beside 0.991 read from the measured top mass of 172.60 GeV. Why the smoothness of the settling matters for the beginning: Chapter 9, section "Smooth Changes, with No Bubbles".

The Higgs particle, past measurement. *The Particular Evidence*, Case 14, "The Last Field", section III, "The Resolution: 4 July 2012": a new particle at about 125 GeV, seen in decays to two photons, two Z bosons and two W bosons; spin zero by 2013; couplings to the top quark, the bottom quark and the tau measured by 2018, each in proportion to mass.

Three light neutrino species. *Side by Side, Volume One*, Appendix B, Prediction 31: the invisible width of the Z counts N_ν (read "N sub nu") = 2.9963 ± 0.0074 light neutrino species (Janot and Jadach 2020), 0.5 standard deviations from three; the data are those of CERN's earlier collider of electrons and positrons.

Four measured inputs: Newton's constant, the strength of light, the strength of color, and the vacuum value of the Higgs field. *Side by Side, Volume One*, Appendix B, section IX, "What the Books Do Not Predict" (citing *The Gravitational Passage*, chapter 13; *The Strengths of the Kinds*, chapter 1, section VI; *The Measures of the Kinds*, chapter 13, section X). The reason one value fixes the scale: *Two Theories, One World*, chapter 15, "Where the Two Descriptions Meet", section "Its Role and Its Size" ("the axioms are structural, not dimensional"; Newton's constant known to about two parts in a hundred thousand); that book pictures the same division with a surveyor's map and the one line of its legend read off the ground, where the chapter uses a photograph of a building. The reason the strength of light stays measured: *The Strengths of the Kinds*, chapter 1, "What Is Measured", section VI, "Why the Strength of Light Stays Measured": the gauge group of light, $U(1)$, leaves one positive number, the coupling, undetermined by the symmetry; on a finite lattice the strength enters the weight of an arrangement only through the bounded action of the links, so the possible, the impossible and the certain arrangements are the same at every value of α_{em} (read "alpha sub e-m", the strength of light); "So no argument from the comparison structure, of any length, fixes αem". The same section states the scope: the independence is "thus derived for the comparisons the argument examined, with the circle's rule as its stated hypothesis", and the independence of the three couplings is derived against two routes only (interchangeability of the three kinds, and a crossing between basic sets); measured value $1/\alpha_{em} = 137.035999177$.

The unit of a mass and the ratios of masses are two questions; the books that carry the ratios hold that none is a number with no reason. *Why These Numbers*, chapter 1, "Numbers Physics Cannot Explain", section "Two Questions in One"; it retells *The Strengths of the Kinds* and *The*

Measures of the Kinds (section "The Two Books This Book Retells"), which state where measurement has not yet chosen between two readings (*Side by Side, Volume One*, Appendix B, section IX). Greene's masses are given in multiples of the proton mass (*The Elegant Universe*, chapter 1, section "The Universe at Its Smallest: What We Know about Matter", Table 1.1).

The tau from the electron and the muon. *The Measures of the Kinds*, chapter 8, "The Charged Leptons", section II, "The Amplitude and the Balance", and section III, "The Tau from the Electron and the Muon": the balance is Koide's quantity $Q = (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ (read "Q, the sum of the three masses of the electron, the muon and the tau divided by the square of the sum of their square roots, equals two thirds"), which that book derives by two routes, given its stated identification that each part is measured by its share of the squared size. From the measured masses $m_e = 0.51099895069$ MeV and $m_\mu = 105.6583755$ MeV (CODATA 2022), with the three square roots of one sign, the balance gives $m_\tau = 1776.969$ MeV; the ratio to the electron is 1776.969/0.51099895069, about 3477.4, beside the measured ratio 3477.37 (the PDG 2024 tau over the CODATA 2022 electron, section V). The same chapter, section V, "The Muon from the Electron: the Ladder and the Turnover": the lepton ladder from the electron alone gives the bare ratio $m_\tau/m_e = 2^{(209/18)s} = 3468.88$, with $s = 1.0128448$; section VI, "The Tau's Two Readings": the balance's 1776.969 MeV and the ladder's 1777.146 MeV, 0.177 MeV apart, which a tau mass measured to ± 0.045 MeV separates at 3.9 standard deviations. Greene's "about 3,520" (*The Elegant Universe*, chapter 1, section "The Universe at Its Smallest: What We Know about Matter") is his figure, told here as his.

An argument about the world is held up as a cable, by many strands from independent sources; a court weighing circumstantial evidence reasons in the form of the cable. *Physics Is Not Math*, chapter 12, "Chains and Cables", section I, "A Chain and a Cable", and section IX, "Summary". The strands of the closed list include the structure of comparison in *The Story of the Particles*, and the two routes of *The Branches of the Kinds* to the absence of any carrier joining a quark to a lepton, one through the sym-

metries of the space of states and one through the algebra of the particles, neither using the other. The two routes in symbols: the isometries PU(3) × SO(3) of the internal space read as a space of states (read "the projective unitary group of degree three times the rotation group of three dimensions"), and the unitaries of the finite algebra (*The Branches of the Kinds*, chapter 2, section I and Box 2.1).

Chapter 7. Why Ten Dimensions?

Kaluza's fifth dimension and Einstein's replies. Greene, *The Elegant Universe*, chapter 8, sections "Kaluza's Idea and Klein's Refinement" and "Unification in Higher Dimensions" (Einstein to Kaluza, 21 April 1919, as quoted in Pais 1982, page 330; the letter about a week later; the letter of 14 October 1921, "I am having second thoughts about having restrained you from publishing your idea on a unification of gravitation and electricity two years ago"). Primary papers: Kaluza, T. (1921), "Zum Unitätsproblem der Physik", Sitzungsberichte der Preußischen Akademie der Wissenschaften zu Berlin, Physikalisch-Mathematische Klasse, 966 to 972; Klein, O. (1926), "Quantentheorie und fünfdimensionale Relativitätstheorie", Zeitschrift für Physik 37, 895 to 906. Klein's size: Greene, same chapter, section "Kaluza's Idea and Klein's Refinement": Klein's calculations "indicated that the additional circular dimension might be as small as the Planck length". The same history in the collection: *At the Edge of Reality*, chapter 3 "Why Ten Dimensions?", section "A Fifth Dimension, Curled Small" (with the garden hose, Greene's picture).

The Planck length. Value: the Planck length l_{Pl} (read "l sub Planck") is about 1.6×10^{-35} meters, about 10^{-33} centimeters, which the chapter says in words as a millionth of a billionth of a billionth of a billionth of a centimeter. *Two Theories, One World*, Key Terms, "Planck length": the bound of every comparison made from a place inside the world, and not a least length. *At the Edge of Reality*, chapter 8, section "The Planck Scale", gives it as about a hundred billion billion times smaller than a proton, with the exact value in its Appendix C.

Greene's ten, the Calabi-Yau shapes, the families, the fluxes, the landscape. Told here as Greene's account: *The Elegant Universe*, chapter 8, sections "More Dimensions and String Theory", "Some Questions", "The Physical Implications of Extra Dimensions", "What Do the Curled-Up Dimensions Look Like?"; chapter 9, section "The Road to Experiment" (tens of thousands of known Calabi-Yau shapes, with 3, 4, 5, 25 and up to 480 holes); *The Fabric of the Cosmos*, chapter 12, section "String Physics and Extra Dimensions" ("at least hundreds of thousands" of shapes; the question and answer on a fatal flaw; "determining the form of the extra dimensions is a prime—and in my opinion attainable—objective"); chapter 13, section "Gravity and Large Extra Dimensions"; *The Hidden Reality*, chapter 4, section "The Dimensions of Space" (the factor written there as "(D − 10) times (Trouble)", with D read "the number of spacetime dimensions"); chapter 5, sections "Sticky Branes and Gravity's Tentacles" and "In Flux" (the flux dressings of Raphael Bousso, Joseph Polchinski, Steven Giddings, Shamit Kachru and others; about 10^{500}, read "ten to the five hundred", forms). Greene's word for the generations is families (*The Elegant Universe*, chapter 9, section "The Road to Experiment": "the patterns of string vibrations will fall into multiple families"). Greene on the number nine of space dimensions, *The Elegant Universe*, chapter 8, section "Some Questions": "no one has an intuitive, nontechnical explanation for the particular number that emerges". The superstring's count, as *The Gravitational Companion*, "Other Programs That Obtain General Relativity", section VIII, gives it: cancellation of the worldsheet central charges gives (3/2)D = 15, so D = 10 (Polyakov 1981b); the internal part at four extended dimensions is a superconformal theory of central charge 9.

M-theory and branes. Greene, *The Elegant Universe*, chapter 12, sections "Duality" (Witten's lecture at Strings '95, which "gave evidence for a new, profound kind of duality") and "A Summary of the Second Superstring Revolution" (M-theory, "tentatively called M-theory", with ten dimensions of space and one of time, and with membranes and three-branes beside strings); *The Fabric of the Cosmos*, chapter 13; *The Hidden Reality*, chapter 4. M-theory and its branes are Greene's terms, told here as his.

The collection records only the eleventh dimension, as a qualification of the count of ten: *The Gravitational Companion*, "Other Programs That Obtain General Relativity", section VIII, "At strong coupling, one of the superstrings has an eleventh dimension (Witten 1995)". Greene's account of branes beyond strings, of the five string theories of 1985, and of the joining of the five into one theory goes beyond that qualification. Told here as Greene's account; the collection gives no reply on it.

Ten dimensions, from comparison: one of time, three of space, six of possibility. *At the Edge of Reality*, chapter 3 "Why Ten Dimensions?", sections "One, Three and Six" and "The Answer of This Book". The six ideas of comparison (the six intrinsics: sameness and difference, content and context, the subjective and the objective), with the thermometer read in the morning and in the evening: *At the Edge of Reality*, chapter 4 "What Every Comparison Contains", section "Six Ideas in Every Comparison" ("There is the comparing, called the subjective, and there is what is compared, called the objective"; "There are six, no more and no fewer"; "A unit, a scale or a number is built from these six and comes after them"). In Chapter 7 I name the third pair by its members, the subjective and the objective, as that section does. At the Edge of Reality, chapter 5, section "The Numbers Comparison Supplies", names the three pairs the verdict, the respect and the stance, and chapter 5, section "The Stance, and What Can Be Followed", says of the third pair: "Together they are the stance from which every comparison is made". A dimension as the number of independent ways in which two findings can differ in a given context, with north-east as a composed way: chapter 5 "What a Dimension Is", section "The Definition". The numbers comparison supplies are 1, 2, 3, 4 and 6, the sizes of the sets of elements in the structure of comparison: the comparing, the members of a pair, the pairs, the followable intrinsics (sameness, difference, content and context, which can be followed apart from the stance), and the six intrinsics (section "The Numbers Comparison Supplies"). The minimality principle, with its reason (section "The Minimality Principle"): "of the numbers comparison supplies, a context takes the least that meets every requirement comparison places on it"; "A fur-

ther way, with nothing to register it as distinct, adds nothing that comparison could find". Time one, from the single stance: chapter 6 "One Dimension of Time, Three of Space", section "Time Has One Dimension". Space not one, because a context has no feature but its form of difference and space must differ from time: section "Space Is Not One". Space three: the doubling (every difference is placed in a context and by a comparing; chapter 5, section "What the Count Can Use": "Every finding is placed twice: it is found in a context, and it is found by a comparing. The context and the comparing are two, and neither is the finding"); the placing rule (the distinction between two ways of a context is placed in that context exactly when the context has a way not composed from those two), section "The Placing Rule" (the flat page is that section's picture); two fails and three holds, and four and six also meet the doubling, section "Two Fails, Three Holds"; three by the minimality principle, section "Three by the Minimality Principle". Possibility six: chapter 7 "Six Dimensions of Possibility", section "Possibility Has Six". The second road to one of time and three of space, from the three transitions and one direction of the cycle of Axiom II: *At the Edge of Reality*, chapter 7, section "Why We Never See Them", paragraph "A Second Road to Ten"; *The Gravitational Passage*, chapter 10, section I ("The total is therefore $4 + 6 = 10$").

The six measures. *At the Edge of Reality*, chapter 4, section "Six Measures" (the content row pattern, force, probability; the context row space, time, possibility; the columns the three roles, the omniscient, the immanent and the transcendent, which the chapter calls the structure, the act and the stance; each content measured in its own context); *The Gravitational Passage* calls the six the six metrics. The reason for the grid's shape, same section: "There are two rows because content and context are a pair, and three columns because every comparison has three roles"; and its first sentence, "When comparison is made in physics, its pairs take a definite form". That the six measures come from the six intrinsics, one measure for each: *At the Edge of Reality*, chapter 7, section "A Space of Possibilities" ("The six measures come from the six intrinsics, one measure for each"). For the act of measurement, *At the Edge of Reality*, chapter 4, section "Three

Roles and Their Order", sets each pair in one role: "sameness and difference are immanent, because the finding is the act; content and context are omniscient, because they are the finding laid out as structure; and the subjective and the objective are transcendent, because they are the stance from which the measurement is made and what it is about".

The identification of the two sixes. *At the Edge of Reality*, chapter 7, section "A Space of Possibilities": "I identify the two sixes as one set"; both sixes are indexed by the same six intrinsics, and a difference that no comparison can register is no difference. The first condition by which the grid is translated into geometry (one direction for each measure) is the identification I state in Chapter 7 as "I give each measure a direction of its own there". The sources are *What the Beginning Left Behind*, chapter 3 "The Shape and Size of the Six", section "From the Grid to a Shape" (where the six small dimensions are "the setting in which the content's comparisons, in all six measures, take place"), and *At the Edge of Reality*, chapter 7, section "The Shape of the Six". In Chapter 7 I write "context" for the "setting" of What the Beginning Left Behind.

The six as a context and as content. *At the Edge of Reality*, chapter 7, section "A Space of Possibilities": "The one set plays two roles, and they do not conflict. Content and context are assigned within each comparison. In physics the internal space stands on the side of content, held in the large world of space and time as its context. Here possibility is a context, in which each finding is the content".

The two tens count different things. *The Gravitational Companion*, "Other Programs That Obtain General Relativity", section VIII, paragraph "Ten": the superstring's ten solves $(3/2)\mathrm{D} = 15$ from worldsheet central charges; the ten of *The Gravitational Passage* is $4 + 6$ from the six metrics of comparison and the cycle of Axiom II; no quantity of either count enters the other; the string's count does not fix the split into 4 and 6, which "is chosen to match observation"; and $\mathrm{M}^4 \times \mathbb{C}\mathrm{P}^2 \times \mathrm{S}^2$ (read "four-dimensional spacetime times the complex projective plane times the two-sphere") is not a vacuum of the superstring at leading order, since $\mathbb{C}\mathrm{P}^2$ admits no spin

structure (Hawking and Pope 1978), its Ricci curvature is positive, and with a de Sitter external factor two-derivative supergravity has no compactification (Maldacena and Núñez 2001). The ten of three kinds: *Measurement*, chapter 11 "The Ten-Dimensional Context", section XII "Questions" ("Four are traversed by located comparers, and six are met only as what could be found").

Multiplicity as the difficulty predicted for string theory's first step. *The Unquantized Arena*, chapter 14 "Four Criteria and Ten Programs", section III "The Three Predicted Difficulties", Proposition 14.1(a) ("A program that starts from a discontinuously symmetric framework and requires the arena to follow from it meets a difficulty of multiplicity: many arenas fit the framework, and no principle within the framework selects one"), with its argument from the Incommensuration Theorem; section IV "The Ten Programs", entry "1. String theory" ("The predicted difficulty is of multiplicity, and the record shows it"; estimates of flux vacua about 10^{500}, Douglas 2004); *Two Theories, One World*, chapter 22 "What Each Program Found", section "String theory" ("A discrete starting point, however rich, has fixed no single smooth world; that is what multiplicity means"). The chapter's two sentences of reason are the argument of Proposition 14.1(a) in plain words: "a discontinuously symmetric framework fixes invariants, discrete labels that are the same in every context. A continuously asymmetric arena is specified by data that vary without repetition from place to place"; "So the map from the framework to arenas is many-valued, and the selection of one arena must come from outside the framework". The step between them in that argument rests on the Incommensuration Theorem: for the framework to fix one arena, its findings "would have to carry the arena's continuous asymmetry perfected together with their own discontinuous symmetry, in the same respect, which P9 excludes" (P9 there is the theorem).

Duality as one system with two descriptions. Greene's definition: *The Elegant Universe*, chapter 12, section "Duality". *Two Theories, One World*, chapter 23 "Two Descriptions That Translate", section "Neither Side Comes First" ("A duality that is exact in both directions does not privilege

either side"; "Emergence is a one-way relation"; "If one thing emerges from another, the second is the more fundamental of the two"; "This is correspondence, not emergence"), on the correspondence of Maldacena (1998, received 1997); the translation of a poem is the picture of section "What the Correspondence Shows", used in Chapter 5 of this book with the same meaning.

The six are never traversed; they present as compact and closed. *At the Edge of Reality*, chapter 7, sections "Why We Never See Them" and "What This Means for You" (the next room, yesterday's diary, the outcome that did not happen). *What the Beginning Left Behind*, chapter 1 "Where the Last Book Left Us", section "One Space, Taken Two Ways" ("here the six are not directions of space, and nothing ever travels through them"; and on the word for the holding: "so the word does not describe six directions of space that were once spread out and then curled up"). *The Compactification*, Preface, section I "The Subject", and chapter 1 "The Re-Forming of the Internal Space", section II "Ten Dimensions of Three Kinds, and the Two Instances" ("The six were never directions of space, and the account never has them traversed"): "compactification" there means the six of the context instance held compact as content of the large four-dimensional factor taken as their context. "Closed" in the chapter is the compact of the collection: finite, with no edge (*At the Edge of Reality*, chapter 3, section "A Fifth Dimension, Curled Small"). The sculptor's block and its grain: *At the Edge of Reality*, chapter 3, section "What This Means for You".

The extent of the six shows only through its effects, among them the strength of gravity; no bulk for a brane. *At the Edge of Reality*, chapter 7, section "Why We Never See Them" ("Their extent shows only through its effects, among them the strength of gravity, which is tied to the volume of the internal space"). *What Comes After the Dark*, chapter 11 "Has This Happened Before?", section "Colliding Worlds" ("So no brane has anywhere to drift"; the ekpyrotic universe of Khoury, Ovrut, Steinhardt and Turok 2001 and the cyclic model of Steinhardt and Turok 2002 are set out there).

The moduli: what they are, and their masses. A modulus as a number that sets the size or the shape of the six and that, varying from place to place, would show in four dimensions as a field: *What the Beginning Left Behind*, chapter 5 "One Vacuum", section "Two Readings, Two Meanings of the Word". *The Compactification*, chapter 2 "One Vacuum", section V "The Floor": moduli masses 2.25 M_{Pl} (mostly size) and 3.09 M_{Pl} (mostly shape), with M_{Pl} (read "M sub Planck") the full Planck mass, 1.221 × 10^{19} GeV, about 22 micrograms, from a first-order computation of one mechanism at the boundary; at the derived point the stable minimum is the only one in shape for ratios of the radii between $\sqrt{3}$ and $\sqrt{12}$, with its barrier at $R_O = 1.79\ l_{Pl}$; masses of two to three Planck masses give the moduli's force a range shorter than the Planck length, about thirty orders of magnitude below the shortest range any test reaches. *What the Beginning Left Behind*, chapter 5, section "One Point Holds Both": "The hold rests on the argument; the computation shows one way in which the hold is stiff". The Planck mass and the grain of sand: *At the Edge of Reality*, chapter 8, section "The Planck Scale".

Past measurements of gravity at short range, of a long-range scalar force, and of the count of dimensions. *The Compactification*, chapter 2, section V (a modulus of small mass would carry a long-range scalar force, excluded by the Cassini measurement at the strength such a modulus would carry): Kapner, D. J., Cook, T. S., Adelberger, E. G., Gundlach, J. H., Heckel, B. R., Hoyle, C. D. and Swanson, H. E. (2007), "Tests of the gravitational inverse-square law below the dark-energy length scale", Physical Review Letters 98, 021101, https://doi.org/10.1103/PhysRevLett.98.021101 (the law holds down to about 56 micrometers, about a twentieth of a millimeter); Bertotti, B., Iess, L. and Tortora, P. (2003), "A test of general relativity using radio links with the Cassini spacecraft", Nature 425, 374 to 376, https://doi.org/10.1038/nature01997 (the post-Newtonian parameter γ, read "gamma", equals $1 + (2.1 \pm 2.3) \times 10^{-5}$). *Side by Side, Volume One*, section "How Many Dimensions Has Space?": Pardo, K., Fishbach, M., Holz, D. E. and Spergel, D. N. (2018), "Limits on the number of spacetime

dimensions from GW170817", Journal of Cosmology and Astroparticle Physics 2018(07), 048, https://doi.org/10.1088/1475-7516/2018/07/048 (4.02 and 3.98 dimensions of spacetime).

Three large because the context factor has three directions of space. *The Gravitational Companion*, "Comparisons at Single Steps", section III "Universes Inside Black Holes, and Strings Grown Large" ("the number of large spatial directions is the number of spatial directions of the context factor (Chapter 10, Section IV)"); *At the Edge of Reality*, chapter 8, section "No Content Without a Context" (the six are compact as content of a world of four large dimensions), and section "How Small the Six Are", of the four directions of space and time: "They are large, they have no edge in any direction we can reach, and they make up the spacetime we observe". Greene's account of Brandenberger and Vafa (*The Elegant Universe*, chapter 14, section "Why Three?") is told here as Greene's account. *The Gravitational Companion*, "Comparisons at Single Steps", section III, and *At the Edge of Reality*, Notes, Part IV, name the model and set beside it the reason of this account for three large directions, the number of directions of space of the context factor ("in this account three is the number of directions of space of the large factor", *At the Edge of Reality*, Notes, Part IV). The primary paper is Brandenberger, R. and Vafa, C. (1989), "Superstrings in the early universe", Nuclear Physics B 316, 391 to 410.

One internal space, $\mathbb{C}P^2 \times S^2$. *At the Edge of Reality*, chapter 7, section "The Shape of the Six" (three identifications translate the grid into geometry; with homogeneity, the Euler characteristic χ, read "chi", is 3 for the piece carrying the three roles and 2 for the piece carrying the two rows; the only compact homogeneous surface with $\chi = 2$ is S^2, the only compact homogeneous four-dimensional space with $\chi = 3$ is $\mathbb{C}P^2$); section "A Space of Possibilities" ($\mathbb{C}P^2$ is the space of states of a three-way alternative, S^2 of a two-way alternative; Wootters 1981 on the natural distance). The source is *What the Beginning Left Behind*, chapter 3, in five sections. Section "A Lamp and a Globe" gives the lamp found red, green or blue, and the tin can as a product. Section "From the Grid to a Shape" gives the three con-

ditions, each an identification. Section "Counting Pieces" gives the Euler count, the surface of a ball as a point and a disc drawn shut, and, for shapes built only from even-dimensional pieces, the count as the number of pieces. Section "Every Point Alike" gives homogeneity, a fourth condition and an identification, with the reason for homogeneity: "the setting in which content is compared cannot itself introduce an unevenness: an unevenness of that setting would be a difference of context that no content carries". In Chapter 7 I write "context" for "setting". Section "The List Leaves One" gives four results. Odd-dimensional closed shapes have count zero. A closed surface has count at most two, so the share is four and two. The sphere is the only homogeneous closed surface with count two. The complex projective plane is the only homogeneous closed four-dimensional shape with count three. *The Gravitational Passage*, chapter 9 "The Internal Space and Its Uniqueness", opening ("The curvature of $\mathbb{C}P^2 \times S^2$ is positive, so it is not a Calabi–Yau space") and section VIII "The Curvature Is Positive": the first Chern class does not vanish, so it admits no Ricci-flat Kähler metric; $\mathbb{C}P^2$ with the Fubini-Study metric has $R_{ij} = 6g_{ij}/R_O{}^2$, read "the Ricci tensor equals six times the metric divided by the square of the radius". *The Gravitational Companion*, "Comparisons at Single Steps", section I "String Theory" ("Where string theory has many internal geometries, the Passage has one".) and section II "Witten's Spaces" (Witten 1981: the seven-dimensional spaces M^{pqr} are circle bundles over $\mathbb{C}P^2 \times S^2$).

Two routes to the one space. *Non-Commutative Geometry and the Immanent Metaphysics*, chapter 12 "The Algebra of Comparison Is $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$", section I "The Main Theorem", Theorem 12.1 (read "the complex numbers, plus the quaternions, plus the three-by-three complex matrices"); *The Gravitational Passage*, chapter 9 (the classification of compact homogeneous spaces). *What the Beginning Left Behind*, chapter 5 "One Vacuum", section "One Algebra, One Space" (the two climbers; the theorem of *Non-Commutative Geometry and the Immanent Metaphysics* is proved for a structure of comparison meeting its postulates, "each of which is an

identification", and "The theorem holds given its postulates"; the states of the three-by-three tables form $\mathbb{C}P^2$, the states of the quaternion piece form S^2); *The Compactification*, chapter 2, Abstract ("One algebra and one internal space, reached by two routes, leave no landscape and no selection problem"), and section IX "One Algebra, One Space". The postulates, as *Non-Commutative Geometry and the Immanent Metaphysics*, "How to Read This Book", defines them: "Postulates are the identifications that the theorems use as hypotheses; each gives a clause of the Immanent Metaphysics the form of a condition on the triple"; in the chapter "The Three-Carrier Theorem", "Two postulates translate the axioms". (P-III reads Axiom III, and P-I reads the conjugacy clause of Axiom I).

What the six hold: two scopes. (i) *At the Edge of Reality*, chapter 8, section "How Small the Six Are": the small factor holds the possibilities that content can take, "as the shape and size of a drum decide which notes it can sound". (ii) *What the Beginning Left Behind*, chapter 1, section "One Space, Taken Two Ways" (the clock face; gravitation, the cosmological constant and the dark matter rest on the context instance); *The Kinds of Particle*, chapter 7 "The Three Generations", section II "What the Geometry and the Algebra Do Not Produce" ("The content instance carries no field and no Kaluza-Klein modes"); *The Compactification*, chapter 1, section II: on the content instance $\mathbb{C}P^2 \times S^2$ is a space of states with no size, no radius and no fields, and the kinds and values of the particles are read there; on the context instance it has radii, and gravitation, the cosmological constant and the dark matter rest on it. The three things the chapter names on the context instance: the strength of gravity tied to the volume of the internal space (*At the Edge of Reality*, chapter 7, section "Why We Never See Them"; *What the Beginning Left Behind*, chapter 5, section "Two Readings, Two Meanings of the Word", the volume modulus setting the Planck mass as measured in four dimensions); the vacuum's energy summed up to the Planck scale, the Planck density, less the bare energy of the curvature of the six: $1 - 3/(8\pi)$, about 0.8806 of the Planck density, which *What the Beginning Left Behind* calls "the energy of empty space at

the boundary" and this book's Chapter 10 calls the trim (*What the Beginning Left Behind*, chapter 3, section "One Step Sets the Size", and chapter 5, section "One Point Holds Both"); the dark matter made of gyrations, conserved excitations of the six held internal dimensions (*The Weight of the Unseen*, chapter 3).

Three generations. That every particle is a comparison: *The Kinds of Particle*, chapter 7, section I ("Every particle is a comparison and so carries three pairs of aspects"); the Root Tautology, "Comparison, interaction and relation are one act, approached three ways": *At the Edge of Reality*, chapter 4, section "One Act, Three Names". The order of the three roles (stance, then act, then structure, which shapes the next stance): *At the Edge of Reality*, chapter 4, section "Three Roles and Their Order", and Axiom II. *The Kinds of Particle*, chapter 7 "The Three Generations", section I "Three Rotations of the Content": three positions in the directed cycle of Axiom II admit three rotations of the content, and a fourth is the first again; bounded above (of the six ways the three pairs can stand at the three positions, the three rotations of one assignment keep the direction of the cycle, and each of the other three exchanges two pairs and runs the cycle backward) and below (Axiom III holds of instances, so no pair is left never foremost); section II: the Euler characteristic $\chi(\mathbb{C}P^2) = 3$ and the index of the Dirac operator (one) do not set the count. Measurement: *The Kinds of Particle*, chapter 7, Chapter Overview, and *Side by Side, Volume One*, Appendix B, Prediction 31, "Measured": N_ν (read "N sub nu", the number of light neutrino species) $= 2.9963 \pm 0.0074$; Janot, P. and Jadach, S. (2020), "Improved Bhabha cross section at LEP and the number of light neutrino species", Physics Letters B 803, 135319, https://doi.org/10.1016/j.physletb.2020.135319; the original combination is ALEPH, DELPHI, L3, OPAL, SLD Collaborations and others (2006), "Precision electroweak measurements on the Z resonance", Physics Reports 427, 257 to 454, https://doi.org/10.1016/j.physrep.2005.12.006.

One vacuum: two moduli, one held point, no flux. *What the Beginning Left Behind*, chapter 5 "One Vacuum", sections "A Ball Among Valleys", "Only Two Settings" (homogeneity, an identification, with a theorem of geometry: a shape that is the same at every point, and the same in every direction at a point, admits only one shape of measuring rule up to its size, so the product has two moduli, the volume modulus and the shape modulus), "One Point Holds Both" (the Einstein condition fixes $R_O = \sqrt{6}\ R_S$, read "the radius of the complex projective plane is the square root of six times the sphere's radius"; the identification that every direction reaches the Planck curvature fixes $R_S = 1\ l_{Pl}$ (chapter 3, section "One Step Sets the Size": "The amount of bending is set by one identification"); the modal boundary is set by Newton's constant, so "the internal space has the same size and the same shape at every point of the exterior"), "No Threads to Count" (the gravitational action on the ten-dimensional product "is built from the metric alone"), and the key idea ("String theory's threaded fields give its six some ten to the five hundred valleys; the six of this account have one"). The radius of the complex projective plane, R_O (read "R sub O"), is the radius of the larger sphere from which $\mathbb{C}P^2$ is built; measured without that convention, each sphere of two of the three alternatives inside $\mathbb{C}P^2$ has one and a half times the area of S^2 (*What the Beginning Left Behind*, chapter 3, section "One Step Sets the Size"; *At the Edge of Reality*, chapter 8, section "How Small the Six Are"). *The Compactification*, chapter 2, sections III "Why Two Moduli", IV "The Einstein Point and the Ratio $\sqrt{6}$", VI "No Flux, No Flux Integers" ("So the account has no flux sectors, and the landscape of flux vacua has no counterpart in it"). *What Comes After the Dark*, chapter 4 "Could Empty Space Change Its Nature?", section "No Threads, No Landscape" (the ring with nothing wound through its hole).

Chapter 8. Why Does Time Have a Direction?

The opening scene: Loschmidt's reversal, 1876, and Boltzmann's count, 1877. Loschmidt's argument is published as J. Loschmidt, "Über den Zustand des Wärmegleichgewichtes eines Systems von Körpern mit Rücksicht auf die Schwerkraft", *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften, Wien, Mathematisch-Naturwissenschaftliche Classe* 73 (1876), 128 to 142. *What Cannot Be*, Chapter 13, "Why Time Has a Direction", section "Why the Equations Hide the Gap", tells it ("In 1876 Josef Loschmidt raised an objection that is an early sign of the same gap, the gap between lawful reversal and faithful reversal"), with the picture of the gas crowded in one corner and every velocity reversed. Boltzmann's answer by counting is L. Boltzmann, "Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung, respektive den Sätzen über das Wärmegleichgewicht", *Sitzungsberichte* 76, Abt. II (1877), 373 to 435. *What the Beginning Left Behind*, Chapter 6, "Two Arrows of Time", section "Counting the Ways", tells it ("The standard answer was given by Ludwig Boltzmann in 1877, and it rests on counting").

Greene's telling of the arrow. In Chapter 8 I tell the arrow in Greene's terms. Those terms come from *The Fabric of the Cosmos*, chapter 6, "Chance and the Arrow" (sections "The Puzzle", "Time-Reversal Symmetry", "Tennis Balls and Splattering Eggs", "Principle and Practice", "Entropy", "Entropy, the Second Law, and the Arrow of Time", "Entropy: Past and Future", "Following the Math", "A Quagmire", "Taking a Step Back", "The Egg, the Chicken, and the Big Bang", "Entropy and Gravity", "The Critical Input", "The Remaining Puzzle") and notes to chapter 6, note 2; from chapter 7, section "Quantum Mechanics and the Arrow of Time"; and from *Until the End of Time*, chapter 2 (sections "From This to That" and "Boltzmann and the Big Bang", and note 7) and chapter 3 (section "The Big Bang and the Second Law", and note 7). In *Until the End of Time*, notes to chapter 2, note 7, Greene adds that for more exotic systems the re-

versed sequence is allowed only together with charge conjugation and parity reversal: "The laws of physics, as currently understood, necessarily respect the conjunction of all three of these reversals, something known as the CPT theorem (with C standing for charge conjugation, P for parity reversal, and T for time reversal)". In *Until the End of Time*, notes to chapter 3, note 7, Greene names the past hypothesis after David Albert ("one named the past hypothesis by philosopher David Albert"). In Chapter 8 I write the name with capitals, as the collection does. Greene's tombstone formula, entropy equals k times the logarithm of W (k Boltzmann's constant, W the number of arrangements), is in *The Fabric of the Cosmos*, chapter 6, section "Entropy". Greene's gravitational step (when gravity matters, clumping is the state of high entropy) is told here as Greene's account; *What the Beginning Left Behind*, Chapter 7, "A Clean Slate, Counted", section "Penrose's Two Findings and One Proposal", reaches the same result as established physics and as Penrose's finding, and Chapter 9 of this book takes it up.

The laws allow lawful reversal (the same result as Greene's). *What the Beginning Left Behind*, Chapter 6, sections "A Film Run Backward" ("The equations of mechanics, of electromagnetism and of general relativity allow, for nearly every lawful process, the reversed process as lawful too".) and "Lawful and Faithful" ("A reversed process that obeys the laws is a lawful reversal").

Entropy is a count relative to a context. *What the Beginning Left Behind*, Chapter 6, section "Counting the Ways" (the deck of cards; "the entropy of a thing is a count of its states that a context cannot tell apart"), and Chapter 7, section "Every Count Has a Context" ("A change of context is a change of what is counted, and a count made relative to one context is not a count relative to another"). In symbols (*The Matter We Know, Volume Two*, Chapter 2, "The Counting of Ways", Section III): $S = k \ln W$, read "the entropy S equals Boltzmann's constant k times the natural logarithm of W, the number of microstates that no available measurement tells

apart". Greene's statement of the physics definition, quoted in the chapter in one word, is in *The Fabric of the Cosmos*, chapter 6, section "Entropy" (the rearrangements that leave the "big-picture" properties unchanged).

The residual entropy of ice (a past measurement). *The Matter We Know, Volume Two*, Chapter 2, Section XVI, "The Coldest State". Pauling's count: with two hydrogens near each of N oxygens, the number of placings is 2 to the power 2N times (6/16) to the power N, which is (3/2) to the power N; the residual entropy is $N_A k \ln(3/2)$ (Avogadro's number times Boltzmann's constant times the natural logarithm of three halves), 3.37 joules for each kelvin and each mole. Giauque and Stout measured 0.82 ± 0.05 calories for each degree and each mole, which is 3.4 ± 0.2 joules for each kelvin and each mole. In Chapter 8 I state the agreement in words ("within the precision of their work").

The counting alone runs both ways; a condition at one end, the Past Hypothesis (the same result as Greene's). *What the Beginning Left Behind*, Chapter 6, section "The Gap in the Counting" ("From any given moment, the same counting predicts higher entropy in the future, and also in the past"; "decks leave the factory in order"; "In 2000 the philosopher of physics David Albert called that condition the Past Hypothesis"; "The gap and the condition that fills it are established physics: the standard statement of where the statistical account gets its one direction"). Albert's book: D. Z. Albert, *Time and Chance* (Harvard University Press, 2000).

The entropic arrow needs an orderly beginning (the same result as Greene's). *Physics Is Not Math*, Chapter 8, "Physics Has the Arrow", Section II, "The Arrow of Time" ("That arrow is a matter of very many parts, and it needs a beginning in which things were less mixed than they are now"); *What the Beginning Left Behind*, Chapter 6, section "The Arrow of Many Parts" ("That the entropic arrow is statistical, and that it needs a beginning of low entropy, is established physics").

Lawful reversal and faithful reversal. *What Cannot Be*, Chapter 13, section "Two Ways to Run Backward" ("Symmetry of the laws guarantees lawful reversal. It does not guarantee faithful reversal"; a faithful reversal

needs "a description of the state with no gaps and no ambiguities, one that leaves no part of the system unspecified and free to go its own way", which is continuity in the theorem's sense). The same distinction is quoted from *The Basis of Physics*, Essay 11, in *Side by Side, Volume Two*, the Prigogine chapter, Section IV. In Chapter 8 I write "a state that fixes all of these motions exactly" for the section's "A full specification of the state, with no gaps and no ambiguities".

The Incommensuration Theorem, stated plainly. *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". The four characters (aphorism 1.43-1): continuity, a sameness of content with a sameness of context; discontinuity, a difference of content with a sameness of context; symmetry, a sameness of content with a difference of context; asymmetry, a difference of content with a difference of context. *What the Beginning Left Behind*, Chapter 6, section "Lawful and Faithful", names the theorem's standing in these words: "The theorem is a structural fact about the nature of knowing". In Chapter 8 I state that the theorem "holds of every comparison, and so of anything any comparison can find", after the same section's "the theorem concerns anything any comparison can find".

A perfect time-reversal is ruled out. *What the Beginning Left Behind*, Chapter 6, sections "Lawful and Faithful" and "Summary" ("So a perfect time-reversal is ruled out; this is a consequence of the theorem, and it must be so"); the limit "is one of principle, and therefore of all practice" (section "Lawful and Faithful"). *The Basis of Physics*, Essay 11, "The Structural Arrow of Time", Abstract: "Perfect time-reversal is therefore precluded, and the preclusion depends on no entropy, no boundary condition, no Past Hypothesis, and no particular formalism".

Loschmidt's gas answered. *What Cannot Be*, Chapter 13, section "Why the Equations Hide the Gap": Boltzmann's answer "concedes, without saying so, that what keeps the gas from gathering itself back is the need to specify the state with every velocity exactly reversed, and not the symmetry of the laws"; "The specification of the state cannot be perfected, be-

cause perfecting it alongside the symmetry of the laws would perfect continuity and symmetry together, the pair the theorem forbids"; "The reversed path exists in the mathematics, as a solution of the laws; a gas in the faithfully reversed state never occurs in the world".

The measured violation of time symmetry in the weak interaction (a past measurement). *The Basis of Physics*, Essay 11, Section VII, "Consistency with Physical Law": the laws show time-reversal symmetry "to an extraordinary degree"; CP violation (Christenson, Cronin, Fitch and Turlay, 1964) entails T violation by the CPT theorem, and T violation has been observed directly (CPLEAR, 1998; BaBar, 2012). CP is charge conjugation (the exchange of matter for antimatter) combined with parity (mirror reflection); T is time reversal; the CPT theorem holds that the three together are an exact symmetry of any local quantum field theory (same essay, "On CPT and the Structural Arrow"). The same section states the reach of the argument: "The argument says that some asymmetry must exist. It does not say how large, in which interaction, or through what mechanism". The step from the theorem to the laws: *What Cannot Be*, Chapter 13, section "What Physics Already Shows" (the reason given is the theorem's words "absolutely and together"; "Within any finite domain, symmetry and continuity are both present, each to a finite degree"; "What the argument says is that the symmetry cannot be perfect"), with the same reading of the kaon measurements ("and I read the known physics as an instance of the argument"; Summary: "and I read it as an instance of the requirement; its size is found by measurement"); *The Basis of Physics*, Essay 11, Section VII ("There is a residual asymmetry: small, difficult to measure, but structurally required"). *Side by Side, Volume One*, part "Where the Accounts Part", Section XIII, "Where Does the Arrow of Time Come From?" ("The laws are symmetric in time to an extraordinary degree and never perfectly"; "The measured violation of the symmetry of time in the weak interaction stands with the collection's claim that the symmetry cannot be perfect"). Greene reports CPLEAR and KTeV at Fermilab in *The Fabric of the Cosmos*, notes to chapter 6, note 2, with his judgment: "To many physicists, including me, it seems unlikely that the time nonreversal

invariance evidenced by these particles plays a role in answering the puzzle of time's arrow, so we shall not discuss this exceptional example further. But the truth is that no one knows for sure". Appendix A sets these measurements beside the statement that no world is purely and completely deterministic.

The spin echo, and the second law in two cases (a past measurement; the same result as Greene's for an isolated system, a stronger result for an open one). *The Matter We Know, Volume Two*, Chapter 2, Section V, "Why Entropy Does Not Fall": when two particles strike, "each comes to carry a trace of the other: from then on, the state of the one is tied to the state of the other"; the relations of phase among the first particle's alternatives pass into the joint state of the two, which is decoherence. The runners and the pistol shot; Hahn (1950), Carr and Purcell (1954), Rhim, Pines and Waugh (1971). For a system open to its surroundings, "the entropy of the system together with what has left it never falls, with no exception". For an isolated system, a measurement would have to act on every particle the traces have reached and restore the relations among all their joint alternatives, whose number "grows as an exponential of the number of particles the traces have reached"; "A return is found only where the traces have reached few particles, and never where they have reached many". In the same section of *The Matter We Know, Volume Two*, I then name the law: "This is the second law of thermodynamics: the entropy of an isolated system does not decrease". The chapter's "every way those molecules can stand together" is the section's joint alternatives, and the deck's orders (each card added multiplies their number) stand for the exponential growth.

The order of every comparison, and the direction of time. The second axiom, in the words of *An Immanent Metaphysics* as *What the Beginning Left Behind*, Chapter 6, section "Position, Click, Print", quotes it: "Axiom II. A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". The gloss of "axiom" is Chapter 6's (the basic statements from which the rest of the

Immanent Metaphysics is worked out). The transcendent is the role the chapter calls the stance, the immanent the act, and the omniscient the structure, of which a record is one instance; a class is a type standing ready as a whole, and an instance one particular case of it, here and now. The identification of the one-way order of findings with the direction of time is made in Chapter 2 of this book. In Chapter 8 I recall that identification. Its sources: *What the Beginning Left Behind*, Chapter 6, section "The Direction in Which Comparison Proceeds" ("I identify the one-way order of findings, the order of Axiom II, with the direction of time"), with the observation that every record ever found follows the act it records; *Incommensuration*, Chapter 8, Section X, "Many-to-One, the Arrow of Time and the Bound Triple" ("The non-entropic arrow of time is the structural precedence of Axiom II applied to the physical domain"); *Physics Is Not Math*, Chapter 8, Section II ("This order needs no large number of parts and no special beginning of the world: it is present in one comparison, made once"). *The Basis of Physics*, Essay 11, Section VI, holds the direction grounded and not primitive.

The structural arrow, reached by two routes, and the entropic arrow as its showing at our scale. *What the Beginning Left Behind*, Chapter 6, section "The Gap in the Counting", states that the structural arrow "is reached by two routes, from two different starting points: one through the Incommensuration Theorem, and one through the order in which every comparison proceeds". The same chapter, section "The Direction in Which Comparison Proceeds": "The structural arrow of time is the impossibility of a faithful time-reversal, which the Incommensuration Theorem requires, together with the direction of the cycle of Axiom II, identified with the direction of time". *What Cannot Be*, Chapter 13, section "An Arrow Beneath the Arrow of Heat" ("The entropic arrow is how the structural arrow shows itself at our own scale, in systems of many parts, where reasoning by chances applies"). The name, same section: "I call it the structural arrow"; "The structural arrow supplies a one-way character that the

structure of comparison requires and that nothing can remove". In Chapter 8 I say in words that "structural" there names how every comparison is built, and not the third role alone.

Quantum mechanics and the arrow. *Measurement*, Chapter 12, "Time", Section VIII, "Questions" ("The evolution is deterministic in the formalism, which exists as structure, and not in the findings, which are real"; "So the determinism of the formalism is the symmetry of the law at its own level, and the findings are causal and not determined"). *Side by Side, Volume One*, part "Where the Accounts Part", Section XIII: the arrow lies "in the order of findings: no reversal of evolution removes a finding from that order". The deterministic equation is Schrödinger's equation, which carries the state forward and is unchanged under time reversal combined with complex conjugation. Two further passages of the collection bear on the same point: *The Matter We Know, Volume Two*, Chapter 2, Section VI ("So the arrow of the count is the order in which every trace follows the act that makes it, counted over very many particles".) and *Why the Quantum Is the Way It Is*, Chapter 17, section "Why Collapse Is Not Physical" ("The cycle has a direction, and so the passage it describes cannot be reversed").

Every record joins the two arrows. *Measurement*, Chapter 12, Section VII, "The Two Arrows" ("Every record follows the act it records, and so does every increase of entropy that a record registers"; "So each act of recording joins the two arrows in one event: the order of the act, and a growth of entropy in the record"); the second sentence also in *Physics Is Not Math*, Chapter 8, Section II; *What the Beginning Left Behind*, Chapter 6, section "Where the Two Arrows Meet". Greene's word "quagmire" is in *The Fabric of the Cosmos*, chapter 6, section "Taking a Step Back", and again in section "The Remaining Puzzle".

The Past Hypothesis is a condition on the entropic arrow. *What the Beginning Left Behind*, Chapter 6, section "What the Past Hypothesis Is For" ("The Past Hypothesis is a condition on the entropic arrow", with *The Basis of Physics*, Essay 11, quoted there: it "is not the source of the ar-

row but a statement about initial conditions"); *What Cannot Be*, Chapter 13, section "An Arrow Beneath the Arrow of Heat" ("The Past Hypothesis is not the source of the arrow"). The same section of *What the Beginning Left Behind* states the reach of the structural arrow: "The structural arrow fixes one thing: that time has a direction".

Where the Past Hypothesis stands. *The Compactification*, a technical book of this collection, states where the condition stands once that answer is given: the Past Hypothesis is "neither an axiom nor a brute law added to physics", because the low entropy of the beginning has a reason of its own, which belongs to the next chapter. The passage is *The Compactification*, Chapter Three, "The Past Hypothesis at the Turn", Section VIII, "The Magnitudes", subsection "Where the Past Hypothesis stands". ("The Past Hypothesis is then, on this account, neither an axiom nor a brute law added to physics"), resting on the subsection "The improbability estimate". (the low gravitational entropy of the beginning "is the count that a new context starts with"). A brute law is a law taken as given, with no reason behind it. The reason, the reset at the turn, is set out in Chapter 9 of this book.

The chapter's result. *What the Beginning Left Behind*, Chapter 6, section "What This Means for the Beginning" ("We now know that the direction of time does not depend on the beginning of the world; only the growth of entropy does").

Chapter 9. What Banged?

The microwave background and its temperature. *At the Edge of Reality*, Chapter 2, "What Came Before the Big Bang?", section "The Afterglow"; *What the Beginning Left Behind*, Chapter 11, "The Oldest Light", sections "The Light That Reaches Us" and "What the Measurements Find". The temperature today is $T_0 = 2.72548$ K (T-nought, the temperature of the glow now; Fixsen 2009); the redshift of last scattering is $z = 1089.92 \pm$

0.25 (z, the factor by which distances have grown, less one), so the light at last scattering was at 2.72548 × 1090.92 ≈ 2,973 K. Penzias and Wilson's excess was about 3.5 K at 4080 Mc/s (Penzias and Wilson 1965).

The expansion: Friedmann and Hubble. *What Comes After the Dark*, Chapter 2, "Will It All Fall Back?", section "A Letter from Petrograd" (Friedmann's letter of 3 June 1922; his paper received 29 June 1922; Einstein's notes received 18 September 1922 and 31 May 1923); Chapter 1, "What Happens to the Sky We Can See?", section "Twenty-Four Islands" (Hubble, January 1929: 24 galaxies with distances, 46 with radial velocities; a roughly linear relation of velocity to distance).

Greene's cosmic time from the homogeneity of space. *The Fabric of the Cosmos*, chapter 8, section "Symmetry and Time". Told here as Greene's account; the collection gives no reply on it.

Greene's repulsive gravity from negative pressure. *The Fabric of the Cosmos*, chapter 10, section "Einstein and Repulsive Gravity": pressure, like mass and energy, is a source of gravity, and a pressure that is negative enough makes gravity repulsive. The collection states the equation of state w (the ratio of pressure to energy density) and the value $w = -1$ for the energy of empty space (*What the Beginning Left Behind*, Chapter 8, section "Fixed Once"). *What Comes After the Dark*, Chapter 3, "Will the Expansion Tear Us Apart?", section "What a Rip Needs", states the same result as Greene for the energy of empty space: a pressure of minus one times the density is a tension, and a tension that fills all of space makes the expansion speed up. Greene's general claim, that pressure gravitates, is argued in no book of the collection and disputed in none. Told here as Greene's account; the collection gives no reply on it.

Greene's inflation: expansion by a factor of 10^{30} or more in about 10^{-35} s; the twenty-pound nugget of inflaton field; the magnetic monopole problem. *The Fabric of the Cosmos*, chapter 10, sections "Of Jumping Frogs and Supercooling" and "Inflation"; chapter 11, sections "Creating a Universe" (a nugget about 10^{-26} cm across, weighing about twenty pounds) and "Boltzmann Redux". Told here as Greene's account; the col-

lection does not treat the energy bookkeeping of inflation, and its account has no inflaton (*What the Beginning Left Behind*, Chapter 13, "No Inflation Needed", section "A Patch of Dough the Size of a City").

Greene's string cosmology by name: the Brandenberger and Vafa model and the pre-big bang scenario of Gasperini and Veneziano. *The Elegant Universe*, chapter 14, sections "Cosmology and Superstring Theory", "Why Three?" and "Before the Beginning?". *The Gravitational Companion*, "Comparisons at Single Steps", Section III, and *At the Edge of Reality*, Notes, Part IV, name the Brandenberger and Vafa model (1989) and set beside it the account's reason for three large directions (the number of directions of space of the context factor). The second model, the pre-big bang scenario of Gasperini and Veneziano, is named in no book of the collection. Told here as Greene's account; the collection gives no reply on it.

COBE: evenness to one part in a hundred thousand. *At the Edge of Reality*, Chapter 2, section "The Afterglow"; *What the Beginning Left Behind*, Chapter 11, section "A Faint Pattern": $\Delta T/T \approx 10^{-5}$ ($\Delta T/T$, the difference of temperature from place to place divided by the temperature), about 27 μK on 2.7255 K (Smoot and others 1992).

For gravity, clumping raises the entropy; the low entropy of the beginning was the entropy of gravity. *What the Beginning Left Behind*, Chapter 7, "A Clean Slate, Counted", section "Penrose's Two Findings and One Proposal"; Chapter 10, "The Counts of an Epoch", section "Gravity Turns the Order Around". The content (light and neutrinos) carries a count of about 10^{90} in the region now within the particle horizon; one black hole of four million solar masses has about 1.7×10^{90}; all the black holes of the observable universe hold about 10^{104} (computed in *The Compactification*).

The light elements. *The Coming of the Kinds*, Chapter 12, "The First Nuclei", Chapter Overview and Section I, "A Quarter of the Matter". About two minutes into the history about one neutron is left for every seven protons, and the reactions that start from the deuteron carry nearly every neutron into helium-4. For x neutrons per proton the helium-4 mass

fraction is $Y = 2x/(1 + x)$ (Y, the share of the mass of ordinary matter in helium-4; for $x = 1/7$, $Y = 0.25$); with the neutrons counted in Chapter 11 of that book, $Y = 0.233$ to 0.247, beside the measured $Y_p = 0.245 \pm 0.003$ (Particle Data Group 2026, review "Big-Bang Nucleosynthesis", Eq. (24.3)).

The history traced back reaches no first moment as an event. *At the Edge of Reality*, Chapter 17, "Why the Big Bang Looks Like a Beginning", sections "Yes, as Our History Finds It" and "The Limit Rendered as a Point" (the quotation is from the first of these).

The bucket and the oldest light (past measurements, not claims of the collection). The principle the bucket carries: *What Comes After the Dark*, Chapter 6, "What Happens to Someone Who Is Still There?", sections "Mach and the Bucket" and "A Standard That Needs Something There" ("a standard of not rotating is fixed only by matter within reach"). The dipole: amplitude $(3.5 \pm 0.6) \times 10^{-3}$ K, read as a motion of the Earth of 390 ± 60 km/s relative to the radiation (Smoot, Gorenstein and Muller 1977). The test of an unevenly expanding universe, rotation (vorticity, the vector mode) included: $(\sigma_V/H)_0 < 4.7 \times 10^{-11}$ at 95 percent (σ_V/H, the shear of the vector mode in units of the expansion rate, today), and odds of 121,000 to 1 against anisotropic expansion with all degrees of freedom together (Saadeh, Feeney, Pontzen, Peiris and McEwen 2016).

Inside a black hole the end is a moment, not a place; the outside world empties; two ends for every bound thing; the ends of all interiors are one end. *What the Beginning Left Behind*, Chapter 1, "Where the Last Book Left Us", section "How an Epoch Ends, in Eight Steps" (first step: the singularity is "a moment and not a place: the future end of every path inside", approached within a finite time and never reached; second step: the funnels, and the principle that a difference no comparison can register is no difference; third step: the emptying of the outside world over about a hundred billion to a trillion years). *What Comes After the Dark*, Chapter 6, section "Two Ways to Be Left" (inside a black hole, or carried away alone);

At the Edge of Reality, Chapter 11, "Every Ending Is the Same Ending", section "The Ends Are One"; *The Gravitational Passage*, Chapter 18, "The Interior Passage", Section II, "All Interiors Are One at Their End".

What holds the six; the six released; content needs a context; three open while six close. *What the Beginning Left Behind*, Chapter 1, section "What Holds the Six": the compactification conditions of *The Gravitational Passage* (the large world present as a context for the six, as a domain of the smooth description, with a reference for inertia fixed in it); while they hold, each of the six directions stands where the curvature it meets is the Planck curvature ("That is the hold"); sixth and seventh steps of "How an Epoch Ends, in Eight Steps". In Chapter 9 I give the hold the gloss that Chapter 16 gives the hold. That gloss is the large world of space and time keeping the six at their one size and shape, and with the six the forms of matter. The three facts of Chapter 9 are these conditions, which the large world supplies while the large world keeps the six. *At the Edge of Reality*, Chapter 16, "The Turn", sections "What Comes After the End", "Content Needs a Context", "The Six Are Released, and the Split Re-forms" and "The Same Arc, Read Away From the End". On the Kasner arc, read away from the limit, each of the three directions grows as $t^{5/9}$ and each of the six contracts as $t^{-1/9}$ (t, the time counted from the limit in the new epoch's own history); the exponents p_i satisfy $\Sigma p_i = 1$ and $\Sigma {p_i}^2 = 1$ ($3 \times 5/9 + 6 \times (-1/9) = 1$; $3 \times 25/81 + 6 \times 1/81 = 1$); *What the Beginning Left Behind*, Chapter 2, "Three Open While Six Close". The Planck length is about 1.6×10^{-35} m and the proton's width about 1.7×10^{-15} m, so the ratio is about 10^{20}.

The turn; the epoch; no comparison registers the turn as an event. *What the Beginning Left Behind*, Chapter 1, "Where the Last Book Left Us", section "The Turn", and Terms ("Turn, the"); *At the Edge of Reality*, Chapter 17, section "No Comparison Across the Turn".

The identification: the Big Bang is the turn as our epoch's history renders it. *At the Edge of Reality*, Chapter 17, section "The Limit Rendered as a Point" ("I identify the Big Bang of the equations of cosmology with

that appearance, and I call the appearance the projection"); Chapter 16, section "The Identification"; *What the Beginning Left Behind*, Chapter 1, section "A History Made from What Arrives Now", and Terms ("Big Bang"). The picture of the history drawn as size against time, heading for a line it never reaches, is *At the Edge of Reality*, Figure 10.

The turn is not a creation from nothing; what came before is the collapse of the interiors of the epoch before. *What the Beginning Left Behind*, Chapter 1, section "The Turn"; *At the Edge of Reality*, Chapter 2, section "The Answer of This Book", and Chapter 16, section "What the Turn Is Not".

Three large directions because the context factor has three directions of space; the same internal shape re-forms. *The Gravitational Companion*, chapter "Comparisons at Single Steps", Section III, "Universes Inside Black Holes, and Strings Grown Large" (quoting *The Gravitational Passage*, Chapter 18, Section VII); *At the Edge of Reality*, Chapter 16, section "The Six Are Released, and the Split Re-forms": the internal space $\mathbb{C}P^2 \times S^2$ (the complex projective plane times the sphere).

Beyond the Planck scale no description takes over; the axioms and the theorem hold there; there is no initial state. *What the Beginning Left Behind*, Chapter 1, section "The Turn" ("the three axioms and the Incommensuration Theorem hold beyond the Planck scale as everywhere"; "The structure of comparison holds on both sides while the descriptions reach the edges of their ranges, so the turn is not a breakdown at which the equations fail"); Chapter 7, section "What the Axioms Do Not Do" ("It is not the output of an axiom"). Beyond the Planck scale no description takes over, and the account builds none there; the structure of comparison, and the Incommensuration Theorem with it, hold on both sides of the turn.

One origin: the horizon problem answered from the turn. *What the Beginning Left Behind*, Chapter 12, "Four Marks of the Pattern", section "One Origin, One Cloth" (the ends of the old interiors are one domain; the tiled floor and the woven cloth); Chapter 13, section "The Sameness

and the Flatness, Answered From the Turn"; *At the Edge of Reality*, Chapter 19, "The Faint Pattern in the Sky", section "What This Says About Inflation"; source: *The Gravitational Passage*, Chapter 18, Questions. In the standard history, directions of the sky more than about one degree apart were never within reach of a signal from each other before last scattering.

Space is flat because no length of the old epoch passes the turn. *What the Beginning Left Behind*, Chapter 12, section "Flat Space"; *The Gravitational Passage*, Chapter 18, Section IX, "The Departures from Uniformity", paragraph "Spatial flatness"; *Side by Side, Volume One*, part "Where the Accounts Part", Section VIII, "Is Space Flat?". The curvature k enters the Friedmann equation as a constant of integration; the state at the turn holds the length of the turn, $\ell \approx \ell_{\mathrm{Pl}}$ (the Planck length, about 1.6×10^{-35} m), and pure numbers of order one; a radius built from ℓ_{Pl} and the de Sitter radius (about 17.5 billion light years) would need the de Sitter radius in the state, or a pure number of about 10^{30}, and neither is there; a radius equal to the de Sitter radius would leave $\Omega_{\mathrm{K}} \approx 10^{-64}$ today (Ω_{K}, the share of the curvature in the total density). *What the Beginning Left Behind* adds that this argument "uses nothing of the cosmological constant: neither its value nor even its sign". Measured: $\Omega_{\mathrm{K}} = 0.001 \pm 0.002$, from the microwave background with baryon acoustic oscillations (Aghanim and others 2020), as *Side by Side, Volume One*, Prediction 9, reports it ("Measured"); *What the Beginning Left Behind*, Chapter 11, gives 0.0007 ± 0.0019 from the same paper. No measurement can test the flatness of the smoothed model more finely than about 10^{-5} (Waterhouse and Zibin 2008).

A flat space can close. *Side by Side, Volume One*, part "Where the Accounts Part", Section VIII, "Is Space Flat?" ("a flat space can also close, as a torus does, and a flat space closed on a scale beyond every horizon meets both his condition and Predictions 9 and 23"); *The Weight of Possibility*, Chapter 3, Section II ("So the open space counts as one, whatever it is beyond the horizon"). Measurement has found space flat within all that

has reached us; a flat space can also close on itself, as the surface of a ring-shaped doughnut closes, and a flat space closed on a scale beyond everything that has reached us is flat within it.

The identification: the first ripples are the form of the content carried through the turn, averaged and not erased. *What the Beginning Left Behind*, Chapter 11, "The Oldest Light", section "What Passes the Turn" ("What passes the turn is the form of the content: which alternatives the content has taken, the specification of what has been selected").

The theorem requires the departures. *What the Beginning Left Behind*, Chapter 7, section "Even From the Start", quoting *The Gravitational Passage*, Chapter 18, Section VIII ("perfect isotropy would make symmetry and continuity both perfect, which the Incommensuration Theorem forbids"); *The Compactification*, Chapter 3, Section VI, "The Even Beginning". The Incommensuration Theorem in full: nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous (*An Immanent Metaphysics*, aphorism 1.43-2).

Empty space of the new epoch adds almost nothing to the ripples. *What the Beginning Left Behind*, Chapter 11, section "Empty Space Adds Almost Nothing" (computed in *The Gravitational Passage*): an observed length lay about 10^{27} times beyond the horizon near the turn; the vacuum's power there is below 10^{-51}, against an observed amplitude $A_s \approx 2.10 \times 10^{-9}$ (A-sub-s, the power of the first ripples at the reference length), a shortfall of more than 10^{42}. Two consequences follow: the first is computed, and the second is argued from the one domain.

Linked across the whole sky, nearly the same strength at every length; strength and tilt measured. *What the Beginning Left Behind*, Chapter 12, section "One Origin, One Cloth"; Chapter 11, sections "What the Measurements Find" and "What the Pattern Must Show". Measured: spectral index $n_s = 0.9649 \pm 0.0042$ (n-sub-s, the index of the power spectrum; one means the same power at every length), $A_s = 2.10 \times 10^{-9}$ (Agh-

anim and others 2020); both are inputs of the account. The strength of the ripples, about one part in a hundred thousand, and the slow change of that strength with length are taken from measurement.

Gravity grows the departures into galaxies. *What the Beginning Left Behind*, Chapter 10, section "Gravity Turns the Order Around" ("So, as the small departures from evenness grow into galaxies, stars and clusters, the count of the whole rises").

No inflation needed; no monopoles. *What the Beginning Left Behind*, Chapter 13, "No Inflation Needed", opening and section "The Sameness and the Flatness, Answered From the Turn"; *The Gravitational Passage*, Chapter 18, Questions ("The account therefore needs no inflationary phase for the horizon and flatness problems"); *At the Edge of Reality*, Chapter 19, section "What This Says About Inflation". No monopoles, strings or walls: no grand unification ("In this account the forces were never one larger force, so nothing breaks in the way that makes monopoles, and the monopole problem does not arise"), and the Higgs field's set of lowest states, a three-sphere, has every loop and sphere shrinking to a point (Kibble 1976); *What the Beginning Left Behind*, Chapter 13, section "No Monopoles, No Strings, No Walls". The two moduli weigh about 2.25 and 3.09 Planck masses, far too heavy for slow-roll inflation (*The Gravitational Companion*, chapter "Comparisons at Single Steps", Section IX, "Inflation").

A long inflation is excluded, on the identification of the first ripples. *What the Beginning Left Behind*, Chapter 13, opening (the early horizon: "the greatest distance a signal could have crossed since the turn, in the first moments of our history") and section "A Long Inflation Is Excluded"; *Side by Side, Volume One*, Appendix B, Prediction 13. A shorter inflation, one that leaves the observed lengths outside the early horizon the whole time, "is neither required nor excluded by the account" (*The Gravitational Passage*, as *What the Beginning Left Behind*, Chapter 13, quotes it); the starting temperature bounds it.

The first waves. *What the Beginning Left Behind*, Chapter 12, section "Ripples of Density, and Waves Only Later" (the stretched sheet and the children who press straight down; the ripples that pass the turn are scalar, and tensor modes arise only at second order, so the ratio of first waves to first ripples is far below one millionth). Measured: tensor-to-scalar ratio r < 0.036 at 95 percent (r, the power of the first waves divided by the power of the first ripples at the reference length; BICEP/Keck with Planck and WMAP, Ade and others 2021), below 0.032 jointly (Tristram and others 2022).

A second ground from the dark matter. *What the Beginning Left Behind*, Chapter 13, sections "How Hot the Beginning Could Have Been" and "A Second Reason, From the Dark Matter" (quoting *The Weight of Possibility*); *The Weight of the Unseen*, Chapter 6, 6.6. A second ground reaches the same result from the dark matter, which Chapter 9 meets in its section "The Weight Nobody Sees". An inflation that made the ripples would make first waves in step with its energy, so that bound caps the heat such an inflation could hand on; and with heat so capped, dark matter of the kind this account finds could not have been made in the measured amount. The temperature needed to make gyrations in the measured amount is about one thirtieth of a gyration's mass, some 2 to 5×10^{17} GeV.

Entropy is a count relative to a context; the reset. *What the Beginning Left Behind*, Chapter 7, section "Every Count Has a Context", and Terms ("Reset, the"); Chapter 10, section "The Emptying, and the Reset"; source: *The Gravitational Passage*, Chapter 18, Section VIII, "One Process, Two Sides".

The even beginning; Penrose's Weyl curvature hypothesis follows. *What the Beginning Left Behind*, Chapter 7, section "Even From the Start"; *The Gravitational Passage*, Chapter 18, Section VIII ("This is Penrose's Weyl curvature hypothesis, reached here as a consequence of the restoration and not as a separate postulate"). The Weyl curvature is the part of the

curvature that distorts shapes, as distinct from the part that changes volumes; the new geometry begins conformally flat (Weyl curvature zero) up to the departures, about 10^{-5}.

Penrose's figure is no draw; the Past Hypothesis. *What the Beginning Left Behind*, Chapter 10, section "One Draw Among Many?" (the lottery; "It treats the beginning as one draw from all the states the matter could have taken, and it weighs each of those states equally"); *The Compactification*, Chapter Three, "The Past Hypothesis at the Turn", Section VIII, "The Magnitudes". Penrose's figure: one part in $10^{10^{123}}$; the largest count used is that of one black hole of all the mass of the observable universe, about 10^{123}. On this account the Past Hypothesis is "neither an axiom nor a brute law added to physics".

Both changes of state are crossovers; neither raises the count. *What the Beginning Left Behind*, Chapter 10, section "Smooth Changes, with No Bubbles" (a first-order transition raises the count of the content and its colliding bubbles make gravitational waves; "No change of state raises the count of the content in a comoving region in bulk"); *The Coming of the Kinds*, Chapter 6, "The Higgs Field Settles", Section III, "The Field Takes Its Value". The Higgs crossover is centered at 159.5 ± 1.5 GeV, width about 5 GeV (D'Onofrio and Rummukainen 2016), for $m_H \approx 125$ GeV, far above the endpoint of a first-order transition near 72 GeV; the quark crossover is near 156.5 MeV (Aoki and others 2006).

The evidence for the dark matter. *The Weight of the Unseen*, Chapter 1, "What Is Seen", 1.1, 1.2 and 1.3 (about 5.4 times the ordinary matter, the same proportion everywhere the microwave background measures it, to within about one percent).

The dark matter is made of gyrations, of about the Planck mass, made at the turn; dark because instruments register content. *The Weight of the Unseen*, Chapter 3, "What the Dark Matter Is", 3.1, 3.2 and 3.3 ("A vibration of the internal space is a distortion of a context, not an item of content. It has energy, so it has weight. It is not content, so nothing that registers content registers it"); Chapter 6, "The Cycle of Epochs". The six as

a context: *The Weight of the Unseen*, Chapter 2, 2.1 to 2.3 (the contexts are space, time and possibility; probability is measured in possibility; "A content is what a measurement registers"; "A context is where the measurement registers it"; "The dark matter is a vibration of the six small internal dimensions, which are, in plain mathematics, a space of possibilities"). The two placements of the six, each relative to its own comparisons, are those of Chapter 7 of this book, after *At the Edge of Reality*, Chapter 7, "Six Dimensions of Possibility", section "A Space of Possibilities" ("Content and context are assigned within each comparison"; "In physics the internal space stands on the side of content, held in the large world of space and time as its context"; "Here possibility is a context, in which each finding is the content"). In Chapter 9 I state the dark matter's placement relative to the comparisons every instrument makes, the comparisons that register content. How the weight is seen: *The Weight of the Unseen*, Chapter 1, 1.1 and 1.2 ("The unseen matter has energy, so it pulls, and its pull shows in the bending of light, in the spinning of galaxies and in the pattern of the oldest light"; "Every piece of evidence for it is evidence of its weight alone, and none of the evidence shows what it is made of"); gravity as the shape of the arena, the arena being the setting in which every comparison is made, is the claim of Chapter 1 of this book. The lightest lasting vibrations: about 1.4 Planck masses in the simplest calculation, about 0.6 with the first correction (3.2). The phonon as a quantum of the description of a held structure's vibration: *What the Beginning Left Behind*, Chapter 7, section "Gravity Was Never Born".

The ratio of dark to ordinary matter. *The Weight of the Unseen*, Chapter 5, "How Much There Is", 5.1 to 5.4 (the six counts the independent kinds of shape of the internal space, the basic pieces from which it can be built, three for the complex projective plane times two for the sphere; the ordinary matter counts one, the ball within the horizon: "A ball has one independent kind"; two divided by pi is the part of the bending of the complex projective plane that abides, which that book calls "the abiding"). The part taken away, in the steps of 5.4: a bending of a given strength sets a horizon, and the part is three of that horizon's degrees of freedom

among all of them; the count is $3\pi/2$ for the complex projective plane, which gives $3/(3\pi/2) = 2/\pi$; 3π for the sphere, which gives $1/\pi$; π for the whole internal space, which gives $3/\pi$. In words: for the bending of the complex projective plane the count of those ways is one and a half times pi, and three out of one and a half times pi is two divided by pi. The count of a horizon's degrees of freedom is the count of Bekenstein and Hawking, the area of the horizon divided by four Planck areas, so it need not be a whole number: a horizon of radius L Planck lengths has the count πL^2 (*The Weight of Possibility*, Chapter 3, "The Registered Ratio", Section III, "The Share of a Curvature": "That is the count A/4 of Bekenstein and Hawking for a sphere of radius L"). A count that already includes the sphere's own kinds is not reduced again by the sphere's bending; the proportion's count includes both kinds of the sphere, so only the bending of the complex projective plane abides, and the part is $2/\pi$; the count of the cosmological constant's rule includes one kind of the sphere, so there the whole internal bending abides, and the part is $3/\pi$ (the rule $8 - 3/\pi$ of Chapter 10). The independent kinds of shape are what mathematicians call classes (5.2); a solid ball can be shrunk to a point and has one class, as a point has; for these shapes the count equals the Euler number: $\chi(\mathbb{C}P^2) = 3$, $\chi(S^2) = 2$, and for a solid ball $\chi = 1$ (chi, the Euler number). The value: *Side by Side, Volume One*, part "Where the Accounts Part", Section IX, "What Is the Dark Matter?", and Appendix B, Prediction 16: $\Omega_c/\Omega_b = 6 - 2/\pi = 5.3634$ (Ω-sub-c over Ω-sub-b, the density of the dark matter divided by the density of the ordinary matter), beside 5.364 ± 0.065 (Planck alone).

Chapter 10. Why Is the Energy of Empty Space So Small?

The gap between the summed energy of the vacuum and the measured value. Greene's figures: the sum stopped at the Planck length gives about 10^{94} g cm^{-3}, one Planck mass per Planck volume (*The Hidden Reality*, chapter 6, section "Explaining Zero"), with the Planck mass put at about 10^{-5} grams (same chapter, section "The Cosmological Constant";

the value is about 2.18×10^{-5} g), and the measured value is about 10^{-123} in Planck masses per cubic Planck length (same chapter, section "The Cosmological Constant"; his note 9 to chapter 6 gives 1.38×10^{-123}). The gap is regarded as the largest by which a successful theory has missed a measurement: *The View from Inside*, Chapter 1, "The Emptiest Space Is Not Empty", section "The Number That Shouldn't Exist".

The supernova measurements of 1997 and 1998. The scene of the chapter's opening (autumn 1997, Berkeley; Riess working for Schmidt's team; the first fits matched only by a faster expansion; Schmidt the first told) is told in *What Comes After the Dark*, Chapter 1, "What Happens to the Sky We Can See?", section "Faster, Not Slower" ("The expansion is speeding up"); see also *The View from Inside*, Chapter 1, section "A Strange Discovery". Riess and others (1998), *Astronomical Journal* 116, 1009: the distances of the high-redshift supernovae are on average 10 to 15 percent larger than in a universe of low mass density with no cosmological constant. Perlmutter and others (1999), *Astrophysical Journal* 517, 565: forty-two high-redshift supernovae. Nobel Prize in Physics, 2011, to Perlmutter, Schmidt and Riess.

Einstein's constant of 1917, its return in 1998, and Hubble's expansion of 1929. Greene's telling: *The Hidden Reality*, chapter 2, sections "Taxing Gravity" and "The Primeval Atom"; chapter 6, sections "The Return of the Cosmological Constant" and "The Cosmological Constant". Told here as Greene's account; the collection tells the same history in *The View from Inside*, Chapter 1, section "The Cosmological Constant" (the constant term "later read as an energy density spread uniformly through space"). In the field equations the constant enters as Λ (lambda, a geometric quantity with units of inverse length squared); the energy density of empty space that it corresponds to is ρ_Λ (rho lambda), and $\rho_\Lambda = \Lambda\, c^4/(8\pi G)$, where c is the speed of light and G is Newton's constant.

The measured value. $\rho_\Lambda \approx 2.52 \times 10^{-47}$ GeV^4 (an energy to the fourth power is an energy per volume in units with the speed of light and Planck's constant set to one); about 5.8×10^{-27} kg m^{-3}, the mass of three

or four hydrogen atoms in a cubic meter. *What the Beginning Left Behind*, Chapter 8, "The One Edge That Sets the Vacuum", section "The Number, Seen Two Ways"; *The View from Inside*, Chapter 1, section "The Cosmological Constant". Greene's figure: just under 10^{-29} g cm^{-3} (*The Hidden Reality*, chapter 6, section "The Cosmological Constant").

Seven tenths dark energy, three tenths matter; Greene's link with inflation. Told here as Greene's account (*The Fabric of the Cosmos*, chapter 10, section "The Missing 70 Percent"; chapter 14, section "Dark Matter, Dark Energy, and the Future of the Universe"). In the collection the flatness that makes the fractions sum to one comes from the turn: *What the Beginning Left Behind*, Chapter 12; *The Gravitational Passage*, Chapter 18, Section IX.

The hum of the vacuum; quantum fields force a vacuum energy; the Casimir force. Greene's telling: *The Hidden Reality*, chapter 6, section "Explaining Zero"; *The Fabric of the Cosmos*, chapter 12, section "Quantum Jitters and Empty Space" (Lamoreaux 1997, "to an accuracy of 5 percent"). The collection, same result: *The View from Inside*, Chapter 1, section "What Is 'Empty' Space?" (Casimir 1948; Lamoreaux 1997, within about five percent), and Chapter 10, "Why the 10^{123} Never Arises", section "Back to the Numbers" ("Every quantum field hums, even when it is as quiet as it can be, and that hum is energy"). Casimir, H. B. G. (1948), *Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen* 51, 793; Lamoreaux, S. K. (1997), *Physical Review Letters* 78, 5.

The size of the mismatch, and the two ways of writing it. The Planck density $M_{Pl}{}^4$ (the Planck mass, $M_{Pl} = (\hbar c/G)^{1/2} \approx 1.221 \times 10^{19}$ GeV, to the fourth power, where $\hbar$, h-bar, is Planck's constant divided by two pi) is about 2.22×10^{76} GeV^4; the ratio to the measured density is about 10^{123}. With the reduced Planck mass $M_{Pl}/(8\pi)^{1/2}$ the ratio is about 10^{120}, smaller by $(8\pi)^2 \approx 632$; this is the figure of Greene's "some 120 orders of magnitude", written before the speeding-up was measured and set against what experiment then allowed (*The Elegant Universe*, chapter 9, section "Some Longer Shots"). The collection uses the full Planck mass, because the meeting scale of the two descriptions is formed from the three con-

stants alone (*The View from Inside*, Chapter 9, section "The Modal Boundary"). *The Concept Branches*, entry G-003 ("The Mismatch"); *The View from Inside*, Chapter 1, section "The Number That Shouldn't Exist", and Chapter 10, section "Back to the Numbers".

Supersymmetry does not cancel the total; within the content, the line between content and context is drawn again, and on the first such line the family of matter and the family of the carriers stand on its two sides. Greene's telling: *The Hidden Reality*, chapter 6, section "Explaining Zero" (the total cancellation needs partners of the same mass; "Experimental data have ruled this out"). The line drawn again within the content is the result Chapter 6 of this book states from *The Story of the Particles*. The collection: *The View from Inside*, Chapter 1, section "What Went Wrong?", and Chapter 3, "Why Nothing Else Has Worked", section "Strategy One: Find a Symmetry" (residual of order $M_{break}{}^4$, the supersymmetry-breaking scale to the fourth; with M_{break} above about 1 TeV the residual is at least 10^{12} GeV^4, some 59 to 60 orders of magnitude above the measured value); *The Story of the Particles*, Level 9, Section IV; *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Proposition 19.8 ("The field space of the Standard Model admits no supersymmetry").

String theory and the mismatch. Told here as Greene's account (*The Elegant Universe*, chapter 9, section "Some Longer Shots": he asks, "Can calculations in string theory improve on this mismatch and explain why the cosmological constant is zero", and goes on to the case of a small value that is not zero).

The anthropic proposal, and quintessence against a true constant. Told here as Greene's account (*The Hidden Reality*, chapter 6, section "Explaining Zero" and following, which Chapter 11 takes up; *The Fabric of the Cosmos*, chapter 14, section "Dark Matter, Dark Energy, and the Future of the Universe").

Eddington's instability of Einstein's static universe. *The View from Inside*, Chapter 1, section "The Cosmological Constant" (the pencil on its point). Eddington, A. S. (1930), "On the Instability of Einstein's Spherical World", *Monthly Notices of the Royal Astronomical Society* 90, 668–678. Greene gives the same step in his notes: *Until the End of Time*, notes to chapter 3, note 2 (Eddington); *The Hidden Reality*, notes to chapter 2, note 7 (de Sitter).

The total is real, of the size quantum theory gives, and curves nothing. *The View from Inside*, Chapter 10, "Why the 10^{123} Never Arises", sections "Back to the Numbers" ("The total is there, and it curves nothing"), "What Curves Space" ("What no comparison finds places no strain on the arena, and so bends nothing"; "If no sofa ever pushes, the rug lies flat"), "The Share and the Total" ("The energy of the vacuum is real, and it is of the size quantum theory gives".) and the chapter's Key idea.

Why space curves: the rug between the wall and the sofa. *The View from Inside*, Chapter 8, "Content, Context, and Why Space Curves", sections "Two Demands That Cannot Both Be Met" (the comparing must be continuous; the content must be symmetric; the Incommensuration Theorem forbids perfecting both in one character) and "The Rug Between the Wall and the Sofa" (the content is the wall, the comparing the sofa, the context the rug; the context carries the continuity of the comparing, gives up symmetry, and bends). The bending goes with energy because energy is the rate at which findings can be made (same chapter, sections "Energy Is the Rate of Finding" and "In Proportion to What Can Be Found"; the Margolus and Levitin bound, at most $2E/(\pi\hbar)$ distinguishable states per second). The form of the field is fixed by Lovelock's theorem (same chapter, section "From Here and Now to Everywhere at Once"). The Incommensuration Theorem in the words of *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous"; "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous".

First reason: no measurement takes in the total; the identification of what a measurement receives. *The View from Inside*, Chapter 10, section "A Cup from the Sea": "I identify what a measurement receives of the vacuum's energy with what is held by the degrees of freedom it engages"; "No measurement ever made, and none that could be made from inside, takes in the vacuum's energy as a whole". The number of degrees of freedom within the particle horizon is $S \approx 2.29 \times 10^{123}$ (S, the count of the horizon, read "S").

Second reason: the total taken whole would be perfectly symmetric and perfectly continuous at once. *The View from Inside*, Chapter 10, section "The Shape That No Comparison Finds". The sameness for every comparer in steady motion is the Lorentz invariance of the vacuum state (the symmetry of special relativity). The argument holds whatever the size of the total, and whichever way the sum is cut off. In the words of the same section, "This holds whatever the total's size, and whichever way of doing the sum is preferred": a cutoff at the Planck energy gives about 10^{76} GeV^4, other ways of doing the sum give less, and the argument does not depend on which.

The cosmological constant is the accessible share (identification). *What Comes After the Dark*, Chapter 2, "Will It All Fall Back?", section "The First Way: Space Is Flat": "I identify the cosmological constant with the part of the energy of empty space that a measurement made from inside the universe can take in". The same identification: *The View from Inside*, Chapter 11, "The Problem of Being Inside", section "The Share We Find"; *What the Beginning Left Behind*, Chapter 8, section "Measuring Empty Space from Inside". In Chapter 10 I write "the part of the vacuum's energy" for that book's "the part of the energy of empty space". So in Chapter 10 "the vacuum's energy" names the total, and "the energy of empty space" names the energy that drives the speeding-up.

The count of an edge, and why the particle horizon carries it. The count of a horizon of radius R: $S = A/(4\ l_{Pl}{}^2) = \pi\ R^2/l_{Pl}{}^2$ (A, the area $4\pi R^2$; l_{Pl}, the Planck length, about 1.6×10^{-35} m), the Bekenstein and

Hawking count, in nats. Bekenstein, J. D. (1973), *Physical Review D* 7, 2333; Hawking, S. W. (1975), *Communications in Mathematical Physics* 43, 199. The count grows with the area because the count made from a place, through what crosses the boundary, is the more basic of the two natural counts of a region, by the first axiom: *What the Beginning Left Behind*, Chapter 8, section "What the Count of an Edge Counts". The count was carried to the edge of the observable universe by Fischler and Susskind (1998): *The View from Inside*, Chapter 12, "Counting the Horizon", section "Information Written on a Surface". The count is taken on the particle horizon by an identification: "the edge of what has been compared is the edge of the information open to the comparison" (*The View from Inside*, Chapter 12, section "The Edge of What Has Reached Us"); "the bound of what has been compared is the bound of the information available to the comparison" (*What the Beginning Left Behind*, Chapter 8, section "Why the Edge of What Has Reached Us"). For the particle horizon, R ≈ 46.1 billion light-years and S ≈ 2.29×10^{123}.

The share: three tiles out of the count of the particle horizon. *What the Beginning Left Behind*, Chapter 8, section "Three Tiles Out of All the Tiles"; *The View from Inside*, Chapter 11, sections "Tiles on the Horizon" and "The Equation, Piece by Piece", and Chapter 12, section "Why Three". In symbols: $\rho_\Lambda = 3 \text{ f } M_{Pl}^4/S$ (rho lambda equals three, times f, times the Planck mass to the fourth, divided by S). The standing of each step, as *What the Beginning Left Behind* gives it: the channels are identified with the three contents a measurement registers ("I identify the channels through which the vacuum reaches a measurement with the three contents a measurement registers, one in each of the three roles"); each channel takes one tile, and a different one, by the third axiom ("I take each channel to take in one tile, and a different tile for each channel, because the three contents the channels carry are distinct (the third axiom: distinct, inseparable, and none able to do another's work)"); and "The share itself, three parts out of the count, is an identification: mine, made in the three steps above".

The three channels, and which role each goes with. The grid of six measures: pattern measured in space, force in time, probability in possibility; "pattern and space are omniscient (what is laid out), force and time are immanent (what happens), and probability and possibility are transcendent (what could be)" (*The View from Inside*, Chapter 8, section "Six Measures"). The omniscient, immanent and transcendent are the formal names of the three roles the chapters call the structure, the act and the stance. The context row supplies no channel of its own, "as a page is where a word is written and not a second word" (*The View from Inside*, Chapter 12, section "Why Three"). The same three appear by a second route, in the shape of the six small dimensions of Chapter 7: the larger of their two parts, the complex projective plane, which is the space of the states of a three-way choice, has exactly three basic pieces, one for each role. That count is the Euler number of the complex projective plane $\mathbb{C}\mathrm{P}^2$, $\chi(\mathbb{C}\mathrm{P}^2) = 3$ (chi of CP-two equals three): *The View from Inside*, Chapter 9, sections "Counting Pieces" and "The Number Three"; *What the Beginning Left Behind*, Chapter 8, section "Three Tiles Out of All the Tiles".

The trim. $\mathrm{f} = 1 - 3/(8\pi) \approx 0.8806$ (one less three divided by eight pi), from the classical energy of the bare curvature of the six at their derived size, which has the sign that draws space together: *What the Beginning Left Behind*, Chapter 8, section "Measuring Empty Space from Inside"; *The View from Inside*, Chapter 9, section "A Small Trim" (the sheet of spring steel rolled into a coil; the calculation is the one Einstein's equations give when ten dimensions are reduced to four, carried in full in *The Gravitational Passage*). The 3 in the trim comes from that calculation and is not the count of channels (same section).

The result. 2.56×10^{-47} GeV^4 against 2.52×10^{-47} measured, within two percent. "The energy of empty space is small because the edge of what has reached us is large". (*What the Beginning Left Behind*, Chapter 8, section "Three Tiles Out of All the Tiles".)

The share is found as a count, so the theorem allows it. *The View from Inside*, Chapter 10, section "The Share and the Total" (the share is discontinuously symmetric: found in countable steps and the same for every comparer).

Why the particle horizon. *What the Beginning Left Behind*, Chapter 8, section "Why the Edge of What Has Reached Us"; *The View from Inside*, Chapter 12, sections "Four Edges", "The Edge of What Has Reached Us" and "A Second Reason, Quite Separate". First reason (in the chapter): it is the only edge a present measurement holds; the cosmological event horizon is drawn by what could reach the observer in the future, and the Hubble radius and the de Sitter radius by rates and by idealized universes. Second reason (here only): on the de Sitter radius R_{dS} (the edge of a universe holding nothing but the energy of empty space, about 17.53 billion light-years for ours), established physics gives $\rho_\Lambda = (3/8)\ M_{Pl}{}^4/S_{dS'}$ so the share there is $8f \approx 7.05$ times the constant, and every smaller edge (the Hubble radius, the cosmological event horizon) gives more; the share equals the constant only on an edge of $(8f)^{1/2} \approx 2.65$ de Sitter radii, and the particle horizon stands at 2.63 today.

The holographic horizon proposals came closest. *The View from Inside*, Chapter 3, section "A Clue at the Horizon" ("This is a real clue"). Fischler, W., and Susskind, L. (1998), "Holography and Cosmology", arXiv:hep-th/9806039; Cohen, A. G., Kaplan, D. B., and Nelson, A. E. (1999), "Effective Field Theory, Black Holes, and the Cosmological Constant", *Physical Review Letters* 82, 4971. Those proposals set the vacuum energy equal to the Planck density divided by the horizon's count, times a number of order one, and needed a reason for three choices: which horizon, which coefficient (the number in front), and why the result stays constant (same section). Those proposals left the edge, the number in front and the constancy to be chosen; in this account the edge is the particle horizon, the number in front is three channels and the trim, fixed by the structure of comparison before any measurement is consulted, and the constancy fol-

lows from Einstein's equations. The account's reasons for each: *The View from Inside*, Chapter 11, section "Is This Just a Horizon Proposal with a Three in Front?", and Chapters 12 to 15.

The two surveyors: the share and Friedmann's equation give one identity and the matter fraction. *What the Beginning Left Behind*, Chapter 8, section "The Number, Seen Two Ways"; *The View from Inside*, Chapter 13, "One Equation, One Number", sections "The First Way: The Share", "The Second Way: The Astronomers' Way" and "Setting the Two Equal". Friedmann's equation for a flat universe gives the critical density $3H^2/(8\pi G)$ (H, the present expansion rate, the Hubble constant); the vacuum's part is Ω_Λ (omega lambda) of it, and the matter fraction is Ω_m (omega m). The steps, in units with $\hbar = c = 1$, where $G = 1/{M_{Pl}}^2$ and $l_{Pl} = 1/M_{Pl}$: the count is $S = \pi R^2 {M_{Pl}}^2$; the share is $\rho_\Lambda = 3f{M_{Pl}}^2/(\pi R^2)$; with the particle horizon at $R = I/H$ (I, the horizon's distance in Hubble radii, about 3.18; the Hubble radius is $1/H$), $\rho_\Lambda = 3f{M_{Pl}}^2 H^2/(\pi I^2)$; the astronomers' way gives $\rho_\Lambda = \Omega_\Lambda \times 3H^2{M_{Pl}}^2/(8\pi)$. Setting the two equal cancels 3, H^2, ${M_{Pl}}^2$ and π, and gives the identity $I^2\,\Omega_\Lambda = 8f = 8 - 3/\pi \approx 7.05$ (I squared times omega lambda equals eight times the trim, which is eight less three over pi). The 8 is the 8π of Einstein's equations divided by the π of the horizon's count; the three channels cancel the 3 of the critical density, which comes from the three dimensions of space. I depends only on how the energy has been divided between matter, radiation and empty space over the history of the expansion, so the identity has one unknown, Ω_m; the solution is $\Omega_m = 0.311$ ($\Omega_\Lambda = 0.689$), against 0.315 ± 0.007 measured (Aghanim and others 2020). The density, 2.56 against the measured 2.52, and the matter fraction, 0.311 against the measured 0.315, are "one measured agreement, seen two ways" (same section of *What the Beginning Left Behind*). Friedmann, A. (1922), "Über die Krümmung des Raumes", *Zeitschrift für Physik* 10, 377.

Within one geometry the cosmological constant is one number; no quintessence. *What Comes After the Dark*, Chapter 2, section "The Second Way: One Number for Our Whole Epoch" ("So, within one geometry, the cosmological constant is one number, where one geometry means the one

space and time of our epoch"; "That is a theorem about Einstein's equations"); *What the Beginning Left Behind*, Chapter 8, section "Fixed Once"; *The View from Inside*, Chapter 15, "One Constant, One History", section "One Map, One Scale". The theorem: the contracted Bianchi identity, $\nabla^{\mu}G_{\mu\nu} = 0$ (the divergence of the Einstein tensor vanishes), with the conservation of the energy-momentum tensor, $\nabla^{\mu}T_{\mu\nu} = 0$, sets $\partial_{\nu}\Lambda = 0$ (the gradient of lambda is zero) within one geometry; Lovelock's theorem leaves room for exactly one term in the field equations that is the same at every place and time (*The View from Inside*, Chapter 10, section "The Share and the Total").

The constant is fixed once, by the present measurement. *What the Beginning Left Behind*, Chapter 8, section "Fixed Once" ("A law exists; a measurement is real"); *The View from Inside*, Chapter 15, sections "Which Measurement Fixes It" and "An Astronomer Long Ago". The share worked out on paper for an astronomer at half the present size (redshift 1) is 7.049 times the measured constant; for our horizon a billion years ago, 1.224 times; a billion years from now, 0.852 times (*What the Beginning Left Behind*, Chapter 8, section "Fixed Once"). Her records, in the reconstructed history, carry the one constant of that history, with a matter fraction of about 0.79 in her day (same section). Her own measurement: "Her measurement was a real act in her day. What reaches us of it is her record, a present impression in our reconstructed history, and as a record it carries the one constant of that history". (*What the Beginning Left Behind*, Chapter 8, section "Fixed Once"); the stance from which the account is stated: "The account is stated from the stance of the one who measures, as every finding is". (*The View from Inside*, Chapter 15, section "An Astronomer Long Ago"). What she would have found from her own stance, in her own present, is a question about an act that is not ours; the account is stated from the stance of the one who measures and makes no claim about it (*The View from Inside*, Chapter 15, section "An Astronomer Long Ago": "No measurement we make can reach her present, and this account makes no claim about it").

The equation of state is exactly minus one. *What the Beginning Left Behind*, Chapter 8, section "Fixed Once"; *The View from Inside*, Chapter 15, section "Exactly Constant" (w, the ratio of pressure to energy density, equals -1). Measured: $w = -1.03 \pm 0.03$, from the microwave background with baryon acoustic oscillations and type Ia supernovae (Aghanim and others 2020), as *Side by Side, Volume One*, Appendix B, Section IV, Prediction 8, "Measured", records it. The other measurements that bear on the equation of state are in Appendix A of this book.

The far future of a true constant. Greene's sentence is from *The Fabric of the Cosmos*, chapter 10, section "The Missing 70 Percent"; the far future in the collection: *What Comes After the Dark*, Chapters 1 to 8 (Chapters 16 to 19 of this book).

The small number measures the observer's place. *The View from Inside*, Chapter 16, "The Observer Inside", section "The Fish Again" ("The most embarrassing number in physics is a measure of the observer's own place in the world"); the one value with no selection: *The Compactification*, Chapter 2, Section IX ("The cosmological constant is then not selected from a distribution of vacua").

Chapter 11. Is Our Universe the Only One?

The opening scene. Brandon Carter's paper "Large Number Coincidences and the Anthropic Principle in Cosmology" was given at Symposium No. 63 of the International Astronomical Union (Copernicus Symposium II), Kraków, 10 to 12 September 1973, and printed in M. S. Longair (editor), *Confrontation of Cosmological Theories with Observational Data* (Dordrecht: Reidel, 1974), pages 291 to 298, DOI https://doi.org/10.1017/S0074180900235638. The sentence quoted in the chapter is Carter's, from that paper: "Although our situation is not necessarily central, it is inevitably privileged to some extent". Greene tells the scene in *The Hidden Reality*, chapter 6, section "Cosmological Anthropics". Told here as Greene's account, with Carter's words from the primary paper.

Greene's nine varieties of other universe and where he says they stand: *The Hidden Reality*, chapter 1, sections "Varieties of Parallel Universes", "Universe and Universes" and "The Cosmic Order"; chapter 7, section "Accessible Multiverses" (the count of five before the chapters on quantum theory); chapter 11, section "Is the Copernican Pattern Fundamental?". Greene's Quilted Multiverse: chapter 2, sections "Infinite Space and the Patchwork Quilt", "Finite Possibilities" (the fly in the bedroom, and the principle that changes too small to be measured are no changes), "Cosmic Repetition" and "What to Make of This?". His count of arrangements in one horizon-sized patch is about ten to the power (ten to the power one hundred and twenty-two), a one followed by ten to the power one hundred and twenty-two zeros, written by him as $10^{10^{122}}$. Greene's horizon-sized universes in *Until the End of Time*: chapter 10, section "Thought and the Multiverse".

Eternal inflation and the bubbles: *The Hidden Reality*, chapter 3, sections "Eternal Inflation", "Changing Perspectives" (the "indirect yet compelling case" for the second version of parallel worlds, since a great many versions of inflation are eternal), "Swiss Cheese and the Cosmos" and "Space in a Bubble Universe" (each bubble finite from outside and infinite from inside). Greene tells the same idea in *The Fabric of the Cosmos*, chapter 11, section "Boltzmann Redux", and in *The Elegant Universe*, chapter 14, section "Cosmological Speculation and the Ultimate Theory", where he reports Andrei Linde's proposal that inflation may happen repeatedly in isolated regions.

Branes and the cycle: *The Hidden Reality*, chapter 5, sections "Branes and Parallel Worlds", "Time, Cycles, and the Multiverse" (the other member universes "are in our past and future", and ours "the latest in a temporal series"; the time scale for a full cycle, as Steinhardt and his co-workers estimated it, "about a trillion years") and "The Past and Future of Cyclic Universes"; *The Fabric of the Cosmos*, chapter 13, section "Cyclic Cosmology". The landscape: *The Hidden Reality*, chapter 5, section "In Flux" (about ten to the power five hundred forms, written 10^{500}); chapter 6, sections "The String Landscape", "Quantum Tunneling in the Landscape"

and "The Rest of Physics?". Carter, the shoe shop and Weinberg: chapter 6, sections "Cosmological Anthropics" and "Life, Galaxies, and Nature's Numbers"; Weinberg's paper is S. Weinberg, "Anthropic Bound on the Cosmological Constant", *Physical Review Letters* 59, 2607 (1987), DOI https://doi.org/10.1103/PhysRevLett.59.2607. Greene's question and his line: chapter 6, section "Is This Science?"; chapter 7, section "Science and the Inaccessible I"; chapter 11, section "Can We Test the Multiverse Theories We've Encountered?" ("I draw the line at ideas that have no possibility of being confronted meaningfully by experiment or observation").

The principle of comparison, which the chapter sets beside Greene's "Changes that are too small to be measured, even in principle, are not changes at all". (*The Hidden Reality*, chapter 2, section "Finite Possibilities"): *What Comes After the Dark*, Chapter 6 "What Happens to Someone Who Is Still There?", section "A Standard That Needs Something There"; the same principle in *The Gravitational Companion*, "One Mass, Three Elevators", Section II. The standard of not rotating "is fixed only by matter within reach" (same section of *What Comes After the Dark*). Greene reaches his sentence from quantum theory, for the position and speed of an object ("be it a fly or an electron"); the collection states the principle of every distinction and reaches it from what a comparison is.

The far patches as further parts of one world: *Side by Side, Volume Two*, "The Structure and the Act" (Max Tegmark), Section VI "The Four Levels, Level by Level", Level I (the sentence quoted, and "On Level I the two accounts agree in substance"). The principle of existence: *Incommensuration*, Chapter 9 "Each Axiom Derived Again", Section I "Axiom I: The Asymmetry of the Relational", resting on the principle of identity ("that which is indistinguishable must be the same").

The search for matching circles: P. Petersen, Y. Akrami, C. J. Copi, A. H. Jaffe, A. Kosowsky, D. P. Mihaylov, G. D. Starkman, A. Tamosiunas, J. R. Eskilt, Ö. Güngör, S. Saha and Q. Taylor, "Cosmic topology. Part I. Limits on orientable Euclidean manifolds from circle searches", *Journal of Cosmology and Astroparticle Physics* 2023 (01), 030, DOI https://doi.org/10.1088/1475-7516/2023/01/030: the shortest closed path through our loc-

ation that cannot be shrunk to a point is longer than 98.5 percent of the comoving diameter of the last scattering surface. Reported in *The Weight of Possibility*, Chapter 3, Section II "The Count: Six Against One", and in *Side by Side, Volume One*, "The Sealed Record", Prediction 23 ("Measured"). Greene's report that no repeated images of stars or galaxies had been found, and that this "doesn't prove that space is infinite": *The Hidden Reality*, chapter 2, section "Reality in an Infinite Universe".

The curvature of space: Ω_K (read "omega K", the share of the expansion's budget carried by the curvature of space) $= 0.001 \pm 0.002$, from the microwave background with baryon acoustic oscillations: N. Aghanim and others (Planck Collaboration), "Planck 2018 results. VI. Cosmological parameters", *Astronomy and Astrophysics* 641, A6 (2020), DOI https://doi.org/10.1051/0004-6361/201833910. Reported in *Side by Side, Volume One*, "The Sealed Record", Prediction 9 ("Measured"). The flatness of space, derived from the state at the turn: *The Gravitational Passage*, Chapter 18, Section IX; *Side by Side, Volume One*, "Is Space Flat?"; stated in Chapter 9 of this book.

Space infinite for every comparison, and the open space counted as one: *The Weight of Possibility*, Chapter 3, Section II "The Count: Six Against One". The region within the particle horizon is a ball, with one cohomology class; the horizon "is defined by a limit of comparison", and "Within the limit of comparison, that ball cannot be told apart by any comparison from a ball of an infinite flat space"; by the principle of identity, "for a registration, space is infinite up to the limit of distinguishability", and so "the open space counts as one, whatever it is beyond the horizon". *The Weight of Possibility*, on the dark matter, uses this step to count the independent kinds of shape a measurement meets in the three large dimensions of space, which that book calls the open space; a solid ball has one such kind, and Chapter 9 of this book uses the count of one for the ordinary matter. The open space is the three dimensions of space that are not compact, as against the six internal dimensions (same book, Terms, "open dimensions"). The same step in plain words: *The Weight of the Unseen*, Chapter 5, 5.3 ("A ball has one independent kind").

Where a test applies: *Side by Side, Volume Two*, "The Open Future" (George Ellis), Section V "The Integrity of Physics", quoting *Physics Is Not Math*, Chapter 19, Section III. The same section states what kind of claim the collection makes about the world: "Every claim the collection makes about the world is of that kind, and each is stated beside the measurement that bears on it". The collection and Ellis on the multiverse: "The two accounts reach the same verdict on different grounds" (*Side by Side, Volume Two*, "The Registered Predictions Beside George Ellis", Section V, entry "The multiverse"). The chain and the cable: *Physics Is Not Math*, Chapter 12 "Chains and Cables", Section I "A Chain and a Cable".

The anthropic principle as a selection effect: *The Geometric Route*, Chapter 32 "Self-Descriptive Closure in the Physics", Section IV "The Physical Analogue", where the selection effect stands beside the self-descriptive closure, "a uniqueness theorem applied to the structure of comparison itself". The collection's own use of selection reasoning, for the absolute epoch at which comparers exist: *The Gravitational Companion*, "Open Questions as Research", Section I "Why This Epoch, Asked from Outside", citing Dicke (1961) and Carter (1974); the absolute epoch is about ten to the power sixty-one Planck times (10^{61} t_{Pl}, where t_{Pl} is read "the Planck time"); the era of star formation runs from about 10^6 to about 10^{14} years (Adams and Laughlin 1997); the facts answer the question "to an order of magnitude". The same section states that a selection account stated from outside gives a distribution of epochs, and a distribution singles out no epoch as real (also *The Gravitational Passage*, Chapter 13, "Questions" after Section VII). One world, with its constants fixed: *Side by Side, Volume Two*, "The Open Future" (George Ellis), Section VI "The Multiverse" ("the structure of comparison yields one world, with its constants fixed").

No long inflation, so no eternal inflation and no bubbles: *Side by Side, Volume Two*, Tegmark chapter, Section VI, Level II ("It needs no inflationary phase and excludes a long one, so there is no eternal inflation to make the regions"); *The Compactification*, Chapter 6 "The Primordial Depar-

tures", Section VIII "Inflation"; *What the Beginning Left Behind*, Chapter 13, section "A Long Inflation Is Excluded" ("given that the first ripples are the carried form, a long inflation cannot have happened"). The identification of the first ripples with the form of the content carried across the turn: *What the Beginning Left Behind*, Chapter 11; made in Chapter 9 of this book.

One internal space, so no landscape: the internal space is $\mathbb{C}P^2 \times S^2$ (read "the complex projective plane times the two-sphere"), with the ratio of its radii the square root of six and its scale a few Planck lengths. It is reached by two routes: the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ (read "the complex numbers, plus the quaternions, plus the three-by-three complex matrices") of *Non-Commutative Geometry and the Immanent Metaphysics*, Theorem 12.1, and the classification of compact homogeneous spaces in *The Gravitational Passage*, Chapter 9. Recalled in the chapter from Chapter 7 of this book, where the two routes are told. *The Compactification*, Chapter 2 "One Vacuum", Section VI "No Flux, No Flux Integers" (no form field, no flux integer; the integers the internal space carries are its own topological counts, the Euler characteristic $3 \times 2 = 6$ and $c_1 = 3$) and Section IX "One Algebra, One Space" ("Neither route chooses the space from a set of candidates, so there is no landscape of internal spaces and no problem of selecting one"). Why no field is threaded through the six: *What Comes After the Dark*, Chapter 4 "Could Empty Space Change Its Nature?", section "No Threads, No Landscape" (a threaded field would add an energy of its own to the energy of empty space, changing the matter fraction, and its strength would be a second number beside the strength of gravity). The pictures: the ring, same section; the ball among valleys, *What the Beginning Left Behind*, Chapter 5 "One Vacuum", section "A Ball Among Valleys".

No bulk for branes: *What Comes After the Dark*, Chapter 11 "Has This Happened Before?", section "Colliding Worlds"; *At the Edge of Reality*, Chapter 7 "Six Dimensions of Possibility", section "Why We Never See Them".

The two cycles, what they share: *What Comes After the Dark*, Chapter 11, section "Colliding Worlds" (no first turn; the speeding-up empties each observer's region; the transition is no place where physics fails). Where they differ: same section (the brane cycle ends each cycle with a field whose energy turns negative; in the collection the dark energy is the cosmological constant, with equation of state w exactly minus one, read "the ratio of pressure to energy density is minus one", so the acceleration of our epoch never ends, and "the acceleration does not prevent" the ends of the black hole interiors; the turn is a limit, with no temperature and no duration); *What Comes After the Dark*, Appendix A, entry for Chapter 11; *At the Edge of Reality*, Chapter 20 "The Cycle Without a First Turn". The epoch as one cycle of one world: *Side by Side, Volume Two*, "The Open Future" (George Ellis), Section I; *At the Edge of Reality*, Chapter 20, section "What Is New Here" ("the epochs form a single sequence, and not a branching population"). Smolin and the one next epoch: *Side by Side, Volume Two*, "Time Is Real" (Lee Smolin), Section V "Cosmological Natural Selection and the Turn", quoting *At the Edge of Reality*, Chapter 11 (the ends of all interiors are one); Greene names Smolin in *The Elegant Universe*, chapter 14, section "Cosmological Speculation and the Ultimate Theory".

The first waves: Greene's statement that a brane collision would make first waves "almost certainly" too weak to leave a lasting signal, and that early gravitational waves would count against the cyclic model, is in *The Hidden Reality*, chapter 5, section "The Past and Future of Cyclic Universes". The bound of 2021, told in Chapter 9: P. A. R. Ade and others (BICEP/Keck Collaboration), "Improved Constraints on Primordial Gravitational Waves Using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season", *Physical Review Letters* 127, 151301 (2021), DOI https://doi.org/10.1103/PhysRevLett.127.151301. The bound of 2022: the tensor-to-scalar ratio r (read "r", the power of the first waves divided by the power of the first ripples) is below 0.032 at 95 percent confidence: M. Tristram and others, "Improved limits on the tensor-to-scalar ratio using BICEP and Planck data", *Physical Review D* 105, 083524 (2022),

DOI https://doi.org/10.1103/PhysRevD.105.083524; reported in *Side by Side, Volume One*, "The Sealed Record", Prediction 10 ("Measured"). The collection's first ripples are scalar, sourced by departures of density, and waves of the metric arise from them only at second order (*At the Edge of Reality*, Chapter 19 "The Faint Pattern in the Sky", section "Only Departures of Density"; the sealed record gives the ratio as far below 10^{-6}). The coming measurements that bear on the two cycles are in Appendix A (*Side by Side, Volume One*, "The Sealed Record", Predictions 8 and 10).

The anthropic argument at its strength: *The View from Inside*, Chapter 3 "Why Nothing Else Has Worked", section "Strategy Two: Accept the Landscape" ("This reasoning is called the anthropic principle. It is logically impeccable", and, if the value is selected by our existence, "It is not a consequence of the laws of nature but an accident of location"). The value without selection: *The View from Inside*, Chapter 16 "The Observer Inside", section "Two Answers" (three degrees of freedom, "three tiles out of all the tiles on the edge of what has reached us"); *The Compactification*, Chapter 2, Section IX (the constant fixed by the identity of Chapter 4, from the count of channels $c_1 = 3$ (read "c one equals three": the three channels in which a measurement registers, counted as the independent classes of the complex projective plane, *The Weight of Possibility*, Chapter 3, Section II), and the particle horizon of the present measurement). The share and the total: *The View from Inside*, Chapter 10 ("So the share enters the equations as that constant. That constant is the cosmological constant". and "The total is there, and it curves nothing"). The identification of the cosmological constant with the accessible share is made in Chapter 10 of this book. Matter fraction $\Omega_m = 0.311 \pm 0.001$ (read "omega m"), with $\Omega_\Lambda = 0.689$ (read "omega lambda"): *Side by Side, Volume One*, "The Sealed Record", Prediction 7; measured 0.315 ± 0.007 (Aghanim and others 2020). In Chapter 11 I say these two matter fractions in words as 31.1 percent and 31.5 percent. The cosmological constant Λ (read "lambda"): 2.56 against 2.52 measured, in units of 10^{-47} GeV^4 (*What the Beginning Left Behind*, Chapter 8). Greene's "deeper explanation" question for a shop with one size: *The Hidden Reality*, chapter 6, section "Cosmological Anthropics".

Derived pure numbers and the four measured inputs: *Side by Side, Volume One*, "Where the Accounts Part", "Can the Constants Be Derived?"; the ratio of dark to ordinary matter $6 - 2/\pi$ (read "six less two divided by pi"), about 5.3634 (Prediction 16); the four measured inputs, Newton's constant, the strength of light, the strength of color and the vacuum value of the Higgs field (*Side by Side, Volume One*, "The Sealed Record", Section IX). The closed list of kinds: *The Kinds of Particle*, Chapter 6, Section IV; the three generations as the three rotations of the content, *The Kinds of Particle*, Chapter 7, Section I. The structure of comparison, recalled in the words of Chapter 6 of this book (the three roles of comparison and the axioms that relate them, its content and its context, and the theorem): *The Story of the Particles*, "The Scientists", section I, "An Empty World" ("the structure of the act by which anything can be known"), as the notes to Chapter 6 give it; the same structure in *The Kinds of Particle*, Chapter 1 "Comparison and Its Counts", Summary ("six intrinsics in three inseparable pairs, three positions related by three axioms", and the theorem that limits their compounds).

Greene on calculating the properties of the particles: *The Hidden Reality*, chapter 6, section "The Rest of Physics?" ("at least in our current understanding of string theory").

Greene's measure problem: *The Hidden Reality*, chapter 7, section "Dividing Up Infinity"; notes to chapter 7, note 12 ("My view, one held by a small but growing number of researchers, is that the measure problem is tapping into a deep problem at the very root of physics, one that may require a substantial overhaul of foundational ideas"). Told here as Greene's account. The rule set beside it: the lottery, *What the Beginning Left Behind*, Chapter 10 "The Counts of an Epoch", section "One Draw Among Many?"; every count has a context, *The Compactification*, Chapter 3 "The Past Hypothesis at the Turn", Section V "Every Count Has a Context"; the vantage no comparison can occupy, *Side by Side, Volume Two*, Ellis chapter, Section VI "The Multiverse". No book of the collection discusses the

measure problem of eternal inflation by name. In Chapter 11 I set the rule beside Greene's finding and state nothing of the collection's about the measure problem.

The collection's case for one world, strand by strand, with the book that carries each: the far patches (*Side by Side, Volume Two*, Tegmark chapter, VI); the first ripples (*What the Beginning Left Behind*, Chapters 11 and 13); the one internal space (*The Compactification*, Chapter 2, IX); the dimensions of possibility (*At the Edge of Reality*, Chapter 7); the one end of all interiors (*At the Edge of Reality*, Chapter 11); the value of the cosmological constant (*The View from Inside*, Chapter 16). The strands come from separate books on separate premises: the far patches joined to their neighbors; the first ripples, which leave no room for a long inflation; the one shape of the six, reached by two routes; the six as dimensions of possibility, with no bulk; one end of all the interiors, and so one next epoch; and the energy of empty space given without selection. Each of the chapter's answers to the bubbles, the landscape, the branes and the cycle stands on its own premises. Each strand ties the collection's account to something a measurement finds, as the matter fraction of 0.311 stands beside the measured 0.315.

A region no path can reach is no place at all: *What Comes After the Dark*, Chapter 10 "Could Anyone Escape?", section "No Other Sky Along Any Path".

Greene's telling of two points: that each bubble is finite from outside and infinite from inside (*The Hidden Reality*, chapter 3, section "Space in a Bubble Universe"), and Weinberg's bound of 1987, by name (chapter 6, section "Life, Galaxies, and Nature's Numbers"). Told here as Greene's account; the collection gives no reply on it.

Chapter 12. Are There Other Worlds in the Quantum, the Hologram, the Computer?

Opening scene, Everett, Wheeler and Bohr, 1956 to 1957; DeWitt, 1970. Greene, *The Hidden Reality*, chapter 8, section "Quantum Reality". Primary papers: H. Everett III, "'Relative State' Formulation of Quantum Mechanics", *Reviews of Modern Physics* 29 (1957), 454. B. S. DeWitt, "Quantum Mechanics and Reality", *Physics Today* 23, no. 9 (1970), 30.

Greene's account of the Quantum Multiverse: *The Hidden Reality*, chapter 8, sections "Not So Fast", "Many Worlds", "A Tale of Two Tales", "When Is an Alternative a Universe?", "Uncertainty at the Cutting Edge", "Probability and Many Worlds" (Everett's weighting of worlds by wave height, "meant to address this mismatch", of which Greene writes "But this is perplexing. And controversial"; the agreement of those who have studied the thesis that "he didn't convincingly spell out how to achieve it"; the Oxford argument of David Deutsch, Simon Saunders, David Wallace and Hilary Greaves, on how a gambler who holds the Many Worlds approach should bet; "Some are convinced that this completes Everett's program. Some are not"), "Predictions and Understanding", and note 11 ("I'd like the Many Worlds approach to be right. At the same time, I maintain a healthy skepticism, fueled by the difficulties of integrating probability into the framework, so I'm fully open to alternative lines of attack"); *The Elegant Universe*, chapter 4, section "Waves of What?"; *The Fabric of the Cosmos*, chapter 7, section "Prochoice". Linearity, in symbols: for Schrödinger's evolution U (read "U", the operator that carries a state at one time to the state at a later time), $U(a\ \psi_1 + b\ \psi_2) = a\ U\psi_1 + b\ U\psi_2$, where ψ_1 and ψ_2 (read "psi one", "psi two") are two alternatives and a and b are complex numbers, the amplitudes. Greene's note 12 to chapter 8 (a reversal experiment that would tell Copenhagen from Many Worlds) is told in Chapter 4; the statement of the collection that bears on it is *Side by Side, Volume One*, "The Sealed Record", Prediction 28 ("a reversal of evolution, by an inverse in a formalism or by a pulse in a laboratory, does not

remove a finding from the order of findings"), with its application to Deutsch's reversible observer in *Side by Side, Volume One*, Appendix D, Section IV.

Greene's account of black-hole entropy and the Holographic Multiverse: *The Hidden Reality*, chapter 9, sections "The Second Law and Black Holes", "Hawking Radiation", "Entropy, Hidden Information, and Black Holes", "Locating a Black Hole's Hidden Information" (Susskind's proposal: the freely falling observer and the distant observer), "Beyond Black Holes" (the bound on the information a region can hold, less than the area of its surface in Planck units), "String Theory and Holography" (Maldacena's results "established a mathematical proving ground in which ideas regarding holographic universes could be made explicit and investigated quantitatively"; Witten's black hole as a hot quantum field theory on the boundary, with the note to that section: "conclusive evidence that all information that falls into a black hole is ultimately available to the outside world"), "Parallel Universes or Parallel Mathematics?" ("is deemed conjectural"; "a great many stringent mathematical tests"; Greene speaks of "Maldacena's more abstract argument"), "Coda: The Future of String Theory" ("a firm formulation"; "if Maldacena's insights are fully valid, as all available evidence attests, string theory and quantum field theory may very well be two sides of the same coin"), and note 7. The entropy Greene reports is $S = k_B\, c^3\, A \,/\, (4\, G\, \hbar)$, read "S equals Boltzmann's constant times the speed of light cubed times the horizon area A, divided by four times Newton's constant times the reduced Planck constant"; in Planck units, $S/k_B = A \,/\, (4\, l_P{}^2)$, where l_P (read "el sub P"), the Planck length, is about 1.6×10^{-35} meters. The proton's measured charge radius is about 0.84×10^{-15} meters, so its width is about 1.7×10^{-15} meters, and the ratio of that width to the Planck length is about 10^{20}, which the chapter writes as "roughly a hundred billion billion". Maldacena's setting is string theory on five-dimensional anti-de Sitter space times a five-sphere, $AdS_5 \times S^5$, equivalent to four-dimensional supersymmetric Yang-Mills theory on its boundary (J. Maldacena, *Advances in Theoretical and Mathematical Phys-*

ics 2 (1998), 231). Susskind's proposal of the freely falling observer and the distant observer, which physicists call black-hole complementarity, is reported in the section "Locating a Black Hole's Hidden Information". Told here as Greene's account; the collection gives no reply on it.

Greene's account of the Simulated Multiverse: *The Hidden Reality*, chapter 10, sections "The Stuff of Thought" ("The possibility of artificial sentience clearly relies on a functionalist viewpoint"; "The assumption could be wrong"; "Let's take a functionalist viewpoint, and see where it leads"), "Simulated Universes", "Are You Living in a Simulation?" (Bostrom's argument, "assuming that realistic simulations prove possible"), "Seeing Beyond a Simulation" (the errors of rounding; a simulator who "might reset the program and erase the inhabitants' memory of the anomalies"; and "So it would seem a stretch to claim that a simulated reality would reveal its true nature through glitches and irregularities"), "Simulating Babel", "The Roots of Reality". Bostrom: N. Bostrom, "Are We Living in a Computer Simulation?", *Philosophical Quarterly* 53 (2003), 243.

Greene's account of the Ultimate Multiverse and of mathematics: *The Hidden Reality*, chapter 10, sections "The Library of Babel" ("the most expansive possible"), "Multiverse Rationalization" ("Cooked up specifically to address one issue, the Ultimate Multiverse lacks the independent rationale characterizing the multiverses discussed in earlier chapters"; his preference for a process that generates a multiverse; simulated worlds as "a mechanism for generating a particular privileged part of the Ultimate Multiverse"), "Simulating Babel" ("a scaled-down version of the Ultimate Multiverse that includes only universes arising from computable functions", which "can emerge from a process", spawned by "An army of future computer users"); chapter 11, section "Should We Believe Mathematics?" ("I've titled this book in the singular to reflect the unique and uniquely powerful theme that underlies them all"). Dirac and Anderson, as Greene tells them (chapter 10, section "The Library of Babel"): P. A. M. Dirac, *Proceedings of the Royal Society A* 133 (1931), 60; C. D. Anderson, *Physical Review* 43 (1933), 491.

The account uses the same unitary evolution as the Everett program: *Side by Side, Volume One*, chapter "The Wave Function and the One Who Finds" (Carroll), Section III "The Branches"; the credit to the program (one linear dynamics, applying to closed systems and the universe as a whole, a branching structure of quasi-classical worlds with decoherence) in *The Unquantized Arena*, Chapter 12 "The Interpretations as Modal Strategies", Section III "Everett", as quoted in *Side by Side, Volume One*, chapter "Tasks, Runs and Worlds" (Deutsch), Section V "The Multiverse". Unitary means U†U = 1 (read "U dagger U equals one": the evolution preserves the total probability and can be inverted). The glosses of "formalism" and "third-person" are those of this book's Chapters 8 and 14.

Decoherence prepares the records, and the act completes the measurement with one finding: *Why the Quantum Is the Way It Is*, Chapter 17, section "What the Surroundings Do" ("Decoherence is not collapse"; "Decoherence prepares the conditions for the measurement, by selecting the pointer basis"; "Decoherence prepares the records, and the act completes the measurement with one outcome"); *The Unquantized Arena*, Chapter 11, Section III, Proposition 11.2.

The smooth rule held in the tests: molecules of more than 25,000 atomic mass units still showed interference (*Why the Quantum Is the Way It Is*, Chapter 18, section "Objective Collapse"; Y. Y. Fein and others, "Quantum superposition of molecules beyond 25 kDa", *Nature Physics* 15 (2019), 1242); the underground search for the radiation of gravity-related collapse ruled out the Diósi and Penrose model with no free parameter (same section: "they ruled out its natural version, the one with no adjustable constant"; *Side by Side, Volume One*, "The Sealed Record", Prediction 2, "Measured"; S. Donadi and others, "Underground test of gravity-related wave function collapse", *Nature Physics* 17 (2021), 74, at the Gran Sasso National Laboratory). Diósi: L. Diósi, *Physics Letters A* 120 (1987), 377, and *Physical Review A* 40 (1989), 1165; Penrose: R. Penrose, *General Relativity and Gravitation* 28 (1996), 581. Greene's test, as he states it: Greene names a break in

that rule as one finding that would count against Everett's worlds (*The Hidden Reality*, chapter 11, section "Can We Test the Multiverse Theories We've Encountered?").

Where the difficulty of odds comes from: *Why the Quantum Is the Way It Is*, Chapter 18, section "Many Worlds" ("When every outcome occurs, a probability needs an agent who faces outcomes and weighs them"); *The Unquantized Arena*, Chapter 12, Section III ("the decision-theoretic derivations take such an agent and its choices as premises"). The collection begins with the comparer, the act and the range of what could be found, "so it has nothing to bring back": *Side by Side, Volume One*, Carroll chapter, Section III. Born's rule is derived through a theorem that Andrew Gleason proved in 1957. Two steps of that derivation show what it starts from. A weight, the odds of an outcome, is the steady frequency with which the outcomes of the same comparison, made on the same state in the same context, recur. And a finding has one weight in every measurement that contains it; *Side by Side, Volume One* gives the reason a finding keeps one weight: "a difference of weight with no difference of content would be a difference no comparison of the content could register". Born's rule, derived through Gleason's theorem, whose premises *The Quantum Passage* finds in comparison: *The Quantum Passage*, Chapter 7, Sections II and IV, as *Side by Side, Volume One*, Carroll chapter, Section III, and "The Sealed Record", the prediction resting on *The Quantum Passage*, Chapter 7, report it. The four steps there: a measurement is a comparison, and by the Incommensuration Theorem no lawful description fixes every finding; a weight is the stable relative frequency of the same comparison, performed on the same state in the same context ("This is an identification, and it is stated as one"); Section IV supplies the two premises of Gleason's theorem, one of which, that a finding has one weight in every measurement that contains it, rests on "a difference of weight with no difference of content would be a difference no comparison of the content could register"; the state is the whole of what bears on the weights. A. M. Gleason, "Measures on the Closed Subspaces of a Hilbert Space", *Journal of Mathematics and*

Mechanics 6 (1957), 885. Born's rule in symbols: the probability of outcome k is $|\langle k|\psi\rangle|^2$, read "the squared size of the amplitude of the state ψ along the outcome k".

Explaining is more than predicting: *Side by Side, Volume One*, Deutsch chapter, Section VI "Good Explanations".

Black-hole entropy is a physical count, and its dependence on area is the count made from outside, through the horizon: *The Unquantized Arena*, Chapter 17 "Black-Hole Information and the Page Curve", Section III "The Premise Removed" ("The count is real".) and Section IV "Why Area, Not Volume" ("So the physical count of a horizon's information is the count made from a place, and it goes by area"), where the step rests on Axiom I. That book makes the identification in its own words ("I therefore identify the dependence of equation (17.2) on area as the immanent count"). I give the argument in Chapter 6 of this book, "The Count of a Black Hole", and Chapter 12 recalls the argument without making the identification again. The identification makes no claim on the coefficient one quarter, which it takes from Hawking (1975) (same section; *The Compactification*, Chapter 4, Section II).

Information is not destroyed in a black hole: *The Unquantized Arena*, Chapter 17, Section III, Proposition 17.3; *Two Theories, One World*, Chapter 25. Information in two senses (the form of the content, "the specification of what the matter is and of which alternatives it holds, carried with the matter wherever the matter goes"; a count of states relative to a context, of which the entropy of Bekenstein and Hawking is one): *The Unquantized Arena*, Chapter 17, Section III; *At the Edge of Reality*, Chapter 18. For a hole that evaporates while an outside world remains, the Page curve, with quantum mechanics unmodified: *Two Theories, One World*, Chapter 25; *What the Beginning Left Behind*, Chapter 9; *The Unquantized Arena*, Chapter 17, Section V, where the Page curve follows as a result under that section's condition (C) and premises. For a hole formed from a star (evaporation time over 10^{67} years for a solar mass), the information is carried with the content to the turn and re-emerges in the beginning of the new epoch, as

its departures from uniformity; it is not accessible from the outside world of the original epoch, and it is not destroyed: *At the Edge of Reality*, Chapter 18, section "Information Is Not Destroyed" ("It is not accessible from the outside world of the original epoch: it is lost to every observer who remains outside the horizon, and it is not destroyed"); *The Gravitational Passage*, Chapter 18, Section XIII. Of the outside world of the original epoch, *The Unquantized Arena*, Chapter 17, Section VII, states the scope: "The account asserts neither the return of this information to the outside world nor its loss to it". For a hole formed from a star the collection's answer differs from the one Greene reports from Witten (*The Hidden Reality*, chapter 9, note to the section "String Theory and Holography": "conclusive evidence that all information that falls into a black hole is ultimately available to the outside world"); Chapter 18 of this book, section "Two Things Called Information", sets out the difference. The first ripples as the form of the content carried through the turn: *What the Beginning Left Behind*, Chapter 11, identified in Chapter 9 of this book, section "The Worn Stairs".

The freely falling body meets nothing at the horizon: *The Unquantized Arena*, Chapter 17, Section VIII "Questions".

The correspondence accepted as exact, the clearest example of two descriptions of one system: *Two Theories, One World*, Chapter 23 "Two Descriptions That Translate", section "The Strongest Objection" ("It is two descriptions of one system, one in each of the two characters the theorem allows, joined by an exact dictionary. Read that way, it is the clearest example of this book's thesis that physics has found"). Greene reports that Maldacena's result "is deemed conjectural" and that the holographic ideas have come through "a great many stringent mathematical tests" (*The Hidden Reality*, chapter 9, section "Parallel Universes or Parallel Mathematics?"); the collection accepts the correspondence as exact, which goes further than Greene's report.

The quark-gluon plasma at the Relativistic Heavy Ion Collider, Brookhaven National Laboratory, as Greene tells it, with his two conditions (the boundary theory does not contain gravity; read conservatively, the calculation is a string-based mathematical tool): *The Hidden Reality*, chapter 9, section "Parallel Universes or Parallel Mathematics?". The holographic value is $\eta/s = \hbar / (4\pi k_B)$, read "the shear viscosity divided by the entropy density equals the reduced Planck constant over four pi times Boltzmann's constant" (P. Kovtun, D. T. Son and A. O. Starinets, *Physical Review Letters* 94 (2005), 111601). Experimental summaries: J. Adams and others (STAR), *Nuclear Physics A* 757 (2005), 102; K. Adcox and others (PHENIX), *Nuclear Physics A* 757 (2005), 184.

A fixed program shows itself to a fine enough measurement: *Causality Without Determinism*, Chapter 5 "Computation, Consciousness and Simulation", Section VIII "Simulation"; stated as *Side by Side, Volume One*, "The Sealed Record", Prediction 29 (Appendix A of this book, entry "A fine enough measurement shows a rule's least step"). That a description reaches what it describes as closely as one wishes without ever containing it is the cross-modal principle of *Measurement*, as *Physics Is Not Math*, Chapter 8, Section VII, states it. Greene's rounding argument and his judgment that a simulator could hide the errors: *The Hidden Reality*, chapter 10, section "Seeing Beyond a Simulation".

Mathematics keeps its necessity, its exactness and its power to hold structure in view as a whole, and is the finest instrument for what is laid out: *Physics Is Not Math*, Chapter 11 "Why the Order Surprises", Section VII "What Each Discipline Keeps"; *Comparison; The Necessary Basis*, Chapter 10, Section IX.

The chess table (the game tonight is real; the rules and the score sheet exist): *What Cannot Be*, Chapter 4 "Process, Pattern and Possibility", section "Interaction Before Existence". "A law exists; a measurement is real": *Comparison; The Necessary Basis*, Chapter 10, Section II. The glosses of "exist" and "real" are those of this book's Chapter 2.

The universal state exists; for every comparer one outcome is real; the many-worlds account takes the structure that exists for the whole of what is real: *Side by Side, Volume One*, Carroll chapter, Section III "The Branches" ("The other branches are never given in any act of that comparer, and the many-worlds account takes the structure that exists for the whole of what is real").

The alternatives are possibilities of one world; the quantum computer's work is done among the possibilities of one world: *Side by Side, Volume One*, Deutsch chapter, Section V "The Multiverse", which tells Deutsch's argument from *The Fabric of Reality* (a quantum computer factoring a large enough number would, by its interference, do more computation than the whole visible universe could hold) and states the standing of possibility: "Possibility is real in its own way: it is the transcendent, the range from which every finding is one selection, and it has six dimensions in the collection's geometry". and "Possibility is the transcendent role of every comparison, distinct from the act and from the record and inseparable from both, and not a lesser kind of actuality; so no further worlds are needed to hold the computation". In Chapter 12 I keep the word "real" in the sense of this book's Chapter 2 (given in an actual act of comparison). In Chapter 12 I state the standing of possibility in that section's other words: the range from which every finding is one selection, distinct from the act and from the record and inseparable from both. Also *Why the Quantum Is the Way It Is*, Chapter 13, section "A Second Route, from Possibility" ("Superposition is the default, and definite outcomes are the exception, produced only when a specific comparison is performed"). In the collection's formal vocabulary, possibility is the transcendent role of every comparison. Greene's question on the sum over histories: *The Fabric of the Cosmos*, chapter 7, section "Prochoice".

Many worlds, scope (i), as a description of a whole world, excluded: *Side by Side, Volume One*, Deutsch chapter, Section V ("As a description of a whole world, it is excluded"); *Physics Is Not Math*, Chapter 8 "Physics Has the Arrow", Section VII "Three Grounds". The Incommensuration

Theorem in the words of *An Immanent Metaphysics*, aphorism 1.43-2, as *Home in the World*, "For the Specialist", section "The Incommensuration Theorem", quotes them: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". Symmetry is a sameness of content with a difference of context; continuity a sameness of content with a sameness of context; asymmetry a difference of content with a difference of context; discontinuity a difference of content with a sameness of context (aphorism 1.43-1). In a whole world fixed by one law, the law the same at every place and step is symmetry perfected, and every state fixed with no gap is continuity perfected.

Many worlds, scope (ii), as an account of knowing, limited: *Side by Side, Volume One*, Deutsch chapter, Section V ("As an account of knowing, it is limited"); *Incommensuration*, Chapter 15 "The Theorem Among Other Results", Section III, subsection "The Bound Triple Across the Interpretations" ("This limits what many-worlds can claim, without refuting it"; the three absent features: no subjectivity, since there is no fact of which branch I find myself in; no hard randomness, since the evolution is deterministic; no arrow of time, since the fundamental equation is symmetric in time); *Why the Quantum Is the Way It Is*, Chapter 18 "The Interpretations, Each Missing a Role", section "What the Interpretations Were Doing", in its own words: "The reason the theorem gives limits what many worlds can claim, and it does not refute many worlds". The three features the formalism lacks (the side a comparison is made from, the one-way order, hard randomness) are, in the collection's formal name, the bound triple; they are features of continuity with asymmetry, the character of context, and a description holding only symmetry with discontinuity, the character of content, cannot contain them (same places). The map and the walk: *Why the Quantum Is the Way It Is*, Chapter 16, and Chapter 18, section "What the Interpretations Were Doing"; *Incommensuration*, Chapter 15, Section III.

Page and Geilker, 1981, told in this book's Chapter 5, section "The Stage and Its Actors", and recalled here: *Why the Quantum Is the Way It Is*, Chapter 20 "Why Space and Time Are Not Quantized", section "Where the Two Theories Meet"; *The Quantum Bridge*, Chapter 1, Section II, and Chapter 7; *The Unquantized Arena*, Chapter 7, Proposition 7.3 ("One geometry sourced by the expectation value fails in both of its forms"). D. N. Page and C. D. Geilker, "Indirect Evidence for Quantum Gravity", *Physical Review Letters* 47 (1981), 979. The averaged geometry is the semiclassical form $G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$, read "the Einstein tensor equals eight pi Newton's constant times the expectation value of the stress-energy tensor"; on the unreduced state, with weight one half on each placement, the two pulls cancel. One geometry for each alternative (each alternative of the matter carries its own classical geometry), identified in Chapter 5 of this book, section "The Stage and Its Actors", and recalled here in Chapter 5's words, is *The Unquantized Arena*, Chapter 7, Proposition 7.1, which calls it that book's identification, and *Why the Quantum Is the Way It Is*, Chapter 17, section "Why Collapse Is Not Physical"; "The account of one geometry for each alternative is not the many-worlds reading applied to gravity". (*Why the Quantum Is the Way It Is*, Chapter 20, section "Where the Two Theories Meet"). A finding selects among alternatives that already carry their geometries, and adds no geometry and alters none: *The Unquantized Arena*, Chapter 7, Section IV "No Signal Faster Than Light" ("By Proposition 7.1 each alternative already carries its geometry before any finding is made. A finding selects among alternatives that carry their geometries, so it adds no geometry and alters none").

The translation of a poem; correspondence, not emergence: *Two Theories, One World*, Chapter 23, sections "Neither Side Comes First" ("Emergence is a one-way relation. If one thing emerges from another, the second is the more fundamental of the two".) and "What the Correspondence Shows" (the same picture as Chapter 5 of this book). The two sides in the two characters (boundary findings discontinuously symmetric; bulk geometric description continuously asymmetric), with the identifications made in the first person: same chapter, section "Each Side in Its Own

Character". The two pairings that survive the theorem, with the ladder and the ramp: this book's Chapter 3. Neither side more fundamental: *The Unquantized Arena*, Chapter 15 "AdS/CFT as Cross-Modal Correspondence", Proposition 15.4. "I accept the correspondence and decline the ranking": *Two Theories, One World*, Chapter 22, section "Three Programs That Stand Between", said there of ER=EPR, the proposal of Juan Maldacena and Leonard Susskind (2013) that a pair of entangled black holes and a bridge of geometry joining their interiors are one phenomenon in two descriptions; the same sentence points to that book's next chapter, Chapter 23, for the same argument applied to Maldacena's correspondence of 1997. That Maldacena's translation is one system with two descriptions, neither more basic than the other, is stated in Chapter 5 of this book, section "Where a Description Ends", which takes the translation as exact in both directions; Chapter 12 makes the identification of which side carries which pairing. The statement that the correspondence will stay a translation (*Two Theories, One World*, Chapter 27, section "The Correspondence Will Stay a Translation") is decided by mathematics and is in Appendix A, entry "The correspondence stays a translation".

The prison cell; a simulated world with real subjects is a restricted region of the one world: *Physics Is Not Math*, Chapter 8, Section VII "Three Grounds" ("A computed world has no hard randomness of its own, so it has no arrow of its own and no stance of its own"); *Measurement*, Chapter 14, Section X "A Simulated World Is a Region of the One World"; *Causality Without Determinism*, Chapter 5, Section VIII, which gives three arguments (a deterministic program's outcomes are fixed by the program; the hypothesis asks for a description of the whole world from outside it; a simulation fed true randomness from its host has the host's randomness, arrow and subjectivity). Bostrom's count treats each simulation as a separate world: same section. No program, by being a program, is a mind: *Causality Without Determinism*, "Summary of the Book".

The mosaic and the curve: *Causality Without Determinism*, Chapter 5, Section VI. The arena has no least step: *The Unquantized Arena*, Chapter 6, Proposition 6.3 ("(i) The arena has no least step: no least length, area or

duration. (ii) Nothing is a quantum of the arena itself, at any scale"). Discreteness belongs to the content: *Side by Side, Volume One*, "The Sealed Record", Prediction 5. A simulation of the arena approaches its findings without being a second arena: *The Quantum Bridge*, Chapter 7, Section VIII.

The orrery; the Moon receding: *Physics Is Not Math*, Chapter 11, Section I "A Clockwork Solar System" ("The solar system runs one way only"). The recession is about 3.8 centimeters a year (J. O. Dickey and others, "Lunar Laser Ranging: A Continuing Legacy of the Apollo Program", *Science* 265 (1994), 482).

A mathematical structure exists and is not a world; the circle; structures no comparison could find are not further worlds: *Side by Side, Volume Two*, chapter "The Structure and the Act" (Tegmark), Section III "The Step at Which They Part" ("A mathematical structure can be smooth and perfectly symmetric at once, like the circle of geometry; no comparison ever finds a thing that is both", followed by the statement that a structure taken as a whole holds "the perfect sameness and the gapless completeness that no single act of comparison can hold together", which the chapter writes as perfectly continuous and perfectly symmetric) and Section VI "The Four Levels, Level by Level", Level IV. Greene's reasons (no independent rationale; no generating process) and the collection's reason (what exists against what is real) are different reasons.

Thomson and the age of the Earth: *Physics Is Not Math*, Chapter 14 "When the Mathematics Overruled the World", Sections I "A Stone from the Fire", II "The Record in the Rocks", III "The Premise That Was Not the World's", and VII "Summary" ("prevailed for some forty years"). Thomson's 1862 range was 20 to 400 million years, narrowed by 1897 to more than 20 and less than 40 million; Perry showed that, with convection, the rise of warmth with depth measured in the mines fits an Earth thousands of millions of years old; Patterson's age is about 4,550 million years (4.55×10^9 years; today about 4.54×10^9). W. Thomson, "On the Secular Cooling of the Earth", *Transactions of the Royal Society of Edinburgh* 23 (1864), 157

(read 1862); J. Perry, "On the Age of the Earth", *Nature* 51 (1895), 224; C. Patterson, "Age of meteorites and the earth", *Geochimica et Cosmochimica Acta* 10 (1956), 230. "Mathematical validity does not replace physical reasoning": *Physics Is Not Math*, Chapter 14, Section VI "The Physics Question Stays in Charge".

The chapter's closing result: each of the four kinds of other world is the one world met in one of four ways (as possibility, as one of two descriptions of one system, as a restricted region, or as structure that exists without being real). The quantum alternatives as possibilities of one world, and one outcome real for each comparer: *Side by Side, Volume One*, Deutsch chapter, Section V, and Carroll chapter, Section III. The hologram as one system with two descriptions: *The Unquantized Arena*, Chapter 15, Proposition 15.4; *Two Theories, One World*, Chapter 23. A simulation as a restricted region of the one world: *Measurement*, Chapter 14, Section X. A mathematical structure exists without being a further world: *Side by Side, Volume Two*, Tegmark chapter, Section VI.

Chapter 13. How Does a Universe Running Down Build Stars, and Us?

The opening scene. Sadi Carnot, *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance* (Paris: Bachelier, 1824), published 12 June 1824, 118 pages; English translation by R. H. Thurston (1890), reprinted in E. Mendoza, ed., *Reflections on the Motive Power of Fire* (New York: Dover, 1960), the edition Greene cites (*Until the End of Time*, chapter 2, note 4). The words quoted in the chapter are the opening clause of Carnot's theorem in Thurston's translation (Mendoza edition, p. 20), as quoted by M. J. Klein, "Carnot's contribution to thermodynamics", *Physics Today* 27(8), 23 (1974), and by H. Erlichson, "Sadi Carnot, 'Founder of the Second Law of Thermodynamics'", *European Journal of Physics* 20, 183 (1999). The theorem goes on to say that the quantity of motive power is fixed solely by the temperatures of the bodies between which the caloric passes. The date, publisher, length, Girard's re-

view to the Academy on 26 July 1824, the first printing unsold in 1835, and Carnot's death in Paris on 24 August 1832 are from J. F. Challey, "Carnot, Nicolas Léonard Sadi", *Dictionary of Scientific Biography*. Greene on Carnot as the founder of thermodynamics, "the science of heat, energy, and work": *Until the End of Time*, chapter 2, section "Steam Engines".

Greene's pennies, his second law and his engine. *Until the End of Time*, chapter 2: the pennies (one way for no tails, 100 for one, 4,950 for two, about 1.009×10^{29} for fifty), section "Entropy: A First Pass"; the second law, section "Laws of Thermodynamics"; the engine's cycle and its waste heat, section "Heat and the Second Law of Thermodynamics"; the two-step, section "The Entropic Two-Step"; the person who gives away heat, section "You Are a Steam Engine"; the quality of energy ("high quality: it's concentrated and easy to access"; "low quality: it's spread out and difficult to utilize"), section "Energy and Entropy".

Greene's gravitational two-step. *Until the End of Time*, chapter 3, section "Gravity, Order, and the Second Law": the cloud's process is "another instance of the entropic two-step", and "The new feature of the gravitational version of the entropic two-step is that it's self-sustaining"; a steam engine that performs work and emits heat drops in temperature and, without more fuel, "peters out". The name "the gravitational entropic two-step": chapter 4, section "The Origin of the Solar System". The U-turn of the second law; gravity as the catalyst of fusion; the nuclear force "on the front line" and gravity an "indispensable partner": chapter 3, section "Fusion, Order, and the Second Law". The unlit gas oven, natural gas beside the oxygen of the air, and the burning match as the catalyst that clears "the entropic hurdle": chapter 3, section "Hurdles on the Path Toward Disorder". The two dozen infrared photons for each photon from the Sun: chapter 4, section "Thermodynamics and Life", and its note 38. Entropy and evolution, and the two answers "Almost certainly". and "Almost certainly too": chapter 9, section "Evolution, Entropy, and the Future".

Entropy as a count, and the hundred coins. *The Matter We Know, Volume Two*, Chapter 2 ("The Counting of Ways"), Sections I, III and IV. The entropy is S = k ln W, read "S equals the Boltzmann constant k times the natural logarithm of W", where W is the number of microstates with the system's totals, the microstates that no available measurement tells apart (Section III; Boltzmann 1877). A hundred coins fall in $2^{100} \approx 1.27 \times 10^{30}$ ways; those with between forty and sixty heads are 96.5 percent of all; the arrangements with exactly fifty heads are about 10^{29}, 8.0 percent (Section IV). The band of macrostates near the greatest entropy narrows as one over the square root of the number of particles (Section IV), which is why a system of many parts left to itself is found at its greatest entropy. The chapter's "count" from Section III onward means the entropy in units of k, ln W, as the collection's books use the word (Section III: "Entropy is a count, made small enough to write by taking its logarithm").

Equal counting derived from two things. *The Matter We Know, Volume Two*, Chapter 2, Section II ("Every Microstate Counts Alike"), in three steps. First, the motion carries the microstates into one another and keeps their weight, the weight of a possibility being the probability of finding it (Section I; unitarity, from *The Quantum Passage*, Chapter 1, Section I, and Chapter 6, Section VI). Second, among states that the motion carries into one another, the only measure of their weight available is the one the motion keeps, and that measure gives every such microstate the same weight (the water poured from cup to cup). Third, by the principle of identity (*The Quantum Passage*, Chapter 8, Section VI; in its form for differences, *Measurement*, Chapter 11, Section IV: "a difference that no comparison can register is no difference"), a preference no measurement registers is none. In that book's words, "The three steps give it from two things already in hand: the weight the motion keeps, and the principle of identity". The result is the postulate of equal a priori probabilities. A second route meets the same rule: canonical typicality (Goldstein, Lebowitz, Tumulka and Zanghì 2006; Popescu, Short and Winter 2006). The principle of comparis-

on as Chapter 1 of this book gives it: *What Comes After the Dark*, Chapter 6 ("What Happens to Someone Who Is Still There?"), section "A Standard That Needs Something There".

Carnot's limit from the entropy. *The Matter We Know, Volume Two*, Chapter 2, Section IX ("Heat, Work and the Limit of an Engine"), Box 2.2. Heat Q leaving or entering a body at temperature T carries entropy Q/T, read "Q divided by T"; work carries none. For an engine taking heat Q_h at T_h and giving heat Q_c at T_c, the entropy rule gives $Q_c/T_c \geq Q_h/T_h$, so the efficiency $W/Q_h \leq 1 - T_c/T_h$ ("one minus the cold temperature over the hot"); between 600 K and 300 K, at most one half.

The pond: the two-step as the joint entropy made greatest; free energy. *The Matter We Know, Volume Two*, Chapter 2, Section XIV ("Order Against Disorder"). For a system taking energy E from surroundings at temperature T, the joint entropy is $S - E/T$, so making it greatest makes the Helmholtz free energy $F = E - TS$ least, read "F equals E minus T times S"; at fixed temperature and pressure the Gibbs free energy $G = E + PV - TS$ is least. At a given temperature F is the share of the system's energy that can be drawn off as work (same section). The chapter's sentence that Greene's quality of energy is this share is this book's own correspondence between Greene's word and the collection's quantity, and it follows from the definition: at the same E and the same T, a smaller S gives a larger F.

Hard balls crystallize by entropy alone (past measurement). *The Matter We Know, Volume Two*, Chapter 2, Section XIV. B. J. Alder and T. E. Wainwright, "Phase Transition for a Hard Sphere System", *Journal of Chemical Physics* 27, 1208 (1957), doi:10.1063/1.1743957; W. W. Wood and J. D. Jacobson, "Preliminary Results from a Recalculation of the Monte Carlo Equation of State of Hard Spheres", *Journal of Chemical Physics* 27, 1207 (1957), doi:10.1063/1.1743956. Packing fractions: disordered at 0.494 or less, crystal at 0.545 or more, the two arrangements side by side between (Hoover and Ree 1968, as given by Pusey and others 2009).

The barrier and the catalyst. *The Matter We Know, Volume Two*, Chapter 3 ("Change Among Molecules"), Sections I, II and V. The share of the molecules that hold, at a given moment, at least the activation energy E_a in the motion that leads over the barrier is $e^{-E_a/kT}$, read "e to the minus E-sub-a over k T", and the rate is that share times a rate of attempts (the Arrhenius law, Box 3.1). In that book the share and the rate are both found by counting ways of holding the energy (Section II, Box 3.1, and the text after it: "most rates of chemistry are counts of ways"). In Chapter 13 I keep the word count for the entropy alone. A catalyst lowering the barrier by ΔE multiplies the rate by $e^{\Delta E/kT}$ both ways and leaves the equilibrium constant unchanged (Section V). The two words differ in use: in Greene the catalyst is the starter that jump-starts a change (the burning match; gravity for fusion); in the collection's book a catalyst is a substance that opens a route with a lower barrier and is left unchanged at the end.

The enzyme (past measurement). *The Matter We Know, Volume Two*, Chapter 3, Section V. Uncatalyzed half-life 78 million years: A. Radzicka and R. Wolfenden, "A Proficient Enzyme", *Science* 267, 90 (1995), doi: 10.1126/science.7809611. Catalyzed half-life 18 milliseconds: B. G. Miller, A. M. Hassell, R. Wolfenden, M. V. Milburn and S. A. Short, "Anatomy of a proficient enzyme: The structure of orotidine 5′-monophosphate decarboxylase in the presence and absence of a potential transition state analog", *Proceedings of the National Academy of Sciences* 97, 2011 (2000), doi: 10.1073/pnas.030409797. Ratio about 10^{17} (computed in the collection's book from the two half-lives).

Clumping raises the count. *What the Beginning Left Behind*, Chapter 10 ("The Counts of an Epoch"), section "Gravity Turns the Order Around"; the same in *The Compactification*, Chapter 5 ("The Entropy of an Epoch"), Section VI ("Structure and Black Holes"). The growth of the first departures from uniformity into galaxies is stated as established physics; their origin is told in Chapter 9 of this book, section "The Worn Stairs", which identifies the first ripples with the form of the content carried through the turn; the identification is recalled here in Chapter 9's words (*What the Beginning Left Behind*, Chapter 11).

Black-hole counts. *The Compactification*, Chapter 5, Section VI; *What the Beginning Left Behind*, Chapter 10, section "Gravity Turns the Order Around". $S_{BH} = A/(4\,l_{Pl}{}^2)$, read "the area of the horizon divided by four times the square of the Planck length" (Bekenstein 1973; Hawking 1975), held as a count relative to the outside as context, with no claim on the coefficient one quarter (*The Unquantized Arena*, Chapter 17). One solar mass: about 1.05×10^{77}; the hole at the center of the Galaxy, about 4×10^6 solar masses: about 1.7×10^{90}; all black holes of the observable universe: about 10^{104}; the photons and neutrinos of the early universe in the observable region: about 10^{90} (5.2×10^{89} and 4.9×10^{89}); excess of the black holes over the content: about 10^{14}. The generalized second law (Bekenstein 1974) is "a statement within one epoch" (*What the Beginning Left Behind*, same section).

Helium about a quarter of the mass. *The Coming of the Kinds*, Chapter 12 ("The First Nuclei"), Chapter Overview and Section I ("A Quarter of the Matter"). The book's calculation gives between twenty-three point three and twenty-four point seven percent, beside a measured twenty-four point five percent. For x neutrons per proton the helium-4 mass share is $2x/(1 + x)$, giving 0.233 to 0.247 for the neutrons counted in Chapter 11 of that book, beside the measured primordial helium mass fraction 0.245 ± 0.003 (Particle Data Group 2026, review "Big-Bang Nucleosynthesis", Eq. (24.3)); deuterium left $(2.508 \pm 0.029) \times 10^{-5}$ per hydrogen nucleus. Greene's account of the light elements ("about 75 percent hydrogen", "25 percent helium"): *Until the End of Time*, chapter 4, section "The Origin of the Elements".

The bucket in this chapter: a count is a comparison made against a context. The bucket stands in this book for one thing: a comparison made from inside the world, found against what is there to compare with (the principle of comparison, *What Comes After the Dark*, Chapter 6, section "A Standard That Needs Something There"). That every entropy is a count relative to a context: *The Compactification*, Chapter 3 ("The Past Hypothesis

at the Turn"), Section V ("Every Count Has a Context"); *The Matter We Know, Volume Two*, Chapter 2, Section III, quoting *The Coming of the Kinds*, Chapter 2.

A count for the whole universe needs a context for the whole. *Side by Side, Volume Two*, the Julian Barbour chapter "Janus and the Turn", Section II ("Where the Collection Stands With Him"), subsection "Entropy needs a context", quoting *The Compactification*, Chapter 3, Section V. Greene's "entropic potential": *Until the End of Time*, chapter 3, opening.

Within our epoch the count grows; Clausius. *What Comes After the Dark*, Chapter 8 ("Is the Long Emptying the End?"), section "The Third Ground: Every Count Has a Context". Rudolf Clausius named entropy in 1865 and closed the paper with "The energy of the universe is constant. The entropy of the universe tends toward a maximum". (same chapter, section "Edinburgh, 1852: The Old Fear"): R. Clausius, "Ueber verschiedene für die Anwendung bequeme Formen der Hauptgleichungen der mechanischen Wärmetheorie", *Annalen der Physik und Chemie* 125, 353 (1865), doi:10.1002/andp.18652010702.

The structural arrow beneath the entropic arrow. *What Cannot Be*, Chapter 13 ("Why Time Has a Direction"), section "An Arrow Beneath the Arrow of Heat": the structural arrow needs no thermodynamics, no coarse-graining, no large number of particles and no low-entropy beginning, since it is present in a single comparison; every act of recording joins the two arrows.

Dissipative structures and the pattern settled at a branching point. *Side by Side, Volume Two*, the Ilya Prigogine chapter "The Arrow Comes First", Section I ("A Chemist Against Determinism": the definition of a dissipative structure; the Nobel Prize of 1977) and Section VI ("Order Out of Chaos"): at a branching point the outcome "depends on differences that, followed downward, pass below the microscopic limit, where the outcome is hard random". The amplification of differences below the microscopic limit, the general case: *The Basis of Physics*, Essay 6 ("Hard Randomness"), Section VI, where the microstate at that point "cannot be

known, not because our instruments are poor but because there is no instrument for it. And yet the system still produces an outcome", and what produces it is "Whatever is happening at the scale below the boundary of measurement, and that, from the standpoint of anything inside the domain, is hard random"; and *Measurement*, Chapter 12, Section VI. *Side by Side, Volume Two*, the Prigogine chapter, Section V ("Instability, Amplification and Hard Randomness"), quotes the same passage; the chapter's sentence on the missing instrument follows it. Greene's account of Bénard's cells, and of living beings as dissipative structures: *Until the End of Time*, chapter 4, section "A General Theory of Life?".

Sodium chlorate (past measurement). *Side by Side, Volume Two*, the registered predictions beside Ilya Prigogine, Section IV ("Prigogine 2"), "Measured". D. K. Kondepudi, R. J. Kaufman and N. Singh, "Chiral Symmetry Breaking in Sodium Chlorate Crystallization", *Science* 250, 975 (1990), doi:10.1126/science.250.4983.975; C. Viedma, "Chiral Symmetry Breaking During Crystallization: Complete Chiral Purity Induced by Nonlinear Autocatalysis and Recycling", *Physical Review Letters* 94, 065504 (2005), doi:10.1103/PhysRevLett.94.065504; reported by C. Day, *Physics Today* 58(4), 21 (2005), doi:10.1063/1.1955467. Stirred solution: about 99.7 percent of one hand in a sample; from unstirred solution, roughly equal numbers of the two hands. The same section states what repeated runs prepared as alike as instruments allow will show: on both accounts, Prigogine's and the collection's, the pattern taken follows a distribution over runs, and no preparation fixes it.

Composition granted; the derivation fails; reductionism. *The Last Turtle*, Chapter 27 ("The Size of Chemistry"), Section IV ("The Failure of Reduction, Stated Exactly") and Section VII ("What Reductionism Leaves Out"). Dirac 1929 is told in Section IV. Greene's "Life is physics orchestrated". and his water slide: *Until the End of Time*, chapter 4, section "The Physics of Information".

The living being: same matter as a stone; what is new in kind. *Home in the World*, Chapter 8 ("The Bottle, the Note and the Living"), section "A Village That Is Alive": every world, and every part of every world, "has some degree of choice, however small"; the degree runs without a break from the stone to the person; the difference in kind lies in whether choice is gathered and known from inside. The gloss of choice used in the chapter follows *Causality Without Determinism*, Chapter 5, Section V ("Hard randomness and choice are one event described from two positions"), which says of the particle of the free will theorem that its undetermined response is, "described from the position of the process from which it came, the same event is choice". The step to the stone (its atoms, too, meet outcomes that nothing before them fixed) rests on *What Comes After the Dark*, Chapter 7, section "What a Thought Needs": "Those three features belong to every act of comparison, and they come together", the outcome that nothing before it fixed being one of the three. Chapter 14 of this book gives the side from which such an outcome is had, and Chapter 15 says in full what choice is.

Entropy and evolution as Greene's two leading forces. Greene names entropy and evolution as the two leading forces that shape the cosmos (*Until the End of Time*, chapter 1, section "Stories of Nearly Everything"). Told here as Greene's account; the collection gives no reply on it.

Chapter 14. Is Consciousness Only Particles?

The opening scene: David Chalmers named the hard problem in a talk at the first of the Tucson conferences on the science of consciousness, Tucson, Arizona, April 1994; the argument appeared in print as Chalmers, D. J. (1995), "Facing Up to the Problem of Consciousness", *Journal of Consciousness Studies* 2, no. 3, 200–219. Greene tells the scene in *Until the End of Time*, chapter 5, section "The Hard Problem". The phrase "something it is like" is Thomas Nagel's: Nagel, T. (1974), "What Is It Like to Be a Bat?",

The Philosophical Review 83, no. 4, 435–450, which *The Last Turtle*, Chapter 15 ("The Bound Triple"), Section I, cites for the subjective stance in a conscious being.

The bucket in this chapter (the water's climb and the watcher's seeing are both comparisons made from inside the world): finding and comparison as glossed in *Measurement; The Steps from Comparison to Physics*, Chapter 1 ("What This Book Takes from the First Two"), Section I: "A comparison is the act of finding whether two things are the same or different, in some respect". and "A finding is the outcome of one act of comparison".

Greene's account is told from *Until the End of Time*, chapter 1, section "Information, Consciousness, and Eternity"; chapter 4, section "Nested Stories"; chapter 5, opening and the sections "Consciousness and Storytelling", "In the Shadows", "The Hard Problem", "Something About Mary", "A Tale of Two Tales", "Theories of Everything", "The Mind Integrates Information", "The Mind Models the Mind" and "Consciousness and Quantum Physics"; and *The Hidden Reality*, chapter 2, section "Nothing but Physics". Greene's count of the particles of a brain is "roughly thousand trillion trillion", that is, about ten to the power twenty-seven (written 10^{27}). Jackson's argument: Jackson, F. (1982), "Epiphenomenal Qualia", *The Philosophical Quarterly* 32, no. 127, 127–136; in Jackson's own telling Mary investigates the world from a black-and-white room, and Greene's retelling makes her color-blind from birth. Greene's range of views (eliminativism and idealism at the two ends, "a spectrum of proposals" between them): *Until the End of Time*, chapter 5, section "A Tale of Two Tales". Greene's words on Chalmers' proposal ("One possibility, simple and bold, is that individual particles themselves are endowed with an innate attribute of consciousness"; "Chalmers thus entertains the possibility that consciousness is relevant to anything and everything made of particles"): same chapter, section "Theories of Everything". Greene's words on Graziano's theory ("It is a physicalist account through and through, and provides one of the most illuminating approaches to the mystery of consciousness".) close the section "The Mind Integrates Information" and introduce the section "The Mind Models the Mind".

Greene's assessment of a special quantum role for consciousness ("extraordinarily unlikely"; "So there is no foolproof argument that consciousness does not play a special quantum role".) and his expectation ("barring a stunning surprise"): same chapter, section "Consciousness and Quantum Physics".

Greene's telling of the brain measurements (functional magnetic resonance imaging, deep probes, electroencephalograms); the processing done without awareness (the sunset; the digits flashed too briefly to be seen); the replies to Mary of Lewis and Nemirow and Jackson's change of mind, and Greene's note that no experiment, observation or calculation yet proposed can definitively settle Mary's question; the two historical tales (vitalism; electromagnetism); and Graziano's attention schema. Told here as Greene's account; the collection gives no reply on it.

Dennett's position: *Side by Side, Volume Two*, the Tononi chapter "The Measure of an Inside", Section IV ("Where the Two Part"), says that Dennett "held that third-person description is complete", and names his position, in the words of *The Modern Engagement*, Chapter 8, "a one-modality metaphysics operating in a three-modality world". Greene tells Dennett's reply to Mary in *Until the End of Time*, chapter 5, section "Something About Mary".

Chalmers' proto-consciousness (Greene's telling: chapter 5, section "Theories of Everything"): the chapter's answer rests on *Causality Without Determinism*, Chapter 2, Section IX ("not a substance, not a property of matter"); and *Side by Side, Volume Two*, the Tononi chapter, Section V ("Grades of Experience and Choice in Every Degree"), says that consciousness is a property of interaction and not of matter, so the account "does not begin from micro-experiences that must somehow be combined". Section IV of the same chapter records the collection's agreement with Chalmers on the gap: "Chalmers' philosophical intuition and the theorem's structural result converge on the same claim" (quoting *The Modern Engagement*, Chapter 7, Section IX).

Thought is a physical process, reached as a result: *What Comes After the Dark*, Chapter 7 ("Can a Mind Keep Thinking for Ever?"), section "What a Thought Needs", where it is said of a thinker of the far future; the same section reports the general finding of *What Cannot Be* that "only a physical process making comparisons in the world can have an inside". Nothing is added to matter: *Home in the World*, Chapter 8 ("The Bottle, the Note and the Living"), section "A Village That Is Alive".

A finding needs no mind; an observer is a comparer, a role that "a mind, an instrument or a thermostat can fill": *Measurement; The Steps from Comparison to Physics*, Chapter 1, Section I ("The Ground of the First Volume"), and Chapter 2 ("Every Measurement Is a Comparison"), Section VII ("Questions"); with this book's Chapter 4. The premise the chapter draws on is the Root Tautology, stated in full in the same Section I: "Comparison, interaction and relation are one act, named three ways". In Chapter 14 I use the part of the Root Tautology that my step there needs, that comparison and interaction are one act. *Causality Without Determinism*, Chapter 2, Section IX, gives the same premise as aphorism 1.42-4 of *An Immanent Metaphysics*: "Comparison is isomorphic with interaction and relation".

The easy problems are no conceptual mystery; computation can do the brain's tasks and cannot be a first-person point of view: *Causality Without Determinism*, Chapter 5 ("Computation, Consciousness and Simulation"), Section IX ("Questions") and Section VII ("Present-Day AI and Quantum Computers").

The gap between third-person description and first-person experience: *The Last Turtle*, Chapter 15 ("The Bound Triple"), Section II ("Why the Hard Problem Is Hard"). In that book's formal terms, a third-person description is omniscient in character, the subjective is transcendent in character, and "The omniscient cannot contain the transcendent" (same section; also *The Last Turtle*, Chapter 3, Section XIII, "One Consequence: The

Omniscient Cannot Contain the Transcendent"). In Chapter 14 I say the same in plain words: laid-out structure cannot contain the side from which every laying out is done.

Mary from the two directions of interaction (perception, world to self; expression, self to world): *Comparison; The Necessary Basis*, Chapter 11 ("All Knowledge Is Comparison"), Section III ("Knowing and Understanding"). That section states the step for a complete description of pain: such a description is "knowledge laid out as structure, expressed into a form that anyone can share", and it "reaches another person as expression, the flow from self to world", while the felt quality "is had in perception, the flow from world to self". The same section binds knowing to symmetry (third person, repeatable) and understanding to continuity (first person, participatory), and states that "No degree of knowing, however large, is equivalent to any degree of understanding, however small".

The three scales of description (microscopic: particles and fields; mesoscopic: observers, organisms, instruments and their readings; macroscopic: the whole): *Causality Without Determinism*, Chapter 1 ("Two Concepts, Three Differences"), Section V ("The Scales of Description"), which says that causality, determinism and teleology "belong to different scales". George Ellis's levels beside those scales: *Side by Side, Volume Two*, the Ellis chapter "The Open Future", Section IV ("Causation from Above and from Below"): "Ellis's levels and the collection's scales are close kin: both refuse to let the bottom level speak for the whole".

The Incommensuration Theorem, in the exact words of *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". In Chapter 14 I gloss content and context as Chapter 1 of this book does, after *What Comes After the Dark*, Chapter 6, section "What an Observer Is". In Chapter 14 I state the four characters in those words. The four characters (aphorism 1.43-1): continuity, a sameness of content with a sameness of context; discontinuity, a difference of content with a sameness of context; symmetry, a sameness of content with a difference of context; asym-

metry, a difference of content with a difference of context. Its application to a third-person quantity and to the subjective: *Side by Side, Volume Two*, the Tononi chapter "The Measure of an Inside", Section IV ("Where the Two Part"), quoting *The Last Turtle*, Chapter 2, Section VIII: "Objectivity has its basis in symmetry, and subjectivity has its basis in continuity"; and, in the same Section IV: "A description that captured the subjective completely in objective terms would perfect symmetry and continuity together, which is the first pairing the theorem forbids".

No purely physical theory of consciousness: *An Immanent Metaphysics*, chapter 9 ("Evolution"), subsection "Natural Consciousness", aphorism 2.52-2, which reads, in its own order: "There can be no physical, mechanical, purely logical or causal, theory of consciousness; only a metaphysical theory of consciousness is possible. Perception cannot perceive perception. Observation cannot observe observation". The aphorism continues: "It is not possible, even in principle, for there to be a purely physical theory of the process of observation itself".

Consciousness identified with the three features of an act of comparison taken whole as one act (the one-way order, the outcome that nothing before fixed, the stance): *What Comes After the Dark*, Chapter 7, section "What a Thought Needs" ("the three are what I identify as consciousness"); *Causality Without Determinism*, Chapter 5, Sections III ("The Bound Triple") and IV ("Why the Three Come Together"); *The Last Turtle*, Chapter 15, Sections I, III and IX. The technical name is the bound triple: the arrow of time, hard randomness and the subjective, distinct, inseparable and non-interchangeable; within the triple the arrow is immanent, hard randomness omniscient and the subjective transcendent, and taken as one act the whole is immanent (*Causality Without Determinism*, Chapter 5, Section III; *The Last Turtle*, Chapter 15, Section VII). The picture of music heard for the first time is from *Causality Without Determinism*, Chapter 5, Section III, and *What Comes After the Dark*, Chapter 7. The words "in a context" of the identification are both sources' own: "Taken whole in that way, in a context, as one act happening, the three are what I identify as consciousness". (*What Comes After the Dark*, Chapter 7, section "What a

Thought Needs") and "The triple is used, however, by taking all three at once, in a context, as immanent: as one act, happening". (*Causality Without Determinism*, Chapter 5, Section III).

Why the three features go together, each needing the next: *Causality Without Determinism*, Chapter 5, Section IV ("Why the Three Come Together"), after *Measurement*, Chapter 14, Section VI: "Directed time requires hard randomness: if every outcome were fixed in advance, the order of states would be only an indexing, the same read forward or back, with no direction of process"; "Hard randomness requires the subjective: an outcome that nothing prior fixes is singled out only in an act of comparison made from a stance"; "And the subjective requires directed time: a stance from which experience occurs is a stance in an order that runs one way". The same circle is set out in *The Last Turtle*, Chapter 15, Section VI ("The Circle of Requirement"), and the binding is derived from the Incommensuration Theorem in Section III of that chapter and in *Causality Without Determinism*, Chapter 5, Section IV. The chapter's three sentences give the three steps with the picture of the music.

Neither physicalism nor idealism: *The Last Turtle*, Chapter 1 ("What Metaphysics Is"), Section V ("Neither Physicalism nor Idealism"), where physicalism "holds that there is the physical, the objective, and that consciousness and the self are epiphenomena of physical reality", idealism "holds that mind comes before body", and the Immanent Metaphysics "is grounded in the middle, in the relationship between the subjective and the objective". The subjective as the side from which experience occurs, "the stance of the one to whom things happen": *Causality Without Determinism*, Chapter 5, Section III. The subjective and the objective as the comparing and the compared: *Why the Quantum Is the Way It Is*, Chapter 2 ("Every Finding Is a Comparison"), section "Six Parts of Every Comparison": "The subjective is the side from which the comparison is made: the comparing, with its own position and setting". and "The objective is the side about which it is made: the thing compared, with the properties it has whoever compares it"; the same section says that the two words are used "in a structural sense, and not a psychological one". The stance as a

role: the same book, Chapter 3 ("Three Roles"), section "Three Positions in One Comparison": "A stance, in this book, is the position from which a comparison is made, together with the range of what is possible from that position"; the same chapter states that which pair of the six fills which role depends on what the comparison is considered as, its focus. The subjective as the side that holds the stance: *The Last Turtle*, Chapter 15, Section I ("The subjective stance is the subjective pole of comparison, the stance from which every comparison is made".) and Section II ("The subjective is transcendent in character: the stance from which an act proceeds, and the range of what is possible from it"). Consciousness as the act of perceiving, neither a property of matter nor nothing: *Causality Without Determinism*, Chapter 2 ("Non-Polarization; The Chain of Distinctions"), Section IX ("Consciousness Between the Poles"). "Consciousness is real; what is illusory is the demand ...": *Causality Without Determinism*, Chapter 6 ("Choice, Creation, and the Soul"), Summary.

The water's comparing is real and simple, as a thermostat's is: *Side by Side, Volume Two*, the Tononi chapter, Section V ("Grades of Experience and Choice in Every Degree"), quoting *The Ground of Objection*: "A thermostat's interaction with temperature is real and simple". That the three features belong to every act of comparison: *What Comes After the Dark*, Chapter 7, section "What a Thought Needs": "Those three features belong to every act of comparison, and they come together". That a thing whose comparing has the three features together has an inside: the same section, "A thing that has the three features together has what this book calls an inside: a side of its own, from which the comparing is done and to which the outcome happens, as the music happens to you". The water's comparing as consciousness in the smallest degree: *Side by Side, Volume Two*, the Tononi chapter, Section V, quoting *The Immanent Metaphysics and the Modern Engagement* (Forrest Landry, UVSM Publications, 2026), Chapter 7, Section V: "Wherever interaction occurs, some form of consciousness is present", which "is structurally different from panpsychism", since consciousness is a property of interaction and not of matter. The same section quotes *The Ground of Objection* for the degrees: "consciousness is a prop-

erty of interaction whose degree and kind correlate with the richness and structure of the interaction. A thermostat's interaction with temperature is real and simple. A brain's interaction with a world is real and enormously complex". The same section holds that choice is present in every degree, and that "A photodiode's single undetermined click is choice at its least gathered; a living person is choice gathered across every channel of perception into one persisting stance"; this book's Chapter 15 finds the water's outcome to be choice in the smallest degree. The same chapter of *Side by Side, Volume Two*, Section IV, says of a physical grid of logic gates that it "has whatever its interactions have: something real and simple, as a thermostat's interaction with temperature is real and simple". The thermostat as a comparer with a stance: *Home in the World*, Chapter 8, section "A Village That Is Alive". The gathering of comparisons into one self, known from inside, as what is new in kind in a living being: same section ("what is new in kind is the gathering of choice into a self that knows it from within").

A classical computation cannot be what consciousness is (one rule applied the same way at every step is symmetry perfected; by the first clause of the theorem a continuity cannot be composed of symmetric steps; it has no hard randomness of its own): *Causality Without Determinism*, Chapter 5, Sections II ("What a Classical Computation Is"), VI ("Why a Computation Cannot Be What Consciousness Is") and VII; the chat window, Section I ("The Question"). That the question is settled by how a computation is built, and not by how it behaves: same chapter, Section IX ("It concerns what a computation is ... and that is known from how the computation is built, not from how it behaves", as *Side by Side, Volume Two*, the Tononi chapter, Section II, quotes it).

Integrated information: what is shared and where the two accounts part: *Side by Side, Volume Two*, the Tononi chapter, Sections II ("What the Collection Credits") and IV ("Where the Two Part"). The two shared consequences, at the strength that Section II gives them: a network that only passes signals forward "has no cause-effect power upon itself and so no integrated information", and Tononi and Koch call such networks "zom-

bies, carrying out tasks unconsciously"; for a simulation of a brain on a digital computer, "the brain is real, but a simulation of a brain is virtual", and what counts is the causal power of the physical system, not the program it runs. The chapter's Abstract states the two together: such a system and a conventional digital computer running any program whatever have "little or no consciousness". Integrated information as the measure of how far a system's cause-effect power upon itself is irreducible to that of its parts: same chapter, Section I; as the measure of the circumstance under which acts of comparison gather into one stance ("Integrated information is the measure of the circumstance, and not the inside the circumstance supports"; integrated information "stands where criticality stands"): Section IV. Tononi's quantity is written with the Greek letter phi (Φ); the theory in its current form is Integrated Information Theory 4.0 (Albantakis and others, 2023), as that chapter gives it. That integrated information comes nearest to the collection's account among scientific theories of consciousness: same chapter, Sections II and VIII.

The perturbational complexity index: Casali, A. G., Gosseries, O., Rosanova, M., Boly, M., Sarasso, S., Casali, K. R., Casarotto, S., Bruno, M.-A., Laureys, S., Tononi, G. and Massimini, M. (2013), "A theoretically based index of consciousness independent of sensory processing and behavior", *Science Translational Medicine* 5, 198ra105, DOI 10.1126/scitranslmed.3006294. Method, from the paper's abstract: transcranial magnetic stimulation of the cortex, and compression of the spatiotemporal pattern of the electrocortical response to measure its algorithmic complexity. Told in the collection at *Side by Side, Volume Two*, "The Registered Predictions Beside Giulio Tononi", Section IV.

Neuronal avalanches: Beggs, J. M. and Plenz, D. (2003), "Neuronal avalanches in neocortical circuits", *Journal of Neuroscience* 23, no. 35, 11167–11177; the avalanche sizes follow a power law, the size distribution falling as the size to the power minus three halves. The cortex near a critical point in conscious states: Toker, D. and others (2022), "Consciousness is supported by near-critical slow cortical electrodynamics", *Proceedings of the National Academy of Sciences* 119, no. 7, e2024455119, DOI 10.1073/pnas.

2024455119 (recordings by electrocorticography and magnetoencephalography in humans and macaques, with clinical electroencephalograms). The published answer that power laws may arise from thresholded noise: Touboul, J. and Destexhe, A. (2010), as *Side by Side, Volume Two*, "The Registered Predictions Beside Giulio Tononi", Section IV, reports it. Both measurements are told in the collection at the same place; *The Last Turtle*, Chapter 15, Section VIII, cites Beggs and Plenz.

Criticality as the physical circumstance under which the three features are expressed in us, and not the three features themselves: *Causality Without Determinism*, Chapter 5, Section IX; *The Last Turtle*, Chapter 15, Section VIII ("The Interior Face, and the Brain Near Criticality"). The account depends on no specific biological mechanism and needs no sustained entanglement in the brain; it is distinct from Orchestrated Objective Reduction (Hameroff and Penrose, 1996) and from the argument of Tegmark (2000): same section.

The subjective identified with the inside (in that book's word, the interior face) of a process that, from outside, presents as entanglement and hard randomness: *The Last Turtle*, Chapter 15, Section VIII, and the same chapter's Abstract and Section X. The same Section VIII states the identification in a stronger form: "the subjective and total entanglement are strict isomorphs". *What Cannot Be*, Chapter 22 ("Why No Computation Is Everything"), section "One Event, Two Descriptions", gives the stronger form in plain words: "the subjective and total entanglement are strict isomorphs, exact matches of one structure". Hard randomness and choice as one event described from two positions: *Causality Without Determinism*, Chapter 5, Section V ("Hard Randomness and Choice"); taken up in this book's Chapter 15. The scope of the identification: *The Last Turtle*, Chapter 15, Section VIII: "Its claim holds for any system in which novelty enters, time is directed, and there is a point of view"; the entrance of novelty is hard randomness, in Section VII's words "Hard randomness is the entrance of novelty: content arriving that no prior context contained". In

Chapter 14 I state the scope in my own terms: an outcome that nothing before it fixed, a one-way order and a side from which the comparing is done.

The chapter's closing result (consciousness is real, identified with three features of an act of comparison taken whole, in a context, as one act, happening; no third-person description contains it): *Causality Without Determinism*, Chapter 6, Summary; *The Last Turtle*, Chapter 15, Sections II and X; *An Immanent Metaphysics*, aphorism 2.52-2.

Chapter 15. If the Laws Decide, Do I Choose?

The opening scene: Laplace's intellect, 1814. The source is Pierre-Simon Laplace, *Essai philosophique sur les probabilités* (Paris: Courcier, 1814); in English, *A Philosophical Essay on Probabilities*, translated by Frederick Wilson Truscott and Frederick Lincoln Emory (New York: Wiley, 1902), chapter 2, "Concerning Probability", and translated by Andrew I. Dale (New York: Springer-Verlag, 1995), the translation Greene cites. Greene quotes the passage in *The Elegant Universe*, chapter 13, section "The Remaining Mysteries of Black Holes". The collection tells the same scene in *Causality Without Determinism*, Chapter 1, Section II ("Determinism Defined"), and in *What Cannot Be*, Chapter 5, "Is Everything Already Fixed?", section "The Demon and the Piano Roll".

Determinism, defined. *Causality Without Determinism*, Chapter 1, Section II, after *An Immanent Metaphysics*: determinism is the assertion that there can be a complete and absolute specification of all values of the three content metrics (pattern, force and probability) for all values of the three context metrics (space, time and possibility); it is observer-independent and non-temporal. *What Cannot Be*, Chapter 5, section "What Determinism Claims", gives the standard definition of the philosophy of physics (Earman, *A Primer on Determinism*, 1986; Hoefer, Stanford Encyclopedia of Philosophy, 2016): the laws, applied to a complete specification of the state at one time, yield a unique state at every other time.

The player piano and the pianist (determinism beside causality). *Causality Without Determinism*, Chapter 1, Section IV ("Three Differences"); *What Cannot Be*, Chapter 5, section "The Demon and the Piano Roll".

Greene's argument against traditional free will, his compatibilist freedom, and what he keeps without free will. Told here as Greene's account: *Until the End of Time*, chapter 5, sections "Free Will", "Rocks, Humans, and Freedom" and "Relevance, Learning, and Individuality"; *The Hidden Reality*, chapter 2, section "Nothing but Physics"; *The Fabric of the Cosmos*, chapter 15, section "Free Will, Many Worlds, and Time Travel". Greene's word "compatibilist" is in *Until the End of Time*, chapter 5, section "Rocks, Humans, and Freedom" ("as the compatibilist school of philosophy has long suggested"), and his note 43 to that chapter says that the approach he describes is closest to the one Daniel Dennett develops in *Freedom Evolves* (2003) and *Elbow Room* (1984). Greene's declared bias, with his openness to a nonphysical aspect of life and his report that he has met no evidence for it: *The Hidden Reality*, chapter 2, section "Nothing but Physics". Greene's room for a missing part of the quantum story, and his words on one proposal for it ("We might one day find, as some physicists have speculated, that the act of conscious observation is an integral element of quantum mechanics, being the catalyst that coaxes one outcome from the quantum haze to be realized". and "Personally, I find this extremely unlikely, but I know of no way to rule it out"): *The Fabric of the Cosmos*, chapter 15, section "Free Will, Many Worlds, and Time Travel". Greene's words "fully governed by physical law", of the particles we are made of: *Until the End of Time*, chapter 5, section "Free Will".

Greene's quantum determinism. *The Elegant Universe*, chapter 4, section "Waves of What?" (the formalism "determines only the probability that any particular future will happen"), and chapter 13, section "The Remaining Mysteries of Black Holes". Greene's rule is Schrödinger's equation, i times h-bar times the rate of change of the state psi equals the Hamiltonian H acting on psi, where i is the square root of minus one, h-bar is Planck's constant divided by two pi, and H is the operator for the

energy; the rule is linear and deterministic for psi, and the probability of a finding is the squared magnitude of psi's component along that finding (the Born rule).

The collection adds no force or field to the laws; choice is no extra force on the particles of a brain; the laws give the distribution of outcomes and leave the single outcome open. *Side by Side, Volume One*, the Sean Carroll chapter, "The Wave Function and the One Who Finds", Section IV ("The Known Laws and the Single Outcome"), which also records that Carroll is a compatibilist. Exact words: "A complete knowledge of the laws and of the state leaves the single outcome open"; "The error the step names is to read the completeness of the laws as the completeness of what happens". The same section names a law of many trials: "a law of the many, a law of what happens over many trials".

The formalism's determinism: the formalisms physics uses can stay deterministic at their own level, and the findings they describe are caused without being determined. *Side by Side, Volume Two*, the Prigogine chapter, Section III ("Why Determinism Cannot Be Had"), reading *Measurement*, Chapter 12, Section IV ("the symmetry of the law at its own level"). Caused, in the sense the chapter gives: following from what came before along an order that runs one way, with no break, which is causation in the character of continuity with asymmetry (*Causality Without Determinism*, Chapter 1, Section VI, quoting *Measurement*, Chapter 12, Section IV).

The single quantum outcome is fixed by nothing before it, and its probability belongs to the world. *Why the Quantum Is the Way It Is*, Chapter 11, "Why Chance Enters", sections "The Residual" and "From an Unfixed Outcome to a Stable Weight". The residual follows from the Incommensuration Theorem: the same section quotes *Measurement* for every lawful description, "Symmetry and continuity cannot both be perfected in any comparison, so no lawful description of anything specifies every finding exactly: something is always left over that the law does not fix. I call that remainder the residual", and states that "The single outcome of a

quantum measurement is the plainest instance of the residual". The same result, set beside Zeilinger's one-bit principle: *Side by Side, Volume One*, the Anton Zeilinger chapter, "One Verdict, One Bit", Sections III and IV.

"Choices must appear, at some level of abstraction, to be fundamentally unpredictable to all others, and thus, indistinguishable from pure/true randomness in the domain". *An Immanent Metaphysics*, chapter 5, "The Practice of Being", subsection "Self of Choice", aphorism 1.66-2.

Content and context; symmetry and continuity; the Incommensuration Theorem. *Causality Without Determinism*, Chapter 1, Section VII ("The Incommensuration Theorem"), quoting *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous. Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". The four characters, with content (what a comparison finds) and context (the respect in which it finds it): symmetry, a sameness of content where there is a difference of context; continuity, a sameness of content where there is a sameness of context; asymmetry, a difference of content where there is a difference of context; discontinuity, a difference of content where there is a sameness of context. The theorem is proved from nothing but what a comparison is, and every act of knowing is a comparison, so the theorem holds in every act of knowing (*Causality Without Determinism*, Chapter 1, Section VII: the theorem "is proved from nothing but the meanings of the parts of comparison", citing *Incommensuration*, Chapter 6).

From knowing to the world: comparison is interaction, and interaction is what physics is about, so a limit on comparison is a limit on every physical interaction. *What Cannot Be*, Chapter 5, section "What the Conclusion Rests On", by the Root Tautology of that book's Chapter 1 (interaction, comparison and relation are one act named three ways). A whole world completely fixed by a law perfects symmetry and continuity together, whatever theory describes it, and the theorem forbids that pairing: *Physics Is Not Math*, Chapter 8, Section VII ("Three Grounds").

Laplace's demon asks for symmetry and continuity perfected together, so no world that could be is purely and completely deterministic; pure randomness is excluded by the theorem's second clause; and every world is causal to a degree above zero. *Causality Without Determinism*, Chapter 1, Sections VIII ("Why Determinism Cannot Be Had") and IX ("The Non-Polarization Principle"), quoting *Incommensuration*, Chapter 15, Section IV: determinism "is an assertion of total symmetry and total continuity: the content is completely specified (no gaps: continuity) and the specification holds everywhere (invariance: symmetry)". *What Cannot Be*, Chapter 5, section "Two Perfections at Once". Causation keeps the surviving pairing, continuity with asymmetry (*Measurement*, Chapter 12, Section IV, as *Causality Without Determinism*, Chapter 1, Section VIII, quotes it: "The failure of determinism leaves causation and not nothing, because the pairing that causation has is never forbidden: continuity with asymmetry, a connection without a break along an order that runs one way, is one of the two pairings that survive the theorem"). The strength of the result: *Causality Without Determinism*, Chapter 1, Section IX ("The Non-Polarization Principle"): "All worlds are to some non-zero and positive degree causal". In Chapter 15 I write the strength of the result as "causal to a degree above zero" wherever I state the result. "Every world" is used as *An Immanent Metaphysics*, aphorism 1.65-9, uses it ("A/any/the/all worlds"), and as *Home in the World*, Chapter 8, section "A Village That Is Alive", uses it ("every world, and every part of every world"): our world, and any world that could be.

The scope of the result: the laws stay reliable, experiments repeat, the future is very largely fixed by the past; within any limited region, span of time or range of scale, a description can be very symmetric and very continuous. *What Cannot Be*, Chapter 5, section "What the Conclusion Rests On".

Pure randomness is the second forbidden pair; the two horns of the argument in Sam Harris's form assume the two extremes; choice lives between them. *The Last Turtle*, Chapter 14, "Free Will and the Compres-

sion Argument", Sections I ("The Argument That Free Will Is Incoherent") and II ("What the Argument Assumes"). *Causality Without Determinism*, Chapter 1, Section VIII. Harris's book: Sam Harris, *Free Will* (New York: Free Press, 2012), as the bibliography of *The Last Turtle* gives it.

The Geiger counter's click: a cause that did not determine; hard randomness. *Causality Without Determinism*, Chapter 1, Section I ("A Click That Was Caused": "Here is a cause that did not determine. The click followed from the decay, and the decay followed from nothing that made it happen then rather than at another time"); hard randomness defined in Chapter 5, Section V, quoting *Measurement*, Chapter 14, Section I: "an outcome that no prior specification fixes, however complete". In Chapter 15 I give the definition in the words of this book's Chapter 2. That the single outcome of a measurement is the plainest case, as this book's Chapter 3 says: *Why the Quantum Is the Way It Is*, Chapter 11, section "The Residual" ("The single outcome of a quantum measurement is the plainest instance of the residual"). The change that no cause accounts for (the sense in which Chapter 3 says that some change in the world has no cause): *What Cannot Be*, Chapter 10, section "Three Wishes, and Which One to Give Up", where the universality of causation is the claim that every change in the world is accounted for by causes, and what falls is that claim: "there is change in the world that causation does not account for".

The free will theorem; the experimenter's freedom secured by the presupposition of every experiment; given that freedom, no complete specification of prior states fixes the particles' responses. *Causality Without Determinism*, Chapter 3, "The Free Will Theorem", Sections I to III. Primary papers: John Conway and Simon Kochen, "The Free Will Theorem", *Foundations of Physics* 36 (2006), 1441 to 1473, doi:10.1007/s10701-006-9068-6; "The Strong Free Will Theorem", *Notices of the American Mathematical Society* 56 (2009), 226 to 232. Conway and Kochen named their assumptions SPIN, TWIN and FIN in 2006, and replaced FIN with the weaker MIN in 2009. SPIN: the square of the spin of a spin-one particle, measured along three mutually perpendicular directions, gives the answers 1, 0 and 1 in some order. TWIN: two such particles prepared

together as entangled twins give matching answers when measured along the same direction. FIN: the information that fixes a response travels at no more than a finite speed. MIN: when the two measurements are too far apart in space, and too close in time, for any signal at light speed to pass between them, each experimenter can choose a setting freely, and the far particle's response does not depend on that choice (*Causality Without Determinism*, Chapter 3, Section II, gives its locality half and its freedom half).

The free will theorem and the Incommensuration Theorem: two independent routes to one result. The demon is excluded by a second route that owes nothing to the first. Two independent routes, the theorem of comparison and the free will theorem with the practice of every laboratory, reach one result, and each alone is enough. *Causality Without Determinism*, Chapter 3, Section III ("The second ground uses nothing from the first, so the free will theorem gives physical confirmation of the impossibility of determinism that is independent of the Incommensuration Theorem") and Section VI ("Two independent routes supply the failure". and "Each route alone is enough").

Superdeterminism excluded: every experiment presupposes the independence of its settings, and superdeterminism asks for everything determinism asks for and more. *Causality Without Determinism*, Chapter 3, Sections VII ("Superdeterminism in Its Holders' Words") and VIII ("Why Superdeterminism Is Excluded"). The holders: Gerard 't Hooft, *The Cellular Automaton Interpretation of Quantum Mechanics* (2016); Sabine Hossenfelder and Tim Palmer, "Rethinking Superdeterminism", *Frontiers in Physics* 8 (2020), 139. The coming experiment of Hossenfelder and Palmer, and what the collection says it will find, are in Appendix A.

The cosmic Bell tests: settings drawn from Milky Way starlight (a common cause would reach back about six hundred years) and from quasar light (billions of years). *Side by Side, Volume One*, the Anton Zeilinger chapter, "One Verdict, One Bit", Section I ("The Experiments and the Principle"). Primary papers: J. Handsteiner and others, "Cosmic Bell Test:

Measurement Settings from Milky Way Stars", *Physical Review Letters* 118 (2017), 060401, doi:10.1103/PhysRevLett.118.060401; D. Rauch and others, "Cosmic Bell Test Using Random Measurement Settings from High-Redshift Quasars", *Physical Review Letters* 121 (2018), 080403, doi:10.1103/PhysRevLett.121.080403. In each test the Clauser, Horne, Shimony and Holt combination S of four correlations exceeded the bound of two that every account with answers laid out in advance obeys; quantum theory allows S up to two times the square root of two.

Greene's count of the particles of a body and brain, and his answer to the proposal that so many particles escape the laws of each one: *Until the End of Time*, chapter 5, section "Free Will". Told here as Greene's account.

Lorenz's convection model, amplification, and the catastrophic theorem of determinism: a scale cannot be completely specified while the scales that compose it are not, so one failure of determinism anywhere is a failure of determinism at every scale; an outcome at the microscopic scale that nothing prior fixes is magnified by the same processes and can reach the scale of the experimenters. *Causality Without Determinism*, Chapter 3, Section VI ("The Catastrophic Theorem of Determinism"), with *Measurement*, Chapter 12, Section VI (microstate amplification). Primary paper: Edward N. Lorenz, "Deterministic Nonperiodic Flow", *Journal of the Atmospheric Sciences* 20 (1963), 130 to 141, doi: 10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2. Lorenz's model is three coupled equations for the rates of change of x, y and z: sigma times (y minus x); x times (r minus z) minus y; and x times y minus b times z, where sigma, r and b are fixed numbers of the convecting fluid (Lorenz used 10, 28 and eight thirds).

The friend's next word: hard randomness and choice are one event described from two positions. *Causality Without Determinism*, Chapter 5, Section V ("Hard Randomness and Choice").

The raised left hand and the compression program; "From outside, randomness; from inside, choice: one event, two descriptions, both correct, neither complete". *What Cannot Be*, Chapter 7, "Choice, Change and

Cause", section "From Outside, Chance; From Inside, Choice"; *The Last Turtle*, Chapter 14, Section III ("The Compression Argument"). *What Cannot Be*, same section, names the passage from one position's description of an event to another's a modal rotation.

The principle of identity, and its relation to the principle of comparison: the principle of identity says that what no comparison can distinguish, in any way at all, even in principle, is one thing and not two; stated for differences, it is "a difference that no comparison can register is no difference". *The Matter We Know, Volume Two*, Chapter 2, "The Counting of Ways", Section II ("Every Microstate Counts Alike"), quoting *The Quantum Passage*, Chapter 8, Section VI, and *Measurement*, Chapter 11, Section IV.

Meaningful to the process from which it came: the choice carries the nature of that process, and its meaning belongs to that process alone. *Causality Without Determinism*, Chapter 5, Section V ("The meaning of a choice belongs to that process alone: it cannot be extracted, communicated in its specificity, or reproduced by any other process", after *Measurement*, Chapter 14, Section VIII); *What Cannot Be*, Chapter 7, section "From Outside, Chance; From Inside, Choice" ("carrying the character of the process it came from").

The inference from "randomness is meaningless" to "choice is meaningless" reversed by the principle of identity; meaning is not something an outside description can evaluate. *The Last Turtle*, Chapter 14, Section IV ("The Inference Reversed").

Choice is subjectively meaningful, and true randomness is not; choice must have consequences. *An Immanent Metaphysics*, chapter 5, "The Practice of Being", subsection "Self of Choice", aphorism 1.66-2 and its note [1].

Choice as potentiality, selection and consequence; the place of choice is the single outcome that a law of many trials does not fix. *Causality Without Determinism*, Chapter 6, Section III ("Decision, Choice and Creation"); *Side by Side, Volume One*, the Sean Carroll chapter, Section IV.

Decision and choice; two ways to spend an evening; decision engages causality, since it weighs what is known to follow from what. *Causality Without Determinism*, Chapter 6, "Choice, Creation, and the Soul", Section I ("Two Ways to Spend an Evening") and Section III; Chapter 1, Section XI (compatibilists and libertarians both take determinism to be a possible state of the world).

The water in the bucket: its comparing is real and simple, consciousness in the smallest degree (this book's Chapter 14, after *Side by Side, Volume Two*, the Giulio Tononi chapter, Section V), and each of its acts of comparison has the three features of every act of comparison, an outcome that nothing before it fixed among them; that outcome is choice in the smallest degree, in the wide sense of the word. *What Comes After the Dark*, Chapter 7, and *Side by Side, Volume Two*, the Giulio Tononi chapter, Section V (a simple comparing is real and simple); *What Cannot Be*, Chapter 10, section "Three Wishes, and Which One to Give Up" (choice "in the general sense that An Immanent Metaphysics gives the word, a sense far wider than a person's choosing"); *An Immanent Metaphysics*, aphorism 1.65-9 (every world contains some degree of selfness, and these selves always have some positive degree of effective choice).

A stone and a person differ in degree of choice; what is new in kind is the gathering of choice into a self that knows it from within. *Home in the World*, Chapter 8, "The Bottle, the Note and the Living", section "A Village That Is Alive".

The self: "Self is the appearance of the aggregation of that aspect of interaction which is choice", neither purely physical nor purely non-physical. *An Immanent Metaphysics*, chapter 5, "The Practice of Being", subsection "Self of Choice", aphorism 1.67-2; *Causality Without Determinism*, Chapter 6, Section VI ("The Self and the Soul"), which quotes the definition and adds that the self is observable and the soul is not.

Responsibility: because the world is causal, what is chosen has effects; because it is not determined, the one who chose could have chosen otherwise. *Causality Without Determinism*, Chapter 6, Section VIII ("Questions").

Chapter 16. Will the Sky Go Dark?

Opening scene: Krauss and Scherrer's far-future observer. Lawrence M. Krauss (Case Western Reserve University) and Robert J. Scherrer (Vanderbilt University), "The Return of a Static Universe and the End of Cosmology", posted 2 April 2007 (arXiv:0704.0221), published in General Relativity and Gravitation 39 (2007): 1545 to 1550. Their text reads "such an observer will conclude the existence of a static 'island universe'". In Chapter 16 I quote the words as *What Comes After the Dark*, Chapter 1, "Astronomers Who Will See No Expansion", prints the words. The timescale (all structures beyond the observer's bound cluster driven outside the de Sitter event horizon in about a hundred billion years) is reported in *What Comes After the Dark*, Appendix A, "Chapter 1. What Happens to the Sky We Can See?", table of reported values.

Greene's building: floor N stands for ten to the power N years since the big bang (read: the number of the floor is the base-ten logarithm of the time in years). Today, at about 1.38×10^{10} years (read: one point three eight times ten to the power ten), lies just above floor 10; the observation deck is floor 86; the top landing is floor 102. *Until the End of Time*, chapter 9, section "An Empire of Time". Where Greene places an event on a floor, the chapter gives his floor; where he places none, the floor is the whole number part of the count of zeros in the number of years.

The floors the chapter gives, each Greene's own, in *Until the End of Time*, chapter 9 unless another chapter is named: the sun fades to black a few steps above floor 10 ("The Black Sun"); the rip of the example of Caldwell, Kamionkowski and Weinberg comes before floor 11 ("The Big Rip"); the Local Group has likely merged by floor 11, and the oldest light can likely no longer be detected by floor 12 ("The Cliffs of Space"); the

distant galaxies have disappeared beyond the horizon on floor 12 ("A Final Thought on Thought"); star formation ends by floor 14 ("The Twilight of Stars"); galaxies are depleted of their stars by ejection between floors 19 and 20 ("The Twilight of Astronomical Order"); planets still in orbit spiral into their stars by floor 23, and central black holes sweep most galaxies clean by floor 30 ("Gravitational Waves and the Final Sweep"); protons decay from floor 38 on Greene's working value of the lifetime ("The Fate of Complex Matter"); a black hole of one solar mass has radiated away by about floor 68 (chapter 10, "The Disintegration of Black Holes").

The floor is the cosmological decade of Adams and Laughlin. *What Comes After the Dark*, Appendix A, "Chapter 1. What Happens to the Sky We Can See?", table of reported values: η is the base-ten logarithm of the time in years (read: eta, the cosmological decade); normal star formation ends by $\eta \approx 14$; the disruption of the galaxy unfolds over $15 < \eta < 25$; evaporation of black holes over $60 < \eta < 100$ (Adams and Laughlin 1997, Reviews of Modern Physics 69, 337). *What Comes After the Dark* reports several of its far-future spans from their review: the end of star formation, the coming apart of a galaxy and the evaporation of black holes (Chapter 5, "The Last Glow of the Black Holes"; Appendix A, Chapters 1, 5 and 7). Its span for the emptying of the sky comes from the standard calculation and from Krauss and Scherrer (2007), and its lifetime of a black hole of one solar mass from Page (1976) (Chapter 1, "Clumping and Departing" and "The Slow Glow of a Black Hole"; Appendix A, Chapter 8).

Greene's assumptions for the climb (the known laws, acting in their undirected way, as the dominant influence; changing laws or constants, and intelligence taking control at the scale of galaxies, set aside; his suspicion that future discoveries will not force a wholesale rewriting, which he calls "A bold assumption, perhaps"), and his account of the sun, the acceleration, the big rip as a branch he judges unlikely (with "I, and many other physicists, consider this an unlikely possibility", chapter 9, "The Big Rip"), the cliffs of space, the twilight of stars, ejection, the gravitational-wave spiral, the final sweep and proton decay at floor 38, with his note 31 to chapter 9 on the alternative (iron after about 10^{1500} years; the sphaler-

on process within the standard model at roughly 10^{150} years). *Until the End of Time*, chapter 9, sections "Evolution, Entropy, and the Future" to "The Fate of Complex Matter", and chapter 9, note 31; *The Hidden Reality*, chapter 6, "Cosmic Acceleration"; *The Fabric of the Cosmos*, chapter 10, "The Missing 70 Percent", and chapter 14, "Dark Matter, Dark Energy, and the Future of the Universe". Told here as Greene's account.

Every finding is a comparison; a comparer is whatever compares and registers what it finds. *What Comes After the Dark*, Chapter 6, "What an Observer Is", and Chapter 1, "The Edge of What You Can Ever See".

The cosmological horizon and the outside world; content beyond the horizon is no part of an observer's outside world. *What Comes After the Dark*, Chapter 1, "The Edge of What You Can Ever See"; *At the Edge of Reality*, Chapter 12, "The Bound of What Can Ever Be Compared"; the source argument in *The Gravitational Passage*, Chapter 18, Section III, "The Exterior Empties" ("The exterior is not eternal"). *What Comes After the Dark*, Appendix A, Chapter 1, gives the horizon as reported physics, and derives from the definition of the outside world that content beyond the horizon is no part of it. Greene's name for the same edge is "cosmological horizon" (*Until the End of Time*, chapter 9, "A Final Thought on Thought": "You can picture the distant cosmological horizon as an enormous glowing sphere"); the particle horizon, the edge of everything whose light has already reached an observer, is named in Chapter 10 of this book.

Within the horizon matter clumps into black holes or departs; every observer's outside world empties; the populated sky is a passing stage of our epoch. *What Comes After the Dark*, Chapter 1, "Clumping and Departing" and "A Passing Stage"; *The Gravitational Passage*, Chapter 18, Section III.

The two spans on the building. The emptying of the sky beyond the Local Group takes 10^{11} to 10^{12} years; Greene places the disappearance of the distant galaxies on floor 12. *What Comes After the Dark*, Chapter 1, "Clumping and Departing" and "The Slow Glow of a Black Hole" ("The emptying comes first, by an enormous margin"), and Appendix A,

Chapter 1. For a black hole of one solar mass, Greene places the end at about floor 68, updating Page's result, and his note 10 to chapter 10 says that his numbers update Page's calculation "based on more recent assessments of particle properties, especially nonzero masses for neutrinos" (*Until the End of Time*, chapter 10, "The Disintegration of Black Holes", and note 10). Page's paper of 1976 is titled "Particle emission rates from a black hole: Massless particles from an uncharged, nonrotating hole" (Physical Review D 13, 198); *What Comes After the Dark*, Chapter 1, "The Slow Glow of a Black Hole", and Appendix A, Chapter 1, report its lifetime as $2.16 \times 10^{66}\ (M/M_{\odot})^3$ years (read: two point one six times ten to the power sixty-six years, times the cube of the hole's mass in units of the Sun's mass), about $10^{66.3}$ years for one solar mass. *The Unquantized Arena*, Chapter 17, Section II, gives 2.1×10^{67} years for a hole of one solar mass emitting photons only, and *The Gravitational Passage*, Chapter 18, gives about 10^{67} years. In Chapter 16 I place the event on Greene's floor, about floor 68. On every one of these figures the emptying comes first by more than fifty powers of ten.

The privileged age: all knowledge of the past arrives as evidence reaching the observer now; Krauss and Scherrer's end of cosmology is the emptying seen from an astronomer's desk. *What Comes After the Dark*, Chapter 1, "Astronomers Who Will See No Expansion"; *The View from Inside*, Chapter 2, "What the Cosmological Constant Tells Us" ("The sky will darken").

Epoch and turn: an epoch is one era of the world, from one turn to the next; the turn is the passage at the end of the interiors of the black holes of the era before, in which the ten-dimensional geometry restores a large world. *What Comes After the Dark*, Chapter 1, "A Passing Stage", and Chapter 4, "Where the Hold Ends"; *What the Beginning Left Behind*, Terms.

The bucket in this chapter: a standard of not rotating is fixed only by matter within reach; matter beyond an observer's cosmological horizon fixes no standard for that observer. *What Comes After the Dark*, Chapter 6,

"Mach and the Bucket" and "A Standard That Needs Something There"; the principle of comparison in the same section ("A difference that no comparison can register is no difference").

The cosmological constant is one number within one geometry. *What Comes After the Dark*, Chapter 2, "The Second Way: One Number for Our Whole Epoch", with Appendix A, Chapter 2: within one geometry Λ (read: lambda, the cosmological constant) does not vary, by the contracted Bianchi identity (the identity of the geometry that makes the divergence of the Einstein tensor vanish) with the conservation of the stress-energy of the content; source *The Gravitational Passage*, Chapter 13, Section VI ("So within one geometry Λ does not vary. This is a theorem of the geometry, and it uses no measurement and no value of w"). Chapter 10 of this book makes the identification of the constant with the accessible share, the part of the vacuum's energy that a measurement made from inside the universe can take in, and glosses the vacuum as empty space with every field at its lowest energy (*What Comes After the Dark*, Chapter 2, "The First Way: Space Is Flat"); the map with its scale in one corner is Chapter 10's picture of the same theorem.

A kind of particle has the same properties wherever and whenever it is found, within our epoch; so the particles that make an atom, and the forces by which they hold to one another, are the same at every later time of our epoch. *What Comes After the Dark*, Chapter 3, "Binding Lives Where the Matter Is".

No field whose energy changes drives the speeding up (Greene's quintessence branch has no place in this account). *What Comes After the Dark*, Chapter 3, "One Number Through Our Whole Epoch"; Appendix A, Chapter 3 (the list of kinds holds one scalar field, the Higgs field, whose energy is constant after the electroweak crossover); *What the Beginning Left Behind*, Chapter 8, "Fixed Once" ("For the energy of empty space the ratio is exactly minus one: no field that changes over time drives the acceleration of the expansion").

No recollapse. The Friedmann equation $H^2 = (8\pi G/3)\rho - kc^2/a^2 + \Lambda c^2/3$ (read: the square of the expansion rate H equals eight pi times Newton's constant G, over three, times the energy density rho, less the curvature term, k times the square of the speed of light c over the square of the scale factor a, plus the cosmological constant times the square of the speed of light, over three). With $k = 0$ (space flat, Chapter 9 of this book), ρ not negative and Λ positive, H^2 is at least $\Lambda c^2/3$ at every time, so H never reaches zero for any density. *What Comes After the Dark*, Chapter 2, "The First Way: Space Is Flat" ("Our epoch does not fall back"), and Appendix A, Chapter 2, Form A. Measured: the curvature share $\Omega_K = 0.001 \pm 0.002$ (Planck Collaboration, Aghanim and others 2020).

No rip. The equation of state $w = p/\rho$ (read: w, the pressure p divided by the energy density rho); $w = 0$ for dust, $w = 1/3$ for radiation, $w = -1$ for a cosmological constant. Caldwell, Kamionkowski and Weinberg (2003, Physical Review Letters 91, 071301): a rip occurs for $w < -1$ (phantom energy); for $w = -3/2$ and $H_0 = 70$ kilometers per second per megaparsec, the end comes 22 billion years from now; for $w = -1$ exactly, "any system that is currently gravitationally bound ... will herafter remain so" (spelling as printed in their arXiv text, as *What Comes After the Dark*, Appendix A, Chapter 3, reports it). In this account $w = -1$ exactly, at every time of the epoch: *What Comes After the Dark*, Chapter 3, "What a Rip Needs" and "One Number Through Our Whole Epoch"; source *The Gravitational Passage*, Chapter 13, Section V.

A constant term shifts the balance of a bound thing once, by a fixed amount. *What Comes After the Dark*, Chapter 3, "A Steady Wind on the Step", and Appendix A, Chapter 3: at the Milky Way halo's edge $R_{200} = 217$ kiloparsecs, with $M_{200} = 1.08 \times 10^{12}$ solar masses (Cautun and others 2020), the outward acceleration $\Lambda c^2 r/3$ is 0.0069 of the inward acceleration GM/r^2 (read: lambda times the square of the speed of light times the radius, over three, set beside Newton's constant times the mass over the square of the radius); in hydrogen, at the Bohr radius, the outward force of Λ on the electron is 1.9×10^{-69} of the electric force (computed in that

appendix). Bound things stay bound: Chapter 3, "Binding Lives Where the Matter Is" (the raisin bread), with *The Weight of Possibility*, Chapter 2, Section II, as quoted there.

The measured equation of state: $w = -1.03 \pm 0.03$ (Planck 2018 with supernovae and baryon acoustic oscillations; Planck Collaboration, Aghanim and others 2020, Astronomy and Astrophysics 641, A6). The coming measurements of w at every era are in Appendix A of this book (*Side by Side, Volume One*, Appendix B, "The Registered Predictions", Section IV, Prediction 8; *What Comes After the Dark*, Appendix B, "The Energy of Empty Space at Every Era").

Exclusion derived from the principle of identity; degeneracy pressure needs no heat. *The Matter We Know, Volume Two*, Chapter 13, "Matter Pressed Together", Section II, "Pressure Without Heat", with exclusion from *The Matter We Know, Volume One*, Chapter 1, Box 1.2. Box 13.1 of Volume Two: a filled Fermi sea presses with $P = (3\pi^2)^{2/3}\hbar^2 n^{5/3}/(5m)$ for particles moving slowly against c, and $P = (3\pi^2)^{1/3}\hbar c n^{4/3}/4$ for particles moving near c (read: P the pressure, ħ the reduced Planck constant, n the number of particles in each unit of volume, m the particle's mass, c the speed of light); neither contains the temperature. The neutron star adds the repulsive core of the force between nucleons: Section VI.

Sirius B weighed: 1.018 ± 0.011 solar masses, orbital period 50.13 years (Bond and others 2017, The Astrophysical Journal 840, 70); radius about 5,600 kilometers (computed from Joyce and others 2018); mean density about 2.8 tonnes per cubic centimeter. *The Matter We Know, Volume Two*, Chapter 13, Section I, "A Star Smaller than the Earth"; *What Comes After the Dark*, Chapter 5, "A Star That Has Burned Its Fuel".

Chandrasekhar's voyage, 31 July 1930, and the Chandrasekhar mass, about 1.4 solar masses, made of ħ, c, the constant of gravitation and the nucleon's mass alone (Chandrasekhar 1931, The Astrophysical Journal 74, 81). *What Comes After the Dark*, Chapter 5, "A Voyage in 1930"; *The Matter We Know, Volume Two*, Chapter 13, Section IV and Box 13.2.

A cooled remnant keeps its size; a black dwarf. *What Comes After the Dark*, Chapter 5, "Held Up Without Heat". The Sun will leave a white dwarf: *The Matter We Know, Volume Two*, Chapter 13, Section III (standard accounts of the evolution of stars).

Star formation ends near $\eta \approx 14$; stars evaporate from the galaxy at $\eta \approx$ 19 to 20, the floors on which Greene places both; galaxy disruption as a whole over $15 < \eta < 25$; from 1 to 10 percent of a galaxy's stars fall to the central black hole and the rest are evaporated from the galaxy (Adams and Laughlin 1997). *What Comes After the Dark*, Chapter 5, "Two Roads for Every Remnant" and "The Last Glow of the Black Holes", and Appendix A, Chapters 1, 5 and 7 (Chapter 7: "Normal star formation ends by $\eta \approx 14$, and stars evaporate from the galaxy at $\eta \approx 19$ to 20").

Every remnant's path ends inside a black hole or carried away alone. *What Comes After the Dark*, Chapter 5, "Two Roads for Every Remnant"; Appendix A, Chapter 1 (derived from the reported dynamics of clumping and departing); source *The Gravitational Passage*, Chapter 18, Section III ("Stars form, burn out and leave remnants; remnants accrete and merge; bound structures end in black holes").

Hulse and Taylor's binary pulsar (orbital period 7.75 hours), its orbit shrinking as general relativity requires for loss of energy to gravitational waves (Hulse and Taylor 1975, The Astrophysical Journal 195, L51; Taylor, Fowler and McCulloch 1979, Nature 277, 437). *The Matter We Know, Volume Two*, Chapter 13, "In Use".

The Event Horizon Telescope's picture of the black hole at the center of the Milky Way, about 4×10^6 solar masses (Event Horizon Telescope Collaboration 2022, The Astrophysical Journal Letters 930, L12). *What Comes After the Dark*, Chapter 5, "Two Roads for Every Remnant".

The proton lasts. The gauge group $SU(3) \times SU(2) \times U(1)_Y$ (read: the special unitary groups of three and of two dimensions, and the unitary group of one dimension for hypercharge) is a product of three factors; no gauge boson carries both color and electroweak labels; so every operator that changes baryon number has a zero coefficient. *The Branches of the*

Kinds, Chapter 2, "The Stable Proton", Chapter Overview and Sections I and II ("Two independent routes converge on the absence of mixed carriers": the direct-product symmetry of the state space, and the gauge group of the finite algebra); *What Comes After the Dark*, Chapter 5, "Does the Proton Last?" (the book and the cup), and Appendix A, Chapter 5; *Side by Side, Volume One*, Appendix B, "The Registered Predictions", Section VII, Prediction 33. *The Branches of the Kinds* shows by two independent routes, from different premises, that no carrier joins color to the weak charge, and neither route uses the other. The proton is the lightest whole of baryon number one: a measured input.

The count of baryons is kept by every process built from single transformations; it changes only through a collective process. *The Branches of the Kinds*, Chapter 2, Section II, "Baryon Number": "Every process built from single transformations keeps the count of baryons" and "The baryon count can change only through a collective process, in a universe hot enough, that changes the counts of baryons and leptons together". The chapter's "no process in which a carrier turns one particle into another" is that book's "every process built from single transformations".

The sphaleron channel (Greene's note 31 to chapter 9, about 10^{150} years in his text, which he gives as far shorter than the time for matter to settle into iron) decays no single proton, and at zero temperature its rate is effectively zero. *The Branches of the Kinds*, Chapter 2, Section III, "The Closed Channels": "The zero-temperature sphaleron rate is of order 10^{-161}: effectively zero". and "No single-proton decay proceeds by this channel; sphalerons need multi-particle initial states and temperatures above the electroweak scale". *What Comes After the Dark*, Appendix A, "Chapter 5. What Becomes of the Stars and Their Ashes?", states the scope of that result: "The zero-temperature electroweak sphaleron is stated as exponentially suppressed and unable to decay a single proton (*The Branches of the Kinds*, Chapter 2, Section III); its action among the many nucleons of cold dense matter is not argued".

Super-Kamiokande, watch begun at midnight on 1 April 1996; no proton decay seen: $\tau/B(p \to e^+\pi^0) > 2.4 \times 10^{34}$ years and $\tau/B(p \to \mu^+\pi^0) > 1.6 \times 10^{34}$ years at 90 percent confidence (read: the lifetime divided by the share of decays into a positron, or a muon, and a neutral pion exceeds the value given) (Takenaka and others, Super-Kamiokande, 2020, Physical Review D 102, 112011). *What Comes After the Dark*, Chapter 5, "Does the Proton Last?", and Appendix A, Chapter 5. The coming searches (Hyper-Kamiokande and the Deep Underground Neutrino Experiment) are in Appendix A of this book.

The electron lasts: it is the lightest charged particle, as measured, and charge is conserved. Borexino: no electron decay seen, $\tau \geq 6.6 \times 10^{28}$ years at 90 percent confidence for $e \to \gamma\nu$ (read: an electron going to a photon and a neutrino) (Agostini and others, Borexino, 2015, Physical Review Letters 115, 231802). *The Branches of the Kinds*, Chapter 2, Section IV; *What Comes After the Dark*, Chapter 5, "Does the Proton Last?", and Appendix A, Chapter 5.

Caplan's heavy black dwarf: pycnonuclear fusion processes the composition to iron-56; isolated dwarfs above about 1.2 solar masses collapse after about 10^{1100} years; no proton decays in that calculation (Caplan 2020, Monthly Notices of the Royal Astronomical Society 497, 4357). *What Comes After the Dark*, Chapter 5, "A Heavy Dwarf Alone", and Appendix A, Chapter 5 (reported physics, not argued in the collection).

The six and the hold: the six are held small as the content of the large world of space and time, which is their context; they are held at one size and one shape while that large world is there around them, with things in it to compare against; the forms of matter are fixed while the six are held; the hold ends at the end of a black hole's interior and for a body carried away alone; the matter does not vanish into nothing. *What Comes After the Dark*, Chapter 4, "The Six Small Dimensions" and "Where the Hold Ends"; Chapter 5, "The End of Each Road" (the ice sculpture, and the two places set out one at a time) and Key idea ("Matter does not wear away: it keeps its forms until the hold that gives it those forms ends"); Appendix

A, Chapter 5 (derived from the third axiom with the principle of comparison and the modal boundary); source *The Gravitational Passage*, Chapter 18, Sections VI and VII and Questions. The first place: inside a black hole's horizon the direction toward the center is the direction of time, the end of the interior comes after a finite time on the clock of a body that falls in, and that end lies at no moment of the outside world's time (*What Comes After the Dark*, Chapter 4, "Where the Hold Ends", and Chapter 5, "The End of Each Road": "At that end the outside world has no moment at which it could be there around the six, and nothing is there to compare against"); the reason given in the chapter, that no signal from inside the horizon ever reaches the outside world, is reported general relativity, which Chapter 17 of this book sets out with a friend watching from far away. The six read as the range of states: *What Comes After the Dark*, Chapter 4, "The Six Small Dimensions" ("Read in another way, the same six are the space of all the states that a particle of matter can be in"). The radii of the six: the sphere at one Planck length, the larger factor at $\sqrt{6}$ (read: the square root of six) Planck lengths, about 2.45 (*What Comes After the Dark*, Chapter 4, "The Six Small Dimensions").

Chapter 17. Can a Mind Keep Thinking Forever?

The opening scene. Freeman Dyson gave four lectures at New York University in the autumn of 1978, the James Arthur Lectures on Time and its Mysteries, printed as F. J. Dyson, "Time without end: Physics and biology in an open universe", *Reviews of Modern Physics* 51, 447 (1979), DOI 10.1103/RevModPhys.51.447. The quotations of Weinberg and of Dyson ("pointless", "I prefer the eighteenth", "fried in a closed universe or frozen in an open one", "a no, a yes, and a maybe", "subjectively identical to the original creature") are as *What Comes After the Dark*, Chapter 7, sections "Autumn 1978: Time Without End" and "Slower, Colder, and Asleep", gives them from Dyson's paper. Greene cites the same paper (*Until the End of Time*, chapter 9, note 32).

Greene's assumption beneath the Thinker. Told here as Greene's account. Greene's sentence on "some other arrangement of basic particles" follows his statement that life and mind are taken to be "collections of integrated processes", not dependent on cells, bodies and brains (*Until the End of Time*, chapter 9, section "The Fate of Complex Matter").

Greene's plan of slowing, in his symbols. Told here as Greene's account. In Greene's notes (*Until the End of Time*, chapter 9, notes 33 to 36), Dyson's rate of dissipation D (read: the energy the Thinker must give off per unit of time) for a Thinker of complexity Q (read: the entropy it makes per thought) at temperature T is $D \propto QT^2$ (read: D grows as Q times the square of T). With T falling as t to the power minus p (t read as time), the total energy is finite for p greater than one half, and the total number of thoughts is infinite for p less than one, so for p between one half and one the Thinker has infinitely many thoughts on a finite store. The largest rate of shedding waste, by radiation from electrons, goes as T^3 (read: the cube of T), and the energy given off goes as T^2, so below some temperature waste builds faster than it can leave. Dyson's figure for the whole store needed by a society as complex as ours is of the order of 6×10^{37} erg (read: six times ten to the thirty-seven ergs), about what the Sun radiates in eight hours (*What Comes After the Dark*, Appendix A, "Chapter 7. Can a Mind Keep Thinking for Ever?", table of reported values; Dyson 1979). Greene's words to the Thinker, "we assure the Thinker that with this rhythm thought can continue forever", are in chapter 9, section "Thinking Slow".

Greene's lowest temperature of the sky, his condition, and his time of the last thought. Told here as Greene's account. Greene gives the horizon's temperature, "based on the measured rate of accelerated expansion", as about 10^{-30} kelvin (read: ten to the power minus thirty kelvin), and the last thought by about 10^{50} years (read: ten to the power fifty years), the fiftieth floor (*Until the End of Time*, chapter 9, section "A Final Thought on Thought"). His conclusion carries a stated condition: "The conclusion rests on the assumption that the accelerated expansion of

space will persist unchanged. No one knows if this will prove to be the case". (same section). If the acceleration decreases, Greene writes, that "would obviate a cosmological horizon, turn off the distant heat lamps, and allow the temperature of the universe to decrease indefinitely" (same section). Floor N of Greene's Empire State Building is 10^N years (read: ten to the power N years) since the beginning (Greene, chapter 9). Greene's branch in which the acceleration decreases rests on K. Freese and W. Kinney, "The ultimate fate of life in an accelerating universe", *Physics Letters B* 558, 1 (2003) (Greene, chapter 9, notes 39 and 40).

Clearing a memory gives off heat: same result, separate ground. The structural arrow (the one-way order inside every act of comparison, act before record) and the growth of entropy meet in every act of recording: *What Cannot Be*, Chapter 13, "Why Time Has a Direction", section "An Arrow Beneath the Arrow of Heat" ("A record is itself a change that cannot be undone, and making a record raises the entropy of the instrument that holds it"). *What Comes After the Dark*, Appendix A, "Chapter 7", sets that sentence beside Krauss and Starkman's point about heat. Clearing one bit gives off heat of at least kT ln 2 (read: Boltzmann's constant k times the temperature T times the natural logarithm of two); at 300 kelvin that is 0.0179 electron volts, or 2.87×10^{-21} joules (read: two point eight seven times ten to the power minus twenty-one joules), the chapter's "about three billionths of a trillionth of a joule" (*The Matter We Know, Volume Two*, Chapter 2, "The Counting of Ways", Section IX, "Heat, Work and the Limit of an Engine"; R. Landauer, "Irreversibility and Heat Generation in the Computing Process", *IBM Journal of Research and Development* 5, 183 (1961)).

Maxwell's sorter. The being at the door between two halves of a box of gas: J. C. Maxwell, *Theory of Heat* (London: Longmans, Green, and Co., 1871). That the sorter must keep what it finds of each molecule until it acts: L. Szilard, "Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen", *Zeitschrift für Physik* 53, 840 (1929). The accounting, that the sorter gives back to the surround-

ings, at least in full, the entropy it takes from the gas when it clears its memory, at kT ln 2 of heat for each bit cleared: *The Matter We Know, Volume Two*, Chapter 2, Section IX.

The measurement of Landauer's heat. A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider and E. Lutz, "Experimental verification of Landauer's principle linking information and thermodynamics", *Nature* 483, 187 (2012), DOI 10.1038/nature10872; told in *The Matter We Know, Volume Two*, Chapter 2, Section IX, from which the quotation "the mean dissipated heat does saturate at the Landauer limit" is taken.

The warm walls of the horizon, reported in part. The account's part of Greene's second step is that the outside radiation thins toward the horizon's own temperature, which *What Comes After the Dark*, Appendix A, "Chapter 7", gives as reported: "On the temperature floor, the account states that the outside radiation thins toward the horizon's own temperature (reported); the cost of cooling below that floor is not argued in the collection". The room whose walls stay faintly warm, the refrigerator that costs more energy than it saves, and Krauss and Starkman's "the de Sitter temperature fixes a minimum temperature below which life cannot operate" are told in *What Comes After the Dark*, Chapter 7, section "The Reply: A Horizon and a Floor", as the second part of Krauss and Starkman's reply, beside Gibbons and Hawking (1977). The temperature is $T = \hbar H / (2\pi k_B)$ (read: the reduced Planck constant times the expansion rate H, divided by two pi times Boltzmann's constant), which for the horizon the epoch approaches, with $H_0\sqrt{\Omega_\Lambda} = 55.8$ kilometers per second per megaparsec (read: today's expansion rate times the square root of the share of the energy of empty space), is about 2.2×10^{-30} kelvin (read: two point two times ten to the power minus thirty kelvin), computed from the Planck values (*What Comes After the Dark*, Appendix A, "Chapter 7", table of reported values). The rate of the constant alone, about 55.8 kilometers per second per megaparsec, lies below the measured present rate of 67.4 (*What Comes After the Dark*, Appendix A, "Chapter 2"). Greene's about 10^{-30} kelvin is computed from the measured present rate; the two agree to

their power of ten. Sources: G. W. Gibbons and S. W. Hawking, "Cosmological event horizons, thermodynamics, and particle creation", *Physical Review D* 15, 2738 (1977), DOI 10.1103/PhysRevD.15.2738; L. M. Krauss and G. D. Starkman, "Life, the Universe, and Nothing: Life and Death in an Ever-expanding Universe", *The Astrophysical Journal* 531, 22 (2000), DOI 10.1086/308434, which Greene cites (*Until the End of Time*, chapter 9, note 39, and bibliography); Planck Collaboration, Aghanim and others (2020).

A thought is a finding made against something there; a thinker is a comparer of a particular kind. *What Comes After the Dark*, Chapter 7, section "What a Thought Needs"; the observer as a role: *What Comes After the Dark*, Chapter 1, "What Happens to the Sky We Can See?", section "The Edge of What You Can Ever See". The water's comparing is real and simple, consciousness in the smallest degree, and what a thinker has besides is many comparisons gathered into one self, known from inside: this book, Chapter 14, section "What Awareness Is", on *What Comes After the Dark*, Chapter 7 ("Those three features belong to every act of comparison, and they come together").

What makes a comparer a thinker: many acts of comparison gathered into one self that persists. *Side by Side, Volume Two*, chapter "The Measure of an Inside", Section V, "Grades of Experience and Choice in Every Degree" ("A photodiode's single undetermined click is choice at its least gathered; a living person is choice gathered across every channel of perception into one persisting stance"; and the first two theorems of being, "Subjective context is continuous: the persistence of self-identity. Subjective content is discontinuous: the diversity of thoughts and perceptions"); *Causality Without Determinism*, chapter "Choice, Creation, and the Soul", Section VI, "The Self and the Soul" ("Self is an intrinsic of choice"). The one listener who goes on through the whole piece while the notes change is this book's picture of those two theorems.

The three features of every act of comparison, taken whole as one act, identified with consciousness. *What Comes After the Dark*, Chapter 7, section "What a Thought Needs" ("Taken whole in that way, in a context,

as one act happening, the three are what I identify as consciousness"; and the inside: "A thing that has the three features together has what this book calls an inside"); *What Cannot Be*, Chapter 22, "Why No Computation Is Everything", section "The Bound Triple, Again". The identification is made in Chapter 14 of this book; Chapter 17 points back to it.

The water's comparing is consciousness in the smallest degree. This book, Chapter 14, section "What Awareness Is", where the step is made; *Side by Side, Volume Two*, chapter "The Measure of an Inside", Section V, "Grades of Experience and Choice in Every Degree" ("Wherever interaction occurs, some form of consciousness is present", quoted there from *The Modern Engagement*, Chapter 7, Section V; and "A photodiode's single undetermined click is choice at its least gathered; a living person is choice gathered across every channel of perception into one persisting stance").

Thought is a physical process, as a result; a count of computations is not a count of thoughts. *What Cannot Be*, Chapter 22, section "What Computation Can Do" ("never depends on which program it runs"); *What Comes After the Dark*, Chapter 7, section "What a Thought Needs".

Greene's sentence on other arrangements of particles, in its two senses. Same, where the other arrangement is a physical process of another material (*What Comes After the Dark*, Chapter 7, "What a Thought Needs": "of whatever material"); different, where it means running the processes as a program (*What Cannot Be*, Chapter 22, section "A Person Is Not a File": "A person is not a very large file").

The smallest comparison needs a second thing, a lasting record and a signal. *Why the Quantum Is the Way It Is*, Chapter 2, "Every Finding Is a Comparison", section "Five Particles in an Empty Universe" ("a comparison needs an actual interaction, not a possible one"); as told in *What Comes After the Dark*, Chapter 7, "What a Thought Needs".

Energy is a capacity to take part in comparisons, so the store does not decide. *Comparison; The Necessary Basis*, Chapter 4, "The Six Intrinsics of Comparison", Section III, "Why Exactly Six". *What Comes After the Dark*, Appendix A, "Chapter 7", states the sentence on energy as an identification, and the need of a target and a record as derived.

Time comes with comparison; thinking forever means an endless run of findings. *What Cannot Be*, Chapter 14, "Three Kinds of Time", section "Three Times" ("Time comes with the comparison, not before it"); *What Comes After the Dark*, Chapter 7, "What a Thought Needs".

The two ends of every road; inside a black hole's horizon the way inward is the way forward in time; the fall lasts a finite time on the falling clock. *What Comes After the Dark*, Chapter 7, section "What the Emptying Leaves a Thinker", and Chapter 6, section "Two Ways to Be Left". The likeness of moving inward and moving toward tomorrow: *At the Edge of Reality*, Chapter 1, "What Happens at the Bottom of a Black Hole?", section "The Center Is in the Future" ("Inside, moving toward the center is no longer a choice of direction, just as moving toward tomorrow is no longer a choice", a statement of general relativity for a black hole that does not rotate, shown there by the leaning light cones; the same chapter adds that general relativity expects the end inside a rotating hole to be of the same kind). The distant friend who never sees the crossing, and the end of an interior placed "before, at or after no moment of the outside world": *At the Edge of Reality*, Chapter 9, "The Singularity Is Not Now", sections "Two Clocks, One Structure" and "The View from Outside" ("And no signal from inside the horizon ever reaches the outside"). For a hole of one solar mass the end of the interior comes at most about 15 microseconds after the crossing, by the standard calculation of general relativity (*What Comes After the Dark*, Chapter 7, same section; Appendix A, "Chapter 6"; *At the Edge of Reality*, Chapter 9, section "The View from Inside"). The emptying of every observer's outside world within 10^{11} to 10^{12} years (read: a hundred billion to a trillion years): *What Comes After the Dark*, Preface, "The Answer, in Brief"; on Greene's building the emptying

is done by floor 12, the floor on which he places the disappearance of the distant galaxies (*Until the End of Time*, chapter 9, section "A Final Thought on Thought").

The hold, and the forms of the thinker's matter at both ends. The six are held at their one size and shape "While the large world of space and time is there around them, with things in it to compare against": *What Comes After the Dark*, Chapter 4, "Could Empty Space Change Its Nature?", sections "The Six Small Dimensions" and "Where the Hold Ends"; the six as the content of the large world, which is their context, as Chapter 7 of this book gives it. At both ends the six are no longer held and the forms of matter end with the hold: *What Comes After the Dark*, Chapter 7, "What the Emptying Leaves a Thinker" ("A lone particle in the far future stops being matter in the form our epoch gives it. What it was made of does not stop being".) and Chapter 5, "What Becomes of the Stars and Their Ashes?", section "The End of Each Road" ("The matter does not vanish into nothing"; the ice sculpture, with its limit: what goes on past the end of each road is no longer protons, nuclei or electrons); ground in *The Gravitational Passage*, Chapter 18, Section VI (as Appendix A of *What Comes After the Dark* records). The six are held a few Planck lengths across; the Planck length is about 1.6×10^{-35} meters (read: one point six times ten to the power minus thirty-five meters).

No thinker makes an endless run of findings, and the conclusion uses neither the store nor the temperature: same result as Greene's, on separate ground. *What Comes After the Dark*, Chapter 7, sections "What the Emptying Leaves a Thinker" and "Dyson's Three Questions" ("Krauss and Starkman reached the same answer from a finite store and a floor of temperature. This physics reaches it without either: the store and the temperature do not enter").

Dyson's copy is not shown by the match to be the same subject. *What Comes After the Dark*, Appendix A, "Chapter 7", section "Dyson and Krauss and Starkman, set beside the account" ("it holds that a copy matching the original part for part is not shown, by the match, to be the

same subject"); *What Cannot Be*, Chapter 22, section "A Person Is Not a File" ("because being a person is not exhausted by any exterior description").

A clock certain to wake the sleeper after endlessly many sleeps would be a deterministic process. *What Comes After the Dark*, Appendix A, "Chapter 7", same passage, which states the step as derived given the identification of a certain clock with a deterministic process; *Measurement*, Chapter 14, Section II ("no world is purely and completely deterministic"), where determinism needs "the same law carrying every state into its successor everywhere and at every time, which is symmetry perfected" and "every state specified at every point of the context without a gap", which is continuity perfected. The theorem in the words of *An Immanent Metaphysics*, aphorism 1.43-2, as *Home in the World* quotes it: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". Krauss and Starkman's "Alarm clocks are eventually guaranteed to fail": *What Comes After the Dark*, Chapter 7, "The Reply: A Horizon and a Floor".

No chain of societies passes a message beyond the sender's horizon. *What Comes After the Dark*, Chapter 7, section "Dyson's Three Questions" (the runner on the lengthening track); Appendix A, "Chapter 7" ("derived given the stated identification" of the speed of light with the edge of the domain of comparison).

The constant is one number, so the horizon grows toward one fixed size, and the branch of Kinney and Freese does not arise. *What Comes After the Dark*, Chapter 3, "Will the Expansion Tear Us Apart?", section "One Number Through Our Whole Epoch"; Chapter 2, "Will It All Fall Back?", section "The Second Way: One Number for Our Whole Epoch". The equation of state is exactly minus one ($w = -1$, read: the pressure of the energy of empty space equals minus its density); the coming measurements of it are in Appendix A of this book.

No act of comparison spans the turn, so no thinker passes into the next epoch as a thinker. *What Comes After the Dark*, Chapter 7, section "Dyson's Three Questions"; *The Gravitational Companion*, "Two Formal Triples of a Domain", Section III ("No comparison registers the turn as an event, so no act spans it, and no finding made in one epoch is real in the other", as Appendix A of *What Comes After the Dark* quotes it).

Thinking does not end: in each epoch that follows, comparison begins again; the next epoch's thinking continues no thinker of this one. *What Comes After the Dark*, Chapter 7, section "Where Thinking Begins Again" and Key idea; *At the Edge of Reality*, Chapter 21, "The Story Whole", section "The Turn" ("Then the new epoch makes stars, and the stars make black holes"). I speak of the epochs one after another, and never of all of the epochs gathered into one finished whole. The collection states what carries across one turn and makes no claim about a completed totality of all epochs (*What Comes After the Dark*, Appendix A, "Chapter 7"; *The Gravitational Companion*, "Two Formal Triples of a Domain", Questions).

Our thinking belongs to this epoch as the sky does; comparison itself is not used up. *What Comes After the Dark*, Chapter 7, section "What This Means for You". Beside Greene's "rare, wondrous, and precious" (*Until the End of Time*, chapter 9, section "A Final Thought on Thought").

Chapter 18. What Is Left When the Black Holes Are Gone?

The opening scene. Hawking first announced black-hole radiation at a meeting on quantum gravity at the Rutherford Laboratory on 15 and 16 February 1974 (Carr, Ellis, Gibbons, Hartle, Hertog, Penrose, Perry and Thorne, "Stephen William Hawking: a biographical memoir", Biographical Memoirs of Fellows of the Royal Society, 2019; arXiv:2002.03185). The paper: S. W. Hawking, "Black hole explosions?", *Nature* 248, 30–31, issue of 1 March 1974, received 17 January 1974, DOI 10.1038/248030a0. Greene tells Hawking's first view of Bekenstein's proposal ("ludicrous") in *Until the End of Time*, chapter 10, section "The Disintegration of Black Holes".

Greene's floors for the black holes. *Until the End of Time*, chapter 10, section "The Disintegration of Black Holes": taking the mass of the Sun as representative of the holes left by dying stars, "Page's result shows that by about floor 68 of the Empire State Building, 10^{68} years after the bang, such black holes will have radiated away" (read: ten to the sixty-eight years). Greene's note 10 to chapter 10 cites D. N. Page, "Particle emission rates from a black hole: Massless particles from an uncharged, nonrotating hole", *Physical Review D* 13, 198–206 (1976), DOI 10.1103/PhysRevD.13.198, and says: "The numbers quoted update Page's calculation based on more recent assessments of particle properties, especially nonzero masses for neutrinos". Section "The Disintegration of Extreme Black Holes": a hole of about one hundred billion solar masses lasts until floor 102. Section "The Disintegration of Black Holes": holes heavier than the Moon are colder than the microwave background, and "even the smallest black holes so far discovered by astronomical observations are much more massive than the moon". Section "An End of Time": the view "from the 102nd floor"; if the dark energy drains away, particles "may accumulate into even larger black holes"; if it persists, they "will rarely if ever encounter one another", and "such clumping will be extraordinarily unlikely". The name dark energy is the one in common use, which Greene adopts: "dark energy has become the de facto explanation for the accelerated expansion of space" (chapter 9, section "The Big Rip").

The floors this chapter gives, one for each event. Greene's own floor where *Until the End of Time* prints one; otherwise the whole number part of the count of zeros in the number of years. Today, about 1.38×10^{10} years after the beginning: about floor 10. The cooling of the microwave background below the temperature of a hole of one solar mass, 10^{11} to 10^{12} years (*The Gravitational Passage*, Chapter 18, Section X): done by about floor 12, the floor at the end of that span, since Greene places this event on no floor. The emptying of every observer's outside world, 10^{11} to 10^{12} years (*What Comes After the Dark*, Preface, "The Answer, in Brief"): done by floor 12, the floor on which Greene places the disappearance of the distant galaxies (*Until the End of Time*, chapter 9, section "A Final

Thought on Thought"). The glowing away of a hole of one solar mass: floor 68 (Greene's). The glowing away of a hole of about 10^{11} solar masses: floor 102 (Greene's). The end of the interior of a black hole lies at no moment of the outside world's time and so on no floor.

The glow is a comparison of two empty states, on a smooth geometry. *The Unquantized Arena*, Chapter 17 "Black-Hole Information and the Page Curve", Section II "Hawking Radiation on a Classical Geometry"; *What the Beginning Left Behind*, Chapter 9 "Black Holes Within an Epoch", section "The Faint Glow". The two sets of modes are related by a Bogoliubov transformation fixed by the classical geometry, and the vacuum of the early modes, restricted to the late modes outside, is thermal at the Hawking temperature $T_H = \hbar c^3/(8\pi G M k_B)$ (read: h-bar, the reduced Planck constant, times the cube of the speed of light c, divided by eight pi times Newton's constant G times the hole's mass M times Boltzmann's constant k_B). The picture of pairs created at the horizon is a heuristic that the computation does not use (*The Unquantized Arena*, Chapter 17, Section II: "The account of the radiation is the computation, and the heuristic is not used here"); *The Gravitational Passage*, Chapter 18 "The Interior Passage", Section X "The Hawking Question", calls the popular picture misleading. Greene's words for the pair picture: "Briefly, here is the essence of Hawking's reasoning". (*Until the End of Time*, chapter 10, section "The Disintegration of Black Holes").

The glow does not rest on what happens beyond the Planck scale. *The Unquantized Arena*, Chapter 17, Section II; *The Gravitational Passage*, Chapter 18, Section X. The modes that become the glow are traced back to wavelengths far below the Planck length (trans-Planckian modes). *The Gravitational Passage*, Chapter 18, Section X: "The Planck scale is the modal boundary, where each existing description reaches the edge of its domain and no description takes over", and "The Incommensuration Theorem says the description does not hold at all scales". In Chapter 18 I state that sentence in the sense that the smooth description does not hold at every scale, the Planck scale being where the range of the smooth description ends. In Chapter 18 I also recall the probe of Chapter 5 for why the edge

lies at the Planck scale. Modified dispersion: Unruh (1995); Corley and Jacobson (1996); lattice: Corley and Jacobson (1998); the thermal spectrum survives whenever the surface gravity is far below the cutoff. Measured: J. R. Muñoz de Nova, K. Golubkov, V. I. Kolobov and J. Steinhauer, "Observation of thermal Hawking radiation and its temperature in an analogue black hole", *Nature* 569, 688–691 (2019), DOI 10.1038/s41586-019-1241-0, a Bose–Einstein condensate with a sonic horizon. *The Gravitational Passage*: "Hawking radiation is real and negligibly slow for astrophysical black holes".

The Incommensuration Theorem, as the chapter states it. In the exact words of *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". Its bearing on the smooth description: *What the Beginning Left Behind*, Chapter 9, section "The Faint Glow" ("the smooth description does not hold at every scale"). The step the chapter gives from the theorem to that edge: at the Planck scale the two descriptions are relevant together, and "A single description that carried both would need a forbidden pairing" (*The Gravitational Passage*, Chapter 17, Section II, "The Modal Boundary"); the pairing such a description would need is named in the same book, Chapter 3, Section V, "The Physical Reading", for a quantized arena: "The one factor would then carry continuity with symmetry, or asymmetry with discontinuity, and the Incommensuration Theorem excludes both". The glosses of symmetric and continuous, with the square turned a quarter round and the dimmer knob, are those of Chapter 3 of this book.

Black-hole entropy goes by area, on the ground of Axiom I. *The Unquantized Arena*, Chapter 17, Section III "The Premise Removed" (equation 17.2) and Section IV "Why Area, Not Volume". $S_{BH} = k_B c^3 A/(4G\hbar)$ (read: the entropy is Boltzmann's constant times the horizon area A in units of the Planck area, divided by four), established physics (Bekenstein 1973; Hawking 1975). Axiom I (*An Immanent Metaphysics*, aphorism 1.3-1): "The immanent is more fundamental than the omniscient and/or

the transcendent", where "and/or" means of either one and of both together (*Home in the World*, "For the Specialist"). In Chapter 18 I use the part of the axiom that my argument there uses, the act over the structure. The count of interior cells is the omniscient count (a structure laid out as a whole); the count of what crosses the boundary is the immanent count (the act, made from a place). By Axiom I the count of what crosses the boundary is the more fundamental. The sense of "more fundamental" the chapter gives, that a structure is made of acts: *The Unquantized Arena*, Chapter 2 "The Three Axioms and the Two Levels of a Theory", Section II "The Three Axioms" ("Axiom I is met in the act of comparison being prior to the compounds formed from it"; "In plain words, Axiom I says that the act comes first, and that structure and stance are conjugate"); *Home in the World*, chapter "Three Parts in Every Finding", section "The Borromean Rings" ("The timetable and the platform are both made of acts"). The chapter's three roles are the stance (transcendent), the act (immanent) and the structure (omniscient). What the entropy counts, recalled from Chapter 6 of this book: *Two Theories, One World* ("I read the count as a count of content"); *What Comes After the Dark*, Chapter 8 "Is the Long Emptying the End?", section "The Third Ground: Every Count Has a Context" ("I identify what that entropy counts as the states of the hole's interior that no comparison from outside can tell apart": and "a count made against the outside world as its context"); *What Comes After the Dark*, Chapter 9, section "Two Things Called Information" ("So it is a count made against the outside world as its context"); *The Unquantized Arena*, Chapter 17, Section III. Scope: the identification makes no claim on the coefficient one quarter, which equation 17.2 takes from Hawking (1975) (*The Unquantized Arena*, Chapter 17, Section IV; *The Compactification*, Chapter 4, Section II).

The count of a hole of one solar mass. About 1.05×10^{77} (in units of Boltzmann's constant), computed from the measured constants: *What the Beginning Left Behind*, Chapter 9, section "A Bell Struck Far Away".

The area law checked in a merger. The area theorem (the total horizon area does not decrease): *What the Beginning Left Behind*, Chapter 9, section "The Count of a Merger" ("The area theorem requires a ratio of at least one"). GW150914, recorded 14 September 2015 (B. P. Abbott and others, LIGO Scientific Collaboration and Virgo Collaboration, "Observation of gravitational waves from a binary black hole merger", *Physical Review Letters* 116, 061102, 2016, DOI 10.1103/PhysRevLett.116.061102). M. Isi, W. M. Farr, M. Giesler, M. A. Scheel and S. A. Teukolsky, "Testing the black-hole area law with GW150914", *Physical Review Letters* 127, 011103 (2021), arXiv:2012.04486: the area increased, at a probability of about 97 percent. Carried in *What the Beginning Left Behind*, Chapter 9, section "The Count of a Merger", and *The Compactification*, Chapter 4, Section VI.

The times of the glow, and why the sources' figures differ by one or two floors. $T_H \approx 6.2 \times 10^{-8}$ kelvin for one solar mass (*The Unquantized Arena*, Chapter 17, Section I; *What the Beginning Left Behind*, Chapter 9, "The Faint Glow"); the background cools below it in about 10^{11} to 10^{12} years, since in accelerated expansion the radiation temperature falls by a factor e about every seventeen billion years (*The Gravitational Passage*, Chapter 18, Section X). The evaporation then takes about a one followed by sixty-seven zeros of years, as *The Gravitational Passage* computes it (Chapter 18, Section X: about 10^{67} years). For one solar mass emitting photons only, 2.1×10^{67} years (*The Unquantized Arena*, Chapter 17, Section II; the same figure in *What the Beginning Left Behind*, Chapter 9, "The Faint Glow"). Page's rate, with the neutrinos taken as massless: lifetime $\approx 2.16 \times 10^{66}\ (M/M_\odot)^3$ years (read: two point one six times ten to the sixty-six years, times the cube of the mass in solar masses), Page (1976), as *What Comes After the Dark*, Appendix A "For the Specialist", Chapter 1, reports; the same book, Chapter 5, section "The Last Glow of the Black Holes", gives "about two times ten to the sixty-six years". Greene's floor 68, from Page's calculation updated for nonzero neutrino masses (his note 10 to chapter 10), is the floor by which such holes have radiated away. The figures lie between floor 66 and floor 68; a hole that can give off fewer kinds of massless particle loses its mass more slowly, which is why the figure for

photons only (2.1×10^{67}) is longer than Page's figure with massless neutrinos (2.16×10^{66}). Fred Adams and Gregory Laughlin place the evaporation of black holes between a one followed by sixty zeros and a one followed by a hundred zeros of years, which on Greene's building runs from floor sixty to floor one hundred (Adams and Laughlin 1997, "A dying universe: the long-term fate and evolution of astrophysical objects", *Reviews of Modern Physics* 69, 337, as *What Comes After the Dark*, Chapter 5, section "The Last Glow of the Black Holes", reports: "between ten to the sixty and ten to the hundred years"). Holes of 10^9 to 10^{10} solar masses evaporate in 10^{93} to 10^{96} years (*What Comes After the Dark*, Appendix A, Chapter 10).

The black holes are the last of the ashes; the emptying comes first. *What Comes After the Dark*, Chapter 5, section "The Last Glow of the Black Holes" ("So the black holes are the last of the ashes"; "A remnant taken into a black hole is not given back by the hole's glow within the times over which the emptying runs"); Chapter 1 "What Happens to the Sky We Can See?", section "The Slow Glow of a Black Hole". Every observer's outside world empties within 10^{11} to 10^{12} years (same book, "The Question of the Far Future", section "The Answer, in Brief"). The epoch and the turn, as glossed: *What the Beginning Left Behind*, Terms.

A body falling freely meets nothing at the horizon. *The Unquantized Arena*, Chapter 17, Section VIII "Questions": "a body falling freely through the horizon meets there nothing that a body falling freely anywhere else would not meet". The account keeps the smooth horizon against the firewall argument of Almheiri, Marolf, Polchinski and Sully (2013) by denying that the partner modes behind the horizon are independent of the early radiation (past the Page time the island belongs with the radiation).

The end of the interior; no last tick. *What Comes After the Dark*, Chapter 6 "What Happens to Someone Who Is Still There?", sections "Two Ways to Be Left" (inside the horizon the direction toward the center is the direction of time; for a hole of the Sun's mass the end comes at most about fifteen millionths of a second after the crossing, on the falling clock; the

end lies at no moment of the outside world's time) and "No Last Tick" ("No clock records a last tick beyond which it ticks no more"; the marks at half the distance to a line), and Key idea ("The end of the sky is the end of what an observer can compare with, not the end of time").

Information in two senses. *The Unquantized Arena*, Chapter 17, Section III, Proposition 17.3 ("Information is not destroyed in a black hole"): in the first sense, the form of the content, "the specification of what the matter is and of which alternatives it holds, carried with the matter wherever the matter goes"; in the second sense, a count of states relative to a context. The letter and the post office: *What Comes After the Dark*, Chapter 9 "What Crosses Over, and What Stays Behind?", section "Two Things Called Information"; the road off the edge of the map, same section.

The loss Hawking found sits at the edge of a description's range. *The Unquantized Arena*, Chapter 17, Section III, Propositions 17.1 and 17.2: premise (Q), that geometry and matter form one quantum state, is the category error of Chapter 6 of that book; premise (W), that the classical geometry describes the whole history, places the loss at the singularity, where the classical description reaches the edge of its domain. Hawking (1976), "Breakdown of predictability in gravitational collapse", *Physical Review D* 14, 2460, DOI 10.1103/PhysRevD.14.2460; Hawking (2005), "Information loss in black holes", *Physical Review D* 72, 084013, DOI 10.1103/PhysRevD.72.084013 (*What Comes After the Dark*, Chapter 9, section "A Paper Sent from Pasadena"). Greene's report of Witten's result: *The Hidden Reality*, chapter 9, section "String Theory and Holography", and its footnote ("Witten's result, by establishing an equivalence between a black hole and a physical situation that does not destroy information (a hot quantum field theory), supplied conclusive evidence that all information that falls into a black hole is ultimately available to the outside world"). Greene on the standing of Maldacena's result: *The Hidden Reality*, chapter 9, section "String Theory and Holography" (Maldacena considered "a universe whose shape differs from ours", and "Maldacena argued convincingly that everything taking place within the specified universe is a reflection of laws and processes acting themselves out on the boundary"),

and section "Parallel Universes or Parallel Mathematics?" ("Maldacena's result, and the many others it has spawned in the years since, is deemed conjectural. Because the mathematics is tremendously difficult, fashioning an airtight argument remains elusive. But the holographic ideas have been subject to a great many stringent mathematical tests"). The words "As of this writing" are those of *The Elegant Universe*, chapter 13, section "The Remaining Mysteries of Black Holes", in the W. W. Norton text whose copyright line reads 2003, 1999. The wager, in *The Elegant Universe*, chapter 13, section "The Remaining Mysteries of Black Holes": "Hawking and Thorne bet that the information is forever lost, while Preskill has taken the opposite position and bet that the information re-emerges as the black hole radiates and shrinks". The answer that the laws of quantum theory never destroy information, and the name information paradox, are told in *What Comes After the Dark*, Chapter 9, section "A Paper Sent from Pasadena".

For a hole that evaporates while an outside world remains: the Page curve. *The Unquantized Arena* derives that rise and fall from two conditions it states, with quantum theory left as it is. *The Unquantized Arena*, Chapter 17, Section V "The Page Curve for a Hole That Evaporates", Propositions 17.4 and 17.5; *What the Beginning Left Behind*, Chapter 9, sections "The Rise and Fall of the Radiation's Link to the Hole" and "Where the Information Goes". Proposition 17.4 holds under two conditions and two premises of established physics: condition (C) (the whole formed by what fell in, the content inside the horizon and the radiation began pure and stays pure, as described from one outside world, with no geometry put into the state); condition (H) (the entanglement entropy of the content inside the horizon cannot exceed S_{BH}); P22 (established physics: the entanglement entropy of the radiation cannot exceed its coarse-grained entropy S_{emit}); and P23 (established physics with an identification; Page 1993: the joint state is one of very many that no comparison from outside tells apart, and each counts alike). Then $S_{rad}(t) = \min[S_{emit}(t), S_{BH}(t)]$ to within half a nat (read: the entanglement entropy of the radiation is the smaller of the entropy emitted and the entropy the hole can hold). The turning point comes before the hole has lost half its entropy, no later than 0.646 of the

evaporation time; for photons and the small waves of the geometry with greybody factors (Page 2013) at 0.538. Page (1993) found the shape. The island computations (Penington 2020; Almheiri, Engelhardt, Marolf and Maxfield 2019) give the same shape; the account compares the two as shapes and declines the reading of the saddles as a superposition of geometries (*The Unquantized Arena*, Chapter 17, Section VI). Which holes these are: "Only holes far lighter than any star evaporate while an outside world remains around them. A hole of less than about 5×10^{11} kilograms (five hundred billion kilograms) formed in the early universe would have evaporated by now". (*What the Beginning Left Behind*, Chapter 9, "The Faint Glow"); *Two Theories, One World*, Chapter 25, section "Small Holes and the Page Curve" ("Take first a hole far lighter than any star"). The gloss of entangled is that of Chapter 3 of this book.

For a hole formed from a star: not destroyed, not accessible from outside while it stands, carried to the turn. *The Unquantized Arena*, Chapter 17, Section VII "Holes Formed from Stars"; *At the Edge of Reality*, Chapter 18 "A Clean Slate", section "Information Is Not Destroyed" ("It is not accessible from the outside world of the original epoch: it is lost to every observer who remains outside the horizon, and it is not destroyed"; "The information paradox rests on the premise that information must be recoverable in the original outside world, and this account rejects that premise"); *The Gravitational Passage*, Chapter 18, Section XIII "Predictions"; *What Comes After the Dark*, Chapter 9, sections "What Stays Behind" and "What Crosses". The form passes the turn as one form (the principle of comparison: no comparison registers a difference between the ends of the interiors) and re-emerges as the departures from uniformity of the next epoch, which this book calls the first ripples; no count reckoned against the old outside world passes. The worn stairs: Chapter 9 of this book. Scope: "The account asserts neither the return of this information to the outside world nor its loss to it". (*The Unquantized Arena*, Chapter 17, Section VII; *What the Beginning Left Behind*, Chapter 9, "Where the Information Goes"). In

Chapter 18 I state that nothing of the information reaches the outside while the hole stands. In Chapter 18 I also state that such a hole stands longer than the outside world of the hole's epoch lasts.

The identifications recalled in this chapter. The Big Bang with the turn: Chapter 9 of this book (*At the Edge of Reality*, Chapter 17; *What the Beginning Left Behind*, Terms; *What Comes After the Dark*, Chapter 8, section "One End, and the Turn"). The first ripples with the form of the content carried through the turn: Chapter 9 of this book (*What the Beginning Left Behind*, Chapter 11). The cosmological constant with the part of the vacuum's energy that a measurement made from inside the universe can take in: Chapter 10 of this book (*What Comes After the Dark*, Chapter 2, section "The First Way: Space Is Flat"). What a black hole's entropy counts, identified with states of its content: Chapter 6 of this book.

Greene's Boltzmann-brain argument. *Until the End of Time*, chapter 10, opening ("And if there truly is no end date for time, then any and all outcomes not strictly forbidden by the quantum laws ... can rest assured that sooner or later they will be given their moment to shine".) and section "Boltzmann Brains"; the horizon temperature about 10^{-30} kelvin (Gibbons and Hawking 1977, *Physical Review D* 15, 2738, DOI 10.1103/PhysRevD.15.2738; computed as about 2.2×10^{-30} kelvin from the Planck values in *What Comes After the Dark*, Appendix A, Chapter 7); Greene's estimate $10^{10^{68}}$ years (his note 23 gives $e^{10^{68}}$). His verdict "A conservative assessment is that the jury is still out". closes his paragraph on a weakening of the dark energy; the more radical escapes, in which the universe ends early, follow in the next paragraph and in the section "Is the End Near?". His note 27 to chapter 10 names two further approaches: drifting constants (S. Carlip 2007) and the argument of Carroll and collaborators. Greene's own bibliography lists the work in which Kimberly Boddy, Sean Carroll and Jason Pollack argued in 2014, as *Side by Side, Volume One* reports, that "de Sitter space settles into a quiescent vacuum in which there are no dynamical quantum fluctuations". The year 2014 is that of the preprint (K. K. Boddy, S. M. Carroll and J. Pollack, "De Sitter Space Without Dynamical Quantum Fluctuations", arXiv:1405.0298); the journal paper,

which Greene's note 27 and his bibliography cite, is *Foundations of Physics* 46, no. 6, 702 (2016). *Side by Side, Volume One*, chapter "The Wave Function and the One Who Finds" (Sean Carroll), Section VIII "Where the Predictions Part, and Why", subsection "Whether our epoch began with inflation", reports the paper. Told here as Greene's account. The collection does not discuss Boltzmann brains by name; its statements on the three premises of the argument follow.

The account's statements on the three premises of that argument. Faithful reversal: *What Cannot Be*, Chapter 13 "Why Time Has a Direction", section "Why the Equations Hide the Gap" ("The reversed path exists in the mathematics, as a solution of the laws; a gas in the faithfully reversed state never occurs in the world"), a statement about faithful reversals, which that book does not carry to brains. The epochs, one after another, each with its own outside world: *What Comes After the Dark*, Chapter 7 "Can a Mind Keep Thinking for Ever?", section "Where Thinking Begins Again" ("each epoch is followed by a next, and in each one comparison begins again"); and the account speaks of the epochs one after another, never of all of them gathered into one finished whole (same section: "this book speaks of the epochs one after another, never of all of them gathered into one finished whole"). Every road ends inside a black hole or carried away alone: same book, Chapter 6, "Two Ways to Be Left"; Chapter 5, "Two Roads for Every Remnant". The lone particle: same book, Chapter 7, section "What the Emptying Leaves a Thinker" ("A lone particle in the far future stops being matter in the form our epoch gives it"). The constant is one number in one geometry: same book, Chapter 2 "Will It All Fall Back?", section "The Second Way: One Number for Our Whole Epoch". The improbability and its context: *What the Beginning Left Behind*, Chapter 10 "The Counts of an Epoch", section "One Draw Among Many?"; the lottery ticket: Chapter 11 of this book. The largest count of an epoch counted against its own outside world, and the next epoch's own count: *What Comes After the Dark*, Chapter 8, section "The Third Ground: Every Count Has a Context" ("When that world has emptied, the largest value has nothing left to be the largest value of, and the next epoch begins

its own count against its own context"). The chapter's closing sentence of that section joins these statements and compares Greene's estimate, $10^{10^{68}}$ years, with the emptying, 10^{11} to 10^{12} years; it states no claim of the collection about Boltzmann brains.

Chapter 19. Is the Long Emptying the End?

Clausius's two sentences, 1865. Read before the Naturforschende Gesellschaft in Zürich on 24 April 1865; printed in the *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich* 10 (1865), pages 1 to 59, and in *Annalen der Physik und Chemie* 125 (1865), pages 353 to 400 (Clausius 1865, DOI 10.1002/andp.18652010702). The English translation quoted in the chapter, and the name "entropy" from the Greek word for a transformation (Greek *tropē*, a turning or transformation), are those of *What Comes After the Dark*, Chapter 8 "Is the Long Emptying the End?", section "Edinburgh, 1852: The Old Fear".

Entropy is a count relative to a context. *What Comes After the Dark*, Chapter 8, section "The Third Ground: Every Count Has a Context"; *What the Beginning Left Behind*, Chapter 7; *The Compactification*, Chapter 3, Section V "Every Count Has a Context".

The heat death, as a picture. *What Comes After the Dark*, Chapter 8, section "Edinburgh, 1852: The Old Fear": a final state of one temperature, greatest entropy, and no further change.

Greene's last floors, his two branches, and his three earlier ends. *Until the End of Time*, chapter 10: section "An End of Time" (floor 102, the mist; if the dark energy has drained away, particles may gather into larger black holes that radiate more slowly; if it persists, clumping "will be extraordinarily unlikely"; "Not that time stops"; rare events that "may punctuate oblivion"); section "The Disintegration of Emptiness" (the Higgs field at 246 in the units physicists conventionally use, as Greene writes; the champagne flute; physicists "cannot pinpoint" when the jump might come, and it "can only be predicted probabilistically"; current data "suggest" that the Higgs field is "likely" to tunnel between 10^{102} and 10^{359}

years, read: ten to the power one hundred and two, and ten to the power three hundred and fifty-nine; the timescale that "gives little cause for anxiety", with the chance that the tunneling "could happen today"); section "Boltzmann Brains" (an end of the universe "long before the timescale that would make the production of Boltzmann brains likely", about $10^{10^{68}}$ years, read: ten to the power of ten to the power sixty-eight); section "Is the End Near?" (the jump of the dark energy to a negative value and the big crunch; the cycle of Steinhardt, Turok and Ijjas, each cycle on the order of hundreds of billions of years; the climb cut short near the eleventh or twelfth floor; Tolman's buildup of entropy; cyclic cosmology as "a main competitor to the inflationary theory"; truth in science "determined by experiments, observations, and evidence"; inflation as the foremost theory among researchers; the gravitational waves, with note 33 on why the case would not be clear-cut). *The Fabric of the Cosmos*, chapter 13, sections "Cyclic Cosmology" and "A Brief Assessment"; chapter 14, section "Cosmic Origins" ("the cyclic model predicts an absence of primordial gravitational waves"). *The Hidden Reality*, chapter 5, sections "Time, Cycles, and the Multiverse" and "The Past and Future of Cyclic Universes" (Tolman's papers of 1931 to 1934, and Zanstra's argument of the 1950s that only finitely many cycles can precede ours). Told here as Greene's account.

The emptying is real, over a hundred billion to a trillion years. *What Comes After the Dark*, Preface, section "The Answer, in Brief" (10^{11} to 10^{12} years, read: ten to the power eleven to ten to the power twelve; on Greene's building the emptying is done by floor 12, the floor on which he places the disappearance of the distant galaxies, *Until the End of Time*, chapter 9, section "A Final Thought on Thought"); Chapter 8, section "The Dark Sky Is Not Where the Story Stops" (the quotation "That dark sky is real, as far as it goes"); *The View from Inside*, Chapter 16 "The Observer Inside", section "Context Must Be Restored".

Time does not stop; no last tick. *What Comes After the Dark*, Chapter 6 "What Happens to Someone Who Is Still There?", sections "The Order Inside Each Comparison" and "No Last Tick". For a hole of the Sun's mass

the end of the interior comes at most about fifteen millionths of a second (1.5×10^{-5} seconds) after the crossing, on the falling clock (*What Comes After the Dark*, Chapter 6, section "Two Ways to Be Left").

Within our epoch the count grows, as Clausius said. *What Comes After the Dark*, Chapter 8, section "The Third Ground: Every Count Has a Context".

The bucket at either end: nothing to measure its turning against; the Mach principle fixes no frame. *What Comes After the Dark*, Chapter 6, sections "Mach and the Bucket", "A Standard That Needs Something There" (the principle of comparison) and "Two Ways to Be Left" (the bucket sentence); *The Gravitational Passage*, Chapter 18 "The Interior Passage", Section IV "The Mach Principle Unpins"; *At the Edge of Reality*, Chapter 13 "When Nothing Is Left to Compare Against".

Two roads for every observer. *What Comes After the Dark*, Chapter 5, section "Two Roads for Every Remnant", and Chapter 6, section "Two Ways to Be Left"; *The Gravitational Passage*, Chapter 18, Section III "The Exterior Empties".

Ten dimensions; the six held small; the hold ends at either end. *At the Edge of Reality*, Chapter 3 ("one of time, three of space, and six of possibility"); *What Comes After the Dark*, Chapter 4 "Could Empty Space Change Its Nature?", sections "The Six Small Dimensions" and "Where the Hold Ends"; Chapter 8, section "What Is Left When the Sky Is Empty". Until the hold ends, matter keeps its forms: "Matter does not wear away: it keeps its forms until the hold that gives it those forms ends". and "The proton lasts". (*What Comes After the Dark*, Chapter 5 "What Becomes of the Stars and Their Ashes?", Key idea and text); Greene's working lifetime of the proton, about 10^{38} years (read: ten to the power thirty-eight), is set beside it in Chapter 16 of this book (*Until the End of Time*, chapter 9, section "The Fate of Complex Matter"). The internal space is the complex projective plane times the two-sphere, with radii of one Planck length (the sphere) and the square root of six Planck

lengths, about 2.45 (the larger factor), fixed at one point (*The Compactification*, Chapter 2, Section IV; *The Gravitational Passage*, Chapter 9, Section VIII).

The content does not cease at the end. *What Comes After the Dark*, Chapter 8, section "What Is Left When the Sky Is Empty"; *At the Edge of Reality*, Chapter 16 "The Turn", section "Content Needs a Context".

First ground: no content without a context; the ten-dimensional geometry restores a large outside context. *What Comes After the Dark*, Chapter 8, section "The First Ground: No Content Without a Context"; *At the Edge of Reality*, Chapter 16, section "Content Needs a Context" (the quotation "When the conditions fail, the ten-dimensional geometry restores a large outside context. It must"); *The Gravitational Passage*, Chapter 18, Section VII "De-Compactification as the Restoration of Context". The third axiom, in the words of *An Immanent Metaphysics*, aphorism 1.3-1, is given in the opening section of this appendix: it is stated of the three modalities, which the chapters call the act, the structure and the stance, and Chapter 6 of this book gives it in those plain words. The sentence that the third axiom holds content and context distinct and inseparable is that of *What Comes After the Dark*, Chapter 8, section "The First Ground: No Content Without a Context": "The third axiom holds content and context distinct and inseparable": followed by the words the chapter quotes.

The seesaw: the three shrink while the six grow, and the reverse in the new world. *What Comes After the Dark*, Chapter 8, section "The First Ground: No Content Without a Context"; Appendix A, entry for Chapter 11. On the Kasner arc the three large directions shrink as the time remaining to the power five ninths $(5/9)$, while each of the six grows as that time to the power minus one ninth $(-1/9)$; read away from the turn, the three grow and the six contract (*The Gravitational Passage*, Appendix B, Section IV; *The Compactification*, Chapter 1, Section VI).

Second ground: a featureless emptiness is never found. *What Comes After the Dark*, Chapter 8, section "The Second Ground: A Featureless Emptiness Is Never Found"; *The View from Inside*, Chapter 16, section

"Context Must Be Restored" (both quotations). The Incommensuration Theorem in the exact words of *An Immanent Metaphysics*, aphorism 1.43-2: "Nothing anywhere at all can be both absolutely symmetric and absolutely continuous". and "Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous". The new world begins even except for small departures, which the theorem requires (*What Comes After the Dark*, Chapter 8; *The Gravitational Passage*, Chapter 18, Section XI).

Third ground: every count has a context; the reset. *What Comes After the Dark*, Chapter 8, section "The Third Ground: Every Count Has a Context" (the census; the closing sentence on the heat death); *The Gravitational Passage*, Chapter 18, Section VIII "One Process, Two Sides" ("the count has no reference left"; the sentence on the reset); *The Compactification*, Chapter 3, Sections V and VII. In Chapter 6 of this book I identify what a black hole's entropy counts: a count of what the hole holds, which no comparison from outside can tell apart (*The Unquantized Arena*, Chapter 17, Section IV; *The Compactification*, Chapter 4, Section II). The entropy of a black hole is $S = k_B A / (4 l_P^2)$ (read: Boltzmann's constant times the horizon's area A, divided by four times the square of the Planck length), the formula of Bekenstein (1973) and Hawking (1975); by the no-hair theorem a settled hole is registered from outside by its mass, charge and spin alone.

Three grounds, each standing alone: the third axiom, the Incommensuration Theorem, and what an entropy counts. *What Comes After the Dark*, Chapter 8, section "Three Strands"; Appendix A, entry for Chapter 8. The three grounds come from three places: the third axiom, the Incommensuration Theorem, and what an entropy counts. They are strands of the cable of Chapter 6, each spun from a different fiber, and each answers the old fear by itself.

The ends of all interiors are one end, the turn; the lone body comes to the same end. *What Comes After the Dark*, Chapter 8, section "One End, and the Turn" (the funnels, which "stay distinct along their walls"); *The Gravitational Passage*, Chapter 18, Section II "All Interiors Are One at Their

End"; *At the Edge of Reality*, Chapter 11 "Every Ending Is the Same Ending"; *The View from Inside*, Chapter 16, section "Context Must Be Restored" (both exits lead to the turn).

The turn is approached from both sides and reached from neither; it is not an event in time and is on no floor. *What Comes After the Dark*, Chapter 8, section "One End, and the Turn" (each history "is like Chapter 6's marks at half the distance to a line, then half of what is left"); Chapter 6, section "No Last Tick"; *At the Edge of Reality*, Chapter 16; *The Gravitational Passage*, Chapter 18, Section VIII. Duration, temperature and rate apply to the approach on each side, and none of them applies to the turn itself.

The Big Bang of the new epoch stands to the turn as ours does: identified with the turn as that epoch's history renders it. The identification is made in Chapter 9 of this book; *What Comes After the Dark*, Chapter 8, section "One End, and the Turn"; *The Gravitational Passage*, Chapter 18, Section VII; *At the Edge of Reality*, Chapter 16, section "The Identification".

The cosmological constant does not keep the turn from coming; the history is a cycle. *What Comes After the Dark*, Chapter 8, section "One End, and the Turn" (the quotation "Since the end of one epoch is the beginning of the next"), and Chapter 11 "Has This Happened Before?", section "A Cycle With No First Turn" (the constant belongs to the relation between an observer and that observer's horizon, and at the end of an interior, in that book's words, "the constant has no hold there", which the chapter states as having no effect there); *At the Edge of Reality*, Chapter 20 "The Cycle Without a First Turn", section "The Expansion Does Not Stop It"; *The Gravitational Passage*, Chapter 18, Section XII "The Cycle". The identification of the constant with the accessible share is made in Chapter 10 of this book, in the words "the part of the vacuum's energy that a measurement made from inside the universe can take in"; *What Comes After the Dark* calls the same total "the energy of empty space". The constant Λ (read: capital lambda) is one constant term of Einstein's equations within one geometry (*What Comes After the Dark*, Chapter 2, section "The Second Way: One Number for Our Whole Epoch").

The cars of one train: one new epoch from all the interiors of an epoch. *What the Beginning Left Behind*, Chapter 21 "What the Beginning Left Behind", section "A Cycle with No First Turn".

The cycle of epochs has no first turn. *What Comes After the Dark*, Chapter 11, section "A Cycle With No First Turn".

The six have one vacuum; the turn is no bubble. *What Comes After the Dark*, Chapter 4, sections "Valleys and Bubbles" (the false vacuum and tunneling), "The Six Small Dimensions" (the two quantities that could change, which that book calls settings: "their overall size, and the ratio of the two radii"), "No Threads, No Landscape", "Where the Hold Ends" and "Coleman's Question, Answered for the Six"; *The Compactification*, Chapter 2, Sections III to VII; *What the Beginning Left Behind*, Chapter 5 "One Vacuum", section "Nowhere to Tunnel". Two moduli remain, the volume and the shape (homogeneity with Schur's lemma), fixed at one point with the ratio of the radii the square root of six; no flux and no flux integer. The criterion for decay by tunneling is reported from Coleman (1977) and Coleman and De Luccia (1980).

The Higgs field: in the curve derived at the energies where the Higgs boson is measured, the lowest state is the present state; the standard calculation, followed to far higher energies, finds the present state long-lived. *What Comes After the Dark*, Chapter 4, section "The Higgs Field" (the wine bottle; the self-coupling worked out at the energies of the weak force; the standard calculation, in which the strength falls to small or negative values at very high energies); Appendix A, entry for Chapter 4, "The vacuum of the Higgs field". The potential is $V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$ (read: V of phi equals minus mu squared times the squared size of the field phi, plus lambda times the fourth power of its size), with the tree-level self-coupling $\lambda = 1/8$ (read: lambda equals one eighth) at the weak scale (*The Measures of the Kinds*, Chapter 13, Sections II and IV; *What the Beginning Left Behind*, Chapter 5, "The Vacuum of the Content"). In $x = |\Phi|^2$ the potential is $-\mu^2 x + \lambda x^2$, with one minimum at $x = \mu^2/(2\lambda)$, the ring of the present condensate; the set of lowest states is one connected piece (*The*

Compactification, Chapter 7, Section IX). In the words of *What Comes After the Dark*, Chapter 4, section "The Higgs Field": "This physics gives the field's energy a floor, and, in the two-part curve it derives, that floor is the present state. It does not follow the field's strength out to very high energies. The standard calculation, which does, finds the present state long-lived, far beyond the age of the universe". The scope: the collection holds the running of the self-coupling in form and does not compute it out to field values far above the weak scale; *The Compactification* states "This chapter makes no claim about the stability of the content's vacuum under such running". Reported values: $\lambda(M_t) = 0.12604$ and a zero crossing near 10^{10} GeV (Buttazzo and others 2013); lifetime of the electroweak vacuum about 10^{139} years, and more than 10^{58} years at 95 percent confidence (Andreassen, Frost and Schwartz 2018). On Greene's building the standard calculation's figure, about floor 139, lies far above floor 12, by which every observer's outside world has emptied; the comparison is arithmetic on the two figures, and the collection's statement on the Higgs field is the one quoted above, at its scope. Greene's range, 10^{102} to 10^{359} years, is his (*Until the End of Time*, chapter 10, section "The Disintegration of Emptiness"); his note 19 to that sentence cites the same paper of Andreassen, Frost and Schwartz (2018).

No crunch: our epoch does not fall back. *What Comes After the Dark*, Chapter 2 "Will It All Fall Back?", sections "The First Way: Space Is Flat" (the constant is positive, since the energy of empty space is a positive amount and the accessible share is a fraction of it; that book's "energy of empty space" is there the vacuum's total energy of Chapter 10 of this book), "The Second Way: One Number for Our Whole Epoch" and "The Third Way: The Collapse That Does Happen". Friedmann's equation, which *What Comes After the Dark*, Chapter 2, states in words, is in symbols $H^2 = (8\pi G/3)\rho - k/a^2 + \Lambda/3$ (read: the square of the expansion rate H equals eight pi times Newton's constant G times the density rho, over three, minus the curvature term k over the square of the scale factor a, plus capital lambda over three); with $k = 0$ (flat) and $\Lambda > 0$ constant, $H^2 > \Lambda/3$ for every $\rho \geq 0$, so H never reaches zero. The step from a state that

carries over no length to flat space is that of Chapter 9 of this book, section "Why Space Is Flat" (*What Comes After the Dark*, Chapter 2, section "The First Way: Space Is Flat"). The measured curvature share is $\Omega_K = 0.001 \pm 0.002$ (Planck Collaboration, Aghanim and others 2020).

The colliding worlds share real ground with this account. *What Comes After the Dark*, Chapter 11, section "Colliding Worlds" (no first turn; the acceleration empties each observer's region; the transition has a finite temperature and density and is no place where physics fails). Rival models in their own terms: Khoury, Ovrut, Steinhardt and Turok (2001); Steinhardt and Turok (2002); Ijjas and Steinhardt (2019).

Where the cycles part: what ends a cycle, the acceleration, the turn's lack of temperature and duration, the extra dimensions. *What Comes After the Dark*, Chapter 11, section "Colliding Worlds"; Appendix A, entry for Chapter 11 (the seven points, including "Steinhardt and Turok and Ijjas and Steinhardt expect the present acceleration to end"); *At the Edge of Reality*, Chapter 20.

Tolman's question: no count crosses the turn; each epoch begins its own count. *The Compactification*, Chapter 3, Section VII "Tolman's Objection and the Cycle"; *The Gravitational Passage*, Chapter 18, Section VIII ("Each epoch begins its own count, and the cycle has no first iteration"); *What Comes After the Dark*, Chapter 11, section "Does Disorder Pile Up?" (the new notebook); *What the Beginning Left Behind*, Chapter 7 "A Clean Slate, Counted". Tolman (1931; 1934).

Penrose's aeons come nearest. *What Comes After the Dark*, Chapter 11, section "Penrose's Aeons" ("Set beside the cycle of this book, Penrose's cycle comes closer than any other"; the conformal change of scale, which keeps the shapes of things, every angle and every path of light); Appendix A, entry for Chapter 11; *Side by Side, Volume One*, "Two Roads to Reality", Section V (six shared features and six differences). Penrose (2006; 2010; 2018); Penrose gives the erebons a lifetime of perhaps about 10^{11} years within each aeon (Penrose 2018).

First waves: measured faint; in this account far below a millionth of the power of the density ripples. *What Comes After the Dark*, Appendix B, section "Primordial Gravitational Waves" (waves arise "only at second order: as a side effect of the departures acting on one another, so that their strength goes as the square of the departures' small size"); *The Gravitational Passage*, Chapter 18, Section XIII; *Side by Side, Volume One*, Appendix B, Prediction 10. Measured: ratio of tensor to scalar power $r_{0.05} < 0.036$ at 95 percent confidence (read: r at the scale 0.05 per megaparsec is below 0.036) (BICEP/Keck Collaboration 2021). The account: departures adiabatic and scalar at first order, tensor modes only at second order, r far below 10^{-6}.

The equation of state is exactly minus one at every era. *What Comes After the Dark*, Appendix B, section "The Energy of Empty Space at Every Era"; *What the Beginning Left Behind*, Chapter 8; *Side by Side, Volume One*, Appendix B, Prediction 8. $w = p/\rho$ (read: w equals the pressure p divided by the energy density rho); measured $w = -1.03 \pm 0.03$ (Planck Collaboration, Aghanim and others 2020, with baryon acoustic oscillations and supernovae).

The dark matter is made of gyrations, made at the turn and kept until the next; it is measured to last. *The Weight of the Unseen*, Chapter 3; *The Weight of Possibility*, Chapters 1 and 4; *What Comes After the Dark*, Appendix B, section "The Lifetime of the Dark Matter". Measured: $f\Gamma < 6.3 \times 10^{-3}$ per billion years (read: the decaying fraction f times the decay rate capital gamma), a lifetime beyond about 160 billion years for the whole component (Poulin, Serpico and Lesgourgues 2016).

The chapter's closing result. *What Comes After the Dark*, Chapter 8, section "What This Means for You" ("The dark sky is the last view of one epoch") and Key idea ("The emptying ends in a turn, not in a final dark"); Appendix A, entry for Chapter 8.

Chapter 20. Does Anything of Us Go On?

The Voyager records, 1977. Voyager 2 launched on 20 August 1977 and Voyager 1 on 5 September 1977, both from the Kennedy Space Center at Cape Canaveral (NASA, Voyager mission pages). Each spacecraft carries a twelve-inch gold-plated copper disk. The contents were chosen by a committee headed by Carl Sagan of Cornell University, and they are given in full in Sagan, Drake, Druyan, Ferris, Lomberg and Salzman, *Murmurs of Earth: The Voyager Interstellar Record* (New York: Random House, 1978). President Carter's statement placed on the spacecraft is dated 29 July 1977 (The American Presidency Project, "Voyager Spacecraft Statement by the President").

Greene's two forms of poetic immortality; symbolic immortality, real, and on the scale of civilization. *Until the End of Time*, chapter 8, section "Poetic Immortality": the light picture first, then "Back here on earth, poetic immortality takes a different form", then Sappho, the change of scale "from eternity to the duration of civilization", described by Greene as "a significant cost", and Shakespeare. Told here as Greene's account; the collection gives no reply on it. The collection does not discuss symbolic immortality by name; the statement of the collection set beside it in the chapter, that no finding and no record of our epoch is real in the next, is in the note below on the thinking of the next epoch.

Greene's thinker and the material of mind. *Until the End of Time*, chapter 9, section "The Fate of Complex Matter". The reply is told in Chapter 17 of this book (*What Comes After the Dark*, Chapter 7 "Can a Mind Keep Thinking for Ever?"; *What Cannot Be*, Chapter 22 "Why No Computation Is Everything", section "A Person Is Not a File").

Greene's capsule and the copies of our history in an infinite space. *Until the End of Time*, chapter 10, section "Thought and the Multiverse". Greene states the assumption that space is infinite, following "the dominant inflationary perspective", whose most studied formulations, he reports, imply that space is endless. His note 37 to that chapter cites J. Gar-

riga and A. Vilenkin, "Many worlds in one", *Physical Review D* 64 (2001) 043511 (the finite number of histories in a region the size of the horizon), and J. Garriga, V. F. Mukhanov, K. D. Olum and A. Vilenkin, "Eternal inflation, black holes, and the future of civilizations", *International Journal of Theoretical Physics* 39 (2000) 1887–1900 (the far future of civilizations and the message sent to other regions). Greene names insights from David Deutsch in the conclusion that the plan is hopeless. Told here as Greene's account; the collection gives no reply on the count of histories in an infinite space or on the fake capsules. The statements of the collection set beside the capsule in the chapter (the far regions are further parts of one world; nothing escapes as itself; a message is a record, and the new world inherits no record and no mark from the old one) are in the notes below.

Greene on descendants, with Scheffler, Wolf and Frankfurt. *Until the End of Time*, chapter 11, section "Descendants"; S. Scheffler, *Death and the Afterlife* (New York: Oxford University Press, 2016, the edition Greene cites in his notes 7 to 9 to chapter 11; first published 2013), with the replies of Susan Wolf and Harry Frankfurt in the same volume. Greene marks the reaction as his own, which he suspects is common, quotes Scheffler's conclusion as resonating with his own musings, and reports Frankfurt's different view. Told here as Greene's account. The collection bears on it through two statements set beside it in the chapter, and the question of where the worth of what we do comes from is taken up in Chapter 21 of this book: the conditions under which watchers arose in our epoch are present again in the next, and nothing of us is in that next epoch as itself (*What Comes After the Dark*, Chapter 12 "What Comes After the Dark?", section "What This Means for You").

Greene on teleportation and identity. *The Fabric of the Cosmos*, chapter 15, section "Teleportation in a Quantum World": a being in exactly the same quantum state as his own would be him, as his own judgment; a near match "could very well be" him; identity, in his belief, "all resides in the physical". Told here as Greene's account; the collection gives no reply

on it for a person rebuilt within our epoch. In Chapter 20 I set beside Greene's account the statement of the collection on identity across the turn (below).

No finding of our epoch is real in the next; the thinking of the next epoch continues no thinker of this one. *What Comes After the Dark*, Chapter 7 "Can a Mind Keep Thinking for Ever?", section "Where Thinking Begins Again". The ground, that no act of comparison spans the turn: *The Gravitational Companion*, "Two Formal Triples of a Domain", Section III; *What the Beginning Left Behind*, Chapter 21, "A Cycle with No First Turn".

The turn: one end of all the interiors, approached from both sides. *What Comes After the Dark*, Chapter 8 "Is the Long Emptying the End?", section "One End, and the Turn"; told in Chapter 19 of this book with the picture of the funnels.

Regions beyond our particle horizon are further parts of one world. *Side by Side, Volume Two*, "The Structure and the Act" (Max Tegmark), Section VI, Level I; told in Chapter 11 of this book.

A message sent beyond the cosmological horizon never arrives. *What Comes After the Dark*, Chapter 7, sections "Dyson's Three Questions" and "The Reply: A Horizon and a Floor"; the cosmological horizon is defined in Chapter 1 "What Happens to the Sky We Can See?", section "The Edge of What You Can Ever See"; told in Chapter 17 of this book.

Nothing escapes as itself, and there is no other universe that a journey could reach; any region a path can reach is part of our world, and any region none can reach is no place relative to us; no path runs from any place in our epoch to any place in the next. *What Comes After the Dark*, Chapter 10 "Could Anyone Escape?", Key idea, and section "No Other Sky Along Any Path" (the road country). Space as the pattern of causal connection: *What Cannot Be*, Chapter 12 "The Shape of What Can Reach What".

Every structure that remains ends inside a black hole or is carried away alone; matter keeps its forms until the hold ends. *What Comes After the Dark*, Chapter 12, section "The Sky Thins"; Chapter 5 "What Becomes of the Stars and Their Ashes?", section "Two Roads for Every Remnant" and Key idea. The six as the range of possibilities content can take, "as the shape and size of a drum decide which notes it can sound": *At the Edge of Reality*, Chapter 8, "How Small the Six Are".

A message is a record; the new world inherits no record and no mark from the old one. *What Comes After the Dark*, Chapter 10, section "No Message, No Record, No Identity"; Chapter 9 "What Crosses Over, and What Stays Behind?", section "What Stays Behind". Source of the step: *What the Beginning Left Behind*, Chapter 21, "What Passed the Turn, and What Did Not"; *The Gravitational Passage*, Chapter 18, Sections VII and VIII.

Nothing in the next epoch is the same thing as anything in ours, by any finding anyone could make. *What Comes After the Dark*, Chapter 10, section "No Message, No Record, No Identity". Identity is evaluated by comparison across contexts: *Comparison; The Necessary Basis*, Chapter 12, Section I.

A person is not a very large file. *What Cannot Be*, Chapter 22 "Why No Computation Is Everything", section "A Person Is Not a File"; quoted first in Chapter 17 of this book.

Information in two senses; the form of the content crosses the turn as one averaged form; no count crosses. *What Comes After the Dark*, Chapter 9, sections "Two Things Called Information", "What Stays Behind" and "What Crosses"; *The Unquantized Arena*, Chapter 17, Proposition 17.3; *The Compactification*, Chapter 3, Section IX; *At the Edge of Reality*, Chapter 18, "Information Is Not Destroyed".

The first ripples, identified with the form of the content carried through the turn. The identification is made in Chapter 9 of this book: *What the Beginning Left Behind*, Chapter 11; *The Gravitational Passage*, Chapter 18, Section IX; *What Comes After the Dark*, Chapter 9, section "The Faint Pattern". The measured amplitude of the temperature differences of

the oldest light, about one part in a hundred thousand (that is, about 10^{-5}, read: ten to the power minus five, of the mean temperature): G. F. Smoot and others, "Structure in the COBE differential microwave radiometer first-year maps", *The Astrophysical Journal Letters* 396 (1992) L1, DOI 10.1086/186504.

Everything of you that ends inside a black hole is kept, as a share of the form of the content; nothing of you crosses as you. *What Comes After the Dark*, Chapter 9, section "What This Means for You". The form of a message dropped into a black hole (in that chapter, the seed of information of Michio Kaku's last hope) arrives as no message and as the instructions of no one: Chapter 10, section "No Message, No Record, No Identity". For a body carried away alone, the books of the collection follow the form of the content to the turn; for the content that ends inside the black holes, they follow it into the one averaged form of the new sky (*What Comes After the Dark*, Chapter 9, section "What Crosses").

The law returns in every epoch and is not carried across the turn (the play performed from one script). *What Comes After the Dark*, Chapter 9, sections "What Stays Behind" and "What Returns"; *At the Edge of Reality*, Chapter 20, "What Carries Over".

Under the sky of the next epoch the conditions under which watchers arose are present again; nothing of us is in that picture as itself. *What Comes After the Dark*, Chapter 12, section "What This Means for You". The scope of the statement, in the words of *What Comes After the Dark*, Appendix A, entry for Chapter 7: "The collection derives no biology, so the arising of minds in the next epoch is not argued; what the collection establishes is that the next epoch begins with the same structure and the same clean slate under which minds arose in ours".

Immortality and timelessness, in the foundation of this account. *An Immanent Metaphysics*, General Appendix, section "The Metaphysics of Science and Religion", the second of its three resolutions, with the list of resolutions just before it. In *An Immanent Metaphysics* I take up the old hope of immortality "by regarding the concept of immortality as an iso-

morph of timelessness (without time or atemporality)". An isomorph, in that sentence, is a thing of the same form, and the book calls the relation an identity ("With this identity, ..."): immortality, taken in its form, is being without time. In *An Immanent Metaphysics* I then set two triples, lists of three, side by side: no time (atemporality), one time (temporality) and all time (immortality); and the self, the world, and the relation between them. Its three lines of the second resolution read: "That soul is atemporal (without time)"; "The relation between self and world is temporal"; "The essential nature of substance (particle matter) is also atemporal (exists/lasts forever)". The soul, in that passage, is the inner self: the first resolution reads "That inner self/soul is chosen (free), the outer self/body and world is causal (fixed/limited) and that between them, as foundation, is change". The three lines pair the members of the two triples: the self, called the soul, with no time ("without time"); the relation between self and world with one time ("temporal"); and the world, in the essential nature of its substance, with all time ("exists/lasts forever"), which the line also calls atemporal, by the book's identity of immortality with timelessness. The far-future chapters of this book followed the second line to its end. Every road of the emptying reaches, for each comparer, the end of a world to compare with; that end is the end of a relation between a self and a world, and the line "The relation between self and world is temporal". places that relation in time. The third line meets what the far-future chapters found of matter, in the words of *What Comes After the Dark*, Chapter 9, section "What Crosses": "the content does not cease at the end, and so a new context forms around it". Each statement holds of its own object. The statements of Chapter 20 that "Nothing of you crosses as you" (*What Comes After the Dark*, Chapter 9) and that "nothing in the next epoch is the same thing as anything in ours, by any finding anyone could make" (Chapter 10) are said of what crosses the turn and of identity as a finding could establish it across the turn. The first line of the second resolution is said of the soul, the inner self, which the same passage sets apart from the outer self, the body, and from the world, and it places the soul without time.

Chapter 21. Where Does Meaning Come From?

The opening scene. The debate between Bertrand Russell and Frederick Copleston, broadcast on the BBC Third Programme on the evening of 28 January 1948: Greene, *Until the End of Time*, chapter 2, opening, and its note 1, which gives the time of the broadcast as 9:45 p.m. and says that the debate took place in the year before. Russell's words are quoted as Greene prints them, from Bertrand Russell, *Why I Am Not a Christian* (New York: Simon and Schuster, 1957), pages 32 and 33 (Greene's note 2). Only Russell's sentence on universal death and purpose is taken up; the subject of the debate is not.

Greene's conclusion. Told here as Greene's account, from *Until the End of Time*, Preface; chapter 1, sections "Stories of Nearly Everything" and "Reflections on the Future"; chapter 9, section "A Final Thought on Thought"; chapter 11, sections "Order and Significance", "Mortality and Significance", "Descendants" and "Meaning"; *The Hidden Reality*, chapter 1, section "The Cosmic Order"; *The Elegant Universe*, chapter 15, section "Reaching for the Stars". Greene on the reach of mortality as an explanation (Preface): "I have come to see my awareness of my own inevitable end as having considerable influence but not providing a blanket explanation for everything I do".

Language, story, myth, religion and art. Told here as Greene's account, from *Until the End of Time*, chapter 6 ("First Words", "Storytelling and Intuition", "Storytelling and Other Minds", "Mythic Tales"), chapter 7 ("Evolutionary Roots of Religion", "Take One for the Team", "Individual Adaptation and Religion", "A Sketch of Religious Roots", "The Urge to Believe", "Belief, Confidence, and Value") and chapter 8 ("Sex and Cheesecake", "Imagination and Survival", "Art and Truth"). The Armstrong phrase behind the sentence on myth: myths are "nearly always rooted in the experience of death and the fear of extinction", and Greene adds that we may soften "nearly always" to "often" (chapter 6, "Mythic Tales"). The Harburg line: "Words make you think a thought. Music

makes you feel a feeling. But a song makes you feel a thought". (chapter 8, "Art and Truth"). Greene on confidence: "my confidence in the existence of God is low because of the paucity of rigorous supporting data". (chapter 7, "Belief, Confidence, and Value"). Greene on religion without literal belief: "Such varieties of religious engagement do not require a belief in the factual nature of religious content; they reflect a belief in the value of such content, regardless of whether the content is veridical". (same section). Greene on Hindu and Buddhist thought and physics: "I have never encountered more than a metaphorical resonance between distinct ideas vaguely construed". and "Invoking such translations as the basis for contact with other disciplines will almost never rise above the level of a poetic alliance". (chapter 7, "A Sketch of Religious Roots"). Greene on language and thought: "once the human mind had language, our species' engagement with reality was poised for radical change". (chapter 6, "Why We Spoke"). Greene on the shortness of life and the rarity of minds that reflect on themselves: "Yet our moment is rare and extraordinary, a recognition that allows us to make life's impermanence and the scarcity of self-reflective awareness the basis for value and a foundation for gratitude". (chapter 11, "Descendants"). Armstrong's point behind the sentence on myth and belief: those who carried out the rites of the Eleusinian mysteries "would have been puzzled if they had been asked whether they believed that Persephone really had descended into the earth, in the way that the myth described". (chapter 7, "Belief, Confidence, and Value").

Comparison as the ground of every finding. *What Comes After the Dark*, chapter 6, section "What an Observer Is". The collection works out the physics of the world from the act of comparison: *What Comes After the Dark*, Preface. A thought is a finding of the same kind as the water's hollow: this book, Chapter 17.

Purpose is individual; no single purpose for all of life. *An Immanent Metaphysics*, chapter 7 ("Aesthetics"), section "Meaning, Purpose, and Value", aphorisms 2.22-9 and 2.22-25. Aphorism 2.22-9: "There is no single purpose, or any single value for, all of life, all of consciousness, all of be-

ingness, in any world or domain". and "The purposes, values, and meaningfulness of each life, consciousness, and beingness, are always plural". The same result as Greene's "no grand design", read as no single purpose set for all of life, reached from the act of comparison.

The universe has no favorites. *An Immanent Metaphysics*, chapter 6 ("Ethics"), section "The Basal Motivations", aphorism 2.11-3. The same as Greene on "no favorites"; different on whether significance is in the world, by the aphorism's next line: "Everything (all eventities in all domains) is/are intrinsically significant".

Morality as the rules of a domain. *An Immanent Metaphysics*, chapter 6, section "Distinguishing Ethics and Morality", aphorism 2.12-1: "Morality refers to an externally defined set of rules in a particular domain (generally applicable to all selves), and the degree that the choices of self adhere to those rules". The same result as Greene's for morality. The gloss of *domain* as a world or setting in which things happen follows aphorism 1.45-9 (below).

What crosses into the next epoch, and what does not. *What Comes After the Dark*, chapter 9, section "What This Means for You", and chapter 10, Key idea; this book, Chapter 20. The populated sky is a passing stage of our epoch: *What Comes After the Dark*, chapter 1; this book, Chapter 16.

Meaning cannot be removed. The three steps (no edge of meaning drawn from outside meaning; the fixed point; no definition of meaninglessness has a case of its own), the definition of "meaningful" used, and the sentence on declaring that nothing matters: *What Cannot Be*, chapter 7 ("Choice, Change and Cause"), section "Meaning Cannot Be Removed". The source states the first step as: "A comparison is made from a stance, and making it is a meaningful act". The chapter's steps for the first step apply the source's own definition (the outcome "carries that process's character") to the outcome of a comparison made from a stance, using the stance of this book's Chapter 3 (the direction a detector is turned to, and the yes or no it could hear). The source states the second step as: "The idea that choice is meaningful is itself an idea with definite content: it

identifies the residual, seen in the first person, as a specific outcome carrying the character of its process". and "That idea is meaningful, so it is itself a case of meaning". The chapter's steps for the second step join definite content to meaning through the first step's own premise, "every distinction is made by comparison", and the premise that a comparison made from a stance is a meaningful act; the source's third step uses the same join for a definition that has content. The gloss of choice in those steps is this book's, from Chapter 15 (an outcome that nothing before it fixed, seen from inside by the one whose outcome it is), and "every part of every world has some degree of choice" is Chapter 15's sentence, resting on *An Immanent Metaphysics*, aphorism 1.65-9, and, for the water, on *What Comes After the Dark*, Chapter 7, as the note to that chapter gives them. The chapter's sentence of scope (what every comparison carries is a share in value, and not a message anyone could write down) states positively the same section's sentence: "it is not the claim that every event has a significance someone could write down".

The same result in the words of *An Immanent Metaphysics*. In Chapter 21 I quote the conclusion, aphorism 2.53-3, and gloss purpose and value by the pairing of aphorism 2.22-10. The steps that lead to the conclusion are given here. Everything has some degree of meaningfulness, purpose and value: *An Immanent Metaphysics*, chapter 9 ("Evolution"), section "PanPsychism", aphorism 2.53-3; and chapter 7, section "Meaning, Purpose, and Value", aphorism 2.22-8, "Everything (every perception and expression) has (cannot not have) significance (some degree of meaningfulness)". Aphorism 2.22-8 also says "Significance (meaningfulness) is zero positive"; the next line, "has (cannot not have) significance", states that significance is never zero. The triple: aphorism 2.22-1, "Meaning, purpose, and value are distinct, inseparable, and non-interchangeable", with "To truly have any one of these, so must some part of each of the other two also be present". The pairing of purpose with expression and of value with perception: aphorism 2.22-10, "There is no expression that is without purpose". and "There is no perception or experience that is without value". The formal names that aphorism 2.22-2 gives

the triple (purpose, the omniscient; value, the transcendent; meaning, the immanent) are kept here and not in the chapter. The two statements of meaning wherever comparison occurs start from different places: one from the structure of comparison, in *What Cannot Be*, a book on physics; the other from what *An Immanent Metaphysics* says of meaning, purpose and value. They reach one result, as the strands of a cable share one load, so that the cable holds when one strand is cut (this book, Chapter 6).

The degree of choice in a stone. The degree of choice in a stone is small, and it runs without a break to the degree in a person: *Home in the World*, chapter 8 ("The Bottle, the Note and the Living"), section "A Village That Is Alive"; this book, Chapter 13.

Randomness from outside, choice from inside. One event, described from two positions; from the outside position the outcome that nothing fixed shows as a gap with no pattern, and from the position of weighing worth it is a meaningful choice: *What Cannot Be*, chapter 7, section "From Outside, Chance; From Inside, Choice". That section names the two positions the omniscient position (description in the third person) and the transcendent position (engagement in the first person), and the passage from one position's description to the other's a modal rotation. No third-person description contains the first person: *The Last Turtle*, chapter 15, Section II; this book, Chapter 14.

The bedrock is the act of comparison. The particles and fields are worked out from the act of comparison: *What Comes After the Dark*, chapter 7 ("Can a Mind Keep Thinking for Ever?"), section "Where Thinking Begins Again", and Preface.

The order of presupposition. *Physics Is Not Math*, chapter 5 ("What Depth Means"), Section V ("The Order of the Four"), for the order and for each step of it ("Mathematics presupposes computation, because every proof is checked by a computation"; "Computation presupposes physics, because every computation that is carried out is a physical process"; "Physics presupposes the act of comparison"). The definition of presupposition quoted in the chapter: the same book, chapter 5, Section II

("Depth Is the Order of Presupposition"). "Mathematics works as if only the objective existed": chapter 3 ("Truth Has a Frame"), Section VII ("Mathematics Works as if Only the Objective Existed"), with Section VIII on the generalized assumption. That book pictures the four with a kitchen, and the chapter pictures the first two steps with a sum checked on paper.

The meaning of a work and the coherence of its choices. *An Immanent Metaphysics*, chapter 7, section "The Meaning of Art", aphorism 2.24-1: "The degree of meaningfulness associated with an eventity (such as a work of art or a play) is proportional to the degree of coherent selectivity that is defined by selective choice(s) that compose that eventity". and "The more the details of an eventity are known as being causality, deterministically, or logically defined, the less meaningful (significant) it will be". Notes and the sonata: *Home in the World*, chapter 8, section "A Village That Is Alive"; this book, Chapter 13, with the same idea.

Meaning gathered in a chooser. A living being is a place where choosing is gathered and known from inside, and in it "true choice is gathered and known from inside, and with it, meaning": *Home in the World*, chapter 8, section "A Village That Is Alive". The same section: "Meaning lives only where there is choice". and "So meaning, known from inside, lives only where life is". The self as the gathering of choice: *An Immanent Metaphysics*, chapter 5 ("The Practice of Being"), section "Self of Choice", aphorism 1.67-2, "Self is the appearance of the aggregation of that aspect of interaction which is choice"; this book, Chapter 15. In Chapter 21 I say that meaning is gathered and known from inside in a chooser, in those words. In Chapter 21 I give no ranking of degrees of meaning beyond the measure of aphorism 2.24-1 for a work.

Creation an aspect of all events. *An Immanent Metaphysics*, essay "The Origins of Worlds", aphorism 1M8 (item 6), "True creativity, and creation, is, and must be, inherent in all choice", and its note 3.

Life is its own purpose, value and meaning. *An Immanent Metaphysics*, chapter 7, section "Meaning, Purpose, and Value", aphorism 2.22-24, with its next line, "There is no greater purpose of life than to live", and its note 3: "There is no life that is without significance, and there is no significance that is not alive".

Meaning and permanence; meaning between domains. Meaningfulness is not defined by persistent future states, and it is found "in between: in the interchange among three domains": *The Last Turtle*, chapter 14 ("Free Will and the Compression Argument"), Section V ("Where Meaning Is Found"). The condition the chapter finds in Greene's first premise (that a meaning given by the world would have to last) is my reading of Greene's step; Greene does not state it.

Language and being in a domain. *An Immanent Metaphysics*, chapter 1 ("Foundations"), section "Simplicity, Clarity, and Metaphor", aphorism 1.45-9, both lines.

Fact, truth and myth. *An Immanent Metaphysics*, chapter 7, section "The Meaning of Art", aphorism 2.24-3. The aphorism's words: "Truth has the modality of the immanent". "Fact has the modality of the omniscient". "Myth has the modality of the transcendent". In Chapter 21 I give the three modalities in the names of the three roles that this book uses (Chapter 3; *Home in the World*, Appendix, "The Three Names and the Three Roles"): the omniscient is the structure, the immanent the act, the transcendent the stance. The same aphorism: "Eventities which have mythic, mystic, or mysterious qualities will tend to have greater significance than eventities (arts) which are purely factual and structural". Its first line speaks of seeming: "The degree of seeming comprehensibleness of an eventity in factual terms is inversely proportional to the degree of seeming meaningfulness of that eventity". The chapter's clause of scope, that comprehending in terms of laws and facts is the third-person description, which weighs no worth, joins the aphorism to *What Cannot Be*, chapter 7, section "From Outside, Chance; From Inside, Choice" (the note on randomness from outside and choice from inside, above).

A root for creation stories beside Greene's. *An Immanent Metaphysics*, essay "The Origins of Worlds", aphorisms 1M1 and 1M3. The essay finds that "The popular notions of the Big Bang, as developed by cosmological science, and the myth of the event of creation, as developed within many historical religions" share one shape: "They each implicitly posit the idea that all of creation happened once, very long ago, in a place very far away from the here and now". It finds the reason in a fear: "many people fear the unknown and the unknowable, and anything else which is regarded as capricious, arbitrary, or random". The popular notion meant is the notion of the Big Bang as one event of creation, long ago and far away; the Big Bang is the one I identified in Chapter 9 with the turn as our epoch's history renders it.

The root of public religion, and its central concern. *An Immanent Metaphysics*, chapter 8 ("Mind"), section "Gentleness", aphorism 2.48-5: "Any consciousness with self reflective properties which is in vivid and intimate contact with nature will tend to develop a higher level of spiritual awareness". and "When such consciousness has social aspects as well (human civilization being an example) such personal spiritual awareness eventually becomes reflected into public religions whose central concern (either directly or indirectly) is about suffering and its resolution (a mythos of redemption)". In the aphorism, the root of public religions is the personal spiritual awareness of a self-reflective consciousness in vivid and intimate contact with nature, and suffering and its resolution are the central concern of those religions. Greene, in *Until the End of Time*, chapter 7, "Individual Adaptation and Religion", first reports "Clearly, there is as yet no consensus on why religion arose nor on why it has so tenaciously remained", and then gives his own leaning: "I remain partial to religion's relevance to our singular recognition of our finite lives". In Chapter 21 I set the aphorism's root, spiritual awareness, beside the root Greene leans toward, and I find that the two roots agree in kind: each root lies in what a mind lives through and knows from inside. That finding is the chapter's

own reading of the two roots, joined to the chapter's result that meaning in a chooser is gathered and known from inside (*Home in the World*, chapter 8, section "A Village That Is Alive").

Belief and faith. On belief, *An Immanent Metaphysics* writes: "A belief is an unconscious or subconscious choice". (General Appendix, section "Belief", aphorism 2.75-1). Greene, too, finds that many of our beliefs are formed below the threshold of awareness, by a mind shaped for survival (*Until the End of Time*, chapter 7, "Belief, Confidence, and Value"). On faith: "To have faith is to embrace the mythic, rather than to be rejecting the factual". (chapter 6, section "Faith", aphorism 2.17-9). That sentence stands near Greene's religion read for its value and not as literal fact.

Every event in a living thing is a physical process. Greene's sentence on art has two clauses: the physical facts can fully address how the particles interact and behave, and they shed "only monochrome light" on how we live (*Until the End of Time*, chapter 8, "Art and Truth"). The first clause agrees with the first half of the finding: every event in a living thing is a physical process (*Home in the World*, chapter 8). The second clause stands with the second half: no description, however refined, contains everything it describes (*The Last Turtle*, chapter 27, Sections IV and VII; this book, Chapter 13).

Ethics. Ethics as the principles of effective choice, common to all of consciousness, practiced uniquely by each individual: *An Immanent Metaphysics*, chapter 6, section "Distinguishing Ethics and Morality", aphorism 2.12-1, and section "The Two Principles of Ethics", aphorisms 2.12-6 and 2.12-8. The two principles in full (2.12-6): the Symmetry Ethics, "Where the omniscient/objective/external context is different, and where the transcendent/subjective/internal context is the same, the content of expression (immanent) shall be the same"; the Continuity Ethics, "Where the omniscient/objective/external content is different, and where the transcendent/subjective/internal content is the same, the context of expression (immanent) shall be the same". The friendship picture, the channel between any chooser and the world, and the sentence on what a chooser

most deeply wants: *Home in the World*, chapter 8, section "Relative, Not Arbitrary", where I write "The relation between any chooser and the world is a channel of that kind" and "Keep it whole, and choosing goes well; break it, and the chooser loses the very thing choosing is for". The words "the ethics binds without being a rule laid on anyone from outside" are from the same section.

The three edges of reality. Every act of comparison made from inside the world lies within the three edges of reality (the cosmological event horizon, the Planck scale and the turn): *What the Beginning Left Behind*, chapter 21 ("What the Beginning Left Behind"), section "Where You Stand". The same sentence, "Your reading of this sentence is one such act", is in *At the Edge of Reality*, which also names the three edges. *What the Beginning Left Behind* describes the second edge in these words: "One is the Planck scale, where the smooth description of space and time reaches the edge of its domain; the six are held there, and the dark matter rings there". In Chapter 21 I describe the Planck scale in the words of this book's Chapter 5 ("At the Planck scale itself neither description governs, and both are needed").

Greene's accounts told as his. When and why language began, and its openness: told here as Greene's account; the collection gives no reply on it. Storytelling as an adaptation, the flight simulator: told here as Greene's account; the collection gives no reply on it. The three evolved leanings behind myth: told here as Greene's account; the collection gives no reply on it. The evolutionary accounts of religion (Boyer, Wilson, Durkheim, Bering, Becker): told here as Greene's account (the root that *An Immanent Metaphysics* gives public religion is in the note on the root of public religion and its central concern, above). That links between Hindu or Buddhist thought and physics stay metaphor: told here as Greene's account (what the collection says of Buddhist thought is in the next note). Confidence and belief, and a literal claim against established law held false: told here as Greene's account (what *An Immanent Metaphysics* says of belief and faith is in the note on belief and faith, above). The evolutionary accounts of art: told here as Greene's account; the collection gives no

reply on it. That limited time gives our choices their value: told here as Greene's account (what the collection says of meaning and permanence is in the note on meaning and permanence, above). Significance in the depth of understanding (*The Hidden Reality*): told here as Greene's account; the collection gives no reply on it. The search as a human drama (*The Elegant Universe*): told here as Greene's account; the collection gives no reply on it. Why there is something rather than nothing, which Greene raises in chapter 11, is not taken up in this book.

Buddhist thought beside the collection. *Incommensuration*, chapter 13 ("Reasoning, Relation and Focus"), Section V, subsection "The Four Negations": "The theorem has a counterpart in logic: the four negations, or catuskoti, of classical Buddhist logic". *The Last Turtle*, chapter 3 ("The Three Modalities and the Three Axioms"), Section VI ("Axiom II: Precedence and Convergence"), Abstract: "The Buddhist teaching of dependent origination comes close to Axiom II, but at the level of phenomena, not at the level of the structure of process itself". Both passages set Buddhist thought beside the theorem and the axioms of the collection; neither speaks of Hindu thought, and the chapter does not take them up.

Chapter 22. What Do I Now Hold?

The announcement of the first-year COBE maps. 23 April 1992, at a meeting of the American Physical Society in Washington, by George Smoot of the Lawrence Berkeley Laboratory: *What Comes After the Dark*, Chapter 12, "What Crosses"; Chapter 9, "The Faint Pattern". The team's paper, by Smoot and his colleagues, followed that September in *The Astrophysical Journal Letters*. The primary paper is G. F. Smoot, C. L. Bennett, A. Kogut, E. L. Wright and others, "Structure in the COBE differential microwave radiometer first-year maps", *The Astrophysical Journal Letters* 396, L1 (1 September 1992), doi:10.1086/186504. COBE was launched on 18 November 1989. The Differential Microwave Radiometers measure the difference of sky signal between two horn antennas with fields of view of 7 degrees, whose axes are 60 degrees apart. The paper gives the root mean

square variation of the sky, smoothed to 10 degrees, as 30 $\pm$ 5 microkelvin (millionths of a kelvin) at galactic latitudes above 20 degrees; against the mean temperature of about 2.725 kelvin this is a fraction of about 1.1 $\times$ 10^{-5} (read: ten to the power minus five, one part in a hundred thousand), which the chapter states in words.

Greene's figures for the size of the variations, and how he says the pattern can be explained. *The Fabric of the Cosmos*, chapter 11, section "The Golden Age of Cosmology": 2.7249 and 2.7250 kelvin, and the pattern that "can be explained by attributing them to the same mechanism that has been suggested for seeding galaxy formation: quantum fluctuations stretched out by inflation". The gloss of inflation is the one the Preface of this book gives in Greene's terms; the gloss of quantum fluctuations is the one Chapter 20 gives. Told here as Greene's account (what the collection says of the source of the ripples is in the notes on the first ripples and on the Big Bang identified with the turn, below).

The 1977 flight. Smoot, Gorenstein and Muller, the motion of the Earth against the oldest light, told in Chapter 9 of this book, with its primary paper there (G. F. Smoot, M. V. Gorenstein and R. A. Muller, "Detection of anisotropy in the cosmic blackbody radiation", *Physical Review Letters* 39, 898 (1977)).

The first ripples, identified with the form of the content. The identification is made in *What the Beginning Left Behind*, Chapter 11, "What Passes the Turn"; "averaged and not erased" is in *The Gravitational Passage*, Chapter 18, Section VIII. The quoted sentence, "The faint pattern in the microwave sky is the averaged form of what those black holes took in", is in *What the Beginning Left Behind*, Chapter 21, "Where You Stand". The sentence before it there names the black holes meant: "You are inside an epoch whose beginning is the common end of the interiors of the black holes of the epoch before". *What Comes After the Dark*, For the Specialist, "Chapter 9. What Crosses Over, and What Stays Behind?", lists "The departures from uniformity of the new epoch are the form of the content, the old distinctions averaged and not erased". as an identification. In

Chapter 18 I recall that identification each time as the identification made in Chapter 9 of this book. The glosses of the turn, the epoch and the form of the content, and the words of the identification ("with the form of the content carried through the turn"), are Chapter 9's: the turn and the epoch in section "What Banged: The Turn", the form of the content and the identification in section "The Worn Stairs". The form of the content, as the chapter glosses it, is the first sense of information: *The Unquantized Arena*, Chapter 17; *At the Edge of Reality*, Chapter 18.

Epoch and turn, as glossed. *What the Beginning Left Behind*, Terms ("Turn, the"; "Reset, the"). The turn has no date, temperature or duration (*What Comes After the Dark*, Chapter 6, "No Last Tick").

The closing passages of Greene's four books. *The Elegant Universe*, chapter 15, section "Reaching for the Stars"; *The Fabric of the Cosmos*, chapter 16, section "Inner and Outer Space"; *The Hidden Reality*, chapter 11, section "Should We Believe Mathematics?" (with Greene's condition: "If some or all of the mathematics that's compelled us to think about parallel worlds proves relevant to reality"); *Until the End of Time*, chapter 11, section "Meaning". The closing of *The Elegant Universe*, the climb of each generation on the one before: told here as Greene's account; the collection gives no reply on it. The closings of the other three books: told here as Greene's account (the collection's statements on the pattern in the sky, on one world and on meaning are in the notes below).

One world, and one value for each constant. *Side by Side, Volume Two*, Ellis chapter, Section VI, states it as "the structure of comparison yields one world, with its constants fixed", set against the anthropic reading in which a constant is "an accident of location"; "fixed" there means that each constant has one value in the one world and is not a variable selected by where an observer stands. In Chapter 22 I state the result, one world and one value for each constant, in those plain words. Some of the values are derived (the matter fraction, the ratio of dark to ordinary matter), and

four are taken from measurement: Newton's constant, the strength of light, the strength of color and the value of the Higgs field in empty space (*Side by Side, Volume One*, Appendix B, Section IX).

Comparison, content, context, finding; the principle of comparison; the standard of not turning. *What Comes After the Dark*, Chapter 6, "What an Observer Is" and "A Standard That Needs Something There" ("A difference that no comparison can register is no difference"); *The Gravitational Companion*, "One Mass, Three Elevators", Section II. The glosses, the standard fixed by the matter within reach, and the principle are given in the words of Chapter 1 of this book, sections "Every Finding Is a Comparison" and "What the Water Turns Against".

The Incommensuration Theorem and its four words. *An Immanent Metaphysics*, aphorism 1.43-2 (quoted, the first of its last two lines); the four characters, aphorism 1.43-1, as *Home in the World*, "For the Specialist", gives them: continuity, a sameness of content with a sameness of context; discontinuity, a difference of content with a sameness of context; symmetry, a sameness of content with a difference of context; asymmetry, a difference of content with a difference of context. The four pictures (the dimmer, the switch, the square, the coast) and the two survivors (the ramp and the ladder) are Chapter 3's, from *Home in the World*, Chapter 4. The proof uses nothing but the meanings of sameness, difference, content and context, and is given in Chapter 3 of this book; the theorem has no physical premise (*Home in the World*, "For the Specialist", "The Incommensuration Theorem").

Every measurement an instance of the theorem. *Physics Is Not Math*, Chapter 19, "A Right Method of Evaluation", Section X, "Summary": "By both of the ways that validate truth the Incommensuration Theorem is confirmed: every measurement is an instance of it and it is proved, so it is the most fully confirmed truth there is, and the very act that would test it confirms it". The same book, Chapter 21, Section X, "The Truth on Which the Four Rest", repeats it: "every test of it is already an instance of it".

The bucket's returns recalled in the chapter, each with the one meaning of the image. Chapter 4 (a finding needs no watcher, in Chapter 4's words); Chapter 5 (the water's molecules blur its surface long before the bound; the bound on placing any body is about one Planck length, $l_P \approx 1.6 \times 10^{-35}$ m, read: the Planck length, about one point six times ten to the power minus thirty-five meters, which is about 10^{-33} cm, the "millionth of a billionth of a billionth of a billionth of a centimeter" of the chapter; the arena has no least length, *The Unquantized Arena*, Chapter 6, Proposition 6.3, and Chapter 9, "A limit on localization is a different thing from a least length"); Chapter 14 (the water's comparing is real and simple, and is consciousness in the smallest degree, in Chapter 14's words, section "What Awareness Is": *What Comes After the Dark*, Chapter 7, and *Side by Side, Volume Two*, Tononi chapter, "The Measure of an Inside", Section V, "Grades of Experience and Choice in Every Degree", which quotes *The Modern Engagement*, Chapter 7, Section V, "Wherever interaction occurs, some form of consciousness is present", and *The Ground of Objection*, entry T3.19, "A thermostat's interaction with temperature is real and simple"); Chapter 16 (the astronomer of the far future); Chapter 19 (*What Comes After the Dark*, Chapter 6, "Two Ways to Be Left": "Mach's bucket, set spinning at either end, would have nothing to measure its turning against").

Space is no container; gravity is the shape of the arena. *What Cannot Be*, Chapter 12, "What Space Is"; *Measurement*, Chapter 3, Section VII; *Two Theories, One World*, Chapter 9, "The Identification". The identification is made in Chapter 1 of this book, section "Let Go of a Pen", in the words "So I identify gravity with the shape of the arena, and not with a force acting within the arena". In Chapter 22 I recall that identification as the one made in Chapter 1, with Chapter 1's gloss of the shape of the arena ("the way space and time are bent, from one place to the next") and Chapter 1's picture of the falling pen, in Chapter 1's words. The same words, "the shape of the arena", name gravity where the chapter sets the two arguments side by side.

Exists and real; the loaf exists; the past given, the future not yet given. *The Gravitational Passage*, Chapter 5, Section II; *The Basis of Physics*, Essay 11, Section VI. The two loaves and the glosses of "exist" and "real" are Chapter 2's; the result recalled is Chapter 2's arrival, in its words.

The Planck scale; the two pairings in physics; the two descriptions meet and stay two. The two perfected pairings (content discontinuously symmetric, context continuously asymmetric): *An Immanent Metaphysics*, aphorism 1.83-5, as *Home in the World* quotes it. The Planck scale as the meeting of a body's gravitational size and its quantum size, with no least length: *Two Theories, One World*, Chapter 15; *The Unquantized Arena*, Chapter 9. The glosses of the two sizes are Chapter 5's; the gravitational size there is about half the length within which the mass, squeezed, would be a black hole. The two surviving pairings, in the theorem's terms: Chapter 3 of this book, which asks for two of the four characters at once, each held perfectly. The identification of quantum theory with the description of the content of physics, and of general relativity with the description of its context, is made in Chapter 5 of this book, section "Where the Two Descriptions Meet". In Chapter 22 I recall that identification as the one made in Chapter 5, with Chapter 5's arrival: Chapter 5 says "as I identified them above" and "the two", and Chapter 22 says "as I identified them in Chapter 5" and "the two theories". *Home in the World*, "For the Specialist", "The Two Kinds of Perfected Comparison": "I identify quantum theory as the description of the content of physics, and general relativity as the description of its context, the arena of space and time". The glosses of quantum theory and general relativity are those of Chapters 21 and 5. The theorem allows a common source and forbids a merger: *Two Theories, One World*, Chapter 24, Key Terms.

One theory of everything, of the railway's kind; the three forces; the closed list. The arrival recalled is Chapter 6's, in its words. The four values taken from measurement are named as Chapter 6 names them, section "Four Numbers from the World": Newton's constant, which sets the strength of gravity; the strength of light; the strength of color, the strength

of the strong force; and the value of the Higgs field in empty space. The three forces besides gravity are glossed as Chapter 6 glosses them in section "Strings That Play Every Note"; the gloss of the Higgs field is Chapter 19's. *Two Theories, One World*, Chapter 24, "One World, One Account, Two Descriptions": "There is one account of everything, and it is made of two descriptions joined together". The three forces stay three: *The Story of the Particles*, Level 9, Section IV; *The Strengths of the Kinds*, Chapter 3, Section III. The closed list of kinds: *The Kinds of Particle*, Chapter 6, Section IV.

Ten dimensions of three kinds; one shape for the six; one vacuum and no landscape. *At the Edge of Reality*, Chapter 3. The internal space $CP^2 \times S^2$ (read: the complex projective plane times the two-sphere), the spaces of states of a three-way and a two-way choice, with radii in the ratio $\sqrt{6}$ (the square root of six), one Planck length and $\sqrt{6}$ Planck lengths: *The Gravitational Passage*, Chapter 9, Section VIII. No fluxes and no flux integers: *The Compactification*, Chapter 2, Section VI. One vacuum and no landscape, reached by two routes: *The Compactification*, Chapter 2. The drum: *At the Edge of Reality*, Chapter 8, "How Small the Six Are". The fluxes of string theory, each hole of the shape threaded a whole number of times: Chapter 7 of this book, beside Greene's *The Hidden Reality*, chapter 5, section "In Flux". Greene's count of ten, "nine space dimensions and one time dimension" (*The Elegant Universe*, chapter 8, section "More Dimensions and String Theory"), and this account's ten count different things: *The Gravitational Companion*, "Other Programs", Section VIII. In Chapter 22 I give this account's ten in Chapter 7's words and leave the comparison of the two tens to Chapter 7. The six as everywhere you are, held small as content of the large world, and the glosses of a vacuum for the six and of the landscape, are Chapter 7's, in its words.

The agreement of entangled particles; locality in two senses. *What Cannot Be*, Chapter 10; *Why the Quantum Is the Way It Is*, Chapter 15. No message can be sent by the correlation: *Why the Quantum Is the Way It Is*, Chapter 15; *Measurement*, Chapter 13. The narrow sense of locality is given

in Chapter 3's words, "no signal travels faster than light", and Bell's broad sense in Chapter 3's gloss. The result recalled is Chapter 3's arrival, in its words.

One outcome in each act, no mind needed, no reach into the past. *Why the Quantum Is the Way It Is*, Chapter 17; *The Quantum Passage*, Chapter 11. The result recalled is Chapter 4's arrival, in its words, with Chapter 4's description of the state of each electron.

The direction of time; two arrows; entropy as a count relative to a context. The identification of the one-way order of findings with the direction of time is made in Chapter 2 of this book. *What the Beginning Left Behind*, Chapter 6, "What This Means for the Beginning"; *Physics Is Not Math*, Chapter 8, Section II; *Measurement*, Chapter 12, Section VII. Entropy as a count relative to a context: *What the Beginning Left Behind*, Chapter 7. The result recalled is Chapter 8's arrival, in its words.

The Big Bang identified with the turn; no inflation needed. The identification: *At the Edge of Reality*, Chapter 17, "The Limit Rendered as a Point"; *What the Beginning Left Behind*, Terms. "The account answers every puzzle inflation was introduced to solve, from the turn and from the structure of the forces, so it needs no inflation": *What the Beginning Left Behind*, Chapter 13. *Side by Side, Volume One*, Appendix B, Prediction 13, words the same result as "The horizon and flatness problems are answered by the turn from the previous epoch, with no inflation". A shorter phase of inflation "is neither required nor excluded" (*What the Beginning Left Behind*, Chapter 13). The result recalled is Chapter 9's arrival; Chapter 9 says "I have identified", and Chapter 22 says "I identified in Chapter 9".

The cosmological constant: one number in one geometry, and the accessible share. *What Comes After the Dark*, Chapter 2, "The Second Way: One Number for Our Whole Epoch" and "The First Way: Space Is Flat" (where the identification is made). The matter fraction $\Omega_m = 0.311$ (read: omega sub m, the matter fraction) beside 0.315 ± 0.007 measured by the Planck satellite: *What the Beginning Left Behind*, Chapter 8. The identifica-

tion recalled is Chapter 10's ("the part of the vacuum's energy that a measurement made from inside the universe can take in"), and so is the arrival, which in Chapter 10 also has the words "as I identified it above", pointing to Chapter 10's own text. The vacuum's energy as the energy that the quantum activity of empty space carries: Chapter 1 of this book; that no measurement made from inside the world takes it in whole: Chapter 10, section "What the Small Number Says".

One world; regions beyond the horizon; no selection; no long inflation and no bulk. *Side by Side, Volume Two*, Tegmark chapter, Section VI; *The Compactification*, Chapter 2, Section IX; *The Compactification*, Chapter 6; *What Comes After the Dark*, Chapter 11. The glosses of a brane, the bulk and the particle horizon, and the result recalled, are Chapter 11's, in its words.

Greene's remaining other worlds as the one world met another way. *Side by Side, Volume One*, Deutsch chapter, Section V; *The Unquantized Arena*, Chapter 15, Proposition 15.4; *Measurement*, Chapter 14, Section X; *Side by Side, Volume Two*, Tegmark chapter, Section VI.

Order built by passing disorder on; clumping under gravity. $F = E - TS$ (read: the free energy equals the energy less the temperature times the entropy) least at fixed temperature, which is the joint entropy made greatest: *The Matter We Know, Volume Two*, Chapter 2, Section XIV. Clumping raises the count for matter bound by its own gravity: *The Compactification*, Chapter 5, Section VI. The result recalled is Chapter 13's arrival, and in Chapter 22 the word "because" joins it to its question.

Consciousness identified with three features of an act of comparison taken whole. The identification is recalled in Chapter 14's words. The sentence after it, on the water's comparing as consciousness in the smallest degree, restates Chapter 14, section "What Awareness Is", in its words; its sources are in the note on the bucket's returns, above. The sentence after that is the second sentence of Chapter 14's arrival; Chapter 14 says "it", and Chapter 22 says "consciousness". The map of a walk and Greene's bag of particles are Chapter 14's, sections "A Gap Built into Every Comparison" and "Only Particles?", in its words. *Causality Without Determin-*

ism, Chapter 5, and Chapter 6, Summary; the identification is made in Chapter 14 of this book. The structural gap between the third person and the first: *The Last Turtle*, Chapter 15, Section II.

Every world causal and never completely determined; choice and hard randomness; responsibility. *Physics Is Not Math*, Chapter 8, Section VII; *Causality Without Determinism*, Chapter 1, Section VIII; Chapter 5, Section V; Chapter 6, Section VIII. The result recalled is Chapter 15's arrival, in its words.

The populated sky a passing stage; the proton lasts; the hold. *What Comes After the Dark*, Chapter 1, Chapter 5 and its Key idea, and Preface, "The Answer, in Brief". The expansion never turns around and contracts: "Our epoch does not fall back". (*What Comes After the Dark*), as Chapter 16 of this book gives it, from the sum that sets the rate of expansion with a flat space and a positive constant. The gloss of the hold is Chapter 16's, word for word, and so is the gloss of the outside world, which Chapter 16 quotes from *What Comes After the Dark*. The result recalled is Chapter 16's arrival, in its words. The two roads: "So every remnant's path ends in one of two ways: inside a black hole, or carried away alone". (*What Comes After the Dark*, as Chapter 16 of this book quotes it).

No one thinker thinks forever; thinking begins again. *What Comes After the Dark*, Chapter 7, Key idea. The warm room: the warmth that reaches a thinker from outside thins toward the glow of the cosmological horizon, and no lower, as Chapter 17 of this book states it; Gibbons and Hawking (1977), and Krauss and Starkman (2000), as Chapter 17 gives them. The scope of that part, in *What Comes After the Dark*, For the Specialist, "Chapter 7. Can a Mind Keep Thinking for Ever?": "On the temperature floor, the account states that the outside radiation thins toward the horizon's own temperature (reported); the cost of cooling below that floor is not argued in the collection". Every observer's road and its two ends are given in the words of Chapter 17, section "Where Every Road Ends". The arrival recalled is Chapter 17's, in its words.

The black holes are the last of the ashes; Greene's floors; the form of what falls in. *What Comes After the Dark*, Chapter 5; *The Gravitational Passage*, Chapter 18, Section X (evaporation of a hole of one solar mass in about 10^{67} years); *The Unquantized Arena*, Chapter 17, Proposition 17.3. Greene's floors: *Until the End of Time*, chapter 10, section "The Disintegration of Black Holes" (floor 68 for a hole of the Sun's mass) and section "An End of Time" (floor 102).

The emptying ends in a turn; each epoch begins its own count; a cycle with no first turn. *What Comes After the Dark*, Chapter 8 and Chapter 11; *The Gravitational Passage*, Chapter 18 ("Each epoch begins its own count, and the cycle has no first iteration").

What crosses is the form of the content; nothing crosses as itself. *What Comes After the Dark*, Chapter 9, "What Crosses" and "What This Means for You", and Chapter 10, Key idea.

Meaning present wherever comparison occurs; morality and ethics. *What Cannot Be*, Chapter 7, "Meaning Cannot Be Removed" (both quotations). Morality and ethics: *An Immanent Metaphysics*, Chapter 6, aphorisms 2.12-1 and 2.12-8; the commonness belongs to the principles ("The absolute principles of ethics are common to all of consciousness (all individuals)"), and the practice is "particular and unique to each individual". The arrival recalled is Chapter 21's, in its words.

The two arguments and their starting points. Greene's argument starts from the theories of physics; the argument of the collection starts from the act of comparison. So wherever the two arrive together, the place is held from two sides at once.

The four agreements checked by measurement. The waves of the arena: B. P. Abbott and others (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of gravitational waves from a binary black hole merger", *Physical Review Letters* 116 (2016), 061102, doi:10.1103/PhysRevLett.116.061102; the arena ripples, and its ripples are those of a smooth arena: this book, Chapter 5. No grain detected: A. A. Abdo and others (Fermi LAT and Fermi GBM Collaborations), "A limit on the vari-

ation of the speed of light arising from quantum gravity effects", *Nature* 462 (2009), pages 331 to 334, doi:10.1038/nature08574; the bound on placing and the arena with no least length: this book, Chapter 5, with its notes. The sameness and the flatness: the oldest light, as told in Chapter 9 of this book; the curvature of space from N. Aghanim and others (Planck Collaboration), "Planck 2018 results. VI. Cosmological parameters", *Astronomy and Astrophysics* 641, A6 (2020), doi:10.1051/0004-6361/201833910, as Chapter 11's notes give it; both explained by the turn: *What the Beginning Left Behind*, Chapter 13 (above). The speeding up of the expansion: A. G. Riess and others, *Astronomical Journal* 116, 1009 (1998), and S. Perlmutter and others, *Astrophysical Journal* 517, 565 (1999), as Chapter 10's notes give them; the constant as one number in one geometry: *What Comes After the Dark*, Chapter 2 (above).

The two string results, recognized and not reached. *Two Theories, One World*, Chapter 22, "Four Programs That Quantize the Arena", subsection "String theory" (quoted): the graviton among the string's vibrations, and the count of Strominger and Vafa. The same subsection: "Where its gravitons describe small waves on a smooth background, they are quanta of a description, which the third question allows; they are never quanta of the arena". No graviton among the kinds of particle, because gravity is the shape of the arena: Chapter 6 of this book, section "A Closed List of Kinds". The primary paper for the count is A. Strominger and C. Vafa, "Microscopic origin of the Bekenstein-Hawking entropy", *Physics Letters B* 379, 99 (1996), doi:10.1016/0370-2693(96)00345-0.

Greene's statements set beside this account where the two arguments meet and part. *The Fabric of the Cosmos*, chapter 3, section "Spacetime in the Third Millennium" ("Although the issue is still debated, as we've now seen, the most straightforward reading of Einstein and his general relativity is that spacetime can provide such a benchmark"); chapter 5, section "The Persistent Illusion of Past, Present, and Future" (the condition of the loaf, "if you buy the notion that reality consists of the things in your freeze-frame mental image right now, and if you agree that your now is no more valid than the now of someone located far away in space who

can move freely, then reality encompasses all of the events in spacetime", then "The total loaf exists". and "The only thing that's real is the whole of spacetime"); chapter 6, section "The Egg, the Chicken, and the Big Bang". *The Elegant Universe*, chapter 1, section "String Theory as the Unified Theory of Everything". *The Hidden Reality*, chapter 6, section "Cosmological Anthropics". *Until the End of Time*, chapter 5, section "Consciousness and Quantum Physics"; chapter 5, section "Relevance, Learning, and Individuality" ("We do make choices", in the human-level story, with the next lines: "We need to set aside the notion that our choices and decisions and actions have their ultimate origin within each of us"); chapter 9, section "A Final Thought on Thought" ("rare, wondrous, and precious"); chapter 10, section "An End of Time" ("Gazing on the universe from the 102nd floor, we will not see much beyond a diffuse mist of particles wafting through space", with Greene's two conditions: "If the dark energy has drained away and the rapid expansion of space has diminished, it is possible that particles may accumulate into even larger black holes that will radiate yet more slowly, yielding yet longer lifetimes. But if the dark energy persists, particles will be driven apart increasingly quickly by the accelerated expansion, ensuring that they will rarely if ever encounter one another", and "Physicists sometimes liken this future era to the end of time. Not that time stops"), sections "The Disintegration of Emptiness" (the Higgs field's leap to a lower value) and "Is the End Near?" (the big crunch and the bounce; the glosses of the two are Chapter 19's, which calls the bounce a cyclic cosmology); chapter 11, section "Meaning". Told here as Greene's account (what the collection finds on each question is in the table's third column and in the notes above).

The faint pattern shows what comes after the dark of an epoch; the next sky. *What Comes After the Dark*, Chapter 12, "What This Means for You" (quoted). The picture of a world of the next epoch with a watcher on it is in the same chapter and section. In *What Comes After the Dark* I give the same picture without the bucket.

The structure of knowing is the same for every comparer; what each comparison finds is given only in the act. *Home in the World*, Chapter 9, "Me Facing You", section "Home, with a Floor" ("content is uncertain while the structure of knowing is certain and the same for every perspective"), and section "Above the Hao River" ("every perspective is a comparison").

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Glossary

Every word below is explained in plain words in the chapter named beside the word. Each entry here is a short reminder of the chapter's explanation.

Absolute space. Newton's *absolute space*, as Greene describes Newton's idea: a space standing behind everything, the same everywhere, which nothing acts on and which is there whether or not anything is in it. In this book, space is the pattern of which events are causally connected with which (which events can affect which), and space is no container (Chapter 1).

Absolute zero. The coldest temperature there can be, from which temperatures in kelvins are measured upward (Chapters 9 and 13).

Absolutely. In the exact words of the Incommensuration Theorem, *absolutely* means perfectly: held exactly, at the foundation, with no exception at any scale (Chapter 3).

Accessible share. The part of the vacuum's energy that a measurement made from inside the universe can take in. In Chapter 10 I identify the cosmological constant with the accessible share, and I find the accessible share to be three tiles out of all the tiles of the particle horizon, where the surface of the particle horizon is pictured as tiled like a floor, one tile for each degree of freedom within the horizon (Chapter 10).

Act. The happening itself, as the happening happens. In a detector, the act is the click (Chapter 3).

Algebra. A stock of quantities with rules for adding and multiplying the quantities (Chapter 7).

Anthropic principle. The rule of the physicist Brandon Carter, which Greene describes: what we observe must be read as observed by beings who could exist where and when they observe it (Chapter 11).

Antihydrogen. The twin of hydrogen, built from particles with the opposite electric charges to the particles of hydrogen (Chapter 1).

Antimatter. Matter whose particles carry the opposite charges to the particles of ordinary matter (Preface).

Arena. Greene's words are the *fabric of the cosmos* and *spacetime*. This book's word is the *arena*: space and time taken together, as the setting in which everything that is measured is measured. The shape of the arena is gravity (Preface; Chapter 1).

Arrow of time. The one-way direction from past to future (Preface; Chapter 8). This book tells two arrows of time apart, the *structural arrow* and the *entropic arrow* (Chapter 8).

Asymmetry. A different content across a different context, as no cove of a rocky coast repeats the cove before it. The adjective *asymmetric* means different from place to place (Chapters 2, 3 and 6).

Atomic clocks. Clocks that count the vibrations of atoms (Chapter 2).

Atomic mass unit. A unit of mass, each unit about the mass of one hydrogen atom (Chapter 4).

Axiom. One of three basic statements from which the rest of the Immanent Metaphysics is worked out. Each axiom holds of every comparison. The first axiom: the act is the most fundamental of the three roles of a comparison (the stance, the act and the structure). The second axiom gives the order of the three roles: the stance before the act, the act before the structure, and the structure before the next stance. The third axiom: the three roles are distinct, inseparable and non-interchangeable (Chapters 6, 8 and 19). The third axiom also holds content and context distinct and inseparable, so that there is no content without a context (Chapter 19).

Barrier. An arrangement of higher energy that a change must pass through on its way. Greene calls such obstacles hurdles (Chapter 13).

Basic pieces. The pieces from which a shape is built. The surface of a ball is built from two basic pieces: a point, and a disc whose rim is drawn together to that point (Chapters 7 and 9).

Beam splitter. In an interferometer, the device that brings the two paths together (Chapter 4).

Benchmark. Greene's word for the standard against which a motion is judged to be accelerated, whether the motion is speeding up, slowing or turning (Chapter 1).

Big bang. In Greene's sense, the hot, dense, expanding state in which the history of our universe begins (Preface). In Chapter 9 I identify the Big Bang with the turn, as the turn appears in the history of our epoch. The turn is the passage at the end of the black-hole interiors of the epoch before ours, in which a large world is restored.

Big rip. The tearing apart of galaxies, planets and atoms that the push of empty space would bring about if the push in time overcame every force that holds things together. A big rip could be made only by an energy that grows denser as space grows (Chapter 16).

Bit. The smallest piece of information, one choice between two (Chapter 17). Two bits are a pair of yes-or-no answers (Chapter 3).

Black dwarf. A white dwarf that has cooled until the white dwarf no longer glows: a dark, solid ball about the size of the Earth, still holding the mass of a star (Chapter 16).

Black hole. A region whose gravity is so strong that nothing, not even light, comes back out of it (Preface). The edge of a black hole is the black hole's horizon (Chapter 6).

Block universe. Greene's conclusion: the view that past, present and future are all equally there, laid out together, like the slices of one loaf (Chapter 2).

Boltzmann brains. Greene's term, given here in Greene's sense: brains that, over spans of time beyond imagining, would gather by chance from particles of the glow of the cosmological horizon, with memories of a life never lived. The brains are named after the physicist Ludwig Boltzmann, who first read entropy as a count of arrangements (Chapter 18).

Born's rule. The rule by which physicists get the odds of each finding from the smooth description of the quantum state (Chapter 4; Chapter 12).

Bounce. A collapse that rebounds into a new expansion (Chapters 19 and 22).

Bound triple. The name that my book *Causality Without Determinism* gives to the three features of every act of comparison held together: a one-way order, from before to after; an outcome that nothing before the act fixed; and someone to whom the outcome happens (Chapter 14).

Branch. In the many-worlds interpretation, a branch is one of the alternatives into which the state branches. Because of decoherence, each alternative looks like the everyday world. In the many-worlds interpretation every outcome occurs, each outcome in a branch of its own (Chapters 4 and 12).

Brane. Greene's term, given here in Greene's sense: an object spread out in two, three or more dimensions, as a sheet is spread out in two (Chapter 7). Our three directions of space, as one brane, would float in a larger space that Greene calls the *bulk* (Chapter 11).

Braneworld. Greene's term for a picture in which our universe lies within a three-dimensional brane in a larger space of more dimensions (Chapter 7).

Bubble universe. Greene's word for a region where the burst of expansion of eternal inflation stops and the region settles down (Chapter 11).

Calabi-Yau shapes. Greene's term for a family of six-dimensional shapes, named for the mathematicians Eugenio Calabi and Shing-Tung Yau. The equations of string theory allow the six further dimensions of string theory to be curled up in a Calabi-Yau shape (Preface; Chapter 7).

Carried away alone. Said of a remnant or a body: carried by the expansion beyond the reach of all other matter (Chapter 16).

Carriers. The particles through which a force acts. Carriers pass between the particles of matter, as the photon passes between electric charges (Chapter 6).

Catalyst. A substance that opens a new route for a change, with a lower barrier, and is left unchanged at the end (Chapter 13).

Category mistake. Placing a thing in the wrong kind (Chapter 2).

Causal. Said of a world whose findings follow from what came before along an order that runs one way, with no break (Chapter 15).

Caused. Said of a finding that follows from what came before along an order that runs one way, with no break. A finding is *determined* when what came before fixes the finding in full. The findings that physics describes are caused without being determined (Chapter 15).

Channel. One of the three kinds of thing that a measurement registers: pattern, registered in space; force, registered in time; and probability, registered among the possibilities. Every measurement registers one of each of the three channels (Chapter 10).

Character. One of the four kinds of finding: symmetry, discontinuity, continuity and asymmetry. A finding's content can come out the same or different, and so can the finding's context, and the four ways the two can come out are the four characters (Chapters 3 and 6).

Choice. Greene's phrase is *free will in the traditional sense*, which would be the power of a mind to rise above the laws that govern the mind's particles (Preface; Chapter 15). This book's word is *choice*: an outcome that nothing before the outcome fixed, described from the position of the process from which the outcome came (Chapter 13), and seen from inside by the one whose outcome the choice is (Chapters 15 and 21). In full, a choice has three parts: a range of what is open, a selection that no prior condition fixes, and the consequences of the selection (Chapter 15).

Classical computation. The kind of computation that ordinary digital machines carry out: one rule applied the same way at every step, with each state fixed by the state before (Chapter 14).

Classical laws. The laws of the physics before quantum theory (Chapter 4).

Closed universe. In the physicist Freeman Dyson's use, a universe whose expansion stops and turns into a collapse. By contrast, an *open universe* is one that expands forever (Chapter 17).

Clumping raises the count. Greene's phrase is the *gravitational two-step*. This book's sentence is: for matter held together by its own gravity, clumping raises the count of arrangements (Chapter 13).

Collapse. Greene's word for the change of a measured particle's probability wave, everywhere at once, to fit the outcome just found. This book's phrase is the *update of the record* (Chapters 3 and 4).

Collapse models. Interpretations of quantum mechanics, begun by the physicists GianCarlo Ghirardi, Alberto Rimini and Tullio Weber in 1986, that change the smooth rule of quantum mechanics itself, so that a superposition of a large thing ends of its own accord (Chapter 4).

Color. The charge of the strong force, which quarks carry (Chapters 6 and 16).

Common source. Greene hopes for a single theory of all physical phenomena. This book's *common source* is set beside Greene's hope: two descriptions that arise from the same act of comparison and apply to the same world, each perfect in one of the two surviving pairings (symmetry with discontinuity, or continuity with asymmetry), joined side by side (Chapter 6).

Comparer. Greene's word is *observer*, whoever does the measuring, and this book uses Greene's word for whoever or whatever does the measuring (Chapter 2). This book's word is *comparer*: whatever makes a comparison and registers what it finds. The word *comparer* names a role, and in this book the word never means a mind (Chapter 1).

Comparison. The act of finding whether two things are the same or different, in some respect. A comparison is made, for example, when two readings of a thermometer, one in the morning and one in the evening, are set beside each other and found to differ (Preface; Chapter 1).

Compatibilism. The view that freedom is compatible with laws that fix everything that happens. Greene follows compatibilism (Chapter 15).

Complex numbers. Numbers that carry both a size and a clock-hand setting (Chapter 7).

Complex projective plane. A shape of four dimensions. All the states of a three-way choice together form the complex projective plane (Chapters 7 and 11).

Computation. One rule applied the same way, step after step. A program is such a rule written out (Chapter 17; Chapter 21).

Condensate. Greene's phrase is the *Higgs ocean*. This book's word is the *condensate*: the settled value of the Higgs field, the same everywhere in space (Chapter 6).

Conduction. The passing of heat through material that stays where it is (Chapter 12).

Cone, the. Greene's term is the *light cone*. This book's term is the *cone*: at each event, everything that the event can affect and everything that can affect the event (Chapter 2).

Conformal. Said of a change of scale that keeps every shape, every angle and every path of light, while every size changes. The cycle of the mathematical physicist Roger Penrose joins one history of the universe to the next by a conformal change of scale (Chapter 19).

Conjecture. A proposal not yet proved (Chapter 2).

Consciousness. In Chapter 14 I identify consciousness with the three features of an act of comparison taken whole, in a context, as one act: the one-way order of the act, its outcome that nothing before it fixed, and its stance. The water's comparing in Newton's bucket is consciousness in the smallest degree, because the water compares and every act of comparison has the three features (Chapter 14).

Content. What a comparison finds (Chapter 1).

Context. The respect in which a comparison finds what it finds: what makes the two things compared two, as the two times of day make two readings of a thermometer two (Chapter 1).

Continuity. The same content where the context is the same, as a dimmer knob turned by a hair changes the light by a hair. The adjective *continuous* means joining without a break (Chapters 2, 3 and 6).

Convection. The carrying of heat by a fluid that flows, warm fluid rising and cool fluid sinking (Chapters 12 and 15).

Copenhagen interpretation. The view of the physicist Niels Bohr's school in Copenhagen, which puts the finding first: on the Copenhagen view, every experiment ends in a record that can be read, and between findings the state is a tool of calculation and nothing more (Chapter 4).

Copernican principle. The working rule, from the lesson of the astronomer Nicolaus Copernicus, that we hold no central place in the universe (Chapter 11).

Correlation. A pattern of agreement found by setting two records side by side (Chapter 3).

Correspondence. Greene's word is *duality*: a match between two descriptions that look different and describe the same physics (Preface). This book's word is *correspondence*: two descriptions with an exact translation between them, neither made of the other (Chapter 5).

Cortex. The outer layer of the brain (Chapter 14).

Cosmic microwave background. Astronomers' name for the *oldest light* (Chapter 9).

Cosmological constant. Einstein's entry in his equations, as Greene describes the entry: one number, which physicists later read as an energy of empty space, the same in every cubic meter of space and not thinning out as space grows. In Chapter 10 I identify the cosmological constant with the accessible share, the part of the vacuum's energy that a measurement made from inside the universe can take in (Chapter 10).

Cosmological decade. Greene counts the ages of the universe by the floors of his Empire State Building of time, and the number of a floor of Greene's building is the same count as a cosmological decade. *Cosmological decade* is the name that the astrophysicists Fred Adams and Gregory Laughlin give to the count: the number of zeros in the count of years since the beginning (Chapter 16).

Cosmology. The study of the universe as a whole: its history, its contents and its shape (Chapter 8).

Count. This book's word for the entropy itself, as the books of my physics collection use the word: the count of arrangements, written as the count's logarithm (Chapter 13).

Couples. A force couples to the things that the force acts on, and most forces couple to some things and not to others (Chapter 1). A particle's *coupling* to a field is how strongly the particle and the field act on each other (Chapter 6).

Criticality. The balance between order and disorder in which a small disturbance can cascade to any size (Chapter 14).

Crossover. A smooth change of state, with no jump, no bubbles and no walls between regions (Chapter 9).

Crystal. A regular pattern repeated in every direction (Chapter 13).

Curvature. Of space: whether on the largest scales space bends like the surface of a ball or lies flat like a tabletop (Chapters 9 and 11). A closed curvature is a bending of space as a whole that would close space on itself (Chapter 16).

Cut, the. The boundary between what is described by the state and the instrument that records the finding (Chapter 4).

Cyclic cosmology. A history in which regions of space like ours expand, then contract, then rebound into a new expansion, again and again (Chapter 19).

Dark energy. Whatever drives the speeding up of the expansion of the universe. Dark energy is called dark because it gives off no light, and energy because it acts on the expansion as an energy does (Chapter 10).

Dark matter. The unseen matter whose pull is measured in every galaxy. Dark matter gives off no light and shows itself only by its weight (Chapters 6 and 19). In this account, dark matter is made of *gyrations*, lasting vibrations of the six small dimensions of possibility (Chapter 9).

De Sitter temperature. The name that the physicists Lawrence Krauss and Glenn Starkman give to the faint warmth of the horizon of an accelerating universe (Chapter 17).

Decay. Said of a particle: to break up into lighter particles (Chapter 16). Said of an atom of a radioactive lump: to break apart at a moment of the atom's own (Chapter 2).

Decision. Greene's phrase is *human freedom*. This book's word is *decision*: a selection within a set already laid out, by weighing (Chapter 15).

Decoherence. The washing out of interference between the alternatives of a thing, as the thing's surroundings strike the thing and record the thing's alternatives (Chapter 4).

Degeneracy pressure. The pressure a crowd of electrons exerts because exclusion forbids them all to settle into the lowest states. Degeneracy pressure needs no heat at all (Chapter 16).

Degree of freedom. An independent way in which something can hold energy or vary, as each string of a piano can ring on its own (Chapter 10).

Density. The amount of matter and energy in each cubic meter (Chapter 16).

Determined. Said of a finding that is fixed in full by what came before; set beside *caused* (Chapter 15).

Determinism. The claim that a complete description of the world at one moment, together with the laws, fixes every detail of every other moment, earlier and later (Chapter 15). In this book, *total determinism* is every change fixed in advance, step by unbroken step at every scale, which is continuity perfected (Chapter 3).

Dimension. An independent way in which things can differ: we know three dimensions of space and one of time (Preface; Chapter 5). The number of dimensions is the count of those independent ways, and this book's ten dimensions are one of time, three of space and six of possibility (Chapter 7).

Discontinuity. A different content where the context is the same, as a light switch changes a room all at once. The adjective *discontinuous* means coming in separate, countable steps (Chapters 2, 3 and 6).

Discrete. Made of separate steps, with nothing between the steps (Chapter 12).

Dissipative structure. An ordered pattern that exists only because energy and matter keep flowing through it, far from equilibrium (Chapter 13).

Distribution. Of outcomes over many trials: how often each outcome comes (Chapter 15).

Domain. My book *An Immanent Metaphysics* uses the word *domain* for a world or setting in which things happen (Chapter 21).

Double slit. Two narrow openings side by side (Chapter 4).

Duality. Greene's word for a match between two descriptions that look different and describe the same physics (Preface). This book's word for a duality is *correspondence* (Chapter 5).

Edge of a pair. The line that divides one member of a pair of particles of matter from the other member (the electron from the electron's neutrino, the up quark from the down quark). The weak force crosses the edge of a pair. The edge of a pair is no place in space, and the Higgs field lives on the edge of a pair (Chapter 6).

Edge of comparison. Greene's phrase is the *speed of light*. This book's phrase is the *edge of comparison*, and in Chapter 2 I identify the speed of light with the edge of comparison: the greatest speed at which any signal can carry a finding from one place to another, the same for every comparer (Chapters 2 and 3).

Electron biprism. A device that divides an electron's wave in two, as a double slit does (Chapter 4).

Electronvolt. The energy an electron gains in crossing one volt, a small unit of energy. A proton weighs a little under a billion electronvolts (Chapters 5 and 6).

Eliminativism. The view that dismisses consciousness as an illusion (Chapter 14).

Emergence. In Greene's sense, one thing arising out of the joint behavior of other, more basic things, as the hardness of a cannonball emerges from the cannonball's atoms (Chapter 5). Emergence is a one-way relation:

when one thing emerges from another, the second thing is the more fundamental of the two. An exact translation in both directions, a *correspondence*, sets neither description below the other (Chapters 7 and 12).

Empire State Building. Greene's picture of the timeline of the universe as a building, each floor standing for ten times as many years as the floor below, so that a floor is a count of years since the beginning, written as a power of ten (Preface; Chapter 16). The building has no floor for the turn, the passage between one epoch and the next (Chapter 19).

Energy of empty space. In Greene's account, and in this book's sentences on the speeding up of the expansion of the universe, the energy that speeds the expansion. In Chapter 10 I find the energy of empty space to be the share of the vacuum's energy that a measurement made from inside the universe takes in (Chapters 10 and 11).

Ensemble. A whole collection of universes, in the word of my book *Side by Side, Volume Two* (Chapter 11).

Entangled. Greene's word is *entanglement*, his word for the link between far particles. This book's phrase is *entangled particles*: two particles whose outcomes, measured far apart, agree in a pattern that no separate instructions carried by each could give them (Chapter 3).

Entropic arrow. The growth of entropy in things of very many parts. The entropic arrow is how the structural arrow (the one-way order inside every comparison, act before record) shows itself at our scale, and the entropic arrow needs an orderly beginning (Chapter 8).

Entropic two-step. Greene's term, in his words: "any process in which the entropy of a system decreases because it shifts a more than compensating increase in entropy to the environment" (Chapters 13 and 17). This book's sentence: a thing gains order by passing more disorder to its surroundings, and settles where its entropy and theirs together are greatest (Chapter 13).

Entropy. In Greene's sense, the number of ways the parts of a thing can be rearranged without changing how the thing looks overall (Preface). In this book, entropy is a count of the arrangements of a thing's parts that some context cannot tell apart (Chapters 6 and 8).

Enzyme. A large molecule of a cell, shaped to hold reacting molecules in the arrangement they must pass through (Chapter 13).

Epiphenomenon. A by-product that does nothing (Chapter 14).

Epoch. One era, or cycle, of the world, from one turn to the next. Our epoch is the epoch we live in (Chapters 9, 13, 16, 17, 19, 20 and 22).

Equation of state. The ratio of a kind of energy's pressure (the energy's push on its surroundings) to the amount of the energy in each unit of volume (Chapter 10). When an energy stays the same in every cubic meter as space grows, the energy has an equation of state of exactly minus one (Chapter 19).

Equilibrium. The settled state in which nothing further changes on average (Chapters 9 and 13).

Equivalence principle. Within a small enough room, the push you feel from gravity cannot be told apart from the push you feel when the room speeds up (Chapter 1). In this book, turning, speeding up and gravity each push every body in proportion to the body's inertia. Turning, alone of the three, is registered at a point (Chapter 1).

Eternal inflation. Greene's term for inflation that, once begun, goes on for ever in some regions and ends in others (Chapter 11).

Ethics. Set beside morality: the study of, and keeping to, the principles of effective choice, the conditions under which choosing goes well, for any chooser at all. The principles of ethics are common to every chooser (Chapter 21).

Evaporation. The slow shrinking of a black hole by the black hole's own glow, until in the end nothing of the black hole is left (Chapter 18).

Event. A single happening at one place and one moment (Chapters 1 and 2).

Event time. The order in which findings are laid down as records. A counter's clicks mark event time (Chapter 2).

Exclusion. Greene's phrase is the *Pauli exclusion principle*, named for the physicist Wolfgang Pauli. This book's word is *exclusion*: the rule that no two electrons can be in the same state, so two electrons cannot share both the same place and the same motion (Chapter 16).

Exist. To stand as structure: timeless, the same in every instance, able to be held in view as a whole; set beside *real* (Chapter 2).

Extra dimensions. String theory's six further dimensions beyond the three of space and the one of time, kept too small to see. This book's word for the extra dimensions is the *six* (Chapters 6 and 7).

Fabric of the cosmos. Greene's name for space and time as the setting of everything. This book's word is the *arena* (Chapter 1).

Faint glow. Greene's phrase is *Hawking radiation*. This book's phrase is the *faint glow*: the steady warm flow moving outward from a black hole's horizon (Chapter 18).

Faithful reversal. A reversed process that retraces exactly the states the original process passed through (Chapter 8).

False vacuum. A settled state of empty space that holds for now but is not the lowest state (Chapter 19).

Families. The two kinds into which the particles fall by their spin: the family of matter, and the family of the carriers of force, to which the particle of the Higgs field also belongs (Chapters 6 and 10). Greene also uses the word *families* for the three *generations* of the particles of matter (Chapter 7).

Field. Something that has a value at every place, as the wind has a strength and a direction at every place across a country (Chapters 1 and 6).

Finding. The outcome of one act of comparison, laid down as a record (Chapter 1).

First person. A first-person description (Greene's first-person account) is had from inside, by the one to whom the thing described happens. A *third-person* description is one that anyone can check from outside, told of a thing as "it" (Chapters 14 and 15).

First ripples. The faint differences of density with which an epoch begins and from which its galaxies grow. In Chapter 9 I identify the first ripples with the form of the content carried through the turn (Chapter 9; Chapter 18).

First waves. Gravitational waves present from the beginning of an epoch, ripples of the arena itself (Chapter 9).

Fixed point. My book *What Cannot Be* uses the words *fixed point* for something that applies to itself, so that no description can take that one case away (Chapter 21).

Flatness problem. The question of why space began so flat (Chapter 9).

Fluctuation. A rare chance swing out of disorder (Chapter 8).

Flux. In string theory, a field threaded through the holes of a shape, or wound through the shape's loops, each a whole number of times (Chapters 7 and 11).

Form of the content. Information in its first sense: what the matter is, and which of its possibilities the matter has taken, carried with the matter wherever the matter goes. The form of the content is carried through the turn with the content (Chapters 9 and 18).

Formalism. The mathematical form in which physics is written (Chapters 8 and 15).

Formalism's determinism, the. Greene's phrase is *quantum determinism*. This book's phrase is the *formalism's determinism*: in the description, the quantum state changes by exact rules, and those rules do not fix the findings themselves (Chapter 15).

Frame. What counts as standing still and not turning. By the Mach principle, a frame is fixed by the matter there is to compare with (Chapter 19).

Free energy. At a given temperature, the share of a thing's energy that can be drawn off as work. Greene's *quality of energy* is the same share, seen in Greene's terms (Chapter 13).

Free will. Greene's phrase is *free will in the traditional sense*, which would be the power of a mind to rise above the laws that govern the mind's particles. Greene holds that no mind has that power. This book's word is *choice* (Preface; Chapter 15).

Functionalism. The view that a mind is a pattern of processes and functions, whatever material carries them (Chapter 12).

Fusion. The joining of light nuclei into heavier nuclei, which gives out energy (Chapter 13).

Gamma rays. Light of the highest energies (Chapter 5).

General relativity. Einstein's theory of gravity, in which gravity is the shape of space and time, the way space and time are bent from one place to the next (Preface; Chapters 2 and 5).

Generations. Greene's word is *families*. This book's word is *generations*: the three copies of every particle of matter, the same in kind and charge, differing in mass. The three generations are the three rotations of the content, the three ways of turning a particle's three pairs of ideas round the fixed order of stance, act and structure, each turning putting a different pair in front (Chapters 6 and 7).

Geometric time. The time of the arena: the time along a path through space and time, which runs at different rates in different places (Chapter 2).

Gluons. The carriers of the force that binds the quarks inside protons (Chapter 12).

Grain. In the picture that Greene reports, space is made of least pieces like the squares of a grid, so that no distance is shorter than the Planck length. In this account the arena has no grain: no least length, no least area and no least duration (Chapter 5).

Grand unification. The joining of the three forces other than gravity into one force, which a grand unified theory would make. In this account the three forces were never one larger force (Chapters 6 and 9).

Gravitational size. The length at which a mass's own gravity would become strong: squeezed inside about twice that length, the mass would be a black hole (Chapter 5).

Gravitational wave. A change in the shape of the arena, traveling outward at the speed of light (Chapter 1).

Graviton. The particle long proposed as the quantum of gravity, with no mass and no charge. In this book the word *graviton* names at most a quantum of a description, never a piece of space and time (Chapters 5 and 6).

Gravity. Greene's phrase is the *warping of space and time*. This book's word is *gravity*, and in Chapter 1 I identify gravity with the shape of the arena, and not with a force acting within the arena (Chapter 1).

Ground. Said of an argument: the starting point of the argument, the things that a person who makes the argument takes as given (Preface).

Gyrations. Greene's phrase is *dark matter*. This book's word is *gyrations*, of which the dark matter is made: lasting vibrations of the six held dimensions, each of about the Planck mass, made at the turn (Chapter 9).

Gyroscope. A spinning wheel on a mount that keeps the wheel's axis pointing one way (Chapter 1).

Hard problem. The question that the philosopher David Chalmers named: why is any of the brain's doing felt? The *easy problems* ask how the brain does what the brain does (Chapter 14).

Hard randomness. An outcome that nothing before it fixes, however completely what came before is known (Chapters 2 and 3).

Hawking radiation. Physicists' name, after the physicist Stephen Hawking, for the *faint glow* of a black hole (Chapter 18).

Heat death. A final state in which everything has one temperature, the entropy of everything has reached its largest value, and nothing ever happens again (Chapter 19).

Held small. Greene's phrase is *curled up*. This book's phrase is *held small*, said of the six (the six small dimensions of possibility): held a few Planck lengths across, as the content of the large world of four dimensions. The large world of four dimensions is the context of the six (Chapter 7).

Hibernation. The physicist Freeman Dyson's name for thinking in bursts and resting between the bursts, shedding the heat of each burst while resting (Chapter 17).

Hidden variable. A quantity beneath the state that would settle an outcome in advance (Chapter 3).

Higgs field. The field whose settled value gives many particles their masses (Chapters 9 and 11). The *Higgs boson* is a small ripple of the Higgs field (Chapter 19).

Higgs ocean. Greene's word for the settled value of the Higgs field filling space. This book's word is the *condensate* (Chapter 6).

Hold, the. The large world of space and time keeping the six at their one size and shape, and with them the forms of matter. The hold lasts while the large world is there around the six, with things in the large world to compare against (Chapters 9, 16 to 20 and 22).

Hologram. In the everyday sense, a flat film that shows a scene in depth when the film is lit (Chapter 12). Greene's hologram is his picture of a universe mirrored exactly on a distant boundary (Preface). Greene calls the paired worlds, the world inside a region and the world on the region's boundary, Holographic Parallel Universes (Chapter 12). The *holographic* ideas describe what happens inside a region on the region's boundary, as Greene reports the ideas (Chapter 18).

Horizon. In this book the word *horizon* always says which edge the word names. The *cosmological horizon* is the edge beyond which nothing can ever send a comparer a signal (Chapter 1). The *early horizon* is the greatest distance a signal could have crossed since the turn, in the first moments of our history (Chapter 9). The *particle horizon* is the edge of everything whose light has reached us since the expansion began (Chapter 10). The

horizon of a black hole, also called the black hole's event horizon, is the black hole's edge: the surface from inside which nothing returns (Chapters 6 and 12).

Horizon problem. The question of why regions of the sky that never exchanged a signal agree so closely (Chapter 9).

Hubble radius. The distance at which the expansion now carries things away from us at the speed of light. The distance to our particle horizon is a little over three Hubble radii (Chapter 10).

Hypercharge. In the hot young universe, before the Higgs field settled, the charge that played the part electric charge plays now (Chapter 6).

Idealism. In Greene's words, the view "that consciousness is the only quality of the world that is real". My book *The Last Turtle* describes idealism as the view that mind comes before body (Chapter 14).

Immanent. In the name *Immanent Metaphysics*, *immanent* means within: the study is worked out from inside the act of comparison, as the act happens, and never from a place outside the world (Preface).

Incommensurable. Said of two things when no common measure fits both things (Chapter 3).

Incommensuration Theorem. Nothing can be both perfectly symmetric and perfectly continuous, and nothing can be both perfectly asymmetric and perfectly discontinuous. The theorem uses nothing but the meanings of sameness, difference, content and context, so the theorem holds of every comparison. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapters 3 and 8).

Inertia. How hard a body is to set moving (Chapter 1).

Inflation. Greene's word and the word of most cosmologists for a brief early burst of very fast expansion of space in the earliest moments (Preface; Chapter 9).

Inflaton. In Greene's account, the field that drives inflation (Chapter 9).

Information. A word with two senses. In the first sense, information is the form of the content: what the matter is, carried with the matter. In the second sense, information is a count of states relative to a context (Chapter 18).

Infrared light. Light too red to see (Chapter 13).

Inside. My book *What Comes After the Dark* describes an inside as "a side of its own, from which the comparing is done and to which the outcome happens". A thing has an inside when the thing's comparing has the three features of every act of comparison together (Chapter 14).

Integrated information. The quantity of the neuroscientist Giulio Tononi, as Greene describes the quantity. Tononi characterizes conscious awareness as information that is highly integrated and highly differentiated, and integrated information is the amount of such information: a number computed from a third-person description of a system, the system's states, and the chance of each next state given each present state (Chapter 14).

Interference. The adding of two waves where crest meets crest, and the canceling of the two waves where crest meets trough, which draws bands of bright and dark (Chapter 4).

Interferometer. An apparatus that splits a beam into paths and brings the paths back together (Chapter 4).

Interior. Said of a black hole: the region inside its horizon, where every path leads forward to an end (Chapter 19).

Internal space. The one small shape that the six (the six small dimensions of possibility) form together: the complex projective plane times the sphere, the spaces of all the states of a three-way and of a two-way choice, side by side (Chapter 7).

Interpretations. The rival views of quantum mechanics (Chapter 4).

Ion. An atom with one of the atom's electrons taken away (Chapter 2).

Joule. The everyday unit of energy (Chapter 17).

Kaon. A short-lived particle, the neutral kaon, made in collisions at an accelerator (Chapter 8).

Kelvin. The degree of the temperature scale that starts from absolute zero, the coldest anything can be (Chapters 13 and 17).

Label. A property that sorts particles into kinds and is exactly the same for every particle of one kind, as electric charge is (Chapter 1).

Landscape. Greene's word for the many-valleyed country of string theory's forms, in which our world would be one valley picked out of a vast number of valleys. The bottom of each valley is a settled state with the least energy nearby. In this book, the six have one vacuum: one size and shape that the laws of motion keep steady (Chapter 7).

Laplace's demon. The imagined knower of the mathematician Pierre-Simon Laplace: a knower that knows, at one instant, every force acting in nature and the position of every thing in nature (Chapter 15).

Lattice computations. Computations made by computer on a fine grid of points in space and time (Chapter 9).

Law. In this account, the structure of the world that the physics of this account derives, including the ten dimensions and the one small shape of the six (Chapter 20).

Law of many trials. A law of what happens over many trials, such as the quantum rule for how often each outcome comes. A law of many trials fixes the range of outcomes and the weights of the outcomes (Chapter 15).

Lawful reversal. A reversed process that obeys the laws (Chapter 8).

Lawfulness. The same procedure, under the same conditions, gives the same result in this room, on the Moon, and in any age (Chapter 3).

Lepton. One of the particles of matter that feel no strong force: the electron, its two heavier cousins, and the neutrinos (Chapters 6 and 16).

Light-year. The distance light covers in one year (Chapters 1 and 2).

Linear. Said of a rule of change: when the state is a sum of alternatives, each alternative changes by the rule as if the alternative were alone, and the state at the later time is the sum of the changed alternatives. So the result of a linear rule always contains all of the alternatives, and never one alone (Chapter 12).

Locality. Greene's word is *local*: that you can directly affect only things that are next to you (Preface). This book's word is *locality*, in two senses. In the broad sense, causes that travel from place to neighboring place carry every agreement between what is found in two places. In the narrow sense, no signal travels faster than light (Chapter 3).

Logarithm. A measure of a number's size that adds where the numbers multiply, as the number of digits in a written number does (Chapter 13).

Macrostate. What is found of the whole, such as the count of heads among coins, or the temperature and the pressure of a gas. A *microstate* is one complete arrangement, every coin or every molecule given (Chapter 13).

Magnetic monopole. A heavy particle that carries a single magnetic pole, a north with no south. Greene names a *magnetic monopole problem*: theories joining the forces predict far more magnetic monopoles than the measured expansion of the universe allows (Chapter 9).

Many-worlds interpretation. Greene's many worlds: the interpretation in which the smooth rule governs everything, and every outcome occurs, each outcome in a branch of its own (Chapter 4). By the Incommensuration Theorem, the many-worlds view is excluded as a description of a whole world, and the theorem limits what the many-worlds formalism can claim as a description of knowing (Chapter 12).

Mark of which path, the. A record of which way a particle went (Chapter 4).

Mathematical structure. A pattern of relations laid out whole and the same for everyone (Chapter 12).

Matter fraction. The part of the universe's energy that is matter, ordinary and dark together (Chapters 10 and 11).

Meaning. In the words of my book *What Cannot Be*, meaningful means significant to the process an outcome came from. In the triple of meaning, purpose and value, *purpose* goes with what is expressed or done, and *value* goes with what is perceived or undergone. Each of the three is its own, and none of the three can replace another (Chapter 21).

Measure. A kind of quantity that a measurement finds. There is one measure for each of the six ideas of every comparison: sameness and difference, content and context, and the subjective and the objective (Chapter 7).

Measure problem. The problem of saying how common each kind of universe is in an infinite collection of universes (Chapter 11).

Measurement problem. The puzzle of how a measurement gives one outcome (Chapters 2 and 3), and physicists' name for the question of how one definite outcome comes out of the list of possibilities (Chapter 15). Greene calls the measurement problem the quantum measurement problem (Chapter 4). In this book the name has two senses: a clash between two rules (the smooth rule of change and the rule of the sudden jump at a measurement), and the gap between the smooth description and the single finding (Chapter 4).

Membranes. Objects that spread out in two or more directions, as a sheet spreads out in two, where a string spreads out in one (Chapter 6).

Merger. One description that requires the directions of space and time themselves to be smooth and different from place to place, and also to come in exact, repeating steps. The Incommensuration Theorem forbids a merger (Chapter 6).

Metaphysics. In this book, the account of the act of comparison itself (Chapter 21).

Moduli. The dials that set the size and the shape of the six (Chapter 7).

Momentum. A body's quantity of motion; at everyday speeds, momentum is the body's mass times the body's velocity (Chapter 3).

Morality. In the words of my book *An Immanent Metaphysics*, "an externally defined set of rules in a particular domain": the rules a culture holds, in its place and age, for everyone in it (Chapter 21).

M-theory. Greene's term, given here in Greene's sense: the one theory that the dualities join, with eleven dimensions, ten of space and one of time (Preface; Chapter 7).

Multiverse. Greene's word for a collection of universes of which ours would be one. Greene also calls the members of a multiverse parallel universes or parallel worlds (Preface; Chapter 11). Greene's names for the kinds of multiverse include the Quilted, Brane, Cyclic and Landscape Multiverses (Chapter 11) and the Quantum, Simulated and Ultimate Multiverses (Chapter 12).

Muon. A heavier cousin of the electron, made high in the air when particles from space strike the top of the atmosphere (Chapter 2).

My account. The Immanent Metaphysics together with the physics I work out from it in my books. The appendices call it *this account* (Preface).

Neuronal avalanches. Cascades of activity in the cortex, the outer layer of the brain, whose sizes follow the pattern of a system poised at criticality (Chapter 14).

Neutrinos. The lightest particles of matter, with no electric charge. Each generation, or family, has one kind of neutrino (Chapters 6 and 7).

Neutron star. The dense collapsed core that some large stars leave behind (Chapter 1): a ball some twenty-five kilometers across, packed as tightly as the core of an atom (Chapter 16).

Neutrons. The particles of the core of an atom that carry no charge (Chapters 9 and 16).

Nihilism. The view that nothing is meaningful (Chapter 21).

Non-causal correlation. An agreement that is real and is carried by no influence. My book *What Cannot Be* calls a non-causal correlation "real, and not an influence" (Chapter 3). The physicist Nicolas Gisin calls such correlations nonlocal (Chapter 3).

Nucleus. The tiny core of an atom (Preface; Chapter 8).

Objective, the. What is compared; set beside *the subjective*, the comparing (Chapters 7 and 14).

Observer. Greene's word for whoever does the measuring. In this book an observer is whoever or whatever does the measuring, a *comparer* (Chapters 2 and 16).

Oldest light. Greene's phrase is the *cosmic microwave background*. This book's phrase is the *oldest light*: light left from the early universe, stretched by the expansion of the universe until the light now has a temperature of about 2.7 degrees above absolute zero (Chapter 9).

Orrery. A working model of the solar system: a brass Sun, planets on arms, and beneath them a train of gears that carries each planet around at its proper pace (Chapter 12).

Outside world. An observer's outside world is, in the words of my book *What Comes After the Dark*, "the whole of what can ever reach that observer and be set beside the rest of what it receives" (Chapters 9 and 16).

Particle physics. The study of the smallest pieces of matter and of the forces that act between them (Preface).

Passage. The step from one finding to the next. Each passage runs one way (Chapter 4).

Past Hypothesis. The condition that the universe began with very low entropy, named by the philosopher David Albert (Chapter 8).

Perturbational complexity index. A number computed from how the brain's electrical response to a brief magnetic pulse spreads through the cortex (Chapter 14).

Phantom energy. A substance whose ratio of pressure to density lies below minus one, so that phantom energy grows denser as space grows (Chapter 16).

Phase. Of a wave: how far along its swing the wave has come (Chapter 3).

Phenomenon. In the physicist John Wheeler's use, after Niels Bohr: a happening that becomes a fact only when an irreversible act (an act that cannot be undone) brings the happening to a close (Chapter 4).

Phonon. One step of energy in one way a crystal lattice can vibrate: a step in the description of how the atoms move together (Chapter 5).

Photon. A particle of light (Chapters 2 and 3).

Physicalism. The view that the physical, objective world is all there is at bottom, so that methods using only the physical properties of matter will explain consciousness in the end (Chapter 14).

Pion. A light particle, named in the search for the decay of the proton (Chapter 16).

Planck area. A square one Planck length on a side (Chapter 18).

Planck density. The density built from the constants of gravity, of the quantum and of light alone (Chapter 10).

Planck energy. The energy a Planck mass holds at rest (Chapter 5).

Planck length. About a millionth of a billionth of a billionth of a billionth of a centimeter, roughly a hundred billion billion times smaller than the width of a proton (Preface; Chapters 5 and 12). The Planck length is a length built from the constants of nature alone (Chapter 5), and the finest scale at which any comparison made from inside the world can place a body (Chapter 9). The Planck length is a limit on placing a body, and the arena has no least length (Chapter 5).

Planck mass. About twenty-two millionths of a gram, about the mass of a grain of quartz sand a quarter of a millimeter across (Chapter 5); a mass built from the constants of gravity, of the quantum and of light alone (Chapter 10).

Planck scale. The scale at which a body's gravitational size equals the body's quantum size. At the Planck scale both descriptions of physics, quantum theory and general relativity, are needed, and the two descriptions stay two separate descriptions (Chapter 5).

Planck time. A moment so early that gravity and quantum theory must be described together at that moment (Chapter 9).

Planck's constant. The number that fixes the size of every quantum step (Chapter 3).

Polarization. Said of light: the way the light is lined up, which a polarizing filter tests (Chapters 3 and 19).

Positron. The electron's twin of opposite charge (Chapters 6 and 16).

Possibility. The range of what a comparison could find, every outcome it could have (Preface; Chapter 7).

Postulates. In my book *Non-Commutative Geometry and the Immanent Metaphysics*, the conditions from which the book's theorems start. Each postulate puts one statement of the Immanent Metaphysics into the form of mathematics (Chapter 7).

Power. Said of a ripple: the ripple's strength multiplied by itself (Chapter 9).

Power of ten. One more zero on the end of a number (Chapter 10).

Prime number. A whole number that no whole number divides exactly except one and the number itself (Chapter 2).

Principle of comparison. A difference that no comparison can register is no difference: every distinction is made by comparison (Chapter 1).

Principle of identity. What no comparison can tell apart, in any way at all, even in principle, is one thing and not two. Stated for differences, the principle of identity is the principle of comparison (Chapters 13 and 15).

Probability wave. Greene's word for the description that carries the chances of each finding. This book's word is the *state* (Chapter 3).

Proton. The particle at the core of every atom that carries a positive electric charge (Chapters 9 and 16).

Pulsar. A spinning neutron star that sweeps a beam past the Earth like a lighthouse (Chapter 16).

Pure numbers. Numbers with no unit, such as a ratio (Chapter 11).

Pure randomness. A world with no pattern carried anywhere and nothing joined to anything (Chapter 15).

Quality of energy. Greene's phrase. This book's phrase is *free energy*, the share of a thing's energy that can be drawn off as work (Chapter 13).

Quantized. Described in countable steps (Chapter 5).

Quantum. One of the separate steps in which matter and light trade energy (Chapters 5 and 6).

Quantum computer. A machine that computes with the alternatives of a quantum state while the alternatives still interfere (Chapter 12).

Quantum corrections. The small changes that quantum theory makes to the answers Einstein's equations give. The quantum corrections grow with the bending of space and time (Chapter 5).

Quantum determinism. Greene's phrase. This book's phrase is the *formalism's determinism* (Chapter 15).

Quantum field theory. A quantum description of particles as ripples of fields, with no gravity in it (Chapter 12).

Quantum foam. The physicist John Wheeler's name, which Greene uses, for a froth below the Planck length in which left and right, before and after, lose their meaning (Chapter 5).

Quantum gravity. The joining of quantum physics with gravity (Chapter 18).

Quantum measurement. The finding of one outcome among the possibilities that the quantum state describes (Chapter 14).

Quantum mechanics. Also called quantum theory or quantum physics: the theory of atoms and of the particles smaller than atoms (Preface; Chapter 3).

Quantum of a description. In the description of small waves of the arena by quanta, one step of energy. A quantum of a description is what the word *graviton* names at most, never a piece of space and time (Chapter 5).

Quantum size. The length within which quantum theory will not let a mass be pinned down without spending enough energy to make new particles of the mass's kind (Chapter 5).

Quantum variable. A quantity that quantum theory describes with the jitter set by the uncertainty principle (Chapter 5).

Quarks. The particles of which protons and neutrons are made, three quarks in each proton or neutron. The up and down quarks are the two kinds of quark of which protons and neutrons are made (Chapters 6, 11 and 16).

Quasars. The blazing centers of far galaxies (Chapter 3).

Quaternions. A four-part number system (Chapter 7).

Quintessence. The name physicists give, and Greene uses, for a field that slowly changes and mimics a constant for billions of years (Chapter 10).

Radioactivity. The giving off of energy by certain atoms as the atoms change (Chapter 12).

Radiometers. The receivers for microwaves carried by COBE, the Cosmic Background Explorer satellite (Chapter 22).

Radius of curvature. Said of a curved space: the radius of the ball whose curve the space would follow (Chapter 9).

Real. To be given in an actual act of comparison: here, now, with the future still open; set beside *exist* (Chapter 2).

Recollapse. Greene's phrase is the *big crunch*. This book's word is *recollapse*: the expansion of space as a whole stopping, turning around, and contracting (Chapter 16).

Record. A finding laid down and held after the act that made it: the structure that one act lays down (Chapter 8).

Red dwarfs. The small, dim, long-lived stars that are likely most of the stars there are (Chapter 16).

Redshift. The stretching of light into longer, redder waves (Chapter 16).

Reductionism. The claim that the description at the lower level exhausts the thing at the higher level (Chapters 13 and 14).

Referent. The thing a word names (Chapter 14).

Remnants. What burned-out stars leave behind (Chapter 16).

Reset, the. This book calls the dropping of the old counts at the turn the reset: at the turn, no count reckoned against the old world is carried across, while the content and the form of the content are carried across (Chapter 9).

Residual. What is left over in every finding that the law does not fix (Chapter 4).

Rest mass. The mass a body has when the body is still (Chapter 19).

Reversibly. Said of the steps of a computation: in steps that could each be undone, which need give off no heat (Chapter 17).

Roles, the three. The three roles that every comparison has: the stance, the act and the structure (Chapter 3).

Run of findings. One finding after another by one thinker, each made against something actually there, and each kept in a lasting structure of the thinker's own (Chapter 17).

Schrödinger's equation. The rule by which the state changes in time when nothing is measured. The physicist Erwin Schrödinger wrote the rule down (Chapter 4).

Second law. The second law of thermodynamics, the science of heat, energy and work. In Greene's telling, the second law is the tendency of things with many parts to go toward higher entropy (Chapter 8). In this book, the second law says that for a thing open to the world, counted with all that the thing has given out, the entropy never falls (Chapter 8).

Selection effect. Greene's phrase is the *anthropic principle*. This book's phrase is *selection effect*: the shaping of what we observe by the conditions for our being there to observe it (Chapter 11).

Self. The gathering of choice in a person. My book *An Immanent Metaphysics* describes the self as "the appearance of the aggregation of that aspect of interaction which is choice" (Chapter 15).

Settings. Said of a detector: the direction the detector is turned to (Chapter 3).

Shear viscosity. The drag of one flowing layer on the next layer (Chapter 12).

Signal. Anything that passes from one place to another and lets a record at the second place depend on an act at the first place (Chapter 3).

Silica. The stuff of glass (Chapter 17).

Singularity. A place where, by general relativity's own equations, the bending of space and time grows without limit: the point at which the smooth description of space and time reaches the edge of the range in which the smooth description holds (Chapters 5 and 18).

Six, the. Greene's phrases are *extra dimensions* and *curled-up dimensions*. This book's word is the *six*: six dimensions of possibility, held a few Planck lengths across, which are never traveled through and are met only as the range of what could be found (Chapter 7; Chapters 9 and 11).

Sodium chlorate. A salt whose crystals come in two mirror-image forms, left-handed and right-handed, as a pair of gloves does (Chapter 13).

Space of states. Said of a pair of particles, or of any system: the whole range of the states the pair or the system can be in (Chapter 3).

Spacetime. Greene's word for space and time joined into one four-dimensional whole. This book's word is the *arena* (Preface; Chapters 1 and 2).

Special relativity. Einstein's theory of 1905 of motion at a steady speed in a straight line and of light, and of how lengths and durations compare for observers moving steadily relative to one another (Chapters 1 and 2).

Speed of light. In Chapter 2 I identify the speed of light with the *edge of comparison*: the greatest speed at which any signal can carry a finding from one place to another (Chapter 2).

Sphere. The surface of a ball (Chapter 7).

Spin. A turning built into every particle, the same amount for every particle of one kind (Chapter 6). For an electron, spin is a direction that can be measured (Chapter 3), and for a nucleus, spin is a turning of the nucleus's own that makes the nucleus a small magnet (Chapter 8).

Stance. The side a comparison is made from, with the range of what could come. For a detector, the stance is the direction the detector is turned to and the yes or no the detector could hear (Chapter 3).

Standard Model. The theory of the known particles and of the three forces other than gravity: electromagnetism, the weak force and the strong force (Preface; Chapter 6).

State, the. Greene's phrase is the *probability wave*. This book's word is the *state*: the complete description of what could be found, and of how likely each finding is (Chapter 3).

Statistics. Said of a program: the pattern of how often each of the program's outcomes comes up (Chapter 12).

String theory. The proposal that the smallest pieces of matter are not points but tiny vibrating strands of energy, called strings, each kind of particle being one way a string can vibrate (Preface; Chapter 5).

Structural arrow. The one-way order inside every comparison, act before record. No reversal can undo the structural arrow, and the structural arrow needs no special beginning (Chapter 8).

Structure. What is laid out whole and stays the same for everyone who looks at it, as the sliced loaf does (Chapter 2). In a comparison, the structure is what is laid down, which is the same for everyone who reads the structure afterward: for a detector, the structure is the line in the detector's log, the detector's record (Chapter 3).

Structure of comparison. The three roles of comparison and the axioms that relate the roles, the content and the context of comparison, and the Incommensuration Theorem, taken together. In the name *structure of comparison* the word *structure* has its everyday sense, the way a thing is built, and does not mean the third role (Chapters 6 and 11).

Subject. One who compares from a side of its own (Chapter 12).

Subjective, the. The comparing, the side from which a comparison is made. The subjective holds the stance, one of the three roles of every comparison. In the same pair, *the objective* is what is compared (Chapters 7 and 14).

Subjective time. In the physicist Freeman Dyson's use, time as the creature itself lives through time (Chapter 17).

Sum over paths. The rule of the physicist Richard Feynman, as Greene describes the rule: a particle travels from one place to another along every possible path at once, with a number given to each path (Chapter 3).

Superdeterminism. The proposal that the settings of the instruments and the states of the particles share one deterministic past, so that the settings only seem free (Chapter 15).

Supernova. A star that blows itself apart, and for a few weeks can shine as brightly as the whole galaxy the star lives in (Chapter 10).

Superpartners. The partners that supersymmetry would give every particle, each partner with a spin that differs from the particle's own spin by half a unit. In this account the reason that no superpartner has been found is that a supersymmetry cannot exist for the fields of the Standard Model, as my book *Non-Commutative Geometry and the Immanent Metaphysics* proves (Chapter 6).

Superposition. A combination of the state for one alternative and the state for the other alternative, with an amount of each state (Chapter 4).

Supersymmetry. A proposed pairing that would give every particle a partner whose spin differs from the particle's own spin by half a unit, so that each particle of matter would have a partner among the carriers of force, and each carrier a partner among the particles of matter (Chapters 6 and 10).

Surface of simultaneity. Greene's phrase is *time slice*: all of space at one moment as one observer reckons the moment. This book's phrase is *surface of simultaneity*, the physicist's name for a time slice, and no comparison registers a surface of simultaneity (Chapter 2).

Surviving pairings, the two. The two pairings that the Incommensuration Theorem allows: continuity with asymmetry, like a smooth ramp, and symmetry with discontinuity, like a ladder (Chapters 3, 5, 6 and 12).

Symbolic immortality. Greene's term for a life that goes on in its achievements and its works after the one who lived it has died (Chapter 20).

Symmetry. The same content across a different context, as a square turned a quarter of the way round is the same square (Chapters 2, 3 and 6). A symmetry of the laws is the sameness of the laws when a laboratory is turned to face another way, and Greene also gives the name *symmetry* to the change itself, the turning of the laboratory (Chapter 6).

Teleportation. The moving of an unknown state from a sender to a receiver who share an entangled pair of particles (Chapter 3).

Theorem. A statement shown to follow, step by step, from what has already been established (Chapter 3).

Theory of everything. Greene's phrase for one theory of every force and every piece of matter. In this book the phrase can mean two things, a *common source* or a *merger*, and the Incommensuration Theorem allows a common source and forbids a merger (Chapter 6).

Thinker. Greene's word is *Thinker*, with a capital letter. This book's word is *thinker*: a comparer of a particular kind, of whatever material (Chapter 17).

Third person. A third-person description is one that anyone can check from outside, told of a thing as "it" (Chapters 14 and 15): a description given as one describes someone else, from outside, the same for every reader, with no one's side in the description (Chapter 21).

Three scales of description. The scale of particles and fields, the scale of comparers (organisms, instruments and their readings), and the scale of the whole, each scale with its own kind of description. The three scales of description are the counterpart of Greene's nested stories, the story of particles, then of life, then of mind, then of culture (Chapter 14).

Three times, the. Geometric time; event time; and evolution time, the time along which the description of what could be found, and of how likely each finding is, changes between findings. None of the three times can do the work of another (Chapter 2).

Time loop. A path through space and time that returns to its own starting event (Chapter 2).

Time slice. Greene's word for all of space at one moment as one observer reckons the moment. The physicist's phrase is *surface of simultaneity* (Chapter 2).

Time-reversal symmetry. The property of the laws that the laws stay the same when the order of time is turned around (Chapter 8).

Top quark. The heaviest known elementary particle (Chapter 19).

Total generality. The same law everywhere, which is symmetry perfected (Chapter 3).

Trace. What a particle carries of another particle after the two particles strike each other: from then on, the state of the one particle is tied to the state of the other particle (Chapter 8).

Trim. The part of the Planck density that remains once the curve of the six (the six small dimensions of possibility) has taken the curve's share, about eighty-eight hundredths. The shape and size of the six set the trim (Chapter 10).

Tunneling. The passing of a thing held behind a wall through the wall without climbing over the wall (Chapter 19).

Turn, the. The passage at the end of the black-hole interiors of one epoch, in which the ten-dimensional geometry restores a large world and the next epoch begins (Chapter 9). The turn is the limit at which the interiors of all black holes end, approached from both sides and reached from neither, and the turn is not an event in time (Chapter 19).

Ultraviolet catastrophe. The prediction, by the physics of around 1900, of an infinite amount of energy at the shortest wavelengths of the light inside a hot oven (Chapter 5).

Uncertainty principle. The rule of quantum mechanics that a particle cannot have both a definite position and a definite velocity. The limit comes from the spread of waves, and the uncertainty principle is an instance of the Incommensuration Theorem (Chapters 3 and 5).

Unitary evolution. The physicist's name for the smooth, linear change of the state (Chapter 12).

Update of the record. Greene's word is *collapse*: in a collapse, the wave jumps, all at once, to the one outcome found. This book's phrase is the *update of the record*: at a finding, the shared record of what has been found changes, the same for everyone who has the outcome (Chapter 4).

Vacuum. A word in three senses, each sense marked where the word is used. In the first sense, *the vacuum* is empty space with every field at the field's lowest energy (Chapters 1 and 10). In the second sense, *a vacuum* is a settled state with the least energy nearby, which for the six is one size

and shape that the laws of motion keep steady (Chapter 7). In the third sense, the everyday sense, a vacuum is a chamber pumped nearly empty of gas (Chapter 4).

Wave function. Greene's word for the quantum state (Chapter 15).

Weak charge. The label on which the weak force acts (Chapter 16).

Weak force. The force that changes one kind of particle into another in radioactive decay (Chapters 6 and 8).

Weight. Of a microstate: the likelihood of finding the gas in that microstate (Chapter 13). The *weights* of a range of outcomes are how likely each outcome is (Chapter 15).

White dwarf. The very dense ball of carbon and oxygen that a star like the Sun ends as, held up by the degeneracy pressure of the white dwarf's electrons (Chapter 16).

Work. The energy that moves a piston as a whole in one direction (Chapter 13).

World. In this book's own sentences, the world means everything that is. When this book speaks of other universes, the word *universes* is always Greene's word, in sentences that tell what Greene says (Chapter 11).

Wormhole. A short tunnel joining two distant places (Chapter 2).

X and Y bosons. The carriers that a grand unified theory would need (Chapter 6).

Z particle. A carrier of the weak force (Chapter 7).

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