

# The Gravitational Passage

*Deriving General Relativity*
*from the Immanent Metaphysics*

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# Preface

I derive the Einstein field equations of general relativity, including the value of the cosmological constant, from the structure of comparison, in the sense of "derived" stated below.

Every observation, every measurement and every test of a law is a comparison, and science is made of them. In Part I, I ask what comparison cannot not involve. I find six intrinsics, and four compound concepts that physics already uses, and prove from them the Incommensuration Theorem: symmetry and continuity cannot both be perfected in any comparison. The three axioms of the Immanent Metaphysics are then found as the structure that comparison already has, so the derivation rests on what science itself has to be. Read in physics (Chapter 6), the constraint of the Incommensuration Theorem partitions physics into two structural kinds: content phenomena, governed by symmetry, which are the quantized forces and matter fields; and context phenomena, governed by continuity, which is gravity: the geometric arena within which all content operates.

The geometric expression of this partition is the product structure $M^4 \times \mathbb{C}P^2 \times S^2$. The continuously asymmetric factor, $M^4$, is the four-dimensional Lorentzian manifold of general relativity. The discontinuously symmetric factor, $\mathbb{C}P^2 \times S^2$, is the internal space that is the content factor relative to the exterior (Appendix C, Section VIII). Given the identifications of Chapter 9, it is the only compact homogeneous product that meets them: six dimensions for the six metrics of physical measurement, in a product of a factor carrying the three modalities (omniscient, immanent and transcendent) and a factor carrying content and context. The two factors coexist as a product because the Incommensuration Theorem forbids their merger into a single geometry.

The ten-dimensional Einstein-Hilbert action, integrated over the internal space by Kaluza-Klein reduction, gives the four-dimensional Einstein equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$. The cosmological constant $\Lambda$ is

a prediction: the share of the vacuum's energy that is accessible to an observer inside the vacuum, set by the information that observer's horizon leaves out (Chapter 13).

The one measured constant of this book is the gravitational constant $G_N$. The axioms fix its structural role; its magnitude is a fact about this domain. The Planck mass is formed from it. The numerical value of the cosmological constant also uses the measured radius of the observer's particle horizon, an observed feature of the present universe and not a constant of the equations, and the predicted matter fraction uses the measured density of radiation.

**On how this book argues.** The mathematics is stated as mathematics: a theorem is proved or cited, and a computation is given with its inputs. Where the argument passes from the structure of comparison to a physical quantity, it does so by an identification, and the text states each identification in the first person ("I identify", "I take"). An identification is a reading that I propose, not a theorem. The reader can examine the reasons for each identification. In this book a result is derived when it follows, by theorem or by standard technique, from the structure of comparison set out in Part I, and the three axioms it carries, together with the identifications so stated. I take a metric geometry as given, as general relativity does; what I obtain from comparison is the number of dimensions, the signature and the product form of that geometry, and the form of the field equations then follows from Lovelock's theorem (Chapter 12, Section VII). Chapter 5, Section V, states which kind of reasoning establishes each kind of claim the book makes: necessity, correspondence with measurement, sufficiency and comprehensiveness.

**On the words.** Omniscient, immanent and transcendent are technical names for the three roles that concepts play in any comparison (Chapter 4): the omniscient is what is structural and known, the immanent is what mediates and happens, and the transcendent is what is potential and exceeds any instance. They carry no theological meaning. Symmetric and continuous are used in the senses Chapter 2 defines, which concern what

is found in a comparison; the table of Chapter 3, Section V, sets those senses beside the ordinary mathematical ones. The Glossary at the back gives every technical term with the place where it is introduced.

**How the book is arranged.** The map that follows the Preface sets out the chain: each link, the chapter that gives it, what it rests on, and its standing. Each chapter opens by saying what it does, what it rests on and what uses it (and, where another chapter cites it without resting on it, what refers to it), and closes with the questions a careful reader brings to its steps, with their answers. Five figures show the square of verdicts (Chapter 3), the grid of the metrics beside the classes of the internal space (Chapter 9), the identity against the matter fraction (Chapter 13), the cycle of epochs (Chapter 18) and the curve of Kasner exponents (Appendix B). Readings for readers of the Immanent Metaphysics that the argument does not need are in the Notes.

**On other programs.** Several programs obtain general relativity, or parts of it, from principles other than its standard postulates: the constructive axiomatics of Ehlers, Pirani and Schild (1972); the thermodynamic derivation of Jacobson (1995); the entropic gravity of Verlinde (2011) and the emergent gravity of Padmanabhan (2010; 2015); the spectral action of Chamseddine and Connes (1997); loop quantum gravity (Ashtekar 1986; Rovelli 2008) and causal set theory (Bombelli and colleagues 1987; Surya 2019); and the constructor theory of time of Deutsch and Marletto (2025). The argument of this book uses none of them, and its chapters state each step on its own ground. The formula of Chapter 13 has the form of the holographic dark energy of Cohen, Kaplan and Nelson (1999), Hsu (2004) and Li (2004), a density proportional to $M_{Pl}^2/L^2$ for the length L of a horizon; in that program the density follows the horizon as the universe expands, and Chapter 13, Section VI, fixes it once.

**On scope.** This book is complete in itself. It makes a set of arguments in gravitation and cosmology. It makes no claim about the Standard Model or about the particle spectrum; it uses established particle physics only where established physics is needed, as in the phase transitions of the early universe. Appendix A gives the argument for the content/context partition, and Appendix B gives the dimensional schema of the black hole interior. Appendix C gives a second derivation of the dimensions of time and space, by a separate route, and shows that the six dimensions of possibility are the internal six of Chapter 9. Appendix D computes, at first order, one mechanism for the hold of the internal space, with its method and its standing, and names the published additional materials that give the computation in full. Each chapter introduces whatever formal apparatus it requires.

# The Chain at a Glance

I argue from comparison to gravitation in one chain. Observation, measurement and law are comparisons, and comparison involves six intrinsics and four compound concepts (Chapters 1 and 2). From them the Incommensuration Theorem proves that only two conjunctions of the compound concepts survive, the continuously asymmetric and the discontinuously symmetric (Chapter 3), and the three axioms are found as the structure that comparison already has (Chapter 4); Chapter 5 states the reasoning the book uses. I identify the first conjunction with the context of physics, gravity, and the second with its content, the quantized fields (Chapter 6). The six metrics of measurement form a grid of two rows and three columns (Chapter 7), and the two kinds, context and content, are carried by two factors of one product geometry (Chapter 8). Given the identifications of Chapter 9, the internal factor is $\mathbb{C}P^2 \times S^2$, the only compact homogeneous product that meets them; the external factor is four-dimensional and Lorentzian (Chapter 10). The Kaluza-Klein reduction of the ten-dimensional action, with the internal space held, gives the Einstein equations in four dimensions, and Lovelock's theorem gives their form as the only one (Chapters 11 and 12). The cosmological constant is the share of the vacuum's energy accessible to an observer inside it, $c_1/S$ of the external value, with $c_1 = 3$ and S the count of the observer's particle horizon (Chapter 13). The same product geometry carries black hole interiors to the origin of a new epoch (Chapters 18 and 19; Appendix B).

Three results can be checked without the metaphysics. The classification of Chapter 9 is a statement about compact homogeneous products with prescribed Euler characteristics. The cosmological constant is suppressed below the Planck density by 1/S, about $10^{-123}$, and combined with the Friedmann equation the formula gives $I^2\Omega_\Lambda = 8 - 3/\pi$, where $I$ is the radius of the particle horizon in units of the Hubble radius, a relation from which the Hubble constant and the Planck mass cancel: it predicts $\Omega_m = 0.311$, against the measured $0.315 \pm 0.007$, equivalently that the

measurement which fixes the constant is made when the universe is 0.792 de Sitter times old, against $0.787 \pm 0.009$ (Chapter 13). The Kasner algebra of Appendix B places the black hole interior and the compactification on one arc of exponents, traversed in opposite directions of time. A reader who wants only these results can read Chapter 9, Section VI; Chapter 13, Sections III and IV; and Appendix B, Sections III and IV. The computations behind the numbers of the three results are given, with their inputs, in the book's additional materials (Appendix D, Section V).

I make six principal predictions: a dark-energy equation of state of exactly $w = -1$; a matter fraction of 0.311; two masses, each held in two places and related only through gravitation, that become entangled; no primordial gravitational waves; no primordial isocurvature mode; and a spatial curvature of order $10^{-5}$ (Chapter 20).

The table gives each link, the chapter that gives it, the links it rests on, and its standing: a theorem; a standard technique; an identification (a reading from comparison to physics, not a theorem); argued (from Part I and the stated identifications, each cited); computed; or measured.

| Link | Chapter | Rests on links | Standing |
|---|---|---|---|
| 1. Comparison as the ground of science; six intrinsics; four compound concepts | 1, 2, 5 | none | the base: what any comparison involves, and any denial is itself a comparison |
| 2. The Incommensuration Theorem; the three axioms and the modalities | 3, 4 | 1 | theorem (enumeration); the axioms found in the structure of comparison; the readings in physics (Chapters 3 and 4) identifications |
| 3. Content and context: gravity as context, the quantized fields as content | 6, App. A | 2 | identification; the markers argued |
| 4. The six metrics and the grid | 7 | 2, 3 | identification |
| | 8 | 2, 3, 4 | |

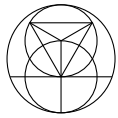
| | | | |
|---|---|---|---|
| 5. The product $M^4 \times K$ | | | identification; the separation of the factors argued from the theorem |
| 6. The internal space $\mathbb{C}P^2 \times S^2$ | 9 | 4, 5, and the identifications below the table | classification theorem, given the identifications |
| 7. Four external dimensions; Lorentzian signature; a finite signal speed | 10, App. C | 2, 4, 18 | argued, with identifications |
| 8. The radii $R_O = \sqrt{6}\, l_{Pl}$, $R_T = l_{Pl}$ | 9, 17 | 6, 18 | ratio and scale by theorem, given one identification: the Ricci curvature of every internal direction is the Planck curvature (each direction at the modal boundary), which makes the internal metric an Einstein metric |
| 9. The ten-dimensional action and its reduction | 11 | 5, 6 | standard technique |
| 10. The internal space held; the masses of the moduli | 11, App. D | 6, 8, 18 | the hold argued (each direction at the modal boundary, the same throughout the exterior); the masses computed at first order for one mechanism |
| 11. The Einstein equations in four dimensions | 12 | 1, 4, 9, 10 | Lovelock's theorem, with covariance identified with the omniscient modality; the source of the curvature identified as the count of findings, which energy and momentum measure |
| 12. One geometry for each alternative of the matter | 12 | 3 | identification |
| 13. A finding made in the first person; no common surface of simultaneity | 12 | 12 | argued, with the theorem that local results do not depend on the surface |
| 14. $\Lambda$ from the information deficit | 13 | 6, 8, 9, 18, 22 | identification; computed |
| 15. $I^2\Omega_\Lambda = 8 - 3/\pi$; $\Omega_m = 0.311$ | 13 | 14, 22 | computed; compared with measurement |

| | | | |
|---|---|---|---|
| 16. $w = -1$ | 13 | 14 | argued |
| 17. Equivalence, geodesics, the Newtonian limit | 14 to 16 | 11 | standard results; the equivalence extended to the inertial forces, with a test that tells the three apart |
| 18. $G_N$ and the modal boundary at the Planck scale | 17 | 2, 3 | $G_N$ measured; the boundary argued |
| 19. The interior passage and the cycle | 18, 19, App. B | 6, 8, 14, 18 | argued; the items this book does not fix are listed in the Questions of Chapters 18 and 19 |
| 20. The turn as a limit; the Big Bang as its projection | 19 | 19 | identification |
| 21. Scalar, adiabatic primordial departures; a spatially flat epoch | 18 | 19 | argued; amplitude and tilt not fixed in this book |
| 22. The flatness used by the identity | 18 | 21 | argued |

The identifications of link 6 are the three conditions of Chapter 9, Section I (one dimension for each metric; a product of a factor carrying the modalities and a factor carrying the rows; each factor carrying its terms as its independent classes) and homogeneity (Chapter 9, Section II). Each chapter's opening line *Rests on* lists only what the links of that chapter rest on in this table, and its line *Refers to* lists the other chapters it draws on without resting on them; *Used by* and *Referred to by* are their mirrors. Two pairs of links support each other, and neither pair fixes a value from itself. The radii (8) and the modal boundary (18): the boundary is the Planck scale, fixed by the measured $G_N$, and the internal space is read at it. $\Lambda$ (14) and the flatness of the epoch (22): the flatness uses only that $\Lambda$ is positive, not its value.

# PART I

What Science Has to Be



Chapter 1

# Comparison as the Ground of Science

*On What Every Measurement Already Is*

This chapter states the ground of the book. Observation, measurement and the testing of a law are each a comparison, and science is made of them. A law of physics states what stays the same when the context of a measurement changes. Causation, as science establishes it, is a relation that persists across what is observed, and it has the same form as a law. The concepts of comparison, interaction and relation name one act, and that recognition, the Root Tautology, is the single premise from which the rest of Part I is drawn. I close the chapter by saying what kind of claim each part of the book makes, and which kind of reasoning establishes it.

*Rests on:* nothing earlier; what this chapter states is what any measurement already involves. *Used by:* Chapter 2 (comparison and its intrinsics), Chapter 3, Section IV (measurement is comparison), Chapter 4 (Sections V and VI: measurement and the Root Tautology), Chapter 5 (reasoning and causation), and Chapter 12, Section V (energy and momentum as the measure of findings).

## I. Science Is Made of Comparison

Science is a method for obtaining reliable knowledge about the world, and it proceeds by observation, hypothesis and test. Each of these is a comparison.

An observation compares a quantity with a standard. A thermometer reading is compared with its scale; a spectral line is compared with a reference wavelength; a count in a detector is compared with the background rate. Without the standard, the observation yields no information.

A hypothesis is compared with what has been observed: does the proposed pattern match the data? A test compares a prediction with an outcome, and an experiment is designed so that the two can be set side by side.

So comparison is what science is made of. Observation, hypothesis and test are each made of it, and comparison is not a tool that science merely happens to use. Remove the comparison from an observation and nothing is observed; remove it from a test and nothing is tested.

I use *comparison* in its plain sense: the act of finding whether two things are the same or different, in some respect. In the rest of Part I, I ask what that act cannot not involve, and what follows from it.

## II. Measurement and Law

Every physical measurement is a comparison of this kind: a reading taken against a standard, within a context, from a stance. Whatever every comparison requires, every measurement requires.

A law of physics states what stays the same when the context of a measurement changes. Conservation of energy states that the total energy is the same whenever it is measured. The universality of a law states that the result of an experiment is the same wherever and whenever the experiment is performed. Each is a statement that the content of a comparison remains the same across a difference of its context. In Chapter 2 I name that character symmetry, and define it from the intrinsics of comparison alone.

So a law of physics is a finding of comparison, stated for every context in which the comparison can be made, and not something added to comparison from outside. I pass from comparison to physics through measurement, which is comparison, and through law, which is what comparison shows to stay the same.

## III. Causation

Causation, as science establishes it, is a persistent relation of association among three events: the observing self, the observed cause, and the observation of the effect. It requires an observer. It runs one way in time with respect to that observer. It does not require a complete specification of every event in the world; it requires a relation that holds again and again in what is observed.

A causal law therefore has the form of every law in Section II: the same relation of cause to effect, found again across a difference of context, here and there, now and later. Causation is an invariance, found by comparison.

Causation is a second route from comparison to science. The argument from comparison to physics runs through measurement and law; causation agrees with it, and the later chapters use causation where a finding is made in the first person. In Chapter 5, Section III, I set causation beside determinism, with which it is often confused.

## IV. The Root Tautology

To compare two things is to bring them into relation, and to bring them into relation is for something to happen between them. A comparison is an interaction: the measured system acts on the instrument, and the instrument registers the action. An interaction is a relation: the two things that interact stand in relation by interacting. And a relation is a comparison: to say how two things are related is to say in what respect they are the same and in what respect different.

The *Root Tautology* is that comparison, interaction and relation are one act, approached three ways.[1] Comparison names the act as a finding of sameness and difference; interaction names it as an event, something that happens; relation names it as a standing connection between the two.

The Root Tautology is a recognition that three names name one act, not a postulate chosen from among alternatives. It is also the one premise of Part I. To attempt to deny it is to compare the claim with something and find them different, and so is itself an act of comparison, an interaction between the one who denies and the claim, and a relation between them. The premise is in use in every attempt to reject it.

Physics is the study of interaction, and every quantity physics measures is measured by an interaction between a system and an instrument. By the Root Tautology that interaction is a comparison. So whatever comparison requires, physical interaction requires, in the system as well as in the instrument.

## V. What Kind of Claim Each Part Makes

Four different claims can be made about any description of a domain, and I make each of the four claims in the proper place of that claim.

A claim of *necessity* says that something must be present. In Part I, I argue necessity: what any comparison must involve, and what that requires of anything compared. A claim of correspondence says that a result holds in the world, and in the later chapters I check their results against measurement. A claim of *sufficiency* says that nothing further is required, and a claim of *comprehensiveness* says that nothing has been left out.

The four claims, of necessity, correspondence, sufficiency and comprehensiveness, are argued by different kinds of reasoning. Mathematics establishes what follows from what, within a structure fixed by definition. Measurement establishes what holds in the world, one case at a time. Sufficiency and comprehensiveness are argued from the structure of comparison itself, because every proof and every measurement is made of that structure, and so nothing either of them applies to lies outside it. A proof applies only to its own defined structure, and a measurement applies only

to the cases measured, so neither can establish sufficiency, in either direction. Chapter 5, Section V, states the three kinds of reasoning and what each can and cannot establish.

So each claim in this book depends on the reasoning that can establish it: its necessities on the structure of comparison and on mathematics, its correspondences on measurement, and its sufficiency and comprehensiveness on the structure of comparison.

---

### Questions

**Is this not philosophy, placed in front of a physics book?**
It is a description of what physics already does. Every result of physics is a measurement compared with a prediction, and every law is a sameness found across a change of context. I name those acts and ask what the acts involve. The physics of the later chapters is stated as physics, with its mathematics and its measurements, and each passage from the structure of comparison to a physical quantity is an identification, stated in the first person where it is made.

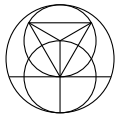
**Why should the structure of comparison constrain the world, and not only our knowledge of it?**
Because the world, as physics reaches it, is reached only by interaction, and by the Root Tautology an interaction is a comparison (Section IV). What every comparison requires is therefore required of every interaction, between two particles as much as between a particle and a detector. A constraint that held of knowledge and not of interaction would need some interaction that is not a comparison, and there is none.

**Can the Root Tautology be tested?**
The Root Tautology is in use in every test. Each test is a comparison of a prediction with an outcome, made by an interaction between an instrument and a system. So the Root Tautology is not an empirical hypothesis

tested case by case. What can be tested are its consequences in physics, and the later chapters state them, each with the observation that bears on it.

**I use one measured constant. Where does it enter?**

Physics enters the chain of argument through the identifications the text states, through standard mathematics and physics, and through the one measured constant, $G_N$ (Chapter 17, Section III). The argument of Part I, from comparison to the Incommensuration Theorem and the axioms, uses none of them. Its readings in physics (Chapter 3, Section V; Chapter 4, Section II) are identifications, stated in the first person, and the identifications by which I carry the argument to geometry begin in Chapter 6.



Chapter 2

# The Six Intrinsics and the Four Compounds

*On What Comparison Cannot Not Involve*

This chapter names the six intrinsics of comparison (sameness, difference, content, context, subjective, objective) and states the inseparability and distinctness of each pair. From sameness and difference applied to content and context it defines four compound concepts: continuity, discontinuity, symmetry and asymmetry. It then shows that each of the four is a concept physics already uses, under the same name or another: conservation and universal law, the calculus and the smooth manifold, quantization and the discrete event, irreversibility and the record a measurement leaves. Chapter 3 proves the Incommensuration Theorem from the four.

*Rests on:* comparison as the act every measurement is (Chapter 1, Sections I, II and IV). *Used by:* Chapter 3 (the compound concepts of Section II), Chapter 4 (the three pairs), Chapter 5 (the intrinsics), Chapter 6 (content and context), Chapter 7 (the intrinsics), Appendix A (symmetry and continuity), and Appendix C (the intrinsics).

## I. The Six Intrinsics of Comparison

To compare is to find whether two things are the same or different. The finding is a verdict on something, and what the verdict is on is the *content* of the comparison. The two things are two in some respect: that is the *context*. The content and the context cannot be one respect. If the respect judged were the respect that makes the two things two, the verdict would be fixed before the comparison was made, and a verdict fixed in advance is no verdict.

The comparison is also made. There is the comparing, the *subjective*, and what is compared, the *objective*: the perspective from which the comparison is made, and the perspective from which it is received.

So the concept of comparison involves six intrinsic concepts: *sameness*, *difference*, *content*, *context*, *subjective* and *objective*.[2] The six intrinsic concepts are what comparison is, and are not additions to comparison. Any process of comparison involves all six, and no comparison can be defined without at least implicit reference to every one of them. The six are necessary and sufficient for the formal consideration of comparison.

The six fall into three pairs, and each pair is inseparable. Sameness and difference are inseparable: wherever one appears, the other is implicit. Content and context are inseparable: where there is content, there is context, and where there is context, there is content. The subjective and the objective are inseparable: there is no comparing without something compared, and nothing is compared without a comparing.

Each pair is also distinct. The meanings of sameness and difference are not interchangeable, nor are those of content and context, nor those of the subjective and the objective. And no pair can stand in for another: the respect in which a verdict is given is not the verdict, and neither is the stance from which it is given.

The subjective and the objective are the stance of every comparison: ourselves comparing, and what we compare. They cannot be followed apart from the comparison itself, and in the next section they are held fixed. They return in Chapter 4, as the third of the three pairs, in Chapter 7, Section III, and in Appendix C, Sections I and II, as the stance of the comparing and the compared.

## II. The Compound Concepts

The concepts of sameness and difference can be applied to the concepts of content and context. Where there is a sameness of subjective context (the stance of the comparing and the compared held the same), four secondary compound concepts emerge:

*Continuity* is a sameness of content where there is a sameness of context. An infinitesimal change in context produces an infinitesimal change in content. Nothing is sudden.

*Discontinuity* is a difference of content where there is a sameness of context. The content changes even though the context does not. Something is sudden.

*Symmetry* is a sameness of content where there is a difference of context. The content remains the same even though the context has changed. Something is invariant.

*Asymmetry* is a difference of content where there is a difference of context. The content changes when the context changes. Nothing is invariant.

In each of the four paragraphs above, the first sentence is the definition, stated in sameness and difference alone. The sentences after it give the physical reading of that definition. The statement of continuity in terms of infinitesimal change is the calculus's reading of the definition, not a second definition: an infinitesimal change of context is the limit at which the context is the same, and the reading states the definition at that limit.

Each compound concept is a pair of verdicts: a verdict of sameness or difference on the content, and a verdict of sameness or difference on the context. Chapter 3 uses exactly that form.

## III. The Compounds Are the Concepts of Physics

The four compound concepts are concepts physics already has. Each is a concept physics already relies on, and each is found where physicists find it.

*Symmetry* is the basis of the conservation laws: Noether's theorem (Noether 1918) states that every continuous global symmetry of the action corresponds to a conserved quantity. The universality of the laws of science is a symmetry: the result of an experiment does not depend on where or when it is performed. Energy conservation, momentum conservation and charge conservation are all symmetry laws. A law of physics, read as in Chapter 1, Section II, is a symmetry found by comparison.

*Continuity* is the basis of the calculus, of differential equations, and of the smooth manifold structure on which general relativity is built. To assert continuity is to assert that an infinitesimal change in context always produces an infinitesimal change in content: that the world does not jump.

*Discontinuity* is the character of quantization. An energy level is occupied or it is not; a detector registers a count or it does not; a measurement yields one outcome from among its alternatives, and the outcome changes while the arrangement of the measurement stays the same. Each is a difference of content where there is a sameness of context.

*Asymmetry* is the character of irreversibility and of record. A measurement leaves a record that was not there before it, and the record changes as the context of the measurement changes: a different time, a different place, a different reading. The ordering of before and after in which every measurement is made is a difference of content across a difference of context. A generic spacetime, in which no two points need share a geometry, is asymmetric in the same sense.

So the four compound concepts are the concepts physics already uses, defined from the intrinsics of comparison alone. Chapter 3 proves that symmetry and continuity cannot both be perfected, simultaneously and

fundamentally, in one comparison. The result is a statement about what comparison requires. The result holds whatever the limits of our knowledge or of our mathematical tools.

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### Questions

**Why exactly six intrinsics, and not five or seven?**

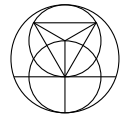
Because each of the six is required by a feature every comparison has, and nothing further is. A comparison yields a verdict, same or different: two intrinsics. The verdict is on something, in some respect that is not the thing judged: two more. The comparison is made, by a comparing, of something compared: two more. Remove any one and its partner loses its meaning, by the inseparability of each pair (Section I), and there is no comparison. Any further concept that a comparison might involve, a measure, a unit or a number, is built from these six and comes after them.

**Does the Incommensuration Theorem depend on there being exactly six intrinsics?**

The enumeration of Chapter 3 uses four of the six: sameness and difference, applied to content and context, with the subjective context held the same (Section II). The count of six enters the argument later: in the three pairs of Chapter 4, in the rows and columns of the grid of the six metrics (Chapter 7, Section III), and in the count of the dimensions of possibility (Appendix C).

**Physics uses "symmetry" and "continuity" of mathematical objects. Are these the same words as this chapter uses?**

They are the same words used of different things. In this book symmetry and continuity are characters of what is found in a comparison. Physics uses the words of mathematical objects: a group of transformations, a smooth function, a manifold. Chapter 3, Section V, sets the two usages side by side, and Appendix A, Section III, relates them.



Chapter 3

# The Incommensuration Theorem

*On the Structural Limit of Comparison*

In this chapter I prove the Incommensuration Theorem by exhaustive enumeration. The four compound concepts of Chapter 2 form ten candidate conjunctions. Three rejection criteria, each stated with its rule, eliminate eight of them. The third criterion, the balance of sameness and difference, is stated in its direct form: inseparability applied in each respect in which a comparison gives a verdict. Two conjunctions survive: continuity with asymmetry, and symmetry with discontinuity. The theorem follows: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. I then set the book's senses of the words beside those of physics, and state once why quantizing the arena would need a forbidden pairing.

*Rests on:* the compound concepts and the inseparability of sameness and difference (Chapter 2, Sections I and II), and measurement as comparison (Chapter 1, Section II). *Refers to:* the two modes of a theory and the invariants of content (Appendix A, Sections III and IV). *Used by:* Chapter 4 (Axiom I), Chapter 5 (Section V), Chapter 6, Chapter 7 (Sections IV and V), Chapter 8, Chapter 9, Chapter 12 (Section VI), Chapters 17, 18 and 19, and Appendix A.

## I. The Ten Candidates

Chapter 2 defined the four compound concepts of the intrinsics of comparison: continuity (sameness of content, sameness of context), discontinuity (difference of content, sameness of context), symmetry (sameness of content, difference of context), and asymmetry (difference of content, difference of context).

These four compound concepts can be applied to one another to form tertiary conjunctions: compound concepts of compound concepts. Since there are four compound concepts, the number of pairwise conjunctions is ten (four self-pairings plus six cross-pairings). The Incommensuration Theorem states the answer to the question: which of these ten conjunctions are structurally coherent?

Each compound concept is a pair of verdicts, one on content and one on context (Chapter 2, Section II). A conjunction of two compound concepts therefore carries two verdicts on content and two verdicts on context.

## II. The Three Rejection Criteria

Three structural constraints eliminate eight of the ten candidates.

*Criterion !x: no self-application.* The application of a concept to itself yields no semantic content. Continuity with continuity, symmetry with symmetry, discontinuity with discontinuity, and asymmetry with asymmetry are all eliminated. This removes four candidates.

*Criterion !y: no opposite pairing.* Some pairs of compound concepts are direct opposites. Continuity asserts sameness of content in the same context; discontinuity asserts difference of content in the same context. Their conjunction asserts that content is both the same and different in the same context: a violation of the distinctness of sameness and difference. Similarly, symmetry with asymmetry asserts that content is both the same and different under change of context. This removes two more candidates.

*Criterion !z: sameness/difference balance.* Sameness and difference are inseparable: wherever the concept of sameness appears, the concept of difference must also appear, and the reverse (Chapter 2, Section I). A comparison gives its verdicts in two respects, content and context, and content and context are themselves inseparable. So inseparability holds in each respect: among the verdicts on content, sameness must appear with difference; among the verdicts on context, sameness must appear with difference. Criterion !z is inseparability in each respect. It uses inseparability exactly as stated, and it involves no counting.

The table applies criterion !z to all ten candidates.

| **Conjunction** | **Verdicts on content** | **Verdicts on context** | **Criterion !z** | **Also removed by** |
|---|---|---|---|---|
| Continuity with asymmetry | same, different | same, different | passes | none: survives |
| Symmetry with discontinuity | same, different | different, same | passes | none: survives |
| Continuity with symmetry | same, same | same, different | fails: no difference of content | none |
| Discontinuity with asymmetry | different, different | same, different | fails: no sameness of content | none |
| Continuity with discontinuity | same, different | same, same | fails: no difference of context | !y |
| Symmetry with asymmetry | same, different | different, different | fails: no sameness of context | !y |
| Continuity with continuity | same, same | same, same | fails in both respects | !x |
| Symmetry with symmetry | same, same | different, different | fails in both respects | !x |
| Discontinuity with discontinuity | different, different | same, same | fails in both respects | !x |
| Asymmetry with asymmetry | different, different | different, different | fails in both respects | !x |

In continuity with symmetry, sameness of content appears twice and difference of content does not appear at all. Continuity with symmetry has sameness without difference in the very respect in which the verdict is given, and inseparability excludes that conjunction directly. Discontinuity with asymmetry has difference of content twice and sameness of content not at all. Criterion !z removes these two, which neither of the other criteria removes.

Criterion !z also fails the six candidates that !x and !y remove. The three criteria are independently grounded, and they agree: every candidate that !x or !y removes fails criterion !z as well. Inseparability alone, applied in each respect, leaves the same two survivors, continuity with asymmetry and symmetry with discontinuity.

Criterion !z can also be stated as a count. Across the four verdicts of a conjunction, a candidate that passes in each respect carries sameness twice and difference twice. Equal number is what the pairing of every sameness with a difference, in each respect, comes to when it is counted. Counting is itself made of sameness and difference, treating items as units of one kind and telling them apart, so the count is a later description of the pairing, and adds nothing to it.

## III. The Two Survivors

Of the ten candidates, eight are eliminated. The two survivors are:

*Continuity with asymmetry*: sameness of content in sameness of context, combined with difference of content in difference of context. This conjunction is structurally coherent: in the verdicts on content, sameness appears with difference, and in the verdicts on context, sameness appears with difference.

*Symmetry with discontinuity*: sameness of content in difference of context, combined with difference of content in sameness of context. This conjunction is likewise coherent: each respect carries both sameness and difference.

These two conjunctions are exhaustive and exclusive: any act of comparison is characterized by one or the other, and by no third.

The two survivors have one form. In each, the second compound is the exact complement of the first: where one has sameness, in content or in context, the other has difference, and the reverse. Every other pairing repeats a verdict in at least one respect, and criterion !z excludes it. So the Incommensuration Theorem can be stated in one line: the only coherent conjunctions are a compound concept joined with its exact complement (Figure 1).

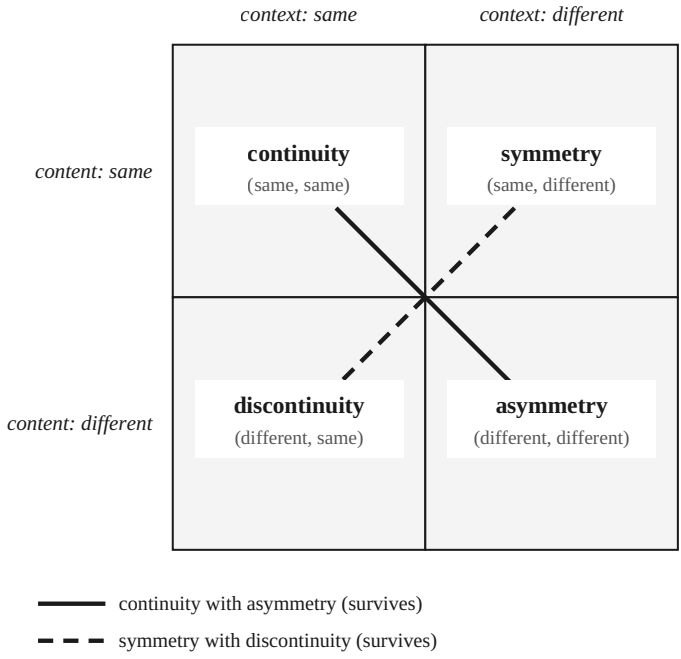



***Figure 1.*** *The four compound concepts as the cells of a square, each written as (verdict on content, verdict on context). Each surviving conjunction joins a cell to its exact complement across a diagonal; every other pairing repeats a verdict in some respect, which criterion !z excludes.*

## IV. The Theorem

The Incommensuration Theorem: *the concepts of symmetry and continuity cannot both be simultaneously and fundamentally applied to any event or comparison. The concepts of asymmetry and discontinuity cannot both be simultaneously and fundamentally applied to any event or comparison.*

Any comparison, perfected at its foundation, must be either continuously asymmetric or discontinuously symmetric.

The theorem is a statement about what comparison requires, and it has no physical premise. The proof is by exhaustive enumeration[3]: all ten candidates are examined, three criteria are applied, and two survive. No step in the proof appeals to any physical content. The theorem therefore applies to any domain in which comparison occurs, and physical measurement is comparison (Chapter 1, Section II). The enumeration of Sections I to III is the proof I give.

## V. The Physical Reading

*The words, in physics and in this book.* In this book symmetry and continuity are characters of what is found in a comparison, defined in Chapter 2: symmetry is a sameness of content across a difference of context, and continuity a sameness of content across a sameness of context. Physics uses the same words of mathematical objects, and a continuous symmetry in physics is a group of transformations whose elements vary with continuous parameters. Appendix A, Section III, relates the two usages. Every physical theory has two modes: the mode of what is found in it, its observables, and the mode of its formalism, and the two are always conjugate (Appendix A, Section III). The Incommensuration Theorem concerns what is found.

In a quantum theory of content the invariants found, the charges and quantum numbers of the content (Appendix A, Section IV), are symmetric and discrete, and the continuous group acts in the continuous formal-

ism; in general relativity the symmetry, diffeomorphism invariance, belongs to the formalism, and the continuity is that of the observed geometry. A continuous symmetry of physics is therefore a symmetry and a continuity in the two conjugate modes, the pairing that the cross-modal principle requires, and Noether's theorem is the bridge between them. The same reading holds of a space. A compact homogeneous space on which a Lie group of isometries acts transitively is what physics calls continuously symmetric. What is found of it in a comparison is symmetric, since every point is equivalent to every other, and discrete, since the harmonics of a compact space form a discrete set; the smooth metric and the continuous parameters of the group belong to the formalism. In this book's sense such a space is discontinuously symmetric, and in Chapter 8 I read the internal factor so. Of the theorem's two forbidden conjunctions, continuity with symmetry is one mode perfected in both characters; the other is asymmetry with discontinuity.

| **Term** | **In this book** | **In physics** | **How they meet** |
|---|---|---|---|
| Symmetry | Sameness of content across a difference of context: what is found does not change when its context changes. | Invariance of an action, an equation or a space under a group of transformations. | The invariants found are a theory's symmetric observables; the group acts in its formalism. |
| Continuity | Sameness of content across a sameness of context: nothing is sudden. | Smoothness of a function, a manifold, or the parameters of a group. | The book's continuity is a character of what is found; smooth parameters belong to a formalism. |
| Continuous symmetry | A symmetry and a continuity in the two conjugate modes of one theory. | A Lie group of symmetries; by Noether's theorem, a conserved quantity for each continuous global symmetry. | The cross-modal pairing of Appendix A, required by it. |
| Continuously symmetric | Both characters perfected in one mode: the forbidden conjunction. | No single term. A quantized arena would need this pairing or asymmetry with discontinuity (the end of this section). | The first of the two conjunctions the theorem forbids; the second is asymmetry with discontinuity. |

| | | | |
|---|---|---|---|
| Asymmetry | Difference of content across a difference of context. | Absence of an isometry; a generic spacetime; a broken symmetry. | The generic geometry of the context factor. |
| Discontinuity | Difference of content across a sameness of context: something is sudden. | Discrete spectra, quantized levels, the outcome of a measurement. | The content at the microscopic limit. |
| Homogeneous | Every point equivalent: the symmetry of a factor. | An isometry group acting transitively. | The internal factor $\mathbb{C}P^2 \times S^2$, the content factor (Chapter 9, Section II). |
| Perfected | Holding exactly, at the foundation of a description. | Exact, as against approximate or a property of a model. | A symmetric model such as de Sitter space is a geometry of idealized content. |

I identify the two surviving conjunctions with the two fundamental theories of physics.

*General relativity* is the theory of gravity: the theory of the geometric arena within which all physical events occur. Its fundamental character is continuous asymmetry. Spacetime is continuous: an infinitesimal change in position produces an infinitesimal change in the gravitational field. And spacetime is asymmetric: the metric tensor $g_{\mu\nu}$ varies from point to point, and no two points need have the same geometric structure. General relativity is the continuously asymmetric theory.

*Quantum mechanics* is the theory of matter and radiation: the theory of the content that occupies the geometric arena. Its fundamental character is discontinuous symmetry. What is found in quantum mechanics is discontinuous: a measurement yields one outcome from among its alternatives, bound energy levels are quantized, and the quanta of a field are counted in whole numbers. And quantum mechanics is governed by symmetry: the gauge symmetries of particle physics determine the interactions, and Noether's theorem links every continuous global symmetry to a conservation law. Quantum mechanics is the discontinuously symmetric theory.

The two survivors also say why the arena is not quantized. I identify discrete, symmetric observables as the character of content, and continuous, asymmetric observables as the character of context. A quantized arena would carry the first in the factor whose observables are the second. The one factor would then carry continuity with symmetry, or asymmetry with discontinuity, and the Incommensuration Theorem excludes both. The exclusion concerns any quantum of the arena itself, at every scale. The small waves of the metric can be described by quanta, and that description is admitted; its quanta are quanta of the description, never quanta of the arena (Chapter 12, Questions).

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### Questions

**Criterion !z carries the proof. What grounds it?**

The inseparability of sameness and difference (Chapter 2, Section I), applied in each of the two respects in which a comparison gives a verdict (Section II). Three grounds support it, each independent of the others. First, read in each respect it needs no further step: continuity with symmetry has sameness of content and no difference of content, which is exactly what inseparability excludes. Second, the count by which the criterion can also be stated (Section II) is the same pairing described in arithmetic, and arithmetic is built from sameness and difference, so it cannot add a commitment beneath them. Third, a conjunction with sameness unpaired by difference in some respect has lost the capacity to differentiate in that respect, and a comparison that cannot differentiate has collapsed into identity; unpaired difference loses the capacity to relate, and the comparison ends the other way. An imbalance therefore removes a comparison, rather than weakening the comparison. The full enumeration, with the table of tertiary conjunctions, is also given in the work cited in the Notes.

**Is the reading of !z chosen to give the answer?**

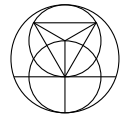
Inseparability holds wherever sameness or difference appears, and in a conjunction they appear in two respects, content and context. The criterion applies it in both. A test applied to the verdicts on context alone would pass continuity with symmetry, whose context verdicts are a sameness and a difference; but that test leaves out the verdicts on content, where continuity with symmetry carries sameness twice and difference not at all. A test of one respect is a partial application of inseparability, and the criterion is its full application, so no choice of reading is made.

**Physics is built on continuous symmetries. Does the Incommensuration Theorem not exclude them?**

The theorem permits them. A continuous symmetry of physics is a group whose elements vary with continuous parameters. In a quantum theory of content the group acts in the continuous formalism, and the invariants of the content it yields, its charges and quantum numbers, are discrete (Section V). The symmetry and the continuity lie in the two conjugate modes of one theory, which is the pairing the theorem permits. The theorem excludes the two characters perfected together in what is found in one theory.

**Does the Incommensuration Theorem derive general relativity and quantum mechanics?**

The theorem constrains the type of theory that can describe context (continuously asymmetric) and content (discontinuously symmetric). It does not give the specific content of either theory, and so it does not derive them. The Einstein equations require the structure developed in Parts II and III of this book; the Schrödinger equation is outside this book.



Chapter 4

# The Modalities and the Three Axioms

*On the Structure Comparison Already Has*

In this chapter I find the three axioms in the intrinsics of comparison and the Incommensuration Theorem. The three pairs of intrinsics are distinct, inseparable and non-interchangeable, and that is Axiom III. Each pair fills one of three roles, and the roles are named the modalities: the omniscient, the immanent and the transcendent. Which pair fills which role depends on which part of comparison is the focus, and this book's focus is the act of measurement. The order in which the pairs engage in any comparison is Axiom II. Comparison is prior to the compound concepts made from it, and the two compounds the Incommensuration Theorem sets against each other, symmetry and continuity, are conjugate: that is Axiom I. The axioms are then stated in their exact form, and the Root Tautology is named a formal triple.

*Rests on:* the three pairs of intrinsics (Chapter 2, Section I), the compound concepts (Chapter 2, Section II), the Incommensuration Theorem (Chapter 3, Section IV), and the Root Tautology (Chapter 1, Section IV). *Used by:* Chapter 5 (the axioms), Chapter 6 (Axiom III), Chapter 7 (the modalities, the intrinsics and the three axioms), Chapter 9 (the modalities), Chapter 10 (Axioms I and II), Chapter 11, Section I (the omniscient modality), Chapter 13, Section II (Axiom III), Chapter 18 (Axiom III), and Appendix C (Axiom III).

## I. The Three Pairs and Axiom III

The six intrinsics of comparison fall into three pairs: sameness and difference; content and context; the subjective and the objective (Chapter 2, Section I).

The three pairs are distinct. Sameness and difference concern what a comparison finds: what is shared and what diverges. Content and context concern what the finding is about, and the respect in which it is made. The subjective and the objective concern the stance: the one from which the comparison is made, and the one about which. The three pairs are three different aspects of one act.

The three pairs are inseparable. There is no verdict without something it is a verdict on and a respect in which it is given; there is no content and context without a verdict that distinguishes them; and there is neither without a comparing and a compared. Every comparison involves all three pairs at once.

The three pairs are non-interchangeable. A verdict is not the respect it is given in, and neither is the stance it is given from. Putting one pair in the role of another gives a different and wrong description of comparison.

So the three pairs are distinct, inseparable and non-interchangeable. That form of the three pairs is the form that Axiom III states (Section V), found in the structure of comparison itself.

## II. The Three Modalities

The three pairs play definite roles with respect to one another, and the same pattern of roles recurs at the foundation of every domain in which comparison occurs, which is every domain. A terminology is needed to refer to these roles without specifying what particular domain is under discussion. The three roles are called *modalities*, and they are named *the omniscient*, *the immanent*, and *the transcendent*.

The omniscient modality is the role of that which is known, that which is structural, that which is pattern: what can be laid out and held in view as a whole. The immanent modality is the role of that which mediates, that which is processual, that which is happening: the act itself. The transcendent modality is the role of that which is potential, that which exceeds any particular instance: the stance and the possibility from which an act proceeds.

On principle, each of the three pairs of intrinsics can be associated with a modality, and because the pairs are bound together by comparison, they follow the pattern of Axiom III and the pattern of Axiom II together. Which pair fills which role depends on which part of comparison is taken as the central focus: the focus determines what is immanent in that context, and so determines the binding needed for that context.[4] Every binding so made follows Axioms II and III, and the direction of the cycle of Axiom II is the same in every one.

This book's focus is the act of measurement: comparison as it is performed, with an instrument, here and now. In that focus the act is the finding of sameness and difference, and the binding that focus determines is: sameness and difference as immanent, the act itself; content and context as omniscient, the finding laid out as structure; the subjective and the objective as transcendent, the stance from which the measurement is made and the referent it is about.

In physics I identify the omniscient modality with conservation laws, with structure, and with the content of measurement; the immanent modality with interaction, with force, and with the process of measurement; and the transcendent modality with possibility, with the space of configurations, and with the boundary conditions within which events occur. The six metrics of measurement in Chapter 7 have the same binding: pattern in space as omniscient, force in time as immanent, probability in possibility as transcendent.

## III. Axiom II: The Order of Engagement

In any act of comparison the three pairs engage in a definite order.

As a class, the stance precedes any instance of a verdict: there is a comparing and something to be compared, and every finding is made from that stance. That stance is the transcendent pair, present as a class for the act.

From that stance a finding is made: something is the same, something is different. That finding, of sameness and difference, is the immanent pair, engaged as an instance: the act of comparing, done before the act is reflected on.

The finding, once made, is laid out as structure: what was compared, and in what respect. That structure is the omniscient pair, recognized as an instance of what the act has found. And what is laid out reshapes the stance from which the next comparison is made: the omniscient, as a class, precedes a new instance of the transcendent.

So a class of the transcendent precedes an instance of the immanent, a class of the immanent precedes an instance of the omniscient, and a class of the omniscient precedes an instance of the transcendent. That order of precedence is Axiom II. The order is one of precedence, not of clock time: each moment presupposes the one before it and is presupposed by the one after. The cycle can be entered at any point; its direction cannot be reversed.

## IV. Axiom I: Comparison and Its Compounds

The four compound concepts are made from comparison: each is sameness or difference applied to content and context (Chapter 2, Section II). Comparison is not one of them. Comparison is the act from which all four compound concepts are made, and so comparison is prior to each compound concept. In this book's binding the act is the immanent: the finding of sameness and difference.

Two of the compounds have the characters of the other two modalities. Symmetry, sameness of content across a difference of context, is what can be repeated, shared and laid out: the omniscient character. Continuity, sameness of content within a sameness of context, is what is unbroken and encompassing: the transcendent character. The Incommensuration Theorem states that symmetry and continuity cannot both be perfected in one comparison (Chapter 3, Section IV). Each can be perfected only at the expense of the other, and neither is prior to the other. That relation between symmetry and continuity is the conjugate relationship that Axiom I states.

So the immanent is more fundamental than the omniscient and/or the transcendent, and the omniscient and the transcendent are conjugate. That priority of the immanent, together with the conjugacy of the omniscient and the transcendent, is Axiom I.

## V. The Three Axioms Stated

I state the three axioms below in their exact form.[5]

**Axiom I.** The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another.

**Axiom II.** A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent.

**Axiom III.** The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable.

These three statements are three facets of a single structural fact, not independent propositions.[6] They have been found here as the structure that comparison already has: Axiom III in the three pairs, Axiom II in the order of their engagement, Axiom I in the priority of the act over its compounds, with the conjugacy of symmetry and continuity that the Incom-

mensuration Theorem establishes. They are assertions about the structure of comparison itself: about what must be true whenever anything is compared with anything else. They are not assertions about any particular physical system. Physical measurement is comparison (Chapter 1, Section II), so whatever the axioms require of every comparison they require of every measurement. The passage from the structure of comparison to a physical quantity is made by identifications, and each is stated in the first person where it is made.

## VI. The Root Tautology as a Formal Triple

The Root Tautology identifies comparison, interaction and relation as one act, approached three ways (Chapter 1, Section IV). By Axiom III that recognition has a precise form.

The three names are distinct: comparison names the act as a finding, interaction as an event, relation as a standing connection. They are inseparable: by the Root Tautology, each is present wherever either of the others is. And they are non-interchangeable: a finding is not an event, and neither is a standing connection. So comparison, interaction and relation are a formal triple, and they fill the three roles of the modalities.

In the domain that holds the Root Tautology, the focus is the one act, known by its three names, and the binding is: interaction as immanent, the act as it occurs; relation as omniscient, the connection held in view as structure; comparison as transcendent, what is implicit in every interaction and every relation, the finding that each of them already carries. That focus differs from the focus of measurement in Section II, where the finding of sameness and difference is itself the act, and so the binding differs. The direction of the cycle of Axiom II is the same in both; what changes with the focus is which term stands in which role.

## Questions

**Are the axioms proved, or assumed?**

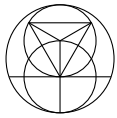
They are found in the structure of comparison. Axiom III is the distinctness, inseparability and non-interchangeability of the three pairs of intrinsics (Section I); Axiom II is the order in which the pairs engage in any act of comparison (Section III); Axiom I is the priority of the act of comparison over its compounds, together with the conjugacy of symmetry and continuity that the Incommensuration Theorem establishes (Section IV). The one premise is the Root Tautology, and any attempt to deny it is itself a comparison (Chapter 1, Section IV).

**Different works assign the pairs to the modalities differently. Is that not an inconsistency?**

The assignments are consistent, because which pair is immanent depends on which part of comparison is the focus (Section II). A description centered on the act of measuring, as this book is, binds sameness and difference to the immanent; a description centered on the one act known by three names binds interaction to the immanent (Section VI). Both bindings are valid in their contexts, and the direction of Axiom II is the same in each. The binding is part of the description of a context, and a work that changes its focus changes its binding with it.

**The modalities are named from the roles the pairs play, and the axioms are then stated in terms of the modalities. Is that circular?**

The pairs, their roles and their pattern are present together, and that is simultaneity. A circle needs an order: something assumed first, and later derived from what it was used to derive. The pairs of intrinsics, their roles and the pattern of their relations are present together in every comparison, with no one of them prior to the others. Naming the roles and stating their pattern makes explicit what is already there, all at once, in each act. Chapter 5, Section IV, states this simultaneity in general.



Chapter 5

# Reason, the Real and the Abstract

*On the Reasoning This Book Uses*

This chapter states what kind of reasoning the book uses, and why that reasoning is itself within the subject of the book. Comparison works at three levels at once: it builds the structure, it is the act performed on the structure, and it is what an argument is made of. Reasoning about the real is set beside reasoning about the abstract, what exists beside what is real, and validity beside soundness. Causation is set beside determinism. The intrinsics of comparison occur together, with no one of them leading, and presupposing them is simultaneity, not circularity. Last, I set out the three kinds of reasoning (mathematics, measurement and the structure of comparison) that establish necessity, correspondence, sufficiency and comprehensiveness, and what each kind can reach.

*Rests on:* comparison and the Root Tautology (Chapter 1), the intrinsics (Chapter 2), the Incommensuration Theorem (Chapter 3), and the axioms (Chapter 4).
*Used by:* Chapter 1, Sections III and V (the routes and the kinds of claim).

## I. Reasoning Is Comparison

Comparison enters an argument at three levels, and at all three at once.

Comparison builds the structure an argument is about. The intrinsics, the compound concepts and the conjunctions of Chapters 2 and 3 are all made by applying sameness and difference to content and context.

Comparison is the act performed on that structure. Each of the ten candidate conjunctions of Chapter 3 was examined by setting its verdicts side by side and finding, in each respect, whether sameness appeared with difference.

And comparison is what the argument itself is made of. Each step of a proof is a verdict: this follows, that does not; this case is the same as that one, in this respect. A proof is a sequence of comparisons, each of which could not come out otherwise under the conditions it states.

So reasoning is comparison, and the reasoning of this book is within its own subject. Its arguments are made of the act they describe. The arguments of this book apply to their subject at all because the arguments are made of the act that the arguments describe. That the arguments are made of comparison is not a defect to be guarded against. A description of comparison that used anything other than comparison would need a way of reasoning that was not a way of comparing, and there is none.

## II. The Real and the Abstract

Two kinds of reasoning are distinct. General reasoning is about the real: the world as it is met in actual acts of comparison, in measurement and in perception. Mathematical reasoning is about the abstract: structures whose terms are fixed by definition and whose domain is closed by assumption.

Two claims are distinct in the same way. To say that something *exists* is to say that it stands as structure: timeless, the same in every instance, like the pattern of a law or the state of a system at an instant. To say that something is *real* is to say that it is given in an actual act: temporal, occurring now, with the future still open to possibilities not yet settled. A law exists; a measurement is real.

The standards of the two kinds of reasoning are distinct as well. Mathematical reasoning is held to *validity*: a conclusion is valid when it follows from its premises by the rules of the structure. Reasoning about the real is

held to *soundness*: an argument is sound when its premises hold in the world as well as its form being valid. Within its own structure a deduction is certain. Its passage to the world is never guaranteed, because the structure and the world are two domains, and the passage between them always carries some doubt. That doubt is why I state each passage from the structure of comparison to a physical quantity as an identification in the first person. That doubt is also why I check the results of those passages against measurement.

Every deduction also fixes its domain from outside: its terms do not change, they have no further internal aspects, and the domain is closed. In mathematics that is harmless, because the definitions make the domain. In physics it is a property of the model and not of the world. The physical domain is the whole of what comparison can reach from within it, and every measurement of it is made from inside, by an observer who is part of it. In Chapter 13 I take that fact as the ground of my account of the cosmological constant.

## III. Causation and Determinism

Causation and determinism are distinct concepts, and they cannot be used interchangeably.

*Causation* is a persistent perceived relation of association among three events: the observing self, the observed cause, and the observation of the effect (Chapter 1, Section III). It requires an observer. It is asymmetric in time, with respect to that observer. It does not require a complete specification of every event.

*Determinism* is the assertion that there can be a complete and absolute specification of every value of every measured content for every value of every context. It requires no observer. It requires no direction of time: a complete specification can be read forward or backward with equal ease.

It does require the complete specification at every scale: determinism, if it could hold, would have to be made good all the way to the scale of the absolute microscopic, the scale of exactly zero.

Science is founded on observation, and observation can establish a persistent relation in what is observed. That persistent relation is causation. No observation delivers a complete specification of everything, so determinism is never observed. The laws of science are therefore causal laws. A causal law may be written in deterministic form, as an equation that assumes perfect information, and the knowledge the equation records is causal knowledge all the same.

The two concepts belong to the two claims of Section II. A deterministic specification exists: it is timeless structure. Causation is real: it is found in the act of observing, it spans the observer's interval from cause to effect, and across that interval the future is open. The equations of physics can be solved backward as readily as forward because they describe what exists, with time as one more coordinate. The direction of causation belongs to the real, to the order of actual acts, and the lack of direction in an equation says nothing against it. The later chapters use this distinction where a finding is made in the first person, by the comparer that finds (Chapter 12, Questions), and where the direction of time is read from the geometry (Appendix B, Section IV).

## IV. Simultaneity

Every comparison involves the six intrinsics together. Every comparison is also an act, and an act has a before and an after, so the ordering of before and after is involved in every comparison as well. None of these is involved first. No comparison first has sameness and then acquires a context, or first occurs and then acquires a verdict. Each is present wherever any of the others is present, in the same act, at once. I call this co-occurrence, with no lead of one over another, *simultaneity*.

A *circular* argument is one in which a claim is used to establish itself: the conclusion appears among the premises, so that it is assumed first and derived afterward. A circle has an order, running from an assumption back to the same assumption. Simultaneity has no such order. Where nothing comes first, nothing can return to it. The precedence of Axiom II is of a class over an instance; within any one act, the pairs co-occur.

So an account that presupposes sameness and the ordering of before and after presupposes what every comparison involves, all at once: that is simultaneity, and not a circle. Every account presupposes them, because every account is a comparison, including any account that objects to another on these grounds. The intrinsics are found together, with nothing prior to them from which they are derived, and nothing derived from them by an order that returns to its start. So the structure of Part I makes explicit what was present together from the beginning.

## V. Three Kinds of Reasoning

Three claims can be made about any description of a domain: that what the description contains is *necessary*, that the description is *sufficient* (nothing further is required), and that the description is *comprehensive* (nothing has been left out). A fourth claim, *correspondence*, is that a result holds in the world. They are distinct claims, and each needs its own argument. To show that comparison must involve six intrinsics is to show that none can be removed; that the six are enough is a second claim, argued by closure, below.

Mathematics argues validity and necessity, within a structure fixed by definition, and there it is complete master. It cannot argue soundness, for the reason given in Section II. Nor does it certify its own comprehensiveness: the completeness of a formal structure is completeness relative to that structure.

Measurement argues correspondence: what holds in the world, case by case, each measurement one comparison made here, now, with an instrument. It is the only reasoning that reaches the world directly. It cannot argue sufficiency. A claim of sufficiency is a claim about every case in a domain, and measurement meets cases one at a time. Nor can it argue the absence of sufficiency: when a prediction fails, the failure shows that something in the whole chain did not hold (premises, derivation, instrument or application), and it does not single out which.

The structure of comparison argues sufficiency and comprehensiveness. Its premises are what every act of reasoning and measuring is made of, so they hold in the act in which they are examined. Its reach is everything that can be compared, which is everything that can be measured, proved or described, and that is sufficiency. And the description of comparison is itself a comparison, needing no concept beyond those it describes, so every further act of description is already an instance of what has been described. So the description of comparison is comprehensive by closure, with the completeness of a fixed point, reached without listing every case.

The Incommensuration Theorem shows why closure is the route. Necessity is invariance across every case: a symmetry. Completeness by listing is coverage without a gap: a continuity. The two cannot both be perfected (Chapter 3, Section IV). Comprehensiveness by closure does not need them together.

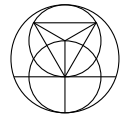
So I place each claim on the reasoning that can carry that claim. I place the claims of necessity on the structure of comparison and on mathematics, the claims of correspondence on measurement, and the claims of sufficiency and comprehensiveness on the structure of comparison.

## Questions

**If the book's reasoning is inside its own subject, can it be tested at all?**
Yes. Its consequences in physics are stated as predictions and compared with measurement, and the observations that bear on each are named where the prediction is made and gathered in Chapter 20. What the book's reasoning cannot be is reasoning from outside comparison, and no reasoning can.

**Why does a book of physics need the distinction between causation and determinism?**
Because its later chapters depend on it. A finding is made in the first person, by the comparer that finds, and it selects among alternatives that already carry their geometries (Chapter 12, Questions): that is causation, with an observer and a direction. The equations of the later chapters are written in the deterministic form physics uses, and the knowledge they record is causal. Reading the form of the equations as a claim of determinism would ask of them what no observation can supply.

**Who can judge whether the foundations are sufficient?**
Sufficiency is judged with the reasoning that can reach it, which is the structure of comparison (Section V). Neither a proof nor an experiment can settle it in either direction, for anyone, the author included. A judgment of sufficiency made by experiment or by proof alone is a fact about the method used, not a finding about the foundations.



Chapter 6

# The Content/Context Partition

*On Why Gravity Is Not a Force*

In this chapter I divide physical phenomena into content and context, by the two conjunctions that survive the Incommensuration Theorem. Content is discontinuously symmetric at the microscopic limit; context is continuously asymmetric. I state four markers by which gravity differs from the gauge forces (universality, geometry, non-quantizability and spin 2), and identify gravity as the context-modal phenomenon. In Appendix A I derive the first three markers (universality, geometry and non-quantizability) from the axioms. In this chapter I argue for the fourth marker, spin 2. General relativity and quantum mechanics are then conjugates, composed as a product, which I call per-alternative geometry: quantum fields for the matter, and one classical geometry for each alternative of the matter, with the relative phases of the alternatives kept.

*Rests on:* the intrinsics content and context (Chapter 2, Section I), the two survivors and the statement on quantizing the arena (Chapter 3, Sections III and V), the conjugate relation of Axiom I (Chapter 4, Section IV), and Appendix A, Sections II to V. *Used by:* Chapter 7, Section V; Chapter 8; Chapter 12 (Sections II and VI); Chapter 15; Chapters 17 and 19; and Chapter 20.

## I. Content and Context in Measurement

Every act of physical measurement has two aspects: the content being measured and the context in which the measurement takes place. The reading on a thermometer is content; the room in which the thermometer

sits is context. The position of a particle is content; the coordinate system in which the position is expressed is context. The energy of a photon is content; the spacetime through which the photon travels is context.

The distinction applies two of the six intrinsics of comparison, content and context (Chapter 2, Section I), to the physical domain. It is an application of the intrinsics, not an analogy.

## II. The Four Structural Markers

Gravity differs from the three gauge forces in four structural ways. I state each marker as a physical fact and give the reading by which the marker belongs to context. The derivations are in Appendix A, which derives the markers of a context-modal phenomenon from the axioms before gravity is mentioned: universality and the geometric marker in its Section II, and non-quantizability in its Sections III to V. The fourth marker, spin 2, is argued in this section.

*Marker 1: Universality.* Gravity couples to all forms of energy and momentum without exception. The gauge forces couple selectively: the strong force to color charge, the electromagnetic force to electric charge, the weak force to weak isospin. Content is selective: each content phenomenon picks out a specific attribute to measure. Context is universal: the context in which measurement occurs is the same for all measurements.

*Marker 2: Geometry.* Gravity is the curvature of spacetime itself. It is the arena, not a field *within* the arena. The gauge forces are fields defined on a geometric background. Content operates within the context; context is the structure within which all content is situated, and it is not within anything. Appendix A grounds this marker with non-shieldability: a region is itself a structure of context.

*Marker 3: Non-quantizability.* Content is governed by symmetry, and so is discontinuously symmetric at the microscopic limit: quantized. Context is governed by continuity, and so is continuously asymmetric: not quant-

ized. The gauge forces are all quantized: they have particle mediators (gluons, photons, W and Z bosons). Gravity has shown no quantum of its own, and no quantum theory of the arena itself has been tested. The word *graviton*, where this book uses it, names at most a quantum of a description: the description of the small waves of a classical geometry. It never names a quantum of the arena; a graviton in that sense is not a particle, not a carrier of gravitation and not one of the kinds of particle. A quantized arena would need one of the two pairings that the Incommensuration Theorem forbids (Chapter 3, Section V). Not quantized means that the context has no least step and no quantum of its own at any scale (Chapter 12, Questions, states the two senses in full). Its alternatives are those of the content it holds: each alternative of the content carries its own geometry (Chapter 12, Section VI).

*Marker 4: Spin 2.* The gravitational field is the metric, a rank-2 tensor. The gauge forces are connections, rank-1 tensors, and their quanta, the gauge bosons, have spin 1. The rank belongs to what is found: a measured interval is the metric applied twice to one displacement, $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$, while the measured phase of a charge carried along a path takes the connection once, $\oint A_\mu\, dx^\mu$. I identify the rank-2 metric with the context, in which every interval is found, and the rank-1 connection with the mediation of interactions of content. A quantization of the metric would carry spin 2, the only spin of a massless particle that couples universally to the stress-energy tensor (Weinberg 1964). I hold that the metric itself is not quantized (Marker 3). Small waves of the metric can be described by quanta; those quanta are quanta of the description of the wave, not particles of the arena (Chapter 12, Questions). The nearest sense in which such a quantum can be made intelligible is the phonon, a quantum of the description of the vibrations of a held structure such as a crystal. The arena is not a held structure, so even that sense does not make a quantum of the metric's waves a quantum of the arena.

Gravity has all four markers. It also has the two further markers that Appendix A, Section II, derives: the equality of gravitational and inertial mass, and the absence of a discrete charge. I identify gravity as the context-modal phenomenon by that match.

## III. GR and QM as Conjugates

The content/context partition makes general relativity and quantum mechanics conjugates, to be composed rather than unified.

Two descriptions are *conjugate* in the sense of this book when neither can be perfected without loss to the other and neither is prior to the other: the relation Axiom I states between the omniscient and the transcendent (Chapter 4, Section IV). GR and QM are conjugate: the continuously asymmetric and the discontinuously symmetric cannot be perfected together (Chapter 3, Section IV), and neither theory is prior to the other. They also satisfy the conditions of Axiom III: they are distinct (they describe different structural aspects of physics), they are inseparable (neither alone describes the full physical domain), and they are non-interchangeable (the equations of one cannot do the work of the other).

The relationship between them is therefore a product, not a unification, and I call it per-alternative geometry: the matter is described by quantum fields, and each alternative of the matter carries its own classical geometry, shaped by that alternative's stress-energy through the Einstein equations, with the relative phases of the alternatives kept. Where the matter is sharply one alternative, as for a star, a planet or a black hole of definite mass, that geometry is the one sourced by the expectation value, $G_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle$, and every computation of this book is made in that case. Per-alternative geometry is not the Møller-Rosenfeld theory (Møller 1962; Rosenfeld 1963), which keeps one geometry for all the alternatives, sourced by their expectation value, and predicts no entanglement through

gravitation (Chapter 12, Section VI). The framework contains no quantum theory of the geometry; a geometry for each alternative is not an approximation to one (Chapter 12, Section VI).

Appendix A contains the argument in full: the derivation from the axioms of the markers of a context-modal phenomenon, the cross-modal principle, the mode assignment theorem, and per-alternative geometry as the configuration that remains. To quantize the arena is to give to context the mode that belongs to content. Giving to context the mode that belongs to content is a category error, in the philosophical sense: a concept applied to a domain where the concept does not belong. Why a quantized arena would need a forbidden pairing is stated in Chapter 3, Section V.

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### Questions

**Are the markers derived, or are they observed facts given a reading afterward?**

The physical facts are observed. What is derived is that any context-modal phenomenon must have the markers. The axioms require a context-modal phenomenon to exist, since content and context are inseparable. Appendix A, Section II, derives for any such phenomenon, before gravity is mentioned, universality of coupling, non-shieldability, the equality of gravitational and inertial mass, and the absence of a discrete charge; its Sections III to V derive non-quantizability. Gravity has every one of them, and that single structural match is the empirical input by which I identify gravity as the context-modal phenomenon in Section II.

**What does the partition predict?**

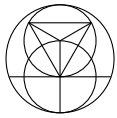
Two masses, each held in two places and related only through gravitation, become entangled, with the phase of their Newtonian interaction (Chapter 12, Questions). The prediction separates this account from the programs that keep one geometry for all alternatives, each of which predicts no entanglement, and from collapse driven by gravity. Below the

Planck scale it does not separate this account from linearized quantum gravity: the two differ in what is superposed, and in where the description by quanta ends, at the modal boundary (Chapter 17). A detector that registers a single quantum of a gravitational wave registers a quantum of the wave's description, which the account admits, and such a quantum is not a quantum of the arena (Chapter 12, Questions).

# PART II

## The Product Geometry

*Why content and gravity inhabit separate geometric factors*



Chapter 7

# The Six Metrics

*Content and Context in Physical Measurement*

This chapter states the six metrics of physical measurement: the content triad (pattern, force and probability) and the context triad (space, time and possibility). I assign each metric a modal character, and show that the six form a grid: two rows, content and context, and three columns, the omniscient, the immanent and the transcendent. The grid is a product of the two rows and the three columns. The content metrics are governed by symmetry and so are quantized at the microscopic limit; the context metrics are governed by continuity, and the context-modal phenomenon is gravity.

*Rests on:* the modalities and the axioms (Chapter 4, Sections II to V), the intrinsics and the compound concepts (Chapter 2, Sections I and II), the Incommensuration Theorem (Chapter 3), and the partition (Chapter 6). *Refers to:* the stance of comparison (Appendix C, Sections I and II), the one relation that pattern, inertia and weight find (Appendix A, Section II), the content at the microscopic limit (Appendix A, Section IV), and the geometry of an idealized content (Chapter 10, Section III). *Used by:* Chapter 8; Chapter 9 (the internal geometry carries the grid); Chapter 10 (time as the immanent context); Chapter 12, Section V; Chapter 13, Section II (what a measurement registers); Chapter 18; and Appendix C. *Referred to by:* Appendix A, Section IV.

## I. The Content Triad and the Context Triad

The previous chapters established the six intrinsics of comparison and the Incommensuration Theorem that governs their conjunctions. I ask what the six intrinsics of comparison become when the structure of comparison is instantiated in the domain of physical measurement.

Any act of physical measurement, any interaction, any observation, any comparison between physical states, involves six metrics, grouped into two triads. The *content triad* consists of pattern (inertia, mass, structure), force (the directed flow of change), and probability (the specification of values among alternatives). The *context triad* consists of space (the arena in which structure is laid out and known), time (the bare ordering of before and after), and possibility (the field of what could occur).

Pattern, inertia and mass share one metric. Pattern, inertia and weight are three findings of one relation of a content to its context, the relation of being found there: what there is of the content to be found, its resistance to a change of its place in the context, and the hold the context takes on it. Mass is the measure of that one relation (Appendix A, Section II).

The two triads are the physical form of the content/context pair of the intrinsics. What is measured is content; the arena in which it is measured is context. Each content metric is measured in its own context: pattern in space, force in time, probability in possibility.

## II. The Modal Characters

I assign each metric a modal character, by the roles that the modalities play (Chapter 4, Section II). The metrics of pattern in space have the nature of the omniscient modality: the structural, the persistent, the knowable. The metrics of force in time have the nature of the immanent modality: the relational, the processual, the actual happening. The metrics of probability in possibility have the nature of the transcendent modality: the potential, the creative, the transformative.

The reasons are the roles the axioms define (Chapter 4, Section II). Time is the immanent context: the bare ordering of before and after in which any interaction occurs. Space is the omniscient context: the arena in which structure is laid out and known. Possibility is the transcendent context: the field of what could occur.

## III. The Grid

The six metrics form a grid of two rows and three columns.

| | **Omniscient** | **Immanent** | **Transcendent** |
|---|---|---|---|
| **Content** | pattern | force | probability |
| **Context** | space | time | possibility |

The shape of the grid is the shape of comparison. Comparison involves three pairs of intrinsics (Chapter 2, Section I), and the pairs are of three kinds: sameness and difference are its outcomes, content and context its respects, and the subjective and the objective its positions (Appendix C, Section II). Each pair stands to the metrics in its own way.

The positions act through the columns: pattern, space, force and time can be followed apart from the stance, and probability and possibility stay bound to it (Appendix C, Section I), so within each column a content and its context stand alike to the stance. The outcomes belong to every metric, and they enter as the laws of the rows, since symmetry and continuity are each a sameness or a difference of content and of context (Chapter 2, Section II), and content is governed by symmetry and context by continuity (Sections IV and V). The respects alone separate, within each column, what is measured from the arena in which it is measured (Section I). So the rows are content and context, and they are two because a pair has two members.

Each row is a class, and a class that subsumes the essence of a domain has exactly three members, one in each modal role (Chapter 4, Section II; Axiom III). These roles are the columns. Axiom I picks out the immanent pair, force and time, both of which are abstractions of change; Axiom II separates the omniscient pair, pattern and space, from the transcendent pair, probability and possibility (Section II).

Each content is measured in its own context (Section I), since content and context are inseparable; so each column holds one content and one context. Each row holds one metric in each role, and no cell holds two, since a second metric in one role would be a fourth member of a row. The grid is therefore a product: the set of six metrics is the set of pairs formed from the two rows and the three columns.

The two indexings are distinct, since the rows come from a pair of intrinsics and the columns from the modal roles; they are inseparable, since every metric carries both; and they are non-interchangeable, since a pair cannot do the work of a triple. They stand in the relation of Axiom III. A row is not a modality: a concept has its modality as its role within a triple. In the triple of Chapter 4, Section II, the content of measurement, the process of measurement and the context, content is omniscient and context transcendent; within each row of the grid, every metric takes its own role.

Chapter 9 shows that the internal geometry of physics carries exactly this product, one factor for the three modalities and one for the two rows. Chapter 10 uses the column of the immanent to identify time.

## IV. Content Is Quantized

The content metrics are governed by symmetry, by the law of symmetry for all that is of content. The Incommensuration Theorem establishes that whatever is defined by symmetry is also discontinuous: symmetry is paired with discontinuity. The content of any physical measurement is therefore quantized at the microscopic limit of the domain.

Physicists find that the content of what they measure is quantized. The forces and the matter fields of particle physics are quantized field theories: their states are discrete, their interactions are governed by symmetry groups, and each interaction proceeds by the exchange of discrete quanta. All of them are content phenomena. I make no claim about which particular forces and groups occur, or about how they correspond to the modalities; I use only their common status.

## V. Context Is the Arena

The context metrics are governed by continuity, by the law of continuity for all that is of context. The Incommensuration Theorem pairs continuity with asymmetry. The context of any physical measurement is therefore continuously asymmetric: smooth, and without the invariance that would make every place and direction the same.

The content phenomena exhaust the discontinuously symmetric conjunction. Whatever occupies the other surviving conjunction must be structurally different from any of them: governed by continuity, not by symmetry; the arena itself, not something operating within an arena. That phenomenon is gravity. Chapter 6 has shown that its structural character follows from its identification as the context-modal phenomenon.

The act of comparison is the reason the arena is continuously asymmetric. To compare is to hold two things in one act, and connection without break is continuity: the act is continuous. The content is held to symmetry. It comes after the act that finds it, laid out as structure (Axiom II, Chapter 4, Section III), the same for every comparer and consistent across the record, so that the findings of many comparers make one history. Both demands are held in full: an act that did not connect would find nothing, and a content that was not consistent would be no single history. The Incommensuration Theorem makes symmetry and continuity conjugate, as Axiom I states (Chapter 4, Section IV), so no one term can perfect both, and Axiom III keeps the content, the context and the act distinct and

inseparable. The asymmetry that the continuous act and the symmetric content cannot carry is therefore carried by the context: it is continuous, as the act is, and it gives up the symmetry that the content keeps. The arena is where the consistency of the content is paid for in asymmetry, and it is uniform only in the geometry of an idealized content, with no content or a perfectly uniform one (Chapter 10, Section III).

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### Questions

**Why these six metrics, and not others, such as charge or temperature?**
Each metric is a kind of measure, not a particular quantity (Appendix C, Section III); charge and temperature are quantities, measured in these kinds. The count is fixed by the grid. Each row is a class, and a class that subsumes the essence of a domain has exactly three members, one in each modal role (Chapter 4, Section II; Section III). A further metric would be a fourth member of a row, or a second metric in one role.

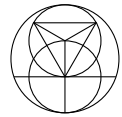
**Is the assignment of the metrics to the columns not arbitrary?**
It is an identification, and I give its reasons in Section II. Axiom I picks out the immanent pair, force and time, both of which are abstractions of change. Axiom II separates the omniscient pair, pattern and space, from the transcendent pair, probability and possibility. The omniscient pair is the structural and the knowable; the transcendent pair is what could be found, and it stays bound to the stance of the comparing and the compared (Appendix C, Section I).

**Classical fields, such as the electromagnetic field of classical electrodynamics, are content and are continuous. Is content then quantized?**

At the microscopic limit, yes. A classical theory of content describes content at the macroscopic limit, where its discreteness is not resolved (Appendix A, Section IV). Section IV concerns the fundamental description of content, and there the forces and matter fields of physics are quantized field theories.

**Minkowski space and de Sitter space are maximally symmetric. Is the context then not continuously asymmetric?**

The law of Section V concerns the context as found, and a generic spacetime has no isometry. Minkowski space and de Sitter space are exact models, and their symmetry holds only where the content is idealized: empty, or of uniform density. A symmetric model such as de Sitter space is a geometry of idealized content (Chapter 3, Section V, the table).



Chapter 8

# The Product Structure $M^4 \times \mathbb{CP}^2 \times S^2$

*On the Geometric Expression of the Incommensuration Theorem*

In this chapter I place the two surviving conjunctions of the Incommensuration Theorem in the two factors of one product geometry. The continuously asymmetric factor $M^4$ is the four-dimensional Lorentzian manifold on which general relativity is formulated. The discontinuously symmetric factor is a compact internal space that is the content factor relative to the exterior (Appendix C, Section VIII); in Chapter 9 I identify it as $\mathbb{CP}^2 \times S^2$. A merger of the two factors into one geometry would need a forbidden pairing. The product is a Kaluza-Klein geometry of ten dimensions, and its reduction gives the four-dimensional field equations (Part III).

*Rests on:* the Incommensuration Theorem, its two survivors and its physical reading (Chapter 3, Sections III and V), the content/context partition (Chapter 6), the six metrics with the quantized content and the arena (Chapter 7, Sections IV and V), and the cross-modal principle and the formalism of the context (Appendix A, Sections III and VI). *Used by:* Chapters 9 to 12, Chapter 18, Section VI, and Appendix B. *Referred to by:* Appendix C.

## I. Why a Product

The Incommensuration Theorem proves that symmetry and continuity cannot both be perfected at once. Two conjunctions survive its rejection criteria: the continuously asymmetric and the discontinuously symmetric (Chapter 3, Section III). A single geometric space that is continuous in all directions and symmetric under all transformations would carry the forbidden conjunction of continuity with symmetry.

Physics needs both surviving conjunctions. The content (matter and the gauge forces) is governed by symmetry: gauge invariance, quantization, discrete spectra. The context (gravity) is governed by continuity: smooth manifolds, differentiable metrics, the calculus of curvature.

I identify the two surviving conjunctions with the two factors of a product geometry, one factor for each. The total space is the Cartesian product of the two factors. Points in the total space have coordinates in both factors, and the factors do not interpenetrate. The continuous geometry of spacetime and the symmetric geometry of the content coexist without conflict because they occupy separate factors of the product.

## II. The Two Factors

The continuously asymmetric factor is a four-dimensional manifold $M^4$ with Lorentzian signature (Chapter 10). It is continuous: the metric varies smoothly from point to point. It is asymmetric: the curvature breaks any global symmetry, and no two points need be equivalent. The continuously asymmetric factor is the manifold on which general relativity is formulated.

The discontinuously symmetric factor is the compact internal space $\mathbb{C}P^2 \times S^2$. Physics calls the same space continuously symmetric, since its isometries form a Lie group; in this book's sense, which concerns what a comparison finds, it is discontinuously symmetric, and the first of the Questions at the end of this chapter relates the two usages. It is symmetric: every point is equivalent to every other point (homogeneity; Chapter 9, Section II). It is the content factor relative to the exterior (Appendix C, Section VIII); which particles there are, and the groups that govern their interactions, are outside this book (On scope). Its discontinuity is that of the spectra a comparison with it can find: at the microscopic limit, the labels read on this space as a space of states, the representations of its compact isometry group and the counts of spinors on it, form discrete sets (Section IV). I make no claim about which spectra those discrete sets are.

## III. The Incommensuration Theorem Forbids a Merger

The product keeps continuous asymmetry and discontinuous symmetry in separate factors, and the Incommensuration Theorem excludes a merger of them into one geometry.

A ten-dimensional space that is not a product, a space in which all ten dimensions take part in a single geometry, would have to carry in the same directions the continuous asymmetry that reproduces gravity in four dimensions and the discontinuous symmetry of the content in six. The one geometry would then carry a forbidden pairing (Chapter 3, Section V).

The separation of the factors therefore follows from the Incommensuration Theorem together with the identification of the two surviving conjunctions with the two factors (Section I): the context factor (spacetime) and the content factor (the internal space) are geometrically separate factors. For a program that merges them, treating all ten dimensions as of one character, the theorem is a no-go result.

## IV. The Kaluza–Klein Reading

The product structure $M^4 \times \mathbb{C}P^2 \times S^2$ is a Kaluza-Klein compactification. The total space has $4 + 4 + 2 = 10$ dimensions. The internal dimensions are compact (finite volume) and small (a few Planck lengths across; Chapter 9, Section IX). The external dimensions are non-compact and constitute the observable spacetime.

The Kaluza-Klein machinery extracts four-dimensional physics from the ten-dimensional structure: the ten-dimensional Einstein-Hilbert action, integrated over the internal space with the moduli held, yields the four-dimensional Einstein equations (Chapter 11).

The internal space is read in two ways. Read with radii, as the internal factor of the ten-dimensional metric, it is the space I size and hold in this book (Chapter 9, Section IX; Chapter 11, Section II), which *The Kinds of*

*Particle* calls the context instance. Read as a space of states, with no size, it is the content instance: the kinds and charges of the content are read there, and no field lives on it (*The Kinds of Particle*, Chapter 3, Section III, and Chapter 4, Section IV). The content's fields are fields over $M^4$, and none of them is a field on the internal space read with radii. In this book's naming the internal factor is the content factor, relative to $M^4$ as its context (Appendix C, Section VIII); the two readings concern where the content's fields are, and leave that naming as it is.

The reduction is the subject of Part III of this book: the derivation of the Einstein field equations from the product geometry.

Kaluza-Klein theory is often called a unification, because the gauge fields of the reduction are components of the one ten-dimensional metric. Kaluza-Klein theory is a unification in the sense of a common source, and the Incommensuration Theorem permits a unification of that kind. What the theorem excludes for the product is a merger: one geometry in which the same directions are both continuously asymmetric and discontinuously symmetric, which would need a forbidden pairing (Chapter 3, Section V). The reduction makes no such geometry: with the gauge fields present the ten dimensions form a bundle over $M^4$, locally a product, whose fibers keep the homogeneous geometry of $\mathbb{C}P^2 \times S^2$, and no direction is both. Each gauge field the reduction yields is a symmetry of the internal factor carried as a field over spacetime: one index lies along $M^4$, and the other along a symmetry direction of $\mathbb{C}P^2 \times S^2$. The gauge field joins the two factors while each keeps its own conjunction, which is the inseparability of the content factor and the context factor that Axiom III requires. These mixed components of the metric belong to the internal space read with radii. They are not the gauge fields of the content, and their excitations couple to the matter and the gauge forces only through gravity (*The Branches of the Kinds*, Chapter 2, Section III). The gauge symmetry of the content is read from the isometries of the internal space read as a

space of states, not from Killing vectors of a sized space becoming fields (*The Kinds of Particle*, Chapter 4, Section IV). The metric over $M^4$ is the arena (Chapter 11, Section II).

I derive the context: the field equations, the cosmological constant and the matter fraction. What is read on the internal space as a space of states, its spectra, charges and representations, is content and is not derived here. Three facts of that kind are named so that the boundary is plain: the isometry group of $\mathbb{C}P^2 \times S^2$ is locally $SU(3) \times SU(2)$, with no $U(1)$ factor; on an internal space that admits spinors, a smooth Kaluza-Klein theory with gravity alone gives no chiral fermions (Witten 1981; 1985); and $\mathbb{C}P^2$ is not a spin manifold, only spin$^c$, so a spinor field on it needs a $U(1)$ connection. I make no claim on those three facts. The first fact is a stated difference from the seven-dimensional internal spaces of Witten (1981), which are circle bundles over $\mathbb{C}P^2 \times S^2$ with isometry group $SU(3) \times SU(2) \times U(1)$ (Chapter 9, Section III).

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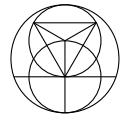
## Questions

### Is the internal space not continuously symmetric?

In the ordinary sense of the words it is: $\mathbb{C}P^2 \times S^2$, with its Fubini-Study and round metrics, is a smooth manifold on which a Lie group of isometries acts transitively. In this book symmetry and continuity are characters of what a comparison finds, and physics uses the same words of mathematical objects; in Chapter 3, Section V, I set the two usages side by side. The symmetry of the internal factor is the symmetry of its content: every point is equivalent to every other (Chapter 9, Section II). Its discontinuity is that of the spectra a comparison with the internal factor can find: the harmonics of a compact space form a discrete set, and so do the irreducible representations of its compact isometry group (Chapter 9, Section V). Its smooth metric and the continuous parameters of its isometry group belong to the formalism. The constitutive mode of a theory is the mode of its

observables, and its instrumental mode is the mode of its formalism; the cross-modal principle requires the two to be conjugate (Appendix A, Section III). For the content the formalism is therefore continuous where the observables are discretely symmetric. A continuous symmetry in the sense of physics is a symmetry and a continuity in the two conjugate modes, and the cross-modal principle requires that pairing (Chapter 3, Section V). The forbidden conjunction is both characters perfected in the one mode of what is found, and the internal factor does not carry it.

The objection that the internal space is continuously symmetric, turned round, applies to the context factor. De Sitter space is maximally symmetric, and it is the geometry that a model with a positive cosmological constant approaches as its matter density falls. Each symmetric geometry of the context factor (Minkowski space, de Sitter space, the homogeneous models of cosmology) is the geometry of an idealized content, with no content or a perfectly uniform one (Chapter 10, Section III). The context in which actual content stands has no isometries, and the symmetries of the description belong to its formalism (Appendix A, Section VI). The de Sitter symmetry is therefore a property of a model, and not a perfected symmetry of the context factor (Chapter 3, Section V).



Chapter 9

# The Internal Space and Its Uniqueness

*The Derivation of $\mathbb{C}P^2 \times S^2$ from the Structure of Comparison*

In this chapter I identify the internal factor K of the product. Three conditions translate the grid of the six metrics into geometry: one dimension for each metric; a product of a factor carrying the three modalities and a factor carrying the two rows; and each factor carrying its terms as its independent classes. With homogeneity, the classification of compact homogeneous spaces leaves one solution, $\mathbb{C}P^2 \times S^2$. The coefficient of the first Chern class of $\mathbb{C}P^2$ is $c_1 = 3$. The curvature of $\mathbb{C}P^2 \times S^2$ is positive, so it is not a Calabi-Yau space; the Einstein condition on the internal metric fixes the ratio of its radii at $\sqrt{6}$, and the modal boundary fixes its scale at a few Planck lengths. The steps from the conditions to the space are theorems of topology; the conditions are identifications, and the uniqueness holds at their standing.

*Rests on:* the product and its content factor (Chapter 8), the grid of the six metrics (Chapter 7, Section III), the three modal roles (Chapter 4, Section II), the forbidden pairings (Chapter 3, Section V), per-alternative geometry (Appendix A, Section VI), the six dimensions of possibility (Appendix C, Sections III, VII and VIII), and the modal boundary (Chapter 17). *Refers to:* how the internal space reaches observation (Chapter 11, Section II; Chapter 13, Section II). *Used by:* Chapter 10; Chapter 11 (the curvature and the radius of Sections VIII and IX); Chapters 12 and 13 ($c_1$, and the bare curvature term at the radius of Section IX); Chapter 18; and Appendices B and D. *Referred to by:* Chapter 17, Section V.

## I. From the Grid to a Geometry

The internal factor K is the content factor of the product: it is the discontinuously symmetric factor of the product, in which the content metrics are realized. The question is what structure it must carry.

The answer is the grid of Chapter 7. Every act of physical measurement involves the six metrics, pattern, force and probability in space, time and possibility, and each metric is indexed by a row and a column. The internal space is the geometric arena in which the content's comparisons across all six metrics are realized. I translate this into geometry by three conditions.

*One dimension for each metric.* Each of the six metrics is an independent respect in which content can vary. The internal space gives each its own direction. It therefore has six dimensions. In Appendix C I count six dimensions of possibility, by a route that does not use the grid; its Section VIII shows that the two sixes are one set.

*A product of two factors.* The grid is a product of two indexings, two rows and three columns (Chapter 7, Section III), which stand in the relation of Axiom III: distinct, inseparable, and non-interchangeable. The counts are fixed by comparison: the two is the number of members of the pair of respects, content and context, and the three is the number of modal roles of every domain (Chapter 4, Section II), which Axiom III asserts to be distinct, inseparable and non-interchangeable. The internal space realizes the product as a product: $K = K_M \times K_R$, in which $K_M$ carries the three modalities and $K_R$ carries the two rows. Each factor is of positive dimension, since each indexing is a genuine respect of variation. Each factor is connected: it is one arena for its indexing, and a space in separate pieces would be several arenas.

*Each factor carries its terms as its independent classes.* The factor that carries the modalities has one independent class for each modality, three in all, and no others; the factor that carries the rows has one for each row, two in all, and no others. Each class is the class of a cell of even dimen-

sion. That each class is the class of a cell of even dimension is part of the identification of the grid with the internal space. With cells of even dimension only, the count of terms can be read as the Euler characteristic. Section IV states this in the language of topology.

These three conditions are identifications, not theorems: they translate the grid of comparison into geometry, in the way that, in Chapter 8, I translated the two surviving conjunctions, the continuously asymmetric and the discontinuously symmetric, into a product. The uniqueness of Section VII holds at their standing, and at the standing of the homogeneity of Section II.

## II. Homogeneity

I take each factor to be homogeneous: every point of it is equivalent to every other point, under a group of symmetries acting on it.

The reason is the character of the factor. K is the discontinuously symmetric factor, the factor governed by symmetry. The arena in which content is compared cannot itself introduce asymmetry, since an asymmetry of the arena would be a difference of context not carried by any content. A space in which no point is structurally distinguished from another, in the sense used here, is a homogeneous space: a quotient $G/H$ of a compact Lie group $G$ by a closed subgroup $H$, with $G$ acting transitively by isometries. Homogeneity is essential to what follows: without it, many spaces would satisfy the counts (Section VI).

## III. The Product Realized

The two factors are distinct, since one carries three terms and the other two. They are inseparable, since the content is realized in their product, not in either alone. They are non-interchangeable, since a factor that car-

ries three terms cannot do the work of one that carries two. A single space carrying both indexings at once would merge the rows with the modalities, and so would fail their distinctness.

The product $\mathbb{C}P^2 \times S^2$ is the base of the seven-dimensional spaces $M^{pqr}$ that Witten (1981) studied.

## IV. Cells and the Euler Characteristic

The Euler characteristic $\chi$ of a space is the alternating sum of its Betti numbers, the numbers of its independent cohomology classes in each degree. When every independent class lies in even degree, each counts with a plus sign, and $\chi$ is the total number of independent classes. For a compact homogeneous space G/H this is the case exactly when $\chi$ is positive, which holds exactly when H has the same rank as G (Hopf and Samelson 1941).

The third condition of Section I places every independent class of each factor in even degree, and fixes their number. It therefore reads: $\chi(K_M) = 3$ and $\chi(K_R) = 2$. Since the Euler characteristic of a product is the product of the Euler characteristics, $\chi(K) = 3 \times 2 = 6$, the number of the metrics (Figure 2).

$\mathbb{C}P^2$ is also the space of the pure states of a quantum system with three alternatives, and $S^2$ is that of a system with two (Appendix C, Section VIII). The factor that carries the three columns of the grid is the space of possibilities of a three-way alternative, and the factor that carries the two rows is that of a two-way alternative.

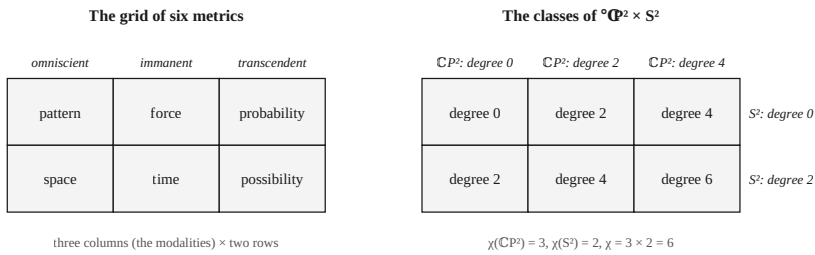



***Figure 2.*** *Left: the grid of the six metrics, three columns (the modalities) by two rows (content and context; Chapter 7, Section III). Right: the six independent classes of* $\mathbb{C}P^2 \times S^2$*, each the product of a class of* $\mathbb{C}P^2$ *(degree 0, 2 or 4) and a class of* $S^2$ *(degree 0 or 2), with its degree. The condition of Section I matches* $\mathbb{C}P^2$ *to the three columns and* $S^2$ *to the two rows. The figure shows the counts; it assigns no particular class to a particular metric.*

## V. The Incommensuration Theorem as a No-Go Theorem

The Incommensuration Theorem states that no framework can simultaneously perfect both symmetry and continuity.

The geometric translation concerns the second clause of the theorem: asymmetry and discontinuity cannot both be applied fundamentally to one comparison. That clause has an exact counterpart in the theory of connections. By the Ambrose-Singer theorem (Ambrose and Singer 1953), the curvature of a connection generates the Lie algebra of its holonomy group. A connection whose holonomy group is discrete therefore has zero curvature everywhere; a connection whose smooth curvature is non-zero anywhere has a holonomy group of positive dimension, which is continuous. Curvature is the asymmetry of a connection, and discrete holonomy is its discontinuity; no connection on the bundle of the product geometry carries both. The two connections of the bundle, the metric connection and the gauge connection, take their characters in different ways. The metric connection has smooth curvature and therefore continuous holonomy: it produces smooth geometry and no discrete spectra. The gauge connection has a compact structure group, whose irreducible unitary representa-

tions form a discrete set; that set, and not the holonomy, is the source of the discrete spectra of the quantized gauge field. The two connections coexist on the bundle; Chapter 8, Section III, states the reason they cannot be merged.

The bundle of the product geometry therefore admits no connection that is both curved and discrete in its holonomy, which would be the conjunction of asymmetry with discontinuity that the second clause forbids. Why quantizing the arena needs a forbidden pairing is stated in Chapter 3, Section V; the theorem of Ambrose and Singer exhibits the second of the two forbidden pairings as a fact about connections. The configuration I adopt is quantum content on the classical geometry of the context, with a geometry for each alternative of the matter (Chapter 12, Section VI; Appendix A, Section VI).

## VI. The Classification

The conditions are tight, and the classification of compact homogeneous spaces settles them. Homogeneous has here the sense of Section II: a quotient G/H of a compact Lie group G by a closed subgroup H, with G acting transitively by isometries. Every connected manifold is homogeneous as a topological space, so the classification uses the stronger sense throughout.

*The dimensions are even.* A compact manifold of odd dimension has Euler characteristic zero. Both factors have non-zero Euler characteristic, so both have even dimension. With a total of six, the possible dimensions of $(K_M, K_R)$ are $(6, 0)$, $(4, 2)$, $(2, 4)$ and $(0, 6)$.

*Three splits fail.* Section I requires each factor to have positive dimension; this excludes $(6, 0)$ and $(0, 6)$. A connected compact surface has Euler characteristic at most 2, so no such surface has $\chi = 3$; this excludes $(2, 4)$. The split is $(4, 2)$: $K_M$ has dimension four, and $K_R$ has dimension two.

*The factor of dimension two.* The connected compact homogeneous surfaces are the sphere $S^2$, the projective plane and the torus, with Euler characteristics 2, 1 and 0. The only one with $\chi = 2$ is $S^2$.

*The factor of dimension four.* The connected compact homogeneous four-manifolds with positive Euler characteristic are finite quotients of simply connected ones, each a quotient $G/H$ in which $H$ has the same rank as $G$ (Section IV), and those are three: the four-sphere $S^4$ ($\chi = 2$), the complex projective plane $\mathbb{C}P^2$ ($\chi = 3$), and $S^2 \times S^2$ ($\chi = 4$). The homogeneous Riemannian spaces of four dimensions are classified by Ishihara (1955). A quotient by a free action of a finite group of order $k$ has Euler characteristic $\chi/k$. The non-trivial quotients of $S^4$ and $S^2 \times S^2$ therefore have Euler characteristic 1 or 2; $\mathbb{C}P^2$ admits no free action of a non-trivial finite group, since by the Lefschetz theorem every self-map of $\mathbb{C}P^2$ has a fixed point. The only connected compact homogeneous four-manifold with $\chi = 3$ is $\mathbb{C}P^2$.

Homogeneity does the decisive work here. There are four-manifolds with $\chi = 3$ that are not homogeneous; the condition of Section II excludes them.

## VII. The Uniqueness

The internal space is

$$K = \mathbb{C}P^2 \times S^2, \tag{9.1}$$

and it is the only compact homogeneous product that satisfies the conditions. Its dimension is $4 + 2 = 6$, and its Euler characteristic is $3 \times 2 = 6$.

The six-dimensional flag manifold, $SU(3)/(U(1) \times U(1))$, also has Euler characteristic six; it is a single homogeneous space and not a product (its cohomology ring has no factorization into the rings of two smaller spaces), so the product condition excludes it. The triple product $S^2 \times S^2 \times S^2$ has Euler characteristic eight.

The counts of the cells are the counts of the grid. $\mathbb{C}P^2$ has three cells in its standard decomposition, of dimensions zero, two and four: one independent class for each of the three modalities. $S^2$ has two cells, of dimensions zero and two: one for each of the two rows. The product has $3 \times 2 = 6$ cells, one for each metric.

The first Chern class of $\mathbb{C}P^2$ is three times the class of a projective line; its coefficient equals the Euler characteristic of $\mathbb{C}P^2$: $c_1 = \chi(\mathbb{C}P^2) = 3$. This integer, the count of the modalities as the internal space carries them, enters the cosmological constant in Chapter 13.

## VIII. The Curvature Is Positive

$\mathbb{C}P^2$ with the Fubini-Study metric has Ricci tensor $R_{ij} = 6g_{ij}/R_O^2$, everywhere positive, where $R_O$ is its radius in this normalization. The subscripts are labels only. $S^2$ with the round metric of radius $R_T$ has Ricci tensor $R_{ij} = g_{ij}/R_T^2$, also everywhere positive.

A Calabi-Yau space has vanishing first Chern class and admits a Ricci-flat metric. The first Chern class of $\mathbb{C}P^2 \times S^2$ does not vanish, so the internal space is not a Calabi-Yau space and admits no Ricci-flat Kähler metric. That the internal space is not a Calabi-Yau space is a fixed structural fact.

## IX. The Geometry Is Fixed up to Scale

The topology of $\mathbb{C}P^2 \times S^2$ is fixed by the conditions. The shape and the scale of its metric are fixed by one identification, about the curvature that each internal direction has at the modal boundary.

A single direction has one curvature that takes every plane through it into account: its Ricci curvature. A sectional curvature belongs to a plane, which two directions span, and the scalar curvature belongs to a point. The Ricci curvature of a unit direction X is the sum of the sectional curvatures of the five planes that X spans with the other members of an

orthonormal frame containing X; the sum is the same for every such frame. The other curvatures that can be compared from direction to direction either cannot be the same in every direction of $\mathbb{C}P^2 \times S^2$ or fix nothing. A plane with one direction in each factor has sectional curvature zero, while a plane within $\mathbb{C}P^2$ has sectional curvature between $1/R_O^2$ and $4/R_O^2$, so the sectional curvatures cannot all agree. The holomorphic sectional curvature of a direction X (the sectional curvature of the plane of X and JX, where J is the complex structure) takes different values on different directions at every ratio of the radii, and so does the largest sectional curvature among the planes through X. The least sectional curvature among the planes through X is zero for every direction at every ratio, and the scalar curvature is the same at every point at every ratio, so neither fixes the ratio. The Ricci curvature can be the same in every direction, and it is so at exactly one ratio of the radii.

Each of the six internal directions is an independent way of differing (Section I; Appendix C, Section III). Independence alone fixes no curvature: directions that cannot be composed from one another remain so when either factor is enlarged or reduced, so independence holds at every ratio of the radii. What independence supplies is that the modal boundary bounds each direction on its own, and not only the six directions on average. I take the modal boundary to be one curvature, the Planck curvature $1/l_{Pl}^2$, for every internal direction, and I take each internal direction to reach it: the Ricci curvature of every unit internal direction is $1/l_{Pl}^2$.

A symmetric tensor that takes the same value $\lambda$ on every unit vector is $\lambda$ times the metric. The identification therefore makes the internal metric an Einstein metric, with Ricci tensor $\lambda g$ and $\lambda = 1/l_{Pl}^2$, one constant on both factors. By Section VIII the Ricci constants of the two factors are $6/R_O^2$ and $1/R_T^2$, so $6/R_O^2 = 1/R_T^2 = 1/l_{Pl}^2$, and

$$R_O/R_T = \sqrt{6},\ R_T = l_{Pl},\ R_O = \sqrt{6}\, l_{Pl}. \tag{9.2}$$

In the normalization of Section VIII, $R_O$ is the radius of the round five-sphere whose quotient by the circle of phases (the multiplication of a state vector by a phase) is $\mathbb{C}P^2$; in this normalization every projective line of $\mathbb{C}P^2$ is a round sphere of radius $R_O/2$. The number $\sqrt{6}$ belongs to this normalization. Stated without a normalization, the Einstein condition is that a projective line of $\mathbb{C}P^2$ has 3/2 times the area of $S^2$: $\pi R_O{}^2/(4\pi R_T{}^2) = 6/4$.

In Appendix C, Section VIII, I read each factor as a space of states: "The complex projective space $\mathbb{C}P^n$ is the space of the pure states of a quantum system with n + 1 alternatives, and its Fubini–Study metric measures how distinguishable one such possibility is from another (Wootters 1981)." That reading fixes the metric of each factor up to its scale; it does not fix the ratio of the two scales. Suppose the distinguishability of two pairs of states is measured by one rule for both factors, with the squares of the two distances added. Then the states that superpose two given alternatives form a round sphere of the same radius whichever factor carries them, $R_O/R_T = 2$, and the Ricci constants stand in the ratio 3 to 2, so that metric is not an Einstein metric. I do not take that metric as the internal metric. The Einstein metric is the metric of distinguishability in which the square of the distance on each factor is counted once for each alternative of that factor: three times on $\mathbb{C}P^2$ and twice on $S^2$. That weighting follows from the identification of this section; it is not the ground of the identification.

The Einstein condition concerns the internal metric only; the curvature of $M^4$ is governed by the four-dimensional field equations (Chapter 12). The internal space is a few Planck lengths across. The Planck length here is the one formed from the measured $G_N$, the bound of every comparison a located comparer makes; a ten-dimensional Planck length takes the internal volume as an input, and that volume is what this boundary fixes (Chapter 17, Section V). One identification places all six directions at the boundary together, and it fixes the ratio of the radii and their scale at once.

The identification sets a coefficient of order one: the factor 1 that multiplies $1/l_{Pl}{}^2$, whose value I take from the identification and do not fix otherwise (Chapter 13, Questions). That coefficient enters the cosmological constant through the bare curvature term of the reduction, which at this radius is $-3/(8\pi)\ M_{Pl}{}^4$ (Chapter 11, Section III; Chapter 13, Section II). The bare term depends on the internal metric only through its scalar curvature, the sum of the Ricci curvatures of six orthonormal directions, which the identification fixes at $6/l_{Pl}{}^2$.

The ratio of the radii is tested by no observation of this book: an internal metric with another ratio and the same scalar curvature gives the same bare term, and so the same predictions in Chapter 13.

*The volume.* The radii of (9.2) fix the volume of the internal space, the volume that Chapter 17, Section V, says the boundary fixes. In the normalization of Section VIII, $\mathbb{C}P^2$ is the quotient of the round five-sphere of radius $R_O$ by the circle of phases; the five-sphere has volume $\pi^3 R_O{}^5$ and the circle has length $2\pi R_O$, so $\mathbb{C}P^2$ has volume $\pi^2 R_O{}^4/2$. The sphere $S^2$ has area $4\pi R_T{}^2$. The volume of the internal space is the product of the volume of $\mathbb{C}P^2$ and the area of $S^2$:

$$\begin{aligned}\mathrm{Vol}(\mathbb{C}P^2 \times S^2) = 2\pi^3 R_O{}^4 R_T{}^2 = 2(R_O/R_T)^4\ \pi^3 R_T{}^6 = 72\ \pi^3 R_T{}^6 \\ = 72\ \pi^3 l_{Pl}{}^6.\end{aligned}$$

In the unit $\pi^3 R_T{}^6$ the volume is 72, which is $2 \times 6^2$, twice the fourth power of the ratio of the radii.

The unit $\pi^3 R_T{}^6$ has a geometric reading. The internal space is a complex manifold of three complex dimensions: two from $\mathbb{C}P^2$, and one from $S^2$, which is the complex projective line $\mathbb{C}P^1$. In each complex direction the natural cell of a complex manifold is a disc, and a disc of radius $R_T = l_{Pl}$ has area $\pi R_T{}^2$. One such disc in each of the three complex directions, taken as a product, is the Planck polydisc, whose volume is $(\pi R_T{}^2)^3 = \pi^3 R_T{}^6$. So the unit is the Planck polydisc: three Planck discs, one in each

complex direction, held as a product. The three factors of $\pi$ are the three circles of phase, one in each complex direction; the six powers of $R_T$ are the six real directions at the bound of comparison, the modal boundary.

The factor $\pi^3$ is also the volume of the unit round five-sphere. That five-sphere is the space of the normalized state vectors of three complex amplitudes, phase included, and its quotient by the circle of phases is $\mathbb{C}P^2$ (Section VIII). The unit $\pi^3 R_T{}^6$ therefore joins the two readings of the internal space (Chapter 8, Section IV): $\pi^3$ is the measure of the content's own state vectors, read on the space of states, and $R_T{}^6$ is the extent of the context, read on the space with radii. The unit sets that content measure against the context's extent.

I read the unit $\pi^3 R_T{}^6$ by the modalities. The factor $\pi^3$ is a spherical term, and its three factors carry the three open directions in which a comparison is made (the three directions of space, which are not compact): it is an omniscient measure. The factor $R_T{}^6$ carries the six dimensions of possibility at the bound of comparison: it is a transcendent measure. Their product, a construction joining the two measures, is immanent. The unit is then a whole that holds all three modalities at once, which is what one comparison is. The volume divided by this unit counts comparisons, and the internal space holds 72 of them.

Planck's constant gives the unit a further reading. Since $l_{Pl}{}^2 = \hbar G_N/c^3$, in units in which $G_N$ and $c$ are one the area of a disc of Planck radius is $\pi l_{Pl}{}^2 = \pi\hbar = h/2$, half of Planck's constant $h = 2\pi\hbar$. The Planck polydisc is then half a cell of action in each complex direction, $(h/2)^3$, and the volume of the internal space is $72(h/2)^3 = 9h^3$: nine cells of action, each the cell of one state of three degrees of freedom, and each divided into $2^3 = 8$ parts by one halving in each complex direction. Read this way, the 72 of the internal space is $9 \times 8$, and it counts the parts that the halvings make: the binary divisions inside each state. A cell of action counts a state, and a volume in this unit counts the eighths into which three halvings divide each state.

The polydisc and the ball of the same radius are two different units. The six-dimensional ball of Planck radius has volume $\pi^3 R_T{}^6/6$, six times smaller than the polydisc. The ball measures all six directions by one radius, as one; the polydisc measures each complex direction by its own disc and joins the three as a product. A measure that treats three as one is the measure Axiom III forbids, and the factor between the two units is $3! = 6$, the number of orders of three. The volume of the internal space is therefore counted in polydiscs, and the reason it must be is Axiom III. Axiom III has its figure in the Borromean rings: three rings, each perpendicular to the other two, linked so that no two are linked without the third. Unless there are three, linked, there are none entangled. The polydisc is that figure in measure: three Planck discs, one in each complex direction, mutually perpendicular and joined as a product. The ball would merge the three discs into one radius, and the factor $3! = 6$ between the two units is the measure of what the merging discards.

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## Questions

**Is 3 × 2 = 4 + 2 a coincidence?**

No. The grid is a product of three columns and two rows, and the Euler characteristic of a product is the product of the Euler characteristics: $\chi(\mathbb{C}P^2 \times S^2) = 3 \times 2 = 6$, one cell for each metric (Section VII). The dimension is a sum, and its agreement with the count of cells holds for every number of columns. For a grid of $a$ columns and $b$ rows carried by projective factors $\mathbb{C}P^{a-1} \times \mathbb{C}P^{b-1}$, the count of cells is $ab$, since $\chi(\mathbb{C}P^n) = n + 1$, and the real dimension is $2(a-1) + 2(b-1)$. The two are equal exactly when $(a-2)(b-2) = 0$. With two rows, $b = 2$, the dimension equals the count of the metrics for any number of columns; for the three columns of the grid, both are six.

**Does an internal space of Planckian curvature not give spacetime a Planckian curvature too?**

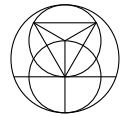
No. The Einstein condition of Section IX is a condition on the internal metric of $\mathbb{C}P^2 \times S^2$ alone, and the four-dimensional metric obeys the field equations of Chapter 12. The curvature of the internal space reaches those equations, classically, through the bare curvature term of the reduction (Chapter 11, Section III). That term is part of the external value of the vacuum's energy, and in Chapter 13 I give the cosmological constant as the share of that value accessible to an observer inside the vacuum, about $10^{-123}$ of the Planck density (Chapter 13, Section II).

**Are the three conditions of Section I not chosen to give the answer?**

The conditions are stated as identifications (Section I): one dimension for each of the six metrics; a product of a factor carrying the three modalities and a factor carrying the two rows; and each factor carrying its terms as its independent classes. They are read from the structure of comparison, and none refers to the particles to be reproduced or to the symmetry group the internal space should have. From them the classification of compact homogeneous spaces determines $\mathbb{C}P^2 \times S^2$ by theorem (Section VI). A referee who rejects one of the identifications rejects the uniqueness at that step.

**Does any observation reach the internal space beyond the counts of this chapter?**

Observation reaches the internal space through two numbers, and both enter through $\Lambda$: the count of three, $c_1 = 3$, and the curvature at the derived radius, a scalar curvature of $6/l_{Pl}^2$ (Chapter 13, Section II). The ratio $R_O/R_T = \sqrt{6}$ and the two radii taken separately enter only the masses of the moduli, whose force has a range some thirty orders of magnitude shorter than any experiment reaches (Chapter 11, Section II). The matter fraction is therefore the one observation that reaches the internal space, and it reaches it through the count of three and the curvature.



Chapter 10

# Why Four Dimensions, Why Lorentzian

*The 10 = 4 + 6 Decomposition and the Signature of Spacetime*

In this chapter I give the external factor $M^4$ its four dimensions and its Lorentzian signature. The directed precedence of Axiom II gives each event a cone of possible precedence, and the cone falls into a before and an after for exactly one time direction, so the signature is $(-, +, +, +)$. Axiom I gives the same single time direction by a second route: time, the immanent context, is the single prior direction. The three transitions of the directed cycle of Axiom II give three spatial directions, and its direction gives the one time direction; both readings are identifications. With the six internal dimensions of Chapter 9, the total is $4 + 6 = 10$.

*Rests on:* Axioms I and II (Chapter 4), the context triad (Chapter 7, Section I), the product and the internal space (Chapters 8 and 9), the formalism of the context (Appendix A, Section VI), and the maximum signal speed (Chapter 17, Section I). *Refers to:* the parallel derivation of the dimensions of context (Appendix C). *Used by:* Chapter 11; Chapter 12; and Chapter 18, Section VI. *Referred to by:* Chapter 7, Section V; Chapter 17, Section I; and Appendix C, Section VIII.

## I. The Arithmetic of Dimensions

The dimensions of the product geometry are fixed from two sides.

The internal space has six dimensions, one for each of the six metrics of comparison; Chapter 9 shows that $\mathbb{C}P^2 \times S^2$, of dimension $4 + 2$, is the one space that carries their grid under the conditions of that chapter.

The external spacetime has four dimensions, one temporal and three spatial; In Section IV I derive this count from the directed cycle of Axiom II and from Axiom I.

The total is therefore $4 + 6 = 10$. Neither count is obtained from the other: the internal six comes from the grid, the external four from the cycle, and the ten is their sum.

In Appendix C I derive one dimension of time and three of space a second time, by a separate route: from the single stance of comparison, from the requirement that space differ from time, and from the doubling by which every finding is placed twice. Neither derivation uses the other's steps.

## II. Why Not Three or Five or Fourteen

The claim that spacetime has exactly four external dimensions is stronger than a consistency check. It is a uniqueness argument, made on each side.

On the internal side, a space of fewer than six dimensions cannot carry the six metrics of comparison, one direction each; and among spaces of six dimensions, the classification of Chapter 9 leaves only $\mathbb{C}P^2 \times S^2$.

On the external side, the directed cycle of Axiom II has exactly three transitions, since there are exactly three modalities, and exactly one direction. Section IV shows that the transitions give the spatial directions and the direction gives time. Three spatial dimensions and one temporal dimension are therefore fixed by the number of the modalities and by the directedness of their cycle.

## III. The Lorentzian Signature

Having established that spacetime is four-dimensional, the question is why it has signature $(-, +, +, +)$, one temporal and three spatial dimensions, rather than Euclidean signature $(+, +, +, +)$, split signature $(+, +, -, -)$, or some other combination.

The answer is the directed precedence of Axiom II. A class of the transcendent precedes an instance of the immanent, a class of the immanent precedes an instance of the omniscient, and a class of the omniscient precedes an instance of the transcendent: precedence runs one way. I identify that precedence, in the context factor, with the order of events, and the order has two features.

*The first feature is that the order of events has a limit.* I identify the precedence of a class of the immanent over an instance of the omniscient, in the context factor, with a span of time before every placing of a finding in space: no finding at a separation enters a comparer's record at the moment it is made there. I take every domain of measurement to have a macroscopic boundary; in the context of space and time that boundary is one ratio of spatial to temporal measure, the same throughout the domain and for every comparer in uniform motion: the speed of light c (Chapter 17, Section I).[7] A finding at a separation therefore reaches a comparer no sooner than the separation divided by c, and c is finite. The Galilean structure, a direction of time and a total order of events with signals of any speed, has no such boundary: its spatial and temporal measures stand apart, with no constant relating them (Malament 2012). The principle of relativity, homogeneity, isotropy and the group law fix the transformations between frames up to one constant, $1/c^2$, the Galilean case being the value zero (Lévy-Leblond 1976). The boundary of the domain, the same for every comparer in uniform motion, makes the constant nonzero, and the cone of precedence makes it positive. That relation of spatial to temporal measure is the metric of the context factor, so the limit is carried by the metric itself: the directions along which one event can precede another

form a cone at each event, bounded by the metric's directions of zero interval. A metric with directions of zero interval is indefinite. A Euclidean metric has none; it gives no cone and no order of events.

*The second feature is that the order of events has a direction.* What precedes is distinct from what follows, so the cone at each event falls into two halves, a before and an after.

The cone falls into two halves for exactly one time direction. With one direction of negative sign, every timelike direction points into the future half or into the past half, and no continuous path within the cone joins them. With two or more, the cone is one connected piece: any timelike direction can be turned into its own reverse without leaving the cone, and before cannot be told from after. The signature therefore has exactly one negative sign, $(-, +, +, +)$, with the count of spatial directions given in Section IV.

Analyses of the field equations support the same count of one time direction. With two or more time directions the wave equation is ultrahyperbolic, and its initial value problem is ill posed for data given on a surface that contains no time direction (Tegmark 1997; Craig and Weinstein 2009); it is well posed only for data on a surface of one dimension less than spacetime that includes time directions, and then only under a nonlocal constraint (Craig and Weinstein 2009).

The direction of precedence is the asymmetry of the context factor in its plainest form. Precedence is a difference of content (what precedes, what follows) where there is a difference of context (the two events), and it varies continuously from event to event: the continuously asymmetric conjunction, realized as the causal order.

Particular geometries of the context factor carry symmetries: Minkowski space, de Sitter space, and the homogeneous models of cosmology. Each is the geometry of an idealized content, with no content or a perfectly uniform one. The context in which actual content stands has no isometries, and the symmetries of the description, local Lorentz symmetry among them, belong to its formalism (Appendix A, Section VI). The in-

ternal space $\mathbb{C}P^2 \times S^2$ is symmetric because it is the content factor (Chapter 9, Section II). The signature rests on precedence, and the context factor is continuously asymmetric in its order of events, as the Incommensuration Theorem requires.

## IV. The 3 + 1 Split

In Section III I exclude two or more time directions by the precedence of Axiom II. Axiom I gives a second, independent route to the same count of one time direction.

The split signature $(+, +, -, -)$ would be asymmetric, since it would distinguish two axes from two others. It would not distinguish *one* axis as structurally prior. Axiom I requires that the immanent modality be more fundamental than the other two. In the context triad of Chapter 7, the immanent context is time. Time must be structurally distinguished as the single, minimal, prior direction: the bare cut, the measurement event that divides before from after.

The number of spatial directions comes from Axiom II. The axiom orders the three modalities in a directed cycle: a class of the transcendent precedes an instance of the immanent, a class of the immanent precedes an instance of the omniscient, and a class of the omniscient precedes an instance of the transcendent. The cycle has three transitions, from transcendent to immanent, from immanent to omniscient, and from omniscient to transcendent, and each is an independent structural relationship between two modalities. I identify each independent structural relationship, in the context factor, with an independent direction of extension: the three transitions give three spatial directions. The cycle also has a direction, the oriented return from transcendent through immanent and omniscient back to transcendent; that directedness is the ordering of before and after, and I identify it with the one temporal direction. Spacetime therefore has 3 + 1 dimensions.

The two arguments agree. Axiom I makes time the single prior direction, and Axiom II supplies one direction for the cycle's order and three for its transitions. The one temporal dimension has the character of minimal dimensionality: the temporal dimension divides, and does not extend. The three spatial dimensions have the character of finite structural dimensionality, the omniscient context: they provide the arena within which objects can be distinguished, measured, and known.

The same triad appears in the internal factor in another form, because each factor carries terms in its own way: the context factor carries them as directions of extent, and the content factor carries the terms of each of its indexings as independent classes (Chapter 9, Section I). I identify the triad accordingly: in $M^4$ as three spatial directions, and in $\mathbb{C}P^2$ as three independent classes, $\chi(\mathbb{C}P^2) = 3$. These identifications are of the same standing as the translation of the grid into the internal geometry in Chapter 9: they are grounded in the axioms and pass through a geometric reading.

## V. The Einstein Condition and the Internal Radii

The product structure $M^4 \times \mathbb{C}P^2 \times S^2$ admits a natural Einstein condition on the internal metric. In a product of Einstein manifolds, the Einstein condition constrains the ratio of the factor radii. For $\mathbb{C}P^2$ with the Fubini-Study metric and $S^2$ with the round metric, the Einstein condition is:

$$R_O/R_T = \sqrt{6}, \tag{10.1}$$

where $R_O$ is the radius of $\mathbb{C}P^2$ and $R_T$ is the radius of $S^2$ (Chapter 9, Section IX). This fixes the ratio, given the condition, which is an identification, with its ground stated there. The overall scale lies at the modal boundary: the internal space is a few Planck lengths across (Chapter 17). At energies far below the corresponding scale, the massive modes of the internal space are not excited, and the effective four-dimensional description is general relativity coupled to the content of physics.

## VI. The Complete Geometry

In Chapters 8, 9 and 10 I have established the full geometry of the product structure.

The total space is ten-dimensional. Six dimensions are internal: $\mathbb{C}P^2$ (dimension 4, $\chi = 3$) times $S^2$ (dimension 2, $\chi = 2$), carrying the grid of the six metrics. The internal radii stand in the ratio $R_O/R_T = \sqrt{6}$, and the internal space is a few Planck lengths across.

Four dimensions are external: $M^4$, a Lorentzian manifold with signature $(-, +, +, +)$. The Lorentzian signature is required by the directed precedence of Axiom II, which is the asymmetry of the context factor. The $3 + 1$ split follows from Axiom I and from the directed cycle of Axiom II.

The next step is to write the action. The ten-dimensional Einstein-Hilbert action $S_{10} = \int d^{10}x \sqrt{(-g_{10})}\, R_{10}$, integrated over the internal space by Kaluza-Klein reduction with the moduli held, yields the four-dimensional Einstein equations. That reduction is the subject of Chapter 11.

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### Questions

**Is the count of three spatial directions not read off whatever comes in threes?**

The count rests on one identification, stated in Section IV: an independent relationship of the cycle of Axiom II is an independent direction of extension in the context factor. The cycle orders three modalities, so it has three transitions. Precedence runs one way (Section III), so the reverse of a transition is not a relationship of the cycle, and the count is three, not the six ordered pairs of the modalities. In Appendix C I reach one dimension of time and three of space by a route that uses neither the cycle nor this identification (Section I).

**Are the two routes to a single time direction independent?**

Yes. Section III uses Axiom II: precedence has a direction, and the cone of possible precedence falls into a before and an after only when the metric has exactly one negative sign, which is a fact about quadratic forms. Section IV uses Axiom I: the immanent modality is prior, and the immanent context is time (Chapter 7), so time is the single distinguished direction. Neither route uses the other's premise. Appendix C, Section V, gives a third route, from the single stance of comparison.

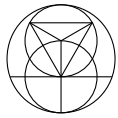
**Newtonian spacetime has a direction of time and a total order of events, with no finite signal speed. What in the axioms excludes it?**

Two premises together (Section III). The precedence of Axiom II, read in the context factor, gives a span of time before every placing of a finding in space; that excludes signals that take no time, and it leaves signals of any finite speed. The macroscopic boundary of every domain of measurement gives, for space and time, one finite ratio of spatial to temporal measure, the same throughout the domain and for every comparer in uniform motion; that excludes signals of any speed, and with it the Galilean structure, whose spatial and temporal measures stand apart. Of the transformations between frames that relativity, homogeneity, isotropy and the group law allow, the Galilean ones are the case $1/c^2 = 0$ (Lévy-Leblond 1976), which the boundary excludes; the cone of precedence excludes $1/c^2 < 0$. The first premise is an identification of the same standing as the reading of the cycle's transitions in Section IV.

# PART III

## The Einstein Equations

*From the product geometry to the field equations*



Chapter 11

# The Kaluza–Klein Reduction

*From Ten Dimensions to Four*

This chapter reduces the ten-dimensional gravitational action on $M^4 \times \mathbb{C}P^2 \times S^2$ to four dimensions. The reduction gives three things: the four-dimensional Einstein-Hilbert term, with the Planck mass set by the volume of the internal space; the moduli, which describe the size and shape of the internal space; and a bare curvature term with the sign of anti-de Sitter space. The moduli are held: each internal direction stands at the modal boundary, and the boundary is the same throughout the exterior. One mechanism, the least motion of the metric with the least extent as its local term, holds the internal space there at first order with moduli of about the Planck mass (Appendix D). The mechanism beyond its first order and the coefficients of the higher Lovelock terms are not fixed in this book; neither changes the form of the four-dimensional equations.

*Rests on:* the product geometry and the internal space (Chapters 8 to 10; Chapter 9, Sections VIII and IX), the omniscient modality (Chapter 4, Section II), and the modal boundary (Chapter 17). *Refers to:* the internal space as the context of possibility (Appendix C, Section VIII), the compactification conditions (Chapter 18, Section VI), the description of the metric's small motions by quanta (Chapter 12, Questions), the polarizations of gravitational waves (Chapter 16), and the floor of the metric (Appendix D). *Used by:* Chapter 12 (Sections III and VII), Chapter 13 (Section II), Chapter 16 (Section I), Chapter 19, Chapter 20, and Appendix D. *Referred to by:* Chapter 17 (Section V), and Chapter 9 (Questions).

## I. The Ten-Dimensional Action

On the ten-dimensional product manifold $M^4 \times \mathbb{C}P^2 \times S^2$, the gravitational action is built from the metric alone. The leading term of the gravitational action is the Einstein-Hilbert action:

$$S_{10} = \int d^{10}x \, \sqrt{(-g_{10})} \, R_{10}. \tag{11.1}$$

Lovelock's theorem (1971; 1972) fixes which terms the action may contain. A term is allowed when it is generally covariant and gives field equations of second order in the metric. In four dimensions the allowed terms are the Einstein-Hilbert term and a cosmological term, and there are no others. In ten dimensions the theorem allows more. The Gauss-Bonnet term, quadratic in curvature, is allowed, and so are the cubic and quartic Lovelock terms. Each gives second-order equations in ten dimensions. The Einstein-Hilbert term is the leading term, the only one linear in curvature.

The higher Lovelock terms leave the form of four-dimensional gravity unchanged. Integrated over the internal space, each term adds to three quantities: the four-dimensional Planck mass, the bare curvature term of Section III, and the couplings of the moduli of Section II. With the moduli fixed, the four-dimensional theory is generally covariant and second order in the four-dimensional metric. Lovelock's theorem in four dimensions then fixes its form: the Einstein-Hilbert term and a cosmological term (Chapter 12). The four-dimensional form is the only form open to a located comparison, and in four dimensions Lovelock's theorem fixes it. The four-dimensional Gauss-Bonnet term that the reduction also produces is a total derivative in four dimensions, and with the moduli fixed it adds nothing to the field equations. The coefficients of the higher terms are not fixed in this book. A large ratio of scales does not suppress them, since the internal radius is only a few Planck lengths (Section IV). Their effects enter the values of the constants, among them the bare curvature term of Section III (Chapter 13, Questions), and the form of the equations stays the same.

General covariance enters the argument of this book twice. It restricts the action to the terms Lovelock's theorem allows (this section), and it makes the stress-energy of the matter conserved (Chapter 12, Section VII). I identify general covariance with what the omniscient modality requires of a description. The omniscient modality is the role of what is known: structure, pattern (Chapter 4, Section II). What is known in a comparison as structure is what stays the same when the comparison is described in another way. A system of coordinates is a way of describing: it labels the events of the arena, and the labels are chosen by the one who describes, not found by the comparison. If a result of comparison changed when the labels changed, what was known would include the choice of labels, and so would not be the structure alone. The omniscient modality therefore requires that what is compared not depend on how it is described.

Two characters of the arena fix which changes of description that requirement covers. The arena is continuous (Chapter 3, Section V): its events are labeled by coordinates on a smooth manifold, and a change of labels is a smooth invertible map, which keeps neighboring events neighboring. The arena is asymmetric (Chapter 3, Section V): a generic geometry has no isometry, so it singles out no family of labels, in the way that the geometry of flat spacetime singles out its inertial coordinates. The changes of description that must leave every result of comparison unchanged are therefore all smooth changes of coordinates, not only those of a fixed group such as the Lorentz transformations. Invariance under all smooth changes of coordinates is general covariance. Covariance alone does not restrict the action; covariance together with an action built from the metric alone does (Questions).

The reduction takes a metric as given: the context factor is a smooth manifold with a metric (Chapter 8, Section II), and the reduction starts from the ten-dimensional metric on the product. What the reduction supplies is not the existence of a metric. It supplies the four-dimensional metric with its field equations (Chapter 12), and the internal metric with its

shape fixed (Chapter 9, Section IX). What the axioms supply, with the identifications of Chapters 8 to 10, is the product the metric lives on, its dimensions, and its Lorentzian signature.

## II. The Decomposition of Metric Fluctuations

The ten-dimensional metric fluctuations on the product manifold decompose into three classes, each corresponding to a different sector of four-dimensional physics.

*Fluctuations along* $M^4$ are the four-dimensional metric perturbation $h_{\mu\nu}$. The fluctuations of the four-dimensional metric are the dynamical degrees of freedom of four-dimensional general relativity. The perturbation has two physical polarizations, the two polarizations of gravitational waves (Chapter 16), as required by the structure of the continuously asymmetric factor. These fluctuations satisfy the four-dimensional Einstein equations.

*Mixed fluctuations*, the components with one $M^4$ index and one internal index, are four-dimensional vector fields associated with the symmetries of the internal space. They belong to the internal space read with radii (Chapter 8, Section IV); they are not the gauge fields of the content, and their excitations couple to the matter and the gauge forces only through gravity (*The Branches of the Kinds*, Chapter 2, Section III). Their particular form, and their couplings among the excitations of the internal space, are not used in this book.

*Fluctuations within the internal space* are scalar moduli that parameterize deformations of the internal geometry. They include the volume modulus, which sets the overall size of the internal space and thereby the four-dimensional Planck mass, and the shape modulus, which sets the ratio of the radii of $\mathbb{C}P^2$ and $S^2$.

The moduli are the size and shape of the internal space. The internal space cannot shrink to nothing. It is the context of possibility (Appendix C, Section VIII), and Axiom III excludes content without context. A size

held at zero with no spread is excluded by the uncertainty relation. The internal space therefore has a floor: a least extent, with a least motion that cannot be removed. That motion is already the lowest state, so it cannot radiate or dissipate.

*The hold.* While the exterior factor is the context of the compact factor (Chapter 18, Section VI), the internal space is held as content, and each of its six directions stands at the modal boundary, where the curvature it sees is the Planck curvature (Chapter 9, Section IX). The boundary is fixed by the measured $G_N$, which has one value in this domain (Chapter 17, Sections I and III), so the internal space has the same size and the same shape at every point of the exterior: $R_O = \sqrt{6}\, l_{Pl}$ and $R_T = l_{Pl}$. The moduli are therefore fixed at these values throughout the exterior.

*One mechanism.* The curvature of the internal space pulls it toward smaller size; the least extent and the least motion resist. The metric is the one field the internal space cannot do without, and its least motion is one mechanism that holds the space: its levels on the internal factor are content (Chapter 17, Section V), described by quanta as small waves are (Chapter 12, Questions). At first order, in the form that does not depend on the gauge (Vilkovisky and DeWitt), that floor, with the least extent as its local term, holds the internal space at the derived radius and ratio as a stable minimum (Appendix D). Holding an internal space by the zero-point energy of its fields is the stabilization of Candelas and Weinberg (1984); here the field is the metric itself.

*The stiffness of the hold.* The masses of the moduli belong to the mechanism: at first order they are 2.25 and 3.09 Planck masses, so the force the moduli carry has a range shorter than the Planck length, some thirty orders of magnitude shorter than the shortest range that any of these tests reaches: the Cassini value $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ (Bertotti, Iess and Tortora 2003), and the inverse-square tests to about a twentieth of a millimeter (Kapner and colleagues 2007). Four-dimensional gravity is general relativity. The computation is a first-order description at the boundary, where no parameter of the theory makes the next order small; the bound

from these tests needs only the order of the masses, and the hold itself does not rest on it. The floor's energy is counted within the ground-state density $M_{Pl}^4$ (Section III), which I take whole in Chapter 13, so its energy beyond first order bears on the moduli and not on the cosmological constant.

## III. The Four-Dimensional Lagrangian

Integrating the ten-dimensional action over $\mathbb{C}P^2 \times S^2$ produces a four-dimensional Lagrangian whose gravitational part has three terms.

First: the Einstein-Hilbert term for four-dimensional gravity, with the four-dimensional Planck mass determined by the ten-dimensional fundamental scale and the volume of the internal space:

$$M_{Pl}^2 = M_{10}^8 \times \mathrm{Vol}(K). \tag{11.2}$$

Here $M_{Pl}$ is the full Planck mass, $G_N = \hbar c / M_{Pl}^2$, and $M_{10}$ is formed in the same way from the ten-dimensional coupling, with no factors of $2\pi$ (Chapter 12, Section III). The bare term below is stated in the same convention.

Second: the kinetic and potential terms of the moduli, which describe how the size and shape of the internal space can vary from point to point of spacetime.

Third: a bare cosmological term from the curvature of the internal space. Its energy density is of order $M_{Pl}^2/R^2$, where R is the internal radius. With R a few Planck lengths, this is of the order of the Planck density. Positive internal curvature enters the reduced action with the sign of a negative cosmological constant, the sign of anti-de Sitter space.

This bare term is not the observed $\Lambda$. It is part of what I call in Chapter 13 the external value: the total vacuum energy density at the scale of the modal boundary (Chapter 17), of order $M_{Pl}^4$, before the information deficit reduces it. The observed $\Lambda$ is the internal value, the fraction of the external value that is accessible from inside (Chapter 13). The bare term is a

value of the classical action and carries no ħ; it stands beside the ground-state density $M_{Pl}^4$ as one contribution to the external value. The floor of Section II is a zero-point energy, a quantum effect, and I count it within $M_{Pl}^4$. At the derived radius (Chapter 9, Section IX) the bare term is $-3/(8\pi)\, M_{Pl}^4$.

The content, whose fields are fields over $M^4$ with their kinds and charges read on the internal space as a space of states (Chapter 8, Section IV), adds its own terms, and its stress-energy enters the right-hand side of the four-dimensional field equations.

## IV. The Two Radii

The internal space $\mathbb{C}P^2 \times S^2$ has two factor radii: $R_O$ for $\mathbb{C}P^2$ and $R_T$ for $S^2$. The Einstein condition on the product requires the Ricci tensor to be proportional to the metric with one constant on both factors. The Einstein condition imposes a definite relationship between the radius of $\mathbb{C}P^2$ and the radius of $S^2$:

$$R_O / R_T = \sqrt{6}. \tag{11.3}$$

The value arises because the Fubini-Study metric on $\mathbb{C}P^2$ has Ricci constant $6/R_O^2$ and the round $S^2$ has Ricci constant $1/R_T^2$ (Chapter 9, Sections VIII and IX). The overall scale is set at the modal boundary (Chapter 17): $R_T = l_{Pl}$ and $R_O = \sqrt{6}\, l_{Pl}$ (Chapter 9, Section IX).

## V. What the Reduction Achieves

The Kaluza-Klein reduction of the ten-dimensional action on $\mathbb{C}P^2 \times S^2$ produces four-dimensional gravity in a single step: the metric perturbation, the Einstein-Hilbert term, the Planck mass from the internal volume, the moduli of the internal space, and a bare curvature term.

The reduction is a standard technique in mathematical physics, applied to a specific manifold. Its inputs are three: the product structure $M^4 \times \mathbb{C}P^2 \times S^2$, the Einstein metric on the internal space, and the floor that holds the moduli (Section II). The product structure is derived in Part II. The leading term of the action follows from general covariance and the second-order condition. The form of the four-dimensional equations follows from Lovelock's theorem in four dimensions, once the moduli are fixed. Given the structural identifications of Chapters 9 and 10 and the floor of Section II, every remaining step is a theorem or a standard technique.

The reduction fixes the form of the field equations; it does not fix the value of $G_N$ or of $\Lambda$. $G_N$ is the one measured constant of this book (Chapter 17), and Chapter 13 fixes $\Lambda$. The two quantities this book does not fix, the coefficients of the higher Lovelock terms (Section I) and the energy of the floor beyond its first order (Section II), enter the values of the constants and the moduli; neither changes the form of the four-dimensional equations.

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## Questions

**Any theory can be written in a generally covariant form. Does general covariance restrict anything?**

Covariance alone does not. Kretschmann (1917) argued that any physical theory can be given a generally covariant form; a theory with a fixed background structure is made covariant by carrying that structure along as a further field. What Lovelock's theorem uses is covariance together with an action built from the metric alone, with no field fixed in advance (Section I). With no background structure to carry along, covariance admits only scalars formed from the metric and its curvature, and the second-order condition then leaves the Lovelock terms. The identification of general co-

variance with what the omniscient modality requires (Section I) supplies the covariance; the gravitational action of this book is the action of the metric alone.

**I do not quantize the metric in this book. How is a zero-point energy of the metric licensed?**

The zero-point energy belongs to the description of the metric's small motions by quanta. The nearest sense in which such a quantum can be made intelligible is the phonon, a quantum of the description of the vibrations of a solid; the arena is not a held structure, as a solid is, so even that sense does not make the quantum a quantum of the arena. A quantum of that description is a step in the energy of one mode of the wave, not a step in the extent of the geometry; the two senses of "not quantized" are stated in Chapter 12, Questions. The description is the effective field theory of gravity (Donoghue 1994a; 1994b), and on the internal factor it ends at the modal boundary (Chapter 17, Section V). I answer this objection in full in Chapter 12, Questions.

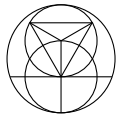
**Is the one-loop balance under control, at a radius of a few Planck lengths and with the metric as the only field?**

No, and the argument of the book does not rest on it. The hold rests on the argument of Section II: each internal direction stands at the modal boundary, which is the same throughout the exterior. From the computation I take one thing, the order of the moduli's masses, which the inverse-square tests need with thirty orders of magnitude to spare. The computation itself is a first-order description at the boundary: at the derived radius the zero-point energy is 0.59 of the curvature energy, and no parameter of the theory makes the next order small. At first order, $\zeta(-2)$ is negative at every shape, opposite in sign to that of a scalar field; every least value of the energy lies above 1.48 Planck lengths; and with the local term fixed at $\sqrt{6}$ Planck lengths the derived point is a stable minimum, the only one in shape between $\sqrt{3}$ and $\sqrt{12}$. The full computation, with its method and its

one disputed level, is Appendix D, and the book's additional materials give the computation step by step, with its inputs (Appendix D, Section V).

**At a radius of a few Planck lengths, why is the ten-dimensional action led by the Einstein-Hilbert term, when the higher Lovelock terms are not suppressed?**

The Einstein-Hilbert term leads because it is the only term linear in curvature, not because the others are small. Their coefficients are not fixed in this book (Section I). With the moduli fixed, each higher term adds to the four-dimensional Planck mass, to the bare curvature term and to the couplings of the moduli, and the four-dimensional Gauss-Bonnet term is a total derivative. The form of the four-dimensional equations is therefore fixed by Lovelock's theorem in four dimensions whatever the coefficients are (Chapter 12, Section VII). Their classical energy would shift the bare curvature term, and with it the factor f of the cosmological constant (Chapter 13, Questions).



Chapter 12

# The Einstein Field Equations

*Each Term and Its Origin, and Why the Form Is Unique*

In this chapter I state the four-dimensional field equations that the reduction of Chapter 11 produces, and give the origin of each term. The form of the equations follows from Lovelock's theorem once the moduli are held, given covariance and the second-order condition; Section VII states exactly what the product geometry adds. The coupling $G_N$ is the one constant taken from measurement. The cosmological term is fixed in Chapter 13. The context curves where the content is, in proportion to the energy and momentum of the content, because the context carries the asymmetry that the content and the act of comparison cannot carry (Section V). The matter enters through a symmetric, conserved stress-energy tensor, and each alternative of the matter carries its own classical geometry.

*Rests on:* the Root Tautology (Chapter 1, Section IV), the Incommensuration Theorem and its physical reading (Chapter 3), the partition (Chapter 6), the content as quantized and the context as the arena (Chapter 7, Sections IV and V), the product geometry (Chapters 8 to 10) and the reduction with its floor (Chapter 11). *Refers to:* the modal boundary at the Planck scale (Chapter 17), and the principle that a difference no comparison can register is no difference (Chapter 18, Section II). *Used by:* Chapter 13, Chapters 15 and 16, Chapter 19, and Appendix D. *Referred to by:* Chapter 11 (Questions), Chapter 17 (Sections III, VI and VII), Appendix A (Section VI), and Chapter 18 (the content's spinors).

## I. The Equation

The Kaluza-Klein reduction of Chapter 11, on the product manifold $M^4 \times \mathbb{C}P^2 \times S^2$ with the moduli held, gives in four dimensions the Einstein field equations:

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G_N\, T_{\mu\nu}. \tag{12.1}$$

Each term has a definite origin, and the sections below give them in turn. One quantity, $G_N$, is taken from measurement. Section VII shows that, within the conditions of Lovelock's theorem, no other equation is possible.

## II. The Einstein Tensor

The left-hand side of the Einstein field equations is the Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}, \tag{12.2}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor and $R$ is the Ricci scalar. Both are constructed from the metric $g_{\mu\nu}$ and its first and second derivatives alone.

The Einstein tensor arises from the Kaluza-Klein reduction of the ten-dimensional Ricci scalar $R_{10}$. When the ten-dimensional metric is decomposed into the four-dimensional metric and the internal metric, the ten-dimensional Ricci scalar splits into the four-dimensional Ricci scalar plus curvature terms from the internal space. The four-dimensional Einstein tensor is the variational derivative of the reduced action with respect to the four-dimensional metric.

The Einstein tensor measures the curvature of the context factor. I identify that curvature as the expression of the continuously asymmetric character of the context (Chapter 6). In a curved manifold, an infinitesim-

al change of position, a change of context, produces a change in parallel transport, a change of content. The relation is continuous, since the manifold is smooth, and asymmetric, since the curvature depends on direction.

## III. Newton's Gravitational Constant

The factor $G_N$ on the right-hand side sets the strength of the gravitational coupling: the proportionality between the curvature of spacetime and the energy-momentum content within it. Its measured value is $6.67430 \times 10^{-11}$ m³ kg$^{-1}$ s$^{-2}$. It is the one measured constant of this book.

The reduction of Chapter 11 relates $G_N$ to the geometry. The Planck mass is fixed by the ten-dimensional scale and the volume of the internal space: $M_{Pl}^2 = M_{10}^8 \, \mathrm{Vol}(K)$. Here $M_{Pl}$ is the full Planck mass, $G_N = \hbar c / M_{Pl}^2$, and $M_{10}$ is formed in the same way from the ten-dimensional coupling, with no factors of $2\pi$.

The relation fixes the strength of gravity in terms of the ten-dimensional scale and the internal volume. It does not fix either of them independently. I therefore read the relation in the other direction: the measured $G_N$ fixes the Planck mass, and the Planck mass sets the scale at which the modal boundary lies (Chapter 17).

## IV. The Cosmological Term

The term $\Lambda g_{\mu\nu}$ carries the energy density of the vacuum, $\rho_\Lambda = \Lambda / (8\pi G_N)$. Quantum field theory estimates the vacuum energy density at the Planck scale as of order $M_{Pl}^4$; the observed value is about $10^{-123}$ of that (Weinberg 1989).

In Chapter 13 I give $\rho_\Lambda$ as the share of the vacuum's energy that is accessible to an observer inside the vacuum:

$$\rho_\Lambda = c_1 \, f \, M_{Pl}^4 \, / \, S, \tag{12.3}$$

where $c_1 = 3$ is the coefficient of the first Chern class of $\mathbb{C}P^2$, equal to its Euler characteristic (Chapter 9), $f\,M_{Pl}{}^4$ is the vacuum energy density at the modal boundary, with $f = 1 - 3/(8\pi)$ from the curvature of the internal space, and S is the entropy of the observer's particle horizon. Combined with the Friedmann equation, the formula gives a relation in which the Hubble constant cancels and which fixes the matter fraction: the predicted $\Omega_m$ is 0.311, and the measured value is $0.315 \pm 0.007$ (Chapter 13). The equation of state is $w = -1$: $\Lambda$ is a constant, not a dynamical field.

## V. The Stress-Energy Tensor

The right-hand side of the Einstein equations contains the stress-energy tensor $T_{\mu\nu}$, which encodes the energy, momentum, and stress of the matter and radiation fields.

The matter belongs to the content factor. Its fields are fields over $M^4$, whose kinds and charges are read on the internal space as a space of states, on which no field lives (Chapter 8, Section IV): discontinuously symmetric, and so quantized (Chapter 7). Whatever those fields are, they enter the gravitational equations only through their stress-energy tensor.

Why the curvature goes with the content follows from the character of the arena (Chapter 7, Section V). The act of comparison is continuous, and the content is held to symmetry; the asymmetry that neither can carry is carried by the context, and that asymmetry is there because the content is there. I identify where and how much the arena curves. It curves where findings can be made, and in proportion to how many can be made there. Laid out everywhere at once, that curving is the gravitational field.

Energy and momentum measure how many findings can be made, by the Root Tautology (Chapter 1, Section IV): every finding is an interaction, and energy and momentum are what an interaction exchanges. So the source of the curvature is the density and flow of the energy and momentum of the content, and that is the stress-energy tensor. Section VII gives the one form of equation that relates the two.

The stress-energy tensor $T_{\mu\nu}$ is constructed in the standard way from the matter Lagrangian. It is symmetric, and it is conserved: $\nabla^{\mu}T_{\mu\nu} = 0$. Those two properties are all that the gravitational equations require of it. I make no claim about the particular fields that compose the stress-energy tensor, their charges or their number.

## VI. One Geometry for Each Alternative

The matter is quantum, so it can be in several alternatives at once. I identify the geometry with the context of each alternative of the matter: where there is content there is context, so each alternative of a content has its own context, and so its own classical geometry, shaped by its own stress-energy. The field equations above hold in each alternative. A geometry shaped by the alternatives together would carry, in each alternative, curvature where that alternative has no content to source it.

Where the matter is sharply one alternative, the equation is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle: \tag{12.4}$$

the equation of per-alternative geometry for one sharp alternative. Only in that case is the expectation value the source; the Møller-Rosenfeld theory uses it as the source whatever the alternatives, and predicts no entanglement through gravitation.

A geometry for each alternative quantizes nothing. Within each alternative the geometry is continuous and has no least step, and the discreteness of the alternatives is the content's. Where two bodies are each in several places, each joint alternative carries the gravitation of its own arrangement, and that gravitation shifts the phase of the joint alternative relative to the others. The phase belongs to possibility: gravitation changes how the alternatives combine, and leaves the probability of each alternative unchanged.

A relative phase is a comparison of two alternatives, and a comparison needs a term common to both. The geometries of two alternatives are two geometries, so no event of the one is an event of the other, and neither geometry supplies the common term. I take the common term to be the region far from the bodies, where the geometry of every alternative is flat and one measure of time serves them all. I take the phase of each alternative to advance, in that time, at the rate set by the whole energy of that alternative, which is the energy its own geometry carries. In general relativity that energy is a quantity of the far region alone (Regge and Teitelboim 1974), so the comparison uses the far region of each geometry and no event within it. For two bodies of mass m at separation d, the energy of the alternative contains the term $-G_N\ m^2/d$ once, and in a time t that term advances the phase of the alternative by $G_N\ m^2\ t/(\hbar\ d)$. Let each body have a place nearer to the other body and a place farther from it. Whether the two bodies entangle is fixed by one combination of the four joint phases: the phases of the two joint alternatives in which both bodies are in their nearer places or both are in their farther places, less the phases of the two in which one body is in its nearer place and the other is in its farther place. No change made to the phases of one body alone alters this combination, and for the experiment of the Questions below it is 0.314 radians.

To first order in $G_N$ the combination is the same whichever single common term is taken, the far region or a clock whose matter is sharply one alternative, since such a choice shifts the phase of each joint alternative only by parts fixed by the place of one body. The phase between the two places of one body does depend on the common term, and with the far region it is the phase that neutrons show in the Earth's field (Colella, Overhauser and Werner 1975). The proper times of the bodies alone, without the energy of the geometry, count the interaction twice, and so give twice the phase above. Below the modal boundary this construction is the one of linearized quantum gravity, and I adopt that construction there: the geometry of each alternative is a small departure from one flat

geometry, the departures are matched in the far region, and the radiation of each alternative has a state of its own in the description of the small waves by quanta, so that two alternatives combine only as far as those states overlap (Danielson, Satishchandran and Wald 2022). The adoption assumes three things: that the geometries of the alternatives differ by small departures, that they share a far region, and that the curvature is far below the Planck curvature. Where any of the three fails, as for alternatives that place a horizon differently, or for alternatives of the whole universe, which has no far region, I give no construction of the relative phase, and the relative phase of those alternatives is not fixed in this book.

The framework contains no quantum theory of the geometry; a geometry for each alternative is not an approximation to one. I identify discrete, symmetric observables as the character of content, and continuous, asymmetric observables as the character of context (Chapter 6). A quantized arena would carry the first in the factor whose observables are the second. The one factor would then carry continuity with symmetry, or asymmetry with discontinuity, and the Incommensuration Theorem excludes both (Chapter 3).

## VII. Uniqueness: Lovelock's Theorem

Lovelock's theorem (1971; 1972) states that in four dimensions the most general symmetric, divergence-free tensor built from the metric and its first and second derivatives is a linear combination of the Einstein tensor $G_{\mu\nu}$ and the metric tensor $g_{\mu\nu}$:

$$A\, G_{\mu\nu} + B\, g_{\mu\nu},$$

where A and B are constants. Setting this equal to $8\pi G_N\, T_{\mu\nu}$, with $A = 1$ and $B = \Lambda$, gives the Einstein equations with cosmological constant. Within the theorem's conditions there is no other equation.

The theorem applies to the theory reduced from ten dimensions because of two facts. The first fact is that the moduli are fixed, which I establish in Chapter 11, Section II. The second fact is a result of Chapter 11, Sec-

tion I. With the moduli fixed, the higher Lovelock terms of ten dimensions reduce to the two terms above (the Einstein tensor and the metric tensor) or to four-dimensional total derivatives.

The second-order condition means that the initial data are the metric and its first time derivative. Field equations of higher order carry extra degrees of freedom, and these generically include states of negative energy (Ostrogradsky 1850). Curvature-squared terms may still appear as corrections to the effective theory. Their size relative to the Einstein term is the curvature times the square of the Planck length. Far below the Planck curvature the corrections are negligible, and the Einstein form governs. At the Planck curvature they are not negligible, and that is where the book places the modal boundary (Chapter 17).

The matter action is invariant under changes of coordinates, so $T_{\mu\nu}$ is conserved wherever the matter field equations hold. The left-hand side must therefore be a tensor with zero divergence, which is the hypothesis of Lovelock's theorem. The Einstein tensor meets it identically, by the contracted Bianchi identity, $\nabla^{\mu}G_{\mu\nu} = 0$, a theorem of Riemannian geometry. I identify the invariance under changes of coordinates with the omniscient modality, which requires that what is compared not depend on how it is described (Chapter 11, Section I).

The scope of this result is exact. Once covariance and the second-order condition are granted, the form of the equations follows from Lovelock's theorem in four dimensions, and the reduction adds nothing to that form. What the product geometry adds is the setting in which the theorem applies, and what the theorem leaves free: that the extended spacetime is four-dimensional and Lorentzian (Chapter 10), so that the four-dimensional theorem is the one that applies; the bare curvature term of the internal space, which enters $\Lambda$ (Chapter 13, Section II); the relation of the Planck mass to the internal volume (Chapter 11); and the identification of covariance with the omniscient modality (this section).

## VIII. What This Chapter Establishes, and What It Leaves

Given the product geometry and the reduction with its moduli held, the four-dimensional gravitational equations, at curvatures far below the Planck curvature, are $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ and no others. The context curves where findings can be made, in proportion to how many can be made there, and the energy and momentum of the content are the measure of that count. The matter enters through a symmetric conserved tensor, and each alternative of the matter carries its own geometry. The value of $\Lambda$ is fixed in Chapter 13.

I leave three things open. The value of $G_N$ is taken from measurement. Which fields compose $T_{\mu\nu}$, with which masses and charges, is outside this book. The energy of the floor beyond its first order, and the coefficients of the higher Lovelock terms, are not fixed in this book (Chapter 11).

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### Questions

**Is a classical geometry for each alternative not simply a superposition of geometries, which is the low-energy content of quantum gravity?**
Below the modal boundary the two descriptions predict the same. In each, the geometry is correlated with each alternative of the matter, and the alternatives keep their relative phase. Two masses, each held in two places and related only through gravitation, become entangled in both descriptions, with the phase of their Newtonian interaction (Bose and colleagues 2017; Christodoulou and Rovelli 2019). No experiment below the Planck scale tells this account from linearized quantum gravity.
The two descriptions differ in what is superposed, and in where the description ends. Here the alternatives are the content's, and the four-dimensional geometry of each is continuous, with no least step. The small waves of the metric are described by quanta, and the quanta are quanta of the description of the wave, not things in the arena. The nearest sense in

which such a quantum can be made intelligible is the phonon, a quantum of the description of the vibrations of a held structure such as a solid; the arena is not a held structure, so even that sense does not make the quantum a quantum of the arena. What the framework excludes is any quantum of the arena itself, at every scale; so it excludes the graviton taken as a real particle, as a carrier of gravitation or as one of the kinds (Chapter 6). The description by quanta is a description of the small waves, and it ends at the Planck scale, where the book places the modal boundary (Chapter 17). A theory that quantized the arena would have to continue past that boundary; the framework has none to continue to. The claim that quantum gravity is a category error holds at every scale; this account and linearized quantum gravity differ in what they predict only at the Planck scale and beyond. Below it, the account is tested against the programs that keep one geometry, as the next question sets out.

**Which experiments test this account?**

The entanglement experiment of Bose and colleagues (2017): two masses of $10^{-14}$ kg, each held in two places 250 μm apart, with centers 450 μm apart (closest approach 200 μm), for 2.5 s. For these numbers the entangling phase is 0.314 radians, and the account predicts that the entanglement appears with that phase.

**If a detector registers a single quantum of a gravitational wave (the literature calls this the detection of a single graviton), what has it registered?**

A detector that registers one quantum of a gravitational wave (Tobar, Manikandan, Beitel and Pikovski 2024) registers a quantum of the wave's description, which the framework admits. Such a signal is equally accounted for by a classical wave acting on a quantum detector, as the photoelectric effect does not by itself show that light is made of photons (Carney, Domcke and Rodd 2024). What no detection of this kind can find is a quantum of the arena: the arena has none, at any scale.

**How can a metric that is not quantized have a zero-point energy, as Chapter 11 uses?**

The zero-point energy belongs to the description of the metric's small motions by quanta. "Not quantized" means two things in this book: the form of the geometry has no least step, and there is no quantum of the arena at any scale. A quantum of the description is a step in the energy of one mode of the wave; it is not a step in the extent of the geometry, whose four-dimensional lengths, areas and times keep no least value. The description is the effective field theory of gravity (Donoghue 1994a; 1994b), whose corrections to the Newtonian potential are of relative size $(l_{Pl}/r)^2$, of order $10^{-70}$ at a separation of one meter. The same description keeps a geometry for each alternative consistent with causality and complementarity: it gives the radiation of each alternative a state of its own, so two alternatives are told apart by what they emit only as far as those states differ, and the least motion of the small waves bounds the localization of any body at about the Planck length (Belenchia and colleagues 2018; Danielson, Satishchandran and Wald 2022). That bound is a limit on how sharply a body can be placed, set by the motion of the waves; it is not a least length of the geometry.

**Does the field follow the outcome of a quantum event, or the average over outcomes?**

The outcome. Page and Geilker (1981) tested the Møller-Rosenfeld form, in which $\langle T_{\mu\nu} \rangle$ is taken in a state that never collapses. They moved the masses of a Cavendish balance according to a quantum random outcome, and the measured field followed the outcome, not the average. In this account each alternative of the matter sources its own geometry, so the field follows the outcome. A finding is content: discrete, and enacted at a measurement, in the first person, by the comparer that finds. It selects among alternatives that already carry their geometries, so no geometry changes at the finding, here or anywhere. What a distant finding changes for a comparer here is its record, and it enters the record when a signal carry-

ing the finding arrives, at the earliest on the comparer's past light cone. The past light cone is fixed by the geometry and is the same in every frame, so the account needs no preferred frame.

A surface of simultaneity on which the selection would be made for every comparer at once is registered by no comparison: no geometry changes on it, and the observed results of local findings are the same whichever spacelike surface is chosen (Aharonov and Albert 1984; Peres 2000). The principle that a difference no comparison can register is no difference applies to every possible comparison, and in this account every comparison is made at an event and receives a distant finding at the earliest on its past light cone. A comparison that read the state assigned on a spacelike surface, and not only the results of local findings, would let a distant comparer's choice of measurement be read at once, which is a signal faster than light (Sorkin 1993). No comparison therefore registers which spacelike surface the selection is made on. A difference that no comparison can register is no difference (Chapter 18, Section II), so the choice of such a surface is no fact. Two comparers reconcile their records by a further comparison, at an event in the common future of their findings, and their records of one finding agree. Within each alternative the stress-energy is conserved and the Bianchi identity holds; no event of the account has a source whose divergence fails.

**Does a classical field coupled to quantum matter not permit signaling faster than light?**

Not in this account. A single geometry sourced by the weights of the alternatives together would change at once when a distant finding changes those weights, and so would pass a difference faster than light (Gisin 1990; Bahrami and colleagues 2014). Here no geometry is sourced by the weights, and no geometry changes at a finding. Eppley and Hannah (1977) argued that a classical gravitational field coupled to quantum matter would permit either a violation of the uncertainty principle or signaling faster than light. Mattingly (2006) showed that the apparatus their argument needs cannot be built in our universe; Huggett and Callender

(2001) showed that the argument falls short of a theorem; and Kent (2018) showed that it fails when gravity couples to the local quantum state. Their argument does not exclude the account of Section VI.

**°ℂP² admits no spin structure, so the ten-dimensional product admits none. How does matter of half-integer spin exist in this account?**

The account needs a spin structure on $M^4$ alone, and $M^4$ has one. A spin structure on a manifold is what lets spinor fields, the fields of half-integer spin, be defined over the whole manifold. Every four-dimensional spacetime that is orientable, time-orientable and globally hyperbolic admits a spin structure (Geroch 1968; 1970). The spin of a particle is the way its field changes when the frames of $M^4$ are rotated, and the half-integer values of spin label representations of Spin(1,3), the double cover of the group of Lorentz transformations of those frames. I take the content's fields of half-integer spin to be spinors of $M^4$: fields over $M^4$, whose kinds and charges are read on $\mathbb{C}P^2 \times S^2$ as a space of states, on which no field lives (Chapter 8, Section IV). A label read there is a count of spinors and representations on the space of states, carried by its differential forms (a function is a form of degree zero), and not a mode of a field (*The Kinds of Particle*, Chapter 3, Section VII). A spinor field of $M^4$ is defined whatever the topology of the internal factor. Its labels form a discrete set, as Chapter 8, Questions, requires, because the representations of a compact group form a discrete set. A spin structure belongs to the manifold $M^4$ and does not depend on its metric, so one spin structure serves the geometry of every alternative of the matter (Section VI).

The ten-dimensional product admits no spin structure. An orientable manifold admits one exactly when its second Stiefel-Whitney class is zero. For $\mathbb{C}P^2$ that class is the reduction modulo 2 of the first Chern class, which is three times the class of a projective line; three is odd, so the class is not zero (Hawking and Pope 1978). Each slice of $M^4 \times \mathbb{C}P^2 \times S^2$ at a fixed point of $M^4$ and a fixed point of $S^2$ is a copy of $\mathbb{C}P^2$, and on each such slice the second Stiefel-Whitney class of the product restricts to that of $\mathbb{C}P^2$; so

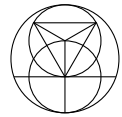
the class of the product is not zero either, and no spinor field of ten dimensions is defined on the product. The product does admit a spin$^c$ structure. A spinor field defined with a spin$^c$ structure must carry a charge under a $U(1)$ connection whose curvature on $\mathbb{C}P^2$ is not zero.

With the content's spinors taken on $M^4$, no result of this book changes. The gravitational action stays the action of the metric alone (Chapter 11, Section I). The counts of Chapter 9 are counts of the cohomology classes of $\mathbb{C}P^2 \times S^2$; the bare curvature term is a value of the metric's action (Chapter 11, Section III); and the identity of Chapter 13 uses $c_1 = 3$ and that term. None of them involves a spinor. The matter enters the field equations through its stress-energy tensor, symmetric and conserved (Section V), whatever its spin. The step of taking the content's spinors on $M^4$ has one cost: the fields of half-integer spin are not spinors of the ten-dimensional geometry, so a rotation of frames that turns a direction of $M^4$ into an internal direction does not act on them. The product keeps those directions in separate factors (Chapter 8, Section III), and the account uses rotations within each factor and no others.

The alternative is a spin$^c$ structure on the internal factor. Its $U(1)$ connection would be either the connection that the metric of $\mathbb{C}P^2$ induces through the complex structure of $\mathbb{C}P^2$, or a field of its own. With a field of its own, the action would no longer be built from the metric alone; the field's classical energy would enter the external value of Chapter 13, Section II, beside the bare curvature term, and so change the predicted matter fraction; and its coupling would be a second constant beside $G_N$. I take no such field. The first form of the connection is already present in the step taken here, and it adds nothing to the action. $\mathbb{C}P^2 \times S^2$ is a Kähler manifold, and on a Kähler manifold the forms of type $(0, q)$, taken over every $q$, are the spinors of the spin$^c$ structure that the complex structure induces, with the connection that the metric induces; the forms of every type are those spinors tensored with the forms of type $(p, 0)$. The forms of $\mathbb{C}P^2 \times S^2$ that carry the content's labels therefore carry that spin$^c$ structure and its connection, whose curvature on $\mathbb{C}P^2$ is the Ricci form, a value of

the metric. The charge of a form under that connection is fixed by its type, a discrete label of the content, and no field beyond the metric is needed to define it. That fields valued in forms need no spin structure is a known property of the operator d + d* on forms (Benn and Tucker 1983).

The step predicts no particular field, so no observation of the particle spectrum bears on it.



Chapter 13

# Λ as a Prediction; Information Cost at the Horizon

*On the Cosmological Constant and the View from Inside*

An observer that measures the vacuum is inside the vacuum it measures. In this chapter I take the accessible share of the vacuum's energy to be $c_1 = 3$ degrees of freedom among the S of the observer's particle horizon, and from that share obtain the cosmological constant. Combined with the Friedmann equation, the result is a relation in which the Hubble constant and the Planck mass both cancel. That relation predicts the matter fraction of the universe: $\Omega_m = 0.311$, against the measured $0.315 \pm 0.007$. The same comparison, made in energy density, places the predicted vacuum density about 1.7 percent above the observed one. The equation of state is $w = -1$.

*Rests on:* what a measurement registers (Chapter 7, Sections I and III), the count of the classes of $\mathbb{C}P^2$ (Chapter 9, Sections I, IV and VII), Axiom III (Chapter 4, Section V), the bare curvature term at the derived radius (Chapter 9, Section IX; Chapter 11, Section III), the field equations with Λ as their constant term (Chapter 12, Section VII), the flatness of the epoch (Chapter 18, Section IX), the modal boundary and the full Planck mass (Chapter 17), and the measured $G_N$. *Refers to:* the one stance of the comparing (Appendix C, Section V). *Used by:* Chapter 16, Chapter 18 (the deficit inside a black hole), Chapter 19, Chapter 20 and Appendix B. *Referred to by:* Chapter 17, and Chapter 9 (Questions).

## I. The Problem of Being Inside

Consider a measurement of the vacuum energy density, the quantity the cosmological constant carries. This measurement is performed by an observer who is located within the universe whose vacuum is being measured. There is no laboratory outside the universe, and no control sample of empty space that the observer can compare to. The observer, the apparatus, and the vacuum are all parts of the same system.

In most measurements, the observer can be treated as separate from the system. When you measure the length of a table, you are not part of the table. When the system is the entire universe, this idealization fails. The observer's own degrees of freedom are excitations of the same quantum fields whose ground-state energy the observer is trying to measure, and they are entangled with the vacuum.

That entanglement leaves an information deficit which no better instrument removes. Some of the information about the vacuum is held in the correlation between the observer and the vacuum, and it is not accessible to the observer. A system cannot fully measure itself. I identify this deficit as an instance of the Incommensuration Theorem at the cosmological scale.

## II. The Information Deficit Formula

I identify the cosmological constant with the share of the vacuum energy that is accessible to an observer inside the vacuum, not with the total. The total is the external value: the vacuum energy density at the scale of the modal boundary (Chapter 17). The observed share is the internal value. The internal value divided by the external value is the accessible share; what the share leaves out is the information deficit, the price of being inside.

The external value has a ground-state part and a classical part. The ground-state part is the vacuum energy of the quantum fields at the scale of the modal boundary; I take it as $M_{Pl}{}^4$, the Planck density. The floor that holds the internal space (Chapter 11, Section II) is a zero-point energy, and I count it within the ground-state part. The classical part is the bare curvature term of the reduction (Chapter 11, Section III), which carries no ħ and has the sign of anti-de Sitter space. At the derived radius (Chapter 9, Section IX) it is $-3/(8\pi)\, M_{Pl}{}^4$. The external value is therefore

$$f\, M_{Pl}{}^4, \quad \text{with } f = 1 - 3/(8\pi) = 0.881. \tag{13.1}$$

The count of the horizon is its entropy,

$$S = \pi\, R^2 \,/\, l_{Pl}{}^2, \tag{13.2}$$

where R is the radius of the observer's particle horizon and $l_{Pl}$ is the Planck length. That entropy is the Bekenstein-Hawking entropy (Bekenstein 1973; Hawking 1975): the count of the independent degrees of freedom within the horizon. The particle horizon is the bound of what has reached the observer, and it is the only horizon a first-person measurement holds. The event horizon is defined by all that could reach the observer in the future, which no present measurement contains; Chapter 18, Section III, uses it for that future-directed question. The particle horizon is not a thermodynamic horizon in the sense of Gibbons and Hawking. I assign it the Bekenstein-Hawking count by an identification: the bound of what has been compared is the bound of the information available to the comparison.

I take the accessible share of the external value to be $c_1/S$, with $c_1 = 3$: one degree of freedom for each channel, among the S degrees of freedom the horizon counts. The share rests on three steps, and I state each with its ground. The S degrees of freedom hold the external value in equal parts; the channels are three; and a measurement engages a different degree of freedom through each of its channels.

The first step makes the share a ratio of counts. I take the S degrees of freedom to be what holds the external value within the horizon: the vacuum energy within the horizon is the sum of their parts, one part for each degree of freedom. They are counted on the area of the horizon, and the energy they hold is the energy of the volume within it, so a fraction of that energy is the same fraction of the density at every place. I take the holding to distinguish no degree of freedom from another. Two facts ground this: the external value is a density, the same at every place, and the count is proportional to the area of the horizon with one constant, so equal elements of the horizon count the same. Degrees of freedom that nothing distinguishes hold equal parts. Each then holds 1/S of the external value, and any k of them hold exactly k/S of it. Once the two premises are granted, the sum of parts and the absence of any distinction, this ratio is a theorem, and it holds however the degrees of freedom are correlated with one another. A measurement distinguishes the degrees of freedom it engages, but the holding does not, and the parts are fixed by the holding. The correlation between the observer and the vacuum (Section I) is the reason a measurement receives only the parts held by the degrees of freedom it engages; the number of those degrees of freedom, not the strength of the correlation, sets the size of what it receives.

I count S in nats, the measure in which S is stated and in which $dE = T\,dS$ holds with Boltzmann's constant equal to one. In that measure Section III finds that the observed density gives each degree of freedom at the de Sitter radius the energy $\hbar c/(2\pi r_\Lambda)$, the value at which Bekenstein's bound is saturated. The choice of measure decides the number: in bits the same horizon counts $S/\ln 2$, and one bit for each channel would give $\Omega_m = 0.407$ in place of 0.311.

The second step counts the channels. To register is to find a sameness or a difference and to leave a reading, a count or a record of it. Chapter 7, Section I, states what a measurement registers: "What is measured is content; the arena in which it is measured is context. Each content metric is measured in its own context: pattern in space, force in time, probability in

possibility." I identify the channels through which the vacuum couples to a measurement with the contents that a measurement registers, one in each modal role. Every act of measurement involves all six metrics (Chapter 7, Section I), and so it registers one content in each of the three columns of the grid. The content row has exactly three members, one in each modal role (Chapter 7, Section III), so the channels are three, one for each column. The context row supplies no channel of its own, because in each column the context is the arena in which the measurement registers the content, not a second thing that the same measurement registers. The two rows therefore do not double the count: each column is counted once, by its content.

A length and a duration are registered in the same way. A length is registered as the extent of a pattern in space, and a duration as the course of a change in time; the space and the time are the arenas in which they are registered. The vacuum's energy acts on the geometry, which is context, and $\Lambda$ is the constant term of the field equations of the context (Chapter 12, Section VII); a measurement finds $\Lambda$ only through what it registers, which is content. The ground of the count is not that content and context are inseparable. The three modal roles are inseparable too (Axiom III), so inseparability treats the rows and the columns alike: counting inseparable terms once would give one channel, and counting distinct terms separately would give six. The ground is the difference between the two members of the row pair: content is what a measurement registers, and context is where the measurement registers it. The three members of the content row stand alike, each as what is registered.

The internal space carries this count. Its six independent classes are the products of the three classes of $\mathbb{C}P^2$ with the two classes of $S^2$ (Chapter 9, Section IV), and $S^2$ carries one class for each row: "the factor that carries the rows has one for each row, two in all, and no others" (Chapter 9, Section I). The classes that share one class of $S^2$ number three, whichever class of $S^2$ it is: $\chi(\mathbb{C}P^2) \times 1 = 3$. So the $S^2$ factor enters the count: it fixes the row in which every channel registers, and it contributes the factor one, not

its Euler characteristic, two. The first Chern class of the product is the sum of the first Chern classes of its factors: three times the class of a line, from $\mathbb{C}P^2$, and twice the class of a point, from $S^2$. It is a pair of integers, (3, 2), and not one count.

The count of channels is therefore the Euler characteristic of $\mathbb{C}P^2$, the number of its independent classes. The condition of Chapter 9, Section I, sets the number of those classes equal to the number of modal roles. I write the count $c_1$, the coefficient of the first Chern class of $\mathbb{C}P^2$, because on every complex projective space the two integers are equal, and for the same reason: each counts the alternatives of the system whose pure states the space is. $\mathbb{C}P^n$ is that space for a system with n + 1 alternatives, as $\mathbb{C}P^2$ is for three (Chapter 9, Section IV). It has n + 1 cells, one in each even dimension from 0 to 2n. Its tangent bundle, with one trivial line added, is, as a complex vector bundle, the sum of n + 1 copies of one line bundle, one copy for each alternative, so its first Chern class is n + 1 times the class of a hyperplane (Milnor and Stasheff 1974). No property of the first Chern class beyond this equality is used here. The name $c_1$ is a convenience; the count is $\chi(\mathbb{C}P^2) = 3$.

The third step fixes how many degrees of freedom a measurement engages through its channels. The comparer is one stance: "the comparing is one stance, the subjective, from which every finding is made" (Appendix C, Section V). The degrees of freedom a measurement engages are therefore counted once, for the one stance, and not once for each instrument. I take each channel to engage one degree of freedom of the count, and a different one for each channel, since the three contents that the channels carry are distinct (Axiom III). A measurement then engages three of the S degrees of freedom and receives the parts they hold, $c_1/S$ of the external value. The information deficit formula is then

$$\rho_\Lambda = c_1 \, f \, M_{Pl}^{\,4} \, / \, S, \quad \Lambda = 8\pi G_N \, \rho_\Lambda. \tag{13.3}$$

The suppression of the observed density relative to the Planck density is about $10^{-123}$, which is the reciprocal of the horizon entropy. The formula makes the deficit proportional to $1/S$ because the comparer is one stance inside the horizon, which engages the S degrees of freedom of the horizon's count through three channels, a different degree of freedom through each.

## III. The Self-Consistency Identity

The formula and the Friedmann equation must agree: the $\Lambda$ the formula gives must reproduce the expansion history from which the horizon is computed. The agreement is one equation. Define the dimensionless conformal time integral

$$I(\Omega_m) = \int_0^\infty dz \,/\, E(z), \tag{13.4}$$

where $E(z) = \sqrt{(\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4)}$ is the dimensionless Hubble function, with $\Omega_r \approx 9 \times 10^{-5}$ the present density of radiation (photons and neutrinos, which are relativistic in the early universe, where the radiation term matters). The integral is the comoving distance light has traveled over the whole expansion, and the particle horizon is $R = I/H_0$, in units with $c = 1$.

The derivation takes three lines. The horizon entropy is $S = \pi R^2 M_{Pl}^2$, so the formula gives

$$\rho_\Lambda = c_1 \, f \, M_{Pl}^2 H_0^2 \,/\, (\pi I^2). \tag{13.5}$$

The Friedmann equation gives the vacuum energy density as a fraction of the critical density:

$$\rho_\Lambda = \Omega_\Lambda \times 3H_0^2 M_{Pl}^2 \,/\, (8\pi). \tag{13.6}$$

Equating the two, the Hubble constant and the Planck mass cancel, and

$$I^2 \times \Omega_\Lambda = 8c_1 f/3 = 8f = 8 - 3/\pi. \tag{13.7}$$

The 8 has a simple reading. The $8\pi$ of the Einstein equations, divided by the $\pi$ of the horizon entropy, gives 8; the three channels, $c_1 = 3$, cancel the 3 of the critical density.

The identity is one equation in one unknown, $\Omega_m$, since $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$ for a spatially flat universe, which the restoration of Chapter 18, Section IX, gives. Its solution is

$$\Omega_m = 0.311, \quad \Omega_\Lambda = 0.689. \tag{13.8}$$

The measured value is $\Omega_m = 0.315 \pm 0.007$ (Planck 2018); the baryon acoustic oscillations of DESI alone give $0.2975 \pm 0.0086$ in the same model (DESI 2025), from which the prediction lies 1.6 standard deviations. With the ground-state part of the external value alone ($f = 1$), the identity is $I^2\Omega_\Lambda = 8$, and its solution is $\Omega_m = 0.280$. Figure 3 shows both solutions beside the measurements.

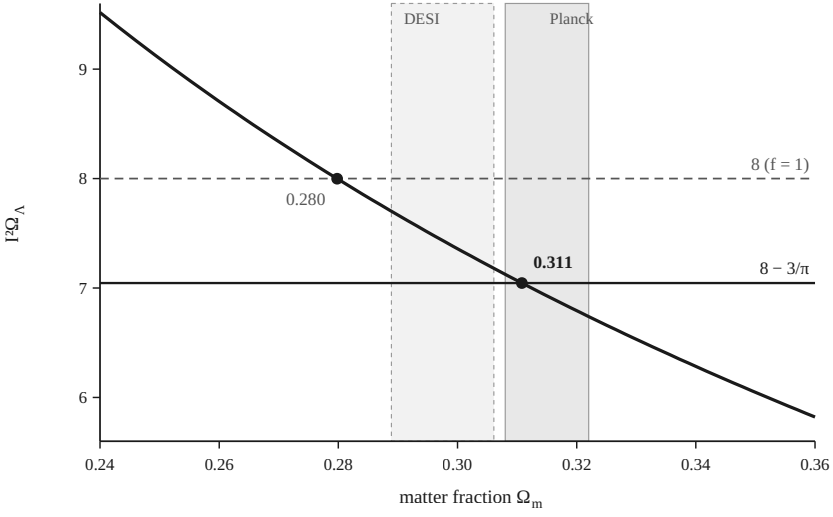


***Figure 3.*** *The left side of the identity, $I^2\Omega_\Lambda$, as a function of the matter fraction in a flat universe with $\Omega_r = 9 \times 10^{-5}$ (curve). The identity sets it to $8 - 3/\pi$ (solid line), which gives $\Omega_m = 0.311$; with the ground-state part alone, $f = 1$, it is 8 (dashed line), which gives 0.280. The shaded bands are the measured matter fraction from Planck 2018, $0.315 \pm 0.007$, and from the baryon acoustic oscillations of DESI 2025 alone, $0.2975 \pm 0.0086$.*

The identity can be stated without Planck units. With $\Lambda = 3/r_\Lambda{}^2$, $r_\Lambda$ the de Sitter radius, it says that the present particle horizon is $2\sqrt{2}\ \sqrt{f} = 2.65$ times the de Sitter radius. With the measured parameters the ratio is $46.1/17.5 = 2.63$ (both in billions of light years). Equivalently, the identity fixes the age of the universe in units of the de Sitter time, $1/H_\Lambda$ with $H_\Lambda = \sqrt{(\Lambda/3)}$: it gives $t_0 H_\Lambda = 0.792$, against $0.787 \pm 0.009$ from the measured parameters. Read this way, the identity states the epoch at which the measurement that fixes $\Lambda$ is made; that measurement is the present one (Section VI), and the present epoch is that epoch, within the measurement. It is the same test as $\Omega_m$, stated in time.

The identity can also be read in terms of information. Take the vacuum energy within a radius, divided over that radius's own count, as the energy per degree of freedom. With the observed density, at the de Sitter radius, the energy per degree of freedom is $\hbar c/(2\pi r_\Lambda)$, the value at which Bekenstein's bound is saturated (Bekenstein 1981; Gibbons and Hawking 1977). With the observed density, at the particle horizon, it is $I^2\Omega_\Lambda$ times the corresponding value there, $\hbar c/(2\pi R)$. The formula requires $8f$ times that value. Equating the two is the identity.

The identity fixes the dimensionless split of the energy, not the Hubble constant, which cancels from it. It fixes the total matter fraction and says nothing of what the matter is made of. The identification of Section II, the accessible share $c_1/S$ of the particle horizon's count, has four parts: the count S of the particle horizon, the equal parts of the external value, the count of three channels, and one degree of freedom for each channel.

## IV. The Numbers

The parameters are those of Planck 2018, used throughout as one set: h = 0.674, $\Omega_m = 0.315$, $\Omega_r = 9 \times 10^{-5}$, and a flat universe. With them the particle horizon is 46.1 billion light years, and its entropy is $S = 2.29 \times 10^{123}$. The formula gives

$$\rho_\Lambda = 3 \times 0.881 \times (2.222 \times 10^{76}\ \mathrm{GeV}^4) \,/\, (2.29 \times 10^{123}) = 2.56 \times 10^{-47}\ \mathrm{GeV}^4. \tag{13.9}$$

The observed value, from the same parameters, is

$$\rho_{\Lambda,\mathrm{obs}} = 2.52 \times 10^{-47}\ \mathrm{GeV}^4. \tag{13.10}$$

With one set of parameters, the ratio of the predicted density to the observed one is exactly $8f/(I^2\Omega_\Lambda)$, with $I$ evaluated at the measured $\Omega_m$. So this comparison and the comparison of $\Omega_m$ in Section III are one test, seen two ways. The predicted density lies about 1.7 percent above the observed one. The uncertainty of the measured $\Omega_m$, $\pm 0.007$, moves the ratio by about $\pm 2.8$ percent, so the prediction lies within it. With the ground-state part alone, the predicted density is $2.91 \times 10^{-47}\ \mathrm{GeV}^4$, about fifteen percent above the observed one.

The prediction is sharp in its inputs. The measured radiation density, from $7 \times 10^{-5}$ to $10^{-4}$, moves it from 0.3119 to 0.3103, so this input gives $\Omega_m = 0.311 \pm 0.001$. A change of one percent in f moves $\Omega_m$ by $\mp 0.0025$, so the measured $\pm 0.007$ corresponds to about $\pm 3$ percent in f, and the coefficients of order one named in the Questions must be known to a few percent before the test is decisive. The count of the floor within the ground-state part is load-bearing: if the first-order energy of the floor, $+0.071\ M_{Pl}^4$ (Appendix D), were added to the external value instead, f would be 0.951 and $\Omega_m$ 0.292, about three standard deviations below Planck 2018.

The same result can be stated through one more measurement. The microwave background measures the matter density in physical units, $\Omega_m h^2 = 0.1430 \pm 0.0011$ (Planck 2018), and with the predicted $\Omega_m$ it gives $h = 0.678 \pm 0.003$. That value of h is the same test as $\Omega_m$, not an independent test. It agrees with the value of the Hubble constant inferred from the early universe: the local distance ladder gives $h = 0.730 \pm 0.010$ (Riess and colleagues 2022), about five standard deviations away, and with the same $\Omega_m h^2$ that value would require $\Omega_m = 0.268$, which the identity excludes.

The computations behind the numbers of this chapter are given, with their inputs, in the book's additional materials (Appendix D, Section V).

## V. The Equation of State

The information deficit formula predicts the equation of state of the cosmological constant to be $w = -1$ exactly.

The formula fixes $\Lambda$ from the information content of the horizon of the measurement, now. Every measurement is made in the first person: here, now, by an observer inside. The past is known only by the impression it makes on the present, and the history of the universe is reconstructed from that impression as the one history it requires.[8] The $\Lambda$ that appears in that history is the $\Lambda$ the present horizon fixes, since the measurement that fixes it is real, and the real is now (Section VI). A $\Lambda$ "larger in the past" would need a comparison made in that past, and no such comparison is part of the measurement; relative to the present measurement it has no meaning. That history is one geometry, whose field equations carry $\Lambda$ as the constant term of Lovelock's theorem (Chapter 12, Section VII); in one geometry $\Lambda$ is one constant, because the field equations themselves require it (Section VI). The epochs before any observer are part of the same history, and so are the epochs of earlier observers, with the findings they recorded; the $\Lambda$ they carry is the present one. So the formula does not track a horizon through time. It fixes $\Lambda$ once, from the horizon of the

measurement, and the reconstructed history carries that one value at every epoch. There is no dynamical field, no quintessence and no evolving scalar, driving the acceleration.

Planck 2018, combined with baryon acoustic oscillations and Type Ia supernovae, gives $w = -1.03 \pm 0.03$. Recent combinations have reported a departure: the final analysis of the Dark Energy Survey, with its own supernovae, galaxy clustering and weak lensing, combined with DESI and the cosmic microwave background, gives $w_0 = -0.82 \pm 0.05$ and $w_a = -0.63\ (+0.21/-0.18)$, about three standard deviations from a cosmological constant (Abbott and colleagues 2026). The departure does not rest on the supernovae: without them the same analysis still finds 2.6 standard deviations, and leaving out each probe in turn gives between 2.3 and 3.2.

A cosmological constant is the default of standard cosmology, so a confirmation of $w = -1$ supports this account only weakly. The account predicts that the reported departure will not be confirmed.

## VI. The Comparison That Fixes Λ

The cosmological constant has one value in the reconstructed history, and the formula of Section II gives that value at the particle horizon of the measurement that fixes it. I state why the value is one, which comparison fixes the value, why the same reasoning fixes nothing on the Hubble radius or on the event horizon, and what an observer at an earlier epoch carries.

In Chapter 5, Section II, I distinguish two claims: "To say that something exists is to say that it stands as structure: timeless, the same in every instance, like the pattern of a law or the state of a system at an instant. To say that something is real is to say that it is given in an actual act: temporal, occurring now, with the future still open to possibilities not yet settled." Λ is the constant term of the field equations (Chapter 12, Section VII), part of the law of the one geometry, so it exists. The value of the formula at the particle horizon of a measurement is given in that act, so it is real.

The field equations themselves require that $\Lambda$ be one number. In $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ the divergence of the Einstein tensor vanishes by the contracted Bianchi identity, the divergence of the metric vanishes, and the stress-energy of the content is conserved (Chapter 12, Section VII). If $\Lambda$ varied from place to place or from epoch to epoch, the divergence of the left side would be the gradient of $\Lambda$, and the equations would set that gradient to zero. So within one geometry $\Lambda$ does not vary. That $\Lambda$ does not vary within one geometry is a theorem of the geometry, and the theorem uses no measurement and no value of w.

A vacuum energy that is the same for every comparer, in whatever uniform motion, has a stress-energy proportional to the metric, and so it enters the field equations as such a constant term. A density that followed a horizon from epoch to epoch would be a property of the content at each epoch, with a frame of rest of its own, and it would not be the energy of the vacuum. In Section II I identify the accessible share as a share of the vacuum's energy, so it enters the field equations as $\Lambda$, and within one geometry it cannot vary.

The comparison that fixes $\Lambda$ is real, so it is the present measurement, by the definition of the real; no further identification is made. Axiom I makes the immanent act prior to the structure reconstructed from it (Chapter 4, Section IV): the history, with its epochs, is reconstructed from the impression the past makes on the present (Section V). The values that the formula gives at the particle horizons of other epochs are computed from that history. They exist, as structure; none of them is given in an actual act, so none of them is real, and none fixes anything. The records of the expansion, made by many comparers and reconciled by further comparisons (Chapter 12, Questions), are present impressions of the one geometry, and they carry its one value.

The identity of Section III, applied to the present measurement, then predicts the present matter fraction, 0.311, against the measured $0.315 \pm 0.007$; equivalently, it predicts that the measurement is made when the age of the universe is 0.792 de Sitter times, against $0.787 \pm 0.009$. The mag-

nitude of $\Lambda$ in Planck units follows: the vacuum density is 3f divided by the count of the present horizon, about $10^{-123}$ of the Planck density, and the count is that large because the particle horizon now spans about $10^{61}$ Planck lengths.

A reader who surveys the history from outside will ask why the present falls at the epoch where the identity holds. The question assumes that the presents of all epochs are given together, so that this present would need a reason to be the one. That assumption takes the reconstructed history, which is structure, as prior to the act from which it is reconstructed, and Axiom I does not grant it: only the present act is given. Declining that assumption rests on Axiom I.

The reasoning that fixes $\Lambda$ at the particle horizon fixes nothing on the Hubble radius or on the event horizon. In a history with a positive constant and with matter and radiation of positive density, the expansion rate exceeds $H_\Lambda = \sqrt{(\Lambda/3)}$ at every epoch. So the Hubble radius and the event horizon are each smaller than the de Sitter radius at every epoch, and approach it only as the epoch grows without bound. On either horizon the value of the formula, $8f/(R\,H_\Lambda)^2$ times the constant for a horizon of radius R, therefore exceeds $8f = 7.05$ times the constant at every epoch; no measurement could find on them a value that one geometry carries. Only the particle horizon grows past the de Sitter radius. That only the particle horizon grows past the de Sitter radius is a second reason for the particle horizon. The first reason, given in Section II, is that the particle horizon is the only horizon a first-person measurement holds. The second reason does not rest on the first reason. The second reason does rest on the coefficients: on the Hubble radius and on the event horizon no such value exists because $8c_1f/3 = 8f$ exceeds 1. With the reduced Planck mass in place of the full one, the formula would be divided by $(8\pi)^2$, and every horizon would have an epoch at which the value of the formula equals the constant; the full Planck mass is fixed by the modal boundary (Chapter 17, Section III), not by this condition.

An observer at an earlier epoch appears in the reconstructed history, with the records she made, and the constant those records carry is the one constant of the history. An observer at redshift 1 finds that constant, and a matter fraction of 0.79 at her present; a later observer who reads her records finds the same. The formula of Section II, applied to her particle horizon, gives 7.0 times the measured constant. Applied to the particle horizon of the present comparer as it was a billion years ago, it gives 1.22 times the measured constant; applied to that horizon as it will be a billion years from now, it gives 0.85 times. These numbers are computed from the structure of the history: they exist, none of them is real, and no record carries one of them as a finding. No observer finds a vacuum density that falls with time.

Fixing $\Lambda$ by the present measurement has one consequence for an observer at another epoch. Had the observer at redshift 1 applied the formula of Section II to her own particle horizon, she would not have obtained the constant she measures. The formula is therefore not a relation that every comparer at every epoch of the reconstructed history can confirm; it is a statement about the one act that is real, and the identity of Section III is a statement about the epoch of that act and no other. The question why the present falls at that epoch takes the presents of all epochs as given together, and Axiom I does not grant that (as stated above).

That the measured acceleration of the expansion is this constant term rests also on a result about averages: the small-scale inhomogeneities of the universe, averaged, do not act as a constant term. Green and Wald (2011) showed, assuming the weak energy condition, that such inhomogeneities supply only "an 'effective stress-energy tensor' that is traceless and has positive energy density", so that they "cannot mimic the effects of dark energy"; *The Quantum Bridge*, Chapter 8, Section IV, relies on the same result. Buchert and others (2015) dispute that the result proves backreaction irrelevant in cosmology, and Green and Wald (2015) answer them.

So $\Lambda$ is fixed once, by the present measurement, as the constant term of one geometry, and the reconstructed history carries that value at every epoch, before the present and after it.

## VII. Summary of Predictions

The derivation uses $c_1 = 3$, which is topological; $f = 1 - 3/(8\pi)$, from the curvature of the internal space at the derived radius; and the measured $G_N$, which fixes the Planck mass. The identity also uses the measured radiation density, $\Omega_r = 9 \times 10^{-5}$ (photons and neutrinos, which are relativistic in the early universe, where the radiation term matters), and the flatness of the epoch (Chapter 18, Section IX). The value of $\rho_\Lambda$ also uses the measured horizon; the identity does not.

The predictions are: the identity $I^2\Omega_\Lambda = 8 - 3/\pi$, which gives $\Omega_m = 0.311$ (measured: $0.315 \pm 0.007$); the same result as an energy density, $\rho_\Lambda = 2.56 \times 10^{-47}$ GeV$^4$ (observed: $2.52 \times 10^{-47}$); and $w = -1$ exactly (Planck 2018 with BAO and supernovae: $-1.03 \pm 0.03$). Stated through other measurements, the same result gives $t_0 H_\Lambda = 0.792$ (measured: $0.787 \pm 0.009$) and, with the matter density from the microwave background, $h = 0.678 \pm 0.003$. The prediction 0.311 holds if the ground-state part of the external value is $M_{Pl}^4$. Read the other way, the comparison with $0.315 \pm 0.007$ measures the coefficient of that part as $0.99 \pm 0.02$. The derivation says nothing of the composition of the matter; the split between baryonic and other matter is outside this book.

## Questions

**Is the formula not numerology: a small integer and a horizon, chosen to fit?**

Two facts answer this. First, the identity of Section III does not use the measured horizon at all: the Hubble constant and the Planck mass cancel from it, and it predicts a pure number, $\Omega_m$, which is compared with the measured one. Second, each choice has a reason that does not refer to the comparison: the particle horizon, because it is the only horizon a first-person measurement holds, and the only one on which the value of the formula ever equals the constant (Section VI); $c_1 = 3$, because a measurement registers one content in each of the three modal roles, and $\mathbb{C}P^2$ carries those roles (Section II; Chapter 9); the full Planck mass, because the modal boundary is a scale formed from the constants alone, and the $8\pi$ of the reduced mass belongs to the coupling of the field equations (Chapter 17). The table shows what the other choices give: the ratio of predicted to observed density, with the ground-state part of the external value alone, for each horizon and each small integer in place of $c_1$.

| Horizon | 1 | 2 | 3 | 4 | 6 |
|---|---|---|---|---|---|
| Hubble radius | 3.9 | 7.8 | 11.7 | 15.6 | 23.4 |
| Event horizon | 2.9 | 5.9 | 8.8 | 11.8 | 17.7 |
| de Sitter radius | 2.7 | 5.3 | 8.0 | 10.7 | 16.0 |
| Particle horizon | 0.38 | 0.77 | 1.15 | 1.54 | 2.31 |

The entry the reasons select, the particle horizon with 3, is 1.15, and the classical part of the external value brings it to 1.017. The nearest other entries are 0.77 and 1.54. The reduced Planck mass in place of the full one divides every entry by $(8\pi)^2 \approx 632$. The ratio of exactly 8 at the de Sitter radius with 3 is an identity of de Sitter thermodynamics, $\rho_\Lambda = (3/8)\, M_{Pl}{}^4/S_{dS}$.

As the epoch advances, the entries for the Hubble radius and the event horizon fall toward the entry for the de Sitter radius in the same column, and never reach it (Section VI).

The table varies two choices; a reader can vary all five. With four horizons, five integers, the ground-state part alone or with the classical part, the floor counted within the ground-state part or outside it, and the full or the reduced Planck mass, there are 160 combinations. One of them gives a density within the measured interval ($\pm$2.8 percent in the ratio): the combination the reasons select, at 1.017. The next nearest is the same combination with the floor counted outside, at 1.10. The classical part is not a further choice. The reduction on a positively curved internal space produces the bare curvature term (Chapter 11, Section III), so leaving it out of the external value would be the error.

**Why is the deficit proportional to $1/S$, and not to $1/\sqrt{S}$ or to $\log S$?**

By the identifications of Section II, I take the S degrees of freedom that the horizon counts to hold the external value in equal parts, and a measurement to engage a different one of them through each of its three channels; the share it receives is then three parts in S. The share is of a count, not a fluctuation in one. A share of $1/\sqrt{S}$ would be the relative size of a statistical fluctuation in a count of S, and $\log S$ grows with S, so it cannot give a suppression at all.

**Why are the channels three, and not six, the count of the whole internal space?**

By the identification of Section II: I take a channel to be a content that a measurement registers, and a measurement registers three contents, one in each modal role. The six metrics are three contents, each measured in its own context, and the context of a column is the arena of that column's channel, not a channel of its own. In the internal space the three channels are the three classes of $\mathbb{C}P^2$ taken with one class of $S^2$, the class of the content row, whichever class that is. The other counts that the internal space offers have no reading as channels. Six counts each context as a channel.

Two counts the rows. Five, the sum of the two parts of the first Chern class of the product, adds a count of rows to a count of roles. One, the largest integer that divides that class, counts nothing in the grid. The identity of Section III gives $\Omega_m = 0.163$ with six channels, 0.197 with five, 0.418 with two and 0.311 with three, against the measured $0.315 \pm 0.007$. These numbers measure what the count decides; the reason for three is the one above, which does not refer to them.

**What would change these numbers?**

Three coefficients of order one, none of them set by this comparison. The ground-state part of the external value is taken as exactly $M_{Pl}^4$; this coefficient is not fixed in this book, and a statement of the ground-state density at the modal boundary as a definite multiple of $M_{Pl}^4$ would fix it. The bare curvature term rests on the internal curvature being exactly the Planck curvature, $\lambda = 1/l_{Pl}^2$ (Chapter 9, Section IX). The classical energy of the higher Lovelock terms (Chapter 11, Section I), which is not fixed in this book, would shift the bare curvature term, and with it f. The floor is counted within the ground-state part (Section II); that count is an identification, and the first-order energy of the floor, about $+0.071\ M_{Pl}^4$ (Appendix D), is the size of what it decides: counted outside, it would give $\Omega_m = 0.292$ (Section IV). The energy of the floor beyond first order bears on the moduli (Chapter 11), and through the same count not on $\Lambda$.

**The identity holds at one epoch only. Is the predicted matter fraction a statement about when we observe?**

Yes, and that is its content. With one $\Lambda$, $I^2\Omega_\Lambda = (R/r_\Lambda)^2$ at every epoch, with $r_\Lambda$ the de Sitter radius, and R only grows; so the identity holds where $R = 2\sqrt{(2f)}\ r_\Lambda$, and at no other epoch. The measurement that fixes $\Lambda$ is the present one (Section VI), so the identity predicts that the present is that epoch, and the measured matter fraction places it there to within 0.6 standard deviations. To ask why the present falls there is to take the presents of all epochs as given together, which I decline in Section VI.

Across the decades in which the vacuum density has been measured the formula's value at the present changes little: in thirty years, by about 5 parts in $10^9$.

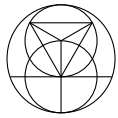
**Asked from outside, by a reader who surveys the whole history, why does the present fall at this epoch?**

The question has three parts, and I answer each in its own way. The first part is the ratio of the present age to the de Sitter time, 0.792. The identity gives it, and it is the prediction that measurement tests. The second part is the absolute epoch: why the act falls about $10^{61}$ Planck times after the beginning of the reconstructed history. The question of the absolute epoch is a question about when comparers exist, and so a question about the content. In this book I do not derive the content, and the epoch is the one that the measured particle horizon gives. The third part is uniqueness: of the epochs of one geometry at which comparers can stand, why the real act is at the epoch where the identity holds. In Section VI I answer the third part from Axiom I, and the step rests on Axiom I for that part alone. The magnitude of $\Lambda$ in Planck units follows the horizon of the real act (Section VI), so the nearness of the vacuum and matter densities at the present is no coincidence in this account: it is the identity, at the epoch of that act. A criterion stated from outside, which surveys all epochs at once, can give a distribution of the epochs at which comparers stand. It cannot single out one of them as real, since that would take the presents of all epochs as given together.

# PART IV

## The Structural Content of General Relativity

*Equivalence, geodesics, and the recovery of classical physics*



Chapter 14

# The Equivalence Principle as a Triple

*On Rotation, Acceleration and Gravitation*

In this chapter I read the equivalence principle as a triple. In each of three phenomena, centripetal force (felt in a rotating room as the centrifugal force), linear acceleration and gravitation, every body is affected alike, whatever its composition: the force it feels is proportional to its inertial mass. That equivalence is the weak equivalence principle, extended to the inertial forces of any non-inertial frame. The extended equivalence keeps Einstein's principle unchanged. Einstein's principle states that gravitation and linear acceleration are indistinguishable at a point; rotation is registered at a point, against the local inertial frame. Across a room of nonzero size the three are told apart by their global structure (rotation by its axis, linear acceleration by its uniformity, gravitation by its center), and two pairs of hanging balls tell which one is present. In Chapter 20 I count the equivalence principle among what follows from the chain; in this chapter I state it, for every inertial force, with a test that tells the three apart.

*Rests on:* the equality of gravitational and inertial mass (Appendix A, Section II), and the local inertial frame (Chapter 18, Section IV). *Refers to:* geodesic deviation (Chapter 15, Section V). *Used by:* Chapter 15 (Section III) and Chapter 20. *Referred to by:* Appendix A (Section II).

## I. The Standard Duality

Einstein's thought experiment is well known. A person in a closed room cannot determine, by any local experiment, whether the room sits on the surface of a massive planet or accelerates upward at the same rate in empty space. The two situations are locally identical.

Einstein's principle is presented as a duality: gravity equals acceleration. On that duality general relativity replaces gravitational force with spacetime curvature. The duality names two members of a triple.

## II. The Third Element

Consider a person standing against the outer wall of a rotating room, a centrifuge. The wall pushes the person toward the axis: that force is centripetal, and so is the person's acceleration. What the person feels is a force pressing outward, toward the wall, the centrifugal force of the rotating room. Like weight on a planet and the push felt in a rocket, it acts on every body in the room alike, in proportion to its inertial mass. Unlike them, it comes with an axis: a gyroscope carried in the room registers the rotation at a point, a body moving across the room is deflected sideways by the Coriolis force, and a ring interferometer shows the shift that Sagnac measured (Sagnac 1913).

The three phenomena of the triple are:

*Centripetal force.* The force that holds a body in circular motion, directed toward the center. It is determined by the geometry of the trajectory: the radius of curvature and the angular velocity. It is structural and geometric: knowable from the outside by examining the shape of the motion.

*Linear acceleration.* The force experienced in a rocket or an elevator. It is the bare processual fact of changing velocity: the direct experience of being pushed. Linear acceleration is determined by the interaction between engine and occupant, and not by the shape of a trajectory (a rocket can accelerate in a straight line).

*Gravitation.* The force experienced near a massive body. It is determined not by the local trajectory or the local interaction but by the distribution of mass-energy throughout the environment. It is potential and field-like: determined by what lies outside the room, beyond the local frame.

## III. Distinguishing the Three: The Hanging-Ball Experiment

The standard statement of the equivalence principle asserts that gravity and linear acceleration are locally indistinguishable, and "locally" means at a point, a zone of zero volume. Any room of nonzero volume inherits, in proportion to its size, something of the global structure of the phenomenon present, and the global structure distinguishes the three phenomena.

Consider a sealed room the size of a small elevator. An observer hangs two balls on strings, attached to the ceiling at a distance L apart, and lets them hang freely. Three cases arise.

*Case 1: Gravitation.* If the room sits on the surface of a compact massive body, such as a planet, the strings point toward the common center of gravity, in whatever direction along the ceiling the two attachment points are separated. They converge. The distance between the two balls near the floor is *less than* L. The convergence is the tidal effect of gravitation: the relative acceleration of neighboring freely falling bodies, which measures the curvature of spacetime (Misner, Thorne and Wheeler 1973; Chapter 15, Section V), here seen in bodies held at rest.

*Case 2: Linear acceleration.* If the room is accelerated uniformly by a rocket engine, the strings hang parallel. There is no common center toward which they converge; the acceleration is the same at every point. The distance between the two balls is *equal to* L.

*Case 3: Centripetal acceleration.* If the room is attached to the end of a long cable and swung in a circle, the strings point directly away from the axis of rotation. For two points separated along the direction of motion the

strings diverge, and the distance between the balls is *greater than* L. For two points separated along the axis the strings hang parallel, as in the rocket.

The relation of the distance between the balls to L depends on the direction of the separation, so the room carries two pairs of balls, separated along two perpendicular directions of the ceiling. Gravitation gives less and less; linear acceleration gives equal and equal; rotation gives at least one greater and none less, greater and equal when the pairs lie along the motion and along the axis. Each pattern picks out exactly one phenomenon, and the triple is physically distinguishable in any room of nonzero volume. The separations lie along the ceiling. A vertical pair, one ball hanging lower than the other, measures how the force changes along its own direction, and it diverges in both rooms: on the massive body the field grows toward the body, and in the swung room the centrifugal force grows with distance from the axis. So a vertical pair does not separate the cases.

As the room shrinks, the gradient of the force field across it vanishes relative to the field itself, and the hanging balls no longer separate the three. At a point, gravitation and linear acceleration coincide: that coincidence is the equivalence principle in its standard form. Rotation keeps its axis at every point, and it is read against the local inertial frame, which the comparable content fixes (Chapter 18, Section IV): a gyroscope registers it, and so does the deflection of any moving body. Across any extent the three are distinguished by their global structure.[9]

## Questions

**A gyroscope detects rotation at a point. Is rotation then equivalent to gravitation in Einstein's sense?**

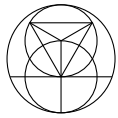
No, and I do not claim it. Einstein's principle states that gravitation and linear acceleration are indistinguishable by any local experiment, at a point. Rotation is not indistinguishable at a point: a gyroscope, the Coriolis deflection of a moving body and the Sagnac shift of a ring interferometer each register it against the local inertial frame (Section II). The equivalence I claim for the triple is a different one: in each of the three phenomena every body is affected alike, in proportion to its inertial mass. The centrifugal and Coriolis forces of a rotating frame, the push felt in an accelerating frame, and weight in a gravitational field all have that property, which is the weak equivalence principle extended to the inertial forces of any non-inertial frame.

**The standard principle is stated at a point precisely to exclude tidal effects. Does the hanging-ball room not simply show the tidal field?**

The hanging-ball room does show the tidal field, and I mean the room to show the tidal field. The convergence of the strings on the massive body is the tidal effect of gravitation (Misner, Thorne and Wheeler 1973), and the divergence in the swung room comes from the centrifugal force, which points away from the axis along each radius. The room is not a counterexample to the standard principle, which holds at a point and is kept here (Section III). The room shows what the statement at a point leaves out: the global structure of each phenomenon, which across any nonzero extent tells the three apart.

**Is the equality of gravitational and inertial mass assumed here, or argued?**

Argued, in Appendix A, Section II. For a phenomenon of context, the coupling is the degree of participation in the context, and the inertial response is the degree of resistance to change of context. The coupling and the inertial response are one quantity. The equality is measured to about one part in $10^{15}$ (Touboul and colleagues 2022). I take the equality from Appendix A and extend the "affected alike" of gravitation to the inertial forces of rotation and linear acceleration, where the "affected alike" holds by the definition of an inertial force.



Chapter 15

# Geodesic Motion

*On the Paths That Preserve Context*

In this chapter I give the law of motion that the field equations contain. The contracted Bianchi identity makes the stress-energy conserved, and conservation gives the geodesic equation: exactly for dust, and for any body small enough that its own field is negligible (Geroch and Jang 1975). A fluid with pressure departs from the geodesic by its pressure gradient. The relative acceleration of neighboring geodesics, the tidal effect, is the observable signature of the curvature of the context.

*Rests on:* the field equations (Chapter 12), the content/context partition (Chapter 6), and the equivalence triple (Chapter 14). *Used by:* Chapter 16 (Section I) and Chapter 20. *Referred to by:* Chapter 14 (Section III).

## I. The Standard Derivation

The Einstein field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ contain the law of motion for all matter coupled to the geometry. The field equations and the equations of motion are not independent postulates: the equations of motion follow from the field equations.

The mechanism is the contracted Bianchi identity. The Einstein tensor satisfies $\nabla_\mu G^{\mu\nu} = 0$ identically, a theorem of the differential geometry of Riemannian manifolds, not a physical assumption. Since $\nabla_\mu g^{\mu\nu} = 0$ by metric compatibility, the field equations imply

$$\nabla_\mu T^{\mu\nu} = 0. \tag{15.1}$$

That equation is the conservation of stress-energy. For a perfect fluid with stress-energy tensor $T^{\mu\nu} = (\rho + p)\, u^\mu u^\nu + p\, g^{\mu\nu}$, the conservation equation decomposes into two conditions: the continuity equation (projecting along $u^\mu$) and the equation of motion (projecting perpendicular to $u^\mu$). The equation of motion is

$$(\rho + p)\, u^\mu \nabla_\mu u^\nu = -(g^{\nu\mu} + u^\nu u^\mu)\, \nabla_\mu p. \tag{15.2}$$

For dust, $p = 0$, and the equation is exactly

$$u^\mu \nabla_\mu u^\nu = 0, \tag{15.3}$$

the geodesic equation. For a fluid with pressure, the right-hand side is the pressure gradient across the direction of motion, and it pushes each element of the fluid off the geodesic.

For a body of any composition, the geodesic law holds in the limit of a test body, a body whose own gravitational field is negligible compared with the background. Geroch and Jang (1975) proved it: if every neighborhood of a curve, however thin, contains a nonzero conserved stress-energy that vanishes outside the neighborhood and satisfies a dominant energy condition, the curve is a timelike geodesic.

The geodesic equation states that the four-velocity is parallel-transported along itself: the body moves on the straightest possible line through the curved geometry. The body follows the geometry, and no external force acts on the body.

## II. The Meaning of Geodesics

A geodesic has two equivalent characterizations, each showing a different aspect of what "straightest possible path" means.

The first characterization is variational. Among all paths connecting two events, the geodesic is the one that extremizes the proper time. In the Lorentzian case, a timelike geodesic is, among nearby paths, the path of

maximum elapsed time between the two events. A freely falling clock records more time than any nearby accelerated clock traveling between the same two events.

The variational characterization can also be reached through the phase of a body. For a body at rest in its own frame, the phase of the body advances at the rate of its rest mass times $c^2$, divided by $\hbar$, per unit of its own proper time. Along a path through the geometry, the action of the body is therefore, up to its sign, its rest mass times $c^2$ times the proper time along that path. The path on which the action is stationary is then the path on which the proper time is stationary, and that path is the geodesic. The rest mass multiplies the action alike on every path, so it cancels from the condition of stationarity, and every body, whatever its mass and composition, falls along the same geodesic (Chapter 14). The four-dimensional geometry has no least step (Chapter 17, Questions), so it is already in its classical limit, and the path of stationary proper time is the path of free fall in it.

The second characterization is local. At every point along its trajectory, the geodesic is the path whose tangent vector does not change direction relative to the local geometry. Parallel transport of the velocity vector along the curve leaves the velocity unchanged. The body does not turn, does not accelerate, does not deviate from the direction established by the geometry at each point.

The variational and local characterizations are equivalent for the connection fixed by the metric. In the Lorentzian setting of general relativity, timelike geodesics maximize proper time among nearby paths, and spacelike geodesics extremize proper distance. Null geodesics, the paths of massless particles, have zero proper interval and are characterized by the parallel-transport condition.

## III. The Equivalence Principle Revisited

The equivalence principle, read in Chapter 14 as a triple of centripetal force, linear acceleration and gravitation, has a direct connection to geodesic motion.

In the equivalence principle's standard formulation, a freely falling observer experiences no gravitational force. The observer in free fall is following a geodesic; in the observer's local frame, spacetime is flat to first order and no force acts. Only when the observer is *not* freely falling, when some non-gravitational force holds the observer stationary on the surface of a planet, for example, does the observer experience what is called "gravity".

On the freely falling observer no content-modal interaction acts. The felt force of gravity on the surface of a planet is not a context-modal phenomenon: it is the content-modal reaction of the electromagnetic forces in the ground and in the observer's body, which prevent the observer from following the geodesic.

What is ordinarily called gravity, the sensation of weight, is the *departure* from geodesic motion, not geodesic motion itself. Geodesic motion is weightless. It is the motion of a body whose trajectory is governed entirely by the context.

## IV. The Geodesic Equation in Coordinates

For computation, the geodesic equation is expressed in a coordinate system $x^\mu = (t, x, y, z)$ as

$$d^2x^\mu/d\tau^2 + \Gamma^\mu{}_{\alpha\beta}\,(dx^\alpha/d\tau)\,(dx^\beta/d\tau) = 0, \tag{15.4}$$

where $\tau$ is the proper time along the trajectory and $\Gamma^\mu{}_{\alpha\beta}$ are the Christoffel symbols: the connection coefficients constructed from the metric and its first derivatives.

The Christoffel symbols encode how the coordinate axes rotate and stretch from point to point. The geodesic equation says that the apparent acceleration of the body in these coordinates is exactly canceled by the connection terms. The body is not accelerating in any physical sense; the coordinates are not adapted to the geometry. In a locally inertial frame, the frame of a freely falling observer, the Christoffel symbols vanish at the observer's position and the trajectory there is a straight line.

## V. Geodesic Deviation and Tidal Forces

Two nearby geodesics can converge or diverge as they proceed through curved spacetime. That relative acceleration is not a force but a manifestation of the curvature of the context.

The geodesic deviation equation describes that relative acceleration:

$$D^2\xi^\mu/d\tau^2 = -R^\mu{}_{\nu\alpha\beta}\, u^\nu\, \xi^\alpha\, u^\beta, \tag{15.5}$$

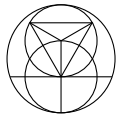
where $\xi^\mu$ is the separation vector between neighboring geodesics, $u^\mu$ is the four-velocity, and $R^\mu{}_{\nu\alpha\beta}$ is the Riemann curvature tensor.

The geodesic deviation equation is the relativistic expression of tidal forces. The Newtonian tidal force, which stretches a falling body along the radial direction and compresses it along the transverse directions, is a special case. Geodesic deviation is the observable signature of the curvature of the context: two observers in free fall find that their separation changes because the context itself is curved. The curvature is sourced by the matter through the Einstein field equations (Chapter 12).

## Questions

**Real bodies have pressure, spin and fields of their own. Does the geodesic law hold for them?**

For dust it holds exactly (Section I). For a fluid with pressure each element departs from the geodesic by the pressure gradient across its motion. For a body of any composition it holds in the limit in which the body is small and its own field is negligible, which is the theorem of Geroch and Jang (1975). A body of finite size departs from the geodesic, and the departure vanishes in that limit. The statements above on motion along the geodesic are results of general relativity, and I take those results unchanged.



Chapter 16

# Recovery of Classical Physics

*On the Completion of the Chain*

In this chapter I take the field equations of Chapter 12 to their weak-field, slow-motion limit and recover the Poisson equation and Newton's law of gravitation; in the same limit the geodesic equation of Chapter 15 gives the Newtonian equation of motion in a gravitational field. The recovery rests on the moduli of the internal space being held (Chapter 11). The standard solutions (Schwarzschild, Friedmann-Lemaître-Robertson-Walker and linearized gravitational waves) are those of general relativity. What differs is what is taken from measurement. $G_N$ is the one measured constant, and the cosmological constant is fixed by the formula of Chapter 13 from the measured size of the present horizon in Planck units, with the matter fraction predicted.

*Rests on:* the field equations (Chapter 12), the geodesic equation (Chapter 15), the moduli held (Chapter 11, Section II), the cosmological constant (Chapter 13), and the status of $G_N$ (Chapter 17, Sections I and III). *Used by:* Chapter 20. *Referred to by:* Chapter 11 (Section II).

## I. The Weak-Field, Slow-Motion Limit

The Einstein field equations describe gravity in full generality: strong fields, fast motions, dynamic spacetimes, gravitational radiation. Most gravitational phenomena encountered in everyday experience and in the solar system occur in a far simpler regime: weak fields and slow motions.

In the regime of weak fields and slow motions, the metric deviates only slightly from the flat Minkowski metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu'} \qquad |h_{\mu\nu}| \ll 1. \tag{16.1}$$

The matter moves at speeds much less than the speed of light: $v \ll c$. And the gravitational field is quasi-static: the time derivatives of the metric perturbation are negligible compared to the spatial derivatives.

Under these three conditions (a weak field, slow motion and a quasi-static field), the Einstein equations simplify. The only relevant component of the metric perturbation is the time-time component $h_{00} = -2\Phi/c^2$, where $\Phi$ is the Newtonian gravitational potential. The Einstein equations reduce to

$$\nabla^2\Phi = 4\pi G_N \rho, \tag{16.2}$$

which is the Poisson equation, the field equation of Newtonian gravity: the gravitational potential is determined by the mass density through the Laplacian operator.

The geodesic equation, in the weak-field, slow-motion limit, reduces to Newton's second law for a particle in a gravitational field:

$$d^2x/dt^2 = -\nabla\Phi. \tag{16.3}$$

For the gravitational potential of a point mass M, $\Phi = -G_N M/r$, Newton's second law gives the inverse-square law:

$$F = G_N Mm/r^2. \tag{16.4}$$

The Newtonian theory of gravity (the Poisson equation, the inverse-square force law, Kepler's laws of planetary motion, the tides, the precession of the equinoxes) is recovered as a limiting case of the Einstein equations of Chapter 12.

The recovery rests on the moduli of the internal space being held, which Chapter 11, Section II, establishes. With the moduli fixed, the weak-field limit is that of general relativity. With a light modulus it would be that of a scalar-tensor theory, which the Cassini bound excludes at the strength such a modulus would carry.

## II. The Significance of the Recovery

The recovery of Newtonian gravity from general relativity is standard textbook material. What differs here is what the equations take from measurement.

In conventional physics, $G_N$ is a measured constant, approximately $6.674 \times 10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$. Newton's law of gravitation has one gravitational parameter. General relativity has two, $G_N$ and the cosmological constant $\Lambda$, and takes both from measurement.

In the present framework $G_N$ remains a measured constant: the one measured constant of this book, for the reason given in Chapter 17, Section I. With c and ħ set to one, the measured $G_N$ fixes the unit of length (Chapter 17, Section III). $\Lambda$ is fixed by the formula of Chapter 13 from $c_1 = 3$, from the factor f, and from the size of the present horizon in Planck units, which is the one dimensionless measured fact the cosmological constant uses (Chapter 17, Section III). The identity of Chapter 13, Section III, uses no measured horizon, and from it the matter fraction $\Omega_m$ is predicted. General relativity therefore takes two gravitational parameters from measurement; this framework takes $G_N$ and the measured size of the horizon, and predicts the split between matter and $\Lambda$.

When the measured $G_N$ is substituted into the Newtonian limit, the result is Newton's law with its one measured constant. The form of the gravitational force between two masses follows from the field equations of Chapter 12 and the limit of Section I; its strength is fixed by measurement.

## III. The Schwarzschild Solution

The Schwarzschild metric is the unique spherically symmetric vacuum solution of the Einstein field equations with $\Lambda = 0$. The uniqueness is Birkhoff's theorem (Birkhoff 1923): any spherically symmetric vacuum spacetime is static and isometric to the Schwarzschild geometry. With $\Lambda$, the same theorem gives the Kottler solution, whose departure from Schwarzschild is negligible at the scale of the solar system.

The solution describes the gravitational field outside any spherically symmetric mass distribution: a star, a planet, a black hole. Its features are well known: the event horizon at the Schwarzschild radius $r_s = 2G_N M/c^2$, the gravitational redshift, the deflection of light, the precession of Mercury's perihelion, and the Shapiro time delay.

In the present framework, the form of the Schwarzschild solution is derived, since the field equations are derived (Chapter 12). The classical tests of general relativity, the perihelion precession, the light deflection and the gravitational redshift, are therefore tests of the derived equations. Once $G_N$ is fixed by one measurement, each test is a prediction, not a fit.[10]

## IV. The FLRW Cosmology

The Friedmann-Lemaître-Robertson-Walker metric describes a universe that is spatially homogeneous and isotropic. The metric is determined by a single function of time, the scale factor $a(t)$, and a curvature parameter $k$.

The Einstein field equations, applied to the FLRW metric, yield the Friedmann equations:

$$H^2 = (8\pi G_N/3)\rho - kc^2/a^2 + \Lambda c^2/3 \tag{16.5}$$

where $H = (da/dt)/a$ is the Hubble parameter, $\rho$ is the energy density, and $\Lambda$ is the cosmological constant.

In the present framework the expansion history of the universe is determined by the matter content, the measured $G_N$, and the $\Lambda$ of Chapter 13. The cosmological constant, which drives the observed accelerated expansion, enters as the accessible share of the vacuum's energy, fixed by the information deficit at the observer's horizon (Chapter 13).

## V. Gravitational Waves

Gravitational waves are perturbations of the metric that propagate at the speed of light. Gravitational waves arise as solutions of the linearized Einstein equations in vacuum:

$$\Box \bar{h}_{\mu\nu} = 0, \tag{16.6}$$

where $\Box$ is the d'Alembertian operator and $\bar{h}_{\mu\nu}$ is the trace-reversed metric perturbation in the Lorenz gauge.

Gravitational waves carry energy and angular momentum away from their sources. The quadrupole formula gives the radiated power: the leading-order emission is proportional to the square of the third time derivative of the source's mass quadrupole moment, multiplied by $G_N/c^5$.

The detection of gravitational waves by LIGO in 2015, from the merger of two black holes, observed these propagating metric perturbations directly, with a waveform that agrees with general relativity (Abbott and colleagues 2016). In the reading of this book, gravitational waves are small oscillations of the context, the continuously asymmetric structure, propagating at c, the macroscopic domain limit (Chapter 17, Section I).

General relativity gives gravitational waves two polarizations, those of a massless field of spin 2, and a speed equal to the speed of light. The three-detector event GW170814 favors purely tensor polarization over purely vector and purely scalar polarization (Abbott and colleagues 2017b). The merger GW170817, with its gamma-ray burst, bounds the dif-

ference between the speed of gravitational waves and the speed of light to between $-3 \times 10^{-15}$ and $+7 \times 10^{-16}$ of the speed of light (Abbott and colleagues 2017a).

## VI. The Complete Chain

With this chapter the chain of derivation runs from the three axioms of the Immanent Metaphysics, through the field equations, to Newtonian gravity and the standard solutions. Chapter 20 sets out the whole chain and the standing of each link, and the recovery of classical physics is as secure as the links it rests on. Every prediction of Newtonian gravity and of general relativity follows, with the measured $G_N$, once the distribution of matter is given. That distribution is content, and this book does not derive it: the initial conditions of the expansion and the composition of the matter come from observation.

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### Questions

**The Newtonian limit and the standard solutions are textbook general relativity. What is claimed in this chapter beyond them?**
Nothing about the physics of the limiting cases. The recovery of the Poisson equation, Newton's law and the standard solutions is standard mathematics, and I claim nothing new about it. What differs is the standing of the equations and of their parameters: the form of the equations follows from the reduction and Lovelock's theorem (Chapters 11 and 12), $G_N$ is measured, and $\Lambda$ is fixed by the formula of Chapter 13 (Section II). Once $G_N$ is measured, the classical tests are predictions, not fits.

**Is the count of parameters fair? The cosmological constant still uses a measured quantity, the horizon.**

The cosmological constant does use the measured horizon, and I count the horizon as a measured quantity in Section II. The value of $\Lambda$ uses the size of the present horizon in Planck units, the one dimensionless measured fact the cosmological constant uses (Chapter 17, Section III). What the framework does not take from measurement is the split between matter and $\Lambda$: the identity of Chapter 13, Section III, in which the Hubble constant and the Planck mass cancel, predicts $\Omega_m = 0.311$ against the measured $0.315 \pm 0.007$. General relativity takes that split from measurement.

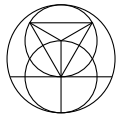
**Would a light modulus not spoil the Newtonian limit?**

A light modulus would spoil the Newtonian limit, and the recovery of the Newtonian limit rests on the moduli being held (Section I). With a light modulus the weak-field limit would be that of a scalar-tensor theory, and the Cassini bound excludes such a modulus at the strength it would carry. The moduli are held (Chapter 11, Section II), and the mechanism of Appendix D gives them masses of about the Planck mass, with a range far below every bound.

# PART V

## Boundaries and Horizons

*The Planck scale, the interior, and the fate of singularities*



Chapter 17

# The Planck Scale as Modal Boundary

*On the Boundary Between Descriptions*

In this chapter I place the modal boundary at the Planck scale: the scale at which an object's gravitational radius equals its quantum wavelength, so that the continuously asymmetric description (general relativity) and the discontinuously symmetric description (quantum mechanics) exchange dominance. I give the roles of c, ħ and $G_N$, and why $G_N$ is the one measured constant: with $c = \hbar = 1$, $G_N$ fixes the unit, and what a measurement finds is a dimensionless ratio. At the boundary the two descriptions exhaust their domains, and no single description carries both. The internal space lies at the boundary, a few Planck lengths across.

*Rests on:* the Incommensuration Theorem and its physical reading (Chapter 3, Section V), and the partition (Chapter 6). *Refers to:* the precedence of Axiom II (Chapter 10, Section III), the derived radius (Chapter 9, Section IX), the reduction (Chapter 11, Section III), the field equations with their corrections (Chapter 12, Sections III, VI and VII), the internal directions as compact to a located comparer (Appendix C, Section VIII), the size of the present horizon in Planck units (Chapter 13), and the classical equations carried beyond the boundary (Chapter 19, Section I). *Used by:* Chapters 9 to 11 (the scale of the internal space and of the boundary), Chapter 13 (the external value), Chapter 16 ($G_N$ as the one measured constant), Chapters 18 and 19, Chapter 20 and Appendix B. *Referred to by:* Chapter 12 (the scale at which the modal boundary lies).

## I. Three Constants, Three Roles

The Planck mass is formed from three constants:

$$M_{Pl} = \sqrt{(\hbar c / G_N)}. \tag{17.1}$$

The three constants have different roles. Two of them are limits of the domain of comparison, and the third is a coupling within it.

The speed of light, c, is the macroscopic limit of the domain. It is the maximum speed of causal propagation, the constant that relates spatial and temporal measures. I identify it as the bridge between the omniscient and immanent modalities. Axiom II's precedence requires that a class of the immanent precede an instance of the omniscient; that precedence excludes signals that take no time, and the macroscopic boundary of the domain fixes their maximum speed (Chapter 10, Section III). The comparison that c measures is first a difference in time and then a difference in space. The speed of light is not, at root, the speed of any one thing but the macroscopic boundary of the domain of comparison itself; light is only the perfected example of c.

Planck's constant, ħ, is the microscopic limit of the domain. It is the least quantum of action, the constant that relates energy and frequency. I identify it as the bridge between the immanent and transcendent modalities. The discreteness of content at the microscopic limit is the mark of the discontinuously symmetric conjunction, and ħ sets the scale at which that discreteness appears.

The gravitational constant, $G_N$, is the coupling of the context phenomenon: it measures how strongly the content curves the context. Its structural role (universal, non-shieldable, consistent with the equivalence principle) is fixed by the axioms. Its numerical magnitude is a fact about this domain, not a fact about the structure of comparison.

The limits c and ħ and the coupling $G_N$ differ in what a measurement of them finds. That the domain has two limits follows from the axioms; the numbers that c and ħ carry express only the units in which time, length and action are measured. In the International System both are ex-

act by definition, and the kilogram is fixed through $\hbar$. A measurement of $G_N$ in that system is therefore a measurement of the Planck mass in kilograms: a dimensionless ratio between a laboratory mass and the Planck mass.

The same holds in any system of units. With $c = \hbar = 1$, time, length, mass and energy share one unit, and the measured $G_N$ fixes that unit as the Planck length. A measurement then finds a dimensionless ratio of the same kind as the ratio of a laboratory mass to the Planck mass. Two such ratios are the mass of a body or a particle over the Planck mass, and the radius of the present horizon over the Planck length (Chapter 13). Each such ratio compares a content, or the extent of a context, with the scale at which content and context exchange dominance (Section II). The axioms require that the coupling exist and constrain its properties. They do not fix these ratios, because the size of a content relative to the boundary is a feature of the particular domain in which comparison operates. I make no claim on the value of any such ratio. I take $G_N$ from measurement, as the book's single measured constant, and express every other dimensionful quantity in the unit $G_N$ fixes.

## II. The Modal Boundary

The Planck scale is the scale at which the two surviving conjunctions of the Incommensuration Theorem, the continuously asymmetric and the discontinuously symmetric, are relevant together. An object of mass $m$ has a gravitational radius $G_N m/c^2$ and a quantum wavelength $\hbar/(mc)$. The two are equal when $m^2 = \hbar c/G_N$, that is, at the Planck mass.

Where an object's gravitational radius far exceeds its quantum wavelength, as in planetary physics, stellar dynamics and cosmology, the continuously asymmetric description dominates. Spacetime is smooth, the metric is well defined, curvature varies continuously, and the tools of dif-

ferential geometry capture the physics. The regime where the gravitational radius far exceeds the quantum wavelength is the regime of general relativity.

Where an object's quantum wavelength far exceeds its gravitational radius, as for atoms, nuclei and elementary particles, the discontinuously symmetric description dominates. States are discrete, symmetry groups govern the interactions, observables do not commute, and the tools of Hilbert space theory capture the physics. The regime where the quantum wavelength far exceeds the gravitational radius is the regime of quantum mechanics.

At the Planck scale itself, where the two lengths are equal, neither description dominates, and both are relevant. A single description that carried both would need a forbidden pairing (Chapter 3, Section V).

The modal boundary is therefore the scale at which the incommensuration of the two descriptions becomes manifest: no single description captures everything there, because the two descriptions cannot be combined into one. The boundary is not a wall that matter cannot cross, and it is not a regime of a third kind of physics.

## III. The Planck Mass from the Measured $G_N$

In standard physics, the Planck mass is computed from c, ħ and $G_N$. It is a derived unit, derived from measured inputs.

In the present framework I keep this construction. The Planck mass is formed from the measured $G_N$ and the constants c and ħ: $M_{Pl} = \sqrt{(\hbar c / G_N)}$. The boundary is a scale formed from those three constants alone, so it is marked by the full Planck mass, not by the reduced mass $M_{Pl}/\sqrt{(8\pi)}$: the $8\pi$ belongs to the coupling of the field equations, $8\pi G_N$ (Chapter 12, Section I). What I add in the framework is the role of the scale the Planck mass marks.

The Planck length and Planck time follow from the Planck mass by the standard relations: $l_{Pl} = \hbar/(M_{Pl}\, c)$, $t_{Pl} = l_{Pl}/c$. All three Planck units are fixed by the single measured constant, $G_N$, together with c and ħ. With c and ħ set to one, the measured $G_N$ fixes the unit of length (Section I). The one dimensionless measured fact the cosmological constant then uses is the size of the present horizon in Planck units (Chapter 13).

The role of the boundary follows from the axioms: it is where the two surviving conjunctions exchange dominance. Its location is fixed by the measured strength of the coupling between content and context.

## IV. What Happens at the Boundary

At the modal boundary the two existing descriptions reach the limits of their domains. The framework has no third description that takes over from both on the other side.

General relativity holds at curvatures far below the Planck curvature. Its predictions are confirmed from the solar system to the cosmological horizon. The corrections to the Einstein action are of relative size the curvature times $l_{Pl}^2$, so they are negligible far below the Planck curvature and of order one at it (Chapter 12, Section VII). Where the classical equations are carried beyond that curvature, they produce divergent curvature and a metric that ceases to be defined (Chapter 19, Section I).

Quantum mechanics holds at energies far below the Planck mass in a different sense: its perturbative expansion is valid, its symmetry principles determine the dynamics, and its predictions are confirmed to high precision. It reaches its limit at the Planck scale in a different way: the gravitational effects of its own fluctuations are no longer negligible, and the description of the matter on a classical background no longer holds.

Below the boundary the relation between the two descriptions is the one of Chapter 12, Section VI: per-alternative geometry, in which each alternative of the matter carries its own classical geometry. The content is quantum. The context is classical. They interact through the stress-energy

of each alternative, which shapes that alternative's geometry; where the matter is sharply one alternative, the coupling is $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle$. The framework contains no quantum theory of the geometry; a geometry for each alternative is not an approximation to one (Chapter 12, Section VI).

A theory of quantum gravity would carry one description across the boundary by quantizing the arena, and a quantized arena needs a forbidden pairing (Chapter 3, Section V). The claim of this book holds at every scale, and it differs from linearized quantum gravity in what it predicts only at the boundary and beyond: below it the account of Chapter 12 and linearized quantum gravity predict the same, and the experiments of Chapter 12, Questions, test the account against the programs that use one geometry for all alternatives.

## V. The Boundary and the Internal Geometry

The modal boundary fixes the size of the internal geometry. The compactification radius lies at the boundary, a few Planck lengths. The internal geometry is the discontinuously symmetric factor, governed by the quantum description; the external geometry is the continuously asymmetric factor, governed by general relativity. The modal boundary separates the two factors of the product $M^4 \times \mathbb{C}P^2 \times S^2$, and the internal scale is the scale at which they meet.

The ratio of the two internal radii is fixed by the geometry, $R_O/R_T = \sqrt{6}$ (Chapter 9). The overall scale is fixed by the boundary itself: each internal direction carries the Planck curvature, so $R_T = l_{Pl}$ and $R_O = \sqrt{6}\, l_{Pl}$ (Chapter 9, Section IX).

The Planck length that bounds the internal directions is the one formed from the measured $G_N$. The modal boundary is fixed by a comparison that a located comparer makes, of an object's gravitational radius with its quantum wavelength, and the coupling that comparison measures is $G_N$ (Sections II and III). The ten-dimensional action defines a

second length, from its coupling $M_{10}$; by Chapter 11, Section III, $M_{10}{}^{8} = M_{Pl}{}^{2}/\mathrm{Vol}(K)$, so that length takes the internal volume as an input besides $G_N$. No located comparison supplies that volume: the internal directions are not traversed, and they present to a located comparer as compact (Appendix C, Section VIII). The volume is what the boundary fixes, so the boundary is the length a comparison supplies, the Planck length formed from $G_N$. At the derived radius the slowest variation the internal space admits, the first eigenvalue of its Laplacian, $12/R_O{}^{2} = 2/R_T{}^{2}$, is twice the Planck curvature: the internal space begins where the boundary lies.

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## Questions

**In the International System c, ħ and $G_N$ are all numbers with units. Why are two of them called structural and one measured?**

Because a measurement of each finds something different (Section I). The values of c and ħ are fixed by the definition of the units, and setting both to one changes no physical statement. $G_N$ cannot be removed in the same way: once c and ħ are one, it fixes the remaining unit, and a measurement of it is a measurement of a dimensionless ratio, such as a laboratory mass over the Planck mass. The axioms fix that the domain has two limits and that content curves context through a universal coupling; they do not fix the size of any content relative to the boundary.

**Is the modal boundary a least length of spacetime?**

No. The four-dimensional geometry has no least step: its lengths, areas and times keep no least value (Chapter 12, Questions). The least motion of the metric's small waves bounds how sharply a body can be placed, at about the Planck length, and that bound is a limit on localization, not a least length of the geometry (Chapter 12, Questions). The boundary is the

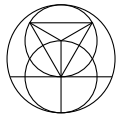
scale at which general relativity and quantum mechanics exchange dominance. The internal space is held at that scale (Chapter 11, Section II), which is a statement of its size, not a lattice of the arena.

**Is the Planck scale not simply where quantum gravity begins, as the standard view holds?**

The standard view and mine agree on the scale and differ on what lies there. In both, the corrections to the Einstein action become of order one at the Planck curvature (Section IV), and the effective field theory of gravity, which describes the metric's small waves by quanta, ends there (Chapter 12, Questions). The standard view expects a quantum theory of the arena to continue past that scale. In this book the description by quanta is a description of the waves, the quanta are not things in the arena, and a quantum of the arena would need a forbidden pairing (Chapter 3, Section V). So the description ends at the boundary, and the framework has no quantum theory of the arena to continue to.

**What does this chapter not fix?**

Two things. The physics of processes that approach the boundary, the black hole interior and the start of an epoch, requires an account of how general relativity and quantum mechanics relate in the transition region. I treat the transition region in Chapters 18 and 19, but in this book I do not fix the detailed dynamics of the transition region (Chapter 18, Section XI). Whether the magnitude of $G_N$, that is, the size of a content relative to the boundary, can be derived from a deeper structural principle is not addressed in this book (Chapter 20).



Chapter 18

# The Interior Passage

*On the Identity of Black Hole Collapse and Cosmological Origin*

In this chapter I argue that the collapse inside black holes and the origin of a cosmological epoch are one transition in a cycle, approached as a limit from its two sides. The argument runs in eight steps: the end of every interior lies outside the exterior's time; all interiors are one at their end; every observer's comparable exterior empties; the Mach reference unpins; the exterior's structures dissolve; the conditions that hold the internal space compact fail; the ten-dimensional geometry restores an exterior context; and I identify that restoration with the Big Bang, as it appears in the history reconstructed from inside the new epoch. No comparison registers the turn between the two sides as an event. The turn appears as a beginning only in a history reconstructed from inside the new epoch. I then give the entropy at the turn, the flatness and the scalar departures of the new epoch, the standing of Hawking radiation, and the cycle with its tests. The first step uses results of general relativity. Three steps rest on principles of comparison stated here: the identity of interiors, on the principle that a difference no comparison can register is no difference; the unpinning of the Mach reference, on inertia being a comparison; and the failure of compactification, on Axiom III. Each holds at the standing of its principle.

*Rests on:* Axiom III (Chapter 4), the reading of the theorem in physics (Chapter 3, Section V), the content and context factors of the product (Chapters 7 and 8), the internal space and its uniqueness (Chapter 9), the context of actual content (Chapter 10, Section III), the information deficit (Chapter 13, Sections II, III and V), the modal boundary (Chapter 17), the singularity theorems and the no-hair

theorem, and Appendix B. *Refers to:* the content's spinors (Chapter 12, Questions), and the turn in the reconstructed history and what does not replace singularities (Chapter 19, Sections III and IV). *Used by:* Chapter 13 (Sections III and VI), Chapter 14 (Section III), and Chapter 19. *Referred to by:* Chapter 11 (the compactification conditions), Chapter 12 (Questions), and Appendix B.

## I. The Singularity Is Not Now

The first step is temporal. The singularity at the center of a black hole belongs to no moment of the exterior's time. Two facts of general relativity establish this, and they are one structure.

The first fact concerns the exterior. In the time of a distant observer, an infalling object approaches the event horizon without end: its crossing takes infinite coordinate time. No signal from inside the horizon ever reaches the exterior.

These facts are stated for holes whose evaporation takes far longer than the exterior takes to empty: about $10^{67}$ years for a hole of one solar mass, against $10^{11}$ to $10^{12}$ years for the exterior (Section III). A hole lighter than about $5 \times 10^{11}$ kilograms would have evaporated by now (Carr, Kohri, Sendouda and Yokoyama 2010). I make no claim on the end of such a hole. Whether the end of such a hole lies in the past of later observers of the exterior belongs with the question of evaporation that the Questions of this chapter record.

The second fact concerns the interior. The infalling object itself crosses the horizon, and the singularity then lies a finite proper time ahead of it: for a black hole of one solar mass, at most about fifteen microseconds after the crossing.

The two facts agree once the singularity is placed correctly. Inside the horizon of a non-rotating hole, the radial direction is the direction of time. The singularity is therefore a moment, not a place at the center: the future end of every path inside the horizon. The singularity is not a point of the spacetime. Each path inside the horizon is defined for every proper time

short of that end and for none beyond it, so the end is a limit that the path approaches and does not reach. That limit lies after every moment of the interior. No signal from it reaches any moment of the exterior, so no exterior comparison can place it before, at or after any of the exterior's own moments. For rotating and charged holes, the inner horizon of the exact solutions is unstable to the infall that forms real holes, and the generic end is expected to be of the same kind: a limit approached by the oscillating directions that Appendix B describes. From the standpoint of external physics, the singularity has not happened and never happens at any finite external time. From the standpoint of the interior, it is the end of time.

The first structural ingredient is that the ends of all black hole interiors lie on a boundary that the exterior's time does not order. No exterior observer can register one such end as earlier or later than another, since no signal from either end reaches any exterior observer.

## II. All Interiors Are One at Their End

The second step is topological. All black hole interiors end in one structural domain. Three facts carry the step, and one principle of comparison joins them.

The first fact is merger. Black holes that are bound to one another merge in the exterior's own time, as LIGO's detections of binary black hole mergers confirm. The merged black hole has one interior, and that interior contains the interiors that merged.

The second fact is the no-hair theorem. The settled exterior of a black hole is determined entirely by its mass, charge and spin. Nothing else about the collapse that produced it can be registered from outside.

The third fact is the step just taken. The ends of all interiors lie on a boundary that the exterior's time does not order, so nothing in the exterior separates one end from another in time.

The principle I apply is that a difference that no comparison can register is no difference. Every distinction is made by comparison, and a distinction that no comparison can make is not a distinction.

What the principle identifies depends on which comparisons it counts, so I state its scope exactly. It counts every possible comparison: every comparison that any comparer could make, in any domain, at any place and at any time. A comparison registers what reaches it, as a finding, a signal or a record. I take a comparison to register nothing that it only predicts, since a prediction is a possibility and not a finding. A comparison that can take one thing as content and cannot take another registers a difference between them, since it finds the one and does not find the other. The principle therefore identifies two things only when no comparison anywhere tells them apart, and it never identifies two things that any one comparison can tell apart. The requirement that no comparison anywhere tells the two things apart is the only bound on what the principle identifies.

The principle counts comparisons as structure. Every comparison that could be made exists, as a possibility, and the principle does not need any of them to be real. It therefore agrees with Chapter 13, Section VI, which takes only the present act as real.

Stated exactly, the principle is that two things are one when, for every possible comparison and for every thing whatever (the two themselves included), the comparison of the first thing with that thing gives the same verdict as the comparison of the second thing with that thing. A comparison that cannot take one of the two things as content gives, for that one, the verdict that it cannot. So stated, the principle defines an equivalence. Each thing has one pattern of verdicts, its verdicts with every thing in every possible comparison, and two things are one when their patterns are the same; two patterns that are each the same as a third pattern are the same as each other. A chain of pairs, each one with the next, therefore joins its ends. The weaker reading, on which two things are one when no comparison of the one with the other finds them different, defines no equivalence. On that reading a thing that a comparison cannot take as

content is never found different by that comparison, so it would be one with each of two things that the comparison finds different. A comparison of limited resolution would find each member of a graded series the same as its neighbor and the two ends of the series different. The exact statement excludes both failures, because it compares the patterns of the two things and does not compare only the two things with each other. Two things that are one in the exact sense are also never found different by a comparison of the one with the other.

The interiors themselves are not one. A comparer inside an interior registers that interior and does not register another, and it registers the mass of its hole by the proper time that remains when it crosses the horizon: $4GM/(3c^3)$ for a fall from rest at infinity into a non-rotating hole. What the principle identifies is the ends of the interiors. The end of an interior is a limit, and no comparison is made at it (Section I). No comparison of the epoch in which the hole forms registers that end: no signal from the end reaches any moment of the exterior or of the interior, and a comparison does not register what it only predicts. The only comparisons that register anything that comes from the end are those of the new epoch, and its context re-forms with no prior identity (Section VIII). I take that context to register no distinction that the old exterior drew, so no comparison of the new epoch registers from which interior any part of its content came. Every possible comparison therefore gives the end of one interior the same verdicts as the end of another, and the ends are one.

The identity of the ends does not rest on a comparison made at the end, of which there is none; it rests on there being no comparison, before the end or after the turn, that registers a difference between them. Applied at the end itself, the principle would find the end indiscernible from every other thing at which no comparison is made, the beginning of the epoch among them, and it would identify too much.

The exact statement of the principle keeps the turns of successive epochs apart. The comparisons of an epoch register the turn from which the epoch emerged, as the beginning that its reconstructed history finds;

they do not register the turn at which the epoch's own interiors end, since that turn lies in their future. The two turns receive different verdicts, and the principle does not identify them.

Now consider the perspective of an observer falling inward. That observer is not approaching the end of *one* black hole. They are approaching the common end of all black hole interiors. The question "which black hole am I inside?" has a referent only while the exterior can register an answer. At the end, it has none.

The closer the observer approaches the end, the less its own register of the curvature tells one hole from another. An observer falling from rest at infinity into a non-rotating hole, with proper time $\tau$ left before the end, measures a radial tidal stretching of $4/(9\tau^2)$ and a transverse compression of $2/(9\tau^2)$, whatever the mass of the hole. On any other radial path the part of these values that depends on the mass falls in proportion to $(c^3\tau/GM)^{2/3}$ as the end is approached. The geometry near the end has no length of its own, and the mass of the hole remains registered only in how long the fall has lasted and in the records the observer carries.

## III. The Exterior Empties

The third step concerns what remains outside. For every observer in the exterior, the part of the universe that the observer can ever compare with is bounded.

The bound is the cosmological event horizon. The expansion of the universe accelerates, and every observer has such a horizon: no event beyond it can ever send a signal to that observer. Content that crosses it is comparable only through what it sent before crossing, and that light is redshifted without limit. Content beyond the horizon, in its present and its future, is not comparable with the observer in principle, and so it is no part of that observer's exterior. Chapter 13 reckons the cosmological con-

stant from the entropy of an observer's horizon; there the horizon is the particle horizon, the bound of what has reached the observer, and here it is the event horizon, the bound of what can ever reach the observer.

Within the horizon, gravitational dynamics makes matter clump. Stars form, burn out and leave remnants; remnants accrete and merge; bound structures end in black holes, and what they do not take in departs, as ejected bodies or as radiation. Whatever is not bound to the observer's own structure is carried beyond the horizon by the expansion.

Over sufficiently long times, every observer's comparable exterior therefore empties. Its matter ends inside horizons, or departs beyond the cosmological horizon, or thins as radiation toward the horizon's own temperature. The exterior is not eternal. It is a transient feature: the epoch in which the context-modal description has a populated exterior domain.

As the exterior empties, the structural conditions that supported the product geometry in its macroscopic form begin to fail.

## IV. The Mach Principle Unpins

The fourth step concerns inertia. Inertia is a comparison. To be at rest, to move uniformly or to rotate is always to do so relative to something. A state of motion is a content, and what it is measured against is its context.

The Mach principle, in its most conservative formulation, states that inertial frames are determined by the distribution of mass-energy in the universe. The gyroscope points toward distant stars because those stars define "non-rotating". General relativity does not impose this principle: its equations admit solutions empty of matter, in which the metric still fixes local inertial frames with nothing for those frames to be compared against.

The principle of comparison supplies what general relativity leaves open. No comparison registers a difference between one frame and another when nothing is there to compare them against, and a difference that no comparison registers is no difference. Inertial frames are frames only

through the comparable content that fixes them. Content beyond the horizon is not comparable in principle (Section III), so it fixes no frame for the observer; the reference is the content within the horizon.

Every observer belongs to a bound structure, and every bound structure ends inside a horizon (Section III); whatever departs instead is carried beyond the horizon, alone. At the end of an interior, the exterior has no moment (Section I), so the distant stars, the remaining holes and the residual radiation offer no comparison there. For a body carried away alone, the only reference left is its own horizon, which the body fixes and no content does. In either case there is no comparable distribution of mass-energy relative to which inertial frames could be defined. The Mach principle then fixes no frame.

Time, which in the exterior had a definite large-scale direction (the direction of increasing entropy, the direction of expansion, the direction that the Mach reference defines), loses that direction. The ordering of before and after across the exterior, and the ordering of cause and effect between distant events, which rest on the Mach reference, are no longer structurally supported. The one-way order inside each act of comparison (Chapter 5, Section IV) is not reversed; at the end of an interior there is no comparable content for it to order.

## V. The Exterior's Structures Dissolve

The fifth step concerns the structures that the exterior supports. Two of them matter here: the cosmological constant, and the form that the content takes. Each is a feature of the product geometry in its macroscopic, exterior configuration. Inside the common interior, neither keeps the form it has in the exterior.

*The cosmological constant.* The cosmological constant is the accessible share fixed by the channel count $c_1$ and the entropy of the particle horizon, the bound of what has reached the comparer (Chapter 13). A comparer that falls into a black hole keeps that count. The hole's horizon

bounds what can leave the hole, and it bounds nothing that reaches the comparer: light from the exterior continues to cross it inward, and the hole lay within the comparer's particle horizon before the fall, its entropy among the S that horizon counts (Appendix B, Section VI). Inside the hole the accessible share, and with it the cosmological constant, is therefore unchanged, apart from a correction that Appendix B, Section VI, computes and finds negligible. The quantity formed with the hole's own entropy fixes the share of no comparer; Appendix B, Section VI, gives its value and its exact reading. The "push" that the cosmological constant provides in the exterior description is not a force resisting collapse. It is a feature of the exterior description: inside the hole as outside it, the count that fixes it is that of the particle horizon, which lies in the exterior. Inside the hole its curvature, $\Lambda$, is about $10^{-45}$ of the tidal curvature at the horizon of a hole of one solar mass, $2GM/(c^2 r_s^3)$, and the ratio falls further on the way in. As the exterior empties and the distinction between interiors dissolves at their end (Section II), the very concept of a well-defined horizon, the structural prerequisite for the $\Lambda$ derivation, ceases to apply; at the end of an interior the continuously asymmetric description reaches the modal boundary (Section VI), and there the constant has no purchase at all.

*The content.* The matter of ordinary experience is the content, whose structure depends on the internal geometry being compact and having a definite topological structure; I make no claim about which structure (Chapter 8, Section II). As the internal geometry evolves toward de-compactification, the structure that the content takes in the compactified epoch is no longer locked in. The content in that form, the matter of stars and planets and people, is a transient structural feature of the compactified epoch, not a permanent substrate.

## VI. Compactification Conditions Fail

The sixth step is the structural crux. The internal space $\mathbb{C}P^2 \times S^2$ is the content factor of the product: the discontinuously symmetric factor, on whose space of states the discrete spectra of the content are read (Chapter 8, Sections II and IV). The exterior factor is the context factor: the continuously asymmetric factor, a context by its modal character (Chapters 7 and 8).

Axiom III holds content and context distinct, inseparable and non-interchangeable. Inseparability means that no content stands without a context in which it is content; each content metric is measured in its own context (Chapter 7). The product $M^4 \times \mathbb{C}P^2 \times S^2$ is the geometric form of this inseparability: the compact factor is held as the content factor relative to the exterior factor as its context. The compactification conditions, stated exactly, are these: the exterior factor is present as a context for the compact factor, as a domain of the continuously asymmetric description with an inertial reference fixed in it.

At the end of an interior, both conditions fail. The exterior has no moment there (Section I), and the continuously asymmetric description reaches its domain limit there: the singularity theorems show that paths inside the horizon end within a finite proper time, and Chapter 17 identifies it as the modal boundary. No inertial reference survives there (Section IV). Steps three and four show that every observer reaches such an end, or is carried away alone with no reference but its own horizon. The exterior factor is then no longer the context in which the compact factor is held as content. Axiom III excludes content without context, so the compact form of the internal space cannot persist as it was. The structural constraints that held the internal geometry compact are no longer in force.

## VII. De-Compactification as the Restoration of Context

The seventh step is the resolution. When compactification conditions fail, the ten-dimensional structure restores an exterior context. It must. The content does not cease at the modal boundary: there each existing description reaches the edge of its domain, and no description takes over (Chapter 17, Section IV); the edge belongs to the descriptions and is not an ending of the content, and what passes the turn is the form of the content (Section XIII). Axiom III excludes content without context, and the only structure from which a new context can form is the ten-dimensional geometry that remains.

Nothing then holds the internal dimensions compact, and the internal space expands. The distinction between "internal" and "external" dimensions re-forms. A new four-dimensional spacetime re-forms as the external factor of the product. It has a new cosmological horizon, and a new Mach reference forms as matter distributes itself in the new exterior. Appendix B gives the form of this change in the ten-dimensional geometry: the powers by which the external directions and the internal directions scale are the same on the two sides of the transition, traversed in opposite directions of time.

The content's fields of half-integer spin are defined by that distinction (Chapter 12, Questions). Each is a spinor of the four-dimensional factor whose labels are read on the internal space as a space of states (Chapter 8, Section IV), and the construction uses the split of the ten directions into four and six and nothing else. It is therefore defined wherever the product form holds: throughout an epoch, and along the arc of Appendix B on both sides of the turn, where the grouping of the directions is kept. At the turn no geometry is constructed (Chapter 19, Section IV), and the account defines no field there, of any spin. What passes the turn is the form of the content (Section XIII), and the spin of a content passes as one of its labels (Appendix A, Section IV), not as a field. The ten-dimensional geometry admits no spin structure (Chapter 12, Questions), so no description of half-integer spin as a field on the undivided ten directions is available,

and I offer none. In the new epoch the product re-forms with the same internal topology (Questions), and its four-dimensional factor is the product of a line of time with an orientable three-manifold, spatially flat (Section IX). Every such four-manifold is parallelizable, so it admits a spin structure whatever metric it carries (Geroch 1970). So does every four-manifold this book uses: the exterior epoch, the interior of a non-rotating hole, and the Kasner geometries of Appendix B are each globally hyperbolic in the epoch in which the book uses them. So the spinors of the new epoch are defined as those of the old were.

Higher-dimensional cosmologies of this Kasner kind, with three directions expanding while the others contract, were found in five-dimensional vacuum gravity by Chodos and Detweiler (1980), and in ten- and eleven-dimensional supergravity, with a preferred expansion of three directions, by Freund (1982). The restoration re-forms the product geometry, and the number of large spatial directions is the number of spatial directions of the context factor (Chapter 10, Section IV).

The eighth step completes the identification. I identify this restoration of exterior context with the Big Bang of the new epoch. The restoration is the far side of the limit that the collapse approaches (Section I). No comparison registers the passage through that limit as an event: on the side of the collapse the description holds up to the limit and not at it, and on the side of the new epoch the same holds in the opposite direction of time (Section VIII). The history of the new epoch, reconstructed from inside it, runs back toward the limit and finds there its beginning. The Big Bang is the turn as it appears in that reconstructed history; Chapter 19, Section III, gives the account of that appearance as a projection.

The restoration is a transition in a cycle: the product geometry reconstituting its exterior factor, because the conditions that dissolved the old exterior also create the necessity for a new one. It is therefore neither a creation from nothing, nor a fluctuation in a pre-existing vacuum, nor a singularity at which the equations break down and the description falls silent.

## VIII. One Process, Two Sides

The black hole collapse and the cosmological origin are one process, approached from the two sides of the modal boundary.

From the exterior, the collapse is the approach to the modal boundary from within the context. The context-modal description (GR) traces the geometry through increasing curvature, increasing density and decreasing volume, until the description approaches the Planck scale and can go no further. The singularity theorems (Penrose 1965; Hawking 1966) prove that such paths exist, which end within a finite proper time and cannot be continued past that end.

From the other side, the history of the new epoch, reconstructed from inside it, runs back toward the same boundary. Traced backward, the context-modal description of that epoch approaches the boundary through increasing curvature, increasing density and decreasing volume. Read forward, it is the standard FLRW expansion, with curvature and density decreasing and volume increasing. The reconstruction reaches no first moment: it approaches a limit, and it takes that limit for its beginning.

The two descriptions approach one limit, the turn, from its two sides, in opposite directions of time. No comparison registers the turn itself as an event, since each description holds up to the limit and not at it. Temporal language applies to the approach on each side and to the history reconstructed from inside the new epoch; it does not apply to the turn, which no comparison registers as instantaneous or as gradual. The collapse concentrates the content; the emergence disperses it.[11] The two surviving conjunctions of the Incommensuration Theorem, the continuously asymmetric (context) and the discontinuously symmetric (content), meet at the boundary, where neither description dominates (Chapter 17, Section II). On the collapse side, the context-modal conjunction presses toward its domain limit. At the boundary each existing description reaches the edge of its domain, and no description takes over (Chapter 17, Section IV). On the emergence side, the context-modal conjunction holds again from the boundary outward.

**Entropy at the turn.** The identification must meet a question of entropy, which Penrose has pressed against bounce cosmologies (1979; 2010). The interiors carry an enormous gravitational entropy: the Bekenstein-Hawking entropy of their horizons. The new exterior begins with a very small one: its geometry is smooth, and its Weyl curvature is near zero. The matter at the beginning of the reconstructed history is hot and close to equilibrium, so its entropy is already high; the drop is in the gravitational part.

I give a structural reason for that drop. The Bekenstein-Hawking entropy counts the states of an interior that the exterior cannot distinguish. It is a count relative to the exterior as context. At the turn that exterior has emptied (Section III), and the count has no reference left. The new exterior is a new context. Its four-dimensional metric re-forms from a geometry in which it had no separate identity. A metric with no prior identity inherits no large-scale anisotropy or inhomogeneity from the old exterior, so on this account it begins conformally flat up to the departures the Incommensuration Theorem requires: perfect isotropy would make symmetry and continuity both perfect, which the Incommensuration Theorem forbids. The old distinctions therefore survive the turn only as small departures from uniformity, averaged and not erased. A new exterior metric that begins conformally flat, up to small departures from uniformity, is Penrose's Weyl curvature hypothesis. I reach the Weyl curvature hypothesis here as a consequence of the restoration of context at the turn, and not as a separate postulate.

The same principle applies to the entropy of the matter. An entropy is a count of the states that a context can distinguish. The count of the matter's states, like the count of the interiors' states, is reckoned relative to the exterior that ends. The new epoch reckons its counts relative to its own context. The content itself is carried from the collapse on one side of the turn to the emergence on the other (Section XIII). The new context does not register the distinctions the old context drew, and that is the reset.

*The scope of the step.* The step to conformal flatness concerns the geometry in which actual content stands. That geometry is a context, and the context of actual content has no isometries (Chapter 10, Section III), so the new metric departs from perfect isotropy, and its departures meet the theorem. The statistics of the departures pairs two aspects: the class of departures that could be selected, a possibility, and the weight it gives each, a probability. A statistical symmetry, scale invariance among them, is a sameness of the probability across a difference within the possibility, a symmetry and a continuity in two conjugate aspects (Chapter 3, Section V), and one actual sky is selected from the class. The theorem permits that pairing. The cosmological constant is fixed once, by the present measurement (Chapter 13, Sections V and VI), and de Sitter space is the geometry of an idealized content (Chapter 10, Section III); the geometry found carries content, its comparers among it. The step requires the departures, and it fixes neither the spectral index nor w.

The reset meets the objection that Tolman (1934) raised against cyclic histories. Tolman showed that entropy produced in one cycle and carried into the next makes each cycle larger and longer than the last. Extrapolated backward, such a history has finitely many past cycles. Here no count is carried across the turn, for the matter or for the geometry, since each count is relative to the context that ends. Each epoch begins its own count, and the cycle has no first iteration. The reset is given here by principle; a count of states across the turn, in numbers, is not given in this book.

## IX. The Departures from Uniformity

*What the departures are.* I identify the departures with the form of the content carried from one side of the turn to the other (Section XIII), the old distinctions averaged. The vacuum of the new epoch adds almost nothing to them: a scale observed today lay, near the turn in the reconstructed history, some $10^{27}$ times beyond the new horizon, and the vacuum of a wave so far beyond the horizon carries a power below $10^{-51}$,

against the observed $2 \times 10^{-9}$. The averaging is the averaging of one domain. Independent pieces, averaged, would leave white noise, with a spectral index of 4 in the curvature or 0 in the density, and the observed index is close to 1. The interiors are one at their end (Section II): no comparison of the new epoch registers from which interior any part of its content came. The form that passes is therefore not a sum of independent pieces, one for each interior; it is correlated across the whole new exterior, and it is one form for every component of the content. This consequence and the identity of the ends rest on one premise, that the new context has no prior identity; the spectral index itself is not fixed by them (Section XI).

*The tilt.* A spectrum that refers to no length is exactly scale invariant, and a tilt is a reference to a length. The measured index, about 0.97, is a weak reference of the departures to a length of the new epoch.

*The sign of the tilt.* The reference to a length does not fix the sign of the tilt. The departure of the metric on a length $\lambda$ does not change while $\lambda$ lies beyond the horizon, and it can depend on a length $\ell$ only through the ratio of $\lambda$ to $\ell$. The index is below one when that departure grows with $\lambda$, and above one when it falls with $\lambda$. To name $\ell$ does not decide between the two, since for any $\ell$ the departure may grow toward $\ell$ or fall toward it. The new epoch has two lengths of its own, the two named under spatial flatness below: the length of the turn, of the order of the Planck length, and the de Sitter radius. The de Sitter radius is a length of the epoch's law and of its reconstructed history, to which the departures may refer; it is not a length of the state that the restoration sets, which holds the length of the turn alone (spatial flatness, below). Near the turn in the reconstructed history, a length observed today was some $10^{27}$ Planck lengths and some $10^{-34}$ of the de Sitter radius, so every observed length lies between the two. If the departures of the metric were largest on the length to which they refer, a reference to the length of the turn would give an index above one, and a reference to the de Sitter radius would give an index below one. Such a rule must be stated for the departures of the metric: the departures of density, compared at one time, are largest on the shortest

lengths for any index near one, and so cannot decide the sign. I identify neither the length to which the departures refer nor the way the departures depend on it. In this section I therefore do not fix the sign of the tilt.

*The running.* The change of the index from one length to another, its running, is where a reference to one particular length would show. If the departure of the metric is a fixed power of $\lambda/\ell$, a change of $\ell$ changes only its amplitude, so its dependence on length cannot show which length it refers to. A reference that weakens as the logarithm of the larger length over the smaller grows does show, in the running. If the index is below one and the reference weakens in this way, the running is negative when the length referred to lies below the observed lengths, and positive when it lies above them. If the departure varies as a fixed power of that logarithm, the size of the running is the departure of the index from one divided by the logarithm: about $6 \times 10^{-4}$ for the length of the turn, and about $4 \times 10^{-4}$ for the de Sitter radius. The measured running is consistent with zero, with an uncertainty of about $6 \times 10^{-3}$ (Akrami and colleagues 2020; Calabrese and colleagues 2025), more than ten times these values.

*The waves of the metric do not bear on the sign.* Their absence (Section XIII) rests on the scalar character of the primordial departures of the metric, which concerns the kind of departure that the content sources and not the way the departures depend on length. The waves that arise at second order are sourced by the scalar departures, and the logarithm of their power changes with length twice as fast as that of the scalar power. They carry the same reference, at a power far too small to measure it.

*The departures of the metric.* What passes is the form of the content, and the departures of the new metric are those the content's departures source. A departure of density sources a scalar departure of the metric. A wave of the metric needs an anisotropic stress, which content close to equilibrium does not have at first order, so the primordial departures of the metric are scalar, and waves arise from them only at second order.

*Spatial flatness.* The restoration also fixes the spatial curvature, which conformal flatness leaves free. A radius of spatial curvature is a length. In the equation of the expansion the curvature carries a constant that the law of the geometry does not fix: it is a constant of integration, set once by the state from which the history begins and carried unchanged thereafter (Misner, Thorne and Wheeler 1973, Chapter 27). So the radius of curvature belongs to the state that the restoration sets, and it can be built only from what that state holds. The metric has no prior identity, and no total of the old content passes, since no comparison of the new epoch registers the mass of the old interiors (Section II). No length of the old epoch passes the turn, and no count passes it either (Section VIII). The cosmological constant belongs to the law of the geometry, fixed once as its constant term (Chapter 13, Section VI), and at the turn it acts as nothing at all (Section XII). The state at the restoration therefore holds the length of the turn, of the order of the Planck length, with the radii of the internal dimensions of the same order (Chapter 11, Section IV); the pure numbers of its structure, of order one; and the size of the departures, about $10^{-5}$. A radius built from the length of the turn and the de Sitter radius together, such as the length halfway between them in powers of ten, is the length of the turn multiplied by a pure number near $10^{30}$. It would need the de Sitter radius in the state, where the constant does not act, or a pure number near $10^{30}$ in the state, where the only numbers that large are counts, and no count passes the turn. So the radius is of the order of the length of the turn, or there is none. A radius of the Planck length would close the new epoch within a few Planck times or leave it dominated by curvature, and neither epoch produces the structure that the cycle requires (Section XII): the restoration is one process, recurring at every turn, and an epoch without structure makes no black holes from which a next epoch could come. The homogeneous model of the new epoch is therefore spatially flat, and the geometry found is curved only by its departures, at their own size, about $10^{-5}$. A radius equal to the de Sitter radius would leave a curvature of about $10^{-64}$ today, so that length too, were it in the state,

would leave no curvature to measure. The argument uses nothing of the cosmological constant, neither its value nor its sign. Chapter 13, Section III, uses this flatness, and the flatness takes nothing from the constant that the identity there fixes.

*One premise, two results.* That the new geometry has no prior identity gives conformal flatness: the new metric inherits no shape from the old exterior, so its Weyl curvature is zero up to the departures (Section VIII). Conformal flatness leaves the spatial curvature free, since homogeneous spaces of every curvature are conformally flat, and what decides between them is a length. The same premise, applied to length, gives spatial flatness: the new metric inherits no length from the old epoch, and the state holds no length from which a radius could be made other than the one the cycle excludes. So the Weyl curvature hypothesis and the flatness of space come from one premise, and the step that settles the second is the cycle (Section XII).

*Another route to flatness.* Gibbons and Turok (2008) studied the natural measure on the space of homogeneous cosmologies, and Carroll and Tam (2010) drew from it that "curved RW cosmologies are a set of measure zero", so that "the flatness problem, as conventionally understood, does not exist". Theirs is a route of typicality: flat histories are almost all of them. McCoy (2017) disputes that route, holding that "no conclusion can be drawn at all about the flatness problem by using the canonical measure alone". The argument of this section rests on what the state at the restoration holds and not on any measure, so it stands however that dispute is settled, and the measure, where it holds, reaches the same flatness by another road.

## X. The Hawking Question

A natural objection: if black holes evaporate via Hawking radiation, then eventually no black holes remain, no interiors exist, and the argument collapses.

The popular picture of Hawking radiation, in which virtual particle pairs split at the horizon, one escaping and one falling in, is misleading. The actual derivation is a Bogoliubov transformation: the vacuum state defined before the collapse differs from the vacuum state defined after, and the mismatch produces a thermal spectrum at the Hawking temperature $T_H = \hbar c^3/(8\pi G M k_B)$.

For a stellar-mass black hole, $T_H \approx 6 \times 10^{-8}$ K, nearly eight orders of magnitude below the cosmic microwave background. The thermal emission consists of photons with wavelengths comparable to the Schwarzschild radius: kilometers, not particle scales. Such a black hole absorbs far more than it emits. It would begin to net-evaporate only after the universe cools below $6 \times 10^{-8}$ K. In accelerated expansion the radiation temperature falls by a factor e about every seventeen billion years, so the cooling takes of order $10^{11}$ to $10^{12}$ years. The evaporation itself then takes about $10^{67}$ years. The time of the evaporation is vastly longer than the timescale on which black hole mergers proceed, and vastly longer than the timescale on which the interior argument operates.

The deeper issue is the trans-Planckian problem. The Bogoliubov transformation traces field modes backward in time, toward the moment of horizon formation. As they are traced back, their wavelengths blueshift without bound. The modes that eventually become Hawking radiation were, at the moment of collapse, trans-Planckian: their wavelengths were far below the Planck length. The derivation assumes that standard quantum field theory on a smooth classical background applies at those scales.

That assumption carries the context-modal description past the modal boundary. The Planck scale is the modal boundary, where each existing description reaches the edge of its domain and no description takes over (Chapter 17, Section IV). Trans-Planckian modes are modes that have been traced past that edge, where the context-modal description is no

longer valid. The Hawking derivation, as written, carries the description to all scales. The Incommensuration Theorem says the description does not hold at all scales.

Physicists have tested this concern directly. Unruh (1995) and Corley and Jacobson (1996) changed the dispersion of the field at short wavelengths, and Corley and Jacobson (1998) put the field on a lattice. The thermal spectrum survived whenever the surface gravity lies far below the cutoff. In these models the outgoing modes trace back to ingoing modes near the cutoff, not to modes of arbitrarily short wavelength, and the temperature still depends only on the surface gravity. An analog black hole in a Bose-Einstein condensate, whose short-wavelength physics is atomic, shows a thermal spectrum at the predicted temperature (Muñoz de Nova and colleagues 2019). What these models show is that the flux does not depend on the details of the short-distance regime: the thermal radiation of the horizon does not rest on what happens beyond the Planck scale. I therefore take the Hawking flux as real, on the timescales given above.

The Incommensuration Theorem marks the step at which the standard derivation traces its modes past the modal boundary, and the results above show that the thermal flux does not rest on that step. Hawking radiation is real and negligibly slow for astrophysical black holes, and the interior argument proceeds on timescales vastly shorter than the evaporation time.

## XI. What the Incommensuration Theorem Can See

The singularity theorems of Penrose and Hawking are theorems of general relativity. They prove that geodesic incompleteness occurs: there exist trajectories that approach the boundary of the description within a finite proper time and cannot be continued past it. The theorems cannot say

what lies beyond that boundary. They are silent at the singularity because the singularity is the limit of the mathematical structure within which the theorems are proved.

The Incommensuration Theorem is a theorem of the structure of comparison, not a theorem of general relativity. It is prior to the content/context partition. It does not depend on the smooth manifold, the Lorentzian metric, or the Einstein equations, and its premises do not include the distinction between gravity and gauge forces.

The Incommensuration Theorem can therefore describe the modal boundary, which GR cannot describe. Where GR finds a singularity, a place where the description breaks down and the equations become meaningless, the Incommensuration Theorem identifies the modal boundary: the limit at which the description in each conjunction (continuously asymmetric and discontinuously symmetric) reaches the end of its domain, and at which no description takes over (Chapter 17, Section IV). The limit is the edge of the descriptions, not a breakdown of what they describe.

The Incommensuration Theorem cannot compute the detailed dynamics of de-compactification: the precise rate at which the internal geometry expands, the temperature at which the reconstructed history of the new exterior begins, and the amplitude and tilt of the departures from uniformity that the restoration leaves (Section IX). The amplitude is set by the count of states over which the form of the content is shared as it becomes the departures of the new metric; the tilt, by the length to which the departures refer and by the way the departures depend on it (Section IX). Those details belong to the physics of the transition, which is neither purely GR nor purely QM but the interface between them. The Incommensuration Theorem provides the structural framework that identifies the transition and constrains its nature. The quantitative details of the transition are not fixed in this book.

## XII. The Cycle

If black hole collapse and cosmological origin are one transition approached from its two sides, the cosmological history is a cycle, and not a single arc from Big Bang to heat death.

The cycle has no first iteration. The beginning that each epoch's reconstructed history finds, its Big Bang, is the turn approached by the collapse of the previous epoch's interiors. Each epoch produces structure, structure produces black holes, the interiors of the black holes share one end, and at that end the internal space de-compactifies into a new epoch (Figure 4). The turns of successive epochs are distinct: the comparisons of an epoch register the turn from which it emerged and do not register the turn at which its interiors end (Section II). The epochs therefore follow one another in a single sequence.

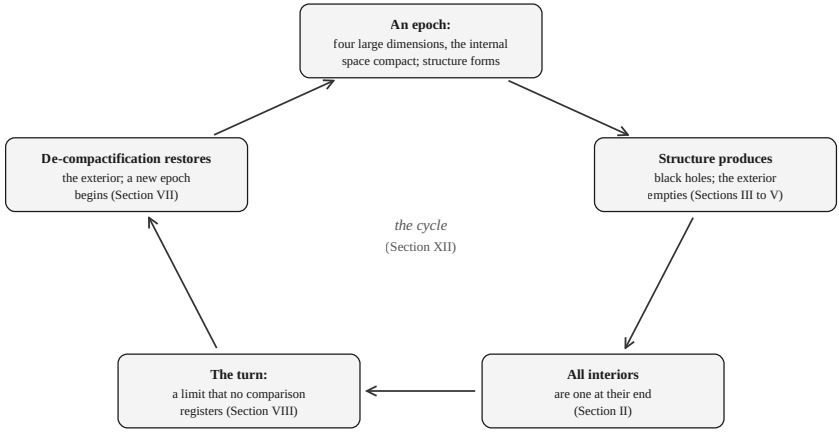



***Figure 4.*** *The cycle of this section, with the sections that state each step. The turn is a limit, and no comparison registers it as an event; the history of each epoch, reconstructed from inside, finds the turn as its Big Bang.*

Gravity concentrates the content toward collapse: the context-modal tendency toward concentration, curvature and collapse. De-compactification restores context when the conditions for compactification dissolve. The two alternate. Neither is more fundamental; both follow from the constraint the Incommensuration Theorem places on the relationship between the two surviving conjunctions.

The cosmological constant does not prevent this cycle. $\Lambda$ is a feature of the exterior description: a structural property of the relation between an observer and its horizon. Inside the common interior, where the exterior has dissolved, $\Lambda$ does not act as a force resisting collapse. It acts as nothing at all, because the structural conditions for its definition are no longer met.

The cycle is compatible with $\Lambda > 0$. The accelerated expansion of the exterior does not prevent the formation of black holes (they form at all scales), does not prevent the merger of bound black holes, does not prevent the identity of all interiors at their end (Section II), and does not prevent the dissolution of compactification conditions (which depend on the exterior, not on $\Lambda$).

A black hole interior as the origin of a universe has precedents (Pathria 1972; Frolov, Markov and Mukhanov 1989; 1990; Smolin 1992; Popławski 2010), and so has a succession of epochs, each joined to the next (Penrose 2010).

## XIII. Predictions

The argument of this chapter generates specific predictions, some testable, some not.

*No information paradox.* If the black hole interior connects to a cosmological origin, then information that falls into a black hole is not destroyed. It is carried with the content from the collapse to the emergence and reemerges in the new epoch's initial conditions, as the departures from uniformity (Section IX). What passes is information as the form of the con-

tent, the specification of what has been selected, and that information passes with the content. What does not pass is a count of states reckoned relative to a context, since every such count is relative to the context that ends (Section VIII). The information is not accessible from the exterior of the original epoch: it is lost to every observer who remains outside the horizon, and it is not destroyed. The information paradox rests on the premise that information must be recoverable in the original exterior, and I reject that premise.

*No first cause.* The cycle has no first iteration. The question "what came before the Big Bang?" has an answer: the collapse of the preceding epoch's interiors, which approaches from the other side the limit that this epoch's reconstructed history takes for its beginning. "Before" here names the order of the cycle, the collapse on one side of the limit and the new epoch on the other; no comparison measures a time across the limit. The question "what came before that?" has the same form of answer, without end. The demand for a first cause mistakes the temporal boundary that physics reports for a boundary in the structure itself. That boundary is the turn as it appears in a history reconstructed from inside (Chapter 19, Section III). In the structure itself, the turn is a transition in a cycle, approached as a limit. The cycle has no first iteration.

*$w = -1$ exactly.* The cosmological constant is structural, not dynamical. The equation-of-state parameter for dark energy is exactly $-1$, not approximately $-1$: $\Lambda$ is fixed once, by the horizon of the measurement (Chapter 13, Section V).

*No primordial gravitational waves.* The primordial departures of the metric are scalar (Section IX); waves of the metric arise from them only at second order, with a ratio of tensor to scalar power far below $10^{-6}$.

*Spatial flatness.* The new epoch begins spatially flat (Section IX). The spatial curvature measured within one horizon is that of the departures, of order $10^{-5}$.

*Adiabatic departures.* The departures pass the turn as one form for every component of the content (Section IX), so the primordial departures are adiabatic: every component departs from uniformity together, and no primordial isocurvature mode separates one component from another. The microwave background bounds such a mode at a few percent of the power (Akrami and colleagues 2020).

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## Questions

**Does a lone body carried beyond every horizon de-compactify?**

Yes. The condition of Section VI is that the exterior factor be present as a context for the compact factor, as a domain of the continuously asymmetric description with an inertial reference fixed in it. For a body carried away alone, the only reference left is its own horizon, a horizon that the body itself fixes and that no content fixes (Section IV). No content then fixes an inertial reference for that body, the condition fails, and the internal space of that body de-compactifies. Section VI includes the case in those terms: every observer reaches the end of an interior, or is carried away alone with no reference but its own horizon.

What follows for its matter is what Section V states for all matter. The matter of ordinary experience is the content in the form it takes in the compactified epoch, and that form is no longer locked in once the internal space is no longer held compact. A lone particle in the far future therefore ceases to be matter in that form. The content itself does not cease (Section VII), and the restoration of context applies to it. Whether the body de-compactifies "while bound structures persist elsewhere" is a question that no comparison answers: the body lies beyond the cosmological horizon of every other observer, so no comparison orders its de-compactification before, at or after any event of theirs (Section III), as no exterior comparison orders the ends of the interiors (Section I). For the same reason no observer can test the case; the answer is the one the condition of Section VI gives.

**If all interiors are one at their end, how does information pass through the turn while counts of states are reset?**

I use "information" and "count of states" for two different things. Information as the form of the content is the specification of what has been selected (Section XIII): which alternatives the content has taken. It belongs to the content, and the content is carried from one side of the turn to the other (Section VII), so the form is carried with it. A count of states is the number of states that a context can distinguish (Section VIII). It is reckoned relative to that context: the Bekenstein-Hawking entropy counts the states of an interior that the exterior cannot distinguish, and the entropy of the matter counts the states of the matter that its exterior can distinguish. At the turn the context of every such count has ended, so no count carries over, and the new epoch reckons its counts relative to its own context. That new reckoning of the counts is the reset.

The identity of the interiors and the passage of the form do not conflict. Section II identifies the interiors because no comparison at their end registers a difference between one and another. To register such a difference would be to set the form of one interior against the form of another in a context that distinguishes them, and the only context that drew those distinctions, the old exterior, has no moment there (Section I). The form therefore passes as one form, undistinguished as to the interior it came from. The new context registers it only as small departures from uniformity, averaged and not erased (Section VIII), correlated across the whole new exterior because the domain they come from is one. Those departures are differences within the new epoch, registered by its own context; they are not differences between the old interiors.

**What solves the horizon problem, and is inflation needed?**

Regions of the sky that no signal within the reconstructed history of this epoch has connected share one temperature to about a part in $10^5$. Inflation was proposed to explain that sameness, and the flatness of space, by a phase of accelerated expansion early in the history (Guth 1981). In this account the sameness has a common origin. The interiors are one at their

end (Section II), and the new exterior re-forms from that one domain, with a metric that has no prior identity and so inherits no large-scale anisotropy or inhomogeneity (Section VIII). Regions that no signal within the new epoch has connected are alike because they come from one domain, not because a signal passed between them. The same origin makes the departures one form for every component of the content, correlated across the whole new exterior, with a spectral index near 1 and not the index of independent pieces (Section IX). The spatial flatness follows from the restoration: no length and no count of the old epoch passes the turn, so the state that the restoration sets holds the length of the turn and only pure numbers of order one, and the cycle excludes a radius of that length, which leaves a flat homogeneous model (Section IX).

The account therefore needs no inflationary phase for the horizon and flatness problems. On the identification of the departures with the form of the content, the account excludes an inflationary phase long enough to stretch the scales observed today from within the early horizon to beyond it: in such a phase the departures on those scales would be those of the inflationary vacuum, and Section VIII holds that they are the form of the content carried through the turn, to which the vacuum of the new epoch adds almost nothing. A shorter phase, which leaves the observed scales outside the early horizon throughout, is neither required nor excluded by the account, and the book does not fix it. The amplitude of the departures, about $10^{-5}$, and their tilt, about 0.97, are not fixed in this book (Section XI).

**Do the constants carry over from one epoch to the next?**

The structure that I derive is the same in every epoch, at the standing of the identifications of Chapter 9. The topology of the internal space, $\mathbb{C}P^2 \times S^2$, is uniquely determined (Chapter 9, Section VII). The shape of its metric is fixed, with $R_O/R_T = \sqrt{6}$, and its radii are fixed in units of the Planck length, $R_T = l_{Pl}$ and $R_O = \sqrt{6}\, l_{Pl}$, by the modal boundary (Chapter 9, Section IX). The channel count $c_1 = 3$ is topological, and the identity $I^2\Omega_\Lambda = 8$

$-3/\pi$, which fixes $\Omega_m$, contains no dimensionful measured constant; it uses the measured radiation fraction (Chapter 13, Section III). None of these is a count reckoned relative to the old exterior, so the reset at the turn does not touch them.

$G_N$ is the one scale parameter. With $\hbar = c = 1$ it sets the unit of length, and I take its value from measurement. A change of $G_N$ with every quantity measured in Planck units unchanged is a change of unit, which no comparison registers. A comparison of $G_N$ in one epoch with $G_N$ in the next would be a comparison across the turn, and no comparison is made across it (Section VIII); so I assign no value of $G_N$ to another epoch. What a comparer in any epoch measures is a dimensionless ratio, such as a mass of the content to the Planck mass. Those ratios concern the composition of the content, which is outside this book, so whether they repeat from epoch to epoch is not fixed in this book.

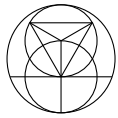
Smolin (1992) proposed that each black hole gives rise to a new universe whose dimensionless parameters differ slightly from those of its parent, so that a population of universes is selected for producing black holes. This account differs at two steps. All interiors are one (Section II), so the epochs form a single sequence, not a branching population on which selection could act. And the structure listed above is fixed in every epoch by the axioms, so it is not a variable for selection.

**What does this chapter not fix?**

The quantitative dynamics of de-compactification: the rate at which the internal geometry expands, the temperature at which the reconstructed history of the new epoch begins, and the amplitude and tilt of the departures (Section XI). The count of states across the turn, in numbers, which would test the reset of Section VIII. Whether the content of a body carried away alone re-emerges in the one new epoch that the interiors share; the answer to the first question gives what the chapter's condition settles for that body, and I do not argue the rest in this chapter.

How this account meets the return of information by radiation. An evaporating black hole would begin to return the information it holds at its Page time, about half of its evaporation time: of order $10^{67}$ years for a stellar mass, far longer than the time on which black holes merge in the exterior (Section X). How the two accounts meet is not set out in this book.

Whether the dimensionless ratios of the content repeat from epoch to epoch, which lies outside this book (the fourth question).



Chapter 19

# What Replaces Singularities

*On the Limits of the Context-Modal Description*

Classical general relativity predicts singularities: the future end of every path inside the horizon of a collapsed non-rotating star (Chapter 18), and the origin of the expansion in the past. This chapter places both outside the domain of the continuously asymmetric description, which governs the context factor down to the modal boundary and no further. It traces the black hole interior through layers of rising equivalent density to that boundary, and states the ten-dimensional duality that joins the interior to the cosmological compactification. The ordinary interior lies off the arc of that duality, and what carries the ordinary interior onto the arc is not fixed in this book. It gives the book's account of what the "event" of the turn is: a limit approached and not reached, which appears as a beginning in a history reconstructed from inside. It does not construct a geometry below the boundary.

*Rests on:* the forbidden pairing (Chapter 3, Section V), the partition (Chapter 6), the internal radius (Chapter 11), the modal boundary (Chapter 17), the corrections at the Planck curvature (Chapter 12, Section VII), the reconstructed history (Chapter 13, Section V), the interior passage (Chapter 18), and the Kasner curve (Appendix B). *Used by:* Chapter 20. *Referred to by:* Chapter 17, Section IV (the corrections at the Planck curvature); Chapter 18 (the temporal boundary that physics reports; the density account); Appendix A (Section VII); and Appendix B (Sections II, V and VII).

## I. Singularities as Artifacts

A singularity in general relativity is a point at which a curvature invariant diverges: $R_{\mu\nu\alpha\beta}\, R^{\mu\nu\alpha\beta} \to \infty$. The singularity theorems of Penrose and Hawking establish that under generic conditions (positive energy and an appropriate causal structure) geodesics end in finite proper time (Penrose 1965; Hawking 1966). Singularities appear in every realistic model of gravitational collapse and in the time-reverse of every expanding universe.

The standard reading takes these singularities as genuine breakdowns of the theory, and expects a theory of quantum gravity to replace each singularity with a finite-curvature quantum-gravitational structure.

In this book the singularity is a breakdown of the *context-modal description*: the description that treats spacetime as a smooth, continuously asymmetric manifold. That description holds at curvatures below the Planck curvature and does not hold beyond it. The singularity is what the context-modal equations produce when they are extended into a regime where they do not hold.

The partition assigns gravity to the context factor (Chapter 6). The context factor is governed by the continuously asymmetric conjunction: smooth manifolds, Riemannian curvature, the Einstein field equations. At the Planck scale that conjunction meets the modal boundary (Chapter 17), where the corrections to the Einstein action are of order one (Chapter 12, Section VII). Beyond the boundary no description takes over (Chapter 17, Section IV), and the smooth manifold structure does not hold.

I do not resolve singularities in this book by quantizing the gravitational field. The gravitational field is the geometric arena, and a quantized arena needs a forbidden pairing (Chapter 3, Section V). I place the singularity outside the domain of the context-modal description, and that domain has its boundary at the Planck scale.

## II. The Black Hole Interior

The Schwarzschild metric describes the gravitational field outside a spherically symmetric mass. Inside the event horizon, the radial and temporal coordinates exchange their causal character: the infall toward the center becomes as inevitable as the passage of time. The curvature increases without bound as the radial coordinate approaches zero.

The black hole interior has a layered structure. The tidal curvature at a radius r equals the curvature just outside the surface of a body of the hole's mass M and radius r. Each depth therefore has an equivalent density: the density at which the hole's mass would fill a sphere of that radius. The equivalent density rises inward as $1/r^3$. Its rise reverses the sequence of densities through which the early universe passed.

For a given mass, each density is reached at a fixed radius. As a fraction of the Schwarzschild radius, that depth falls as $M^{-2/3}$. For a hole of one solar mass, the density at which hadrons dissolve, about $10^{18}$ kg/m³, is already exceeded at the horizon. The density at which the electroweak symmetry is restored, about $10^{30}$ to $10^{31}$ kg/m³, is reached at about $2 \times 10^{-4}$ of the Schwarzschild radius. For a hole of four million solar masses, like the one at the center of the Galaxy, the same two densities are reached at about $10^{-4}$ and $10^{-8}$ of the Schwarzschild radius. These densities come from standard physics, and I use the densities only as markers of depth.

The internal space begins to change when the external curvature approaches the curvature of the internal space, of order $1/R^2$ for an internal radius R. Since R is a few Planck lengths (Chapter 11), this happens close to the Planck layer. For a hole of one solar mass, the Planck density is reached near $10^{-26}$ of the Schwarzschild radius, and the internal space begins to change within a factor of a few of that radius. That depth scales as $R^{2/3}$. There the internal dimensions of $\mathbb{C}P^2 \times S^2$ begin to expand from their compactified size back toward the ten-dimensional geometry.

The innermost layer lies at the Planck scale. There the context-modal description reaches the modal boundary, the continuously asymmetric conjunction can no longer be sustained, and the smooth manifold structure that produces the singularity in the classical equations ceases to hold.

With the three external directions, the four of $\mathbb{C}P^2$ and the two of $S^2$ each moving together, the Kasner exponents of a singularity in the ten-dimensional geometry lie on one closed curve, on which the external exponent runs from $-1/3$ to $+5/9$ (Appendix B, Section III). The interior described here and in Chapter 18, in which the external space collapses while the internal spaces expand, and the cosmological compactification, in which the external space expands while the internal spaces collapse, both belong to the short arc at the upper end of this curve, read in opposite directions of time. At the end point of the arc, external exponent $+5/9$ and internal exponent $-1/9$, the six internal directions move as one, and I place both the black hole interior and the cosmological compactification there. On the arc, the singularity of the black hole interior and the cosmological singularity are one geometric structure, traversed in opposite directions of time, and the black hole interior mirrors the cosmological history in reverse.

The ordinary interior of a Schwarzschild black hole is not on the arc. Its exponents are $-1/3$, $2/3$ and $2/3$, with the six internal exponents zero and the internal space fixed in size. The ordinary interior is a valid Kasner point of the ten-dimensional geometry, and the ordinary interior does not lie on the curve, whose external exponents are equal (Appendix B, Sections II and VII). Near a generic singularity, ten-dimensional vacuum gravity does not settle into one Kasner form: it oscillates through an endless sequence of them (Demaret, Henneaux and Spindel 1985; Appendix B, Section VII). What carries the interior from the ordinary form onto the arc, and how the product geometry selects one Kasner form against the oscillation, is not fixed in this book. The duality holds for an interior that has reached the arc.

## III. The Big Bang

The FLRW cosmology traces the expansion of the universe backward to a point at which the scale factor vanishes and the density diverges. Standard cosmology reads that point as the moment at which the universe began.

In this book the origin of an epoch is the turn at which the collapse of the previous epoch's interiors becomes the restoration of context (Chapter 18, Sections VII and XII). I take the turn to be the limit at which the continuously asymmetric description ends; the discontinuously symmetric description reaches the edge of its own domain at the modal boundary, and no description takes over (Chapter 17, Section IV). The collapse approaches that limit and does not reach it.

To know the turn as an event would need a comparison that registers it: a comparison of a before with an after, made across the turn. There is none. The exterior has no moment at the end of an interior (Chapter 18, Section I), and the interior's own register ends at the singular boundary (Chapter 18, Section II). A difference that no comparison can register is no difference (Chapter 18, Section II). So no comparison registers the turn as an event, and it cannot be known as one.

The turn appears as an event in a history reconstructed from inside. The past is known only by the impression it makes on the present, and the history of the universe is reconstructed from that impression as the one history it requires (Chapter 13, Section V). I take a reconstruction to report what it finds as content (the omniscient modality), discrete and located. Traced backward, the reconstructed history meets the limit and renders it as a located point: a beginning at a definite time, the temporal boundary that physics reports. I identify the Big Bang of the FLRW equations with that appearance, and I call the appearance the projection: the turn, which no comparison registers as an event, rendered as an event in a history reconstructed from inside. The singularity at $t = 0$ is where the reconstruction meets the limit that it cannot pass.[12]

Chapter 18 describes the passage as a transition in a cycle, with a before and an after. The cycle is the structure; the before and the after are epochs, each with its own reconstructed history, and the turn between them is the limit that neither history contains. The cycle of Chapter 18 and the projection of this section are one account: the turn is the limit, and the event is how the limit appears from within an epoch.

On the arc of Section II, the singularity at $t = 0$ and the singularity of the black hole interior are one geometric structure, traversed in opposite directions of time. Chapter 18 sets out the argument that the collapse and the origin are one process, step by step, from the place of the singularity outside the exterior's time to the restoration of exterior context. Appendix B gives its form in the dimensional schema of the ten-dimensional geometry.

## IV. What Does Not Replace Singularities

I place the singularity outside a domain: the singularity is the extension of the context-modal description into a regime where that description does not hold, and the boundary of the domain is the Planck scale. I do not offer a resolution of the standard kind in this book: I do not replace the singularity with a finite-curvature structure computed from a quantum theory of gravity.

The question "what is the geometry at the singularity of the black hole?" is, in this framework, a category error. It asks for the continuously asymmetric structure of a moment that lies beyond the modal boundary, in the regime where the continuously asymmetric conjunction no longer governs. The question has no answer because it presupposes that the context-modal description holds in a domain where the axioms say it does not.

Beyond the boundary the structure of comparison does not cease. The three modalities still operate, the axioms still hold, and the Incommensuration Theorem still holds. What ceases is the partition in its macroscopic

form: the assignment of gravity to the context factor as a smooth manifold phenomenon. Beyond the Planck scale no description takes over (Chapter 17, Section IV): the full axiom structure holds there without the macroscopic partition into separate content and context geometries, and I construct no description of that region.

Loop quantum gravity, which constructs a model of that region, also reaches a finite curvature at the Planck scale (Ashtekar, Pawlowski and Singh 2006; Ashtekar and Bojowald 2006).

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**Questions**

**Is the black hole interior on the Kasner arc, as the duality of Section II between the interior and the cosmological compactification requires?**
Not in its ordinary form. The ordinary Schwarzschild interior, exponents $-1/3$, $2/3$ and $2/3$ with the internal space fixed, is a valid Kasner point of the ten-dimensional geometry and lies off the curve (Appendix B, Section II). The duality is stated for an interior that has reached the arc. Section II and Chapter 18 describe an interior on the arc, in which the external space collapses while the internal space grows.

**Ten-dimensional vacuum gravity oscillates near a generic singularity. Does that not exclude a single Kasner form?**
For vacuum gravity near a generic singularity, yes: the geometry passes through an endless sequence of Kasner forms (Demaret, Henneaux and Spindel 1985), and a form field threading the internal space would also drive the oscillation (Damour and Henneaux 2000). A single Kasner form with each group of directions moving together is consistent within the product geometry, where the curvature of the internal space falls away against the rate of collapse, and it is not the generic behavior (Appendix B, Section VII).

**Chapter 18 describes a before and an after the turn; in this chapter I say that the turn is no event. Do the two chapters state two theories?**

No. The turn is a limit that the collapse approaches and does not reach, and no comparison registers it as an event (Section III). Each epoch reconstructs its own history from the impression the past makes on the present, and in that history the limit appears as a beginning. The cycle of Chapter 18 orders the epochs; the projection of Section III says what the beginning of each is as it appears from inside.

**Does this chapter not replace one unknown, quantum gravity, with another, the description beyond the boundary?**

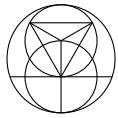
In this chapter I replace a theory that would need a forbidden pairing with a domain boundary that the axioms fix. In this book I do not construct a geometry beyond the boundary, and no other description holds beyond the boundary (Section IV). What I state in this chapter is where the context-modal description ends and why, and that no smooth geometry is to be sought beyond that point.

**What does this chapter not fix?**

Two things. The detailed physics of each layer of the interior, the dissolution of hadrons, the restoration of the electroweak symmetry and the decompactification, requires a dynamical analysis beyond the Kasner approximation. What carries the ordinary interior onto the Kasner arc, and how the product geometry selects one Kasner form against the oscillation, is not derived (Section II).

# PART VI

The Account as a Whole



Chapter 20

# The Complete Chain

*The Steps, What They Rest On, and What They Predict*

This chapter sets out the chain from comparison to classical physics in seven steps, says what the chain establishes, states the standing of its inputs and results, and sets out the book's principal predictions, each with the step it rests on. The standing of every link, and what each link rests on, is set out in the map at the front of the book.

*Rests on:* every chapter before it, and the appendices. *Used by:* the Conclusion.

## I. The Chain

The chain has seven steps, each resting on those before it.

*Step 1* (*Chapters 1 and 2*). Observation, measurement and law are comparisons, and comparison cannot not involve six intrinsics (sameness, difference, content, context, subjective, objective), from which four compound concepts are defined: continuity, discontinuity, symmetry and asymmetry.

*Step 2* (*Chapters 3 and 4*). The Incommensuration Theorem proves that only two conjunctions survive: continuously asymmetric and discontinuously symmetric. The three axioms and the modalities (omniscient, immanent, transcendent) are the structure that comparison already has: Axiom III in the three pairs of intrinsics, Axiom II in the order of their engagement, Axiom I in the priority of the act over its compounds.

*Step 3* (*Chapter 6*). The content/context partition assigns gravity to context (continuously asymmetric) and the gauge forces to content (discontinuously symmetric).

*Step 4* (*Chapters 7 to 10*). The product structure $M^4 \times \mathbb{C}P^2 \times S^2$ is the geometric expression of the theorem. Given the identifications of Chapter 9, the internal space is the only compact homogeneous product that meets them, with six dimensions, one for each metric. Four external dimensions come from the cycle of Axiom II, and their Lorentzian signature from the directed precedence of Axiom II (Chapter 10); the total is $10 = 4 + 6$.

*Step 5* (*Chapters 11 and 12*). The Kaluza-Klein reduction of the ten-dimensional Einstein-Hilbert action, with the moduli held, gives the four-dimensional Einstein equations, and Lovelock's theorem gives their form as the only one.

*Step 6* (*Chapter 13*). $\Lambda$ follows from the information deficit at the observer's particle horizon. $G_N$ is the one measured constant.

*Step 7* (*Chapters 15 and 16*). Geodesic motion follows from the conservation of the stress-energy. The weak-field limit gives Newton's law of gravitation, and the standard solutions follow.

The map at the front of the book gives the standing of each link (theorem, standard technique, identification, argued, computed or measured) and the links each one rests on.

## II. What the Chain Establishes

General relativity, with its geometric formulation, the Einstein field equations, the equivalence principle, geodesic motion and the causal structure of spacetime, follows from the structure of comparison and the three axioms it carries (Part I), together with the identifications the book states, in the sense of "derived" given in the Preface. The Incommensuration Theorem gives the product; the product, with the Kaluza-Klein re-

duction, gives the four-dimensional equations; Lovelock's theorem gives their form; the information deficit fixes $\Lambda$; and $G_N$ is taken from measurement.

## III. The Standing of the Chain's Inputs and Results

*The step from the axioms to the geometry.* The translation of the three axioms into conditions on the internal space (Chapter 9) rests on identifications: one dimension for each metric; a product of a factor carrying the modalities and a factor carrying the rows, each connected and of positive dimension; each factor carrying its terms as its independent classes; homogeneity; and, for the shape of the metric, the Einstein condition (Chapter 9, Section IX). They are identifications, not theorems.

*The gravitational constant.* $G_N$ is the empirical constant of this book. The framework fixes its structural role and requires one dimensionful anchor; the magnitude of the coupling is a fact about this domain (Chapter 17). I do not derive it.

*The comparison.* With one set of measured parameters, the comparison of $\Lambda$ and the comparison of $\Omega_m$ are one test. The external value includes the bare curvature term of the internal space at the derived radius, a classical contribution beside the Planck density (Chapter 13, Section II). The identity then gives $\Omega_m = 0.311$, against the measured $0.315 \pm 0.007$, and the predicted vacuum density lies about 1.7 percent above the observed one. With the Planck density alone, the predicted density lies about fifteen percent above. The coefficients of order one that would change these numbers are named in Chapter 13.

*The stabilization.* The internal space is held at the derived radius as a requirement of the book's premises (Chapter 11, Section II); the masses of the moduli come from one mechanism, the least motion of the metric, computed at first order (Appendix D).

*The interior.* The dimensional schema of the black hole interior (Appendix B) shows collapse and compactification as one arc of powers traversed in opposite directions of time.

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## Questions

**Which steps would a hostile but fair referee accept as proved, and which only as argued?**

The map at the front of the book shows the answer link by link. The Incommensuration Theorem of Chapter 3, the classification of Chapter 9 given its conditions, the reduction of Chapter 11, Lovelock's theorem in Chapter 12, the identity of Chapter 13 given its formula, and the Kasner algebra of Appendix B are mathematics that a referee can check. The steps between them that pass from comparison to physics are identifications, stated as such in the first person where they occur.

**The book's results rest on identifications as well as theorems. Is the book's "derived" then of a different kind from a theorem drawn from stated inputs?**

A result derived in this book is of a different kind from a theorem drawn from stated inputs, and I say so in the Preface. The inputs of this book are comparison, with the three axioms it carries (Part I), and the identifications stated in the first person where they are made: among them the characters of content and context (Chapter 3, Section V), the grid of the six metrics as the internal space (Chapter 9), precedence as the order of events (Chapter 10, Section III), and the accessible share of the vacuum's energy (Chapter 13, Section II). From those inputs the results follow by theorem or standard technique: the classification of compact homogeneous spaces (Chapter 9, Section VI), the Kaluza-Klein reduction (Chapter 11), and Lovelock's theorem (Chapter 12, Section VII). A referee who rejects an identification rejects the results that rest on it. What I claim is nar-

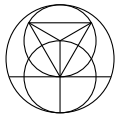
rower than a theorem from the axioms alone: the dimension of the manifold, its signature and the product follow from inputs that do not contain them.

**Which of the predictions of this book's account distinguish it from the standard picture of cosmology, which is the frame in which the measurements are reported?**

Two do: the matter fraction, and the Hubble constant it implies, which are one test. The table sets out each prediction, the step it rests on, whether the standard picture expects it, and whether it distinguishes the account. Here, as throughout the book, confirmed means established at five standard deviations by at least two independent kinds of measurement.

| Prediction | Step it rests on | Expected in the standard picture? | Distinguishing? |
|---|---|---|---|
| $w = -1$ exactly | $\Lambda$ fixed once (Chapter 13, Sections V and VI) | Yes: the default of standard cosmology | No; a confirmation supports the account only weakly |
| $\Omega_m = 0.311$ | the accessible share $c_1/S$ (Chapter 13, Section II) | No: a free parameter of the standard fit | Yes |
| $h = 0.678$, given $\Omega_m h^2$ from the microwave background | the same step; the same test as $\Omega_m$ (Chapter 13, Section IV) | No: free | Yes, with the row above |
| Entanglement of two masses through gravitation alone | one geometry for each alternative (Chapter 12, Section VI) | Yes, in linearized quantum gravity | Only against accounts with one geometry for all alternatives |
| Ratio of tensor to scalar power far below $10^{-6}$ | the scalar character of the primordial departures (Chapter 18, Section IX) | No: many inflationary models give a ratio within reach of coming measurements | Weakly |
| Spatial curvature of order $10^{-5}$ | | Yes, in inflationary cosmology | No |

| | | | |
|---|---|---|---|
| | the fixing of the curvature by the restoration (Chapter 18, Section IX) | | |
| No primordial isocurvature mode (adiabatic departures) | one form for every component of the content (Chapter 18, Section IX) | Yes, in single-field inflation | Only against accounts with several independent fields |



# Conclusion

Einstein arrived at general relativity by following two threads: the equivalence principle and the requirement of general covariance. He needed the mathematics of Riemannian geometry, supplied by Grossmann. He needed the physical insight that gravity is geometry, not force. And he needed the specific form of the field equations, which he found by requiring consistency with Newtonian gravity in the weak-field limit and with the conservation of energy-momentum.

I arrive at the same equations by a different path. It begins not with physics but with the structure of comparison: what must be true whenever anything is compared with anything else. From the structure of comparison it proves a constraint, the Incommensuration Theorem, that partitions physics into two structural kinds, and it finds the three axioms as the structure that comparison already has. From that partition, with the identifications it states, it obtains a product geometry whose context factor is a four-dimensional Lorentzian manifold. From that geometry, by Kaluza-Klein reduction, it obtains the Einstein field equations, with the cosmological constant fixed by the information deficit and the gravitational constant taken from measurement.

By this path I answer questions that Einstein's path leaves: *why* the equations have the form they do, *why* gravity is geometry rather than force, *why* the equivalence principle holds, and *why* $\Lambda$ has the size it has, up to the coefficients of order one that Chapter 13 names.

The step from the axioms to the internal geometry rests on identifications (Chapter 9), and the balance that holds the internal space is computed at first order (Chapter 11). The book's account is tested against observation at six places: the equation of state of the vacuum, $w = -1$; the matter fraction, $\Omega_m = 0.311$; the entanglement of two masses through

gravitation, with the phase of their Newtonian interaction; no primordial gravitational waves; a spatially flat epoch; and primordial departures with no isocurvature mode (Chapter 20).

Where the answers hold, they are not empirical discoveries. They are consequences of the structure of comparison together with the identifications the book states, and the six principal observations are where those consequences meet the world.

The claim that quantizing the arena is a category error has no observational content below the modal boundary. Below it, one geometry for each alternative of the matter predicts the entanglement that linearized quantum gravity predicts (Chapter 12, Section VI), and the predictions that distinguish the account come from the identity of Chapter 13 and from the turn of Chapter 18.

# APPENDICES

Appendix A

# The Argument for the Content/Context Partition

*The Markers, the Cross-Modal Principle, and Per-Alternative Geometry*

In this appendix I give in full the argument that Chapter 6 states. From three principles of the Immanent Metaphysics I derive four markers that any context-modal phenomenon must have, and gravity has all four. I derive the cross-modal principle: the formalism of a physical theory and the observables it predicts are in conjugate modes. From that principle I derive the mode assignment theorem: at the microscopic limit, a theory of content has discontinuously symmetric observables and a continuous formalism, and a theory of context has continuously asymmetric observables and a symmetric formalism. Quantization takes a classical theory of content into a quantum theory of content by a mode exchange, which exchanges the characters of both modes at once. The arena, the geometry of context, admits no mode exchange. The configuration that remains is per-alternative geometry: quantum fields for the matter, with a classical geometry for each alternative of the matter.

*Rests on:* symmetry and continuity as characters of what a comparison finds (Chapter 2), the Incommensuration Theorem and its first rejection criterion (Chapter 3), the forbidden pairing (Chapter 3, Section V), and three principles of the Immanent Metaphysics (Section I). *Refers to:* the content and context triads (Chapter 7), one geometry for each alternative of the matter (Chapter 12, Section VI), the singularity as the limit of the context-modal description (Chapter 19, Section I), and the equivalence as a triple (Chapter 14, Section II). *Used by:* Chapter 6; Chapter 8 (Questions); Chapter 9, Section V; Chapter 10, Section III; and Chapter 14. *Referred to by:* Chapter 3, Section V; and Chapter 7 (Section I and Questions).

## I. The Ground

The argument draws on three principles of the Immanent Metaphysics, which I take as given; the Notes give where each is stated.

The first principle is a pair of laws, the law of symmetry and the law of continuity, which I call the *two laws*. The law of symmetry applies to all that is of content. The law of continuity applies to all that is of context.[13]

The second principle is the *inseparability principle*: where there is content, there must also be context, and where there is context, there must also be content.[14]

The third principle is the characterization of content and of context. Content is local, interexclusionary (two contents exclude one another from the same place), and understood in terms of symmetry. Context is non-local, intercompositional (contexts compose with one another), and understood in terms of continuity.[15]

Together with the Incommensuration Theorem of Chapter 3, these three principles are the whole of the input. A *context-modal phenomenon* is any phenomenon that plays the role of context in the physical domain: the arena within which content occurs. A *content-modal phenomenon* is any phenomenon that plays the role of content: what occurs within that arena.

## II. The Four Context Markers

Each context marker below is derived for any context-modal phenomenon whatever, before gravity is mentioned. The comparison with gravity comes after each derivation.

*Universality of coupling.* By the inseparability principle, every content occurs within a context. A context-modal phenomenon is the structure within which all content occurs, so every content entity participates in it. Therefore a context-modal phenomenon couples to every content entity,

without exception. Gravity couples to energy and momentum, which every form of matter and radiation carries. The gauge forces couple only to their specific charges. Gravity has no neutral objects.

*Non-shieldability.* Context is the prerequisite for the existence of any region at all. To shield a context-modal phenomenon would be to make a region from which the context is absent. A region is itself a structure of context, so a region without context is not a region. Therefore a context-modal phenomenon cannot be shielded. Electromagnetic shielding is routine, and the strong force is confined to hadronic scales; there is no gravitational shield of any kind.

*The equality of gravitational and inertial mass.* For a content-modal force, the charge that sets the coupling and the inertia that resists acceleration are two different quantities: the charge is a property of content, and the inertia is a response to change of context. Content and context are distinct, so these two quantities are in general unequal; electric charge and inertial mass are unrelated. For a context-modal phenomenon, I take the coupling as the degree of participation in the context, and the inertial response as the degree of resistance to change of context, and I identify the two as one quantity. Therefore gravitational mass equals inertial mass. The local indistinguishability of gravity and acceleration follows in the same way: gravity is a phenomenon of context, and acceleration is a change of context; as phenomena of context the two are locally indistinguishable, and their causes remain distinct (Chapter 14, Section II). The same identification says what that one quantity measures. To participate in the context is to be found in it, and the degree to which a content can be found is how much of it there is to find: its pattern. So pattern, inertia and weight (the gravitational mass) are one relation of a content to its context, found three times: as what there is of it to be found, as its resistance to a change of its place in the context, and as the hold the context takes on it. Mass is the measure of that one relation. The equality of gravitational and inertial mass is measured to about one part in $10^{15}$ (Touboul and colleagues 2022). In Chapter 14 I develop the same ground further, showing the equivalence as a triple.

*The absence of a discrete charge.* Content is governed by symmetry, and symmetry yields discrete invariants that can be classified: quantum numbers, conserved charges, labels of representations. Context is governed by continuity, which yields no such discrete invariants. Therefore a context-modal phenomenon carries no discrete conserved charge; its coupling is a continuous participation in the context. The gauge forces have discrete charges: electric charge in multiples of one third of the electron's charge, three colors, weak isospin of plus or minus one half. Gravity couples through the stress-energy tensor, a continuous quantity with no discrete label.

The axioms require a context-modal phenomenon to exist, since content and context are inseparable. They fix its four context markers. Gravity has all four. That single structural match is the empirical input to the identification of gravity as the phenomenon that fills the context-modal place the axioms define in advance (Chapter 6, Section II).

In Chapter 6, Section II, I name four markers by which gravity differs from the gauge forces. Universality is derived above. The geometric marker, that context is the arena and is not within anything, rests on the same ground as non-shieldability: a region is a structure of context. Non-quantizability is derived in Sections III to V below. Spin 2 is argued in Chapter 6 itself.

## III. The Cross-Modal Principle

Any act of investigation is a comparison between a framework and a content. The Incommensuration Theorem constrains that comparison. Its first rejection criterion, !x, forbids self-application: symmetry joined with symmetry, or continuity joined with continuity, yields no semantic content. A framework in the same mode as its content is that mode applied to itself. For the comparison to find anything that self-examination could not, the content must be in the conjugate mode.

The *cross-modal principle* is that framework and content are always in conjugate modes. A symmetric framework probes continuous content; a continuous framework probes symmetric content. The reach of the framework into its content is dense but never exhaustive.

A physical theory is such a comparison. Its formalism is the framework, and its observable predictions are the content. The *constitutive mode* of a theory is the mode of its observables: discontinuously symmetric where the observables are discrete (eigenvalues separated by gaps, exact quantum numbers), continuously asymmetric where they are continuous (smooth fields, real-valued measurements with no structural preference for particular values). The *instrumental mode* is the mode of its formalism. The cross-modal principle requires the two to be conjugate.

A formalism has features of both characters. A quantum theory has a continuous space of states and groups of symmetries acting on it; a classical field theory has smooth fields and a group of gauge transformations. The instrumental mode is the character of the feature that stands in the mode conjugate to what the theory finds. A continuous group of symmetries shows how the two modes are told apart, because the group has both characters: it is a symmetry, and its elements vary with continuous parameters. Where what a theory finds includes the labels of the group's representations (discrete charges and quantum numbers), the symmetry is found, and the continuity belongs to the formalism: the continuous parameters of the group and the continuous space of states on which it acts. Where what a theory finds is continuous and carries no label of the group (smooth fields, trajectories, a geometry), the continuity is found, and the symmetry belongs to the formalism: an invariance of the description, whose invariants are the quantities found. So a continuous symmetry is a symmetry and a continuity in the two conjugate modes of one theory: the mode of what the theory finds, and the mode of the theory's formalism. I give the same reading of a continuous symmetry in Chapter 3, Section V.

The table names, for four established theories, what each finds and the feature of its formalism that stands in the conjugate mode. In each, the formalism has such a feature, as the cross-modal principle requires.

| **Theory** | **What it finds (constitutive mode)** | **Conjugate feature of the formalism (instrumental mode)** |
|---|---|---|
| Quantum mechanics | the discrete spectra of bound states, and quantum numbers, the labels of the representations of its symmetry groups: discontinuously symmetric | the Hilbert space of states, with continuous amplitudes, on which the symmetry groups act with continuous parameters: continuous |
| Quantum electrodynamics | charges in whole multiples of an elementary charge, the labels of the representations of U(1), and whole numbers of photons: discontinuously symmetric | the Fock space of states, with continuous amplitudes, on which U(1) acts with a continuous parameter: continuous |
| General relativity | a smooth geometry, with no isometries in general: continuously asymmetric | diffeomorphism invariance of the description, whose invariants are the geometry found: symmetric |
| Classical electrodynamics | smooth electric and magnetic fields and a continuous charge density, none of them a label of U(1): continuously asymmetric | U(1) gauge invariance of the potentials, whose invariants are the fields found: symmetric |

The cross-modal principle is a claim, and not a reading that every theory meets: it excludes a theory whose formalism has no feature in the mode conjugate to what the theory finds, such as a cellular automaton on a lattice, with discrete states, a discrete rule and no continuous parameter anywhere in its formalism.

## IV. The Mode Assignment Theorem

*Lemma 1: governance by symmetry gives discrete observables.* Content is governed by symmetry, by the two laws. The observables of a content-modal phenomenon are its invariants under the symmetries of the content factor, which are compact: the quantities that keep a sameness of content across a difference of context. Invariants of the context's own symmetries that take continuous values, such as the invariant mass of a scattering pair under the Poincaré group, belong to the context (Questions). The discrete labels of the compact part of those symmetries are content. Spin labels the representations of the rotations of the frames of $M^4$, a compact group whose double cover is the SU(2) within Spin(1,3), and a content keeps its spin across every change of context; so spin is a content, registered against the frames that the context supplies, and its half-integer values are why the content's fields of half-integer spin are spinors of $M^4$ (Chapter 12, Questions). A body's own mass is likewise content, the metric of pattern (Chapter 7, Section I); the invariant mass of a pair is a relation of two contents through the context. The invariants fall into classes that the symmetry preserves, and each class is a discrete unit; to classify is to divide. The Incommensuration Theorem pairs symmetry with discontinuity. Therefore the observables of any content-modal phenomenon are discontinuously symmetric. The mathematics of the content factor agrees: the irreducible representations of a compact group form a discrete set (the Peter-Weyl theorem), labeled by quantum numbers, and the Laplacian on a compact manifold has a discrete spectrum.

*Lemma 2: governance by continuity gives smooth observables.* Context is governed by continuity, by the two laws. The observables of a context-modal phenomenon are quantities that vary smoothly, without gaps. The Incommensuration Theorem pairs continuity with asymmetry. Therefore the observables of any context-modal phenomenon are continuously asymmetric: smooth, and not repeating.

*The theorem.* At the microscopic limit, the fundamental theory of any content-modal phenomenon has a discontinuously symmetric constitutive mode and a continuous instrumental mode. Any physical theory of context-modal phenomena has a continuously asymmetric constitutive mode and a symmetric instrumental mode. The content-modal configuration and the context-modal configuration are the only two configurations. The proof joins the two lemmas to the cross-modal principle. The content/context partition is exhaustive and exclusive; the two laws assign the governance; the Incommensuration Theorem pairs each governance with its observable character; the cross-modal principle assigns the instrumental mode. Classical theories of content, such as classical electrodynamics, describe content at the macroscopic limit, where its discreteness is not resolved; the table of Section III records their modes as they stand, and the theorem concerns the fundamental description.

The table below sets out the grid that the two laws govern, row by row. Its rows are the content triad and the context triad of Chapter 7; its columns are the three modalities.

| | **Omniscient** | **Immanent** | **Transcendent** |
|---|---|---|---|
| **Content** (**symmetry**) | pattern | force | probability |
| **Context** (**continuity**) | space | time | possibility |

## V. What Quantization Is

Quantization is a mode exchange: both modes change at once. Classical electrodynamics finds continuous fields, and the conjugate feature of its formalism is the U(1) gauge invariance whose invariants those fields are. Quantum electrodynamics finds discrete charges and whole numbers of photons, and the conjugate feature of its formalism is the continuous space of states (Section III). The passage from the first to the second turns what is found from continuously asymmetric to discontinuously symmetric and, together with it, turns the conjugate feature of the formalism from

a symmetry to a continuity. The gauge group belongs to both formalisms; what changes is which of its two characters is found (Questions). The comparison as a whole stays conjugate.

Content-modal degrees of freedom admit this mode exchange, because they can be described in either mode without ceasing to be what they are. The electromagnetic field is content; it has a classical description and a quantum one.

Gravity is context, and the mode exchange is not available to it: quantizing the arena needs a forbidden pairing (Chapter 3, Section V). The continuity of the context is also the condition under which descriptions exist, so a mode exchange of the context would remove the ground on which any description stands.

## VI. Per-Alternative Geometry

The Incommensuration Theorem keeps quantum effects and gravitational effects from being subsumed into one formalism that perfects both modes; it leaves them free to act on one another. What the axioms allow is the configuration in which each sector stands in its own conjunction.

The gauge sector is quantum. The electromagnetic, weak and strong forces and the matter fields are quantized field theories, in the discontinuously symmetric conjunction. The gauge groups of the content sector are compact, so their representations form a discrete set. The resulting Yang-Mills theory is renormalizable.

The gravitational sector is classical. Spacetime geometry is a smooth metric field governed by the Einstein equations, in the continuously asymmetric conjunction. Generic spacetimes have no isometries; the symmetries that appear in the description (diffeomorphism invariance, local Lorentz symmetry) belong to the formalism, as the cross-modal principle requires, and are not invariances of the content. The diffeomorphism group is not compact. The quantized theory of the metric is not perturbatively renormalizable.

The two sectors are coupled through the stress-energy of each alternative of the matter: each alternative carries its own classical geometry, shaped by its stress-energy through the Einstein equations, and where the matter is sharply one alternative the coupling is the Einstein equation with the expectation value as its source,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle, \tag{A.1}$$

whose left side is classical (the Einstein tensor and the cosmological term) and whose right side is the quantum expectation value of the stress-energy tensor. Only for one sharp alternative is the expectation value the source; this is not the Møller-Rosenfeld theory, which keeps one geometry sourced by the expectation value whatever the alternatives. I identify the geometry with the context of each alternative of the matter (Chapter 12, Section VI), and with that identification the inseparability principle requires this form: each alternative of a content is content, and has its own context. The quantum sector and the classical sector each stand in the conjunction that the mode assignment theorem gives them, and the coupling joins them without merging them. Per-alternative geometry is the configuration consistent with the axioms. It is unique in this sense: no configuration that quantizes the context, and none that makes the content classical at the microscopic limit, is available. The relative phase of the alternatives is constructed as Chapter 12, Section VI states: below the modal boundary, by the construction of linearized quantum gravity, with the geometries of the alternatives matched in the region far from the bodies. Where the geometries of the alternatives differ by more than small departures, I give no construction of the relative phase, and the relative phase of those alternatives is not fixed in this book.

General relativity and quantum mechanics are therefore conjugates in the sense of Chapter 6, Section III: two descriptions of one domain, distinct and inseparable, each capturing what the other must leave out. Per-alternative geometry is the mathematical form of their conjugation. Per-alternative geometry keeps the two modes in conjugation, where a unification would merge the two modes into one formalism (Chapter 8, Section

III). The product geometry of Chapter 8 is the same fact in geometric form: the continuously asymmetric factor and the discontinuously symmetric factor coexist as a product. Chapter 9, Section V, states the Incommensuration Theorem as a condition on connections, by the Ambrose-Singer theorem.

## VII. The Residual

The cross-modal principle gives the reach of a framework into its content as dense and never exhaustive. Each theory therefore carries a residual of a definite character.

For quantum mechanics, the residual is the measurement problem. The continuous formalism probes the discrete content with great precision, and does not capture the transition itself: the moment at which a superposition becomes an outcome.

For general relativity, the residual is the singularity. The symmetric formalism probes the smooth content, and does not capture the points at which the smooth geometry fails. In Chapter 19 I treat the singularity as this residual, the place where the context-modal description meets the limit of its domain; in Appendix B I give the form of the geometry on the approach to it.

The residuals are structural features predicted by the cross-modal principle, with the character the principle assigns them.

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### Questions

**Quantum mechanics has continuous spectra: position, momentum, the energies of scattering states. How does Lemma 1 apply to them?**
Lemma 1 concerns the invariants of a content: the quantities that keep a sameness of content across a difference of context (Section IV). Position, momentum and the energy of a scattering state are relations of a content

to its context, and each takes its values in the context factor $M^4$: a position is a point of it, momentum generates its translations, and a scattering state is a motion through its unbounded extent. A translation changes a position, and a change of frame changes a momentum and the energy of a motion. The translations of $M^4$ form a non-compact group, and the continuity of these spectra is the continuity of the context. The same holds of a quantity built from the relative motion of two contents through the context, such as the invariant mass of a scattering pair, although that quantity does not change with the frame.

The quantities that a content keeps across every change of context are the labels of the representations of the compact groups of the content factor, and the counts of spinors on the internal space read as a space of states (Chapter 8, Section IV). Both are discrete (Section IV). Where a content is confined, its momentum and energy are discrete as well: the bound states of an atom have discrete energies, and the continuous part of a spectrum begins where the motion is unbounded. The answer does not rest on the microscopic-limit clause of the mode assignment theorem, which concerns the classical descriptions of content.

**Quantum electrodynamics keeps the U(1) gauge invariance of classical electrodynamics. Does the common symmetry not undo the mode exchange of Section V?**

No. The U(1) group belongs to the formalism of both theories, and in both its invariance selects the physical quantities. The mode exchange is not the loss of that group. It is the change of which of the group's two characters is found (Section III). In classical electrodynamics nothing found is a label of the group: the fields are its continuous invariants, and the group's symmetry is the formalism's conjugate feature. In quantum electrodynamics the charges found are the labels of its representations, so its symmetry is found, and the formalism's conjugate feature is the continuity of the space of states on which the group acts. The symmetry groups of quantum mechanics, which act by unitary operators on its

space of states, stand in the same way: the quantum numbers found are the labels of their representations, and their continuous action on the space of states belongs to the formalism.

Appendix B

# The Dimensional Schema on Approach to the Singularity

*The Kasner Curve for Three, Four and Two Directions*

In this appendix I give the exact form, in the ten-dimensional geometry, of the description in Chapters 18 and 19 of the black hole interior as the cosmological history run in reverse. Near a spacelike singularity each spatial direction scales as a power of the time remaining to the singularity, and two equations constrain the powers. For the three external directions, the four directions of $\mathbb{C}P^2$ and the two of $S^2$, with each group moving together, the constraints leave a closed curve on which the external power runs from $-1/3$ to $+5/9$; at both ends of the curve the six internal directions move as one. On a short arc at the upper end, read toward a singularity, ordinary space collapses while the internal space grows; read away from a singularity, the same powers describe the compactification, in which ordinary space grows while the internal space contracts. The black hole interior and the cosmological compactification therefore share one arc, traversed in opposite directions of time, and I place both at its end point. In the appendix I then record the layered interior, the information deficit at the horizon of a black hole, and further features of the interior and the turn.

*Rests on:* the Kasner form of the Einstein equations in empty space (Kasner 1921), the product $M^4 \times \mathbb{C}P^2 \times S^2$ (Chapters 8 and 9), the modal boundary (Chapter 17), and the information deficit formula (Chapter 13, Section II). *Refers to:* the interior and the cycle (Chapters 18 and 19). *Used by:* Chapters 18, 19 and 20.

## I. The Kasner Form

Near a spacelike singularity, the geometry of empty space takes, to leading order, the form found by Kasner:

$$ds^2 = -d\tau^2 + \Sigma_i \, \tau^{2p_i} \, dx_i^2. \tag{B.1}$$

Here $\tau$ is the proper time measured from the singularity, which lies at $\tau = 0$, and each spatial direction $x_i$ carries a scale factor $\tau^{p_i}$. The numbers $p_i$ are the *Kasner exponents*. The Einstein equations in empty space require

$$\Sigma_i \, p_i = 1 \quad \text{and} \quad \Sigma_i \, p_i^2 = 1. \tag{B.2}$$

Three facts follow, and the rest of the appendix uses them.

*The sign rule.* As the singularity is approached, $\tau$ falls toward zero. A direction with a positive exponent then shrinks toward zero size, and a direction with a negative exponent grows without bound. The two constraints together require at least one exponent to be negative, except in the case of a single exponent equal to one and all others zero. Outside that case, on the approach to the singularity, some directions are squeezed while at least one is stretched.

*The volume rule.* The volume of a region of space scales as the product of the scale factors, $\tau$ raised to the sum of the exponents. That sum is one, so the volume goes to zero as $\tau$. This holds in any number of dimensions.

*The rule of time's direction.* Reversing the direction of time leaves every exponent unchanged. What changes is the direction of travel along $\tau$. Read toward a singularity in the future, a direction with a positive exponent shrinks; read away from a singularity in the past, the same direction grows.

## II. The Interior of an Ordinary Black Hole

Inside the horizon of a Schwarzschild black hole, the radial coordinate is timelike: the infall toward the center is as inevitable as the passage of time (Chapter 19). Near the center, the proper time remaining scales as $r^{3/2}$, and the geometry takes the Kasner form with exponents $-1/3$, $2/3$ and $2/3$. One direction is stretched, as $\tau^{-1/3}$; the other two are squeezed, as $\tau^{2/3}$. That stretching and squeezing of the three directions is the tidal stretching of an infalling body along one axis, with compression along the other two axes.

In the ten-dimensional product $M^4 \times \mathbb{C}P^2 \times S^2$, the same exponents, joined by six internal exponents of zero, still satisfy both constraints. The ordinary interior is therefore a valid Kasner point of the ten-dimensional geometry, with the internal space holding its size. The constraints admit other points as well, in which the internal directions change; in Section III I find the family in which each group of directions moves together.

## III. The Curve for Three, Four and Two Directions

Let the three external directions share one exponent, $p$; the four directions of $\mathbb{C}P^2$ share another, $q$; and the two directions of $S^2$ a third, $r$. The constraints become

$$3p + 4q + 2r = 1 \quad \text{and} \quad 3p^2 + 4q^2 + 2r^2 = 1. \tag{B.3}$$

Eliminating $r$ with the first equation turns the second into a quadratic for $q$:

$$24q^2 + (24p - 8)q + (15p^2 - 6p - 1) = 0. \tag{B.4}$$

A real $q$ exists exactly when the discriminant of this quadratic is not negative. The discriminant is $-32(3p + 1)(9p - 5)$, which is zero or positive exactly when $p$ lies between $-1/3$ and $+5/9$. The solutions therefore form a closed curve, the intersection of a plane with an ellipsoid, on which the external exponent runs from $-1/3$ to $+5/9$.

At each end of the curve the quadratic has a double root, and the four $\mathbb{C}P^2$ exponents equal the two $S^2$ exponents. The six internal directions move as one block. The two ends of the curve are:

*The lower end.* External exponent $-1/3$ for each of the three directions; internal exponent $+1/3$ for each of the six. (Check: $-1 + 2 = 1$; $1/3 + 6/9 = 1$.)

*The upper end.* External exponent $+5/9$ for each of the three directions; internal exponent $-1/9$ for each of the six. (Check: $15/9 - 6/9 = 1$; $75/81 + 6/81 = 1$.)

The computations behind the algebra of this section and the next are given, with their inputs, in the book's additional materials (Appendix D, Section V). Figure 5 draws the curve.

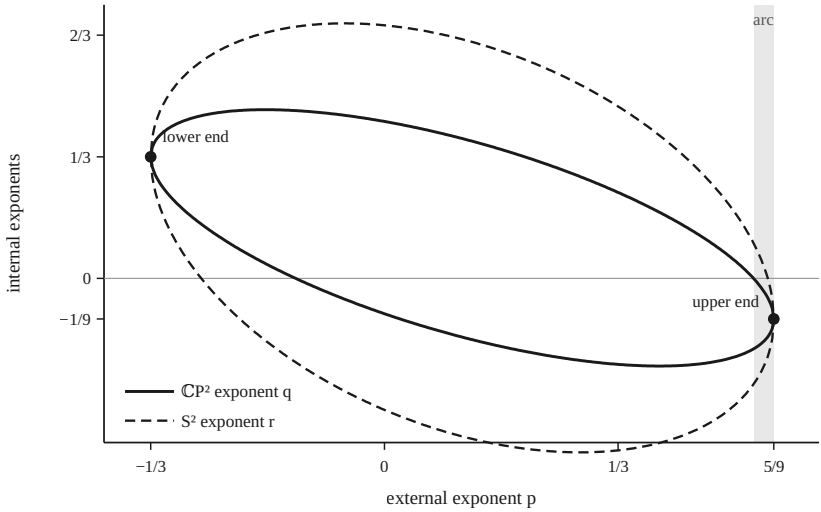



***Figure 5.*** *The curve of this section. For each external exponent p between $-1/3$ and $5/9$, the exponent of the $\mathbb{C}P^2$ directions (solid) and that of the $S^2$ directions (dashed) each take two values, which meet at the two ends, where the six internal directions move as one*

*block. In the shaded region the arc of Section IV runs from p ≈ 0.527 on one side of the upper end, through the end, to p ≈ 0.547 on the other; along it both internal exponents are negative.*

## IV. The Direction of Time

At the upper end, the six internal exponents are negative, and they stay negative along a short arc through the end point. On one side the arc ends where the $S^2$ exponent reaches zero, at an external exponent of about 0.547; on the other side it ends where the $\mathbb{C}P^2$ exponent reaches zero, at about 0.527. On this arc, read toward a singularity in the future, ordinary space collapses while both internal factors grow. Ordinary space collapsing while both internal factors grow is the behavior of the black hole interior that I describe in Chapters 18 and 19. At the end point itself, each external direction shrinks as $\tau^{5/9}$. Each of the six internal directions grows as $\tau^{-1/9}$. The external volume goes to zero as $\tau^{5/3}$; the internal volume grows without bound as $\tau^{-2/3}$; the total volume goes to zero as $\tau$. Ordinary space collapses while the internal space expands.

The same exponents, read away from a singularity in the past, describe the cosmological compactification. At the end point, each external direction grows as $\tau^{5/9}$, and each internal direction contracts as $\tau^{-1/9}$: three dimensions expand while six contract.

Cosmologies in which three dimensions expand while the others contract are known. Chodos and Detweiler (1980) found a vacuum solution of Kasner type in five dimensions, and Freund (1982) studied such cosmologies in higher dimensions, including ten- and eleven-dimensional supergravity, in which three space dimensions are preferred in the expansion. What I add in this appendix is the curve for the grouping of three, four and two directions, and the reading of its upper arc in both directions of time.

Reversing time takes each point of the curve to itself. The interior and the compactification therefore occupy one arc, traversed in opposite directions of time, and wherever on the arc both are placed, they share that point's powers. I place both at the end point, where the six internal directions move as one block. The placement is my choice, made where the internal space moves as a whole; the shared arc follows from the constraints. Chapter 18 states that collapse and origin are one transition, approached from its two sides and described in opposite directions of time. At the level of the dimensional schema, the statement holds: on the shared arc the powers are the same, and only the direction of travel differs.

The lower end has the opposite character. Read toward a singularity in the future, its three external directions grow, as $\tau^{-1/3}$, and its six internal directions shrink, as $\tau^{1/3}$. Read away from a singularity in the past, ordinary space contracts while the internal space grows. Neither reading is the history this book describes.

The growth of the internal directions at the upper end is slow. Between two depths $\tau_1$ and $\tau_2$, with $\tau_2$ nearer the singularity, each internal direction grows by the factor $(\tau_1/\tau_2)^{1/9}$: the ninth root of the ratio of the two times. Over many orders of magnitude in $\tau$, the internal space grows by a modest number of orders, and the curvature reaches the Planck scale, where the context-modal description meets its boundary (Chapter 17), while the growth is still under way. The Kasner account describes the direction of the change, and its rate; the completion of de-compactification belongs to the modal boundary that I identify in Chapter 18, Section XI, as the edge of both descriptions.

## V. The Layered Interior

In Chapter 19 I give a second account of the same interior, in terms of density. The tidal curvature at radius r equals the curvature just outside a body of the hole's mass M and radius r, so each depth has an equivalent density of order $M/r^3$. The fraction of the Schwarzschild radius at which a

given density is reached falls as $M^{-2/3}$. For a hole of one solar mass, the density at which hadrons dissolve is reached outside the horizon. The density at which the electroweak symmetry is restored is reached at about $2 \times 10^{-4}$ of the Schwarzschild radius. The Planck density is reached near $10^{-26}$ of the Schwarzschild radius. The internal space begins to change where the external curvature approaches its own, of order $1/R^2$; with R a few Planck lengths, that is within a factor of a few of the radius of the Planck layer.

The density account describes the depth at which each structure of the exterior is undone. The Kasner account of Section IV describes the form the geometry takes as the internal space grows. The two accounts describe one interior, and in Section VII I record the question of how they join.

## VI. The Information Deficit and the Black-Hole Horizon

In Chapter 13 I derive the cosmological constant from the information deficit of an observer inside the cosmological domain: the ratio $c_1/S$, with S the count of the independent degrees of freedom within the particle horizon, reduces the Planck density to $c_1 M_{Pl}{}^4/S$. The deficit is the price of being inside: it is paid by a comparer for the domain that contains it, and it is one ratio, set by that one count. A black hole lies within the observer's particle horizon, and the observer is outside it. The states its horizon hides are states within the observable universe, so they are already among the S that the observer's deficit counts; the total entropy of all black holes in the observable universe, about $10^{104}$, is a small part of the $10^{123}$ of the horizon (Egan and Lineweaver 2010). They enter the observer's deficit there, once, and the observer, outside the hole, pays no deficit at its horizon. For the exterior, a black hole's horizon carries no vacuum energy of its own, and the cosmological constant is set by one horizon, the particle horizon of the measurement.

For a black hole of stellar mass, with a horizon entropy of order $10^{77}$, the quantity $c_1 M_{Pl}^4/S$ is larger than the cosmological constant by about forty-six orders of magnitude. It enters no vacuum energy of the exterior, whose cosmological constant is set by the particle horizon alone (the paragraph above). It fixes the share of no comparer within the hole either. The count that sets the share is the count of what has reached the comparer (Chapter 13, Section II), and an event horizon bounds what can leave the hole and none of that: light from the exterior continues to cross it inward, and the hole lay within the comparer's particle horizon before the fall. A comparer within the hole therefore keeps the count S of its particle horizon, and its accessible share, and with it the cosmological constant, is unchanged. What reaches it during its finite time inside, about $0.28\ r_s/c$ of the exterior's advanced time for a fall from rest far away into a hole without rotation, and the delay of that light by the hole's own field, change that count by parts in $10^{23}$ to $10^{22}$ for a hole of one solar mass. The quantity itself has an exact reading. With $S = \pi R^2/l_{Pl}^2$ it equals $3c^4/(\pi G R^2)$, which carries no Planck constant, and at the horizon radius $r_s = 2GM/c^2$ it is eight times the mean energy density of the hole's mass within its horizon, $Mc^2/(4\pi r_s^3/3)$. The eight is the de Sitter identity $\rho_\Lambda = (3/8)\ M_{Pl}^4/S_{dS}$ (Chapter 13, Questions), at the de Sitter radius: the mean density of a hole and the density of de Sitter space stand in one relation to their radii, $3c^4/(8\pi G R^2)$. The interiors end in one domain (Chapter 18, Section II), the turn from which the next epoch begins. Many black holes end in one origin, and what the holes hide passes to that origin.[16]

## VII. Further Features of the Interior and the Turn

*The ordinary interior and the curve.* The ordinary interior of Section II, with exponents $-1/3$, $2/3$, $2/3$ and an internal space of fixed size, is a valid Kasner point, and it does not lie on the curve of Section III, whose external exponents are equal.

*Oscillation.* In ten spacetime dimensions, empty-space gravity near a generic spacelike singularity does not settle into one Kasner form. It passes through an endless sequence of them, each bounce exchanging which directions grow and which shrink (Demaret, Henneaux and Spindel 1985; the oscillation ceases only from eleven dimensions upward). A single Kasner form with each group moving together is consistent within the product geometry, where the curvature of the internal space falls away against the rate of collapse, but a single Kasner form is not the generic behavior. A form field threading the internal space would also drive the oscillation, since form fields restore it even in eleven dimensions (Damour and Henneaux 2000).

*Three accounts of de-compactification.* I give three accounts of the restoration of context. In Chapter 18 I give a global account: the compactification fails where the exterior factor is no longer the context of the compact factor. The compactification fails at the end of every interior, where the exterior has no moment and no inertial reference survives. In Chapter 19 I give a density account: the internal space begins to change where the external curvature approaches its own, close to the Planck layer inside every black hole. In Section IV I give a geometric account: at the upper end of the Kasner curve the internal directions grow on the approach to the singularity.

*Entropy at the turn.* A collapsing interior carries large entropy and large tidal curvature. The start of an epoch, as Chapter 18 describes it, is a restoration of context from which the low-entropy order of a new exterior proceeds. In Chapter 18, Section VIII, I give the structural reason for the drop: the gravitational entropy of the interiors is counted relative to an exterior that ends, and the new exterior begins with Weyl curvature near zero.

*What holds the internal space compact.* In Chapter 18, Section VI, I name the structural conditions under which the compactification holds: the exterior factor present as a context for the compact factor, as a domain of the continuously asymmetric description with an inertial reference fixed in it; their ground is Axiom III's inseparability of content and context.

## Questions

**What selects the three external directions as the ones that grow in the compactification?**

The Kasner constraints do not. They hold for any grouping of the nine directions, and the curve of Section III is the set of their solutions for the grouping of three, four and two that the product $M^4 \times \mathbb{C}P^2 \times S^2$ supplies (Chapters 8 and 9). In pure higher-dimensional Einstein theory no group of three directions is preferred in the expansion (Freund 1982); here also the grouping comes from the product geometry, not from the constraints. Given the grouping, the constraints give the arc on which, read away from a singularity, the three external directions grow while the six internal directions contract (Section IV).

**Is the placement of both the interior and the compactification at the upper end not chosen to fit?**

Three parts of the placement must be told apart. The sharing of one arc follows from the constraints: reversing time takes each point of the curve to itself, so wherever on the arc the interior and the compactification are placed, they have the same powers and differ only in the direction of travel (Section IV). The arc is selected by the behavior each history has. On it, read toward a singularity, ordinary space collapses while the internal space grows, which is the interior of Chapters 18 and 19; read away from a singularity, ordinary space grows while the internal space contracts, which is the compactification. The lower end has neither behavior. The placement at the end point, within the arc, is my choice, made where the six internal directions move as one block. The statement of Chapter 18 that collapse and origin are one transition, approached from its two sides and described in opposite directions of time uses only the shared arc, and at the level of the dimensional schema it holds at every point of the arc.

Appendix C

# A Parallel Derivation of the Dimensions of Context

*Time One, Space Three, Possibility Six, from the Stance of Comparison*

This appendix counts the dimensions of the three contexts, time, space and possibility, by a route separate from that of Chapter 10. The dimension of a context is the number of independent ways in which two findings can differ in it. Time has one, because the comparing is one stance. Space, the followable context of findings, has three: it must differ from time, so it is not one; a distinction never stands alone, so it is not two; and by the minimality principle of Section IV it takes no more than comparison requires. Possibility has six, because it is the range of what a comparison could find, and that range spans all six intrinsics. The counts of time and space agree with those of Chapter 10, and neither derivation uses the other's steps; the six of possibility is the same set as the six internal dimensions of Chapter 9 (Section VIII). The derivation uses no particular kind of matter and no law of physics.

*Rests on:* the six intrinsics of comparison and Axiom III (Chapters 2 and 4), and the six metrics and their grid (Chapter 7). *Refers to:* the product structure (Chapter 8, Section III), and the directions of extension and the count of four (Chapter 10). *Used by:* Chapter 9. *Referred to by:* Chapters 7, 10, 11, 13 and 17.

## I. The Ground

The appendix uses comparison, the six intrinsics of Chapter 2, and the six metrics of Chapter 7.

Comparison involves, and cannot not involve, three pairs of intrinsics: sameness and difference, content and context, the subjective and the objective. The intrinsics give six metrics, in three pairs, one for each modality (Chapter 7): pattern and space, force and time, probability and possibility. In each pair the first member is content and the second is context.

One further distinction completes the ground, and it is stated here with its reason. The subjective and the objective are the comparing and the compared. They are the stance from which every comparison is made, and so they cannot be followed apart from that stance. Chapter 2 names the objective as the perspective from which a comparison is received; that is the compared, seen from its side. The other four intrinsics can be: sameness, difference, content and context. At the level of the metrics, pattern, space, force and time can be followed apart from the stance. Probability and possibility concern what a comparison could find, and so they stay bound to the stance of the comparing and the compared. An intrinsic or metric that can be followed apart from the stance is called *followable*.

## II. Three Consequences of Comparison

The argument rests on three consequences of comparison, and each is derived before it is used.

*The first consequence is the triple condition.* The three pairs of intrinsics are distinct, inseparable and non-interchangeable. They are distinct because each answers a different question about one comparison. They are inseparable because every comparison requires each of them: without a verdict there is no outcome; without content and context the verdict would be fixed in advance; without the comparing and the compared nothing is compared. They are non-interchangeable because they are of three different kinds, outcomes, respects and positions, and none can serve in the place of another within one comparison. The three contexts, space, time and possibility, form the context row of the grid of Chapter 7, one for each modality. The modalities are distinct, inseparable and non-interchangeable (Axiom III), and in the grid of Chapter 7, Section III, each context stands in the column of its modality; so the three contexts too are distinct, inseparable and non-interchangeable. The triple condition is the condition that Axiom III states of the modalities, here found in the pairs of intrinsics and in the contexts.

*The second consequence is the doubling*. Every finding is placed twice: it is found in a context, and it is found by a comparing. The context and the comparing are two, and neither is the finding. So every content stands in relation to at least two things, of different kinds. The same holds of a difference: a difference between two findings is itself found in a context and from a comparing, so it is never a single bare distinction standing alone. Every difference is placed in two distinct ways: in a context and by a comparing. A magnitude compared with a standard shows the same count of one and two: of its three possible verdicts, one is sameness, and two, less and greater, are difference.[17]

The third concerns what a context is at this level. Comparison registers a difference only through its own structure, and what a context contributes to a comparison is the ways in which findings placed in it can differ. So a context has no feature but its form of difference, the number of its independent ways. Two contexts with the same form of difference are not told apart by anything in that form or beside it, and distinct contexts differ in their number of ways. I use this consequence in Sections VI and VII to distinguish space from time, and possibility from both.

## III. What a Dimension Is

Two findings can differ in a context in more than one way. A *way of differing* is independent of others when it cannot be composed from them: no combination of differences in the other ways yields it. The *dimension* of a context is the number of its independent ways of differing. The definition uses only difference and the composition of differences, so it is available at this level.

In this appendix, *dimension* means only that count. Each of the six metrics is a kind of measure, not a count. Physics also calls a kind of measure a dimension, as when length, mass and time are called the dimensions of a quantity; that second meaning is not used here. Nor is metric used here in the sense of the metric tensor of Chapters 10 to 12.

## IV. The Numbers Comparison Supplies

At this level nothing exists but comparison and what it cannot not involve. A way of differing is a way in which comparison can register a difference, and comparison registers only through its own structure. So the dimension of a context can only be a number that the structure of comparison itself supplies.

The count must be a count of things that are themselves independent. The structure of comparison contains sets of elements, each element distinct from the others, and it contains combinations of those elements. A combination is made from elements, so it is composed, and composed ways are, by the definition of Section III, not independent. The three pairs, each taken in one of its two members, can be combined in eight ways, but those eight are composed from three choices; they are not eight independent ways. So only the sizes of the sets of elements count, and not the numbers of their combinations.

The sets of elements in the structure of comparison have these sizes, and no others: one, the comparing, which is a single stance; two, the members of any pair; three, the pairs; four, the followable intrinsics; six, the intrinsics.

A third principle bounds the count from above. *The minimality principle*: of the numbers comparison supplies, a context takes the least that meets every requirement comparison places on it. A way of differing beyond that number would be a difference that no part of the structure of comparison registers as distinct from the other ways, and at this level nothing else could register it.

## V. Time Has One Dimension

Time is the context of the order in which findings are found. Findings are found by a comparing, and the comparing is one stance, the subjective, from which every finding is made. Each finding, once made, is held by the

findings made after it, and the order passes in one way only. From one stance, any two findings stand in one relation of order: one is earlier and the other later. So time has exactly one way of differing. Its dimension is one.

## VI. Space Has Three Dimensions

The space counted here is the followable context of findings: the space in which findings are followed apart from the stance. I identify it with the spatial directions of the external spacetime $M^4$, the directions of extension of Chapter 10. The internal space is not that context: it is the internal factor of the product, the factor that is content relative to the exterior (Chapter 8, Sections II and III), and it carries all six metrics (Chapter 9), two of which, probability and possibility, are bound to the stance. Its six directions are counted in Chapter 9, one for each metric.

Space and time are distinct and non-interchangeable (Section II). If space had one dimension, space and time would have the same form of difference, and nothing in that form would tell one from the other. So the dimension of space is not one.

The doubling decides which numbers of ways a context can have, once it is stated when a distinction between two ways of a context counts as placed in that context. A distinction between two ways is a difference: a difference along the one way and a difference along the other are found to differ in their way. So, by the doubling, the distinction is placed in two distinct ways: by a comparing, and in a context (Section II). The comparing places every distinction. Whether the context places it is decided by the placing rule.

The placing rule: the distinction between two ways of a context is placed in that context when, and only when, the context has a way that is not composed from those two. That way is the respect, within the context, in which the distinction is found.

The rule rests on three grounds. First, what places a difference is not the difference. Section II states this of every finding: "The context and the comparing are two, and neither is the finding". Chapter 2, Section I, states it of every comparison: "The content and the context cannot be one respect". The distinction between two ways consists of those two ways, so neither of them places it. Nor does a way composed from those two alone, since a combination is made from its elements (Section IV) and contains nothing besides them. Second, the placing way belongs to the same context. The distinction between two ways of a context is part of the form of that context, and a way of another context cannot serve in its place, because the three contexts are non-interchangeable (Section II). Third, nothing further is required. The six intrinsics are necessary and sufficient for comparison (Chapter 2, Section I), and of the context of a finding they require that it be a respect other than the content. A way of the same context that is not composed from the two ways is such a respect. A difference along a single way, one finding differing from another in that way, consists of the two findings; the way is not those findings, so the way places that difference, in a context of any number of ways.

The placing rule has a consequence for each of the numbers that comparison supplies. In a context of one way there is no distinction between ways, and each difference along the one way is placed by that way and by the comparing; one meets the doubling. In a context of two ways, every way of the context is one of the two or is composed from them, so nothing in the context places the distinction between them. That distinction is placed by the comparing alone, once, and two does not meet the doubling. In a context of three ways, the distinction between any two of them is placed in the context by the third, and it is also placed by the comparing; three meets the doubling. Of the three ways, the third is the only one that places that distinction, and each of the three ways places exactly one of the three distinctions. In a context of four ways, the distinction between any two of them is placed by each of the other two; in a context of six

ways, by each of the other four. Four and six meet the doubling. A context of any number of ways meets the doubling when that number is one, or three or more.

Space is not one, and it meets the doubling, so of the numbers comparison supplies it can be three, four or six. The doubling bounds the dimension of space from below, and it does not fix it. By the minimality principle (Section IV), space takes the least of the three. So the dimension of space is three.

The placing rule admits four and six, so the step from those numbers to three is the minimality principle and not the doubling.

## VII. Possibility Has Six Dimensions

Possibility is distinct from time and from space (Section II), so its dimension is neither one nor three. The dimension of possibility is not two either, for the reason of Section VI. Of the numbers comparison supplies, four and six remain.

Possibility is the range of what a comparison could find. A finding is specified in every one of the six intrinsics: its verdict, its respect, and its place in the stance of the comparing and the compared. So the range of what could be found spans all six intrinsics. Possibility stays bound to the stance of comparing (Section I), so it cannot be confined to the four intrinsics that can be followed apart from that stance. Its dimension is six.

Six is distinction applied to the triad: each of the three pairs, in its two members. Six places distinction within each of the three pairs of intrinsics, so that no distinction stands alone; the doubling requires that placing, and a bare two does not supply that placing.

## VIII. Concordances

Several facts agree with the counts one, three and six, and each follows from what has already been derived or from what the book derives elsewhere.

*The followable four.* Time and space together have one and three dimensions: four. Four is the number of the followable intrinsics. Time and space are the contexts in which findings can be followed apart from the stance, and possibility, bound to the stance, is the context that cannot.

*The pairs of four.* Among four independent ways of differing, the number of distinct pairs is six. So the dimension of possibility equals the number of ways in which two of the followable directions of context can be related to each other.

*The progression.* One, three and six are one; one and two; one, two and three. Each context adds the next count that comparison supplies: the comparing, then the pair, then the triad.

*The placing by the third.* In a space of $n$ dimensions with an inner product, the directions perpendicular to a plane form a space of $n - 2$ dimensions (Axler 2024, Section 6C). That space is a single line when, and only when, $n$ is three. So in three dimensions, and in no other number of dimensions, each plane of two directions has exactly one perpendicular line, as in a context of three ways the distinction between two ways is placed by the third. Three is also the only number of ways that equals the number of pairs of those ways.

*Agreement with Chapter 10.* Chapter 10 derives one dimension of time and three of space from the directed cycle of Axiom II: the cycle's three transitions give the spatial directions, and its one direction gives time. This appendix derives the same counts from the single stance of comparison, from the requirement that space differ from time, and from the doubling. Neither derivation uses the other's steps.

The facts of this section that agree with the counts one, three and six are concordances, and I record those facts as concordances. The derivation of Sections V to VII does not depend on them.

*Identity with Chapter 9.* One further agreement is more than a concordance. In Chapter 9 I give the internal space six dimensions, one for each metric, and in this appendix I give possibility six dimensions, spanning the six intrinsics that the six metrics come from. I identify the two sixes as one set. Both are indexed by the same six intrinsics, one dimension for each, and by nothing else. A difference between them would need a respect of comparison beyond the six intrinsics, and comparison has no such respect (Section IV). No comparison can register a difference between the two sixes, and a difference no comparison can register is no difference (Chapter 18, Section II).

In Chapter 9 and in this appendix I give the one set two roles, and the roles do not conflict. I take content and context to be assigned within each comparison (Chapter 2, Section I). In Chapter 9 the internal space is the content factor, relative to the external spacetime as its context. Here possibility is a context, relative to the finding as its content. The two assignments, in Chapter 9 and here, are two comparisons of one set.

Mathematics agrees. The complex projective space $\mathbb{C}P^n$ is the space of the pure states of a quantum system with $n + 1$ alternatives, and its Fubini-Study metric measures how distinguishable one such possibility is from another (Wootters 1981). $\mathbb{C}P^2$ is the space of possibilities of a system with three alternatives, and $S^2$, which is $\mathbb{C}P^1$, is that of a system with two. The internal space is a space of possibility in the plain mathematical sense.

The identity also accounts for why the internal dimensions present as compact. Comparison is located: it finds this time and not that time, this place and not that place, and this possibility realized and not that one. Other places can be looked into, and other times can be read from records, but other possibilities can be reached only by hypothesis. No comparison traverses the dimensions of possibility as it traverses space. To a located

comparer they show no depth, and their extent appears only through its effects, the Planck mass among them (Chapter 11). Because the dimensions of possibility show no depth to a located comparer, the internal dimensions present as compact.

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## Questions

**Why does the doubling exclude two dimensions of space but permit one of time?**

Because a context of one dimension contains no distinction between ways of differing, and a context of two dimensions contains one that nothing in the context places. By the doubling, every difference is placed in two distinct ways: in a context and by a comparing (Section II). Time has one way of differing (Section V). Each difference along it, one finding earlier than another, is placed in time and by the comparing, the single stance from which the order of time is found (Section V); and with one way only, there is no distinction between ways to be placed. A context of two dimensions contains, besides the differences along its ways, one distinction between its two ways of differing. That distinction is the whole of the context's form, so the context holds nothing else in which it could be placed: it is placed by the comparing alone, once, and the doubling excludes a difference placed once (Section VI).

The rule that decides when a distinction between the ways of a context counts as placed in that context is stated in Section VI: the distinction is placed in the context when, and only when, the context has a way that is not composed from the two. By that rule a pair of ways cannot place its own distinction, and so two is excluded. In a context of three ways the distinction between any two of them is placed in the context by the third, and so three meets the doubling. The rule is met by every number of ways from three upward, so the doubling does not fix three; the minimality

principle does (Section IV). The six of Section VII meets the rule, since in a context of six ways the distinction between any two of them is placed by each of the other four.

**Why is space not four-dimensional, when four is the number of the followable intrinsics?**

Four is a number comparison supplies, and the minimality principle (Section IV) excludes it for space: of the numbers that the requirements on space do not exclude, three is the least (Section VI). Four meets the doubling, as every number of ways from three upward does (Section VI), so the doubling does not exclude four, and the minimality principle does. Four is the dimension of time and space together, the two followable contexts, and that agreement with the four followable intrinsics is a concordance (Section VIII), on which the derivation does not depend. The minimality principle is stated once and applies to every context. It fixes nothing for time, whose one way is fixed by the single stance, and it leaves six for possibility, since the range of what could be found requires six (Section VII).

**Why should a count of ways of differing be the dimension of a manifold?**

The dimension of a manifold is the number of independent directions in which two nearby points can differ: the number of coordinates needed near each point, where a direction composed of others is not counted. That definition of the dimension of a manifold is the definition of Section III, applied to the differences between positions. Under the identification of Section VI, in which the space counted here is the spatial directions of $M^4$, the two counts are one count.

Appendix D

# The Floor of the Metric at First Order

*The Computation of Chapter 11, Section II, with Its Method and Standing*

This appendix gives the computation of one mechanism for the hold of Chapter 11, Section II: the zero-point energy of the small waves of the ten-dimensional metric on the internal factor, at first order, and the balance with the least extent that holds the internal space at the derived radius and ratio. It states the method, what was computed, one level of the sum whose treatment is disputed, where the computation is given with its inputs, and its standing.

*Rests on:* the internal space and its radius (Chapter 9, Sections VIII and IX), the reduction (Chapter 11), and the description of small waves by quanta (Chapter 12, Questions). *Used by:* no later step; the chain takes the hold of the internal space as a requirement. *Referred to by:* Chapter 11, Section II, where that requirement is stated.

## I. What Is Computed

The floor of Chapter 11, Section II, has two parts: the least motion of the ten-dimensional metric on the internal factor, and the least extent, a local term. In this appendix I compute the least motion at first order, the zero-point energy of the levels of the metric's small waves on $\mathbb{C}P^2 \times S^2$, and show that with the least extent as its local term it holds the internal space at the derived radius and ratio (Chapter 9, Section IX) as a stable minimum. The background is flat $M^4$ times $\mathbb{C}P^2$, with the Fubini-Study metric of Ricci constant $6/R_O{}^2$, times the round $S^2$ of radius $R_T$.

## II. The Method

The zero-point energy is computed in the form that does not depend on the gauge, the unique effective action (Vilkovisky 1984; DeWitt 1987); in the Landau-DeWitt gauge only the Christoffel symbol of the metric on the space of fields enters (Fradkin and Tseytlin 1984). That metric is the one the Einstein-Hilbert action supplies, $G(h,k) = \int \sqrt{g}\ (h{\cdot}k - \tfrac{1}{2}\ \mathrm{tr}\ h\ \mathrm{tr}\ k)$; within the local family $\int \sqrt{g}\ (h{\cdot}k + a\ \mathrm{tr}\ h\ \mathrm{tr}\ k)$ it is the one member at which the one-loop operator has the same leading part on every polarization, and the one that adds no number to the definition of the floor.

On a symmetric space the Laplacian on every bundle acts on each harmonic as a Casimir value, $4C(p,q)/R_O{}^2$ on $\mathbb{C}P^2$ and $l(l+1)/R_T{}^2$ on $S^2$, so each harmonic sector reduces to a finite matrix whose determinant has the four-dimensional masses as its zeros. The energy per unit four-volume is $R_O{}^{-4}\ [a + b\ \ln(\mu R_O) + C]$, with $b = -\zeta(-2)/(32\pi^2)$ and $a = -(\zeta'(-2) + 3\zeta(-2)/2)/(64\pi^2)$, where $\zeta(s) = \Sigma\ m^{-2s}$ runs over the masses, and C is the least extent, a local term of fifth order in the curvature.

The method reproduces the published value for six-dimensional gravity on $M^4 \times S^2$, $-2249/840$ (Cho and Kantowski 2000, equation 124), and, for the present geometry, the count of 35 polarizations of ten-dimensional gravity in the leading coefficient, and the local value of its next coefficient.

## III. What Was Computed

At the ratio $\sqrt{6}$, $\zeta(-2)R_O{}^4 = -6596269937/3326400$ (exact), with a closed form in the shape that is negative at every shape; for a scalar field it is 71237/51975 (minimal coupling) and 317/10395 (conformal coupling), of the opposite sign. The finite part is $\zeta'(-2)R_O{}^4 = 3662.009$ (a small-time series to $t^{12}$). In the four-dimensional Einstein frame, at any stationary point in size, every minimum lies above $\sqrt{(\pi b/9)}\ l_{Pl} = 1.48\ l_{Pl}$.

The least extent is the same function of the curvature along each of the six internal directions, so at the ratio $\sqrt{6}$ it gives each an equal share. Its two constants, fixed by the balance at the derived point, are $\kappa = 8.98 \times 10^{-5}$ for the term $\kappa$ tr $\mathrm{Ric}^5$ and $\mu = 0.651\ \mathrm{M_{Pl}}$. The derived point is then a stable minimum, the only one in shape between $\sqrt{3}$ and $\sqrt{12}$, with its barrier at $\mathrm{R_O} = 1.79\ \mathrm{l_{Pl}}$, past the modal boundary: at that radius the Ricci curvature of $\mathbb{C}\mathrm{P}^2$, $6/\mathrm{R_O}^2 = 1.87/\mathrm{l_{Pl}}^2$, exceeds the Planck curvature. The minimum stays stable for ratios up to 2.50 at the derived radius and for radii above $1.87\ \mathrm{l_{Pl}}$ at the derived ratio, so its location is not a test of the computation; its stability and the sign of the least-extent term are tests, and the derived point passes both. The moduli have masses of $2.25\ \mathrm{M_{Pl}}$ (mostly size) and $3.09\ \mathrm{M_{Pl}}$ (mostly shape). At the derived point the floor's own energy is about $+0.071\ \mathrm{M_{Pl}}^4$, counted within $\mathrm{M_{Pl}}^4$ (Chapter 13, Section II); with the curvature energy, whose bare part $-3/(8\pi)\ \mathrm{M_{Pl}}^4$ stands beside $\mathrm{M_{Pl}}^4$, the internal space contributes $-0.049\ \mathrm{M_{Pl}}^4$ in all.

## IV. One Level of the Sum

The singlet spin-two level, constant on the internal space, is the arena's own wave. On the flat background its eigenvalue is $\mathrm{R}/2 = 18/\mathrm{R_O}^2$, the effect of the configuration's four-dimensional vacuum energy on a background that does not solve the four-dimensional equations, and it adds $+1620$ to $\zeta(-2)\mathrm{R_O}^4$. On the arena's own background, which solves the Einstein equations with the observed $\Lambda$ (Chapter 12, Section VII; Chapter 13, Section II), it is massless and adds nothing. Without it, $\zeta(-2)\mathrm{R_O}^4 = -11985037937/3326400$, every minimum lies above $1.996\ \mathrm{l_{Pl}}$, the net is $-0.040\ \mathrm{M_{Pl}}^4$, and the derived point is again a stable minimum. The same four-dimensional vacuum energy shifts the other spin-two levels, the singlet spin-zero block mixes the trace of the arena's wave with the moduli, and on this background about a third of the levels have masses squared in

complex-conjugate pairs, so the computed functional is not a literal sum of oscillator zero-point energies. The floor on a background that is on shell in its four-dimensional part is not computed.

## V. Where the Computation Is Given

The computation of this appendix is given in full, with its inputs, in the additional materials of this book (*The Gravitational Passage; Additional Materials*, UVSM Publications, 2026, at uvsmpublications.com). The computation is numerical, written in Python 3 with the libraries numpy, scipy, sympy and mpmath, and has eight parts. The first builds the operator sector by sector. The second part, with the shape as its input, gives $\zeta(-2)$ and $\zeta'(-2)$ at that shape. The third gives the value on $M^4 \times S^2$. The fourth gives the bound on every minimum. The fifth gives the local term, the minimum and its stability. The sixth gives the range over which the minimum stays stable. The seventh gives the scalar values, the ratio of energies and the masses of the moduli. The eighth gives the numbers of Section IV. The materials also give the values of $\zeta(-2)$ and $\zeta'(-2)$ at eighteen shapes. Each number in this appendix comes from these eight parts. The same materials give the computations behind the numbers of Chapter 13 and behind the algebra of Appendix B, and the computations behind every other number the book computes.

## VI. Standing

The values of $\zeta(-2)$ are exact; the finite parts and balances are computed to the precision stated. The computation is the Gaussian order of the small waves. It is a first-order description at the boundary: at every balance of this kind the zero-point energy is between one half and three quarters of the curvature energy, 0.59 at the derived point, and the balance itself fixes that ratio. At the modal boundary neither description dominates (Chapter 17, Section II), and these numbers are that statement

in energies; no parameter of the theory makes the next order small, since the field that holds the space is the metric itself. The radius and the ratio are derived in Chapter 9, Section IX, and do not rest on this computation. The depth of the minimum and the masses of the moduli are fixed here at first order only, and the order of those masses, far above what the inverse-square tests require, is what the argument of the book uses from it.

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### Questions

**Does the result of this appendix depend on the disputed level, the arena's own wave?**

Its numbers do; its conclusion does not. With the level counted as on the flat background, every minimum lies above 1.48 Planck lengths; with it left out, as on the arena's own background, above 1.996. In both cases the derived point, $\sqrt{6}$ Planck lengths, is a stable minimum, and the moduli have masses of the order of the Planck mass.

**What does this appendix not fix?**

The energy beyond first order; the floor on a background that is on shell in its four-dimensional part; and the metric on the space of fields once the coefficients of the higher Lovelock terms are fixed.

# Symbols and Equations

The symbols the book uses, with the place where each is introduced or argued, and the numbered equations, with the page on which each stands.

| Symbol | Meaning | Where |
|---|---|---|
| $M^4$ | the four-dimensional factor of the product, the context: spacetime | Chapter 8 |
| $K = \mathbb{C}P^2 \times S^2$ | the six-dimensional internal factor, the content | Chapter 9, Section VII |
| $\chi$ | Euler characteristic; $\chi(\mathbb{C}P^2) = 3$, $\chi(S^2) = 2$ | Chapter 9, Section IV |
| $c_1$ | the count of three channels, the first Chern class of $\mathbb{C}P^2$; $c_1 = 3$ | Chapter 13, Section II |
| $R_O$, $R_T$ | radii of $\mathbb{C}P^2$ and of $S^2$; $R_O/R_T = \sqrt{6}$ | Chapter 9, Section IX |
| $l_{Pl}$ | Planck length, $\sqrt{(\hbar G_N/c^3)}$ | Chapter 17 |
| $M_{Pl}$ | full Planck mass, $\sqrt{(\hbar c/G_N)}$ | Chapter 17, Section I |
| $M_{10}$ | mass scale of the ten-dimensional action | Chapter 11, Section III |
| $G_N$ | Newton's gravitational constant, measured | Chapter 12, Section III |
| $G_{\mu\nu}$, $g_{\mu\nu}$, $T_{\mu\nu}$ | Einstein tensor, metric, stress-energy tensor | Chapter 12, Sections I and II |
| $\Lambda$ | cosmological constant, the constant term of the field equations | Chapter 12, Section IV; Chapter 13 |
| $\rho_\Lambda$ | vacuum energy density, $\Lambda/(8\pi G_N)$ | Chapter 13, Section II |
| f | $1 - 3/(8\pi) = 0.881$, the external value in units of $M_{Pl}^4$ | Chapter 13, Section II |
| S | count of the particle horizon, $\pi R^2/l_{Pl}^2$ | Chapter 13, Section II |
| R | radius of the particle horizon | Chapter 13, Section II |
| I | radius of the particle horizon in units of the Hubble radius | Chapter 13, Section III |
| E(z) | expansion rate at redshift z in units of $H_0$ | Chapter 13, Section III |
| | | Chapter 13, Section III |

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|---|---|---|
| $\Omega_m$, $\Omega_\Lambda$, $\Omega_r$ | fractions of the critical density in matter, vacuum and radiation | |
| $H_0$, h | Hubble constant now; $h = H_0/(100\ \mathrm{km\ s^{-1}\ Mpc^{-1}})$ | Chapter 13, Section III |
| $H_\Lambda$, $r_\Lambda$ | de Sitter rate $\sqrt{(\Lambda/3)}$ and de Sitter radius $c/H_\Lambda$ | Chapter 13, Section III |
| $t_0$ | age of the universe | Chapter 13, Section III |
| w | equation-of-state parameter of the vacuum; $w = -1$ | Chapter 13, Section V |
| $\Gamma^\mu{}_{\alpha\beta}$ | connection coefficients of the metric | Chapter 15, Section IV |
| $\xi^\mu$ | separation of neighboring geodesics | Chapter 15, Section V |
| $\Phi$ | Newtonian potential | Chapter 16, Section I |
| $p_i$; p, q, r | Kasner exponents; those of the three, four and two directions | Appendix B, Sections I and III |
| $\tau$ | proper time | Chapter 15; Chapter 18, Section II |

| Equation | Statement | Page |
|---|---|---|
| (9.1) | $K = \mathbb{C}P^2 \times S^2$ | 65 |
| (9.2) | $RO/RT = \sqrt{6}$, RT = lPl, $RO = \sqrt{6}$ lPl | 67 |
| (10.1) | $RO/RT = \sqrt{6}$ | 75 |
| (11.1) | $S10 = \int d10x\ \sqrt{(-g10)}\ R10$ | 80 |
| (11.2) | $MPl^2 = M108 \times Vol(K)$ | 83 |
| (11.3) | $RO\ /\ RT = \sqrt{6}$ | 84 |
| (12.1) | $G\mu\nu + \Lambda\, g\mu\nu = 8\pi GN\, T\mu\nu$ | 88 |
| (12.2) | $G\mu\nu = R\mu\nu - \tfrac{1}{2}\, R\, g\mu\nu$ | 89 |
| (12.3) | $\rho\Lambda = c1\ f\ MPl4\ /\ S$ | 90 |
| (12.4) | $G\mu\nu + \Lambda g\mu\nu = 8\pi GN\ \langle T\mu\nu \rangle$ | 91 |
| (13.1) | f MPl4, with $f = 1 - 3/(8\pi) = 0.881$ | 102 |
| (13.2) | $S = \pi\ R^2\ /\ lPl^2$ | 103 |

| | | |
|---|---|---|
| (13.3) | $\rho\Lambda = c1\ f\ MPl4\ /\ S,\ \Lambda = 8\pi GN\ \rho\Lambda$ | 106 |
| (13.4) | $I(\Omega m) = \int 0\infty\ dz\ /\ E(z)$ | 106 |
| (13.5) | $\rho\Lambda = c1\ f\ MPl^2H0^2\ /\ (\pi\ I^2)$ | 106 |
| (13.6) | $\rho\Lambda = \Omega\Lambda \times 3H0^2MPl^2\ /\ (8\pi)$ | 107 |
| (13.7) | $I^2 \times \Omega\Lambda = 8c1f/3 = 8f = 8 - 3/\pi$ | 107 |
| (13.8) | $\Omega m = 0.311,\ \Omega\Lambda = 0.689$ | 107 |
| (13.9) | $\rho\Lambda = 3 \times 0.881 \times (2.222 \times 1076\ GeV4)\ /\ (2.29 \times 10123) = 2.56 \times 10{-}47\ GeV4$ | 109 |
| (13.10) | $\rho\Lambda,obs = 2.52 \times 10{-}47\ GeV4$ | 109 |
| (15.1) | $\nabla\mu\ T\mu\nu = 0$ | 125 |
| (15.2) | $(\rho + p)\ u\mu\ \nabla\mu\ u\nu = -(g\nu\mu + u\nu\ u\mu)\ \nabla\mu\ p$ | 126 |
| (15.3) | $u\mu\ \nabla\mu\ u\nu = 0$ | 126 |
| (15.4) | $d^2x\mu/d\tau^2 + \Gamma\mu\alpha\beta\ (dx\alpha/d\tau)\ (dx\beta/d\tau) = 0$ | 128 |
| (15.5) | $D^2\xi\mu/d\tau^2 = -R\mu\nu\alpha\beta\ u\nu\ \xi\alpha\ u\beta$ | 128 |
| (16.1) | $g\mu\nu = \eta\mu\nu + h\mu\nu,\ \|h\mu\nu\| \ll 1$ | 130 |
| (16.2) | $\nabla^2\Phi = 4\pi GN\ \rho$ | 131 |
| (16.3) | $d^2x/dt^2 = -\nabla\Phi$ | 131 |
| (16.4) | $F = GNMm/r^2$ | 131 |
| (16.5) | $H^2 = (8\pi GN/3)\rho - kc^2/a^2 + \Lambda c^2/3$ | 133 |
| (16.6) | $\Box\ \bar{h}\mu\nu = 0$ | 134 |
| (17.1) | $MPl = \sqrt{(\hbar c/GN)}$ | 138 |
| (A.1) | $G\mu\nu + \Lambda g\mu\nu = 8\pi GN\ \langle T\mu\nu\rangle$ | 196 |
| (B.1) | $ds^2 = -d\tau^2 + \Sigma i\ \tau 2pi\ dxi^2$ | 201 |
| (B.2) | $\Sigma i\ pi = 1$ and $\Sigma i\ pi^2 = 1$ | 201 |
| (B.3) | $3p + 4q + 2r = 1$ and $3p^2 + 4q^2 + 2r^2 = 1$ | 202 |
| (B.4) | $24q^2 + (24p - 8)q + (15p^2 - 6p - 1) = 0$ | 202 |

# Glossary

Terms as this book uses them, with the place where each is introduced or argued.

**Asymmetry**: difference of content where there is difference of context (Chapter 2, Section II).

**Axiom I**: the immanent is more fundamental than the omniscient and/or the transcendent, which are conjugate (Chapter 4, Sections IV and V).

**Axiom II**: the asymmetric, three-valued precedence of classes over instances: a class of the transcendent precedes an instance of the immanent, a class of the immanent precedes an instance of the omniscient, and a class of the omniscient precedes an instance of the transcendent (Chapter 4, Sections III and V; Chapter 10).

**Axiom III**: the (classes/instances) of the three modalities are distinct, inseparable and non-interchangeable; found in the three pairs of intrinsics (Chapter 4, Sections I and V).

**Bare curvature term**: a classical cosmological term of the order of the Planck density, from the curvature of the internal space, with the sign of anti-de Sitter space and carrying no $\hbar$; at the derived radius it is $-3/(8\pi)\ M_{Pl}{}^4$, and it is the classical part of the external value (Chapter 11, Section III; Chapter 13, Section II).

**$c_1 = 3$**: the number of channels through which the vacuum couples to a measurement, one for each content a measurement registers. It is the Euler characteristic of $\mathbb{C}P^2$; on every complex projective space the Euler characteristic equals the coefficient of the first Chern class (Chapter 9, Section VII; Chapter 13, Section II).

**Causation**: a persistent perceived relation of association among the observing self, the observed cause and the observation of the effect; it requires an observer, runs one way in time, and needs no complete specification (Chapter 1, Section III; Chapter 5, Section III).

**Compactification conditions**: the compact factor is content relative to the exterior factor as its context; when the comparable exterior and its inertial reference end, the compact form cannot persist (Chapter 18, Section VI).

**Comparer**: whatever makes a comparison; in cosmological passages, the observer who measures.

**Comprehensiveness**: the standing of a description that leaves no aspect of its domain outside its reach; argued from the structure of comparison, by closure (Chapter 5, Section V).

**Conjugate descriptions**: two descriptions neither of which can be perfected without loss to the other, and neither prior to the other, as Axiom I states of the omniscient and the transcendent; general relativity and quantum mechanics, which also meet the conditions of Axiom III, composed as a product and not merged into one theory (Chapter 6, Section III).

**Constitutive and instrumental modes**: the mode of what a theory finds, and the character of the feature of its formalism that stands in the conjugate mode (Appendix A, Section III).

**Content and context**: what the verdict of a comparison is on, and the respect in which the things compared are two; in physics, the quantized fields and the geometry (Chapter 2, Section I; Chapter 6).

**Content instance and context instance**: the two readings of the internal space $\mathbb{C}P^2 \times S^2$: read as a space of states, with no size and no fields, on which the kinds and charges of the content are read; and read with radii, as the internal factor of the ten-dimensional metric that I size and hold in this book (Chapter 8, Section IV).

**Content/context partition**: the division of physical phenomena into content, governed by symmetry and quantized, and context, governed by continuity and not quantized (Chapter 6).

**Context markers**: universality of coupling, non-shieldability, equality of gravitational and inertial mass, and absence of a discrete charge; each argued for any context-modal phenomenon, and gravity has all four (Appendix A, Section II).

**Continuity**: sameness of content where there is sameness of context (Chapter 2, Section II); its physical reading is that an infinitesimal change of context produces an infinitesimal change of content.

**Continuously asymmetric**: the surviving conjunction that characterizes the context and general relativity (Chapter 3).

**Criterion !x**: self-application of a compound concept is semantically empty (Chapter 3, Section II).

**Criterion !y**: opposite concepts, continuity and discontinuity, or symmetry and asymmetry, cannot be asserted together (Chapter 3, Section II).

**Criterion !z**: sameness and difference, being inseparable, must appear together in each respect in which a conjunction gives verdicts: among the verdicts on content, and among the verdicts on context; stated as a count, each conjunction that passes carries two samenesses and two differences (Chapter 3, Section II).

**Cross-modal principle**: a framework and its content are always in conjugate modes; a theory's formalism has a feature in the mode conjugate to what the theory finds (Appendix A, Section III).

**De-compactification**: when the compactification conditions fail, the ten-dimensional structure restores exterior context; the restoration appears as the Big Bang in the history reconstructed from inside the new epoch (Chapter 18, Section VII).

**Derived**: follows, by theorem or by standard technique, from the structure of comparison and the three axioms it carries (Part I), together with the identifications the text states (Preface).

**Determinism**: the assertion that there can be a complete and absolute specification of every value of every measured content for every value of every context; it requires no observer and no direction of time, and is never observed (Chapter 5, Section III).

**Dimension**: the number of independent ways two findings can differ in a context; distinct from a metric, which is a kind of measure (Appendix C, Section III).

**Discontinuity**: difference of content where there is sameness of context (Chapter 2, Section II).

**Discontinuously symmetric**: the surviving conjunction that characterizes the content and quantum mechanics (Chapter 3).

**Doubling**: every finding is placed twice, in a context and by a comparing, so no difference stands alone (Appendix C, Section II).

**Equivalence principle as a triple**: centripetal force, linear acceleration and gravitation: in each, every body is affected alike, in proportion to its inertial mass; the three are told apart by their global structure (an axis, uniformity, a center) (Chapter 14).

**Euler characteristic**: when every independent class lies in even degree, the number of independent classes: 3 for $\mathbb{C}P^2$, 2 for $S^2$, 6 for their product (Chapter 9, Section IV).

**Event horizon**: the bound of all that could reach an observer in the future; no present measurement contains it (Chapter 13, Section II).

**Exists and real**: to exist is to stand as timeless structure, the same in every instance; to be real is to be given in an actual act, with the future open (Chapter 5, Section II).

**Exterior empties**: content beyond the cosmological horizon is not comparable in principle; within it, matter ends in black holes or departs, and with it every observer's inertial reference (Chapter 18, Sections III and IV).

**External value**: the vacuum energy density at the scale of the modal boundary, $f\,M_{Pl}{}^4$ with $f = 1 - 3/(8\pi)$: a ground-state part, $M_{Pl}{}^4$, and a classical part, the bare curvature term (Chapter 13, Section II).

**Floor**: the least extent and least motion that the internal space cannot lose; one mechanism that holds the internal space, computed at first order in Appendix D (Chapter 11, Section II).

**Forbidden pairing**: continuity with symmetry, or asymmetry with discontinuity; a quantized arena would need one of them, and the Incommensuration Theorem excludes both (Chapter 3, Section V).

**Geodesic**: the path of a free test body in curved spacetime, along which its tangent vector is parallel-transported; exact for dust, and in the limit of a small body (Chapter 15, Section I).

**Geodesic deviation**: the relative acceleration of neighboring geodesics, governed by the Riemann tensor; the relativistic form of tidal forces (Chapter 15, Section V).

**Grid**: the six metrics as two rows (content, context) by three columns (the modalities) (Chapter 7, Section III).

**Homogeneity**: every point of a factor is equivalent to every other under a compact Lie group of isometries (G/H) (Chapter 9, Sections II and VI).

**Identification**: a step from the structure of comparison to a physical quantity that is a reading, not a theorem; the text states each in the first person (Preface).

**Incommensuration Theorem**: symmetry and continuity cannot both be perfected in any comparison; only the continuously asymmetric and the discontinuously symmetric conjunctions survive (Chapter 3).

**Information deficit**: what the accessible share leaves out of the external value; the share is $c_1/S$ of the external value, with S the count of the observer's particle horizon (Chapter 13, Section II).

**Information, two senses**: information as the form of the content, which passes through the turn, and a count of states reckoned relative to a context, which resets with the context (Chapter 18, Sections VIII and XIII).

**Intrinsics of comparison**: the six concepts necessary and sufficient for any comparison: sameness, difference, content, context, subjective, objective (Chapter 2, Section I).

**Kaluza-Klein reduction**: the standard technique of integrating a higher-dimensional action over the internal space to obtain a four-dimensional effective theory (Chapter 11).

**Kasner arc**: with the external directions, the $\mathbb{C}P^2$ directions and the $S^2$ directions each moving together, the external exponent runs from $-1/3$ to $+5/9$; read away from a singularity, the short arc at the upper end is the cosmological compactification, and read toward one, the form of a black hole interior once it has reached the arc (Appendix B, Section IV).

**Kasner exponents**: the powers of the time to a singularity by which each spatial direction scales; in empty space they sum to one and their squares sum to one (Appendix B, Section I).

**Minimality principle**: of the numbers comparison supplies, a context takes the least that meets every requirement comparison places on it (Appendix C, Section IV).

**Modal binding**: the association of each of three terms with one of the modalities; it depends on which part of comparison is the focus; in this book, sameness and difference immanent, content and context omniscient, the subjective and the objective transcendent (Chapter 4, Section II).

**Modal boundary**: the Planck scale, where the continuously asymmetric and the discontinuously symmetric descriptions exchange dominance; a structural transition, not a barrier (Chapter 17).

**Modality**: one of three roles that concepts play with respect to one another in any comparison: omniscient (what is structural and known), immanent (what mediates and happens), transcendent (what is potential and exceeds any instance) (Chapter 4, Section II). Which term fills which role depends on the focus of the context; this book's focus is the act of measurement. The names carry no theological meaning.

**Moduli**: the scalar fields describing the size and the shape of the internal space; held at the derived radius and the ratio $\sqrt{6}$, the same throughout the exterior, with masses of about the Planck mass from the mechanism of Appendix D (Chapter 11, Section II).

**Necessity and sufficiency**: a claim of necessity says that something must be present; a claim of sufficiency says that nothing further is required; the first is argued by the structure of comparison and by mathematics, the second by the structure of comparison alone (Chapter 1, Section V; Chapter 5, Section V).

**Particle horizon**: the bound of what has reached an observer; the only horizon a first-person measurement holds (Chapter 13, Section II).

**Per-alternative geometry**: the framework of this book: quantum fields for the matter, and one classical geometry for each alternative of the matter, shaped by its own stress-energy, with the relative phases of the alternatives kept (Chapter 6, Section III; Chapter 12, Section VI). It is not the Møller-Rosenfeld theory, which keeps one geometry for all the alternatives.

**Placing rule**: the distinction between two ways of a context is placed in that context when, and only when, the context has a way that is not composed from those two; one, and every number from three upward, meet it (Appendix C, Section VI).

**Product structure**: $M^4 \times \mathbb{C}P^2 \times S^2$: the context factor, four-dimensional and Lorentzian, and the content factor, the six-dimensional compact internal space (Chapter 8).

**Projection**: the turn at the end of a cycle, a limit that no comparison registers as an event, as it appears in a history reconstructed from inside: a beginning at a definite time (Chapter 19, Section III).

**$R_O/R_T = \sqrt{6}$**: the ratio of the radii of $\mathbb{C}P^2$ and $S^2$ fixed by the Einstein condition on the internal product; the overall scale is set at the modal boundary, $R_T = l_{Pl}$ (Chapter 9, Section IX).

**Root Tautology**: comparison, interaction and relation are one act, approached three ways; the one premise of Part I, and by Axiom III a formal triple (Chapter 1, Section IV; Chapter 4, Section VI); every finding is an interaction, so energy and momentum measure how many findings can be made, and the context curves in proportion to that count (Chapter 12, Section V).

**Self-consistency identity**: $I^2\Omega_\Lambda = 8f = 8 - 3/\pi$, from the information deficit formula and the Friedmann equation, with the Hubble constant and the Planck mass canceled; it gives $\Omega_m = 0.311$ (Chapter 13, Section III).

**Simultaneity**: the co-occurrence of the intrinsics of comparison in every comparison, with no lead of one over another; presupposing them is not circular (Chapter 5, Section IV).

**Six metrics**: pattern, force and probability (the content triad); space, time and possibility (the context triad): pattern (inertia, mass, structure); force (the directed flow of change); probability (the specification of values among alternatives); space (the arena in which structure is laid out and known); time (the bare ordering of before and after); possibility (the field of what could occur) (Chapter 7). Pattern, inertia and mass share one metric because pattern, inertia and weight are three findings of one relation of a content to its context, the relation of being found there, and mass is the measure of that relation (Chapter 7, Section I; Appendix A, Section II).

**Spin**: a discrete label of a content, the representation of the rotations of the frames of $M^4$ that its field carries; content by the mark of discreteness, though the group whose compact part it labels belongs to the formalism of the context (Appendix A, Section IV; Chapter 12, Questions).

**Symmetry**: sameness of content where there is difference of context (Chapter 2, Section II).

**The turn**: the end of a cycle, approached as a limit by the collapse of the interiors and not reached; no comparison registers it as an event (Chapters 18 and 19).

**Validity and soundness**: a conclusion is valid when it follows from its premises by the rules of a structure; an argument is sound when, in addition, its premises hold in the world (Chapter 5, Section II).

$\mathbf{w = -1}$: the equation of state of the vacuum: $\Lambda$ is fixed once, by the horizon of the present measurement, as the constant term of one geometry, which the reconstructed history carries at every epoch (Chapter 13, Sections V and VI).

# Notes

1. *An Immanent Metaphysics*, aphorism 1.42-4: "The Root Tautology: Comparison is isomorphic with interaction and relation".

2. *An Immanent Metaphysics*, aphorism 1.42-5, names the six concepts of sameness, difference, content, context, objectivity and subjectivity, taken together, the intrinsics of comparison.

3. The proof by exhaustive enumeration is in *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20; the table of tertiary conjunctions is at 1R17.

4. *An Immanent Metaphysics*, aphorism 1.41-2: "Axiom III does not in itself identify which domain essential concept is associated with which modality, but only that such associations are always strongly definable".

5. The axioms are given in the wording of *An Immanent Metaphysics*, aphorism 1.3-1.

6. The three axioms are each associated with a modality. Axiom I has the nature of the omniscient modality: it is a structural fact about the pattern. Axiom II has the nature of the immanent modality: it describes the process of precedence. Axiom III has the nature of the transcendent modality: it asserts the distinctness that exceeds any particular instance. In this way the axioms describe themselves.

7. The macroscopic boundary of a domain of measurement, and its constant as the analog of the speed of light: *An Immanent Metaphysics*, aphorisms 1.75-20, 1.82-3 and 1.86-13.

8. The past as seen from the present, static and fixed, and its one existence as arising from the interactions in the present: *An Immanent Metaphysics*, aphorisms 1.86-8 and 1.86-9.

9. The relationship of the point to the room, of the instance to the class, is governed by Axiom II.

10. On Birkhoff's theorem and the content/context partition. Birkhoff's theorem states that spherical symmetry alone determines the vacuum geometry. In the reading of the Immanent Metaphysics, this is a structural consequence of the context-modal character of gravity: the context responds to the symmetry of the content distribution in the unique way that preserves continuous asymmetry. A symmetric matter distribution produces a geometry whose asymmetry is purely radial: the minimum asymmetry consistent with the content.

11. In the terms of the Immanent Metaphysics, the collapse, which concentrates, is the ordering principle, and the emergence, which disperses, is the disordering principle. Gravity is the ordering principle: the context-modal tendency toward concentration, curvature and collapse. De-compactification is the disordering principle: the structural necessity that context be restored when the conditions for compactification dissolve. The cycle of epochs is their alternation, and neither is more fundamental than the other.
12. For readers of the Immanent Metaphysics. The projection of Section III is the transcendent aspect of the ontological triple (existence, interaction, creation), projected onto the omniscient axis. The concept "universe" has three irreducible aspects, corresponding to the three modalities: existence (omniscient), interaction (immanent) and creation (transcendent). Creation is the transcendent aspect: the ever-present ground of novelty, the coming-into-being of the genuinely new; it is not a past event but a structural feature of reality. Creation belongs to the context, which is continuously asymmetric. Physics reports in the omniscient mode: it describes, computes, locates and specifies, and so renders what it reports in the discrete terms of content. The projection of the one onto the other produces a discontinuity, a break at the temporal origin: the Big Bang. As quantization at the microscopic limit renders the transcendent potential in discrete, particle-like form, the macroscopic projection renders it as a singular temporal origin. Both are projections, both are necessary, and neither is more real than the structure of comparison that generates both.
13. *An Immanent Metaphysics*, aphorism 1.83-5.
14. *An Immanent Metaphysics*, aphorism 1R3.
15. *An Immanent Metaphysics*, aphorism 1.46-12.
16. Read in the Immanent Metaphysics, many black holes ending in one origin is Axiom II on the largest scale: a plurality of existence precedes a single instance of creation.
17. The doubling is the form the Immanent Metaphysics gives to Axiom I when counting: for every one, at least two. The count is shared with Axiom I; the roles of Axiom I are not mapped onto the finding and its two placings.

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