The Strengths of the Kinds



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The Couplings, Their Crossings, and the Scales of the Proton and of Gravity
What the Structure Sets and What Measurement Supplies, Volume I

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Contents

Preface

The Measured Values Used in This Book

Chapter 1. What Is Measured

Chapter Overview

I. The Kinds of Number

II. The Units and the Pure Numbers

III. The Electron as the Unit

IV. The Strength of Light as the Atom Registers It

V. The Strength of Light as Measured

VI. Why the Strength of Light Stays Measured

VII. The Settings, and the Count of the Numbers

Summary

Key Concepts

Chapter 2. The Couplings and Their Running

Chapter Overview

I. What a Coupling Is

II. What Is Measured and Where It Is Read

III. The One-Loop Rates

IV. The Thresholds

V. Beyond One Loop, and the Matching at the Top

VI. The Couplings Across the Span

VII. Relations Among the Couplings

VIII. Light at the Proton: the Restated Relation

Summary

Key Concepts

Chapter 3. Where the Couplings Cross

Chapter Overview

I. The Law of Place

II. The Spacing Ratio

III. Where the Crossings Are

IV. The First Crossing and the Weak Angle

V. The Level and the Place of the First Crossing

VI. The Gaps at the Span's End

VII. What Is Derived, What Lands, What Decides

Summary

Key Concepts

Chapter 4. The Weak Angle and the Masses of W and Z

Chapter Overview

I. The Angle Defined

II. The Masses at Tree Level

III. The Departure from $\rho = 1$: Its Form, Then Its Size

IV. The Weak Angle at Measured Scales

V. The Z Width and the Pole Observables

VI. What the Carriers' Masses Do

Summary

Key Concepts

Chapter 5. The Span to the Planck Mass

Chapter Overview

I. The Measured Span

II. The Count and the Test of the Count

III. The Remainder at First Order

IV. The Second Order: the Constant k

V. The Third Order and What a Measurement of G Can Decide

VI. The Second Route to the Span, through the Couplings

VII. What Other Chapters and Books Carry

Summary

Key Concepts

Chapter 6. The Binding Scale of the Proton

Chapter Overview

I. What Kind of Mass the Proton's Is

II. Color's Scale from Color's Strength

III. Color's Whole Is the Proton's Mass

IV. The Floor $2m_p$

V. The Proton's Place and the Ratio m_p/m_e

VI. The Proton's Size

Summary

Key Concepts

Notes

Terms

Bibliography

Index

Preface

This book says how strong the couplings of the particles are. It is the first of two volumes that carry the numbers of the particles, under the common title What the Structure Sets and What Measurement Supplies; the second, The Measures of the Kinds, carries the masses and the mixings. The second volume rests on this one, and this one rests on nothing in the second. The two volumes follow *The Kinds of Particle*, which says what kinds there are and why the structure that carries them must be as it is, and *The Branches of the Kinds*, which carries what the same structure then allows and forbids. The strengths of this book's title are the strengths of the couplings: of light, of the weak force, of color, read through the binding of the proton, and of gravity, read through the span from the electron to the reduced Planck mass.

A number here stands in one of four ways. Some are derived: the structure sets them, and each is given with its argument and set beside its measurement. Some are derived given a stated identification: they follow by argument once an identification is made, and the identification is stated as the question the value carries. Some are measured inputs: the electron's mass, which is the unit of mass; the strength of light; the strength of color; the vacuum value; together with the settings at which numbers are read. And some are measured agreements: values that agree with measurement for a reason that is not yet stated as an argument. Each is given with its value, its measurement and the distance between them, and with its question, why it lands, and the clues the book has for that question; it is never presented as derived.

What the structure sets is a *whole*: the anchor of a measure, such as the equal traces with the weak angle 3/8 at them, or the count of 72 halvings against which the span is read. What measurement gives is a *value*, read against its whole with its residue. So where another book of the series says that the couplings are measured, it speaks of values, and this book reads them as values too; the wholes are this book's. Two readings recur and are set out once: the Planck mass meant throughout is the reduced one, against

which the span of 72 halvings is read (Chapter 5, Section I); and light's strength is read at the proton's mass, the mass of what the comparison is made against (Chapter 2, Section II).

The six chapters take, in order: what is measured and why it stays measured, the couplings and their running, where the couplings cross, the weak angle and the masses of the W and the Z at tree level, the span of seventy-two halvings from the electron to the reduced Planck mass, and the binding scale of the proton. Chapters 7 to 13 are in The Measures of the Kinds, which continues this volume's numbering of chapters. Where this volume names one of those chapters, it names where a matter is carried; no argument of this volume rests on it.

Every measured value is given with its uncertainty and its source, and the values used in this volume are collected in one table at its front. A distance from measurement is given in standard deviations, as the stated value less the measured value, divided by the combined uncertainty; that distance is called the *pull*.

A reader may ask how a number written in the small integers of the structure is told from a number that merely lands near a measured value. Independent routes, resting on different premises, meet on one value: the span's integer from the count and from the couplings (Chapter 5), the place of the first crossing from the running couplings and from light's relation at the proton (Chapter 3), and the proton's place from color's way and from light's way (Chapter 6). And every value that lands is stated with its question, why it lands, and its clues.

The internal space has two instances. On its content instance it is a space of states, with no size, no radius and no fields; on its context instance it has radii, and that instance is the subject of The Gravitational Passage. Every statement this book makes about the internal space is made on the content instance.

The book cites, and does not repeat, what other books establish. *The Kinds of Particle* is cited by chapter and section. The Story of the Particles, cited as the zoo, is the authority on which particles exist and of what kind; it fixes no value of a mass, a mixing or a coupling beyond the few its structure sets (the zero masses of the photon and the gluons, the rank order of the couplings, the

top's coupling of one and the strong force's own angle of zero). The Unquantized Arena and Two Theories, One World state the ground: comparison, its intrinsics, its axioms and the Incommensuration Theorem. Non-Commutative Geometry and the Immanent Metaphysics, cited as the NCG book, is cited for its mathematics.

Each chapter is written in two layers. The main text says what is measured, what is derived and from what, and how the two compare, in plain and exact prose that an interested reader can follow. Derivation boxes carry the derivations; each is complete in itself, with its premises, its steps and its result, so that a physicist can check a chapter by reading its boxes in order. Each chapter opens with a chapter overview and closes with a summary and its key concepts. The terms of the book are collected at the back, with the notes, the bibliography and the index.

The Measured Values Used in This Book

The table collects the measured values the chapters of this volume use, each with its uncertainty and its source. The digits in parentheses are the standard uncertainty in the last digits. Where two editions of the Particle Data Group's Review of Particle Physics are given, each chapter names the edition it uses at the place it uses it. The settings at which values are read (the energy of a running strength, and the scheme that defines it) are stated with each value in the chapters.

The measured values used in this book.

Quantity	Value	Source
Electron mass, m_e (the unit)	0.51099895069(16) MeV	CODATA 2022
Inverse fine-structure constant, $1/\alpha_{em}(0)$	137.035999177(21)	CODATA 2022
Light's MS-bar inverse strength at the Z mass, $1/\hat{\alpha}_{em}(M_Z)$	127.951 ± 0.009	Particle Data Group 2022, electroweak review
Light's MS-bar inverse strength at 1.035 GeV, $1/\hat{\alpha}_{em}$	134.07 ± 0.12	no experiment named (Chapter 2, Table II-1)
Width of the Z, Γ_Z	2.4955 ± 0.0023 GeV	Particle Data Group 2024
The seven pole observables of the Z (the forward-backward asymmetries of leptons, bottom and charm; A_l; R^0_{had}, R^0_b and R^0_c)	as listed in Chapter 4, Table V-1	the combination of the LEP and SLD experiments, Physics Reports 427, 257, 2006
Hadronic vacuum polarization at the Z mass, $\Delta\alpha_{had}$	0.02783	Particle Data Group 2024, electroweak review
	0.23122 ± 0.00006	

Quantity	Value	Source
Weak mixing angle at the Z mass, MS-bar, $\sin^2\theta_W(M_Z)$		Particle Data Group 2026
Color's strength at the Z mass, $\alpha_s(M_Z)$	0.1180 ± 0.0009	Particle Data Group 2024
Fermi constant, G_F, and the vacuum value $v = (\sqrt{2}\, G_F)^{-1/2}$	$1.1663785(6) \times 10^{-5}$ GeV^{-2}; $v = 246.21967 \pm 0.00006$ GeV	Particle Data Group and CODATA 2022
W mass, m_W	80.3625 ± 0.0077 GeV (2026); 80.3692 ± 0.0133 GeV (2024)	Particle Data Group 2026 and 2024
Z mass, m_Z	91.1880 ± 0.0020 GeV	Particle Data Group 2024
Top mass, direct	172.60 ± 0.27 GeV (2026); 172.57 ± 0.29 GeV (2024)	Particle Data Group 2026 and 2024
Higgs mass, m_H	125.13 ± 0.11 GeV (2026); 125.20 ± 0.11 GeV (2024)	Particle Data Group 2026 and 2024
Proton mass, m_p	938.27208943(29) MeV	CODATA 2022
Proton charge radius, r_p	0.84075(64) fm	CODATA 2022
Newton's constant, G, and the span $E = \log_2(M_{Pl,red}/m_e)$	$6.67430(15) \times 10^{-11}$ m^3 kg^{-1} s^{-2}; $E = 72.0132112 \pm 0.0000162$	CODATA 2022
Muon to electron mass ratio, m_μ/m_e	206.7682827(46)	CODATA 2022



Chapter 1

What Is Measured

CHAPTER OVERVIEW

A reader comes to this chapter with two questions: which numbers of the particles does measurement supply, and why does the strength of light stay measured when so much else is derived? This chapter names the numbers that the particle side of physics (the account of the kinds of particle and their numbers) takes from measurement, and it states the standing of each. It first sorts every attribute of a particle into one of three kinds: one number read by one comparison, a relation among such numbers, or something every measurement rests on and none yields. The electron's mass is the unit of mass: it is the one dimensionful measured input of the particle side, and everything else about the electron is derivable on the ladder of the charged leptons' masses (Chapter 7). The strength of light, the fine-structure constant, is a ratio with no unit, one comparison that each of six kinds of measurement registers; it is measured, and its independence of comparison is derived for the comparisons the argument examined, with a stated hypothesis (Section VI). The energy and the convention at which a number that changes with energy is read are fixed by stating them. The chapter closes with the numbers of the Standard Model that this volume carries, each with its chapter and its standing there.

I. The Kinds of Number

Comparison is the act of holding one thing against another: there is a thing compared, an act of comparing, and a thing compared against (*The Kinds of Particle; Why the Structure Must Be So*, here called *the book of kinds*, Chapter 1, Section I). The *particle side* of physics is the account of the kinds of particle and of their numbers; its measured input is the electron, as Newton's constant is the measured input of the side of gravitation and cosmology. *The zoo* is *The Story of the Particles*, the account of which particles exist and of what kind; this book cites it and does not repeat it. A *generation* is one of the three copies of the sixteen states of matter, each with the same charges (the book of kinds, Chapter 6, Section I).

Every attribute of every elementary particle (its electric charge, its spin, its mass, the strength of the force it feels, a mixing angle, a rate of decay) is exactly one of three things. It is a *single measure*: one number read by one comparison, such as the mass of the electron in some unit. It is a *law*: a relation among measures, such as the rule that the charges of one complete generation sum to zero, or the rule by which the strength of a force changes with the energy at which it is read. Or it is a *non-measure*: what every measurement rests on and none yields, namely the choice of unit, the settings of the apparatus, and the particular history of this universe.

The three parts stand at the three positions of comparison, which the book of kinds, Chapter 1, Section II, names the omniscient, the immanent and the transcendent; in its assignment of the known, the unknown and the unknowable to them (the book of kinds, Chapter 1, Section IV), the omniscient is the known, the immanent the unknown and the transcendent the unknowable, with the immanent also the comparing act, which mediates between the other two. In the frame of what is measured about a particle, a single measure is the *actual* (what is found) and stands at the omniscient. A law, taken as the transformation that carries one measure into another, stands at the immanent; taken as the structure that holds in every instance, it is omniscient, as Comparison, Chapter 10, Section VIII, and Measurement, Chapter 6, Section IV, place it. A non-measure is the *potential* (what could be found) and stands at

the transcendent. Together the three are called the *span of measure*. The assignment holds in this frame, as every assignment of the three positions holds in the frame of its own argument.

Box I-1. The span of measure.

Premise 1. Every attribute of a particle is exactly one of a single measure, a law, or a non-measure (the unit, the settings, the history of this universe).

Premise 2. Only what is actual can be measured objectively; what is potential, and anything of its kind, can never be measured objectively (note 1).

Premise 3. Measurement, interaction, the transformation of states, force and strength have the nature of the immanent (note 1).

Premise 4. A law relates measures and carries one into another.

Result. A single measure is the actual, the omniscient. A non-measure is the potential, the transcendent. A law, being a transformation of states, is in this frame the immanent; the law as structure is omniscient (Measurement, Chapter 6, Section IV). In the frame of what is measured about a particle, the three parts of the span of measure stand at the three positions of comparison.

A second sorting applies to numbers as they are written. Every number in physics belongs to one of three *tiers*. A number of the first tier is fixed by structure: it is exact, it is the same in every instance, and it owes nothing to measurement. A number of the second tier is a *value of the instance* (the *instance* being this world, in which the structure is realized): a *pure number* (a ratio of like to like, which carries no unit) to be measured, as precisely as its measurement reaches. A number of the third tier is a unit or a convention: a choice of bookkeeping with no fact behind its number.

A value relates *terms*, the quantities of which it is a ratio or a relation; a *form* is one concept of the structure, and a form supplies a value's terms when all of them belong to it. A number of the first tier never needs measuring; it needs only checking. A number of the second tier can be derived only where a form is found that supplies its terms. A number of the third tier can never be derived.

A number written in a chosen unit splits into a pure number and a unit factor. The unit factor is a choice of bookkeeping, of the third tier; the pure number is a value of the instance. So no number falls outside the three tiers: the unit part is of the third tier, and the remainder is of the first or of the second.

Box I-2. The three tiers.

Premise 1. A number is fixed by structure only where one form supplies all its terms.

Premise 2. A unit or a convention is a choice of bookkeeping with no fact behind its number.

Premise 3. The numeral of an inertia written in a chosen unit is a value of the instance multiplied by a conversion fixed by the choice of units.

The first tier. It holds the charges in thirds; the three generations; the strong force's own angle of zero (Section VII); the rates $41/10$, $-19/6$ and -7 at which the three inverse strengths change with energy (in the unification normalization and the sign convention of Chapter 2); the two of the electron pair in a superconductor; the whole numbers of steps of magnetic flux through a ring and of steps of charge passed sideways across a sheet in a magnetic field (Section II); the odd denominators of the fractional sideways steps; and the numbers of mathematics, such as π and the whole-sphere factor 4π. The balance $2/3$ among the charged leptons' masses is derived, given a stated identification: that each of its two parts, the part the three masses share and the part by which they differ, is measured by its share of the squared size (*The Measures of the Kinds*, Chapter 8).

Result. Every number is fixed by structure, a value of the instance, or a unit or convention. A choice cannot be derived; a value can be derived only where a form is found that supplies its terms; a number fixed by structure is exact and needs only checking.

Every number of the Standard Model is to reach one of two ends. At the first end the number is derived: it is obtained through its law. At the second end a proof of independence shows that the structure supplies no law for the number; such a number can never be derived, and it is carried as a reference,

read once, as a unit is. A value shown not derivable is closed under reason as fully as a derived value: it has an argument that states exactly why no law fixes it, as a derived value has an argument that fixes it.

The target for the Standard Model therefore has two parts. The values that a law fixes will be derived, each with its argument. Every other value will be shown to be of a kind already named as not derivable: a non-measure (the unit; a setting, which is a choice fixed by stating it, such as the energy at which a number that changes with energy is read and the convention that defines it there; a matter of history), a value shown independent of comparison, as α_{em} is (Section VI), or a selection by chance within a distribution whose form is itself derived. The *groups of measure* sort values by what fixes them: a value of group 1 is fixed by a law, and a selection by chance is of the third group.

The limit of the target for the Standard Model is the one the three positions fix: the unknown, at the immanent, can be driven toward the known, at the omniscient, and the unknowable, at the transcendent, stays. The boundary between the unknown and the unknowable is fixed, and no completeness of argument moves it. That boundary is the ground of every argument of non-derivability: it is what allows a value to be shown not derivable, and a value so shown is closed under reason. So the limit of the target for the Standard Model is also its most exact result.

A value that the present measurements leave unresolved stays within reach in principle: more digits of measurement can settle it. Every parameter this book writes down is one that some interaction among the finite list of kinds could register or some reasoning could fix; a parameter that neither could ever reach is not written down at all.

A parameter that no interaction among the kinds could register and no reasoning could fix is left out because of the principle of identity: that which is indistinguishable must be the same (Comparison, Chapter 11, Section V); anything that cannot be distinguished, in any way at all, even in principle, is the same thing (the zoo, Level 0, Section III). An interior constant that cannot be seen or known in any way, by any conceivable measurement or reason, cannot be told apart from its absence, and no one of its values can be told apart from another; so such a constant is, by the principle of identity, nothing.

No fundamentally inaccessible parameter is therefore defined, anywhere, ever. Either the value of a parameter is fixed by reason of argument, or by basis of the potential scope of the allowable types of interaction, or it is not fixed at all. Either a coupon (a second, independent place among the patterns of interaction at which the value would have to show itself, and show the same value; in engineering, a coupon is a test piece on which a property of a material is measured apart from the structure) is at least theoretically possible, given the patterns of interaction, or the value is available through the finite modes of reason and argument allowed, or it is not. The guard is the finiteness, and it shows up in two ways: once in the degrees of reason available from the primitive of the concept of comparison (the axioms, the Incommensuration Theorem and their pure theorems), and once again in the finite list of particle types (the kinds) themselves set by that reason. If no particle type can reach a quantity, and no notion of reasoning about comparison can reach it, then there is no fixity to it; it is purely transcendent and unknowable, and therefore not contributive to the real, which is knowable.

A value is fixed by a law, and a law is fixed by reasons, the arguments that establish it. So a shortage of derived values is, at bottom, a shortage of reasons, and the remedy runs the same order in reverse: more reasons give more laws, and more laws bring more values within reach. The alphabet of the particles, the list of the kinds that exist, has been reasoned into existence from comparison, forward in *The Story of the Particles* and in reverse in *Geeze, Wublets, and the Higgs*. An alphabet closed under reason at the level of existence implies values closed under reason at the level of measure: each value either follows, or is shown to be of a kind that cannot follow.

A choice between two values is sorted by one test. It is a *setting* when swapping its two values is a symmetry of the whole description, as with the sign of charge. It is a *value of group 1*, a value that a law fixes, when measurement can tell its two values apart, as with $\bar{\theta}$, the angle by which the strong force could distinguish matter from its mirror image (Section VII).

A number of this universe's history is not an attribute of any kind of particle, since a kind of particle has the same attributes wherever and whenever it is found. The excess of matter over antimatter, η_B, is such a number. It belongs to the initial conditions, to the history of the universe, and not to the structure that fixes the kinds.

These words name the standings of a number in this book. A number is *derived* when an argument gives it. It is *derived, given a stated identification* when it follows by argument once an identification is made, and the identification is stated with the question it carries, whether it is true of the world. It is a *measured agreement* when it agrees with measurement and the argument does not state why it agrees. Each measured agreement carries that question, why it lands, and the book gives the clues to it beside the value. It is a *measured input* when the book takes it from measurement. It is *not fixed* when this book does not fix it, and it stands at a *fork* when two arguments for it are carried side by side until measurement decides between them. The standing *not fixed* is relative to this book: a number with this standing is one that this book does not fix, and it is not thereby a parameter that is not fixed at all, by any measurement or reason, which the principle of identity shows is nothing (above). Where a number is compared with measurement, the comparison is stated as a *pull*: the stated value minus the measured value, divided by the combined standard deviation.

The standings are different relations between a number and the world, and not degrees of confidence. Each standing claims something different of its number, and each leaves something different unclaimed. Keeping the standings apart is most of what it means to know what one knows.

II. The Units and the Pure Numbers

Mechanics has three independent kinds of magnitude: mass, length and time. Three constants, the quantum of action $\hbar$, the speed of light c and Newton's constant G_N, together make one standard of each: the Planck mass, the Planck length and the Planck time. Dividing every magnitude by these standards makes it a pure number. So three units suffice, and what remains of the con-

stants of physics is a list of pure numbers. The first of them is α_{em}, the *fine-structure constant*, which measures how strongly light acts on charge; this book also calls it the strength of light.

A change of a unit alone leaves the verdict of every comparison as it was. Suppose the unit of action were changed so that the number $\hbar$ carries in it is larger, while every pure number keeps its value. Every comparison returns its verdict (its finding of same or different, more or less) on a ratio of like to like; the ratios are unchanged, so every verdict would be as before. So the number a unit carries has no value for comparison to fix: it is a choice of bookkeeping, of the third tier (Section I). The same reason shows why no unit is derived: a derivation ends in a verdict, and no verdict depends on a unit alone. The change alters only the number written for $\hbar$, and not the relation $\hbar$ stands for, the relation of potentiality to actuality, which this section sets out below.

The Planck mass built this way is $M_{Pl} = (\hbar c/G_N)^{1/2}$. Wherever else this book speaks of the Planck mass, it means the *reduced Planck mass*, $M_{red} = M_{Pl}/(8\pi)^{1/2}$, and each use below names which of the two is meant. In units of $\hbar$ and c the electron's charge e is a pure number; its square, divided by 4π for the whole sphere of directions, is $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$.

This book calls c, $\hbar$, α_{em} and G_N the four *relations*, since each relates ideas of comparison (c, for example, relates space to time). Of the four, three, c, $\hbar$ and G_N, fix a unit for every kind of magnitude, which is why the units are defined from them. α_{em} is the one pure number among the four: with c and $\hbar$ in fixed units it is a ratio of like to like, and it has no dimension.

α_{em} and G_N share two of their three terms. Each begins from a relation of force and distance, and each passes through c, the relation of space to time. They differ in the third: α_{em} joins the shared terms to $\hbar$, and G_N joins them to inertia. That one difference is the whole difference between the kinds of term the two relate. c and G_N lie wholly among space, time, force and inertia; $\hbar$ is the one relation that reaches beyond them. So α_{em} is the one relation of the four that joins relations lying wholly among space, time, force and inertia to $\hbar$.

The six *intrinsics* of comparison are the six aspects present in every comparison (the book of kinds, Chapter 1, Section I). Four of them (sameness, difference, content and context) can be followed apart from the *stance* of the comparing and the compared, the standpoint from which the comparison is made; the other two (the subjective and the objective) cannot. The six measures stand the same way: inertia, space, force and time can be followed apart from the stance, and probability and possibility stay bound to it (the book of kinds, Chapter 1, Sections VII and X). Member by member, in the *focus of comparison as relation* (the binding of the intrinsics to the positions that the zoo and the book of kinds use), the objective is read as force and the subjective as time, sameness as inertia and difference as space, content as probability and context as possibility (the book of kinds, Chapter 7, Section IV). ħ joins actuality, the relation of force to time, to potentiality, the relation of probability to possibility; in the frame of the intrinsics ħ is the relation of potentiality to actuality. So, of the four relations, α_{em} is the one that joins what can be followed apart from the comparing and the compared to what cannot.

Among the six measures, whose columns keep their positions in every focus, ħ is the bridge between the immanent and the transcendent, between time and possibility, and it marks the microscopic limit of the domain, which is the transcendent limit. *c* is the bridge between the omniscient and the immanent, and it marks the macroscopic limit, which is the omniscient limit. No constant joins the omniscient and the transcendent directly: as conjugates they meet only through the act (Measurement, Chapter 7, Sections IV and VII; note 2). Each assignment is stated with its frame.

The least step of spin is ħ. Intrinsic circulation is found only when a comparing finds it, as one of the outcomes the comparing could find, and its step is the relation of potentiality to actuality, which physics writes as ħ. The number that ħ carries in the international units expresses the unit in which action is measured, so it stands in the third tier, and like every unit it is not derived (Measurement, Chapter 7, Section IV).

The pattern of shared and differing terms above suggests that *c*, ħ, α_{em} and G_N may be the complete set of relations that tune a world (that fix the values its laws take), with α_{em} passing through both *c* and ħ, and G_N through *c* alone.

Among the electron's charge e, $\hbar$ and c, with ε_0 carrying the unit of charge, the counting of units leaves exactly one pure number, $e^2/(\varepsilon_0 \hbar c)$. The factor 4π and ε_0 are conventions of the international unit of charge: in Gaussian units the same number is written $e^2/(\hbar c)$. $\hbar$, and not $h = 2\pi\hbar$, appears because phase is measured in radians. Which function of the one pure number is called the strength is fixed by the atom. α_{em} is the first power, $e^2/(4\pi\varepsilon_0 \hbar c)$, because that is the number the atom registers as the speed of hydrogen's ground-state electron over c, and because its inverse is the charge at which Dirac's equation can no longer bind an electron in its lowest state.

Since 2019 the international system of units has been defined by fixing the values of seven constants: the frequency of the caesium atom's hyperfine splitting, 9,192,631,770 Hz; c; Planck's constant h; the elementary charge e; the Boltzmann constant; the Avogadro constant; and the luminous efficacy of radiation at 540 THz, 683 lm/W.

None of the seven is a relation that comparison fixes, and none of the seven defining numbers, as written in the international units, could be derived. c and h give two of the three units. The units of charge, of warmth, of amount (the count of entities) and of brightness to the eye are units or conventions, and they add no kind of magnitude. The seventh, the rhythm of the caesium atom's splitting, is a value of the instance (a pure number times the scale of inertia), chosen as the standard of time because every atom of a kind is alike.

Counted along the chain by which the base units are built, the constants are five: the second is fixed by the caesium frequency, the meter by c = 299,792,458 m/s, the kilogram by $h = 6.62607015 \times 10^{-34}$ J s, the ampere by $e = 1.602176634 \times 10^{-19}$ C, and the kelvin by the Boltzmann constant, 1.380649×10^{-23} J/K. Each step introduces one constant and one degree of freedom. A characteristic mass or length of particular objects, such as the mass of a particle, is a further scale that must be specified in addition.

The count of five and the sorting of seven count different things: the chain counts the constants by which the five base units are built in turn, and the sorting classifies all seven by kind.

With the elementary charge, c and h fixed exactly, the constants of empty space that relate charge to c and h, the permeability μ_0 and the permittivity ε_0, can no longer also be set. Their values follow from α_{em}, as $\mu_0 = 2\alpha_{em}h/(e^2c)$ (Box II-1): they carry α_{em}, and they are found by measurement.

A sheet of charge in a strong magnetic field passes charge sideways in whole multiples of e^2/h; the step of that sideways passing is h/e^2, the von Klitzing constant R_K. Set against the impedance of empty space, $Z_0 = \mu_0 c$, the step is $R_K/Z_0 = 1/(2\alpha_{em})$. Measuring the one against the other is therefore a way of finding α_{em}.

In the steps of sideways passing and of magnetic flux, the step ($\hbar$ or h) and the charge enter as values. What is fixed by structure is the ratios in which they enter, and those ratios are the same in every material.

The sorting of the seven constants, and the place it leaves to α_{em}, agree with the international system as defined since 2019. With e and h fixed, μ_0 and ε_0 are no longer exact; they are measured through α_{em}, by $\mu_0 = 2\alpha_{em}h/(e^2c)$, and α_{em} is a pure number that the system measures and does not define. The reason for the agreement is the two results above: the seven are units, conventions or a value chosen as a standard, and fixing e, c and h leaves α_{em} to the constants of empty space. Duff, Okun and Veneziano (2002) debated whether three, two or no fundamental dimensionful constants are needed; their common ground is that only dimensionless numbers are measurable facts.

Box II-1. The constants of empty space and the step of sideways passing.

Premises. $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$; $\varepsilon_0 = 1/(\mu_0 c^2)$; $h = 2\pi\hbar$; $Z_0 = \mu_0 c$; $R_K = h/e^2$. e, c and h have exact defined values.

Step 1. Substituting ε_0 gives $\alpha_{em} = \mu_0 e^2 c/(4\pi\hbar) = \mu_0 e^2 c/(2h)$. So $\mu_0 = 2\alpha_{em}h/(e^2c)$, and $\varepsilon_0 = e^2/(2\alpha_{em}hc)$.

Step 2. $R_K/Z_0 = h/(e^2\mu_0 c) = h/(e^2c) \cdot e^2c/(2\alpha_{em}h) = 1/(2\alpha_{em})$.

Result. With e, c and h exact, each of μ_0, ε_0 and R_K/Z_0 is fixed by α_{em} alone, so a measurement of any one of them is a measurement of α_{em}.

III. The Electron as the Unit

The electron's mass in absolute units is the one dimensionful measured input of the particle side, and it is the unit of measurement. It is not derivable. Its measured value is $m_e = 0.51099895069(16)$ MeV (CODATA 2022, the values recommended in 2022 by the Committee on Data, CODATA), where the digits in parentheses are the standard uncertainty in the last digits. Every other mass in this book is read in this unit.

Three routes reach the claim that the electron's mass is the unit and is never derived. They start from different premises, and they are kept side by side. The first two are set out here; the third asks the six questions of the electron, answers five of them by the ladder of the charged leptons' masses, and is set out with that ladder (Chapter 7, Section II).

The first route starts from the span of measure. A unit is a non-measure: it is not a measure, and it has no law. So the unit itself is never derived, which agrees with the known result that the scale of a unit cannot be fixed by relations among quantities. The electron's mass is the unit; it is therefore the non-measure of the particle side, with no law and no measure.

The second route starts from dimension. A dimensionful scale cannot be derived from dimensionless structure: the structure fixes dimensionless numbers, and one dimensionful unit must be supplied by measurement. Dimensionless ratios come from the *kernel* (the integers 3 and 2 of the book of kinds, Chapter 2, and the numbers built from them); the unit does not. The electron's mass is that unit, the single dimensionful measured input, and every other dimensionful quantity is expressed in terms of it.

The electron's mass is the unit, never derived, and no law sets it; it is the one dimensionful input. It sets the absolute scale of energy: it converts dimensionless ratios into units such as the GeV, and it selects no universe from a family of consistent possibilities. The dimensionless quantities (ratios of couplings, ratios of masses, mixing angles) follow from the topology and the stated premises, together with constants fixed by agreement with measurement where an argument uses them (Section VII).

The electron is massive because it carries what the Higgs condensate reads. The *Higgs field* is the one scalar field of the Standard Model, and its *condensate* is the value v that it holds throughout space. A particle's left-handed and right-handed parts are its two parts of opposite handedness (chirality), and the lepton *weak doublet* is the pair of states that the weak force turns into each other: the neutrino and the electron's left-handed part. The electron's left-handed part sits on one side of the lepton weak doublet, its right-handed partner stands outside the doublet, and the condensate joins the two; that joining is the electron's mass. How strong the joining is, a coupling y_e of about three parts in a million, is not derived; the electron's mass is taken as the unit against which the other masses are stated.

Given gravity, the electron carries exactly two independent pure numbers. One is α_{em}, which sets how strongly light acts between two electrons. The other is m_e/M_{Pl}, the electron's place on the Planck scale, with $M_{Pl} = (8\pi)^{1/2} M_{red}$ the unreduced Planck mass. The ratio of the electric force to the gravitational force between two electrons is $\alpha_{em}/(m_e/M_{Pl})^2 = 8\pi\alpha_{em}/(m_e/M_{red})^2$, so it is not a third number (Chapter 5 evaluates it). The strength of gravity between two electrons, α_G, contains the electron's mass and not its charge; α_{em} and α_G are distinct, and neither determines the other. So α_{em} and the three units are not a complete set of independent constants: m_e/M_{Pl} is a second measure, which needs its own test of independence.

Box III-2. The electron's two pure numbers.

Premise 1. The electron's own quantities are its mass m_e, its charge e and the quantum of action $\hbar$; the speed of light c and Newton's constant G_N tie them.

Premise 2. These five dimensional quantities stand in three independent dimensions (mass, length, time), with charge fixed through ε_0.

Step. By the counting of dimensions (Buckingham's theorem), five quantities in three independent dimensions leave $5 - 3 = 2$ independent pure numbers. Two such are $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$ and m_e/M_{Pl} with $M_{Pl} = (\hbar c/G_N)^{1/2}$.

Result. Every other pure number of the five is a function of these two; the ratio of the electric to the gravitational force between two electrons is $\alpha_{em}/(m_e/M_{Pl})^2$, and with $M_{Pl}{}^2 = 8\pi M_{red}{}^2$ it is $8\pi\alpha_{em}/(m_e/M_{red})^2$.

α_{em} is the electron's second pure number, the one its force supplies. The other, the electron's place below the reduced Planck mass, is carried on the particle side by a count: 72 *halvings* (factors of two), the count 8×9 of eight first-order states (2^3) times nine joins (3^2), each free cell one comparison, as the book of kinds, Chapter 2, Section VII, sets out, and each comparison one halving, as this book reads the count (Chapter 5). Nothing carries α_{em} in the same way. The measured span from the electron to the reduced Planck mass is set beside the count as a test in Chapter 5.

The structure fixes the strong force's own angle in $\bar{\theta}$ at zero (Section VII). The span from the electron to the reduced Planck mass is carried on the particle side by the count of 72 halvings, whose identification with the measured span is tested (Chapter 5). The masses that the argued relations fix from the electron's mass, the muon's on the ladder and the tau's by Koide's relation, are carried with the ladder (Chapters 7 and 8).

The Table of the Electron and the Vacuum

The electron's two pure numbers and the constants that make them stand in one table of three rows and three columns. The first row holds the electron's own constants, the content of every measurement made on the electron: its mass (m_e), its charge (e) and the quantum of action ($\hbar$). The second row

holds the constants of empty space, the context in which the electron is found: Newton's constant (G_N), the Coulomb constant, which is one over 4π times the permittivity of empty space ($1/(4\pi\varepsilon_0)$), and the speed of light (c). The third row holds each column's pure number: gravity's strength on the electron (α_G), the strength of light (α_{em}), and the unit.

Table III-1. The table of the electron and the vacuum

	Gravitational column	**Electric column**	**Column of limits**
Content: the electron's own	the electron's mass, m_e	the electron's charge, e	the quantum of action, $\hbar$
Context: empty space's	Newton's constant, G_N	the Coulomb constant, $1/(4\pi\varepsilon_0)$	the speed of light, c
The column's pure number	gravity's strength on the electron, $\alpha_G = 1.7518 \times 10^{-45}$	the strength of light, $\alpha_{em} = 1/137.035999177$	the unit, 1

Three forms relate the nine entries. Gravity's strength on the electron is Newton's constant times the square of the electron's mass, divided by the quantum of action times the speed of light. The strength of light is the Coulomb constant times the square of the electron's charge, divided by the quantum of action times the speed of light. The unit is the quantum of action times the speed of light, divided by itself. Every other relation among the nine entries follows from these three.

Each form is a comparison of two strengths, and a strength here is an energy times a distance. Two electrons at a distance attract by gravity with an energy equal to Newton's constant times the square of the mass, over the distance, and repel by light with an energy equal to the Coulomb constant times the square of the charge, over the distance. One quantum whose wavelength is 2π times a distance carries an energy equal to the quantum of action times

the speed of light, over that distance. Multiplying each energy by its distance removes the distance, so the three strengths are compared at every distance at once.

Energy falls as one over distance, so that energy times distance is the same at every distance, only in three directions of space. Light acts by turning the phase of charged things, and the charge it acts on is conserved (Section VI): a charge is a label of its content, the same in every comparison. So every closed surface around a charge is crossed by the whole of the charge's action, counted once; this is Gauss's law. In three directions of space the closed surface at a distance from a point is a sphere whose area is 4π times the square of the distance, so the action through each unit of area falls as one over that area, and the force falls as one over the square of the distance. The 4π of the sphere is the factor that the international unit of charge places inside the Coulomb constant (Section II), and Newton's constant stands where the Coulomb constant stands, with the same 4π already divided out, as Gauss's law for gravity shows. The three directions of space are derived by two roads that share no step (Measurement, Chapter 4; The Gravitational Passage, Chapter 10).

The table has two kinds of column. The gravitational column and the electric column are columns of comparison. Each holds a content of the electron, a context of empty space, and the pure number that compares the strength those two set between two electrons with the strength of one quantum. The third column is the column of the limits of the domain. Section II places the quantum of action at the transcendent limit and the speed of light at the omniscient limit, and states that the two meet only through the act. The act of comparing is the third limit, the immanent, and it carries no constant: a constant relates measures that findings lay out, and the act that makes a finding is not among the things it lays out (Measurement, Chapter 7). So the unit stands in the place of the third column's pure number.

The two kinds of column enter the forms in two ways, and each way has its reason. The electron's mass and charge enter to the second power, because each comparison is between two electrons. Newton's constant and the Coulomb constant enter to the first power, because the two electrons are compared in one context. The quantum of action and the speed of light enter to the first power, because their product is the strength of one quantum, the unit

against which both comparisons are made. The numerical factor of each form is one, with the 4π of the sphere inside the context constant. The unit is the strength of one quantum compared with itself, and a comparison of a thing with itself adds nothing to it. These are the eleven features of the three forms: five in each of the first two and one in the third.

The columns relate in three ways. Divided one by the other, the first two forms give the ratio of the two pure numbers: gravity's strength on the electron, over the strength of light, equals Newton's constant over the Coulomb constant, times the square of the electron's mass over its charge. Regrouped, the same relation says that Newton's constant over the square of the charge and the Coulomb constant over the square of the mass are quantities of one kind, in the ratio of the two pure numbers. Read for the column of limits, each form gives the product of the two limits: the quantum of action times the speed of light equals Newton's constant times the square of the mass over gravity's strength, and equals the Coulomb constant times the square of the charge over the strength of light. The product of the two limits is fixed by the two comparisons, and that fixed product is the conjugate relation of Section II in the table. Their ratio, the quantum of action over the speed of light, has the unit kilogram meter, so its number is set by the choice of the units of mass and length, which the international system makes by fixing the values of h and c (Section II).

No combination of the constants of one row, or of the constants of one column, is a pure number, so the table is fixed only as a whole. The six constants with units carry four kinds of unit (mass, length, time and charge), so by Buckingham's theorem they hold exactly two pure numbers, the same two that Box III-2 counts with the Coulomb constant taken into the unit of charge. In atomic units, where the electron's mass, its charge, the quantum of action and the Coulomb constant are each one, the speed of light is one over the strength of light, and Newton's constant is gravity's strength on the electron over the strength of light.

The rows are not a second set of columns. Each row's third member is a limit, while its first two are constants that relate measures, and no combination of a row's own constants is a pure number. A row read as a column would set the quantum of action equal to the electron's charge times the

square of its mass over the two pure numbers, and the speed of light equal to the Coulomb constant times the square of Newton's constant over the same two; each sets a quantity equal to one of other units, so each would change with the choice of units. What holds in the place of a row relation is the product of the two limits.

The two pure numbers take their values in two ways. Gravity's strength on the electron is carried by the count: the span from the electron's mass to the reduced Planck mass is minus one half of the base-two logarithm of 8π times gravity's strength on the electron, 72.0132 halvings, and the count of 72 with the first-order remainder $11/(80\pi^2)$ is set beside it in Chapter 5. The strength of light is a measured input, and Section VI gives the reason no concept of comparison fixes its value.

Box III-3. The table of the electron and the vacuum.

Premise 1. A pure number compares two things of one kind.

Premise 2. In three directions of space, energy between two charges or two masses falls as one over distance (Gauss's law), so energy times distance is a strength, the same at every distance.

Premise 3. The quantum of action times the speed of light is the strength of one quantum; the two are the transcendent and the omniscient limits, which meet only through the act (Section II).

Step 1. Gravity's strength on the electron is Newton's constant times the square of the electron's mass over the strength of one quantum; the strength of light is the Coulomb constant times the square of the electron's charge over the strength of one quantum; the unit is the strength of one quantum over itself.

Step 2. Content enters squared (two electrons), context once (one context), each limit once (one quantum); each numerical factor is one, with the 4π inside the context constant.

Step 3. The product of the two limits is fixed by either comparison; their ratio, in kilogram meters, is set by the units of mass and length.

Result. The table is two columns of comparison and one column of limits; it holds exactly two pure numbers, gravity's strength on the electron (carried by the count of 72, Chapter 5) and the strength of light (a measured input, Section VI).

IV. The Strength of Light as the Atom Registers It

α_{em} is one comparison, and every one of six kinds of measurement (force, inertia, time, space, probability and possibility) registers it (Table IV-1; note 3). The force, inertia and space readings use the electron's *own length*, its reduced Compton wavelength $\lambda\!\!\!^{-}_e = \hbar/(m_e c)$.

Table IV-1. The six readings of α_{em} and the exactness of each.

Kind of measurement	**What equals α_{em}**	**Exactness**
Force	The force between two electrons at $\lambda\!\!\!^{-}_e$, over the unit of force built from m_e, $\lambda\!\!\!^{-}_e$ and the time $\lambda\!\!\!^{-}_e/c$	Exact, by the definition of the comparison
Inertia (energy)	The energy between two electron charges at $\lambda\!\!\!^{-}_e$, over the rest energy $m_e c^2$	Exact, by the definition of the comparison
Time	The speed of hydrogen's ground-state electron over c	Exact in Schrödinger's account of hydrogen; finer accounts add terms of order α_{em}^2
Space	The step between the electron's three lengths	Exact, given the definitions of the three lengths
Probability	The chance that a charge emits or absorbs one more photon in an encounter	At leading order (about α_{em} times a factor of order one)
Possibility	The inverse, $1/\alpha_{em}$ (about 137), is the largest point charge to which Dirac's equation binds an electron in its lowest state	Exact, as a property of Dirac's equation

An argument that fixes α_{em} from the standpoint of one kind of measurement fixes it for all six, and it has to agree with each. The six readings are therefore a standard for any derivation of α_{em}.

Probability reads α_{em} as a chance, and possibility reads it as a range, $1/\alpha_{em}$; the two readings multiply to one. The pair of probability and possibility thus registers α_{em} as a chance within a range whose size is its inverse.

The size of hydrogen's least state, the *Bohr radius* a_0, is reached by three routes, kept side by side (Box IV-1).

The first route balances two costs. A charge held within an extent r gains from the hold of light in proportion to $\alpha_{em}\hbar c/r$. Confining it within r costs, through the widening of its possible motion, in proportion to $\hbar^2/(mr^2)$. Minimizing the sum gives $r = \hbar/(mc\alpha_{em})$, the own length over α_{em}, up to a number set by mathematics that this route does not fix. The momentum is then about $mc\alpha_{em}$, so the electron's speed is about $\alpha_{em}c$: the atom shows α_{em} as the ratio of two motions, the least motion of the content over the speed of light, a ratio of like to like.

The second route counts units. With one pure number α_{em} among the three units, each of the electron's lengths is its own length times a power of α_{em}, and the unit that a length omits fixes the power. Two definitions and one result of quantum mechanics fix every numerical factor at one. The own length is taken with $\hbar$ (phase in radians). The length of the charge, the *classical electron radius* r_e, is defined by the equality of energies $e^2/(4\pi\varepsilon_0 r_e) = m_e c^2$. The quantum mechanics of non-relativistic hydrogen gives $a_0 = \lambda\!\!\!^{-}_e/\alpha_{em}$ exactly.

The third route is the standard result. The Schrödinger equation for hydrogen gives the ground-state radius $a_0 = \hbar/(m_e c\alpha_{em})$, about 0.529×10^{-10} m: with the own length read as $\hbar/(m_e c)$, the number that the first route does not fix is exactly one.

Box IV-1. Three routes to the Bohr radius.

Route 1 (*the balance of costs*). $E(r) = \hbar^2/(mr^2) - \alpha_{em}\hbar c/r$, each term fixed in proportion. The least E lies at $r = \hbar/(mc\alpha_{em})$ up to a number of mathematics, which this route does not compute. Then the momentum is about $\hbar/r = mc\alpha_{em}$ and the speed about $\alpha_{em}c$.

Route 2 (*the counting of units*). Premises: one pure number α_{em}; the own length $\lambda\!\!\!^{-}_e = \hbar/(m_e c)$; the classical radius defined by $e^2/(4\pi\varepsilon_0 r_e) = m_e c^2$; the result $a_0 = \lambda\!\!\!^{-}_e/\alpha_{em}$ of non-relativistic quantum mechanics. Then $r_e = \alpha_{em}\lambda\!\!\!^{-}_e$ and $\lambda\!\!\!^{-}_e = \alpha_{em}a_0$.

Route 3 (*the standard result*). The Schrödinger ground state of hydrogen has $a_0 = \hbar/(m_e c\alpha_{em})$: the number of mathematics is one.

Result. The three routes meet on $a_0 = (\hbar/(m_e c))/\alpha_{em}$. The first gives the form up to a number of mathematics; the second fixes the powers by counting units and takes the factor one from the quantum mechanics of hydrogen as a premise; the third is the solution of Schrödinger's equation, in which the number is one.

The electron's three lengths stand in exact steps of α_{em}: $r_e : \lambda\!\!\!^{-}_e : a_0 = \alpha_{em}^2 : \alpha_{em} : 1$. Each length omits one of the three units. The classical radius, about 2.818 fm, is built from charge and light and omits $\hbar$. The own length, about 386.2 fm, is built from $\hbar$ and light and omits charge. The Bohr radius, about 52,918 fm, is built from charge and $\hbar$ and omits light. The smallness of α_{em} is the width of this triad: nearly nineteen thousand ($1/\alpha_{em}^2$) across the whole.

A flow that carries the electron's spin has a length of its own. A flow of mass m_e circulating at radius r, at a speed that cannot exceed c, has angular momentum at most $m_e cr$; to carry the spin $\hbar/2$ it needs $r \geq \hbar/(2m_e c) = 1.93 \times 10^{-13}$ m, half the own length. Ohanian (1986) places the circulating flow of the Dirac field at about the Compton wavelength $h/(m_e c) = 2.43 \times 10^{-12}$ m. Both are radii of a flow that carries the spin; the own length of the triad is $\hbar/(m_e c)$, by its definition above.

Physics rejects spin as a small body rotating: a body as small as the classical electron radius would need its surface to move far faster than light. One standing reading in physics treats spin instead as a circulating flow within the field of the content itself, and Ohanian's flow is that reading. To carry the spin at no more than the speed of light, the flow needs a radius of at least $\hbar/(2m_e c)$, half the electron's own length, as the bound above states. That reading is the intrinsic circulation of Section II: circulation intrinsic to the content, not the rotation of a part of it.

The scale of the atom depends on α_{em}. When an electron and a positron at rest vanish into two photons, conservation of energy and momentum gives each photon one electron rest energy, 511 keV (less the binding energy of the pair). Hydrogen's ionization energy is $(\alpha_{em}{}^2/2)m_e c^2$, and the photon energies between levels n and k are $(\alpha_{em}{}^2/2)m_e c^2(1/k^2 - 1/n^2)$, with small corrections for recoil and fine structure. The rational part $1/n^2$ is fixed by the symmetry of the bound pair, and the scale by $\alpha_{em}{}^2$. So the energy of a photon inside hydrogen is $\alpha_{em}{}^2$ times rational numbers of the levels times the electron's rest energy.

A photon's energy has no fixed ratio to the electron's mass. The energy of a photon is a property of its state (its frequency, as read in a given frame), not of its kind, and two photons of one kind differ in energy without limit. A ratio exists only where something else fixes a state's energy: conservation, in annihilation, and binding, in the atom.

The number the atom does not fix, the strength of light α_{em}, is the same kind of number as the one the proton does not fix, the strength of color α_s (color being the charge of the strong force; Chapter 6): in each case, the strength of a force at a stated resolution.

Coulomb's law takes the form $F = \alpha_{em}\hbar c/r^2$. The hold falls as $1/r^2$ because the flux spreads over the sphere; that α_{em} joins $\hbar c$ as a product is a premise of the form.

An atom built with a muon or a tau in place of the electron has a least extent $\hbar/(mc\alpha_{em})$ smaller in proportion to $1/m$; for each kind of atom, that extent is the same everywhere in the instance.

The derived scalings agree with hydrogen as measured (CODATA 2022): the Bohr radius, about 0.529×10^{-10} m; the electron's speed, $\alpha_{em} c$; and the binding energy, about 13.6 eV, which the form gives as $\alpha_{em}{}^2 m_e c^2/2 = (1/137.036)^2 \times 511$ keV$/2 = 13.6$ eV. They agree because they are the forms of Schrödinger's solution, whose number for the Bohr radius is exactly one, and because the measured α_{em} and m_e are their only inputs. The zoo fixes the photon's mass at zero by structure, so on the zoo's account Coulomb's inverse square is exact. Measurement places the photon's mass below about 10^{-18} eV (the Particle Data Group's limit), and the inverse square holds to high precision: every measured bound agrees with it.

The layers of hydrogen's spectrum stand beside measurement. Levels of the same principal number n and orbital number l split by total angular momentum at relative order $\alpha_{em}{}^2$ (the *fine structure*), as derived. The measured shift between the 2S and the 2P levels (the levels with $n = 2$ and $l = 0$ and 1), about 1058 MHz (Lamb and Retherford, 1947), is of relative order $\alpha_{em}{}^3$ times a logarithm. Set beside it is the part of that shift this book derives, the vacuum-polarization part (the Uehling term, Chapter 2, Section IV), which falls on the S states, the states whose wave reaches the proton. That part lowers the 2S level, and it is the smaller part of the measured shift: the larger part, the self-energy, carries the logarithm. Physics describes the cause of the measured shift as the fluctuations of the electromagnetic field in empty space.

The electron's magnetic moment registers α_{em} as well. The *g-factor* is the ratio of a particle's magnetic moment to its spin, in the natural unit; at lowest order the electron's is 2, Dirac's value (1928). The field the electron sends out acts back on the electron, and this is taken as a premise. It shifts g above 2: the *anomalous moment* $a_e = (g - 2)/2$ is $\alpha_{em}/(2\pi)$ at first order, Schwinger's term (1948). The corrections come through the strength of the coupling, not through motion, so they come in all powers of α_{em}, not only in even ones.

At first order, with $\alpha_{em} = 1/137.036$ at zero momentum, $\alpha_{em}/(2\pi) = 0.0011614$. The measured value is $a_e = 0.00115965218059(13)$ (Fan, Myers, Sukra and Gabrielse, 2023). The first-order term stands 0.15 percent above it; the remainder is the second-order term of the standard series, which this derivation does not state.

V. The Strength of Light as Measured

Measured, the strength of light is $1/\alpha_{em} = 137.035999177$, with uncertainty 21 in the last two digits (2022 recommended values). Eddington's whole number 137 is excluded, because $1/\alpha_{em}$ is not a whole number. Wyler's value 137.03608 stands 0.000080823 above the measured value, a pull of +3849 standard deviations.

Five routes determine α_{em}, and they are kept side by side (Table V-1).

Table V-1. Determinations of α_{em}, each with what it rests on.

Determination	**What it rests on**	**What is stated of its value**
Recoil of caesium atoms (2018)	A measurement of atomic recoil	Differs from the rubidium determination by more than five times their combined stated uncertainty
Recoil of rubidium atoms (2020)	A measurement of atomic recoil	Differs from the caesium determination as above
The electron's anomalous moment	The measured $a_e = 0.00115965218059(13)$ (Fan, Myers, Sukra and Gabrielse, 2023), inverted through the series of quantum electrodynamics (the quantum theory of light and charge), computed to fifth order in α_{em}	α_{em} to 0.1 parts per billion; $g - 2$ measured to about one part in 10^{13} (2023)
The step of sideways passing against the impedance of empty space	The identity $R_K/Z_0 = 1/(2\alpha_{em})$ (Box II-1)	A measurement of it is a measurement of α_{em} (Box II-1)
The constants of empty space	The identity $\mu_0 = 2\alpha_{em}h/(e^2c)$ with e, c and h exact (Box II-1)	A measurement of it is a measurement of α_{em} (Box II-1)

The two recoil determinations differ by more than five times their combined stated uncertainty; the difference is a measured fact. The electron's $g - 2$ gives a third determination. Measured to about one part in 10^{13} (2023), it agrees with quantum electrodynamics to the precision with which α_{em} is known.

The series of quantum electrodynamics for the electron's anomalous moment is a function of α_{em}, and it agrees with the measured a_e to all measured digits. The comparison is therefore used in reverse: the measured a_e fixes α_{em}, to 0.1 parts per billion. Every calculation of $g - 2$, the muon's included, takes α_{em} from measurement; the muon's is carried in Chapter 2.

Box V-1. The measured value against two formulas.

Measured. $1/\alpha_{em} = 137.035999177 \pm 0.000000021$ (2022 recommended values).

Eddington. 137, a whole number; the measured value is not a whole number, so 137 is excluded.

Wyler. $137.03608 - 137.035999177 = 0.000080823$; divided by 0.000000021, the pull is +3849.

VI. Why the Strength of Light Stays Measured

The symmetry of light fixes the kinds and leaves one number. Light acts by turning the phase of charged things, and the turns of a phase form a circle, the group U(1), which is light's *gauge group* (the group of turns of phase by which it acts). For a circle group of turns, a theorem about compact groups (groups that are closed and bounded, as the circle is) leaves exactly one positive number undetermined by the symmetry: the *coupling*, the strength with which light acts. Everything the symmetry fixes (the circle of phases, the whole-number charges, the conservation of charge, every symmetry of the laws) is the same at every value of α_{em}.

The *comparison structure* of a world is its kinds of finding, which contents recur across contexts, and its possible verdicts. That structure is the same at every value of α_{em}: the symmetry fixes the kinds, the charges and the symmetries apart from α_{em}, and on a finite lattice (a finite grid of places on which arrangements are counted) the possible, the impossible and the certain arrangements do not change with it. The reason is a bound on the weights that the lattice gives its arrangements. In the lattice form of a gauge theory (Wilson 1974), the field of light on each link between neighboring places is an

angle, a turn of phase, and charge sits on the places. The weight of an arrangement carries the strength of light through one factor only: the exponential of minus the action of the links, divided by the square of the coupling. The action of the links has one term for each elementary square of links; each term is bounded, because it depends only on angles; and a finite lattice has finitely many squares, so the action of the links lies between zero and a fixed largest value. So the weights that two finite, nonzero strengths of light give one arrangement stay within fixed positive multiples of each other: an arrangement with some weight at one strength has some weight at every other, and an arrangement with no weight at one strength has none at any. The two strengths therefore agree exactly on which arrangements are possible, impossible and certain. Varying α_{em} changes only how often and how strongly findings occur, which the world, and not comparison, settles. One thing is independent of another when differences in the first produce no necessary difference in the second. So α_{em}, as a measure with a definite numerical value, is independent of comparison.

The step that carries the load, that the comparison structure is the same at every value of α_{em}, also fixes how far the independence of α_{em} extends. A valid argument keeps its verdicts wherever its premises hold, and premises drawn from the comparison structure hold at every value of α_{em}, since the structure is the same at every value. An argument from the comparison structure that ended at one value of α_{em} would end in a verdict true in the world of that value and false in the worlds of every other value, and no valid argument does that. So no argument from the comparison structure, of any length, fixes α_{em}. The result that no argument from the comparison structure fixes α_{em} covers every argument whose premises are drawn from the comparison structure as the argument examined it: the kinds, the symmetries and the possible verdicts.

The *circle's rule* closes the argument: a value is fixed by concepts alone only when one form supplies all the terms that the value relates. The rule takes its name from the circle, where a concept does fix a value. The concept of a circle in a flat plane relates the circle's circumference to its diameter, and both terms come from that one figure, so the concept alone fixes their ratio, π. A figure whose circumference stands to its diameter in any other ratio is not a circle: a

different value breaks the concept. The flat plane is meant because the plane supplies no length of its own. A curved surface, such as a sphere, supplies one, its radius of curvature (for a sphere, its radius, the length that sets how sharply the surface bends); on the sphere the ratio of a circle's circumference to its diameter, both measured along the surface, is less than π, and it depends on the circle's size measured against that radius, coming nearer to π the smaller the circle is. There the circle's concept no longer supplies every term of the ratio, and, as the rule says, it no longer fixes the ratio. α_{em}'s terms stand on all three pairs of intrinsics, and no single form among the intrinsics supplies them together. With a different value, α_{em} still relates the same intrinsics in the same way, so the concept of α_{em} stays whole under a different value, whereas the concept of the circle breaks under a different ratio. Whatever could fix α_{em} is its own concept, the concept of something else in the structure, or several concepts together; by the circle's rule none of these fixes it. The independence of α_{em} is thus derived for the comparisons the argument examined, with the circle's rule as its stated hypothesis.

The comparison of worlds is a test, and it applies to any constant: vary the constant, and compare the comparison structures of worlds that differ only in its value. Three results are possible. If the structure stays the same over a range of values, the constant is independent of comparison over that range; that is the result for α_{em}. If the structure changes at isolated values only, those values are excluded and the rest stay free; excluding some values does not fix one. If a world has the comparison structure that the concept of the constant requires at one value of the constant only, and at no other, the constant is fixed by comparison, and the test also says how to compute it: the value is the one at which a world has that structure, as π is the one ratio at which a figure in the flat plane is a circle. A procedure that finds the value at which a world has that structure is a procedure for the constant's digits, as there are procedures for the digits of π. The first two results end at independence and the third at derivation, the two ends of Section I.

Box VI-1. The independence of α_{em}.

Step 1. The gauge group of light is the circle U(1); for a compact circle group the symmetry leaves exactly one positive number, the coupling, undetermined.

Step 2. Charge quantization, charge conservation and the symmetries of the laws are the same at every value of α_{em}.

Step 3. The kinds, the symmetries and the possible verdicts are the same at every value of α_{em}; varying α_{em} leaves the comparison structure the same. On a finite lattice the strength enters the weight of an arrangement only through the action of the links, which is bounded, so the weights that two strengths give one arrangement stay within fixed positive multiples of each other, and the possible, the impossible and the certain arrangements are the same at every value. This is the step that carries the load.

Step 4. By the definition of independence (differences in the first produce no necessary difference in the second), α_{em} as a measure with a definite value is independent of comparison. A valid argument keeps its verdicts wherever its premises hold, and premises drawn from the comparison structure hold at every value of α_{em} (Step 3); so no argument from the comparison structure, of any length, ends at one value of α_{em}.

Step 5. Hypothesis (the circle's rule): a value is fixed by concepts alone only when one form supplies all its terms. α_{em}'s terms stand on all three pairs, and no single form supplies them together.

Result. No concept or set of concepts fixes the value of α_{em}; its independence is derived for the comparisons the argument examined, under the circle's rule.

No size fixes the strengths. The *internal space* on which the kinds are read is the space of states of the color label and the doublet label (the book of kinds, Chapter 3); in its *content instance* it is a space of states with no size and no radius (Chapter 3, Section III). There is no size to fix, so the couplings cannot be derived from a size. The zoo states the same: the structure gives no computation of the values of the strengths, because the finite space of states on which it rests has no size.

At the level of the zoo (the level of found things, as distinct from the intrinsics) the strengths are three, one for each kind of turning: g' for the phase (hypercharge, $U(1)_Y$), g for the set of two (weak isospin, $SU(2)_L$), and g_s for the set of three (color, $SU(3)_c$). α_{em} is one function of two of them. The strength of the electric field is $e = g \sin \theta_W = g' \cos \theta_W$, where θ_W is the *weak angle*, the angle by which the hypercharge and weak-isospin fields mix into the photon and the Z boson, and $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$ relates e to c and $\hbar$; so α_{em} is fixed by e together with c and $\hbar$, and it is not identical to e. The strength of the other combination of g and g', the weak angle between them, and the strength of color are further values at the level of the zoo. The count of three strengths does not reopen the four kinds of relation of Section II, which may be the complete set.

The independence of the three couplings is derived against two routes by which a principle could relate or equate them: interchangeability of the three kinds, and a crossing between basic sets. That no principle of another kind could relate or equate them (for example, a relation of the kind that ties the ratio of the masses of the W and Z bosons, the carriers of the weak force, to the weak angle) is not shown by that argument; the independence is derived against those two routes only.

No argued relation fixes α_{em}, and the measured strengths are not derived.

α_{em} therefore stands in this book as a measured input. It is a measured input, a premise that no argument of this book fixes: a pure number with a part from the continuum, as every value containing π has, which measurement supplies ($1/\alpha_{em} = 137.035999177$).

This book also holds that no pure number among the particles is a *brute constant* (a number with no reason). Every pure number not yet derived carries the question that settles it: the law that fixes it, or the proof that no law does (Section I). What remains of a pure number after the unit, the setting and the history are removed is fixed by its law, or it is shown independent of comparison, as α_{em} is.

VII. The Settings, and the Count of the Numbers

A setting is fixed by stating it: a strength is stated at a given energy, and the statement fixes the setting. A *running* number is one whose value changes with the energy at which it is read; the scale at which it is read, and the *scheme* (the convention by which it is defined at that scale), are settings. A two-valued choice is a setting when swapping its values is a symmetry of the whole description (Section I).

A comparison's strength is read at the mass of the thing that the comparison is made against, so the strength of light is read at the proton's mass (Chapter 2).

The Standard Model as first written has nineteen adjustable parameters: nine masses of charged fermions, three quark mixing angles and one phase, three strengths of force, the Higgs mass and its vacuum value, and the strong force's angle $\bar{\theta}$; $9 + 3 + 1 + 3 + 2 + 1 = 19$. The masses and mixing of the neutrinos add seven (three masses, three angles, one phase), for 26. The electron's mass is one of the nineteen; after it, eighteen remain.

The seven added numbers are those of a *Dirac* neutrino, whose mass joins two distinct components of one state, as every mass does; it has no *Majorana* mass, a mass that would join a state to its own reversal (the zoo; the book of kinds, Chapter 6, Section I; The Branches of the Kinds, Chapter 1, Sections II and VII).

Some of these numbers are fixed by structure. The electric charges and the hypercharges are fixed (the book of kinds, Chapter 5). The ratio $\rho = m_W{}^2/(m_Z{}^2 \cos^2\theta_W)$ is one, that is, $m_W = m_Z \cos\theta_W$. The strong force's own angle is zero (the zoo, Level 9, Section V, and note 31). The number of generations is three, by the rotation argument (the book of kinds, Chapter 7, Section I).

$\bar{\theta}$ is the strong force's own angle plus a phase that comes from the quark masses. The structure sets the first to zero (The Branches of the Kinds, Chapter 3, Section V). The second, the phase from the quark masses, is carried as zero rather than π; that is a measured agreement, since measurement places $\bar{\theta}$ below about one part in ten billion. This book carries $\bar{\theta} = 0$.

Counted as dimensionful inputs, the electron's mass is the one input: every other mass, v among them, is read in its unit. Of the strengths, α_{em} and α_s are measured inputs, and the weak angle at the Z mass is a measured agreement (Chapters 3 and 4). What measurement supplies to the masses and the mixings is set out in two accounts in The Measures of the Kinds (Chapter 13, Section X). Among the pure numbers, measured information also enters through constants fixed by agreement with measurement; each number such a constant touches is a measured agreement, and Table VII-1 gives its standing.

Of the inputs, α_s is the one that Chapter 6 joins to the ratio of the proton's mass to the electron's, m_p/m_e, returning each from the other. The settings beside the inputs are non-measures, fixed by stating them.

Chapter 2 states a relation that lands on measurement, $1/\alpha_{em}(m_p) = 3\pi(1/\alpha_2 - 1/\alpha_1)(E)$, with the gap read at the end of the measured span E; read the other way, it returns α_{em} at the proton's mass from α_2 and α_1. Chapter 6 returns α_s and m_p/m_e each from the other.

Table VII-1 sets out the numbers of the Standard Model that this volume carries, each with its chapter and its standing there. The three strengths of force are written as α_{em}, the weak angle θ_W and α_s.

Table VII-1. The numbers of the Standard Model that this volume carries, with the chapter that carries each and its standing there.

Number	How many	Chapter	Standing there
Electron mass m_e	1	1	Measured input: the unit.
Strength of light α_{em}	1	1 and 2	Measured input.
Weak angle θ_W	1	3 and 4	$\sin^2\theta_W = 3/8$ at the equal traces (the equal weights the structure gives the three forces over the states) is the book of kinds' value (the book of kinds, Chapter 5); its value at the Z mass is a measured agreement, read with light's relation at

Number	How many	Chapter	Standing there
			the proton's mass (Chapter 4, Section IV) and restating the place of the first crossing (Chapter 3).
Strength of color α_s	1	2 and 6	Measured input; Chapter 6 gives it from m_p/m_e, and m_p/m_e from it.
Strong angle $\bar{\theta}$	1	1	Zero: the strong force's own angle by structure; the phase from the quark masses, carried as zero, is a measured agreement.
Carried here	5		Five of the twenty-six; the other twenty-one, the masses and the mixings, are carried by The Measures of the Kinds, Chapters 7 to 13, whose Table X-3 sets out all twenty-six.

Summary

Every attribute of a particle is a single measure, a law or a non-measure, and every number is fixed by structure, a value of the instance, or a unit or convention. Every number of the Standard Model is to reach one of two ends: derivation through its law, or a proof that it stands at a join with no law, after which it is carried as a reference read once. No parameter is written down that no measurement and no reasoning could ever reach: by the principle of identity, such a parameter is nothing.

Three units, $\hbar$, c and G_N, make every magnitude a pure number, and α_{em} is the first of the pure numbers that remain. None of the constants that define the international units is fixed by comparison, and with e, c and h fixed exactly, the constants of empty space carry α_{em} and are measured.

The electron's mass is the unit of mass and the one dimensionful measured input of the particle side, reached by three routes: from the span of measure, from dimension, and from the six questions (the third set out with the ladder, Chapter 7, Section II). Given gravity, the electron carries two pure

numbers, α_{em} and its place below the Planck mass; its place below the reduced Planck mass is carried on the particle side by the count of 72 halvings, whose identification with the measured span is tested (Chapter 5).

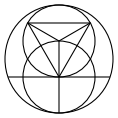
α_{em} is one comparison registered by all six kinds of measurement, and its six readings are a standard for any derivation of it. Measured, $1/\alpha_{em} = 137.035999177(21)$ (2022 recommended values), and five routes that determine it are kept side by side. Its independence of comparison is derived for the comparisons the argument examined, with the circle's rule as hypothesis, and it stands in this book as a measured input, which measurement supplies. No argument from the comparison structure, of any length, fixes its value, since that structure is the same at every value of α_{em}.

The scale and the scheme at which a running number is read are settings. The Standard Model has nineteen adjustable parameters, twenty-six with the neutrinos'; after the electron's mass, eighteen of the nineteen remain, and Table VII-1 gives the chapter and the standing of each that this volume carries. The two accounts of what measurement supplies to the masses and the mixings are set out in Chapter 13, Section X.

The question each measured agreement of this chapter carries is why it lands. The phase from the quark masses, carried as zero and measured below one part in ten billion, carries it in this form: why do the quark masses add no angle to the strong force's own angle, whose zero the structure sets?

KEY CONCEPTS

- **Span of measure.** A single measure (the omniscient), a law (the immanent) and a non-measure (the transcendent), in the frame of what is measured about a particle.
- **Three tiers.** Fixed by structure; a value of the instance; a unit or convention. Every number falls in one of them.
- **Two ends.** Derivation through a law, or a proof of independence; a value shown not derivable is closed under reason.
- **The unit.** The electron's mass: the one dimensionful measured input of the particle side, never derived.
- **Six readings of α_{em}.** Each of six kinds of measurement (force, inertia, time, space, probability, possibility) registers the one comparison.
- **Circle's rule.** A value is fixed by concepts alone only when one form supplies all its terms.
- **Principle of identity.** That which is indistinguishable must be the same; so a parameter that no measurement and no reasoning could ever reach is nothing, and none is written down (Section I).
- **The comparison of worlds as a test.** Vary a constant and compare the comparison structures: the same structure over a range, independent over that range; changes at isolated values only, those values excluded and the rest free; the structure at one value only, fixed by comparison and computable (Section VI).
- **Setting.** Fixed by stating it: the scale and scheme at which a running number is read.
- **The table of the electron and the vacuum.** Three rows (the electron's constants, empty space's constants, the pure numbers) and three columns (gravitational, electric, limits); two columns of comparison and one column of limits (Section III).
- **The product of the two limits.** The quantum of action times the speed of light, fixed by either comparison; their ratio is set by the units of mass and length (Section III).



Chapter 2

The Couplings and Their Running

CHAPTER OVERVIEW

Three gauge couplings set how strongly the three ways of comparison act: color, the weak doublet and the phase. Light's coupling, the fine-structure constant, is a combination of the weak and phase couplings. Their values at any one resolution are measured; this chapter states where each is measured and at what mass each is read. How the couplings change with resolution, their running, is fixed at first order by the *closed list of contents*: three generations of every kind of matter particle, and the one Higgs doublet.

The three one-loop rates, $b_1 = 41/10$, $b_2 = -19/6$ and $b_3 = -7$, are reached by two routes, the count of the zoo's contents and the standard count of the sources, and color's rate also by a third, its writing in the *kernel integers* ($c_1 = 3$, $n_C = 2$, and the triangle's sides O = 3, N = 4, H = 5). The rates change only at thresholds, the masses of the contents and of the W and Z bosons, and the fermions add the same amount to every rate, so they never separate the forces. Light's own rate in the unbroken phase is 11/3.

Three-loop running reproduces the published couplings at the Planck scale, and the matching at the top is the largest remaining uncertainty. Light's inverse strength is the weighted mean of the two electroweak inverse strengths, with the weights 5 and 3, at every scale. One relation lands on measurement at two loops in both matchings and at three loops in one: light's inverse strength at the proton's mass is 3π times the gap between the two electroweak inverse strengths at the end of the measured span; across the two matchings at two and three loops the pull of 3π lies between -2.04 and $+1.00$. Five candidate relations among the couplings do not hold, and each is stated with its pull or its miss.

I. What a Coupling Is

A *coupling* is the strength with which comparison performs one of its three transformations on a state, at a given resolution. The chance of the transformation in an encounter grows with the coupling, roughly as its square. Each coupling is written g; its *strength* is $\alpha = g^2/(4\pi)$, a pure number, and its *inverse strength* is $1/\alpha$.

The three transformations are the three *ways*, and each way has one gauge factor (the book of kinds, Chapter 4, Sections I to III). Color is SU(3), with coupling g_3 and strength α_3, also written α_s. The weak doublet is SU(2), with coupling g_2 and strength α_2. The phase is U(1), the hypercharge, with coupling g'.

The hypercharge coupling has two normalizations, and this chapter uses both, each named where it is used. In the normalization Y, with electric charge $Q = T_3 + Y$, the strength is $\alpha_Y = g'^2/(4\pi)$. In the *unification normalization* the strength is $\alpha_1 = (5/3)\,\alpha_Y$. Where no normalization is named, α_1 means the unification normalization.

The *resolution* is the separation at which a comparison is made, or equivalently the energy (a mass, in units where the speed of light is one) that reaches that separation. The *running* of a coupling is its change with resolution. Running is a change of the resolution of the lens through which the three ways are seen; it is not a change of the *kernel triangle*, the right triangle with legs 3 and 4 and hypotenuse 5 from which the book of kinds draws its whole numbers, which stays fixed (the book of kinds, Chapter 2).

The running is often described as forces changing with energy. In the picture of the lens the triangle does not change; what changes, as the energy is lowered, is the lens through which the triangle is observed, the resolution of the probe. At the finest resolution the triangle is seen directly: no force stands before another, and the equal traces give $\sin^2\theta_W = 3/8$ (Section VII). At lower resolution the lens deforms the image, and the three forces appear to separate.

At low energy the strengths stand in the order strong, then weak, then electromagnetic. The zoo (The Story of the Particles, the authority on which particles exist and of what kind) fixes this order from what each force acts on and from whether its carriers act on one another. The order does not fix the values; it leaves them to the chain of calculation, and the zoo states that the values of the three strengths, at any scale, are read from measurement.

The strengths of the three ways are the first numbers of the calculation to compute: force comes first. The couplings of the Higgs field belong to the calculation of mass, which comes after. Gravity enters as the scale of the arena,

not as a strength among the three (The Branches of the Kinds, Chapter 4, Section III). The strength of the one force at the finest resolution, where the three ways are not separated (Section VII), is a number that joins force to probability, since the chance of a transformation grows with its strength.

II. What Is Measured and Where It Is Read

Light's coupling found from afar is the electron's own number: its value at the electron's mass and below. Its value at zero momentum, about 1/137.036, is the photon's coupling as atomic measurements find it. It is a single definite number because no charged particle is lighter than the electron: the zoo, which is the authority on which particles exist, states that the electron is the lightest charged particle, and stable; that nothing among the charged leptons lies below it is structural, and that nothing of another kind carrying charge lies below it is measured (The Branches of the Kinds, Chapter 2, Section IV). Beyond the electron's own length the value does not change; at shorter distance it grows, to about 1/128 at the mass of the Z boson.

The electron's own length is its *reduced Compton wavelength*, $\hbar/(m_e c)$. Inside that length charged contents cover and the coupling changes; beyond it no charged content remains to cover, so the coupling no longer changes.

Two bounds hold at every resolution. Light's strength is positive and less than one, $0 < \alpha_{em} < 1$. It is also less than the weak strength, $\alpha_{em} < \alpha_2$, because $\alpha_{em} = \alpha_2 \sin^2\theta_W$ and the *weak mixing angle* θ_W has $\sin^2\theta_W < 1$.

The photon is the combination of the weak and phase carriers that the *Higgs condensate*, the value the Higgs field keeps throughout the vacuum, leaves unbroken (the book of kinds, Chapter 5, Section VI). Its inverse strength is the sum $1/\alpha_{em} = 1/\alpha_2 + (5/3)(1/\alpha_1)$, the standard electroweak matching with the hypercharge in the unification normalization. The factor 5/3 is H/O, the hypotenuse of the kernel triangle over its short leg (the book of kinds, Chapter 5, Section V), so the sum is also $1/\alpha_{em} = 1/\alpha_2 + (H/O)(1/\alpha_1)$.

From zero momentum to the mass of the Z boson, $1/\alpha_{em}$ falls by about 8 to 9 units, from 137.036 to about 128. About 4.3 of those units come from the charged leptons, the electron, the muon and the tau. The remainder comes from the quarks and is computed from measured data or from lattice calculation, both of which take color's strength as given. So light's coupling at the electron depends on every charged mass and on the strength of color.

The same two kinds of input serve the hadronic part of the muon's anomalous magnetic moment: measured cross sections of electron and positron annihilating into hadrons, and lattice calculation. The cross section measured by the CMD-3 experiment at VEPP-2000 in Novosibirsk, for annihilation into two charged pions, is higher than the earlier BaBar and KLOE measurements. It gives a hadronic vacuum polarization consistent with the lattice value and in tension with BaBar and KLOE. The tension among the electron and positron data sets is a significant issue, and it is unresolved. Lattice calculations are independent of those data altogether, and they now provide the primary input to the Standard Model prediction of the muon's moment.

The Brookhaven experiment E821 (2006) measured the muon's anomalous moment 3.7 standard deviations from the Standard Model prediction of that time. The final result of the Fermilab experiment E989 (June 2025) is $a_\mu = 116\,592\,070.5\ (11.4)(9.1) \times 10^{-11}$, statistical and systematic uncertainties in that order, a precision of 127 parts per billion and four times finer than Brookhaven's. The measured moment agrees with the most recent lattice-based prediction (the Muon g − 2 Theory Initiative, 2025).

A comparison's strength is read at the mass of what the comparison is made against. In a relation that reads the strength of a comparison as a whole, the strength is read at that mass. Light is the comparison made against the proton: in the zoo's opening triple the photon is the act of comparing, the electron the thing compared and the proton the thing compared against. So light's strength in a relation is read at the proton's mass, m_p.

The *MS-bar scheme* is the standard convention for removing the infinite parts of loop calculations at a chosen mass; a strength stated in it is written with a hat, $\hat{\alpha}$, and its value depends on the mass at which it is read.

In the MS-bar scheme, light's inverse strength is 134.07 ± 0.12 at 1.035 GeV, consistent with the crossing relation, the formula of Chapter 3 that places, in halvings, the scales at which two inverse strengths are equal.

Table II-1 states the measured values this chapter uses, with the uncertainty and the source that each carries. For each quantity the value is the most recent one in the source named; one row ($\Delta\alpha_{had}$) carries no uncertainty, and one value, light's inverse strength at 1.035 GeV, is given with no experiment named.

Table II-1. Measured values used in this chapter

Quantity	Value	Source as named
Light's inverse strength at zero momentum, $1/\alpha_{em}(0)$	137.035999177, uncertainty 21 in the last two digits	2022 recommended values (CODATA 2022)
Light's MS-bar inverse strength at the Z mass, $1/\hat{\alpha}_{em}(M_Z)$	127.951 ± 0.009	Particle Data Group 2022, electroweak review
Weak mixing angle at the Z mass, MS-bar, $\sin^2\theta_W(M_Z)$	0.23122 ± 0.00006	PDG 2026
Color's strength at the Z mass, $\alpha_s(M_Z)$	0.1180 ± 0.0009	PDG 2024
Hadronic vacuum polarization at the Z mass, $\Delta\alpha_{had}$	0.02783	Particle Data Group 2024, electroweak review
W mass, M_W	80.3625 ± 0.0077 GeV	PDG 2026
Light's MS-bar inverse strength at 1.035 GeV, $1/\hat{\alpha}_{em}$	134.07 ± 0.12	no experiment named
Proton mass, m_p	938.27208943(29) MeV	CODATA 2022
Measured span from the electron's mass to the reduced Planck mass, E	72.0132112 ± 0.0000162 halvings	CODATA 2022

A *halving* is a factor of two in mass: a mass x halvings above the electron's mass is $2^x\, m_e$. The *measured span* $E = \log_2(M_{Pl,red}/m_e)$ is the number of halvings from the electron's mass to the reduced Planck mass $M_{Pl,red}$; the reduced Planck mass is meant throughout this book. Chapter 5 treats the span; this chapter uses its measured value only.

No input of this chapter gives measured values of $1/\alpha_1$ and $1/\alpha_2$ at the Z mass directly. They follow from the measured $1/\hat{\alpha}_{em}(M_Z)$ and $\sin^2\theta_W(M_Z)$ by the definitions of the electroweak relation, as Box II-1 computes.

Box II-1. The two electroweak inverse strengths at the Z mass.

Definitions. $\alpha_{em} = \alpha_2 \sin^2\theta_W$ and $\alpha_{em} = \alpha_Y \cos^2\theta_W$, with α_Y in the normalization Y; in the unification normalization $\alpha_1 = (5/3)\,\alpha_Y$. So $1/\alpha_2 = \sin^2\theta_W/\alpha_{em}$ and $1/\alpha_1 = (3/5)\cos^2\theta_W/\alpha_{em}$, all read at M_Z in the MS-bar scheme.

Measured inputs. $1/\hat{\alpha}_{em}(M_Z) = 127.951 \pm 0.009$ (Particle Data Group 2022, electroweak review); $\sin^2\theta_W(M_Z) = 0.23122 \pm 0.00006$ (Particle Data Group 2026).

Result. $1/\alpha_2(M_Z) = 0.23122 \times 127.951 = 29.5848 \pm 0.0080$. $1/\alpha_1(M_Z) = (3/5) \times 0.76878 \times 127.951 = 59.0197 \pm 0.0062$. In the normalization Y, $1/\alpha_Y(M_Z) = 98.3662$. The uncertainties combine the two measured uncertainties in quadrature; the angle's uncertainty is the larger part of each. *Check.* $1/\alpha_2 + (5/3)(1/\alpha_1) = 29.5848 + 98.3662 = 127.951$, the matching of the paragraph above.

III. The One-Loop Rates

At first order in the loops, the *one-loop* order, each inverse strength changes linearly with the logarithm of the resolution: $1/\alpha_i(\mu) = 1/\alpha_i(M_Z) - (b_i/2\pi)\ln(\mu/M_Z)$, for i = 1, 2, 3 and a mass μ. The number b_i is the one-loop *rate* of the way i (in the standard term, its one-loop beta coefficient). This chapter uses the standard sign convention, which is also the zoo's (The Story of the Particles, note 19): a negative rate makes the strength fall at shorter distance, and that fall is *asymptotic freedom*.

The three rates are $b_1 = 41/10$ for the hypercharge in the unification normalization ($41/6$ in the normalization Y; the factor between them is $3/5$), $b_2 = -19/6$ for the weak doublet, and $b_3 = -7$ for color.

In the opposite convention, in which asymptotic freedom is positive, the same rates read $b_3 = 7$, $b_2 = 19/6$ and $b_1 = -41/10$. Chapter 6 writes color's rate in that convention, as $b_0 = -b_3 = 7$, as the literature of color does; every other rate in this book is in the convention of this section.

Two effects make a rate, and this book's terms name them. A *turning* is the transformation one way performs; a content's *label* in a turning is the charge it carries under that way (color, weak isospin or hypercharge); a content *answers* a turning when it carries a label in it; a *sense* is a handedness, left or right. *Covering* (in the standard term, screening) is the part of a source's label taken up by pairs of contents near it, which makes the strength seen from farther away smaller. *Spreading* (antiscreening) is the part of a source's label that labeled carriers take up and carry outward, which acts against covering.

The carriers of the doublet (the W bosons) and of color (the gluons) carry the labels of their own turnings, and their fields are their own sources. Such labeled carriers spread with coefficient $(11/3)\,N$ for SU(N): $22/3$ for the doublet and 11 for color. The photon, the carrier of the phase, carries no label and does not spread, so the phase strength only grows toward short distance.

Each sense of a content that answers a turning covers by $2/3$ of the square of its label in that turning, counting a member of a set as one half; the joiner, the Higgs doublet, covers by $1/3$. The list of contents is closed: three generations of every kind of matter particle, each kind being a joining of labels (the book of kinds, Chapter 6, Section I), and the one joiner. Box III-1 sums the covering and the spreading over that list for each way.

Box III-1. The three rates by the count of the zoo's contents.

The phase, in the normalization Y. Per generation the sum of Y^2 over the Weyl states is: the quark doublet $6 \times 1/36 = 1/6$; the up-type antiquark $3 \times 4/9 = 4/3$; the down-type antiquark $3 \times 1/9 = 1/3$; the lepton doublet $2 \times 1/4 = 1/2$; the charged antilepton 1. The sum is $10/3$; over three generations it is 10; times $2/3$ it gives $20/3$. The Higgs doublet (two complex components, $Y = 1/2$) gives $(1/3) \times 2 \times 1/4 = 1/6$. No carrier spreads. So $b_1 = 20/3 + 1/6 = 41/6$, which grows toward short distance.

The right-handed neutrino, the sixteenth state of each generation (the book of kinds, Chapter 6, Section I), and the carriers that are their own opposites carry no phase label and add nothing.

The phase, in the unification normalization. Since $g_1^2 = (5/3)\, g'^2$, the rate is $(3/5)(41/6) = 41/10$.

The weak doublet. The W bosons spread by $22/3 = (11/3) \times 2$. Only the left-handed sense of each content answers the doublet: per generation, one lepton doublet and one quark doublet in each of three colors, four doublets, twelve over three generations, each covering by $1/3$, for a total covering of 4. The Higgs doublet covers by $1/6$. Net rate: $22/3 - 4 - 1/6 = 44/6 - 24/6 - 1/6 = 19/6$ toward spreading, so $b_2 = -19/6$.

Color. The gluons spread by $11 = (11/3) \times 3$, three being the Casimir number C_2 of SU(3). Six kinds of quark (the up-type and the down-type in each of three generations) answer color in both senses, each sense covering by $1/3$: $6 \times 2 \times 1/3 = 4$. The leptons and the joiner carry no color. Net rate: $11 - 4 = 7$ toward spreading, so $b_3 = -7$.

The standard count of the sources gives the same three numbers by another bookkeeping, Box III-2: the gauge bosons, the three generations of fermions and the one Higgs doublet each contribute to each rate, and the three contributions add.

Box III-2. The rates by the standard count, and color's rate in the kernel integers.

Premises (*standard physics*). One-loop running as in this section; the Standard Model with three generations and one Higgs doublet; the hypercharge in the unification normalization. Contributions: gauge bosons (0, −22/3, −11); three generations of fermions, 4/3 per generation to each rate, (4, 4, 4); one Higgs doublet (1/10, 1/6, 0).

Result. $b_1 = 0 + 4 + 1/10 = 41/10$; $b_2 = -22/3 + 4 + 1/6 = -19/6$; $b_3 = -11 + 4 + 0 = -7$.

Color's rate in the kernel integers. With $n_f = 6$ kinds of quark, the magnitude of color's rate is $11 - 2n_f/3 = 11 - 4 = 7 = 9 - 2 = c_1^2 - n_C$, with $c_1 = 3$ and $n_C = 2$ (the book of kinds, Chapter 2, Section I).

Asymptotic freedom of color. With three colors the magnitude of color's rate, written in thirds, is $33 - 2n_f$, which is positive (color asymptotically free) when n_f is at most 16. Six kinds of quark, two in each of three generations (the book of kinds, Chapter 7), give $33 - 12 = 21$, so color's strength falls at short distance.

Table III-1. The one-loop rates, in this chapter's convention, by three routes

Way	The zoo's count (Box III-1)	The standard count (Box III-2)	Kernel writing
Phase, unification normalization	41/10 (41/6 in the normalization Y)	41/10	none stated
Weak doublet	−19/6	−19/6	none stated
Color	−7	−7	magnitude $7 = c_1^2 - n_C$

The routes rest on different premises. The zoo's count rests on the zoo's list of contents and on the named mathematics of one-loop covering and spreading. The standard count rests on the Standard Model's content, as standard physics states it. The kernel writing of color's rate rests on the kernel integers

c_1 and n_C together with the count of six kinds of quark. The zoo's count and the standard count give the same three rates, and the kernel writing gives the same magnitude for color's rate.

The three one-loop rates are fixed by structure: they are the first numbers about the couplings that the structure fixes, since the closed list of contents supplies every term of the one-loop count and no free number enters at that order. A change in a rate at any mass other than that of a known content would reopen the closed list, so the rates test the list. The strengths themselves, at any one separation, remain values that the structure does not fix.

The derived rates agree with the measured running: the running of light's strength from zero momentum to the Z mass, and of color's strength from the tau mass to the TeV scale, is described by the same rates, and no measured running has been found to differ from them. No single number states this test, so the agreement is stated as agreement in form.

The fermions do not separate the forces. Each generation adds 4/3 to each of the three rates, so the fermions' part of every difference $b_i - b_j$ is zero, for any number of complete generations; the gauge bosons, not the matter, separate the three couplings. Box III-3 gives the reason.

Box III-3. The fermions drop out of the differences of the rates.

Premises. The charges and colors of one generation (the book of kinds, Chapter 5, Section I) fill complete multiplets $5 + 10$ of SU(5), with the hypercharge in the unification normalization; the sixteenth state is a singlet of SU(3), SU(2) and U(1). A complete multiplet of a simple group contributes equally to the one-loop running of every coupling of its subgroups (standard physics).

Result. Each generation adds $4/3$ to each of b_1, b_2 and b_3, and the singlet adds nothing, so the fermions' part of $b_i - b_j$ is $4/3 - 4/3 = 0$ per generation, for every pair i, j. A fourth complete generation would add $4/3$ to each rate and leave every difference's fermion part at zero.

Remark, stated without a calculation of the broken case. Removing a quark leaves an incomplete multiplet, and altering the charges changes which multiplets are filled; either would in general make the fermion part of the differences nonzero.

Scope. The statement is arithmetic about the charges; it makes no claim that SU(5) is a gauge group of the theory. It is a property of this content, not of anomaly freedom as such: a quark that is the same in both senses is free of anomalies and shifts only b_3 and b_1.

The sixteenth state, the right-handed neutrino, does not enter the running: it carries none of the three labels. Its existence is taken from the zoo and from the book of kinds, Chapter 6, Section I, where the neutrino is Dirac.

Light has its own rate in the unbroken phase, the phase before the Higgs condensate, read as the same weighted sum that composes its inverse strength: $b_{em} = (H\,b_1 + O\,b_2)/O = 11/3$ exactly. Its carrier belongs to the phase row, which is immanent in the focus of comparison as relation (the book of kinds, Chapter 1, Section I, the binding the zoo's frame of measurement uses; Chapter 4, Section III); that carrier has no self-action, and its rate is exactly $11/3$ times the inradius $r = 1$ of the kernel triangle. Box III-4 gives the count.

Box III-4. Light's own rate.

Premises. The rates $b_1 = 41/10$, $b_2 = -19/6$, $b_3 = -7$ (Boxes III-1 and III-2); the composition $1/\alpha_{em} = (H/\alpha_1 + O/\alpha_2)/O$ (Section II); each inverse strength linear in $\ln \mu$ with its rate. Kernel integers $O = c_1 = 3$, $N = 4$, $H = 5$, $n_C = 2$, inradius $r = O - n_C = 1$ (the book of kinds, Chapter 2, Sections I to III).

The rate. $b_{em} = (5/3)(41/10) - 19/6 = 41/6 - 19/6 = 22/6 = 11/3 = (11/3)(O - n_C) = (11/3)\, r$.

Its parts. The charged matter loads light's rate by 11: the three generations add 4 to each of b_1 and b_2 (Box III-2), which gives light's rate $(H \times 4 + O \times 4)/O = (N/O)(H + O) = (4/3)(H + O) = 32/3$, and the one doublet adds $(H \times 1/10 + O \times 1/6)/O = 1/3$; so $11 = O^2 + n_C = (N(H + O) + r)/O$. The pair's two charged carriers take away $(11/3)\, n_C = 22/3$. The rate is $11 - 22/3 = 11/3$. It equals the gap between the weak and color rates of the carriers alone, $(-22/3) - (-11) = 11/3$, and color's self-action per unit of its Casimir number, $11/3$.

Where the identity holds. $N(H + O) + r = O(O^2 + n_C) = 33$ holds only at $c_1 = 3$. Along the triangle family $O = c$, $N = c + 1$, $H = c + 2$, with unit inradius $r = 1$ and so $n_C = c - 1$ (the book of kinds, Chapter 2, Sections I to III), the two sides differ by $2(c + 1)^2 + 1 - c(c^2 + c - 1) = -(c - 3)(c + 1)^2$, which vanishes for positive c only at $c = 3$.

IV. The Thresholds

The rates change only at thresholds set by masses: those of the contents in the list, and those of the *held carriers*, the W and Z bosons, which the Higgs condensate gives mass (next paragraph). A content of mass m contributes to a rate only at separations within its own length $\hbar/(mc)$, that is, at energies above its mass; beyond that length it drops out, which is *decoupling* (the theorem of Appelquist and Carazzone). A mass at which a rate changes is a *threshold*. Below the electron's mass, light's coupling stops running.

The rates change also at the masses of the held carriers, the W and the Z. A carrier that rotates between states the vacuum tells apart acquires mass, and a carrier with mass acts only within its own length; the spreading of the W and Z enters the weak rate, so the weak rate has thresholds at their masses as well as at the masses of the contents.

The atom shows the running. The least state of hydrogen lies at the electron's own length divided by α, far beyond that length, so atomic measurements find the zero-momentum value. Inside the electron's own length the coupling is larger, and the states of the atom that reach the proton find it so. That part of their shift lowers them: the vacuum-polarization part of the Lamb shift, the Uehling term, lowers the 2S level of hydrogen.

Vacuum polarization is the covering of a charge by the pairs of charged contents that the field makes near it; the *Lamb shift* is the separation of the 2S and 2P levels of hydrogen, which the simplest theory of the atom puts at one energy.

V. Beyond One Loop, and the Matching at the Top

At the next order, two loops, the rates take in the strengths themselves and the amounts of the joinings. So beyond one loop the rates are fixed in form and take the values of the strengths as inputs.

Run at three loops from the published couplings at the top quark's mass, the three couplings reproduce the published couplings at the Planck scale within rounding: $g_1 = 0.61537$, $g_2 = 0.50546$ and $g_3 = 0.48732$, against the published 0.6154, 0.5055 and 0.4873. At two loops $g_2 = 0.50539$, outside rounding. The published values are those of Buttazzo and collaborators (their equations 57 to 60 at the top, equation 61 at the Planck scale), given at $M_{Pl} = 1.2209 \times 10^{19}$ GeV, the unreduced Planck mass; this chapter names that mass only as the published point of comparison.

The three-loop coefficients of Mihaila, Salomon and Steinhauser, converted to the normalization used here, reproduce the published three-loop terms in the fourth powers of the couplings exactly: $-388613/24000\, g_1^4$, $324953/1728\, g_2^4$ and $65/2\, g_3^4$. From two to three loops, with the matching of

Martin and Robertson used in Section VIII, the measured unit of that section (the ratio of light's inverse strength at the proton's mass to the electroweak gap at the end of the measured span) changes by +0.0052. The estimated four-loop terms, 0.0003 to 0.001, lie below the uncertainty band of every comparison that uses the running.

The *matching at the top* is the step that fixes the MS-bar strengths of the three ways at the top quark's mass from measured quantities, so that the running can start there. Two published matchings are carried side by side, without a choice between them. The matching of Martin and Robertson (their program SMDR; arXiv:1907.02500) starts from the Fermi constant G_F, the Z mass, $\alpha_{em}(0)$ and $\Delta\alpha_{had}$, and returns M_W as an output. The matching of Buttazzo and collaborators (arXiv:1307.3536) takes M_W as an input.

The two matchings differ in their electroweak strengths at the top, and the difference has two parts. Stated in the *level* (the common inverse strength at the place where $1/\alpha_1$ and $1/\alpha_2$ are equal, which Chapter 3 treats), a difference of 0.0157 splits into +0.0071 from the two W masses (80.3648 GeV in the benchmark of Martin and Robertson, 80.3692 GeV in the formula of Buttazzo and collaborators) and +0.0086 from carrying the newer $\Delta\alpha_{had}$ into one matching at fixed $\sin^2\theta_W$. The top mass, α_s and the Higgs mass together contribute less than 0.0001.

Box V-1. The two parts of the difference between the matchings.

Premises. Buttazzo's own formula, given M_W = 80.3692 GeV, reproduces the benchmark couplings of Martin and Robertson at M_W = 80.3648 GeV (standard physics). Carrying the hadronic vacuum polarization into the matching of Martin and Robertson at fixed $\sin^2\theta_W$ shifts $1/\hat{\alpha}$ by −0.026. At fixed G_F and Z mass, the tree relation gives, for that shift, changes (−0.0170, +0.0026) in ($1/\alpha_1$, $1/\alpha_2$); through the Standard Model's M_W it gives (−0.0160, +0.0025).

Result. The difference of 0.0157 in the level is +0.0071 from the W masses and +0.0086 from the shift at fixed $\sin^2\theta_W$. That shift moves ($1/\alpha_1$, $1/\alpha_2$) by (−0.0120, −0.0060), against the tree relation's (−0.0170, +0.0026): its change in $1/\alpha_2$ has the wrong sign, −0.0060 against the tree relation's +0.0026. The top mass, α_s and the Higgs mass contribute less than 0.0001.

With the shift of the tree relation. Started with the shift of the tree relation, the two matchings differ by 0.0176 in the level and by 0.0356 in the measured unit of Section VIII, three of that unit's standard deviations.

With the running taken to three loops, the matching at the top is the largest remaining uncertainty of the level and of the measured unit.

VI. The Couplings Across the Span

Run up from their measured values with the Standard Model's rates, the three inverse strengths come near one another but do not become equal. The zoo states the same: with the zoo alone the measured couplings meet at no scale. On the way down, the rank order strong, weak, electromagnetic comes out right for any upper end of the running between 2.4×10^{18} and 1.2×10^{19} GeV.

At the end of the measured span, the reduced Planck mass, color's inverse strength run at three loops is $1/\alpha_3(E) = 51.17$.

No coupling runs from a scale of the internal space. The *content instance* of the internal space, the internal space read as a space of states (the book of kinds, Chapter 3, Section III), has no size, no radius and no scale, so there is no scale of the internal space from which a coupling could run.

With the coupling g itself taken as the measure, and $g = 1$ as unit strength, only color reaches unit strength inside the span. The weak coupling is $g_2 = 0.652$ at the Z mass and falls with rising resolution; the hypercharge coupling g' is 0.357 at the Z mass and 0.476 at the Planck scale. That color is the only way to reach unit strength inside the span holds in every convention; the scale at which it does so depends on the convention.

The scale at which color's strength "is one" depends on what is called one and on the loop order, Box VI-1. The top of the span does not bound it.

Box VI-1. Where color's strength is one.

Definitions. Three meanings of unit strength for color: $\alpha_s = 1$; $g_s = 1$, with $\alpha_s = g_s^2/(4\pi)$; and $g_s^2/(16\pi^2) = 1$. *Input.* $\alpha_s(M_Z) = 0.118$, run with the standard running of color at one loop and at higher loop order.

Results. $g_s = 1$ at about 3378 GeV. $\alpha_s = 1$ at 0.29 to 0.67 GeV, depending on the loop order. $g_s^2/(16\pi^2) = 1$ at about 0.15 GeV. The factor 4π between the conventions and the loop order move the scale; the top of the span does not bound it.

The strength of the phase force (the force that turns the phase of each content in proportion to its hypercharge, the charge of the phase; its coupling is g') is the one of the three that grows toward short distance, and it stays far from any limit across the whole span. At one loop its inverse strength, 59.0197 at the Z mass in the unification normalization (Box II-1), falls by $b_1/2\pi = 41/(20\pi) = 0.653$ for each step of one in the natural logarithm of μ/M_Z (Section III). It would reach zero, so that the strength grew without limit (in the standard term, the phase force's Landau pole), only where $\ln(\mu/M_Z) = 2\pi \times 59.0197/4.1 = 90.45$, near $\mu = 1.7 \times 10^{41}$ GeV, about 76 halvings above the reduced Planck mass, the end of the span. The Planck scale is where each existing description reaches its edge, the scale past which that description no

longer holds, and no description takes over beyond it (*The Gravitational Passage*, Chapter 17, Section IV; *The Unquantized Arena*, Chapter 9, Section V, Proposition 9.3). So the scale at which one-loop running would make the phase force's strength grow without limit lies outside the domain of every description; and since the strengths of the weak force and of color fall toward short distance (Section III), none of the three strengths, run with the closed list of contents, grows without limit toward short distance anywhere inside the span.

VII. Relations Among the Couplings

Light's inverse strength is, at every scale, the H : O weighted mean of the two electroweak inverse strengths: $1/\alpha_{em} = (H/\alpha_1 + O/\alpha_2)/O$. In the shares of one generation's charge squared, the quark share $S_q = 5/8$ and the lepton share $S_\ell = 3/8$, the same relation is $1/\alpha_{em} = (S_q/\alpha_1 + S_\ell/\alpha_2)/S_\ell$. Where $\alpha_1 = \alpha_2 = \alpha$, it reduces to $1/\alpha_{em} = ((O + H)/O)/\alpha$. Box VII-1 gives the steps.

Box VII-1. Light's inverse strength as a weighted mean.

Premises. The matching $1/\alpha_{em} = 1/\alpha_2 + (5/3)/\alpha_1$ at every scale (Section II). The equal traces over the finite space of states, which hold at the finest resolution by two routes, the structure giving no force a greater weight than another because the three positions are distinct and none stands before the others in the cycle of the second axiom (the zoo, Level 7), and the spectral action's single trace over the whole finite space tying the couplings to one common measure at its cutoff scale (the NCG book, Chapter 13); the book of kinds, Chapter 5, Section IV, gives both: $g_3^2 = g_2^2 = (5/3)\, g'^2$, with g' the hypercharge coupling in the normalization Y (the sources write g_1 for g' in this equation); in the unification normalization of this chapter, $g_3^2 = g_2^2 = g_1^2$. The trace ratio $\mathrm{Tr}\, Y^2/\mathrm{Tr}\, T_3^2 = (\text{quark charge squared})/(\text{lepton charge squared}) = 5/3 = H/O$; the shares of one generation's charge squared, up 1/2, down 1/8, charged lepton 3/8, with $S_\ell = \sin^2\theta_W$ at the traces = 3/8 and the up share equal to the down share plus the lepton share, $1/2 = 1/8 + 3/8$ (the book of kinds, Chapter 5, Section V).

Steps. $1/\alpha_{em} = 1/\alpha_2 + (H/O)/\alpha_1 = (H/\alpha_1 + O/\alpha_2)/O$. The quark share is $S_q = 1 - S_\ell = 5/8$, so $S_q/S_\ell = 5/3 = H/O$ and $1/\alpha_{em} = (S_q/\alpha_1 + S_\ell/\alpha_2)/S_\ell$. At $\alpha_1 = \alpha_2 = \alpha$: $1/\alpha_{em} = (1 + 5/3)/\alpha = ((O + H)/O)/\alpha = (8/3)/\alpha$.

The 5/3 has its source in the *equal traces* at the finest resolution: the weights of the three ways, computed as traces over the finite space of states, are equal there. Through the *spectral action* (the action computed from the spectrum of the geometry's Dirac operator), the algebraic route gives the relations among the couplings at the cutoff scale: $g_3^2 = g_2^2 = (5/3)\, g'^2$ (in the unification normalization, $g_3^2 = g_2^2 = g_1^2$), which is $\sin^2\theta_W = 3/8$ (the book of kinds, Chapter 5, Sections IV and V). The weak angle 3/8 has its reason: the trace ratio and the triangle's ratio are one formula, and 3/8 is the leptons' share of charge squared. The running of the angle from 3/8 to its measured value is Chapter 3's.

At the finest resolution the forces are not separated; at lower resolution they appear separate. The equality the structure asserts there is not an equality of measured strengths: it is the equality of the three traces, a relation among the weights the structure gives the three forces. The first candidate below tests a different equality, one of the running couplings themselves.

The three inverse strengths can be read as one vector whose part orthogonal to the direction (1, 1, 1) is the values read from measurement, with the equal traces lying on the whole direction (1, 1, 1) (Chapter 3 states this reading). That step uses two things. The first is the three ways as three orthogonal axes, which this book's premise that any triple is an orthonormal set (three mutually orthogonal directions of unit length) supplies. The second is the three inverse strengths as the coefficients of one vector on those axes, which that premise does not supply; it is a premise in its own right, here called the *coefficient premise*. Given the coefficient premise, the split into the whole direction (1, 1, 1) and a two-dimensional spread is linear algebra.

For the electroweak pair the coefficient premise is realized by the photon's composition. Light's inverse strength is the weighted sum $1/\alpha_{em} = 1/\alpha_2 + (H/O)/\alpha_1$, and its rate is the same weighted sum, $b_{em} = (H\,b_1 + O\,b_2)/O$ (Box III-4): the inverse strength is additive across ways that compose into one carrier, as coordinates are under a linear map. For color, which is unbroken and composes with nothing, the coefficient premise is realized only by the running's additivity, each inverse strength being linear in $\ln \mu$ with its rate. What the premise still needs for color is that additivity along the running alone makes color a coordinate of the same vector.

Candidate relations among the couplings are tested against measurement, and a candidate with a large pull is held as a hypothesis with its pull stated. Five candidates do not hold; Table VII-1 lists them. One further form holds at a single scale only, and it is stated after them.

The first is the equality of the three couplings at the finest resolution, as a candidate for fixing light's strength. Set equal at about 9.7×10^{16} GeV (67.4 halvings above the electron's mass), with the phase in the normalization of the equality, and carried down at one loop with the one-loop rates, the three give $1/\alpha_{em}(M_Z) = 145.6$ against the measured 128.0 in the same convention,

a miss of fourteen percent, and $\sin^2\theta_W = 0.203$ against 0.231. So this candidate does not fix light's strength. The same tension is known for the minimal spectral model and for minimal unification.

The second is the trace of the three inverse strengths at the end of the measured span, set equal to light's inverse strength read at the proton's mass. The sum $1/\alpha_1 + 1/\alpha_2 + 1/\alpha_3$ at E is 133.628 ± 0.064; light's inverse strength at the proton is $T(m_p) = 134.157 \pm 0.12$ (Section VIII). The pull is -3.9 standard deviations at two loops; by scheme it lies between -3.1 and -4.5, and at one loop the candidate does not hold either. So no law on record holds for the free coordinate of the coupling vector: its component along $(1, 1, 1)$, which is the electroweak level, the common inverse strength at the place where $1/\alpha_1 = 1/\alpha_2$.

The third is $1/\hat{\alpha}_{em}(M_Z) = 2^7 = 128$. Against the measured value it has a pull of $+5.4$ standard deviations.

The fourth relates the weak strength to the other two as their geometric mean, $\alpha_2^2 = \alpha_1\alpha_3$. It is 32 percent off at the Z mass: with $\alpha_1 = 0.0169$ and $\alpha_3 = 0.118$ there (Box II-1 and Table II-1), $\sqrt{(\alpha_1\alpha_3)} = 0.0447$ against $\alpha_2 = 0.0338$.

The fifth is the product form $\alpha_2 \sim \alpha_1 \times \alpha_3$ (α_2 of the size of α_1 times α_3). It does not hold where the three strengths come close: with each near $1/40$, the right side is $1/1600$ and the left side $1/40$.

The harmonic form, $2/\alpha_2 = 1/\alpha_1 + 1/\alpha_3$, holds under one-loop running at one scale only, 3×10^8 GeV (39.1 halvings above the electron's mass).

Table VII-1. Candidate relations that do not hold

Candidate	Stated value	Measured or compared value	Pull or miss
The three couplings equal at the finest resolution, taken at about 9.7×10^{16} GeV, carried down at one loop	$1/\alpha_{em}(M_Z) = 145.6$; $\sin^2\theta_W = 0.203$	128.0 (no uncertainty given); 0.23122 ± 0.00006 (PDG 2026)	14 percent in $1/\alpha_{em}$; -470 in $\sin^2\theta_W$
$1/\alpha_1 + 1/\alpha_2 + 1/\alpha_3$ at E equals $T(m_p)$	133.628 ± 0.064	134.157 ± 0.12	-3.9 (-3.1 to -4.5 by scheme)
$1/\hat{\alpha}_{em}(M_Z) = 2^7$	128	127.951 ± 0.009 (Particle Data Group 2022, electroweak review)	+5.4
$\alpha_2^2 = \alpha_1\alpha_3$ at M_Z	$\sqrt{(\alpha_1\alpha_3)} = 0.0447$	$\alpha_2 = 0.0338$ (Box II-1)	32 percent
$\alpha_2 \sim \alpha_1\alpha_3$ where each is near 1/40	$\alpha_1\alpha_3 = 1/1600$	α_2 near 1/40 (each strength near convergence, not a measurement)	a factor of 40

VIII. Light at the Proton: the Restated Relation

Three quantities enter the restated relation. $T(m_p) = 1/\hat{\alpha}_{em}(m_p)$ is light's MS-bar inverse strength read at the proton's mass. $\Delta_{21}(E) = (1/\alpha_2 - 1/\alpha_1)(E)$ is the gap between the two electroweak inverse strengths read at the end of the measured span, the reduced Planck mass, E halvings above the electron's mass. The *measured unit* is their ratio, $U = T(m_p)/\Delta_{21}(E)$.

The restated relation is $1/\hat{\alpha}_{em}(m_p) = 3\pi\,(1/\alpha_2 - 1/\alpha_1)(E)$, that is, $U = 3\pi = 9.42478$. Light is read at the proton's mass, and E is the measured span. The measured unit lands on 3π at two loops in both matchings at the

top and at three loops in the matching of Martin and Robertson; at three loops in the matching of Buttazzo and collaborators the pull of 3π is −2.04, outside the band of two standard deviations by 0.04.

Light's inverse strength at the proton's mass is $T(m_p) = 134.157 \pm 0.12$: the measured 134.07 ± 0.12 (MS-bar, at 1.035 GeV) carried down 0.142 halvings to m_p with the electron, the muon and the up, down and strange quarks active.

At three loops, with the matching of Martin and Robertson moved to the updated inputs (top mass 172.57 GeV and Higgs mass 125.20 GeV, both PDG 2024; $\alpha_s = 0.1180 \pm 0.0009$, PDG 2024; $\Delta\alpha_{had} = 0.02783$, Particle Data Group 2024, electroweak review; and M_W the Standard Model's output, 80.3498 GeV), $\Delta_{21}(E) = 14.2522$ and $U = 9.41309 \pm 0.0117$; the pull of 3π is +1.00. Under the matching of Buttazzo and collaborators, $U = 9.4487$, and the pull of 3π, against the same $\sigma_U = 0.0117$, is −2.04, outside the band of two standard deviations by 0.04.

At two loops, with the two matchings at next-to-next-to-leading order (two orders of loop correction beyond the simplest calculation), $U = 9.4299$ (Martin and Robertson; $\Delta_{21}(E) = 14.2268$), with 3π at −0.44, and $U = 9.4435$ (Buttazzo and collaborators), with 3π at −1.60. One standard deviation of M_W, taken as ±0.0133 GeV in that calculation, moves U by ±0.048.

At two loops with the matching at the top taken at tree level (with no loop corrections in the matching), $\Delta_{21}(E) = 14.2355 \pm 0.0123$ and $U = 9.4241 \pm 0.0117$, with 3π at +0.06 (other treatments of the top threshold give +0.04 and −0.33). At one loop $U = 9.376$, and 3π lies at +4.2. So the relation is a relation of the two-loop running.

Table VIII-1. The measured unit U by loop order and matching; $3\pi = 9.42478$; every pull is $(3\pi - U)/0.0117$

Running and matching	$\Delta_{21}(E)$	U	Pull of 3π
Three loops, Martin and Robertson, updated inputs	14.2522	9.41309	+1.00
Three loops, Buttazzo and collaborators	not stated	9.4487	−2.04
Two loops, Martin and Robertson, next-to-next-to-leading order	14.2268	9.4299	−0.44
Two loops, Buttazzo and collaborators, next-to-next-to-leading order	not stated	9.4435	−1.60
Two loops, matching at tree level	14.2355 ± 0.0123	9.4241	+0.06
One loop	not stated	9.376	+4.2

Box VIII-1. The measured unit and its uncertainty.

Inputs. $T(m_p) = 134.157 \pm 0.12$, read at the proton's mass. $\Delta_{21}(E)$ from the running of the measured couplings to $E = 72.013$ halvings above the electron's mass; at two loops with tree matching, 14.2355 ± 0.0123, the uncertainty carrying that of $\sin^2\theta_W(M_Z) = 0.23122 \pm 0.00006$ (PDG 2026).

The ratio. $U = 134.157/14.2522 = 9.41309$ (three loops, Martin and Robertson); $134.157/14.2268 = 9.4299$ (two loops, Martin and Robertson); $134.157/14.2355 = 9.4241$ (two loops, tree matching).

The uncertainty. The uncertainty of T contributes $0.12/14.25 = 0.0084$ to U; that of $\Delta_{21}(E)$ contributes $9.42 \times 0.0123/14.24 = 0.0081$. In quadrature they give $\sigma_U = 0.0117$, the standard deviation with which every pull of this section is computed.

The one-loop writing. At one loop, $\Delta_{21}(E) = ((b_1 - b_2)/2\pi)\ln 2\,(E - X_{12})$, where X_{12} is the place, in halvings above the electron's mass, at which $1/\alpha_1 = 1/\alpha_2$ (Chapter 3), and $b_1 - b_2 = 41/10 + 19/6 = 109/15$. The relation then reads $1/\hat{\alpha}_{em}(m_p) = (3/2)(b_1 - b_2)\ln 2\,(E - X_{12}) = (109/10)\ln 2\,(E - X_{12})$. That writing is exact only as a one-loop writing; the agreement of Table VIII-1 needs the two-loop terms, about −0.07 in $\Delta_{21}(E)$ and −0.7 in $3\pi\Delta_{21}(E)$.

The unit has a second writing, in light's own rate $b_{em} = 11/3$ (Box III-4) in place of the gap's rate $b_1 - b_2$. The identity $3(b_1 - b_2) = (327/55)\, b_{em} = 6(1 - 1/110)\, b_{em}$ converts the one rate into the other exactly. So the one-loop writing in light's own rate, $1/\hat{\alpha}_{em}(m_p) = 3(1 - 1/110)\, b_{em} \ln 2\, (E - X_{12})$, is the writing in the electroweak gap restated, $(109/10) \ln 2\, (E - X_{12})$, and it adds no content to it.

Across the five values beyond one loop, the pull of 3π is $+1.00$, -0.44 and $+0.06$ in three of them, -1.60 in the fourth and -2.04 in the fifth. The unit 3π is a measured agreement, and its question is which law fixes it. Three clues stand beside it: $3\pi = 4\pi/b_e$, with $b_e = 4/3$ the one-loop coefficient of one Dirac fermion of unit charge (Chapter 5, Section VI); $3\pi = 6 \times \pi/2$, the Gauss measure of six unit points (Chapter 5, Section III); and $3\pi = 2\pi \times 3/2$, one full turn of phase times the half-twist $3/2$ by which a fermion's label is shifted on the space of states (the book of kinds, Chapter 3: $q = k + 3/2$), the half-twist being half the first Chern number $c_1 = 3$ of $\mathbb{C}P^2$, so that 3π is also half of $2\pi \times c_1 = 6\pi$.

Read the other way, the same relation returns quantities that belong to the neighboring chapters. Solved for the span, it returns a value of E from the measured couplings alone; that value, and its comparison with the count of 72 halvings, are Chapter 5's, where the measured span is a test. Solved with color's running, it bears on color's strength and the proton's mass; those are Chapter 6's. The common inverse strength at the place where $1/\alpha_1 = 1/\alpha_2$ is Chapter 3's.

Summary

Light's coupling at zero momentum is the electron's number, constant beyond the electron's own length and larger at shorter distance; light's strength is read at the proton's mass, the mass of what light is compared against. The two electroweak inverse strengths at the Z mass follow from light's measured strength and the measured weak angle by the electroweak definitions: $1/\alpha_1 = 59.0197 \pm 0.0062$ and $1/\alpha_2 = 29.5848 \pm 0.0080$.

The one-loop rates $41/10$, $-19/6$ and -7 are fixed by the closed list of contents, reached by two routes (and color's by a third), and change only at the masses of the contents and of the held carriers. The fermions never separate the forces, and light's own rate is $11/3$.

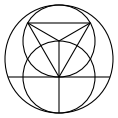
Three-loop running reproduces the published couplings at the Planck scale, and the matching at the top is the largest remaining uncertainty. Light's inverse strength is the $5 : 3$ weighted mean of the two electroweak inverse strengths at every scale. The vector reading of the three inverse strengths rests on the coefficient premise, realized for the electroweak pair by the photon's composition and for color only by the running's additivity; what the premise still needs for color is that this additivity alone makes color a coordinate of the same vector. The equality of the three running strengths and the trace candidate do not hold, with their pulls stated.

Light's inverse strength at the proton's mass lands on 3π times the electroweak gap at the end of the measured span within two standard deviations in four of the five computations beyond one loop: the pull of 3π is $+1.00$ (Martin and Robertson) and -2.04 (Buttazzo and collaborators) at three loops, -0.44 and -1.60 at two loops, and $+0.06$ with tree matching.

The question each measured agreement of this chapter carries is why it lands: the unit 3π of the restated relation, with its clues the coefficient $4\pi/b_e$ of the coupling route, the Gauss measure $6 \times \pi/2$ (Chapter 5), and one turn times the half-twist, $2\pi \times 3/2$ (the book of kinds, Chapter 3).

KEY CONCEPTS

- **Coupling and strength.** g, the strength of one of the three transformations at a given resolution; $\alpha = g^2/(4\pi)$.
- **Running.** The change of a coupling with resolution; a change of the lens, not of the triangle.
- **The one-loop rates.** $b_1 = 41/10$, $b_2 = -19/6$, $b_3 = -7$; asymptotic freedom negative.
- **Thresholds.** The rates change only at the masses of the contents and of the W and Z.
- **Light's own rate.** $b_{em} = (H\,b_1 + O\,b_2)/O = 11/3$.
- **The weighted mean.** $1/\alpha_{em} = (H/\alpha_1 + O/\alpha_2)/O$ at every scale.
- **The coefficient premise.** The three inverse strengths are the coefficients of one vector on the axes of the three ways.
- **The measured unit.** $U = T(m_p)/\Delta_{21}(E)$, light's inverse strength at the proton's mass over the electroweak gap at the end of the measured span.
- **The restated relation.** $U = 3\pi$; the pull of 3π lies between -2.04 and $+1.00$ across the two matchings at two and three loops.



Chapter 3

Where the Couplings Cross

CHAPTER OVERVIEW

This chapter states where the running inverse strengths of the three gauge forces cross one another, the law that places each crossing, and the gaps that the running leaves between the three at the end of the span from the electron's mass to the reduced Planck mass.

The law of place of a crossing is a form that uses no measured value. Two of the three crossing places are free, and the third is fixed by them, because their three distances from the Z mass obey one linear relation. The spacing of the three crossings is fixed by the particle content: in the one-loop form the ratio of the two spacings is 218/115, set by the gauge forces' own action and the one Higgs doublet, with the generations canceling. The one-loop formula is the form of the law of place; its values are stated of the three-loop places.

The places are read from the measured couplings run upward: at three loops they lie at 54.259, 58.200 and 65.521 halvings above the electron's mass (one halving being one doubling of energy), with spacing ratio 1.8581 $\pm$ 0.0006. The gauge group is the product of the three groups of the three forces at every scale, with no unifying group (no single simple group containing the three), and the measured couplings meet at no scale.

The first crossing is the one scale at which the running weak angle equals the structural value 3/8. The common inverse strength there, 42.25163 $\pm$ 0.00390 at three loops, is read from measurement. At the span's end the ratio of the two splits between the inverse strengths is measured as 5.093 $\pm$ 0.128; a law of that ratio is a law of the crossing places, and the chapter carries the two values 5 and 21/4 side by side with their pulls in each determination.

I. The Law of Place

A gauge force's strength at a given resolution is its coupling, written α; its *inverse strength* is $1/\alpha$. The *resolution* of a probe is its energy μ: a higher energy resolves a shorter length. The strength a force shows depends on the resolution, and the change of its inverse strength with resolution is its *running* (Chapter 2). The subscripts 1, 2 and 3 name, throughout this chapter, the *phase force* (the force of hypercharge, taken in the *unification normalization*, $\alpha_1 = (5/3)\,\alpha_Y$), the weak force, and *color* (the strong force).

Resolution is counted in *halvings*: an energy μ stands at $h = \log_2(\mu/m_e)$ halvings above the electron's mass m_e, so that one halving is one doubling of the energy. The count starts at the electron because the electron's mass is the measured input from which the particle side of this book reckons masses, and it is the unit of mass. On this count the reduced Planck mass stands at 72.013 halvings; that place is the *span's end*, $E = \log_2(M_{Pl,red}/m_e)$, the measured span, which this book uses as a test only (Chapter 5).

A running computed *at one loop* keeps the first term of the perturbative expansion of the rates of change; a running *at three loops* keeps the terms through the third order, in which the rates of the three forces depend on one another and on the couplings of the top quark and the Higgs field.

A *crossing* is a scale at which two inverse strengths are equal, and its *place* is its position in halvings: the crossing of forces i and j has the place X_{ij}. The *rate* b_i of force i is its one-loop coefficient: at one loop, $1/\alpha_i$ changes by $-b_i/2\pi$ per unit of the natural logarithm of the resolution. The *gap* between forces i and j at a scale is $D_{ij} = 1/\alpha_i - 1/\alpha_j$.

With the running taken at one loop, two inverse strengths cross at $X_{ij} - X_Z = 2\pi\, D_{ij}(M_Z)/((b_i - b_j)\ln 2)$ halvings, where X_Z is the place of the Z mass M_Z and $D_{ij}(M_Z)$ is the gap at the Z mass. This formula is the *law of place*, the law that places a crossing. It uses no measured value; the measured gaps at the Z mass enter only when a place is computed from it.

The rates of the Standard Model are $b = (41/10, -19/6, -7)$ for the phase force, the weak force and color, in the convention $1/\alpha_i(\mu) = 1/\alpha_i(M_Z) - (b_i/2\pi)\ln(\mu/M_Z)$; Chapter 2 carries them.

The gap between forces 1 and 3 is the sum of the gap between 1 and 2 and the gap between 2 and 3. So the three distances $L_{ij} = X_{ij} - X_Z$ of the crossing places from the Z mass obey one linear relation, $(b_1 - b_3)\, L_{13} = (b_1 - b_2)\, L_{12} + (b_2 - b_3)\, L_{23}$: two of the three crossing places are free, and the third is fixed by them.

With the rates fixed and the masses of the charged states measured, three numbers fix all three couplings at every resolution. One choice of the three is the common strength of color and the weak force where they meet, the place of that meeting, and the phase force's strength there. Another choice is two crossing places and the inverse fine-structure strength at the Z mass, $1/\alpha_{em}(M_Z)$.

This book takes as a premise that any triple is an orthonormal set (three mutually orthogonal directions of unit length). On that premise the three inverse strengths are the components of one vector, $(1/\alpha_1, 1/\alpha_2, 1/\alpha_3)$. The vector's part along (1, 1, 1) carries the structural equality of the three traces (stated in *The Kinds of Particle; Why the Structure Must Be So*, the book of kinds, Chapter 5, Section IV); the values obtained from measurement are its part orthogonal to (1, 1, 1); and the crossings are the points at which the line traced by the running vector cuts the three planes $1/\alpha_i = 1/\alpha_j$.

The one-loop formula gives the form of the law of place, and the three-loop running gives its values. With the couplings of the PDG 2024 compilation, the one-loop places (54.16, 58.72 and 67.36 halvings) differ from the three-loop places (54.259, 58.200 and 65.521 halvings) by −0.10, +0.52 and +1.84 halvings, the larger differences falling on the two crossings at which color is one of the pair. The *spacing ratio*, the ratio of the two distances between consecutive crossings (Section II), is 218/115 = 1.8957 at one loop and 1.8581 ± 0.0006 at three loops, a difference of 2 percent. The difference is the content of the two-loop and three-loop terms, and its size varies with color's two-loop coefficient. Every law of place in this chapter is therefore stated of the three-loop places.

Box I-1. The law of place, and the one linear relation

Premises.

(1) One-loop running, standard physics: $1/\alpha_i(\mu) = 1/\alpha_i(M_Z) - (b_i/2\pi)\ln(\mu/M_Z)$. (2) Halvings, by definition: $h = \log_2(\mu/m_e)$, so $\ln(\mu/M_Z) = (h - X_Z)\ln 2$.

Steps.

The gap $D_{ij} = 1/\alpha_i - 1/\alpha_j$ runs as $D_{ij}(\mu) = D_{ij}(M_Z) - ((b_i - b_j)/2\pi)\ln(\mu/M_Z)$. The crossing is the zero of the gap: $\ln(\mu_{ij}/M_Z) = 2\pi\, D_{ij}(M_Z)/(b_i - b_j)$. In halvings: $X_{ij} - X_Z = 2\pi\, D_{ij}(M_Z)/((b_i - b_j)\ln 2)$. Since $D_{13} = D_{12} + D_{23}$ at every scale, $(b_1 - b_3)\, L_{13} = (b_1 - b_2)\, L_{12} + (b_2 - b_3)\, L_{23}$, with $L_{ij} = X_{ij} - X_Z$.

Result.

Each crossing is placed by one gap at the Z mass and one rate difference, and the third place is fixed by the other two. With the rates fixed, two places and $1/\alpha_{em}(M_Z)$ fix the three couplings at every scale.

Table I-1. The crossing places at one loop and at three loops, PDG 2024 couplings, in halvings above the electron's mass.

Crossing	One loop	Three loops	One loop minus three loops
$\alpha_1 = \alpha_2$ (X_{12})	54.16	54.259	−0.10
$\alpha_1 = \alpha_3$ (X_{13})	58.72	58.200	+0.52
$\alpha_2 = \alpha_3$ (X_{23})	67.36	65.521	+1.84
Spacing ratio	1.8957 (218/115)	1.8581 ± 0.0006	2 percent

II. The Spacing Ratio

The spacing ratio of the three crossings is the distance from the middle crossing to the last, divided by the distance from the first to the middle: $(X_{23} - X_{13})/(X_{13} - X_{12})$. At one loop it equals the ratio of rate differences $(b_1 - b_2)/(b_2 - b_3)$ for any couplings at which the three crossings are not all at one

place ($X_{12} \neq X_{23}$). So the spacing ratio is fixed by what the *content* (the kinds of particle and their charges) contributes to the rates. It has two writings, set side by side below with the source of each.

The standard writing

The gauge bosons alone contribute the rates (0, −22/3, −11), from −11N/3 for the group SU(N) and zero for U(1). Their pairwise differences are 22/3 between the phase force and the weak force, 11/3 between the weak force and color, and 11 between the phase force and color, in the proportion 2 : 1 : 3. So the gauge-only spacing ratio is exactly 2.

The fermions add the same amount to each of the three rates, so they drop out of every difference (Chapter 2).

The one Higgs doublet, with hypercharge 1/2 (quantum numbers that the algebra fixes, The Branches of the Kinds, Chapter 4, Section II), adds (1/10, 1/6, 0) to the three rates; it couples to the weak force and to the phase force and not to color. So $b_1 - b_2 = 22/3 - 1/15 = 109/15$ and $b_2 - b_3 = 11/3 + 1/6 = 23/6$, and the spacing ratio becomes $(109/15)/(23/6) = 218/115 = 1.8957$, a departure of 5 percent from 2.

The kernel writing

The book of kinds fixes two integers by comparison: $c_1 = 3$, the count of the positions of comparison (the omniscient, the immanent and the transcendent, which its Chapter 1 defines), and $n_C = 2$, the count of the poles, the conjugate pair content and context, the two aspects into which comparison partitions what is compared (its Chapter 2, Section I). They fix the right triangle with legs $O = c_1 = 3$ and $N = c_1 + 1 = 4$, hypotenuse $H = c_1 + n_C = 5$ and inradius $r = 1$ (the book of kinds, Chapter 2, Sections I and II). These are the *kernel integers*.

In the kernel writing, the gauge Casimirs of U(1), SU(2) and SU(3), the numbers (0, 2, 3) that set each group's self-action, are the kernel's sizes r, n_C, c_1 with the first size, r, replaced by zero, because U(1), the way of the phase force, has no self-interaction. Their consecutive differences are $n_C - 0 = 2$ and

$O - n_C = 1 = r$, so the gauge-only ratio is $n_C/r = 2$. The ratio equals 2 exactly where the inradius r equals 1, since $r = 1$ holds if and only if $n_C = c_1 - 1$. The condition $n_C/(O - n_C) = 2$ is linear in c_1 and has the one root 3.

Each generation adds $N/O = 4/3$ to every one of the three one-loop rates, because one generation contributes equally to the three forces: $\mathrm{Tr}\, T_8^2 = \mathrm{Tr}\, T_3^2 = (3/5)\, \mathrm{Tr}\, Y^2$, the equal traces. The three generations add $N = 4$ to each rate, and they cancel from every difference. So 218/115 is fixed by the gauge self-actions $-(0, n_C, O) \times 11/3$ and the one Higgs doublet's $(1/10, 1/6, 0)$ alone.

In kernel integers, $218/115 = n_C(1 - 1/110)/(1 + 1/22)$, with $110 = n_C\, H\, (O^2 + n_C) = 2 \times 5 \times 11$ and $22 = n_C(O^2 + n_C) = 2 \times 11$. The departure of the ratio from n_C is the Higgs doublet's alone. Both correction factors, $(1 - 1/110)$ and $(1 + 1/22)$, carry the factor $11 = O^2 + n_C$, color's self-action coefficient; the first also carries H, and the second does not.

The kernel writing writes the same coefficients as the standard writing, in kernel integers. The two are one count written twice, and they are kept side by side with their sources: the standard rates of the Standard Model for the first, and the kernel integers of the book of kinds for the second.

Box II-1. The spacing ratio from the content

Premises.

(1) Standard physics: a pure gauge group SU(N) contributes $-11N/3$ to its one-loop rate; U(1) contributes zero. (2) Standard physics: the Higgs doublet of hypercharge 1/2 contributes (1/10, 1/6, 0) to (b_1, b_2, b_3). (3) Each generation contributes $N/O = 4/3$ to every rate (its equal contribution to the three forces). (4) The kernel integers $c_1 = 3$ and $n_C = 2$ (the book of kinds, Chapter 2, Section I), and the triangle's sides O = 3, N = 4, H = 5 and inradius r = 1 (the book of kinds, Chapter 2, Section II).

Steps.

Gauge bosons: $(0, -22/3, -11)$; differences $b_1 - b_2 = 22/3$, $b_2 - b_3 = 11/3$; ratio 2. Kernel form of the same: $-(0, n_C, O) \times 11/3$, differences $n_C \times 11/3$ and $r \times 11/3$, ratio $n_C/r = 2$. Generations: 4/3 added to each rate, three times; zero in every difference. Higgs doublet: $b_1 - b_2 = 22/3 - 1/15 = 218/30 = 109/15$; $b_2 - b_3 = 11/3 + 1/6 = 115/30 = 23/6$. Ratio: $218/115 = 1.89565$. Kernel form: $2 \times (109/110) \times (22/23) = 218/115$, with $110 = n_C\, H\, (O^2 + n_C)$ and $22 = n_C(O^2 + n_C)$.

Result.

The one-loop spacing ratio is $218/115 = n_C(1 - 1/110)/(1 + 1/22)$, fixed by the gauge self-actions and the one Higgs doublet, for any couplings at which $X_{12} \neq X_{23}$.

The gap that remains at the first crossing

At the first crossing, where $\alpha_1 = \alpha_2$, the gap between the weak and color inverse strengths is $D_{23}(X_{12}) = D_{23}(M_Z) - (115/218)\, D_{12}(M_Z)$. Between the Z mass and the first crossing, the weak and color gap shrinks by $115/218 = 0.5275$ of the initial gap between the phase force and the weak force. The coefficient 115/218 is the ratio of rate differences $(b_2 - b_3)/(b_1 - b_2)$, and it depends on the rates alone (and on no measured coupling).

The share of the weak and color gap at the Z mass that remains at the first crossing depends on the couplings: it is 0.2645 at one loop and 0.2301 at three loops. What is exact is the rate ratio 115/218.

Box II-2. The gap that remains at the first crossing

Premises.

(1) One-loop running of a gap, Box I-1: $D_{ij}(\mu) = D_{ij}(M_Z) - (B_{ij}/2\pi)\ln(\mu/M_Z)$, with $B_{ij} = b_i - b_j$. (2) $B_{12} = 109/15$ and $B_{23} = 23/6$ (Box II-1). (3) At the first crossing $D_{12} = 0$, so $\ln(\mu_{12}/M_Z) = 2\pi\, D_{12}(M_Z)/B_{12}$.

Steps.

$D_{23}(\mu_{12}) = D_{23}(M_Z) - (B_{23}/2\pi)(2\pi\, D_{12}(M_Z)/B_{12}) = D_{23}(M_Z) - (B_{23}/B_{12})\, D_{12}(M_Z)$. $B_{23}/B_{12} = (23/6)/(109/15) = 115/218 = 0.52752$. The remaining share $D_{23}(\mu_{12})/D_{23}(M_Z) = 1 - (115/218)\, D_{12}(M_Z)/D_{23}(M_Z)$ contains the measured gaps.

Result.

The coefficient 115/218 is fixed by the content; the remaining share is 0.2645 at one loop and 0.2301 at three loops, a value of the measured couplings.

The modal reading and what the ratio tests

In the focus of comparison as relation the three forces carry the three modalities, which the book of kinds defines in its Chapter 1 as the three positions of comparison: color the omniscient, the weak force the transcendent, the phase force the immanent (the book of kinds, Chapter 4, Sections I and III). These bindings hold on both sides of the Higgs condensate, the constant value of the Higgs field that breaks the weak symmetry: color the omniscient, the weak force the transcendent, the phase force the immanent. Every modal assignment is specific to its frame, and the frame here is the focus of comparison as relation. In that frame the Higgs field breaks the transcendent's symmetry, SU(2), and leaves the omniscient's symmetry, SU(3), intact. It therefore changes the transcendent's running more than the omniscient's, and it lowers the spacing ratio from 2 to 218/115, a change of about 5 percent.

Since 218/115 is a standard one-loop result of the Standard Model's content, the agreement of the measured crossings with it is consistency with that content, and it holds equally for the Standard Model and for the structure.

In this chapter the *pattern* of the mismatch among the three couplings is the part that follows from the content alone: the spacing ratio 218/115, the coefficient 115/218, and the place of the $\alpha_1 = \alpha_3$ crossing between the other two. The pattern is determined by the content, and the magnitude of the gaps depends on the measured couplings at the Z mass. The order of μ_{12} and μ_{23} and the absence of a common point follow from the measured ratio D_{12}/D_{23} at the Z mass (Section III), and so they lie outside the pattern in this sense.

III. Where the Crossings Are

The places of the crossings are values: they are read from the measured couplings run upward. Run at three loops from the couplings of the PDG 2024 compilation, the three crossings lie at 54.259 halvings ($\alpha_1 = \alpha_2$), 58.200 halvings ($\alpha_1 = \alpha_3$) and 65.521 halvings ($\alpha_2 = \alpha_3$) above the electron's mass. Their spacing ratio is 1.8581 ± 0.0006, where 0.0006 bounds its change across the measured ranges of the couplings; the one-loop ratio is 1.8957.

At one loop, with inverse strengths at the Z mass of 59.0 (phase force, in the unification normalization), 29.6 (weak force) and 8.5 (color), rounded from Box II-1 of Chapter 2 and $\alpha_s(M_Z) = 0.1180$, the crossings lie near $10^{13.0}$, $10^{14.4}$ and $10^{17.0}$ GeV, in the order $\mu_{12} < \mu_{13} < \mu_{23}$, spread over nearly four orders of magnitude, and the three couplings do not meet at a single scale. Converted with the electron's mass, the three scales stand at about 54.1, 58.8 and 67.4 halvings, near the one-loop places of Table I-1 (54.16, 58.72 and 67.36).

The laws of this chapter are stated of the three-loop places.

The middle place follows from the content alone. At one loop $X_{13} = (218\ X_{12} + 115\ X_{23})/333$, the weighted mean of the other two places, so μ_{13} lies between μ_{12} and μ_{23} for every value of the couplings.

Which of μ_{12} and μ_{23} is the lower, and whether the three meet, follow from the measured couplings. The first crossing lies below the last exactly when D_{12}/D_{23} at the Z mass is smaller than 218/115, and the inverse strengths at the Z mass stated in this section give $D_{12}/D_{23} = 29.4/21.1 = 1.39$. The directions of change are fixed by the signs of the rates; whether two couplings meet depends on their starting values, which are read from measurement and are not fixed by structure. This book reads "not fixed by structure" as "not fixed by the counts of the structure", and not as "not fixed by reason": no parameter is fundamentally inaccessible, and every value of the Standard Model reaches one of the two ends of Chapter 1, Section I, so the values are closed under reason.

Box III-1. The order of the three places

Premises.

(1) $X_{ij} - X_Z = 2\pi\, D_{ij}(M_Z)/(B_{ij} \ln 2)$, Box I-1. (2) $B_{12} = 109/15$, $B_{23} = 23/6$ (Box II-1); $B_{13} = B_{12} + B_{23} = 111/10$. (3) Inverse strengths at the Z mass, as Section III states them: 59.0, 29.6, 8.5.

Steps.

From the linear relation of Box I-1: $X_{13} - X_Z = (B_{12}\, L_{12} + B_{23}\, L_{23})/B_{13}$, and $B_{12} + B_{23} = B_{13}$, so $X_{13} = (218\, X_{12} + 115\, X_{23})/333$, a weighted mean. $X_{12} < X_{23}$ exactly when $D_{12}/B_{12} < D_{23}/B_{23}$, that is, when $D_{12}/D_{23} < 218/115$. Gaps at the Z mass: $D_{12} = 29.4$, $D_{23} = 21.1$, $D_{13} = 50.5$; $D_{12}/D_{23} = 1.39$. Logarithms of μ_{ij}/M_Z: $2\pi \times 29.4/7.267 = 25.4$; $2\pi \times 50.5/11.1 = 28.6$; $2\pi \times 21.1/3.833 = 34.6$.

Result.

The middle place is fixed by the content; the order $\mu_{12} < \mu_{13} < \mu_{23}$ and the absence of a common point follow from the measured gaps.

The measured couplings draw close and stay apart

Run upward with the Standard Model's own rates, the three measured couplings draw close to one another, and they meet at no scale. The rank order of the three at low energy (color strongest, then the weak force, then elec-

tromagnetism) is fixed by structure, and the structure needs the three only nearly equal at the finest resolution (as distinct from equal) (*The Story of the Particles*, Level 7, Section V, and note 19).

Axiom III gives a reading of this in two of its clauses: the three positions of comparison, and so the three forces that carry them in the focus of comparison as relation (Section II), are inseparable and distinct (*An Immanent Metaphysics*, aphorism 1.3-1, which Chapter 5, Section III, quotes). Because the three forces are inseparable, their strengths draw close; because they are distinct, they never become one force. Inseparability produces nearness; distinctness produces the gap.

Run from their measured values at the Z mass to 5.3×10^{18} GeV with the Standard Model's rates, the three inverse strengths are about 34 (phase force, in the unification normalization), 49 (weak force) and 51 (color). The scale 5.3×10^{18} GeV lies 1.2 halvings below the unreduced Planck mass (the Planck mass without the factor $1/\sqrt{(8\pi)}$); the Planck mass meant everywhere else in this book is the reduced one (Chapter 1, Section II, and Chapter 5), and the scale lies about 1.1 halvings above it, by the arithmetic of the two masses. The spread of the three is below about twenty units of inverse strength, the spread the rank order at low energy needs when the running is taken downward from that scale; and the three are not equal.

Closing the gap of 17.6 units between the phase force's and color's inverse strengths over the 1.2 halvings below the unreduced Planck mass would need a difference of rates of about 133, whereas the whole content supplies 11.1: the difference of rates there would have to be about twelve times the difference that the whole content supplies at every lower scale.

In one-loop Standard Model running, the gauge couplings g_2 and g_3 of the weak force and of color (with $\alpha = g^2/4\pi$) agree to 0.007 percent at 10^{17} GeV.

Two statements hold throughout this chapter: the gauge group has no unifying group, and the measured couplings meet at no scale. The gauge group at every scale is the product of the three factors U(1), SU(2) and SU(3), with their twelve gauge bosons and no others, and no simple Lie group is the gauge group at any scale (the book of kinds, Chapter 4, Section I). *The Story of the Particles*, this book's authority on which particles exist, holds exactly three

forces, one for each position ("Three positions, three forces"), and no single transformation that turns a quark into a lepton; so no added particles of a unifying theory exist, and none is needed.

The Story of the Particles states the same of the meeting: with the particles it lists, and no others, the measured couplings meet at no scale. The equality the structure asserts is the equality of the three traces over the finite space of states, $g_3^2 = g_2^2 = (5/3)\, g'^2$, with g' the hypercharge coupling in the normalization Y (in the unification normalization of this chapter, $g_3^2 = g_2^2 = g_1^2$), a relation among the weights the structure gives the three forces (as distinct from an equality of measured strengths).

The spacing ratio 218/115, the coefficient 115/218 of the remaining gap and the order $\mu_{12} < \mu_{13} < \mu_{23}$ follow, with no adjusted parameter, from the content and the measured couplings, and the crossing places computed from the measured couplings agree with them. The three couplings do not meet in the minimal Standard Model (the Standard Model with one Higgs doublet and no added particles), the order is as stated, and at the first crossing the remaining mismatch is about 5 to 6 units of inverse strength (5.58 at one loop, with $1/\hat{\alpha}_{em}(M_Z) = 127.951$, $\sin^2\theta_W(M_Z) = 0.23122$ and $\alpha_s(M_Z) = 0.1179$, the Particle Data Group 2022 value). These values agree with measurement because of standard one-loop running with the measured couplings.

The book of kinds derives the content, and the pattern is a prediction of that content, at the scope of consistency with Standard Model running: the ratio follows from the derived content, and the magnitude of the remaining gap follows from the measured couplings.

Table III-1. The crossing scales and places.

Crossing	One-loop scale	One-loop place	Three-loop place
$\alpha_1 = \alpha_2$	$10^{13.0}$ GeV	54.16	54.259
$\alpha_1 = \alpha_3$	$10^{14.4}$ GeV	58.72	58.200
$\alpha_2 = \alpha_3$	$10^{17.0}$ GeV	67.36	65.521

IV. The First Crossing and the Weak Angle

The *weak angle* θ_W is the angle of the mixing by which the hypercharge boson and the neutral weak boson combine into the photon and the Z. At every scale $\alpha_2 = \alpha_{em}/\sin^2\theta_W$ and $\alpha_1 = (5/3)\ \alpha_{em}/\cos^2\theta_W$, where α_{em} is the fine-structure strength (Chapter 2). The *finest resolution* is the limit of resolution at which the structure gives no force a greater weight than another.

At the finest resolution the structural weak angle, the angle the equal weights give, has $\sin^2\theta_W = 3/8$: the equal traces over the finite space of states, ${g_3}^2 = {g_2}^2 = (5/3)\ g'^2$, with g' the hypercharge coupling in the normalization Y (in the unification normalization of this chapter, ${g_3}^2 = {g_2}^2 = {g_1}^2$), give it, and it carries no measured number (the book of kinds, Chapter 5, Sections IV and V). Its reason is that the trace ratio and the triangle's ratio are one formula, and that 3/8 is the leptons' share of charge squared (the book of kinds, Chapter 5, Section V). This chapter states where the running weak angle meets it.

The place of the first crossing depends on the weak angle alone. Since $1/\alpha_2 - 1/\alpha_1 = (8/5)(\sin^2\theta_W - 3/8)/\alpha_{em}$ at every scale, this gap depends on the couplings only through $(\sin^2\theta_W - 3/8)/\alpha_{em}$, with the constant factor 8/5, and the first crossing, where the gap is zero, is the scale at which $\sin^2\theta_W = 3/8$.

The first crossing X_{12} is therefore the one scale at which the running weak angle equals the structural one. The structural 3/8 is the squared cosine of the fixed angle between the charge direction $\hat{Q}$ and the isospin direction $\hat{T}_3$ in the charge space of one generation. So the first crossing is where the running $\sin^2\theta_W$, the running squared direction cosine of the light way (the electromagnetic combination), equals the fixed squared cosine of charge against isospin, 3/8.

At the first crossing $\tan^2\theta_W = \mathrm{Tr}\ {T_3}^2/\mathrm{Tr}\ Y^2 = O/H = 3/5$. So $\sin^2\theta_W = 3/8$ and $\cos^2\theta_W = 5/8$ are O and H over their sum, O + H = 8.

At the first crossing, where $\alpha_1 = \alpha_2 = \alpha$, the fine-structure strength is $1/\alpha_{em} = 1/\alpha_2 + (5/3)/\alpha_1 = ((O + H)/O)/\alpha = (8/3)/\alpha$. With the three-loop value of the common inverse strength at the first crossing (Section V), $1/\alpha = 42.25163 \pm 0.00390$, this gives $1/\alpha_{em} = 112.671 \pm 0.010$ there.

The factor 8/3 has three writings: $(O + H)/O$, the triangle's writing, from the normalization of the traces; Tr Q^2 over one generation counted as Dirac fields (each charged particle and its antiparticle counted as one field); and Tr Q^2/Tr ${T_3}^2$ counted as Weyl fields (each handedness counted as its own field), which is the inverse of the leptons' share of charge squared. The three coincide because Tr ${T_3}^2$ over a generation equals $n_C = 2$, which is the one condition $N_c + 1 = 2 \times 2$ on the number of colors N_c (the book of kinds, Chapter 5, Section V). The trace ratio and the triangle's ratio are one formula (the book of kinds, Chapter 5, Section V), so the three writings agree for one reason; they are kept side by side with their sources, the normalization of the traces for the first and the charges of one generation for the second and third.

Box IV-1. The first crossing and the weak angle; the values at the crossing

Premises.

(1) Standard physics: $\alpha_2 = \alpha_{em}/s^2$ and $\alpha_1 = (5/3)\ \alpha_{em}/(1 - s^2)$, with $s^2 = \sin^2\theta_W$. (2) Standard physics: $1/\alpha_{em} = 1/\alpha_2 + (5/3)/\alpha_1$ at every scale. (3) The traces of one generation: Tr Y^2/Tr ${T_3}^2 = 5/3 = H/O$; Tr $Q^2 = 16/3$ (Weyl); Tr ${T_3}^2 = 2$ (Weyl) (the book of kinds, Chapter 5, Section V). Counted as Dirac fields, as this chapter counts them, Tr $Q^2 = 3 \times 4/9 + 3 \times 1/9 + 1 = 8/3$ (the up quark in three colors, the down quark in three colors, the charged lepton; the neutrino has no charge).

Steps.

$1/\alpha_2 - 1/\alpha_1 = [s^2 - (3/5)(1 - s^2)]/\alpha_{em} = (8/5)(s^2 - 3/8)/\alpha_{em}$. The zero of this gap is $s^2 = 3/8$; the gap depends on the couplings only through $(s^2 - 3/8)/\alpha_{em}$, with the constant factor 8/5. $\tan^2\theta_W = (3/5)\ \alpha_1/\alpha_2 = 3/5$ at $\alpha_1 = \alpha_2$; $\sin^2 = 3/8$, $\cos^2 = 5/8$. $1/\alpha_{em} = (1 + 5/3)/\alpha = (8/3)/\alpha$; $(16/3)/2 = 8/3 =$ Tr Q^2(Weyl)/Tr ${T_3}^2$, which needs Tr ${T_3}^2 = (N_c + 1)/2 = 2$. With $1/\alpha = 42.25163 \pm 0.00390$: $1/\alpha_{em} = 112.671 \pm 0.010$.

Result.

The running weak angle equals 3/8 at the first crossing and at no other scale; there $\tan^2\theta_W = O/H$ and $1/\alpha_{em} = (8/3)/\alpha$.

The occurrences of 3/8

The number 3/8 occurs in nine statements, in four writings, and it is one number in all nine. The four writings are $O/(O + H)$, A/N^2 (with $A = 6$ the triangle's area), $O/{n_C}^3$ and $c_1/({c_1}^2 - 1)$, which coincide only at $c_1 = 3$; the last is the number of colors over the number of gluons, and it equals Tr ${T_3}^2$/Tr Q^2 over one generation in the equal-trace normalization. Three of the nine statements are this chapter's: the first crossing, as the zero of $3/8 - \sin^2\theta_W(\mu)$; the squared cosine of the angle between the charge and isospin directions of a generation; and the lepton doublet's appearances, 3 of 8, in the projections of the content at the third scale of *The Story of the Particles*, whose

partition is color. The ratio O/(O + H), the charged lepton's share of a generation's charge squared and the weak angle at the equal traces are in the book of kinds, Chapter 5, Section V; the ratio P_l/P_q of the step costs of the mass ladder, the charged leptons' share of the appearances of content in one generation at three colors, and the lepton share $S_l = O/8$ carried by the Higgs edge are in Chapter 7.

The running downward from 3/8

Starting the running from 3/8 at the first crossing, near 10^{13} GeV, returns $\sin^2\theta_W = 0.2314$ at the Z mass. That value restates the place of the crossing: the first crossing is defined by $\alpha_1 = \alpha_2$, where the angle is 3/8 by the relation of Box IV-1, so the value at the Z mass and the place of the crossing are one fact.

The measured $\sin^2\theta_W$ at the Z mass is 0.23122 ± 0.00006 (PDG 2026, MS-bar). Set side by side with no running computed between them, 3/8 (the ratio of equal weights) and the measured 0.2312 at the Z mass are the two ends that the running of this section joins; the book of kinds, Chapter 5, Section V, gives the 3/8 its reason.

The ratio 3/8 is group-theoretic, and it holds on the content instance, the internal space read as a space of states with no size (the book of kinds, Chapter 3, Section III). In unified theories 3/8 is the boundary value of the weak angle at the unification scale. The 3/8 of the equal traces is, in the words of *The Story of the Particles* and of the book of kinds (Chapter 5, Section IV), a relation among weights (as distinct from an equality of measured strengths); the structure imposes no grand-unification relation ($\sin^2\theta_W = 3/8$ at a unification scale).

At the resolution where color and the weak force meet, the phase force's share, the running $\sin^2\theta_W = \alpha_{em}/\alpha_2$ at that resolution, is about 0.436 at one loop with measured inputs, against $3/8 = 0.375$ for equal strengths. Given that share together with the strength and the place of the meeting, α_{em}, α_s and the weak angle follow through the rates. The share is read from measurement.

V. The Level and the Place of the First Crossing

The *level* of the first crossing is the common inverse strength $1/\alpha_1 = 1/\alpha_2$ there, written $1/\alpha(X_{12})$. A *matching* is the step that fixes the couplings of the running, in the MS-bar scheme (written with a hat, as in $1/\hat{\alpha}_{em}$), at the top quark's mass from measured quantities. The program SMDR (Standard Model in Dimensional Regularization, of Martin and Robertson) takes the W mass as an output of the Standard Model; the matching of Buttazzo and collaborators takes the W mass as an input; both include corrections through next-to-next-to-leading order (NNLO), and a *tree matching* includes no loop corrections. $\Delta\alpha_{had}$ is the contribution of hadrons to the running of the fine-structure strength up to the Z mass.

The *block* is three laws taken together: light's inverse strength at the proton's mass equals 3π times the split $\Delta_{21}(E)$ at the span's end (the relation of Chapter 2, with its unit 3π); the ratio of the splits is $\rho = H$ (Section VI); and color's binding join with slot $c = H$, the law $1/\alpha_3(E) = (7 \ln 2/2\pi)(E - x_p + c)$ with $x_p = \log_2(m_p/m_e)$ the proton's place (Chapter 6). A *closure* of the block is the joint solution of its laws: in the *first closure* the proton-to-electron mass ratio is measured; in the *second closure* a law of the level is added, so that nothing in the block is measured. The *window* of a quantity is the range of its values for which the block's outputs stay within $|\text{pull}| \leq 2$; a value *passes* when every output it gives lies within $|\text{pull}| \leq 2$, and *fails* otherwise.

The level is read from measurement. Run at three loops from the measured couplings, with SMDR started from the updated inputs and with the W mass taken as the Standard Model's output, 80.3498 GeV, the level is 42.25163 $\pm$ 0.00390. The couplings entered, the updated inputs, are $1/\hat{\alpha}_{em}(M_Z) = 127.951 \pm 0.009$ (MS-bar, Particle Data Group 2022, electroweak review), $\sin^2\theta_W(M_Z) = 0.23122$ (MS-bar, Particle Data Group 2026) and $\alpha_s(M_Z) = 0.1180 \pm 0.0009$ (PDG 2024), with $\Delta\alpha_{had} = 0.02783$ (the same review) and, from PDG 2024, $M_t = 172.57$ GeV and $M_h = 125.20$ GeV.

The other determinations stand beside it: 42.25358 with SMDR started from its reference inputs, before the updated value of $\Delta\alpha_{had}$; 42.26925 with the Buttazzo matching; and 42.24 at two loops. The difference of +0.0157

between the Buttazzo matching and SMDR at its reference inputs splits into +0.0071 from the W mass and +0.0086 from the change of the $\Delta\alpha_{had}$ input; the top mass, α_s and the Higgs mass each contribute less than 0.0001. The part of the difference that belongs to the two matchings is carried by the W mass.

The level moves by +0.0161 for each 10 MeV of the W mass. The gap between the measured W mass of the PDG 2024 compilation, 80.3692 $\pm$ 0.0133 GeV, and the Standard Model's 80.3498 GeV is worth +0.0311 in the level. Against the PDG 2026 measured W mass, 80.3625 $\pm$ 0.0077 GeV, the same sensitivity gives +0.0204. At equal W mass the formulas of the two matchings differ by -0.0135 in the level, about 8.4 MeV of W mass, and reading the masses of the Buttazzo matching as pole masses (the masses of free particles) moves its level by +0.033 to +0.041, further from SMDR.

Table V-1. Determinations of the level $1/\alpha(X_{12})$.

Running and matching	**Inputs**	**Level**
Three loops, SMDR	Updated inputs; M_W the Standard Model output 80.3498 GeV	42.25163 $\pm$ 0.00390
Three loops, SMDR	reference inputs, before the updated $\Delta\alpha_{had}$	42.25358
Three loops, Buttazzo	M_W = 80.3692 GeV as input	42.26925
Two loops	measured couplings at M_Z	42.24

What fixes the level

The level is read from measurement; no law of this book fixes it. The laws of scale of the block are two independent laws (the relation of Chapter 2 and the law of ρ) together with this level. *The Story of the Particles* (cited in the quotation as "the zoo") fixes no value of a gauge coupling ("none of these are produced by the zoo"), so the level does not take its value from it.

The value $169/4 = (13/2)^2 = 42.25$ stands at -0.42σ against the measured level 42.25163 ± 0.00390, and the second closure of the block (Section VI) takes it as its law of the level.

The place of the first crossing

The place of the first crossing has two routes, kept side by side. The first is the measured couplings run upward at three loops: $X_{12} = 54.259 \pm 0.014$ halvings, a *displacement* (a distance above a whole-number fraction of the count $72 = 8 \times 9$ of Chapter 5) of $+0.259 \pm 0.014$ above $54 = (O/N) \times 72$, three quarters of 72. The second is the relation of Chapter 2 for light's inverse strength at the proton, with its unit 3π and the span's end E, solved for X_{12}; light's strength is read at the proton's mass (Chapter 2, Section II), and the relation's left side T is light's MS-bar inverse strength $1/\alpha_{em}$ taken at 1.035 GeV, 0.142 halvings above the proton's mass, the scale at which $1/\alpha_{em}$ equals the relation's right side (Chapter 2). At one loop the gap $1/\alpha_2 - 1/\alpha_1$ grows from zero at X_{12} by $(109/15)\ln 2/(2\pi)$ per halving, so $T = 3\pi \times (109/15)\ln 2/(2\pi) \times (E - X_{12}) = 7.5553\,(E - X_{12})$. With $T = 134.07 \pm 0.12$ and $E = 72.0132$, this gives $X_{12} = 54.268 \pm 0.016$, a displacement of $+0.268 \pm 0.016$. The two displacements agree at 0.4σ. The two routes rest on different measurements: the first on the couplings measured at the Z mass, run upward; the second on light's strength measured near the proton's mass and the span from the electron to the reduced Planck mass, read through the relation of Chapter 2. Two independent routes meet on one place.

With the three-loop places the same relation gives $T = 134.14$, a pull of $+0.6\sigma$ against 134.07 ± 0.12, and with $E = 72$ exactly it gives 134.04, a pull of -0.3σ. The span's remainder of 0.0132 halvings is worth 0.10 in light's inverse strength, below the precision of the block.

Box V-1. Two routes to the place of the first crossing

Premises.

(1) Route of the running: $X_{12} = 54.259 \pm 0.014$ halvings at three loops (PDG 2024 couplings). (2) Route of the relation (Chapter 2): $T = 3\pi\ (1/\alpha_2 - 1/\alpha_1)\ (E)$, where $T = 134.07 \pm 0.12$ is light's MS-bar inverse strength at 1.035 GeV, near the proton's mass. (3) One-loop growth of $1/\alpha_2 - 1/\alpha_1$ from zero at X_{12}: $(b_1 - b_2) \ln 2/(2\pi)$ per halving, $b_1 - b_2 = 109/15$. (4) $E = \log_2(M_{Pl,red}/m_e) = 72.0132$ (Chapter 5).

Steps.

$T = 3\pi \times (109/15) \times \ln 2/(2\pi) \times (E - X_{12}) = (109/10) \ln 2 \times (E - X_{12}) = 7.5553\ (E - X_{12})$. $E - X_{12} = 134.07/7.5553 = 17.745$; $X_{12} = 54.268 \pm 0.016$. Displacements above 54: $+0.259 \pm 0.014$ and $+0.268 \pm 0.016$; difference 0.009, combined standard deviation 0.021, 0.4σ.

Result.

The running and the relation place the first crossing at the same displacement above three quarters of 72, within 0.4σ.

The first crossing lies 0.259 halvings above 54 = 3/4 of 72, and the last crossing lies 0.479 halvings below 66 = 11/12 of 72.

The map of wholes of Chapter 13 sets three scales beside whole-number fractions of the count 72, and the first crossing's displacement is one of its three residues, the distances of those scales from their fractions. The condensate's whole (the scale of the Higgs field's constant value divided by $\sqrt{2}$, 18.378 halvings above the electron's mass) lies 0.378 halvings above the quarter point 18 = 72/4; the first crossing lies 0.259 above the three-quarter point 54; the span lies 0.0132 above the whole 72. Their signed sum is the difference between the condensate's whole above the electron's mass and the first crossing's distance below the span's end. With the three-loop place the distance is 17.754 and the sum is $0.378 + 0.259 - 0.0132 = 0.624$. The sum equals the difference $18.378 - 17.754 = 0.624$, and the equality of the sum and the difference is an identity of the map's definitions, so it holds at any place of the first crossing; the map uses the three-loop place 54.259, as every law of place of this chapter does.

VI. The Gaps at the Span's End

At the span's end $E = 72.013$ halvings, the three inverse strengths leave two *splits*, the gaps at that place: $\Delta_{21} = 1/\alpha_2 - 1/\alpha_1 = D_{21}(E)$ and $\Delta_{32} = 1/\alpha_3 - 1/\alpha_2 = D_{32}(E)$. Their ratio is written $\rho = \Delta_{21}(E)/\Delta_{32}(E)$.

A law of the ratio and a law of the places are one law. At one loop the split $\Delta_{ij} = 1/\alpha_i - 1/\alpha_j$ at any place X is $(b_j - b_i)(X - X_{ij}) \ln 2/(2\pi)$, so $\Delta_{21} = (b_1 - b_2)(X - X_{12}) \ln 2/(2\pi)$ and $\Delta_{32} = (b_2 - b_3)(X - X_{23}) \ln 2/(2\pi)$, and Δ_{21}/Δ_{32} at X equals $(218/115)(X - X_{12})/(X - X_{23})$. A law $\rho = H$ at the span's end is therefore the law of places $(E - X_{12})/(E - X_{23}) = 115H/218$ in the one-loop form.

Stated with the three-loop spacing ratio, as Section I requires, the law $\rho = H$ is written $(E - X_{12})/(E - X_{23}) = H/1.8581 = 2.691 \pm 0.001$. The measured ratio of the distances below the span's end is $17.754/6.492 = 2.735 \pm 0.068$, so the law has a pull of -0.65σ; the one-loop form $575/218 = 2.638$ has a pull of -1.43σ. Stated as a ratio of splits, the three-loop ratio at the span's end, 5.093 ± 0.128, gives $H = 5$ a pull of -0.73σ.

Box VI-1. The law of the ratio as a law of places

Premises.

(1) Box I-1: $\Delta_{ij}(X) = 1/\alpha_i - 1/\alpha_j = (b_j - b_i)(X - X_{ij}) \ln 2/(2\pi)$ at one loop, the split being zero at the crossing. (2) Spacing ratio: 218/115 at one loop; 1.8581 at three loops (Section III). (3) Three-loop places 54.259 and 65.521; E = 72.0132.

Steps.

$\Delta_{21}/\Delta_{32} = [(b_1 - b_2)(X - X_{12})]/[(b_2 - b_3)(X - X_{23})] = (218/115)(X - X_{12})/(X - X_{23})$. $\rho = H$ at $X = E$: $(E - X_{12})/(E - X_{23}) = 115 \times 5/218 = 575/218 = 2.638$ (one loop); $H/1.8581 = 2.691$ (three loops). Measured: $E - X_{12} = 17.754$, $E - X_{23} = 6.492$; ratio 2.735 ± 0.068. Pulls: $(2.691 - 2.735)/0.068 = -0.65\sigma$; $(2.638 - 2.735)/0.068 = -1.43\sigma$. Written back as a ratio of splits with the three-loop spacing ratio: $1.8581 \times 2.735 = 5.08 \pm 0.13$.

Result.

A law of ρ is a law of the two places measured from the span's end; $\rho = H$ stands at -0.65σ in its three-loop form.

Table VI-1. Determinations of $\rho = \Delta_{21}(E)/\Delta_{32}(E)$, with the pulls of H = 5 and of 21/4.

Determination	ρ	Pull of 5	Pull of 21/4
Three loops, PDG 2024 couplings	5.093 ± 0.128	-0.73σ	$+1.23\sigma$
SMDR, the ratio alone	5.150 ± 0.12	-1.2σ	$+0.83\sigma$
Two loops, by scheme	5.03 to 5.17	below the range	above the range
From the splits 14.2 and 2.75	5.16	H 3.2 percent below	21/4 1.7 percent above
Three-loop places, 1.8581 × 2.735	5.08 ± 0.13	-0.65σ (as places)	$+1.31\sigma$

In Table VI-1 the pulls of 21/4 in rows 1, 2 and 5, the comparison of 21/4 with row 4, and the ratio of row 5 follow by arithmetic from the values stated in the rows.

The three-loop determination uses the couplings of the PDG 2024 compilation, and its uncertainty of ± 2.5 percent comes mainly from α_s. The ratio equals $H = 5$ at $\alpha_s(M_Z) = 0.11725$, a pull of -0.83σ against the measured 0.1180 ± 0.0009 (PDG 2024).

The block and its closures

As a test of the measured ratio alone (5.150 ± 0.12 under SMDR), $\rho = H$ passes at -1.2σ. The first closure of the block closes with $\rho = 5.2792$ under SMDR or 5.2849 under the Buttazzo matching, each within a window of ± 0.044, and with 5.169 to 5.265 under the tree matchings. Used as an exact law in the first closure, $\rho = H$ gives $1/\hat{\alpha}_{em}(M_Z) = 127.614$ (-13.8σ) and $\sin^2\theta_W(M_Z) = 0.23071$ (-7.9σ) under SMDR, and it fails in all five schemes considered, among them the NNLO matchings SMDR and Buttazzo and the two tree matchings.

At two loops, in the first closure, the block gives $1/\hat{\alpha}_{em}(M_Z) = 127.753$ (-8.1σ), $\sin^2\theta_W(M_Z) = 0.23089$ (-5.1σ) and $\alpha_s(M_Z) = 0.11907$ ($+1.2\sigma$); in the second closure, with the law of the level added, it fails in every scheme ($m_p/m_e = 1863$ or 1757). The law whose exact use produces the pulls in $\sin^2\theta_W$ and $1/\alpha_{em}$ is $\rho = H$.

A finite choice that a contradiction or a large pull rules out as an exact law is carried as a hypothesis, with its outcomes stated. So $\rho = H$ is carried as a hypothesis, with its two outcomes stated: it passes as a test of the measured ratio alone in every determination of Table VI-1 that carries an uncertainty or a range (rows 1, 2, 3 and 5), and it fails as an exact law of the block in every scheme.

The count 21/4

The count 21/4 stands beside H. It is written $21/4 = \dim K \times (D - 3)/(D - 2)$ at $D = 10$, which equals $c_1(2c_1 + 1)/(c_1 + 1)$; it is a count of the product, not a radius, where $\dim K = 6$ is the dimension of the internal space $\mathbb{C}P^2 \times S^2$ and $D = 10$ is the dimension of its product with spacetime; the count is this book's, and it uses no radius, which the content instance does not have (the book of kinds, Chapter 1, Section XIII). It has a further writing, $21/4 = |b_3|/b_e = 7/(4/3)$, color's one-loop rate over the one-loop coefficient 4/3 of one Dirac fermion of unit charge, which is also $3\pi \times 7/(2\pi)/2$.

The chapter carries H and 21/4 side by side as values of ρ, with their pulls in each determination (Table VI-1, Table VI-2), and chooses neither; no statement of this book identifies the count 21/4 with ρ.

Table VI-2. H = 5 and 21/4 side by side.

Test	ρ = 5	ρ = 21/4
Measured ratio, three loops, 5.093 ± 0.128	-0.73σ	$+1.23\sigma$
Measured ratio, SMDR, 5.150 ± 0.12	-1.2σ	$+0.83\sigma$
First closure, SMDR	$1/\hat{\alpha}_{em}$ at -13.8σ; $\sin^2\theta_W$ at -7.9σ	closes
First closure, Buttazzo	fails	$1/\hat{\alpha}_{em}$ at -2.8σ
Second closure, $\alpha_s(M_Z)$ against 0.1180 ± 0.0009	fails in every scheme	0.11888 ± 0.00011, $+0.97\sigma$

In Table VI-2 the pulls of 21/4 against the measured ratio in rows 1 and 2 follow by arithmetic from the stated values, as in Table VI-1.

The second closure with 21/4

The second closure with 169/4 and 21/4 gives $m_p/m_e = 1827.7$ ($x_p = 10.8358 \pm 0.0102$, -0.65σ), with pulls of -1.3σ in $1/\hat{\alpha}_{em}$, $+0.7\sigma$ in $\sin^2\theta_W$ and $+0.9\sigma$ in α_s under SMDR.

With 21/4 the second closure predicts $\alpha_s(M_Z) = 0.11888 \pm 0.00011$, a pull of $+0.97\sigma$ against 0.1180 ± 0.0009 (PDG 2024).

The matching changes which values pass. SMDR takes the W mass as the Standard Model's output, and the Buttazzo matching takes it as an input; the two are carried side by side, each with the values that pass under it.

The statements of this chapter use the gauge couplings and the masses of the electron, the proton, the W, the Z, the top quark and the Higgs boson, and no neutrino mass or lepton mixing; the two alternatives this book carries for the neutrino masses and for lepton mixing therefore bear on none of them.

VII. What Is Derived, What Lands, What Decides

The chapter derives, from the content and with no measured value, the form of the law of place, the one linear relation that leaves two crossing places free, the spacing ratio 218/115 with the generations canceling, the middle place of the $\alpha_1 = \alpha_3$ crossing, and the coefficient 115/218 of the remaining gap. It derives that the place of the first crossing depends on the weak angle alone, so that the running angle equals the structural 3/8 there and at no other scale, with $\tan^2\theta_W = O/H$ and $1/\alpha_{em} = (8/3)/\alpha$ at that place.

The places of the crossings, the level of the first crossing, the order of the first and last crossings, the absence of a common point and the ratio ρ at the span's end are read from the couplings run at three loops. The three-loop places govern every law of place in the chapter.

The agreement of the measured pattern with 218/115 tests consistency with Standard Model running. The agreement of the two routes to the place of the first crossing, at 0.4σ, tests the relation of Chapter 2 written as a place. The law $\rho = H$, restated in three-loop places, stands at -0.65σ against the measured places; the count 21/4 stands at $+1.23\sigma$ against the three-loop ratio and closes the first closure under SMDR.

The chapter's reach is the gauge sector at the scales of the crossings. The neighboring chapters carry the rest: the rates and the relation at the proton's mass in Chapter 2, the weak angle at measured scales in Chapter 4, the span in Chapter 5 and the binding scale of the proton in Chapter 6.

Summary

On the premise that any triple is an orthonormal set, the three inverse strengths of the gauge forces are the three components of one running vector, and the places where two of them become equal carry the content in three ways. The spacing of the places carries the gauge forces' own action and the one Higgs doublet; the first place carries the weak angle of the equal traces, 3/8; and the gaps at the span's end carry the two places measured from that end, so that a law of the gaps and a law of the places are one law. The values themselves, the places, the level and the ratio of the gaps, are read from measurement, and the chapter states each with the running, the inputs and the matching that give it.

The question each measured agreement of this chapter carries is why it lands: the ratio of the splits at the span's end, near H = 5 and near the count 21/4 of the product, whose clue is that a law of the ratio is a law of the two places; and the first crossing 0.259 halvings above three quarters of 72, which two independent routes place at one point, whose clue is the map of wholes of Chapter 13.

KEY CONCEPTS

- **Crossing.** A scale at which two running inverse strengths are equal; its place is X_{12}, X_{13} or X_{23} in halvings.
- **Law of place.** $X_{ij} - X_Z = 2\pi\, D_{ij}(M_Z)/((b_i - b_j)\ \ln 2)$; two places free.
- **Spacing ratio.** $(X_{23} - X_{13})/(X_{13} - X_{12}) = 218/115$ at one loop; 1.8581 ± 0.0006 at three loops.
- **First crossing.** The one scale at which the running weak angle equals 3/8.
- **Level.** The common inverse strength at the first crossing, 42.25163 ± 0.00390 (three loops, SMDR, updated inputs).
- **Splits and ρ.** $\Delta_{21}(E)/\Delta_{32}(E)$ at the span's end; a law of ρ is a law of the places.
- **H = 5 and 21/4.** Two values of ρ carried side by side.



Chapter 4

The Weak Angle and the Masses of W and Z

CHAPTER OVERVIEW

This chapter states what the weak mixing angle is, how large it is at the scales where it is measured, and how large the masses of the W and Z bosons are. The angle is defined first, as the share of the electric strength in the weak strength, because every later section uses it. At the leading order of calculation (tree level), one Higgs doublet gives $m_W = m_Z \cos \theta_W$, so that the ratio $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W)$ equals one. The departure of ρ from one has its form from the way each carrier is corrected by the fermion partners of the weak doublets, and its size from calculation; the measured departure is the electroweak correction that tree level omits, almost all of it the top quark's loop. The carriers' share of the squared vacuum value v^2, and the carriers' law for the sum of their squared masses, belong with the division of v^2 (Chapter 13, Section V). At tree level the asymmetries of Z decays measured at the Z's mass depend on the angle alone, and light's relation of Chapter 2, read at the proton's mass, gives the angle at the Z within two standard deviations of measurement. The chapter closes with what the carriers' masses do: they make the weak force short-ranged and the first step of fusion in a star of protons exceedingly rare.

A word on comparisons, used throughout. Each measured value is stated with its source, and with its uncertainty where one is given; where the source names no compilation, the text says so.

I. The Angle Defined

The electroweak *gauge factors*, the groups SU(2) and U(1) of the book of kinds, Chapter 4, carry two couplings. The *weak coupling* g is the strength of the *turnings* of the weak doublet, the SU(2) transformations that rotate a doublet's upper and lower members into each other; the *hypercharge coupling* g' is the strength of the phase, the group U(1) of hypercharge Y. The neutral weak carrier is W_3 and the hypercharge carrier is B; the label T_3, the third component of weak isospin, is $+1/2$ for a doublet's upper member and $-1/2$ for its lower member. The *condensate* is the nonzero vacuum value of the Higgs field. The Story of the Particles (the *zoo*, this book's authority on which particles exist) states that the condensate gives mass to the W and Z bosons and leaves the photon massless. The photon is the combination of W_3 and B that leaves the condensate unchanged: the combination whose labels add to the electric charge, $Q = T_3 + Y$.

The photon's combination is a rotation of the pair (W_3, B) by an angle θ_W, the *weak mixing angle*: the photon is $A = \sin\theta_W W_3 + \cos\theta_W B$, and the tangent of the angle is the ratio of the couplings, $\tan\theta_W = g'/g$. The photon's coupling, the *electric coupling* e, is then $e = g \sin\theta_W = g' \cos\theta_W$. The inverse squares add: $1/e^2 = 1/g^2 + 1/g'^2$, so e is smaller than both g and g'. Given g and g', the electric coupling is fixed, so it adds no third independent strength. The Z carries the other combination, $Z = \cos\theta_W W_3 - \sin\theta_W B$, at right angles to the photon's, and its strength is $\sqrt{(g^2 + g'^2)}$. Among measured quantities the fine-structure constant $\alpha_{em} = e^2/(4\pi\hbar c)$ is fixed by e together with c and $\hbar$, and so it is a function of two of the three gauge strengths, g and g' (the third is the strong coupling).

Box I-1. The electric coupling from the photon's combination.

Premises. (1) The photon carries the combination of T_3 and Y with $Q = T_3 + Y$, which leaves the condensate unchanged (the zoo, Level 8). (2) g is the coupling of SU(2) and g′ the coupling of hypercharge (definitions). (3) The angle θ_W of that combination has $\tan \theta_W = g'/g$ (definition of θ_W). (4) The Z carries the orthogonal combination (standard physics).

Steps. With $\tan \theta_W = g'/g$, the photon's coupling is $e = g \sin \theta_W = g' \cos \theta_W$. Then $\sin^2 \theta_W = e^2/g^2$ and $\cos^2 \theta_W = e^2/g'^2$. Adding, $\sin^2 \theta_W + \cos^2 \theta_W = 1$ gives $1/e^2 = 1/g^2 + 1/g'^2$. The orthogonal combination has strength $\sqrt{(g^2 + g'^2)}$.

Result. $e = g \sin \theta_W = g' \cos \theta_W$; $1/e^2 = 1/g^2 + 1/g'^2$; $e < g$ and $e < g'$; the Z's strength is $\sqrt{(g^2 + g'^2)}$.

Written in strengths, $\alpha_i = g_i^2/(4\pi)$, with α_2 the weak strength and $\alpha_Y = g'^2/(4\pi)$ the hypercharge strength in the normalization in which $Q = T_3 + Y$, the same relation reads $1/\alpha_{em} = 1/\alpha_2 + 1/\alpha_Y$, and it gives the angle as a share. The weak angle is the share of the electric strength in the weak one: $\sin^2 \theta_W = \alpha_{em}/\alpha_2 = 1/(1 + \alpha_2/\alpha_Y)$. Chapter 2 states the same relation as a weighted mean of the two electroweak inverse strengths.

At the *equal traces*, the equality of the three forces' weights as traces over the finite space of states, $g_3^2 = g_2^2 = (5/3)g'^2$ with g′ the hypercharge coupling in the normalization Y (in the unification normalization, $g_3^2 = g_2^2 = g_1^2$), the angle has the value $\sin^2 \theta_W = 3/8$ (the book of kinds, Chapter 5, Section V). The value has its reason: the trace ratio and the ratio of the *kernel triangle* (the right triangle with sides 3, 4 and 5 of the book of kinds, Chapter 2) are one formula, and, at three colors, 3/8 is the leptons' share of the sum of the squared charges of one generation. The *running* of a coupling is its change with the scale at which it is evaluated; the running of the angle away from 3/8 is Chapter 3's.

II. The Masses at Tree Level

Tree level means the leading order of the calculation, before any correction from virtual states. The *vacuum value* v is the size of the condensate; measured through the Fermi constant, $v = (\sqrt{2}\, G_F)^{-1/2}$. The condensate is one Higgs doublet, a scalar on the *edge* (the one crossed edge of the structure), with two internal components and no color (the zoo, Level 8). From one doublet the carriers' masses at tree level are $m_W = g\, v/2$ for the charged carriers W^+ and W^-, and $m_Z = \sqrt{(g^2 + g'^2)}\, v/2$ for the Z.

So $m_W = m_Z \cos\theta_W$ at tree level, and the parameter $\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$ equals one. The reason is the doublet's symmetry: its two halves have equal standing, so the charged and the neutral carriers receive the same strength factor from it. Since $\cos\theta_W < 1$, the Z is heavier than the W.

Box II-1. $m_W = m_Z \cos\theta_W$ and $\rho = 1$.

Premises. (1) The condensate is one Higgs doublet, and its vacuum value gives mass to the W and Z (the zoo, Level 8). (2) The doublet's two halves have equal standing, so the charged and neutral carriers receive one strength factor (standard physics: the custodial symmetry of one doublet at tree level). (3) $e = g \sin\theta_W = g' \cos\theta_W$, and the Z is the orthogonal combination with strength $\sqrt{(g^2 + g'^2)}$ (Box I-1).

Steps. The mass table of one doublet's vacuum value is $m_W = g\, v/2$ and $m_Z = \sqrt{(g^2 + g'^2)}\, v/2$. Their ratio is $m_W/m_Z = g/\sqrt{(g^2 + g'^2)} = \cos\theta_W$.

Result. $m_W = m_Z \cos\theta_W$ and $\rho = 1$ at tree level; $m_Z > m_W$.

The carriers' masses and the couplings fix each other. The Z mass depends on v and on the couplings, and the couplings depend on the Z mass, at which they are read, through their running. On the content instance (the space of states, which has no size; the book of kinds, Chapter 3, Section III) this mutual fixing is a fixed-point iteration among numbers: take a Z mass, compute the couplings there, compute a new Z mass from v and the Z's coupling, and re-

peat; the iteration converges. Its starting scale is a parameter of the iteration; how the step of the mass ladder enters the Z mass it returns is set out with the ladder (Chapter 7, Section IX).

The structure fixes a relation among the three forces' weights, not an equality of their measured strengths (the zoo). Equal weights, normalized by their traces and taken as equal strengths at the finest resolution, do not give the Z mass: that assumption sets the inverse hypercharge strength at the upper end of the running equal to the other two inverse strengths, within their spread of 49 to 51, while on this book's relations the Z mass would need it near 36, and it gives m_Z near 89.2 GeV, against the measured value of about 91 GeV. The exact relation $m_W = m_Z \cos \theta_W$ at tree level stands.

III. The Departure from ρ = 1: Its Form, Then Its Size

A *radiative correction* is the change in a quantity that comes from virtual states. A W or Z boson, while it travels, briefly turns into a pair of fermions and back; that closed path, a *loop*, adds to the carrier's inertia, and so corrects its mass. In this book's terms, a carrier with mass is answered also by the least states of the fermions that answer its turning.

The charged carriers $W^{\pm}$ turn the upper member of a doublet into the lower one (an up-type quark into a down-type quark, a neutrino into a charged lepton), so the loops that correct them hold an upper and a lower partner together. The Z changes neither member into the other, so the loops that correct it hold each member with itself. Where an upper member and its lower partner have the same mass, the two corrections stand to each other as the condensate's own contributions to m_W^2 and m_Z^2 do, in the ratio $\cos^2 \theta_W$, and ρ stays one. Where the partners' masses differ, the two corrections differ, and ρ departs from one.

The departure of ρ from one is therefore a relation between m_W, m_Z and the mass differences between weak-doublet partners. It is largest for the heaviest generation, top and bottom, and it would vanish if every doublet's partners had equal mass. The departure is closed in form by the argument of this section; its size is calculation given the values of the masses.

Box III-1. The form of the departure from $\rho = 1$.

Premises. (1) $m_W = m_Z \cos \theta_W$ ($\rho = 1$) from the condensate alone (Box II-1). (2) The W and Z have inertia because the condensate answers them at every place (the zoo, Level 7, Section III, and Level 8, Section III: a carrier that rotates between states the vacuum tells apart acquires mass). (3) A carrier with mass is answered also by the least states of the fermions that answer its turning (a loop correction; standard physics). (4) The W turns an upper part into a lower one and is answered by both partners together; the Z is answered by each part with itself (standard physics). (5) Answers by partners of equal mass stand in the condensate's ratio; answers by partners of unequal mass do not (mathematics).

Steps. From (1), $m_W^2/(m_Z^2 \cos^2 \theta_W) = 1$ from the vacuum value alone. From (3) and (4), the fermion loops add to m_W^2 through pairs that mix upper and lower masses, and to m_Z^2 through each fermion with itself. From (5), the two additions keep the ratio one when the partner masses are equal and change it otherwise; the heaviest generation, top and bottom, changes it most.

Result. ρ departs from one through the partner mass differences, most for top and bottom, and not at all for equal partners; the size of the departure is a calculation given the masses.

The Standard Model states the same form by its own calculation: the departure of ρ from one is dominated by the difference between the top and bottom masses, its formula vanishes when the two masses are equal (Veltman, 1977), and smaller contributions come from the Higgs. The two statements rest on different sources: the first on comparison, the foundation the book of kinds builds from (its Chapter 1), applied to the carriers answered by the least states of their partners; the second on the Standard Model's loop calculation. The second is set beside the first and is used by nothing in it; the two are kept side by side as two routes to one form.

The size is measured. The *on-shell* definition reads the angle from the physical masses, $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$; the *MS-bar* definition reads it from the couplings renormalized by minimal subtraction at the Z mass. With the measured masses, $m_W \approx 80.37$ GeV and $m_Z \approx 91.19$ GeV, the ratio $m_W/$

$m_Z = 0.8814$ gives the on-shell $\sin^2\theta_W \approx 0.223$, against ≈ 0.231 from the couplings at the Z. The tree relation therefore holds to about one percent, the size depending on the definition of the angle.

The tree relation $m_W = m_Z \sqrt{(1 - \sin^2\theta_W)}$, evaluated with the measured $m_Z = 91.1880 \pm 0.0020$ GeV (PDG 2024) and the measured MS-bar angle 0.23122, gives $m_W = 79.95$ GeV. That is 0.52 percent below the measured 80.369 ± 0.013 GeV (PDG 2024, the Particle Data Group's Review of Particle Physics of 2024), and 0.51 percent below 80.3625 ± 0.0077 GeV (PDG 2026). The shortfall is the size of the electroweak correction that the tree relation omits.

Standard physics writes the MS-bar relation as $m_W^2 = \hat{\rho}\, m_Z^2\, \hat{c}^2$, where $\hat{c}^2$ is the MS-bar $\cos^2\theta_W$ and $\hat{\rho}$ is about 1.01, almost all of it the top's loop $\rho_t = 3G_F m_t^2/(8\sqrt{2}\,\pi^2)$; the electroweak review of the Particle Data Group gives $\hat{\rho} = 1.01016 \pm 0.00009$ (Particle Data Group 2024, electroweak review). On the inputs of the previous paragraph the measured m_W asks $\hat{\rho} = 1.0104$ (PDG 2024) and 1.0103 (PDG 2026). The departure of about one percent in m_W^2 and the half percent in m_W are one fact read two ways.

With the known top and bottom correction removed, the measured value is $\rho_0 = 1.00031 \pm 0.00019$ (Particle Data Group 2024, electroweak review), and the tree value $\rho = 1$ stands at -1.6σ against it. What remains of the departure, once top and bottom are removed, is three parts in ten thousand.

Table III-1. The size of the departure from $\rho = 1$.

Quantity	Stated	Compared with, and source (measured unless marked)	Comparison
$\sin^2\theta_W$, on-shell against MS-bar	0.223 (from $m_W/m_Z = 0.8814$)	≈ 0.231 from the couplings at the Z (Particle Data Group 2026: 0.23122 ± 0.00006)	about one percent, depending on the definition of the angle
m_W at tree level	79.95 GeV	80.369 ± 0.013 GeV (PDG 2024); 80.3625 ± 0.0077 GeV (PDG 2026)	0.52 and 0.51 percent below
$\hat{\rho}$ asked by the measured m_W	1.0104 (PDG 2024); 1.0103 (PDG 2026)	1.01016 ± 0.00009 (a Standard Model value, not a measurement: Particle Data Group 2024, electroweak review)	the omitted correction, almost all the top's loop
ρ_0, top and bottom removed	1 (tree)	1.00031 ± 0.00019 (Particle Data Group 2024)	-1.6σ

IV. The Weak Angle at Measured Scales

At the Z the angle is measured by the pole asymmetries. The *pole* is the peak of the Z resonance, at the Z's mass, and a *pole asymmetry* is an asymmetry of Z decays measured there. The asymmetry parameter A_f of a fermion f compares the Z's couplings to the fermion's left-handed and right-handed states (its two *chiralities*); the *forward-backward asymmetry* A_{FB}^{f} compares the fermions emitted forward and backward of the incoming electron's direction. They are pure ratios of the Z's couplings, so the overall normalization of those couplings (the ρ parameter) cancels out of them, and each asymmetry is a function of $\sin^2\theta_W$ alone at tree level. The asymmetries therefore probe the angle directly, through the ratio of each fermion's *vector coupling* v_f to its *axial coupling* a_f.

Box IV-1. The pole asymmetries depend on the angle alone.

Premises. (1) $A_f = 2v_f a_f/(v_f^2 + a_f^2)$, with the vector coupling $v_f = T_3 - 2Q_f \sin^2\theta_W$ and the axial coupling $a_f = T_3$; at the pole $A_{FB}^{\,f} = (3/4)A_e A_f$ (standard physics). (2) An overall normalization of the Z's couplings, the ρ parameter, multiplies v_f and a_f by one factor.

Steps. A_f is homogeneous of degree zero in (v_f, a_f), so a common factor cancels. With $r = v_f/a_f = 1 - 2Q_f \sin^2\theta_W/T_3$, $A_f = 2r/(1 + r^2)$, which for each fermion is a function of $\sin^2\theta_W$ alone.

Result. Every pole asymmetry is, at tree level, a function of $\sin^2\theta_W$ only; ρ cancels.

Two extractions of the angle from single asymmetries disagree with each other. From the forward-backward asymmetry of the bottom quark alone, $\sin^2\theta_W = 0.23223 \pm 0.00029$; from the lepton asymmetry A_l measured at SLD (the SLAC Large Detector), 0.23098 ± 0.00026 (the combination of the LEP and SLD experiments, Physics Reports 427, 257, 2006). They differ by 3.2σ.

This book evaluates each coupling at the mass of the thing it is compared against: a comparison's strength is read at the mass of what it is made against, and light, compared against the proton, is read at the proton's mass. That is the reason light's relation is read at m_p, its *reading scale*. Read there, the relation gives $\sin^2\theta_W(M_Z) = 0.23127$ to 0.23130, against the measured 0.23122 ± 0.00006 (Particle Data Group 2026): a residue of $+0.00005$ to $+0.00008$, which is 0.83σ to 1.33σ.

The residue lies within the uncertainty of the reading scale itself. At 0.00034 in $\sin^2\theta_W$ per halving of the reading scale, the residue equals 0.15 to 0.24 halvings, so the scale that would remove it lies 0.15 to 0.24 halvings above m_p. The reading scale is known within a window of ± 0.19 halvings about 10.98 halvings above the electron's mass, where m_p is at 10.842. Its two determinations are 1.035 GeV (0.14 halvings above m_p) and 1.111 GeV (0.24 halvings above m_p); the second carries throughout the hadronic part $\Delta\alpha_{had} = 0.02766$, the contribution of quarks bound in hadrons to the running of the electric coupling. The scale that would remove the residue lies inside

that window, and so does the scale $m_p \times 2^{1/4} = 1.116$ GeV, a quarter of a halving above m_p. The window depends on how the hadronic part $\Delta\alpha_{had}$ is carried.

The closure of *The Measures of the Kinds*, Chapter 13, Section V, gives a second value of the angle through the same relation: $\sin^2\theta_W(M_Z) = 0.231164 \pm 0.000036$, at -0.80σ against PDG 2026. The two values come from one relation, light's relation with its coefficient 3π (Chapter 2), and they are kept side by side, each with its premises.

V. The Z Width and the Pole Observables

The *Z width* Γ_Z is the spread of energies over which the Z is produced, which measures how fast it decays; each light neutrino opens one channel of decay and widens it. The width's main structural input is the count of light neutrinos, three, one per generation (the book of kinds, Chapter 7; the zoo, Level 7).

The values of this section are the Standard Model's own formulae evaluated at $\sin^2\theta_W = 0.23127$, which is their input. That input lies 0.00005 above the measured 0.23122: 0.83σ at the Particle Data Group 2026 uncertainty, ± 0.00006. The other inputs are $\alpha_s = 0.1179$, $m_t = 172.88$ GeV and $M_Z = 91.18$ GeV.

The tree-level Z width with its QCD and QED factors (the corrections of quantum chromodynamics, the theory of the strong force, and of quantum electrodynamics) is 2.487 GeV, 0.3 percent below the measured 2.4955 ± 0.0023 GeV (Particle Data Group 2024). The Standard Model's own electroweak form factor (its full electroweak correction to the partial widths), a net of about +0.3 percent once the larger $\Delta\rho$ of about one percent (the departure of ρ from one, Section III) is counted with the vertex terms (the corrections where the Z meets the fermion pair), closes the gap. The width is the Standard Model's tree formula at these inputs, so it lands where the Standard Model's own tree width lands, and the 0.3 percent is the electroweak form factor that the tree formula omits.

Of the seven pole observables, five lie within 2σ of measurement. The ratios R^0 are ratios of the Z's partial widths: R^0_{had} is its hadronic width over its width to one charged lepton, and R^0_b and R^0_c are its widths to bottom and to charm pairs over its hadronic width. The forward-backward asymmetry of the bottom quark is +3.4σ, the excess that the Standard Model also has. R^0_b is +2.0σ, from the incomplete vertex correction of the Z to a bottom pair: the full vertex gives 0.2170 (+1.0σ), and the Standard Model itself gives 0.2158 (−0.8σ). These are the Standard Model's own formulae at the angle used, so they land where it lands and miss where it misses. The MS-bar angle in the tree formulae, in place of the effective leptonic angle (the angle that the Z's corrected couplings to leptons define), raises A_e and with it the bottom quark's forward-backward asymmetry.

Table V-1. The Z width and the seven pole observables, Standard Model formulae at $\sin^2\theta_W = 0.23127$ (measured values: the combination of the LEP and SLD experiments, Physics Reports 427, 257, 2006; Γ_Z, Particle Data Group 2024).

Quantity	Stated	Measured	Pull
Γ_Z, tree with QCD and QED factors	2.487 GeV	2.4955 ± 0.0023 GeV	0.3 percent below
$A_{FB}(l)$	0.01667	0.0171 ± 0.0010	−0.4σ
$A_{FB}(b)$	0.10462	0.0992 ± 0.0016	+3.4σ
$A_{FB}(c)$	0.07474	0.0707 ± 0.0035	+1.2σ
A_l	0.14908	0.1513 ± 0.0021	−1.1σ
R^0_{had}	20.750	20.767 ± 0.025	−0.7σ
R^0_b	0.2177	0.2163 ± 0.0007	+2.0σ
R^0_c	0.1709	0.1721 ± 0.0030	−0.4σ

VI. What the Carriers' Masses Do

A carrier with mass relates contents only within its own length, $\hbar/(mc)$. The weak interaction is therefore short-ranged, reaching only about the Compton wavelength of the W and the Z, while the range of the electromagnetic interaction, whose carrier has no mass, is unbounded.

The first step of fusion from protons alone, two protons to a deuteron, needs one proton to be turned into a neutron while the two meet, because two protons do not hold together. Only the weak turnings turn a proton into a neutron, and at the energies of a star they are feeble because their carriers are held, that is, have mass. So the first step is exceedingly rare, and a star of protons burns for a very long time.

The zoo's running carries the measured weak coupling up to the inverse value 49 at 5.3×10^{18} GeV, the endpoint of that running, with the Standard Model's one-loop coefficient of the weak running, $b_2 = -19/6$. Run back down with the same coefficient, it gives $1/\alpha_2(80\ \text{GeV}) = 29.48$, $g = 0.653$ and $m_W = g\,v/2 = 80.3$ GeV with $v = 246$ GeV (band 79.1 to 81.6 GeV, which contains PDG 2026's 80.3625 ± 0.0077 GeV): a round trip of one law, which returns the measured W mass and is not an independent test.

Summary

The weak angle is the share of the electric strength in the weak strength, $\sin^2\theta_W = \alpha_{em}/\alpha_2$, with $e = g \sin\theta_W = g' \cos\theta_W$; it is 3/8 at the equal traces. One Higgs doublet gives $m_W = m_Z \cos\theta_W$ and $\rho = 1$ at tree level. Comparison and the Standard Model give one form for the departure of ρ from one, ruled by the partners' mass differences and largest for top and bottom; its measured size, half a percent in m_W, is the omitted electroweak correction. At tree level the pole asymmetries measure the angle alone, and light's relation read at the proton's mass gives it within two standard deviations. The carriers' mass makes the weak force short-ranged and the burning of a star of protons slow.

The question each measured agreement of this chapter carries is why it lands: the angle at the Z from light's relation read at the proton's mass (clue: the reading scale's window about m_p).

KEY CONCEPTS

- The weak angle is the share of the electric strength in the weak strength: $\sin^2 \theta_W = \alpha_{em}/\alpha_2$.
- One doublet with two halves of equal standing gives $m_W = m_Z \cos \theta_W$ and $\rho = 1$ at tree level.
- ρ departs from one where weak-doublet partners differ in mass; the size is calculation given the masses.
- At tree level the pole asymmetries are functions of the angle alone; ρ cancels from them.



Chapter 5

The Span to the Planck Mass

CHAPTER OVERVIEW

The electron is the unit of mass of this book. The reduced Planck mass is the mass M at which $8\pi G\, M^2/(\hbar c)$ equals one, $8\pi G$ being gravity's coupling in the field equations. A halving is a factor of two in mass, and the span between the two masses, counted in halvings, is measured: $E = \log_2(M_{Pl,red}/m_e) = 72.0132112 \pm 0.0000162$. The integer part is a count, $72 = 8 \times 9$: eight first-order states times nine joins. The identification of the span with that count is assumed, and the measurement is its test. Seventy-two halvings above the electron land 0.9 percent below the reduced Planck mass. The measured span exceeds the count by a remainder $r = E - 72 = 0.0132$.

A count measured against a continuous magnitude leaves a residue at every such measurement, by the Incommensuration Theorem. The first order of that residue is the one mismatch $\varepsilon = 11/(80\pi^2)$. The measured remainder is $0.948\ \varepsilon$. The second order is a constant k, measured at $k = 3.7120 \pm 0.0835$ once ε is affirmed. The third order, c_3, enters the remainder together with k. So a measurement of Newton's constant fixes k only together with a law for c_3. A second route, through the measured gauge couplings, returns the span within its own uncertainty.

I. The Measured Span

A *halving* is a factor of two in mass. The *halving count* of a mass M is $\log_2(M/m_e)$, its position above the electron in halvings, so the electron sits at 0. The *halving map* is the line of halving counts, with every mass placed at its own. The electron is the measured input of the particle side, the books on the particles and their measures, and it is the unit of mass of this book (Chapter 1, What Is Measured). This book cites The Kinds of Particle; Why the Structure Must Be So, which derives what kinds of particle there are, as "the book of kinds".

The *reduced Planck mass* is $M_{Pl,red} = (\hbar c/(8\pi G))^{1/2}$, with G Newton's constant. It is the mass M at which $8\pi G\, M^2/(\hbar c)$, gravity's coupling in the field equations taken on that mass, equals one; gravity's strength $G\, M^2/(\hbar c)$ equals one at the unreduced Planck mass, $\sqrt{(8\pi)}$ times larger, which The Gravitational Passage takes as the boundary of the two descriptions. The reduced Planck mass is meant throughout this chapter.

The *span* is the halving count of the reduced Planck mass, $E = \log_2(M_{Pl,red}/m_e)$. Rebuilt from the measured Newton's constant and the measured electron mass, $E = 72.0132112 \pm 0.0000162$ halvings; the span is given as CODATA 2022, and the whole uncertainty comes from G. In the test reading of the measured span, read upward from the electron, the count of 72 halvings runs from the electron's mass to the reduced Planck mass, and the measured span exceeds 72 by 0.0132 halvings.

Box I-1. The span from Newton's constant and the electron

Premises.

Newton's constant $G = 6.6743 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, with uncertainty 1.5×10^{-15} in the same unit, so $\sigma_G/G = 2.2 \times 10^{-5}$ (CODATA 2022). The electron mass $m_e = 0.51099895069(16)$ MeV (CODATA 2022). $M_{Pl,red} = (\hbar c/(8\pi G))^{1/2} = 2.435323 \times 10^{18}$ GeV from the measured G.

Steps.

$E = \log_2(2.435323 \times 10^{18}$ GeV / $5.1099895069 \times 10^{-4}$ GeV$) = 72.013211$. $M_{Pl,red}$ varies as $G^{-1/2}$. So the uncertainty of E is $(1/2)(1.5 \times 10^{-15}/6.6743 \times 10^{-11})/\ln 2 = 0.0000162$ halvings. The uncertainty of the electron mass adds nothing at this precision.

Result.

$E = 72.0132112 \pm 0.0000162$. The remainder is $r = E - 72 = 0.0132112 \pm 0.0000162$ halvings. Every quantity of this chapter derived from the span is computed from this E.

The unreduced Planck mass, $M_{Pl} = \sqrt{(8\pi)}\ M_{Pl,red}$, sits higher on the map, at 74.339 halvings above the electron, since $\log_2\sqrt{(8\pi)} = 2.326$. Against the unreduced Planck mass the count of 72 halvings misses by a factor of about five: $m_e \times 2^{72}$ is 0.198 of M_{Pl}, and it is 0.99 of $M_{Pl,red}$. The relation of 72 halvings holds with the reduced Planck mass.

Written as a strength, the electron's mass against the Planck mass is gravity's strength on the electron, $\alpha_G = G\ m_e^2/(\hbar c) = (m_e/M_{Pl})^2$, with the unreduced Planck mass. With $m_e/M_{Pl} = 4.19 \times 10^{-23}$, $\alpha_G = 1.75 \times 10^{-45}$. The fine-structure constant is $\alpha_{em} = 1/137.036 = 7.297 \times 10^{-3}$. So $\alpha_{em}/\alpha_G = 4.16 \times 10^{42}$, the ratio of the electric to the gravitational force between two electrons.

The test is written upward, from the electron. Seventy-two halvings above the measured electron reach $m_e \times 2^{72} = 2.4131 \times 10^{18}$ GeV, against the reduced Planck mass 2.4353×10^{18} GeV; the count lands 0.9 percent low. Writ-

ten as a value of Newton's constant, 72 exact halvings put the unreduced Planck mass at 1.2098×10^{19} GeV against 1.2209×10^{19} GeV. Since G varies as M_{Pl}^{-2}, they put G 1.8 percent above its measured value.

The particle side keeps the count 72 and uses the measured span as a test only. The chain of this book does not rest on any bridge from the electron to Newton's constant. The ground fixes the role of Newton's constant and not its magnitude (The Unquantized Arena, Proposition 9.4), and Newton's constant is the measured input of the side of gravitation. Where the best accuracy is wanted, the measured reduced Planck mass is used in place of the count.

The *vacuum value* v is the value at which the Higgs field stands in the vacuum (Chapter 13, The Vacuum Value, the Higgs and the Top). The ratio $v/M_{Pl,red}$ is a pure number. Its smallness, the *hierarchy*, is not derived in this book. This chapter states the span from the measured electron to the measured reduced Planck mass, and tests the count against it.

Table I-1. The measured span and the quantities read from it.

Quantity	Value	Measured inputs and source
Span $E = \log_2(M_{Pl,red}/m_e)$	72.0132112 ± 0.0000162 halvings	the span is given as CODATA 2022; m_e and G CODATA 2022
Remainder $r = E - 72$	0.0132112 ± 0.0000162 halvings	the same
Unreduced Planck mass, $\log_2(M_{Pl}/m_e)$	74.339 halvings ($E + 2.326$)	the same, with $M_{Pl} = \sqrt{(8\pi)}\, M_{Pl,red}$
$m_e \times 2^{72}$ against $M_{Pl,red}$	2.4131×10^{18} against 2.4353×10^{18} GeV (0.9 percent low)	m_e; $M_{Pl,red}$ from G (CODATA 2022)
G from 72 exact halvings	1.8 percent above the measured G	$M_{Pl} = 1.2209 \times 10^{19}$ GeV; $G = 6.6743 \times 10^{-11}$ m^3 kg^{-1} s^{-2}; both from CODATA 2022
α_G; α_{em}/α_G	1.75×10^{-45}; 4.16×10^{42}	$m_e/M_{Pl} = 4.19 \times 10^{-23}$; $1/\alpha_{em} = 137.036$; both from CODATA 2022

II. The Count and the Test of the Count

Stand on a mountain and look down. The houses in the valley below are small, not because they are small (each is as large as a house anywhere) but because they are far away. The distance between the mountain and the valley makes the houses look tiny. Now imagine someone in the valley looking up at the mountain. The figure on the ridge is small too, barely visible. The same distance that makes the houses look small from above makes the figure on the ridge look small from below. The distance is real. The smallness is perspective.

Asking "why is the electron so light?" is like asking "why is the valley so deep?" One answer is: count the steps. There are 72 of them, and a little more.

The count changes the shape of a problem. The span between the reduced Planck mass and the electron becomes a count: 72, eight first-order states times nine joins, read as halvings. That the measured span is this count is an identification, and measurement tests it: 72 halvings above the electron land 0.9 percent below the reduced Planck mass, and the remainder is 0.948 ε. This is a different relationship to mystery than the one physics is accustomed to. Physics is used to solving problems: finding mechanisms, building models, computing answers. Here a question about a number (why is this number what it is?) turns into a question about a structure (how many steps are in the staircase?).

The integer of the span is a count. The *kernel integers* are $c_1 = 3$, the count of the three positions of comparison, and $n_C = 2$, the count of the conjugate pair, content and context, the two aspects into which comparison partitions what is compared (the book of kinds, Chapter 2, Section I). A *kernel number* is a number written in the kernel integers and the sides of the kernel triangle; a *kernel rational* is a rational kernel number. In the kernel integers, $72 = 8 \times 9 = n_C{}^3 c_1{}^2$. The 8 counts the *first-order states*, two outcomes at each of the three positions; the 9 counts the *joins*, three positions against three across the pair. Each cell of the count is one comparison (the book of kinds, Chapter 2, Section VII), and one comparison is one bit (Comparison, Chapter 1, Section IV: "So one comparison is one bit"); this book reads that bit, in mass, as one halv-

ing. The word “first-order” in “first-order states” names these states; it is distinct from the first order of the remainder in Section III. The factor 2^{72} is a pure count, and 72 is a pure kernel number.

From the mountaintop, the valley looks deep. The depth can be counted. It is seventy-two steps, and a little more. Each step is a halving, one comparison, read in mass.

Two exercises make the count concrete. The first counts the steps. Find a staircase. Count the steps. The height from the bottom of the staircase to its top is determined by that count. Twenty steps, each about 7 inches, give a height of about 12 feet. Now imagine a staircase with 72 steps, where each step down halves the altitude. After 10 halvings: a factor of 1,024. After 20: a million. After 40: a trillion. After 72: 4.7×10^{21}. That is the span from the electron to within 0.9 percent of the reduced Planck mass. It is a staircase, not a mystery. The kernel integers give the count of steps; measurement tests whether the span is that count.

The second exercise is a multiplication. Compute 8×9. That is 72. The 9 comes from the joins: three positions against three across the pair, $3^2 = 9$. The 8 comes from the first-order states: two outcomes at each of three positions, $2^3 = 8$. The twenty-two orders of magnitude between the reduced Planck mass and the electron are written, to within 0.9 percent, in the same two integers that set the sides of the triangle.

The identification of the span with this count is the assumed part, and measurement is its test. The ratio of the reduced Planck mass to the electron is identified with a count of 72 halvings; the identification is assumed, and the measured span tests it. The test has the result of Section I: seventy-two halvings above the electron land 0.9 percent below the reduced Planck mass.

The identification rests on one further premise, the endpoints: the span runs from the content's unit, the electron's mass, to the limit of comparison, where the count ends at the reduced Planck mass, and it counts every cell between them once.

The 0.9 percent is the remainder itself. The reduced Planck mass taken down 72 halvings is 5.1570×10^{-4} GeV, and its ratio to the measured electron is $1.0092 = 2^{0.0132}$. So the factor by which the count falls short is two raised to the remainder r.

The remainder is positive: r = +0.0132 halvings. This is a measured result, and no step of the argument of this chapter uses it as a premise.

The count is a count of cells, one comparison to each and one halving to each comparison, so it is $8 \times 9 = 72$. The number $8^9 = 2^{27}$ counts the configurations of nine joins, each in one of eight states, and is not a count of halvings. The measured span, about 72 halvings, agrees.

Each cell of the count is one comparison with two outcomes, a binary degree of freedom, so the 72 cells carry 72 bits and, read in mass, are 72 halvings. That answers the objection that a count of 72 cells is 72 states, which would carry only $\log_2 72 = 6.2$ bits.

Box II-1. The count and the factor of the remainder

Premises.

$n_C = 2$ and $c_1 = 3$ (the book of kinds, Chapter 2, Section I). Eight first-order states, $n_C{}^{c_1} = 2^3$; nine joins, $c_1{}^{n_C} = 3^2$; each cell one comparison (the book of kinds, Chapter 2, Section VII); each comparison one halving (this chapter's reading). $m_e = 5.10999 \times 10^{-4}$ GeV. $M_{Pl,red} = 2.4353 \times 10^{18}$ GeV.

Steps.

$72 = 2^3 \times 3^2 = n_C{}^3 c_1{}^2$. $2^{72} = 4.7224 \times 10^{21}$. $m_e \times 2^{72} = 2.4131 \times 10^{18}$ GeV, which is 0.9909 of $M_{Pl,red}$. $M_{Pl,red}/2^{72} = 5.1570 \times 10^{-4}$ GeV. $5.1570/5.10999 = 1.0092 = 2^{0.0132}$. $8^9 = 2^{27}$ counts the configurations of nine joins in eight states each, not cells.

Result.

The integer of the span is the count $n_C{}^3 c_1{}^2 = 72$. The mismatch of 0.9 percent is the factor 2^r of the remainder.

III. The Remainder at First Order

A measure belongs to one of three groups by the number of selections it involves. A value selected among a finite set of exact values involves none, and it is of the first group. One constant selected from a continuum involves one,

and it is of the second group. A distribution governed by continuous numbers together is of the third group. A count is of the first group. A *continuum whole*, one magnitude read as continuous (here the reduced Planck mass), is of the second group. The two have no common measure.

The *Incommensuration Theorem* states: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (Incommensuration; Measurement). To *read a count by a continuum whole* is to measure the count in that continuous magnitude. By the Incommensuration Theorem (the book of kinds, Chapter 1, Section III), reading a count by a continuum whole leaves a non-zero residue at every such reading. The span is such a reading: the count of 72 halvings read against the reduced Planck mass. So the span has a remainder, and the remainder is not zero.

The circle is the plainest picture of a count read by a continuous whole. Draw a circle. Measure its diameter. Call it 1. Now measure the circumference: the distance around the circle. It is not an integer. It is not a fraction. It is π, 3.14159265..., a number that never terminates, never repeats, and cannot be expressed as the ratio of two whole numbers. This is a fact about the relationship between straight lines and curves. A straight line (the diameter) and a curved line (the circumference) are measured in the same units, but they are not *commensurate*: they do not have a common measure. However finely the diameter is divided into equal parts, no collection of those parts exactly equals the circumference. The two lengths are incommensurable. This incommensurability is a structural fact about the relationship between the discrete (integer, countable) and the continuous (curved, flowing), and not a deficiency of the tools of measurement. Straight and curved are different kinds of geometry, and the conversion between them always involves π.

Imagine trying to tile a sphere with square tiles. It cannot be done. The curvature of the sphere means that the tiles do not fit together perfectly. However small the tiles are made, there will always be gaps or overlaps at the places where the curvature bends the surface away from flatness.

An exercise makes the same mismatch tangible. Find a ball: a tennis ball, an orange, a globe. Imagine covering its surface with small, flat, square pieces of paper, or try it, with scissors and tape. The squares do not fit together smoothly. They bunch up. They wrinkle. They gap. This is curvature, made tangible. The squares are flat (integer, discrete). The sphere is curved (continuous, irrational). The mismatch between them is the physical experience of what the step factor s (the number of halvings that one step of the mass ladder spans; Chapter 7, The Mass Ladder) corrects for.

There is a way of understanding the incommensuration that goes beyond the technical. Comparison measures. To measure, it needs a ruler: a standard, a unit, a basis for counting. The triangle {3, 4, 5} is comparison's ruler. Its integers are the tick marks. But at the veil (the finite scale below which there is no comparator smaller than the thing compared), the thing being measured has the form of the thing doing the measuring. Comparison measures itself. The ruler and the ruled are one form. And the ruler is flat (integer, discrete), while the thing being measured is curved (continuous, π-dependent). Can the ruler measure itself perfectly? No. The ruler is a count, what it measures is continuous, and the Incommensuration Theorem says that the two cannot both be perfected in one comparison. The flat ruler and the curved thing it measures are both aspects of comparison (one is its count, the other its continuum), but they do not perfectly coincide. At first order they differ by ε, the one mismatch, about 1.4 percent. This is a structural fact about the nature of knowing, and not a failure. Here the measurer and the measured have one form, and measuring more finely does not remove the residue. The residue is the irreducible cost of a count reading a continuum. Its existence is derived; the power of π in it is argued; its fraction, 11/5, is stated as the kernel writing gives it; and the measured span meets it, as the remainder 0.948 ε. The imperfection has a shape, and the shape is simple.

The one mismatch also says how a definite number is reached. The integers are countable: they can be listed, one after another, without end. The curve is continuous: between any two of its points lie more points than any list can hold. Every position on the mass ladders (Chapter 7, The Mass Ladder) is a count written in the triangle, multiplied by s, so no mass is reached

from the integers alone. Each is carried across the curve, and arrives, finite and measurable, on the far side. The factor s is where that crossing happens, and it happens on every step.

A second exercise gives the cost of self-measurement. Measure a person's height with that person's own foot as the ruler. How many foot-lengths equal the height? The answer is a number, but it is not a clean integer, and it is not perfectly precise, because the ruler (the foot) is part of the thing being measured (the body). The incommensuration is the structural version of this difficulty: comparison measuring itself with itself as the ruler, a count laid against a continuum. The residue ε, about 1.4 percent, is the irreducible cost of that reading. Perfection is not possible when the measurer and the measured are one, and the imperfection has a definite form.

Box III-1. Why there is a remainder

Premises.

The Incommensuration Theorem: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Axiom I: the immanent is more fundamental than the omniscient and/or the transcendent; the omniscient and the transcendent are in conjugate relationship to one another (An Immanent Metaphysics, aphorism 1.3-1; the book of kinds, Chapter 1, Section II). Axiom III: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable" (An Immanent Metaphysics, aphorism 1.3-1; the book of kinds, Chapter 1, Section II). (The book of kinds, Chapter 1, states the axioms and the three modalities.) The dimension of a measure is the number of selections it involves: 0 for a value selected among a finite set of exact values (the first group); 1 for one constant selected from a continuum (the second group); 2 for a distribution governed by continuous numbers together (the third group). A count is discrete (first group). A continuum whole is continuous (second group). The two have no common measure.

Step.

By the Incommensuration Theorem, reading a count by a continuum whole leaves a non-zero residue at every such reading; Axioms I and III place the count and the whole in the two groups.

Result.

A residue exists at every reading of a count by a continuum whole. The span's remainder r is one such residue.

The size of the residue at first order is fixed by one rule. Every small leftover between a counted measure and its continuous reading, anywhere in the particle domain, equals the one mismatch ε times a whole-number ratio fixed by the kind of reading, and it is not a separate small number of its own. For readings counted in halvings the ratio is one at first order. The rule names four cases. The first is the span's remainder (this chapter). The other three are carried with the masses they belong to: the step's excess, the amount by which the step s of the mass ladder exceeds one halving (Chapter 7, The Mass

Ladder); the top's shortfall, the amount in halvings by which the top quark lies below the condensate's whole $v/\sqrt{2}$ (Chapter 13, The Vacuum Value, the Higgs and the Top); and the floor's residue, the residue at the floor $2m_p$, twice the proton's mass (Chapter 9, The Quark Masses).

The *one mismatch* is $\varepsilon = (c_1^2 + n_C)/(N^2H\pi^2) = 11/(80\pi^2) = 0.013932$, with $N = 4$ and $H = 5$ the even leg and the hypotenuse of the kernel triangle $\{3, 4, 5\}$ (the book of kinds, Chapter 2, Section II).

The measured remainder is $r = E - 72 = 0.013211 = 0.948\ \varepsilon$. At the precision of the measurement the first order alone does not match it. The first-order span $72 + \varepsilon = 72.013932$ stands 44.5 standard deviations above the measured span. The rule fixes the ratio one at first order, and the measurement resolves a second order.

The span excludes one law for the second order. Let $K = r/(72\varepsilon^2)$, the number of ε^2 of remainder per halving of the count. The span carries $K = 0.9454 \pm 0.0012$. The law "one ε^2 per unit of count" asks $K = 1$, which stands 47 standard deviations above the measured K (pull +47 on K). The law is excluded at the span.

The two precisions give two statements, and both hold. At the level of magnitude, the remainder 0.013 is consistent with the tree-level count, the count of 72 halvings at first order, and $s - 1$ is of the same magnitude as the remainder; the tree-level formula is correct to its expected accuracy. At the precision of the measurement, the first order misses by the pulls stated above.

Box III-2. The first order against the measured span

Premises.

E = 72.0132112 ± 0.0000162 (Box I-1). $\varepsilon = 11/(80\pi^2)$.

Steps.

$\varepsilon = 0.0139317$. $\varepsilon^2 = 1.9409 \times 10^{-4}$. $r/\varepsilon = 0.0132112/0.0139317 = 0.948$. Pull of $72 + \varepsilon$ on E: $(72.013932 - 72.013211)/0.0000162 = +44.5$. $K = r/(72\varepsilon^2) = 0.0132112/0.0139746 = 0.94537$. The uncertainty of K is $0.0000162/0.0139746 = 0.00116$. Rounded, $K = 0.9454 \pm 0.0012$. Pull of the law $K = 1$ on K: $(1 - 0.94537)/0.00116 = +47$.

Result.

The remainder is 0.948 ε. The first-order span $72 + \varepsilon$ misses at +44.5 standard deviations on E. One ε^2 per unit of count misses at +47 on K.

The form of epsilon

The one mismatch has three writings, and they are writings of one number. In kernel integers, $\varepsilon = (c_1^2 + n_C)/(N^2H\pi^2)$. In the gauge self coefficient of color, $\varepsilon = b_{self}(SU(3))/(16\pi^2H) = (11/3)(O/H)/(16\pi^2)$, with $b_{self}(SU(3)) = 11$ the one-loop coefficient with which the carriers of color antiscreen (Chapter 2, The Couplings and Their Running), and $O = 3$ the odd leg of the kernel triangle. In the Euler weight of the spectral action, $\varepsilon = 72 \times 11/(5760\pi^2)$. The *spectral action* is the action read from the spectrum of the Dirac operator of the product geometry of the NCG book; the *Euler weight* $11/(5760\pi^2)$ is the coefficient of the Euler density per Dirac spinor in its scale-free term (the book of kinds, Chapter 3, Section V). So the third writing is seventy-two Dirac spinors, each with the Euler weight. It is the arithmetic $72/360 = 1/H$ restated.

The *product geometry* is four-dimensional spacetime (the manifold factor) times the finite space of states (the finite factor); the figure of the finite factor is the *cube*, a figure of eight vertices (the book of kinds, Chapter 3, Section V). The argument from the product geometry gives the power of π: every factor π comes from the manifold factor, and every rational comes from the finite factor. It gives the power of π in ε, in the correction $1/(24\pi^2)$ to the Higgs

self-coupling, in the Higgs's loop unit $\kappa_H = b_e/(16\pi^2) = 1/(12\pi^2)$ (Chapter 13), and in 3π. Here $b_e = 4/3$ is the one-loop coefficient of one Dirac fermion of unit charge. The rationals are carried by the kernel writings: 11/5 by the writing of ε in kernel integers, and 3/4 by the share of the cube stated next.

The share 3/4 recurs three times: $\varepsilon = (72/96) \times 11/(60\pi^2)$; $3\pi = (3/4) \times 4\pi$; and $u = (3/4)\kappa_H$, with $u = 1/(16\pi^2)$ the one-loop factor. On the rationale that the cube is one generation's eight fermions of one chirality, with six of its eight vertices colored, 3/4 is the share of the colored vertices, and 3π is the Gauss measure of the six unit points: the curvature the six colored vertices carry, $\pi/2$ each.

Box III-3. The three writings of epsilon

Premises.

$c_1 = 3$, $n_C = 2$, N = 4, H = 5, O = 3 (the book of kinds, Chapter 2, Sections I and II). $b_{self}(SU(3)) = 11$ (this book, Chapter 2). The Euler weight per Dirac spinor is $11/(360 \times 16\pi^2) = 11/(5760\pi^2)$ (the book of kinds, Chapter 3, Section V). The finite space of states of the spectral triple has dimension 96 (the book of kinds, Chapter 2, Section VII).

Steps.

$(c_1^2 + n_C)/(N^2 H\pi^2) = 11/(16 \times 5 \times \pi^2) = 11/(80\pi^2)$. $(11/3)(3/5)/(16\pi^2) = 11/(80\pi^2)$. $72 \times 11/(5760\pi^2) = 792/(5760\pi^2) = 11/(80\pi^2)$, since $72/360 = 1/5 = 1/H$. $(72/96) \times 11/(60\pi^2) = (3/4) \times 11/(60\pi^2) = 11/(80\pi^2)$. $16\pi^2\varepsilon = 11/5$.

Result.

$\varepsilon = 11/(80\pi^2) = 0.013932$ in each writing. Its power of π is argued. Its rational 11/5 is stated as the kernel writing gives it.

IV. The Second Order: the Constant k

The remainder is written to second order as $r = \varepsilon - k\varepsilon^2 + O(\varepsilon^3)$, and k, the span's *second-order coefficient*, is defined by this writing. With $\varepsilon = 11/(80\pi^2)$, the measured span gives $k = 3.7120 \pm 0.0835$. A *measured agreement* is a value

that agrees with measurement and whose reason is not stated in this book's derivations. The remainder's form $r = \varepsilon(1 - k\varepsilon)$, with k about 3.71, is a measured agreement: its first order has its reason in the rule of Section III, and its second order is measured.

The span measures one combination of ε and k. So affirming ε and measuring k are one statement. Affirming $\varepsilon = 11/(80\pi^2)$ is the same as measuring $k = 3.7120 \pm 0.0835$; affirming a kernel rational k measures ε to 0.13 percent.

The rational 11/5 is carried by the writing of ε in kernel integers (Section III).

Box IV-1. One equation, two unknowns

Premises.

$r = \varepsilon - k\varepsilon^2 + O(\varepsilon^3)$ (the definition of k). $r = 0.0132112 \pm 0.0000162$ (Box I-1).

Steps.

Given ε: $k = (\varepsilon - r)/\varepsilon^2$, with uncertainty σ_r/ε^2. At $\varepsilon = 0.01393166$: $k = 0.00072046/1.94091 \times 10^{-4} = 3.7120$, with uncertainty $0.0000162/1.94091 \times 10^{-4} = 0.0835$. Given k: solve $k\varepsilon^2 - \varepsilon + r = 0$, so $\varepsilon = (1 - \sqrt{(1 - 4kr)})/(2k)$, with uncertainty $\sigma_r/\sqrt{(1 - 4kr)}$, about 0.0000181, which is 0.13 percent of ε.

Result.

The span fixes one combination of ε and k. Given $\varepsilon = 11/(80\pi^2)$, $k = 3.7120 \pm 0.0835$.

The rule of *one incommensuration per domain per kind of reading* states that within one domain each kind of reading has one mismatch and no second. This rule is the reason carried for k being a rational. Read within the span as one domain, the rule says that the second-order coefficient at the count's whole, the reduced Planck mass at which the count ends, cannot be a second constant from the continuum. So k is a measure of the first group, a kernel rational: a law gives it, and this book does not state that law. The Story of the Particles, the book that names which particles exist, fixes what the forces are, how many carriers each has and the order of their strengths, and it does not

fix how strong each force is. So the value of k is carried by the two volumes on the numbers: by its measurement here, and by the lepton ladder's prediction (Chapter 7, Section X).

V. The Third Order and What a Measurement of G Can Decide

The remainder r is one number, and the second-order coefficient k and the third-order coefficient c_3 enter it together. So experiment alone cannot decide k. A measurement of Newton's constant sharpens r, and r fixes k only together with a law that fixes c_3. The uncertainty of the span from G is $(1/2)(\sigma_G/G)/\ln 2$ halvings (Box I-1), so each factor by which G is sharpened sharpens the span by the same factor.

VI. The Second Route to the Span, through the Couplings

A second route reaches the span with no input from Newton's constant. The *restated relation* of Chapter 2, The Couplings and Their Running, joins light's strength at the proton's mass to the split between the weak and the hypercharge inverse strengths at the span: $\alpha_{em}(m_p) \cdot \Delta_{21}(E) = 1/(3\pi)$, with $\Delta_{21}(E) = (1/\alpha_2 - 1/\alpha_1)(E)$. Light's strength is read at the proton's mass, the mass of the thing light is compared against (Chapter 2). With the gauge couplings measured and two-loop running, the relation solved for E returns $E = 72.0120 \pm 0.0221$ halvings. That value lies 0.05 standard deviations below the span from Newton's constant (pull -0.05 on E). A *matching scheme* is the convention by which measured inputs are carried into the running couplings; over the matching schemes, the route spreads from 72.0120 to 72.0204.

The reach of this route is the integer. The bare count 72 lies 0.54 of the route's standard deviation below the route's value. So the route cannot see the remainder $r = 0.0132$; to see it, the route would need an uncertainty of about ± 0.005. At one loop the route fails: it returns 71.920, 4.2 standard deviations below the span (pull -4.2 on E).

Two further values of the same route are carried with the matchings their computations name. The first is E = 72.0325. Its value is the same under the two matchings its computation names, and the two matchings carry different uncertainties into the comparison: ± 0.0159 under the matching of Martin's program SMDR (Standard Model in Dimensional Regularization), where its pull against the span is +1.21 standard deviations, and, under the matching of Buttazzo and collaborators, an uncertainty of $0.0193/2.83 = 0.0068$, the one its pull of +2.83 implies ($0.0193 = 72.0325 - 72.0132$, the distance of the value above the span). The second, with the SMDR matching, the PDG 2024 inputs and three-loop running, is $E = 71.9912 \pm 0.0159$. It stands 1.38 standard deviations below the span (pull -1.38 on E), and the bare 72 lies 0.55 of that standard deviation above it. The uncertainty of each of these values, ± 0.0159 and ± 0.0068, is above the ± 0.005 that the remainder would need, so none of them resolves the remainder.

The two routes to E are independent in their inputs. The count route rests on the count $72 = n_C{}^3 c_1{}^2$ and the remainder $r = \varepsilon(1 - k\varepsilon)$ with k about 3.71, and it is compared with the span from Newton's constant and the electron. The coupling route rests on the measured gauge couplings, the proton's mass and the relation's unit $3\pi = 4\pi/b_e$, and it uses neither Newton's constant nor the kernel integers. The coupling route is a convergent route to a measured number.

Table VI-1. The routes to the span E.

Route to E	Value (halvings)	Pull on E against 72.0132112
Newton's constant and the electron (the measurement)	72.0132112 ± 0.0000162	(reference)
Restated relation, measured couplings, two loops	72.0120 ± 0.0221 (matching schemes 72.0120 to 72.0204)	−0.05
The same, one loop	71.920	−4.2
The same relation, a further value	72.0325 (± 0.0159 SMDR; ± 0.0068 Buttazzo and collaborators, as its pull implies)	+1.21 (SMDR); +2.83 (Buttazzo and collaborators)
The same, SMDR matching, PDG 2024 inputs, three loops	71.9912 ± 0.0159	−1.38

VII. What Other Chapters and Books Carry

This chapter states the span as a measured test of the count, and its remainder in three orders. Each neighboring matter is carried where its own premises are. The electron as the unit of mass, and the fine-structure constant as a measured input, are Chapter 1's, What Is Measured. The restated relation and its unit 3π are Chapter 2's, The Couplings and Their Running. The crossing at 54.259 and its displacement are Chapter 3's, Where the Couplings Cross. The rest is carried by The Measures of the Kinds. The step s, its coefficient $c_2 = 28/5$, the use of ε on the ladder and the lepton ladder's route to k are Chapter 7's, The Mass Ladder; so is the lepton offset, the charged leptons' offset in the step formula, with its one ε^2 per unit. Chapter 7 cites this chapter for the form of ε. The floor's residue is Chapter 9's, The Quark Masses. The condensate's whole at 18.378, the four wholes of the count, the top's shortfall below the condensate's whole and the top read from the remainder are Chapter 13's, The Vacuum Value, the Higgs and the Top.

Every reading of the 72 or of the Planck mass through a geometry of the internal space with lengths belongs to The Gravitational Passage. In the Passage's normalization, with $R_O^2 = 6\, l_{Pl}^2$ and $R_T^2 = l_{Pl}^2$, the volume of $\mathbb{C}P^2 \times S^2$ is $(\pi^2 R_O^4/2)(4\pi R_T^2) = 72\pi^3\, l_{Pl}^6 = 72(\pi\, l_{Pl}^2)^3$: counted in Planck polydiscs, the volume of the context instance carries the same 72 as the count of this chapter. In units in which Newton's constant and the speed of light are one, the Passage's Planck length gives $l_{Pl}^2 = \hbar$, so one Planck disc, of area πl_{Pl}^2, is $\pi\hbar = h/2$, half a cell of action h. The Planck polydisc, one such disc in each of the three complex directions, is therefore $(h/2)^3$, and the volume $72(h/2)^3 = 9h^3$ is nine cells of action h^3, each halved once in each of the three complex directions: the same 9×8 as the count. This book does not state why the volume of the context instance carries the count of the content instance; it gives the clues to the reason. The clues are these: in the unit $\pi^3 R_T^6$ the volume is $2(R_O/R_T)^4$, and the Einstein condition of the context instance gives $(R_O/R_T)^2 = 6$, which is also the Euler number $c_1 n_C = 3 \times 2$ of the space of states, the product of the Euler numbers of $\mathbb{C}P^2$ and S^2 (the book of kinds, Chapter 3, Sections III and IV); so the context form of the number is $2(c_1 n_C)^2$ and the content form is $n_C{}^{c_1} c_1{}^{n_C}$, and the two agree at $c_1 = 3$, $n_C = 2$. The two forms give the same integer at that value of c_1, and only there: with $n_C = 2$ the context form is $8c_1^2$ and the content form is $2^{c_1} c_1^2$, and they are equal only where $2^{c_1} = 8$, at $c_1 = 3$ (at $c_1 = 2$ they give 32 and 16; at $c_1 = 4$, 128 and 256). The value $c_1 = 3$ is the one at which the triangle with sides c_1, $c_1 + 1$, $c_1 + 2$ has a right angle, by the identity $c^2 + (c+1)^2 - (c+2)^2 = (c-3)(c+1)$; the condition at which the two forms meet, $2^{c_1} = 8$, and the equation that selects the kernel triangle {3, 4, 5} pick out the same value, $c_1 = 3$. A third form, $2A^2 = 2n_C{}^2 c_1{}^2 = 72$ with A = 6 the triangle's area, has the same factorization, and $2n_C{}^2 = 2^{c_1} = 8$ holds only at $c_1 = 3$. The book of kinds finds a third 72, the anticanonical degree of the fibration $P(O \oplus O(3))$ over $\mathbb{C}P^2$, with the count's factors $2^3 \times 9$ (Chapter 3, Section V). One more fact belongs beside these: the Passage's Planck length is the one formed from Newton's constant without the factor 8π (The Gravitational Passage, Chapter 9), while the

span's 72 is read against the reduced Planck mass (Section I), so the two 72s stand on scales a factor $\sqrt{(8\pi)}$ apart. This chapter reads the internal space $\mathbb{C}P^2 \times S^2$ only as the content instance, a space of states with no size.

Any form of the 72 that joins the electron to Newton's constant, a bridge from the measured input of the particle side to the measured input of the side of gravitation and cosmology, is argued on the particle line, where the 72 itself is argued: the book of kinds gives the count, and this chapter sets the span measured from Newton's constant beside it as its test. The partition follows the scope of each book. The particle side works on the content instance; it needs a count of comparisons, and the count form gives one with no size, no radius and no reference to Newton's constant, so the particle side proceeds without waiting on any question of gravity. The side of gravitation works on the instance with radii, which The Gravitational Passage secures, and the volume form lives there with them; its measured end is Newton's constant. The bridge carries the count of the particle side up to the measured end of the side of gravitation. *Two Theories, One World* and *The Unquantized Arena*, the two books on how the quantum and the gravitational descriptions stand to each other, rule on how the two descriptions meet, and not on the particle spectrum; so the 72 of the bridge is argued with the spectrum, and each measured import stays within its own side: the electron on the particle side, Newton's constant on the side of gravitation.

Summary

The span from the measured electron to the reduced Planck mass is $E = 72.0132112 \pm 0.0000162$ halvings. Its integer is the count $72 = n_C{}^3 c_1{}^2$, eight first-order states times nine joins. The identification of the span with that count is assumed, and the measurement tests it, written upward from the electron: 72 halvings land 0.9 percent below the reduced Planck mass.

A count read by a continuum whole leaves a residue at every reading. The residue's first order is the one mismatch $\varepsilon = 11/(80\pi^2)$, and the measured remainder is $0.948\ \varepsilon$. The argument gives the power of π in ε; the rational 11/5 is stated as the kernel writing gives it.

The second order is $k = 3.7120 \pm 0.0835$, measured once ε is affirmed. Read within the span as one domain, k is a kernel rational: a law gives it, and this book does not state that law. The third order c_3 enters with k, so a measurement of G fixes k together with a law for c_3.

The measured gauge couplings, through the restated relation, return the span's integer with no input from Newton's constant.

The question each measured agreement of this chapter carries is why it lands: the remainder's form $\varepsilon(1 - k\varepsilon)$ with k about 3.71 (clues: the lepton ladder's route to k, Chapter 7, Section X, and the rule of one incommensuration per domain); the coupling route's return of the integer 72 with no input from Newton's constant (clue: the relation's unit $3\pi = 4\pi/b_e$); and the same 72 in the volume of the context instance (clue: the radii of The Gravitational Passage).

KEY CONCEPTS

- **The span.** $E = \log_2(M_{Pl,red}/m_e) = 72.0132112 \pm 0.0000162$ halvings, measured from Newton's constant and the electron.
- **The count.** $72 = 8 \times 9 = n_C{}^3 c_1{}^2$, first-order states times joins; the span tests the identification of the span with this count.
- **The remainder** $r = E - 72 = 0.0132 = 0.948\,\varepsilon = \varepsilon - k\varepsilon^2 + O(\varepsilon^3)$.
- **The one mismatch.** $\varepsilon = 11/(80\pi^2) = 0.013932$, the first-order size of every small leftover of a reading counted in halvings.
- **The constant k.** The span's second-order coefficient, 3.7120 ± 0.0835 measured.
- **The constant c_3.** The third-order coefficient, which enters the remainder together with k.



Chapter 6

The Binding Scale of the Proton

CHAPTER OVERVIEW

The proton's mass is a mass of binding: about ninety-nine percent of it is the energy of the color field that holds its quarks, and about one percent is the quarks' own. Color's strength, which has no unit, sets that mass by running: it grows from about one fiftieth at the reduced Planck mass to order one about sixty-one halvings lower, near the proton. This book places color's whole at the proton's mass, reads light's strength there, and places the pair floor $2m_p$, the least mass at which a whole of color is made as content, at which the light quarks are read (Chapter 9). The form of the ratio m_p/m_e is argued: it is the exponential of an inverse strength of color times numbers fixed by structure. A law fixes the ratio's value, and this book does not state that law; several routes return the ratio as values that agree with measurement. The proton's charge radius, in the unit of its own reduced Compton length, is 3.9977 ± 0.0030 against the integer 4.

I. What Kind of Mass the Proton's Is

The proton's mass is a whole of binding, a *binding mass*: the mass of a whole that comes from the energy of what holds its parts. About ninety-nine percent of it comes from the color field (the field of the strong force, whose charge is called color), the motion and binding of the quarks and the gluons; the three quarks supply about one percent, about 9 MeV of 938 MeV.

The electron's mass is set otherwise. It is set by the small coupling y_e of the *edge field* (the Higgs field) to the electron, about 3×10^{-6}, through $m_e = y_e v/\sqrt{2}$ with $v = 246.22$ GeV. So the ratio m_p/m_e compares a mass set by the edge field with a mass of binding: the two masses are set by two different laws.

A whole's rest mass is its total energy, seen from its rest frame, divided by c^2. That energy includes the rest energies of the parts, their motion within the whole, and the field energy of what holds them. So the proton's mass includes the energy of the gluon field and of the quarks' motion. For the same reason a bound whole is lighter than its separated parts, by its binding energy divided by c^2: hydrogen is lighter than a free electron and a free proton by 13.6 eV/c^2. Box I-1 sets out the step.

Box I-1. The mass of a bound whole

Premise 1 (special relativity, the invariant mass of a system): a whole's rest mass is its total rest-frame energy divided by c^2.

Premise 2 (standard physics): that energy is the sum of the parts' rest energies, their kinetic energies within the whole, and the binding or field energy.

Step: $m_{whole} = (\Sigma \text{ rest energies} + \Sigma \text{ kinetic energies} + \text{field energy})/c^2$.

Result for the proton: the gluon field's energy and the quarks' motion are contributions to m_p. The three valence quark masses supply about 9 MeV of $m_p \approx$ 938 MeV, about one percent; the rest, about ninety-nine percent, is the color field.

Result for a bound pair: a negative binding energy lowers the mass; hydrogen's binding energy is 13.6 eV.

The electron, for comparison: $m_e = y_e v/\sqrt{2}$, so $y_e = \sqrt{2}\, m_e/v = \sqrt{2} \times 0.511$ MeV/246.22 GeV $\approx 2.9 \times 10^{-6}$, about three parts in a million.

The argument from comparison (an argument from the structure of comparison alone, with no measured number) gives the holding, the binding of the quarks by the color field, a place in the proton's inertia. The holding is held within the proton and cannot pass out of it; it is found again with the

proton each time the proton is met; so it is part of the proton's own inertia. That argument makes no claim that the holding is the larger part. Standard physics gives the share: the three valence quark masses supply about 9 MeV of 938 MeV, about one percent, and the rest, about ninety-nine percent, is the color field. The two statements rest on different bases, comparison for the first and standard physics for the second, and both hold.

The Story of the Particles, which this book calls *the zoo*, is the authority on which particles exist and of what kind. The zoo gives mass three meanings, one for each of the three carriers of the frame of measurement (the comparison in which the photon is the act of comparing, the electron the thing compared and the proton the thing compared against; the book of kinds, Chapter 6, Section I): rest mass for the electron, exchange mass for the photon, confinement mass for the proton. So m_p/m_e, about 1836, is a ratio across meanings, a confinement mass over a rest mass; m_μ/m_e is a ratio within one meaning, rest over rest, and is a different kind of number. For the proton-to-electron ratio the joining law (the law that joins the two masses of the ratio) is the binding of the strong force, and that law turns the ratio into the strength of the strong force at one resolution.

Color's strength changes with the resolution at which it is read, so it carries no single value of its own; where it grows strong it sets a mass, the scale of the proton. In the focus of comparison as relation color's force is the omniscient force (in the zoo, the pattern-holding force), and the omniscient's measure of content is inertia. So color belongs with mass among the units of measurement, where it supplies a second scale beside the electron's. The ratio of the color-set scale to the electron's is the proton question: the question of what fixes m_p/m_e.

The strong force is confined: nothing colored escapes to the scale of a hand. Its carriers, the gluons, carry color themselves and attract one another, so the field between two quarks gathers into a narrow tube whose energy grows with its length, until making a new pair of quarks costs less than pulling the old ones apart. Only the color-neutral leftover acts outside the proton: the nuclear force, which reaches about the width of a proton.

Lattice QCD is quantum chromodynamics (QCD, the quantum theory of color) computed on a four-dimensional grid of points by random sampling of the field's configurations (Monte Carlo sampling), with the grid spacing taken to zero and the volume to infinity. It computes the proton's mass to better than one percent. It reproduces the pion's mass and decay constant to better than one percent, and it computes the rho meson's mass, the light baryon spectrum and the glueball masses with uncertainties of one to five percent (Flavour Lattice Averaging Group Review 2024; for the glueballs, Morningstar and Peardon, Physical Review D 60, 034509, 1999). When the pion's mass, the kaon's mass and either the Omega baryon's mass or the Upsilon splitting (the mass difference between two states of the bottom quark bound to its antiquark) set the lattice scale and the bare quark masses (the quark masses entered into the calculation), every other hadron mass is a prediction of the lattice calculation.

The proton's mass is taken from lattice QCD, given the strength of color and the masses of the quarks; the kernel triangle (the right triangle with sides 3, 4 and 5 of *the book of kinds*, The Kinds of Particle, Chapter 2, Section II) does not write it. A law writes m_p/m_e from the structure, and this book does not state that law; Section V states what is known of the law.

The ratio m_p/m_e is a value, and every measure of the atom carries it: the atom's size and steps depend mainly on the electron's inertia, and on the proton's only through the small recoil of the reduced mass, a residue of about one part in 1836. The proton nearly stays while the electron moves because the proton has more inertia; how nearly it stays depends on how much more, which is that value. The effect of the proton's intrinsic circulation (its spin, in the book of kinds' term, Chapter 1, Section VIII) on the differences between the atom's states carries the same ratio of inertias; that effect is not derived here.

The proton's magnetic moment (the strength of its response as a small magnet) is smaller than the electron's by about the ratio of their inertias. The doubling that gives spin one half (the factor 2 in the g-factor of a point particle of spin one half, the g-factor being the ratio of the magnetic moment

to the value its spin alone would give) is not enough by itself to fix the proton's. The proton's measured g-factor is about 5.586, far from 2; standard physics ascribes the difference to the proton's being composite.

The electromagnetic share of the nucleon masses rests on one premise: that the energy of the electromagnetic field within a whole adds to the whole's inertia. With that premise both nucleons carry the share, since the proton and the neutron each hold charged parts that the electromagnetic force relates to one another, though the neutron carries no net charge. The argument fixes no order between the two shares.

II. Color's Scale from Color's Strength

Color's strength, written α_s, falls at finer resolution and rises at coarser resolution. The rate is set by the one-loop coefficient b_0 of color's running: $b_0 = 11 - (2/3)n_f$, eleven-thirds for each of the three colors (color acting on itself) less two-thirds for each kind of quark, n_f in number. With the six kinds of quark, $b_0 = -b_3 = 7$, where $b_3 = -7$ is color's rate in the convention of Chapter 2; a positive b_0, a negative b_3, is what makes color weaken at fine resolution.

Read downward from a fine resolution M, color's strength reaches order one at a scale Λ. At one loop (the first order of the calculation of the running in powers of the strength; higher loops are the higher orders) the number of natural-log units between them is $\ln(M/\Lambda) = 2\pi/(b_0\alpha_s(M))$. A strength with no unit thereby sets a mass; standard physics calls this *dimensional transmutation*. The proton's mass is a fixed multiple of Λ. So m_p/m_e is the exponential of an inverse strength of color, times numbers fixed by structure: $m_p/m_e = (M/m_e) \times (\text{a fixed multiple}) \times \exp(-2\pi/(b_0\alpha_s(M)))$. That is why the ratio is large.

A *halving* is one factor of two in mass; a distance in halvings is the base-two logarithm of a ratio of masses. The span E from the electron to the reduced Planck mass (the mass scale of gravitation in its reduced convention, about 2.4×10^{18} GeV) is 72.013 halvings, measured; its count of 72 and its re-

mainder are Chapter 5's. The proton sits 10.84 halvings above the electron, since $\log_2(m_p/m_e) = 10.8425$. So the proton lies $72.013 - 10.84 = 61.17$ halvings below the reduced Planck mass, which is 42.4 natural-log units.

The measured strength of color, run upward by the one-loop law, is about $1/50$ at the reduced Planck mass. Run down from there with $b_0 = 7$, a strength near $1/50$ gives $2\pi \times 50/7 = 44.9$ natural-log units, of the same size as 42.4, and so it becomes of order one near the proton's scale. The gap from the proton to the reduced Planck mass is $2^{61.17}$, about 2.6×10^{18}, about e^{42}; the one-loop estimate from a strength of $1/50$ is e^{45}. Box II-1 sets out the arithmetic.

Box II-1. Color's scale and the span to the reduced Planck mass

Premise 1 (standard physics): the one-loop coefficient of color's running is $b_0 = 11 - (2/3)n_f$; with $n_f = 6$, $b_0 = 7$.

Premise 2 (standard physics): $\ln(M/\Lambda) = 2\pi/(b_0\alpha_s(M))$, where M is the starting resolution, $\alpha_s(M)$ color's strength there, and Λ the scale where the strength becomes of order one.

Premise 3 (standard physics): m_p is a fixed multiple of Λ.

Premise 4 (measured): $m_p/m_e = 1836.15267343$, the ratio of the CODATA 2022 masses 938.27208943(29) MeV and 0.51099895069(16) MeV.

Premise 5 (measured, Chapter 5): the span $E = \log_2(M_{Pl,red}/m_e) = 72.013$ halvings, $M_{Pl,red}$ the reduced Planck mass.

Premise 6 (measured, one-loop extrapolation): $1/\alpha_s(M_{Pl,red}) \approx 50$.

Step 1: $x_p = \log_2(1836.15267343) = 10.8425$ halvings.

Step 2: $E - x_p = 72.013 - 10.8425 = 61.17$ halvings; $61.17 \times \ln 2 = 42.40$ natural-log units.

Step 3: $2\pi \times 50/7 = 44.88$ natural-log units, of the same size as 42.40.

Step 4: by Premises 2 and 3, $m_p = (\text{fixed multiple}) \times M \exp(-2\pi/(b_0\alpha_s(M)))$, so $m_p/m_e = (M/m_e) \times (\text{fixed multiple}) \times \exp(-2\pi/(b_0\alpha_s(M)))$.

Step 5: $M_{Pl,red}/m_p = 2^{61.17} = 2.6 \times 10^{18} = e^{42.4}$; the one-loop estimate of Step 3 is $e^{44.9}$.

The sensitivity is steep. One unit of $1/\alpha_s$ at the reduced Planck mass changes m_p by a factor of about 2.5. The ratio m_p/m_e is measured to about one part in sixty billion; to derive it to its measured digits, the inverse strength of color at the starting resolution must be known to about twelve significant figures. So those figures must come from a law: a value of color's strength chosen to agree with the measured ratio cannot supply them. Box II-2 gives the steps.

Box II-2. The precision the ratio m_p/m_e asks of color's strength

Premise 1 (standard physics): $\Lambda = M \exp(-2\pi(1/\alpha_s(M))/b_0)$ with $b_0 = 7$.

Premise 2 (measured): m_p/m_e is measured to about one part in sixty billion, a relative precision of 1.7×10^{-11}.

Step 1: $d \ln \Lambda = -(2\pi/7)\, d(1/\alpha_s)$; one unit of $1/\alpha_s$ moves Λ, and with it m_p, by the factor $\exp(2\pi/7) = \exp(0.898) = 2.45$.

Step 2: a relative precision of 1.7×10^{-11} in m_p asks $d(1/\alpha_s) = 1.7 \times 10^{-11}/0.898 = 1.9 \times 10^{-11}$; at $1/\alpha_s \approx 50$ that is about twelve significant figures.

A formula for the ratio written in powers of π shows how far five figures of agreement stand from the twelve figures a law must supply. The number $6\pi^5 = 1836.118$ (Lenz, 1951) agrees with the measured ratio, 1836.15267343, to about two parts in a hundred thousand; at the measured precision $6\pi^5$ stands 0.0346 below the measured ratio, and against the ratio's standard uncertainty, 1.7×10^{-11} of 1836.15, that is 3.1×10^{-8}, the pull of $6\pi^5$ is about -1.1×10^6 standard deviations. No argument joins π^5 to the binding of color.

Color's strength and m_p/m_e are one value: the proton's mass in units of the electron's is fixed by color's strength through dimensional transmutation, so a law for either is a law for both. A law fixes that one value, and this book does not state that law. Read backward from the measured m_p/m_e, color's way (the transmutation law of Section V, with color's slot c = H) gives $\alpha_s(M_Z)$ 0.26 standard deviations from its measured value in one tree-level scheme (a matching of the measured inputs that keeps no loop corrections); under SMDR matching (the conversion of the measured inputs into MS-bar couplings with Martin's code SMDR, Section V) color's way asks $\alpha_s(M_Z) = 0.11898$, +1.1 standard deviations.

The scale Λ itself depends on the order of the running and on the scheme in which the strength is defined. At one loop with five kinds of quark, $\Lambda_{QCD} = 87$ MeV, from $\alpha_s(M_Z) = 0.1179$ and $M_Z = 91.18$ GeV; at four loops in the *MS-bar* scheme (the modified minimal subtraction scheme, the standard way of defining a running coupling) the measured Λ_{QCD} is 210 MeV.

Box II-3. The one-loop five-flavor Λ

Premise 1 (standard physics): $\Lambda = M_Z \exp(-2\pi/(b_0 \alpha_s(M_Z)))$ at one loop, with $b_0 = 11 - 10/3 = 23/3$ for five kinds of quark.

Premise 2 (measured, Particle Data Group): $\alpha_s(M_Z) = 0.1179$, the value this computation uses (the 2024 value is 0.1180 ± 0.0009), and $M_Z = 91.18$ GeV.

Step: $2\pi/(7.6667 \times 0.1179) = 6.951$; $\exp(-6.951) = 9.57 \times 10^{-4}$; $91.18 \text{ GeV} \times 9.57 \times 10^{-4} = 87$ MeV.

Comparison: the four-loop MS-bar Λ_{QCD} is 210 MeV (measured).

III. Color's Whole Is the Proton's Mass

Every observable state that takes part in the strong force is a complete comparison in color, with no net color. Three quarks, one of each color, form a color-neutral whole, a *baryon*; this book calls such a whole a *trine*. A quark with an antiquark forms a color-neutral pair, a *meson*. *Color's whole* is the mass at which the strong force's pattern is complete. This book places color's whole at the proton's mass, 0.938 GeV, and not at the pair floor $2m_p$ (twice the proton's mass, the least mass at which a proton is made with its antiproton; Section IV). Three routes reach that placement, each on its own basis. The first and the third are set out here; the second, the boundary between the quark columns, rests on the measured quark masses and the ladder of Chapter 7, and it is set out with the quark masses (Chapter 9, Section I).

The first route: what the whole is

In the zoo the omniscient position is the thing fully available to knowing, "the pattern held whole in view". On the reading of "held" as stable as a free particle, the proton is the one color-neutral whole that is held, since it is stable and the lightest baryon. Two wholes in comparison are not one whole. So, on that reading, color's whole is the proton's mass, 0.938 GeV, and not twice it.

A second reading of color's whole, the least whole of color that is content, gives the same proton, since no process that can be produced changes the count of baryons (the earliest moments of a universe are the exception; the zoo, Level 4, Section IV). Both readings exclude the meson: the zoo calls three quarks "a color-neutral whole" and a quark with its antiquark "a color-neutral pair", and a pair is not a whole.

The third route: where light's strength is read

A comparison's strength is read at the mass of what it is made against; light's strength is read at the proton's mass. On the rule that a context's whole sits where its coupling to its source has unit strength (context in the zoo's first sense, the side of the bosons, the carriers of force), light's whole lies in another context: the inverse strength of light is at most 137.036 at every scale and reaches only 127.951 at the mass of the Z, so light's strength is at most 1/128 anywhere in the domain and never one. The weak force has $1/\alpha_2 \geq 29.6$ in the domain, and neither its strength nor the hypercharge force's is ever one. So light is read at the whole of another context. The premise that makes this route bear on color is that the whole light reads is a whole of color.

In values, light's reading scale picks the proton. Light's *reading scale* is the scale at which light's relation of unit 3π holds (Chapter 2: the inverse strength of light there equals 3π times the gap between the inverse strengths of the weak and hypercharge forces at the reduced Planck mass): 1.035 GeV, 10.98 halvings above the electron (Chapter 2), with an uncertainty of ± 0.19 halvings that this chapter carries from the measured couplings run to that scale. Against it, m_p stands at -0.72 standard deviations; the scale μ_1 where color's strength is one stands at -3.50; the floor $2m_p$ at $+4.54$; and the two wholes of the Higgs condensate (the constant value $v = 246.22$ GeV that the Higgs field takes throughout space; its wholes are Chapter 13's), $v/2$ and $v/\sqrt{2}$, at about $+36$ and $+39$. The neutron's mass stands at -0.71, beside the proton's at -0.72: the reading scale picks a nucleon's mass, which agrees with the placement of color's whole at m_p and does not test the reading of "held". Box III-2 and Table III-1 give the numbers.

Box III-2. Light's reading scale against the candidate wholes

Premise 1 (measured couplings run to the scale): light's reading scale is 1.035 GeV, 10.98 halvings above m_e (Chapter 2); its uncertainty, ± 0.19 halvings, is this chapter's, from the measured couplings run to that scale.

Premise 2 (definition): the place of a mass m is $x(m) = \log_2(m/m_e)$, with $m_e = 0.51099895069$ MeV (CODATA 2022).

Premise 3 (measured): $m_p = 938.27208943$ MeV (CODATA 2022); $m_n = 939.56542194$ MeV (CODATA 2022); $v = 246.22$ GeV. The scale $\mu_1 = 0.651$ GeV is a convention-dependent scale, not a measurement (Chapter 2).

Step 1: $x(m_p) = 10.842$; $x(m_n) = 10.844$; $x(\mu_1) = 10.315$; $x(2m_p) = 11.842$; $x(v/2) = 17.878$; $x(v/\sqrt{2}) = 18.378$.

Step 2: pull $= (x(m) - 10.98)/0.19$: m_p -0.72; m_n -0.71; μ_1 -3.50; $2m_p$ $+4.54$; $v/2$ $+36$; $v/\sqrt{2}$ $+39$.

The three routes rest on different bases. The first rests on the zoo's kinds (the proton stable and the lightest baryon; the baryon count unchanged) and on a reading of one word. The second, in Chapter 9, rests on the measured quark masses and the columns of the ladder. The third rests on the measured couplings of light run to its reading scale. All three place color's whole at m_p.

Why the whole is a trine

Binding mass that stays without the Higgs condensate belongs to the trines, not to every color-complete whole. Without the condensate the quarks are massless. The transformations that rotate massless quarks of either handedness (the two senses of a quark's spin along its motion) among themselves carry anomalies (quantum effects that break their conservation laws in a fixed, computable way), which the color-complete wholes must reproduce ('t Hooft anomaly matching). So either some whole of spin one half is massless, or those rotations are hidden (the ground state lacks the symmetry the laws have) and massless wholes of spin zero appear (Goldstone's theorem). With the trines keeping their binding mass, the lightest pairs are the wholes that are massless without the condensate.

Geeze, Wublets, and the Higgs states two things of the color-complete wholes of light quarks: that the mass of every such whole is binding mass, ζ times the binding scale, with ζ a pure number that one loop cannot fix; and that binding mass kept without the condensate holds for the trines and not for every whole. The two cannot both hold of the lightest pairs. The second holds, by the argument above; so the binding mass and its factor ζ are statements about the trines, and this book takes ζ at the proton, a trine.

The lightest quarks are light beside that binding scale wherever it lies. The same book places the binding scale between about 0.03 GeV and 2 GeV.

The scale where color's strength is one

The scale μ_1 at which $\alpha_s = 1$ depends on the convention and on the loop order: it lies between 0.29 and 0.67 GeV (Chapter 2), and the value 0.651 GeV (range 0.65 to 0.67 GeV) belongs to the top of that range. The factor $\sqrt{2}$ between μ_1 and m_p is measured and depends on the scheme, and it is taken against a scale that depends on a convention.

The *two-powers reading* assigns the two wholes of one context the share $a = 1/2$ at the first power and a^2 at the second, by position (the upper whole a, the lower a^2). That reading holds when the upper and lower wholes stand in opposite order for the edge field and for color. On the two-powers reading, $\mu_1 = m_p/\sqrt{2} = 0.66346$ GeV: 0.027 halvings above 0.651 GeV and 0.008 halvings below the one-loop value 0.6673 GeV, inside the range 0.65 to 0.67 GeV. On that reading, the scheme in which $\alpha_s = 1$ is the one in which it holds at exactly $m_p/\sqrt{2}$. The scale $\sqrt{2}\, m_p = 1.3269$ GeV sits 0.060 halvings (4.2 percent, 12.0 standard deviations of the charm's measured error) above $m_c(m_c) = 1.2729 \pm 0.0045$ GeV.

Two ratios between color's scales land on simple numbers. The ratio $\zeta = m_p/\mu_1$ is 1.441, $\log_2\zeta = 0.527$; over μ_1 from 0.65 to 0.67 GeV, $\log_2\zeta$ runs from 0.530 to 0.486, and $\sqrt{2}$ ($\log_2 = 0.5$) lies inside. At the one-loop scale, $m_p/\Lambda^{(3)} = 2^{1.50} = 2\sqrt{2}$ to 0.1 percent, with $\Lambda^{(3)}$ the three-flavor MS-bar Λ. *Geeze, Wub-*

lets, and the Higgs gives the mass of a trine as a pure number times the scale at which color grows strong, and does not fix that number; taken with μ_1 as that scale, the number is ζ, the first of these ratios, m_p/μ_1.

Light's reading scale carries three simple candidates within one standard deviation. The data's residue for light's reading lies above m_p, by +0.15 to +0.24 halvings, while the scale where color's strength is one lies 0.53 halvings below m_p; by that sign the reading scale is not the scale μ_1 at which color's strength is one. Inside the window of the reading scale, three candidates stand within one standard deviation of its unrounded place, 10.984 halvings (1.035 GeV): $m_e 2^{11} = 1046.5$ MeV, with $11 = O^2 + n_C$ halvings above the unit ($O = 3$, the odd leg of the kernel triangle), at +0.08; m_p at −0.75; and $m_p 2^{1/4}$ (11.09 halvings) at +0.57. The data select none of the three; the reading of light at the proton's mass selects m_p. The scale μ_1 lies 0.669 ± 0.19 halvings below the reading scale's unrounded place; of that, 0.527 halvings is the distance from μ_1 to m_p, and the remaining $+0.142 \pm 0.19$ halvings, $\log_2(1.035\ \text{GeV}/m_p)$, is light's residue above m_p.

Table III-1. Light's reading scale, 10.98 ± 0.19 halvings above m_e, against candidate masses (pull: candidate minus reading scale, over 0.19; the last two rows are taken against the unrounded 10.984)

Candidate	Place (halvings above m_e)	Pull
m_p, color's whole	10.842	−0.72
m_n	10.844	−0.71
μ_1, where $\alpha_s = 1$	10.315	−3.50
$2m_p$, the floor	11.842	+4.54
$v/2$	17.878	about +36
$v/\sqrt{2}$	18.378	about +39
$m_e 2^{11}$	11	+0.08
$m_p 2^{1/4}$	11.09	+0.57

IV. The Floor $2m_p$

The *pair floor* is the least mass at which a whole of color is made as content: $2m_p = n_C m_p = 1.8765$ GeV. *Made as content* means made as matter: at the first scale of the zoo, content means fermion (a particle of half-integer spin, a particle of matter) and context means boson (a particle of whole-number spin, a carrier of force). Two routes reach the floor, an argued route and a data route. The argued route is set out here; the data route uses the measured light quark masses and the ladder of Chapter 7, and it is set out with the quark masses, which are read at the floor (Chapter 9, Section I).

The argued route

A whole of color is a trine: the whole pattern of comparison held inside content, the three positions together, available at once. No process that can be produced changes the count of baryons (the earliest moments of a universe are the exception; The Branches of the Kinds, Chapter 2, Section II), and the proton is stable. An antibaryon counts minus one, so a trine is made only together with its conjugate. The proton is, as measured, the lightest baryon. The conjugate pair has multiplicity two, which is n_C. So the least whole of color made as content is the proton, made together with its antiproton, and the least mass at which it is so made is $2m_p = n_C m_p = 1.8765$ GeV; the zoo gives a proton-antiproton pair as about 1,876 MeV. The proton's being the lightest baryon is enough by itself to exclude the neutron's pair, $n_C m_n$. The floor is the pair of wholes: n_C times color's whole.

Content at the first scale means fermion, for composites as for elementary particles. A color-complete state is a fermion exactly when its count of baryons is odd; a meson's count is zero, so every meson is a boson (the pion has spin zero), and no meson is content. So no meson is a whole of color made as content, and the least mass at which a whole of color is made as content is the pair $2m_p$, and not a meson's mass.

Box IV-1. The least whole of color made as content

Premise 1 (the zoo, Level 7, Section II, and Level 9, Section I): a whole of color is a trine, three quarks forming a color-neutral whole.

Premise 2 (standard physics; the zoo, Level 0, Section IV): no process that can be produced changes the count of baryons (the earliest moments of a universe are the exception; the zoo, Level 4, Section IV); the proton is stable.

Premise 3 (standard physics; the zoo, Level 4): an antibaryon counts minus one, so a trine is made only together with its conjugate; a pair costs twice the rest mass.

Premise 4 (measured; the zoo, Level 0, Section IV): the proton is the lightest baryon.

Premise 5 (an identification of this chapter): the trine and its antiparticle partner are counted as a pair of multiplicity $n_C = 2$, the count of the conjugate pair of the book of kinds (Chapter 2, Section I).

Premise 6 (the zoo, Level 6, Section VI; standard physics): content at the first scale means fermion, for composites too; a color-complete state is a fermion exactly when its baryon count is odd; every meson is a boson.

Step 1: by Premises 1 to 3, the least color-complete content made from nothing is a baryon with its antibaryon.

Step 2: by Premise 4, the least such pair is the proton with its antiproton, so the least whole of color made as content is the proton, made at the cost $2m_p$; $n_C m_n$ is excluded by Premise 4 alone.

Step 3: by Premise 6, no meson is a whole of color made as content.

Result: the floor is $2m_p = n_C m_p = 2 \times 938.27208943$ MeV $= 1.87654$ GeV (CODATA 2022 proton mass). Among the multiples of m_p by a kernel ratio i/j, with i and j in {1, 2, 3, 4, 5, 12, 60}, only the multiple 2 lies inside the data's window 1.865 to 2.050 GeV.

V. The Proton's Place and the Ratio m_p/m_e

The *proton's place* is its position above the electron in halvings, $x_p = \log_2(m_p/m_e) = 10.8425$. Its form is argued (Section II): m_p/m_e is the exponential of an inverse strength of color, times numbers fixed by structure. Its value is measured; a law fixes that value, and this book does not state that law. The argument from comparison gives the order of the two inertias, the proton's the greater; the amount is a value of the instance (a value of this world that the structure of comparison does not fix) and is not derived. Since color's strength and m_p/m_e are one value, a law for the proton's place is a law for color's strength, and the converse.

A near equality stands beside the ratio m_p/m_e: $m_e/m_p \approx \alpha^2 m_p/\Lambda_{QCD}$, with $\alpha = 1/137.036$ the fine-structure constant. With the one-loop $\Lambda = 87$ MeV it gives 5.74×10^{-4}; with the four-loop MS-bar $\Lambda = 210$ MeV it gives 2.38×10^{-4}; the measured m_e/m_p is 5.446×10^{-4}, and equality needs $\Lambda = 91.7$ MeV. In the one-loop five-flavor convention ($\Lambda = 87.8$ MeV for $\alpha_s(M_Z) = 0.1180$), $\Lambda = 91.7$ MeV corresponds to $\alpha_s(M_Z) = 0.11875$, +0.83 standard deviations from the measured 0.1180 ± 0.0009 (Particle Data Group, PDG, 2024).

Color's way and light's way

Color's way writes color's one-loop transmutation between the proton and the reduced Planck mass in halvings: $1/\alpha_s(M_{Pl,red}) = (b_0 \ln 2/2\pi)(E - x_p + c)$, with $b_0 = 7 = O^2 - n_C$. Here c is a number of halvings, color's slot in the law. The three-loop running of the measured $\alpha_s(M_Z)$ gives $1/\alpha_s(M_{Pl,red}) = 51.17 \pm 0.10$; reaching that value asks $c = 5.09 \pm 0.13$ halvings. With $c = H = 5$, the hypotenuse of the kernel triangle (the book of kinds, Chapter 2, Section II), the law gives 51.099, −0.71 standard deviations. The residue $c - H = 0.092 \pm 0.129$ halvings is a measure of the second of the three groups of measure, the group of one constant read from a continuum; here the continuum is the thresholds of the quark kinds and the higher loops. Box V-1 sets out the arithmetic.

Box V-1. Color's slot c

Premise 1 (standard physics, one-loop transmutation in halvings): $1/\alpha_s(M_{Pl,red}) = (b_0 \ln 2/2\pi)(E - x_p + c)$, $b_0 = 11 - (2/3)6 = 7$.

Premise 2 (measured, Chapter 5): $E = 72.013$ halvings.

Premise 3 (measured): $x_p = \log_2(m_p/m_e) = 10.8425$.

Premise 4 (derived from measurement by three-loop running): $1/\alpha_s(M_{Pl,red}) = 51.17 \pm 0.10$.

Step 1: $7 \ln 2/(2\pi) = 0.7722$ per halving; $E - x_p = 61.17$.

Step 2: $c = 51.17/0.7722 - 61.17 = 5.09$, with uncertainty $0.10/0.7722 = 0.13$ halvings.

Step 3: $c = H = 5$ gives $0.7722 \times 66.17 = 51.099$; $(51.099 - 51.17)/0.10 = -0.71\sigma$.

Step 4: $c - H = 0.092 \pm 0.129$ halvings.

Two routes with different measured inputs return the proton's place. Color's way takes color's slot $c = H$ and the measured α_s, with the top decoupled from color below its mass: it returns 10.746 ± 0.081 halvings, $m_p/m_e = 1717$, -1.19 standard deviations; without the top's threshold in color it returns 10.845, $+0.03$. Light's way solves light's relation of unit 3π, $1/\alpha_{em}(m_p) = 3\pi\Delta_{21}(E)$ (Chapter 2; $\Delta_{21}(E)$ is the gap between the inverse strengths of the weak and hypercharge forces at the reduced Planck mass), for its reading scale, from the measured strengths of light and of the weak and hypercharge forces: it returns 10.83 ± 0.19 halvings, $m_p/m_e = 1818$ (1593 to 2074), -0.08 standard deviations. The two ways agree at 0.4 standard deviations. They use different measured couplings, α_s for color's way and α_{em}, α_2, α_1 for light's way, and share the span E and the three-loop running. The proton-to-electron ratio has these two consistent routes; a law fixes the ratio, and this book does not state that law.

The matching scheme moves color's way. Under SMDR matching color's way returns 10.755 ± 0.081 halvings, $m_p/m_e = 1728$, -1.08 standard deviations, and Buttazzo and others' matching returns 1765, -0.70. Read the other

way, from the measured m_p/m_e, the same law asks $\alpha_s(M_Z) = 0.11898$ under SMDR, +1.1 standard deviations; with the PDG 2024 inputs ($M_t = 172.57$ GeV, $\Delta\alpha_{had} = 0.02783$, $\alpha_s = 0.1180$, $M_h = 125.20$ GeV) it asks 0.11897, +1.08. The 0.26 standard deviations of Section II belongs to one tree-level scheme. Each value holds in its own scheme.

What the closures return

The closures of the force block, whose laws (the electroweak level $1/\alpha(X_{12}) = 169/4$, the inverse strength at the crossing X_{12}, and the ratio 21/4 among them) are Chapter 3's, also return the proton. Their pulls are taken against the closure's own propagated uncertainty, 0.0102 halvings (0.7 percent in the ratio), and not against the measurement, which is known far more finely (Section II). The second closure of Chapter 3, with this chapter's values $\alpha_s = 0.118877$ and a proton of 932.709 MeV, gives $x_p = 10.8339 \pm 0.0102$ halvings, $m_p/m_e = 1825.3$, −0.84 standard deviations of that uncertainty, and so a floor of 1.8654 GeV. At Chapter 3's inputs, with nothing in the block measured, the second closure with the electroweak level 169/4 and the ratio 21/4 gives $x_p = 10.8358 \pm 0.0102$ halvings, $m_p/m_e = 1827.7$, −0.65 (Chapter 3); at those inputs every ratio in its window passes alike. With the closure's own top and its span of Chapter 5, the measured proton excludes the ratio 47/9 of the four candidates in the window (47/9 at −2.76 standard deviations in x_p) and leaves three, 21/4 among them; and the electroweak level is not free: with 21/4, the electroweak levels 167/4 and 171/4 put x_p at +77 and −79 standard deviations. The number 21/4 is a count of the product, not a radius, $\dim K \times (D-3)/(D-2)$ at $D = 10$ with $\dim K = 6$; the count is this book's, and it uses no radius, which the content instance does not have (the book of kinds, Chapter 1, Section XIII).

The joint closure solves four laws together for four unknowns, $1/\hat{\alpha}(M_Z)$ (the inverse MS-bar strength of light at the Z mass), $\sin^2\theta_W$ (the weak mixing angle), x_p and α_s, and returns $x_p = 10.40 \pm 0.41$ halvings, and 10.58 with α_s held at its measured value. The joint closure has two solutions; both pass from the SMDR start, and from the start of Buttazzo and others the joint clos-

ure fails ($1/\hat{\alpha}(M_Z)$ at -8.23 standard deviations). The first gives $x_p = 10.580 \pm 0.202$ halvings, $m_p/m_e = 1531$, -1.30 standard deviations, with $1/\hat{\alpha}(M_Z) = 127.9470$ (-0.44), $\sin^2\theta_W = 0.231213$ (-0.12) and $\alpha_s(M_Z) = 0.1161 \pm 0.0022$ (-0.80). The second, with the electroweak strengths at the top fixed from light and the W mass the Standard Model's output, gives $x_p = 10.398 \pm 0.411$ halvings, $m_p/m_e = 1349$, -1.08, with $1/\hat{\alpha}(M_Z) = 127.9327$ (-0.94), $\sin^2\theta_W = 0.231244$ ($+0.32$) and $\alpha_s(M_Z) = 0.1142 \pm 0.0044$ (-0.85). For the first solution, and for the second with α_s held at the measured 0.1180, x_p moves 47.9 halvings per unit of the electroweak level, and the window of the electroweak level for $|\text{pull of } x_p| \leq 2$ is 42.2555 ± 0.0084.

Box V-2. The closures' proton in halvings and in ratio

Premise 1 (the second closure of Chapter 3): with this chapter's values ($\alpha_s = 0.118877$, a proton of 932.709 MeV) it returns $x_p = 10.8339 \pm 0.0102$ halvings; at Chapter 3's inputs it returns $x_p = 10.8358 \pm 0.0102$ halvings, $m_p/m_e = 1827.7$ (Chapter 3).

Premise 2 (measured): $x_p = 10.8425$.

Step 1: $2^{10.8339} = 1825.3$; $(10.8339 - 10.8425)/0.0102 = -0.84\sigma$ of the closure's uncertainty.

Step 2: $\log_2 1827.7 = 10.8358$; $(10.8358 - 10.8425)/0.0102 = -0.65\sigma$.

Step 3: $m_e \times 1825.3 = 932.7$ MeV; twice that is 1.8654 GeV.

Step 4: $2^{0.0102} - 1 = 0.7$ percent, the closure's uncertainty in the ratio.

Step 5 (joint closure): $2^{10.580} = 1531$, $(10.580 - 10.8425)/0.202 = -1.30\sigma$; $2^{10.398} = 1349$, $(10.398 - 10.8425)/0.411 = -1.08\sigma$.

Table V-1. Routes to the proton's place $x_p = \log_2(m_p/m_e) = 10.8425$ (measured; $m_p/m_e = 1836.15267343$, CODATA 2022 masses)

Route	x_p (halvings)	m_p/m_e	Pull, and against what
Color's way, c = H	10.746 ± 0.081	1717	−1.19, the route's uncertainty
Color's way, SMDR matching	10.755 ± 0.081	1728	−1.08, the route's uncertainty
Light's way	10.83 ± 0.19	1818	−0.08, the route's uncertainty
Second closure of Chapter 3, this chapter's values	10.8339 ± 0.0102	1825.3	−0.84, the closure's uncertainty
Second closure of Chapter 3, electroweak level 169/4 and ratio 21/4, Chapter 3's inputs	10.8358 ± 0.0102	1827.7	−0.65, the closure's uncertainty
Joint closure, first solution (SMDR start)	10.580 ± 0.202	1531	−1.30, the closure's uncertainty
Joint closure, second solution (SMDR start)	10.398 ± 0.411	1349	−1.08, the closure's uncertainty

VI. The Proton's Size

The proton is composite, so it has an extent. Its charge radius is finite, measured at about 0.84 fm; the strong holding sets it, and it is not derived. Inside the proton the charge is spread over the parts, and the electric hold departs from its inverse-square form; so the atom's states that reach the proton, the S states, are shifted by a term proportional to $(r_p/a_0)^2$, with a_0 the Bohr radius.

The proton's own length is its *reduced Compton length*, $\hbar/(m_p c) = 0.2103$ fm. In that unit the charge radius is $m_p r_p/(\hbar c)$, a relation of the proton's size to its mass that uses $\hbar$, c and m_p and does not involve the electron. The rela-

tion $m_p r_p/(\hbar c) = 4$ is a *value that lands*: it agrees with measurement, and this book states no reason for it. With the CODATA (the Committee on Data of the International Science Council) 2022 radius, 0.84075 ± 0.00064 fm, the value is 3.9977 ± 0.0030, and 4 stands at +0.76 standard deviations. With the CODATA 2018 radius, 0.8414 ± 0.0019 fm, it is 4.0008 ± 0.0090, and 4 stands at −0.09. With the CODATA 2014 radius, 0.8751 ± 0.0061 fm, it is 4.1610 ± 0.0290, and 4 stands at −5.55. The CODATA 2022 value has a third of the CODATA 2018 uncertainty.

Box VI-1. The extent in the proton's Compton unit

Premise 1 (measured, CODATA 2022): m_p = 938.27208943(29) MeV; $\hbar c$ = 197.3269804 MeV fm (exact); r_p = 0.84075(64) fm; the proton's magnetic moment 2.79284734463 nuclear magnetons, so κ_p = 1.79284734463.

Premise 2 (measured): r_p = 0.8414 ± 0.0019 fm (CODATA 2018); 0.8751 ± 0.0061 fm (CODATA 2014); 0.84060(66) fm from the muonic Lamb shift alone.

Premise 3 (standard physics): in the atomic convention used by CODATA the radius is the slope of the Sachs electric form factor, $\langle r^2\rangle = 6\hbar^2\, dG_E/dq^2$ at $q^2 = 0$, and the Darwin-Foldy term $3\hbar^2/(4m_p{}^2c^2)$ stays in the atomic theory; $G_E = F_1 - (Q^2/4m_p{}^2)\kappa_p F_2$, so the Dirac radius satisfies $\langle r^2\rangle_{F1} = \langle r^2\rangle_{GE} - 3\kappa_p/(2m_p{}^2)$ in natural units.

Step 1: $x = m_p r_p/(\hbar c)$ = 938.27208943 × 0.84075/197.3269804 = 3.9977, with uncertainty 0.00064 × 4.7549 = 0.0030.

Step 2: with 0.8414 fm, 4.0008 ± 0.0090; with 0.8751 fm, 4.1610 ± 0.0290; with 0.84060 fm, 3.9970 ± 0.0031.

Step 3: with the Darwin-Foldy term added, $\sqrt{(x^2 + 3/4)}$ = 4.0904; for the Dirac radius, $\sqrt{(x^2 - 3\kappa_p/2)}$ = 3.6459.

Step 4: pulls of 4, in the book's convention (4 minus the measured value, over its uncertainty): +0.76 (2022), −0.09 (2018), −5.55 (2014), +0.96 (muonic), −30 (Darwin-Foldy convention), +106 (Dirac radius, uncertainty 0.0030 × 3.9977/3.6459 = 0.0033).

The landing belongs to one radius and rests on one set of data. It belongs to the *Sachs electric radius* in the atomic convention (the radius from the slope of the proton's electric form factor at zero momentum transfer, the form factor being the proton's response to a probe as a function of the momentum transferred): the same CODATA 2022 radius gives 4.0904 in the convention that adds the Darwin-Foldy term, where 4 stands at −30 standard deviations, and the *Dirac radius* (from the slope of the Dirac form factor F_1) gives 3.6459 ± 0.0033, where 4 stands at +106. It rests on the data of muonic hydrogen (hydrogen with a muon, the electron's heavier copy, in place of the electron): the muonic data alone give 3.9970 ± 0.0031 (4 at +0.96); the electronic hydrogen and deuterium data alone give 4.0555 ± 0.0207 (4 at −2.7); and the 2026 measurement of the hydrogen 2S-6P transition alone (Maisenbacher and others, arXiv:2602.14980, $r_p = 0.8406(15)$ fm) gives 3.9970 ± 0.0071 (4 at +0.42). The CODATA 2022 adjustment run without the muonic Lamb-shift data (the Lamb shift being the splitting of the 2S and 2P states) gives $r_p = 0.8529$ fm, with relative uncertainty 51×10^{-4}.

Standard physics gives the number its kind. QCD has one scale of mass besides the quark masses (dimensional transmutation), so $m_p r_p/(\hbar c)$ is a pure number of QCD, apart from a small correction from QED, fixed by the number of colors N_c, the number of light flavors and the quark masses in QCD's own unit. It depends on the quark masses: in the chiral limit (the light quark masses taken to zero) the pion cloud (the virtual pions around the proton) makes the charge radius diverge logarithmically while the mass stays finite, so the product diverges there, and a 4 at the physical point is a statement about QCD at the physical quark masses. At large N_c and fixed quark masses the product grows linearly in N_c at leading order, so any reading linear in N_c, a multiple of N_c plus a constant, is the form QCD allows. Lattice QCD at the physical pion mass gives the electric radius 0.820(14) fm and the magnetic radius 0.8111(89) fm, so $m_p r_p/(\hbar c) = 3.90 \pm 0.07$, 1.5 standard deviations below 4 and below the measured 3.998; QCD has not yet been computed finely enough to tell 4 from its neighbors.

A standard-physics route gives a 4 of a different radius. In light-front holographic QCD (an approximation to QCD built from a correspondence with a five-dimensional theory of gravity) with massless quarks, the nucleon's mass is 2κ and its Dirac radius is $2/\kappa$, for any value of the confinement scale κ, so their product is exactly 4 in units of $\hbar c$, the 4 being 2×2 (de Téramond, arXiv: 1602.04899, 2016). That 4 depends on placing the poles of the form factor (the squared momenta at which it diverges) at the masses of the vector mesons, $4\kappa^2(n + 1/2)$; with the poles at $4\kappa^2(n + 1)$ it is 3. The same framework gives 4.32 for the Sachs radius. So its 4 belongs to the Dirac radius, whose measured value is 3.6459 in these units, while the measured 4 belongs to the Sachs radius; the two are 4s of different radii, set side by side.

Box VI-2. The holographic 4

Premise 1 (light-front holographic QCD, massless quarks): baryon masses $M^2 = 4\kappa^2(n + L_B + 1)$; the nucleon ($n = 0$, $L_B = 0$) has $M_N = 2\kappa$.

Premise 2: the proton's Dirac form factor is $F_1 = 1/((1 + Q^2/{M_0}^2)(1 + Q^2/{M_1}^2))$, with poles at the vector mesons ${M_n}^2 = 4\kappa^2(n + 1/2)$.

Premise 3 (standard physics): $\langle r^2 \rangle = -6F'(0)$, and for a product of monopoles $-6F'(0) = 6\Sigma 1/{M_n}^2$.

Step 1: $\langle {r_1}^2 \rangle = 6(1/(2\kappa^2) + 1/(6\kappa^2)) = 4/\kappa^2$, so $r_1 = 2/\kappa$ and $M_N r_1 = 2\kappa \times 2/\kappa = 4$ for every κ.

Step 2: with the poles at $4\kappa^2(n + 1)$, $6(1/4 + 1/8)/\kappa^2 = 9/(4\kappa^2)$, and $M_N r_1 = 3$.

Step 3: adding the Pauli term normalized by $\kappa_p = 1.79284734463$, $M_N r_E = \sqrt{(16 + 3\kappa_p/2)} = 4.323$.

Table VI-1. The proton's extent in its own reduced Compton length, against 4 (pull: 4 minus the value, over its uncertainty)

Radius and data	$m_p r/(\hbar c)$	Pull of 4
Sachs electric radius, CODATA 2022	3.9977 ± 0.0030	+0.76
Sachs electric radius, CODATA 2018	4.0008 ± 0.0090	−0.09
Sachs electric radius, CODATA 2014	4.1610 ± 0.0290	−5.55
Muonic hydrogen data alone	3.9970 ± 0.0031	+0.96
Electronic hydrogen and deuterium data alone	4.0555 ± 0.0207	−2.7
Hydrogen 2S-6P (2026) alone	3.9970 ± 0.0071	+0.42
With the Darwin-Foldy term, CODATA 2022	4.0904	−30
Dirac radius, CODATA 2022	3.6459 ± 0.0033	+106
Lattice QCD, electric radius (a calculation)	3.90 ± 0.07	4 lies 1.5 above it

Summary

The proton's mass is a mass of binding, set by color's strength through dimensional transmutation; the electron's is set by its coupling to the edge field. Their ratio is the exponential of an inverse strength of color times numbers fixed by structure, and color's strength and the ratio are one value. Color's whole is the proton's mass, reached by three routes on three bases; light's strength is read there; and the pair floor $2m_p = n_C m_p = 1.8765$ GeV is the least mass at which a whole of color is made as content, reached here by an argued route; the light quarks are read there (Chapter 9).

The ratio m_p/m_e is measured; a law fixes it, and this book does not state that law. Color's way and light's way return it from different measured couplings and agree at 0.4 standard deviations; the closures of Chapter 3 return it against their own uncertainty. The proton's charge radius in its own reduced Compton length is 3.9977 ± 0.0030, and 4 stands at $+0.76$ standard deviations; the relation is a value that lands, read on the Sachs electric radius and the muonic data.

The question each value that lands in this chapter carries is why it lands: color's slot at $c = H$, the hypotenuse of the kernel triangle (clue: $b_0 = 7 = O^2 - n_C$); the one-loop Λ of 91.7 MeV that the near equality $m_e/m_p \approx \alpha^2 m_p/\Lambda$ asks (clue: it corresponds to $\alpha_s(M_Z) = 0.11875$, $+0.83$ standard deviations); and the proton's charge radius at four of its own reduced Compton lengths (clues: the Sachs radius, the muonic data, and the holographic 4, which belongs to the Dirac radius).

KEY CONCEPTS

- **Binding mass.** The mass of a whole that comes from the energy of what holds its parts; about ninety-nine percent of the proton's mass.
- **Halving.** One factor of two in mass; a distance in halvings is the base-two logarithm of a ratio of masses.
- **Dimensional transmutation.** A running strength with no unit fixing a mass: the scale Λ where color's strength becomes of order one.
- **Color's whole.** The mass at which the strong force's pattern is complete; this book places it at the proton's mass.
- **The floor.** The pair floor $2m_p = n_C m_p = 1.8765$ GeV, the least mass at which a whole of color is made as content; the light quarks are read there.
- **Reading scale.** The scale at which a running quantity is compared with the law that predicts it.
- **The proton's place.** $x_p = \log_2(m_p/m_e) = 10.8425$, the proton's place above the electron in halvings.
- **Value that lands.** A value that agrees with measurement and for which this book states no reason; it is stated with its measurement and its pull.

Notes

Chapter 1. What Is Measured

1. The two premises of Box I-1 correspond to An Immanent Metaphysics, aphorism 1.75-19 (only things of actuality can be measured or determined objectively; potentiality can never be) and aphorism 1.87-3 (measurement, interaction, the transformation of states, force, power and strength, and observation itself, have the nature of the modality of the immanent).

2. An Immanent Metaphysics, aphorisms 1.89-1 and 1.89-2, binds the two limits of a domain: the speed of light as the omniscient (macroscopic) limit, and Planck's constant as the transcendent (microscopic) limit. Each limit joins the immanent to the position of its own limit: c joins the immanent to the omniscient, and ħ joins the immanent to the transcendent (Measurement, Chapter 7, Section VII).

3. The six kinds of measurement pair as An Immanent Metaphysics, aphorism 1.85-15, pairs them: force in time (the immanent), inertia in space (the omniscient), probability in possibility (the transcendent). The scale is anchored on the electron (as distinct from Newton's constant), and the anchoring is a matter of measurement (metrology, as distinct from ontology, the account of what exists).

Chapter 2. The Couplings and Their Running

Section I. Force and probability are different measures of a measurement: in the terms of An Immanent Metaphysics, force goes with time and the immanent, probability with possibility and the transcendent. The strength of the one force at the finest resolution stands at the join of the two.

Section III. The modalities (omniscient, immanent, transcendent) are the three positions of comparison of the book of kinds, Chapter 1, Section II. In the focus of comparison as relation each row keeps its modality on both sides of the Higgs condensate: color omniscient, the weak doublet transcendent, the phase immanent.

Chapter 3. Where the Couplings Cross

1. The three modalities named in Section II (omniscient, transcendent, immanent) are those of *An Immanent Metaphysics*, which the book of kinds, Chapter 1, Section II, defines for this book as the three positions of comparison.

2. The running tools named in this chapter are SMDR (Martin and Robertson), the matching of Buttazzo and collaborators, and the three-loop gauge rates of Mihaila, Salomon and Steinhauser; they are listed in the Bibliography.

Chapter 4. The Weak Angle and the Masses of W and Z

1. The W and Z belong to the weak row of the gauge factors, which is transcendent in the focus of comparison as relation; the row keeps its modality before and after the condensate, the gauge factor SU(2) before it and the W and Z after it (the book of kinds, Chapter 4, Section III). The three modalities, omniscient, transcendent and immanent, are the book of kinds' terms, defined in its Chapter 1 as the three positions of comparison.

Chapter 5. The Span to the Planck Mass

The Incommensuration Theorem used in Section III corresponds to An Immanent Metaphysics, Technical Appendix, aphorisms 1R1 to 1R20; the axioms of comparison correspond to its aphorism 1.3-1.

That the reduced Planck mass is the natural unit of gravity's coupling is stated in The Unquantized Arena, notes to Chapter 9, note 4.

That the values of the couplings are measured, and not fixed by the kinds of particle, is stated in The Story of the Particles, Level 9.

Chapter 6. The Binding Scale of the Proton

Section III and Section IV. The doubling of color's whole into the pair floor, and the question whether the scale μ_1 is a second whole of color, correspond to An Immanent Metaphysics, aphorism 1.47-4: for every part there are always at least two wholes.

Section I. The three carriers of the frame of measurement (the photon at the act of comparing, the electron at the thing compared, the proton at the thing compared against) and the three meanings of mass correspond to the three modalities of An Immanent Metaphysics, which the book of kinds, Chapter 1, Section II, defines as the three positions of comparison.

Terms

Actual, potential (*actuality, potentiality*)

The actual is what is found (actuality: the relation of force to time); the potential is what could be found (potentiality: the relation of probability to possibility). In the split of the charged-lepton triple the actuality is carried by the static aspect, the common part that the rotations leave unchanged, and the potentiality by the dynamic aspect, the differing part. (Chapter 1)

Anchor

In Chapter 4, the ideal part of the carriers' law, $v^2/4$; in Chapter 11, the frame from which a mixing block's mixing starts before any correction: the identity for the quark block, the sameness frame for the lepton block; in Chapter 13, the whole fixed by the structure, the first stage of a measure. (Chapter 4)

Anomalous moment

The amount $a_e = (g - 2)/2$ by which the electron's g-factor exceeds 2. (Chapters 1 and 2)

Anomaly

A quantum effect that breaks the conservation law of a symmetry in a fixed, computable way; 't Hooft anomaly matching asks the bound wholes to reproduce the anomalies of their parts. (Section III) (Chapters 2 and 6)

Asymptotic freedom

The fall of a strength at shorter distance; in Chapter 2's convention, a negative rate. (Chapter 2)

Axial coupling

The Z's coupling $a_f = T_3$ to the axial current of a fermion f. (Chapter 4)

Axiom I

"The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relation-

ship to one another" (An Immanent Metaphysics, aphorism 1.3-1). (Chapter 5)

Balance

The equality $|c|^2 = |d|^2$ of the squared lengths of the common part and the differing part; it is Koide's relation $Q = 2/3$. (Chapter 1)

Baryon

Three quarks, one of each color, forming a color-neutral whole; this book calls it a trine. The proton is the lightest. (Section III) (Chapter 6)

Binding mass

The mass of a whole that comes from the energy of what holds its parts. (Section I) (Chapter 6)

Bit; information in bits

A bit is the base-two logarithm of a ratio. In Chapter 8, a selection carries the base-two logarithm of the ratio of the range of Q to the width of the window, less the base-two logarithm of the number of forms considered; a window of width w on a range of width W carries $\log_2(W/w)$ bits. In Chapter 13, the information an output carries is the base-two logarithm of the measured value over the larger of its measured uncertainty and the distance between the stated and the measured values: the number of binary digits of the measured value that the output gets right. (Chapter 5)

Block

The three laws taken together: light's inverse strength at the proton's mass equals 3π times the split $\Delta_{21}(E)$; $\rho = H$; color's binding join with slot $c = H$. (Chapters 3 and 6)

Bohr radius

The size of hydrogen's least state, $a_0 = \hbar/(m_e c \alpha_{em})$, about 0.529×10^{-10} m. (Chapters 1 and 6)

Book of kinds

The Kinds of Particle; Why the Structure Must Be So, the book that derives which kinds of particle there are and gives the reasons for those kinds, using no measured number. (Chapters 1, 2, 3, 4, 5 and 6)

Boson

A particle of whole-number spin, a carrier of force; at the zoo's first scale, context. (Section IV) (Chapters 1, 2, 3, 4 and 6)

Brute constant

A number with no reason; this book holds that no pure number among the particles is one: every pure number not yet derived carries the question that settles it, the law that fixes it or the proof that no law does. (Chapter 1)

c_1, n_C

$c_1 = 3$ is the count of the three positions of comparison; it is also the count of colors and of generations. $n_C = 2$ is the count of the poles, the conjugate pair content and context, the two aspects into which comparison partitions what is compared (the book of kinds, Chapter 2, Section I). (Chapters 3 and 5)

c_2, c_3

The second-order coefficient of the step s, 28/5; the coefficient of a third-order term in s. (Chapters 1, 5 and 6)

Carriers' clause

The carriers' law written $(v^2/4)(1 + n\kappa_H)$, with $n = -3$. (Chapter 4)

Carriers' law

The relation $m_W^2 + m_Z^2 = v^2(1/4 - 1/(16\pi^2))$: the carriers' quarter of v^2 less one loop unit of v^2. (Chapter 4)

Carry

The placement, in the lepton block, of the right angle of the quark unitarity triangle between the first and third generations' sides on both indices. (Section IV) (Chapters 1, 2, 3, 4, 5 and 6)

χ^2, $\Delta\chi^2$

χ^2 is the sum of the squared pulls of a set of values; $\Delta\chi^2$ is its rise above its least value. A scale passes when $\Delta\chi^2$ is at most 4 (Chapter 6); a step fixed at a named number is judged by the rise it causes (Chapter 7). (Chapter 6)

Chirality

The left-handed or right-handed state of a fermion. (Chapters 1 and 5)

Circle's rule

A value is fixed by concepts alone only when one form supplies all the terms the value relates. The rule is named for the circle of the flat plane, whose concept supplies both terms of the ratio π and so fixes it; on a curved surface the circle's concept no longer supplies every term of the ratio, and it no longer fixes the ratio. (Chapter 1)

Circuit

Probability's triple of three transitions and one circulation: the three transitions close on one another, and the one circulation runs once round them. Its figure in quark mixing is the unitarity triangle, its three sides closing as the three transitions do and its one area answering to the one circulation (*The Measures of the Kinds*, Chapter 10). (Chapter 1)

Classical electron radius

The length r_e at which the energy between two electron charges equals the electron's rest energy. (Chapter 1)

Closed list of contents

Three generations of every kind of matter particle, and the one Higgs doublet (the joiner). (Chapter 2)

Closure; first closure, second closure

A joint solution of a set of laws. In Chapter 3, of the block's laws: in the first closure the proton-to-electron mass ratio is measured, and in the second a law of the level is added and nothing in the block is measured. In Chapter 4, of the carriers' law and light's relation, with no measured electroweak coupling. In Chapter 7, an identity among the constants of two or more sectors whose value is a measure of the kernel triangle. (Chapters 3, 4 and 6)

CODATA

The Committee on Data of the International Science Council, whose adjustments give the recommended values of the fundamental constants, cited by year (CODATA 2022). (Chapters 1, 2, 5 and 6)

Coefficient premise

The premise that the three inverse strengths are the coefficients of one vector on the axes of the three ways. (Chapter 2)

Color

The charge of the strong force; its strength is α_s. (Chapters 1, 2, 3, 4, 5 and 6)

Color's way

The route to the proton's place x_p from color's one-loop transmutation with the slot c = H and the measured α_s. (Section V) (Chapter 6)

Color's whole

The mass at which the strong force's pattern is complete: the proton's mass. (Section III) (Chapter 6)

Comparison

The act of holding one thing against another: a thing compared, an act of comparing, and a thing compared against. (Chapters 1, 2, 3, 4, 5 and 6)

Comparison structure

The kinds of finding of a world, which contents recur across contexts, and its possible verdicts. It is the same at every value of α_{em}: on a finite lattice the weights that two strengths of light give one arrangement stay within fixed positive multiples of each other, so the possible, the impossible and the certain arrangements do not change. (Chapter 1)

Condensate (Higgs condensate); held carriers

The condensate is the constant value v that the Higgs field takes throughout space, 246.22 GeV. The held carriers are the W and Z bosons, to which it gives mass. (Chapters 1, 2, 3, 4, 5 and 6)

Condensate's whole

$v/\sqrt{2}$, with v the vacuum value; at 18.378 halvings above the electron (Chapter 13). (Chapters 3 and 5)

Conjugates

The omniscient and the transcendent, a pair in structural opposition (Axiom I). (Chapter 1)

Content, context

The two aspects into which comparison partitions what it compares, the conjugate pair counted by $n_C = 2$ (Chapter 13). Read at the book of kinds' scales: at the zoo's first scale, content means fermion (matter) and context means boson (the carriers of force) (Chapter 6); at scale 3, the one quark ladder is the content, which is the whole, and the two lepton ladders are the contexts, which pay shares of it (Chapter 7). In the mode reading of the charged leptons, the content is the non-rotating common part of the triple and the contexts its two rotating parts (Chapter 8). In Chapter 3, content names the kinds of particle and their charges, as the book of kinds derives them. (Chapters 1, 2, 3, 4, 5 and 6)

Content instance; context instance

The two instances of the internal space: the content instance is the internal space read as a space of states, with no size, no radius and no fields (the book of kinds, Chapter 3); the context instance is the internal space read with radii, the subject of The Gravitational Passage. Every statement of this book about the internal space is made on the content instance. (Chapters 1, 2, 3, 4, 5 and 6)

Continuum whole

One magnitude read as continuous; here the reduced Planck mass. (Chapter 5)

Copy

One generation of one kind of quark. (Chapter 6)

Count

The integer $72 = n_C{}^3 c_1{}^2$: eight first-order states times nine joins, each comparison one halving. (Chapters 1, 2, 3, 4, 5 and 6)

Count of the product

The number $21/4 = \dim K \times (D - 3)/(D - 2)$ at $D = 10$ with $\dim K = 6$, the dimension of the internal space, and $D = 10$ the dimension of its product with spacetime; a count, not a radius. (Chapters 3 and 6)

Count route; Koide route

The two routes of the neutrino fork. On the count route the neutrino's step cost is given by the sector rule, 9/4, with curvature −41/60 and the nested offset. On the Koide route the neutrinos are in the scope of Koide's relation, with the lightest amplitude negative and step cost 3/2. (Chapter 5)

Count's whole

The reduced Planck mass, at which the count of 72 halvings ends. (Chapter 5)

Coupling

The strength with which a force acts; for light, α_{em}. (Chapters 1, 2, 3, 4, 5 and 6)

Covering; spreading

This book's terms for screening (pairs of contents near a source take up part of its label) and antiscreening (labeled carriers carry part of the label outward). (Chapter 2)

CP phase

The phase by which a mixing table tells matter from its mirror image under charge and parity (CP). (Chapter 1)

Crossing

A scale at which two inverse strengths are equal. (Chapters 1, 2, 3, 5 and 6)

Cube

The figure of the finite factor, with eight vertices; on the rationale of Section III, the vertices are one generation's eight fermions of one chirality. (Chapter 5)

Curvature, d

The coefficient of g^2 in the step formula, which bends the spacing of a ladder from one generation to the next: the extent of the count, half the second difference of the bracket, and the only constant that carries a sign; 16/9 for the charged leptons. (Chapter 5)

Cut, column, ceiling, ground

A cut is a mass at which a ladder is divided, the whole of one class of its members, and the reading of a mass can change there; a ladder is cut only

at the wholes of its members' classes, one cut per class. A column is the part of a ladder between two cuts; on the quark ladder the top stands alone, the charm with the bottom, the strange with the down. The ceiling is the whole of the whole scope: on the quark ladder, the cut at the largest coupling to the Higgs condensate, where the top, with coupling one, stands alone. The ground is the whole of the charged kind, the cut that sets the ladder's lowest rung in a column of its own: on the quark ladder, the cut u | d, at which the reading does not change. (Chapter 3)

Cycle, turn

The cyclic permutation C of the three generations, $C^3 = 1$. (Chapter 2)

Darwin-Foldy term

The term $3\hbar^2/(4m_p{}^2c^2)$ that the atomic convention keeps in the atomic theory and not in the radius. (Section VI) (Chapter 6)

Decoupling; threshold

A content of mass m acts on a rate only above its mass; a threshold is a mass at which a rate changes. (Chapter 2)

$\Delta_{21}(E)$

The gap $1/\alpha_2 - 1/\alpha_1$ read at the end of the measured span. (Chapters 3, 5 and 6)

Derived

Given by an argument. (Chapters 1, 2, 3, 5 and 6)

Derived, given a stated identification

The standing of a value that follows by argument once an identification is made; the identification is stated with the question it carries, whether it is true of the world. The angles $\beta = \pi/8$ and $\gamma = 3\pi/8$, read by three independent arguments, and the Cabibbo angle stand so. (Chapters 1, 2, 3, 5 and 6)

Dimensional transmutation

A strength with no unit setting a mass through its running. (Section II) (Chapter 6)

Dirac neutrino

A neutrino whose mass joins two distinct components of one state; it has no Majorana mass, which would join a state to its own reversal. (Chapter 1)

Dirac radius

The radius from the slope of the proton's Dirac form factor F_1. (Section VI) (Chapter 6)

Direct mass

The top quark's mass read directly from its decay products, not through a hadron. (Chapter 1)

Displacement; clause

A displacement is, in Chapter 3, the distance of a place above a whole-number fraction of the count 72; in Chapter 13, the amount by which a measured share of v^2 differs from its ideal share. A clause states one part's displacement as an integer times κ_H. (Chapters 3 and 5)

Domain; application; term set

A domain is a part of the particle side with its own counted and continuous measures, a range of readings with one degree of incommensuration. An application is a counted measure read against a continuous one with a leftover in powers of ε. A term set is the group of terms by which one quantity is written. (Chapters 1, 5 and 6)

Doublet (weak doublet); sides; weak isospin

A doublet is the pair of states the weak force turns into each other (for the leptons, the neutrino and the electron's left-handed part); its sides are its two members; weak isospin is the weak charge, $+1/2$ and $-1/2$ on the two sides. (Chapters 1, 2, 3 and 4)

Doublet's half, a

$a = 1/2$, the weak isospin of each side of the weak doublet; at tree level the self-coupling is $\lambda = a^2/2 = 1/8$. (Chapter 4)

Dressing (dressing of the rungs, turnover); bare, dressed

The dressing is the first-order factor $1 + \kappa\Phi(g)$ on the rungs of a charged ladder below its head; Chapter 7 calls it the dressing of the rungs, and

Chapter 8, for the charged leptons, the turnover. A dressed value applies it; a bare (undressed) value applies no correction to any rung. On the quarks the first-order dressing pair is $(1 + (10/27)\varepsilon)$ on the charm and the strange and $(1 - (5/27)\varepsilon)$ on the up and the down. (Chapters 1 and 5)

Edge; edge field

The edge is the one crossed edge of the structure, on which the Higgs field sits; the edge field is the zoo's name for the Higgs field, whose couplings set the masses of the elementary fermions (the zoo, Level 8). (Chapters 3, 4 and 6)

Effective leptonic angle

The weak angle that the Z's corrected couplings to leptons define. (Chapter 4)

Electric coupling

The photon's coupling $e = g \sin \theta_W = g' \cos \theta_W$. (Chapter 4)

Electroweak level

The inverse strength $1/\alpha(X_{12})$ at the crossing X_{12} of Chapter 3; the second closure holds it at 169/4. (Section V) (Chapters 2 and 6)

ε, the one mismatch (*epsilon*)

$\varepsilon = 11/(80\pi^2) = 0.0139317$, the one mismatch, or leftover, between a counted measure and its continuous reading, the first-order size of every small leftover of a reading counted in halvings (the book of kinds, Chapter 2, Section VII). (Chapter 5)

Equal traces; spectral action

The equal traces are the equality of the three forces' weights computed as traces over the finite space of states, $g_3^2 = g_2^2 = (5/3)g'^2$, with g' the hypercharge coupling in the normalization Y (in the unification normalization, $g_3^2 = g_2^2 = g_1^2$). The spectral action is the action read from the spectrum of the Dirac operator of the product geometry, four-dimensional spacetime times the finite space of states. (Chapters 1, 2, 3 and 4)

Euler weight

$11/(5760\pi^2)$, the coefficient of the Euler density per Dirac spinor in the scale-free term of the spectral action. (Chapter 5)

Fermi constant G_F

The strength of the weak interaction at low energy, measured in the decay of the muon. (Chapter 2)

Fermion

A particle of half-integer spin, a particle of matter; at the zoo's first scale, content. (Section IV) (Chapters 1, 2, 3, 4, 5 and 6)

Fine structure

The splitting of hydrogen's levels of the same n and l by total angular momentum, at relative order α_{em}^2. (Chapter 1)

Finest resolution

The limit of resolution at which the structure gives no force a greater weight than another. (Chapters 2, 3 and 4)

Fine-structure constant

The strength of light, $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$; measured, $1/\alpha_{em} = 137.035999177(21)$. (Chapters 1, 2, 4, 5 and 6)

First-order states; joins

The $8 = 2^3$ states of two outcomes at each of three positions, and the $9 = 3^2$ joins of three positions against three across the pair; the count $72 = 8 \times 9$. (Chapters 1 and 5)

FLAG

The Flavour Lattice Averaging Group, whose reviews give lattice averages of hadronic quantities, cited by year (FLAG 2024). (Chapter 6)

Flavor

The name of a weak event by its charged lepton: electron, muon or tau. (Section I) (Chapter 6)

Floor (pair floor), $2m_p$

Twice the proton's mass, $2m_p = 1.8765$ GeV: the least energy that makes a proton with its antiproton, the mass of the least whole of color made as content, and the scale at which the light quarks u, d and s are read. (Chapters 5 and 6)

Focus of comparison as relation

The first-person binding of the six intrinsics to the three positions: sameness and difference omniscient, the objective and the subjective immanent, content and context transcendent. It is the binding the zoo and the book of kinds use (the book of kinds, Chapter 1, Section I); described from outside, in the third person, every pair stands one step back round the cycle, which the book of kinds calls the focus of measurement. (Chapters 1, 2, 3 and 6)

Fork

Two arguments for one number, carried side by side until measurement decides between them. (Chapter 1)

Form

One concept of the structure; a form supplies a value's terms when all of them belong to it. (Chapters 1, 2, 3, 4, 5 and 6)

Form factor

The proton's response to a probe as a function of the momentum transferred; its slope at zero gives a radius, and its poles are the squared momenta at which it diverges (Chapter 6). The electroweak form factor is the full electroweak correction to the Z's partial widths (Chapter 4). (Chapters 4 and 6)

Forward-backward asymmetry

The asymmetry between fermions emitted forward and backward of the incoming electron's direction in Z decays, $A_{FB}^{f} = (3/4)A_e A_f$ at the pole. (Chapter 4)

Frame of measurement

The zoo's frame: the comparison in which the photon is the act of comparing, the electron the thing compared and the proton the thing compared against (the zoo, Level 0; the book of kinds, Chapter 6, Section I). Its modal binding is the focus of comparison as relation. (Chapters 2 and 6)

Gap

$D_{ij} = 1/\alpha_i - 1/\alpha_j$, the difference of two inverse strengths at one scale. (Chapters 1, 2, 3, 4 and 6)

Gauge factors

The groups SU(3), SU(2) and U(1) of the forces (the book of kinds, Chapter 4); Chapter 4 uses SU(2) and U(1). (Chapter 4)

Gauge group

The group of turns of phase by which a force acts; for light, the circle U(1). (Chapters 1, 2 and 3)

Generation

One of the three copies of the sixteen states of matter, each with the same charges. (Chapters 1, 2, 3, 4 and 5)

G-factor

The ratio of a particle's magnetic moment to its spin, in the natural unit; 2 for the electron at lowest order. (Chapters 1 and 6)

Gravity's strength on the electron

$\alpha_G = G\, m_e^2/(\hbar c) = (m_e/M_{Pl})^2$, with the unreduced Planck mass. (Chapter 5)

Groups of measure

The sorting of values by what fixes them: a value of group 1 is fixed by a law, values of the first group being fixed to type by counting with no selection from a continuum; a value of the second group is one constant selected from a continuum; a selection by chance is of the third group. (Chapters 1 and 6)

Hadron

A particle made of quarks bound by color; a confined quark is a quark bound in one. (Chapters 2, 3, 4 and 6)

Hadronic part, $\Delta\alpha_{had}$

The contribution of quarks bound in hadrons to the running of the electric coupling. (Chapters 2 and 4)

Halving; halving count; halving map

A halving is a factor of two in mass. The halving count of a mass M is its position $\log_2(M/m_e)$ above the electron; the halving map is the line of

halving counts. The span from the electron to the reduced Planck mass is counted in halvings. (Chapters 1, 2, 3, 4, 5 and 6)

Hierarchy

The smallness of the ratio of the vacuum value to the reduced Planck mass. (Chapter 5)

Higgs field

The one scalar field of the Standard Model; its condensate is the value v. (Chapters 1, 2, 3, 4, 5 and 6)

Hypercharge coupling

The strength g′ of the phase U(1) of hypercharge. (Chapters 2, 3 and 4)

Ideal

The value a quantity takes at the structural shares, before the displacement that the measured values show. (Chapter 4)

Identity; kernel only; kernel values

The scopes of a writing of a constant: an identity holds for every right triangle (or for any legs, where stated); kernel only marks a writing that holds at $c_1 = 3$ and fails along the family of triangles with sides c, c + 1, c + 2; kernel values marks an equality among the kernel's numbers. (Chapters 1, 2 and 3)

Impedance of empty space

$Z_0 = \mu_0 c$, the resistance the electromagnetic field meets in passing through empty space. (Chapter 1)

Incommensuration Theorem

Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. The Incommensuration Theorem is a structural fact about the nature of knowing (*Incommensuration*; *The Unquantized Arena*). On the particle side its consequence is that a count read by a continuum whole leaves a non-zero residue at every such reading, the one mismatch ε. (Chapter 5)

Independence

One thing is independent of another when differences in the first produce no necessary difference in the second. (Chapter 1)

Independent routes

Routes that rest on different premises and meet on one value; they are the strongest evidence the book has for that value, and Table X-2 of Chapter 13 gathers them. (Chapter 3)

Inertia

Mass. (Chapters 1, 4 and 6)

Instance

This world, in which the structure is realized. (Chapters 1, 2, 3, 4, 5 and 6)

Internal space

The space of states of the color label and the doublet label (the book of kinds, Chapter 3). (Chapters 1, 2, 3 and 5)

Intrinsics (aspects); occupant; open configuration

The intrinsics are the six aspects present in every comparison: content and context, sameness and difference, the subjective and the objective (the book of kinds, Chapter 1, Section I). An occupant is the particle that fills a position. An open configuration is an assignment of outcomes to the six verdicts before a consideration resolves. (Chapter 1)

Inverse strength

$1/\alpha$; the quantity that runs linearly in the logarithm of the resolution at one loop. (Chapters 1, 2, 3, 4, 5 and 6)

Joining

The tie of a lepton to the condensate; its strength is a measure, and the mass is that strength. (Chapters 1, 2 and 6)

κ_H, the Higgs's loop factor

$\kappa_H = b_e(4\pi)^{-2} = (4/3)/(16\pi^2) = 1/(12\pi^2)$, with $b_e = 4/3$: the one-loop coefficient per unit charge squared, the photon's one-loop unit. (Chapter 5)

Kernel; kernel integers; kernel triangle; kernel number

The kernel integers are $c_1 = 3$ and $n_C = 2$ of the book of kinds, Chapter 2. They fix the kernel triangle, the right triangle with legs O = 3 and N = 4, hypotenuse H = 5 and inradius r = 1. A kernel number is a number written in these integers and the triangle's parts; a kernel rational is a rational kernel number; a kernel word is a fraction written as a product or ratio of them. The kernel is the integers and the numbers built from them. (Chapters 1, 2, 3, 4, 5 and 6)

Kinds of measurement

Force, inertia, time, space, probability and possibility; each one registers α_{em}. (Chapter 1)

Koide's quantity; Koide's relation (Koide's balance); Koide's tau

Koide's quantity is $Q = (m_1 + m_2 + m_3)/(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3})^2$ for a triple of masses. Koide's relation, the balance, is Q = 2/3 for the charged leptons: the sum of their three masses is 2/3 of the square of the sum of their square roots. Koide's tau is the tau mass computed from the measured electron and muon masses with that balance. (Chapter 1)

Ladder, rung, head; down ladder

A ladder is the sequence of masses of one sector, its three positions counted in halvings below a reference, rung above rung; a rung is one such position, and the head is the ladder's heaviest member. The down ladder is that of the down-type quarks. (Chapters 1, 3, 4, 5 and 6)

Lamb shift; Uehling term

The Lamb shift is the separation of the 2S and 2P levels of hydrogen; the Uehling term is its vacuum-polarization part. (Chapters 2 and 6)

Λ_{QCD}

The scale at which color's strength becomes of order one; its value depends on the loop order and the scheme. (Chapter 6)

Lattice QCD

Quantum chromodynamics computed on a four-dimensional grid of points by random sampling of the field's configurations, with the grid spacing taken to zero and the volume to infinity. (Section I) (Chapter 6)

Law

A relation among measures, carrying one into another. Taken as that transformation, in the frame of what is measured about a particle, the immanent; taken as the structure that holds in every instance, the omniscient (Comparison, Chapter 10, Section VIII; Measurement, Chapter 6, Section IV). (Chapters 1, 2, 3, 4, 5 and 6)

Law of place

The one-loop formula $X_{ij} - X_Z = 2\pi\, D_{ij}(M_Z)/((b_i - b_j)\ \ln 2)$ that places a crossing; its values are stated of the three-loop places. (Chapter 3)

Lepton ladder

The step formula of Chapter 7 read for the charged leptons: $n_g = (391/60 + gP_1 + g^2 d_{lep})$s halvings below a common reference point, with $P_1 = 9/4$ and $d_{lep} = 16/9$. (Chapter 5)

Level

The common inverse strength at the place where $1/\alpha_1 = 1/\alpha_2$ (Chapter 3). (Chapters 1, 2, 3, 4, 5 and 6)

Light's relation

Chapter 2's relation $1/\alpha_{em}(m_p) = 3\pi(1/\alpha_2 - 1/\alpha_1)(E)$. (Chapters 1, 4 and 6)

Light's way

The route to the proton's place x_p from light's relation of unit 3π solved for its reading scale. (Section V) (Chapter 6)

Loop

A closed path of virtual states, such as a carrier turning into a fermion pair and back. (Chapters 2, 3, 4, 5 and 6)

Loop order

The order of a calculation in powers of the coupling: one loop is the first order, higher loops the higher orders. (Section II) (Chapters 2 and 6)

Loop unit, u

The factor $(4\pi)^{-2} = 1/(16\pi^2)$ that one loop carries in four dimensions. (Chapter 5)

Made as content

Made as matter; at the zoo's first scale, content means fermion. (Section IV) (Chapter 6)

Mark

The rung the mass ladder assigns a quark. (Section III) (Chapter 1)

Mass ladder; step formula

The writing of each fermion's mass as $m = m_{ref}\, 2^{-ns}$, with n its count of halvings below a reference mass and s the step: the step formula $n = (\Delta + gP + g^2 d)s$ places each mass a counted number of steps along its ladder. (Chapters 3, 4 and 5)

Matching; matching scheme

The step that fixes the MS-bar strengths at the top quark's mass from measured quantities; the two published matchings are those of Martin and Robertson (the program SMDR, which takes the W mass as an output) and of Buttazzo and collaborators (which takes it as an input). A matching scheme is the convention by which measured inputs are carried into the running couplings. (Chapters 2, 3, 5 and 6)

Measured agreement

A value that agrees with measurement, where the argument does not state why it agrees; it carries that question, why it lands, with the clues the book has for it. (Chapters 1, 2, 3, 4 and 5)

Measured input

A number this book takes from measurement: in the working account, the electron's mass, α_{em}, α_s and v, together with the two offsets of the charged ladders. (Chapters 1, 2, 3, 5 and 6)

Measured unit, U

$T(m_p)/\Delta_{21}(E)$, light's inverse strength at the proton's mass over the electroweak gap at the end of the measured span; its standard deviation $\sigma_U = 0.0117$; it lands at 3π, a measured agreement. (Chapter 2)

Measured values used in this book

The table at the front of this volume that collects the measured values the chapters use, each with its uncertainty and its source. (Front matter)

Meson

A quark with an antiquark, a color-neutral pair; every meson is a boson. (Section III) (Chapter 6)

Mirror

The up ladder reversed from its light end and rescaled by O/H = 3/5. (Chapter 1)

Mixing

The mismatch between the up-type and the down-type mass states. (Chapters 1, 2, 3, 4 and 6)

Modal assignment

An assignment of the three positions of comparison to concepts; it belongs to the frame of each argument. (Chapter 3)

MS-bar

The modified minimal-subtraction scheme, the standard convention for removing the infinite parts of loop calculations at a chosen mass; a hat (as in $\hat{\alpha}$, $\hat{\rho}$) marks a quantity in it. (Chapters 2, 3, 4 and 6)

μ_1

The scale at which $\alpha_s = 1$; it depends on the convention and the loop order (0.29 to 0.67 GeV). (Section III) (Chapter 6)

Non-measure

What every measurement rests on and none yields: the unit, the settings, the history of this universe; the transcendent. (Chapter 1)

Normalization Y; unification normalization

In the normalization Y, $Q = T_3 + Y$ and the hypercharge strength is α_Y; in the unification normalization the hypercharge strength is multiplied by 5/3, $\alpha_1 = (5/3)\alpha_Y$, and in it the phase force's rate is 41/10. (Chapters 2 and 3)

Not fixed

The standing of a number that this book does not fix. (Chapter 1)

One incommensuration per domain per kind of reading

Within one domain, each kind of reading has one mismatch and no second. (Chapter 5)

One loop, two loops, three loops

The orders of the calculation of the running, each adding a further power of the strengths: one loop computes with the first term of the perturbative expansion of the rates of change, three loops with the terms through the third order. (Chapters 2, 3, 5 and 6)

One-loop coefficient

The coefficient of the logarithm of the scale in a coupling's running at one loop: $b_2 = -19/6$ for the weak coupling; $b_e = 4/3$ for one Dirac fermion of unit charge in the running of the electric coupling. (Chapters 2, 3, 4, 5 and 6)

One-P test

The test that the up and down quark ladders return one step cost P_q within two standard deviations (Chapter 7). (Section IV) (Chapter 6)

On-shell

The definition of a quantity from measured pole masses; for the weak angle, $\sin^2\theta_W = 1 - m_W^2/m_Z^2$. (Chapter 4)

Pair mass

Twice a particle's rest mass, the least mass at which the particle is made with its antiparticle. (Section IV) (Chapter 6)

Particle Data Group (PDG)

The collaboration whose Review of Particle Physics gives the measured values of particle physics, cited by edition (PDG 2024, PDG 2026). (Chapters 1, 2, 3, 4 and 6)

Particle side

The account of the kinds of particle and of their numbers; its measured input is the electron. (Chapters 1, 3 and 5)

Pattern

The part of the mismatch among the three couplings that follows from the content alone: the spacing ratio, the coefficient 115/218 and the middle place of the $\alpha_1 = \alpha_3$ crossing. (Chapters 1, 3 and 6)

Phase force

The force of hypercharge, U(1); subscript 1. (Chapter 3)

Place

The position of a scale in halvings above the electron's mass. (Chapters 1, 2, 3, 4, 5 and 6)

Planck mass

$M_{Pl} = (\hbar c/G_N)^{1/2} = 1.2209 \times 10^{19}$ GeV, the unreduced Planck mass, at which gravity's strength $GM^2/(\hbar c)$ equals one. This book means the reduced Planck mass everywhere else. (Chapters 1, 2, 3, 5 and 6)

Pole

The peak of the Z resonance, at the Z's mass. (Chapters 3, 4 and 6)

Pole asymmetry

An asymmetry of Z decays measured at the pole. (Chapter 4)

Pole mass; running mass

The pole mass is a particle's physical mass, defined by its propagation: the mass of a particle as a persisting thing, at the macroscopic limit. The running mass is the MS-bar mass m(Q) read in one interaction at a chosen scale Q; it grows as the scale falls. (Chapter 3)

Position

The number of steps n by which a quark's mass lies below the top's; the mass is $m_t \times 2^{-ns}$. (Chapters 1, 2, 3, 5 and 6)

Positions of comparison (*modalities*)

The omniscient, the immanent and the transcendent (the book of kinds, Chapter 1, Section II): the known, the comparing act and the beyond. In the book of kinds' assignment of the known, the unknown and the unknowable to them (Chapter 1, Section IV) they are the known, the un-

known and the unknowable, with the immanent also the comparing act, which mediates between the other two. (Chapters 1, 3 and 5)

Principle of identity

That which is indistinguishable must be the same (Comparison, Chapter 11, Section V): anything that cannot be distinguished, in any way at all, even in principle, is the same thing. By it, a parameter that no measurement and no reasoning could ever reach is nothing, so no fundamentally inaccessible parameter is defined. (Chapter 1)

Proton's place

The proton's position above the electron in halvings, $x_p = \log_2(m_p/m_e)$. (Section V) (Chapters 3 and 6)

Pull; pass

The pull is the stated value minus the measured value, divided by the combined standard deviation. A value passes a measurement when its pull is at most 2 in size; a block of laws passes when every output it gives does, and fails otherwise. (Chapters 1, 2, 3, 4, 5 and 6)

Pure number

A ratio of like to like, which carries no unit. (Chapters 1, 2, 5 and 6)

QCD, QED

Quantum chromodynamics, the theory of the strong force; quantum electrodynamics, the quantum theory of light and charge. (Chapters 4 and 6)

Quark doublet; lepton doublet

The up and down sectors; the charged leptons and the neutrinos. (Chapter 2)

R^0 ratios

Ratios of the Z's partial widths: R^0_{had}, hadronic over one charged lepton; R^0_b and R^0_c, bottom and charm over hadronic. (Chapter 4)

r_1, r_2; ring of the sectors

$(c_1 - 1)/c_1 = 2/3$ and $c_1/(c_1 + 1) = 3/4$; the cycle of the four sectors around the positions of knowing. (Chapter 6)

Radiative correction

The change in a quantity that comes from virtual states, such as a loop's addition to a carrier's mass. (Chapter 4)

Rate

The one-loop coefficient b_i of the running of $1/\alpha_i$ (the beta coefficient). (Chapters 1, 2, 3 and 6)

Reach

The amplitude $\sqrt{m}$ of a joining whose strength, a measure, is the mass. (Chapters 1, 2, 3, 5 and 6)

Reading scale

The mass at which a coupling is evaluated; light's is the proton's mass. (Chapters 4 and 6)

Reading; writing

A reading is a measured quantity set against the whole that bounds it (Chapter 13). A writing is one form in which a value, a relation or a correction is written, for a constant an expression of its value in the kernel triangle's measures (Chapter 7); two writings of one correction may differ at higher order (Chapter 8). (Chapters 1, 2, 3, 4, 5 and 6)

Reduced Compton wavelength (*own length*)

$\hbar/(mc)$, a particle's own length; for the electron $\lambda\!\!\!^{-}_e = \hbar/(m_e c)$, and 0.2103 fm for the proton. (Chapters 1 and 2)

Reduced Planck mass

$M_{Pl,red} = (\hbar c/(8\pi G))^{1/2}$, the mass M at which $8\pi GM^2/(\hbar c)$ equals one; gravity's strength $GM^2/(\hbar c)$ equals one at the unreduced Planck mass, $\sqrt{(8\pi)}$ times larger. (Chapters 1, 2, 3, 5 and 6)

Relations (c, $\hbar$, α_{em}, G_N)

The four constants, so called because each relates ideas of comparison (c relates space to time). (Chapters 1, 2 and 4)

Remainder

$r = E - 72$, the residue of the count read against the reduced Planck mass. (Chapters 1, 2, 3, 5 and 6)

Residue

A mass's departure from its ladder position. (Chapters 3, 4, 5 and 6)

Resolution

The separation at which a comparison is made, or the energy that reaches it. (Chapters 1, 2, 3, 4 and 6)

Rest, exchange and confinement mass

The mass of a particle at rest; the mass shown to an interaction at a given resolution; the mass of a confined whole. (Chapters 1, 2, 3, 4, 5 and 6)

Restated relation

$\alpha_{em}(m_p) \cdot \Delta_{21}(E) = 1/(3\pi)$: light's strength at the proton's mass times the split of the weak and hypercharge inverse strengths at the span (Chapter 2). (Chapters 2 and 5)

ρ parameter

$\rho = m_W^2/(m_Z^2 \cos^2\theta_W)$, equal to one at tree level for one doublet; $\hat{\rho}$ is its MS-bar form. (Chapter 4)

Ring; move

The ring is the directed cycle of the three positions of comparison, transcendent to immanent to omniscient (Axiom II, the book of kinds, Chapter 1). A move is one passage along the ring from one position to the next, 120 degrees (Chapter 12). (Chapter 1)

Route

One of the three terms z_u, z_c, z_t that join the d column to the b column of V. (Chapters 1, 2, 3, 4, 5 and 6)

Running; running number

Running is the change of a strength, or its inverse, with the energy at which it is read; a running number is one whose value changes with that energy. (Chapters 1, 2, 3, 4, 5 and 6)

Sachs electric radius

The radius from the slope of the proton's electric form factor at zero momentum transfer; the charge radius CODATA quotes. (Section VI) (Chapter 6)

Scale; scheme

The scale is the energy at which a running number is read; the scheme is the convention by which it is defined at that scale. Both are settings. (Chapters 1, 2, 3, 4, 5 and 6)

Second-order coefficient (k)*; third-order coefficient* (c_3)

The constants defined by $r = \varepsilon - k\varepsilon^2 + c_3\varepsilon^3 + \ldots$ (Chapter 5)

Sector

One family of fermions of one kind: up quarks, down quarks, charged leptons or neutrinos; the first three are the charged sectors. (Chapter 3)

Self-coupling λ*; tree level*

The self-coupling λ is the coefficient of the quartic term $\lambda(\Phi\dagger\Phi)^2$ of the Higgs potential. Tree level is the leading order of a calculation, before corrections from virtual states; a tree matching has no loop corrections, and next-to-next-to-leading order adds two orders of loop correction beyond it. (Chapters 2 and 4)

Setting

A choice fixed by stating it: the scale and the scheme at which a running number is read, a convention of description, or a two-valued choice whose swap is a symmetry of the whole description. (Chapter 1)

Single measure

One number read by one comparison; in the frame of what is measured about a particle, the actual, the omniscient. (Chapter 1)

Six questions

What, who, where, when, how and why, asked of one thing. (Chapter 1)

SLD

The SLAC Large Detector, which measured the lepton asymmetry A_l. (Chapter 4)

Slot

The place of a coefficient in the bracket: additive (Δ), multiplicative (the factor of g) or power (the factor of g^2). (Chapters 3 and 6)

SMDR

Standard Model in Dimensional Regularization, the code of Martin and Robertson for the Standard Model's matching. (Chapters 2, 3, 5 and 6)

Spacing ratio

The distance from the middle crossing to the last over the distance from the first to the middle. (Chapter 3)

Span, E (*measured span; span's end*)

The span is $E = \log_2(M_{Pl,red}/m_e)$, the halving count of the reduced Planck mass above the electron: measured at 72.0132112 ± 0.0000162 (CODATA 2022), it is the count of 72 halvings with its remainder. The span's end, the scale E, is used as a test only. (Chapters 1, 2, 3, 5 and 6)

Span of measure

The three parts (single measure, law, non-measure) of what is measured about a particle, at the three positions of comparison. (Chapter 1)

Splits; ρ

The gaps at the span's end, $\Delta_{21} = 1/\alpha_2 - 1/\alpha_1$ and $\Delta_{32} = 1/\alpha_3 - 1/\alpha_2$; $\rho = \Delta_{21}(E)/\Delta_{32}(E)$. (Chapters 1, 2 and 3)

Splitting

A difference of two squared masses; the solar Δm^2_{21} and the atmospheric Δm^2_{31}. (Section I) (Chapters 1 and 6)

Step cost, P

The coefficient of g in the step formula: the halvings one generation adds at first order, the unit of the count; $P_q = 6$ for both quark ladders, $P_l = 9/4$ for the charged leptons. (Chapter 3)

Step of sideways passing

The von Klitzing constant $R_K = h/e^2$; $R_K/Z_0 = 1/(2\alpha_{em})$. (Chapter 1)

Step s (*step factor*); c_2

The common factor of every count, $s = 1 + \varepsilon - c_2\varepsilon^2 = 1.0128448$, with $c_2 = 28/5$: one plus the leftover ε less a second-order term. It is the number of halvings one step spans; a width of w units of s on a ladder is a mass ratio

of 2^{-ws}, and numbers in units of s are counts of halvings divided by s. (Chapter 5)

Strong angle

$\bar{\theta}$, the strong force's own angle plus a phase from the quark masses; carried as zero. (Chapter 1)

Structural value

A value fixed by structure. (Chapter 3)

Structure

What the counts of comparison fix, as the book of kinds sets it out; a value not fixed by the counts is read from measurement. (Chapters 1, 2, 3, 4, 5 and 6)

Sum rule

Chapter 13's rule by which the squared masses of the block share v^2. (Chapter 4)

T_3

The third component of weak isospin: $+1/2$ for a doublet's upper member, $-1/2$ for its lower member. (Chapters 3 and 4)

Terms (*of a value*)

The quantities of which a value is a ratio or a relation. (Chapters 1, 2, 3 and 4)

Tiers

Fixed by structure; a value of the instance; a unit or convention. (Chapter 1)

$T(m_p)$

Light's MS-bar inverse strength read at the proton's mass. (Chapter 2)

Top

The mass of the top quark, which heads the whole ladder of the step formula. (Section II) (Chapters 2, 3, 4, 5 and 6)

Trine

A whole of color: three quarks, one at each position of comparison within the content. (Section III) (Chapter 6)

Turning; label; answer; sense

A turning is a way's transformation (those of the weak doublet, SU(2), rotate a doublet's upper and lower members into each other); a label is the charge a content carries under it; a content answers a turning when it carries a label in it; a sense is a handedness. (Chapters 1, 2 and 4)

Two ends

The two ends every number of the Standard Model is to reach: derivation through a law, or a proof that no law fixes the number; a value shown not derivable, as α_{em} is, is closed under reason. (Chapters 1 and 3)

Two-powers reading

The reading that gives the two wholes of one context the share $a = 1/2$ at the first power and a^2 at the second, by position. (Section III) (Chapter 6)

Unit

A choice of standard in which magnitudes are written; for mass on the particle side, the electron's mass. (Chapters 1, 2, 3, 5 and 6)

Unitarity triangle

The triangle that the quark mixing table closes. (Chapter 1)

Vacuum polarization

The covering of a charge by the pairs of charged contents the field makes near it. (Chapter 2)

Vacuum value, v

The size of the condensate, read from the Fermi constant G_F: $v = (\sqrt{2}\, G_F)^{-1/2} = 246.21967 \pm 0.00006$ GeV. In the working account it is a measured input; in the closure account the electron alone gives it. (Chapters 1, 4 and 5)

Value

A number read from measurement, directly or by running from measured inputs. (Chapters 1, 2, 3, 4, 5 and 6)

Value of the instance

A pure number of this world, to be measured: the second tier. (Chapters 1 and 6)

Value that lands

A value that agrees with measurement and for which this book states no reason: a measured agreement, which carries the question why it lands. (Section VI) (Chapter 6)

Vector coupling

The Z's coupling $v_f = T_3 - 2Q_f \sin^2 \theta_W$ to the vector current of a fermion f. (Chapter 4)

Vertex terms

The corrections at the point where the Z meets a fermion pair. (Chapter 4)

Weak angle (weak mixing angle), θ_W

The angle by which the hypercharge and weak-isospin fields mix into the photon and the Z boson, with $\alpha_{em} = \alpha_2 \sin^2\theta_W$. (Chapters 1, 2, 3 and 4)

Weak coupling

The strength g of the turnings of the weak doublet, SU(2). (Chapters 2 and 4)

Wholes of the halving map

The four positions at 0, 1/4, 3/4 and 1 of the count 72: the unit, the condensate's whole, the crossing $\alpha_1 = \alpha_2$, and the reduced Planck mass. (Chapter 13, in The Measures of the Kinds)

Window

The range of a quantity's values for which the block's outputs stay within $|\text{pull}| \leq 2$. (Chapters 3, 4 and 6)

Z width

The spread of energies over which the Z is produced, which measures its rate of decay. (Chapter 4)

Zero of the dressing

The generation at which the dressing vanishes; for the charged leptons it is placed at the tau, so $D(0) = 1$ there. (Chapter 5)

ζ

The ratio $m_p/\mu_1 = 1.441$ of the proton's mass to the scale at which $\alpha_s = 1$; it is the pure number that *Geeze, Wublets, and the Higgs* does not fix, by which the binding mass of a trine is ζ times the binding scale. (Chapter 6)

Zoo

The Story of the Particles, the authority on which particles exist and of what kind; it fixes no value of a mass, a mixing or a coupling beyond the few its structure sets. (Chapters 1, 2, 3, 4 and 6)

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Index

anomalous moment, 31 , 36 , 52

anomaly, 55 , 145

asymptotic freedom, 55 , 75

axial coupling, 110

Axiom I, 122

bare, 131 , 137

baryon, 137 , 145 , 150 , 156

binding mass, 137 , 145 , 161

Bohr radius, 31 , 156

book of kinds, 14 , 19 , 24 , 38 , 43 , 50 , 52 , 55 , 64 , 66 , 78 , 80 , 85 , 89 , 97 , 104 , 106 , 107 , 112 , 117 , 120 , 122 , 133 , 137 , 150 , 152

boson, 38 , 50 , 52 , 55 , 61 , 80 , 85 , 89 , 97 , 103 , 104 , 107 , 145 , 150

bottom quark, 110 , 112 , 137

brute constant, 38

Buttazzo, 62 , 70 , 73 , 93 , 97 , 131 , 152

carriers' law, 103

charm, 112 , 145

chirality, 24 , 122

circle's rule, 38 , 45 , 47

classical electron radius, 31

closed list of contents, 50 , 55 , 64 , 73

CMD-3, 52

CODATA, 24 , 31 , 52 , 117 , 141 , 145 , 150 , 152 , 156

coefficient premise, 66 , 73 , 75

color,
31 , 38 , 43 , 50 , 50 , 52 , 55 , 64 , 66 , 70 , 73 , 78 , 80 , 85 , 89 , 93 , 97 , 104 , 106 , 122 , 137 , 137 , 141 , 145 , 150 , 152 , 156 , 160 , 161

color's way, 141 , 152 , 160

color's whole, 137 , 145 , 150 , 160 , 161

Comparison, 13 , 14 , 19 , 24 , 31 , 36 , 38 , 43 , 45 , 47 , 50 , 50 , 52 , 55 , 62 , 70 , 80 , 97 , 103 , 107 , 110 , 114 , 120 , 122 , 131 , 137 , 141 , 145 , 150 , 152

comparison, 13 , 14 , 19 , 24 , 31 , 36 , 38 , 43 , 45 , 47 , 50 , 50 , 52 , 55 , 62 , 70 , 80 , 97 , 103 , 107 , 110 , 114 , 120 , 122 , 131 , 137 , 141 , 145 , 150 , 152

comparison of worlds, 38 , 47

comparison structure, 38

condensate, 24 , 52 , 55 , 61 , 80 , 93 , 104 , 106 , 107 , 122 , 133 , 145

condensate's whole, 93 , 122 , 133

confinement mass, 137

conjugates, 19

content instance, 38 , 64 , 89 , 97 , 106 , 133 , 152

context instance, 133 , 135

continuum whole, 122 , 135

count 72, 93 , 117 , 131 , 135

count of the product, 97 , 152

count route, 131

count's whole, 129

coupling, 24 , 31 , 38 , 50 , 50 , 52 , 55 , 61 , 62 , 64 , 66 , 70 , 73 , 75 , 77 , 78 , 80 , 85 , 89 , 93 , 97 , 101 , 104 , 106 , 107 , 110 , 112 , 114 , 116 , 117 , 122 , 131 , 133 , 135 , 137 , 141 , 145 , 152 , 160

covering, 55 , 61

crossing, 38 , 43 , 52 , 77 , 78 , 80 , 85 , 89 , 93 , 97 , 101 , 102 , 102 , 133 , 152

curvature, 122

cycle, 66

Darwin-Foldy term, 156

decoupling, 61

derived, given a stated identification, 14

dimensional transmutation, 141 , 156 , 160 , 161

Dirac neutrino, 43

Dirac radius, 156 , 160

displacement, 93 , 133

doublet, 24 , 38 , 50 , 50 , 55 , 77 , 80 , 85 , 89 , 102 , 103 , 104 , 106 , 107 , 114 , 115

E989, 52

Eddington, 36

edge field, 137 , 145 , 160

effective leptonic angle, 112

electric coupling, 104 , 110

electron, 13 , 14 , 19 , 24 , 31 , 36 , 43 , 45 , 47 , 52 , 61 , 66 , 70 , 73 , 77 , 78 , 85 , 93 , 97 , 110 , 116 , 117 , 120 , 131 , 133 , 135 , 136 , 137 , 141 , 145 , 152 , 156 , 160 , 161

electroweak level, 66 , 152

epsilon, 122

equal traces, 43 , 66 , 80 , 89 , 102 , 104 , 114

Euler weight, 122

exchange mass, 137

Fermi constant, 62 , 106

fermion, 43 , 50 , 55 , 70 , 73 , 80 , 97 , 103 , 107 , 110 , 112 , 122 , 150

fine structure, 31

finest resolution, 50 , 66 , 85 , 89 , 106

fine-structure constant, 13 , 19 , 50 , 104 , 117 , 133 , 152

first closure, 93 , 97 , 101

first-order states, 24 , 116 , 120 , 135 , 136

flavor, 141 , 145 , 152 , 156

floor, 122 , 133 , 137 , 145 , 150 , 152 , 160 , 161

focus of comparison as relation, 19 , 55 , 80 , 137

form factor, 112 , 156

forward-backward asymmetry, 110 , 112

four relations, 19

frame of measurement, 55 , 137

gauge factors, 104

gauge group, 38 , 55 , 77 , 80 , 85

Geeze, Wublets, and the Higgs, 145

generation, 14 , 43 , 50 , 55 , 66 , 77 , 80 , 89 , 101 , 104 , 107 , 112 , 122

G-factor, 31 , 137

gluon, 55 , 89 , 137

gravity's strength on the electron, 117

groups of measure, 14 , 152

hadron, 52 , 93 , 110 , 137

hadronic part, 52 , 110

halving,
24 , 45 , 52 , 66 , 70 , 77 , 78 , 85 , 93 , 97 , 102 , 102 , 110 , 116 , 117 , 120 , 122 , 131 , 131 , 135 , 136 , 137 , 141 , 145 , 152 , 161

halving count, 117

halving map, 117

held carriers, 61 , 73

hierarchy, 117

Higgs, 24 , 43 , 50 , 50 , 52 , 55 , 61 , 62 , 70 , 77 , 78 , 80 , 85 , 89 , 93 , 97 , 102 , 103 , 104 , 106 , 107 , 114 , 117 , 122 , 133 , 137 , 145

Higgs condensate, 24 , 52 , 55 , 61 , 80 , 145

Higgs edge, 89

Higgs field, 24 , 50 , 52 , 78 , 80 , 93 , 104 , 117 , 137 , 145

hypercharge coupling, 50 , 64 , 66 , 85 , 89 , 104

impedance of empty space, 19 , 36

Incommensuration, 116 , 122 , 129 , 135

Incommensuration Theorem, 116 , 122

independent routes, 93 , 102

internal space, 38 , 64 , 89 , 97 , 133

intrinsics, 19 , 38

inverse strength, 14 , 50 , 50 , 52 , 55 , 62 , 64 , 66 , 70 , 73 , 75 , 77 , 78 , 80 , 85 , 89 , 93 , 97 , 102 , 102 , 104 , 106 , 131 , 137 , 141 , 145 , 152 , 160

kernel, 24 , 50 , 50 , 52 , 55 , 80 , 104 , 120 , 122 , 129 , 131 , 135 , 137 , 145 , 150 , 152 , 160

kernel integers, 50 , 55 , 80 , 120 , 122 , 129 , 131

kernel number, 120

kernel triangle, 50 , 52 , 55 , 104 , 120 , 122 , 137 , 145 , 152 , 160

kinds of measurement, 13 , 31 , 45 , 47

Koide's relation, 24

ladder, 13 , 24 , 45 , 89 , 106 , 122 , 129 , 133 , 135 , 145 , 150

Lamb shift, 61 , 156

Landau pole, 64

lattice, 38

lattice QCD, 137 , 156

law of place, 77 , 78 , 93 , 97 , 101 , 102

Lenz, 141

lepton doublet, 55 , 89

lepton ladder, 129 , 133 , 135

lepton offset, 133

light's relation, 43 , 103 , 110 , 114 , 145 , 152

light's way, 152 , 160

loop order, 55 , 64 , 70 , 145

loop unit, 122

made as content, 137 , 150 , 160 , 161

Majorana, 43

Martin and Robertson, 62 , 70 , 73 , 93

mass ladder, 89 , 106 , 122 , 133

matching scheme, 131 , 152

measured agreement, 14 , 43 , 45 , 70 , 73 , 102 , 114 , 129 , 135

measured input, 13 , 14 , 24 , 38 , 43 , 45 , 47 , 52 , 78 , 89 , 117 , 131 , 133 , 141 , 152

measured unit, 62 , 70 , 75

Measurement,
13 , 14 , 19 , 24 , 31 , 36 , 38 , 43 , 45 , 47 , 50 , 50 , 52 , 55 , 61 , 66 , 77 , 78 , 85 , 89 , 93 , 102 , 103 , 107 , 112 , 116 , 120 , 122 , 129 , 131 , 131 , 135 , 137 , 137 , 145 , 152 , 156 , 161
meson, 137 , 145 , 150 , 156
mixing, 14 , 24 , 43 , 45 , 52 , 89 , 97 , 103 , 104 , 152
modal assignment, 80
modalities, 80 , 122
MS-bar, 52 , 62 , 70 , 89 , 93 , 107 , 112 , 141 , 145 , 152
muon, 24 , 31 , 36 , 52 , 70 , 156
NCG book, 66 , 122
neutrino, 24 , 43 , 45 , 55 , 89 , 97 , 107 , 112
neutron, 114 , 137 , 145 , 150
non-measure, 14 , 24 , 43 , 45 , 47
normalization Y, 50 , 52 , 55 , 66 , 85 , 89
offset, 133
one incommensuration per domain per kind of reading, 129
one loop, 62 , 64 , 66 , 70 , 73 , 78 , 80 , 85 , 89 , 93 , 97 , 102 , 131 , 141 , 145
one mismatch, 116 , 122 , 129 , 135 , 136
one-loop coefficient, 70 , 78 , 97 , 114 , 122 , 141
on-shell, 107
own length, 31 , 52 , 61 , 73 , 114 , 156
pair floor, 137 , 145 , 150 , 160 , 161
Particle Data Group, 31 , 52 , 66 , 70 , 85 , 93 , 107 , 110 , 112 , 141 , 152
particle side, 13 , 14 , 24 , 45 , 47 , 78 , 117
phase force, 64 , 78 , 80 , 85 , 89
photon, 31 , 38 , 52 , 55 , 66 , 73 , 89 , 104 , 137
placement, 145
Planck mass, 19 , 24 , 45 , 52 , 62 , 64 , 70 , 77 , 78 , 85 , 93 , 116 , 117 , 120 , 122 , 129 , 133 , 135 , 137 , 141 , 145 , 152
Planck scale, 24 , 50 , 62 , 64 , 73

pole asymmetry, 110

pole mass, 93

positions of comparison, 14 , 80 , 120

potentiality, 19

principle of identity, 14 , 45 , 47

proton, 31 , 43 , 50 , 52 , 61 , 62 , 66 , 70 , 73 , 75 , 93 , 97 , 101 , 103 , 110 , 114 , 114 , 122 , 131 , 137 , 137 , 141 , 145 , 150 , 152 , 156 , 160 , 161

proton's place, 93 , 152 , 161

pull, 14 , 36 , 50 , 66 , 70 , 73 , 75 , 77 , 93 , 97 , 112 , 122 , 131 , 145 , 152 , 156 , 161

pure number, 14 , 19 , 24 , 31 , 38 , 43 , 45 , 50 , 117 , 145 , 156

QCD, 112 , 137 , 141 , 152 , 156

QED, 112 , 156

quark doublet, 55

radiative correction, 107

reading scale, 110 , 114 , 145 , 152 , 161

reduced Compton wavelength, 31 , 52

reduced Planck mass, 19 , 24 , 45 , 52 , 64 , 70 , 77 , 78 , 93 , 116 , 117 , 120 , 122 , 129 , 133 , 135 , 137 , 141 , 145 , 152

remainder, 14 , 31 , 52 , 93 , 116 , 117 , 120 , 122 , 129 , 131 , 131 , 133 , 135 , 136 , 141

rest mass, 137 , 150

restated relation, 70 , 73 , 75 , 131 , 133 , 135

ρ parameter, 110

running, 43 , 45 , 47 , 50 , 50 , 55 , 61 , 62 , 64 , 66 , 70 , 73 , 75 , 77 , 78 , 80 , 85 , 89 , 93 , 101 , 102 , 102 , 104 , 106 , 110 , 114 , 122 , 131 , 133 , 137 , 141 , 152 , 161

running number, 43 , 45 , 47

Sachs electric radius, 156 , 160

second closure, 93 , 97 , 152

second-order coefficient, 129 , 131 , 136

sector, 101

self-coupling, 122

single measure, 14 , 45 , 47

six questions, 24 , 45

SLD, 110 , 112

SMDR, 62 , 93 , 97 , 101 , 102 , 131 , 141 , 152

spacing ratio, 77 , 78 , 80 , 85 , 97 , 101 , 102

span, 14 , 24 , 43 , 45 , 47 , 50 , 52 , 62 , 64 , 66 , 70 , 73 , 75 , 77 , 78 , 93 , 97 , 101 , 102 , 102 , 116 , 117 , 120 , 122 , 129 , 131 , 131 , 133 , 135 , 136 , 141 , 152

span of measure, 14 , 24 , 45 , 47

spectral action, 66 , 122

splitting, 19 , 137 , 156

spreading, 55 , 61

Standard Model, 13 , 14 , 24 , 43 , 45 , 52 , 55 , 62 , 64 , 70 , 78 , 80 , 85 , 93 , 97 , 101 , 107 , 112 , 114 , 114 , 131 , 152

step cost, 89

step formula, 133

step of sideways passing, 19 , 36

strange, 70

strong angle, 43

structural value, 77

tau, 24 , 31 , 52 , 55

The Gravitational Passage, 64 , 117 , 133 , 135

The Kinds of Particle, 13 , 14 , 78 , 80 , 117 , 137

The Story of the Particles, 14 , 50 , 55 , 85 , 89 , 93 , 104 , 129 , 137

The Unquantized Arena, 64 , 117

third-order coefficient, 131 , 136

three loops, 50 , 62 , 64 , 70 , 73 , 75 , 77 , 78 , 80 , 85 , 93 , 97 , 101 , 102 , 131

threshold, 50 , 61 , 70 , 75 , 152

tiers, 14 , 47

top quark, 62 , 78 , 93 , 97 , 103 , 122

tree level, 70 , 103 , 106 , 107 , 110 , 114 , 115

trine, 145 , 150

turning, 38 , 55 , 104 , 107 , 114

two ends, 14 , 45 , 47 , 85 , 89

two loops, 50 , 62 , 66 , 70 , 73 , 93 , 97 , 131

two-powers reading, 145

Uehling term, 31 , 61

unification normalization, 14 , 50 , 52 , 55 , 66 , 78 , 85 , 89

vacuum polarization, 52 , 61 , 62

vacuum value, 43 , 103 , 104 , 106 , 107 , 117 , 122 , 133

value of the instance, 14 , 19 , 45 , 47 , 152

value that lands, 156 , 160 , 161

vector coupling, 110

vertex terms, 112

W boson, 55

weak angle, 38 , 43 , 66 , 73 , 77 , 89 , 101 , 102 , 102 , 104 , 110 , 114 , 115

weak coupling, 64 , 104 , 114

weak doublet, 24 , 50 , 50 , 55 , 103 , 104

weak isospin, 38 , 55 , 104

weak mixing angle, 52 , 103 , 104 , 152

Wilson, 38

Wyler, 36

Z boson, 38 , 50 , 52 , 61 , 103 , 104 , 107

Z width, 112

zoo, 14 , 19 , 31 , 38 , 43 , 50 , 50 , 52 , 55 , 64 , 66 , 93 , 104 , 106 , 107 , 112 , 114 , 137 , 145 , 150

