

What the Beginning Left Behind

The First Ripples, the First Waves and the One Vacuum, in Plain Words

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Before We Begin

Who This Book Is For

Is this book for you, and what will this book find? This book is for a reader who is curious about how the world began and has an ordinary life to live. All the physics you need is the physics you met at school, and all the mathematics you need is arithmetic. You need an evening, and the patience to follow one step after another. Each chapter is meant to be read in one sitting of about half an hour. The section "What This Book Finds", below, answers the second question.

I make four promises to you, and I keep all four promises in every chapter. First, every technical word is explained in plain words at the place where you first read the word. Second, every number is said in words, is explained, and is given its standing, which says where the number comes from. Every standing is one of five: set by the structure, set by the structure once a stated step is taken, measured, measured with its reason still a question, or bounded, with its value not given. Each of the five standings is explained below, in the section "How the Book Speaks of a Number", and so is the one number that keeps a standing of its own. Third, you can follow every step without seeing a figure or reading an equation: where a picture helps, the picture is drawn in words. Fourth, each chapter starts from a question you would ask, and ends by saying plainly what the chapter found.

I follow a simple standard: whatever has been understood rightly can be explained in plain language.

Where the Story Stands

I take up a story at the point where an earlier book of the collection to which this book belongs left the story. The earlier book is *At the Edge of Reality*, which followed a black hole all the way to the black hole's end. What the earlier book found can be told in a page.

A *dimension* is an independent way in which things can differ. The world has ten dimensions: one of time, three of space, and six of possibility, the range of what could be found. You move through the three dimensions of space, and you live through the one dimension of time. The six dimensions of possibility are met only as what could be found, and nothing ever travels through those six dimensions. For short, this book calls the six dimensions of possibility "the six". The Planck length is the length at the edge of the domain of the smooth description of space and time: below the Planck length, the smooth description of space and time no longer holds. The Planck length is about a hundred billion billion times smaller than a proton. The six are held very small, a few Planck lengths across, and remain so while the large world around them serves as their setting.

Inside a black hole, everything falls toward an end that lies ahead in time and is never reached. Over an enormous span of time, the world outside every black hole thins out and empties. Certain conditions keep the six small, and those conditions fail together at the end of the interiors of the black holes. Then the ten-dimensional geometry that remains restores a large outside world: three directions open up while the six close in. In *At the Edge of Reality* I gave the passage from the end of the interiors to the restored outside world a name, *the turn*, and I identified the far side of the turn with the Big Bang of the next era of the world. A comparison is the act of finding whether two things are the same or different. No comparison can register the turn itself as an event. We read our own history backward from the light and the records we have now. Our history runs back toward the turn and begins at the turn.

The earlier book ended at the turn, and so at the opening of a new world: the large outside world that the turn restores. I ask what the opening of that new world left behind.

What This Book Finds

I follow the beginning of an era through what that beginning leaves in the sky and in the world. Part I asks how the six small dimensions come to be held again after the turn. Part I finds that the six are held at one size and one shape, the same every time: there is one *vacuum*, one lowest, settled state of empty space, and there is no range of other possible vacuums among which the vacuum of the world had to be chosen. Part II asks why the new era begins so orderly, and Part II finds the answer in what an entropy is. An *entropy* is a count of the states that a setting cannot tell apart: many ways for the parts to be, all looking alike from outside. So an entropy makes sense only against the setting in which the entropy is counted. Part III reads the faint pattern in the oldest light in the sky. In this book, the account is the account of the world that this book teaches. This book belongs to a collection of books, and in other books of the collection I set the account out in full. Part III finds what the account sets about the faint pattern, which values the account uses as measured, and why the account needs no early burst of expansion to explain the pattern. *Gravitational waves* are ripples in the shape of space and time themselves. Part IV follows the gravitational waves that the beginning leaves, and the instruments that may detect those waves. *Antimatter* is matter made of the twins of the ordinary particles, each twin with the same mass as its ordinary particle and every charge reversed. Part V takes up two things that the turn made. One of the two things that the turn made is the dark matter, the unseen matter whose pull is measured in every galaxy. In the account, the dark matter is made only at the turn. The other of the two things that the turn made is the small excess of matter over antimatter, which the turn leaves and which stays the same through every later change.

In brief, the beginning left five things: one form, one vacuum, the dark matter, a faint hum and the matter we are made of. The one form is the single form that passes the turn. The faint hum is a faint background of gravitational waves from the settling of the six. Part I finds the one vacuum, and Part II finds why the new world begins its own count of entropy. Part III reads the one form in the ripples of the oldest light, and Part IV follows the faint hum. Part V finds the dark matter and the matter we are made of.

How the Book Speaks of a Number

A number in physics can stand on different ground: the number may follow from the shape of the world, may come from an instrument, or may be bounded without a value being given. I always say which ground each number stands on; I call that ground the number's standing, and I use five sorts of standing.

A number may be *set by the structure*: the number follows from the shape of the world as the account derives that shape. The *cosmological constant* is the energy of empty space: the energy that drives the accelerating expansion of the universe, and that appears in the physicist Albert Einstein's equations of gravity as one constant part, the same at every place and time. A count of three appears in the cosmological constant, and the count is set by the structure: the count of three comes from the shape of the six small dimensions, as Chapter 3 shows. A number may be *set by the structure once a stated step is taken*: the number follows only after a named step from the structure of comparison (the structure that every comparison has) to the world. I always name that step at the place where the step is made. A number may be *measured*: the number comes from an instrument, and the account uses the number as measured, as the account does for Newton's constant (named for the physicist Isaac Newton), the measured strength of gravity. A number may be *measured, with its reason still a question*: the account uses the measured value and says what would set the

number, as the account does for the strength of the faint ripples in the oldest light in the sky, which Part III reads. And a number may be *bounded, with its value not given*: the account sets a ceiling on the number but gives no value for the number, as the account does for the temperature at which our history begins.

The steps of an argument come in different sorts too, as numbers do. A *theorem* is a result of mathematics, proved. *Established physics* is a result of physics, taken from the source where the result was established. An *identification* is a step from the structure of comparison to something in the world. I make each identification in my own voice, in the first person. I name each identification as an identification at the place where I make the identification. I also name any observation that bears on an identification. An *argued* step follows from stated premises by a stated argument. When this book says that a result is derived, the book means that the result follows, by a theorem or by a standard method of physics, from the structure of comparison and its three axioms, together with identifications stated in the first person. The three axioms are statements that hold of every comparison whatever. A *computed* value is arithmetic on stated inputs. A *measured input* is a measured value that the account uses. A *measured agreement* is a value that follows from the account and that measurement then finds. Where the account leaves an item unsettled, the book names the item as *not fixed* at the place where the item comes up. When a step is proved within one domain, the book states the step with the word "must": if the premises of the step are true, the conclusion of the step must be true. Each step in this book is of one of the sorts named in this paragraph. I say which sort each step is, at the place where the step is made.

A computed number has the same standing as the inputs from which the number is computed, and I name those inputs. So a computed number is not a sixth standing. For example, a number computed only from measured inputs is itself measured. A few numbers in this book are *computed at first order*: they are worked out for the largest and simplest part of an effect, with the smaller corrections left aside. Each number computed at first

order is set by the structure once a stated step is taken, and the stated step is the first-order computation itself, which the book names at the place where the computation is made.

One number keeps a standing of its own: the ratio of dark matter to ordinary matter is a *registered ratio*, which is the standing that the ratio's source book, *The Weight of Possibility* (described below), gives the ratio. A registered ratio is a ratio fixed by what the present measurement registers, and not by a mechanism at the turn. The principle behind the ratio is the account's own, and so is the rule that turns the principle into a number. The rule was picked out by comparison with the measured ratio, among a small family of candidate rules that are all built the same way. Chapter 14 explains the standing of a registered ratio at the place where the ratio enters the ledger of the sky's numbers, and Chapter 19 walks through the principle and the rule.

What Stands Behind the Book

This book is a teaching book. Every claim in this book comes from other books of the collection to which this book belongs, and those books set the account out in full. I say each claim exactly as firmly as those books of the collection say the claim, never more firmly and never less. Of the five books of the collection that this book names most often, the main source is *The Compactification*. *The Compactification* sets out for physicists how an era of the world begins and what that beginning leaves in the sky. *The Gravitational Passage* derives Einstein's equations of gravity from the structure of comparison and sets the whole account out for physicists, and that book is the source for the turn, the six small dimensions and the cosmological constant. *The Weight of Possibility* is the source for the dark matter. That book identifies the dark matter with a lasting vibration of the six small dimensions. *The Coming of the Kinds* is the source for the ordinary matter that remains. That book tells how the kinds of particle came to be after the Big Bang. *The View from Inside* is a book for the general reader on the cos-

mological constant and on why empty space weighs so little. *The View from Inside* tells parts of the account in plain words, and several pictures in this book come from that book. The Notes at the back of this book give the place in the source books where each claim is made, chapter by chapter.

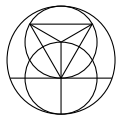
Three appendices contain what a physicist will want and what a reader can pass by. Appendix A gives the formulas, each read aloud in words. Appendix B lists the measurements that will be made in the coming years, and what the account says each measurement will show. Appendix C gathers every number in the book, each said in words, with the number's standing.

One fact about knowing lies underneath everything: the Incommensuration Theorem. Every measurement is a comparison: the act of finding whether two things are the same or different, in some respect. The Incommensuration Theorem uses four words, each in one exact sense. *Symmetry* is sameness through a change, and *continuity* is smoothness, with nothing sudden. *Asymmetry* is unevenness, a difference from one place to another, and *discontinuity* is coming in separate steps. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. When two of the four (symmetry, continuity, asymmetry and discontinuity) are perfect at once at the foundation of a comparison, we say that the two are applied simultaneously and fundamentally. You will see the Incommensuration Theorem at work several times in this book, and most of all in the places where the theorem requires that the beginning was even, but never perfectly even.

We start where *At the Edge of Reality* stopped, at the opening of a new world. What exactly does the opening of that new world leave us with?

PART I

THE SIX HELD AGAIN



Chapter 1

Where the Last Book Left Us

At the Edge of Reality ends with a new world opening out of the one end that all black holes share. What exactly does the opening of the new world leave us with, and what does this book set out to find? If you have not read *At the Edge of Reality*, this chapter contains what you need from that book, and more besides.

Start with a film of our own universe, run backward. The galaxies come apart into thin clouds of gas. Today a faint glow fills the whole sky: the oldest light there is. In the backward film that glow shrinks back into a hot, bright fog. Earlier still, the first nuclei of atoms come apart into loose protons and neutrons, and earlier again everything is a hot soup of particles. Run the film further, and the soup grows denser and space bends more and more sharply, until the film reaches a limit. *Cosmology* is the science of the universe as a whole, and in cosmology the limit where the backward film stops is called time zero.

I begin with three questions about the limit at time zero. What is the limit, and what lies on the far side of the limit? How does the world come to have its present shape at the start: four large dimensions and six small ones, held small? And why does the world begin so orderly, with so little disorder in its geometry, the shape of its space and time? In this chapter I answer the first question, from the findings of *At the Edge of Reality*. The second question is answered in the rest of Part I, and the third question in Part II. The last section of this chapter sets out the questions of Part I in

more detail, with the further questions answered in Parts III, IV and V. Before any of the three questions can be answered, we need the findings of the earlier book, which the next section sets out.

What the Last Book Found

Every observation, every measurement and every test is a comparison, as Before We Begin said: a comparison is the act of finding whether two things are the same or different, in some respect. When a thermometer compares this morning's warmth with this evening's warmth, the thermometer is making a comparison. The *content* of a comparison is what the person or instrument making the comparison finds: for the thermometer, the content is the temperature. The *context* of a comparison is the respect in which the content is found: for the thermometer, the context is the two times of day. Place, time and possibility are the three contexts in which anything is ever found.

Everything that follows in this book depends on one fact about comparison. The fact is the *Incommensuration Theorem*, a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. Every act of knowing is an act of comparison, or is built from acts of comparison, so the Incommensuration Theorem is inherent in every act of knowing. The four words keep the meanings Before We Begin gave them: symmetry is sameness through a change, continuity is smoothness, asymmetry is unevenness, and discontinuity is coming in separate steps. When two of the four are both present, perfect, at once, at the foundation of a comparison, the two are applied simultaneously and fundamentally. A flat map of the whole Earth has a limit of the same kind as the Incommensuration Theorem: a flat map can keep every angle true, or every area true, and no flat map keeps both. True angles and true areas play the part of the two words of one pair in the Incommensuration Theorem. No flat map has both perfect at once.

Under the Incommensuration Theorem, two pairings remain possible: continuity with asymmetry, and discontinuity with symmetry. So the world shows two characters, smooth and uneven on one side, stepwise and symmetric on the other. The *arena* is space and time together, the stage on which everything happens. The arena is smooth and uneven, and the shape of the arena is gravity. The matter and light that fill the arena come in steps. Every step of one kind is the same as every other step of that kind. Physics has a description for each of the two characters. Einstein's general relativity is the smooth description of the arena, and quantum theory is the stepwise description of what fills the arena.

The world has ten dimensions. A dimension is an independent way in which something can differ, as Before We Begin said: a point on a city map differs from another point in two ways, east and north, so the city map has two dimensions. Time has one dimension, space has three, and possibility has six: one and three and six make ten. In this book, *possibility* means the range of what someone making a comparison could find. The structure of comparison itself sets the count of ten, and each step toward that count is argued from the structure of comparison.

The three sorts of dimension are not interchangeable. You can walk to the next room, and you can read yesterday in a diary. You cannot walk to the outcome that did not happen. The four dimensions of time and space are the ones a person or an instrument moves along and records along. The six dimensions of possibility show up only as what could be found, and nothing ever travels through them. The six are present everywhere you are, as the grain of a sculptor's stone is present at every point of the block: the grain decides how the stone can be shaped, while no tape measure ever measures the grain. In the picture, the block is the world around you, the grain is the six, and the shapes the stone can take are what could be found.

The six dimensions of possibility together form one small space, called the *internal space*: the range of what content could be. The shape of the internal space has a plain meaning, because the internal space is two spaces side by side. The first space is the space of all the states a three-way choice

can be in, called the complex projective plane. The second space is the space of all the states a two-way choice can be in, which is an ordinary sphere. (Chapter 3 draws both: a lamp that could be found red, green or blue, held between its three colors, and a globe.) The ten dimensions of the world are the four of time and space and the six of the internal space, so the world is a *product* of two parts: a shape in which every point is a pair, one point taken from each part. One part is large, smooth and four-dimensional, and the other part is the small six-dimensional internal space. The two parts lie side by side as a product. By the Incommensuration Theorem, the two parts cannot merge into a single geometry.

The internal space is very small. The *Planck scale* is the scale at which gravity and the quantum act on equal terms. Every object has two sizes, and comparing the two sizes shows what equal terms means. The first size is the object's gravitational size: the radius to which the object would have to be squeezed to become a black hole. The second size is the object's quantum size. Quantum theory treats every object as having a wave, and the object's quantum size is the *wavelength* of the object's wave, the length of one ripple of the wave, and the wavelength shrinks as the mass grows. The two sizes are equal for one mass, called the Planck mass, about twenty-two millionths of a gram. When one description alone is enough, we say that the description rules. For a planet or a star the gravitational size is far the larger, and the smooth description of gravity rules. For an atom or a single particle the quantum size is far the larger, and quantum theory rules. The Planck length is the length that goes with the Planck mass: about a hundred billion billion times smaller than a proton. Before We Begin introduced the Planck length. One measured number fixes both the Planck mass and the Planck length: Newton's constant of gravitation, the measured strength of gravity, which is the one measured constant on the gravitational side of the account. At the Planck scale neither description rules, and both are relevant. The Planck scale is called the *modal boundary* in *The Gravitational Passage*, the source book that Before We Begin described. At the modal boundary the two characters of the Incommensuration Theorem, smooth and stepwise, trade places as the character that

rules. The range of each description ends at the modal boundary, and beyond the boundary no third description takes the place of the two. The internal space lies at the modal boundary: the internal space is a few Planck lengths across. The large geometry of space and time has no least step, so the modal boundary is not a least length of space and time. The geometry has small waves (ripples in the shape of space and time), and the least motion of the small waves limits only how sharply a body can be placed, to about the Planck length. The limit is a limit on placing a body, and the limit does not divide space into grains. When this book says that the internal space is held at the Planck scale, the statement gives the size of the internal space. The statement does not say that the arena is a lattice, a grid of separate points. The small waves can be described by quanta only up to the modal boundary. The account has no quantum theory of the arena for that description to continue into.

An *epoch* is one era of the world, pieced together from inside the era, in which stars, galaxies and black holes form. *At the Edge of Reality* is the story of a fall into a black hole, and the finding of that book is that the black holes of one epoch share one end. A new large world forms from the shared end of the black holes. When someone traces the history of the new epoch backward, the beginning the tracer finds lies at that shared end. When someone traces a history of the universe backward and finds the history's beginning, that beginning is called the history's *Big Bang*. I take up the story at that beginning, and first I make three things exact: which internal space we are speaking of, what holds the internal space small, and what happens at the end of the black holes.

One Space, Taken Two Ways

Think of the face of a clock. Taken as the pattern of twelve hours, the face has no size at all: three o'clock is three o'clock on a wristwatch and on a tower clock alike, and you read the time from the pattern. Taken as a particular clock, the face has a width: two centimeters on the wrist, several meters on the tower. The face of a clock is one pattern, taken two ways.

The internal space is one pattern taken two ways, and the two ways are called the two *instances* of the internal space. A *field* is something spread through space with a value at every point. The temperature of a room is one example, because the temperature has a value at every point in the room. On the *content instance* the internal space is a space of states, with no size, no radius and no fields. Nothing on the content instance has a length. The kinds of particle and their values are read on the content instance, in the books of this collection about the particles. On the *context instance* the same internal space is the space of possibility in which the content of an epoch is held. On the context instance the internal space has sizes and a *curvature*, a bending, like the bending of the surface of a ball. Gravitation depends on the context instance, and so do the cosmological constant, the energy of empty space that drives the accelerating expansion of the universe, and the dark matter, the unseen matter whose pull is measured in every galaxy. The clock picture fails in one respect: a wall clock hangs in a room, but neither instance of the six is a place. Nothing travels through either instance.

This book is about the context instance. The context instance has every size, every curvature, every way the six can stretch or vibrate, and every wave the six release. I put no field, no particle and no adjustable size or shape on the content instance, and so I put there no energy that depends on an adjustable size or shape. I take each fact about the particles that this book needs, such as how many kinds of particle there are, from the books about the particles, and I say so where I use the fact.

If you have heard that string theory also has ten dimensions, then string theory's ten dimensions differ from the ten of this book. Superstring theory is the string theory that has ten dimensions: in superstring theory the smallest pieces of matter are tiny vibrating strings, and the extra six dimensions are directions of space, curled up small. In this book the six are not directions of space, and nothing ever travels through them.

The word for holding the six small needs the same care as the count of ten. When a space is finite and has no edge, the space is called *compact*. The surface of a ball, for example, has a finite area and no edge. In this book, the six are *held compact*: held small while the large world of four dimensions is their context. The six play two parts at once, and I need both parts. First, the six are the context of possibility: the respect in which the possibilities of content are found. Second, the large world holds the six, small, as the content of the large world, at every point of the large world. In this book, as in *The Gravitational Passage*, the word for that holding is *compactification*. In this book the word compactification means only that holding: the six were never directions of space, so the word compactification does not describe six directions of space that were once spread out and then curled up.

What Holds the Six

What keeps the six small in the world as it is now? Look around the room you sit in. The room has an outside. Space and time are smooth at every scale anyone can measure. And the distant galaxies fix what "not rotating" means, for a spinning gyroscope and for the balance organ of your inner ear. Three plain facts are the conditions that hold the six: the room has an outside, space and time are smooth, and the distant galaxies fix what "not rotating" means.

The conditions are called the *compactification conditions*, and *The Gravitational Passage* states them exactly: the large world is present as a context for the six, as a domain of the smooth description, with a reference for in-

ertia fixed in it. The exact statement contains two conditions, and the first condition has two parts. The first part is that the large world must be present as the context of the six, at moments at which a comparison can be made. The second part is that the large world must lie within the range of the smooth description, which means that the large world must bend less sharply than the *Planck curvature*, the bending of the surface of a ball whose radius is one Planck length. The second condition is that a reference for inertia be fixed in the large world. *Inertia* is the tendency of a body to keep its state of motion, and a body is at rest, or rotates, always relative to something. A spinning gyroscope keeps pointing at the same distant stars, because the distant stars define "not rotating". A *reference for inertia* is the standard of not accelerating and not rotating, fixed by matter within reach, against which motion is compared. A reference for inertia is also called the Mach reference, after the physicist Ernst Mach, who argued that rotation and acceleration have meaning only relative to the matter of the universe.

While both compactification conditions hold, the six are held as content, and each of the six directions lies at the modal boundary, where the curvature in that direction is the Planck curvature. When the six are held in that way, with each direction at the modal boundary, the six are in the state called *the hold*. Newton's constant fixes the modal boundary, and Newton's constant has one value everywhere in the large world, so the internal space has the same size and the same shape at every point of the large world: the size and the shape of the internal space are fixed throughout.

The hold rests on a floor: the internal space cannot shrink to nothing. Two facts show that the internal space cannot shrink to nothing. The first fact comes from the account's three *axioms*, three starting points found in the structure of comparison: each axiom is a statement that holds of every comparison whatever, and every argument of the account is reasoned from the three axioms. The third axiom says that the three roles every comparison has, and the pairs that fill those roles, are distinct and inseparable, and that no role or pair can do the work of another. (Chapter 6

names the three roles.) Content and context form one of the pairs that fill the three roles, so there is no content without a context. The internal space is the context of possibility, so the internal space cannot vanish while there is content. The second fact comes from quantum theory: the size of the internal space cannot be held at exactly zero with no spread at all. So the internal space has a least size, with a least motion that can never be removed. Chapter 4 goes through both facts step by step.

The ten-dimensional geometry itself holds the six. The ten-dimensional geometry moves by a law (a rule that says how the geometry bends and changes), and the law is built from the geometry alone, from the bending of the geometry added up over all ten dimensions. The account adds no further field to hold the six.

The compactification conditions hold in every part of the world you know, so the six are held there, and the forms that the matter of your body can take stay fixed, because the form of matter depends on the six being held.

How an Epoch Ends, in Eight Steps

Can the compactification conditions fail? The conditions can fail, and inside a black hole each condition is tested. A *black hole* is a region where so much matter is squeezed into so small a space that nothing can climb back out, not even light. The boundary of a black hole is the *horizon*: the surface from inside which no signal can reach the outside. The *interior* of a black hole is everything inside the horizon. In *The Gravitational Passage* an interior is followed to its end in eight steps. In *At the Edge of Reality* the same eight steps are walked in plain words. The eight steps follow, in order.

First, the singularity is not now. In a black hole, Einstein's equations give curvature without bound at the *singularity*. Picture a friend watching from far away as you fall toward a black hole. In your friend's time you approach the horizon without end: your light arrives more and more slowly, more and more reddened, and your friend never sees you cross.

On your own watch you do cross the horizon, and then the end lies a short time ahead: for a black hole with the mass of the sun, at most about fifteen millionths of a second, a figure that general relativity gives. Inside the horizon, moving toward the center is no longer a choice of direction, just as moving toward tomorrow is no choice. So the singularity is a moment and not a place: the future end of every path inside the horizon. Every path inside approaches the singularity within a finite time, on the watch of whoever travels the path, and never reaches the singularity. On this account a singularity is what the smooth equations produce when someone carries the equations past the edge of their range. A microscope gives a picture of what a singularity is: the finest detail under a microscope blurs at the limit of the microscope's lenses, while the leaf on the slide stays sharp. In the picture the lenses are the smooth equations, the blur is the singularity, and the leaf is what the equations describe. No signal from the end ever reaches the outside world, so nothing outside can place the end before, at or after any moment of the outside world.

Second, all black-hole interiors are one at their end. Draw each interior as a funnel narrowing toward its end: the funnels stay distinct along their whole length, and the ends of all the funnels are one end. The second step rests on three facts. Black holes that orbit each other merge, and the merged hole has one interior that contains both former interiors. A settled black hole shows the outside only its mass, its spin (how fast the hole turns) and its charge. And, by the first step, nothing outside orders the ends of the interiors in time. One principle joins the three facts: a difference that no comparison can register is no difference. Someone inside one hole registers that hole and not another, so the interiors are distinct while anyone travels through them, but at the common end of the interiors no comparison registers a difference between one interior and another. No signal from the end reaches anyone in the old epoch. Only the comparisons of the new epoch register anything coming from the end. The outside world of the new epoch re-forms with no identity carried over from the outside world of the old epoch. I take a stated premise for the second step: the new world registers no distinction that the old outside world drew. So

in the new epoch no comparison registers from which interior any part of the content came. Therefore, for every possible comparison, the verdicts on the end of one interior are the same as the verdicts on the end of another, and, by the principle that a difference no comparison can register is no difference, the ends are one. Someone falling in is heading for the common end of all black-hole interiors. The question "which black hole am I inside?" has an answer only while the outside world can register which hole you are inside. Near the end, the mass of the hole stops mattering to the stretching and squeezing that a falling body feels: the part of the stretching and squeezing that depends on how heavy the hole is fades away, and what remains depends only on how much time the falling body has left before the end. So the geometry near the end has no length of its own, no size set by the hole: the end of a hole of the mass of the sun and the end of a hole four million times heavier feel nearly the same. The mass of the hole stays registered only in how long the fall has lasted and in the records the falling body carries.

Third, the outside world empties. The expansion of the universe is speeding up, so every observer has an outer edge beyond which nothing can ever send the observer a signal. Within an observer's outer edge, gravity makes matter clump. Stars form and burn out, the remains of stars gather and merge, and bound structures end in black holes, while the expansion carries everything not bound to the observer's own galaxy beyond the observer's outer edge. Over about a hundred billion to a trillion years, the outside world thins out and passes beyond the outer edge of every observer held in a galaxy. Black holes give off a faint glow and slowly evaporate, far too slowly to change the emptying: a black hole with the mass of the sun takes about ten to the power sixty-seven years to evaporate. Both spans come from the known physics of stars, of black holes and of the expansion. Every observer ends inside a black hole, or carried away alone.

Fourth, the reference for inertia comes loose: at the end of an interior, the standard of not rotating is no longer fixed. Einstein's equations do not by themselves require that matter fix what "not rotating" means: Ein-

stein's equations allow worlds empty of matter in which a local standard of not rotating is still fixed, with nothing for the standard to be compared against. What Einstein's equations leave unsettled follows from the principle of comparison. Where there is nothing to compare against, no comparison registers a difference between one standard of not rotating and another, and a difference that no comparison can register is no difference. So a standard of not rotating is fixed only by matter within reach. At the end of an interior nothing can be compared against the distant stars, because the outside world has no moment there. A body carried away alone has nothing left to compare its motion against but its own outer edge. For a body with nothing to compare against, "not rotating" means nothing. At the end of an interior, time loses its large-scale direction as well. In the outside world, time has a definite large-scale direction: the direction in which disorder grows, the direction of the expansion, and the direction that the reference of the distant matter defines. Two orderings rest on the reference of the distant matter: the ordering of "before" and "after" across the outside world, and the ordering of cause and effect between distant events, so at the end of an interior those two orderings have nothing to rest on. Each single act of comparison has a one-way order inside it, and that order stays as it is, unreversed. But at the end of an interior there is no content that can be compared, so the one-way order has nothing to put in order. (Chapter 6 is about the one-way order inside each act of comparison.)

Fifth, the outside world's structures dissolve. The cosmological constant belongs to the outside world. The cosmological constant does not push back against the collapse, and at the end of an interior the cosmological constant has no effect at all. The form that ordinary matter takes depends on the six being held small, and as the internal space moves toward no longer being held, the form of ordinary matter is no longer fixed. The matter of stars and planets and people belongs to the epoch in which the six are held.

Sixth, the compactification conditions fail. At the end of an interior the outside world has no moment, the smooth description has reached the edge of its range at the Planck scale, and no reference for inertia survives. The large world is no longer the context in which the six are held as content. By the third axiom there is no content without a context, so the six cannot stay held as they were. The conditions also fail for a body carried away alone. The lone body's only reference for inertia is its own outer edge, which the body itself fixes and no content fixes, so the conditions fail for the lone body too, and the lone body's six are released. The matter of the lone body then stops being matter in the form the epoch gave it, while the content itself does not cease. No one can tell by any comparison whether the release happens before, at or after any event elsewhere, because the lone body lies beyond the outer edge of every other observer.

Seventh, a context is restored. When the conditions fail, the ten-dimensional geometry restores a large outside world. The geometry must restore a large outside world, for three reasons. First, the content does not cease at the end: at the modal boundary the range of each description ends and no other description takes the place of the two, and the edge belongs to the descriptions, not to the content. Second, there is no content without a context. Third, a new context can form only from the ten-dimensional geometry that remains. So nothing holds the six small any longer, and the split into large and small dimensions forms again. A new four-dimensional space and time forms as the large part, with its own outer edge and, as the new world's matter spreads, its own reference for inertia. The re-forming of a large outside world is called the *restoration of context*.

The eighth step is an *identification*: a step from the structure of comparison to something in the world, made in the first person, with the observation that bears on the step. I identify the restoration of an outside world at the end of the black holes with the Big Bang of the new epoch. The observation that bears on the identification is our own history: read backward, our history runs back to a limit of rising curvature and rising density, and has that limit for its beginning.

Each of the eight steps is of one of the kinds that Before We Begin names. The first step uses established results of general relativity, and the third uses the known physics of stars, of black holes and of the expansion. Four steps rest on three principles of comparison. The second step, that the ends are one, rests on the principle that a difference no comparison can register is no difference. The fourth step, that the reference comes loose, rests on inertia being a comparison. The sixth step (the conditions fail) and the seventh step (a context is restored) rest on the third axiom. I argue each of the four steps from the principle named for it. The fifth step is argued too, from the hold described earlier in this chapter: the form of matter rests on the six being held. The eighth step is the identification.

The Turn

What happens between the end of the old epoch and the start of the new epoch? Draw the history of the old epoch and the history of the new epoch side by side against time. On the left, the collapse inside the black holes runs toward a line. On the right, the new world runs away from the same line. Neither history reaches the line, so draw the line open.

The open line is *the turn*: the limit between the two sides. I take the turn to be the limit at which the range of each description, the smooth one and the stepwise one, ends, and past which no other description takes their place. Taking the turn to be that limit is a stated premise, and *The Gravitational Passage* and *At the Edge of Reality* both take the same premise. The collapse approaches the turn and does not reach the turn. No comparison can register the turn itself as an event. Someone would need to compare a before with an after, across the turn, to know the turn as an event, and no such comparison exists. The outside world of the old epoch has no moment at the end. The clock of anyone falling in ends at the limit the clock approaches. On the side of the collapse, the description holds up to the turn and not at the turn, and on the side of the new epoch the description likewise holds up to the turn and not at the turn, in the opposite

direction of time. So “how long”, “how hot” and “how fast” apply to the approach on each side, and none of the three applies to the turn. No comparison shows the turn to be sudden, and no comparison shows the turn to be gradual.

At the turn no geometry is built, and the account defines no field of any kind at the turn. What passes the turn is the *form of the content*: which alternatives the content has taken, that is, what has been selected. The structure of comparison continues at the turn: the three axioms and the Incommensuration Theorem hold beyond the Planck scale, as they hold everywhere. The axioms and the Incommensuration Theorem hold without the large-scale split of the world, the split in which the content has one geometry and the context a separate geometry, and that split does not hold at the turn. Beyond the Planck scale no description takes the place of the smooth and the stepwise descriptions, and the account contains no such description. To ask for the geometry at the singularity is to ask a description for what lies past the edge of the description’s range. A map of a country has an edge, but the edge is where the map ends, not where the land ends. In the same way the smooth description ends at the Planck scale, and the world does not end there. So the account does not resolve the singularity by quantizing gravity. The account also does not replace the singularity with a finite structure computed from a quantum theory of gravity. A quantized arena would be both stepwise and uneven, and so would need discontinuity and asymmetry together, a pairing that by the Incommensuration Theorem cannot be applied, simultaneously and fundamentally, to any event or comparison. The account places the singularity outside the range of the smooth description, and the edge of that range is the Planck scale.

In the new epoch the product re-forms with the same internal shape as before: the complex projective plane times the sphere. The new space and time carry no identity from the old space and time. The new space and time re-form from a ten-dimensional geometry in which they had no sep-

arate identity. So the large directions of the new world are neither the old small directions nor the old large directions. The new space and time begin as themselves.

The turn is a passage in a cycle: at the turn the ten-dimensional geometry re-forms its large part, because the failure that ended the old outside world also makes a new outside world necessary. Three things follow from the turn being a passage, for the language of this book.

First, there is no *initial state* (the state from which a description of a system is run forward). When someone traces an epoch's history backward, the beginning the tracer finds is the far side of a limit approached from the epoch before, and not an arrangement chosen at a first moment.

Second, the turn is a passage, and three things are true of the turn. The content continues through the turn, so the turn is not a creation from nothing. The outside world itself re-forms, so the turn is not a flicker in an empty space that was already there. The structure of comparison applies on both sides of the turn while the ranges of the descriptions end there, so the turn is not a breakdown at which the equations fail.

Third, the cycle of epochs has no first time round. Each epoch makes stars, the stars make black holes, the ends of the black holes are one end, and the turn restores a large world: the next epoch. That sequence is called *the cycle*. Each epoch begins from the common end of the black-hole interiors of the epoch before, and every earlier epoch began in that way too. Each turn stays distinct from the next turn: an epoch registers the turn the epoch came from, as the epoch's beginning, and does not register the turn the epoch's own black holes will reach, which lies in the epoch's future.

A History Made from What Arrives Now

A photograph of starlight shows the star as the star was when the light left it. Your whole picture of the past is made the same way, from impressions arriving now: the light now reaching us from distant galaxies, the

glow of the oldest light, the amounts of the chemical elements, and the records of every instrument. We piece together one history from the impressions arriving now. The history we piece together is the one history that the present requires. The pieced-together history is called the *reconstructed history*. The reconstructed history is a structure built from what arrives now, and the reconstructed history exists as structure, as a map or a record exists.

When read forward, our reconstructed history is the familiar expansion of the universe: curvature and density falling, volume growing. When traced backward, our reconstructed history shows rising curvature, rising density and shrinking volume, just as the collapse inside the black holes shows them from the other side. The collapse and our history approach one limit, the turn, from its two sides in opposite directions of time.

I take one stated premise about reconstructions: a reconstruction describes everything in it as things at definite times and places, as a record or a map does, and so a reconstruction says "this happened, then, there". A reconstruction has no way to say "this was a limit that nothing reached". So when our history is traced backward, the reconstruction comes to the turn and draws the turn as a point in time: a beginning. The time zero of cosmology is the point where the reconstruction comes to the limit the reconstruction cannot pass. The identification I made in the eighth step states that the Big Bang of the equations of cosmology is the turn as the turn appears in our reconstructed history. The turn, which no comparison registers as an event, is drawn as an event in the history we reconstruct from inside. The observation that bears on the identification is the one named in the eighth step. Our history runs back to a limit and has that limit for its beginning. Stand on a beach and look out to sea: the horizon, the line where the sea meets the sky, is the edge of what you can see, and not the edge of the sea. The Big Bang is an edge of the same kind for the history we can reconstruct: the content continues through the turn, as the sea continues past the horizon. The cycle and the edge are one ac-

count, not two: the cycle is the order of the epochs, and the edge of the reconstructed history is how the beginning of each epoch appears from inside the epoch.

Every count that this book states for the past is a number of one sort, and the reconstructed history fixes that sort. Later chapters state counts "at" eras long past, such as how many states the matter of an era could be in. Each count "at" an era long past is a number within the reconstruction, computed from the structure of the one history, with the measured present as its source.

What This Book Sets Out to Find

Where are we in the story now? We are at the turn, with a new epoch opening out of the turn: a large world of four dimensions, with the six again as its content, in the same shape as before.

A reader of *At the Edge of Reality* comes away with questions, each about something named in that book and not followed further there. How do the six come to be held again, and at what size? Why is the shape of the six the same in every epoch, with no range of alternative shapes? How hot did the new epoch begin? What do the faint ripples in the oldest light record of the beginning? Did the turn leave waves of its own in the geometry of space and time? And what did the turn make that still exists today? Every one of the questions just asked is answered in the five parts of this book.

The account in this book is set out for physicists in *The Compactification*, which rests on *The Gravitational Passage*. Part I follows the six from the turn until the six are held again: how the directions of the new world move as the new world opens out, what shape and size the six settle to, what catches the six at that size and where the energy of their motion goes, and why the six can settle at only one size and shape. Part II asks why the beginning was so orderly and follows the counts of states that each epoch begins afresh through the whole of an epoch. The reconstructed history of

the new epoch begins hot, even and nearly settled. The account bounds the starting temperature of the new epoch from above, but the account does not state the value of that temperature. The bound comes from the measured amount of dark matter, as Chapter 4 shows. Part III reads the oldest light, whose faint ripples record the beginning, and shows why the account needs no phase of very rapid early expansion, and why a long phase of that kind is incompatible with the account. In Part IV we listen for the waves that the beginning left in the geometry of space and time. Part V turns to what the turn made: the dark matter, and the ordinary matter of stars and of us that remains.

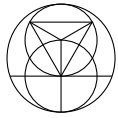
Why does this chapter matter for what the beginning left behind? Every later finding about the sky depends on three facts made exact in this chapter. The six are held small only while a large world is present as their context. At the end of every black-hole interior the large world fails as the context of the six, and the ten-dimensional geometry restores a new context. And what passes the turn is the form of the content, while the new space and time begin with no identity from the old space and time. What the beginning left behind is what the restoration of context made.

> **Key idea.** The six dimensions of possibility are held compact, a few Planck lengths across and the same everywhere, only while a large world is present as their context. At the end of every black-hole interior the large world fails as the context of the six, and the ten-dimensional geometry must restore a large world. By the identification I made in this chapter, the restoration of a large world is the Big Bang of the new epoch. The turn between the epochs is no event, and what passes the turn is the form of the content.

Summary

We now know that the story has reached the turn, and we know what is carried across the turn. The world has one dimension of time, three of space and six of possibility, a count set by the structure of comparison. The internal space of the six has two instances: on the content instance the internal space is a space of states with no size, on which the kinds of particle are read. On the context instance the internal space has sizes, and the context instance is the one this book is about. The six are the context of possibility, and the large world holds the six, small, as its content. In this book compactification means the large world holding the six, and the six were never directions of space. The six stay held while the compactification conditions hold: the large world is present as their context, within the range of the smooth description, with a reference for inertia fixed in it. Each of the six directions then lies at the modal boundary, with the same size and shape everywhere. The six rest on a floor of least size and least motion. Inside every black hole the old epoch comes to its end in eight steps: the conditions fail, and the ten-dimensional geometry restores a large world, as the geometry must, and I identified the restoration with the Big Bang of the new epoch. The turn between the epochs is no event. At the turn no geometry is built and no field is defined, and the form of the content passes the turn. The product re-forms with the same internal shape, with a space and time that carry no identity from before. There is no initial state, and the cycle has no first time round. The history we reconstruct from what arrives now exists as structure, as a map exists. The Big Bang is the turn as the turn appears in the reconstructed history.

When the new world opens out of the turn, how do its directions move: which grow, which shrink, and how fast?



Chapter 2

Three Open While Six Close

When the new world opens out of the turn, how do the new world's directions move? Which directions grow, which shrink, and how fast does each one change? The turn is the limit between the end of the old epoch and the start of the new one, as Chapter 1 described it. The short answer is that three directions open out while six close up. Each of the three grows as the time counted from the limit at the turn, raised to the power five ninths, and each of the six shrinks as the same time raised to the power minus one ninth. Three things fix the numbers five ninths and minus one ninth. The first thing is a pair of rules that Einstein's equations, the equations of gravity, set. The second thing is the shape of the six. The third thing is one stated choice, explained near the end of this chapter: the choice of where, on a curve of allowed numbers, to place both the end of a black hole's interior and the opening of the new world. This chapter walks through the two rules one step at a time, with nothing harder than fractions.

A Balloon and Six Small Rings

Picture a balloon being blown up. The balloon swells in length, in width and in height, all at once and all at the same pace. Now picture six tiny rings at every point inside the balloon, far too small to see, each ring like

the circle around a garden hose seen close up, and as the balloon swells, every one of the six rings tightens. The balloon grows fast, and the rings shrink slowly.

The picture shows one thing: as a new world opens, the three large directions and the six small ones move in opposite ways. The balloon stands for the three directions of space we live in. The rings stand for the six small dimensions of possibility described in Chapter 1. In the world we know, the six are held at a tiny size at every point. The picture has no numbers in it. The numbers come in the rest of this chapter. I show how fast each direction grows or shrinks, and why each rate has its value and no other. I also show why the end of a black hole's interior runs through the numbers of the opening of a new world, in the opposite direction of time.

A Power of the Time

Near a limit like the one at the end of a black hole's interior, or at the start of an epoch, space takes a simple form. The mathematician Edward Kasner found this simple form in 1921, and it is named after him the *Kasner form*. In the Kasner form, each direction of space has a size that changes as the time counted from the limit, raised to a fixed number, and the fixed number is the direction's *exponent*: the power to which the time is raised. Each direction has its own exponent.

A few examples show how an exponent sets the change in a direction's size. A direction with exponent one half has a size proportional to the square root of the time. The square root of a quarter is a half, so when the time left falls to a quarter, the size halves. A direction with exponent one changes in step with the time itself. A direction with exponent zero keeps its size. A direction with a negative exponent goes the opposite way to the time: as the time grows, the direction shrinks, and as the time falls, the direction grows. Exponent minus one, for example, makes the size one divided by the time: double the time, and the size halves.

The time in the Kasner form is the time on the watch of someone carried along in the space the Kasner form describes, counted from the limit. Toward a limit in the future, as inside a black hole, the time is the time remaining. The time remaining falls toward zero. Away from a limit in the past, as at the start of an epoch, the time is the time elapsed. The time elapsed grows from zero. The limit itself is of the kind in Chapter 1: a moment, not a place, which every path approaches and does not reach. Duration, temperature and rate apply to the approach to the turn from either side, and never to the turn itself, so the time in the Kasner form is always a time of the approach.

The Kasner form is a first approximation. The Kasner form describes the largest part of how space behaves near the end of a black hole's interior or the start of an epoch, while one set of exponents holds. The Kasner form is established physics, in this book's sense of a result of physics taken from its source. The Kasner form is a solution of Einstein's equations for empty space, that is, for space with no matter in it. Everything else in this chapter is worked out inside the Kasner form.

Two Rules

Einstein's equations for empty space allow only some sets of exponents. Einstein's equations impose two rules on the exponents. The first rule: the exponents, added together, make one. The second rule: the squares of the exponents, added together, also make one. The square of a number is the number multiplied by itself. The square of one third is one ninth, and the square of minus one third is also one ninth. Einstein's equations set the two rules. Nothing in the two rules is chosen.

Three facts follow from the two rules, and I use all three facts.

The first fact is about which way each direction goes, toward growing or toward shrinking. On the approach to a limit in the future, the time remaining falls toward zero, so a direction with a positive exponent shrinks toward nothing, and a direction with a negative exponent grows without

bound. The two rules require at least one exponent to be negative, with one exception: one exponent equal to one and all others zero. Fractions show the reason. A number between zero and one is larger than its own square: a half is larger than a quarter, the square of a half. Suppose every exponent lay from zero to one. Then the squares would add to less than the exponents add to, unless every exponent were exactly zero or exactly one, and since both sums must be one, every exponent would be exactly zero or exactly one, and only one exponent could be one. So every exponent lies from zero to one only in the one exception: a single exponent of one, with all the others zero. An exponent larger than one cannot occur, because the square of an exponent larger than one would alone be more than one. So, outside the one exception, at least one exponent is below zero. On the approach to a limit in the future, outside the one exception, some directions are squeezed while at least one direction is stretched.

The second fact is about volume. Picture a box whose length doubles while the box's width also doubles: the floor area of the box grows four times, two times two. When sizes multiply, as a length and a width multiply to make an area, the growths of the sizes multiply too. And when two powers of the same time are multiplied, their exponents add: the square root of the time, multiplied by the square root of the time, is the time itself, so one half and one half make one. So the volume of a region of space changes as the time raised to the sum of the exponents of all the region's directions. The first rule makes the sum of all the exponents one. So near a future limit the whole volume shrinks to zero in direct proportion to the time remaining, however many directions there are.

The third fact is about the direction of time. Reversing the direction of time changes only the direction of travel along the history: every exponent stays the same. When the history is read toward a limit in the future, a direction with a positive exponent shrinks. When the history is read away from a limit in the past, the same direction grows, with the same exponent. Of the three facts, I rely on the third fact most. By the third fact, one set of exponents describes both the end of a black hole's interior and the opening of a new epoch.

Nine Directions in Three Groups

The world has ten dimensions: one of time, three of space, and six of possibility. The one dimension of time is the time the clock counts. So near the turn, nine directions are left that can grow or shrink: the three large ones and the six small ones. Both rules apply to all nine exponents together: the nine exponents add to one, and their nine squares add to one.

The six small directions come in two pieces, side by side: a larger piece with four directions, and a sphere with two. The larger piece is the complex projective plane of Chapter 1, the space of all the ways a three-way choice can stand, and the sphere is the space of all the ways a two-way choice can stand. So the nine directions fall into three natural groups: the three large directions, the four, and the two. The grouping comes from the shape of the six, not from the two rules, because the two rules apply equally to every grouping of nine directions. In more dimensions than four, Einstein's equations alone single out no group of three directions to grow. In this account the shape of the six supplies the grouping. Once the grouping is given, the two rules fix the allowed exponents.

Let the directions of each group move together, with one exponent shared by every direction in the group. I call the three shared exponents the exponent of the three, the exponent of the four, and the exponent of the two. The exponent of the two is also called the sphere's exponent. Close to the limit, every direction changes so fast that how sharply the six curl matters little. *The Gravitational Passage* is the book of this collection that sets out the whole account for physicists. *The Gravitational Passage* states that close to the limit, one Kasner form with each group moving together is consistent within this account's ten-dimensional geometry.

One Closed Loop

When the nine directions move in three groups, the two rules become two sums. The first sum says that three times the exponent of the three, plus four times the exponent of the four, plus two times the exponent of the two, makes one. The second sum says that three times the square of the exponent of the three, plus four times the square of the exponent of the four, plus two times the square of the exponent of the two, also makes one.

Picture the sets of exponents that the two sums allow. Treat each set of three exponents as one point, with the exponent of the three measured across, the exponent of the four measured up, and the exponent of the two measured toward you. The points that obey the first sum fill a flat sheet. The points that obey the second sum lie on a closed surface shaped like a melon, a ball stretched more one way than another. A set of exponents that obeys both sums lies on the line where the flat sheet cuts through the surface of the melon. Cut a melon with one straight stroke and look at the cut face: the edge of the rind is one closed loop, and the flat sheet plays the part of the knife's stroke.

When three, four and two directions each move together, the allowed exponents lie where the flat sheet cuts the melon, and so form one closed loop, which this book calls the *Kasner curve*. The Kasner curve has a definite span. On the Kasner curve the exponent of the three can lie anywhere from minus one third to five ninths, and nowhere else, because for an exponent of the three outside that span, no exponents of the four and of the two obey both sums. The working is in Appendix A. For each exponent of the three inside the span, the loop passes that value twice, so there are two choices for the exponents of the four and of the two, one on each side of the loop. At the two ends of the span the two sides of the loop meet. At each end the four and the two share one exponent, so all six small directions move as one block.

The two ends of the Kasner curve have names. At the *lower end* of the curve, the three large directions each have exponent minus one third, and the six each have exponent one third. At the *upper end*, the three each have exponent five ninths, and the six each have exponent minus one ninth. The Kasner curve, its span and its two ends are set by the structure: in the sense of Before We Begin, the curve, the span and the ends follow from the shape of the world as the account derives that shape. Once the Kasner form is taken, the curve, its span and its ends follow from the two rules by *algebra*, the working-out of unknown numbers from the rules those numbers must obey.

Checking the Two Ends

Anyone can check both ends against both rules with nothing but fractions. At each end the four and the two share one exponent, so in the two sums the six count together, as six with one exponent.

Start with the lower end: minus one third for each of the three, one third for each of the six. For the first rule, three times minus one third is minus one, and six times one third is two. Minus one plus two is one. So the lower end satisfies the first rule. For the second rule, the square of minus one third is one ninth, and three such squares make three ninths. The square of one third is also one ninth, and six such squares make six ninths. Three ninths plus six ninths is nine ninths, which is one. So the lower end satisfies the second rule.

Now take the upper end: five ninths for each of the three, minus one ninth for each of the six. For the first rule, three times five ninths is fifteen ninths, and six times minus one ninth is minus six ninths. Fifteen ninths minus six ninths is nine ninths, which is one. So the upper end satisfies the first rule. For the second rule, the square of five ninths is twenty-five eighty-firsts, and three such squares make seventy-five eighty-firsts. The

square of minus one ninth is one eighty-first, and six such squares make six eighty-firsts. Seventy-five eighty-firsts plus six eighty-firsts is eighty-one eighty-firsts, which is one. So the upper end satisfies the second rule.

End of the curve	Exponent of each of the three	Exponent of each of the six	Sum (first rule)	Sum of squares (second rule)
Lower end	minus one third	one third	minus one plus two: one	three ninths plus six ninths: one
Upper end	five ninths	minus one ninth	fifteen ninths minus six ninths: one	seventy-five eighty-firsts plus six eighty-firsts: one

The Arc

At the upper end, the exponents of all six small directions are negative. The six exponents stay negative along a short stretch of the Kasner curve on either side of the upper end. Away from the upper end, the six no longer share one exponent. On one side of the upper end, the sphere's exponent rises toward zero while the exponent of the four sinks further below zero. On the other side, the exponent of the four rises toward zero while the sphere's exponent sinks further below zero.

On the side where the sphere's exponent rises toward zero, the stretch ends where the sphere's exponent reaches zero. Put zero for the sphere's exponent in the two sums, and the two sums fix the exponent of the three and the exponent of the four (Appendix A gives the working). Where the sphere's exponent is zero, the exponent of the three is about 0.547 (zero point five four seven), and exactly, the exponent of the three there is one plus twice the square root of two, all divided by seven. The square root of two is about one point four one four, so twice the square root of two, plus one, is about three point eight three, and a seventh of three point eight three is about zero point five four seven. On the other side, the stretch

ends where the exponent of the four reaches zero, and the same working, with zero put for the exponent of the four, gives the exponent of the three at that end. Where the exponent of the four is zero, the exponent of the three is about 0.527 (zero point five two seven), and exactly, the exponent of the three there is three plus twice the square root of six, all divided by fifteen. The square root of six is about two point four four nine, so twice the square root of six, plus three, is about seven point nine, and a fifteenth of seven point nine is about zero point five two seven. Five ninths is about zero point five five six. So at both ends of the stretch the exponent of the three is a little less than at the upper end. The two ends of the stretch lie one on each side of the loop.

All six small exponents are negative along the Kasner curve from an exponent of the three of about zero point five four seven, through the upper end at five ninths, to about zero point five two seven. The *arc* is the short stretch of the Kasner curve through its upper end on which all six small directions have negative exponents. The arc and its two end points are set by the structure, by the same algebra as the Kasner curve.

How Fast: A Billionfold

The exponents at the upper end of the Kasner curve state how fast each group of directions changes. The volume of a group of directions changes as the time raised to the sum of the group's exponents. For the three large directions together, the sum is three times five ninths: fifteen ninths, which is five thirds, and for the six small directions together, the sum is six times minus one ninth: minus six ninths, which is minus two thirds. For all nine together, the sum is five thirds minus two thirds: three thirds, which is one. So the whole volume changes in step with the time itself, as the first rule requires at every point of the curve.

A worked example with numbers shows what the exponents of the upper end mean. Let the time grow a billionfold, so that the time becomes a billion times larger. A billion is a one followed by nine zeros. The ninth

root of a number is the number which, taken nine times over and multiplied together, gives back the number you started with. Nine tens multiplied together make a billion, so the ninth root of a billion is ten. Each of the six changes as the time raised to the power minus one ninth, which is one divided by the ninth root of the time, so when the time grows a billionfold, each of the six shrinks tenfold. Each of the three changes as the time raised to the power five ninths, which is the ninth root of the time, taken five times over and multiplied together, and since five tens multiplied together make a hundred thousand, each of the three grows a hundred thousand times. The volume of the three grows by a hundred thousand three times over, which is a million billion times, while the volume of the six shrinks by ten six times over, which is a millionfold. A million billion divided by a million is a billion, so the volume of all nine grows by the same billionfold as the time, as the second fact says it must. The tenfold and hundred-thousandfold rates are set by the structure, because the rates are arithmetic on the exponents of the upper end.

Now the balloon and the rings have numbers. When the history is read away from the limit at the turn, the time elapsed grows. Each of the three large directions then grows as the time raised to the power five ninths. Each of the six contracts as the time raised to the power minus one ninth. The balloon swells fast, and the rings tighten slowly.

One Arc, Two Directions of Time

By the third fact, reversing the direction of time leaves every exponent unchanged, so reversing time carries every point of the curve to itself. Whether the arc is read toward a limit in the future or away from a limit in the past, the arc has the same exponents. The two readings travel the one arc in opposite directions of time.

Read the upper end toward a limit in the future, as the time remaining falls toward zero. Each of the three large directions then shrinks as the time remaining raised to the power five ninths. Each of the six grows as

the time remaining raised to the power minus one ninth. The volume of the three goes to zero, as the time remaining raised to the power five thirds, while the volume of the six grows without bound, as the time remaining raised to the power minus two thirds. The volume of all nine goes to zero in proportion to the time remaining. Ordinary space collapses while the internal space of the six expands. *The Gravitational Passage* describes the end of a black hole's interior as ordinary space collapsing while the six expand. Along the whole arc, read toward a limit in the future, ordinary space collapses while both pieces of the six grow.

Now read the upper end away from a limit in the past, as the time elapsed grows from zero. The exponents are unchanged. Each of the three grows as the time raised to the power five ninths. Each of the six contracts as the time raised to the power minus one ninth. Three dimensions open out while six close up, and that motion is the opening of a new epoch.

At the Edge of Reality is the book before this one in the story. In that book, a picture stands for the reading toward a limit in the future: the room you sit in, shrinking in length, width and height together, while six further directions, everywhere in the room and far too small to see, slowly swell. Run the picture backward, like a film in reverse, and a room opens out in three directions while six close up. The balloon of this chapter is the shrinking room run backward. On the arc, one geometric structure is both the end of an interior and the opening of an epoch, traveled in opposite directions of time, so the interior of a black hole mirrors the history of a new world, run in reverse. The likeness between the two histories is in the exponents, the rates at which the directions grow and shrink, and not in any region of space.

Inside a black hole's interior, the growth of the six is slow. Between two moments, each of the six grows by the ninth root of the ratio of the larger time remaining to the smaller one. So while the time remaining falls by many factors of ten, the six grow by only a few factors of ten. Because the six grow so slowly, the growth of the six is still under way when the bending of space and time reaches the Planck scale. The smooth description of space and time reaches the edge of its range at the Planck scale,

which Chapter 1 called the modal boundary. The Kasner form describes the direction and rate of the change. At the modal boundary the release of the six is completed, and the six are no longer held compact. *The Gravitational Passage* describes the modal boundary as the edge of both descriptions, the smooth one and the stepwise one.

At the lower end of the Kasner curve, every direction moves opposite to its motion on the arc, in either direction of time. Read the lower end toward a limit in the future. The three large directions then grow, as the time remaining raised to the power minus one third, and the six shrink, as the time remaining raised to the power one third. When the lower end is read away from a limit in the past, ordinary space contracts while the six grow. Neither reading of the lower end matches the end of an interior or the opening of an epoch.

Where on the Arc

Where on the arc do the end of an interior and the opening of an epoch lie? The answer has three parts, and each part has its own standing, the kind of ground the part rests on.

The first part follows from the two rules. Reversing time takes each point of the curve to itself. So the two histories have the same exponents wherever on the arc they are set, and the two histories differ only in the direction of travel. So the two histories share one arc, a result set by the structure.

The second part is that the two histories lie on the arc, and the ground for the second part is the behavior of each history. On the arc read toward a limit, ordinary space collapses while the six grow, as at the end of an interior. On the arc read away from a limit, ordinary space grows while the six contract, as at the opening of an epoch. The lower end has neither behavior. So each history belongs on the arc, because of what the history does.

The third part is the *placement*: the choice of the one point on the arc at which both histories are set. *The Gravitational Passage* sets both histories at the upper end of the Kasner curve, the point of the arc at which the exponent of the three is largest and the six small directions move as one block. *The Gravitational Passage* says plainly that the placement is a choice made in that book, made at the point where the internal space moves as a whole, and that the shared arc follows from the two rules. *The Compactification*, the main source of this book, uses the placement of *The Gravitational Passage*, and so does this book. So two histories share the exponents five ninths for each of the three and minus one ninth for each of the six. The two histories are the end of every interior that has reached the arc, and the beginning of an epoch. The two histories pass through the upper end in opposite directions of time. The placement is set by the structure once a stated step is taken: the choice made in *The Gravitational Passage*.

The Gravitational Passage states that the collapse inside the black holes and the origin of the new epoch are one transition, approached from the transition's two sides and described in opposite directions of time. The statement in *The Gravitational Passage* uses only the shared arc, so the statement does not depend on the placement, and at the level of how the nine directions grow and shrink, the statement holds at every point of the arc. The placement adds the particular pair of exponents, five ninths and minus one ninth, that this book uses for both histories. One Kasner form with each group moving together is consistent within this account's geometry, but gravity does not in general hold to one Kasner form. Near a general singularity, gravity in ten dimensions with nothing else present passes through an endless sequence of Kasner forms instead of settling into one, and at each bounce from one Kasner form to the next, the directions that grow and the directions that shrink swap. The ordinary interior of a black hole is a valid Kasner form that does not lie on the Kasner curve. In the ordinary interior, one direction is stretched as the time remaining raised to the power minus one third, two directions are squeezed as the time remaining raised to the power two thirds, and the six keep their size. The account does not fix what carries an interior from the or-

dinary Kasner form onto the arc. The account also does not fix how, in the product of the large four-dimensional world and the six, one Kasner form is selected against the endless swings from one Kasner form to the next. So when an interior has reached the arc, the interior shares the one arc with the opening of an epoch.

One Interior, Three Accounts

The Gravitational Passage contains three accounts of how the six come to be released at the end of an interior. The first account, from Chapter 1, is the whole picture: the six stop being held compact where the large world stops being their context, which happens at the end of every interior, where the outside world has no moment and no reference for inertia survives. The second account follows the interior by depth, layer by layer. The third account is the geometry of this chapter: at the upper end of the Kasner curve, the six grow on the approach to the limit.

The account by depth starts inside the horizon, where falling inward is as unavoidable as the passing of time. The interior has layers, one at each depth. Tidal stretching is the difference in the pull of gravity from one part of a body to another, which stretches the body along one direction and squeezes the body along the others. At each depth, the tidal stretching equals the stretching just outside a body that has the hole's mass and is squeezed to the radius of that depth, so each depth has an *equivalent density*: the density at which the hole's mass would fill a sphere of that radius. Halve the radius and the same mass fills an eighth of the volume, so the equivalent density rises eightfold. A body going down through the layers therefore passes backward through the densities that the early universe passed through.

For a hole of a given mass, each density is reached at a fixed depth. A heavier hole reaches a given density at a smaller share of the horizon's radius: with eight times the mass, a given density lies at a quarter of the share. The first marker density is the density at which protons, neutrons

and the other particles made of quarks dissolve, about a billion billion kilograms in each cubic meter, and for a hole of one solar mass (the mass of the sun), the first marker density is passed already at the horizon. Quarks are the smaller particles of which protons and neutrons are made. The second marker density is the density at which the weak force and the electromagnetic force act as one again: about ten to the thirtieth to ten to the thirty-first kilograms in each cubic meter (a one followed by thirty or thirty-one zeros). The weak force acts in some sorts of radioactive decay, and the electromagnetic force is the force of electricity and magnetism. For a hole of one solar mass, the second marker density is reached at about two ten-thousandths of the horizon's radius. The first marker density lies at about a ten-thousandth of the horizon's radius, and the second at about a hundred-millionth, for a hole of four million solar masses like the one at the center of our Galaxy. The two marker densities come from standard physics, and *The Gravitational Passage* uses the two densities only as markers of depth. The markers show that a body falling through the interior passes, layer by layer, through the densities of the early universe in reverse order.

The six begin to change where the curvature of the large space approaches the curvature of the six themselves. The radius of the six is a few Planck lengths, so the depth at which the six begin to change lies close to the *Planck layer*: the depth at which the bending of the large space reaches the Planck curvature. The Planck curvature is the bending of the surface of a ball one Planck length in radius. The *Planck density* is one Planck mass in a cube one Planck length on a side: the density at which the smooth description of space and time reaches the Planck scale. For a hole of one solar mass, the Planck density is reached near one part in a hundred million billion billion of the horizon's radius, and the six begin to change within a factor of a few of that radius. Both depths are computed, that is, worked out by arithmetic on stated inputs, to the nearest power of ten. At the depth where the six begin to change, the six begin to expand from their held size back toward the ten-dimensional geometry. The *classical* equations are the equations of general relativity, which take no account of the

quantum. The innermost layer lies at the Planck scale, where the smooth description reaches the modal boundary and the smooth structure that produces the singularity in the classical equations no longer holds.

The account by depth states the depth at which each structure of the outside world is undone. The Kasner account describes the form the geometry takes as the six grow. The account by depth and the Kasner account describe one interior. Every interior ends in one domain, the turn from which the next epoch begins. Many black holes therefore end in one origin. What passes to the one origin is the form of the content that fell into the black holes: which alternatives the content has taken (Chapter 1).

Lopsided in Every Direction

The opening of a new epoch is strongly lopsided: three directions grow fast while six shrink slowly. Moving at different rates in different directions is called *anisotropy*. How lopsided is the opening, and is the opening on the arc more lopsided than other Kasner forms are? The question matters because the lopsided opening is the first thing the beginning leaves. Is the lopsidedness special to the arc, or is every Kasner form in nine directions just as lopsided?

Lopsidedness has a measure. The two rules fix the value of the measure. The measure is built from the exponents themselves, since each exponent states how fast its direction changes compared with the whole volume. The average exponent of the nine is one ninth, because nine exponents add to one. Now take each direction's departure from the average, square the departure, and add the squares over all nine directions. The sum of the nine squares is the measure of the spread.

The upper end of the Kasner curve shows the value of the spread. Each of the three has exponent five ninths, which departs from the average of one ninth by four ninths. The square of four ninths is sixteen eighty-firsts, and three such squares make forty-eight eighty-firsts. Each of the six has exponent minus one ninth, which departs from the average by

minus two ninths. The square of minus two ninths is four eighty-firsts, and six such squares make twenty-four eighty-firsts. Together the two parts make seventy-two eighty-firsts (forty-eight eighty-firsts plus twenty-four eighty-firsts), which is eight ninths. The lower end gives the same value of the spread as the upper end: there the three depart from the average by minus four ninths and the six by two ninths, and the squares again add to seventy-two eighty-firsts.

The two rules alone fix the spread at eight ninths for every Kasner form in nine directions: at every point of the curve, and at every Kasner form whose nine directions are not grouped three, four and two. The general working is in Appendix A. The two ends above are two cases of the general working. The spread is set by the structure, because the spread follows from the two rules alone. So the arc has the same spread as every other Kasner form. The opening of an epoch is strongly lopsided because every Kasner form in nine directions is strongly lopsided.

What We Now Know

We now know how the directions of a new world move as the new world opens out of the turn: three grow as the time raised to the power five ninths, while six contract as the time raised to the power minus one ninth. Two histories share the exponents five ninths and minus one ninth, traveled in opposite directions of time: the end of an interior and the opening of an epoch.

The results of this chapter matter for the question of this book: what did the beginning leave behind? The first thing the beginning leaves is a world of three large directions growing fast, with six small directions closing slowly at every point. The six stop contracting where the six are held again. Two things in the new world rest on the shape and size at which the six are held: the energy of empty space and the dark matter, as Parts II and V of this book will show.

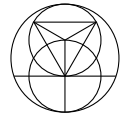
Key idea. Near the limit at the turn, each direction of space changes as the time raised to its own exponent, and two rules fix the exponents: the exponents add to one, and their squares add to one. When three, four and two directions move together, the allowed exponents form one closed curve. On a short arc through the curve's upper end, read away from the turn, each of the three large directions grows as the time raised to the power five ninths, while each of the six small directions shrinks as the time raised to the power minus one ninth. Read toward a limit in the future, the arc through the upper end is the end of a black hole's interior.

Summary

In this chapter we found how the nine directions of space move as a new epoch opens. In the Kasner form, established physics since 1921, each direction changes as the time counted from the limit raised to the direction's exponent. Einstein's equations for empty space require the exponents to add to one and their squares to add to one. The shape of the six sorts the nine directions into three large directions, four and two. For that grouping the allowed exponents form one closed curve, the Kasner curve, with the exponent of the three between minus one third and five ninths. At the lower end of the Kasner curve the three have minus one third and the six one third. At the upper end the three have five ninths and the six minus one ninth. Both ends check against both rules in plain fractions. On the arc all six small exponents are negative, and in the exponent of the three the arc runs from about zero point five four seven, through five ninths, to about zero point five two seven. At the upper end a billionfold change of the time changes each of the six tenfold and each of the three a hundredthousandfold. Reversing time leaves every exponent unchanged, so the end of an interior and the opening of an epoch share the arc. The shared arc follows from the two rules. Each history belongs on the arc because of what the history does. Placing both histories at the upper end is the choice

made in *The Gravitational Passage*. The lower end of the curve is neither history. The account by depth and the Kasner account describe one interior, and many black holes end in one origin. The spread of the rates is eight ninths at every Kasner form in nine directions, so every Kasner form in nine directions is strongly lopsided, and so the opening of an epoch is strongly lopsided too. If the Kasner form is taken, every result of this chapter is set by the structure, except the placement. The placement is set by the structure once a stated step is taken: the choice made in *The Gravitational Passage*.

The six shrink as the new world opens. What shape and what size do the six shrink to, and what fixes the shape and the size?



Chapter 3

The Shape and Size of the Six

As the new world opens, its three directions of space swell and the six small directions shrink. What shape do the six small dimensions have? How big are the six? And what fixes the shape and the size of the six? The answers to all three questions are in this chapter. The shape follows from the structure of comparison, the structure that every comparison has, by a few stated steps and by a list of shapes that mathematicians have already completed. The size follows by one more stated step and a little algebra, and the full derivation of both is in *The Gravitational Passage*, the book of this collection that sets the whole account out for physicists. Along the way we keep track of two sorts of step, both described in Before We Begin. An identification is a step from the structure of comparison to the world, and I make each identification in the first person. A theorem is a result of mathematics, proved. Where a theorem gives a part of the answer, that part rests on mathematics alone. Where an identification gives a part of the answer, that part rests on a step that I state and take.

A Lamp and a Globe

Start with a lamp that could be found red, green or blue. In quantum physics, the lamp, like any thing with three alternatives, can be held between its three alternatives at once, in any proportions, and with any relative timing among the alternatives. Picture a tiny clock hand on each alternative. The *phase* of an alternative is how far its hand has turned com-

pared with the other hands. One shape has a point for every way of holding the three alternatives at once. Now count the dimensions of the shape. The three proportions must add up to the whole, so the proportions leave two free numbers. The clock hands give two more free numbers, because only two hand settings count, each measured against the third hand. So the shape has four dimensions: two from the proportions and two from the clock hands. Mathematicians call the four-dimensional shape of all the ways of holding three alternatives the *complex projective plane*. In this book the complex projective plane is the space of all the ways a three-way choice can stand.

Now take a thing with only two alternatives. Picture a globe, with one alternative at the north pole and the other at the south pole. The globe has a point for every way of holding the two alternatives at once. The proportion between the two alternatives sets how far north or south the point lies, and the relative setting of the two clock hands sets how far around the globe the point lies. So the ways a two-way choice can stand make up the *sphere*, the surface of a ball, with two dimensions. Light is a familiar case of a two-way choice. Light vibrates across the direction in which it travels, and every way light can vibrate, from straight up and down through every tilt and twist to straight across, is one point on the sphere.

The shape of the six small dimensions is exactly the complex projective plane taken side by side with the sphere: the complex projective plane gives four dimensions and the sphere two, six in all. "Side by side" means a product, as Chapter 1 said: a shape whose points are pairs, one point from each of two shapes. Think of the surface of a tin can. Every point on the can's surface is a pair: an angle around the can's circle, and a height along the can's length. So the can's surface is the product of a circle and a line. Within a product, each of the two shapes stays what it was on its own: the circle is the same circle at every height, and the line is the same line at every angle. In the same way, the six are the product of the complex projective plane and the sphere. For short, this chapter calls the complex projective plane the plane.

Chapter 1 found that the six have two instances, which are two ways of taking one pattern. On the content instance, the six are a space of states with no size at all, while on the context instance the six have radii (sizes, like the radius of a ball). The size found in this chapter is the size of the six on the context instance. The shape of the six is the same on both instances. In the rest of this chapter we follow, step by step, how *The Gravitational Passage* derives the shape of the six, and then the size.

From the Grid to a Shape

Every physical measurement involves six measures. Three of the six belong to what is measured, which Chapter 1 called the content: pattern (mass, inertia and structure), force (the directed flow of change), and probability (how values are spread among alternatives). The other three belong to where the thing is measured, which Chapter 1 called the context: space, time and possibility. The six measures can be set out as a grid of two rows and three columns, and the two rows are content and context. There is one column for each of the three roles that every comparison has: what is possible, what is happening, and what is known. *The Gravitational Passage* shows that the list of six measures follows from what every comparison contains. I use the list as *The Gravitational Passage* states the list.

The six small dimensions are the setting in which comparisons of content take place, in all six measures. So I read the grid as a description of the shape of the six. I translate the grid into geometry by three conditions, each of which is an identification.

The first condition is one dimension for each measure. Each of the six measures is a separate way in which the content can vary, so each measure gets its own direction, and the internal space, the one small space that the six form together, has six dimensions. *The Gravitational Passage* also contains a second count of six dimensions of possibility, by a route that does not use the grid, and shows that the six of the grid and the six of the second route are one set.

The second condition is a product of two shapes. The grid is itself a product: two rows taken with three columns, as the can is a circle taken with a line. The structure of comparison fixes both counts. The two rows are the two members of the pair content and context, and the three columns are the three roles. The rows and the roles are distinct, they cannot be separated, and none can do another's work. An axiom is one of the three starting points, found in the structure of comparison, from which the account reasons. The third axiom, which Chapter 1 stated, says the same three things of the roles and of the pairs that fill the roles. In the second condition, I state of the shape what the third axiom says of the roles and the pairs, by way of the grid. So the internal space is a product of two shapes: one that carries the three roles, and one that carries the two rows. Each shape has at least one dimension, because the rows and the roles are each a real way of varying. Each shape is also all of one piece, because each shape is the one setting in which its rows or its roles vary, and a shape in separate pieces would be several settings.

The third condition is that each shape is built from exactly as many basic pieces as it carries rows or roles. A basic piece is a simple block, such as a single point or a disc, from which a shape can be built. So the shape for the three roles is built from exactly three basic pieces, one for each role, and the shape for the two rows is built from exactly two basic pieces, one for each row. Under the third condition, every basic piece also has an even number of dimensions. The next section shows how a shape is built from basic pieces.

The three conditions are identifications, and not theorems: they are stated steps from the structure of comparison to geometry. Once the three conditions are taken, every step that follows from them is mathematics.

Counting Pieces

There is a famous way to count the basic pieces of a shape. Take any solid with flat faces, such as a cube or a pyramid. Count the solid's corners, edges and faces. Corners, minus edges, plus faces, always comes to two. A cube has eight corners, twelve edges and six faces: eight minus twelve plus six is two. A pyramid on a square base has five corners, eight edges and five faces: again two. The total does not depend on which solid is counted, but only on the fact that the solid's surface is, in shape, the surface of a ball. The count is named after the mathematician Leonhard Euler. In this book the count is called the *Euler count*: corners minus edges plus faces. More generally, the Euler count is the number of a shape's basic pieces, counted with signs.

The Euler count can also be found more simply, by counting basic pieces. The surface of a ball can be built from two basic pieces: a single point, and a disc whose whole rim is drawn together to the point, like a drawstring bag pulled shut. Two pieces give an Euler count of two. The general rule is that a piece with an even number of dimensions (a point, a disc) counts plus one, and a piece with an odd number of dimensions (a line, an arc) counts minus one. The shapes of this chapter are built only from even pieces, which are pieces with an even number of dimensions, so for the shapes of this chapter the Euler count is simply the number of basic pieces. I ask for even pieces in the third condition for one reason: when every piece is even, the number of rows or roles that a shape carries is the shape's Euler count.

When two shapes are set side by side, their pieces pair up. Every piece of the product is a piece of the first shape taken with a piece of the second, so the counts of pieces multiply. On the other hand, the dimensions of the two shapes add: the can's circle (one dimension) and the can's line (one dimension) make a surface of two dimensions. So the three conditions require three counts. The shape for the roles has an Euler count of three,

and the shape for the rows has an Euler count of two. The internal space as a whole then has three times two pieces, which is six, one for each of the six measures.

The conditions are satisfied by the plane times the sphere, as a later section shows. In the plane, the three pieces are a point, a piece of two dimensions and a piece of four dimensions, while in the sphere the two pieces are a point and a piece of two dimensions. Each of the six pieces of the product is one piece of the plane taken with one piece of the sphere. The product matches the grid in its counts: three columns to the plane's three pieces, and two rows to the sphere's two pieces, with no particular piece assigned to a particular measure.

Every Point Alike

Every point on a perfect ball is like every other point: a ball has no special spot. An egg has two special spots, its two ends. When every point of a shape is like every other point, in a strict sense, we call the shape *homogeneous*. A symmetry of a shape is a way of carrying the shape onto itself that keeps every distance unchanged. In the strict sense, a homogeneous shape has, for any two of its points, a symmetry that carries the first point to the second.

I take each of the two shapes, the shape for the roles and the shape for the rows, to be homogeneous. My reason is what the six are. The six are the part of the world governed by symmetry, the part in which content is compared. The setting in which content is compared cannot itself add an unevenness, because an unevenness of the setting would be a difference of context that no content carries. Homogeneity is a fourth condition, and like the first three conditions, homogeneity is an identification. Homogeneity is the condition that decides the shape. Many shapes have the right counts, and exactly one of those shapes has every point alike.

The List Leaves One

When a shape is finite and has no edge, like the surface of a ball, we call the shape closed. Mathematicians have listed every closed, homogeneous shape. The conditions require a count of three for the shape of the roles and a count of two for the shape of the rows, so the question "which shapes have exactly these counts?" has a definite answer. The answer comes from the list in four steps.

First, both shapes have an even number of dimensions. When a closed shape has an odd number of dimensions, its Euler count is always zero: a circle, the simplest case, is built of one point (plus one) and one arc (minus one), so its Euler count is zero. Both shapes need counts other than zero, three and two, so both shapes have an even number of dimensions. The six dimensions can then be shared between the two shapes in four ways: the shape for the roles can take six and the shape for the rows none, or four and two, or two and four, or none and six.

Second, three of the four shares fail. Each shape needs at least one dimension, so the shares six and none, and none and six, fail. A closed surface is a shape of two dimensions, and its Euler count is at most two, so no surface can carry three pieces, and the share two and four fails. The share is therefore four and two: the shape for the roles has four dimensions, and the shape for the rows has two.

Third comes the shape of two dimensions, the shape for the rows. Only three closed surfaces have every point alike: the sphere, with count two; a one-sided surface called the real projective plane, with count one; and the torus, the surface of a doughnut, with count zero. Only the sphere has the count of two, so the shape for the rows is the sphere.

Fourth comes the shape of four dimensions, the shape for the roles. Few closed homogeneous shapes of four dimensions have a positive count. The four-dimensional sphere has count two, the complex projective plane has count three, and two ordinary spheres side by side have count

four. The rest are made by folding the four-dimensional sphere, or the two spheres side by side, onto itself, and their counts are one or two. Folding a shape means gluing its points together in groups, and the complex projective plane cannot be folded at all. To fold a shape, we need a way of carrying the shape onto itself that moves every point, but however the complex projective plane is carried onto itself, at least one point stays where it was. So the complex projective plane is the only closed homogeneous shape of four dimensions with count three. Some uneven four-dimensional shapes also have count three, and those uneven shapes fail the condition that every point is alike.

So the internal space is the complex projective plane times the sphere, the only closed homogeneous product that satisfies the conditions. The plane times the sphere has four plus two dimensions, which is six, and its Euler count is three times two, which is six.

The two shapes of the product are related to each other as the rows and the roles are: distinct, inseparable, and unable to do each other's work. The plane and the sphere are distinct, since the plane carries three pieces and the sphere two. The plane and the sphere are inseparable, since comparisons of content take place in the two together and in neither alone. The plane and the sphere cannot exchange their work, since a shape with three pieces cannot do the work of a shape with two. If a single shape carried both the rows and the roles, the shape would blur the rows and the roles together.

You might ask why the count of pieces and the count of dimensions agree, at six each. The agreement has a reason. The count of pieces is a product, three times two, while the count of dimensions is a sum, four plus two. The plane has three pieces and four dimensions, and the space for a four-way choice would have four pieces and six dimensions, so each added alternative adds one piece and two dimensions. With the sphere of a two-way choice (two pieces, two dimensions) for the rows, the whole product has twice as many pieces as there are columns, and also twice as many dimensions, whatever the number of columns. For any product of

two spaces of choice, one for the columns and one for the rows, the two counts agree exactly when the rows or the columns number two. For the grid's three columns and two rows, both counts are six.

The standing of a result is what the result rests on. *The Gravitational Passage* states the standing of the shape in one sentence: "The steps from the conditions to the space are theorems of topology; the conditions are identifications, and the uniqueness holds at their standing." Topology is the mathematics of shape. In plain words, the steps from the conditions to the shape are proved as mathematics, while the conditions themselves are identifications. Once the conditions are taken, the plane times the sphere is the only shape that fits. I read the conditions from the structure of comparison alone.

The same pair of shapes also follows by a second route, from algebra. In the sense used here, an *algebra* is a set of rules by which quantities combine, as numbers add and multiply. The algebra of comparison is built in *Non-Commutative Geometry and the Immanent Metaphysics*, another book of this collection. The algebra of comparison has a part that describes things with three alternatives and a part that describes things with two. The states of the part for three alternatives form the complex projective plane. The states of the part for two alternatives form the sphere. *The Quantum Passage* shows that the states of the two parts form the plane and the sphere. *The Quantum Passage* is the book of the collection that derives quantum theory from the structure of comparison. The grid and the algebra are two routes that start from different premises and end at one shape.

Smooth Bending, Separate Steps

The six now have a definite shape, and the six are part of the geometry of the world. Does the geometry of the world come in separate steps, as matter does? The answer comes from the Incommensuration Theorem, which Chapter 1 stated. The Incommensuration Theorem is a structural fact

about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. In the theorem, symmetry is sameness through a change, continuity is smoothness, asymmetry is unevenness, and discontinuity is coming in separate steps. Asymmetry and discontinuity are the subject of the second half of the Incommensuration Theorem, and the second half has an exact counterpart in geometry. The counterpart shows what kind of geometry the product of the four large dimensions and the six can have.

Carry an arrow around a closed loop on a curved surface, keeping the arrow as straight as the surface allows at every step, and the arrow comes back turned, because the bending of the surface turns the arrow. In 1953 the mathematicians Warren Ambrose and Isadore Singer proved a theorem that states exactly how the bending of a surface turns an arrow. The theorem of Ambrose and Singer means that a geometry that bends anywhere turns arrows by amounts that run through a smooth range, and never only in separate jumps. Bending is the unevenness of a geometry, its asymmetry, while turning arrows only in separate jumps would be the geometry's discontinuity. No geometry of the product therefore has both the asymmetry of bending and the discontinuity of separate jumps. So the geometry of gravity is smooth and produces no steps, in the four large dimensions and in the six alike.

The separate steps of matter come from something other than the geometry. As Chapter 1 said, a field is something spread through space with a value at every point, as the temperature of a room has a value at every point in the room. A *quantum field* is a field whose energy comes in separate steps, and the steps are its particles. The steps of quantum fields come from the *symmetry groups* of the forces, where the symmetry group of a force is the set of changes that leave the force's laws the same. On the product, the symmetry groups exist beside the geometry and cannot be merged with the geometry. To quantize the arena would be to treat the arena, space and time themselves, as coming in separate steps, and quantizing the arena would need a geometry that is both bent and stepwise.

The second half of the Incommensuration Theorem means that no geometry is both bent and stepwise. Instead, in the account the content is quantum, and the geometry of the context is smooth and classical, which here means without quantum steps. Matter held between alternatives has one smooth geometry for each alternative it can be in, and that step is an identification stated in *The Gravitational Passage* and explained in Chapter 9. The account has no quantum theory of the geometry. One smooth geometry for each alternative is not an approximation to a quantum theory of the geometry. Gravitation is not quantized.

How Much a Direction Bends

The shape of the six is now fixed, but a shape can be large or small, and in a product one of the two parts can be large while the other is small. We need to measure bending to find the size of the six, and bending, as Chapter 1 said, is curvature. A basketball bends sharply, while the surface of the Earth bends so gently that a field of wheat looks flat. The *radius* of a sphere is the distance from its center to its surface. A sphere's curvature is one over the square of its radius, so halving the radius makes the sphere bend four times as sharply. The Planck length is the length below which the smooth description of space and time no longer holds. Chapter 1 described the Planck curvature in the same terms: the Planck curvature is the bending of a sphere one Planck length in radius, which is one over the square of the Planck length.

A shape of many dimensions can be bent differently in different directions, and there are several ways to measure its bending. Each flat slice through a point is spanned by two directions and has a bending of its own, and a single direction lies in many slices and feels the bending of every one of them. Mathematicians call the total of the bendings of all the slices through a direction the direction's *Ricci curvature*: how much a dir-

ection curves, taking every slice through it into account. A point also has one number, the total of the Ricci curvatures over all the directions at the point. I call the total at a point the total curvature.

Which of these ways of measuring bending can give the same value in every direction of the product? The bendings of slices differ from one another: slices inside the plane bend by different amounts, and a slice with one direction in the plane and one in the sphere does not bend at all. Apart from the Ricci curvature, every other way of measuring bending either differs from direction to direction at every ratio of the plane's radius to the sphere's, or is the same at every ratio and so fixes nothing. The Ricci curvature is the one way of measuring bending that can be the same in every direction, and it is the same in every direction at exactly one ratio of the two radii. When every direction of a geometry has the same Ricci curvature, we say that the geometry satisfies the *Einstein condition*. Every direction then bends by the same total amount.

Each shape on its own satisfies the Einstein condition. Every direction of the plane bends by six over the square of the plane's radius (measured by a convention explained below), and every direction of the sphere bends by one over the square of the sphere's radius. The product satisfies the Einstein condition exactly when the plane's bending equals the sphere's. The rule of equal bendings is a fact of geometry, and the reason for the rule is that the bending of a product is made of the bendings of its two shapes taken together. Geometry does not fix the value of the shared bending, which is a different question.

The shared bending is positive everywhere: the six bend like a sphere in every direction.

One Step Sets the Size

The modal boundary, which Chapter 1 described, is the scale at which the smooth description of space and time and the stepwise description of content apply on equal terms. Each of the six internal directions is an inde-

pendent way of differing. So the modal boundary limits each direction on its own, and not only the six on average. One identification sets the amount of bending: I take the modal boundary to be one curvature, the Planck curvature, for every internal direction. I also take each internal direction to reach the Planck curvature. So every one of the six directions has a Ricci curvature of exactly the Planck curvature. The measured share of matter in the universe is the observation that bears on the identification of the Planck curvature and on the four conditions before that identification. In the section "What Reaches Our World", below, I show how the measured share of matter bears on the identification of the Planck curvature and on the four conditions before that identification.

The plane's radius and the sphere's radius follow by algebra. The same bending in every direction is the Einstein condition, here with one value, the Planck curvature, on both shapes. The plane's bending is six over the square of its radius, and that bending equals the Planck curvature, while the sphere's bending, one over the square of its radius, also equals the Planck curvature. So the sphere's radius is one Planck length. The square of the plane's radius is six times the square of the Planck length, so the plane's radius is the square root of six Planck lengths: about two and a half (2.45). The ratio of the two radii is the square root of six. The two radii follow from the identification as a theorem, by the algebra of Einstein products, which are products that satisfy the Einstein condition. The two radii are therefore set by the structure once the identification is taken.

The plane's radius is measured by a particular convention. The plane can be made from a larger round shape, a sphere of five dimensions that holds every state of a three-way choice, with the state's overall clock-hand setting included. The overall clock-hand setting is the turning of all three clock hands together by one amount. To make the plane, count as one any two states that differ only in the overall clock-hand setting. By the convention, the plane's radius is the radius of the larger five-dimensional sphere. The square root of six is the plane's radius measured by the convention. The size of the plane can also be stated with no convention at all. Many spheres lie inside the plane, and each of those spheres is made of the ways

of holding just two of the three alternatives: for example, the lamp found red or green, with blue left out. Each sphere inside the plane has one and a half times the area of the sphere of the two rows.

In this chapter, the Planck length is built from Newton's constant, the measured strength of gravity. Newton's constant is the one measured constant on the gravitational side of the account. The Planck length is about a hundred billion billion times smaller than a proton. The modal boundary is set by a comparison that something located in space makes. The comparison sets an object's gravitational size beside its quantum size. The gravitational size of an object is the radius to which the object would have to be squeezed to become a black hole, and an object's quantum size is the wavelength of the object's quantum wave. The comparison of the two sizes measures a strength, and that strength is Newton's constant. Nothing travels along the six, so no comparison made from a place in our space can measure their volume. So the modal boundary is the Planck length, built from Newton's constant, which a comparison does measure. The modal boundary needs no input from the volume of the six, and the modal boundary in turn fixes that volume. The mass that goes with the Planck length is the full Planck mass, about twenty-two millionths of a gram: the mass at which a thing's gravitational size equals its quantum size. Chapter 8 says why the account uses the full Planck mass, and not a smaller one that some physicists use.

One identification therefore fixes both the proportions of the six and their size. The proportions are the ratio of the radii, and the equality of the two bendings fixes them. The size is the overall scale, and the value of the shared bending fixes it. The identification fixes both in the same way at every point of the large world, because Newton's constant sets the modal boundary, and Newton's constant has one value throughout. *The Gravitational Passage* says that "the internal space has the same size and the same shape at every point of the exterior". The exterior, in that sentence, is the large world.

The Planck curvature of every internal direction is a condition on the six alone. The four large dimensions obey Einstein's equations of gravity. The four large dimensions bend only as much as the matter and energy in them require. The bending of the six enters Einstein's equations as one part of the energy of empty space. The total curvature of the six sets the size of the part that comes from the six. The total curvature is the sum of the Ricci curvatures over the six directions. The identification fixes the total curvature at six times the Planck curvature. The cosmological constant is the energy of empty space that drives the accelerating expansion of the universe, and it is the small share of the energy from the bending of the six that a measurement made from inside the large world finds. Chapter 8 counts the size of the share that a measurement from inside finds.

Where the Space Begins

Every closed shape has a gentlest way to ripple. On the surface of a ball, the gentlest ripple raises one half of the surface while the other half sinks. No ripple on the ball is gentler than the half-up, half-down ripple. On a smaller ball, even the gentlest ripple must bend more sharply. The first *eigenvalue* of a shape is the sharpness of the gentlest ripple that the shape allows. The first eigenvalue measures the slowest variation that the shape admits. The scale of a shape is the scale of its slowest variation.

For the six at their radii, the first eigenvalue is twice the Planck curvature. *The Gravitational Passage* states what the first eigenvalue means in one line: "the internal space begins where the boundary lies." In plain words, the six have the scale of the modal boundary, and the scale of the six is the scale of their gentlest ripple. The internal space is a few Planck lengths across. The first eigenvalue is computed from the radii, as is the size of a few Planck lengths, so both are set by the structure once the identification is taken.

The modal boundary separates the two parts of the whole ten-dimensional product: the four large dimensions and the six. The four large dimensions are the smooth and uneven part, ruled by the smooth description, general relativity. The six are the setting in which comparisons of content take place. So the six are the part of steps and symmetry, ruled by the stepwise description, quantum theory. In this chapter we have measured the radii and the bending of the six, and both belong to the context instance of the six. The size of the six is the scale at which the two descriptions apply on equal terms.

Seventy-Two Discs

An area is measured in squares of a length, and a volume in cubes of a length. A volume of six dimensions is measured in sixth powers of a length. The volume of the six is the plane's four-dimensional volume times the sphere's area. With the radii found above, the volume comes to seventy-two times pi cubed times the sixth power of the Planck length, which is about two thousand two hundred thirty (2,230) times the sixth power of the Planck length. The volume is arithmetic on the radii, so the volume is set by the structure once the identification is taken. In the unit of pi cubed times the sixth power of the Planck length, the volume is exactly seventy-two, which is twice six squared: twice the fourth power of the ratio of the radii.

The unit can be pictured. The six directions fall into three pairs: two pairs from the plane and one pair from the sphere, and in each pair of directions a quantity has both a size and a clock-hand setting. In each pair the natural unit to measure by is a flat disc, so take a disc one Planck length in radius in each of the three pairs, and hold the three discs together as a product. The product of the three discs is the Planck polydisc, and its volume is the unit. The three factors of pi are the three circles of clock-

hand setting, one circle in each pair, and the six powers of the Planck length are the six directions at the modal boundary. The six small dimensions hold seventy-two Planck polydiscs.

Pi cubed is also the volume of the round five-dimensional sphere of radius one. The five-dimensional sphere is the space of every state of a three-way choice with its overall clock-hand setting included, and the plane is made from the five-dimensional sphere. So the unit has one factor from each of the two instances of the six. Pi cubed measures the content's own states, on the space of states (the content instance), while the sixth power of the Planck length measures the extent of the context, on the space with radii (the context instance). The unit is the measure of the content's states, pi cubed, multiplied by the extent of the context, the sixth power of the Planck length.

I also read the unit by the three roles of a comparison. The reading by roles is an identification: a step from the structure of comparison to the measure. Pi cubed has three factors of pi. In the reading by roles, the three factors stand for the three directions of space in which a comparison is made, which are not held compact (held small), so pi cubed is a measure of what is known. The sixth power of the Planck length carries the six dimensions of possibility at the bound of comparison, the modal boundary, so the sixth power of the Planck length is a measure of what is possible. The unit is the product of the measure of what is known and the measure of what is possible. The unit is a measure of what happens. So one unit holds all three roles at once, which is what one comparison is, and the volume divided by the unit therefore counts comparisons: the internal space holds seventy-two comparisons. The reading by roles changes none of the values above: the reading says only what the seventy-two counts.

Action is energy multiplied by time. Planck's constant is the size of the least step of action in quantum physics. A further reading of the seventy-two uses Planck's constant. In quantum counting, one state takes up a cell of action, which is a patch of the size of Planck's constant. Physicists can choose units in which the speed of light and Newton's constant are both one, and in those units an area and an amount of action are measured in

the same way. A disc one Planck length in radius then has an area of half of Planck's constant. The polydisc is half a cell of action in each of the three pairs. The volume of the six is nine whole cells of action, each the cell of one state of three independent ways of varying, and each cut into eight parts by one halving in each pair. In the reading by cells of action, seventy-two is nine times eight: seventy-two counts the parts that the halvings make inside the nine states.

Take a ball of six dimensions, one Planck length in radius: the ball has one sixth the volume of the polydisc. The ball measures all six directions by one radius, as one. The polydisc measures each pair by its own disc and joins the three discs as a product. A measure that treats three as one breaks the rule of the third axiom: the three roles are distinct, inseparable, and none does another's work. The factor between the ball and the polydisc is six, the number of ways to put three things in order. So the volume of the six is counted in polydiscs, and the rule of the third axiom is the reason it must be. The rule has its figure in the Borromean rings: three rings, each at right angles to the other two, linked so that no two hold together without the third. When things are joined so that none of them can be described without the others, we call the things *entangled*. As *The Gravitational Passage* puts it, "Unless there are three, linked, there are none entangled." The polydisc is the figure of the Borromean rings made into a measure: three Planck discs, one in each pair, at right angles to one another and joined as a product. The ball would merge the three discs into one radius, and the factor of six measures what the merging throws away.

What Reaches Our World

The complex projective plane is built of three basic pieces, one for each role. The number three here is a count, set by the structure. The count of three enters the cosmological constant as a count of channels: the ways a measurement takes in what it finds, one for each role.

What we observe depends on the six through two numbers, and both enter through the cosmological constant: the count of three, and the total curvature, which is six times the Planck curvature. The six have two settings, which are like two dials. One dial is for the overall size of the six, and one is for the ratio of their radii. Only the masses of the two settings depend on the ratio of the radii and on the two radii taken separately. The force that comes from the two settings has a range some thirty powers of ten shorter than the shortest range that any test of gravity has probed. In Chapter 4 we ask what the two settings are, and how heavy. The cosmological constant in turn fixes the share of matter in the universe that astronomers measure in the sky, as Chapter 8 shows. So the measured share of matter in the universe is the one observation that depends on the internal space, and the measured share depends on the internal space through the count of three and the total curvature. The measured share of matter is therefore the observation that bears on the identifications of this chapter.

What Is Taken and What Follows

The table below lists the whole route, step by step, with the standing of each step.

Step	Standing
Ten dimensions: one of time, three of space, six of possibility	Argued from the structure of comparison; the count is set by the structure
Three conditions translate the grid of six measures into geometry	Identifications
Every point of each shape alike (homogeneity)	Identification
The complex projective plane times the sphere, the only shape that fits	Theorems of topology; given the conditions, this shape is the only one

Step	Standing
Every internal direction bends by the Planck curvature	Identification; the size of the bending, exactly one Planck curvature, is set by that identification
Sphere one Planck length; plane the square root of six Planck lengths	Theorem (algebra of Einstein products), given the identification: set by the structure once the identification is taken
Volume seventy-two polydiscs; gentlest ripple twice the Planck curvature	Computed from the radii: set by the structure once the identification is taken
The Planck length	Measured: built from Newton's constant
The count of three	Set by the structure: the count of the plane's basic pieces

We now know the shape and the size of the six, and which steps are identifications and which are theorems. The shape is the only shape that carries the grid of comparison as I read the grid in the four conditions, and the size is set by the Planck curvature in every direction.

The shape and the size of the six matter for what the beginning left behind. The shape and the size are the same at every point of the large world, and each new epoch (each new era of the world) must re-form the same shape and size when the epoch's six are held again. The shape and the size also carry the two numbers through which what we observe depends on the six: the count of three, and the total curvature.

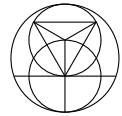
Key idea. By three conditions I translate the grid of six measures into geometry, and by a fourth I make every point of each shape alike. All four conditions are identifications. From the four conditions, the mathematicians' list of shapes leaves exactly one shape: the space of all the ways a three-way choice can stand, side by side with the sphere of a two-way choice. One more identification fixes the size: every internal direction bends by the Planck curvature. So the sphere is one Planck length in radius. The larger shape is the square root of six Planck lengths in radius, about two and a half.

Summary

The six small dimensions are the complex projective plane times the sphere. The complex projective plane is the space of all the ways a three-way choice can stand (the lamp that could be found red, green or blue). The sphere is the space of all the ways a two-way choice can stand (the globe with one alternative at each pole). By three conditions I translate the grid of six measures into geometry, and by a fourth I make every point of each shape alike. The four conditions are identifications. From the four conditions, the steps to the shape are theorems of topology, and once the conditions are taken, the plane times the sphere is the only shape that fits. The same shape also follows by a second route, from the algebra of comparison. The size is fixed by one identification, made in the first person: every internal direction bends by the Planck curvature. The measured share of matter in the universe is the observation that bears on the identifications of this chapter. The sphere's radius is one Planck length, by the algebra of Einstein products. The plane's radius is the square root of six Planck lengths, about two and a half. Without a convention, each sphere inside the plane has one and a half times the area of the sphere of the rows. The internal space begins where the modal boundary lies, a few

Planck lengths across, and its volume is seventy-two Planck polydiscs. The shape and size of the internal space are the same at every point of the large world.

Something must hold the six at their size, as a rim holds a drum skin taut. What holds the six at their size, and what happens to the energy of their shrinking when the six stop shrinking?



Chapter 4

Caught by the Floor

What stops the six from shrinking to nothing, and where does the energy of their motion go? As the new world opens after the turn, the six small dimensions of possibility shrink toward the shape and the size that Chapter 3 found. The six stop at a floor, a least size below which the six cannot go. The six cannot shrink to nothing for two reasons: the six are the context in which every possibility of the world is found, and quantum physics forbids a size held exactly at zero. The energy of the motion of the six goes into a breathing of their size, an even swelling and shrinking, and the breathing passes, almost at once, into ripples of space and time that spread through the large world.

A Drum Held by Its Rim

A drum skin is stretched across a frame and held taut by its rim. Press the middle of the skin, and the skin springs back. You can ask two questions about the drum skin, and the two questions have different answers. Where does the skin sit? The rim fixes where the skin sits: the skin lies across the frame, at the size of the frame. How stiffly does the skin spring back when you press it? The stretch of the skin sets how stiffly the skin springs back, and a drum maker tunes the stretch.

The six are held in the same two ways as the drum skin. Together the six form the internal space, which is everywhere in the world and far too small to see. Chapter 3 answered the first question: where the six sit. The

shape of the six is the complex projective plane times a sphere. The complex projective plane is the space of all the ways a three-way choice can stand, and the sphere is the space of all the ways a two-way choice can stand. The sphere has a radius of one Planck length, while the complex projective plane, the larger of the two factors, has a radius of the square root of six Planck lengths, about two and a half. The second question is the subject of this chapter: what keeps the six at their held size and shape, what pushes them back when something displaces them, and how stiffly they are pushed back.

The Gravitational Passage answers the second question in two steps and keeps the two steps apart. *The Gravitational Passage* is the book of the collection that sets the whole account out for physicists. The first step is an argument, and the hold rests on that argument: the hold, as Chapter 1 named it, is the state in which the six are held at one size and one shape. The second step is a computation of one mechanism for how stiffly the six are held. The hold does not rest on that computation. In the picture of the drum, the argument is the rim, because the argument fixes where the six sit. The computation describes one way the stretch of the skin comes about, because the computation gives how stiffly the six spring back.

Why the Six Cannot Shrink to Nothing

Hang a weight on a string and let the weight settle. In the everyday picture the weight comes to rest exactly at the bottom, perfectly still. Quantum physics says that the weight cannot come to rest in that way. The rule called the *uncertainty relation* says that nothing can have both a perfectly sharp place and a perfectly sharp motion: the more exactly the place of a thing is pinned down, the more widely its motion spreads. If a weight rested exactly at the bottom with exactly no motion, the weight would have both a sharp place and a sharp motion. So even in its lowest state the weight trembles a little about the bottom, and nothing can take the trembling away. The energy of the least trembling is called *zero-point*

energy. Zero-point energy is the energy a thing keeps in its lowest state, the energy of the lowest motion that cannot be removed. For a weight on a string, the zero-point energy is far too small to notice. Every field of physics has a zero-point energy too. A field, met in Chapter 1, is something spread through space with a value at every point, as the temperature of a room has a value at every point in the room. The zero-point energies of all the fields, added up, are the *ground-state energy* of the fields. The ground-state energy is the restless trembling of the fields at their lowest. *The View from Inside* pictures the restless trembling of the fields for the general reader.

Two reasons show that the six cannot shrink to nothing. The first reason comes from comparison itself, and the second reason is the rule behind the trembling of the weight, applied to the six. A comparison is the act of finding whether two things are the same or different, in some respect. Every comparison has a content, what is found in the comparison, and a context, the respect in which the content is found. Take a thermometer that compares this morning's warmth with this evening's. The temperature is the content of the thermometer's comparison, and the two times of day are its context. The six are the context of possibility: the respect in which the possibilities of content are found. The account rests on three axioms, met in Chapter 1: statements that hold of every comparison whatever. The third axiom says that the three roles every comparison has, and the pairs that fill those roles, are distinct, inseparable, and none can do another's work. Content and context are one of the pairs that fill those roles, so there is no content without a context. The world has content, so the context of possibility of the world cannot be absent. The internal space cannot shrink to nothing, because the internal space is the context of possibility.

The second reason is the uncertainty relation. The uncertainty relation forbids the size of the six to be held at zero with no spread, just as the uncertainty relation forbids the weight its perfect rest. So the internal space has a *floor*: a least extent, with a least motion that cannot be removed. The least motion of the floor is the zero-point motion of the six. The zero-point

motion is already the lowest state of the six, so the zero-point motion cannot radiate or dissipate: there is no lower state to which the six could pass the energy of the zero-point motion, and the zero-point motion does not wear down.

Held the Same Everywhere

The floor shows that the six cannot vanish, and the hold shows where the six sit while the large world is their context. When two conditions are met, the six are held compact: held small while the large world of four dimensions is their context. Chapter 1 called the two conditions the compactification conditions. The first condition is that the large world is present as a context for the six, and is smooth enough for the smooth description of space and time: the large world bends less sharply than the Planck curvature. The Planck curvature is the bending of the surface of a ball whose radius is one Planck length. The second condition is that a reference for inertia is fixed in the large world: a standard of not accelerating and not rotating, set by content within reach against which motion is compared. Every part of the world you know satisfies both conditions. The room you sit in has an outside, space and time are smooth at every scale you can reach, and the distant galaxies fix what "not rotating" means for a gyroscope.

While the large world is the context of the six, the six are held as content. Each of the six directions then stands at the modal boundary. The modal boundary is the scale at which the smooth description of space and time and the stepwise description of content act on equal terms, and at the modal boundary the curvature that each direction sees is the Planck curvature. Chapter 1 gave a name to the state in which the six are held as content, with each direction at the modal boundary: the hold. Newton's constant, the measured strength of gravity, fixes the modal boundary, and Newton's constant has one value throughout our world. So the internal

space has the same size and the same shape at every point of the large world: the larger factor has a radius of the square root of six Planck lengths, and the sphere has a radius of one Planck length.

Physicists call the numbers that describe the size and shape of an internal space its *moduli* (one of the numbers is a *modulus*): settings, like the dials of an instrument. The six have two settings. One setting is the volume of the six, their overall size. The other setting is the shape of the six, the ratio of their two radii. The volume of the six sets the strength of gravity measured in the four large dimensions, because the strength of gravity in the four large dimensions is fixed by the strength of gravity in ten dimensions together with the volume of the six. The hold fixes both settings at their values throughout the large world. So the strength of gravity we measure is held fixed along with the two settings.

The hold rests on an argument with two parts: the floor, which follows from the third axiom and the uncertainty relation; and each internal direction standing at the modal boundary while the large world is the context of the six.

How Stiffly: One Mechanism

A tent stands because two forces balance. The fabric of the tent pulls inward, and the poles of the tent push outward. The tent takes the shape at which the inward pull and the outward push are equal.

The six are in the same kind of balance as the tent. The curvature of the six pulls the six toward a smaller size, as the fabric pulls the tent inward. The six resist the pull with their least extent and their least motion, as the poles push the tent outward. *The Gravitational Passage* computes one mechanism for the balance of the six. The geometry of space and time is the one field the six cannot do without, and the geometry, the measure of every distance, has small waves of its own on the six. The small waves have a least motion, and so a zero-point energy. *The Gravitational Passage* computes that zero-point energy *at first order*. The computation covers the

largest and simplest part of the effect of the zero-point energy. A number computed at first order is set by the structure once a stated step is taken, and for the zero-point energy of the small waves the stated step is the computation of one mechanism in its largest and simplest part. *The Gravitational Passage* adds one more part to that zero-point energy: a *term* (one part of a sum) for the least extent. The least-extent term has one part for each of the six directions: each part is the same function of the curvature that its own direction sees, built from the fifth power of that curvature. The six parts are added together. In the mechanism of *The Gravitational Passage*, the geometry itself is the field whose zero-point energy holds the six.

The least-extent term has two constants. The two constants are set so that the balance falls at the derived point. The derived point is the radius and the ratio of the two radii that Chapter 3 derived. One constant is about nine parts in a hundred thousand (0.0000898). The other constant is a scale of about two thirds of a Planck mass (0.651). The Planck mass, met in Chapter 1, is the mass at which the gravitational size of a thing equals its quantum size: about twenty-two millionths of a gram, about the mass of a grain of sand a quarter of a millimeter across. The two constants are set by the structure once a stated step is taken: the step of placing the balance at the derived point. The hold rests on the argument of the floor and the modal boundary, from which the derived point follows. The computation shows how stiffly the six are held at the derived point.

Picture the energy of the six as the height of the ground along a path. Each point of the path stands for one possible size of the six. The bottom of a hollow is a *stable minimum*: step off the bottom in any direction and the ground rises, so anything nudged off the bottom of a hollow is pushed back. When a ridge would have to be crossed to reach other ground, the ridge is a *barrier*. At first order, with the two constants set by the balance, the derived point is a stable minimum. For any shape of the six whose ratio of the two radii lies between the square root of three and the square root of twelve, the derived point is the only stable minimum. The square root of three is about one and seven tenths (1.7). The square root of twelve

is about three and a half (3.5). On the shrinking side of the hollow, the ground rises to a ridge, the barrier. On the growing side, the ground rises gently to level ground, with no ridge. The barrier lies where the larger factor has a radius of about one and eight tenths Planck lengths (1.79), which is smaller than the held radius of the larger factor and lies past the modal boundary. At that radius, the curvature of the larger factor would be about one and nine tenths times the Planck curvature (1.87). So the ridge lies where the bending is sharper than the Planck curvature. Bending that sharp is beyond the range in which *The Gravitational Passage* applies the smooth description. *The Gravitational Passage* computes the stable minimum, the range of ratios and the place of the barrier at first order.

A setting that is pushed back when displaced has a stiffness. In quantum physics the stiffness of a setting shows itself as a mass. A gentle setting swings slowly and acts like a light particle, while a stiff setting swings fast and acts like a heavy particle. The heavier a setting is, the shorter the distance over which the setting can carry a force. At first order, the two settings of the six have masses of about two and a quarter Planck masses (2.25; the swing that is mostly a change of size) and about three and a tenth Planck masses (3.09; the swing that is mostly a change of shape).

Physicists can test the masses of the two settings by measuring gravity. If a setting had a small mass, the setting would carry a long-range force of its own beside gravity, and gravity would then follow a different law from Einstein's wherever gravity is gentle, as gravity is around the Sun. The space probe Cassini measured how the Sun bends radio waves passing near the Sun, and the Cassini measurement shows that no long-range force beside gravity exists at the strength a light setting would carry. Experimenters have tested the law that gravity weakens with the square of distance down to about a twentieth of a millimeter. With masses of two to three Planck masses, the force of the settings has a range shorter than a Planck length, about thirty powers of ten (a million million million million million times) shorter than the shortest distance at which gravity has

been tested. So gravity in the four large dimensions is general relativity, Einstein's theory. The account uses the size of the two masses from the computation: about the Planck mass.

The Energy of the Floor

The floor has an energy of its own. The unit for the energy of the floor is the Planck density, met in Chapter 2. One Planck density is one Planck mass in every cube one Planck length on a side. At the derived point, the energy of the floor itself is about plus seven hundredths of the Planck density (0.071), computed at first order. The energy of the floor is a zero-point energy, a quantum effect. As in Before We Begin, the vacuum is empty space in its lowest state. Nothing is in the vacuum but the least motion of its fields. I take the ground-state energy of the vacuum at the modal boundary to be one Planck density, exactly. I make an identification when I take that ground-state energy to be one Planck density. An identification is a step, made in my own voice, from the structure of comparison to something in the world. I count the energy of the floor within that ground-state energy, and that count is an identification too. The observation that bears on both identifications is the measured share of matter in the universe, which is the subject of Chapter 8. At the modal boundary, the energy of empty space is one whole Planck density, and the energy of the floor is counted inside that Planck density rather than added on top. So if the energy of the floor differs from its first-order value, the difference bears on the settings of the six, and not on the cosmological constant, the energy of empty space that drives the accelerating expansion of the universe.

A strip of spring steel rolled into a coil stores energy in its curl. The curvature of the six stores energy in the same way as the coiled strip of steel. The energy of the curvature is a classical energy, which carries no quantum effect. The energy of the curvature enters the description of the large world as an energy with the sign that makes a space draw itself to-

gether, and physicists call such a space anti-de Sitter. The *bare part* of the curvature energy is the energy of the curl alone, before the energy of the floor itself is added. At the derived radius, the bare part is minus three divided by eight pi (eight pi is about twenty-five) of the Planck density, about minus twelve hundredths (-0.119), and the value of the bare part is set by the structure. With the energy of the floor added, the internal space contributes about minus five hundredths of the Planck density in all (-0.049). The figure of minus five hundredths is computed at first order. So the floor holds the six in a well about one twentieth of the Planck density deep, a dip in the energy like the hollow in the picture of the ground. As the internal space grows, the energy of the internal space rises to zero, with no barrier on the way.

At the modal boundary, the total energy of the vacuum is the ground-state energy plus the bare part. The total is one Planck density, plus the bare part, which is negative. One, less three divided by eight pi, leaves about eighty-eight hundredths of the Planck density (0.8806). The total of about eighty-eight hundredths is set by the structure. The total is one value, fixed by the hold. So there is no range of values from which one value would have to be selected. The observed cosmological constant is the share of the total energy of the vacuum that a measurement made inside the world finds.

Result	What it is	Standing
About nine parts in a hundred thousand (0.0000898); about two thirds of a Planck mass (0.651)	The two constants of the least-extent term	Set by the structure once a stated step is taken: the balance placed at the derived point
A stable minimum	The derived point	Computed at first order
About one and eight tenths Planck lengths (1.79)	The barrier, on the shrinking side, past the modal boundary	Computed at first order

Result	What it is	Standing
About two and a quarter (2.25) and about three and a tenth (3.09) Planck masses	The masses of the size and shape settings	Computed at first order
About plus seven hundredths (+0.071)	The floor's own energy, in Planck densities	Computed at first order
About minus twelve hundredths (−0.119; minus three divided by eight pi)	The bare curvature energy, in Planck densities	Set by the structure
About minus five hundredths (−0.049)	The internal space in all: the depth of the well	Computed at first order
About eighty-eight hundredths (0.8806)	The total vacuum energy at the boundary, in Planck densities	Set by the structure

Caught on the Way In

Run the story forward from the turn. As the new world opens, its three directions of space grow, and the six shrink slowly along the arc. The arc, found in Chapter 2, is the short stretch of allowed rates of change near the turn on which all six small directions shrink. The size of each of the six is in proportion to the time counted from the turn raised to the power minus one ninth, so a billionfold change of the time shrinks each of the six only tenfold. The shrinking of the six ends where the floor holds the six at their held radii.

The Weight of Possibility is the book of the collection that is the source for the dark matter. *The Weight of Possibility* states the account that *The Gravitational Passage* gives of how the new epoch, the new era of the world, begins. In that account, the description of the new epoch begins where the

curvature falls below the Planck curvature, while the internal space, released at the turn, is re-forming: returning to its compact form. The floor then receives the re-forming internal space in its well, one twentieth of the Planck density deep. As the size of the six falls from large sizes downward, the energy of the six falls from zero to the bottom of the well. The energy falls with no ridge on the way. So the floor catches the six: the six come to rest at the bottom of the well, at their held radii, the square root of six Planck lengths for the larger factor and one Planck length for the sphere.

The account fixes the facts of the capture, the end of the re-forming: the well, the fall of the energy with no ridge, and the held radii at which the six come to rest. Two further facts are not fixed, and neither *The Compactification* nor this book fixes them: where on the arc the description of the new epoch begins, and the detailed motion by which the six return to their compact form in the new epoch. *The Gravitational Passage* applies the smooth description at curvatures below the Planck curvature, and not beyond. *The Gravitational Passage* states that the Incommensuration Theorem, a structural fact about the nature of knowing, gives the framework for picking out the turn and for limiting what the turn can be. *The Gravitational Passage* also states that the detailed motion of the release of the six cannot be computed from the Incommensuration Theorem. The release of the six is their ceasing to be held compact at the turn. *The Gravitational Passage* also leaves unfixed the temperature at which the reconstructed history of the new epoch begins. The account bounds the starting temperature of the new epoch from above. The bound appears below, in the first of the three conditions on the turn.

Where the Energy of the Settling Goes

A crystal holds its shape. Because the crystal is held, its vibrations have definite energies: each way the crystal can ring has its own note. One way a crystal can ring is to swell and shrink evenly all over, as a chest does in breathing.

The six are held too, so their vibrations also have definite energies. Above the lowest motion of the six, the size of the six can swell and shrink evenly. We call the even swellings and shrinkings of the size of the six *breathing excitations*. As the six settle into their held size, the energy of their motion goes into the breathing excitations.

The breathing excitations decay almost at once. The ten-dimensional geometry moves by a law built from the geometry alone. The main part of the law is Einstein's, the only part of the law that is in simple proportion to the curvature. David Lovelock proved a theorem about the laws that can be built from a geometry alone, and the theorem allows further parts in ten dimensions, built from higher powers of the curvature, whose sizes *The Gravitational Passage* does not fix. The first further part is the *term of Gauss and Bonnet*, built from the curvature squared. The term is named for the mathematicians Carl Friedrich Gauss and Pierre Ossian Bonnet. At the order that *The Gravitational Passage* computes, the breathing of the six has no link to the waves of the large world. With the term of Gauss and Bonnet in the law, the breathing of the volume of the six is linked to the waves of the geometry of the large world. If the term of Gauss and Bonnet has a natural size, with nothing making the term unusually small, the lowest breathing excitation decays within about a Planck time. A *Planck time* is the time light takes to cross a Planck length, about 5×10^{-44} seconds (a decimal point, forty-three zeros, and then a five).

The decay time of the breathing is an estimate made by step-by-step approximation. Step-by-step approximation is the method in which an effect is computed as a series of ever smaller corrections. The estimate

comes out at about ten to the minus forty-six seconds (a decimal point, forty-five zeros, and then a one). The estimate is about a five-hundredth of the Planck time. Because the estimate comes out shorter than a Planck time, what the estimate shows is that nothing holds the decay back. The breathing excitations made at the turn have long since decayed.

The energy of the breathing excitations leaves as gravitational waves of the four large dimensions. Gravitational waves, met in Before We Begin, are ripples in the shape of space and time themselves. As gravitational waves pass, the waves stretch distances one way and squeeze distances the other way. *The Weight of Possibility* computes, from the depth of the well, that the energy leaving as gravitational waves is between a tenth of the depth of the well and most of the depth of the well. The oldest light of the sky carries a record of how much radiation filled the early universe, and gravitational waves made during the settling of the six count in that record as extra radiation. The lowest motion of the six stays: the lowest motion is already the lowest state, so the lowest motion cannot radiate or dissipate.

What the Account Asks of the Turn

The Weight of Possibility states three requirements that the account places on the turn: the *conditions on the turn*. The conditions on the turn are requirements set by the amounts we measure, and not results that the account derives about the turn. Two of the three conditions concern the dark matter, the unseen matter whose pull is measured in every galaxy. *The Weight of Possibility* identifies the dark matter with *gyrations*: a gyration is a conserved excitation of the six held internal dimensions, a lasting vibration of their geometry, of about the Planck mass.

The first condition concerns the *plasma* of the new epoch: the gas of particles so hot that nothing in the gas holds together. A hot plasma can make, in pairs and through gravity alone, anything that gravitates. If the plasma of the new epoch formed at once, the measured amount of dark

matter requires that the plasma was never hotter than about one thirtieth of the mass of a gyration. In the first condition, temperature is counted in the same units as mass, as physicists count the temperature of very hot plasmas. *The Weight of Possibility* computes the figure of one thirtieth from the published method for making, in a plasma, dark matter that acts on the plasma through gravity alone. The bound of one thirtieth is the ceiling on the starting temperature that Before We Begin promised: a number bounded, with its value not given, and the measured amount of dark matter sets the bound. Suppose instead that the energy of the turn reached the plasma gradually, through a stage that behaved as matter. In a stage that behaves as matter, energy is held in heavy, slow things that later decay into the plasma. The first condition then applies to the end of the stage: the temperature at which the stage hands its energy to the plasma must be below about one hundredth of the mass of a gyration. The plasma is then allowed to have been as hot as about one thirtieth of the mass of a gyration at its hottest moment, because the radiation the stage releases afterward thins out the gyrations made at that hottest moment. Under the first condition, an epoch cannot have begun with a plasma near the Planck temperature. The Planck temperature is the temperature at which a typical particle carries the energy of a Planck mass. A plasma near the Planck temperature would make the gyrations about as numerous as the particles of light.

The second condition concerns the settling of the six. The settling releases energy as gravitational waves, and that energy must stay below about three percent of the energy that the content of the new epoch, its hot matter and light, carried as radiation. The three percent is counted at the time when the heat of the content was shared among about a hundred *species*. A species is one distinct state that a kind of particle can be in, apart from where the particle is and how the particle moves, and in the second condition the species are the states of the particles that share the heat. A measured count sets the three-percent bound. The count is the amount of radiation in the early universe other than light itself, counted in units of the radiation that one kind of *neutrino* carries, and measurements of the

oldest light give the count. The neutrinos are the lightest particles of matter, which barely touch anything else, and there are three kinds of neutrino. Physicists call the measured count the number of *light species*, where "light" means of small mass. *The Weight of Possibility* computes that the energy released by the settling is about forty to three hundred times the largest energy that a plasma cool enough for the first condition can hold. So a plasma formed at once could not outweigh the gravitational waves released by the settling. The waves can then stay below three percent of the radiation in one of two ways. Either the waves are small, or other radiation outweighs the waves. In the first way, the six arrive at their held size displaced by no more than about one part in a thousand of their radius. In the second way, a stage that behaves as matter carries at least as much energy as the waves, and decays into the content and not into gravitational waves, so that the radiation the stage releases outweighs the waves. The figure of one part in a thousand is a bound computed from the three-percent bound. The account contains no value for the displacement itself. A stage that behaves as matter belongs to the content. The breathing of the six cannot be that stage, since the breathing decays at once, and into gravitational waves. The account says that the oldest light will show radiation within the measured bound: the waves stay below the three-percent ceiling, and the account predicts no particular amount of extra radiation below that ceiling.

The third condition concerns the gyrations made directly as the six reform. The re-forming must make about a billion billion billion (ten to the twenty-seven, a one followed by twenty-seven zeros) times fewer gyrations than an abrupt re-forming would make. The measured amount of dark matter sets the factor of a billion billion billion. The factor is a bound, with its value not given by the account. The re-forming must be slow and gentle compared with one swing of the lightest gyration, because only a slow and gentle re-forming makes so few gyrations. The floor must also catch the six slowly and gently at the end of the re-forming. Give a child's swing one sudden shove, and the seat swings on. Lean on the seat slowly, over many of its swings, and the seat hardly swings at all. The sudden

shove is like an abrupt re-forming, the slow lean is like a gentle re-forming, and the swinging of the seat is like the gyrations that the re-forming makes. A change made slowly, compared with one swing, makes few gyrations. When the yield of gyrations falls off exponentially with the smoothness of the change, that is, by the same factor for each further step of smoothness, each swing of the gyration over which the change is spread cuts the yield about five hundredfold (computed: about five hundred thirty-five each swing). Ten factors of five hundred, multiplied together, make about a billion billion billion. So a cut of a billion billion billion needs a change smooth over about ten swings. On the arc each of the six contracts as the time raised to the power minus one ninth. The contraction of the six along the arc is slow compared with one swing of the lightest gyration once the time exceeds about the time light takes to cross the radius of the six. The third condition on the turn is a re-forming smooth over about ten swings of the lightest gyration.

What This Means

Go back to the drum. The rim fixes where the skin sits, and the stretch of the skin sets how stiffly the skin springs back. For the six, the rim is the argument: the six cannot vanish, and they stand at the modal boundary while the large world is their context, with the same size and shape everywhere. The stretch is the mechanism that *The Gravitational Passage* computes. In the mechanism of *The Gravitational Passage*, computed at first order, the settings of the six weigh two to three Planck masses, so the force carried by the settings has a range shorter than even a Planck length, and gravity in the large world is Einstein's.

We now know what holds the six, how stiffly, and where the energy of the settling goes. The floor catches the six as the new world opens, and the energy of the motion of the six rings out as breathing. Within about a Planck time, the breathing passes into gravitational waves of the four large dimensions. The gravitational waves of the settling are one of the

things the beginning left behind, and the waves travel on through the epoch. The least motion of the six stays, and so do the gyrations the turn made. Given the amounts we measure, the three conditions on the turn state what the account requires of the turn: a plasma never too hot, settling waves within the measured bound, and a re-forming smooth over about ten swings of the lightest gyration.

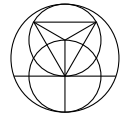
> **Key idea.** The six cannot shrink to nothing, for two reasons. The six are the context of possibility, and there is no content without a context. And the uncertainty relation forbids a size held at zero with no spread. So the six have a floor, a least extent with a least motion that cannot be removed. While the large world is the context of the six, each of their directions stands at the modal boundary. The size and shape of the six are the same everywhere. One mechanism, computed at first order, shows how stiffly the six are held. In the mechanism that *The Gravitational Passage* computes at first order, the settings of the six weigh two to three Planck masses. The six sit in a well one twentieth of the Planck density deep. The hold rests on the argument of the floor and the modal boundary, not on the computation of the mechanism. The energy of the settling leaves as gravitational waves within about a Planck time, and the lowest motion of the six stays.

Summary

The hold of the six rests on an argument. The six cannot shrink to nothing, because the six are the context of possibility and there is no content without a context. The uncertainty relation forbids a size held at zero with no spread. So the six have a floor, a least extent with a least motion that cannot be removed. While the large world is the context of the six, each of the six directions stands at the modal boundary, so the six have the same size and shape at every point of the large world. *The Gravitational Passage* computes one mechanism for how stiffly the six are held, and the hold does not rest on that computation. At first order the derived point is a stable minimum, with its barrier on the shrinking side at about one and

eight tenths Planck lengths, past the modal boundary. The two settings weigh about two and a quarter and about three and a tenth Planck masses, so gravity in the large world is Einstein's. The internal space sits in a well about one twentieth of the Planck density deep. The total vacuum energy at the modal boundary is one value, about eighty-eight hundredths of the Planck density. As the six contract slowly along the arc, the floor catches the six at their held radii. The energy of the settling goes into breathing excitations, which decay within about a Planck time into gravitational waves of the four large dimensions. The account does not fix where on the arc the description of the new epoch begins, or the detailed motion of the return of the six. The account places three conditions on the turn, which are requirements and not results. The first condition is a plasma never hotter than about a thirtieth of the mass of a gyration, which is the ceiling on the starting temperature, or a stage like matter that hands on its energy below a hundredth. The second condition is settling waves below about three percent of the radiation. The third condition is a reforming smooth over about ten swings of the lightest gyration.

The floor caught the six at one size and one shape. Could the six have been caught at some other size or shape, so that a different epoch would have different physics?



Chapter 5

One Vacuum

I call the six small dimensions of possibility the six. Chapter 4 found that the six are caught by their floor and held at one size and one shape. The floor is the least size of the six, with a least motion that cannot be removed. Could the six have settled at some other size or shape, and given a different world? If the six could be caught at another size or another shape, the next epoch might have different physics and a different energy of empty space, and which world an epoch got would then be an accident of the turn that began the epoch. I read the six in two ways, as a space with a size and as a space of states with no size, and in this chapter we ask the question of each way. Both ways give one answer. The six have exactly one held size and shape, and the six take that same size and shape again at every turn.

A Ball Among Valleys

Picture a hilly country with many valleys. Set a ball at the bottom of one valley. Nudge the ball, and the ball rolls a little way up the slope, rolls back and settles where the ball started, because after a small push the slope brings the ball back. The bottom of the valley holds the ball. Now drop a ball onto the country from the sky. The dropped ball can come to rest in any of the valleys. Which valley the ball rests in is a fact about where the ball happened to fall, and no fact about the shape of the country.

In the picture of the hilly country, the height of the ground stands for energy, and the ball stands for a physical system. Physicists use the word vacuum for the state at the bottom of a valley: the state a system settles into when the system has the least energy that the system can have nearby. Before We Begin, the opening section of this book, described a vacuum as one lowest, settled state of empty space. For the six, a vacuum is a held size and shape. A held size and shape is an arrangement of the geometry of the six that the laws of motion hold. At a held arrangement the energy is level, as at the bottom of a valley. After a small push, the six roll back to the held arrangement. If a theory allows many valleys, the world of the theory might settle in any one of the valleys. The valley that holds the world is then a fact of history and not a fact of law.

The question of many valleys is sharpest in string theory. String theory is the theory in which the smallest pieces of matter are tiny vibrating strings. String theory is also described in *At the Edge of Reality*, the earlier book of this collection. String theory is consistent only if the arena (space and time together) has ten dimensions. In string theory, six of the ten dimensions are curled small, into shapes of a family called Calabi-Yau spaces. Chapter 1 explained that a field has one value at every point of space. Temperature, for example, has one value at every point of a room. A Calabi-Yau space has many loops, as a ring has a hole, and certain fields can be threaded through the loops. Each loop then carries a whole number, which counts how many times a field threads the loop, and each allowed choice of the whole numbers on all the loops gives a different valley. Physicists estimate that the number of valleys is as high as ten to the five hundred (a one followed by five hundred zeros), or higher. Each valley has its own energy of empty space and its own physics at the energies our experiments can produce. Physicists call the whole country of valleys the *landscape of vacua* ('vacua' is the plural of 'vacuum'). The landscape of vacua is the whole range of held arrangements that a theory allows, each arrangement a valley of its own. On the landscape of vacua, our world is one valley picked out of the many.

The View from Inside is a book of this collection for the general reader on why empty space weighs so little. Physicists turn to the landscape of vacua for a reason that *The View from Inside* describes. When physicists add up the energy of empty space from the fields one by one, the sum comes out enormous. The measured total is tiny. The enormous sum and the tiny total are like a hundred numbers, each about a hundred and twenty-three digits long, some positive and some negative, that add up to a number with a single digit. The opening section, Before We Begin, described the cosmological constant as the energy of empty space that drives the accelerating expansion of the universe. Chapter 8 is about the cosmological constant. On the landscape of vacua, the cosmological constant has a different value in each valley. If very many of the valleys actually exist somewhere, we find a small value of the cosmological constant for the same reason we find ourselves on a temperate planet: we could not live where the value is large. The value of the cosmological constant is then an accident of location, like the distance of a planet from its star. No law sets the value.

The question of this chapter is therefore plain. Does this account have a landscape of vacua?

Two Readings, Two Meanings of the Word

The question of a landscape of vacua has to be asked twice, because in this book I read the six in two ways, and the word 'vacuum' means a different thing on each reading. Chapter 1 called the two readings the two instances: the content instance and the context instance. In this book, content is what is found in a comparison, and context is the respect in which the content is found. The face of a clock is one thing taken in two ways. The face is one pattern with no size, and on a particular clock the face is also a face with a width. The pattern with no size is like the content instance, and the face with a width is like the context instance. The context instance is the subject of this book. On the context instance, the six have

radii and are the small held factor of the ten dimensions, which are a product of two factors set side by side: the four large dimensions and the six small ones. On the context instance, a vacuum would be a held size and shape of the six. On the content instance, the six are the space of all the states that the choices of matter can take. The space of states has no size. On the content instance, a vacuum is the value that the fields of matter take in their state of least energy. Among the elementary fields, one field has a value other than zero in its state of least energy: the *Higgs field*, which fills all of space and, by settling at a value other than zero, gives many of the elementary particles their masses. The Higgs field belongs to the *Standard Model*, the established account in physics of the particles and of every force but gravity. Physicists call the settled value of the Higgs field, which is the same everywhere, the Higgs *condensate*.

Chapter 4 called the settings of the six their moduli. In this chapter a *modulus* means one precise thing: a number that describes the size or the shape of the six and could differ from place to place in the large world. If a modulus differed from place to place, we would measure the modulus, in our four dimensions, as a field: a quantity that has its own value at every point. A modulus needs a size or a shape to vary.

On the content instance the six have neither a size nor a shape. *The Kinds of Particle* is the book of this collection that reads the kinds of particle on the six as a space of states. *The Kinds of Particle* says plainly that the six have no size on the content instance: the internal space (the one small space that the six form together) "is the product of the carriers' spaces of states: it has no size and no fields live on it." *The Kinds of Particle* places every quantity that needs a size on the context instance, and the moduli of the size are among those quantities. *The Kinds of Particle* also states that the content instance has no vibrations of the six, no scale of size, no modulus and no condition set at a scale of size. So on the content instance nothing could hold more than one value, because the content instance has no size and no modulus. On the content instance, the one ques-

tion of a landscape concerns the Higgs field, which is a field on space and time and not a modulus of the six. We come to the Higgs field later in this chapter, in the section on the vacuum of the content.

The word "modulus" has one more use, which must be kept apart from the use in this chapter. *Non-Commutative Geometry and the Immanent Metaphysics* is another book of this collection. *Non-Commutative Geometry and the Immanent Metaphysics* describes the content by an *algebra*: in the mathematician's sense, a stock of quantities together with rules for adding and multiplying the quantities. The algebra of the content has three pieces. The first piece is the ordinary complex numbers, which are numbers that carry both a size and a clock-hand setting. The second piece is the quaternions, a number system whose numbers have four parts. The states of the quaternion piece form the sphere of a two-way choice. The third piece is the three-by-three tables of complex numbers. The states of the tables form the complex projective plane, the space of a three-way choice. In the description of *Non-Commutative Geometry and the Immanent Metaphysics*, a table of numbers called the finite Dirac operator links the pieces of the algebra. The finite Dirac operator carries the masses of the matter particles and the ways in which the matter particles mix with one another. *Non-Commutative Geometry and the Immanent Metaphysics* calls the entries of the table "the moduli of the finite geometry". The entries of the table are numbers that the algebra leaves free. The entries are not fields, and the entries do not vary over space and time in the sense of this chapter. The entries are not sizes of anything. In this chapter, "modulus" means only a setting of the size or the shape of the context instance.

The context instance has a size. *The Gravitational Passage* is the book of this collection that derives Einstein's equations of gravity from the structure of comparison. *The Gravitational Passage* names two moduli of the context instance: the volume modulus and the shape modulus. The volume modulus sets the overall size of the six, and through the overall size the volume modulus also sets the Planck mass as we measure the Planck mass in four dimensions. The six are made of two shapes side by side: the complex projective plane and the sphere. Each shape has its own radius.

The shape modulus sets the ratio of the two radii. The next section shows that the volume modulus and the shape modulus are the only moduli of the context instance.

Only Two Settings

A perfect ball has no special spot: every point of the surface of the ball is like every other point. An egg has two ends, a blunt one and a pointed one. The two shapes that make up the six are called the two factors of the six. Chapter 3 took each of the two factors of the six to be like the ball. Each factor is homogeneous: every point of the factor is like every other point, under the symmetries of the factor. A symmetry of a shape is a motion of the shape that keeps every distance unchanged. On a homogeneous shape, a symmetry can carry any point to any other point. Homogeneity is an identification: a step from the structure of comparison to something in the world. The ground of the identification is stated with the identification, and the ground is that the setting in which content is compared cannot itself single out a point. The reason is that an unevenness of the setting would be a difference of context carried by no content.

How many numbers are needed to give the shape of a homogeneous factor? A shape is given by a measuring rule: a rule that says how long each small step in each direction is, at every point. Physicists call the measuring rule the *metric*. On a homogeneous factor every point is like every other point. So the measuring rule at one point fixes the measuring rule everywhere. Pick one point of a factor. The symmetries fixing the point can turn any direction at the point into any other direction at the point. On the sphere, the turning is done by turning the sphere about the point, and on the complex projective plane, the turning is done by symmetries that mix all four directions of the plane at the point. A measuring rule that the symmetries leave unchanged must therefore measure every direction at the point alike, and so the measuring rule has one shape and one free number: its overall size.

A theorem of geometry states the count of one shape and one size exactly. The theorem says that a shape which is the same at every point and in every direction at a point allows only one shape of measuring rule, apart from the overall size of the rule. The theorem is mathematics: if the premises of the theorem hold, the conclusion of the theorem must hold. For the ball, the theorem says something you already know. A perfect sphere is fixed by one number, its radius. Once you know how big a sphere is, you know every distance on the sphere. You cannot stretch a sphere toward an egg without giving the sphere ends. The complex projective plane is fixed the same way, by one radius.

The six are the two factors taken together, side by side. Could a measuring rule on the whole of the six tie a direction of the plane to a direction of the sphere? The plane has four directions, and the symmetries of the plane turn the four directions among themselves. The sphere has two directions, and the symmetries of the sphere turn the two directions among themselves. The two factors have different numbers of directions, four and two. The theorem of geometry also shows that a measuring rule left unchanged by the symmetries can have no part that ties a direction of one factor to a direction of the other. So every symmetric measuring rule on the six is the one shape of the plane at some size, beside the one shape of the sphere at some size. Every symmetric measuring rule on the six has two numbers and no others: the radius of the plane and the radius of the sphere. The two radii changed together give the volume modulus, which makes the overall size of the six grow or shrink, and the two radii changed against each other give the shape modulus, which moves the ratio of the two radii.

Two things follow from the count of two numbers. First, among the homogeneous shapes there is no third modulus. In particular, the plane alone cannot be reshaped while the plane stays homogeneous. Second, if a change in the shape of the six differs from point to point, the change is not one of the two settings at all. A change that differs from point to point is a ripple of the geometry across the six: one of the levels of the small waves of the geometry on the six, and the levels have separate, fixed masses. The

Planck curvature is the bending of the surface of a ball whose radius is one Planck length. Chapter 3 found that the gentlest ripple the six can carry is set by twice the Planck curvature. The levels of the ripples therefore have masses of about the Planck mass, the mass that belongs to the Planck scale, because the gentlest ripple is on the Planck scale. Chapter 4 said that the Planck mass is about twenty-two millionths of a gram, the mass of a grain of sand a quarter of a millimeter across, and a mass that is enormous for a single particle. Some levels of the ripples decay quickly and some last (Chapters 4 and 18 describe the levels). No level of the ripples is a light field of our four-dimensional world.

The count of two moduli has two different grounds. First, that each factor is homogeneous is an identification. The identification is made in the first person in *The Gravitational Passage*, and the ground of the identification is stated in *The Gravitational Passage* too. Second, a theorem shows that a homogeneous product of the plane and the sphere has exactly two numbers in its measuring rule. So if the identification is taken, the count is exactly two. Homogeneity is the step on which the count of two depends. If no symmetry is asked of the shapes of a six-dimensional space, the shapes form an endless family, with endlessly many numbers to set, while the homogeneous shapes of the product of the plane and the sphere are set by two numbers.

One Point Holds Both

Both moduli of the six are fixed, and a single step fixes the two moduli together. Each factor of the six already curves by the same amount in every one of its own directions, and geometry shows that the six as a whole curve by one amount in every direction exactly when the two factors curve by the same amount. Chapter 3 called equal curving in every direction the Einstein condition. Geometry does not say which amount the two factors share. The shared amount is set by an identification that Chapter 3 recorded: every direction of the six reaches the Planck curvature, the

curvature of the modal boundary. The modal boundary is the Planck scale, the scale at which the range of the smooth description of space and time ends.

Chapter 3 found that the equal curving of the two factors fixes the shape modulus: the radius of the plane is the square root of six times the radius of the sphere, about two and a half times. The shared value of the curvature fixes the volume modulus. The sphere has a radius of one Planck length, the length at which gravity and the quantum act on equal terms, about a hundred billion billion times smaller than a proton. The plane has a radius of the square root of six Planck lengths, about two and a half Planck lengths. At the held point, the volume of the six is seventy-two Planck polydiscs, the unit of volume of Chapter 3. Once the identification is taken, the structure sets the ratio of the radii, the two radii and the volume. The identification itself sets the size of the shared curvature at exactly one Planck curvature. *The Gravitational Passage* fixes the size in no other way.

One identification fixes both moduli, and the identification fixes the moduli the same at every point of the large world. The strength of gravity sets the modal boundary. The strength of gravity is the one measured constant on the side of the account that concerns gravity. The strength of gravity has one value throughout the epoch. *The Gravitational Passage* says that "the internal space has the same size and the same shape at every point of the exterior", where the exterior is the large world and the internal space is the six.

Chapter 4 showed what keeps the six at the held point. The hold is the state in which the six are held with each of their directions at the modal boundary. The hold rests on an argument from the axioms of the account. The axioms are the three starting points from which the account reasons. First, the third axiom of the account says that there is no content without a context, and the six are the context of possibility, so the six cannot shrink to nothing. Second, the uncertainty relation excludes a size held at zero with no spread. So the six have a floor: a least size, with a least motion that cannot be removed. *The Gravitational Passage* computes one mechan-

ism of the hold to first order (the largest and simplest part of the effect). The computation shows that the held point is a stable minimum, the bottom of a hollow to which the six are pushed back after any small step. The moduli at the held point weigh about two and a quarter Planck masses (2.25, mostly size) and about three Planck masses (3.09, mostly shape). Because the moduli are so heavy, the force that the moduli carry has a range of less than a Planck length. The range is some thirty powers of ten shorter than the shortest range any test of gravity has probed, which is about a twentieth of a millimeter. The hold rests on the argument of the floor. The computation shows one way in which the hold is stiff, that is, how strongly the hold resists a push.

Chapter 4 found that the held six also have one energy of empty space at the modal boundary: the Planck density, less the bare energy of the curvature of the six. The Planck density is one Planck mass in every cube one Planck length on a side: the density at which the smooth description of space and time reaches the Planck scale. The bare energy of the curvature is three divided by eight pi of the Planck density. Eight pi is about twenty-five, so the bare energy is about twelve hundredths of the Planck density. At the modal boundary, the energy of empty space is therefore one, less three divided by eight pi, of the Planck density: about eighty-eight hundredths (0.8806) of the Planck density, set by the structure. Chapter 8 shows how the cosmological constant is the share of the energy at the boundary that a measurement made inside the vacuum takes in. The hold fixes one value of the energy of empty space at the boundary. There is no range of values from which one value would need to be picked.

No Threads to Count

In string theory, the landscape of vacua comes from fields that are threaded through the loops of the six small dimensions of string theory. Physicists call each of the threaded fields a *flux*. Physicists call the whole

number that counts how many times a flux threads one loop a *flux integer*. Picture threads wound through the holes of a ring with many holes. Each hole is wound some whole number of times, and each way of winding all the holes is a different valley. In the picture of the ring, the ring stands for the six of string theory. A hole stands for a loop, and a thread stands for a flux. The spaces that string theory uses commonly have some hundreds of independent loops. Multiplying together the choices for every loop gives the enormous count of valleys.

In this account, the law of gravity on the ten dimensions is built from the geometry alone, and *The Gravitational Passage* states that, on the ten-dimensional product, "the gravitational action is built from the metric alone". The action, met in Chapter 3 as energy multiplied by time, is also the one quantity from which the laws of motion follow. The metric is the measuring rule of the geometry, as the section Only Two Settings explained.

The account has no further field. *The Gravitational Passage* says so in so many words. The complex projective plane needs a phase at each of its points, a setting like the angle of a clock hand, so that some waves on the plane can be defined. Mathematicians call the phase at every point a spin-c structure. The phase can come in either of two ways: from the geometry itself, through the way the plane is built from complex numbers, or from a separate field for the phase. In *The Gravitational Passage* the phase comes from the geometry, and The Gravitational Passage states three reasons against a separate field for the phase. First, if the phase came from a separate field, the law of gravity would no longer be built from the geometry alone. Second, the energy of the separate field would add to the energy of empty space at the boundary, beside the bare energy of the curvature, and so the added energy would change the predicted share of matter in the universe, a number that Chapter 8 computes. Third, the strength of the separate field would be a second constant beside the strength of gravity. *The Gravitational Passage* concludes, speaking of itself: "This book takes no such field."

Since the account has no further field, there is nothing to thread a loop of the six. There is no whole number to count the threading. The whole numbers that the six do carry are counts of the shape of the six. One whole number is the count of the basic pieces of the six, three times two, which is six: the Euler count of Chapter 3. The Euler count is the number of the basic pieces of a shape, counted with signs. The other whole number is the count three for the plane. The count three enters the energy of empty space as the count of channels, and in no other way. The channels are the ways in which a measurement takes in the energy of empty space, one for each of the three roles that every comparison has: what is possible, what is happening, and what is known. Each count has one value, fixed by the shape of the one space, and there is nothing to choose in either count. So the account has no flux and no flux integers, and nothing in the account corresponds to the landscape of vacua.

Nowhere to Tunnel

Go back to the ball among the valleys of the hilly country. Suppose the valley that holds the ball is not the lowest valley. In quantum physics, the ball can then pass through the ridge into a lower valley, without ever climbing over the ridge. Physicists call the passing through a ridge *tunneling*. For a whole world held in one vacuum, tunneling begins in one small region. In the small region, a *bubble* of the lower vacuum forms and grows. The physicists Sidney Coleman and Frank De Luccia showed how to compute the rate at which a bubble of a lower vacuum forms and grows. When a vacuum can decay by forming a bubble of a lower vacuum, physicists call the vacuum *metastable*. A metastable vacuum is held for a long time and is able to pass by tunneling to a lower valley, but tunneling can happen only if there is a second valley, lower than the first, for the bubble to lead to.

Chapter 4 found that the computation in *The Gravitational Passage* shows one valley for the six. At first order and among the homogeneous shapes, the held point is the only minimum in shape for ratios of the radii between the square root of three and the square root of twelve, as Chapter 4 said. The square root of three is about one and seven tenths (1.7), and the square root of twelve is about three and a half (3.5). The computation also shows that no minimum in size can lie at a plane radius smaller than about one and a half Planck lengths (1.48). The plane radius at the held point is the square root of six Planck lengths (about 2.45), so the held point lies above the bound. The computation shows no second valley toward larger sizes either. The held six have an energy of about minus five hundredths of the Planck density (-0.049), and as the six grow, the energy rises to zero with no ridge in the way. So the energy at the held point lies below the energy at every larger size, and nothing in between holds the six anywhere else. The computation shows one ridge, toward smaller size, at a plane radius of about one and eight tenths Planck lengths (1.79), and the ridge lies past the modal boundary: at the ridge the plane curves by about one and nine tenths times the Planck curvature (1.87). The ridge is a high point between sizes, and not a valley. There is one minimum and no second vacuum for a bubble to lead to, at the order computed and within the shapes that the computation covers (ratios from the square root of three to the square root of twelve).

That there is no second minimum is a computed result, at first order, and the result applies to the shapes that the computation covers. The hold itself rests on the argument of the floor, given earlier in this chapter: there is no content without a context, and the uncertainty relation excludes a size held at zero with no spread.

The hold does end, and the hold ends in one way: where the conditions of the hold fail. The conditions of the hold are the two compactification conditions that Chapter 1 named. The first condition is that the large world is present as a context for the six, as a domain of the smooth description of space and time. The second condition is that a reference for inertia is fixed in the large world, as the distant stars fix what "not rotating"

means for a spinning gyroscope. A reference for inertia is the standard of not accelerating and not rotating, fixed by matter within reach. Both conditions fail together at the end of the interior of a black hole, because the outside world has emptied, the smooth description of space and time reaches the Planck curvature, and no reference for inertia survives. The turn is the end of the interior of a black hole. The turn is the passage in which a large outside world is restored and a new epoch begins.

The turn is a failure of the context in which the six are held. The turn is not a bubble of a lower vacuum, because on the far side of the turn there is no second held arrangement for the six to fall into. The sources of this account state that in the new epoch "the product re-forms with the same internal topology". The product is the six, and the same topology means the same shape: the complex projective plane times the sphere, with the same radii in units of the Planck length, because the same identification fixes the radii. Could gravity be stronger in one epoch than in the next? Answering the question would need a comparison made across the turn, and no comparison is made across the turn: no value of the strength of gravity in one epoch is set beside the value in another epoch. A *comparer* is anyone or anything that makes a comparison, such as an observer with an instrument. In any epoch, what a comparer measures is a ratio, such as the ratio of the mass of a particle to the Planck mass, and every measured ratio is found within one epoch.

If there were a landscape, a question about history would arise, and since there is one held point, the question has a plain answer. If there were many valleys, each epoch might settle into a different valley at its turn, and the physics of the epochs would differ. Since there is one valley, the six that re-form at each turn are caught at the same point, in units of the Planck length. The earlier book of this collection is *At the Edge of Reality*. The earlier book states the consequence in a passage that refers back to its own Chapter 7, the chapter that states the conditions fixing the six: "The shape of the internal space, the complex projective plane times the sphere, is fixed by the conditions of Chapter 7. The ratio of its two radii is fixed, and its size is fixed at the Planck scale." Since there is one held shape and

size, every epoch has the same six. Chapter 1 said that gravitation, the cosmological constant and the dark matter rest on the six. So everything that rests on the six rests on the same six in every epoch.

The Vacuum of the Content

The content has a vacuum of its own, and the vacuum of the content is a different thing from the hold of the six. Among the elementary fields of the content, the Higgs field alone has a value other than zero in its state of least energy. The Higgs condensate is the value of the Higgs field in its state of least energy.

The Higgs field is not a change in the six, on either instance. *Non-Commutative Geometry and the Immanent Metaphysics* describes the Higgs field as a small variation of the finite Dirac operator, the table of numbers that links the pieces of the algebra of the content. The algebra of the content has three pieces, and the variation lies across the link between two of the pieces: the quaternion piece, whose states form the sphere of a two-way choice, and the piece made of the ordinary complex numbers. The weak force, met in Chapter 2, is the force at work in some sorts of radioactive decay, and the weak force can turn one kind of particle into another. The Higgs field comes as a pair of fields that the weak force turns into one another, and because the Higgs field comes as a pair, physicists call the Higgs field a doublet. The Higgs field leaves the algebra unchanged, and so the Higgs field leaves the six unchanged. Once the condensate has formed, *The Kinds of Particle* says of the six on the content instance that "the condensate is a value on it, not a change of it". So the condensate is not a position among any moduli, and not a valley of the six.

The energy of the Higgs field depends on the value of the field and has two parts: one part grows as the square of the value, and the other part grows as the fourth power of the value. The two-part form of the energy is derived in *Non-Commutative Geometry and the Immanent Metaphysics*. The strength of the fourth-power part is the *self-coupling*: how strongly the

Higgs field acts on itself, which sets how steeply the walls of the energy valley of the Higgs field rise. Each member of the Higgs pair carries a *weak charge* of one half: a weak charge is the label that the weak force acts on. The weak charge of one half is set out in *The Measures of the Kinds*. *The Measures of the Kinds* is the book of this collection on the measured values of the kinds of particle. Physicists call the simplest level of a calculation, before the smaller corrections, tree level. At tree level, I identify the self-coupling with half the square of the weak charge of one half. The square of one half is one quarter, and half of one quarter is one eighth. So the self-coupling at tree level is one eighth, set by the structure once the identification is taken.

The Higgs boson is the particle that is a small ripple of the Higgs field, and the mass of the Higgs boson follows from the self-coupling and from two further pieces. The first piece is the measured strength of the settled field, about two hundred forty-six billion electron volts (246.22). A billion electron volts, written GeV, is the unit in which particle physicists measure mass and energy, and one electron volt is the energy an electron gains in crossing one volt. The second piece is a small correction from the next level of the calculation: one divided by the product of twenty-four and pi squared, about four thousandths, which brings the self-coupling to about thirteen hundredths (0.1292). The mass of the Higgs boson is the strength of the settled field times the square root of twice the self-coupling. The computed mass is one hundred twenty-five and seventeen hundredths GeV (125.17). The measured mass is one hundred twenty-five and thirteen hundredths GeV (125.13), plus or minus eleven hundredths (0.11). *The Measures of the Kinds* states the standing of the computed mass: "one measured agreement". In this book, a measured agreement is a value that follows from the account and that measurement then finds. In this book, the mass of the Higgs boson is set by the structure once the identification is taken, with one measured input: the strength of the settled field. The computed mass agrees with the measured mass.

The value one eighth holds at the energies of the weak force, the scale at which the Higgs boson is measured, and the value one eighth is a value at that scale, and not a value carried down from far higher energies. The claim of this chapter concerns the context instance: the geometry of the six has one held size and shape.

One Algebra, One Space

The six have no landscape of vacua because the geometry that is held is already one geometry. The six are one space, and not one choice among many, on two grounds. The two grounds come from two routes of argument that start from different premises. Picture two climbers who start from opposite valleys and climb by different paths, and who meet on one summit: the two climbers have found one summit, and neither climber chose the summit from a list. The two climbers stand for the two routes of argument, and the summit stands for the one space of the six.

The first route of argument goes through the algebra of the content. *Non-Commutative Geometry and the Immanent Metaphysics* sets out postulates (starting assumptions), each of which is an identification, and a theorem of *Non-Commutative Geometry and the Immanent Metaphysics* shows that a structure of comparison satisfying the postulates has one algebra. The one algebra is made of the three pieces described earlier in this chapter: the complex numbers, the quaternions, and the three-by-three tables of complex numbers. The states of the three-by-three tables form the complex projective plane, the space of a three-way choice, and the states of the quaternion piece form the sphere, the space of a two-way choice. Together the plane and the sphere make the six. The theorem holds if its postulates hold. The numbers that the theorem leaves free are the entries of the finite Dirac operator. The entries are moduli only in the second sense of "moduli" in this chapter: free numbers in a table, and not valleys.

The second route of argument uses no algebra. *The Gravitational Passage* shows that the same space follows from three identifications together with homogeneity. The three identifications are drawn from the grid of six measures that Chapter 3 set out: pattern, force and probability for the content, and space, time and possibility for the context. Mathematicians have listed every compact homogeneous shape of small dimension (a compact shape is closed and finite in extent), and each shape on the list is checked against the conditions. The result is that the six are "the only compact homogeneous product that satisfies the conditions." The conditions are the three identifications. The steps from the conditions to the space are theorems of topology, the mathematics of shape. The conditions are identifications that restate the axioms through the grid of six measures (Chapter 3 walked through the conditions), and if the conditions are taken, the space is the only one.

The two routes arrive at one space, the complex projective plane times the sphere. On neither route is the space picked from a list of candidates. So there is no landscape of spaces for the six, and no problem of choosing one space.

The context instance has one space, two moduli and one held point, so the context instance has one vacuum. The cosmological constant is then fixed, and there is no spread of vacua from which to pick its value. Chapter 8 shows how three things fix the cosmological constant. The first thing that fixes the cosmological constant is the energy of empty space at the boundary, eighty-eight hundredths of the Planck density. The second thing that fixes the cosmological constant is the count of channels, which is three. The third thing that fixes the cosmological constant is the particle horizon of the present measurement. The particle horizon is the edge of everything whose light has reached us since the expansion began.

What the Turn Gives Back

We now know that the six have one vacuum, held at one point, and the same at every turn.

One vacuum is part of the answer to the question of this book, what the beginning left behind. Whatever else a turn makes, every turn gives back the same six: the same shape, held at the same point in units of the Planck length. Every part of the world you know satisfies the conditions that hold the six. The room you sit in has an outside, space and time are smooth at every scale you can observe, and the distant galaxies fix what "not rotating" means for a gyroscope and for your inner ear. The six are held, so the forms that the matter of your body can take stay fixed throughout our epoch. The account fixes the six that come back at every turn. What could still differ from one epoch to the next? A measured ratio of matter concerns the make-up of the matter itself, and not only the six. One example is the ratio of the mass of a particle to the Planck mass. Do the measured ratios of matter repeat from one epoch to the next? The question is about the make-up of the content and lies outside the scope of *The Gravitational Passage*. I leave the question for another day.

> **Key idea.** A vacuum of the six is a held size and shape, like the bottom of a valley to which a ball rolls back after a small push. String theory threads fields through its six, and the threaded fields give the six of string theory some ten to the five hundred valleys, while the six of this account have one valley. Homogeneity and a theorem of geometry leave two moduli, and one identification fixes both moduli. No flux threads the six of this account. The hold ends only at the turn, and the same six re-form at the same point at every turn.

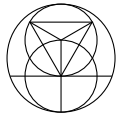
Summary

The six have one vacuum: one held size and shape, the same at every point of the large world and the same at every turn. On the context instance, homogeneity and a theorem of geometry leave two moduli, the volume and the shape, and homogeneity is an identification. One identification fixes both moduli: every direction of the six reaches the Planck curvature, so the sphere has a radius of one Planck length and the plane has a radius of the square root of six Planck lengths. The floor holds the six at the held point, and the hold rests on the argument of the floor. At first order, a computation of one mechanism of the hold shows that the moduli weigh about two to three Planck masses. In the account, gravity is built from the geometry alone, so the six carry no flux and no flux integer. Because the six carry no flux, the account has no landscape of vacua. The whole numbers that the six do carry, six and three, are counts of the shape of the six. The computation shows one minimum and no second minimum to tunnel to, at the order computed and within the shapes from the square root of three to the square root of twelve. The hold ends only where the conditions of the hold fail, at the turn, and after the turn the same six re-form at the same point in units of the Planck length. There is one algebra and one space, found by two routes from different premises, so nothing is left to select. The energy of empty space at the boundary therefore has one value, and the cosmological constant is fixed with no spread of vacua to pick its value from. On the content instance the six have no size and no moduli. The vacuum of the content is the Higgs condensate, a value of the Higgs field and not a change in the six. The self-coupling of the Higgs field is one eighth at tree level, by an identification. The mass of the Higgs boson is about one hundred twenty-five GeV (125.17), one measured agreement with the measured value (125.13). The claim of this chapter concerns the context instance: the geometry of the six has one held size and shape.

So every epoch begins with the same six, caught in the same way. Each new epoch also begins in a state of high order, and from then on everything in the epoch runs down toward disorder. If the six are caught the same way every time, why does each new epoch begin so orderly, when everything since runs down?

PART II

WHY THE BEGINNING WAS ORDERLY



Chapter 6

Two Arrows of Time

Why does time run one way, and does the one-way running of time depend on how the universe began? Chapter 5 ended on a sharper form of the same question. At every turn (the passage from one epoch of the world to the next), the six small dimensions of possibility are caught the same way. Yet each new epoch begins in a state of high order, and everything in the epoch runs down from then on. "Runs down" already names a direction of time, the direction from order toward disorder. So before we ask why each epoch begins orderly, we need to know whether the direction of time itself depends on the orderly beginning.

Physicists call the one-way running of time the *arrow of time*, and they speak of the arrow of time as though there were only one. There are two arrows of time, and the two arrows are two different things that point the same way. The first arrow comes from the structure that every comparison has, and the first arrow needs no special beginning of the world. Before We Begin defined a comparison as the act of finding whether two things are the same or different, in some respect. The second arrow is the growth of disorder in things made of very many parts, and only the second arrow needs an orderly beginning.

A Film Run Backward

Watch a cup slide off the edge of a table. The cup strikes the floor and breaks. The pieces scatter, the tea spreads into a puddle, and a sharp sound fills the kitchen. Now take a film of the fall, and run the film backward. The pieces slide together across the floor, leap up, and join into a whole cup. The tea gathers itself back into the cup, and the cup rises onto the table. You know at once that the film is running backward, because no one has ever seen a broken cup put itself back together.

The puzzle is that no part of the backward film, looked at closely enough, breaks a law of physics. A piece of the cup flying upward follows the same laws as a piece falling down, with the direction of the motion turned around. Far beyond the cup, too, a motion run in reverse follows the same laws as the motion run forward. For nearly every lawful process (a process that obeys the laws of physics), the reversed process is also allowed as lawful by the equations of mechanics, of electromagnetism and of general relativity. Run a film of a planet circling its star backward, and the film shows the planet circling the other way, a perfectly good orbit.

Yet the world runs one way. Heat flows from the hot body to the cold body. A *record* is anything that holds a finding after the act that found the finding, and a record holds the past and not the future. A footprint in wet mud is a record of the step that made the footprint, and nothing in the mud tells of a step still to come. And a broken cup does not reassemble. So the laws of physics run both ways, while the world runs only one way. Where does the one-way running of the world come from, and does the one-way running come from the way the universe began?

Counting the Ways

The physicist Ludwig Boltzmann gave the standard answer in 1877, and Boltzmann's answer rests on counting. Take a new deck of cards, straight from its box and in order: each suit in turn, each suit running from ace to king. Shuffle the deck a few times. The deck becomes mixed, and further shuffling keeps the deck mixed. You could shuffle for the rest of your life, and the chance that the deck would ever come back to the order the deck came in is too small to matter.

The reason the deck stays mixed is a count. Only a handful of the deck's arrangements look "in order" to you. A vastly larger number of arrangements all look the same to you: all of those arrangements look "shuffled". Each shuffle carries the deck from one arrangement to another, and almost every arrangement that a shuffle can produce is one of the shuffled arrangements. So the deck drifts into the large group of shuffled arrangements and stays in that group.

Physicists call a count of arrangements that all look alike an entropy, the word that Before We Begin named. A context is the respect in which a person or an instrument making a comparison finds something. In the words this book uses, the entropy of a thing is a count of the thing's states that a context cannot tell apart: many ways for the parts of the thing to be, all looking alike from outside. For the deck, the context is a glance at the deck, and the states are the arrangements of the cards. High entropy is disorder in the everyday sense, and low entropy is order. A shuffled deck has high entropy, and a deck in its factory order has low entropy.

Boltzmann's count applies to anything made of very many parts: the air in a kitchen, the tea in a cup, the warmth in a floor. A mixed-looking state of the whole thing can be made in far more ways than a tidy-looking state. So a thing left to itself passes, with overwhelming probability, from states of lower entropy to states of higher entropy. The passage from lower to higher entropy is why heat spreads from the hot body into the cold

body, and why the pieces of the broken cup scatter across the floor and stay scattered. Boltzmann's counting is established physics (a result of physics, taken from its source), and Boltzmann's counting is the standard *statistical* explanation of the one-way running of things: an explanation of what happens almost always among a great many possible arrangements, and not of what happens in each single case.

The Gap in the Counting

Boltzmann's counting has a gap, and the gap concerns the direction of time. Suppose someone hands you a deck that is partly in order: the hearts still run from ace to king, and the rest of the cards are mixed. Use the counting to predict what the deck will look like after a few more shuffles. The counting says that the deck will be more mixed, because almost every arrangement that the shuffles can produce is mixed. Now use the counting to work out what the deck looked like a few shuffles ago. The counting says that the deck was more mixed a few shuffles ago too, because almost every arrangement that the shuffles can produce is mixed.

Someone who has only the counting cannot tell the future from the past, because the laws that the counting relies on run both ways. From any given moment, the counting predicts higher entropy in the future, and also higher entropy in the past. You know better about the deck only because you know one more fact: decks leave the factory in order. Factory order is a condition placed at one end of the deck's history, the end where the history starts.

The universe needs a condition of the same sort as factory order. If the counting is to yield one direction of time, a condition must be placed at one end of the history of the universe: the condition that the universe began in a state of extraordinarily low entropy. In 2000 the philosopher of physics David Albert called the condition of a low-entropy beginning the *Past Hypothesis*. Albert argued that the Past Hypothesis should be admitted as a law of nature in its own right, because nothing else in physics

supplies a low-entropy beginning. Established physics includes both the gap in Boltzmann's counting and the Past Hypothesis that fills the gap, and physicists give the gap and the Past Hypothesis together as the standard statement of where the statistical explanation gets its one direction.

The question of this chapter can now be put sharply. Does the direction of time itself rest on how the universe began, or does only the growth of entropy rest on how the universe began? The answer has two parts, one for each arrow. The first arrow of time is the *structural arrow*, the arrow of time that comes from the structure of comparison itself. We arrive at the structural arrow by two routes, that is, by two arguments from two different starting points: one route goes through the Incommensuration Theorem, and the other route goes through the order in which every comparison proceeds.

Lawful and Faithful

Go back to the backward film of the cup, and ask two different questions about the backward film. The first question is whether the backward film is lawful: does each part of the backward film obey the laws of physics? For nearly every process the answer is yes: the falling cup and the circling planet showed that the laws of physics allow the reversed process too. The second question is much more demanding: could the world run back through exactly the states that the world passed through on the way forward, every one of those states, with nothing left out? When a reversed process obeys the laws, we call the reversed process a *lawful reversal*. When a reversed process retraces exactly the states that the original process passed through, we call the reversed process a *faithful reversal*.

The laws themselves guarantee lawful reversal, and the reason is a symmetry of the laws. In the sense that the Incommensuration Theorem uses, a symmetry is a sameness of content across a difference of context: something that stays the same through a change. The content of a comparison is what the person or instrument making the comparison finds, and

the context is the respect in which the content is found. The laws of physics stay the same when time is counted backward instead of forward. So the laws of physics have a symmetry: the laws are the same across one difference of context, counting forward against counting backward. And because the laws have that symmetry, the reverse of every lawful process is lawful too.

The symmetry of the laws is not enough for faithful reversal. A faithful reversal needs something more: the state of the world at the moment of turning back, given without any gap. Think what a faithful reversal of the broken cup would need. Every piece of the cup would have to be sent back along its exact path. Every drop of tea would have to be sent back too, and so would every molecule of air that the falling cup pushed aside, every trace of warmth that the crash left in the floor, and the sound spreading out through the walls. Someone attempting a faithful reversal would have to give each piece, each drop, each molecule, each trace of warmth and the sound in the walls exactly, at every place and to every fineness. Leave a gap anywhere, however small, and the backward run is no longer the forward run retraced. In the sense of the Incommensuration Theorem, continuity is a sameness without a break, with nothing sudden, and a state given without gap has continuity.

A perfect time-reversal is a faithful reversal, so a perfect time-reversal needs two things perfected together: the symmetry of the laws of physics, and the continuity of a state given without any gap. The Incommensuration Theorem forbids exactly that pair. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. Applied to findings, the first half of the Incommensuration Theorem means that no finding of any comparison can be both perfectly symmetric and perfectly continuous. So a perfect time-reversal is ruled out. The limit on time-reversal is a limit of principle, and therefore a limit of all practice: no better

instrument removes the limit, because every instrument makes comparisons, and the Incommensuration Theorem concerns anything found in any comparison.

The Basis of Physics is a book of essays in this collection on the ground of physics, and the essay "The Structural Arrow of Time" in that book states, in one sentence, that perfect time-reversal is ruled out: "Perfect time-reversal is therefore precluded, and the preclusion depends on no entropy, no boundary condition, no Past Hypothesis, and no particular formalism". "Precluded" means ruled out in advance. A boundary condition is a condition placed at one end of a history, as the Past Hypothesis is placed at the start of the history of our universe. A formalism is a particular mathematical way of writing the physics. Perfect time-reversal is ruled out as a consequence of the Incommensuration Theorem. We reached that consequence by valid steps from the Incommensuration Theorem and from what symmetry and continuity mean, so perfect time-reversal must be ruled out.

The argument from the Incommensuration Theorem is the first of the two routes to the structural arrow. The world cannot run back exactly the way the world came, and we reached that result without counting anything or asking how the universe began.

Position, Click, Print

We begin the second route to the structural arrow, the route through the order in which every comparison proceeds, from a picture. Watch a photographer at work. First she takes up a position: she stands somewhere, holds the camera at some height, chooses a lens. From her position many pictures are possible, and she has not yet taken any of those pictures. Then she presses the shutter, and in that instant one picture is taken, and every other picture she might have taken is not. Afterward there is the

print: a fixed thing, laid out on paper, that anyone can pick up and look at, the same for everyone who sees the print. And the print shapes what she does next: she looks at the print, and she moves to a better spot.

Position, click, print: every comparison has these three parts, in this order, and each part plays a role of its own. In the account that this book describes, the three roles have three old names, each used in a new and exact sense. A *stance* is the position from which a comparison is made: the comparing, together with what is compared. The *transcendent* is the role of what is possible: the stance from which an act proceeds, and the range of what could be found, like the photographer's position with all the pictures she might take from that position. The *immanent* is the role of what is happening: the act of comparison itself, as the act occurs, like the click of the shutter. The *omniscient* is the role of what is laid out as structure, which can be held in view as a whole, like a map, a record or the photographer's print. The three words name roles in an act of comparison and nothing more, and none of the three words means a mind or a god.

The second of the account's three axioms states the order of the three roles. Chapter 1 said that an axiom is a statement that holds of every comparison whatever. In *An Immanent Metaphysics* I set out the framework of ideas beneath the whole series of books that this book belongs to. In *An Immanent Metaphysics*, Axiom II, the second of the three axioms of comparison, reads:

"Axiom II. A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent."

Two words in the second axiom need a gloss. A *class* is a type of thing, taken as a whole and standing ready. In general, an *instance* is one particular case of a type, here and now. The second axiom says three things in the photographer's terms. First, the range of positions open to her comes before any one click: the range of positions is a class of the transcendent, and one click is an instance of the immanent. Second, clicking as a type of act

comes before any one print: clicking as a type is a class of the immanent, and one print is an instance of the omniscient. Third, prints as a type come before any one new position she takes: prints as a type of laid-out finding are a class of the omniscient, and one new position is an instance of the transcendent. Put in plain words, the account says that every comparison has first a stance and a range of what could be found, then the act in which one finding is made, then the record of the finding, and that the record shapes the next stance.

The three roles therefore run in a cycle: the transcendent, the immanent, the omniscient, and the transcendent again. We call the one-way cycle of the three roles the cycle of *Axiom II*. The order of the cycle is an order of precedence: an order that says what must come before what, as a question must come before its answer, and not an order of clock time. The cycle can be entered at any point, but the cycle's direction cannot be reversed: no print comes before the click that made the print.

The Direction in Which Comparison Proceeds

Now we take the step from the cycle of Axiom II to time. The book *Incommensuration* establishes the Incommensuration Theorem and derives the account's three axioms from the Incommensuration Theorem. *Incommensuration* states the step to time in two sentences: "The non-entropic arrow of time is the structural precedence of Axiom II applied to the physical domain. The direction of time is the direction in which comparison proceeds". "Non-entropic" means made of no count of states: the non-entropic arrow is the order of Axiom II, and no entropy enters that order. "The physical domain" means the physical world.

I make the step from the order of comparison to the direction of time in the physical world as an identification, that is, as a step from the structure of comparison to something in the world. I identify the one-way order of findings, the order of Axiom II, with the direction of time. One observation bears on the identification: every record ever found follows the

act it records. No one has ever found a print that came before its click, a footprint that came before its step, or a count on a detector that came before the particle that made the count.

Put the two routes to the structural arrow side by side. The first route goes through the Incommensuration Theorem, and the first route shows that no faithful reversal can happen: the conclusion must hold. The second route goes through the second of the account's three axioms, by which the order of every comparison has one direction, from stance to act to record, and never record before act. I have identified that one direction with the direction of time. Together the two routes make the structural arrow, which we can now state in full. The *structural arrow* of time is the impossibility of a faithful time-reversal, which the Incommensuration Theorem requires, together with the direction of the cycle of Axiom II, which I identify with the direction of time.

The structural arrow is present in a single comparison, and the structural arrow needs no crowd of parts to show itself, because one click of one shutter already has a before and an after. *Physics Is Not Math* is the book of this collection on the order among metaphysics, physics, computation and mathematics, and *Physics Is Not Math* says of the structural arrow: "This order needs no large number of parts and no special beginning of the world: it is present in one comparison, made once". So the structural arrow does not depend on how the universe began.

The Arrow of Many Parts

The second arrow of time is the arrow that Boltzmann counted. The *entropic arrow* of time is the growth of entropy in a thing left to itself. The entropic arrow is statistical, in the sense met earlier in this chapter: the entropic arrow describes what happens almost always, among a great many possible arrangements, and not what happens in each single case. So the entropic arrow is found only in things of very many parts. One card cannot be shuffled into disorder, but a deck of fifty-two cards can.

The entropic arrow also needs a beginning. The partly ordered deck showed the reason: the counting alone predicts more mixing toward the past as well as toward the future. The growth of entropy runs one way only from a starting point that was less mixed than what came after the starting point. *Physics Is Not Math* states both features of the entropic arrow in one sentence: "That arrow is a matter of very many parts, and it needs a beginning in which things were less mixed than they are now". Two features of the entropic arrow are established physics: the entropic arrow is statistical, and the entropic arrow needs a beginning of low entropy. The two features come from Boltzmann's counting, together with the gap that Albert's Past Hypothesis fills.

Set the two arrows side by side, and the difference between the two arrows is plain. The structural arrow is in one click of one shutter. The entropic arrow is in the mixing of a crowd of parts, and the entropic arrow shows itself only where there is a crowd to mix and a less mixed start to mix from.

Where the Two Arrows Meet

The two arrows are two different things, and the two arrows point the same way wherever both arrows are found. The record is the reason the two arrows point the same way.

Go back to the photographer, and suppose she uses film. When the shutter opens, light strikes the film and darkens a tiny grain wherever the light strikes. The darkened grain is the record. Or take a particle detector: a particle passes through the detector, and a counter steps up by one. The registered count is the record.

The darkened grain and the registered count are each a change that cannot be undone. The darkened grain does not lighten again of its own accord, and the counter does not step back. Making a record raises the entropy of the instrument that holds the record: the film, or the counter, ends a little more mixed than before. Making a record carries the instru-

ment from a state that few arrangements of the instrument's parts make to a state that many arrangements make, in the way that shuffling carries the deck away from its factory order. Each act of recording does two things at once. First, an act of recording has the order of every comparison: stance, then act, then record. Second, an act of recording is a growth of entropy, in the record. *Measurement* is a foundation volume of this collection on what a measurement is, and *Measurement* states the point in one sentence: "So each act of recording joins the two arrows in one event: the order of the act, and a growth of entropy in the record". *Measurement* argues from what a record is that each act of recording joins the two arrows. Every record is one event in which both arrows are present, and in every record the two arrows point the same way, and so the two arrows agree wherever both arrows are found.

What the Past Hypothesis Is For

Once the two arrows are told apart, we can say exactly where the Past Hypothesis belongs. The Past Hypothesis is a condition on the entropic arrow. The Past Hypothesis is not the source of the structural arrow.

The Basis of Physics states in full what the Past Hypothesis is for. *The Basis of Physics* says that the Past Hypothesis "is not the source of the arrow but a statement about initial conditions, one that fixes how the structural arrow is expressed in the particular history of this particular universe. The argument does not say why that initial condition was what it was. It says only that the direction of time was not waiting on it". The argument that the quotation names is the argument for the structural arrow. Initial conditions are the conditions at the start of a history, as the factory order is the initial condition of the deck.

In plain words, time has its direction in every comparison. The low entropy at the start of our history fixes how the direction of time shows itself across the whole history of our universe: in the long, one-way running

down of things of many parts. Stars burn their fuel, cups of tea cool, and broken cups stay broken, all in the direction that every single comparison already has.

The structural arrow fixes one thing, that time has a direction, and the structural arrow fixes nothing else. The structural arrow does not fix the size of any asymmetry, that is, the size of any difference between how a process runs forward in time and how the process runs backward. The structural arrow does not fix the low entropy of the early universe, and the structural arrow does not fix the *second law* itself, the law that the entropy of a thing left to itself grows. So the structural arrow contains no answer to the question of why the entropy was low at the beginning. The answer is in what the account says about the beginning of an epoch, that is, in what the account says about the turn.

What This Means for the Beginning

The question of this book is what the beginning left behind. The direction of time comes from comparison itself. Every comparison, in any epoch, runs from stance to act to record, because the second axiom holds of every comparison whatever. So the direction of time is present wherever anything is compared, and the direction of time depends on no beginning.

The beginning did leave the low entropy from which the entropic arrow runs. The entropic arrow is at work, running from the low-entropy beginning, in every cup that breaks and stays broken, every star that burns its fuel, and every cup of tea that cools. We now know that the direction of time does not depend on the beginning of the world, and that only the growth of entropy does. The next task is plain: the low entropy of the beginning needs its own reason, and the direction of time does not supply that reason.

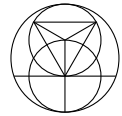
Key idea. Time has two arrows, and the two arrows point the same way. The structural arrow is present in every single comparison. The Incommensuration Theorem rules out a faithful reversal, and by the second axiom every comparison runs from stance to act to record, an order that I identify with the direction of time. The entropic arrow is the growth of entropy in things of very many parts, and the entropic arrow is statistical and needs a beginning of low entropy. Each record joins the two arrows in one event. The Past Hypothesis is a condition on the entropic arrow, and the direction of time does not depend on the Past Hypothesis.

Summary

We have separated the two arrows of time. The laws of physics stay the same when time is counted backward instead of forward, and because the laws have that symmetry, lawful reversal holds: for nearly every lawful process, the reversed process is lawful too. A faithful reversal retraces exactly the states that the original process passed through, and a faithful reversal needs, in addition, a state given without gap. So a faithful reversal would need symmetry and continuity perfected together, and the Incommensuration Theorem, a structural fact about the nature of knowing, forbids that pair. A perfect time-reversal is therefore ruled out, as a consequence of the Incommensuration Theorem, and the ruling-out must hold. By the second axiom every comparison runs one way, from stance to act to record. I identified that one-way order of findings with the direction of time, with the observation that every record ever found follows the act it records. The structural arrow is the two results together: the impossibility of a faithful reversal and the one-way order of Axiom II, present in one comparison, made once, and needing no special beginning of the world. The entropic arrow is Boltzmann's growth of entropy: statistical, found in things of very many parts, and needing a beginning of low entropy, the condition that Albert called the Past Hypothesis. Every record joins the

two arrows in one event, an act with its order and a growth of entropy in the instrument. The Past Hypothesis is a condition on the entropic arrow, and not the source of the structural arrow. The structural arrow fixes that time has a direction and fixes nothing more, so the low entropy of the beginning is a question for what the account says about the turn.

Then what about the beginning itself: why was the entropy of the beginning so low?



Chapter 7

A Clean Slate, Counted

Why was the entropy of the beginning so low, when an enormous entropy had fallen into the black holes of the epoch before? Chapter 6 found that the direction of time, the one way that time runs, does not depend on the beginning of the world. The growth of entropy does depend on the beginning. In Chapter 6 we met entropy as a count of the states of a system that a context cannot tell apart, where a context is the respect in which a comparison finds what it finds. In a system of very many parts the count of states grows. The count grows from a beginning in which the parts of the system were less mixed than they are now. Yet the epoch before ours ends in the interiors of its black holes, and the black-hole interiors carry an enormous entropy of gravity. So the question can be put sharply. How can an epoch begin with a small count of states, when the epoch before it ended with an enormous count?

At the Edge of Reality is the earlier book whose story this book takes up. It answered the question with the picture of a census, and called its answer a clean slate. We recall the census answer first, and then we take the answer further with what *The Compactification*, the main source of this book, adds.

Penrose's Two Findings and One Proposal

Open a bottle of perfume in a closed room. The scent spreads until it fills the room evenly, and the scent never gathers itself back into the bottle. For a gas like the scent of the perfume, spreading out evenly is the most mixed state, the state of highest entropy. Under gravity the order is the other way round: for matter drawn together by its own gravity, clumping raises the entropy. Leave a thin cloud of gas spread through space to its own gravity, and the gas clumps into stars, the stars gather into galaxies, and the densest clumps become black holes. A black hole has the largest entropy that a given mass can have.

The physicist Roger Penrose used the two behaviors, spreading for a gas and clumping under gravity, to put the question of the beginning more sharply. First, Penrose found where the low entropy was. The oldest light in the sky is the faint afterglow of the hot beginning, and it still fills the sky. The oldest light shows that the matter of the early universe was close to thermal *equilibrium*, the settled state in which nothing further changes on average, so the entropy of the matter was already high. A settled gas has a high entropy, as the perfume in the closed room does once the scent has spread evenly. The low entropy was the entropy of gravity: the geometry of space and time was smooth, with almost no clumping. Second, Penrose estimated how improbable a beginning with so smooth a geometry is. Penrose took the largest entropy that the matter of the observable universe could have to be the entropy of one black hole holding all the mass of the observable universe, a count of about ten to the power one hundred twenty-three. From that largest count, Penrose estimated that our actual beginning takes up about one part in ten to the power ten to the power one hundred twenty-three of all the states that the same matter could have taken. The figure is an estimate that Penrose made, and we weigh the estimate in Chapter 10, where we follow the count of states through a whole epoch.

Third, Penrose proposed a condition on the geometry of space and time itself. Picture a ring of small droplets floating in space. Curvature can do two different things to the ring. Curvature can shrink or swell the ring while keeping the ring round, and so change the area inside the ring. Or curvature can squeeze the ring in one direction and stretch it in another. Then the circle becomes an oval, and the shape of the ring is distorted. The *Weyl curvature* is the part of the curvature of space and time that distorts shapes, as distinct from the part that changes volumes. In 1979 Penrose proposed, as a separate principle, that the Weyl curvature was near zero at the beginning. Penrose's proposal is the *Weyl curvature hypothesis*.

Penrose has raised the question of the low-entropy beginning as an objection to every account in which a collapse becomes a new expansion. *The Gravitational Passage* is the source of this book's account of the turn, and in the account of the turn a collapse becomes a new expansion, so *The Gravitational Passage* must answer Penrose's question too. Inside the old epoch's black holes there is an enormous entropy of gravity, the entropy of the black holes' horizons, while the new outside world begins with a very small entropy of gravity. The geometry of the new outside world is smooth, and its Weyl curvature is near zero. The reconstructed history is the one history that we piece together from the light and the records arriving now. At the beginning of the reconstructed history the matter is hot and close to equilibrium, so the entropy of the matter is already high. The entropy drops from the old epoch's end to the new epoch's beginning, and the drop is in the gravitational part. How can an enormous entropy of gravity become a small one?

What the Axioms Do Not Do

Watch the photographer of Chapter 6 again. First she takes up a position, and from that position many pictures are possible. Then she presses the shutter, and one picture is taken. Afterward there is the print, a fixed thing that anyone can pick up and look at. Position, click and print are

pictures of the three roles of every comparison, and Chapter 6 gave the three roles their names. The transcendent is the role of what is possible: the stance from which an act proceeds, and the range of what could occur, like the photographer's position. The immanent is the role of what is happening: the act itself, as it occurs, like the click. The omniscient is the role of what is known and laid out as structure, like the print.

As Chapter 1 said, an axiom is a statement that holds of every comparison whatever. The Immanent Metaphysics has three axioms, and it is the framework beneath this whole series. Each axiom is a statement about the three roles. The question of the low entropy has one tempting answer. On that answer, an account built on the three axioms would fix the beginning of the world by an axiom, so that one axiom would fix a state of the universe from which everything else proceeds. The three axioms fix no starting state of the universe, and I show why from the wording of each axiom.

The book *An Immanent Metaphysics* sets out the framework. In the words of *An Immanent Metaphysics*, the first axiom reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another." Read aloud, "and/or" in the first axiom means either one, or both together, and the word "conjugate" means paired as counterparts. The book *Incommensuration* establishes the Incommensuration Theorem and derives the three axioms from it, and *Incommensuration* calls the first axiom the axiom of asymmetry. *Incommensuration* states the content of the first axiom in concrete terms: "interaction is more fundamental than either existence or creation", and "The relationship between two things is more fundamental than the fact that the two things exist". Existence is the omniscient role: what is laid out as structure, the print. So the first axiom ranks the act of interaction above existence, and an axiom that ranks the act above existence places neither existence nor any state that exists before every act.

The second axiom, which Chapter 6 set out, sets the three roles in a closed cycle of three steps: stance, then finding, then record, and the record shapes the next stance. A cycle has no first member, as a circle has no

first point. The third axiom reads: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable". The third axiom holds when it is read with "classes" and when it is read with "instances". The third axiom states that the three roles are inseparable in every comparison. It contains no moment at which the three roles begin.

Where, then, does a starting state of the world belong, if no axiom fixes one? As Chapter 1 said, an initial state is the state from which a description of a system is run forward. The foundation volume *Comparison* is the book of this collection that sets out comparison itself as the ground of all knowing. *Comparison* places an initial state in the transcendent role. A boundary condition is the setting given at the edge of a description, from which the description runs. *Comparison* says that the transcendent role, in the language of physics, "is the boundary condition: the choice of initial state, the selection of which experiment to perform, the decision about what to measure, and the range of outcomes that a prepared setting allows". So an initial state belongs to the photographer's position, the transcendent role: an initial state is part of how a description is set up, and not the output of an axiom.

The Compactification adds one further step, an argument of its own made from two statements, and the first statement is the Incommensuration Theorem itself, as Before We Begin stated it. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. *The Compactification* states the first half of the theorem for what a comparison finds: nothing that a comparison finds can be both absolutely symmetric and absolutely continuous. For a geometry, symmetry is sameness through a change of direction or of place, so "absolutely symmetric" means perfectly the same in every direction and at every place. "Absolutely continuous" means perfectly smooth, with no gap anywhere. The second statement says what the Incommensuration Theorem applies to, in the words of *Incommensuration*: "an exact symmetry of a model holds of

the model, and the theorem concerns what a comparison finds. The theorem concerns the real, and not what exists". In this account the two words "real" and "exists" mean two different things: what exists stands as structure, laid out the same for everyone who looks, like a map, or like the reconstructed history of Chapter 1. What is real is given in an actual act of comparison, here and now, like the walk you take with the map in hand. A map exists and the walk you take with the map in hand is real, in the same way that a train timetable exists and the train you catch this morning is real. The Incommensuration Theorem is about what a comparison actually finds, and so it is about the real, and not about what exists. From the two statements it follows that one kind of beginning cannot be found. If a beginning were presented as found, perfectly the same in every direction and at every place and also perfectly smooth, the beginning would be a finding that perfects symmetry and continuity together. The Incommensuration Theorem states that no finding perfects symmetry and continuity together.

A perfectly even, perfectly smooth geometry can still be written down as a model. Cosmologists write down one model of a perfectly even, perfectly smooth geometry, called de Sitter space: an expanding universe with exactly the same shape at every place and in every direction. The exact symmetry of de Sitter space belongs to the model on paper, and not to anything a comparison finds. So any beginning that the account can admit departs from perfect evenness. We return to the departures from perfect evenness later in this chapter, because the account requires them.

Gravity Was Never Born

The division of physics into content and context holds in every act of comparison, in every epoch. The content of a comparison is what the comparison finds, and the context is the respect in which the comparison finds the content. The division into content and context has no moment at which it began. Gravity is the context of physics, as this section shows.

The question of the low entropy has a second tempting answer: gravity itself began at some epoch, and began quiet because it had just begun. That answer fails, because the division of physics into content and context never began.

Measurement is the foundation volume of this collection on what a measurement is. *Measurement* defines the division: "The content and context partition is the division of physical phenomena into content, governed by symmetry and found in discrete classes, and context, governed by continuity and found as smooth values". Content comes in separate classes, and each member of a class is the same as the other members of its class, while context is the smooth setting in which content is found. *Measurement* derives the division from premises, and two of the premises show that the division holds in every comparison. The first premise says that the six ideas every comparison contains come in three pairs: sameness and difference, content and context, and the subjective and the objective. Sameness and difference are the verdict of a comparison. Content and context are what the verdict is on and the respect in which the verdict is given. The subjective and the objective are the side from which the comparison is made and the side about which it is made. The two ideas of each pair are inseparable, while the three pairs are distinct and no pair can take the place of another pair. *Measurement* calls the second premise the inseparability principle, and the second premise says: "where there is content there must also be context, and where there is context there must also be content". In each act of comparison a content and a context come together, and the pair comes afresh with each act. So the division holds of every comparison, in every epoch. The division was not formed at any moment of the history of the universe.

Gravity is the context of physics, by an identification. An identification is a step from the structure of comparison to something in the world, which I make in my own voice. In *Measurement* I state this identification: "I identify gravity as the context-modal phenomenon of physics, by the match of all four markers". The context-modal phenomenon is the phenomenon that plays the role of context in physics: the arena within which

content occurs. The four markers are the four marks by which a context is told apart from a content: a context acts on everything, nothing shields a context, a body's weight in a context and the body's resistance to being moved are one and the same mass, and a context carries no charge that comes in steps. Gravity has all four markers. A context is present wherever a content is present, so gravity is not added to the world after the world's content, and gravity does not begin later than the world's content.

Gravity also has no quantum state from which a quiet "ground state" could be picked out to explain the quiet of the early geometry. A quantum state is quantum theory's description of matter. The description contains the alternatives of the matter, the different ways the matter could be found. A ground state is the lowest and quietest quantum state. *The Unquantized Arena* is the book of this series on how quantum theory and gravity fit together. *The Unquantized Arena* sets aside, as a category error, the premise that the geometry and the matter together form one quantum state. A category error is a mistake of category: a concept applied where the concept does not belong. In the words of *The Unquantized Arena*, the premise "applies the concept of a quantum state to the arena". The arena is space and time together, the context in which everything is measured, whose shape is gravity. Each alternative of the matter has its own ordinary, definite geometry. For each way the matter could be found, there is one smooth shape of space and time. *The Compactification* takes one step further: the quiet of the early geometry cannot be the quiet of a quantum ground state of the arena, because the arena has no quantum state to be in.

Gravitation is not quantized. A small wave of the arena can be described by quanta, single steps of energy like the steps of a crystal's vibration described just below, and the quanta belong to the description of the wave, never to the arena. Think again of a crystal: a lattice of atoms held in place by the bonds between the atoms. Strike the crystal, and a wave of motion runs through the lattice. Quantum theory says that each way the lattice can vibrate takes up energy only in equal steps. Physicists call one

of those steps a *phonon*. Nobody concludes that the crystal is made of phonons, because the crystal is made of atoms, and a phonon is a quantum of the description of how a held structure vibrates. A held structure is a structure whose parts bonds hold in place, as the crystal's atoms are held. The word *graviton* names at most the same sort of thing for a small wave of the arena: a quantum of the description of the wave. The nearest sense of the word graviton is the sense of the word phonon, a quantum of a description, and even in that sense a graviton is not a piece of the arena, because the arena is not a held structure. The arena has no parts that bonds hold in place, and the matter and energy in the arena set the shape of the arena. The description of small waves has its own lowest, emptiest state, which physicists call the vacuum of the description, and this use of the word "vacuum" differs from its use in Part I for the held size and shape of the six. The lowest state of the description belongs to the description of the small waves, and it is not a state of the arena.

So both tempting answers are excluded. No axiom fixes the beginning, and gravity had no birth that could explain the quiet of the early geometry. The account's answer is about what an entropy is.

Every Count Has a Context

Think of a census. A census counts people by the categories that its own forms record, and everyone who fits one line of the form is counted on that line, however those people differ in ways the form does not ask about. When a new country is founded with new clerks and new forms, its first census starts its own count, and the new census does not continue the old country's census. The people of the old country live on in the new country, and no person is lost when the new country is founded. The old count does not continue into the new country.

An entropy is a count of the same sort as a census: the number of states of a system that the comparisons available from some context cannot tell apart, as the census form cannot tell apart the people on one line.

The count of states is made relative to that context. So a change of context is a change of what is counted. A count made relative to one context is not a count relative to another context, as one country's census is not another country's census.

The entropy of a black hole is the clearest case of a count made relative to a context. A black hole's horizon is the surface from inside which nothing comes back out. The *Bekenstein-Hawking entropy* of a black hole is one quarter of the area of its horizon, measured in units of the Planck area. The Planck area is the square of the Planck length. The entropy is named for the physicists Jacob Bekenstein and Stephen Hawking. The formula for the Bekenstein-Hawking entropy is established physics: a result of physics, taken from its source. In *The Unquantized Arena* I identify what the formula counts: I read the Bekenstein-Hawking entropy as a count of "the states of the hole that no comparison made from outside the hole tells apart: a count relative to the outside as context". The factor of one quarter comes from Hawking's calculation of 1975, and I take the factor as established physics gives it. I keep the formula and its numbers as established physics gives them, and my reading says what the formula counts.

Now recall the end of an epoch as Chapter 1 told it: the collapse inside the black holes and the beginning of the new epoch are one process. The process is approached from the two sides of one limit, the turn. The collapse gathers the content together, and the emergence of the new epoch spreads the content out. Each of the two descriptions holds up to the limit and not at the limit: the description of the collapse, and the description of the emergence. So no comparison registers the turn itself as an event.

The Gravitational Passage draws the consequence of the turn for the count of states. "The Bekenstein–Hawking entropy counts the states of an interior that the exterior cannot distinguish. It is a count relative to the exterior as context." At the turn the outside world of the old epoch has emptied, and so the count of the interiors' states has no context left relative to which the count is made. The new outside world is a new context. The entropy of the matter is a count of the same sort: the count of the matter's states is reckoned relative to the outside world that ends at the turn, as the

count of the interiors' states is. In the new epoch every count is reckoned relative to the new epoch's own context, the new outside world. The content itself is carried from the collapse on one side of the turn to the emergence on the other, while no count reckoned against the old outside world passes the turn. In the words of *The Gravitational Passage*: "The new context does not register the distinctions the old context drew, and that is the reset". *The reset* is the fact that at the turn no count reckoned against the old outside world is carried across, while the content, and its form, are carried across.

The reset destroys nothing. The reset is the statement that a count of states has a value only relative to the context in which the count is made, applied at the one place where a context ends. The place where a context ends is the turn: at the turn the outside world of the old epoch has emptied, and the new outside world has no history relative to which an old count could continue.

The reset follows from two things: the meaning of an entropy, as a count relative to a context; and the account of the turn, in which the old outside world has no moment left at the end of the interiors. *The Gravitational Passage* gives the reset as a principle, and gives no count of states across the turn in numbers.

Even From the Start

The reset says what is not carried across the turn. The geometry re-forms at the turn, and its re-forming shows what the new epoch begins with. On the far side of the turn, the four-dimensional geometry of the new epoch re-forms from the ten-dimensional geometry. Inside the ten-dimensional geometry, the four-dimensional geometry had no separate identity. Lumpiness is something a geometry carries from its own past: a hollow here, a ridge there, a direction stretched more than the others. Think of a sheet of paper fresh from the mill. The sheet has no creases, because creases come from a sheet's own past of folding. The new four-dimensional geometry

did not exist as a separate geometry before the turn. So it has no past of its own to carry lumps from. The new geometry inherits no lumps from the old outside world either: neither a favored direction nor the old world's lumpiness. In the plain words of *The Compactification*, the geometry of a new context has nothing to be lumpy with.

When the Weyl curvature of a geometry is zero, we call the geometry *conformally flat*: in the picture of the ring of droplets, circles may swell or shrink, but they stay circles. *The Gravitational Passage* concludes that on this account the new geometry "begins conformally flat up to the departures the Incommensuration Theorem requires". So the new geometry begins with its Weyl curvature near zero, and different from zero only to the size of the departures from uniformity, the faint differences of density with which an epoch begins. The departures are about one part in a hundred thousand, a size that is measured in the oldest light.

The Incommensuration Theorem requires some departures from uniformity. Isotropy is sameness in every direction. In the words of *The Gravitational Passage*, "perfect isotropy would make symmetry and continuity both perfect, which the Incommensuration Theorem forbids". The form of the content is carried across the turn, so the distinctions of the old epoch survive the turn only as small departures from uniformity, "averaged and not erased". And now Penrose's separate principle follows as a consequence: "This is Penrose's Weyl curvature hypothesis, reached here as a consequence of the restoration and not as a separate postulate". The restoration named in the quotation is the one met at the start of this book: the ten-dimensional geometry restoring a context, a large outside world, at the end of the interiors.

Penrose's observation has two halves, the low entropy of gravity and the high entropy of the matter, and both halves now have one explanation. The entropy of gravity in the new epoch is small, because the geometry begins even and no count of the old interiors is carried across. The entropy of the matter is high from the start: "The matter at the beginning of the reconstructed history is hot and close to equilibrium, so its entropy is already high; the drop is in the gravitational part". The growth of entropy

needs one part of the beginning to be low, the gravitational part, and on this account the gravitational part is low because the geometry of a new context has nothing to be lumpy with.

How Far the Even Beginning Reaches

Think of a fair die. A fair die has six faces, and each face has the same chance of turning up: the sameness belongs to the chances. The faces themselves differ, and each throw shows one face.

The argument that the new geometry begins even is about the geometry in which actual content is found, and that geometry is a context. *The Gravitational Passage* states what the argument applies to. The context of actual content has no exact symmetries: no shift and no rotation leaves the context exactly as it was. So the new geometry departs from perfect sameness in every direction, and the departures of the new geometry satisfy the Incommensuration Theorem. The pattern of the departures has two aspects: the range of departures that could be selected, which is a possibility, and the weight that the range gives to each departure, which is a probability. Take a sameness in the pattern, such as the same chance of a departure at every size. A sameness in the pattern is a sameness of the probability across a difference within the possibility, as the die's equal chances run across the die's differing faces. So a sameness in the pattern does not perfect a symmetry and a continuity together in one aspect of what is found. A sameness in the pattern sets the symmetry and the continuity in two aspects, the chances and the range of what could be selected, and the two aspects are conjugate, paired as counterparts. In the words of *The Compactification*, "The symmetry belongs to the statistics of the departures": to the chances, as the die's sameness belongs to its equal chances. The continuity then belongs to the other aspect, the range of what could be selected. Under the Incommensuration Theorem, symmetry and continuity cannot both be perfect in one aspect, while they can

be paired across two conjugate aspects. One actual sky is selected from the range, as one throw shows one face, and the one actual sky is not symmetric.

The argument that the new geometry begins even requires the departures from uniformity, and Part III of this book shows what the departures are and what the account fixes of them.

Does Disorder Pile Up?

If one epoch follows another, does disorder pile up from each epoch into the next? In 1934 the physicist Richard Tolman asked that question of any history made of cycles. Tolman's answer was an objection to histories made of cycles. In a closed universe that collapses and expands again, entropy produced in one cycle and carried into the next makes each cycle larger and longer than the last. When a history of that kind is traced backward, it shrinks to only finitely many past cycles, and so the history has a first cycle after all. *Tolman's objection* is the argument that entropy carried from cycle to cycle gives a history of cycles a first cycle.

Tolman's objection depends on one premise: that the entropy of one cycle is carried into the next cycle. In the account of the turn, no count of states is carried across the turn. In the words of *The Gravitational Passage*: "Here no count is carried across the turn, for the matter or for the geometry, since each count is relative to the context that ends. Each epoch begins its own count, and the cycle has no first iteration". So the epochs are not a series of larger and larger repetitions of one count, as the cycles of Tolman's history are. Tolman found his first cycle by tracing a carried count backward, and in this account no count is carried, so a backward tracing has no growing count to follow back to a first cycle.

A related question has the same answer: can the cycle run at all, if each epoch ends with its entropy near a maximum? The cycle can run. The maximum is reckoned relative to the epoch's own outside world. When the epoch's outside world has emptied and the interiors have reached

their common end, the maximum has no context left relative to which the maximum is reckoned, and the next epoch begins its own count from its own context. In the census picture, even when the old country's census has reached its last page, the new country's first census starts at its own first page.

Information in Two Senses

The reset also applies to a second famous question. The *information paradox* is the question how information that falls into a black hole can be recovered in the outside world, since nothing comes back out, and the hole may in time evaporate. This chapter's response to the paradox is the distinction on which the reset depends: the distinction between the content, which is carried across the turn, and a count of states, which is not. The paradox itself is the subject of Chapter 9.

Think of a letter in the post. The letter's wording travels with the letter wherever the letter goes. The post office keeps a different number: how many letters the post office's sorting can tell apart. That number belongs to the post office, and a new post office starts a count of its own.

The word *information* has two senses, and *The Unquantized Arena* keeps the two senses apart. In the first sense, information is the form of the content: the specification of what the matter is and of which alternatives the matter has taken (what has been selected, as Chapter 1 said), carried with the matter wherever the matter goes. In the second sense, information is a count of states relative to a context. The wording of the letter is information in the first sense, and the post office's count is information in the second sense.

For information in the second sense, the reset says that no count passes the turn. Information in the first sense does pass the turn: the content is carried from the collapse to the emergence, and the form of the content is carried with the content. *At the Edge of Reality* draws the consequence for information that falls into a black hole. Information that falls into a black

hole is not destroyed. The information re-emerges in the beginning of the new epoch, as the new epoch's departures from uniformity, and it is not accessible from the outside world of the original epoch.

The form of the content passes the turn as one form. Each black-hole interior carries its own form, and yet the ends of the interiors are one end. A difference between the ends of two interiors could be registered only by comparing the form of one interior with the form of the other, in a context that tells the two forms apart. The old outside world was the only context that drew distinctions between the interiors. The old outside world has no moment at the end. So no comparison can tell which interior the form came from. The new context registers the form only as small departures from uniformity, correlated across the whole new outside world. The departures are correlated because they all come from one domain. The departures are differences within the new epoch, registered by the new epoch's own context, and not differences between the old interiors. I predict that the departures are correlated across the whole sky, and Part III of this book shows what the correlation looks like.

We can now answer this book's own question, what the beginning left behind, as far as order goes. Everything that has fallen into any black hole keeps its form. The form is carried to the beginning of the next epoch. At that beginning the form is averaged into the faint pattern from which the galaxies of the next epoch will grow. Our own epoch had an orderly beginning, which made stars and planets and people possible, and the orderly beginning of our epoch is the clean slate that every new context begins with. The slate is clean of counts, and it carries the form.

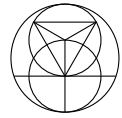
Key idea. Every entropy is a count of the states a context cannot tell apart, reckoned relative to that context. At the turn the old outside world has emptied. So in the reset no count reckoned against the old outside world is carried across, while the content and its form are carried across. The new geometry was not a separate geometry before the turn, so it has no past of its own to carry lumps from. The new geometry begins even, with its Weyl curvature near zero, up to the small departures the Incommensuration Theorem requires. Penrose's Weyl curvature hypothesis follows as a consequence. The matter's entropy is high from the start, and the drop in entropy is in the gravitational part. Because no count is carried across, no disorder piles up from cycle to cycle, and so Tolman's objection to cycles is answered. Information as the form of the content is not destroyed: the information re-emerges as the departures from uniformity.

Summary

We now know why the beginning of an epoch is low in its entropy of gravity, and even in its geometry. The entropy of gravity is low because every count belongs to a context. The old context ends at the turn, and a new context starts its own count. The geometry is even because the new geometry has no past of its own as a separate geometry. So the new geometry has nothing to be lumpy with. The three axioms fix no beginning. An initial state is a setting of a description, in the role of the possible, the transcendent role. Under the Incommensuration Theorem no beginning can be found perfectly even and perfectly smooth, so the account requires small departures from evenness. The division into content and context holds in every act of comparison and was never born at any moment. Gravity is the context, by my identification. The arena, space and time together, has no quantum state, so no quiet ground state of gravity explains the beginning. A black hole's count is one quarter of the area of its horizon in Planck areas, and that count is established physics, while I read the count as a count relative to the outside. The reset follows from the mean-

ing of a count and the account of the turn. The reset is given as a principle, with no count in numbers. Penrose's Weyl curvature hypothesis follows from the even beginning as a consequence. Tolman's objection to cycles fails under the reset, since no count piles up from one cycle to the next. Information that falls into a black hole crosses the turn as form. The information re-emerges as the departures from uniformity, correlated across the whole new sky. As far as order goes, the beginning left behind a clean slate of counts, carrying the averaged form of everything that came before.

Every count needs a context, and the world we live in has edges against which counts are made. Which edge of the world sets the energy of empty space, and how?



Chapter 8

The One Edge That Sets the Vacuum

Every count of states is made against a context (Chapter 7). A black hole's count, for example, is reckoned against the world outside the black hole. Each new epoch starts a count of its own, because the outside world ends at the turn. The world we live in has edges of several sorts, and each edge has a count. Which edge of the world fixes the energy of empty space, and why does only that edge fix the energy of empty space? Why should the count on an edge have anything to do with an energy at all?

In the account, a measurement of the energy of empty space finds only a share of a larger energy, and a count sets the size of the share, for reasons stated in the sections below. One edge supplies the count: the edge of everything whose light has reached us. The count of that edge sets the energy of empty space once. The count gives the value that astronomers measure in the sky.

Edges That Signals Cross One Way

Picture yourself floating at one spot on a wide pond. Stones were dropped all over the pond, long ago and far off. Ripples from the nearer stones have reached you, and ripples from the farthest stones have not yet arrived. In the picture, the pond itself also stretches as the ripples travel. Because the pond stretches, each ripple has farther to go than the ripple had at the moment the ripple set out. Light in an expanding universe is like the ripples on the stretching pond: the stones stand for distant sources of

light, the ripples for the light of those sources, and the stretching of the pond for the expansion of space. Each edge of the world answers a different question about the ripples: which ripples have reached you so far, for example, or which ripples will ever reach you, in all the time to come. Each observer has edges centered on the observer's own spot. On the pond, the ripples that have reached a floater depend on where the floater is.

In physics, a *horizon* is a boundary across which signals pass in one direction only. We met one horizon in Chapter 1. The *event horizon* of a black hole bounds the region from which no signal reaches the outside. *Two Theories, One World* is the plain-words companion of *The Unquantized Arena*, on how quantum theory and gravity fit together, and the companion says of the event horizon: "The horizon is a feature of the shape of the geometry, and no material lies there."

An observer in an expanding universe has edges centered on the observer. We met the particle horizon in Chapter 5: the edge of everything whose light has reached the observer so far, since the expansion began. On the pond, the particle horizon is the place from which the oldest arriving ripples set out. Space has stretched while the light from the particle horizon traveled, so for us the particle horizon is now about forty-six billion light years away (46.1 billion, where a light year is the distance light travels in a year). The distance in light years is more than three times the age of the universe in years, which is about thirteen point eight billion. The *cosmological event horizon* is the edge of everything whose light will ever reach the observer, in all the future. For us, the cosmological event horizon is about sixteen point seven billion light years away today, nearer than the particle horizon, because the cosmological event horizon concerns light sent out from now on. The expansion of space is speeding up, so light that sets out today from beyond about sixteen point seven billion light years will never close the gap to us, however long the light travels.

Two more edges are set by rates of expansion, not by light that has traveled. The *Hubble radius* is the distance at which the expansion carries things away from us at the speed of light, and our Hubble radius is about

fourteen and a half billion light years. On the pond, the Hubble radius is the distance at which the stretching of the water carries a spot away from you exactly as fast as a ripple crosses the water. The *de Sitter radius* is the edge that a universe holding nothing but the energy of empty space would have. Our universe has a de Sitter radius of about seventeen and a half billion light years (17.53), worked out from our energy of empty space. The de Sitter radius is the size toward which the expansion settles. On the pond, the de Sitter radius is the Hubble radius once the stretching has settled to one steady rate. All four distances are measured: astronomers work out each distance from the measured history of the expansion. The shape of the geometry alone fixes a black hole's event horizon. The particle horizon and the cosmological event horizon depend on the shape of the geometry and also on the path the observer travels. So each observer has a particle horizon and a cosmological event horizon of the observer's own, as each floater on the pond has the ripples that have reached the floater's own spot.

Every edge in this chapter lies in the four dimensions of time and space, where comparers stand (a comparer is anyone or anything that makes a comparison, such as an observer with an instrument). Only two numbers in this chapter come from the six small dimensions: the count of three and the trim. Both numbers are explained later in this chapter. Both numbers are read on the context instance of the six, which is the six taken as a space with sizes (Chapter 1).

What the Count of an Edge Counts

Chapter 7 described the count of a black hole: one quarter of the area of the black hole's horizon, measured in Planck areas. A Planck area is a square one Planck length on a side. The physicists Jacob Bekenstein and Stephen Hawking found the count of a black hole in the 1970s, and the count is established physics. For a horizon shaped as a sphere, the count is pi times the radius squared, divided by the Planck length squared. Take a

black hole with the mass of the Sun. The hole's horizon is about three kilometers in radius (2.95), and worked out from the measured constants, the count of the horizon comes to about one followed by seventy-seven zeros (1.05×10^{77}).

Picture the horizon of a black hole as a vast surface tiled like a floor, with one tile for every four Planck areas. Each tile stands for one *degree of freedom* within the horizon: one independent way in which what lies inside the horizon can hold energy or vary, as each string of a piano can ring on its own. The tiles are like a list of every book in a library, written on the library's outer wall, with one entry for each small patch of the wall, so that every book inside is listed outside. The books stand for what lies inside the horizon, the wall for the horizon, and each entry for one tile.

What exactly does the count of a horizon count? In *The Unquantized Arena* I give the answer as an identification: "I read it as the number of states of the hole that no comparison from outside tells apart, a count relative to the outside as context." Chapter 7 kept apart two senses of the word information. In the first sense, information is the form of the matter, which goes with the matter wherever the matter goes. In the second sense, information is a count of states against a context. The count of a horizon is information in the second sense.

Why does the count grow with the area of the edge, and not with the volume inside the edge? A region with a boundary has two natural counts of the information in the region. One way is to count the inside cell by cell, and the first count grows with the volume of the region. The other way is to count what crosses the boundary by interaction, and the second count grows with the area of the boundary. The first count needs the whole region laid out as one structure, with every cell of the inside in view at once. In *The Unquantized Arena* I identify the first count as the omniscient count, in the language of the three roles of Chapter 6. The omniscient role belongs to what is laid out as structure and held in view as a whole, like a map. The second count is made from a place outside the region, through what reaches that place across the boundary. I identify the second count

as the immanent count. The immanent role belongs to what is happening in a comparison: the act itself as it occurs, like the click of a camera. The third role, the transcendent, belongs to what is possible. A comparer outside a black hole can make the second count by standing anywhere outside and registering what reaches the comparer: what falls in, the pull of the hole's mass, and the glow the hole gives off. Every finding in physics is made from a place inside the world. The first of the account's three axioms reads: "The immanent is more fundamental than the omniscient and/or the transcendent." So the immanent count is the more basic of the two counts. For a comparer outside a black hole, the boundary is the horizon. So a black hole's count grows with the area of its horizon. The factor of one quarter comes from Hawking's calculation of 1975, and I take the factor from Hawking as established physics.

In my identification, the words "relative to the outside" say what can change the count of a black hole. The count changes when anything changes that comparers outside can register: the hole's mass, spin or charge, or the area of its horizon. The count stays the same whatever lies behind the horizon, because no comparison from outside registers what lies behind the horizon. Within an epoch, while the outside world lasts, the count is real. At the turn, the outside world ceases to be a context, and so the count has nothing left to refer to. The reset of Chapter 7 is that ending: at the turn, every count reckoned against the old outside world ends.

Measuring Empty Space from Inside

When you measure the length of a table, you are not part of the table. When you measure the energy of empty space, you are part of what you measure. There is no laboratory outside the universe, and no sample of empty space to carry home and set beside a standard. An observer measuring empty space is like a fish trying to measure the water the fish swims in. The fish's instruments are wet, and so is the fish. Likewise, the observer and the observer's instruments are made of the very fields whose low-

est energy the observer is trying to measure. The observer and the instruments are also entangled with the fields, in the sense of Chapter 3: joined to the fields so that part of what is true of each is held only in the relation between the observer and the fields.

So the joining of observer and fields holds part of what there is to know about empty space. The observer does not hold that part. No better instrument can register the part held in the joining, because every instrument is joined to the fields in the same way. A system cannot fully measure itself. I identify the gap in what can be known of empty space from inside as an instance of the Incommensuration Theorem at the scale of the whole universe, where the observer is part of what the observer measures. The observer could hold the whole of empty space in one finding only by finding empty space perfectly symmetric and perfectly continuous at once. Chapter 6 showed that no finding of a comparison is both perfectly symmetric and perfectly continuous. So a knower who is inside the thing known cannot hold the whole of the thing known. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1), and in the measurement of empty space the Incommensuration Theorem applies at the largest scale there is.

The *vacuum energy* is the energy that empty space holds in itself. The vacuum energy is the ground-state energy of the fields, which is the energy that the restless trembling of the fields keeps even when the fields are in their lowest state. The vacuum energy is also the zero-point energy of Chapter 4, summed over all the fields. The cosmological constant is the energy of empty space (Before We Begin). The cosmological constant is the one constant term in Einstein's equations, the same at every place and time. Chapter 5 showed that the six are held in one way only, and so the vacuum energy has one value. When the ground-state energy of the fields is reckoned at the Planck scale, the ground-state energy is larger than the measured energy of empty space by a factor of about a one followed by 123 zeros. I explain the gap between the two energies by one identification, the identification at the center of this chapter.

I identify the cosmological constant with the share of the vacuum's energy that a measurement inside the vacuum engages, and not with the total. The share a measurement engages is the part of the energy that the measurement takes in. The total is the *external value*: the energy of empty space as an observer outside would find that energy, if there were an outside, with the energy taken at the Planck scale. The Planck scale is where the smooth and the stepwise descriptions of the world meet: the modal boundary of Chapter 1. A measurement finds the internal value, a share of the external value. No comparer finds a geometry whose constant is the external value. The share leaves out a part of the external value, and the part left out is the *price of being inside*. The price of being inside is what no measurement made from inside can know about empty space, taken together with the energy that goes with that unknown part of empty space. No measurement from inside can know that part, because that part is held only in the joining of observer and vacuum. The energy withheld from the share is what being inside costs the one who measures. A smaller share means a larger cost of being inside. Every comparer pays the price of being inside the universe that contains the comparer: every measurement that a comparer makes of empty space finds only the share of the vacuum's energy, never the total.

The external value has two parts. The first part is the energy of the quantum fields in their lowest state, at the Planck scale. I take the first part to be the Planck density of Chapter 4, and I count within the Planck density the energy of the floor that holds the six, because the floor's energy is itself an energy of lowest motion. The second part is classical: the energy of the bare curvature of the six small dimensions, at the size that Chapter 3 derived. The second part carries no quantum constant, no Planck's constant (the number that sets the size of the quantum's steps). The second part has the sign that draws space together. Because of that sign, the second part is subtracted: the second part takes away three divided by eight pi of the Planck density. Eight pi is about twenty-five, so the part taken away is about twelve hundredths of the Planck density. So the external value is the Planck density less three divided by eight pi of the

Planck density: about eighty-eight hundredths of the Planck density, or 0.8806 when stated more exactly. The fraction 0.8806 is the *trim*: the share of the Planck density that remains after the bare curvature of the six is subtracted. The trim is set by the structure, because the curvature of the six at their derived size fixes the trim.

Three Tiles Out of All the Tiles

How large is the share? Return to the picture of tiles, and lay the tiles now on the particle horizon. I give my reasons for the particle horizon in the next section. We call the number of tiles on the particle horizon the count of the edge. The size of the share follows from three steps, and each step has its own ground.

First, every tile holds an equal part of the external value. The external value is a density, the same at every place, and the tiles are laid at one rate over the whole edge, so equal patches of the edge hold equal numbers of tiles. Every tile is like every other tile in what it holds. Tiles that are the same in every way hold equal parts. So each tile holds one equal part of the external value: one part out of the count of tiles. Any three tiles hold three of the equal parts. Three tiles hold three parts by plain arithmetic, as with a cake. If a cake is cut into any number of equal slices, whoever takes three slices gets three slices' worth, and in the picture the cake stands for the external value and each slice for one tile. The parts are equal however the tiles are joined to one another. A measurement receives only the parts held by the tiles the measurement takes in, and the reason is the joining of observer and vacuum. A measurement receives an amount set by the number of tiles the measurement takes in, and not by the strength of the joining.

Second, a measurement takes in what the measurement finds through three channels. A channel is a content that a measurement registers (Chapter 3). A content is what is found in a comparison, as the temperature is found by reading a thermometer. Every measurement registers its

content in its own context. The six measures of Chapter 3 form a grid of content and context: pattern in space, force in time, and probability in possibility. I identify the channels through which the vacuum reaches a measurement with the three contents, one in each of the three roles. The grid has one column for each role. In each column, the context is where the content is registered, and the context is not a second thing registered. The context is like a page and the content is like a word, because a page is where a word is written, and a page is not a second word written on the page. A length is registered as the stretch of a pattern in space, and a duration as the course of a change in time. So each column of the grid counts once, by its content, and there are three channels, not six.

The count of three is set by the structure, because the count of three equals the Euler count of the complex projective plane, the larger of the two factors of the six (Chapter 3). The other factor of the six is an ordinary sphere (Chapter 1). The Euler count of a shape is the number of its basic pieces, counted with signs, and the complex projective plane has exactly three basic pieces, one for each of the three roles.

Third, each channel takes in one whole tile, and each channel takes in a different tile. Do a telescope and a thermometer take in six tiles when a comparer uses both together? The two instruments together take in three tiles, not six. The comparer who uses both instruments is one stance: one place from which every finding is made. So the tiles the comparer takes in are counted once, for that one stance, and not once for every instrument the comparer carries. I take each channel to take in one tile, and a different tile for each channel, because the three contents the channels carry are distinct, and by the third axiom the three contents are distinct, inseparable, and none able to do another's work. So a measurement takes in three tiles out of all the tiles on the edge.

By the three steps together, the cosmological constant is the share of the vacuum's energy that a measurement inside the vacuum engages: three parts, out of the count of the particle horizon, of the external value. The external value is 0.8806 of the Planck density, which is the Planck

density less three divided by eight pi of the Planck density. The share is three whole parts of the count: a fixed number of parts, and not a chance fluctuation in the count.

The count of tiles is a number of *nats*, where a nat is the natural unit of information. A bit is the answer to one yes-or-no question, and one nat is about one and forty-four hundredths bits. The same number of states can be stated in bits or in nats, as one distance can be stated in miles or in kilometers. The count of a horizon is stated in nats. In nats, heat and count are related most simply.

Our particle horizon carries a great many tiles: about two and three tenths times ten to the power one hundred twenty-three (2.29×10^{123}). So three tiles hold about one part in a one followed by 123 zeros of the external value. The count is so large because the radius of the particle horizon is now about ten to the power sixty-one Planck lengths. The energy of empty space is small because the edge of what has reached us is large.

Each piece of the share has its own standing, in the sense of Before We Begin. The count of three and the trim are set by the structure. The share is three parts out of the count by an identification, which I made in the three steps of this section. The Planck density comes from Newton's constant, the one measured constant on gravity's side. The radius of the particle horizon is measured.

Why the Edge of What Has Reached Us

Which edge carries the tiles that set the share? The count is taken on the particle horizon, for two reasons, and the second reason is independent of the first.

The first reason uses the principle that the whole account rests on: a measurement is made here and now, from one place. The particle horizon is the edge of everything that has actually reached the observer, which is everything that has by now been compared. So the particle horizon is the only edge that bounds what a present measurement contains. The cosmo-

logical event horizon is set by what could reach the observer in the future, and a present measurement contains nothing from the future. The Hubble radius and the de Sitter radius are set by rates and by idealized universes, not by what has arrived. So, by an identification, I assign the particle horizon the count of Bekenstein and Hawking, and the identification is that the bound of what has been compared is the bound of the information available to the comparison.

The second reason is arithmetic. Our universe holds matter and light besides the energy of empty space. In a universe like ours, the Hubble radius and the cosmological event horizon are smaller than the de Sitter radius at every era, and the two edges creep toward the de Sitter radius, reaching the de Sitter radius only in the infinitely far future. Now we work out the share on each edge in turn. The share grows smaller as the edge grows larger. An edge twice as wide has four times as many tiles, and so gives a quarter of the share. Established physics has a fixed rule for the de Sitter radius, which ties the count of the edge to the energy of empty space, and the rule comes from the established laws of heat and count for a universe of empty space alone. By the rule, the energy of empty space is three eighths of the Planck density, divided by the count of the edge. The eight in the rule is the eight pi of Einstein's equations divided by the pi of the horizon's count. Eight pi is the factor with which Einstein's equations tie energy to the bend of space. The horizon's count is pi times the radius squared, in units of the Planck length squared. The share on the de Sitter radius is three parts of 0.8806 of the Planck density, divided by the same count. When the share and the rule are set side by side, the threes cancel: the share comes to eight times the trim, eight times 0.8806, which is about 7.05 times the measured constant. Any smaller edge gives a larger share, since a smaller edge has fewer tiles and so a larger part for each tile. So on the Hubble radius and on the cosmological event horizon, the share always comes to more than seven times the constant. On either of the two smaller edges, no measurement could find the one value that one geometry carries. The second reason holds because eight times the trim exceeds one. Only the particle horizon grows past the de Sitter radius. For

the share to come down to the constant itself, about seven times smaller, the edge must hold about 7.05 times as many tiles as the de Sitter radius, so the radius of the edge must be larger than the de Sitter radius by the square root of 7.05: about 2.65 times the de Sitter radius. The particle horizon is today about 2.63 times the de Sitter radius. The two reasons come from two directions, and both reasons have the same answer: the particle horizon.

Five Choices, Each with Its Reason

Every choice in the share is fixed by a reason, so nothing in the share is left to adjust. The share depends on five choices, and the reason for each choice comes from the account itself, with no reference to the measured value. First, the edge is the particle horizon, for the two reasons of the section before this one: the particle horizon bounds what a present measurement contains, and only on the particle horizon can the share equal one constant. Second, the count of channels is three, because a measurement registers one content in each of the three roles, and the complex projective plane has one basic piece for each role. Third, the trim is included, because the curved six produce the energy of their bare curvature. Leaving the trim out would be an error. Fourth, I count the floor's energy inside the Planck density part, because the floor's energy is itself an energy of lowest motion. Fifth, the Planck mass in the share is the full Planck mass, about one point two two times ten to the power nineteen GeV (1.221×10^{19}), where a GeV is a billion electron volts (Chapter 5). The share uses the full Planck mass because the modal boundary is a scale formed from the constants of nature alone. Some physicists use a smaller reduced Planck mass, which carries a factor of eight pi. The factor of eight pi belongs to the way Einstein's equations couple energy to curvature.

The Number, Seen Two Ways

Our present particle horizon has a radius of about 46.1 billion light years and a count of about 2.29×10^{123} tiles. With that count of tiles, the share puts the energy of empty space at two and fifty-six hundredths times ten to the power minus forty-seven GeV to the fourth (2.56×10^{-47}). Particle physicists write an energy packed into a volume as an energy raised to the fourth power. So the unit of the share is GeV to the fourth. The measured value is two and fifty-two hundredths in the same unit (2.52×10^{-47}). In everyday units the measured value is about six times ten to the power minus twenty-seven kilograms in each cubic meter (5.8×10^{-27}). The measured value is the mass of three or four hydrogen atoms in each cubic meter of space.

There is a second way to find the same energy, and the two ways must agree. Imagine two surveyors sent to measure the same field, one walking the edge of the field with a measuring chain and the other working from a hilltop with angles and a map. Each method has its own unknowns, but there is one field, so the two answers must agree. When two answers must agree, the agreement itself can tell you something that neither surveyor could find alone. The field stands for the energy of empty space, and the surveyors stand for the two ways of finding the energy.

The first way is the share. The second way is the astronomers' way, through the equation that Alexander Friedmann found from Einstein's theory in 1922. A flat universe is one in which, on the largest scales, space has no overall bend: lines that start out parallel stay parallel. In a flat universe, Friedmann's equation says that the rate of the expansion fixes how much energy the universe must hold in all. The energy of empty space is one part of the total. The *matter fraction* is the part of the total that is matter, ordinary and dark together. In a flat universe the parts fill the whole: matter, the energy of empty space, and a thin sliver of radiation, which is light and other fast particles, about nine parts in a hundred thousand of

the whole today, measured. In the account, space in the epoch is flat because of the way the epoch opens, and the full reason comes in Chapter 12.

Set the two ways equal. Both ways grow with the square of the expansion rate, and both ways contain the strength of gravity in the same way. So the expansion rate cancels, and so does the strength of gravity (the Planck mass). What is left is one statement in pure numbers, and I call that statement the identity. In words, the identity says: the square of the particle horizon's radius, measured in Hubble radii, times the vacuum's fraction of the whole, equals eight, less three divided by pi, which is about 7.05. The number 7.05 is the number found earlier on the de Sitter radius, where the share came to eight times the trim. Eight times the trim equals eight, less three divided by pi, because the trim is one, less three divided by eight pi, and eight times three divided by eight pi is three divided by pi. The eight in the identity is again the eight pi of Einstein's equations divided by the pi of the horizon's count. The three tiles cancel the three in Friedmann's equation.

The particle horizon's size in Hubble radii depends only on how the energy has been divided over the history of the expansion. So the identity is one equation with one unknown, the matter fraction. The identity has one solution. In the solution, matter is 0.311 of the energy of the universe: thirty-one and one tenth percent. The energy of empty space is 0.689. The measured matter fraction is 0.315, give or take 0.007. In this chapter I have now shown one measured agreement, seen in two ways. The one agreement is seen in the density of empty space, 2.56 against the measured 2.52, and in the matter fraction of the universe, 0.311 against the measured 0.315. With one set of measured inputs, comparing the densities is the same comparison as comparing the matter fractions, because the identity is the share and Friedmann's equation set equal to each other. So the density and the matter fraction are two readings of one agreement, as the surveyors' answers are two readings of one field. Every number in the share has the standing given earlier in this chapter, and every number in the share was fixed before the share was compared with the measured values.

The identity fixes how the energy is divided, but not the rate of the expansion, which cancels from the identity. The identity fixes the total matter fraction, and says nothing of what the matter is made of. The identity can also be stated without the Planck scale at all: the particle horizon is 2.65 times the de Sitter radius. The measured distances are 46.1 and 17.5 billion light years, and their ratio is 2.63.

Fixed Once

The particle horizon grows every year, as light from a little farther away arrives. Does the cosmological constant shrink as the count grows? The constant is fixed once, and *The Gravitational Passage* shows why in two steps.

First, Einstein's equations themselves require the constant to be one number within one geometry. Einstein's equations have other terms besides the constant, and those other terms balance on their own, by a property of the geometry called the contracted Bianchi identity (named for the mathematician Luigi Bianchi). So if the constant varied from place to place or from era to era, Einstein's equations would set the variation to zero. The first step is a theorem of the geometry: the first step must be so, and the first step uses no measurement. A road carries one speed limit along its whole length, posted once, and one geometry carries its one constant in the same way, the same at every place and every era.

Second, the comparison that fixes the one value is the present measurement. In the sense of Chapter 7, what exists stands as structure, timeless, like a law. What is real is given in an actual act, here and now. So a law exists, and a measurement is real. The comparison that fixes the constant is real. A real comparison is given in an actual act here and now, by the meaning of the word real, so the comparison that fixes the constant is the present measurement. No further identification is needed for the second step. By the first axiom, the act precedes the structure rebuilt from the act. So the history of the universe is reconstructed from the impression

of the past on the present, as the one history the impression requires. We first have the present act, and the history is rebuilt from what reaches the present act, as a photograph of starlight shows the star as the star was when the light left the star (Chapter 1). We do not first have a history and then find ourselves somewhere in the history. Many comparers made the records of the expansion, and the records are present impressions of the one geometry, so the records carry the one value of that geometry, the cosmological constant. A constant "larger in the past" would need a comparison made in that past, which the present measurement does not contain. So the constant is fixed once, by the present measurement, as the constant term of one geometry, and the reconstructed history carries the one value at every era, before the present and after the present.

Picture an astronomer from when the universe was half its present size. The astronomer appears in the reconstructed history, with the records she made. Her records carry the one constant of the history, which is the constant we measure, and in her day the matter fraction was 0.79. If the share is worked out on paper for her particle horizon, the share comes to 7.049 times the measured constant. For our own particle horizon as our horizon was a billion years ago, the share comes to 1.224 times the measured constant. For our particle horizon a billion years from now, the share comes to 0.852 times the measured constant. *The View from Inside* says of the three numbers 7.049, 1.224 and 0.852: "none of these is a finding; each is a number computed from the structure of the history." The three numbers exist, as structure, but the three numbers fix nothing, because the three numbers come from no actual act. No observer finds a vacuum density that falls with time.

Why does her own measurement, made in her own present, not fix the constant? Her measurement was a real act in her day. What reaches us of her measurement is her record. Her record is a present impression in our reconstructed history, and as a record her record carries the one constant of our reconstructed history. The act that is given is the present act. So the share is a statement about the one act that is real: the present measurement. If the astronomer worked out the share on her own particle horizon,

the share would not give the constant she measures. The particle horizon only grows, so the identity holds at one era only. The identity says that the present is that one era, because the present measurement fixes the constant. Someone could ask, from outside, why the present falls at that era. To ask that question, a person must take the presents of all eras as given together. By the first axiom, only the present act is given. The present act falls about ten to the power sixty-one Planck times after the beginning of the reconstructed history, where a Planck time is the time light takes to cross a Planck length. The date of the present act is a question about when comparers exist, and so a question about the content. In *The Gravitational Passage* I take the era of the present from the measured particle horizon. Comparers of the kind that measure the constant exist while stars shine, and the era of shining stars fixes the date of the present act to within a power of ten. Physicists have long asked why the density of matter and the density of empty space are close to each other today. The question arises because the density of matter thins out as space grows, and the density of empty space does not. The nearness of the two densities today is the identity itself, holding at the era of the real act.

The energy of empty space is the same for every comparer in steady motion, and so the energy of empty space enters Einstein's equations as the one constant term of the equations. Physicists use the letter w for a ratio: the pressure of a form of energy, which is the push of that energy on its surroundings, divided by the amount of that energy in each unit of volume. The ratio tells how a form of energy behaves as space grows. For the energy of empty space, the ratio is exactly minus one. A ratio of exactly minus one means that the energy in each unit of volume stays the same as space grows. So no changing field drives the acceleration of the expansion. The Higgs field does not drive the acceleration: the Higgs field settled long before the expansion began to speed up, through the smooth change near 159.5 GeV that Chapter 10 describes. After settling, the Higgs field kept a constant energy.

One Count, Once

Every black hole in the observable universe lies inside our particle horizon, and we are outside every one of those black holes. So a black hole's horizon hides states within the observable universe, and the count of our particle horizon already includes those states. We pay for the hidden states once, in the one share, and we pay nothing more at the horizon of the black hole. All the black holes in the observable universe have a combined count of about ten to the power one hundred four. The combined count is a small part of the 2.29×10^{123} of the particle horizon, and the black holes enter our count there, once.

The share can still be worked out on a black hole's own count, without the trim, and the result has an exact meaning. Take the quantity three times the Planck density, divided by the hole's count. The quantity is exactly eight times the average density of the hole's mass spread through the ball inside the hole's horizon. The equality is arithmetic: the equality must be so, and Appendix A sets out the arithmetic. In ordinary units, the quantity carries no quantum constant at all. So the quantity is a property of the hole's geometry, not a quantum share of anything. The eight has a reason: the average density of a hole and the density of a universe of empty space alone stand in one and the same relation to the radius of each. So the eight in the quantity is the same eight as in the rule for the de Sitter radius.

For a hole with the mass of the Sun, the average density inside its horizon is about one point eight times ten to the power nineteen kilograms in a cubic meter. Eight times the average density is about one point five times ten to the power twenty kilograms in a cubic meter. The measured density of empty space is about 5.8×10^{-27} kilograms in a cubic meter. Eight times the hole's average density is about two and a half times ten to the power forty-six times the measured density of empty space: about forty-six powers of ten. If each black hole's horizon set a separate energy of empty

space, empty space would be forty-six powers of ten denser near a hole of the Sun's mass than empty space is. Empty space would also be denser near small holes than near large ones. Empty space is not denser near black holes, and not denser near small holes than near large ones. So the quantity enters no energy of empty space outside the hole, and the particle horizon alone sets the constant of empty space.

A comparer that falls into a black hole keeps the share the comparer had before the fall. The count of what has reached the comparer sets the share. A black hole's event horizon limits what can leave the hole, but the horizon does not limit what can enter the hole. Light from outside keeps crossing inward, and the hole lay inside the comparer's particle horizon before the fall, so the comparer inside the hole keeps the count of the comparer's particle horizon, and with that count the comparer keeps the cosmological constant.

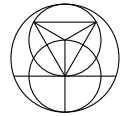
The particle horizon counts the states hidden behind every black-hole horizon within the observable universe. The observer pays for the hidden states once, in the one share, and no horizon within the particle horizon adds a second payment. A black hole's count is a real count, reckoned against the hole's outside, and a black hole's count belongs in the sum of the counts of the black holes and of everything outside the black holes. The sum never falls, by a law of established physics that is the subject of Chapter 9. But a black hole's count enters no energy of empty space.

> **Key idea.** A measurement of empty space is made from inside empty space, and the measurement takes in three tiles out of all the tiles on the edge of what has reached us. The edge of what has reached us is the one edge that sets the constant: the edge of what has reached us is the only edge that bounds what a present measurement contains, and the only edge on which the share can equal one constant.

Summary

We now know which edge sets the energy of empty space. An observer has four edges: the particle horizon, the cosmological event horizon, the Hubble radius and the de Sitter radius. Of those four edges and the event horizons of black holes, the particle horizon alone sets the energy of empty space, and the particle horizon is the edge of everything whose light has reached us. The cosmological constant is the share of the vacuum's energy that a measurement inside the vacuum engages: three parts, out of the count of the particle horizon, of the external value, which is 0.8806 of the Planck density. With the measured horizon the share comes to 2.56×10^{-47} GeV to the fourth, against 2.52 measured. When the share is set equal to Friedmann's equation, the share gives a matter fraction of 0.311, against 0.315, give or take 0.007, measured. The density and the matter fraction are one measured agreement, seen two ways. The constant is one number in one geometry, fixed once by the present measurement, with w exactly minus one. A black hole's count is real, reckoned against the hole's outside, and the particle horizon already counts the states that the hole's horizon hides, so each hidden state is counted once, in one count.

Three findings of this chapter bear on what the beginning left behind. The shape and size of the six set the two numbers that the six supply: the count of three and the trim. The six take the same shape and size again at every turn (Chapter 5). The count of the edge belongs to the present measurement, which is made inside the epoch that the turn began. The black holes' own counts are real, and the black holes' own counts enter no energy of empty space. What happens to the count of a black hole while the black hole grows, merges and evaporates within an epoch?



Chapter 9

Black Holes Within an Epoch

Every black hole has a count, the hole's entropy, reckoned against the world outside the hole, and Chapter 8 showed that the count of a black hole sets no energy of empty space. Yet black holes change. A black hole swallows gas and stars, two black holes can find each other and merge, and every black hole glows very faintly, so the glow slowly carries the hole's mass away. What happens to the count of a black hole as the hole grows, merges and evaporates? And where does the information go that fell into the hole?

The count of a black hole grows when matter falls in. When two holes of equal mass merge, the count grows by about half again. When a hole evaporates, the count passes to the radiation that the hole gives off, and evaporation is so slow that the world outside the hole empties first. Through growth, merger and evaporation, no information is destroyed. The cosmological constant is the energy of empty space. The cosmological constant stays at the value fixed by our present particle horizon, the edge of everything whose light has reached us.

A Bell Struck Far Away

Picture two black holes in orbit around each other, far across the universe. Each time the two holes go around, their motion sends ripples out through space and time, and the ripples carry energy away, so the two holes draw closer together and circle faster. In the last moments the two

holes touch, and one hole is left. The new hole is lopsided at first, and it shakes itself smooth, the way a struck bell rings and then falls quiet. The new hole rings with tones of its own, and the ringing dies away as the hole settles. Physicists call the ringing of the new hole, while the new hole settles into a final shape, the *ringdown*. Physicists call the whole event, from the last circuits to the settled hole, a *merger*.

The ripples are gravitational waves: ripples in the shape of space and time. Detectors on Earth register gravitational waves as a tiny stretching and squeezing of distances. From the waves alone, physicists work out the masses of the two holes, how fast each hole spins, the shape of the orbit of the two holes, and the tones of the final ringing. A struck bell rings, and the ringing tells a listener the bell's size and shape, which belong to the outside of the bell. In the same way, the ringing of the new hole tells the hole's mass and spin to the world outside the hole, and the world outside the hole plays the part of the listener.

Each of the holes has a count. The count of a hole is the hole's entropy: a count of the states that a context cannot tell apart. The horizon of a hole is the surface from inside which no signal can reach the outside. The entropy of a black hole is one quarter of the area of the hole's horizon. The area is measured in Planck areas (squares one Planck length on a side). I read the entropy of a hole as the number of states of the hole that no comparison made from outside can tell apart. The number of states is a count reckoned against the outside world as context. My reading is an identification, the one Chapter 8 made. For a hole of the Sun's mass the count is about 1.05×10^{77}, a one followed by seventy-seven zeros. The count is computed from the measured constants. Horizons change in time: a hole grows when matter falls in, two holes merge into one, and a hole loses mass by radiation. The account is the account of the world taught in this book. To follow how horizons change, the account uses only established physics, and three results in particular.

Three Rules for a Changing Horizon

The first result is a theorem about the horizons of general relativity, which is Einstein's smooth description of space and time, and Stephen Hawking proved the theorem in 1971, under two conditions. First, the matter is of the ordinary kind. The energy that ordinary matter carries is never negative, measured along any ray of light that crosses the matter. Second, every singularity is hidden behind a horizon. A singularity is a place where Einstein's equations give a curvature without bound. When both conditions hold, the area of a black hole's horizon never decreases, and when two holes merge, the area of the final horizon is at least the areas of the two starting horizons added together. Physicists call Hawking's theorem the *area theorem*: the rule that a horizon's area never shrinks in general relativity. Hawking proved the area theorem within general relativity, so within general relativity, and under the two conditions, the area of a horizon must behave as the area theorem says. The count of a hole is one quarter of the area of its horizon, so the area theorem says that the count of a black hole, reckoned against the hole's outside, never falls in any process that general relativity describes without the quantum.

The second result describes a horizon that can be found at one moment. Before the second result, we need a picture of how light moves near a hole. Set off a flash of light at one place and at one moment. The light of the flash spreads out in every direction at once, as a sphere growing at the speed of light, and nothing sent from that place and moment can outrun the sphere. So a signal sent from that place and moment can reach only places within the growing sphere, at that moment or later. Physicists call the reach of a signal sent from a place and moment the *light cone* of that place and moment: all the places, and the later moments, that the signal can reach. (The name comes from the shape that the reach has when physicists draw the reach, but no drawing is needed: the light cone is simply how far the flash can get, and when.) Far from a black hole, the

flash spreads as far outward as inward. Nearer the hole, the whole light cone leans toward the hole: light has less room to move outward, and more room to move inward. At the horizon, light aimed straight outward holds its distance from the hole: the light neither escapes nor falls in. Inside the horizon, even light aimed outward is carried inward.

The horizon in its usual sense is the event horizon: the boundary of the region from which no signal ever reaches places far away. Where the event horizon lies depends on the whole future, because the definition asks whether a signal *ever* reaches far away. At any given moment, no one has the whole future in view. General relativity also contains a horizon that can be found here and now, from the geometry near one surface. Take a closed surface around the hole, like a soap bubble, and send light straight outward from every point of the surface. Far from the hole, the outgoing light spreads apart, as light from a lamp does. Deep inside the hole, even the outgoing light is drawn together. Somewhere between far from the hole and deep inside the hole lies a surface on which the outgoing light neither spreads apart nor draws together. Physicists call a surface on which the outgoing light holds level, while light sent inward draws together, *marginally trapped*.

Stack marginally trapped surfaces one after another through time, and the stack makes a horizon of its own, called a trapping horizon. While a trapping horizon grows, physicists call it a *dynamical horizon*: a horizon found from the geometry near the hole at each moment, rather than from the whole future. While a trapping horizon stays the same, physicists call it an isolated horizon. By definition, the area of a dynamical horizon grows from moment to moment, and the energy flowing in across the dynamical horizon matches the growth. So a physicist can follow the growth of a hole moment by moment while matter falls in, without knowing the whole future. Ordinary matter never carries negative energy, and the energy of ordinary matter never flows faster than light. In 2004 the physicists Abhay Ashtekar and Badri Krishnan showed that, for all ordinary matter, the energy flowing in across a dynamical horizon is never negative. Once

a hole has settled, the isolated horizon and the event horizon are one and the same surface. Established general relativity includes trapping, dynamical and isolated horizons, and the result of Ashtekar and Krishnan.

The third result applies to the one case outside the area theorem: a hole that loses mass by radiation. In 1974 the physicist Jacob Bekenstein proposed a rule about black holes. Bekenstein's rule is that the entropy of the black holes plus the entropy of everything outside the black holes never decreases. Physicists call Bekenstein's rule the *generalized second law*. The generalized second law is the old law that the entropy of a closed system never falls, widened to take in black holes. The generalized second law applies to a hole that loses mass by radiation: as the hole radiates, the area of the hole's horizon falls, while the entropy of the radiation outside the hole rises by at least as much. The sum in the generalized second law is the entropy of the black holes plus the entropy outside the black holes. Of the quantities this chapter uses, the sum of the two entropies is the one quantity that never falls.

All three results are about the count of a horizon, reckoned against the horizon's outside. An epoch is one era of the world, in which stars, galaxies and black holes form. So within an epoch, every change of a horizon is a change of the horizon's count. General relativity and the generalized second law say how each count changes. The cosmological constant does not change with the counts of horizons, because the present particle horizon fixed the cosmological constant once. Chapter 8 showed that the horizon of a black hole carries no energy of empty space. None of the three results needs an energy of empty space.

The Count of a Merger

Now put numbers on the merger that the first section of this chapter pictured as a struck bell. Take two holes of equal mass, neither of them spinning, circling each other on a nearly round orbit. The area of a horizon grows as the square of the hole's mass, so half the mass gives a quarter of

the area. Each starting hole has half the total mass, so each starting hole has a quarter of the horizon area that a still hole (a hole that does not spin) of the whole mass would have. The two starting holes together have half of that area. Physicists find what is left after the merger by *numerical relativity*: general relativity solved by computer, where no formula on paper will do. With numerical relativity, physicists follow the whole merger step by step, by computer solutions of Einstein's equations. For two equal holes that do not spin, numerical relativity shows that the final hole has 0.95162 of the starting mass. The final mass is a little over ninety-five percent of the starting mass. The final hole spins at 0.68646 of the fastest spin that a hole of its mass can have, a little over two thirds (Mark Scheel and colleagues, 2009). Both numbers are established physics, from computer solutions of Einstein's equations, and neither number is a number of this account.

The final hole has nearly all the mass. The spin of the final hole makes its horizon smaller than the horizon of a still hole of the same mass. General relativity has a formula for the horizon area of a spinning hole (the appendix For the Physicist states the formula). Put the two numbers, 0.95162 and 0.68646, into the formula. The final horizon then has about 1.56 times (more exactly 1.564 times) the area of the two starting horizons added together. The count of a hole is one quarter of the area of its horizon, so the count of the final hole, reckoned against its outside, is about one and a half times the two starting counts added together. The count grows by about half again. The area theorem requires a ratio of at least one, and the ratio for this merger, about 1.56, is well above one. At the same time, the mass missing from the final hole leaves as gravitational waves. The missing mass is one minus 0.95162: about 4.8 percent of the starting mass and energy counted together.

The merger takes place within the present particle horizon, and the merger leaves the cosmological constant at the value that the present particle horizon fixed.

Physicists have checked the area theorem in a real merger. The first merger ever detected was registered on 14 September 2015 by LIGO. LIGO is a pair of laser detectors in the United States, each with arms four kilometers long. The merger is named GW150914, for its date. Maximiliano Isi and colleagues (2021) worked out the total area of the two starting horizons from the circling part of the signal, and the area of the final horizon from the ringdown. Isi and colleagues found that the area grew, as the area theorem requires.

The Unquantized Arena is the book of this collection on how quantum theory and gravity fit together. *The Unquantized Arena* predicts that the gravitational waves from a merger behave just as general relativity says. Two measurements show that the waves do behave so. A neutron star is the collapsed core of a large star, packed as densely as the nucleus of an atom. First, the waves travel at the speed of light: in 2017 the waves from two merging neutron stars arrived together with the burst of gamma rays from the same event. Second, the waves stretch and squeeze space in two patterns, the two patterns that general relativity predicts. The measurements of the event GW170814 showed the two patterns. GW170814 was a merger of two black holes, named like the first merger for its date, 14 August 2017.

What does a merger do to the information inside the two merging holes? After the merger, the information inside the two merging holes is all still hidden. The insides of the two starting holes now lie within the inside of the final hole. And the final hole stands: the final hole exists, with its horizon in place. The waves carry the outward features of the merger: the masses, the spins, the orbit, and the ringing of the final hole, and these outward features are everything that a comparison made outside registers of the two holes. The waves carry nothing from behind either horizon, because nothing behind a horizon reaches the outside. In general relativity, which is established physics, no signal from inside a horizon can arrive at any place or moment outside the horizon, so no comparison made outside

registers what is inside. The merger raises the count reckoned against the outside by about half again. Nothing that the count hides becomes reachable from outside, because no signal leaves a horizon.

The Faint Glow

Black holes also lose mass. In 1974 Stephen Hawking found that a black hole glows very faintly, the way any warm body glows, and that the smaller the hole, the warmer the glow. Physicists call the glow *Hawking radiation*, and they call the slow loss of mass that the glow carries away *evaporation*. In quantum theory, matter and light are made of quantum fields, which Chapter 3 introduced: each quantum field fills all of space, and the smallest ripples of a quantum field are the particles. Hawking radiation is real. Hawking radiation is a result for quantum fields on a classical geometry: a smooth geometry of space and time, with no steps in the geometry.

Hawking laid the quantum fields of matter and light on the smooth geometry of a star collapsing to a hole. He then compared two descriptions of the empty state of the fields. The first description is natural long before the hole formed, and the second description is natural long afterward, far from the hole. In the two descriptions, different states count as empty. In the second description, the empty state of the first description contains a steady warm flow moving outward from the horizon, and the steady warm flow is Hawking radiation. Physicists call the temperature of the flow the Hawking temperature. In every step of Hawking's calculation the geometry stays smooth, and no quantum of space or time appears. Hawking's calculation is the comparison of the two descriptions of the empty state. You may have heard the glow told as pairs of particles that appear near the horizon, one falling in and one escaping. The popular picture of particle pairs does not describe Hawking's calculation, because Hawking compared the two empty states instead.

One step of Hawking's calculation uses the smooth description beyond the Planck scale. Follow the waves that become the glow backward in time, toward the moment the horizon formed, and the waves grow shorter without limit. At the moment of collapse, the waves were far shorter than the Planck length. Hawking's calculation, as written, uses the smooth description of fields on a smooth geometry down to lengths far shorter than the Planck length. The Planck scale is the modal boundary: at the Planck scale, each existing description reaches the edge of the domain where that description holds, and beyond the Planck scale no description takes over. The Incommensuration Theorem is a structural fact about the nature of knowing, and the theorem says that the smooth description does not hold at every scale. So the Incommensuration Theorem shows the exact step at which the standard calculation is used past the edge of the smooth description's domain: the step in which the waves are followed back to lengths shorter than the Planck length.

Physicists have tested whether the glow depends on the step that uses the smooth description beyond the Planck scale. Physicists have worked with models that change how a field behaves at the very shortest waves, and the models still predict the warm glow that Hawking found whenever the hole's pull at its horizon is far gentler than the scale at which the models change the field. The temperature of the glow is then still set only by the hole's pull at its horizon. Sound can be trapped in a cloud of atoms chilled almost to absolute zero, the way light is trapped by a hole, and a model black hole of trapped sound glows with sound at the predicted temperature (Muñoz de Nova and colleagues, 2019). So the glow of a horizon does not depend on what happens beyond the Planck scale. *The Gravitational Passage* is the book of this collection that sets the whole account out for physicists. In *The Gravitational Passage*, Hawking radiation is taken as real, on the timescales that follow.

A *kelvin* is a degree of temperature counted up from absolute zero, the coldest anything can be. A hole of the Sun's mass is about three kilometers in radius, and the hole's Hawking temperature is about 6.2×10^{-8}

kelvin: about sixty-two billionths of a degree above absolute zero. Physicists compute the temperature from Hawking's formula and the measured constants. A hole of the Sun's mass glows mostly with light whose waves are kilometers long, about the size of the hole itself. The microwave background is the faint afterglow of the early universe, an afterglow that fills all of space. The temperature of the microwave background is about 2.7 degrees above absolute zero. So a hole of the Sun's mass is colder than the background by a factor of tens of millions (nearly eight powers of ten). The hole is colder than the background, so the hole takes in far more energy from the background than the hole gives off. The hole begins to lose mass on balance only after the background has cooled below the hole's own temperature.

In the accelerating expansion, the temperature of the background falls to a little more than a third of its value about every seventeen billion years. So the background takes a hundred billion to a trillion years (10^{11} to 10^{12} years) to cool below the temperature of a hole of the Sun's mass, as computed in *The Gravitational Passage*. For a hole of the Sun's mass, the evaporation itself then takes about 2.1×10^{67} years. Written out, the number 2.1×10^{67} is twenty-one followed by sixty-six zeros. In *The Unquantized Arena*, the evaporation time is computed from established physics. The evaporation time is vastly longer than the time black holes take to merge.

The outside world empties first, long before a hole of the Sun's mass evaporates. Over a hundred billion to a trillion years, the expansion carries everything that is not bound to an observer's own structure beyond that observer's reach. A hole formed from a star evaporates over a time far longer than a trillion years, so the outside world of the hole's epoch empties long before the hole has evaporated. Only holes far lighter than any star evaporate while an outside world remains around them. If a hole of less than about 5×10^{11} kilograms (five hundred billion kilograms) formed in the early universe, the hole would have evaporated by now. In *The Gravitational Passage*, no claim is made about how so light a hole ends.

The Rise and Fall of the Radiation's Link to the Hole

For a small hole that evaporates while an outside world remains around the hole, the whole history of the hole's glow is gathered in one outside world, so a definite question can be asked about the history of the glow. As the hole glows away, how is the hole's radiation linked to what remains inside the hole?

Picture a coded message and its key: the message alone reads as noise, and the key alone reads as noise. The meaning is in the pair, and none of the meaning is lost while both the message and the key exist, even when only one of the two is in your hands. The radiation of a hole is like the message, and the radiation's partner inside the horizon is like the key. Taken alone, the radiation is a featureless warm glow, because the radiation's partner lies inside the horizon.

When the states of two things are linked so that neither thing can be described alone, the two things are entangled, as Chapter 3 said. Findings made on one of two entangled things are then correlated with findings made on the other more tightly than findings on any two separately described things can be. A quantum state is *pure* when the state is as fully specified as quantum theory allows. Usually, one part of a larger whole has a state less fully specified than a pure state. Physicists measure how far the radiation is entangled with the hole by a count called the *entanglement entropy* of the radiation: how much the radiation and the hole share in quantum correlation.

Under two conditions, the entanglement entropy of the radiation first rises with the ordinary warm-glow entropy that the radiation has carried away, until the warm-glow entropy equals the count that the hole still has, and from then on the entanglement entropy falls with the hole's count. The first condition is about everything that came from what fell in: the content still inside the horizon, and the radiation that has come out. The first condition says that the content inside and the radiation outside, taken

together, began pure and stay pure, as described from one outside world. The pure state in the first condition is the state of the matter alone. An alternative of the matter is one of the several definite possibilities in which the matter can be held at once. Each alternative of the matter carries its own geometry (the last section of this chapter shows how). The first condition includes no geometry in the quantum state. The second condition is that the entanglement entropy of the content inside the horizon cannot exceed the hole's count. Physicists call the hole's count the hole's Bekenstein-Hawking entropy. Physicists call the rise and fall of the entanglement entropy the *Page curve*. The name is after the physicist Don Page, who found the shape of the curve in 1993. In *The Unquantized Arena*, the Page curve is derived as a result under the two conditions. The Page curve holds to within half a nat, half a unit of the natural count: the true entanglement and the Page curve differ by no more than half a nat, while the count of a hole runs to many powers of ten.

Two further steps fix the shape of the Page curve under its two conditions. The first step is established physics. The radiation can share no more entanglement with the hole than the radiation's own warm-glow entropy, which grows as the hole radiates. The second step is established physics read through one identification. In 1993 Page showed that, for nearly every pure state of a large whole made of two parts, the entanglement entropy of a part lies close to the largest value possible: the smaller of the two parts' capacities. The capacity of the radiation is its warm-glow entropy, and the capacity of the hole is its count. I read "nearly every" as a symmetry: states that no comparison made from outside can tell apart count alike, each with the same weight.

Picture a conversation between a person with a large memory and a person with a small memory. The conversation shows Page's result at work. What the two people share can be no larger than the smaller memory, and in a long conversation, what the two people share fills the smaller memory. The two people stand for the radiation and the hole, their memories stand for the two capacities, and what the two people share stands for the entanglement. So the entanglement of the radiation

follows the smaller of the two capacities. Early on, the radiation's capacity is the smaller, and the entanglement rises with the radiation's capacity. Later, the hole's shrinking count is the smaller capacity, and the entanglement falls with the hole's count, reaching nothing when the hole is gone. The turning point of the Page curve is the moment at which the two capacities are equal, and physicists call that moment the Page time.

A hole loses its count slowly at first and quickly at the end, because the smaller a hole becomes, the hotter it is and the faster it radiates. The generalized second law says that the radiation gains at least as much entropy as the hole loses. If the radiation gained exactly as much entropy as the hole lost, the two capacities would become equal when the hole had lost exactly half its count. The radiation in fact gains more, so the radiation's capacity grows faster than the hole's capacity shrinks, and the two capacities become equal earlier. From the law by which a hole evaporates, we can work out when the earlier moment comes. The law of evaporation says that the share of the count that a hole still has is the cube root of the share of the hole's life still to come, multiplied by itself. So a hole has lost half its count only when 0.646 of its life has passed, a little under two thirds, and the other half of the count goes in the little more than a third of the hole's life that remains. The turning point comes before the hole has lost half its count, so the turning point comes no later than 0.646 of the evaporation time. The bound of 0.646 follows of necessity from the Page curve and the law of evaporation. Don Page computed in 2013 that the turning point comes a little past halfway through the life of a hole that gives off light and the small waves of its geometry, at 0.538 of the evaporation time. The Page curve holds under its two conditions, for a hole that evaporates while an outside world remains to receive the hole's glow.

Where the Information Goes

Information is not destroyed in a black hole. In quantum theory, the state of the matter is carried forward on the matter's geometry without loss. In general relativity, whatever falls in is carried forward to the end of the interior of the hole. At the end of the interior, the smooth description reaches the edge of its domain, beyond which the smooth description no longer holds. While the hole stands (while the hole exists, with its horizon in place), the horizon hides the information.

A famous puzzle says that information is destroyed in a black hole, and physicists call the puzzle the *information paradox*. In 1976 Hawking reasoned that a hole that evaporates completely leaves only featureless warm radiation. Then two stars with the same mass, charge and spin would end as the same radiation, even if the two stars differed in every detail of their make-up, but when quantum theory is applied to the whole system, quantum theory says that the end must keep every such difference. The information paradox is the clash between Hawking's reasoning and quantum theory, and besides Hawking's result, the paradox depends on two premises.

The first premise is the premise of the whole history: the smooth geometry describes the whole history of the hole, the hole's singularity included, so whatever the field carries into the singularity is gone. The second premise is the premise of one quantum state: geometry and matter together form one quantum state, and the change of the one quantum state from before the collapse to after the evaporation is one map that keeps every difference. In *The Unquantized Arena*, both premises are set aside. Whoever holds the premise of one quantum state applies the idea of a quantum state to the arena, space and time themselves, and so makes a category error: applying a concept where the concept does not belong, as one would by asking the color of the number seven. The loss that Hawking found occurs at the singularity, where the smooth description

reaches the edge of its domain and no longer holds. The loss is a result of neither description: not of the smooth description of general relativity, and not of quantum theory. Once both premises are set aside, nothing in either description destroys the information. So the claim that opened this section follows: information is not destroyed in a black hole.

The word "information" has two senses in this book. In the first sense, information is the form of the content: what the matter is, and which alternatives the matter has taken, carried with the matter wherever the matter goes. In the second sense, information is a count of the states that a context cannot tell apart. The entropy of a horizon is information in the second sense. Because the word has two senses, there are two separate questions about the information in a black hole. The first question is whether anything destroys the form of the content. Nothing destroys the form of the content. The second question is whether the form of the content can be reached from outside, and when. The answer depends on the kind of hole.

For a small hole that evaporates while an outside world remains, the answer is the Page curve, under its two conditions. For a hole formed from a star, the information is not accessible from outside while the hole stands, and the account states neither the return of the information to the outside world nor its loss to the outside world. Both parts of the statement follow from what this chapter has found: in neither description is the information destroyed, and the horizon lets no signal out. The form of what fell in is carried with the content to the turn, the passage at the end of the interiors of the black holes in which a large outside world is restored. At the turn, the form passes across as one form, and the form then reemerges as the departures from uniformity of the next epoch: the faint differences of density with which the next epoch begins.

Other physicists have recently found the rise and fall of the Page curve by another method. In 2019 and 2020, physicists working separately found the same shape as the Page curve: a rise, a turning point, and a fall with the count of the hole. The physicists were Geoff Penington and, separately,

Ahmed Almheiri and several colleagues. Past the turning point, Penington and Almheiri's group count a region behind the horizon together with the radiation. The region counted with the radiation is called the *island*.

Because of the Page curve, the account states that the horizon is smooth. A body that falls through the horizon finds nothing special at the horizon. Physicists call a wall of energy at the horizon that burns whatever falls in a *firewall*. In 2013 the physicists Almheiri, Marolf, Polchinski and Sully argued that past the turning point the horizon would have to become a firewall. In the account, the answer to the firewall argument follows from the Page curve. Each late piece of radiation has a partner behind the horizon. Past the turning point, the partner of each late piece is encoded in the early radiation, so the link of a late piece to its partner is part of the late piece's link to the early radiation. Because the partners are encoded in the early radiation, no firewall is needed.

One Geometry for Each Alternative

In every result of this chapter, quantum matter is laid on a smooth, classical geometry. How are quantum matter and classical geometry joined? A quantum thing can be held in several definite possibilities at once: a tiny magnet can point up, point down, or be held in both together. Each definite possibility of a quantum thing is called an *alternative*. Each alternative has a phase: a quantity that turns, like the hand of a tiny clock. *The Unquantized Arena* states how quantum matter and geometry are joined, and I call that statement the joining. By the joining, each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by that alternative alone. By the joining also, the relative phases of the alternatives are kept. Where the matter is sharply one alternative, there is one geometry, and the joining is simply Einstein's equation for that matter. A black hole of definite mass is matter that is sharply one alternative, and the hole's geometry is the one geometry that the hole's mass shapes.

In the rival joining, all the alternatives together have one geometry, shaped by the average of the alternatives. The rival joining fails in both of its forms. In the usual telling of quantum theory, the jump of the state to one outcome is the step in which a measurement is made, one alternative is found, and the other alternatives drop out of the state at once. The first form of the rival joining has no jump of the state to one outcome. Without the jump, the rival joining fails the experiment of Page and Geilker. A Cavendish balance is a delicate instrument that measures the pull of gravity between masses. In 1981 Page and Geilker set the masses of a Cavendish balance by a quantum random event. The measured pull followed the outcome, not the average over outcomes. The second form of the rival joining adds the jump of the state to one outcome, and with the jump added, the rival joining would let someone send signals faster than light. A measurement made far away would then change the one averaged geometry here at once, so a sensitive balance here could register which measurement had been made far away, before any light from the measurement could bring the news.

The joining has two consequences for horizons. The first consequence is that a horizon is a feature of a classical geometry, one classical geometry for each alternative of the matter. Space and time have no quantum state at all, so no horizon has a quantum state either. The count of a horizon is therefore the count of the states of the hole that no comparison from outside tells apart, and not a count of quantum states of the geometry. The second consequence is about the small waves of the geometry, such as the waves that a merger sends out. The small waves of the geometry may be described by quanta: the equal steps of energy in which quantum theory describes a wave, as Chapter 7 showed with the phonon, one step of energy in one way the lattice of a crystal can vibrate. The quanta of the small waves are quanta of the description: steps that belong to the way the small waves are described, and not to space and time, which ripple. In this book, as Chapter 7 found, the word *graviton* names at most a quantum of the description: a quantum of the description of the small waves of a classical

geometry, and never a quantum of space and time themselves. Space and time have no least step and no quantum of their own at any scale. Gravitation is not quantized.

The joining predicts the result of an experiment now being built. Hold two tiny specks of diamond, each speck in two places at once, close enough that gravity is the only thing relating the two specks. A joint alternative is one place for the first speck together with one place for the second speck. In each joint alternative the two specks are at one definite distance, and the geometry of that joint alternative is the one that the masses of the specks shape at that distance. So the pull between the two specks differs from one joint alternative to the next. The clock hand of each joint alternative turns at its own rate. The account predicts that the two specks become entangled through gravity alone. The angle that marks the entanglement is called the *entangling phase*: the part of the drift of the clock hands that no adjustment of either speck on its own could produce or undo. A radian is a measure of angle, and a full turn is a little over six and a quarter radians. For the design that the physicist Sougato Bose and colleagues proposed in 2017, the account predicts an entangling phase of 0.314 radians: about a third of a radian, or about eighteen degrees, a twentieth of a full turn of the clock hand.

What the Black Holes Keep

The question of this book is what the beginning left behind. Part of the answer is what happens, in each epoch, to whatever falls into the black holes of that epoch. The counts of the black holes grow, by matter falling in and by mergers, and the generalized second law says that the sum of the counts of the black holes plus the entropy outside the black holes never falls. Every count of a black hole is real, reckoned against the outside world of the hole's epoch. No count of a black hole sets an energy of empty space. When the formula that gives the cosmological constant is applied to the count of a hole, the result is eight times the hole's average

density, as Chapter 8 found. For a hole of the Sun's mass, eight times the average density is about forty-six powers of ten above the density of empty space. Eight times the average density of a hole is a property of the hole's geometry, and not a share of the vacuum. The states a black hole hides are already among the states the particle horizon counts, so the hidden states enter the cosmological constant once, through the one count of the particle horizon.

The outside world empties long before any hole formed from a star evaporates. While the outside world empties, the form of all that fell in is kept: hidden behind the horizons while the holes stand, and carried with the content to the turn.

We now know how the counts of black holes behave within an epoch, and why no count of a black hole changes the vacuum.

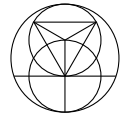
> **Key idea.** A black hole's count is reckoned against the hole's outside. The count never falls in the processes of general relativity, and the count grows by about half again when two equal holes merge. When a hole evaporates, the count passes to the radiation, so slowly that the outside world empties first. Information is not destroyed. For a hole formed from a star, the information is not accessible from outside while the hole stands, and the account states neither the return of the information to the outside world nor its loss to the outside world. No count of a black hole affects the cosmological constant, which the present particle horizon fixes once.

Summary

Within an epoch, every change of a black hole's horizon is a change of the hole's count, and the account uses established physics for those changes: the area theorem, the dynamical horizon and the generalized second law. In a merger of two equal holes the count grows by about half again (about 1.56 times), and about 4.8 percent of the starting mass and energy leaves

as gravitational waves. Hawking radiation is real. For a hole formed from a star, Hawking radiation is so slow that the outside world empties long before the hole evaporates. The Page curve holds under its two stated conditions, for a hole that evaporates while an outside world remains, and the turning point of the Page curve comes no later than 0.646 of the evaporation time. Information is not destroyed. For a hole formed from a star, the information is not accessible from outside while the hole stands, and the account states neither the return nor the loss of the information. Each alternative of the matter carries its own classical geometry. Gravitation is not quantized. The account predicts an entangling phase of 0.314 radians for the design of Bose and colleagues. No count of a black hole sets an energy of empty space.

The black holes are one of several things counted in an epoch, beside the epoch's matter, its radiation and its particle horizon. Put the counts together: how does the whole count of an epoch change, from the beginning of the epoch to its end?



Chapter 10

The Counts of an Epoch

Each epoch of the world starts its own count of entropy. How does the count of our epoch change, from the beginning of our history to the end of our epoch, when the world empties? To answer, we follow one stretch of the world through the whole of our epoch. We count the stretch at every stage. The counts are enormous numbers, and each count comes with its standing in plain words, so that you can tell which numbers physics already holds, which numbers are worked out from measurements, and which numbers rest on the account's own steps. Some counts are established physics, and some are computed from stated measured values. One count rests on an identification, and some counts are argued.

What Is Counted, and Where

Chapter 6 defined entropy as a count of the states that a context cannot tell apart, and the number of states is so large that physicists always give the number by its logarithm. The *logarithm* of a number is the power to which a fixed base must be raised to give the number. In this book the base is the number e, about 2.718. When a count is given as a logarithm to the base e, the count is in nats. Chapter 8 used nats for the count of a horizon. So a count "of ten to the ninety" means that e, raised to the power ten to the ninety, gives the number of states. A logarithm is closely related to the number of digits needed to write a number out: a number with a hundred digits has a logarithm, to the base e, of about two hundred thirty. So

a count of ten to the ninety nats is a number of states so large that writing the number out would take about four times ten to the eighty-nine digits. In a plain sense, every count belongs to a context. Each count is a number of the states that one particular context cannot tell apart, so when the context ends, the count has nothing left to refer to.

Think again of a photograph of starlight. The light arriving tonight shows the star as the star was when the light set out. We know our history in the way we know the star, from what the past leaves in the present: the light now arriving from distant galaxies, the microwave background (the faint afterglow of the early universe's light, reaching us from every direction), the amounts of the light elements, and the records of every instrument. We piece together the reconstructed history of Chapter 1 from the arriving light, the afterglow, the light elements and the records. The reconstructed history is the one history that the present traces require. No other history fits the traces. The reconstructed history exists as structure, as a map or a record exists. So a count "at" some past era is a number computed within the reconstructed history, and the source of the number is the measured present. In this chapter, every count of a past era has that standing: a computed number whose source is the measured present.

To follow a count, you need a stretch of the world to count. Throughout this chapter the stretch is the region that is now the observable universe: a sphere around us whose radius is our particle horizon, the boundary of everything whose light has reached us since the history began. The radius of the particle horizon is 46.14 billion light years, a measured value. The value comes from the measurements of the expansion made by the Planck satellite, a space telescope that mapped the microwave background, and the satellite is named after the physicist Max Planck, as the Planck length is. At an earlier era, "the region" means the matter now in the observable universe, followed back in time. When a region grows with the expansion and always holds the same matter, physicists call the region a *comoving region*.

Where the Count Begins

Followed backward, the reconstructed history comes close to the turn but does not extend back to the turn. The turn is the passage in which a large outside world is restored. Where the reconstructed history begins, the content and the geometry are in opposite conditions. The content is the matter and light that fill the world, and the geometry is the shape of space and time.

The content is hot and close to equilibrium, the settled state in which nothing further changes on average. Every kind of particle is present together with its antiparticle, the particle's twin, with the same mass and every charge reversed, and the pairs are made and unmade in balance, as often one way as the other. So the content of a comoving region already has a count near the largest value that the content's energy allows. The geometry begins even. The geometry has none of the curvature that distorts shapes, except for the small departures from evenness that the Incommensuration Theorem requires. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. Relative to the new context, the outside world that the turn restores, the count of the geometry is small. *The Gravitational Passage* contains the argument for both statements: the count of the content is high, and the count of the geometry is small. The argument rests on the principle that every count belongs to its context, a principle that Chapter 7 followed.

How hot was the content where the history begins? The account sets a ceiling on the starting temperature. The account states no value for the temperature itself. The ceiling comes from the measured amount of dark matter. *The Weight of Possibility* identifies the dark matter with gyrations made at the turn. A gyration is a conserved vibration of the six small dimensions in their held size: the number of gyrations is kept. Chapter 4

showed that a hot enough plasma, the gas of particles so hot that nothing in the gas holds together, can make pairs of anything that has weight, through gravity alone. When two particles of the plasma meet, the two particles can turn some of their energy into a new pair through their gravity alone. Suppose the plasma had been hotter than about one thirtieth of the mass of one gyration. Then the plasma would have made more dark matter through gravity alone than is measured. *The Weight of Possibility* carries two values for the mass of a gyration, from two levels of calculation: the square root of two Planck masses and six tenths of a Planck mass. For a mass of the square root of two Planck masses the ceiling is about 5 times ten to the seventeen GeV, and for six tenths of a Planck mass the ceiling is about 2 times ten to the seventeen GeV. So the content of our epoch was never hotter than about 2 to 5×10^{17} GeV. A GeV, a billion electron volts, is about the energy held at rest in one proton. Physicists give a temperature as the typical energy of motion of the particles. A temperature of one GeV is about eleven and a half million million kelvin. So the ceiling is about 2.3 to 5.8×10^{30} kelvin. The ceiling is between about 1.2 and 2.9 thousand million million times the mass of the top quark, the heaviest kind of particle, so at the start the mass of every kind of particle lies far below the temperature.

The account also states one exact condition on a possible earlier stage of the content. Suppose the energy reached the plasma through an earlier stage of the content that behaved as matter (slow, heavy particles, rather than light). The reheating is the decay that ended the earlier stage and handed the earlier stage's energy to the plasma. The reheating must then lie below about one hundredth of the mass of a gyration. If an earlier stage behaved as matter, the decay of the earlier stage would itself raise the count of the content, and the account fixes neither whether the content had a stage that behaved as matter nor how large the rise in the count would be.

One more contribution to the count has a bound and no value. Chapter 4 showed that, as the six small dimensions settle into their held size, the energy of the motion of the six goes into their breathing vibrations, the even swellings and shrinkings of their size. Within about a Planck time (the time light takes to cross a Planck length), the breathing vibrations decay into gravitational waves of the four large dimensions. In the reconstructed history, the gravitational waves count as radiation, beside the content's own radiation. Chapter 4 introduced the neutrinos, the lightest particles of matter. The neutrinos come in three kinds. Physicists measure how much radiation the early universe held besides its light, and, as Chapter 4 said, physicists count that extra radiation in light species: one light species is the radiation that one kind of neutrino carries. The measurement shows that the gravitational waves held less than about three percent of the content's radiation. The bound applies at the time when about a hundred species shared the heat. As in Chapter 4, the species are the states of particles that share the heat, and the next section shows how many species there are. Today, a share of the radiation at that time appears as extra radiation equal to 2.86 times the share, in light species. So the bound of three percent means less than about 0.09 of one light species. The bound of three percent is measured, and the figure in light species is computed from the bound in *The Compactification*. The count that the gravitational waves carry is small beside the count of the content, since the energy of the waves is at most a few percent of the energy of the content.

A Dance in a Growing Hall

For a long stretch after the start, nearly all the energy of the content is radiation: light, together with every particle that moves so fast that the particle behaves as light does. Physicists call the stretch in which radiation holds nearly all the energy the *radiation era*. In the radiation era, the count

in each unit of volume is a standard result of physics for a hot gas in balance: a fixed number, times the cube of the temperature, times the number of species that share the heat.

Picture a dance in a hall whose walls move slowly outward, with the doors shut and with every dancer holding some tickets. As the hall grows, the dancers spread out and slow down, and nobody enters or leaves. When one group of dancers can no longer keep up and leaves the floor, the group does not carry its tickets out of the hall: the group hands its tickets to the dancers still dancing. The number of tickets in the hall stays the same, while fewer dancers hold the tickets, so each of the remaining dancers holds more tickets.

In the picture, the hall is the comoving region, the tickets are the count, and the dancers are the species. The shut doors are the picture of one plain fact: while the reactions among the particles run far faster than the region grows, no heat enters or leaves a comoving region. When no heat enters or leaves a region as the region expands, physicists call the expansion *adiabatic*, an older word for "with no heat flowing in or out". In an adiabatic expansion, a comoving region keeps the same count of the content, and the rule is established physics. Because the expansion is adiabatic, the region that is now the observable universe holds about the same count of the content today as at the beginning of the radiation era.

How many dancers are there at the start? That is, how many species share the heat? *The Coming of the Kinds* pictures the sharing of the heat as a pot of soup shared among the bowls at a table, with every bowl filled to the same level: the level is the temperature, and the bowls are the states. Each state of a force carrier or of the Higgs field (the particles called bosons) counts one. A *lepton* is the electron, one of the electron's heavier copies, or a neutrino. Quarks and leptons are the particles called fermions. Each state of a quark or a lepton counts seven eighths. The total is the *count of species*: the number of particle states in balance that share the heat, each fermion state counted as seven eighths. At temperatures above the mass of the top quark, every kind of particle is present and light, and the count of species is 106.75: twenty-eight states of the bosons, plus seven

eighths of ninety states of the fermions. The count of 106.75 comes from the list of kinds alone, with nothing measured: the count is set by the structure of the kinds.

One state among the kinds is left out of the count: the right-handed partner of the neutrino. The right-handed partner interacts with the other particles only through the neutrino's tiny mass. The *expansion rate* is how fast every distance grows, as a share of itself, in each unit of time. The reactions of the right-handed partner run at no more than 3.7 ten-billionths of the expansion rate. The limit holds at every temperature from the settling of the Higgs field upward. A particle comes into balance only when the particle's reactions run faster than the expansion. So the right-handed partner never comes into balance and never joins the dance. The right-handed partner's reaction rate is computed in *The Coming of the Kinds*.

As the gas cools, each kind whose mass rises above the temperature leaves the floor. When a kind leaves, the particles and antiparticles of the kind annihilate, meeting and turning into light and other particles, and the energy and the count that the annihilated particles carried pass to the remaining particles and heat the remaining particles. The steps of the cooling are established physics. *The Coming of the Kinds* sets the steps out in the order that follows. As the temperature falls through the tens of GeV, the first kinds leave the floor: the top quark, the Higgs boson, and the W and Z bosons (the carriers of the weak force). The count of species drops from 106.75 to 86.25. The kinds that leave next are the bottom quark, the tau and the charm quark: the bottom and charm quarks are two of the heavier quarks, and the tau is the heaviest copy of the electron. An MeV is a thousandth of a GeV, about eleven and a half billion kelvin. Just above the binding of the quarks, at 156.5 MeV, the count of species is 61.75. The gluons are the carriers of the strong force, which binds the quarks. Just below 156.5 MeV, the quarks and the gluons are bound into protons, neutrons and their relatives, and the count of species is 17.25. The pions are the lightest particles made of quarks, and the pions leave near 14 MeV. The muon, the electron's heavier copy, leaves near 10 MeV. By a few MeV

the count of species is 10.75: two states for the light, four for the electron and the electron's antiparticle, the positron, and two for each of the three neutrinos together with its antineutrino.

Then the neutrinos stop trading energy with the rest of the gas and travel on by themselves: the neutrinos of the muon and the tau go free near 3 MeV, and the electron's neutrino near 2 MeV. In the picture of the hall, the neutrinos step off the floor into a gallery of their own and keep their own tickets. From then on the neutrinos keep a constant count of their own. When the electrons and positrons leave the floor, the electrons and positrons hand their tickets to the light alone. So the light ends warmer than the neutrinos by the cube root of eleven fourths, about 1.40. Today the content's count is held by 3.909 species. The light gives 2, and the three neutrinos, at their lower temperature, give 1.909. Every step of the cooling is established physics, from 106.75 species down to 3.909. In the standard computation, the free neutrinos are worth 3.044 light species, and the computation includes a small correction to the release of the neutrinos. The three kinds of neutrino give three light species, and the small correction gives the rest. The measurements of the oldest light find a value close to 3.044.

A comoving region keeps the same count of the content through every step of the cooling: the steps move the count from species to species and do not raise the count.

Smooth Changes, With No Bubbles

Matter can change its state in two ways, and only one of the two ways raises the count. Picture water chilled below freezing in a very clean bottle. The water stays liquid past the point where ice would form. Then ice appears in sudden crystals that spread and meet, and the bottle warms as the crystals form. The freezing in the clean bottle is a *first-order transition*: the old state persists below the temperature at which the new state would be favored, the new state forms in bubbles, and the bubbles grow,

collide and release a *latent heat*, heat that the old state held hidden. A first-order transition is not adiabatic: the transition raises the count of the content, and the collisions of the bubbles make gravitational waves. Now picture butter warming on a counter. The butter softens steadily, with no moment at which the butter jumps from hard to soft. The softening of the butter is a *crossover*: a smooth change, in which the state follows the temperature continuously, with no persisting old state, no bubbles and no latent heat.

The content's history has two changes of state. A lattice computation is a calculation made by computer on a fine grid of points in space and time. Lattice computations of the Standard Model show that both changes are crossovers, and the result is established physics. The lattice computations are cited in *The Compactification*.

The first change of state is the settling of the Higgs field at its value. The settling switches on the masses of the kinds. For the measured Higgs mass, about 125 GeV, the settling of the Higgs field is a crossover, centered near 159.5 GeV. The settling would be a sudden change only for a Higgs mass below about 72 GeV, so the measured mass lies far above the point where the change would be sudden. The Higgs field is the only field of its type in the content: a field with a single value at each point and no direction. The list of kinds has no further kind that could make the settling sudden.

The second change of state is the binding of the quarks and the gluons into protons, neutrons and their relatives. The binding is a crossover near 156.5 MeV. In 2006 Aoki and others established on the lattice that the binding is a crossover.

So, after the reconstructed history begins, no change in the content's history persists below the temperature at which the new state is favored, forms bubbles, or releases a latent heat. No change of state raises the count of the content in a comoving region in bulk. In this book, a *hum* is a random mixture of gravitational waves arriving from every direction at

once, as Chapter 15 describes. The absence of bubbles also fixes the answer to a question of Chapter 16: neither crossover makes a background hum of gravitational waves.

One number is kept through both changes of state, and the amount of ordinary matter is a share of that number. The small excess of matter over antimatter is carried as a value of *B minus L*: the number of baryons (protons, neutrons and their relatives) less the number of leptons (the electron, the electron's heavier copies and the neutrinos), each number counting particles less antiparticles. The value of B minus L "is a residue the turn leaves, kept by every change after it", in the words of *The Coming of the Kinds*. Among the kinds, only one process changes the number of baryons, and physicists call that process the *sphaleron passages*. Each sphaleron passage carries the field of the weak force from one of the field's lowest states over a ridge to the next lowest state. Each passage changes the baryons and the leptons together, and keeps B minus L. The sphaleron passages run in balance from about ten to the twelve GeV down to 131.7 GeV, and the lower figure is a lattice computation. Below 131.7 GeV the passages stop, and from then on the number of baryons stays fixed. Chapter 20 describes what becomes of the residue after the passages stop.

Nuclei, the First Light, and Today's Count

The later steps of the history leave the count of the content unchanged. Near 0.8 MeV, the ratio of neutrons to protons stops following the value that balance would give. The ratio is left at about one in six. The first nuclei form in the minutes that follow. Much later, the light of the microwave background scattered for the last time, at a temperature of about 2,973 kelvin, when every distance between far-apart things was about 1,091 times smaller than today. Both values, 2,973 kelvin and 1,091, come from measurement. The count of the content does not change appreciably when the nuclei form or when the light last scatters, because the protons and neutrons are too few to matter to the count. Measurement finds 6.13 pro-

tons and neutrons for every ten billion particles of the light of the microwave background (6.13×10^{-10}, give or take 0.04×10^{-10}). The amount of protons and neutrons is a number of history: what happened sets the amount, and the structure of the kinds does not.

Between the forming of the first nuclei and the release of the light, one more change happens: the radiation era ends. When every distance doubles, the energy in each unit of volume falls sixteenfold for the light and the neutrinos, and only eightfold for the matter, ordinary and dark. So at one moment the matter comes to hold as much energy as the radiation, and from then on the matter holds more energy than the radiation and rules the expansion. Physicists call the moment when the two energies are equal the *equality of matter and radiation*. The Planck satellite's measurements show that the equality came when distances were about 3,403 times smaller than today (give or take 26), when the light was at about 9,275 kelvin, an energy of about 0.80 electron volts. The end of the radiation era changes the rate of the expansion and not the count: the expansion stays adiabatic, and the count of the content in a comoving region stays the same through the equality.

We can then work out the count of the light today, step by step. The microwave background is at 2.72548 kelvin, a measured value, and, as an energy, that temperature is about 2.35 ten-thousandths of an electron volt. At that temperature, with two species for the light, the count of the light is about 1,479 in each cubic centimeter. The light holds about 411 particles in each cubic centimeter, so the count is 3.602 times the number of particles: each particle of a warm glow in balance carries about 3.6 units of count, a standard result of physics. The region within the particle horizon has a volume of about 3.48×10^{86} cubic centimeters. So the light of the microwave background carries a count of about 5.2×10^{89}, five point two times ten to the eighty-nine, in the region that is now the observable universe. The three kinds of neutrino carry about 4.9×10^{89}, because the neutrinos' share of the species is 1.909 against the light's 2. Both counts are computed in *The Compactification* from stated measured values: the

temperature of the background and the radius of the particle horizon. Together the light and the neutrinos carry about 10^{90}, ten to the ninety. Ten to the ninety is the count that the content has carried since the beginning of the radiation era, because the expansion of the radiation era was adiabatic and kept the count. If no stage that behaved as matter came before the radiation era, ten to the ninety is also the count that the content has carried since the beginning of the history.

Gravity Turns the Order Around

In the later history, the count of the geometry changes, while the count of the content stays the same. For a gas whose own gravity does not matter, the state of largest count is the even state: matter spread evenly. For matter held together by its own gravity, the order is reversed: clumping raises the count. Picture a cloud of gas collapsing under its own weight. The cloud heats as the cloud falls inward, and the cloud shines. The radiation that the cloud sends away carries more count than the cloud loses. So the count of the whole rises as the small departures from evenness grow into galaxies, stars and clusters. That clumping raises the count is established physics. The clumping sets the direction of the history: toward a growing count.

Of all the clumping, the black hole is the largest step. Chapter 7 told how Bekenstein and Hawking gave a black hole a count: a quarter of the area of the hole's horizon, measured in squares of the Planck length. In *The Compactification*, the count is a count of the hole's states that no comparison from outside tells apart. The count is reckoned relative to the outside as context. A hole of one solar mass has a count of about 1.05×10^{77}, about ten to the seventy-seven. The figure for one solar mass is established physics. The count grows as the square of the mass, so the largest holes hold most of the count of all the black holes. The hole at the center of our Galaxy has about four million solar masses, and the count of that hole is about 1.7×10^{90}, computed in *The Compactification*. Among the giant holes

at the centers of galaxies, the hole at the center of our Galaxy is of middling size, yet that one hole already holds more count than all the light and neutrinos of the early universe in the whole observable region. All the black holes in the observable universe together hold about 10^{104}, ten to the one hundred four. So the count of the black holes exceeds the count of the content by a factor of about ten to the fourteen. Ten to the fourteen is a hundred million million.

The growth of the count obeys one rule. The rule is the generalized second law, met in Chapter 9. The generalized second law says that the count of the matter outside black holes, together with the counts of the holes' horizons, never decreases. The generalized second law is established physics. The law is a statement within one epoch, about counts made relative to the outside world of that epoch.

The Largest Count

The largest count of an epoch is the count of the observer's particle horizon. In *The Gravitational Passage* I identify the count of the particle horizon as a count of the same form as a black hole's count: a quarter of the area of a sphere with the horizon's radius, measured in squares of the Planck length. With the measured radius of 46.1 billion light years, the count is 2.29×10^{123}. In words, the count is two point two nine times ten to the one hundred twenty-three. The horizon's count exceeds the count of all the black holes by a factor of about ten to the nineteen. Every black hole in the observable universe lies within the particle horizon, and the horizon's count includes the states of every black hole. The count of the particle horizon is reckoned relative to one observer, the observer whose measurement the count describes.

The count of the particle horizon is the only count that enters a vacuum energy, the energy that empty space holds in itself. Chapter 8 showed how the count enters. A measurement made from inside the vacuum engages a share of the vacuum's energy, and the share is the part of the en-

ergy that the measurement takes in. The share is the count of channels, three, times the trim, about 0.881, times the Planck density, divided by the count of the particle horizon. The formula gives 2.56×10^{-47} in the units physicists use (GeV to the fourth power), against the observed 2.52×10^{-47}. When the formula is joined with Friedmann's equation, the formula also gives a matter fraction (the part of the energy that is matter, ordinary and dark together) of 0.311. The present measurement fixes the cosmological constant once, as the constant term of one geometry. In the reconstructed history the cosmological constant has the one value that the present measurement fixes, at every era, before the present and after the present, and the argument for the single value is in *The Gravitational Passage*. The number w of a form of energy is the ratio of the energy's pressure to the energy in each unit of volume. Pressure is how hard the energy pushes outward on its surroundings, and a negative pressure is a pull. As Chapter 8 found, the w of the vacuum energy is exactly minus one. So the vacuum energy in each unit of volume stays constant as space grows.

Two other horizons have counts too: the event horizon and a black hole's horizon. Neither of the two counts enters a vacuum energy, because only the edge of what has reached us sets the energy of empty space, as Chapter 8 showed. The event horizon is drawn by what could reach us in the future, and a black hole's horizon bounds what can leave the hole, and not what can enter the hole. The event horizon is the sphere beyond which no signal sent now will ever reach us. Today the event horizon has a radius of 16.68 billion light years. In the future the radius approaches the de Sitter radius, 17.53 billion light years. A sphere of the de Sitter radius has a count of about 3.31×10^{122}, computed in *The Compactification*. Put the count of the de Sitter sphere into the formula in place of the count of the particle horizon. The formula then gives eight times the trim, about 7.05 times the observed constant. A black hole's horizon has a count too. For a black hole, take the formula without the trim: three times the Planck

density divided by the hole's count. The formula without the trim gives eight times the hole's mean density. Eight times the hole's mean density enters no vacuum energy.

Room to Grow

At the start, the count of the content was already high. Did the count of the content have any room left to grow? The count did have room to grow, and the room is measured against a ceiling that gravity sets. Picture a glass being filled from a jug, while the glass itself grows taller faster than the water rises. However full the glass was when you began, the empty space above the water keeps widening. In the picture, the water is the count of the content, and the height of the glass is the ceiling. We call the gap between the count of a region and the region's ceiling the region's *room to grow*.

The ceiling for a region is the count of one black hole made of all the mass and energy in the region. Chapter 7 met that ceiling in an estimate by the physicist Roger Penrose. Take the region within the Hubble radius, which Chapter 8 defined as the speed of light divided by the expansion rate. In the radiation era, the Hubble radius is also the distance a signal has traveled since the history began. The Hubble radius grows in proportion to the time. Now put all the mass and energy within the Hubble radius into one black hole. Friedmann's equation for the expansion, met in Chapter 8, shows that the horizon of that hole has the Hubble radius itself. A black hole's count grows as the square of the hole's radius, so the ceiling of the region grows as the square of the time: a hundred times as much time gives ten thousand times the ceiling. Within the Hubble radius, the count of the content grows more slowly. Two rules of the radiation era set the pace of that growth, and both rules are established physics. While the count of species stays the same, the temperature falls as one over the square root of the time: a hundred times as much time leaves a tenth of the temperature. The count in each unit of volume goes as the

cube of the temperature, so the count in each unit of volume falls a thousandfold while the time grows a hundredfold. Meanwhile the radius of the region grows in proportion to the time. So the volume of the region grows as the cube of the time: a millionfold. So the content's count in the region grows as the time to the power one and a half: a hundred times as much time gives a thousand times the count. The ceiling and the count of the content are both reckoned within the epoch, relative to the epoch's own context. So the ceiling grows faster than the count of the content from the beginning, and the room between the ceiling and the count widens, whatever the count of the content was at the start. The widening of the room is established physics, worked out in *The Compactification*.

How large was the room at the start? The answer is computed in *The Compactification*. With 106.75 species, divide the count of the content within the Hubble radius by the ceiling: the ratio is about 3.64 times the temperature divided by the Planck mass. The Planck mass is about 1.22×10^{19} GeV. The highest starting temperature that the account allows is 2 to 5×10^{17} GeV. That temperature is between about 1.6 and 4.1 hundredths of the Planck mass. At that temperature, the ratio of the count to the ceiling is 0.060 to 0.149. As the epoch cools, the ratio falls in proportion to the temperature. So at the beginning, when the content was hot and close to equilibrium, the content held at most about fifteen percent of the largest count that the region could hold. The largest count of a region is the ceiling set by gravity. The room above the count of the content widened as the epoch grew, and structure and black holes later fill that room.

One Draw Among Many?

In Chapter 7 we met Penrose's figure for how improbable our beginning was: one part in ten raised to the power ten to the 123. The figure is the ratio of two volumes in a space of states: the volume taken by a beginning as smooth as ours, against the volume available to the same matter if the count of the matter's gravity had been near the largest count possible. To

get the figure, Penrose treats the beginning as one draw from all the states the matter could have taken. Penrose weighs each of the possible states equally.

Picture a lottery. To call one ticket a one-in-a-million winner, someone must hold a million tickets and be able to tell the million tickets apart. An improbability works like the lottery: an improbability is a comparison of the actual case with alternatives that some context distinguishes. In Penrose's figure, the winning ticket is our beginning, and the other tickets are the other states the matter could have taken.

The Compactification contains an argument from the reset. The reset is the rule that no count reckoned against the old outside world is carried across the turn. Penrose's figure needs something able to tell all the other possible beginnings apart, and at the turn nothing is there to tell the possible beginnings apart: no comparison makes the count that the figure assumes. The actual beginning can be weighed against the other states the beginning could have taken only if the other states are distinguishable relative to some context. The interiors of the old epoch were counted relative to the old outside world. The old outside world has emptied, and the turn falls at no moment of the old outside world's time. The new outside world has no prior identity, and the new outside world registers none of the distinctions that the old outside world drew. Each other possible beginning is a state of a model of the new geometry: each such state exists as structure, but nothing in the new epoch ever measures the state or tells the state apart from the other states. So nothing in the new epoch finds any of the other possible beginnings. So there is no context relative to which the beginning of the new epoch is one draw from ten raised to the power ten to the 123 distinguishable alternatives. At the beginning, the low count of the geometry is the count that a new context starts with, so the low count is no improbable draw that would need a separate law to compel the low count.

The argument in *The Compactification* rests on the reset. The reset follows from a principle: every count belongs to its context, and the old context has ended.

The Past Hypothesis is the assumption, met in Chapter 6, that the world began in a state of low entropy. We can now settle the standing of the Past Hypothesis, using the reset. On the account, the Past Hypothesis is neither an axiom nor a brute law added to physics. A brute law is a law accepted with no reason behind the law. The claim of the Past Hypothesis splits into parts. As the source of the direction of time, the Past Hypothesis is not needed, because the structural arrow (the arrow of time that comes from the structure of comparison itself) does not depend on the Past Hypothesis. As a statement about the beginning of the epoch, the Past Hypothesis splits again: one part is about the content, and one about the geometry. The count of the content was high where the reconstructed history begins: a high count is the observed state of the content there, and the account does not need the count of the content to be low. The small count of the geometry follows from the restoration of context (the reforming of a large outside world at the turn) and the reset of the count, as *The Gravitational Passage* argues.

The Emptying, and the Reset

How does the count of an epoch end? The accelerating expansion carries everything that is not bound to the observer's own structure beyond the observer's horizon. The emptying is described in *At the Edge of Reality* in these words: "over a hundred billion to a trillion years, every observer's comparable world empties". Black holes form at every scale, and the accelerating expansion does not stop bound holes from merging.

The evaporation of the black holes comes far later. A hole of one solar mass glows at about 6.2 hundred-millionths of a kelvin, colder than the background around the hole, so the hole takes in energy until the background has cooled below the hole's temperature. The evaporation of the hole then takes about 2.1×10^{67} years, computed from established phys-

ics. The emptying of the outside world comes first, by an enormous margin, so, for every hole formed from a star, the outside world of the hole's epoch has emptied long before the hole returns its energy as radiation.

As the outside world empties, the conditions that hold the six small dimensions compact begin to fail. Inside each black hole, the interior approaches its end, and all the interiors are one at their end, since "a difference that no comparison can register is no difference". At the common end of the interiors, the turn is approached, and the history of the next epoch begins from the turn.

At the turn, every count of the epoch has lost its context. Every count of a horizon was reckoned relative to the outside world of the epoch, and so was the count of the content, and that outside world has emptied. As *The Gravitational Passage* argues, no count is carried across the turn, "for the matter or for the geometry, since each count is relative to the context that ends". The argument in *The Gravitational Passage* rests on the principle that a difference no comparison can register is no difference. The form of the content does pass across the turn, and the new context registers the form only as small departures from evenness. Each epoch begins its own count. The physicist Richard Tolman objected, as Chapter 7 told, that entropy carried from cycle to cycle gives a history made of cycles a first cycle. Because each epoch begins its own count, Tolman's objection has its answer. No count piles up from cycle to cycle, so the cycle has no first round.

Two Arrows Across One Epoch

The counts of one epoch run in one direction: from a small count of the geometry and a high count of the content at the beginning, through the growth of structure and of black holes, to the largest counts at the end. The direction in which the counts run is the entropic arrow of time, met in Chapter 6. The entropic arrow is statistical: the arrow belongs to systems of very many parts. The entropic arrow needs a beginning of low entropy

from which the growth proceeds. The low entropy that the entropic arrow needs is the low count of the geometry, which began small. The count of the content began high.

The direction of time itself has another source, the structural arrow of Chapter 6: the order that the second axiom states, the photographer's position, then the click, then the print. *Measurement* identifies the order of the second axiom with the one-way order of findings. In the words of *Physics Is Not Math*, that order "needs no large number of parts and no special beginning of the world". The two arrows point the same way wherever both are found, and each act of recording joins the two arrows: every record follows the act it records, and laying down a record raises the count of the record's surroundings.

The Whole Count in One Table

The table below contains the counts of this chapter, each in nats. Each count is for the region that is now the observable universe, unless the row says otherwise. Each row gives the standing of its count.

Stage or object	What holds the count	Count	Standing
Beginning of the history	the content, hot and close to balance	high: about 10^{90} if no stage that behaved as matter came first; the geometry's part small	argued (*The Gravitational Passage*); the content's count computed
Waves of the settling	radiation of the four large dimensions	below about 3 percent of the radiation; less than 0.09 of one light species	measured bound; its restatement computed
Radiation era	the content, kept in a comoving region	species from 106.75 down to 3.909	established physics

Stage or object	What holds the count	Count	Standing
The two changes of state	nothing added	crossovers at 159.5 GeV and 156.5 MeV	established physics (lattice computations, cited)
Light today	the microwave background	5.2×10^{89}	computed from the measured temperature
Neutrinos today	the free sea of three kinds	4.9×10^{89}	computed from measured values
One hole of one solar mass	its horizon	1.05×10^{77}	established physics
Hole at the center of the Galaxy	its horizon	1.7×10^{90}	computed from measured values
All black holes today	the giant holes at the centers of galaxies	about 10^{104}	established physics, cited
Particle horizon today	the observer's horizon	2.29×10^{123}	identification; computed
Sphere of the de Sitter radius	none that enters a vacuum energy	3.31×10^{122}	computed from measured values
The turn	no count	none carried across	argued (*The Gravitational Passage*)

Every row of the table is said in words in the sections above, in the order of the rows. So the table, read from top to bottom, retraces this chapter.

We now know how the whole count of an epoch changes from the beginning of the epoch to the end. The count begins high in the content and small in the geometry. The count of the content is kept through the radi-

ation era and through every smooth change of state, so the ten to the ninety of today is the count at the start of the radiation era. Gravity then raises the count of the geometry, and black holes come to hold about ten to the one hundred four. The particle horizon holds the largest count, and the count of the particle horizon alone fixes the energy of empty space, once. The epoch ends as the epoch's outside world empties, and the count starts fresh at each turn. The course of the count matters for the question of this book, what the beginning left behind. The course of the count shows what the beginning left behind in the way of order: a content already near the largest count that its energy allows, a ceiling set by gravity that stood far above the count of the content and kept growing faster than the count, and no count carried over from the epoch before.

> **Key idea.** Every count belongs to its context. The count of the content is high from the start, and the count is kept through the radiation era because the expansion is adiabatic and both changes of state are crossovers. Gravity reverses the usual order, in which an even gas has the largest count, so clumping and black holes raise the count. The particle horizon holds the largest count, and the count of the particle horizon is the one count that enters a vacuum energy. At the turn no count is carried across.

Summary

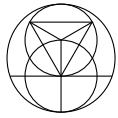
We followed the counts of one epoch through the epoch's reconstructed history, in one comoving region: the matter now within our particle horizon, 46.14 billion light years in radius, followed back in time. At the beginning the content is hot and close to equilibrium, so the count of the content is already high. The geometry is even, so the count of the geometry is small, and both statements are argued. The measured amount of dark matter sets a ceiling on the starting temperature, about 2 to 5 times ten to the seventeen GeV, and the account does not fix the starting temperature itself. If a stage that behaved as matter came first, the decay of that stage

would raise the count, and the account fixes neither whether there was such a stage nor how large the rise would be. The gravitational waves made as the six settle add less than about three percent of the radiation, a measured bound. Through the radiation era the count is kept, because each kind that leaves passes its count to the rest while the count of species falls from 106.75 to 3.909. Both changes of state are crossovers, at 159.5 GeV and 156.5 MeV, and the crossovers add nothing to the count. Today the light and the neutrinos carry about ten to the ninety, computed from measured values. The black holes hold about ten to the one hundred four. The particle horizon holds two point two nine times ten to the one hundred twenty-three, by an identification, and the count of the particle horizon is the one count that enters a vacuum energy and fixes the cosmological constant once. From the start, the ceiling of a region grew faster than the count of the region's content, and the count of the content began at most about fifteen percent of the ceiling. Penrose's improbability needs a context that tells the possible beginnings apart, and at the turn there is no such context, as *The Compactification* argues from the reset. The epoch ends as the epoch's outside world empties, and no count passes the turn.

The beginning was even, but not perfectly even. What were the small departures from evenness at the beginning, and where can we see those departures?

PART III

WHAT THE FIRST LIGHT RECORDS



Chapter 11

The Oldest Light

Turn a radio telescope to any part of the sky, in any direction, by day or by night, and the telescope picks up a faint glow of microwaves. Astronomers call the glow the *microwave background*: light left from the early universe, which the expansion of the universe has stretched until the light's temperature is now about 2.7 degrees above absolute zero. The microwave background is the oldest light there is, the faint afterglow of the beginning of our epoch (our era of the world). Chapter 10 followed an epoch's count of states (the epoch's entropy) from a beginning that was even, though not perfectly even. The microwave background is where we see the small differences with which the beginning started. The question of this chapter is the one you would ask on first hearing of the glow. What is the faint glow in the microwave sky, and what are the tiny differences that the glow carries?

Picture a photograph of the whole sky, taken at one moment long ago: the moment the universe first became transparent. Before that moment, light could not cross any distance in a straight line. After that moment, light traveled freely. Some of the light set free then has traveled ever since and is arriving now. The microwave background is that photograph of the whole sky. Each particle of light in the glow was last turned aside at about that one moment. So the glow shows the universe as the universe was then.

A Fog That Clears

Picture a valley filled with fog on a sunny morning. The fog glows, bright all through, and nothing can be seen across the valley. Each ray of light is turned aside by one droplet after another, so a ray that reaches the eye comes from some droplet close by, and never from the far side of the valley. Then the fine droplets gather into raindrops and fall. The water is all still there, gathered into a few large drops, and the light passes between the drops. The air clears, and the far side of the valley comes into view.

The young universe was like the valley full of fog. A hot crowd of particles and light filled the young universe, so hot that nothing in the crowd held together. Chapter 10 called that crowd a plasma, and here I call the crowd the *gathering*, as *The Coming of the Kinds* does. Chapter 1 met the reconstructed history: the one history we rebuild from what reaches us now. The turn is the passage from the end of the old epoch's black hole interiors to the restored outside world of the new epoch. A few minutes into the reconstructed history, the gathering held the light and the nuclei of hydrogen and helium, with traces of others. The gathering also held the neutrinos, the lightest particles of matter, traveling free, and the dark matter made at the turn, which acted on everything else in the gathering only through the dark matter's weight. The gathering held every part of an atom, but the gathering held no atoms. An atom is a nucleus with electrons bound to the nucleus by electric attraction. Hydrogen, the simplest atom, is one proton holding one electron.

The gathering held one free electron for every proton. The reason is a kind of bookkeeping that electric charge keeps. Picture a cloakroom at a theater, where every coat handed in gets one ticket, and every ticket belongs to one coat. At the end of the evening, however many coats and tickets have come and gone, the coats still hanging match the tickets still out, one to one. Electric charge keeps accounts the way the cloakroom does, and the universe carries no net electric charge. In the picture the coats are

the positively charged protons and the tickets are the negatively charged electrons. The positrons are the opposites of the electrons, and in the first minutes of the reconstructed history the electrons and positrons annihilated: they met in pairs and disappeared, and their energy left as light. Just before the electrons and positrons annihilated, the electrons outnumbered the positrons by exactly the number of the protons, about two extra electrons for every billion positrons, and the annihilation left those extra electrons behind, one for each proton.

A particle of light is called a *photon*, and a free electron turns aside any photon that strikes the electron. Because the gathering held one free electron for each proton, the photons were turned aside again and again. No light crossed any distance in a straight line. The gathering was the fog of the valley. Its free electrons were the droplets. An electron bound in an atom is like water gathered into a raindrop. A neutral atom lets nearly all the light at the temperatures at which atoms form pass by, because nearly every photon of that light carries too little energy to free the electron, or to lift the electron within the atom.

The atoms formed only after a long time, because the light was so plentiful. For every hydrogen nucleus there were about 2.2 billion photons. The count of 2.2 billion is more than the count of 1.6 billion photons for every proton and neutron counted together, because about three protons and neutrons in four are hydrogen nuclei, and the rest are bound in helium. The count of 1.6 billion is Chapter 10's 6.13 protons and neutrons for every ten billion photons, turned the other way up. Among so many photons, a few had enough energy to knock an electron free, and those few kept knocking electrons free long after a typical photon carried far less energy than it takes to free an electron. A second count also favors the free electrons: the count of the states in which an electron can be. A free electron can be anywhere in the gathering and can move in any direction at any speed. A bound electron has only the few states of its atom. Both counts bear on the settled balance: the state in which electrons are bound exactly as often as they are knocked free. So the settled balance favors the free electrons until the temperature has fallen far below the en-

ergy that binds the electron. The physicist Meghnad Saha set out a rule for that balance in 1920. By Saha's rule, half the electrons of the hydrogen would be bound in balance near 3,700 kelvin. A kelvin, met in Chapter 9, is a degree of temperature counted up from absolute zero. A temperature of 3,700 kelvin is about one forty-second of the energy that binds hydrogen's electron. The binding in the gathering ran a little behind even Saha's balance, so some electrons stayed free longer than balance would leave them. In 1968 the physicist James Peebles set out the theory of that lagging binding and of the electrons that the lagging binding leaves free, and by Peebles's theory, at the end, more electrons stay free than balance would leave. Hundreds of thousands of years passed while the gathering cooled to the few thousand kelvin at which the electrons could stay bound.

As the electrons were bound into atoms, the light was turned aside more and more rarely. At last a photon was turned aside one more time and then traveled on without meeting another free electron, and that last turning aside of the light by the free electrons of the gathering is called *last scattering*. Last scattering is often called recombination. About one electron in ten was still free at last scattering. The light was then at about 2,973 kelvin, some fifty-three times below the energy that binds hydrogen's electron. Last scattering came about 370,000 years into the reconstructed history. From last scattering on, the light traveled free. In the picture of the valley, the fog had turned to rain, and we look across to the far side. The light reaching us now from every direction comes from the place, all around us, where the gathering last turned that light aside.

The Light That Reaches Us

As the universe expands and distances grow, the waves of light traveling through space are stretched along with the distances. The stretching of the waves is called *redshift*. When light arrives now from a moment in the past, its waves are longer than the waves it had when it set out, longer by

the factor by which every distance has grown since that moment. The temperature of the light falls by the same factor. Astronomers quote a redshift as the factor of growth less one. The Planck satellite was a space telescope that mapped the microwave background. The Planck satellite measured the redshift of last scattering as 1089.92, give or take 0.25, so every distance in the universe has grown 1,090.92 times since the light of last scattering went free. The light is now at 2.72548 kelvin (Fixsen, 2009). Multiply the growth factor of 1,090.92 by the present temperature of 2.72548 kelvin, and the product is the temperature of the light at last scattering: 2,973 kelvin, give or take about one kelvin. This multiplication is how we know the temperature of 2,973 kelvin given in the section "A Fog That Clears". Space now holds about 411 photons of the microwave background in every cubic centimeter, a number worked out from the measured temperature.

The photons of the microwave background are older than the moment at which they went free. The photons of the microwave background are the photons of the hot gathering itself. They were brought into balance with the gathering far earlier. They were warmed when the electrons and positrons annihilated. Picture two friends in a warm room on a winter evening. One friend steps out onto a glassed balcony and shuts the door behind him. A moment later a log in the fireplace catches and the log's heat fills the room, while the balcony, behind its closed door, is not warmed. Through the night the room and the balcony both cool, each by the same proportion as the other. So the room stays warmer than the balcony all night long, and always by the same proportion: the proportion the log gave the room. In the picture, the light is the room. The neutrinos, which had already gone free, are the friend on the balcony. The night is the expansion, and the log is the annihilation of the electrons and positrons. So the light today is at 2.7255 kelvin and the neutrinos are at 1.945 kelvin: the light is warmer by the cube root of eleven fourths, about 1.40, as Chapter 10 showed by counting. Every photon of the microwave background belongs to the light of a gathering that took in the energy of the electrons and positrons.

After the electrons and positrons annihilated, no kind of particle left in the gathering was heavy enough to drop out of the gathering as the gathering cooled, so the kinds of particle in the gathering were fixed: the light, the free neutrinos, the protons and neutrons, and the electrons. Chapter 10 showed that the matter came to hold more energy than the radiation near 9,300 kelvin, before the atoms formed. The matter has held more energy than the radiation ever since. The first atoms were those of the light elements: about twelve hydrogen atoms for every atom of helium, with traces of a few other elements. When the first atoms formed, the ordinary matter of the universe became neutral matter for the first time: nuclei with their electrons bound to them. So at last scattering the particles of the universe were in these forms: the protons and neutrons in nuclei, the electrons bound in atoms, the light traveling free, the neutrinos free since the first second, and, beside them all, the dark matter made at the turn. I leave aside the later history of the universe's large forms, in which the atoms gathered into clouds, galaxies and stars.

A Faint Pattern

The microwave background is very nearly the same in every direction. The temperature of the glow differs from one direction of the sky to another by about one part in a hundred thousand, a difference first measured in 1992 (Smoot and others). On a sky at 2.7255 kelvin, one part in a hundred thousand is about twenty-seven millionths of a degree. Some patches of the sky are warmer than the average by about twenty-seven millionths of a degree, and others are cooler by as much. The differences of temperature show slight differences of density in the gathering, and the structure of the universe later grew from those differences of density: the galaxies, and the stars within the galaxies.

In *The Compactification*, the main source of this book, these differences of density are called the *departures from uniformity*: the faint differences of density with which an epoch begins. When the subject is the beginning of

an epoch, the departures from uniformity are also called the primordial departures. I also call the departures from uniformity the first ripples, as the book's title does.

I wrote about the faint pattern of the microwave background in *At the Edge of Reality*, in that book's chapter "The Faint Pattern in the Sky". There I explained that the sky is alike in every direction because the new epoch re-forms from one domain, and I read the departures from uniformity as the old distinctions, averaged. The one domain is the interiors of the old epoch's black holes at their end. The interiors are one at their end, because no comparison there registers a difference between one interior and another. The old distinctions are which alternatives the content of the old epoch took. In the rest of this chapter I go further. I set out what each measured quantity of the pattern means and what its measured value is, and I state the one step from the account of the turn to the sky.

How the Ripples Are Described

Astronomers report the first ripples by a small set of quantities. Each quantity has a plain meaning. Every one of the quantities belongs to the standard description, which every cosmologist uses, whatever theory of the beginning he or she holds.

The first quantity is the strength of the ripples. Walk around a hill keeping always at one height, and your path is a contour line: a line joining the places at that one height. Picture a contour line for the young universe, with density in place of height: a contour that joins, through all of space, the places where the total density has one chosen value. The universe thins as it expands, and the universe does not reach the chosen density everywhere at quite the same stage. So along one contour of the chosen density, space has stretched a little more at some places and a little less at other places. Physicists use a quantity called the *curvature perturbation*, which says, place by place along the contour, by what fraction space has stretched more or less than it has on average. Where the stretching dif-

fers from place to place, the contour of one density is itself curved a little, hence the word "curvature" in the quantity's name. The horizon of a given time is the greatest distance a signal could have crossed by that time. When the ripples are of the kind called adiabatic (defined below), the curvature perturbation stays the same, as time passes, on every length larger than the horizon. So an adiabatic ripple wider than the horizon keeps the strength the ripple began with. In Chapter 12 an adiabatic ripple wider than the horizon is called frozen. The typical size of the curvature perturbation is the *strength* of the ripples, about five parts in a hundred thousand. Across the largest patches of the sky, the differences of temperature are smaller than the strength by a factor of about five, a standard result of the description, which the appendix For the Physicist gives. The factor of about five is why the strength is usually quoted as about one part in a hundred thousand.

The second quantity is how the strength of the ripples is spread over lengths. Picture the display on a music system that shows a row of small columns of light, one column for each band of pitch, from the deepest notes at one end of the row to the highest notes at the other. Each column rises as high as its band of pitch is loud. Read across the row, and you can tell at once whether the music is heavy in the bass, bright in the treble, or even throughout. The ripples of the young universe can be shown on a display of the same kind, with bands of length instead of bands of pitch. The strength multiplied by itself is called the *power* of the ripples, and the power is the quantity that the measurements compare. Divide the lengths into bands, each band running from one length up to a fixed multiple of that length, so that every band spans the same factor of length. The *power spectrum* gives the power of the ripples in each band: how strong the ripples are at each length. In the picture of the music display, the power in each band is the height of its column. The *amplitude* is the power at one reference length, which astronomers have chosen by convention.

The third quantity is the *spectral index*, the number that says how the power changes from band to band. An index of exactly one means the same power in every band: the columns of the display stand all at one

height. When a pattern has the same power in every band, the pattern is called *scale invariant*, because no length stands out from any other. An index a little below one means that the power grows slowly toward the longer lengths. An index a little above one means that the power grows toward the shorter lengths. How far the index lies from one is called the *tilt*: the lean of the row of columns. The index itself may change from one end of the row to the other, and that change is called the *running*. When the lean of the row is the same along the row's whole length, the row has no running.

The fourth quantity concerns a second kind of departure from uniformity. Chapter 4 met the gravitational wave, a ripple of the arena itself (of space and time together). As a gravitational wave passes, it stretches distances in one direction and squeezes them in another, and then the reverse. When gravitational waves are present from the beginning of an epoch, they are called *primordial gravitational waves*, and I call those waves the first waves. The first waves have their own power at each length, and the *ratio of first waves to first ripples* is the power of the first waves divided by the power of the first ripples, both taken at the reference length. Physicists call the same ratio the tensor-to-scalar ratio, after the mathematical names of the two sorts of departure, the first waves and the first ripples. A ratio of zero means no first waves at all. A ratio of one means first waves as strong as the first ripples.

The fifth quantity is how the ripples are shared among the ingredients of the universe: the light, the neutrinos, the ordinary matter and the dark matter. Picture a large pot of soup that has not been perfectly stirred, and picture two ways in which the soup can be unevenly mixed. In the first way, a ladle from a thicker spot holds more of everything, broth, beans and carrots alike, in the same proportions as a ladle from anywhere else, and only the amount in the ladle differs. In the second way, one spot holds more beans and less broth, so every ladle weighs the same but the mix differs from spot to spot. In the picture, the broth, the beans and the carrots stand for the ingredients of the universe. A departure of the first kind is called *adiabatic*: every component of matter and radiation departs from

uniformity together, so the number of particles of any one kind, divided by the number of any other kind, is the same everywhere. Chapter 10 used the word "adiabatic" for an expansion that keeps the count of the content in a region the same. For a departure, the word has the meaning just given. A departure of the second kind is called *isocurvature*, from words meaning "the same curvature": one component of the universe departs against another, with the total density unchanged at first.

The last quantity is the *curvature of space* on the largest scale. Draw a triangle on a flat sheet of paper, and its three angles add to exactly half of a full rotation. Draw a large triangle on a ball, and the angles of the triangle add to more than half a rotation. Draw a triangle on a saddle, and its angles add to less. Space on the largest scale can be flat like the paper, or curved like the ball or the saddle. Chapter 8 met the Friedmann equation, named for the physicist Alexander Friedmann: the rule tying the rate of the expansion to what the universe holds. The curvature of space enters the Friedmann equation as one more share of the total density, beside the shares of the matter, of the radiation and of the energy of empty space. Astronomers report the curvature as its share of the total density, which they write Omega K. A share of zero is flat space.

Rules tie the quantities of this section to the temperature of the sky and to the other numbers that describe the universe as a whole. Those rules are established physics: the standard description, cited unchanged. The formulas behind every quantity of this section are set out in the appendix For the Physicist.

What the Measurements Find

The final results of the Planck satellite give the two central numbers of the description: the spectral index and the amplitude. Astronomers read the two numbers from the temperature of the glow, from the direction in which the light of the glow vibrates, and from the slight bending of the paths of that light by the matter the light passed on its way to us. The

spectral index is 0.9649, give or take 0.0042: a little below one. So the tilt, how far the index lies from one, is about three and a half hundredths. The power of the ripples therefore grows slowly toward the longer lengths. The amplitude is about two and one tenth billionths (2.10×10^{-9}), and the square root of the amplitude is the strength of about five parts in a hundred thousand. The running is minus 0.0045, give or take 0.0067, a value consistent with zero: within the spread of the measurement, the row of columns leans by the same amount at one end as at the other.

The measurements set an upper limit on the first waves. The BICEP and Keck telescopes study the microwave background from the South Pole. Astronomers combined the data of the BICEP and Keck telescopes with the data of the Planck satellite and of WMAP, an earlier satellite that mapped the microwave background, and the combined data show that the ratio of first waves to first ripples is below 0.036, about three and a half hundredths, with ninety-five percent confidence. Ninety-five percent confidence is the measurers' way of saying nineteen chances in twenty that the true value lies inside the bound. The polarization data of the Planck satellite are its data on the direction in which the light vibrates. A joint analysis with those polarization data sets the bound lower still: the ratio is below 0.032, about three hundredths (Appendix B). The measurements also set an upper limit on the isocurvature departures. When the measurers allow the two sorts of departure, adiabatic and isocurvature, to mix, an isocurvature departure of the dark matter carries less than 1.3 percent of the power of the sky's differences of temperature. The bound on the isocurvature departure holds at the same ninety-five percent confidence. Astronomers read the curvature share from the glow, together with the bending of the glow's light, and from maps of how the galaxies are spaced. The curvature share is 0.0007, give or take 0.0019: seven ten-thousandths, with a spread nearly three times as large, so the measurement of the curvature share is consistent with flat space.

Quantity	Measured value	Standing
Temperature of the glow today	2.72548 kelvin	measured
Redshift of last scattering	1089.92, give or take 0.25	measured
Temperature at last scattering	2,973 kelvin	computed from the two measured values above
Spectral index	0.9649, give or take 0.0042	measured; an input of the account
Amplitude (power at the reference length)	2.10 billionths	measured; an input of the account
Running	minus 0.0045, give or take 0.0067	measured; consistent with zero
Ratio of first waves to first ripples	below 0.036	measured bound
Isocurvature share of the dark matter	below 1.3 percent	measured bound
Curvature share	0.0007, give or take 0.0019	measured

Every number in the table comes from measurement. The account uses two of the numbers, the amplitude and the index, as its inputs. The other numbers bear on four properties that I argue the first ripples must have. The four properties are named in the last section of this chapter.

Other books of this collection use the pattern too, and those uses show how much can be read from the pattern. *The Coming of the Kinds* uses the pattern in three places. First, the ripples of the pattern are adiabatic, with every component departing together. So the beginning left a small surplus of matter with one sign everywhere: nowhere does antimatter outnumber matter. Second, astronomers weigh the radiation other than light from the shape of the pattern's spectrum, and they find the sea of three kinds of

neutrino. Deuterium is heavy hydrogen, whose nucleus is one proton and one neutron bound together. Third, the pattern gives the number of protons and neutrons for each photon, about six in ten billion (6.13×10^{-10}), and the deuterium made in the first minutes gives the same number again.

What Passes the Turn

What passes the turn is the form of the content: which alternatives the content has taken, that is, the specification of what has been selected. The content is what a comparison finds. An alternative is one of the definite possibilities in which a quantum thing can be held, as a tiny magnet can point up or down. Chapter 1 set out the turn itself, the limit between the collapse of the interiors of one epoch's black holes and the reconstructed history of the next epoch, and no comparison registers the passage through the turn as an event. Chapter 7 set out what stays behind at the turn: every count of states (every entropy) reckoned against the old outside world, for the matter and for the geometry alike. The content passes the turn, and the form of the content passes with the content.

I take one step from what passes the turn to the pattern in the sky, and the step is an identification, which I make here in the first person, with the observation that bears on the identification. I identify the primordial departures from uniformity with the form of the content carried from one side of the turn to the other, the distinctions of the old epoch averaged and not erased. The observation that bears on the identification is the pattern of the first ripples in the sky.

In everyday words, everything that fell into the black holes of the epoch before ours had taken some alternatives and not others. Which alternatives everything that fell in had taken is the form of the content, and that form is carried through the turn. Within the new epoch the form is registered as small differences of density. Those differences of density are

the first ripples. In the passage through the turn, the distinctions of the old epoch are averaged together and kept. The averaged distinctions are the faint pattern in the glow.

How can averaging keep anything? In everyday speech an average throws the details away, but the averaging at the turn is not averaging in that everyday sense. Climb the stone stairs of an old church or castle, and look at the steps. Each step is worn into a shallow curve, lowest where most feet have fallen over hundreds of years. No single footstep can be picked out in the worn curve. Yet no footstep has been wiped out of the curve either: each footstep wore its own small share, and the curve is all the footsteps together, one shape on one stone. The wear is slight beside the size of the step, and still the wear is the shape that every footstep made. The worn step shows what "averaged and not erased" means. In the picture, each footstep is one distinction of the old epoch, and the worn curve is the faint shape the distinctions make together. The distinctions are which alternatives the old epoch's content took, and they are brought together into one faint shape. None of the distinctions is thrown away. Within the new epoch, the form that the old distinctions make is registered only as small differences of density. The old distinctions themselves are not registered. Those small differences of density are the first ripples.

Empty Space Adds Almost Nothing

The identification has a first consequence: the empty space of the new epoch adds almost nothing to the first ripples. In this chapter and in the chapters that follow, every later step assumes that consequence. Why look at empty space at all? The usual story of the beginning is the subject of Chapter 13. In the usual story, the first ripples are made from the trembling of empty space, stretched from tiny lengths up to the lengths we see. So the question for the account is how much the trembling of empty space adds. Empty space is never perfectly still. Every field keeps a lowest mo-

tion that cannot be removed: the zero-point energy of Chapter 4, the restless trembling of the fields at their lowest. The trembling of empty space carries ripples of its own. *The Gravitational Passage* contains the computation of how strong those ripples are on the lengths we observe.

Take a length that astronomers observe today and trace it back in the reconstructed history to near the turn. Near the turn, the observed length was some ten to the twenty-seven times (a billion billion billion times) wider than the horizon of that time, the greatest distance a signal could have crossed since the turn. On a wave so much wider than the horizon, the trembling of empty space carries a power below ten to the minus fifty-one: a one in the fifty-first place after the decimal point. The observed power of the first ripples is about two billionths. So the power of the trembling falls short of the observed power by a factor of more than ten to the forty-two. The powers of ten in this paragraph are computed in *The Gravitational Passage*.

On this account, then, the first ripples are the content's own form, registered by the new context, the new outside world that re-forms at the turn, as small differences within the new epoch. The first ripples are not the trembling of empty space stretched to large lengths, because on the lengths we observe the trembling is smaller than the observed ripples by more than forty-two powers of ten.

What the Pattern Must Show

I argue four properties of the first ripples from the identification together with one premise: the new context has no prior identity. That is, the new context inherits no record, no length and no mark that sets one part of the new context apart from another part. The premise is argued in *The Gravitational Passage* from the way the four-dimensional geometry of the new epoch re-forms: in that book's words, "from a geometry in which it had

no separate identity". Each property follows with the word "must": if the identification and the premise are true, the property holds in the reconstructed history of the epoch.

The first property is that the first ripples are linked across the whole sky and nearly the same in strength at every length, unlike the pattern that independent pieces, each rippling on its own, would leave. The second property is that the first ripples are one form for every component of the content, so they are adiabatic, with no isocurvature departure. The third property is that the first ripples begin as departures of density, and first waves arise only from ripples acting on ripples, so the ratio of first waves to first ripples is far below one millionth. The fourth property is that the large-scale model of the new epoch, which describes the epoch with its ripples set aside, is flat in space.

The account fixes the kind of the first ripples: how the ripples are linked across the sky, and how they are shared among the components. The strength and the tilt are measured inputs: a strength of about one part in a hundred thousand in the temperature of the sky, and an index of about 0.97. The four properties fix neither the strength nor the tilt. So the strength and the tilt are each measured, with the reason for each still a question. What count of states sets the strength, and what length sets the tilt? Even so, the tilt has a meaning in the account. When a pattern refers to no length, so that no length stands out in it, the pattern is exactly the same at every length. So a tilt is a reference to some length. The measured index, a little below one, is a faint reference of the first ripples to a length of the new epoch.

We now know what the oldest light is, what its pattern is made of on this account, and what the account must say about the pattern. What we now know matters for the book's question, because the glow is where what the beginning left behind can first be seen. The first ripples are the averaged form of everything the black holes of the epoch before ours took in, and the galaxies of our epoch, with their stars and planets, grew from the first ripples. The measurements of this chapter are the record against which the four properties are checked.

Key idea. The microwave background is the light of the hot young universe, set free at last scattering, when the first atoms formed and the fog of free electrons cleared, about 370,000 years into the reconstructed history. The microwave background is now at 2.7255 kelvin. Its temperature differs from place to place by about one part in a hundred thousand, and those differences are the first ripples. Astronomers report the first ripples by quantities that are definitions of the standard description. The quantities are the strength, the power at each length, the index, the tilt, the running, the ratio of first waves to first ripples, adiabatic and isocurvature departures, and the curvature of space. On this account the first ripples are the form of the content carried through the turn, the old distinctions averaged and not erased. The empty space of the new epoch adds almost nothing to the first ripples.

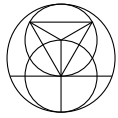
Summary

The faint glow in the microwave sky is the oldest light: the light of the hot gathering of the young universe, warmed when the electrons and positrons annihilated, and held in a fog of free electrons until the first atoms formed. The oldest light went free at last scattering, at about 2,973 kelvin. Last scattering came some 370,000 years into the reconstructed history. Every distance has grown 1,090.92 times since last scattering, and the light is now at 2.7255 kelvin, with about 411 photons in every cubic centimeter. The temperature of the light differs from place to place by about one part in a hundred thousand. Those differences are the first ripples: the departures from uniformity with which our epoch began, and from which the galaxies of our epoch grew.

Astronomers report the first ripples by quantities of the standard description, and each quantity is a definition. The measured values are an index of 0.9649, an amplitude of about two billionths, a running consistent with zero, a ratio of first waves to first ripples below 0.036, an isocurvature

share of the dark matter below 1.3 percent, and a curvature share of 0.0007, give or take 0.0019. At this point I make one identification: the first ripples are the form of the content carried from one side of the turn to the other, the old distinctions averaged and not erased. The first consequence of the identification is computed: on the lengths we observe, the empty space of the new epoch adds a power more than forty-two powers of ten below the observed power. I argue four properties of the ripples from the identification and from the premise that the new context has no prior identity. The strength and the tilt are measured inputs, each with its reason still a question.

If the pattern is the old epoch's form carried through the turn, what must the pattern look like?



Chapter 12

Four Marks of the Pattern

If the faint ripples in the oldest light are the form of the old epoch's content, carried through the turn, what must the pattern of those ripples look like? The answer in this chapter has two parts. The first part is four marks that the pattern must carry. The second part is two numbers for which the account taught in this book uses the values measured in the sky itself.

The answer rests on two things that earlier chapters set down. The first thing is the identification I made in the last chapter. I identified the first ripples with the form of the content carried from one side of the turn to the other. The carried form is the distinctions of the old epoch, averaged and not erased. The first ripples are the departures from uniformity, the faint differences of density with which our epoch begins. The form of the content is the specification of what has been selected: which alternatives the content has taken. The second thing is a premise this book has used since its first part: the new context has no prior identity. The new context is the outside world that re-forms at the turn. The new outside world re-forms "from a geometry in which it had no separate identity", in the words of *The Gravitational Passage*. *The Gravitational Passage* is the book of this collection that sets the whole account out for physicists. The new outside world inherits no record, no length and no mark that sets one part of the new outside world apart from another.

Four marks of the pattern follow from the identification and the premise together. If the identification and the premise hold, each mark must hold in the reconstructed history of our epoch. The reconstructed

history is the history we rebuild from what reaches us now. The first mark is that the pattern is correlated across the whole sky. The second mark is that every ingredient of the universe ripples together. The third mark is that the ripples are ripples of density. Waves of space itself arise from the ripples only later and only faintly. The fourth mark is that space as a whole is flat. Each mark has its picture and its measurement, and this chapter describes the four marks one at a time. After the four marks come two numbers: how strong the ripples are, and how their strength tilts from large sizes to small. The account uses measured values for the two numbers.

One Origin, One Cloth

Look at the microwave sky in one direction, then turn and look in the opposite direction. Measurement shows that the oldest light reaching you from the two directions has one temperature, to about one part in a hundred thousand. Now take two directions of the sky more than about a degree apart, about twice the width of the full Moon: in the standard history, no signal could have passed between those two directions before the oldest light was released. In each of the two directions, the region of the sky lay beyond the horizon of the region in the other direction. The horizon of a region is the greatest distance a signal could have crossed in the time there had been. How could regions that never exchanged a signal agree so closely? Physicists call the question the *horizon problem*.

At the Edge of Reality, the earlier book that this book continues, answers the horizon problem with a picture. If two towns far apart were founded from one place, the two towns may share an accent. The people of the two towns may never have spoken to each other. The regions of the sky are like the two towns. The place the towns were founded from is like the end of the old epoch's black holes. At the end of the interiors of the old epoch's black holes, no comparison registers a difference between one interior and another. So at that end the interiors are one domain. The new

outside world re-forms from the one domain of the ended interiors, with a geometry that has no prior identity. Where no signal has connected two regions of the sky, the regions are alike because the regions come from one domain, so no signal ever had to pass between the regions.

The one domain from which the regions come also fixes one feature of the pattern: how the strength of the ripples changes from large lengths to small. A second picture shows how one domain fixes the change of strength with length. Lay a floor of ten thousand small tiles, each shaded light or dark by a different worker who never saw any other tile. Stand close, and you see plenty of light and dark. Stand far back, and the floor melts into an even gray, because over any wide patch the lights and darks of independent tiles cancel. Now picture a cloth woven on one loom, with one design running through it. The swirls are as strong across the whole width as across a hand's breadth, because every part of the cloth came from one plan.

The tiled floor pictures a world assembled from independent pieces. Each old interior would give one piece, with its own ripples. When independent pieces are averaged, the pieces leave a pattern called *white noise*, in which the ripples die away on large lengths, as the tiled floor goes gray from far off. The last chapter introduced the spectral index, the number that measures how the strength of the ripples changes with size. An index of one means the same strength at every size. By plain arithmetic, the average of independent pieces has an index far from one. The measured index of the sky is close to one, which means nearly the same strength at every length, as with the woven cloth.

The account says which of the two pictures, the floor or the cloth, the sky must match. In the new epoch, no comparison registers which old interior any part of the new epoch's content came from. So the form that passes the turn is one form, correlated across the whole new outside world. The form is not a sum of independent pieces. Therefore the pattern of the sky must be near the same strength at every length, like the cloth, and the pattern cannot be the white noise of independent pieces, like the tiled floor. The first mark of the pattern is that the pattern is correlated

across the whole sky, near the same strength at every length. The exact value of the index is a second matter. The account uses the measured value, and the index comes up again in the part of this chapter on the strength and the tilt.

A pattern nearly the same at every length is a regularity, a kind of sameness. The Incommensuration Theorem applies to every sameness that a comparison finds. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry (sameness through a change) and continuity (smoothness, with nothing sudden) are never both made perfect in what a comparison finds. A well-shuffled deck of cards shows how the sky stays within the limit that the theorem sets. Every order of the deck is equally likely, so the odds of the deck are the same for every order. The deck in your hand lies in one particular order, with its own runs and gaps. The shuffled deck is like the sky: the equal odds of every order are like the odds of the ripples, and the one order in your hand is like the one actual sky. *At the Edge of Reality* says the same of the sky: "A pattern can be the same at every scale in its statistics, which is a sameness of the probabilities across a difference within the possibilities, while one actual sky, with its own particular hot and cold spots, is selected from the range. That pairs a symmetry and a continuity in two conjugate aspects." The quotation names two aspects: the odds of the ripples, which are the probability of the ripples, and the range of patterns the sky could have had, which is the possibility of the ripples. The odds hold the symmetry: the odds are the same at every length. The range holds the continuity. The range holds the differences among the patterns the sky could have had, and the odds are the same across the differences, as the fair die of Chapter 7 has the same chance on each of its six differing faces. As Chapter 7 said, the two aspects are conjugate: paired as counterparts, so that each aspect can be perfected only at the expense of the other aspect, and neither aspect is prior to the other. The Incommensuration Theorem means that symmetry and continuity are never made perfect together in one aspect, and in the sky the symmetry and the continuity are held by two different aspects. So the sky stays within the limit that the Incommensuration The-

orem sets. In the odds of the ripples, the pattern of the sky is the same at every length, across the whole range of patterns the sky could have had, while the one actual sky, a single pick from that range, is uneven in its own particular way.

Every Ingredient Rises Together

Picture a pond with leaves, twigs and a little foam floating on its surface. A ripple runs across the pond. Where the water rises, the leaves, the twigs and the foam rise together. Where the water falls, everything on the surface falls together. Nothing on the surface bobs up while the rest sinks. The account says the first ripples of our epoch are made in the same way as the ripple on the pond. The first ripples are the same for every ingredient of the universe (ordinary matter, dark matter and light), and the ingredients are like the leaves, the twigs and the foam, which all rise and fall together.

The last chapter named two sorts of ripple. In an adiabatic ripple, every ingredient departs from evenness together, as the leaves, the twigs and the foam rise together on the pond. In an isocurvature ripple, one ingredient departs against another, with the total density at first unchanged.

The second mark, that every ingredient ripples together, follows in a few steps from the one form that passes the turn. The form that passes the turn carries no mark of the interior it came from. The form is one form for every ingredient of the content. The old outside world was the only context in which distinctions among the contents of the old epoch were ever drawn. But the end of the interiors falls at no moment of the old outside world's time. So nothing at the turn tells one ingredient from another, and every ingredient of matter and light departs from evenness together. A ripple of one ingredient against another would need the ingredients to carry different forms through the turn, and the different forms would have to be told apart at the turn, by a context that registers the difference. At the turn there is no context that registers the difference.

The account therefore predicts that the first ripples are adiabatic, and that no isocurvature ripple from the beginning will be found. So far, the measurements show the ingredients rising together. The measurements set a limit on each of three sorts of isocurvature ripple that astronomers commonly test. For a ripple of the dark matter against the other ingredients of the universe, the measurements show that the ripple carries less than about one and three tenths percent of the power of the temperature differences of the sky. For either of the other two sorts, the measurements show that the ripple carries less than about one and seven tenths percent of the power of the temperature differences of the sky.

The second mark applies to the dark matter too: the unseen matter whose weight holds the galaxies together. In this account the dark matter is made of gyrations. Gyrations are lasting vibrations of the six dimensions of possibility themselves, each made at the turn. No gyration is a piece of the content, the matter and light that fill the world. Part V of this book describes the gyrations in full. *The Weight of Possibility* is the book of this collection that identifies the dark matter with a lasting vibration of the six. *The Weight of Possibility* fixes the *ratio of dark to ordinary matter* (the mass of dark matter in a region divided by the mass of ordinary matter there) by an identity that is true of every measurement, wherever the measurement is made. The identity is a rule about what a measurement made now, from inside the epoch, registers. Chapter 8 showed that a rule of the same sort as the identity, about the present measurement, fixes the cosmological constant. Chapter 19 states the identity for the dark matter. *The Weight of Possibility* predicts, in its own words, that "no separate pattern of departures from uniformity will be found in the dark component", because the identity is true of every measurement. So the result that the dark matter has no separate pattern follows by two routes. By the first route, I derive the result from the one form that passes the turn. By the second route, *The Weight of Possibility* derives the result from the identity that is true of every measurement. The premises of the two routes are different. The result is the same: the dark matter carries no separate pattern of ripples of its own.

Ripples of Density, and Waves Only Later

Picture a stretched sheet, like a large trampoline. Children stand on the sheet here and there, some heavier and some lighter. Each child makes a dip in the sheet: deeper under a heavier child, shallower under a lighter one. The sheet is bent more in some places and less in others. A wave of the sheet itself runs across the sheet, stretching the sheet one way while squeezing the sheet the other way. Something has to pull on the sheet harder in one direction than in another to set the sheet shaking in a wave. Children standing still press straight down, so the children make dips and no waves.

The geometry of the new epoch behaves like the sheet. What passes the turn is the form of the content. The ripples of the new geometry are the ripples that the ripples of the content (its matter and light) produce. A ripple of density produces a ripple of the geometry of the kind called *scalar*: a change from place to place in the amount of curvature, like the deeper and shallower dips under the children. A wave of the geometry itself needs an *anisotropic stress*: a stress that pulls differently in different directions, like the pull on the sheet that is harder in one direction than in another. Equilibrium is the settled state in which nothing further changes on average, and matter and light close to equilibrium press equally in every direction, as the air in a balloon presses equally on every part of the balloon's skin. So matter and light close to equilibrium have no anisotropic stress at *first order*, which is the largest and simplest part of their effect. The first ripples of the geometry are therefore scalar, and waves of the geometry arise from the scalar ripples only at *second order*: from ripples acting on ripples, a far smaller effect. The third mark of the pattern is that the first ripples of the geometry are scalar, with waves only at second order.

Physicists call waves of the geometry that are present from the beginning of an epoch primordial gravitational waves. The last chapter called the primordial gravitational waves the first waves. The last chapter also compared the power of the first waves with the power of the first ripples. The account predicts that the power of the first waves is far below one millionth of the power of the first ripples. So far, astronomers have found no first waves. The measurements of the astronomers place the power of the first waves below about three hundredths of the power of the first ripples. Two coming instruments will measure much more finely. The first instrument is the Simons Observatory, a set of telescopes in the high desert of northern Chile. The second instrument is LiteBIRD, a satellite. Each instrument is forecast to measure the ratio of first waves to first ripples with an uncertainty of about one or two thousandths. The account says that both instruments will find no first waves. The waves that do arise at second order are real and very faint. Part IV describes how faint the second-order waves are and how the second-order waves compare with the other waves the beginning left.

There are two grounds for the prediction that the first waves carry far below one millionth of the power of the first ripples. The first ground is that the ripples that pass the turn are scalar. The second ground concerns the trembling of empty space, the restless motion that every field keeps at its lowest. *Inflation* is a proposed burst of extremely fast expansion at the very start, and Chapter 13 describes inflation in full. In the usual story of inflation, the burst stretches the trembling of empty space from the tiny lengths inside the horizon of the time near the turn up to the lengths we see today. Nothing in this account stretches the trembling of empty space in that way. The last chapter showed the result: the empty space of the new epoch adds almost nothing to the ripples.

Flat Space

An ant on a marble finds the ground curving away within a few steps. Put an ant on a ball the size of the Earth, and the ant walks for days and finds the ground flat. Both surfaces are curved, and the two surfaces differ in one length: the radius. A length sets a curvature of space in the same way as the radius sets the curve of the marble or the ball: the radius of curvature.

Space on the largest scales is flat, to within the present precision of measurement. In the standard history, if the beginning had any small curvature, the curvature would have grown enormously, so the beginning must have been flat to an extraordinary precision. Physicists call the question of why the beginning was so flat the *flatness problem*. The restoration of context is the re-forming of a large outside world at the turn. In this account, the restoration of context fixes the curvature of space as a whole. The restoration fixes the curvature by the same premise that gave the first three marks: the new context has no prior identity. Chapter 7 showed that the new geometry begins even, without the curvature that distorts shapes. Evenness alone does not fix whether space as a whole is curved. The surface of a ball is the same at every point, and yet the surface is curved. So a space can be the same everywhere and still be curved. The restoration of context fixes whether space as a whole is curved.

A radius of curvature is a length, so the question becomes: which lengths can set the radius of curvature of space as a whole? Einstein's equations do not fix the curvature of space as a whole. In the equation for the expansion of a space that is the same everywhere, the curvature enters as a number that the law does not fix, and the number is set once, by the state the history starts from, and carried unchanged from then on. (Physicists call a number set once in this way, and then carried unchanged, a *constant of integration*.) So the radius of curvature belongs to the state that the restoration of context sets at the turn. The radius of curvature can be

made only from what that state holds. The state set at the turn inherits no length from the old epoch. The geometry of the state has no prior identity, and the total mass of the old content does not pass the turn either, because no comparison made in the new epoch registers the mass of the old interiors. The state holds the one length that the turn has of its own: the length of the turn, about the Planck length. The Planck length is the length at which the smooth description of space and time reaches the edge of its domain. The Planck length is about a hundred billion billion times smaller than a proton. The new epoch has one other length, the de Sitter radius: the radius of the horizon that a universe holding only the energy of empty space would have, today about seventeen and a half billion light years. The de Sitter radius belongs to the law of the geometry and not to the state the turn sets. The de Sitter radius is the radius of the cosmological constant, which appears in the law as one fixed term, and at the turn the cosmological constant "acts as nothing at all, because the structural conditions for its definition are no longer met", in the words of *The Gravitational Passage*.

Could the curvature be set by a length made from the length of the turn and the de Sitter radius together? Take a length halfway between the length of the turn and the de Sitter radius, counting in powers of ten. The curvature share of space is the curvature of space reported as a share of the total density, as the last chapter explained. The halfway length would leave a curvature share today of about one thousandth, right at the edge of what measurement can now see. But a length made from the two lengths needs one of two things in the state set at the turn. The state would need to hold the de Sitter radius, and the de Sitter radius is not in the state. Otherwise the state would need a very large pure number, about a one followed by thirty zeros, to stretch the length of the turn as far out as the halfway length. In this account, every number that large is a count, such as the count of states within the present horizon, and no count passes the turn: each epoch begins its own count (Chapter 7). So the state

holds one length and only ordinary numbers. A radius of curvature is either about the length of the turn, or space has no radius of curvature at all.

A Planck time is the time light takes to cross a Planck length. If the radius of curvature were one Planck length, the new epoch would close, falling back on itself and ending within a few Planck times, or else the new epoch would be ruled by curvature, with its expansion set by the bend of space itself rather than by the matter and light the epoch holds. In each case the new epoch would not build the stars, galaxies and black holes that the cycle requires. The next epoch comes from the black holes of each epoch. A radius of curvature of one Planck length is therefore excluded. The restoration of context is one process, coming round at every turn, so a restoration that set a radius of one Planck length would set that radius in every epoch, and then no epoch would make the black holes from which the next epoch comes. If the de Sitter radius had been available to the state, the de Sitter radius too would leave no curvature to measure. A radius equal to the de Sitter radius leaves, today, a curvature share of space of about ten to the minus sixty-four. Written out, ten to the minus sixty-four is a decimal point, then sixty-three zeros, then a one. Ten to the minus sixty-four is a figure computed from the de Sitter radius, and the figure is far too small to measure.

So no radius of curvature is left, and the smoothed-out model of the new epoch, the model in which every place is like every other, is flat, and *The Gravitational Passage* gives the argument for flatness in full. The fourth mark of the pattern is that space is flat: the geometry that astronomers measure is curved only by its ripples, and only as strongly as the ripples themselves are strong, about one part in a hundred thousand. The measured curvature share of space is consistent with zero, to a precision of about two thousandths. No measurement can ever test the flatness more finely than about one part in a hundred thousand, because the ripples themselves blur the curvature at that level (Waterhouse and Zibin 2008). So the exact flatness of the smoothed model follows from the argument for flatness, and measurement sets a bound on the curvature. The fourth

mark comes from the same premise as the even beginning of Chapter 7: the new geometry has no prior identity. The new geometry therefore inherits no shape from the old epoch, which gives the even beginning, and inherits no length, which gives flat space. The flatness of space enters this book's own account of the energy of empty space: the identity of Chapter 8, from which the share of matter in the universe follows, holds for a flat space. The argument for flatness uses nothing of the cosmological constant: neither its value nor even its sign. So the identity and the flatness support each other without running in a circle, because the identity uses the flatness, the flatness uses nothing from the cosmological constant, and neither one assumes a value that it then fixes.

The Strength and the Tilt

The four marks say what kind of pattern the sky must carry. Two numbers of the pattern remain: how strong the ripples are, and how their strength tilts from large sizes to small. The account fixes the kind of the ripples, their correlation across the whole sky, and their sameness for every ingredient. The account uses measured values for the strength and the tilt of the ripples, and each value has a question beside it. The amplitude is the power of the ripples at one reference length, as the last chapter defined the amplitude. The metric, as Chapter 5 put it, is the measuring rule that says, at every point, how long each small step in each direction is, and in the quotation below the metric is the measuring rule of the new epoch's space and time. *The Gravitational Passage* says what sets the amplitude and what sets the tilt: "The amplitude is set by the count of states over which the form of the content is shared as it becomes the departures of the new metric; the tilt, by the length to which the departures refer and by the way the departures depend on it."

Stir one spoonful of dye into a cup of water, and the water turns a deep color. Stir the same spoonful into a bath, and the water turns pale. The amount of dye is the same in the cup and in the bath. The depth of

color depends on how much water the dye is shared over. In this account, the strength of the ripples works the same way as the dye. The form that passes the turn is one form, like the one spoonful of dye. How strong the ripples come out depends on the count of states over which the form is shared as the form becomes the ripples of the new geometry, and the count of states is like the amount of water. The account says which count sets the strength, and the number of that count is left for later work. So the strength of the ripples is a measured input of this book, with its question beside it: what count of states sets the amplitude? The measured strength of the ripples is about five parts in a hundred thousand in the curvature of space, and in the temperature of the sky the strength is a fifth of the strength in the curvature of space, about one part in a hundred thousand. The power of the ripples is the strength in the curvature of space multiplied by itself, the quantity the measurements compare, and the measured power of the ripples is about two point one billionths.

Why does the account fix the kind of the ripples, yet use a measured value for their strength? The answer depends on what the Incommensuration Theorem can describe. Roger Penrose and Stephen Hawking proved the singularity theorems, which are theorems of Einstein's general relativity, the smooth description of space and time. In general relativity, a singularity is where Einstein's equations give curvature without bound. The singularity theorems prove that some paths reach the edge of general relativity's range within a finite time and cannot be continued past that edge, and the theorems say nothing about what lies past the edge. The Incommensuration Theorem is a theorem of the structure of comparison. The premises of the Incommensuration Theorem include no smooth space, no equation of Einstein's and no split between gravity and the other forces. So the Incommensuration Theorem can describe the place where general relativity gives a singularity: the modal boundary, where the smooth description and the stepwise description each reach the end of their range, and no other description applies beyond it. The edge belongs to the descriptions, not to what the descriptions describe, as the edge of a map of a country is the edge of the map and not of the land (Chapter 1).

The Incommensuration Theorem sets the frame of the turn: the theorem shows what the turn is, and the theorem places limits on what the turn can be like. The numbers of the turn itself belong to the physics of the turn, which is neither purely the smooth description nor purely the stepwise one, and which lies where the two descriptions join. The numbers of the turn are of three sorts: how fast the six dimensions of possibility change at the turn, the temperature at which the new epoch's history begins, and the pair that describes the ripples, their strength and their tilt. *The Gravitational Passage* fixes the frame and leaves the three sorts of number to the physics of the turn, and this book does the same. So the account fixes the kind of the ripples, which follows from the frame, and uses measured values for the strength and the tilt, which are numbers of the turn itself.

The tilt has a meaning of its own, even though its value comes from measurement. Picture a pattern that is strongest at one particular length and grows weaker further from that length. We say that a pattern which is strongest at one particular length refers to that length. When a pattern refers to no length at all, the pattern is exactly the same at every length: scale invariant, with an index of exactly one. A tilt is any departure of the index from one, so a tilt is a reference to a length. In plain terms, the tilt says whether the larger ripples are slightly stronger than the smaller ones, or the reverse. The measured index is zero point nine six four nine, about ninety-six and a half hundredths, a little below one, so the larger ripples are slightly the stronger. In this account, the measured index, a little below one, is a faint reference of the ripples to a length of the new epoch.

The new epoch has two lengths of its own. The length of the turn belongs to the turn, and the de Sitter radius belongs to the law of the epoch's geometry and history. Take a length we observe today and trace the length back to near the turn in the reconstructed history. Near the turn, the length was some ten to the twenty-seven Planck lengths (a one followed by twenty-seven zeros), and some ten to the minus thirty-four of the de Sitter radius (a decimal point, then thirty-three zeros, then a one). Both figures are computed within the reconstructed history. So every observed

length lies between the epoch's two lengths: far above the length of the turn, and far below the de Sitter radius. *The Gravitational Passage* names both lengths, and of the reference of the ripples to a length it says, in the first person: "I identify neither the length to which the departures refer nor the way the departures depend on it." The question written beside the measured index is which length sets the tilt.

The direction of the tilt is a second question, separate from which length the ripples refer to. The horizon of a time is the greatest distance a signal could have crossed since the turn. A ripple of the geometry wider than the horizon of its time stays frozen, which means that the ripple does not change while the ripple is wider than the horizon. The strength of a frozen ripple can depend on the length the ripples refer to only through how many times wider or narrower the ripple is than the length the ripples refer to. Picture the frozen ripples laid out in a row, from the narrowest to the widest. Each ripple keeps the strength it had when it grew wider than the horizon. If the strengths climb along the row as the ripples widen, the wider ripples are the stronger, so the index comes out below one. If the strengths fall along the row as the ripples widen, the wider ripples are the weaker, so the index comes out above one. Whatever length the ripples refer to, the strengths along the row may climb toward that length or fall toward it, so naming the length still does not fix whether the row climbs or falls. The account therefore uses measurement for the direction of the tilt, as well as for its size.

The running is the change of the index from one length to the next. A reference to one particular length would show in the running. The measured running is consistent with zero, to a precision of about six thousandths, and the account uses the measured running as well.

The Marks Beside the Sky

The table below sets the account's statements about the first ripples beside what measurement has found so far. In the column headed Standing, "must" means that the statement must hold, given the identification and the premise that the new context has no prior identity: Before We Begin calls a statement with this standing set by the structure once a stated step is taken, and the stated steps are the identification and the premise. The word "measured" means that the account uses the value from measurement. The word "computed" means a number worked out within the reconstructed history.

What is described	What the account says	Standing	What measurement finds
The pattern across the whole sky	Correlated everywhere, near the same strength at every length; no white noise of independent pieces	must	Index close to one
The spectral index	Taken from the sky, with the direction of its tilt	measured	0.9649
The power of the ripples	Set by a count of states that the account names	measured	About 2.1 billionths
The running	Taken from the sky	measured	Consistent with zero, to about six thousandths
First waves, against the first ripples	Far below one millionth (second order only)	must	Below about three hundredths
One ingredient against another	None	must	Below about 1.3 to 1.7 percent, by sort

What is described	What the account says	Standing	What measurement finds
Curvature of space	Smoothed-out model flat; ripples about one part in a hundred thousand	must	Consistent with zero, to about two thousandths
Power of the new vacuum on observed lengths	Below ten to the minus fifty-one	computed	Not measured separately

Every mark agrees with its measurement. The ripples are correlated across the whole sky, with an index close to one. So far, measurements show every ingredient rising together, no first waves, and flat space, to the precision reached. The last row of the table is the last chapter's finding, kept beside the findings of this chapter. On the lengths we observe, the trembling of the new epoch's vacuum carries a power below ten to the minus fifty-one, against the observed power of the ripples, about two billionths, so the ripples are the form carried through the turn, and the vacuum adds almost nothing to the ripples. *The Gravitational Passage* lists the four marks among its predictions for the sky. Beside the four marks, *The Gravitational Passage* lists a prediction from its account of the energy of empty space: the energy of empty space stays exactly constant at every era of the reconstructed history, fixed once, as Chapter 8 found.

We now know what the pattern of the first ripples must look like. The pattern is one pattern, woven from one domain, near the same strength at every length. The pattern is the same for every ingredient, so the dark matter has no pattern of its own. The pattern is a pattern of density, so the first waves of space are far too faint to find. And the pattern lies on a flat space, curved only by the ripples themselves. The account uses values measured in the sky for the strength and the tilt of the pattern. Each value has its question: what count of states sets the strength, and what length sets the tilt.

The pattern of the first ripples matters for the question of this book, what the beginning left behind. The pattern in the sky is the trace, in our epoch, of the epoch before. The pattern contains the information that fell into the old black holes. The information was carried with the content through the turn, and the information re-emerges as the ripples, averaged and not erased, correlated across the whole sky because the domain the information came from is one. The information is not destroyed, and it is not accessible from the outside world of the old epoch. The question "what came before the Big Bang?" has its answer written across the sky: the collapse of the old epoch's interiors, which approaches, from the other side, the limit that our history takes for its beginning. In this question, "Before" names the order of the cycle, and no comparison measures a time across the limit. The question "what came before that?" has the same form of answer, without end, because the cycle has no first iteration.

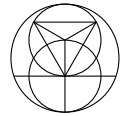
> **Key idea.** The first ripples must carry four marks. One domain: the pattern is correlated across the whole sky, near the same strength at every length, and the white noise of independent pieces is excluded. One form for every ingredient: every ingredient ripples together, and the dark matter has no separate pattern. Ripples of density: the first waves of space arise only at second order, far below one millionth of the ripples' power. Flatness: space as a whole has no radius of curvature, and it is curved only by its ripples. The account uses measured values for the strength and the tilt, and says what sets each: a count of states sets the strength, and a length lying between the turn's length and the de Sitter radius sets the tilt.

Summary

In this chapter I found four marks that the account sets on the first ripples. I also found two numbers for which the account uses measured values. The four marks follow, in the form "must", from two things together. The two things are my identification of the first ripples with the form of the

content carried through the turn, and the premise that the new context has no prior identity. The first mark is one domain. In the new epoch, no comparison registers which old interior any part of the content came from, so the pattern is correlated across the whole sky, near the same strength at every length, and the white noise of independent pieces is excluded. In the first mark, the odds of the ripples hold the symmetry, since the odds are the same at every length. The range of patterns the sky could have had holds the continuity. The one actual sky is a single pick from the range of possible patterns, uneven in its own way, so the sky stays within the limit that the Incommensuration Theorem sets. The second mark is one form for every ingredient: nothing at the turn tells one ingredient from another, so every ingredient ripples together. The dark matter therefore has no separate pattern, and *The Weight of Possibility* derives the same result by a second route. The third mark is that the ripples are ripples of density. Matter close to equilibrium has no stress that pulls differently in different directions at first order, so the first waves of space arise only at second order, far below one millionth of the ripples' power. The fourth mark is flatness. No length of the old epoch passes the turn, and no count passes the turn, so the state the turn sets holds only the turn's own length and only ordinary numbers. The de Sitter radius belongs to the law and not to the state the turn sets, and a radius of the turn's own length is excluded, because the cycle requires black holes in every epoch. So space as a whole has no radius of curvature, and space is curved only by its ripples. Measurement agrees with every mark so far. The account fixes the kind of the ripples, their correlation and their sameness for every ingredient. The account uses measured values for the strength of the ripples, their tilt, the direction of the tilt and their running. The account also says what sets the strength (a count of states) and what sets the tilt (a reference to a length lying between the turn's length and the de Sitter radius).

Most cosmologists explain the sameness of the sky and the flatness of space by an early burst of expansion. Does this account need an early burst of expansion as well?



Chapter 13

No Inflation Needed

Most cosmologists explain the sameness of the sky and the flatness of space by an early burst of expansion called inflation. In this book, the account means the account of the world that I teach. Does the account need inflation? The turn is the passage at the end of the interiors of the black holes of the epoch before ours, in which a large outside world is restored, and our history begins at the turn. The account needs no inflation, because the account has its own answer to every puzzle that inflation was introduced to solve, and the answers come from the turn and from the structure of the forces. The account also says something stronger about a long inflation. The early horizon is the greatest distance a signal could have crossed since the turn, in the first moments of our history. A long inflation is one long enough to stretch the lengths we observe today from inside the early horizon to beyond the early horizon. On the account, a long inflation is excluded, while a shorter inflation is neither required nor excluded. We go through the three statements one at a time: no inflation is needed, a long inflation is excluded, and a shorter inflation is neither required nor excluded. Then we ask what the dark matter of the account says about how hot the beginning could have been.

A Patch of Dough the Size of a City

Picture a baker's patch of dough, small enough to sit in your palm. Every part of the dough is like every other part, because the baker kneaded the dough as one piece. Now stretch the dough, in a moment, until the dough covers a whole city. Any fold or wrinkle the dough had is pulled out flat. Anyone standing anywhere in the city finds smooth dough underfoot, alike in every place. Nobody there could tell that the whole city of dough was once one small kneaded piece.

The stretched dough is the picture behind *inflation*: the proposal, made by the physicist Alan Guth in 1981, that an early phase of ever faster expansion stretched a small region, whose parts had once been in contact with one another, to far beyond everything we can now see. In the picture, the kneaded patch is the small region, and the city is everything we can now see. On Guth's proposal, the sameness of the sky and the flatness of space both follow from the stretching, as the dough alike in every place and the wrinkles pulled flat follow from stretching the dough. The first ripples are the faint differences of density with which our epoch began, and from which our epoch's galaxies grew. Guth's proposal also says where the first ripples came from. On Guth's proposal, the first ripples are the small trembling of the empty space of the inflating phase. The empty space of the inflating phase is called the vacuum of the inflation. The stretching carried the trembling of the vacuum of the inflation out to large lengths. Among cosmologists, inflation is the leading explanation of the early universe, and cosmologists have developed inflation in many versions.

In the usual version of inflation, a field called the *inflaton* drives the expansion. The inflaton is a field spread through space whose energy pushes space apart. While the inflaton pushes space apart, the inflaton drifts slowly down a gentle slope of the inflaton's own energy, as a sled crawls down a long, almost level hill. The sled is the inflaton, and the

height of the hill is the inflaton's energy. Cosmologists count how much an inflation stretched space in steps called *e-folds*. One e-fold is a growth by a factor of about 2.72 (the number mathematicians call e). When the inflaton's energy runs out, the energy has been handed on to ordinary hot matter and light. Cosmologists call that handing on *reheating*. The account contains no inflaton and no count of e-folds. I use the word "reheating" only in one narrower sense, which I give in the section "How Hot the Beginning Could Have Been", below.

Guth proposed inflation to solve three puzzles: the horizon problem, the flatness problem and the monopole problem. The horizon problem and the flatness problem came up in Chapter 12. The horizon problem is the question why regions of the sky that no signal within the history of our epoch could have connected share one temperature, to about one part in a hundred thousand. The sameness of the temperature of the sky is measured. The flatness problem is the question why space on the largest scales has no overall curvature. The monopole problem is explained below, in the section "No Monopoles, No Strings, No Walls". Cosmologists also credit inflation with a fourth result: that the expansion runs the same in every direction. *The Gravitational Passage* is the book of the collection that sets the whole account out for physicists. *The Gravitational Passage* contains the answers to the horizon problem and the flatness problem, and both answers come from the turn. *The Gravitational Companion* is the companion book of the series. *The Gravitational Companion* extends *The Gravitational Passage* and sets *The Gravitational Passage* beside other accounts. *The Gravitational Companion* contains the answers to the monopole problem and to the fourth result. The two books depend on each other in one direction only. *The Gravitational Companion* depends on *The Gravitational Passage*, and nothing in *The Gravitational Passage* depends on *The Gravitational Companion*. We take the three puzzles and the fourth result in order.

The Sameness and the Flatness, Answered From the Turn

Two towns far apart may share an accent although their people never spoke to each other, because settlers from one place founded both towns. The sky is like the two towns. The sky has regions that no signal within our epoch's history has connected, and the regions are alike because the regions come from one domain. The likeness of the regions does not come from a signal passing between them. In the picture, the regions of the sky are the towns, and the one domain is the one place the settlers came from. Every black hole of the epoch before ours has an interior, and all the interiors have one end, and not many ends. The new outside world re-forms from the one end of the interiors, which is the one domain. The new outside world re-forms with a geometry that has no prior identity. The new geometry inherits no record and no mark from the world before. So the new geometry inherits no large-scale unevenness from the world before.

The form of the content is which alternatives the content has taken, that is, what has been selected. The form of the content passes the turn as one form. No comparison at the end of the interiors shows a difference between one interior and another, because the only context that drew distinctions between the interiors was the old outside world, and the end of the interiors falls at no moment of the old outside world's time. So in the new context, the new outside world, the form appears only as small departures from uniformity. The old distinctions are averaged, and not erased. The departures are linked across the whole new sky because the domain the departures come from is one. The departures are differences within the new epoch, registered by the new epoch's own context, and not differences between the old interiors. Because the departures have one origin, the departures have one form for every ingredient of the content. For the same reason, the departures have a spectral index near one (nearly the same strength at every length), and not the spectral index that independent pieces would leave.

The flatness of space also follows from the turn. How strongly space curves is set by a length, the radius of curvature of space. The radius of curvature is set by the state the turn leaves, and no length of the old epoch passes the turn. The state the turn leaves holds one length: the length of the turn itself, about the Planck length. No large count of states passes the turn to stretch the length of the turn into another length. The de Sitter radius is the radius of the horizon that a universe holding only the energy of empty space would have, and the de Sitter radius belongs to the law of the geometry, not to the state the turn leaves. A radius of curvature as small as the Planck length would close the new epoch within a few Planck times. To close the epoch is to make the epoch fall back on itself and end. A Planck time is the time light takes to cross a Planck length. Or else a radius that small would leave the epoch dominated by curvature. Either way, as Chapter 12 showed, the new epoch would make none of the black holes from which the next epoch comes, so a radius of curvature that small is excluded. Even if the radius of curvature were as large as the de Sitter radius, the curvature today would be far too small to measure. So no radius of curvature is left, and, as Chapter 12 showed, the smoothed-out model of the new epoch is flat on the largest scales, curved only by the epoch's own departures from uniformity.

So the horizon problem and the flatness problem have their answers in the account without inflation. The horizon problem and the flatness problem are the two puzzles that made inflation the standard answer. *The Gravitational Passage* states the conclusion in one sentence: "The account therefore needs no inflationary phase for the horizon and flatness problems."

The Same in Every Direction

Cosmologists also credit inflation with making the expansion the same in every direction. The account has two routes to the sameness of the expansion in every direction, two arguments that depend on different premises, and neither route uses inflation.

The first route is the turn itself. The new geometry has no prior identity, so the new geometry inherits no *anisotropy*, no difference between directions, from the old outside world. Chapter 7 showed that the new epoch begins even: without the curvature that distorts shapes, except for the small departures that the Incommensuration Theorem requires. The Incommensuration Theorem, which I set out in the Preface, is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. So the beginning is even, but never perfectly even.

The second route is established physics: the law of an expansion that is the same at every place. The *shear* of an expansion is the part of the expansion that changes the shape of a small region without changing the region's size, as when shear stretches a round region into an egg while keeping the region's volume. Picture an expansion with three properties: the expansion is the same at every place, the space is flat, and the content has no stress pulling differently in different directions. In a universe with those three properties, the rate of expansion is set by the sum of two terms: one term for the energy of the universe's content, and one term for the universe's shear. George Ellis and Malcolm MacCallum are two researchers in general relativity, and in 1969 Ellis and MacCallum showed that the shear dies away fast in an expansion that is the same at every place, flat, and free of any stress pulling differently in different directions. Early on, light carries most of the energy of the content. The shear's term fades much faster than the term of the light, and a worked example shows

how much faster. Each time the universe doubles in size, the shear's term falls sixty-four-fold, while the term of the light falls only sixteen-fold. So the unevenness of direction at the start, however large, soon adds little to the rate of expansion.

The second route needs space to be flat. A universe expands at exactly its escape speed when the energy of the expansion exactly balances the pull of everything in the universe, and the universes that expand at exactly their escape speed are the flat ones. In 1973 the physicists Collins and Stephen Hawking studied universes that are the same at every place, and Collins and Hawking showed that, among those universes, growing toward the same rate in every direction is not the general rule. Collins and Hawking concluded that the flat universes would in general grow toward the same rate in every direction. The new epoch is flat on the largest scales, so the condition of the second route, flat space, holds for the new epoch.

The Gravitational Companion contains a computation of how much the shear fades over one stretch of the history. The computation shows the second route in numbers. The law of Ellis and MacCallum has three conditions, and all three are known to hold across the stretch. Established physics gives the temperature at each end of the stretch. Equilibrium is the settled state in which nothing further changes on average. Hot matter close to equilibrium has no stress that pulls differently in different directions, at first order: in the largest and simplest part of the stress, with the smaller corrections left aside. The hot early mixture of particles stays close to equilibrium until the neutrinos go free. The neutrinos are the lightest particles of matter, which barely touch anything else, and the neutrinos go free when they stop colliding with the rest of the particles and travel on their own, within the first second. Physicists state a very high temperature as the typical energy of one particle of the heat, in electron volts: an electron volt is the energy an electron gains in crossing one volt. The neutrinos go free at a temperature between about one and three million electron volts, and the computation uses three million electron volts. Chapter 10 showed two facts about the count of states in a region that grows with the

expansion: the count stays the same, and a kind of particle that falls out of balance hands its share of the count to the rest of the particles. Chapter 10 also gave the count in each unit of volume: a fixed number, times the cube of the temperature, times the number of kinds of particle that share the heat. So, while light carries most of the energy, the universe grows in size at least by the factor by which the universe's temperature falls. The Higgs field fills all of space and gives many of the elementary particles their masses (Chapter 5). The Higgs field settled at a temperature of about 159.5 billion electron volts. Between the moment the Higgs field settled and the moment the neutrinos went free, the temperature fell to about 1.9 parts in a hundred thousand of its value at the start, so the universe grew in size at least about fifty-three thousand-fold. Suppose the shear had been as large as the whole rate of expansion at the start of the stretch. By the end of the stretch, the shear makes up less than 2.3 parts in a hundred thousand of the rate of expansion then: rounded up, less than three parts in a hundred thousand. The numbers of the computation come from established physics and from *The Coming of the Kinds*, with the measured mass of the Higgs boson. *The Coming of the Kinds* is the book of the collection on how the kinds of particle came to be after the Big Bang.

Under the law of an expansion that is the same at every place, run the other way in time, a collapse grows uneven. The two ends of an epoch differ. The collapses of an epoch end uneven, and the expansion of an epoch grows even. The Kasner form of Chapter 2 is the simple form space takes near a limit such as the end of the interior of a black hole, where each direction of space has a size that changes as a fixed power of the time counted from the limit. Near the center of a black hole the geometry takes the Kasner form with three directions of space: space is stretched along one direction and squeezed along the other two. As the time remaining shrinks, one direction grows with the power minus one third. The other two directions shrink with the power two thirds. The three powers are minus one third, two thirds and two thirds, and the three powers obey two rules from Chapter 2. The first rule is that the three powers add to one: minus one third plus two thirds plus two thirds is one. The second

rule is that the squares of the three powers add to one: one ninth plus four ninths plus four ninths also make one. So the geometry near the center is the Kasner form met in Chapter 2. A collapse inside a black hole therefore grows uneven, while the expansion of an epoch grows even.

There are two routes to an expansion that is the same in every direction, and the two routes depend on different premises. The first route depends on the premise that the new context has no prior identity, and the second route depends on the law of an expansion that is the same at every place. Both routes have one result: the expansion of the epoch is the same in every direction, except for the epoch's departures from uniformity. Neither route needs inflation.

No Monopoles, No Strings, No Walls

The monopole problem is the third puzzle that inflation was proposed to solve, and the monopole problem comes from theories that unite the forces. A *grand unified theory* is a theory in which the strong force, the weak force and electromagnetism are, at very high energy, one force with one larger symmetry, which breaks apart into the three as the universe cools. Grand unified theories predict that the breaking of the larger symmetry leaves behind *magnetic monopoles*. A magnetic monopole is a particle that carries a single magnetic pole, a north with no south. No magnet we know carries a single pole. Grand unified theories predict so many monopoles that the universe would hold more mass than its measured expansion allows. Cosmologists call that excess of monopoles the *monopole problem*. Inflation's answer to the monopole problem is to stretch space so much that the monopoles are diluted, spread too thin to matter.

In the account the forces were never one larger force. So nothing breaks in the way that makes monopoles, and the monopole problem does not arise. The particle books of the collection show that there is no grand unification. The symmetry of the forces is three symmetries side by side, the same at every energy, and there is no force carrier that turns a

quark into a lepton (a lepton is the electron or one of the electron's relatives, the neutrinos among them). Both facts are stated in *The Branches of the Kinds*, the particle book of the series on what the structure allows and forbids. *The Kinds of Particle* is the book of the collection that reads the kinds of particle on the six as a space of states, and *The Kinds of Particle* shows where the three symmetries come from. The six small dimensions are the product of two shapes, the two factors of the six: the complex projective plane and the sphere. In *The Kinds of Particle* I take each factor as a space of states, the states a choice can be in. The symmetry of the strong force is the set of turnings of the complex projective plane, the space of a three-way choice, such as the lamp of Chapter 3 that could be found red, green or blue. The symmetry of the weak force is the set of turnings of the sphere, the space of a two-way choice. The third symmetry is called hypercharge, and hypercharge is tied to electric charge. Hypercharge is a phase over the complex projective plane, the larger of the two factors. The phase is a turning of the tiny clock hand that every particle carries. None of the three symmetries is what is left of a larger symmetry that broke. So no larger symmetry breaks to leave monopoles.

A magnetic monopole would also explain why electric charge comes in fixed steps. Is the account, which has no monopoles, then left without an explanation of the fixed steps? No: the fixed steps of charge have their own reason in the account. The fixed steps follow from the charge rule of *The Story of the Particles*. *The Story of the Particles* is the book of the series on why every particle is required. In *The Story of the Particles*, the charge rule sets the electric charge of each kind of particle, and every charge the charge rule sets is a whole number of one smallest step, a third of the electron's charge.

One breaking of symmetry does take place in the history of the content. The breaking is the settling of the Higgs field, which separates the weak force from electromagnetism. Could the settling of the Higgs field leave anything behind? The answer comes from established physics. When a field settles into its lowest-energy state, the field can trap energy in places, and in 1976 the physicist Tom Kibble showed which sorts of

trapped energy can last. Which sorts last depends only on the shape of the set of lowest-energy states the field can settle into. Physicists call the set of lowest-energy states the vacuum manifold. A *cosmic string* is a thin line of trapped field energy, which lasts only when the vacuum manifold has loops that cannot be shrunk to a point. A *domain wall* is a sheet of trapped energy between regions that settled on different choices. A domain wall forms only when the vacuum manifold falls into separate pieces. A monopole forms only when a sphere drawn on the vacuum manifold cannot be shrunk to a point.

A rubber band shows what "cannot be shrunk" means. Put a rubber band around a beach ball, and you can always slide the band off to one side and shrink the band to nothing. Thread a rubber band through the hole of a bagel and around the bagel's ring, and the band is caught: no sliding shrinks the band. The ball and the bagel stand for two shapes a vacuum manifold could have. The band stands for a loop drawn on the vacuum manifold. A cosmic string is energy caught like the band on the bagel.

A pea wrapped in plastic wrap shows what "cannot be shrunk" means for a sphere. The wrap closes all around the pea, and the wrap cannot shrink to nothing without passing through the pea. If the wrap may not cross the pea, the wrap is caught, as the band is caught on the bagel. A monopole is energy caught like the wrap around the pea. A wrap with no pea inside it can always shrink to a point.

For the Higgs field, the vacuum manifold is a three-sphere: a sphere one dimension up from the surface of a ball. No one can picture a three-sphere, but a three-sphere has three properties that decide what the settling of the Higgs field leaves behind. The three-sphere is all one piece. Every loop drawn on the three-sphere can be shrunk to a point, as on the beach ball, and every sphere drawn on the three-sphere can be shrunk to a point too, as the wrap with no pea inside can. So the settling of the Higgs field makes no lasting strings, no domain walls and no monopoles: a result of established physics. The Standard Model is the established theory of the known particles and of the strong force, the weak force and electro-

magnetism. Suppose the Standard Model is the whole of the content, with no unifying symmetry and no further field of the Higgs sort. Then the history of the content makes no lasting network of strings, and no hum of gravitational waves from a network of strings, where a hum, as Chapter 10 said, is a random mixture of gravitational waves arriving from every direction at once.

Nothing for Inflation to Work With

In *The Gravitational Companion* I ask one more question: does the account need inflation at all? The answer in *The Gravitational Companion* is a three-part test, of method, material and function, which can be put to any proposed mechanism. Think of a saw. A saw works by cutting (the saw's method), and a saw needs wood to cut (the saw's material). Someone must want a board cut (the saw's function). A saw with no wood has nothing to work on, and a job that needs no cutting has no use for a saw. The test has three questions. By what means does the proposed mechanism work (its method)? What does the structure of the account supply for the mechanism's means to act on (the mechanism's material)? What work is the mechanism wanted for (its function)? Then the test has two checks: whether the method matches the material, and whether the function requires the method. In *The Gravitational Companion* I call the second question the form. In this book I call the second question the material, because in this book the word "form" already names the form of the content that passes the turn.

Put to inflation, the three-part test has three answers. The method: inflation is a mechanism of motion, a phase of ever faster expansion, and in the usual version the expansion is driven by a field whose energy has a slope of just the right shape. The material: the six held dimensions provide no field whose energy has a slope of the right shape. The two moduli of the six are the two features of the internal space that could change: the overall size of the internal space, and the ratio of the internal

space's two radii, which sets the internal space's shape. Chapter 4 showed that the two moduli are held stiffly, with masses of about the Planck mass. In *The Gravitational Passage* I identify the first ripples with the form of the content carried through the turn, and not with the trembling of a field. The function: every problem that inflation was designed to solve has another answer in the account. The sameness of the sky comes from the one domain from which the new outside world re-forms. The flatness of space comes from the turn. The state the turn leaves holds no length to set a curvature except the turn's own length, and a curvature set by the turn's own length is excluded by the cycle of epochs, because an epoch with that curvature would make no black holes to begin the next epoch. The expansion is the same in every direction because of the restoration of context at the turn, the first route above. The restoration of context is the re-forming of a large outside world whose new geometry has no prior identity. The expansion is also the same in every direction because of the fading of shear in a flat epoch, the second route. The monopoles never form, because there is no grand unification. So the method does not match the material, and the function does not require the method.

The two moduli are far too heavy to drive an inflation. The usual sort of inflation is called slow-roll inflation, and slow-roll inflation is the sled on the long, almost level hill. Slow-roll inflation needs a field that is very light compared with the rate of the expansion the field drives. Physicists state a rate of expansion in the same units as a mass, so the condition for slow-roll inflation is that the field's mass lie far below the rate of expansion. The two moduli weigh about 2.25 and 3.09 Planck masses (computed at first order in *The Gravitational Passage*). To drive slow-roll inflation, the two moduli would need a rate of expansion far above the Planck mass, and a rate that high lies beyond the modal boundary, the Planck scale, where the description of the large outside world ends. So neither modulus can drive slow-roll inflation.

A Long Inflation Is Excluded

The account says more than that inflation is not needed. In Chapter 11 I identified the first ripples with the form of the content carried through the turn: the old distinctions, averaged. The vacuum of the new epoch adds almost nothing to the first ripples. Near the turn, a length we observe today lay some ten to the twenty-seven times beyond the horizon of that time, the greatest distance a signal could have crossed since the turn. On a wave so far beyond the horizon, the trembling of the new vacuum adds a power below ten to the minus fifty-one. The power observed is about two billionths.

A long inflation is one long enough to stretch the lengths observed today from inside the early horizon to beyond the early horizon. A long inflation would make the ripples on the lengths observed today the trembling of the inflating vacuum. On the account, the ripples on the lengths observed today are the carried form instead. So, on my identification of the first ripples, a long inflation is excluded: given that the first ripples are the carried form, a long inflation cannot have happened.

A shorter inflation is a different case. A shorter inflation is one that leaves the observed lengths outside the early horizon the whole time, and in the words of *The Gravitational Passage* a shorter inflation "is neither required nor excluded by the account". *The Gravitational Passage* does not fix the length of a shorter inflation. A shorter inflation is limited by the starting temperature: the temperature at which the history of the new epoch begins. The starting temperature bounds the lengths the early horizon could contain. So the starting temperature bounds the shorter inflation that the account neither requires nor excludes. The hotter the start, the earlier our history begins, and the earlier our history begins, the smaller the early horizon was then, measured against the lengths we observe today. So the starting temperature says how far outside the early horizon the observed lengths lay when the history began. A shorter inflation must

leave the observed lengths outside the early horizon the whole time, so a shorter inflation can have stretched the observed lengths at most by the factor by which the observed lengths lay outside the early horizon when the history began.

The account has no inflaton, and on the account the first ripples are the carried form, so the account contains none of the quantities of the inflationary description: no count of e-folds, no inflaton that rolls, and no formula for the spectral index or for the ratio of first waves to first ripples in terms of an inflaton. The first waves are gravitational waves present from the beginning of an epoch. The ripples have a measured strength of about one part in a hundred thousand. The ripples also have a measured tilt, a spectral index of about 0.97. As Chapter 12 showed, the account fixes the kind of the ripples. The account uses the measured values of the strength and the tilt of the ripples.

How Hot the Beginning Could Have Been

The history of the new epoch begins with the content of the new epoch hot and close to equilibrium. The geometry of the new epoch begins even. The account does not fix how hot the content was when the history began. In *The Gravitational Passage*, the starting temperature is on the list of things the Incommensuration Theorem cannot compute: "the temperature at which the reconstructed history of the new exterior begins". *The Compactification* is the book of the collection that sets out for physicists how an era of the world begins, and neither *The Gravitational Passage* nor *The Compactification* fixes the starting temperature.

The account bounds the starting temperature from above, through the dark matter. On the account, the dark matter is made of gyrations, which Chapter 4 introduced. A gyration is a lasting vibration of the six held small dimensions, made only at the turn, with no way out afterward. A gyration is a vibration of context, and not an item of content, in the way that a sound in a crystal is a vibration of the crystal and not a thing inside

the crystal: the six are the crystal, and the gyration is the sound. Chapter 18 describes the gyration in full, and here I need only the gyration's mass. At the classical level, each gyration has a mass of the square root of two Planck masses (about 1.41). At the next level of the calculation, the floor is counted too, and the floor is the least extent that holds the six. When the floor is counted at first order, the mass of a gyration is about six tenths of a Planck mass. Both computations are in *The Weight of Possibility*, the source book of the collection for the dark matter. The full Planck mass is about 1.221 times ten to the nineteen billion electron volts, a unit set by the measured strength of gravity. So, by either computation, a gyration weighs about one Planck mass.

Heat makes particles. A plasma is the gas of Chapter 4: a gas so hot that nothing in it holds together. In a hot plasma, colliding particles can make new pairs of anything that gravitates, through gravity alone. The hotter the plasma, the more pairs the plasma makes, but the number of particles made falls off very steeply once a particle's mass is well above the typical energy of the heat. On the account, the turn alone made the gyrations, and the amount of gyrations is measured, because the amount of gyrations is the amount of dark matter. A hot enough plasma could have made more gyrations by gravity, as a plasma makes pairs of anything that gravitates. More gyrations would mean more dark matter than is measured, so the plasma of the new epoch was never hot enough to make more gyrations.

In the words of *The Weight of Possibility*: "If the plasma of the new epoch formed at once, the measured amount of dark matter requires that the plasma never have been hotter than about one thirtieth of the mass of a gyration." Physicists state very high temperatures as the typical energy of one particle of the heat, in billions of electron volts. For a gyration of the square root of two Planck masses, the limit is about five times ten to the seventeen billion electron volts, and for a gyration of six tenths of a Planck mass, the limit is about two times ten to the seventeen. The two numbers are bounds: the account bounds the starting temperature from above, and the value of the starting temperature is not given.

The turn's energy may instead have reached the plasma gradually, through a stage of the content that behaved as matter and then decayed into the content. When the energy arrives through a matter-like stage, the condition on the temperature falls on the end of that stage: "the reheating that ended it must lie below about one hundredth of the mass of a gyration", even if the energy at the start of the matter-like stage was of the Planck density. The plasma may then have been as hot as about one thirtieth of the mass of a gyration at the plasma's hottest moment, because the radiation the matter-like stage releases afterward dilutes the gyrations made at the hottest moment. The matter-like stage goes on pouring out light and particles after the plasma's hottest moment, so the gyrations made at the hottest moment end up spread thin among far more particles than a plasma formed at once would hold. In this book the word "reheating" has only that one sense: the end of a matter-like stage of the content, if there was one. In this book "reheating" names no decay of an inflaton, because the account has no inflaton.

The upper bound lies below the Planck temperature of Chapter 4, the temperature at which the typical energy of one particle of the heat reaches the energy of the Planck mass, so an epoch that began near the Planck temperature is excluded. In 1984 the physicists Edward Kolb and Richard Slansky showed that a start near the Planck temperature would make lasting vibrations of a compact space about as numerous as the particles of light. A compact space is a space held small, as the six are. The bound is one of three conditions on the turn listed in Chapter 4: a condition the account requires of the turn, and not a result derived from the turn. In the standings of this book, the starting temperature is bounded, with its value not given.

A Second Reason, From the Dark Matter

The dark matter is a second ground for excluding a long inflation, separate from my identification of the first ripples. In inflation, the inflating vacuum makes the first waves of the geometry. The strength of the first waves grows with the energy the inflating vacuum holds. In 2021 a measured bound on the first waves was published by the teams of the BICEP and Keck telescopes at the South Pole. So the measured bound limits the energy of the inflating vacuum. The measured bound also limits the heat that the end of the inflation could hand on to the plasma. *The Weight of Possibility* states the consequence: "In an epoch whose observed departures were made by inflation, the bound on primordial gravitational waves would keep the temperature far below this limit, and dark matter of this mass could not be made after that inflation in the measured amount." The limit in that sentence is the upper bound found above, about one thirtieth of the mass of a gyration, some two to five times ten to the seventeen billion electron volts.

So the dark matter, too, requires that the observed ripples were not made by inflation: after an inflation that made the observed ripples, gyrations of about one Planck mass could not have been made in the measured amount. By a separate route, I reach the same conclusion from my identification of the ripples as the carried form: the observed ripples were not made by inflation. The gyrations are made at the turn, and not after an inflation.

Why does the account's answer on inflation matter for what the beginning left behind? The answer matters because the turn itself is the source of the sameness of the sky, the flatness of space and the form of the ripples. The ripples we measure on every observed length are the old epoch's form, carried through the turn, and not the trembling of an inflat-

ing vacuum. The dark matter was made at the turn, and not after an inflation. The dark matter is about five and four tenths times as much as the ordinary matter.

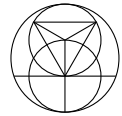
> **Key idea.** Inflation was introduced for three puzzles and is credited with a fourth. On the account, each of the four has its answer without inflation. The sky is the same everywhere because the new world re-forms from one domain, and space is flat because no length of the old epoch passes the turn. The expansion is the same in every direction because of the restoration of context at the turn and the fading of shear. No monopoles, strings or walls form, because no larger symmetry breaks and the Higgs field's set of lowest-energy states is all one piece, with every loop and every sphere on the set shrinking to a point. So no inflationary phase is needed. On my identification of the first ripples, a long inflation is excluded. A shorter inflation is neither required nor excluded, and the starting temperature bounds a shorter inflation.

Summary

We asked in this chapter whether the account needs inflation, the early burst of expansion most cosmologists use. We now know that the account does not need inflation. The horizon and flatness problems have their answers in the turn. The sky is alike because the sky comes from one domain, and space is flat because no length of the old epoch passes the turn and the state the turn sets holds no length from which a curvature could be made. The sameness of the expansion in every direction has two routes, the restoration of context at the turn and the established fading of shear in a flat expansion, and neither route needs inflation. The monopole problem does not arise, because there is no grand unification. The settling of the Higgs field makes no strings, no domain walls and no monopoles, a result of established physics that rests on Tom Kibble's work of 1976. The six held dimensions supply no field that could drive an inflation: the two

moduli of the six weigh about 2.25 and 3.09 Planck masses, far too heavy to drive slow-roll inflation. On my identification of the first ripples as the form of the content carried through the turn, a long inflation is excluded: given that identification, a long inflation cannot have happened. A shorter inflation is neither required nor excluded by the account, and the temperature at which the history begins bounds a shorter inflation. The starting temperature is not fixed, but the dark matter bounds the starting temperature from above, at about one thirtieth of a gyration's mass. For a gyration of the square root of two Planck masses, the bound is about five times ten to the seventeen billion electron volts, and for a gyration of six tenths of a Planck mass, the bound is about two times ten to the seventeen. If instead a matter-like stage carried the turn's energy, the matter-like stage must have ended below about one hundredth of a gyration's mass. A start near the Planck temperature is excluded. The dark matter gives a second ground against a long inflation: after a long inflation, the bound on first waves would keep the plasma far too cool to make gyrations in the measured amount. "Reheating" has only its conditional sense, and the account has no inflaton.

The account now has its answers to the puzzles of the beginning. Each number of the sky has appeared with a standing of its own. When the numbers are put together, which of the universe's numbers does the account set, which does the account use as measured, and which does the account predict?



Chapter 14

The Ledger of the Sky

Which of the universe's numbers does the account set, which does the account use as measured, and which does the account predict? In this book, the account means the account of the world that I teach. Part III has followed the oldest light in the sky, the ripples in that light, and the puzzles that inflation, an early burst of expansion, was proposed to solve. Along the way we met many numbers, among them the shares of the universe's energy, the strength of the first ripples, the rate of the expansion and the energy of empty space. In this chapter I gather the sky's numbers in one place, and beside each number I write how the account came by the number.

A Ledger with Three Columns

Picture a bookkeeper's ledger with three columns. The first column is headed "set by the structure". The second column is headed "taken from measurement". The third column is headed "predicted". Every number of the sky has a row. The bookkeeper writes each number in the column that names the number's source. Where a number has been measured, the number's row also carries the measured value beside the account's own value. A reader can then compare the two values. A careful bookkeeper also writes a note in the margin beside each entry, saying how the entry came to be there: copied from a receipt, worked out from other entries, or promised for a date not yet reached. The margin note is the entry's stand-

ing: the ground on which the number stands, which is how the account came by the number. In the picture, the ledger is this chapter's list of the sky's numbers, and each margin note is one number's standing.

I give every number its standing. In this chapter the margin notes are the notes of the table of the sky's numbers in *The Compactification*, the book of the collection that sets out for physicists how an era of the world begins. Each note has a plain meaning. A *measured input* is copied from measurement: the account uses the measured value and offers no value of its own. In this book, the structure is the shape of the world that the account derives. A *structural value* is set by the structure of the account alone, before any measurement is consulted. An *identification* is a stated step from the structure to a thing in the world, the kind of step Chapter 1 described, and I make each identification in my own voice. A number that depends on an identification is set by the structure once the identification is made. A *computed* value is worked out from other entries of the ledger, by arithmetic and by established physics, which means results of physics taken from their sources. An *argued* value follows from a chain of the account's steps, each step said in words. An entry marked "the same test seen another way" is one result written in a different unit, so the ledger counts the entry once. The registered ratio is one more note, which belongs to a single entry: the ratio of dark matter to ordinary matter. I explain the registered ratio below, at that entry.

The account's numbers of the sky come from three books, while a fourth book tells part of the same account in plain words. *The Gravitational Passage* sets the whole account out for physicists. *The Gravitational Passage* gives the identity for the cosmological constant that Chapter 8 walked through. The cosmological constant is the energy of empty space, and the identity is one statement in pure numbers about the cosmological constant, which I state in full in the section on the matter share. *The View from Inside* tells the identity for the cosmological constant in plain words, for the general reader. The ratio of dark matter to ordinary matter comes from *The Weight of Possibility*, the book of the collection on the dark matter. *The Compactification* gives the argued properties of the first ripples that

Chapters 11 and 12 set out. The first ripples are the faint differences of density with which our epoch began, and from which our epoch's galaxies grew.

The Six Numbers of the Standard Model

Cosmologists describe the smooth universe and its first ripples with a standard model, called *Lambda CDM*. Lambda is the Greek letter that stands for the cosmological constant, and CDM stands for cold dark matter: dark matter that moves slowly. In this chapter the first light is the oldest light in the sky, which traveled free once the first atoms formed (Chapter 11). The model has six *base parameters*: six numbers that cosmologists fit to the first light. Every other number of the model is worked out from the six base parameters. The model also assumes two things without fitting them. The model assumes that space is flat, and the model also assumes that the energy of empty space never thins as space grows. Physicists write the letter w for the ratio of a form of energy's pressure to the amount of that energy in each volume (Chapter 8), and an energy of empty space that never thins has a w of exactly minus one.

The six numbers are written with two terms, the Hubble constant and the critical density, so the two terms come first. The *Hubble constant* is the present rate of the expansion. A distant galaxy moves away from us at a speed in proportion to the galaxy's distance, and the Hubble constant is that speed for each unit of distance: about sixty-seven to sixty-eight kilometers per second for every megaparsec. A megaparsec is about 3.26 million light years. Astronomers often divide the Hubble constant by a hundred and call the result h, so an h of 0.678 means sixty-seven and eight tenths kilometers per second for every megaparsec.

The *critical density* is the total energy in each volume of space that a flat universe, expanding at the present rate, must hold. The Friedmann equation of Chapter 8 states the critical density in terms of the Hubble constant, and Alexander Friedmann found the equation from Einstein's the-

ory of gravity. Picture the universe's energy as a pie, as *The View from Inside* pictures the universe's energy. Some slices of the pie are matter: ordinary atoms, and the dark matter that astronomers find by the pull of the dark matter. A thin sliver is light and other radiation, and the rest of the pie is the energy of empty space. The whole pie is the critical density. Each ingredient's share of the pie is the ingredient's share of the critical density.

The first base parameter is the density of ordinary matter: how much matter made of atoms the universe holds. The first light measures the density of ordinary matter as an amount in each volume of space. A share of the pie depends on the size of the whole pie. The size of the pie goes with the square of the expansion rate. So astronomers multiply the share of ordinary matter by h squared, which turns the share back into an amount in each volume. Measured that way, the density of ordinary matter is 0.02237, a little more than two hundredths. As a share of the pie, the density of ordinary matter comes to about five percent.

The second base parameter is the density of cold dark matter, written in the same way, as a share of the pie multiplied by h squared. The measured density of cold dark matter is 0.1200, twelve hundredths, which as a share of the pie comes to about twenty-six percent.

The third base parameter is the angle of the sound horizon. Before the first atoms formed, the universe was a hot glowing gas of light and charged matter, and sound ran through the gas. By the time the light went free, each sound wave had traveled a definite distance, called the *sound horizon*, as Chapter 11 described. The first light has slightly warmer and slightly cooler spots. The spots come in favored sizes, as a drum sounds favored notes, and the sound horizon sets the favored sizes. In the picture, the favored sizes of the spots are the drum's notes, and the sound horizon is the size of the drum. The third parameter is the angle that the sound horizon spans on our sky today. The angle is a little more than half a degree, a little wider than the full Moon. In the fit, cosmologists write the angle as a hundred times the angle in the unit called a radian (a little over fifty-seven degrees). The measured value is 1.04092.

The fourth base parameter is the *optical depth*. Long after the light went free, the first stars lit. The light of the first stars broke the gas between the galaxies back into charged pieces. Astronomers call that second charging of the gas *reionization*. Reionization freed electrons, and the freed electrons scatter a little of the first light on the way to us, as a thin fog scatters the light of a lamp. The optical depth says how thick the fog of freed electrons is. The measured optical depth is 0.0544, which means that about one photon of the first light in twenty was scattered on the way, where a photon is one particle of light.

The fifth base parameter is the amplitude: the strength of the first ripples, which Chapter 11 introduced. Cosmologists measure the amplitude as the power of the ripples, about two parts in a billion, which is the square of a ripple strength of about five parts in a hundred thousand. In the fit, the amplitude is written in a convenient coded form, 3.044, which decodes to 2.10 billionths.

The sixth base parameter is the index: how the strength of the ripples changes with their length, as Chapter 11 described. An index of one means the same strength at every length. The measured index is 0.9649, a little below one, so the ripples are slightly stronger on the larger lengths.

Five Entries Copied from Measurement

In the account, five of the six base parameters are taken from measurement: the density of ordinary matter, the angle of the sound horizon, the optical depth, the amplitude and the index. The standing of each of the five is measured input: the account uses the values the first light gives, and I write those values in the second column of the ledger.

For the amplitude and the index the account says something more, which Chapter 12 found. The account fixes the kind of the first ripples. The account leaves the strength and the slope of the first ripples to measurement, where the slope is the index. The turn is the passage at the end of the interiors of the black holes of the epoch before ours, in which a

large outside world is restored, and our history begins at the turn. In this book a comparison is the act of finding whether two things are the same or different, and the content is what a person or an instrument finds in a comparison, as a thermometer comparing morning and evening finds the temperature. The form of the content is which alternatives the content has taken, that is, what has been selected. I identify the first ripples with the form of the content carried through the turn: the distinctions of the old epoch, averaged and not erased. From the identification the account sets four marks on the pattern of the first ripples. The four marks are one domain (a pattern linked across the whole sky), one form for every ingredient, ripples of density, and flatness. The strength and the slope are each a measured input, with a question written beside each. For the amplitude the question is: what count of states sets the strength of the ripples? For the index the question is: what length sets the slope?

The sixth base parameter to enter the ledger is the density of dark matter, which enters by another route. The other route needs a margin note of its own.

The Sixth Through One Ratio

The Weight of Possibility gives the amount of dark matter as a ratio to the amount of ordinary matter: six, less two divided by pi (six minus the fraction two over pi). Six, less two divided by pi, is 5.3634, a little more than five and a third, so for every kilogram of ordinary matter there are about five and a third kilograms of dark matter. Part V of this book says what the dark matter is on the account. Part V also walks through where the six and the two divided by pi come from. In the ledger the ratio is one entry, and here I need only the entry's margin note.

The margin note beside the ratio reads *registered ratio*. To register is to find a sameness or a difference and to leave a record of what was found. In the sense of *The Weight of Possibility*, a *registration* is a measurement made inside the world, taken as the act at which a number is set: the num-

ber is what the measurement finds and records, and not the result of some process that made the number. A registered ratio is a ratio set by registration, by what the present measurement registers, and not by a mechanism at the turn. Chapter 8 found the cosmological constant fixed in the same way, by the present measurement. On the account, the dark matter is made of gyrations: lasting vibrations of the six small dimensions of possibility, which are held a few Planck lengths across, and Chapter 13 said that the gyrations are made at the turn. Chapter 13's statement and the registered ratio hold together: the turn makes the gyrations, and the present measurement fixes how much of the gyrations there is, against the ordinary matter. In the words of *The Weight of Possibility*: "The turn makes the gyrations, and the reconstructed history carries the ratio that the present measurement fixes." The reconstructed history is the history of the universe that we reconstruct from the marks the past leaves in the present.

The ratio follows from a principle of counting and from a rule, and both are the account's own principles. The principle is that a registration counts what it registers by basic pieces, as Chapter 8 counted the channels by the basic pieces of the complex projective plane (the pieces of Chapter 3's Euler count). A channel is one of the ways a measurement takes in what it finds. The complex projective plane is the space of all the states of a three-way choice. The complex projective plane is the larger of the two shapes that make up the six small dimensions. The Euler count of Chapter 3 is the number of a shape's basic pieces, counted with signs. The rule says which curvature of the six small dimensions reduces the count, and the rule was chosen by measurement. The measured ratio selected the rule out of a small family of expressions of one form, and twelve expressions were examined in all. *The Weight of Possibility* states the ratio's standing the same way: the account's own principle and rule, with the rule selected by measurement. Chapter 19 walks through the principle and the rule step by step. I keep the ratio's standing under the standing's own name, the registered ratio, as I said in Before We Begin that I would. Before We Begin lists five standings: set by the structure; set by the structure

once a stated step is taken; measured; measured, with its reason still a question; and bounded, with its value not given. I file the registered ratio under none of the five standings.

Multiply the ratio by the measured density of ordinary matter, and the density of dark matter follows. The product, 5.3634 times 0.02237, is 0.1200, give or take eight ten-thousandths. The first light measures 0.1200, give or take twelve ten-thousandths. The dark-matter entry is computed from the ratio and the ordinary-matter entry. So the dark-matter entry restates the ratio at the measured amount of ordinary matter. The dark-matter entry is the ratio written a second time, and not an independent test.

The ratio itself is measured through the first light. Astronomers find the densities of ordinary and dark matter from how strong each favored size of spot in the first light is. Astronomers call the strengths of the favored sizes the heights of the peaks of the pattern. Four things set the heights of the peaks: general relativity, which is Einstein's theory of gravity; sound in the glowing gas of light and ordinary matter; the scattering of light on free electrons; and the forming of hydrogen atoms. The dark matter changes the heights in only two ways: through the dark matter's pull, and through the dark matter's taking no part in the sound. The Planck satellite mapped the first light from space, and the Planck satellite's data show a ratio of 5.364, give or take 0.065. Two telescopes map the first light from the ground: the South Pole Telescope and the Atacama Cosmology Telescope. Combined with Planck, the two telescopes give 5.370, give or take 0.048. Some studies add the distances measured by the Dark Energy Spectroscopic Instrument, called DESI, and those studies give 5.199, give or take 0.035. DESI maps how galaxies are spread through space, and DESI measures distances with a ruler of fixed size: a favored distance that sound in the early universe left in the spread of galaxies. The DESI figure, 5.199, lies below the account's 5.3634. DESI's distances measure how much matter there is in all, and not the dark matter by itself. Where DESI's distances favor less matter in all, the studies that add DESI lower the dark matter to match. The account sets the matter share (the part of the universe's energy that is matter, ordinary and dark together,

which Chapter 8 called the matter fraction) at the value given in the next section. I expect the preference of the DESI studies for less matter to fade as the DESI data grow. Appendix B sets the measurements beside one another in full.

Three things support the rule. The first thing that supports the rule is the rule's form. The second thing that supports the rule is that the measurement excludes the rule's nearest neighbors within the rule's form, as Chapter 19 walks through. The third thing that supports the rule is a number that the rule shares with the identity of Chapter 8. That shared number is the curvature of the six small dimensions in Planck units, and *The Gravitational Passage* identifies that curvature, in Planck units, as one. Planck units are the units built from the speed of light, Planck's constant and Newton's constant of gravity. The shared number appears inside both entries, the registered ratio and the identity. In six, less two divided by pi, the two divided by pi is the part by which the curvature of the complex projective plane reduces the count, and the two divided by pi is worked out from that curvature, counted in Planck units. In the identity of Chapter 8, the trim is one less the curvature energy of the six, counted in Planck units. So two entries of the ledger contain one and the same number of the structure, although the two entries are reached by different routes.

A second kind of measurement finds the amounts of ordinary matter and dark matter without the first light. The second kind of measurement finds what fraction of all the matter is ordinary matter. The account sets that fraction at 0.15644, a little more than fifteen and a half percent. In the account's value, the neutrinos (the lightest particles of matter) are taken at their least masses. The fraction follows from the ratio together with the *matter share*, the slice of the pie that is matter, ordinary and dark together. Chapter 8 called the matter share the matter fraction, and the next section gives the value of the matter share. The light elements made in the first minutes give 0.1587 when combined with the distances of DESI. The light

elements give 0.1542 with a fuller analysis of the DESI map. The hot gas inside clusters of galaxies gives 0.156, and each of the three agrees with the account's value.

One Number Set: The Matter Share

Now picture two surveyors sent to measure the same field, as *The View from Inside* tells the story. One surveyor walks the edge of the field with a measuring chain. The other surveyor works from a hilltop with angles and a map. The two surveyors use different methods, and each method has its own unknowns. But there is one field, so the two answers must agree, and the agreement itself can tell what neither surveyor could find alone.

Chapter 8 set out two ways of finding the energy of empty space, and the two ways are like the two surveyors: the field is the energy of empty space, and each surveyor is one way of finding that energy. The first surveyor works from the inside. The vacuum is empty space in its lowest, settled state. For the first surveyor, the cosmological constant is the part of the vacuum's energy that a measurement made inside the vacuum takes in: three parts out of the count of the particle horizon, each part an equal portion of the trimmed total. The particle horizon is the edge of everything whose light has reached us since the expansion began, and Chapter 8 pictured the particle horizon as a surface tiled like a floor. The count of the particle horizon is the number of tiles. The trim is one, less three divided by eight pi, about 0.8806: the share of the Planck density left once the curvature energy of the six small dimensions is taken away. The trimmed total is the Planck density times the trim. The first surveyor's way depends on the identification Chapter 8 stated. The second surveyor is the astronomers' pie: the energy of empty space is one slice of the critical density. The two answers must agree, and when the two answers are set equal, the expansion rate and the strength of gravity cancel. What is left is the identity: the square of the horizon's size, counted in Hubble radii,

times the vacuum's share of the pie, equals eight, less three divided by pi, about 7.05. A Hubble radius is the distance at which the expansion carries things away at the speed of light. Eight, less three divided by pi, is the same number as eight times the trim.

The horizon's size in Hubble radii depends only on how the energy has been divided among the slices over the history of the expansion, and in a flat universe the slices fill the pie, so the identity is one equation with one unknown: the matter share. The identity uses one measured input, the thin sliver of radiation, about nine parts in a hundred thousand of the pie, and the identity uses the flatness of space that Chapter 12 found. The solution is a matter share of 31.1 percent, and the only spread in the solution, about a tenth of a percent, comes from how precisely the sliver of radiation is known. The margin note reads: identification; computed. In plain words, the matter share is set by the structure once the step of Chapter 8 is taken, so the matter share goes in the first column of the ledger. The Planck satellite measures 31.5 percent, to within seven tenths of a percent either way, and the account's 31.1 lies inside that spread. The DESI measurements of how galaxies cluster, taken alone, give 29.75 percent, to within 0.86 of a percent either way. The account's value lies between the two measurements.

Cosmologists who use the standard model fit the matter share to the sky, while the account sets the matter share. The identity fixes how the energy is split among the slices, and so the identity fixes how much matter there is in all. The identity says nothing about what the matter is made of. The registered ratio splits the matter into dark and ordinary, so the ledger needs both entries, the matter share and the registered ratio.

Once the matter share is set, four more numbers follow from the matter share, and all five are one test, seen five ways. The first of the four is the vacuum's share of the pie: 68.9 percent, against a measured 68.5 percent, give or take 0.7, and the second is the energy of empty space itself. Put the measured size of the particle horizon, about 46.1 billion light years, into the formula of Chapter 8, and the formula gives 2.56 times ten to the minus forty-seven, in the particle physicists' unit of energy in a

volume. The particle physicists' unit is billions of electron volts, raised to the fourth power. An electron volt is the energy an electron gains in crossing one volt. The measured value is 2.52 times ten to the minus forty-seven.

The third of the four numbers that follow from the matter share is the size of the particle horizon measured against the de Sitter radius. The de Sitter radius is the horizon that a universe holding only the energy of empty space would have. The identity says that the particle horizon is the square root of eight times the trim, times the de Sitter radius. The square root of eight times the trim is two and sixty-five hundredths (2.654). The measured particle horizon is about 46.1 billion light years, and the measured de Sitter radius is about 17.5 billion light years, a ratio of two and sixty-three hundredths (2.632). The fourth is the age of the universe counted in de Sitter times. The de Sitter time is the de Sitter radius divided by the speed of light. The identity gives 0.792 of a de Sitter time, a little under eight tenths, while the measured age, about 13.8 billion years, against a de Sitter time of about 17.5 billion years, gives 0.787, give or take 0.009. Read as an age, the identity says when the measurement that fixes the cosmological constant is made: at about 0.792 of a de Sitter time. So the identity places the measurement now, at the age just measured.

The fifth is the Hubble constant, and the Hubble constant needs one more measurement. The first light measures the amount of matter in each volume directly, as the matter share times h squared: 0.1430. Divide 0.1430 by the account's matter share of 0.311, take the square root, and h comes out at 0.678: an expansion rate of sixty-seven and eight tenths kilometers per second for every megaparsec, give or take three tenths. Astronomers measure the rate in two ways today. Astronomers read the rate from the early universe, through the first light, and find 67.4, give or take 0.5. Astronomers also climb a *distance ladder*, and find 73.0, give or take 1.0. The ladder runs from nearby pulsating stars called Cepheids, whose rhythm of pulsing tells how bright each Cepheid is, out to exploding stars farther away. The account's rate agrees with the early-universe value, and the ac-

count also says why the account's rate cannot equal the distance-ladder value: with the amount of matter the first light measures, a rate of 73 would need a matter share of 26.8 percent, which the identity excludes.

Five numbers are one test seen five ways: the vacuum's share, the energy of empty space, the horizon ratio, the age and the Hubble constant. The ledger counts the five numbers once. Each of the five is the agreement of the two surveyors, written in a different unit: a share of the pie, an energy in a volume, a ratio of two distances, an age, and a rate.

Four Predictions of Kind

The third column of the ledger holds four predictions of kind. A prediction of kind is a statement of what sort a thing is, and each of the four follows from its own step of the account. Cosmologists who use the standard model assume or fit each of the four, while the account contains an argument for each.

The first prediction is that the energy of empty space is exactly constant: w is exactly minus one. The cosmological constant is fixed once, from the particle horizon of the measurement made now. We know the past only by the marks the past leaves in the present, and we reconstruct the history of the universe from those marks as the one history the marks require. The reconstructed history is one geometry. In one geometry the cosmological constant is one number, because Einstein's equations themselves require one constant in one geometry. So the reconstructed history has the one value of the constant at every era, and no changing field drives the speeding up of the expansion. The margin note reads: argued. The Planck satellite measures minus one and three hundredths, give or take three hundredths, when the satellite's data are combined with the ruler of galaxy distances and with exploding stars. Appendix B sets out the recent reports on whether the energy of empty space changes, and the rule this book uses to read the reports.

The second prediction is that space is flat. How strongly space curves is set by a length, the radius of curvature. Chapter 12 found that no length of the old epoch passes the turn, so the smooth model of the new epoch has no curvature of its own, and space is curved only by the ripples of the new epoch, by about one part in a hundred thousand. The margin note reads: argued. The measured curvature is seven ten-thousandths, give or take nineteen ten-thousandths: zero, within the measurement.

The third prediction is that the first ripples are of one kind for every ingredient, like ripples on a pond that all rise and fall together. In the picture, every ingredient of the universe rises and falls in density together, as the ripples on the pond do. In the term of Chapter 11 the first ripples are *adiabatic*, and no ingredient carries a separate pattern of its own. What passes the turn is one form, so every ingredient departs from evenness together. The margin note reads: argued. Measurements find no separate pattern. Measurements place a separate pattern of the dark matter below 1.3 percent, and the other two sorts of separate pattern commonly tested below 1.7 percent.

The fourth prediction is that the first waves are far weaker than the first ripples. The first waves are the gravitational waves made at the beginning along with the first ripples. Gravitational waves are ripples in the shape of space and time themselves. Equilibrium is the settled state in which nothing further changes on average. First order means the largest and simplest part of an effect, with the smaller corrections left aside. At first order, matter close to equilibrium at the beginning has no stress that pulls differently in different directions. So the first ripples are ripples of density, and first waves arise only at second order, among the smaller corrections. The power of the first waves is far below one millionth of the power of the first ripples. The margin note reads: argued. Measurements find no first waves. Measurements place the ratio of the two powers below about three and a half hundredths.

The first three predictions follow from the account's own steps. The first three are also what the standard model assumes, or what the usual pictures of inflation expect. On the fourth prediction, many pictures of inflation give first waves strong enough for the coming instruments to find. The account says that the coming instruments will find none.

One more computed entry goes with the matter share: *S eight*, which says how clumped the matter is today. Astronomers take spheres of a fixed size, ask by how much the amount of matter in such a sphere typically differs from the average, and then adjust that typical difference by the matter share. The eight in the name is the radius of the spheres. The radius is eight megaparsecs divided by *h*, about twelve megaparsecs, or nearly forty million light years. *The Weight of Possibility* computes S eight at 0.823, give or take 0.006, from the matter share of 31.1 percent, the registered ratio, and the measured amplitude and index, for the amount of matter at which the sound horizon spans the angle that the first light measures. The ledger carries the value 0.823. With the account's own Hubble constant of 0.678 put in instead, the computation gives 0.827, give or take 0.007. *The Weight of Possibility* gives both values, and the two values differ by four thousandths. The standard model fitted to the same first light gives 0.832, give or take 0.013. The account's value is lower because the account sets the matter share, where the standard model fits the matter share to the sky.

The computation treats the dark matter as perfectly cold, and on the account the dark matter is perfectly cold. Each piece of dark matter loses momentum as the universe expands. So each piece is cold however fast the piece was moving when the turn made the piece. The Planck temperature is the temperature at which the typical energy of one particle of the heat reaches the energy of the Planck mass. Take the most extreme case, hotter than Chapter 13 allows the start of our history to have been: a piece that leaves the turn at very nearly the speed of light, carrying a hundred times its own mass in momentum, when the universe is at the Planck temperature. Even that piece travels less than a meter in the whole history of the universe. So on the scales of the spheres of S eight, the dark matter

clumps exactly as the standard model's cold dark matter does. S eight is computed from the matter share, so the ledger writes S eight beside the matter share as a consequence of the matter share, and not as a fifth prediction of kind.

What Stands Behind the Ledger

The Compactification lists in one place everything that the entries of the ledger depend on, so that a reader can see every item together: what is measured, what the structure sets, which steps are identifications, and what is registered. The measured inputs come first. Newton's constant, the measured strength of gravity, is the one measured constant on the side of gravity, and with the speed of light and Planck's constant, Newton's constant fixes the Planck mass and the Planck density. The identity for the matter share also uses the measured sliver of radiation, nine parts in a hundred thousand, and the energy of empty space, written as a number in the particle physicists' unit, also uses the measured size of the particle horizon. The Hubble constant also uses the measured amount of matter in each volume, 0.1430. And five of the six base parameters are measured inputs.

Two numbers are structural values, set by the structure alone. One is the count of channels, three: one channel for each of the three contents a measurement registers, pattern, force and probability (Chapter 8). Three is also the count of basic pieces of the complex projective plane of Chapter 3, one piece for each role of a comparison. The other structural value is the trim, one, less three divided by eight pi, 0.8806. The trim is the external value of the vacuum's energy (Chapter 8) at the Planck scale, as a share of the Planck density, where the external value is the energy of empty space as the energy would be found from outside, if there were an outside. The trim is the ground-state part less the curvature energy of the six small dimensions, and the ground-state part is the lowest energy of the quantum fields, taken as the Planck density.

Two steps are identifications. One is the identification of Chapter 8: the share of the vacuum's energy that a measurement made inside the vacuum takes in is three parts out of the particle horizon's count, one part for each channel. The other is the identification of Chapter 11: the first ripples are the form of the content carried through the turn. One ratio is registered: dark matter to ordinary matter, six, less two divided by pi.

The cosmological constant depends on five choices, and Chapter 8 gave the reason for each choice before any number was computed. The edge is the particle horizon, the channels are three, and the trim is included. The ground-state part includes the small energy that holds the six small dimensions, and the Planck mass is the full Planck mass. Appendix A sets out a search that varies the five choices all at once. In the search, exactly one combination of the choices gives an energy of empty space within the measured uncertainty, and that combination is the one Chapter 8 gave the reasons for.

The Ledger, Written Out

First I list the three columns of the ledger, entry by entry. The second column, taken from measurement, holds the five measured base parameters. The first column, set by the structure, holds the matter share with the four numbers that are the same test seen other ways. S eight is written beside the matter share as a consequence of the matter share. The third column, predicted, holds the four properties of kind. The registered ratio is written on a line of its own, in none of the three columns. The registered ratio's line also carries the density of dark matter, computed from the ratio. The registered ratio keeps a standing of its own, as Before We Begin said, and the registered ratio is filed under none of the five standings that Before We Begin lists, neither "set by the structure" nor any other. The counting principle and the rule are the account's own principles. The measurement selected the rule from a small family of expressions of one form, twelve examined in all. Where a number in the first column can be

measured, the number is also a prediction in the plain sense of the word: the coming measurements, which Appendix B sets out, will be set beside the number.

The table below gives the whole ledger, with the measured values beside the account's own values. Every number in the table has been said in words earlier in this chapter.

Number	This account	Standing	Measured
Density of ordinary matter	taken from measurement	measured input	0.02237 ± 0.00015
Ratio of dark to ordinary matter	$6 - 2/\pi = 5.3634$	registered ratio	5.364 ± 0.065 (Planck); 5.370 ± 0.048 (three telescopes); 5.199 ± 0.035 (with DESI)
Density of dark matter	0.1200 ± 0.0008	computed from the two rows above	0.1200 ± 0.0012
Angle of the sound horizon	taken from measurement	measured input	1.04092 ± 0.00031
Optical depth	taken from measurement	measured input	0.0544 ± 0.0073
Amplitude (coded form)	not fixed	measured input	3.044 ± 0.014
Index	not fixed	measured input	0.9649 ± 0.0042
Matter share	0.311 ± 0.001	identification; computed	0.315 ± 0.007 (Planck); 0.2975 ± 0.0086 (DESI alone)

Number	This account	Standing	Measured
Vacuum share	0.689	same test as the matter share	0.685 ± 0.007
Energy of empty space (GeV to the fourth)	2.56×10^{-47}	same test	2.52×10^{-47}
Particle horizon over de Sitter radius	2.654	same test	2.632
Age in de Sitter times	0.792	same test	0.787 ± 0.009
Hubble constant, h	0.678 ± 0.003	same test, through one more measurement	0.674 ± 0.005 (Planck); 0.730 ± 0.010 (Cepheids)
w	minus one exactly	argued	-1.03 ± 0.03
Curvature of space	zero in the smooth model	argued	0.0007 ± 0.0019
S eight	0.823 ± 0.006	computed	0.832 ± 0.013 (standard model fit)
First waves against first ripples	far below one millionth	argued	below 0.036
Separate pattern for one ingredient	none	argued	below 1.3 to 1.7 percent, by sort

The ledger sorts what the beginning left behind, which is the question of this book. In kind, the account sets what the beginning left: ripples of one form for every ingredient, a flat space, and first waves far below one millionth of the first ripples. In amount, three things are taken from measurement: the strength of the ripples, the slope of the ripples, and the amount of ordinary matter. The vacuum of the new epoch has one constant, fixed by the measurement made now, and with the one constant come the matter share and the age at which the measurement is made. And the amount of dark matter is one registered ratio times the amount of ordinary matter.

> **Key idea.** Of the six base parameters of the standard model, the account uses five as measured: the density of ordinary matter, the angle of the sound horizon, the optical depth, the amplitude and the index, while the sixth, the density of dark matter, is one registered ratio, six, less two divided by pi, times the measured density of ordinary matter. The account sets one derived quantity, the matter share, at 31.1 percent, and five numbers are that one result seen five ways: the vacuum's share, the energy of empty space, the horizon ratio, the age in de Sitter times and the Hubble constant. The account predicts four properties of kind: an exactly constant energy of empty space, a flat space, ripples of one form for every ingredient, and first waves far below one millionth of the first ripples.

Summary

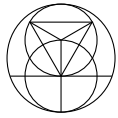
I wrote the numbers of the sky into one ledger, each number with its standing. Cosmologists fit the six base parameters of the standard model of cosmology, Lambda CDM, to the first light, and the standard model assumes a flat space and a constant energy of empty space. In the account, every one of the six base parameters depends on measurement. Five are measured inputs, and the sixth, the density of dark matter, is computed from the measured density of ordinary matter through the registered ra-

tio, six, less two divided by pi, which keeps its own standing. The computed density restates the ratio and is no independent test. The account sets one derived quantity, the matter share, at 31.1 percent through the identification of Chapter 8, and with the matter share and the measured amount of matter, the account also sets the Hubble constant at sixty-seven and eight tenths. Five numbers are one test with the matter share, seen five ways: the vacuum's share, the energy of empty space, the horizon ratio, the age and the Hubble constant. The account predicts four properties of kind, of the first ripples and of the vacuum, that the standard model assumes or fits, each from its own step: w exactly minus one, a flat space, adiabatic ripples, and first waves far below one millionth of the first ripples. The ledger depends on one measured constant of gravity, two structural values (three channels and the trim), two identifications, one registered ratio, and five measured base parameters.

The first ripples come with no first waves at first order, and with only the faintest first waves at second order. What gravitational waves, then, did the beginning leave?

PART IV

THE WAVES THE BEGINNING LEFT



Chapter 15

Ripples of the Arena

What is a gravitational wave, on this account? And which gravitational waves could the beginning of our epoch have left behind? In this book, "this account" means the account of the world that I teach. The first ripples of the sky are the departures from uniformity with which our epoch began. First waves are gravitational waves present from the beginning of an epoch, and "at first order" means in the largest and simplest part of an effect. In Chapter 14 we found that the first ripples carry no first waves of the arena at first order. So if the beginning left waves of the arena behind, some process other than the first ripples made those waves. Before we ask which process made the waves of the arena, we need to know what a wave of the arena is. We also need to know why a wave of the arena can bring us a record straight from a time before the oldest light went free.

A Finger in a Stretched Sheet

Press a finger into a stretched sheet of rubber. The dent is deepest under your finger, but the sheet is pulled for a long way around your finger. Now move your finger quickly up and down. Ripples run out across the sheet, far beyond the place your finger touches.

The sheet stands for the arena of Chapter 1: space and time together, in which everything that is measured is measured. A context is the respect in which a comparison finds what it finds. For a thermometer comparing

morning and evening, the two times of day are the context. The arena is the context of physics, and the shape of the arena is gravity. On this account gravity is the arena itself, the context of every comparison, and not a force acting within the arena. A heavy body bends the arena around the body, as your finger dents the sheet. The bending reaches out into the empty space around the body. Because the bending reaches out, space is curved in the empty region around a star, where no matter is.

When the heavy body moves, the bend in the arena moves with the body, and the change in the bend travels outward at the speed of light, reaching far places later than near ones. A gravitational wave is a traveling change in the bend of the arena: a small ripple of the arena itself, running out from the place where the ripple was made, across regions where nothing at all is found. We met gravitational waves earlier in this book as the ripples that two merging black holes send out. *The Gravitational Passage* is the book of the collection that sets the whole account out for physicists, and there I say what a gravitational wave is in one line: "gravitational waves are small oscillations of the context, the continuously asymmetric structure, propagating at c". In plain words, gravitational waves are small swings of the context, which is smooth and different from place to place, and gravitational waves travel at the speed of light (physicists write that speed as c).

The gravitational waves that reach the Earth are very small. On 14 September 2015 the two LIGO detectors in the United States registered the first gravitational wave ever caught. The wave came from two black holes merging far away. As the wave passed, the wave changed the length of each four-kilometer arm of the detector by far less than the width of a proton, so the first wave caught was a very small ripple on a very smooth background of the arena.

The rubber sheet shows two things well: how a bend spreads, and how a ripple travels. The rubber sheet and the arena differ at one point. A rubber sheet is made of long molecules held together by bonds. The arena is built of no smaller pieces. The arena has no parts like the molecules of the

rubber, because the arena is the context in which every part is found. We will need the difference between the sheet and the arena when we compare the arena with a crystal.

Smooth at Every Scale

Gravitation is not quantized: gravitation comes in no least steps. The arena is smooth at every scale. The arena bends gradually, with no jump anywhere, and the arena keeps bending gradually however closely you look. A quantum, as earlier chapters used the word, is one least step of something that comes only in equal steps, as light comes in photons. *The Unquantized Arena* is the book of the collection on how quantum theory and gravity fit together, and we met that book in Chapter 7. In *The Unquantized Arena* I argue two things about the arena. First, the arena has no least step of extent: no least length, no least area and no least duration. Second, the arena has no quantum of its own, at any scale.

Two Theories, One World is the plain-words companion of *The Unquantized Arena*, and we met the companion in Chapter 8. In *Two Theories, One World* I give the reason the bending of the arena is smooth, in words you already know. Every act of comparing must connect what it compares without a break, since a connection with a break in it connects nothing. Comparing joins the things compared within the context, so the context carries the continuity that comparing brings: the context itself has no breaks. A bend that came in steps would put a break into the context, and at the break the comparing would have nothing to join. So the arena may differ from place to place without limit, but the arena differs gradually.

Two facts from earlier chapters go with the smoothness of the arena. First, the Planck length is the tiny length at which gravity and the quantum act on equal terms. The Planck length bounds how sharply anything can be located, but the Planck length is not a least length of space, and the arena goes on smoothly at lengths shorter than the Planck length. Second, the arena has no quantum state at all. The content is what a com-

parison finds, such as the matter in the arena. Quantum theory describes the content by a quantum state, and on this account that kind of description never applies to the arena. A horizon is the surface of a black hole from inside which no signal can reach the outside. When we followed the black holes of an epoch in Chapter 9, a horizon turned out to be a feature of a classical geometry, in the sense of Chapter 2 (a smooth shape of the arena, described without quanta). There was one classical geometry for each way the matter could turn out, and no horizon was ever put into a quantum state.

What a Phonon Is

Now take a second picture, from a solid you can hold. A crystal is a lattice of atoms held in place by the bonds between the atoms. Strike a crystal, and a wave of motion runs through the lattice. Each atom swings a little about its place and passes the swing to its neighbors. The wave of motion through the lattice is sound.

Quantum theory says that each way the lattice can vibrate takes up energy only in equal steps. Physicists call one step of energy in one way of vibrating a phonon, and we met phonons in Chapter 7. Physicists count and use phonons every day. Phonons carry heat through a crystal, and phonons account for how little heat a very cold solid can hold.

Yet nobody concludes from phonons that a crystal is made of phonons. The crystal is made of atoms. A phonon is a step in the energy of one way the atoms can move together. A phonon belongs to the description of the motion. Nobody concludes either that the spacing between the atoms comes in phonon-sized steps. A phonon counts the energy of a vibration, and a phonon says nothing about a least distance in the lattice.

In Chapter 7 we called a structure whose parts are held in place by bonds a held structure. A crystal is a held structure, because the bonds of the crystal hold the atoms of the crystal in the lattice, so a phonon is a quantum of the description of how a held structure vibrates.

Quanta of a Description

Small waves of the arena can be described by quanta in the same way as the vibrations of a crystal. Physics has a careful description of small ripples riding on a smooth background of the arena, and the description holds within a range that the description itself states: gentle bending and small waves. Within the stated range, physicists apply quantum theory to the description of the waves in the usual way, and in that description each way the arena can ripple takes up energy only in equal steps. The range ends at the Planck curvature, the bending at which gravity and the quantum act on equal terms. At the Planck curvature a ripple would be as large as the geometry the ripple rides on, and the difference between a wave and the background that carries the wave disappears.

The steps in the description of small waves are quanta of the description, the name we gave the phonon in Chapter 7: steps that belong to the way the waves are described, and not to the arena that ripples. In *The Unquantized Arena* I argue that two findings are both true at once. The arena has no least step and no quantum of its own, and the small waves of the arena may still be described by quanta, because the quanta of that description belong to the description and not to the arena. The description by quanta ends at the Planck scale, while the claim that the arena has no quantum of its own holds at every scale.

In this book, as in Chapter 7, the word graviton means a quantum of the description and nothing more: one step in the energy of one way a small wave of the geometry can ripple. The word graviton names at most that one step, and a graviton is nothing more than that step. A graviton is not a quantum of the arena, not a particle, not a carrier of gravitation, and not one of the kinds of particle. The kinds of particle that the content carries are the kinds of the Standard Model, physics' list of the kinds of

particle. *The Kinds of Particle* is the book of the collection on the kinds of particle, and in *The Kinds of Particle* I show that the list of the Standard Model contains no graviton.

The phonon is the nearest sense in which a graviton can be made intelligible, because a phonon too is a quantum of the description of a vibration. A phonon describes the vibration of a held structure, but the arena is not a held structure. The arena has no parts that bonds hold in place, as bonds hold the atoms of a crystal. The shape of the arena is set by the matter and energy in the arena. So even in the phonon's sense, the graviton is not a quantum of the arena. The phonon shows what the word "graviton" means, and the phonon shows no part of the arena to be a quantum.

Light shows why two questions about gravitational waves must be kept apart. Light is a field within the arena: light is a content, and the energy of light is found in countable steps. When physicists give light its quanta, the photons, the steps belong to the field of light. The space the light crosses gets no steps. A gravitational wave is different in kind from light. A gravitational wave is a ripple of the arena, and not a field within the arena. When physicists count the energy of the ripple in steps, the physicists describe the ripple, and the arena that ripples stays smooth. So the first question is whether small waves of the arena can be described by quanta, and the answer is yes, within the range of the description, with the steps belonging to the description. The second question is whether gravitation itself comes in steps, and the answer is no, at every scale.

General relativity is Einstein's smooth description of the arena, and general relativity agrees with everything the detectors of gravitational waves have measured. In the description by quanta, a wave strong enough to register in LIGO is counted as so many gravitons that the count plays no part in what the detectors see. A beam of ordinary light gives a familiar case: the beam carries so many photons that the steps in the energy of the beam play no part in the brightness of the beam. In the chapters that follow, every background of gravitational waves is made of classical ripples of the four open dimensions (the three dimensions of space we

move in, and time). Where the energy of the classical ripples is counted one way of rippling at a time, the energy is counted in the steps of the description.

What a Single Click Would Show

When a detector clicks once for each quantum it absorbs, the detector registers a step of its own. In 2024 the physicists Germain Tobar, Sreenath Manikandan, Thomas Beitel and Igor Pikovski proposed a detector that clicks for single quanta of gravitational waves. The detector is a heavy resonator (a body that rings readily in one way of vibrating, as a bell rings at its note). The resonator is cooled to the resonator's lowest state of vibration. The cooled resonator is watched for single jumps of energy at the moment LIGO registers a passing wave. Tobar and his co-authors argue that the signature of single jumps could be observed in a laboratory experiment.

Also in 2024, the physicists Daniel Carney, Valerie Domcke and Nicholas Rodd asked what a single click of a detector of single quanta would show, and their answer is that a smooth, classical wave acting on a quantum detector gives the same clicks as single quanta would give. The answer of Carney, Domcke and Rodd holds for light too. In the photoelectric effect, light knocks single electrons out of a metal, one click at a time, and physicists can explain the photoelectric effect equally well by a classical wave of light acting on the quantized atoms of the metal. In the explanation by a classical wave, the steps in the clicks are the steps of the atoms of the metal.

Because a classical wave on a quantum detector gives the same clicks as single quanta, I state in *The Unquantized Arena* that a detector which registered a single quantum of a gravitational wave would not show the arena quantized. A detector of single quanta registers a step of the detector's own. The detector's step is a step of content, and the arena is the context of the detector's step. So whatever a detection of a small wave re-

gisters, the detection registers a step of the detector. The arena has no step of its own and no quantum at any scale, and no detection of a small wave can find a step or a quantum of the arena.

Two Patterns, One Speed

Picture a ring of beads floating freely in space, and a gravitational wave passing straight through the middle of the ring. As the wave passes, the ring is squeezed into an oval that is tall and narrow, then into an oval that is short and wide, and back again, over and over. A second kind of wave squeezes the ring into ovals, as the first wave does, but along the diagonals. The diagonals are the lines turned halfway between the tall direction and the wide direction. The two kinds of squeezing are the two *polarizations* of a gravitational wave. Each polarization is one pattern in which the wave stretches and squeezes space across its path.

On this account gravitational waves have exactly the two polarizations of general relativity, and gravitational waves travel at the speed of light. I list both properties among the predictions of *The Unquantized Arena*: "Gravitational waves travel at the speed of light with the two polarizations of general relativity." In *The Gravitational Passage* I argue for both properties from one fact, which we met in Chapter 4 when we found that gravity in the large world is Einstein's gravity. The six small dimensions are six dimensions of possibility, held very small, a few Planck lengths across. The fact is that once the six small dimensions are held, with their two measures (their volume and their shape) kept fixed, the equations of Einstein are the only form that the equations of the four open dimensions can take. Einstein's equations predict waves of exactly the two patterns of the ring of beads, traveling at exactly the speed of light.

The speed of gravitational waves has been measured. On 17 August 2017 two neutron stars merged. A neutron star is the collapsed core of a dead star, heavier than the Sun and about as wide as a city. The detectors registered the gravitational waves of the merger, and telescopes caught a

burst of gamma rays from the merger (gamma rays are light of far higher energy than any the eye can see). After a journey of many millions of years, the gravitational waves and the gamma rays arrived within two seconds of each other. The merger is named GW170817, after the date of the merger. GW170817 shows, from the arrival times of its waves and its gamma rays, that the speed of the waves equals the speed of light, within three parts in a million billion on the slow side (minus 3×10^{-15} of the speed of light) and seven parts in ten million billion on the fast side (plus 7×10^{-16}). So the measured speed is the speed that this account states.

A Hum With No Single Source

Lie awake in the dark and listen to rain on the roof. You cannot pick out a single drop. Yet the sound of the rain tells you a great deal. The sound tells you whether the rain is a drizzle or a downpour, and whether what is falling is rain or hail.

The arena can carry a sound like the rain on the roof. A *stochastic background* of gravitational waves is a random sum of waves of the arena arriving from every direction. No single source in a stochastic background can be picked out. ("Stochastic" means random.) In the picture, the single drops are the single waves, and the sound of the rain is the stochastic background. In Chapter 10 we called a stochastic background a hum. A hum of gravitational waves is the gravitational counterpart of the oldest light, the faint afterglow that fills the microwave sky from every direction.

To say where a hum lies, we need one more word: frequency. Every wave has a *frequency*: the number of times the wave rises and falls each second. Frequency is counted in *hertz*: one hertz is one rise and fall each second. A low note of sound has a low frequency, and a high whistle has a high frequency. The first wave LIGO caught swept upward in frequency from about thirty-five to about two hundred fifty hertz (35 to 250) in a fraction of a second, as measured. That range of frequencies lies within human hearing, so the wave, turned into sound, is a short rising chirp.

Light is a wave too, and the color of light is the frequency of the light. The light your eye sees rises and falls several hundred million million times a second, as measured. Infrared is light of lower frequency than red, which the eye cannot see, and the microwaves in which the oldest light reaches us today are lower still in frequency.

Instruments listen for waves of the arena across a range of frequencies far wider than the range of hearing. Each instrument hears only its own band of frequencies. LIGO hears waves of tens to hundreds of hertz, like its first wave. A detector planned for space will listen near a thousandth of a hertz: one rise and fall in about seventeen minutes. Lower still in frequency, astronomers time the flashes of pulsars. A *pulsar* is a spinning dead star that sweeps a beam of radio waves past the Earth as regularly as a clock. Astronomers have timed many pulsars together and found evidence of a hum near a billionth of a hertz. A billionth of a hertz is one rise and fall in about thirty years. Two facts show what made a hum: where the hum lies along the range of frequencies from a billionth of a hertz to hundreds of hertz, and how loud the hum is there.

Why the Hum Keeps a Record

A hum of gravitational waves keeps a record of whatever made the hum, from whatever time the hum was made. The reason is a difference between the waves of the arena and light: the hot early content scattered light, and the content lets the waves of the arena pass almost freely.

Recall the fog that cleared, from Chapter 11. Before the first atoms formed, the hot content scattered light again and again, as fog scatters the beam of a lamp, but when the first atoms formed, about three hundred seventy thousand years into the reconstructed history, the fog cleared. The content stopped scattering the light, and the light went free. The oldest light was released at that one time, so the oldest light shows us the sky as

the sky stood then. The oldest light tells us of an earlier event only through the effects the event had already had on the sky by the time of the release.

A gravitational wave, once made, passes almost freely through all the content of the universe, so a hum carries a record of the process that made the hum, from whatever time in the history the process belongs to. A hum made before the oldest light went free would bring us a record straight from that earlier time. A hum's frequency and strength tell one maker of a hum from another, as the sound of rain tells hail from drizzle.

So the question of this part of the book can be put exactly. The turn is the passage at the end of the interiors of the black holes of the epoch before ours, in which a large outside world is restored, and the history of our epoch begins at the turn. Which processes in the history of an epoch, from the turn at its beginning to the present, make a hum of gravitational waves in the four open dimensions? At what frequency does the hum of each process that makes a hum peak today? And how strong is the hum, counted as the share of the critical density (the whole energy budget of a flat universe expanding at the present rate) that the waves carry in each band of frequency?

Five Places to Listen

The Compactification is the book of the collection that sets out for physicists how an era of the world begins. *The Compactification* is the main source of this book. In *The Compactification* I examine three candidate makers of a hum, one after another: the settling of the six small dimensions at the turn, the first ripples with which the epoch begins, and a change of state of the matter with bubbles. I place two further possible makers beside the three candidates: a network of cosmic strings, and the pairs of giant black holes at the centers of galaxies. Each maker is a place to listen for a hum, and about each maker there is a question of its own.

The first maker is the settling of the six at the turn. The floor is the fact that the six cannot shrink to nothing. As the new epoch opens, the three directions of space grow while the six shrink, until the six reach the floor and are caught there at their held size. The six are in motion as they settle, and the energy of their motion has to go somewhere. Does the settling of the six make waves of the open arena? If the settling does make waves, how much of the energy of the settling do the waves carry?

The second maker is the first ripples: the departures from uniformity with which the epoch begins, recorded in the oldest light. In Chapters 12 and 14 we found that the first ripples carry no first waves of the arena at first order. Do the first ripples make waves of the arena at all, and if they do, how strong are those waves?

The third maker is a change of state of the matter in which bubbles of a new phase (a new state of the matter) form, grow and collide, as bubbles of steam form in boiling water. The colliding bubbles would make the arena ring with waves, and so would the sound and turbulence that the bubbles stir in the hot plasma. Did the matter of our history ever change its state with bubbles that form, grow and collide?

The fourth maker is a network of cosmic strings: long thin flaws left in a field as the field settles. A network of cosmic strings would hum over a wide band of frequencies. In Chapter 13 we found what the settling of the Higgs field leaves behind: no lasting strings. What does the absence of lasting strings mean for a hum from strings?

The fifth maker is the pairs of giant black holes, millions of times the mass of the Sun or more, that spiral together at the centers of galaxies after the galaxies merge. Across the whole sky, the pairs of giant black holes would together make a hum. The astronomers who time pulsars have found a hum near a billionth of a hertz, and the hum has two candidate makers: a population of pairs of giant black holes, or a process of the early universe. Which of the two candidates made the hum?

For each maker that does make a hum, two questions follow. Where does each maker's hum lie today, and how strong is the hum? And can any instrument hear the hum?

What the Waves Can Tell Us

We now know what a gravitational wave is on this account. A gravitational wave is a small ripple of the arena. The arena is classical and smooth at every scale, with no least step and no quantum of its own. The small waves of the arena can be described by quanta, and those quanta belong to the description. A graviton is at most one step of the description, and a single click in a detector is a step of the detector. Gravitational waves stretch and squeeze space in two patterns. Gravitational waves travel at the speed of light, as measured to a few parts in a million billion. A hum of gravitational waves arrives from every direction at once and keeps a record of what made the hum.

A hum's record matters for the question of the whole book: what did the beginning of our epoch leave behind? The oldest light shows us the sky as the sky stood about three hundred seventy thousand years into the history, while the waves of the arena pass almost freely through everything from the moment the waves are made. If the turn and the settling of the six left a hum, the hum carries a record of the turn and the settling themselves, and the hum is still crossing the sky today.

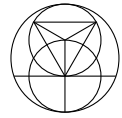
Key idea. Gravitation is not quantized. A gravitational wave is a small ripple of the arena. The arena has no least length, area or duration, and no quantum of its own at any scale. Small waves of the arena can be described by quanta, which are quanta of the description. A graviton is at most one step in the energy of one way a small wave can ripple, never a particle, a carrier of gravitation or one of the kinds of particle. The phonon is the nearest sense of the word graviton, but the arena is not a held structure, so even in the phonon's sense no part of the arena is a quantum. A detector that clicked for single quanta would not show the arena quantized, because a classical wave on a quantum detector gives the same clicks as single quanta. Gravitational waves have two polarizations and travel at the speed of light. A random hum of gravitational waves arrives from every direction and keeps a record of what made the hum. The hum keeps the record because the waves pass almost freely through all the content of the universe.

Summary

On this account a gravitational wave is a small ripple of the arena, the context of every comparison. The wave travels at the speed of light through regions where nothing is found. The arena is smooth at every scale: the arena has no least step and no quantum of its own. Small waves of the arena are described by quanta, and those quanta are quanta of the description, not of the arena. In this book the word graviton names at most one quantum of the description, one step in the energy of one way a small wave of the geometry can ripple, and a graviton is never a particle, a carrier of gravitation or one of the kinds of particle. The phonon is the nearest sense of the word graviton (a phonon is the quantum of the description of a held structure's vibration), but the arena is not a held structure, so even in the phonon's sense the graviton is not a quantum of the arena. A detector that registered a single quantum would register a step of its own, and would not show the arena quantized. Gravitational waves

have the two polarizations of general relativity and travel at the speed of light. The merger of 17 August 2017 shows the speed of gravitational waves equal to the speed of light to within three parts in a million billion on the slow side and seven parts in ten million billion on the fast side. A stochastic background is a hum of gravitational waves from every direction, with no single source. A hum is placed by its frequency, counted in hertz, and by its strength, and a hum keeps a record of the process that made the hum, from whatever time the process belongs to. Five makers of a hum are on the list: the settling of the six at the turn, the first ripples with which the epoch begins, a change of state of the matter with bubbles, cosmic strings, and pairs of giant black holes.

Which of the five makers made waves at the beginning of our epoch, how high in frequency do those waves lie today, and how strong are the waves?



Chapter 16

The Hum of the Settling

Which gravitational waves did the beginning of our epoch leave, how high do they sound today, and how strong are they? In Chapter 15 we found what a gravitational wave is on this account: a small ripple of the arena, the space and time in which everything is measured, traveling at the speed of light. Chapter 15 also named the ways in which the beginning of an epoch could have made such ripples: the settling of the six small dimensions at the turn, the first ripples with which the epoch begins, a change of state that forms bubbles, and a network of cosmic strings. In short, the beginning leaves one background of waves of its own. The background is made as the six settle into their held size. The beginning also leaves a second background, far fainter, which the beginning cannot help making. Three backgrounds that other accounts expect are absent on this account: first waves at first order, a hum from a sudden change in the state of the matter, and a hum from a network of strings.

A Note Stretched by the Expansion

Picture one note played once, at the very start, and recorded on a tape that has been slowly stretched ever since. As the tape stretches, every wave of the note on the tape is drawn out, so the note, played today, sounds lower than when it was played. How low the note sounds now depends on two things only: how high it was played, and how much the tape has stretched since.

The expanding universe stretches every wave in it, as the tape stretches the note. Space is the tape, and each wave in space is a note. The oldest light was set free at about three thousand kelvin. The expansion has since stretched the oldest light until its temperature is under three kelvin. Gravitational waves, the waves of the arena, are stretched in the same way. The beginning made its own waves at a pitch set by the Planck scale, where the description by quanta ends. The growth of the world has been lowering that pitch ever since.

Two words make the picture exact: "frequency" and "pitch". Recall from Chapter 15 that the frequency of a wave is the number of times the wave rises and falls each second, counted in hertz, and that for a sound the frequency is the pitch. As the universe stretches, the length of every wave grows in the same proportion. The frequency of every wave falls in that proportion.

The waves of the settling were made as the six settled into their held size. The settling waves were made everywhere at once, and from no single place, and they arrive today from every direction, overlapping without order, with no single source to point to. Chapter 15 called a mixture of waves from every direction, with no single source, a stochastic background, "stochastic" meaning random. In this book a stochastic background is a hum: the hum of waves from every direction.

What Makes the Settling Waves

As the new world opens after the turn, the three open directions of space grow while the six shrink. The sizes of the open directions and of the six follow the arc, the stretch of the curve of Chapter 2 on which all six small directions shrink together, until the floor catches the six at their held size. The floor, met in Chapter 1, is the limit that keeps the six from shrinking to nothing. The six together form one small space built from two shapes, its two factors. As Chapter 4 found, the larger factor, the complex projective plane, is held at a radius of the square root of six Planck lengths, about

two and a half, and the smaller factor, the sphere, is held at a radius of one Planck length. The motion along the arc makes no ripple of the open world. On the arc the three open directions all grow at one rate, so the open space, taken as a whole, expands the same way in every direction. A space carries no ripple if the space expands the same way in every direction. The arc is strongly lopsided only between the three open directions and the six: the three grow while the six shrink. Seen from the four large dimensions, the shrinking of the six shows up as a change in two numbers, the volume of the six and their shape, which physicists call the moduli of the six. A change in the moduli is no ripple of space and time. *The Compactification* gives the argument that the arc makes no ripple, from the form of the solution that describes the arc and from the way ten dimensions are seen from four.

The settling waves come from what happens to the motion of the six after the floor catches the six. The source books call the one small space of the six the internal space. *The Weight of Possibility* states what happens to the motion. "As the internal space settles into its held size at the turn, the energy of its motion goes into the breathing excitations, and their decay sends that energy into gravitational waves of the open dimensions". A breathing excitation, met in Chapter 4 with the floor, is a swelling and shrinking of the volume of the six that looks the same when every point of the small sphere is swapped with its opposite point. How a vibration behaves under the swap determines whether the vibration can pass its energy to the waves. The waves of the open world also look the same under the swap. So a vibration that changes sign under the swap cannot pass its energy to the waves. The breathing does not change sign, so the swap leaves the breathing free to pass its energy to the waves.

At the order of calculation carried out in *The Gravitational Passage*, the breathing still passes no energy to the waves. The law of the ten-dimensional geometry allows one further term, the term of Gauss and Bonnet, met in Chapter 4 with the floor, and that term lets the breathing pass its energy to the waves. *The Gravitational Passage* does not fix the size of the term. For a term of natural size, neither unusually large nor unusually

small, the lowest breathing vibration turns into waves within about a Planck time. A Planck time is the time light takes to cross a Planck length, about five times ten to the minus forty-four seconds. A Planck time is the briefest interval built from the constants of gravity, quantum theory and light. A Planck time is not a least step of time. So the breathing vibrations made at the turn are long gone: they turned into waves. The dark matter is not among the breathing vibrations. On this account the dark matter is made of gyrations, the lasting vibrations of the six met in Chapter 4, and a gyration changes sign under the swap of opposite points of the sphere. So a gyration cannot pass its energy to the waves, as Chapter 18 sets out. The lowest motion of the six is their state of least energy. The lowest motion is not a gyration, but the lowest motion also stays, for a reason of its own: the lowest motion is already the lowest state, so it cannot give off energy.

The two moduli are the volume and the shape of the six. *The Gravitational Passage* computes the masses of the two moduli at first order, in one mechanism of the hold. The hold is the state in which the floor holds the six small. One modulus is mostly a change of size, and its mass is about two and a quarter Planck masses. The other modulus is mostly a change of shape, and its mass is about three and a tenth Planck masses. I identify the breathing that turns into waves with the modulus mostly of size: a vibration mostly of size, with a mass of about two and a quarter Planck masses.

So the settling hum is a hum of the four open dimensions. The hum is made when vibrations of the volume of the six turn into waves of the large world. The settling hum travels in none of the six, which are directions of possibility, "not spatial and ... never traversed". And the lopsidedness of the arc makes no part of the hum, as the start of this section showed.

How Much Energy the Settling Releases

Two separate things bound the energy that the settling releases: the hold, and measurement.

Recall the picture of the hollow from Chapter 4. A ball that rolls down into a hollow rocks from side to side before the ball comes to rest at the bottom, and the deeper the hollow, the more energy the rocking has to lose. The six come down into the well of the floor in the same way. The well is the dip in energy in which the floor holds the six. The ball is the six, the hollow is the well, and the rocking is the breathing. The energy the rocking loses is the energy the settling releases into waves.

The first bound comes from the hold. The floor holds the six in a well about one twentieth of the Planck density deep, and the settling releases between a tenth of that depth and most of the depth. The Planck density is one Planck mass in every cube one Planck length on a side. *The Weight of Possibility* computes both figures.

The second bound comes from measurement, and *The Coming of the Kinds*, the source book on how the kinds of particle came to be, gives a picture for the measurement. A river runs under a bridge. Upstream, out of sight, several streams feed the river, and from the bridge you can see only one of the streams. Measure how full and how fast the river runs at the bridge, take away the stream you can see, and what remains tells how much the hidden streams bring. The expansion of the early universe is the river. The total energy of all the radiation set the rate of the expansion. The rate shaped the pattern of warm and cool patches in the oldest light. The pattern records two distances at the moment the light went free: how far waves of sound had traveled through the hot gathering of particles and light, and how far the light had wandered among the free electrons, smoothing out the smallest patches. A change in the rate of expansion changes the ratio of the two distances, and the pattern records the ratio. The light itself is the stream we see, and the neutrinos, the lightest

particles of matter, are the hidden streams. So the pattern of the oldest light measures, or weighs, the energy of all the radiation that is not light. Astronomers state the result as the *effective number of light species*, written N eff, the count of light species met in Chapter 4. The effective number is the energy of all the radiation other than light, counted in units of the energy that one kind of neutrino carries. The three kinds of neutrino give three point zero four four, a little over three. The figure is a little over three because the neutrinos were not quite free while the electrons and positrons were annihilating, so the neutrinos took up a small share of the energy that the annihilation released. The Planck satellite mapped the oldest light from space. The Planck satellite measures two point nine nine, give or take seventeen hundredths, when its data are combined with surveys of galaxies. The Atacama Cosmology Telescope maps the oldest light from the ground. The Atacama Cosmology Telescope measures two point eight six, give or take thirteen hundredths, when its data are combined with Planck and further surveys.

The settling waves are one more hidden stream. The settling waves travel with the radiation. The effective number counts the settling waves as radiation beyond the three kinds of neutrino. A species is one distinct state that a kind of particle can be in, apart from where the particle is and how it moves. In the quotation that follows, "the content" means the particles and light of the world. *The Weight of Possibility* states the bound. "The measured number of light species allows those waves less than about three percent of the content's radiation at the time when the content had about a hundred species". The time when the content had about a hundred species is near the start of the history, when the hot mix of particles and light held one hundred six and three quarters species.

Now set the two bounds beside each other. One bound is the energy that the hold lets the settling release. The other is the share that the measured count of light species allows the waves. The measured amount of dark matter sets a limit on the temperature of the plasma, the hot mix of particles and light. A gyration is the dark matter's own vibration of the six. The plasma must never have been hotter than about one thirtieth of

the mass of a gyration. One thirtieth of the mass of a gyration is about two to five times ten to the seventeen GeV. Chapter 10 gave the conversion: one GeV, as a temperature, is about eleven and a half million million kelvin. The energy that the settling releases is "about forty to three hundred times the largest energy that a plasma cool enough" to satisfy the dark matter's limit can hold. So a plasma formed at once beside the waves could not outweigh the waves. A plasma formed at once is formed all at the start, rather than gradually through a stage that behaves as matter. Chapter 4 set out the two cases. If only a plasma formed at once stood beside the waves, the waves would carry far more than three percent of the radiation, and the measured count of light species forbids so large a share.

The account therefore places a condition on the turn: a requirement that the turn must meet. *The Weight of Possibility* states the condition as a requirement, and not as a result. So the waves' share of the radiation is bounded, with its value not given. The account sets a ceiling on the share and gives no value. The condition has two branches. In the first branch, the six arrive at their held size off by no more than about one part in a thousand of their radius. A matter-like stage is a stage in which the content behaves like slow, heavy matter. In the second branch, a matter-like stage carries at least as much energy as the waves, and then decays into the content, not into gravitational waves. So the radiation the stage releases outweighs the waves. The breathing of the six cannot be the matter-like stage, "since the breathing decays at once, and into gravitational waves".

The account therefore predicts a ceiling for the settling hum, and no particular value: "The account predicts no particular excess in the effective number of light species; it requires only that the waves stay within the measured bound".

How High the Hum Sounds Today

How high does the settling hum sound today? *The Compactification* computes the answer itself, from three premises.

The first premise is the energy of each wave at its birth. A breathing vibration turns into two quanta of the description of the waves. Each quantum carries half the mass of the vibration. In this premise, a quantum is one step in which the description of the waves counts the energy of a small wave. The arena itself has no steps. Each quantum then carries about one and an eighth Planck masses, about 1.4 times ten to the nineteen GeV. The figure rests on a mass computed at first order in one mechanism of the hold. The description by quanta also ends at the Planck scale. So the figure is an energy of about the Planck mass: the true figure may be somewhat larger or smaller, by a factor not far from one, and the figure is known no more closely than that.

The second premise is the time of birth. The breathing vibrations turn into waves within about a Planck time of the settling, so *The Compactification* takes the waves to be made at the start of the reconstructed history, when the plasma has its starting temperature. The reconstructed history is the one history that the present requires, pieced together from what arrives now. The account does not fix the starting temperature, but it bounds the temperature from above by the dark matter's limit of about two to five times ten to the seventeen GeV.

The third premise is the stretching since birth. After the start, a region that moves with the expansion keeps the same count of the content's states. So the universe stretches in step with its cooling, corrected for the number of species that carry the count. At the start the number of species is one hundred six and three quarters. Today the number is about three point nine, from the light and from the three kinds of neutrino. The neutrinos are at a lower temperature than the light. The oldest light today is at 2.72548 kelvin. *The Compactification* puts together the two numbers of spe-

cies and the temperature of the oldest light, and finds that every length has grown since the start by about 1.3 times ten to the thirteen for each GeV of the starting temperature.

The pitch today follows from the three premises. The frequency today is about 2.6 times ten to the twenty-nine hertz, divided by the starting temperature in GeV. At the upper bound of the starting temperature, five times ten to the seventeen GeV, the frequency is about 5.2 times ten to the eleven hertz: five hundred twenty billion rises and falls each second. At a starting temperature of two times ten to the seventeen GeV, the frequency is about 1.3 times ten to the twelve hertz. A lower starting temperature gives a higher frequency, because the waves have then been stretched less since the start: at ten to the sixteen GeV the frequency is about 2.6 times ten to the thirteen hertz. So, under the three premises, the settling hum peaks today above about five times ten to the eleven hertz, which are frequencies that light has in the far infrared and above: below the frequencies the eye can see, and above the microwaves.

One possible later stage would lower the frequencies computed above. Suppose a later matter-like stage raised the count of the content's states when the stage decayed. Then the waves would have been stretched more. Every frequency above would be lower by the cube root of the factor by which the stage raised the count, so a thousandfold rise, for example, would lower the pitch tenfold. If the energy of the turn reached the plasma through a matter-like stage, then at the end of the stage the plasma must have had a temperature below about one hundredth of the mass of a gyration. The account fixes neither whether there was a matter-like stage nor how much the stage would raise the count.

A second frequency gives a floor under the pitch. The floor does not depend on the first premise, the energy of each wave at its birth. The horizon of a time is the greatest distance a signal could have crossed since the start. When a process acts in step over a distance no larger than the horizon of its time, a wave the process makes can be no longer than the horizon. At the upper bound of the starting temperature, the longest wave that such a process can make lies today at about 1.3 times ten to the ten

hertz, which is thirteen billion hertz. Today the floor lies below every wave made at the start of the history. The settling waves are made by the decay of quanta of about the Planck mass. So the settling waves are far shorter than the horizon of their time. At the upper bound of the starting temperature, the settling waves lie about forty times higher in frequency than the floor, and the gap grows as the square of how much cooler the start was.

The Compactification does not compute the breadth of the hum, that is, how wide a range of pitches the hum covers. The breadth depends on how spread out in time the waves were made during the settling, which the account does not fix, and on how the energy of the motion of the six was shared among vibrations of different masses. The computation in *The Compactification* states the place of the peak, and the floor below the peak.

How Strong the Hum Is Today

How loud is the settling hum today? The three-percent ceiling fixes how strong the hum can be. *The Compactification* translates the ceiling into the measures astronomers use today, again by its own computation from stated premises.

After the waves are born, they thin out as the expansion spreads their energy, as light does. The light thins in the same way, with one difference. Recall the dance of Chapter 10, in which the dancers who leave the floor hand their tickets to those still dancing: the dancers are the species, and the tickets are the count. Whenever a kind of particle falls out of the hot mix, that kind of particle passes its count to the light, and the light is warmed. The waves receive no tickets. Two effects set the energy of the waves today against the energy of the light. First, at the start the light held only two of the one hundred six and three quarters species sharing the heat. The light held one species for each way light can vibrate. So, measured against the light alone, the waves were many times larger than their share of all the radiation. Second, since the start the light has been

warmed by the tickets of every species that fell out, and the waves have not. Put together, the two effects leave the energy of the waves today at about sixty-five hundredths of the waves' starting share, times the energy of the light. At the ceiling the waves carry about sixty-five hundredths of three percent, or about two percent, of the light's energy.

Counted in the effective number of light species, the waves add to the count their starting share multiplied by about two and eighty-six hundredths. The factor of two and eighty-six hundredths is a multiplier, not a count of species. The multiplier only shares its digits with the count of two point eight six that the Atacama Cosmology Telescope measures. At the three-percent ceiling, the multiplier gives an excess of light species below about nine hundredths. The ceiling of three percent is itself read from the measured counts of light species. *The Compactification* names the measurements on which *The Weight of Possibility* rests the ceiling: later studies that add the data of the Atacama telescope and the measured amounts of the lightest elements, which narrow the bound further. So the excess is a small hidden stream, of the size the measurements leave room for.

Astronomers state the strength of a background in two further ways. The first way is the *energy share per band of pitch*: the share of the critical density (the total density at which space is flat) that the waves carry within one band of frequency. A band is a stretch of frequencies across which the pitch grows by one fixed factor, as an octave doubles the pitch. Physicists write the share with a factor for the present rate of expansion folded in. Every share in this chapter is written in that form. So every share in this chapter can be set beside every other, and beside the figures of Appendix B. Light carries a share of about 2.47 times ten to the minus five. The settling hum, at its ceiling, carries a share below about five times ten to the minus seven: five ten-millionths.

The second way is the unit that detectors use, the *characteristic strain*. The characteristic strain is the fraction by which the passing waves of a background stretch and squeeze a length. An established relation of physics turns a share at a given frequency into a strain. At the ceiling and at five hundred twenty billion hertz, the strain is about 1.7 times ten to the

minus thirty-three. At the ceiling and at 1.3 times ten to the twelve hertz, the strain is about seven times ten to the minus thirty-four. So the strain at the peak is below about two times ten to the minus thirty-three.

Each of the measures of strength above is an upper bound: the excess of light species, the energy share and the strain. If the six settled more gently than the bound requires, the hum is weaker, and if a matter-like stage carried more energy than the waves and then decayed, the stage diluted the hum. The account predicts no particular strength below the bound. The ceiling holds at every pitch, because the effective number of light species weighs the hum whatever its pitch, so the ceiling is valid even though the breadth of the hum is not computed.

The settling hum	Value	Standing
Mass of the breathing vibration that turns into waves	about two and a quarter Planck masses	identified with the vibration mostly of size, whose mass is computed at first order
How long it lasts	within about a Planck time	computed for a Gauss–Bonnet term of natural size
Share of the radiation when the hot mix held about a hundred species	below about three percent	bounded, with its value not given: a condition on the turn, set by the measured number of light species
Peak frequency today	above about five times ten to the eleven hertz	computed by *The Compactification* under its three premises
Floor of frequency today (the size of the horizon)	about 1.3 times ten to the ten hertz, at the upper bound of the starting temperature	computed by *The Compactification*
Excess of light species	below about nine hundredths	computed by *The Compactification* from the three-percent bound
Energy share per band of pitch	below about five times ten to the minus seven	computed by *The Compactification*

The settling hum	Value	Standing
Characteristic strain at the peak	below about two times ten to the minus thirty-three	computed by *The Compactification*

No First Waves at the Start

Chapter 15 also named the first ripples themselves as a possible source of waves. The first ripples are the faint differences of density with which our epoch began, and from which its galaxies grew. The polarization of light is the direction in which the light vibrates. The first waves are gravitational waves present from the beginning of an epoch, on lengths as long as those of the first ripples, and first waves would show as a twist in the pattern of the polarization of the oldest light. Astronomers state the strength of the first waves as the ratio of the power of the first waves to the power of the first ripples.

The Gravitational Passage predicts that the beginning makes no first waves at first order. The first ripples are scalar, and gravitational waves arise from scalar ripples only at second order, with a ratio of first waves to first ripples far below one in a million. The reason is in what the first ripples are. The form of the content passes through the turn. The ripples of the new geometry are the ripples that the ripples of the content make. A difference of density makes a scalar ripple of the geometry: a change in the amount of curvature from place to place. A wave of the geometry needs a stress that pulls differently in different directions, and content close to equilibrium, the settled state in which nothing further changes on average, has no stress of that kind at first order. So first waves arise only at second order, from ripples acting on ripples, and so the ratio lies far below one in a million. Measurements so far place the ratio below about three and a half in a hundred.

The Unquantized Arena is another book of the same collection. *The Unquantized Arena* says what a background of first waves would mean if one were found. The strength of a background of first waves would be set by the lowest, unremovable motion of small waves counted in steps. The steps belong to the description of small waves, and the account allows steps of the description, as Chapter 15 showed. So a finding of first waves would show the steps of a description, not a quantized arena. The account depends on neither outcome of the search.

The Faint Hum Every Epoch Makes

The beginning does make waves at second order. The making of waves at second order is established physics, independent of this account: a scalar ripple of the geometry, acting on itself through Einstein's equations, makes a wave of the geometry in proportion to the square of the ripple's strength. Physicists call a background made this way an *induced*, or second-order, background: the ripples make it as they act on themselves, rather than at the start beside them. While radiation fills the universe, the induced waves carry a share of the radiation's energy in each band of pitch, and the share is about eighty-two hundredths of the square of the power of the ripples.

The power of the first ripples is measured: about two billionths. *The Compactification* takes the power to be nearly the same on all the lengths concerned, and computes the share of the induced hum today at about six times ten to the minus twenty-three to one and a half times ten to the minus twenty-two. The share is stated in the same form as the settling hum's share of five ten-millionths. The range in the share comes from the species that fall out of the hot mix after a given wave has come within the horizon. The share is lowered more for waves that came within the horizon while the mix held about a hundred species than for waves that came within the horizon after the electrons and positrons had annihilated. The pitches of the induced hum run over the whole range of the first ripples,

from the frequencies of the largest lengths observed upward. The account fixes the power of the ripples on short lengths no better than measurement does. The computed share takes the measured power to continue unchanged to short lengths.

An interferometer is a detector that measures tiny changes of distance with light. The induced hum is about nine powers of ten, a billion times, below the strength that the most sensitive planned interferometers reach in their own bands. The induced hum is the one hum of the arena that the account requires wherever scalar ripples exist, at a strength set by their power there, and the induced hum is undetectable.

No Hum from Boiling

Chapter 15 also named a sudden change in the state of the content as a possible source of waves. The content is the particles and light of the world. Chapter 10 gave two pictures of a change of state. Water chilled below freezing in a very clean bottle stays liquid, then turns to ice in sudden crystals that spread and meet: a first-order transition. (In this section the words "first order" name the sudden kind of change, as in Chapter 10, and not the first order of a calculation met earlier in this chapter.) Butter warming on a counter softens steadily, with no jump and no bubbles: a crossover. In a first-order transition of the early universe, bubbles of the new state form, grow and collide. A hum of gravitational waves comes from the collisions of the bubbles, from the sound waves in the plasma and from the churning of the plasma, and the hum peaks at a frequency set by the temperature of the change and by how long the change lasts. For a change at the scale of the weak force, near a hundred GeV, the hum would peak today near a millihertz. A millihertz is one rise and fall in about seventeen minutes, and it lies in the band of *LISA*, a planned detector in space that will listen for slow waves by measuring, with laser light, the changing distances between spacecraft far apart.

The history of the content has no first-order transition. The electroweak transition is the change in which the Higgs field takes its value, met in Chapter 10 near 159.5 GeV. The electroweak transition is a crossover. *The Coming of the Kinds* states the result and the ground for it. The Standard Model is the established account of the particles and of every force but gravity. Physicists have computed the Standard Model on a grid of points, a lattice, and the computations show "that the transition has bubbles and walls only for a Higgs mass below an endpoint near 72 GeV". The measured Higgs mass is 125.13 GeV, give or take 0.11, "far above the endpoint. So the electroweak transition is a crossover". The center of the crossover lies at about 159.5 GeV. The strong force has a transition too, near 156.5 million electron volts. In the transition of the strong force, quarks and gluons are bound into protons, neutrons and other particles made of quarks. The transition of the strong force is likewise "a smooth crossover with no jump".

The kinds of particle are those of the Standard Model, and no further kinds exist to make another transition. The fields of the Standard Model make only two transitions, the electroweak crossover and the crossover of the strong force. So another field of the Higgs field's sort would be needed for a first-order transition at the scale of the weak force or above, and the account has the Higgs field as the only field of that sort. So the history of the content after the turn holds no first-order transition, and no hum from bubbles, sound or churning. The account therefore predicts that LISA will find no hum made by a change of state of the early universe. LISA is expected to hear other hums, from pairs of white dwarf stars in our Galaxy and from the many pairs of black holes, and these hums come from stars and their remains. A white dwarf is the small, dense leftover core of a star like the Sun. The prediction about a change of state leaves the hums of stars and their remains untouched.

Cosmic strings, walls and monopoles make no hum either, for the reason met in Chapter 13 with inflation. A cosmic string is a thin line of trapped field energy. The lowest states of the Higgs field form a set shaped like a three-dimensional sphere, all one piece, on which every

loop and every sphere drawn shrinks to a point. Stable strings need loops that cannot shrink, walls need separate pieces, and monopoles need spheres that cannot shrink. Established physics gives both results: the shape of the set, and what strings, walls and monopoles need. When the Standard Model is the whole of the content, with no larger group of symmetries unifying the forces and no further field of the Higgs field's sort, the history of the content makes no stable network of strings. So the history of the content makes no hum from a network of strings either.

What the Pulsars Hear

One hum of gravitational waves has already been found, at a very low pitch. As Chapter 15 said, a pulsar is a dead, spinning star that sweeps a beam past us once on every rotation, like a lighthouse, and the fastest pulsars keep very steady time. A *pulsar-timing array* is a set of the fastest pulsars timed over years: a slow wave passing between the pulsars and us shifts the arrival times of their pulses, in a pattern shared across the sky. In 2023 the North American Nanohertz Observatory for Gravitational Waves reported evidence for a hum near a nanohertz, a billionth of a hertz: one rise and fall in about thirty years.

Astronomers propose two origins for the nanohertz hum. The first proposed origin is the many pairs of giant black holes at the centers of galaxies, circling each other. The second proposed origin is a process of the early universe: a first-order transition near the scale of the strong force, a network of strings, or first ripples much stronger on short lengths than the ripples measured on long lengths. On this account the early universe supplies none of these processes. The early universe has no first-order transition and no network of strings. At the measured strength of the ripples, the induced hum is some fourteen powers of ten, a hundred million million times, too weak. Only ripples enormously stronger on short lengths would make the induced hum strong enough, and the account neither provides nor excludes such ripples. So, if the power of the ripples

on short lengths is near the measured power, the pulsar-timing hum comes from the pairs of giant black holes. *The Compactification* draws the conclusion about the pulsar-timing hum, under the condition on the ripples, from three findings of this chapter: no first-order transition, no strings, and a faint induced hum.

What the Beginning Left in Waves

We now know the one background that the beginning leaves of its own, the one background that the beginning cannot help leaving, and the three backgrounds that the beginning does not leave. The beginning's own background is the settling hum, sounding today above about five times ten to the eleven hertz. The settling hum has a share below about five ten-millionths. The background the beginning cannot help leaving is the induced hum of the first ripples, with a share of about ten to the minus twenty-two. The beginning leaves no first waves at first order, because the ripples with which an epoch begins are scalar. The beginning leaves no hum from a change of state, because the electroweak and strong transitions are crossovers. No further kind of particle exists to make another transition. The beginning leaves no hum from strings, because the set of the lowest states of the Higgs field is all one piece, with every loop on the set shrinking to a point.

Why do these backgrounds matter for the question of this book, what the beginning left behind? The settling hum is the beginning's own background of waves, and the ceiling on the settling hum places a condition on the turn: either the six arrived at their held size off by no more than about one part in a thousand of their radius, or a matter-like stage, in which the content behaved like slow, heavy matter, outweighed the waves. The first ripples are the form of the content, carried through the turn. The settling hum, by contrast, is made by the new epoch itself, near its beginning, from the energy released as the six were caught. And every wave

named in this chapter is a ripple of an arena that is not quantized. Where the energy of the waves is counted wave by wave, the waves are described by quanta of the description, and never by quanta of the arena.

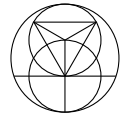
> **Key idea.** As the six are caught at their held size, their motion becomes breathing vibrations that turn at once into gravitational waves. The settling hum has been stretched since the start. Today the hum sounds above about five hundred billion hertz. The measured number of light species caps the energy of the hum at below about three percent of the early radiation: the account predicts the ceiling, and no particular value below the ceiling.

Summary

The beginning of an epoch leaves one hum of gravitational waves of its own. The arc of the re-forming (the return of the six to their compact form) makes no ripple, because the three open directions grow together. The waves come from the settling, when the energy of the motion of the six goes into breathing vibrations of their volume. The vibration that turns into waves is the vibration mostly of size, of about two and a quarter Planck masses. The vibration turns into waves within about a Planck time. The energy released is bounded by the depth of the floor's well and by the measured number of light species, which allows the waves below about three percent of the radiation when the hot mix held about a hundred species. The bound of three percent sets a condition on the turn, not a result. Either the six arrive at their held size off by no more than about one part in a thousand of their radius, or a matter-like stage outweighs the waves. *The Compactification* computes from three stated premises that the hum sounds today at about 2.6 times ten to the twenty-nine hertz divided by the starting temperature in GeV. The starting temperature is at most about five times ten to the seventeen GeV. So the hum sounds above about five times ten to the eleven hertz. The horizon sets a floor under the pitch of

the hum: about 1.3 times ten to the ten hertz at the upper bound of the starting temperature. The computation in *The Compactification* also gives an excess of light species below about nine hundredths, a share below about five times ten to the minus seven, and a strain below about two times ten to the minus thirty-three. The account predicts a ceiling, and no particular value. By established physics, the epoch also makes an induced hum, about a billion times below the strength that the most sensitive planned interferometers reach. The epoch makes no first waves at first order, no hum from a change of state, and no hum from strings. If the ripples on short lengths are near their measured strength, the pulsar-timing hum comes from pairs of giant black holes.

The settling hum sounds at the pitches of the far infrared. The induced hum lies a billion times below the strength that the planned interferometers can register. Can any instrument hear the waves of the beginning?



Chapter 17

Listening for the Beginning

Can any instrument hear the waves that the beginning left, and what will the coming instruments find? Chapter 16 showed that the beginning leaves one background of gravitational waves of its own: the waves of the settling, sent out as the six small dimensions of possibility came to rest at their held size. One measurement already registers the settling waves: the count of light species. The count of light species is the measure of how much radiation the young universe carried. The oldest light records the count. The count is the strong test of the settling waves, because the count registers the whole background at once, in every band of pitch. Instruments have been built to hear gravitational waves at high frequency, and all of them listen below the pitch of the settling waves. The account is the account of the world that this book teaches. An epoch is one era of the world. In the coming decades, new instruments will test the account's other statements about the waves of an epoch, one instrument at a time.

What We Are Listening For

Three quantities describe any background of gravitational waves, and we met all three in Chapter 16. The first quantity is the frequency received today: how many times a second the wave rises and falls as the wave passes. Frequency is counted in hertz, and one hertz is one vibration a second. The second quantity is the energy share: the energy that the waves carry in one band of frequency, as a share of all the energy that a

flat universe (a universe whose space has no overall bend) holds today. The third quantity is the characteristic strain: how much the waves of one band stretch and squeeze a length, as a fraction of that length. Established physics is a result of physics, taken from the source where the result was established. At a given frequency, a standard formula of established physics turns the energy share into the strain, and the strain back into the energy share.

The account makes four statements about the waves of an epoch. Each statement is a target for the instruments. The four statements are about the settling background, the first waves, a faint background from the first ripples, and the matter's changes of state.

The first statement is about the settling hum of Chapter 16, which I call the settling background in this chapter. A breathing excitation is an even swelling and shrinking of the size of the six small dimensions of possibility, which this book calls the six. Each breathing excitation weighs on the order of a Planck mass, so the settling waves were sent out at energies on the order of the Planck energy, the energy that a Planck mass carries. Since then, the whole expansion of the epoch has stretched the settling waves. The frequency of the settling waves today depends on one quantity that the account does not fix: the starting temperature. The starting temperature is the temperature at which the reconstructed history of the epoch begins. The reconstructed history is the history that we piece together backward from the records we have now. *The Compactification* is the book of this collection that sets out for physicists how an era of the world begins. *The Compactification* computes the frequency today from the three premises of Chapter 16: the energy of each wave at its birth, the time of its birth, and the stretching since. The frequency today is about two point six times ten to the power twenty-nine hertz, divided by the starting temperature in GeV. One GeV is a billion electron volts. One electron volt is the energy an electron gains in crossing one volt. The starting temperature has an upper bound of about five times ten to the seventeen GeV, the bound that keeps the hot matter from making gyrations, the pieces of the dark matter, in pairs through gravity. A cooler start gives a higher frequency,

because the frequency is divided by the starting temperature, so the upper bound on the starting temperature gives a lower bound on the frequency: above about five times ten to the eleven hertz. Five times ten to the eleven hertz is five hundred billion vibrations a second, which is half a terahertz. The content is the particles and light of the world. The count of the states of the content is the entropy of the content (Chapter 6). When no later stage of the content raised the count of states of the content, the lower bound on the frequency holds. A stage that behaves as matter is a stage in which energy is held in heavy, slow things that later decay. The turn is the passage, at the end of the black holes' interiors, in which three directions open up while the six close in. If a stage that behaves as matter carried the energy of the turn into the content, and raised the count of states as the stage decayed, then the settling waves were stretched more. When a stage that behaves as matter raised the count, the frequency today is lower by the cube root of the factor by which the stage raised the count, as Chapter 16 showed: a thousandfold rise in the count lowers the frequency tenfold.

The count of light species limits the strength of the settling background. The measured count limits the settling waves to less than about three percent of the radiation of the content at the time when about a hundred species (distinct states of particles, Chapter 4) shared the heat. In the count of light species, a share of three percent is an excess of at most about nine hundredths of one light species. As an energy share today, added over the whole background, the three-percent limit is at most about five parts in ten million. At five hundred billion hertz, the three-percent limit is a strain of about two parts in ten to the power thirty-three. The limit of five parts in ten million is the ceiling of the settling background. The account predicts no particular value below the ceiling. *The Weight of Possibility* is the book of this collection that is the source for the dark matter. In the words of *The Weight of Possibility*: "The account predicts no particular excess in the effective number of light species; it requires only that the waves stay within the measured bound". So the settling background is a target with a ceiling and no floor: the account requires the background to lie below the ceiling, and the background may lie far below the ceiling.

Of the other three statements, two state an absence and one states a faint background. The first ripples are the faint departures of density with which our epoch began. The first waves are the gravitational waves present from the beginning of the epoch. The first ripples were departures of density, so the first waves arise from the first ripples only at second order, through the ripples acting on one another, and the power of the first waves is far below one millionth of the power of the first ripples. On the account, the first waves are so weak because the departures are scalar: a departure of density has a size at each place, and no direction. The first ripples make a faint background of gravitational waves by acting on one another. *The Compactification* computes the energy share of that faint background as about one part in ten to the power twenty-two, spread over the frequencies set by the sizes of the ripples. The fourth statement is that the changes of state of the matter made no background of gravitational waves. The electroweak change happened near a hundred and sixty GeV. In the electroweak change the Higgs field settles at its value, and the masses of the particles switch on. The strong change happened near a hundred and fifty-six and a half million electron volts. In the strong change, quarks became held. Quarks are the particles of which protons and neutrons are made. As Chapter 16 showed, both changes were smooth crossovers. In a crossover the state of the matter follows the temperature continuously, with no sudden jump. The list of kinds of particle is closed: no kinds of particle exist beyond the kinds on the list. So no other field supplies a sudden change. The account therefore has two grounds for saying that no change of state made a background: the results of established physics, by which both changes were smooth, and the closed list of kinds of particle.

The account has a use for each of its four statements about the waves of an epoch, whether or not any instrument can hear down to the level that the statement names. Where no instrument can hear down to the level that a statement names, the account still uses the statement in its reasoning. The faint background that the first ripples make lies far below every instrument, and yet the faint background shows that the hum registered by the pulsar-timing arrays of Chapter 16 cannot come from the

first ripples at their measured strength. Where an instrument can hear down to the level that a statement names, the instrument tests the statement. Of the four targets, only the settling background is strong enough that a detector might register the settling background. So in this chapter we measure the detectors that work at high frequency against the settling background. Each of the other three statements is tested by the instruments that would register its background if the background were there.

The Radio and Its Dial

Picture a radio whose dial runs only across the low stations. Somewhere a station is broadcasting music, far above the top of the dial. You can turn the knob as far as the knob goes, and no music comes. The radio can be perfectly good and still never play the far station, because the station broadcasts at a pitch the radio cannot tune to. A bigger speaker does not help. What the radio lacks is a dial that reaches the station.

Almost every high-frequency detector of gravitational waves is like the radio in the picture. In the picture, the dial is the band of frequencies that a detector can tune to. The far station is the settling background. Almost every high-frequency detector works by *resonance*. A part of the instrument has a natural pitch at which the part likes to swing, and a wave arriving at that pitch builds up the swing, as pushes timed to a child's swing build up the child's motion. In the detectors that work by resonance, the parts that swing are crystals, metal cavities filled with microwaves, magnetic samples and trapped beads. Objects of everyday size have natural pitches at or below about a billion vibrations a second, a gigahertz, while the settling background sounds at half a terahertz or above, five hundred times higher than a gigahertz.

Two gaps measure how far each detector falls short of the settling background. The *frequency gap* is the number of powers of ten by which the top of the detector's band falls below five hundred billion hertz, the lowest frequency of the settling band. The settling band is the band in

which the settling waves sound. The *amplitude gap* is the gap in loudness: the number of powers of ten by which the weakest energy share that the detector can register lies above the ceiling of five parts in ten million. Each gap is counted in powers of ten, so a gap of three is a factor of a thousand.

The radio in the picture is also nearly deaf. The radio could play the far station only if the station broadcast with more power than every transmitter on Earth together. The detectors are deaf as the radio is. A detector's reach is the weakest energy share that the detector can register, and the reach says how deaf the detector is. An energy share of one would be a background that carries as much energy as everything a flat universe holds today. For a reach of ten to the power thirteen, ten trillion, the faintest background that the instrument could register would carry ten trillion times all the energy in the universe. No universe holds a background that large. So a reach far above one is a measure of deafness: the larger the number, the deafer the instrument.

The bands and reaches below are the instruments' own published figures, and *The Compactification* computes each gap from those figures. Every instrument is set beside the ceiling in energy share, because every reach can be stated as an energy share.

Nine Ways to Listen

The Compactification surveys nine sorts of detector, each listening for gravitational waves at high frequency in its own way. For each sort, the paragraphs below say how the detector listens, where the detector's band lies, and whether the detector can hear the settling waves. Three of the nine sorts matter most for this book: the best instrument now running, the method that has come closest to the settling waves in pitch, and the one proposal that listens in the band of the settling waves. Appendix B contains a table of each instrument's exact band, reach, gaps and stage of development.

Quartz crystals. The most developed of the nine sorts is a crystal of very pure quartz mounted between two electrodes. Quartz makes a small electric voltage when it is squeezed, so the electrodes turn the crystal's vibrations into an electric signal. The crystal with its electrodes is a ringing antenna: a passing wave squeezes the crystal, and the crystal rings. The quality factor of a ringing thing is the measure of how slowly its ringing dies away. Crystals of very pure quartz ring for a very long time before the ringing fades: their quality factor is about a billion, approaching ten billion. The leading quartz-crystal instrument is MAGE, at the University of Western Australia. MAGE reads two nearly identical quartz crystals, deeply cooled, with very sensitive magnetic amplifiers. MAGE listens in several narrow bands between five and sixteen million vibrations a second, that is, between five and sixteen megahertz. Quartz crystals could in principle ring at nearly a gigahertz through their higher notes. Even a note of nearly a gigahertz lies about five hundred times below the settling band. In loudness, the faintest background that quartz crystals could register would carry ten trillion times all the energy in the universe, or more. Quartz crystals cannot hear the settling waves.

Superconducting cavities. The second sort is a hollow metal cavity that holds two nearly identical patterns of microwaves. The cavity is cooled until its walls carry current with no resistance. One pattern is filled with microwaves, and the other pattern is kept quiet. A passing gravitational wave moves a little energy from the filled pattern into the quiet pattern. The wave does so by flexing the walls or by acting on the microwaves directly. Precise electronics watch the quiet pattern for any energy arriving in it. The main program today is MAGO 2.0, a joint effort of the DESY laboratory in Hamburg and Fermilab near Chicago. Superconducting cavities listen from about ten thousand vibrations a second (ten kilohertz) up to about a hundred megahertz, and a hundred megahertz is about five thousand times below the settling band. Heat and quantum noise limit how faint a wave the cavities can register. Quantum noise is the small unavoidable jitter that the quantum adds to every measurement. Superconducting cavities cannot hear the settling waves.

Magnon detectors. In a magnetic crystal every atom is a tiny magnet. The tiny magnets can wobble together in a shared wave. One step of the shared wobble is a *magnon*. When the pitch of a gravitational wave matches the pitch of the wobble, the wave can set off magnons by resonance. A magnetic field applied to the crystal tunes the pitch of the wobble, so magnon detectors naturally work at billions of vibrations a second. A related instrument, the haloscope, works in the same band as magnon detectors. The haloscope is a strong magnet around a microwave cavity, built to turn axions (a particle that other theories propose) into microwaves, and a haloscope can turn a gravitational wave into microwaves in the same way. Physicists have read the data of experiments built to search for axions a second time, as a search for gravitational waves. Magnon detectors work between about one and twenty gigahertz, the highest band of any laboratory method now running.

The magnon detector is the best of the instruments now running. The radio picture shows how far the magnon detector is from hearing the settling waves. The magnon detector's band ends at twenty billion vibrations a second, about twenty-five times below the lowest pitch of the settling waves: in the radio picture, the station is still above the top of the dial. In the radio picture, the magnon detector is also deaf. The faintest background a magnon detector can register would carry about a trillion trillion times all the energy in the universe, or ten times as much (ten to the power twenty-four to ten to the power twenty-five). The magnon detector's reach lies thirty to thirty-two powers of ten above the ceiling. Even the best instrument now running cannot hear the settling waves, in pitch or in loudness.

Conversion into light. In 1962 the physicist Mikhail Gertsenshtein described how gravitational waves and light can turn into one another inside a magnetic field. So a magnetic field can turn a passing gravitational wave into a little light. A detector can then catch the light made from the wave. Physicists use the conversion in two ways, which count as two of the nine sorts of detector. The first way uses magnetic fields in space. Clusters of galaxies carry weak magnetic fields, with each field holding its direction

over some thirty thousand light-years, while the thin gas between the galaxies carries a field weaker still. The magnetic fields in space would turn a background of gravitational waves into a little extra radio and microwave light, and astronomers have looked for that extra light with telescopes already built. The second way uses strong magnets in the laboratory, with microwave cavities or guides that collect the light made inside the magnets. To register faint waves, the laboratory way would need large magnets of very high field and the counting of single particles of light, both far beyond present practice at high frequency.

Of all the methods, reading the sky through its magnetic fields has come closest in pitch to the settling waves. The bound set with the fields of clusters lies near a hundred gigahertz, only about five times below the lowest pitch of the settling band: in the radio picture, the station is just above the top of the dial. In the radio picture, the sky reading is deaf as well. In energy share, the cluster bound is about ten thousand trillion (ten to the power sixteen), about twenty-two powers of ten above the ceiling. So the closest approach in pitch came from reading the sky, not from building a detector. Even the sky reading cannot hear the settling waves.

Levitated sensors. In the proposal of levitated sensors, laser light holds up and cools tiny beads or discs of glass inside a cavity between two mirrors. The beads or discs serve as a tunable ringing sensor for waves between about fifty thousand and three hundred thousand vibrations a second. In the band of the levitated sensor, the sensor grows more sensitive as the pitch rises, while a large laser detector such as LIGO, the pair of detectors of Chapter 9, loses sensitivity in that band because of the graininess of its own light. So a compact instrument could do better than LIGO in the band of the levitated sensor, at a length of ten to a hundred meters. Physicists at Northwestern University are building a prototype one meter long. The levitated sensor listens about a million times too low in frequency, and the projected reach of the levitated sensor lies twelve to nineteen powers of ten above the ceiling in energy share. A gap of twelve powers of ten is the smallest stated gap in loudness among the nine sorts of detector. The levitated sensor cannot hear the settling waves.

The Li-Baker concept. The Li-Baker concept would listen between about three and ten gigahertz, and the concept would use a microwave beam of smooth, bell-shaped profile, a steady magnetic field, and patterned reflectors. A passing wave makes a small change in the flow of light in the beam. The detector would read the wave through the change in the flow of light. The Li-Baker concept exists as a proposal. The proponents of the Li-Baker concept claim a reach for the concept, but the claimed reach is not established, so *The Compactification* leaves the claimed reach out of every comparison. The band of the Li-Baker concept lies about fifty times below the settling band.

The Holometer. The Holometer has run and finished. The Holometer was a pair of laser interferometers at Fermilab, side by side, each thirty-nine meters long. An interferometer measures tiny changes of distance with light, as Chapter 16 said. An interferometer splits a laser beam, sends the halves down two arms, and brings the halves back together, so that a tiny change in the length of one arm against the other shows as a change in the light. The Holometer listened between one and thirteen megahertz, and from a hundred and thirty hours of the Holometer's data, its team set the first direct limit on a background at megahertz frequencies. The Holometer's limits are measured, and they lie nineteen to twenty-two powers of ten above the ceiling, while the Holometer's band lies about forty thousand times below the settling band.

Phonon detectors in crystals. A phonon is one step of energy of a vibration in a crystal, as Chapter 15 described. The atoms of a crystal vibrate against one another a trillion times a second and more, far faster than any everyday object can ring. A proposal of 2023 showed that detectors able to register single phonons in crystal targets could search for gravitational waves between about a trillion and a hundred trillion vibrations a second. Between a trillion and a hundred trillion vibrations a second, phonons have energies from about four thousandths to about one tenth of an electron volt.

The phonon proposal is the one proposal whose band contains the settling band, so in the radio picture the phonon detector is the one radio whose dial reaches the station. For a starting temperature between about two and about two hundred times lower than the hottest that the account allows, the peak of the settling waves falls inside the band of the phonon detectors. The proposal exists as a calculation: nobody has mounted an experiment, and no figure yet shows how faint a wave a phonon detector could register. The band of the phonon detectors is the right one, but at the ceiling the settling waves would stretch lengths by a far smaller fraction than any stretch that a laboratory instrument has registered, at any frequency.

The Whole Comparison

When the nine sorts of detector are set side by side, three facts are plain. Eight of the nine sorts work below the settling band in frequency, from about five times to about a million times too low, while every sort with a stated reach lies at least twelve powers of ten above the ceiling in energy share. The one sort whose band contains the settling band exists so far as a calculation. Appendix B contains a table of all nine sorts.

The frequency gap has a reason: the size of the source. The size of the source means the energy at which the source sent out the waves. The settling waves were sent out at energies on the order of the Planck energy, and the expansion has stretched the settling waves since the start of the reconstructed history, by the factor that Chapter 16 computed. Every length has grown by about one point three times ten to the thirteen for each GeV of the starting temperature. So a hotter start means more stretching. Even at the hottest start that the account allows, with no later stage raising the count of states, the settling waves remain above half a terahertz. Many instruments read a wave through a ringing of sound, of microwaves or of magnetism, and every ringing instrument works at or below a gigahertz, the pitches at which objects of everyday size ring. A

detector at half a terahertz or above must use something whose vibrations carry the energy of a wave at that pitch, and so the one proposal in that band uses the fastest vibrations of crystals.

When many excitations decay, the waves they send out are spread in frequency, because the excitations decay over a span of time while the expansion goes on. If each excitation sends its waves out at one frequency, the spread lies at and below the peak, because the waves of the earlier decays are stretched further. So in the megahertz and gigahertz bands, no part of the settling background has an energy share above the ceiling. The gaps in this chapter are taken at the lowest frequency of the settling band, so they are the smallest gaps there are. The account does not compute how wide the spread is, as Chapter 16 said.

The history of gravitational-wave detection gives a scale for the amplitude gap. The ceiling of five parts in ten million is about the same size as the bound that the count of light species places on any background of gravitational waves present before the lightest elements were made, in the first few minutes. The best direct limit at megahertz frequencies is the Holometer's, and the Holometer's limit lies nineteen powers of ten above the ceiling, and no instrument working above a thousand vibrations a second has come near the level of the ceiling. The first resonant bars were large metal cylinders built to ring when a gravitational wave passed, and in strain the first resonant bars were about eight powers of ten less sensitive than Advanced LIGO is, and closing that gap took about fifty years. At a given pitch the energy share goes as the square of the strain, so eight powers of ten in strain are sixteen in energy share. The fifty years of progress were made at the pitches of sound, where stars and black holes supply many sources, so each gain in sensitivity promised discoveries. Above a megahertz no source in stars or black holes is known. The beginning alone gives a reason to listen above a megahertz. Even if the progress of the last fifty years continued at the same rate, the progress would not close a gap of twelve or more powers of ten in energy share together with a gap in frequency.

Weighing the Sealed Box

Picture a sealed box that you may not open. You want to know how much is inside. You cannot look inside, but you can weigh the box. The scale gives the total of everything inside at once, whatever the shape of each thing and wherever each thing sits. Weighing gives the total of the contents, and gives no list of them.

The count of light species weighs the settling background in the way the scale weighs the box. Gravitational waves carry energy. While the wavelengths of gravitational waves are far shorter than the horizon of their time (the Hubble radius of Chapter 8), their energy thins with the expansion just as the energy of light does. So the waves count as extra radiation in their effect on the expansion. More radiation in the young universe means a faster expansion of the young universe, and a faster expansion changes the pattern in the oldest light and changes the amounts of the lightest elements made in the first minutes. The settling waves therefore show as extra radiation both in the oldest light and in the amounts of the lightest elements. Physicists count the extra radiation in the effective number of light species of Chapter 16: the radiation of the young universe beyond light itself, counted in units of the energy that one kind of neutrino carries. The three kinds of neutrino give three point zero four four. In the picture, the sealed box is the young universe, the things inside are all of its radiation beyond light, and the scale is the count of light species.

There is a link of established physics between the energy share of the waves and the count of light species. Today the waves carry a total energy share, over every frequency, of about five point six millionths of the excess of the count above three point zero four four. The link holds for any background present before the lightest elements were made, whatever the frequency of the background. So the ceiling of the settling background matches an excess of nine hundredths of a light species: the ceiling is an energy share of about five parts in ten million.

The count of light species is therefore the *indirect test* of the settling background: the count registers the waves through their effect on the expansion, and not through their effect on a detector. The indirect test is also the strong test, for a plain reason. A direct detector registers the part of a background that falls in its own band. The count of light species registers the energy of the whole background, in every band at once, because the count measures the expansion that the energy produces. The count of light species also sets the ceiling of the settling background. So the count of light species is the first measurement sensitive enough to register a background at the level of the ceiling. No direct detector can register the settling background before the indirect test has bounded the background more closely.

The count of light species has been measured. The Planck satellite was a spacecraft that mapped the oldest light. The Planck team combined the oldest light with the pattern of spacing that sound waves in the early plasma left among the galaxies, and measured two point nine nine, give or take seventeen hundredths. A second measurement gives two point eight six, give or take thirteen hundredths: the Atacama Cosmology Telescope, with Planck and further surveys. Later studies narrow the bound further by adding the amounts of the lightest elements, and the three-percent limit of *The Weight of Possibility* rests on the later studies, as does the excess of at most nine hundredths of a light species. The Simons Observatory is an array of telescopes in the Atacama Desert of Chile, and the observatory is expected to measure the count to within about five to seven hundredths.

The account requires the settling waves to stay within the measured bound. The account predicts no particular excess. The list of kinds of particle is closed, so no further light kind of particle exists to explain an excess in the count. So if a measurement finds the count well over a tenth of a light species above three point zero four four, the excess would measure the share of the radiation that the settling waves carry. The settling waves' share of the radiation would in turn measure how far the six were displaced from their held size when the six arrived at the held size. The

condition on the turn is a requirement that the account places on the turn, and not a result that the account derives. The condition on the turn would then be stated again, with the measured share in place of three percent. For the present bound the condition reads, in the words of *The Weight of Possibility*: “either the internal space arrives at its held size displaced by no more than about one part in a thousand of its radius, or a stage that behaves as matter carries at least as much energy as the waves and decays into the content”. The measured count of light species sets the condition on the turn: the count shows what the turn must have been like. If a measurement finds the count at its three-neutrino value, to within five hundredths, that measurement lowers the ceiling on the settling waves. On the account, the settling waves may then lie anywhere beneath the lower ceiling.

The Other Three Statements

The first waves would show in the oldest light. The oldest light is polarized: the light vibrates more in one direction than in another, and the direction varies across the sky in a pattern. First waves would add a pattern with a curl in it over large patches of the sky. Physicists call the pattern with a curl the B-mode pattern. The power of the first waves is stated as the ratio of the power of first waves to the power of first ripples. Measurements today put the ratio below about three parts in a hundred, and two coming instruments, the Simons Observatory and the Japanese satellite LiteBIRD, will be sensitive to a ratio of about a thousandth. The account says that the ratio is far below one millionth, three or more powers of ten below the reach of both instruments. *At the Edge of Reality* is the book of this collection that this book follows. *At the Edge of Reality* states the prediction for the Simons Observatory and LiteBIRD in these words: “The account says they will find nothing”. The faint second-order background has an energy share of about one part in ten to the power twenty-two. The

second-order background lies at least nine powers of ten below the reach of every instrument in this chapter, at every frequency. No planned measurement aims at the second-order background.

The European Space Agency has adopted LISA, the Laser Interferometer Space Antenna of Chapter 16, and LISA will listen in the band of slow waves that Chapter 16 described, near one vibration in about seventeen minutes. The electroweak change of state was a smooth crossover, so the account says that LISA will register no background from a change of state of the matter. A sudden electroweak change would have put its waves in exactly the band of LISA. LISA will register backgrounds from sources in the sky, among them the many pairs of white dwarf stars in our own galaxy, which we met in Chapter 16. A white dwarf is the dense, burnt-out core of a star like the Sun.

The pulsar-timing arrays of Chapter 16 measure the timing of spinning dead stars, called pulsars, over years. The pulsar-timing arrays listen at billionths of a hertz, around one vibration a year, and the North American and European arrays have both reported evidence of a hum at billionths of a hertz. The strong change, in which quarks became held, near a hundred and fifty-six and a half million electron volts, was a smooth crossover. So the strong change put no background in the band of the pulsar-timing arrays, where a sudden strong change would have put one. The settling background lies about twenty powers of ten higher in frequency than the signal of the pulsar-timing arrays, and the second-order background lies about fourteen powers of ten fainter in energy share than that signal. So the account has no source from the beginning for the signal of the pulsar-timing arrays. On the account, the signal is the summed waves of pairs of supermassive black holes circling one another at the centers of galaxies, each black hole of millions to billions of times the mass of the Sun. Pairs of supermassive black holes are also the leading reading among the pulsar-timing teams. The black-hole reading follows by argument in the account, from the closed list of kinds of particle and the smooth changes of state of the matter.

Every wave that an instrument registers tests two more statements of the account. Both statements are about what a gravitational wave is, so every signal that the coming instruments register checks both statements. In this account, gravitational waves are small ripples of the arena, which is space and time together. The ripples travel at the speed of light, with the two polarizations of general relativity that Chapter 15 described: the two patterns in which a wave stretches and squeezes space across its path. In 2017 two neutron stars merged, and their gravitational waves arrived with the gamma rays of the same event. So the speed of gravitational waves is measured to match the speed of light to within a few parts in a million billion. Every signal also tests the number of polarizations, because detectors on the ground point different ways and LISA has its own geometry. If a detector one day registers single clicks, the clicks would show quanta of the description of the wave, as Chapter 15 found. Quanta of the description are steps that belong to the way small waves of the arena are described, and not to the arena that ripples. The arena would stay unquantized, because a smooth wave acting on a quantum detector also makes single clicks.

Two Masses, One Geometry Each

One test of the account uses no gravitational wave at all. The test belongs among the coming measurements because it tests, in a laboratory, a step that Chapter 9 took: the step that each alternative of the matter carries its own geometry. An alternative is one of the several definite possibilities in which a quantum thing can be held at once. In 2017 two groups of physicists proposed the test: Sougato Bose and colleagues, and Chiara Marletto and Vlatko Vedral. The two groups proposed holding two small masses, each in two places at once, with the two masses linked only through gravity. The experimenters would then ask whether the two masses become entangled, their states linked so that neither state can be described without the other.

In this account each alternative of the matter carries its own ordinary geometry, as Chapter 9 found. Two masses in two places each make four joint arrangements: near and near, far and far, near and far, far and near. Each arrangement has its own geometry, and so its own energy of gravitational pull, and each arrangement has a phase: a tiny clock hand that turns at the rate that the energy of the arrangement sets. The entangling phase, met in Chapter 9, is the one combination of the four phases that no change to the phases of one mass alone can alter. The entangling phase grows with the strength of gravity, with the square of the mass, and with the time for which the masses are held. The entangling phase also depends on the four distances between the masses, each distance taken as one over the distance: the near-and-near and far-and-far arrangements add to the entangling phase, and the two mixed arrangements take from it.

For the numbers that Bose and colleagues proposed, the entangling phase has a definite value. Picture two specks of matter, each a hundred-billionth of a gram, far lighter than a grain of sand. Each speck is held in two places a quarter of a millimeter apart. The two specks are less than half a millimeter from each other. The specks are held in two places for two and a half seconds. *The Gravitational Passage* is the book of this collection that sets the whole account out for physicists. *The Gravitational Passage* states: "For these numbers the entangling phase is 0.314 radians, and the account predicts that the entanglement appears with that phase". In words, the entangling phase is zero point three one four radians. A radian is the measure of angle in which a full turn is a little over six and a quarter, as Chapter 9 said. So the entangling phase is one twentieth of a full turn, or eighteen degrees. The entangling phase is set by the structure once a stated step is taken. The stated step is the identification of one geometry for each alternative, used with the measured strength of gravity. Physicists are building experiments to make the measurement. Physicists have already measured the gravitational pull of masses below a milligram.

How Long the Listening Takes

The spans of time in this section estimate when each measurement will become sensitive enough to test the statement that the measurement is aimed at. The estimates come from the instruments now being built or proposed, such as the Simons Observatory and LISA. The estimates are not results of the account.

Within about twenty years, the Simons Observatory will measure the count of light species to within five to seven hundredths, and the measurement will test how much radiation the settling waves may carry, at about the level of the present bound. The Simons Observatory and LiteBIRD will be sensitive to a ratio of first waves to first ripples of about a thousandth. LISA will listen in its band of slow waves. The pulsar-timing arrays will lengthen their records and map the shape of their signal across frequency. The experiments on entanglement through gravity will aim at the numbers that Bose and colleagues proposed. No instrument will hear the settling background directly: the most hopeful projections for the megahertz and gigahertz bands stay below the settling band in frequency, and in the gigahertz band the most hopeful projections stay many powers of ten above the ceiling.

Look ahead about fifty years, and suppose that detectors at high frequency improve as fast as detectors at the pitches of sound did over the last fifty years. The supposition is a generous guess, because at high frequency no source in the sky drives the work forward, as stars and black holes did at the pitches of sound. The magnon detector is the best instrument now running. The magnon detector lies about thirty powers of ten above the ceiling in energy share. Eight powers of ten in strain are sixteen in energy share, and a gain of that size would leave the magnon detector about fourteen powers of ten above the ceiling. So after fifty years, the reach of detectors at gigahertz frequencies would still lie about fourteen powers of ten above the ceiling of the settling background, and still below

the settling band in frequency. By then the indirect test would have bounded the excess in the count of light species far more closely than any direct detector could.

To hear the settling background directly, an instrument would have to listen at half a terahertz or above. The instrument would also have to register single quanta of excitations in the detector, each of a few thousandths of an electron volt, with every other source of noise held down beyond any present practice. No plan exists for an instrument that could do all three. The settling background is registered all the same, because the count of light species registers the energy of the background, and the account requires only that the settling background stay within the bound that the count of light species sets.

> **Key idea.** The settling waves sound at half a terahertz or above, higher than any working detector can tune. The energy share of the settling waves is at most five parts in ten million, far below what any working detector can sense. So the settling waves are weighed instead of heard: the count of light species registers their whole energy, in every band at once. The count of light species bounds the settling waves before any detector could.

Summary

The count of light species registers the beginning's own waves, the settling background. The count is the indirect test of the settling background, and the strong test. The count weighs the whole background, in every band at once, through the expansion that the energy of the waves produced. The present measurements are two point nine nine and two point eight six, against the three-neutrino value of three point zero four four. The present measurements hold the settling waves to at most nine hundredths of a light species, an energy share of at most five parts in ten million, with no floor. The Simons Observatory will measure the count to

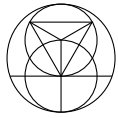
within five to seven hundredths. An excess well above a tenth of a light species would measure how far the six were displaced at the turn. By the detectors' own published figures, eight of the nine sorts of detector at high frequency work below the settling band, from about five times to about a million times too low in frequency. Every sort with a stated reach lies at least twelve powers of ten (and as many as thirty-two) above the ceiling in energy share. So each sort with a stated reach could hear only a background carrying more energy than the whole universe. The closest approach in pitch is a reading of the sky through the magnetic fields of clusters of galaxies. The sky reading is about five times too low. Only one proposal has a band that contains the settling band: phonon detectors in crystals, which exist as a calculation. The reason for the gap in frequency is the size of the source: waves sent out at the Planck energy are still above half a terahertz today. The account's other statements are definite as well. The Simons Observatory and LiteBIRD will search for the first waves. The account says that the two instruments will find nothing. LISA and the pulsar-timing arrays will register backgrounds from sources in the sky, and no background from a change of state of the matter. Every wave that an instrument registers tests the statements that gravitational waves travel at the speed of light and have two polarizations. The account also says that two masses, each held in two places, become entangled through gravity alone. For the numbers of Bose and colleagues, the entangling phase is zero point three one four radians. We now know which statements the coming decades will test, and with which instruments. We also know that the count of light species is the measurement that registers the settling waves.

For the book's question of what the beginning left behind, the finding of this chapter is that the one background of waves that the beginning leaves of its own is already registered. The oldest light carries the weight of the settling waves in its count of light species, and every sharper count sets the condition on the turn more closely.

The settling waves are not the only thing that the turn left behind. What else did the turn make?

PART V

WHAT THE TURN MADE



Chapter 18

Matter Made of Possibility

The world has six small dimensions of possibility, which this book calls the six. The six are never traveled through, and are met only as what could be found. The turn is the passage at the end of the black holes' interiors in which three large directions open up while the six close in. After the turn, the six settled into their held size. The settling left a faint hum of waves in the shape of space and time (Chapter 16). The turn left more behind than the hum. No one has ever seen most of the matter in the universe. What is the unseen matter, and why does it never decay?

Picture two great clusters of galaxies falling into each other. Each cluster is a swarm of galaxies wrapped in a vast cloud of hot gas. As the clusters pass through each other, their clouds of gas crash in the middle, slow down and heat up until the gas glows. Astronomers can weigh the whole scene, because the weight of the scene bends the light of galaxies that lie far behind the scene. Astronomers call one real collision of two clusters the Bullet Cluster. In the Bullet Cluster the weight lies with the galaxies, which have sailed on through, and not with the glowing gas, although the gas holds most of the ordinary matter, the matter made of atoms. Something unseen and heavy passed straight through the crash without colliding, and carried most of the weight of the scene along.

The unseen and heavy something is the dark matter that the Preface, Before We Begin, named: matter that has never been seen to do anything except pull by its weight. In this chapter I say three things about the dark

matter, on the account of the world that I teach in this book (the account, for short): what the dark matter is, where the dark matter was made, and why the dark matter lasts.

What Is Seen

Astronomers find the weight of the dark matter in three places. First, galaxies spin faster than the pull of their visible stars and gas could hold together, and something unseen holds the galaxies together. Second, clusters of galaxies bend the light of more distant galaxies more strongly than the visible matter of the clusters can. Third, the oldest light, the faint afterglow released early in our epoch, carries a pattern that the dark matter set before any star had formed. Our epoch is the era of the world that we live in. Astronomers first inferred the dark matter in 1933 from the Coma cluster, whose galaxies move too fast for the visible galaxies to hold together. Measurements made in 1970 show that the outer parts of the Andromeda galaxy turn about as fast as the inner parts. So the mass of the galaxy keeps growing far from the galaxy's center. Astronomers can weigh the dark matter directly by the bending of light, with no assumption about how the matter moves.

Weight is all that the dark matter shows. No one has ever seen the dark matter bounce off anything, destroy itself, or break down. The Bullet Cluster sets a measured limit on how strongly the dark matter can bounce off itself. Physicists state a limit on bouncing as a target area for each gram: the larger the area, the more readily the matter bounces. For the dark matter, the target area is less than about one square centimeter for each gram. In the gas of galaxies, electric and magnetic forces cannot do the work of the dark matter, because electric and magnetic forces push on charged gas and not on whole stars. A magnetic field bends light only by the weight of its own energy. The magnetic field of a galaxy is about five millionths of a gauss. The gauss is the unit in which physicists state the

strength of a magnetic field. The energy of a galaxy's magnetic field weighs about two parts in a billion of the weight of the dark matter nearby, a computed figure.

The amount of dark matter has stayed the same since the oldest light was released. If the dark matter decayed into anything unseen, the decay would have changed the expansion of the universe since the oldest light was released, and the measured expansion requires a decay into anything unseen to take longer than about a hundred and sixty billion years. Any decay of the dark matter into something we could see has a far tighter limit.

By weight there is about five and four tenths times as much dark matter as ordinary matter, the matter of stars, gas, planets and people. The ratio of dark to ordinary matter is measured, and the ratio is the same in every part of the sky where astronomers can measure the ratio in the oldest light. The first ripples are the faint unevenness with which our epoch began. The first ripples are the same in both kinds of matter. Measurements of the oldest light show no separate pattern of ripples belonging to the dark matter alone: any separate pattern of the dark matter carries less than about one and three tenths percent of the variation in the temperature of the oldest light.

The galaxies themselves need the dark matter, and I show why with a short piece of arithmetic. When the oldest light went free, that is, when the light began to travel freely through space, the temperature of the light varied across the sky by about one part in a hundred thousand. The ripples that astronomers measure squeeze every ingredient of the universe alike, light and matter together, and where every ingredient is squeezed alike, the density of matter varies about three times as much as the temperature. The threefold relation between density and temperature is established physics: a result of physics, taken from its source. The reason for the threefold relation is plain: in a given space, the number of particles of light is in proportion to the cube of the temperature (the temperature multiplied by itself twice). So a squeeze that raises the temperature by one part crowds the light, and the matter squeezed alike with it, by

three parts. When the oldest light went free, the ordinary matter therefore varied in density by about three parts in a hundred thousand. The ordinary matter was caught in the sound waves of the hot mixture of light and matter until the oldest light went free, so the ordinary matter could not gather before then. Matter rules the expansion once matter carries more of the energy of the universe than light does. Once matter rules the expansion, a small excess of density grows in step with the distances between things, and the distances between things have grown about one thousand and ninety times since the oldest light went free. Three parts in a hundred thousand, grown one thousand and ninety times, is about three parts in a hundred. The growth slows once the cosmological constant, the energy of empty space, takes over the expansion, so ordinary matter alone would have reached less than three parts in a hundred. A region stops spreading out and draws together into a galaxy only when the excess of density in the region reaches about one, that is, when the region is about twice as dense as the average. So ordinary matter alone would have made no galaxy at all.

The dark matter took no part in the sound waves, so the pressure of light did not hold the dark matter back. The unevenness of the dark matter grew slowly while light ruled the expansion, and grew in step with the distances once matter ruled. So the dark matter's unevenness was already growing before the light went free. The dark matter built the scaffolding, and the ordinary matter later fell into the wells that the dark matter had made: the dark matter gathered first into dense regions, and the pull of the dense regions later drew the ordinary matter in. The same difference shapes every galaxy today: the ordinary matter trades energy with light, and the dark matter does not. Ordinary gas shines as it falls inward, so it loses energy, sinks and breaks up into stars, while the dark matter gives off nothing, keeps its motion and stays spread out. So each galaxy is a bright core of stars and gas inside a larger dark cloud, called the galaxy's *halo*, and halos gather as smaller halos draw together and merge into larger halos. In the largest clusters of galaxies, the gas cannot shine its energy away fast enough to sink, so the gas stays hot and spread out.

Any account of the dark matter has three facts to explain. First, the dark matter lasts. Second, the dark matter acts only by its weight. Third, the dark matter comes in a fixed ratio to ordinary matter.

A Vibration of the Held Space

Where a thing gives energy to nothing and takes energy from nothing after it is made, the thing can have traded energy with the rest of the world only where conditions were extreme enough for any trade of energy at all. *The Gravitational Passage* is the book of this collection that is the source for the turn and the six. *The Gravitational Passage* shows that the world has two places with conditions extreme enough for a trade of energy: the deep inside of a black hole at the end of an epoch, and the start of the next epoch. The two places are one place seen from two sides. The passage between the two sides is the turn. At the turn, and only at the turn, the six small dimensions and the three large dimensions form again together. The dark matter has never been seen to give or take energy. So the turn is the one place where the dark matter is made.

Think of a crystal: a solid whose atoms are held in a fixed lattice by the bonds between the atoms. Strike a crystal, and the crystal rings. Each way the crystal can ring has a definite energy, because the lattice keeps its shape. The phonon of Chapter 15 is the smallest step of energy in one way of ringing. A phonon is a vibration of the crystal, not an extra thing inside it. Nobody concludes from phonons that the crystal is made of phonons.

A comparison is the act of finding whether two things are the same or different, in some respect. The content of a comparison is what the comparison finds, as a thermometer comparing morning and evening finds the temperature. The context of a comparison is the respect in which the comparison finds what it finds: for the thermometer, the two times of day. An identification is a step from the structure of comparison to something in the world, and I make each identification in my own voice. I identify the dark matter as a conserved excitation of the six held internal dimen-

sions: a vibration of context, not an item of content, made at the turn. The six held internal dimensions are the six, held small. An excitation is a raised state of vibration, a ringing above the quietest state, like the breathing excitations of Chapter 4. Conserved means kept: a conserved excitation, once made, does not go away. The dark matter is made of gyrations, met in Chapter 4. A gyration is a lasting vibration of the six held small dimensions, as a phonon is a vibration of a crystal: a gyration is a vibration of the held space, and not an extra thing inside the held space.

The six can be set beside two different things. When the six are set beside the large dimensions, the six are part of what the large world holds, its content. When the six are set beside what could be found in them, the six are the range of what could be found. So the six are a context. *The Gravitational Passage* shows that the six are a space of possibility in the plain mathematical sense. The six are made of two shapes: the complex projective plane, a shape of four dimensions that is the space of all the states of a three-way choice, and the sphere, a shape of two dimensions that is the space of all the states of a two-way choice. The dark matter belongs to the second way of setting the six: the six as a context, the range of what could be found. The dark matter is a vibration of possibility, the setting of whatever could be found, and not one of the things that a measurement finds.

The six are like a crystal because the six are held. Each direction of the six lies at the modal boundary, the Planck scale, where gravity and the quantum act on equal terms, and the modal boundary is the same throughout the large world. At the modal boundary the smooth character and the stepwise character of the world trade places as the character that rules. So the six keep the same size and the same shape at every place, as a crystal keeps its lattice. Each way the six can vibrate therefore has a definite energy, as each way a crystal can ring does. The floor of Chapter 4 is one mechanism that holds the six at the modal boundary: the floor is a least size of the six, with a least motion that cannot be removed. The large dimensions, space and time together, are called the arena. The arena is not held as the six are held. Gravitation is the bending of the arena, and grav-

itation is not quantized: neither gravitation nor the arena comes in least steps. A "graviton" names at most a quantum of the description of a small wave of the arena, never a quantum of the arena itself. A quantum of the description is one step in the energy of one way a small wave of the geometry can ripple. A gyration is a vibration of a held structure, as the vibrations of a crystal are: a gyration is no quantum of the arena and no quantum of gravitation.

A gyration has energy, so a gyration has weight, and a gyration pulls as all energy pulls. Light, detectors and every instrument register content, and a gyration is not content, so no light, detector or instrument registers a gyration. So astronomers see the dark matter's weight, and nothing else. The dark matter is dark because it is not the kind of thing that light or any detector registers. Nothing is hiding the dark matter. So the account says what every search for the dark matter will find: the dark matter shows itself through its weight alone, and every search for the dark matter through any other way of acting finds nothing.

In the language of physics, a gyration is a heavy vibration of the geometry of the six. Physicists have long studied vibrations of the same kind in theories with extra dimensions, and call such a vibration a particle. I call the vibration a gyration because of its role. In the books of this collection a particle is an item of content, and the dark matter is a vibration of context. I identify the dark matter as a vibration of context, and the identification is a claim about the gyration's role. Physically, a gyration is the kind of vibration physicists already study. Are the gyrations vibrations of the same six on which Part I of this book read the kinds of particle? Yes: the six are the same six, read two ways. Part I read the kinds of particle on the six taken as a space of states, which has no size. The gyrations are vibrations of the six held at their size.

The dark matter is one of three things that bend a context. Everything a comparison finds is placed in three settings: in space, in time and in possibility. The plain shape of a setting is the setting's shape when nothing bends the setting. No setting can keep its plain shape. The Incommensuration Theorem is a structural fact about the nature of knowing: sameness

and smoothness cannot both be perfect in any comparison. In a setting with its plain shape everywhere, sameness and smoothness would both be perfect at once. So space, time and possibility, the three settings, must each be bent. The three axioms of comparison (Chapter 7) are the three statements that hold of every comparison whatever. Under the three axioms of comparison, each setting has a bending of its own. The three bendings go together, none can do the work of another, and there are exactly three. Gravitation bends space locally, around matter: the arena curves around matter, and light follows the curve. The cosmological constant bends time evenly, everywhere. The cosmological constant is the same at every place and every time, and over the whole course of time the cosmological constant widens the distance between every pair of distant places. The dark matter bends possibility: the dark matter is a lasting vibration of the six, the space of possibility, held at the Planck scale, where the six are held. Each of the three bendings is tied to the edge of its own setting: the bending by gravitation, the bending by the cosmological constant, and the bending by the dark matter. The edge of a setting is the limit of what a comparison made from inside the world can register in that setting. Gravitation acts within the cosmological horizon, the edge of the space that an observer can compare with. The cosmological constant is fixed by the time since the turn, the edge of time. The dark matter is held exactly at the Planck scale, the edge of possibility. Gravitation, the cosmological constant and the dark matter are distinct: no amount of one does the work of another. So the dark matter is required: a world with a smooth setting of possibility must carry dark matter, as the world must carry gravitation and a cosmological constant. In the next chapter we ask how much dark matter there is against ordinary matter.

Three Ways Out, All Closed

A way out of a thing is a lower state that the thing can reach through some link that exists, and a way out is closed when there is no lower state to reach, or no link to the lower state. A thing lasts when every way out of the thing is closed. *The Gravitational Passage* already describes one state that lasts because every way out of the state is closed: the lowest motion of the six, the motion that can never be removed, which "is already the lowest state, so it cannot radiate or dissipate". A gyration has three possible ways out: into the content of the world, into waves of the large dimensions, and into evaporation. Each of the three ways out is closed.

The first way out would lead into content: the gyration would have to become an ordinary thing. A vibration of the geometry of possibility and an actual thing are two different sorts of thing, and neither sort turns into the other. The six, the geometry of possibility, form again only at the turn, where the six and the three large dimensions form again together. So no process within an epoch turns a vibration of the six into an item of actual content. Under my identification of the dark matter, a gyration acts on the content of the world through gravity alone. So every search through any other way of acting finds nothing.

The second way out would lead into waves of the large dimensions, the ripples of the arena. A symmetry closes the second way out. Picture a soap bubble. Pair each point of the bubble's skin with the point straight across from it, on the far side of the center. Now swap every point with its partner. When a ripple lifts one side of the bubble while lowering the far side, the ripple comes out of the swap upside down. The ripple flips its sign. When a ripple lifts both sides alike, the ripple comes out of the swap unchanged. The ripple keeps its sign.

The small sphere of the six can be swapped in just the way of the bubble, each point with the point opposite it. The laws of the six contain the geometry of the six alone, with no sense of handedness for the small

space (no difference between a left-handed and a right-handed small space), so the swap leaves the laws unchanged. A swap that leaves the laws the same is called a *parity*. Under the parity of the small sphere, every way of vibrating either keeps its sign or flips its sign. A level of vibration that keeps its sign is called *even*, and a level that flips its sign is called *odd*.

The waves of the large dimensions are even: each wave of the large dimensions is the same all over the six. Every link that the geometry allows respects the swap, so a single odd vibration cannot hand its energy to even waves alone. An odd vibration could fall only to a lighter odd vibration, and the lightest odd level has no lighter odd level below it. So the lightest odd level cannot decay at all. The lightest odd level is the dark matter. In other accounts, which have no symmetry like the swap, heavy matter that acts only by its weight decays into lighter waves. In this book's account, the swap forbids the decay.

The breathing of the six is the swelling and shrinking of the six as a whole (Chapter 4). The breathing is even, and the levels of the breathing decay. The geometry allows a curvature term, the term of Gauss and Bonnet met in Chapter 4. The term of Gauss and Bonnet links the breathing to waves of the large space, and no symmetry forbids the link. A link has its natural strength when nothing tunes the link to be stronger or weaker. When the link has its natural strength, the lowest breathing level decays within about a Planck time, the time light takes to cross a Planck length, about five and four tenths times ten to the minus forty-four of a second. The Planck length is the length at which the smooth description of space and time reaches the edge of its domain. So the breathing levels made at the turn decayed almost at once into the waves of the settling (Chapter 16). Other even levels decay by other routes. The lowest motion of the six stays, for a reason of its own: the lowest motion is already the lowest state. Only the lightest odd level lasts as dark matter.

The third way out would be evaporation, the slow leaking away of mass that a black hole undergoes. A vibration at the Planck scale has no lower level of its kind to leak into. The next scale at which evaporation

could act lies some thirty powers of ten away from the Planck scale, and the floor of the six is the lowest state of its kind. Physicists have held that tiny leftover black holes of about the Planck mass last for the same reason: the evaporation of such a black hole stops at the Planck mass, where no lower state is left to the black hole.

Two odd gyrations can meet and destroy each other, turning together into even waves. The swap allows the meeting, because two flips make no flip. Near the Sun, a given gyration meets another and is destroyed about once in ten to the seventy-eight seconds, a computed figure on the account. Ten to the seventy-eight seconds is so long a span that the meeting of two gyrations is no way out at all. Every way out is closed, so a gyration made at the turn keeps its energy wherever the gyration moves afterward.

One other account also has a dark matter of about the Planck mass. The other account is the conformal cyclic cosmology of the physicist Roger Penrose, and Penrose's account is a cycle too: in conformal cyclic cosmology, the far future of each epoch becomes the beginning of the next epoch. Penrose calls the dark matter of his account erebons. Erebons must decay within each epoch, with a lifetime of the order of a hundred billion years. The gyrations do not decay. The measured expansion already limits any decay of the dark matter into anything unseen to lifetimes beyond about a hundred and sixty billion years. So the measured lifetime of the dark matter is what separates Penrose's account from this book's account, as Appendix B sets out.

Why the Swap Is Exact

A gyration weighs about a Planck mass, and a thing that heavy needs an exact symmetry to keep the thing from decaying. A symmetry of the ordinary kind, called *global*, relates two arrangements that are different and behave alike. Physicists usually expect that in a quantum theory of gravity (a theory that would join gravity with the quantum), effects that grow as a thing's mass approaches the Planck mass wear a global symmetry away.

A gyration's mass is about one Planck mass, so on the usual expectation a global symmetry would not keep a gyration from decaying. The swap of opposite points is a symmetry of another kind. Nothing can wear the swap away, on the account I teach in this book or on the usual one.

The swap is a change in how the geometry of the six is described. Picture two copies of one printed page, alike in every letter. Nothing printed on either page can show which copy you call the first: the first is only a label. When two arrangements of the whole geometry differ only by the swap, the two arrangements are like the two copies of the page: no one can tell the two arrangements apart by any comparison. A difference exists only where someone could register the difference by some comparison, so the two arrangements are one arrangement. When a symmetry holds because the arrangements it relates are one and the same, the symmetry is called *gauged*. A gauged symmetry is exact: nothing can violate it.

The swap makes two whole arrangements of the geometry one arrangement. The swap does not make each point of the sphere one with its opposite. If each point of the sphere were glued to its opposite point instead, the result would be a different space. On the glued space only even vibrations exist, and the odd gyration does not. So the sphere itself stays whole: on this book's account the whole sphere remains, and with the whole sphere the odd gyration remains too.

Three things follow from the fact that the swap is gauged. First, a single gyration carries a sign that no one can detect by a comparison made nearby. Think of an old video game in which the player who walks off the right edge of the screen comes back on the left (Chapter 19 uses the video game again). If space wrapped around like the game's screen, a walk all the way around space could bring the small sphere back swapped. The sign would show only on a walk all the way around: a closed path through the large space. The sign shows itself only around a closed path, and no one can detect the sign at all by a comparison made nearby. Second, in a closed universe (a universe whose space is finite and closes on itself), the gyrations number an even total. The total is even because a closed universe as a whole carries no flip, and flips cancel only in pairs

(two flips make no flip). Within an open region, the total need not be even. Gyrations are made in pairs, from even states, which agrees with an even total. Third, the account requires, as a prediction about the matter of the world, that nothing in the matter of the world couples to (responds to) the handedness of the small sphere. The handedness of the small sphere is which way round the small sphere is, as a left glove differs from a right one. The world's matter may tell left from right only in ways the swap leaves unchanged: left and right in the large space. The weak force is the force at work in some sorts of radioactive decay, and the weak force tells left from right. The weak force fits the requirement, because the weak force tells left from right in the large space, and the swap leaves left and right in the large space exactly as they were.

How Heavy, How Many, How Cold

The six are held small, a few Planck lengths across, so the vibrations of the six are heavy. A gyration weighs about a Planck mass. The Planck mass is the mass at which gravity and the quantum act on equal terms, about twenty-two millionths of a gram: about ten billion billion times the mass of a proton. Every odd level takes its sign from the simplest wave that the sphere can carry, a wave high on one side of the sphere and low on the other side, and the simplest wave varies as slowly as the six allow. Physicists label each family of vibrations of the six by a number called its *spin*. For the gyrations, the spin says only how the vibration is laid out against the directions of the large space. Three values of spin occur: two, one and zero.

There are two computations of the lightest odd level's mass. In the simplest calculation, the floor is treated as an ordinary source of weight. In the simplest calculation, the further curvature terms that the geometry allows are also taken to leave the equations unchanged, and the term of Gauss and Bonnet, met in Chapter 4, is one of the further terms. In the simplest calculation, four families of odd vibration share the lowest mass:

the square root of two Planck masses, about one and four tenths. When the calculation is taken one step further, the floor's quantum response lowers all four families by the same amount, to about six tenths of a Planck mass. The floor's quantum response is the effect of the least motion of the small waves of the geometry, computed at first order (for its largest and simplest part), as in Chapter 4. The rest of the floor's effect lifts the family of spin zero and leaves the other three families where they are. So the lightest odd level weighs between about six tenths and one and four tenths Planck masses, and both values are computed.

Something of about the Planck mass is about as small as a black hole of the same mass would be. The gravitational size of a thing is the size of a black hole of the same mass. A gyration's gravitational size is about one to three Planck lengths. The gravitational size is close to the gyration's own wavelength and to the size of the six that the gyration vibrates. A gyration is a level of vibration of the six, with a definite energy and a definite sign under the swap, and not a collapsed region of the large space.

The candidates for the lightest odd level are three families: one of spin two and two of spin one. The ten-dimensional geometry allows the curvature term of Gauss and Bonnet, with a strength that *The Gravitational Passage* does not fix. The term of Gauss and Bonnet splits the three candidate families, and within the range of strengths that the term may have, the term can put a family of either spin lowest. The lightest of the three families is the family that lasts. Whichever family is lightest is the lightest odd level, so that family cannot decay, and every count in this chapter applies to that family, with the family's own mass.

Could gyrations act on one another by some force other than weight? The small sphere keeps its shape when the sphere is turned, and so does the larger part of the six, the complex projective plane. A turn that leaves a shape unchanged is a turning symmetry. The turning symmetries of the six give rise to forces in the large space, and gyrations carry the charges of the forces. The forces of the turning symmetries are very strong at the Planck scale: about three hundred times the weak and strong forces there. The figure of three hundred is computed. So the forces of the turning

symmetries are forces of the six, separate from the forces of particle physics. Forces as strong as the forces of the six may lock themselves up at the Planck scale, as the strong force locks quarks inside protons, and then never reach across the large space. If the forces of the six lock themselves up, each gyration is wrapped with its force into a neutral bundle that is still odd under the swap, still cannot decay, and at a distance acts only by its weight. The account requires that the forces of the six stay within the Planck scale, or act across the large space only on charges that cancel, so that gyrations act on one another by weight alone.

Gyrations act on one another by weight alone, so they barely touch. Matter in galaxies and clusters moves at speeds from a few hundred to a few thousand kilometers a second. At the speeds of matter in galaxies and clusters, the target area of gyrations for bouncing off one another is between about ten to the minus forty-five and ten to the minus fifty square centimeters for each gram. The target area of gyrations is at least forty-five powers of ten below the measured limit of about one square centimeter per gram that colliding clusters set. The dark matter passes through itself as it passes through everything else.

Each gyration is heavy, so gyrations are few. The ratio of dark to ordinary matter is about five and four tenths (the subject of the next chapter), and at the measured ratio there is about one gyration for every one and a half to three and four tenths billion billion protons and neutrons. There are also about two to four gyrations for every ten billion billion billion particles of the oldest light. Near the Sun, at the measured local amount of dark matter, there is about one gyration in every few tens of thousands of cubic kilometers. The counts of gyrations in this paragraph are computed numbers.

Because gyrations are few, the dark matter is grainy, and the grain is far too fine to have any measurable effect. If each gyration were made independently of every other, the number of gyrations alone would leave an unevenness of about five to eight billion-billion-billionths across a region twenty megaparsecs wide (about sixty-five million light-years). Astro-

nomers quote the first ripples' strength at twenty megaparsecs. The measured first ripples are about one part in a hundred thousand, so the number of the gyrations adds nothing measurable to the first ripples.

Gyrations are also cold in the sense of the cold dark matter of Chapter 14: slow, with no motion from heat that could matter. Gyrations are cold whatever speed they have when the turn makes them, because the momentum of each gyration falls as the universe expands. When the hottest gas that the account allows throws out a gyration, the gyration travels about two centimeters in the whole history of the universe. Picture a gyration that left the turn when the universe was at the Planck temperature (the heat of the Planck scale), with a hundred times its own mass in momentum: even so fast a gyration travels less than a meter. So the gyrations clump exactly as the cold dark matter of standard cosmology does: the gyrations build the scaffolding on which galaxies form, and the gyrations pass through everything. The standard picture of cosmology has two dark ingredients: a constant that drives the expansion apart, and a cold matter that only pulls. So the account contains both dark ingredients of the standard picture of cosmology, and says what each dark ingredient is: the constant is the constant term of the geometry of space and time, and the cold matter is a vibration of the six.

Made at Each Turn

The gyrations are made at the turn, where the geometry of possibility reforms. After the turn, the one other way to make gyrations is a gas so hot that gravity alone makes gyrations in pairs from the gas's heat. The account requires that the new epoch never grow hot enough for gravity to make gyrations in pairs.

Each epoch holds only the gyrations that its own turn makes. Like all other matter, every gyration at last either ends inside a black hole or is carried away alone, and in both cases, where the gyration ends up, the six that the gyration vibrates are released from their held size. *The Gravitation-*

al Passage shows that no count of anything is carried across the turn, so no gyration of one epoch passes into the next epoch as a gyration. The swap, the law that protects the gyrations, returns at each turn: the six of the new epoch re-form with the same shape, so the new epoch has the same odd levels and the same swap. If gyrations were carried from epoch to epoch, the number of gyrations would grow without limit, because the cycle of epochs has no first epoch. So each epoch starts afresh, and the amount of dark matter measured in an epoch counts all the dark matter present in the epoch.

Physicists have proposed dark matter made between epochs before. In Roger Penrose's cyclic account, dark matter is made at each transition from one epoch to the next. Penrose's dark matter has about the Planck mass and acts only by gravity. Penrose's dark matter must decay within each epoch, over about a hundred billion years, so that the far future of each epoch is empty of mass and can join the next epoch. In this book's account the gyrations need no decay, because no gyration is carried across the turn. Physicists have also proposed tiny leftover black holes, and bodies carried through a bounce from an earlier universe, as the dark matter. A gyration lasts for the same reason as the leftover black holes and the bodies carried through a bounce: there is nothing lower for the gyration to fall to. A gyration differs from the leftover black holes and the bodies carried through a bounce in what a gyration is, a vibration of the six, and in where a gyration comes from: a gyration is made at the turn, and not carried through the turn.

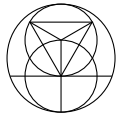
We now know three things about the dark matter on this book's account: what the dark matter is, where the dark matter was made, and why the dark matter lasts. What the dark matter is bears on the question of this book: what did the beginning leave behind? Every galaxy, our own among them, sits inside a halo of vibrations of possibility made at the turn that began our epoch, and none of the vibrations in the halo decays. Of everything the beginning left behind, the gyrations outweigh all the atoms together, about five and four tenths to one.

Key idea. The dark matter is made of gyrations: conserved excitations of the six held dimensions of possibility, vibrations of context and not items of content, made at the turn. Instruments register content, so the dark matter shows itself by its weight alone. A gyration has three ways out, into content, into waves of the large dimensions and into evaporation, and each way out is closed. No process within an epoch turns a vibration of possibility into an actual thing. The swap of opposite points is gauged and so exact, so the lightest odd level cannot decay at all. At the Planck scale there is nothing lower to evaporate into.

Summary

The dark matter is a conserved excitation of the six held internal dimensions: a lasting vibration of the space of possibility, as a phonon is a vibration of a crystal, and not a thing inside the world's content. I identify the dark matter as a vibration of the six, and the identification explains everything that astronomers see of the dark matter: weight, and nothing else. The dark matter is made only at the turn, where the geometry of possibility re-forms. Within an epoch the dark matter cannot become content. The gauged swap of opposite points keeps the lightest odd level from decaying at all, and at the Planck scale there is no lower level for a gyration to evaporate into. The six are held at their size because each direction of the six lies at the modal boundary, and the floor is one mechanism that holds the six at the modal boundary. Each gyration weighs between about six tenths and one and four tenths Planck masses. Gyrations are few, cold and collisionless (gyrations pass through one another), and each epoch holds only the gyrations that its own turn makes.

The turn made the gyrations, and today the gyrations outweigh the ordinary matter about five and four tenths to one. How much dark matter did the turn make, and what does the amount tell us about how hot and how gentle the beginning was?



Chapter 19

How Much the Turn Made

How much dark matter did the turn make, and what does the amount tell us about the beginning? The turn is the passage at the end of the black holes' interiors, in which three directions open up while the six small dimensions of possibility, which this book calls the six, close in. In Chapter 18 I showed what the dark matter is, on the account of the world that I teach in this book. The dark matter is made of gyrations: lasting vibrations of the six held dimensions of possibility, made at the turn. The chapter's two questions have two answers. First, the ratio of dark matter to ordinary matter is a single number: six less two divided by pi, about five and thirty-six hundredths. Second, the gyrations are made only at the turn, so the amount of dark matter sets limits on how hot the beginning could have been, and on how gently the six could have formed again.

A Recipe Card

Picture a recipe card in a kitchen drawer. The card says: for every cup of sugar, a little over five cups of flour. The card fixes the proportion of the two ingredients. The card does not say how much sugar is in your pantry. The card also says nothing about the mill where the flour was ground. The flour was milled long ago and far away, but you read the proportion off the card only when you bake. The mill never needs to know how much sugar you own.

The recipe card pictures one claim only. The proportion of dark matter to ordinary matter is fixed when a measurement is made, and not by any process at the turn. In the picture, the turn is the mill: the turn makes the gyrations, as the mill makes the flour. The ordinary matter is the sugar in the pantry: its amount is whatever it is, and we measure that amount. The card is the rule that fixes the proportion. The rule applies at the moment of measuring, as you read the card when you bake.

The proportion on the card is the proportion between two leftovers of the turn. The turn sends nearly all of its energy into radiation: light, and other particles moving at or near the speed of light. Two leftovers remain beside the radiation. The first leftover is a small excess of ordinary matter over antimatter. Antimatter is the matter made of the twins of the particles of ordinary matter. The second leftover is the dark matter. Each leftover is conserved in number, in the sense that Chapter 18 gave the word: once made, it does not go away. Since the helium of the first few minutes formed, both leftovers have thinned out in the same way as space grew, so the proportion between the two leftovers has not changed since helium formed. When helium formed, the binding of its nuclei lowered the mass of the ordinary matter by about eighteen hundredths of one percent, so before helium formed the proportion was slightly different. The proportion in this chapter is the proportion that has stood since helium formed, which the oldest light in the sky measures. The oldest light is the microwave background, the light left from the early universe.

We met the ratio of dark to ordinary matter in Chapter 12: the mass of dark matter in a region divided by the mass of ordinary matter in the same region. Chapter 14 called the ratio the registered ratio. A registered ratio is a ratio fixed by what the present measurement registers, and not by a mechanism at the turn. In this chapter I show what fixes the value of the ratio of dark to ordinary matter. *The Weight of Possibility* is the book of this collection that is the source for the dark matter, and *The Gravitational Passage* is the book of this collection that sets the whole account out for physicists. *The Weight of Possibility* fixes the ratio in the same way that *The Gravitational Passage* fixes the cosmological constant, the energy of empty

space (Chapter 8): each book fixes its number by an *identity of the present measurement*. An identity of the present measurement is a rule that states what a measurement made now, from inside the epoch, registers. An epoch is one era of the world. *The Weight of Possibility* says of the rule: "The rule acts at registration, as an identity". The ratio keeps the standing that Chapter 14 and Before We Begin gave it: the standing of a registered ratio. The book's opening part, Before We Begin, lists five standings of a number: set by the structure; set by the structure once a stated step is taken; measured; measured, with its reason still a question; and bounded, with its value not given. I keep the standing of the registered ratio under the standing's own name, and I file that standing under none of the five standings. An identity of the present measurement fixes the registered ratio. The reconstructed history is the history that we piece together backward from the records we have now. In the reconstructed history, we carry the value of the ratio back to every time since helium formed, as we carry the one value of the cosmological constant to every era. The gyrations are made at the turn, while the identity fixes how much dark matter there is relative to the ordinary matter.

Suppose a rule is built only from numbers set by the structure, with no count that changes as the epoch ages. Then the rule can apply only to a quantity that stays the same through time. The ratio of dark to ordinary matter is the one proportion in the universe's budget of matter that has stayed the same since helium formed, so the identity applies to the ratio of dark to ordinary matter.

Counting the Classes

The identity of the present measurement depends on a count. A measurement counts the basic pieces of shapes. A measurement does not count the separate directions or ways of waving in which a thing can carry energy. *The Gravitational Passage* already counts by basic pieces. *The Gravitational Passage* counts the basic pieces of the complex projective plane, and

so finds that a measurement registers through three channels. The complex projective plane is the four-dimensional shape of all the states that a three-way choice can be in. We met the basic pieces of a shape in Chapter 3. The surface of a ball is built from two basic pieces: a single point, and a disc whose whole rim is drawn together to the point, like a drawstring bag pulled shut. Mathematicians call each independent basic piece of a shape a *class*. The complex projective plane has three classes. The small sphere among the six has two classes, as the surface of any ball does. One principle of the account is that a measurement counts what it registers by classes, as *The Gravitational Passage* counts its channels.

The rule of counting is one statement, and the statement uses one more word, "factor". The internal space is the one small space that the six form together. The internal space is the complex projective plane times the sphere (Chapter 3). Each point of the internal space is a pair, one point of the plane and one point of the sphere, and each of the two shapes is called a *factor* of the internal space. The rule of counting says that a measurement counts every class of a factor over which the counted thing spreads, and one class of a factor where the counted thing is pinned to one class. The rule of counting applies three times: to the channels of measuring, to the dark matter, and to the ordinary matter.

The first application is to the channels of measuring. The count of the channels takes every class of the complex projective plane and only one class of the sphere, because each channel registers in the row of content, one of the two rows (content and context) that the sphere carries. When someone compares two things, the content is what the comparison shows, and the context is the respect in which the things are compared. When a thermometer compares morning and evening, the temperature is the content, and the two times of day are the context. Three classes times one class is three: the three channels of *The Gravitational Passage*.

The second application is to the dark matter. A gyration is a vibration of the whole internal space, so the dark matter spreads over both factors. The count of the dark matter takes every class of both factors. The count is three classes of the complex projective plane times two classes of the

sphere, which makes six. One principle of the account is that the dark matter spreads over both classes of the sphere, the row of content and the row of context, and two facts support the principle. First, the dark matter is not content, so nothing pins the dark matter to the row of content. The second fact concerns the parity that keeps the gyrations from decaying (Chapter 18). The parity is the swap of each point of the sphere with the point opposite it. The parity treats the two classes of the sphere with opposite signs. Under the parity, one class keeps its sign and the other class reverses its sign. When the parity reverses the sign of a level of vibration, the level is called odd. Chapter 18 found that the lightest odd level is the dark matter. The count of six does not depend on which odd level is the lightest, because the count counts the classes of the space that the gyrations vibrate, and not the notes of a gyration (its levels of vibration). The six classes are a different count from the six dimensions. Four dimensions and two dimensions make six by adding, while three classes and two classes make six by multiplying. Recall the grid of Chapter 3: the six measures set out in three columns, one for each role, and two rows, content and context. Three columns taken with two rows make six cells, and in the same way each class of the plane, taken with each class of the sphere, makes one of the six classes of the whole internal space.

The third application is to the ordinary matter. The ordinary matter is registered in the three large dimensions of space. A measurement registers the large space only within the particle horizon of the place where the measurement is made: the farthest distance from which anything could have reached us since our history began (Chapter 8). The region within the particle horizon is a solid ball. A solid ball is the inside of a ball together with its surface. A solid ball has one class, while the surface alone has two. A comparison is the act of finding whether two things are the same or different. No comparison made within the horizon shows any difference between the ball within the horizon and a ball in a space without end. The principle of identity states that two descriptions are one description when no comparison shows any difference between them. So the large space counts as one class for a measurement, whatever the shape

of the large space beyond the horizon. The ordinary matter is what a measurement registers as actual. The ordinary matter has no spread in possibility, so its count takes no class of the internal space: the count of the ordinary matter is the one class of the ball.

The count of classes is six to one: six classes for the dark matter, and one class for the ordinary matter. Mathematicians have several ways of counting classes, and every way gives the same numbers on the complex projective plane, the sphere and the ball, so the way of counting is not a free choice.

The Share That Holds Fast

One more fact changes the count: the six are curved. The large space is flat, as Chapter 12 found, so the one class counted in the large space carries no curvature, and the count of one for the large space is whole, with nothing taken away. The Planck length is the length at the edge of the domain of the smooth description of space and time, and the Planck length is about a hundred billion billion times smaller than a proton. The Planck curvature is the bending of the surface of a ball whose radius is one Planck length. Each internal direction bends by the Planck curvature. Because the internal space is curved, the count registered on the internal space is reduced.

The Weight of Possibility finds the size of the reduction from the rule that *The Gravitational Passage* gives for the cosmological constant. The curvature of the six enters the equations of the large world as an amount of energy in each unit of space. Any amount of energy spread evenly through space sets a horizon of a definite size, as the de Sitter radius of Chapter 8 showed. A universe holding nothing but an even energy has an edge at a fixed distance. The more energy the universe holds, the nearer the edge. A horizon holds a count, one tile for each of its degrees of freedom, as the tiles on the horizon showed in Chapter 8. A degree of freedom is one independent way in which what lies inside the horizon can

hold energy or vary. There is one tile for every four Planck squares of the surface of the horizon. A Planck square is a square one Planck length on a side. The share of a curvature is three tiles out of all the tiles of the horizon that the curvature's energy sets, and the number is three because every measurement registers through the same three channels of measuring.

For the curvature of the whole internal space, the horizon that its energy sets has a radius of exactly one Planck length. A sphere one Planck length in radius has a surface of four times pi Planck squares. At one tile for every four Planck squares, the surface holds pi tiles. So the count of tiles is pi, and the share of the whole internal space is three divided by pi. The same three divided by pi appears in the rule of *The Gravitational Passage* for the cosmological constant. Each of the six directions bends by the Planck curvature (Chapter 3). The complex projective plane has four of the six directions, so the plane carries four parts in six of the total curvature, which is two thirds, and the sphere, with two directions, carries the other third. So the plane alone gives two thirds of the energy, and less energy sets a wider horizon. For a horizon set by an even energy, the square of the radius goes as one over the energy, so for the plane the square of the radius of the horizon is one and a half times as large. The plane's horizon then has one and a half times the surface, and so one and a half times pi tiles. The plane's share is three tiles out of one and a half times pi tiles, which is two divided by pi, about sixty-four hundredths. The sphere alone gives a third of the energy, so the square of the radius of the sphere's horizon is three times as large, and its count of tiles is three times pi. The sphere's share is three tiles out of three times pi tiles, which is one divided by pi, about thirty-two hundredths. The horizons in this paragraph exist only as steps of the count. No observer has a horizon of the sizes in this paragraph.

To abide is to hold fast. *The Weight of Possibility* uses the word "abide" for one quantity. The share of a factor's curvature is three tiles out of all the tiles of the horizon that the factor's curvature sets. For a factor of the internal space, the *abiding share* is the share of the factor's curvature that

holds fast in a measurement as a term of its own, and the abiding share is subtracted from a count of classes. The abiding share of the complex projective plane is two divided by pi.

Which curvature abides in the count of the dark matter? The answer is one principle of the account, a rule of bookkeeping: a count that already includes the Euler count of a factor is not reduced again by the curvature of that factor. Think of a restaurant bill that already includes the tip: no one adds the tip to the bill a second time. The rule of which curvature abides has support in a theorem of the mathematicians Carl Friedrich Gauss and Pierre Ossian Bonnet. The Euler count of a surface is the number of its basic pieces, counted with signs (Chapter 3). The total curvature of a surface is the bending summed over its directions (Chapter 3). The theorem of Gauss and Bonnet states: add up the total curvature of a surface over the whole surface, divide by four times pi, and the result is the Euler count of the surface, at any size of the surface. So the sphere's two classes already hold the sphere's whole curvature, as the bill holds the tip. The complex projective plane's count of three comes another way. The count of three comes wholly from the plane's curvature taken squared: the bending multiplied by itself, added up over the whole shape. So the plane's count of three holds none of the plane's ordinary curvature, the curvature that changes with the size of the plane. The count of the dark matter is three times two, so the count includes the sphere's two classes. So only the plane's curvature abides in the count of the dark matter, and the count is reduced by two divided by pi. In the rule of *The Gravitational Passage*, the count takes only one class of the sphere, so the curvature of the whole internal space abides in that count, and the reduction is three divided by pi. Both reductions follow from the one rule of bookkeeping. A measurement is one act, so the abiding share is one term, taken once.

Six Less Two Over Pi

Put the count for the dark matter and the count for the ordinary matter together. The count for the dark matter is six classes, less the abiding share of the complex projective plane. The count for the ordinary matter is the one class of the large space. So the ratio of dark to ordinary matter is six less two divided by pi. In figures, the ratio is five and three thousand six hundred thirty-four ten-thousandths (5.3634), about five and thirty-six hundredths. In words, there are about five and thirty-six hundredths kilograms of dark matter for every kilogram of ordinary matter.

The rule for the ratio has the same form as the rule of *The Gravitational Passage* for the cosmological constant: a whole number set by the structure, less an abiding share. In the rule for the cosmological constant, the whole number is eight, less three divided by pi: the share of the whole internal space. In the rule for the ratio, the whole number is six, the count of classes, and the share taken away is two divided by pi, the share of the complex projective plane. The share in the rule for the ratio is a pure number (a number with no unit), taken once.

The two rules have one more thing in common: one coefficient. Each internal direction bends by one over the Planck length squared, as Chapter 3 identified. Write the curvature of each internal direction as some number times one over the Planck length squared, and call the number the shared coefficient. *The Gravitational Passage* identifies the shared coefficient as exactly one. Both rules depend on the shared coefficient in the same way, so two measured quantities measure the one coefficient: the share of matter in the universe, and the ratio of dark to ordinary matter. The measured ratio determines the shared coefficient two to three times more closely than the measured share of matter does, and from the measured ratio the shared coefficient is ninety-nine hundredths, give or take eight hundredths (0.99 ± 0.08). *The Weight of Possibility* claims that both measured quantities agree with the value that *The Gravitational Passage*

identifies, one. The rule for the ratio uses the curvature of the complex projective plane alone, so the measured ratio also shows how the curvature of the six divides between the two factors. When the oldest light is read through the rule for the ratio, the plane's curvature is three and ninety-six hundredths, give or take three tenths. The shape in *The Gravitational Passage* gives the plane a curvature of four: its four directions, each bending by the Planck curvature.

The rule for the ratio fixes the amount of dark matter relative to the amount of ordinary matter. The amount of ordinary matter is about six protons and neutrons for every ten billion particles of light. The amount of ordinary matter is a fact of content, and the account uses the amount as measured. Other accounts of very heavy dark matter set its amount by the details of the early universe, such as the rate of expansion at the end of inflation, a temperature, or the number of early black holes. Inflation is a proposed early phase of ever faster expansion (Chapter 13). None of the other accounts fixes the amount of dark matter relative to the ordinary matter.

The Gravitational Passage gives the share of matter as thirty-one percent of the universe's energy, and with that share two rules divide the whole budget of the universe's energy: the rule for the cosmological constant, and the rule for the ratio. For every one part of ordinary matter there are six less two divided by pi parts of dark matter. The cold matter is the ordinary matter and the dark matter together. So the cold matter holds seven less two divided by pi parts in all, about six and thirty-six hundredths. The ordinary matter is one part of the six and thirty-six hundredths: about sixteen percent of the cold matter. The neutrinos are the lightest particles of matter, and they barely touch anything else. The neutrinos take a small part of the whole: about fourteen ten-thousandths, for the smallest masses that the oscillation experiments allow (the experiments that watch neutrinos change from one kind to another as the neutrinos travel). The change of kind shows that neutrinos have mass. The neutrinos are content, and they are neither of the two leftovers of the turn, the excess of ordinary matter and the dark matter. The neutrinos left the turn as radiation. The

neutrinos kept changing in density against the ordinary matter until the neutrinos slowed, long after the ordinary matter was cold. The identity applies only to a ratio that stays the same through time, so the identity does not apply to the neutrinos. Because the neutrinos are content and the dark matter is not content, the neutrinos are not the dark matter, and a measurement of small galaxies agrees, on a ground of its own (Appendix B). The rest of the thirty-one percent is about four and nine tenths percent of the whole for the ordinary matter and about twenty-six percent for the dark matter, each figure rounded from a pair whose ratio is exactly six less two divided by pi. When the figures are rounded once more, about seven tenths of the universe's energy is the cosmological constant, about a quarter is dark matter, and only about one twentieth is ordinary matter. The cosmological constant and the dark matter both distort the setting: the context of space, time and possibility in which things are found. So most of the energy of the universe is a distortion of the setting, not things in the setting.

A gyration weighs about as much as ten billion billion protons. So the ratio means about one gyration for every one and a half to three and four tenths billion billion protons and neutrons, and about two to four gyrations for every ten billion billion billion particles of the oldest light. So gyrations are far fewer than protons and neutrons. Some other accounts have the opposite: in those accounts, dark and ordinary particles are about equal in number.

What the Oldest Light Finds

Astronomers weigh the dark and the ordinary matter with the oldest light, by the sound that rang in the young universe. Before the light went free, the hot gas of light and ordinary matter rang like a drum, and the favored notes of the drum show in the oldest light as spots of favored sizes across the sky. The heights of the notes are set by general relativity, by sound in the hot gas, by light bouncing off free electrons, and by the

forming of hydrogen atoms. The dark matter enters the heights of the notes in two ways only. The dark matter pulls by its weight, and it takes no part in the sound. So astronomers measure the densities of the dark and the ordinary matter, and their ratio, from the heights of the notes.

The account says that the ratio is six less two divided by pi, five and three thousand six hundred thirty-four ten-thousandths. The Planck satellite was a spacecraft that mapped the oldest light. The Planck satellite alone measures five and three hundred sixty-four thousandths, give or take sixty-five thousandths (5.364 ± 0.065). Two telescopes on the ground also map the oldest light: the South Pole Telescope and the Atacama Cosmology Telescope. Taken together with Planck, the two telescopes measure five and three hundred seventy thousandths, give or take forty-eight thousandths (5.370 ± 0.048). *The Weight of Possibility* computes both ratios from the published densities of the dark and the ordinary matter, and every analysis that uses the oldest light alone agrees with six less two divided by pi.

Does the agreement pick out the rule for the ratio from other rules of the same form? The rule for the ratio has a form: a count of six, less an abiding share. The abiding share is chosen in one of the ways that the quantities of *The Gravitational Passage* allow. Within that form, the abiding share can be taken from no factor, from the sphere, from the whole internal space, or from the complex projective plane, and the first three choices give the three neighbors of the rule. With no abiding share at all, the ratio would be exactly six. With the sphere's share, six less one divided by pi, the ratio would be about five and sixty-eight hundredths, and with the share of the whole internal space, six less three divided by pi, the ratio would be about five and five hundredths. The measurements put the ratio at about five and thirty-six to five and thirty-seven hundredths, within about five to seven hundredths either way. Six lies more than six tenths above the measured value. Five and sixty-eight hundredths lies about three tenths above the measured value. Five and five hundredths lies about three tenths below the measured value. Each of the three neighbors lies far outside the measured range, so the measurement excludes all

three. So the answer is yes. Of the four choices within the form, only one agrees with the measurement: the complex projective plane's abiding share, two divided by pi.

Two parts of the standing of the ratio are principles of the account: the counting principle, and the rule of which curvature abides. The rule for the ratio was selected by the measurement from a small family of expressions of one form. *The Weight of Possibility* examined twelve expressions in all. I give the number twelve so that you know how wide the search was. Three things support the rule for the ratio: its form, which is the form of the rule of *The Gravitational Passage* for the cosmological constant; the exclusion of its three neighbors within that form; and the coefficient that the rule for the ratio shares with the rule of *The Gravitational Passage*. The rule for the ratio is an identity of the present measurement, so the rule needs no derivation from the motion of the turn.

Two Predictions Anyone Can Look For

The identity fixes the ratio for every measurement, wherever the measurement is made, and two consequences follow that anyone can look for.

The first consequence concerns the shape of space on the largest scale. Think of an old video game in which the player who walks off the right edge of the screen comes back on the left. The screen of the game wraps around on itself: a straight walk returns to where the walk began. Space could, in principle, wrap around on itself as the screen does. In a universe whose space wraps around, a path can go all the way around the universe and return to its start, and such a path cannot be pulled tight to a single point. When a path of that kind is shorter than the horizon's diameter, *The Weight of Possibility* calls the path a *short loop*, wherever the path lies. The region within our horizon is a ball, of one class, exactly when no short loop passes through us. The identity holds for every measurement, wherever in the universe the measurement is made, so the count of one holds for every location, and the region within every location's horizon is

a ball. So the account predicts that no short loop exists anywhere in space. Astronomers have searched the oldest light for paths around the universe. The searches find that every path around the universe through our location, of the kind that cannot be pulled tight to a point, is longer than ninety-eight and a half percent of the diameter of the sphere from which the oldest light reaches us. The sphere from which the oldest light reaches us lies a little inside our horizon, so the region within our horizon is a ball out to ninety-six and a half percent of the horizon's radius.

A short loop would change the count, and with the count the ratio. A region with a short loop is not a ball, and it counts at least two classes. If the region within our horizon counted two classes, the ordinary matter, filling the region, would count two, and the dark matter would count twelve, while the abiding share, one term of the measuring act, would still be taken once. The ratio would then be twelve less two divided by pi, all divided by two: six less one divided by pi, about five and sixty-eight hundredths. The measurement already excludes five and sixty-eight hundredths. More classes would give larger values of the ratio, still farther from the measured value.

The second consequence concerns the ripples of the oldest light. Recall the ripples on a pond that all rise and fall together (Chapter 12). Every ingredient of the young epoch departed from evenness in one pattern, as the ripples on the pond rise and fall together. The identity fixes the ratio of dark to ordinary matter everywhere, so the account predicts that no separate pattern of departures will be found in the dark matter. (Chapter 11 called a separate pattern in one component an isocurvature departure: one component departs against another, the total density unchanged at first.) The measurements find no separate pattern, down to about 1.3 percent of the power of the ripples. In the account, the prediction follows by two routes. One route starts from what passes the turn, and the other route starts from the moment of measuring.

The predictions of no short loop and of no separate pattern join the account's other predictions of kind about the dark matter: predictions of what sort of thing the dark matter is. In order, the account predicts that

the dark matter acts only through gravity; that the dark matter does not decay; that the dark matter is perfectly cold, with no cut-off in its structure on small scales; that the dark matter is made of gyrations of about the Planck mass; that no short loop exists anywhere in space; and that the dark matter carries no separate pattern of departures. Astronomers tell cold dark matter from warmer sorts by present bounds, and cold dark matter that does not collide satisfies every bound. The Planck mass is the one mass for which the gravitational size and the quantum size of a thing are equal. Three of the account's predictions belong to the account alone: the ratio, the coefficient that the rule for the ratio shares with the rule of *The Gravitational Passage*, and the absence of a short loop. The account shares the first four predictions in the list with other accounts of a lasting, cold dark matter that acts only through gravity. The account shares the last prediction with every account in which the dark and the ordinary matter vary together from place to place.

The Turn Need Not Know the Proton

Go back to the recipe card. The mill never needs to know how much sugar you own, because the card fixes the proportion of flour to sugar. The turn never needs to know the mass of the proton, as the mill never needs to know how much sugar you own: no process at the turn has to tie the mass of a gyration to the mass of the proton.

Suppose that a process at the turn set the ratio. Then the process would have to tie a gyration of about the Planck mass to the proton's mass, across nineteen powers of ten. The identity applies at the moment of measuring, so the identity needs no tie between a gyration's mass and the proton's mass. The identity needs nothing from the mass of the proton, and nothing from the size of the excess of ordinary matter. The turn makes the gyrations, and we carry back through the reconstructed history

the ratio that the present measurement fixes, just as *The Gravitational Passage* does not need the history to set the value of the cosmological constant.

Some other accounts of dark matter are called asymmetric. In the asymmetric accounts, physicists tie the mass of dark matter for each proton or neutron to the mass of the proton. In the asymmetric accounts, dark and ordinary particles are made in equal excess, so, for the measured ratio, a dark particle must weigh about five protons. The identity fixes only the ratio of the density of the dark matter to the density of the ordinary matter. Under the identity, the proton is tied to neither the number nor the mass of the gyrations.

What the Amount Asks of the Beginning

Could the turn have made more gyrations than the amount that the identity fixes? On the account, if a turn made more gyrations than the identity fixes, the turn belongs to a history no present measurement could reconstruct. The identity fixes what every present measurement registers. So our measurements cannot carry back a history with more gyrations than the identity fixes. The identity is a condition on every history that the present measurement can carry back. The gyrations are made only at the turn. So the identity sets conditions on the turn, through every history that our measurements can carry back. I state those conditions below. The re-forming of the six is their return to their compact form in the new epoch, after the turn. The account therefore predicts that a computation of the re-forming, once someone makes the computation, will produce the registered amount.

When two particles of a hot gas meet, they can make a pair of anything that has weight, through gravity alone. Call the making of pairs in that way *production through gravity*. The number of pairs falls away very steeply once the mass to be made exceeds the typical energy of a gas particle. A gas hot enough would make gyrations in great numbers. The Planck tem-

perature is the temperature at which a typical particle of the gas carries the energy of a Planck mass. A gas that started near the Planck temperature would make gyrations about as numerous as the particles of light, while the measured amount is about two to four gyrations for every ten billion billion billion particles of the oldest light. So the measured amount excludes a gas that started near the Planck temperature.

The turn's energy could have reached the hot gas, the plasma of the young epoch, in two ways: at once, or gradually. If the gas formed at once, the measured amount of dark matter requires that the gas was never hotter than about one thirtieth of a gyration's mass, with a temperature stated as the energy of a typical particle of the gas. The number thirty in the limit runs from about twenty-eight to about thirty-four, depending on the strength of the coupling used in the calculation: how strongly the hot gas and the gyrations act on each other. Chapter 18 computed two masses for a gyration, and the limit depends on the mass. For a gyration of the square root of two Planck masses, the limit is about five times ten to the seventeen billion electron volts. For a gyration of six tenths of a Planck mass, the limit is about two times ten to the seventeen. (A billion electron volts is the unit in which particle physicists state both energies and masses, and a proton weighs a little under one.) The second way is gradual, through a stage that behaves as matter. A stage that behaves as matter is a stage in which energy is held in heavy, slow things that later decay into the content. If the energy reached the gas gradually, the condition set by the measured amount falls on the end of the stage that behaved as matter: when the stage ended, the temperature of the gas must have been below about one hundredth of a gyration's mass. The condition holds even if the energy at the start of the stage was of the Planck density. The Planck density is one Planck mass in every cube one Planck length on a side. The gas may then have been as hot as one thirtieth of a gyration's mass at its hottest moment, because the radiation the stage releases afterward thins out the gyrations made at the hottest moment. The limit of one thirtieth concerns the highest temperature that the epoch reaches.

The first limit concerns temperature, and the second limit concerns how the six form again. Picture a child's swing hanging from a frame. Tilt the frame slowly, over the time of many swings, and the swing simply follows, hanging straight down the whole way. Jerk the frame, and the swing starts swinging. A change spread smoothly over many periods of a vibration sets almost none of the vibration going, as the slow tilt leaves the swing still. A period of a vibration is the time of one swing.

The six re-form at the turn, and the floor catches the six at their held size (Chapter 4). The floor is the fact that the internal space cannot shrink to nothing. A sudden re-forming would set the lightest odd level ringing, and so would make gyrations in vast numbers. For the measured amount, the gyrations made directly as the six re-form must number about a billion billion billion times fewer than a sudden re-forming would make, a fraction of ten to the minus twenty-seven. *The Weight of Possibility* estimates that the number of vibrations set going falls steeply as a change is spread over more swings. A reduction by a billion billion billion times needs a change smooth over about ten periods of the lightest odd level, about ten of its swings, so the re-forming, and the catch by the floor at its end, must be slow and gentle compared with one swing of the lightest odd level. Call a re-forming of that kind a *gentle re-forming*. The arc of *The Gravitational Passage* is the stretch of the history along which all six small directions shrink (Chapter 2). On the arc, each of the six shrinks only as one over the ninth root of the time, so a billionfold change of the time changes each of the six tenfold (Chapter 2). The shrinking on the arc is slow compared with one swing of the lightest odd level, once the time is longer than light takes to cross the internal radius, the radius of the internal space. The description of the new epoch begins when the curvature falls below the Planck curvature, while the six are still re-forming.

Both limits are conditions on the turn, not results: the limit on temperature, and the limit on re-forming. A condition on the turn is a requirement that the account sets on the turn, and not a result that follows from the account. The account sets an upper bound on the temperature: the gas was never hotter than about a thirtieth of a gyration's mass, or a stage that

behaved as matter ended below a hundredth. *The Gravitational Passage* does not fix the temperature at which the new epoch's reconstructed history begins, and the account contains no value for that temperature. Chapter 4 listed three conditions on the turn. The remaining condition is that the waves of the settling stay below about three percent of the radiation. The waves of the settling are the gravitational waves sent out as the six settled at their held size, so the remaining condition concerns the settling of the six, and Chapter 16 follows the remaining condition.

The temperature limit is a second ground against a long inflation, beside the grounds of Chapter 13. The first waves are the gravitational waves present from the beginning of an epoch. In an epoch whose ripples were made by inflation, the measured limit on first waves would require a temperature far below one thirtieth of a gyration's mass, and dark matter of a gyration's mass could then not be made after the inflation in the measured amount. On the account, the gyrations are made at the turn. *The Gravitational Passage* excludes an inflation long enough to stretch the lengths seen today from within the early horizon to beyond the early horizon, and a shorter inflation is neither required nor excluded (Chapter 13). So the amount of dark matter is a second reason, beside the reasons of Chapter 13, why the ripples of our epoch were not made by inflation, because gyrations of about the Planck mass could not otherwise have been made in the measured amount.

Key idea. The ratio of dark to ordinary matter is fixed when a measurement is made, as the cosmological constant is. The dark matter counts six classes, less the abiding share of the complex projective plane (two divided by pi, taken once). The ordinary matter counts one class. So the ratio is six less two divided by pi, about five and thirty-six hundredths. The oldest light measures about five and thirty-six to five and thirty-seven hundredths. Because the turn alone makes the gyrations, the amount of dark matter sets two conditions on the beginning. The first condition is a hot gas never hotter than about a thirtieth of a gyration's mass, or, if the energy of the turn reached the gas gradually, a stage that behaved as matter ending below a hundredth of that mass. The second condition is that the six re-form gently, smooth over about ten swings of the lightest odd level.

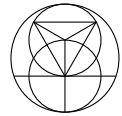
Summary

The turn left two leftovers beside its radiation: a small excess of ordinary matter, and the dark matter. The ratio of the two leftovers has stayed the same since helium formed, and an identity of the present measurement fixes the ratio, as *The Gravitational Passage* fixes the cosmological constant. A measurement counts the basic pieces of shapes, called classes. The dark matter spreads over the whole internal space and counts six, three times two, while the ordinary matter is registered within the horizon, a ball, and counts one. The curvature of the complex projective plane holds fast as a term of its own: the abiding share, two divided by pi, taken once. The sphere's curvature is already inside the sphere's two classes, by the theorem of Gauss and Bonnet. So the ratio is six less two divided by pi, about five and thirty-six hundredths. The count behind the ratio uses two principles of the account: the counting principle, and the rule of which curvature abides. The rule for the ratio was selected by the measurement from a small family of expressions of one form, twelve examined in all. The Planck satellite measures five and three hundred sixty-four thou-

sandths, and three telescopes together measure five and three hundred seventy thousandths. The measurements exclude the rule's three neighbors: six, six less one divided by pi, and six less three divided by pi. The rule for the ratio shares one coefficient with the rule of *The Gravitational Passage* for the cosmological constant. The measured ratio puts the shared coefficient at about one. The identity predicts that no short loop exists anywhere in space, and that the dark matter carries no separate pattern of departures. The turn need not know the proton's mass. No process at the turn ties the mass of a gyration to the mass of the proton. Because the gyrations are made only at the turn, the amount of dark matter sets conditions on the turn. The first condition is a hot gas never hotter than about a thirtieth of a gyration's mass, a fraction that runs from one twenty-eighth to one thirty-fourth with the strength of the coupling (or a stage that behaved as matter ending below a hundredth). The second condition is a gentle re-forming that makes a billion billion billion times fewer gyrations than a sudden re-forming, smooth over about ten periods of the lightest odd level. The two conditions are requirements on the turn, not results. The temperature limit is also a second ground against a long inflation.

For the book's question of what the beginning left behind, the findings of this chapter mean two things. First, the dark matter that the beginning left behind is fixed in amount, against the ordinary matter, at the moment of measuring. Second, the amount of dark matter tells us two things about the beginning: the hot gas of the beginning was never hotter than about a thirtieth of a gyration's mass (or a stage that behaved as matter ended below a hundredth of a gyration's mass), and the six formed again gently.

The dark matter is one leftover of the turn. The amount of dark matter is fixed relative to the other leftover, the small excess of ordinary matter. Why is there any ordinary matter at all, and how much ordinary matter is there?



Chapter 20

The Residue of Matter

The dark matter is a vibration of the six held dimensions of possibility, made at the turn and kept ever since. Now take the ordinary matter, the matter of atoms. Why is there any ordinary matter at all, and how much is there? You are made of ordinary matter, and so are the stars, the planets and the air. Protons and neutrons sit at the core of every atom of your body, of the stars, of the planets and of the air. This chapter shows that the question of how much ordinary matter there is, which sounds like one question, is really two questions: one about history, and one about the kinds of particle. The kinds of particle are the quarks, the electron and its relatives, the neutrinos, the carriers of the forces and the Higgs boson. Each kind carries its own charges. The amount of matter is a *number of history*: a number set by what happened in the earliest moments, and not by the laws of the kinds of particle. The amount of matter is a residue from the earliest moments. No process among the kinds of particle makes the amount. The kinds of particle fix only one thing about the amount of matter: the share of the matter held as protons and neutrons.

A Line of Shells

Walk along a beach after the tide has gone out. The sea has drawn back, and all the water it brought up the sand has gone back with the sea. A thin line of shells and weed is left where the tide stood highest. Through the calm afternoon that follows, the wind lifts the dry sand around the

shells and people step across the shells, and the line of shells stays where the tide left the line. Nearly everything the tide brought went back with the tide. The line of shells is what remained, and nothing that happens on the beach afterward moves the line.

The line of shells is a picture of one idea, a *residue*: what remains when nearly everything else has gone one way. The ordinary matter of the universe is a residue, as the line of shells is. The turn leaves the residue, and every later change keeps the residue. We need a few words to see why the ordinary matter is a residue, and Chapter 10 has already explained some of those words.

A baryon is a proton, a neutron or one of their heavier relatives, as Chapter 10 said. Protons and neutrons are the particles of which the core of every atom is made. Baryons carry nearly all the mass of every atom, so physicists count matter in baryons. As Chapter 10 found, every particle that carries a charge has an antiparticle: a twin of the particle, with the same mass as the particle and every charge reversed. The antiparticle of the electron is the positron, and the antiparticle of a proton is an antiproton. The antiparticles of the baryons are the *antibaryons*. Antimatter is matter made of antiparticles, as the opening of this book, Before We Begin, said. A particle and its antiparticle are always made together, as a pair, and when a particle meets its antiparticle the two annihilate, as Chapter 10 said: both disappear, and their energy leaves as light.

In the first moments of our history, the universe was filled with a hot crowd of particles and light, all striking one another constantly. The crowd was so hot that the energy of its heat was far above the mass of any kind of particle, so every kind was made and unmade in pairs, again and again. The crowd was in *balance*: the settled state in which every reaction runs as often forward as backward. A crowd in balance holds each kind of particle and its antiparticle in equal numbers. The one exception to equal numbers is a crowd that keeps an excess of some count that no reaction changes. I find which count the crowd kept, and I find that the crowd kept

an excess of that count from the start. If balance holds matter and antimatter in equal numbers, the opening question becomes a sharp one: why is there any matter left at all?

Picture the end of a crowded party at which every guest arrived with a partner, and couples leave together as they find each other. At first, partners meet at once and leave in pairs. As the room empties, the remaining guests are spread thin, and finding a partner takes longer. At last the few guests who remain are so far apart that they never meet before the hall is locked. Those few guests stay in the hall. Baryons and antibaryons, with no excess of either, would end the same way as the guests. Each baryon and each antibaryon is a guest, and leaving with a partner is annihilating. As the crowd cooled, the baryons and antibaryons would annihilate in pairs until the growth of space spread the survivors too thin to meet, as the emptying room spreads the last guests thin. A computation of established physics shows what an even annihilation of baryons and antibaryons, with no excess of either, would leave: about one baryon for every billion billion particles of light, with as many antibaryons beside the baryons.

The universe holds far more matter than an even annihilation would leave, and the universe holds no matching antimatter. Physicists count the amount of ordinary matter against the oldest light, the faint glow that fills the sky (Chapter 11). Each particle of the oldest light is a photon (Chapter 11). Every cubic centimeter of space holds about 411 photons of the oldest light. Physicists compute that number from the light's measured temperature of 2.72548 degrees above absolute zero. Physicists state the amount of matter as *baryons per photon*: the number of baryons for every photon of the oldest light. The measured amount is about 6.13 baryons for every ten billion photons. The photons counted in baryons per photon are the photons of the light after the light took in the energy of the electrons and positrons (Chapter 11), so the amount of matter is counted against the warmed light. On average, the measured amount is about one baryon in every four cubic meters of space, which is hundreds of millions of times

more than the even annihilation would have left. The excess of matter over antimatter includes every baryon of the stars, the planets and your own body.

What It Would Take to Make Matter

Could the hot crowd of particles have made the excess of matter as the crowd cooled? *The Coming of the Kinds* is the book of this collection that tells how the kinds of particle came to be. *The Coming of the Kinds* answers the question first with a picture. Take a toothed wheel with a catch, a ratchet of the kind a mechanic's wrench uses, so that the wheel can click forward and the catch stops the wheel from slipping back. Fix a small paddle to the wheel's axle and set the paddle in a box of gas, so that the molecules of the gas strike the paddle at random from every side. You would expect the wheel to creep forward, one click at a time. While the catch and the gas are at one temperature, the wheel does not creep forward. The catch is made of molecules too, and the catch is jostled too. Now and then the catch lifts, and the wheel slips back a tooth. At one even temperature the slips back and the clicks forward come out equal, and the wheel goes nowhere on average. The wheel turns one way only when three things hold together. First, the wheel can turn at all. Second, the wheel's teeth and catch favor one direction. Third, something keeps the whole machine from settling to one temperature, for example a catch kept colder than the gas.

In 1967 the physicist Andrei Sakharov set out three requirements for the making of matter that match the three things the wheel needs. Sakharov's *three conditions* are the three things that any process needs, all together, to make an excess of matter in a universe that starts with none. First, some process must change the number of baryons. Second, the reactions that make the excess must treat matter and antimatter differently: differently under the swap of every particle for its antiparticle, and differently under that swap taken together with the mirror swap of left and right.

Third, the process must run while the universe is out of balance. Otherwise the reverse reaction undoes the excess that the process made. Now we set the kinds of particle beside Sakharov's three conditions, one condition at a time.

The kinds of particle supply Sakharov's first condition. Only one process changes the number of baryons: the sphaleron passages (Chapter 10). A sphaleron passage carries the field of the weak force from one of the field's lowest states, over a ridge, to the next lowest state. A quark is one of the particles of which protons and neutrons are made, three to each (Chapter 2). A lepton is the electron, its two heavier copies, the muon and the tau, or one of the three neutrinos (Chapter 10). Each passage makes nine quarks and three leptons together, or removes nine quarks and three leptons together. Three quarks count as one baryon. So each passage changes the number of baryons by three and the number of leptons by three, in the same direction. As in Chapter 10, I give a temperature as an energy. One GeV, a billion electron volts, is about eleven and a half million million degrees. In the hot crowd the passages ran in balance from about a million million GeV down to 131.7 GeV. The lower temperature, 131.7 GeV, comes from a computation of established physics on a lattice: the grid of points on which a computer solves the equations of the weak force (Chapter 10). Today the passages are suppressed by a factor of about one in ten to the power one hundred sixty-one. A single proton can never make a passage. The passages are the one process that changes the number of baryons, so since the passages stopped, nothing has changed the number of baryons, and the number fixed when the passages stopped is the number still present today.

The kinds of particle supply Sakharov's second condition as well, that the reactions treat matter and antimatter differently. The weak force treats a particle and its mirror image differently, and a particle and its antiparticle differently. The quarks come in three *families of particles*. A family is one set of the particles of matter (two quarks, the electron or one of its heavier copies, and a neutrino), and the three families are alike except in their masses. (In this paragraph the word "family" has a different use

from the families of vibrations of the six in Chapter 18.) The quarks of one family mix with the quarks of the other families through the masses of the quarks: the quark that the weak force pairs with the up quark is a blend of the down, the strange and the bottom quarks. Physicists describe the blend by angles. Physicists may name the states of the quarks in many equivalent ways. A change of names shifts some of the angles of the blend. Picture two roads crossing. The compass bearings of the two roads depend on which direction you choose to call north, and a new choice of north changes the bearings. The angle between the two roads stays the same whatever you call north. Most angles of the blend are like the bearings, because a change of names shifts those angles. With three families, one angle of the blend is like the angle between the roads: in 1973 the physicists Makoto Kobayashi and Toshihide Maskawa found that no renaming removes that one angle. The angle that no renaming removes makes matter and antimatter behave slightly differently. *The Coming of the Kinds* says where the difference between matter and antimatter comes from, following *The Story of the Particles* (the book of the series on why every particle is required). The Incommensuration Theorem limits what is possible, and the theorem leaves possible a difference between a process and the image of the process with every particle swapped for its antiparticle and left swapped for right. The one angle that three families allow makes the possible difference between matter and antimatter actual. The strength of the angle is measured at about three parts in a hundred thousand. At the temperatures where the angle would have to act, the angle acts far more faintly still. Picture three sisters seen across a wide field: up close their faces differ, and from far off the differences are too small against the distance to see. The angle acts only through differences between the masses of the quarks, as the sisters are told apart only by the differences between their faces. Near 160 GeV every quark but the heaviest is light against the temperature: the mass of each of those quarks is small next to the temperature given as an energy, as the sisters' differences are

small against the distance. Near 160 GeV the angle's computed strength is about 4.4 parts in ten thousand billion billion, more than a trillion times smaller than the excess the angle would have to make.

Sakharov's third condition is a process running out of balance, and whether the condition is met depends on how the universe cooled at one moment. Only the sphaleron passages change the number of baryons, and the passages stop at one moment: the cooling through the scale of the weak force. The cooling is the electroweak transition, in which the Higgs field settles at its value and the masses switch on (Chapter 5). Chapter 10 described two ways in which a change like the electroweak transition can go. One way is like ice forming in a chilled bottle, and the other way is like butter softening on a counter. *The Coming of the Kinds* shows the same two ways in a kitchen. Water heated in a pot boils at one temperature: bubbles of steam form, and at each bubble's wall the water is out of balance, turned to steam faster than the water can settle. When a change has bubbles and walls, physicists call the change first-order (Chapter 10), and the forming of the ice is first-order too. Honey cooled on a counter grows thicker steadily, with no jump, no bubbles and no walls, and the honey is settled at every moment. When a change is smooth in the way the thickening of honey is smooth, physicists call the change a crossover (Chapter 10), and the softening of the butter is a crossover too. The Standard Model is the established physics of the kinds of particle and their forces. Computations of the Standard Model show bubbles and walls only for a Higgs boson lighter than about 72 GeV. The mass of 72 GeV is the *endpoint*: the heaviest Higgs boson for which the change still has bubbles. The Higgs boson is measured at 125.13 GeV, far above the endpoint. Because the Higgs boson is far heavier than the endpoint, the electroweak transition is a crossover. The electroweak transition is the whole smooth settling of the Higgs field toward its present value. The settling of the Higgs field is steepest around 159.5 GeV, the center of the settling, over a span of about 5 GeV, and below that span the field goes on growing, more slowly, toward its present value. The sphaleron passages stay in balance through the steepest part of the settling, and then, at 131.7 GeV, well below the center

and while the field is still growing, the passages gradually *fall behind the growth of space*. Falling behind means that the passages begin to run more slowly than space spreads the crowd thinner, so the passages can no longer hold the crowd in balance, like the last guests at the crowded party pictured earlier in this chapter, who no longer meet. While the passages run, no wall ever sweeps through the crowd. So Sakharov's third condition is not met at the electroweak transition, and no excess of matter is made during that transition.

One more fact is needed to show that the kinds of particle make no excess of matter by themselves. Swap every particle for its antiparticle, swap left for right, and run time backward, all at once, and the structure of the kinds is unchanged. The three swaps together are a symmetry of the kinds, and one consequence of that symmetry is that a particle and its antiparticle have exactly equal masses. In balance, the number of particles of each kind is set by four things: the mass of the kind, its count of states, the temperature, and the counts that every reaction keeps. So when a crowd in balance holds no excess of any count that every reaction keeps, the crowd holds each kind and its antiparticle in equal numbers. The numbers stay equal whatever preference the single reactions show for matter or for antimatter, just as the slant of the ratchet's teeth cannot turn the wheel at one temperature. So no excess of matter follows from the structure of the kinds by itself. The excess is a number of history.

The Count That Never Changes

The electroweak transition is one moment, and the sphaleron passages are one process. The particles of the many kinds meet and change in countless other ways, at every temperature from the start of our history to today. Could the excess of matter have come from any of the countless other ways in which particles meet and change? *The Coming of the Kinds* answers with a second picture.

Picture a wall with two tally counters on it, of the kind a doorkeeper clicks to count people coming in. One counter counts baryons, and the other counter counts leptons. Every machine in a large factory is wired to the two counters. Some machines never touch either counter. One machine, when it runs, clicks both counters forward by three together, or both back by three together. No machine clicks one counter without the other. Watch the factory for as long as you like: the readings of the two counters change, but the difference between the two readings never changes. Whatever the difference between the readings is today, the difference was the same when the factory opened.

For the particles, the two readings are the count of baryons and the count of leptons, and the difference between the two counts is called B minus L (Chapter 10). B minus L is the count of baryons less the count of leptons. Each count is taken net: the baryon count is the baryons less the antibaryons, and the lepton count is the leptons less the antileptons. The kinds of particle behave like the factory, and each process among the kinds is a machine. *The Coming of the Kinds* checks every machine.

The ordinary reactions are the first machines to check: a quark turning into another quark, a pair made from light, and a neutron decaying to a proton, an electron and an antineutrino. Each ordinary reaction leaves the baryon count as it was and the lepton count as it was. The sphaleron passages are the machine that clicks both counters by three together, so the passages leave B minus L as it was. A machine that clicked the baryon counter alone would need a carrier of force that turns a quark into a lepton. A carrier would have to carry both *color* and the weak charge to turn a quark into a lepton. Color is the label the strong force acts on, and the weak charge is the label the weak force acts on. The kinds hold no carrier of both color and the weak charge, at any energy: there is no boson, no carrier of force, that joins color to the weak charge. A machine that clicked the lepton counter alone would need a *Majorana mass*, a mass that joins a particle to its own antiparticle. A neutrino with a Majorana mass would be its own antiparticle, and the Majorana mass would change the lepton count by two. A particle's *chirality* is its handedness, left-handed or

right-handed: the two ways the particle can turn about its direction of motion. The kinds hold no Majorana mass, for either chirality: the neutrino's mass is a *Dirac mass*, the usual kind, which joins the left-handed part of a particle to the right-handed part of the same particle. And there is no machine outside the list of kinds, because the list of kinds is closed. The closed list holds no axion, a particle that some physicists propose, and no particle of the dark matter, because the dark matter is a kept vibration of the six held dimensions (Chapter 18), and not one of the kinds.

The factory also stands in a building, and you may ask whether the building could click a counter. In the picture, the building is the arena of space and time. A bending and twisting of the arena can change a count, in the same way that a winding of the field of the weak force lets the sphaleron passages change a count. A bending and twisting of the arena can change a count only where the left-handed and the right-handed particles carry that count unequally. Count B minus L over one family of particles. Each quark counts as one third of a baryon, since three quarks make a proton, and each lepton counts as minus one. The left-handed states are six quark states (three colors of each of the family's two quarks), which count as two, and two lepton states, which count as minus two: a total of zero. The right-handed states are six quark states, which count as two, and the right-handed electron and the right-handed neutrino, which count as minus two: a total of zero as well. The right-handed neutrino is the partner of the neutrino that Chapter 10 met. Without the right-handed neutrino, the right-handed total would not be zero. The left-handed and the right-handed particles carry B minus L equally, so no bending or twisting of space and time changes B minus L either. *The Coming of the Kinds* concludes: every process among the kinds keeps B minus L, at every temperature, from the earliest moment our history reaches to today. So B minus L is *what every change keeps*.

One more step is needed before B minus L explains the excess of matter, and a marble in a bowl shows that step. Push a marble up the side of a bowl and let go. The marble rolls back and settles at the bottom, however far you pushed the marble. The bowl's shape sets where the bottom lies,

and the push does not. While the sphaleron passages run in balance, the baryon count behaves like the marble: the passages roll the baryon count to a settled value, and the settled value is a fixed share of B minus L. Now suppose some process made an excess of baryons while B minus L was zero. A fixed share of zero is zero, so the bottom of the bowl then lies at zero. The passages roll the excess of baryons away. An excess of baryons survives the passages only where B minus L is not zero.

Now follow the steps backward in time, from today. Baryons exist today, and baryons outnumber antibaryons. Nothing has changed the baryon count since the passages stopped at 131.7 GeV, so the baryon count was above zero when the passages stopped. When the passages stopped, the baryon count stood at a fixed share of B minus L. A share of zero is zero, so B minus L was not zero. Every process keeps B minus L, so B minus L had the same value, not zero, at every earlier moment, back to the earliest moment our history reaches. In the words of *The Coming of the Kinds*: "No process among the kinds of particle changes B − L (baryon number less lepton number); so the excess of baryons present today requires a B − L present from the earliest moment our history reaches".

The earliest moment our history reaches is the turn. The excess of matter over antimatter is carried as the value of B minus L present since the turn, and that value is "a residue the turn leaves, kept by every change after it". The residue is the line of shells on the beach: the tide is the turn, and nothing that happens on the sand afterward moves the line. The residue is also the difference between the factory's two counters. The factory is the hot crowd of the kinds, and the factory opened at the turn. How the turn leaves the residue is a question for the account of the turn. The residue is a number of history. The residue is measured through the part of the residue held as protons and neutrons. I say no more of how the turn leaves the residue.

Three Routes With No Starting Point

Physicists have proposed three routes by which the excess of matter could have been made during our history. Each route needs something that three features of the kinds, found in the sections above, do not allow. First, no carrier of force carries both color and the weak charge. Second, every quark and lepton has a Dirac mass. Third, the transition of the weak force is smooth. Picture three keys cut for one lock, each key with one notch more than the lock has room for: none of the three keys turns. The routes are the keys, and the kinds are the lock. Each route rests on one thing more than the three features of the kinds allow.

The first route is *grand unification*: theories in which the strong, weak and electromagnetic forces join into one force at very high energy, inside one larger pattern of symmetry that physicists call a unifying group. Physicists built theories of grand unification to join the forces into one. Theories of grand unification hold heavy carriers of force that carry both color and the weak charge. In theories of grand unification the heavy carriers decay out of balance in the early universe, and the decays change the baryon count directly. The kinds hold no unifying group and no boson that joins color to the weak charge. So the route of grand unification has no carrier among the kinds.

The second route is *leptogenesis*, which physicists proposed as a way to make the excess without grand unification. Leptogenesis needs heavy neutrinos with Majorana masses, far heavier than the three neutrinos of the kinds. On the route of leptogenesis the heavy neutrinos decay out of balance and leave an excess of leptons, and the sphaleron passages carry part of that excess of leptons over into baryons. The kinds hold no Majorana mass of either chirality, so leptogenesis has no Majorana mass among the kinds.

The third route is the making of the excess at the electroweak transition, as the sphaleron passages stop. The electroweak transition is the one moment that could matter: the passages are the one process that changes the number of baryons, so an excess made by the passages would have had to be made at the electroweak transition, as the passages stopped. The third route needs a departure from balance at the electroweak transition. Only a first-order transition would put the crowd out of balance, through the transition's bubbles and walls. The electroweak transition is a crossover, and the passages stay in balance through the steepest part of the crossover, so the third route has no departure from balance among the kinds.

None of the three routes has a starting point among the kinds. So the three routes, too, show that no process among the kinds makes the excess, and that the excess is a residue from the earliest moments.

The Share the Kinds Fix

The kinds of particle fix one thing about the residue: how the residue is shared between baryons and leptons. While the sphaleron passages ran, they shared the residue between baryons and leptons. How much of the residue became baryons?

Picture an estate divided by a will. The will names the shares as fractions of whatever the estate turns out to hold. The size of the estate comes from the history of the estate's owners: what the owners earned, saved and spent over the years. To know what one heir receives, you need both: the size of the estate, which is history, and the fraction the will names, which was written beforehand. The residue is the estate, and the size of the residue is a number of history. The will is the list of kinds, with their charges, and the list of kinds names the *share held as baryons*: the fraction of the residue that ends as protons and neutrons.

Balance has two results, and the share comes from putting them together. When the kept count, B minus L, is zero, a crowd in balance holds each kind and its antiparticle in equal numbers, as the ratchet showed.

When B minus L is not zero, the reactions in balance spread the excess over the kinds that carry B minus L. Each kind then ends up with a little more of the particle than of the antiparticle. Each kind *leans* a little toward particle over antiparticle. The lean appears only when B minus L is not zero, and the size of the lean is in proportion to the residue.

The share held as baryons is a count of the kinds' leans. Picture several vessels standing side by side, joined by pipes at their bases. Pour water into any one vessel, and the levels of all the vessels rise together, because the pipes tie the levels together. The width of each vessel sets how much water the vessel holds. Every reaction that runs in balance is a pipe. The reaction ties the lean of what goes into the reaction to the lean of what comes out. A vessel's level is a kind's lean. A vessel's width is how much of the excess a given lean puts into that kind. The width is more for a kind with more states. A *coupling* to the Higgs field is the strength with which a particle interacts with the Higgs field, and the coupling is what gives the particle its mass once the field has a value. Just above 131.7 GeV, three sets of reactions all run in balance: the sphaleron passages, the reactions of the strong, weak and electromagnetic forces, and the reactions through the couplings of the particles to the Higgs field. The crowd as a whole carries no net charge of any force. Every lean is tied to a single lean, the one free lean, by the reactions in balance and by the zero net charge. The water poured into the vessels is the residue. For example, a passage makes three quarks and one lepton in each family, and in balance the leans of those three quarks and one lepton add up to zero, so the lepton leans three times as far as a quark of one color, the other way. The count of the share needs only the number of families, three, the number of Higgs fields, one, and the charges of the kinds. The right-handed neutrino stays out of the count, because the right-handed neutrino never comes into balance. The count gives the share as one whole number over another, and the arithmetic is short. Twenty-eight is eight times the number of families, plus four times the number of Higgs fields. Seventy-nine is twenty-two times the number of families, plus thirteen times the number of Higgs fields. Before the Higgs field has a value, the share held as baryons is twenty-eight

over seventy-nine: twenty-eight seventy-ninths, a little over a third (about 0.354). Deep in the settled state of the Higgs field, where the field's value is large against the temperature, the share is twelve thirty-sevenths (about 0.324). In between, as the field grows, the share falls steadily from twenty-eight seventy-ninths to twelve thirty-sevenths.

Picture a photograph of the sky taken partway through a sunset. Through the evening the color of the sky moves smoothly from gold toward deep red. Partway through the sunset, a photograph fixes a color between gold and deep red, and nothing changes the color in the print afterward. The share moves smoothly from twenty-eight seventy-ninths toward twelve thirty-sevenths as the Higgs field settles. The moment at which the passages fall behind is the photograph. The passages fall behind at 131.7 GeV, partway through the settling, so the share that froze lies between twenty-eight seventy-ninths and twelve thirty-sevenths. Cristina Benso and colleagues published a computation across the crossover in 2026. The computation by Benso and colleagues gives the share that froze as 0.3328: very close to one third. The structure of the kinds sets all three shares: twenty-eight seventy-ninths and twelve thirty-sevenths are counted from the list of kinds and their charges, and 0.3328 is computed from the kinds across the crossover.

About one third of the residue was held as baryons when the passages stopped, since the share was 0.3328. The leptons carry the rest of the residue with the opposite sign: an excess of antileptons over leptons, about twice the excess of baryons. Since the passages stopped, the excess of baryons has not changed, and the excess of antileptons has not changed either. Astronomers measure the third of the residue held as baryons: about six baryons for every ten billion photons. When the measured amount is divided by the share, the result is the size of the residue itself: about 1.84 units of B minus L for every billion photons of today, about three times the number of baryons. In the words of *The Coming of the Kinds*: "The residue is history; the share is structure".

One Sign Everywhere

If the residue was present from the start, a question of place follows: could some distant region of the sky be made of antimatter? Picture the sea, deep in some places and shallow in others, warm and cold, and carried about by its currents. Draw a liter of water from almost anywhere in the open ocean, and the liter still holds nearly the same weight of salt. The salt and the water were mixed before the currents began, and every current moves the salt and the water together.

The residue was set through the young universe in the same way as the salt through the sea. The young universe departed from evenness in one manner only: where a place held a little more light, the place held a little more of every ingredient, in the same proportion (Chapter 12). So the number of baryons per photon was the same from place to place, and the passages shared the residue alike everywhere. A number that is the same everywhere has one sign everywhere: nowhere in the observable universe do antibaryons outnumber baryons. Astronomers have found, by two independent routes of measurement, that the sky has one sign everywhere. First, the oldest light shows a pattern of warm and cool patches, and the pattern shows the ingredients rising and falling together. Second, gamma rays are light of very high energy, and the gamma rays that reach us from across the sky are far fewer than regions of matter and antimatter would give off where the regions met and annihilated. The observable universe holds no regions of antimatter.

The Cooling in Order

Once the residue is in place, the rest of the story of ordinary matter is the cooling. Picture a ruler of temperature on which each mark is ten times colder than the last. Each event of the cooling is marked on the ruler. *The Coming of the Kinds* draws the ruler as a map, and the events on the map run in order of falling temperature.

Near 159.5 GeV, about nine millionths of a millionth of a second into our history, the Higgs field settles in its crossover, and the masses switch on. At 131.7 GeV, about thirteen millionths of a millionth of a second in, the sphaleron passages fall behind, and the share of the residue held as baryons is fixed at 0.3328. In the 2026 computation that gives 0.3328, Benso and colleagues took the moment at which the passages fall behind from a newer computation of the passages' rate, at about 133.5 GeV. The newer figure of 133.5 GeV lies within the spread of the lattice figure of 131.7, give or take 2.3. An MeV is a million electron volts. Near 156.5 MeV, between about twelve and twenty-four millionths of a second in, the strong force binds the quarks into protons and neutrons, in a second crossover. Just before the quarks were bound, there were only about a billion and four quarks for every billion antiquarks, and that small surplus of quarks is the whole of the baryon count. Then the antibaryons annihilate with the baryons, like vinegar poured on a little more baking soda than the vinegar can use up: the fizzing stops when the vinegar is gone, and the extra soda is left. The antibaryons are the vinegar, and the baryons are the soda. By 30 MeV, between about six and eight ten-thousandths of a second in, the antibaryons have fallen to a few in a million of the baryons, and the baryons left are the excess. The proton is stable, so the number of baryons left then is the number today.

Between about 3 and 1 MeV, in the first second, the neutrinos go free (Chapter 13): the neutrinos stop trading energy with the rest of the hot crowd and travel on. The neutrinos went free while they were still far

lighter than the temperature, so the neutrinos kept the full number that balance had given them, and nothing has removed them since. *The Measures of the Kinds* is the book of this collection that works out the measured values of the kinds of particle. *The Measures of the Kinds* finds the neutrinos' masses by two routes. *The Coming of the Kinds* carries both routes and chooses neither. On both routes every neutrino was more than ten million times lighter than the temperature when the neutrinos went free, so both routes give the same moment of going free and the same sea of neutrinos. The sea of neutrinos fills space today, about 336 neutrinos and antineutrinos in every cubic centimeter. Near 0.8 MeV, about one second in, neutrons and protons stop turning into each other in a two-way exchange, and the count of neutrons freezes at about one for every six protons. Between about three seconds and six minutes the electrons and positrons annihilate. The energy of the electrons and positrons warms the light, and the light ends about 1.40 times warmer than the neutrinos. One electron is left for each proton. At the electron's mass, about 0.5 MeV and some three seconds in, the pairs still hold nine tenths of their share of the energy. By about 0.19 MeV, between twenty and thirty-six seconds in, half of the pairs' share of the energy has passed to the light. By about 0.09 MeV, two and a half minutes in, the pairs have passed all but a tenth of their share of the energy to the light, and by about 0.06 MeV, some six minutes in, all but a hundredth. Near 0.1 MeV, about two minutes in, the *first nuclei* form: the cores of the lightest atoms. By then free neutrons have kept decaying into protons, and about one neutron is left for every seven protons. Nearly every remaining neutron ends in helium, and helium makes up about a quarter of the mass of ordinary matter. You can do the arithmetic yourself: take sixteen particles, two neutrons and fourteen protons. The two neutrons join two of the protons in one nucleus of helium, which holds four of the sixteen particles, a quarter of the mass. The amounts of the light nuclei are fixed by about three minutes in. Heavy hydrogen is the deuterium of Chapter 11, a nucleus of one proton and one neutron. The heavy hydrogen left from the first minutes measures the number of baryons per photon a second time. Picture a bakery that bakes the same batch of

loaves every morning: the loaves left on the shelf at closing tell how crowded the day was. The nuclei of heavy hydrogen are the loaves, and the baryons that meet and burn the nuclei of heavy hydrogen are the customers. The more baryons there are for each photon, the more often a nucleus of heavy hydrogen meets a partner while the first nuclei are being built, and the less heavy hydrogen is left. Astronomers read the amount of heavy hydrogen the way the baker reads the loaves left on the shelf, and the amount gives about 6.04 baryons per ten billion photons, give or take 0.12. The figure from heavy hydrogen agrees with the 6.13 of the oldest light, give or take 0.04, within the uncertainties of the two figures. The two measurements rest on different physics: the nuclear reactions of the first minutes, and the pattern in light set free hundreds of thousands of years later. Yet the two measurements reach one number, so the amount of the residue held as baryons is measured twice and found the same.

Last of all, the *first atoms* form: nuclei holding their electrons, near 2,973 degrees above absolute zero, about 370,000 years in. The forming of the first atoms is the fog clearing of Chapter 11: once the electrons are bound into atoms, nothing turns the light aside, and the light travels free. The light set free then reaches us now as the oldest light.

Each number of the cooling has its own standing, which says where the number comes from. Established physics is the standing of the temperatures and times of the cooling, of the one neutron in six and then in seven, of the quarter of the mass in helium, and of the light's ending 1.40 times warmer than the neutrinos. Two counts are numbers of the cooling, computed from the measured temperature of the light: the 411 photons and the 336 neutrinos in every cubic centimeter. Astronomers read the temperature at which the first atoms formed and the light went free from measurements of the oldest light. The residue and the cooling, together, are the source of the protons and neutrons, the helium and the electrons. The residue itself is the one amount the cooling did not make. So *The Coming of the Kinds* answers the question of ordinary matter in three parts. The kinds of particle are given by structure. The kinds were never made, and the kinds were the same from the first moment of our history. The pres-

ence of each kind was made by the cooling, kind by kind, in order of falling temperature. And the amount of matter is a residue the turn leaves, kept by every change after the turn. The kinds fix only the share of the residue held as baryons.

Two Leftovers of the Turn

The Weight of Possibility is the book of this collection that is the source for the dark matter. *The Weight of Possibility* puts the start of our history in three sentences: "The turn sends nearly all of its energy into radiation. Two residues remain: an excess of ordinary matter over its antimatter, and the dark matter. Both are conserved in number". The dark matter is made of the gyrations of Chapter 18, lasting vibrations of the six held dimensions, made at the turn and kept in number since. The excess of matter is the residue of this chapter, carried as a value of B minus L and kept by every change. As measured, the dark matter outweighs the ordinary matter about 5.36 times. The *matter budget* is the list of how much of each ingredient the universe holds. Only one ratio of the matter budget has stayed constant since the helium of the first minutes formed: the ratio of dark to ordinary matter. Since the helium formed, the dark and the ordinary matter have thinned out alike as space grows. So the ratio of dark to ordinary matter has not changed. A rule for what a measurement made now registers can fix the ratio, because the ratio has stayed constant, as Chapter 19 found.

We now know why there is ordinary matter and how much of it there is. Ordinary matter is a residue the turn leaves, kept by every change after the turn, and the kinds fix only the share of the residue held as baryons, about one third. The residue matters for this book's question, what the beginning left behind, because the residue is one more thing the turn left behind. The turn left the gyrations, the settling waves, and a line of shells of ordinary matter, out of which every star and every person is made.

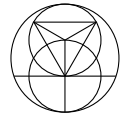
Key idea. The amount of ordinary matter is a residue from the earliest moments: an excess of matter over antimatter carried as B minus L, the count of baryons less the count of leptons. Every process among the kinds of particle keeps B minus L, so no process among the kinds makes the excess. The kinds fix only the share of the residue held as baryons: twenty-eight seventy-ninths while the sphaleron passages run, twelve thirty-sevenths deep in the settled Higgs state, and 0.3328 where the passages froze, near 131.7 GeV. The size of the residue comes from history, and the share of the residue held as baryons is set by the structure.

Summary

Astronomers measure about 6.13 baryons of ordinary matter for every ten billion photons. The universe holds no regions of antimatter. With no excess, the annihilation would have left about one baryon for every billion billion photons. The kinds supply the first two of Sakharov's three conditions. The third condition is not met at the electroweak transition, because the electroweak transition is a crossover and the sphaleron passages stay in balance through the steepest part of the crossover. When the kept count B minus L is zero, each kind and its antiparticle are equal in number in balance, and each kind leans a little toward the particle only when B minus L is not zero. Every process among the kinds keeps B minus L, so the excess present today requires a B minus L present from the earliest moment our history reaches: a residue the turn leaves, kept by every change after the turn, a number of history. Other accounts make the excess by three routes, and none of the three routes has a starting point among the kinds: there is no unifying group and no boson that joins color to the weak charge, no Majorana mass of either chirality, and no first-order electroweak transition. The kinds fix only the share of the residue held as baryons: 0.3328, very close to one third, where the passages froze. The measured amount of ordinary matter is the third of the residue held as

baryons. How the turn leaves the residue is a question for the account of the turn. The cooling then made the protons and neutrons, freed the neutrinos, froze the count of neutrons, built the first nuclei and the first atoms, and let the light go free.

I have now followed the beginning through each of the book's subjects: the six held again in one vacuum, the even start, the first ripples, the waves of the settling, the dark matter and the residue of matter. When we gather the subjects of this book together, what did the beginning leave behind, and what do the leftovers of the beginning mean for our place in the cycle?



Chapter 21

What the Beginning Left Behind

What did the beginning of our epoch leave behind, and what do the things the beginning left show about our place in the cycle of epochs? An epoch is one era of the world, pieced together from inside that era, in which stars, galaxies and black holes form. The cycle is the sequence of epochs: each epoch makes stars, the stars make black holes, the ends of the interiors of the black holes are one end, and the turn restores a large world, which is the next epoch. The turn is the passage at the end of the interiors of the black holes. During the turn, three directions of space open up while the six close in. The six are the six dimensions of possibility, held very small, a few Planck lengths across. In this last chapter we answer both questions in one walk through the whole story: first we tell the story of one epoch in seven scenes, in the order of the cycle that the earlier chapters of this book set out. Then we sort what the beginning left into five groups: what passed the turn, what the turn made, what the six left fixed, what the first light records, and what the coming measurements will find. The first light is the oldest light in the sky, the faint afterglow of the hot beginning, which still fills the sky.

In brief, the beginning left five things: one form, one vacuum, the dark matter, a faint hum and the matter we are made of. A comparison is the act of finding whether two things are the same or different, in some respect. The content of an epoch is what the comparisons of that epoch find: the matter and light that fill the arena of space and time. The one form is the form of the content (Chapter 1): which alternatives the content has

taken, that is, what has been selected. The form of the content is carried across the turn averaged and not erased. The first light records the form of the content as faint ripples in that light. A vacuum is a lowest, settled state of empty space, and the one vacuum is the same in every epoch. The dark matter is the unseen matter whose pull is measured in every galaxy. The dark matter is the gyrations, lasting vibrations of the six. The faint hum is a background of gravitational waves sent out as the six settled. Gravitational waves are ripples in the shape of space and time themselves. The turn leaves a small excess of matter over antimatter. The matter we are made of is the share of the excess held as protons and neutrons. The beginning carried no count of states, no length and no geometry across from the epoch before, so the new world begins its own count of states from its own start. And the beginning of our epoch is one beginning in a cycle with no first turn.

A Play Performed from One Script

Picture a play performed again and again from one script. Each performance builds its own stage and sets out its own props. When the performance ends, the crew strikes the stage and takes the stage apart, and nothing on the stage is kept. The next performance builds a new stage from the same script. No clock runs between the last scene of one performance and the first scene of the next. The only order among the performances is the order of their scenes. What comes back each time is the script.

The picture of the play shows one claim: the law returns from epoch to epoch, and no thing and no count of states returns. In the picture, the stage stands for the outside world of an epoch: the large world outside the black holes of the epoch, with the lengths, the clocks and the counts of states of that world. A count of states is a count of the states that a context cannot tell apart, reckoned relative to that context, and the counts of states are the entropies of Chapter 6. The script stands for the structure that the account derives. The structure is the numbers of dimensions, the shape of

the six and the held size of the six. The account is the account of the world that this book teaches, and that the other books of its collection set out in full. *At the Edge of Reality* is the earlier book whose story this book takes up. *At the Edge of Reality* states that the structure is the same in every epoch. The picture of the play leaves out one thing, and we put that thing back as we walk through one epoch below. In the world, something of the old epoch does reach the new one: the form of the content, carried across the turn as one form. The new outside world is the new stage of the picture. The new outside world registers the form of the content only as faint unevenness.

One Epoch in Seven Scenes

The Compactification is the book of this collection that sets out for physicists how an era of the world begins. *The Compactification* also sets out what that beginning leaves in the sky. *The Compactification* states the results of the account in the order of the cycle. In the words of *The Compactification*, "The order is the order of the cycle and of each epoch's own history; no time is measured across the turn." The seven scenes below are in the order of the cycle: the approach, the turn, the re-forming, the capture and the settling, the start of the reconstructed history, the history of the epoch, and the emptying. In the terms of the play, the first scene is the last act of the old performance. The second scene is the moment the crew strikes the stage. In the scenes after the second, the crew builds the new stage, and the new performance is played on the new stage.

The first scene is the approach. In the epoch before the turn, stars, galaxies and black holes form. The outside world of that epoch empties. The interior of a black hole is everything inside the horizon of the black hole. The horizon is the surface from inside which no signal can get out. The six are held small while certain conditions are met, and at the end of each interior those conditions fail. Chapter 2 calls a short stretch of the collapse, near the end of an interior, the arc. Once an interior has reached the arc,

the three large directions of space shrink and each of the six grows, slowly. As the time remaining before the end falls a billionfold, each of the three large directions shrinks a hundred thousand times. Over the same span, each of the six grows tenfold. The rates are the powers five ninths and minus one ninth of Chapter 2: the size of each large direction goes as the time remaining raised to the power five ninths. The size of each of the six goes as the time remaining raised to the power minus one ninth. Chapter 2 computed the rates from Einstein's equations in ten dimensions, at the end of the arc. *The Gravitational Passage* is the book of this collection that derives Einstein's equations of gravity from the structure of comparison. *The Gravitational Passage* places the end of every interior at the end of the arc. All interiors are one at their end, because no comparison shows a difference between one end and another. The interiors themselves stay distinct, and only the ends of the interiors are one.

The second scene is the turn. The turn is a limit, an end that is approached and never reached, and no comparison registers the turn as an event. At the turn, *The Gravitational Passage* builds no geometry and defines no field. The content carries its form through the turn.

The third scene is the re-forming. When the same arc is read away from the turn, three directions grow while each of the six shrinks. The rates are the rates of the first scene, read the other way. As the time since the turn grows a billionfold, each of the three large directions grows a hundred thousand times. Over the same span, each of the six shrinks tenfold. One process has two sides, the collapse at the end of an interior on one side and the opening of a new epoch on the other, and the process is approached as a limit from each of its two sides. The same arc describes both the collapse and the opening, read in opposite directions of time.

The fourth scene is the capture and the settling. The six are two shapes side by side: the complex projective plane, a shape of four dimensions, and the sphere, a shape of two dimensions. The floor of Chapter 4 is the least extent that the six can have, with a least motion that cannot be removed. The floor catches the six at their held size: a radius of the square root of six Planck lengths (about two and a half) for the complex projective plane,

and one Planck length for the sphere. The breathing vibrations of the six are the even swellings and shrinkings of the size of the six. The breathing vibrations die away within about a Planck time, which is the time light takes to cross a Planck length. The energy of the settling goes out as gravitational waves in the four open dimensions of space and time. The gyrations made at the turn remain.

The fifth scene is the start of the reconstructed history. The reconstructed history of an epoch is the one history that the present requires, pieced together from impressions arriving now. At the start of the reconstructed history, the content is hot and close to equilibrium, the settled state in which nothing further changes on average, at a temperature that the account does not fix. The starting temperature has an upper bound of about one thirtieth of the mass of a gyration, about two to five times ten to the seventeen GeV (a GeV is a billion electron volts, a unit of energy). The geometry is even, up to small departures from evenness. The Incommensuration Theorem is a structural fact about the nature of knowing, and the theorem requires the small departures, so the beginning of the epoch was even, but never perfectly even. When the small ripples are averaged away, the new epoch has flat space (Chapter 12).

The sixth scene is the history of the epoch. The hot gas cools through the electroweak crossover, the binding of the quarks into protons and neutrons, the making of the first nuclei and the release of the microwave background, which is the first light. The electroweak crossover is the smooth change in which the Higgs field settles at its value and the masses of the particles switch on. The small departures from evenness grow into structure. Stars and black holes form.

The seventh scene is the emptying, and the next turn. The outside world of the epoch empties. At the end of the interiors of the epoch's black holes, the conditions that hold the six fail, and the cycle continues. The cycle has no first iteration.

Observers inside an epoch reconstruct the history of their epoch, and the reconstructed history runs back toward the turn, where the observers find the beginning of the history. In Chapter 1 I identified the restoration of an outside world at the turn with the Big Bang of the new epoch, and the seven scenes above, from the approach to the emptying, are that identification followed through one whole epoch.

What Passed the Turn, and What Did Not

Go back to the picture of the play: what leaves one stage and reaches the next? In the world, the question is what passes the turn from one epoch to the next.

No count of states passes the turn, and so no entropy passes the turn, since an entropy is a count of states. Every count of an epoch was reckoned relative to the outside world of the epoch. By the turn, that outside world has emptied. *The Gravitational Passage* states that no count is carried across the turn, "for the matter or for the geometry, since each count is relative to the context that ends". So each epoch begins its own count, as we found in Chapter 7. In Chapter 10 we followed the count through the epoch from its beginning to its end.

No length passes the turn. The new outside world re-forms with no prior identity. So the new outside world inherits no record, no length and no mark that sets one part of the new world apart from another part. The new epoch begins flat, with no overall bend of space, as we found in Chapter 12, because the new outside world inherits no length and no mark that sets one part apart from another.

No geometry and no field pass the turn. At the turn *The Gravitational Passage* builds no geometry and "defines no field there, of any spin". A gyration is a vibration of the geometry of the six. A gravitational wave is a wave of the arena, the space and time of the large world. So no gyration of the old epoch passes into the new epoch as a gyration, and no gravitational wave of the old arena passes into the new epoch as a wave.

What passes the turn is the form of the content: which alternatives the content has taken. The content does not cease, and content needs a context, so the ten-dimensional geometry restores a context for the content: the new outside world. The context of a comparison is the respect in which the content is found: when a thermometer compares this morning's warmth with this evening's, the two times of day are the context. The content is carried from the collapse to the emergence of the new epoch. The form of the content is carried with the content. The distinctions of the old epoch survive the turn only as small departures from uniformity, the faint differences of density with which an epoch begins, which *The Gravitational Passage* describes as "averaged and not erased".

The word "information" has two senses: the form of the content, and a count of states relative to a context. In the first sense, information is the form of the content, so information crosses the turn as the form of the content does. Information as the form of the content is not destroyed in a black hole. A black hole formed from a star outlasts the outside world of its epoch, as we found in Chapter 9, because the outside world empties long before the black hole has evaporated. So whatever fell into the black hole is out of reach of the outside world for as long as the black hole lasts. The account claims neither that what fell in returns to the outside world nor that what fell in is lost to the outside world. The form of what fell in is carried with the content to the turn, where the form passes as one form and re-emerges as the departures from uniformity of the next epoch.

Three features of the beginning of our epoch follow from what did not pass the turn. The new world begins even, because the geometry of the new world inherits no unevenness from the old outside world. The new world begins nearly the same everywhere, because the new world reforms from one domain, the one end in which the interiors of the old epoch's black holes are one. The geometry of the new world also begins with a small count of states, because a new context starts its own count. Roger Penrose, a physicist, put a number on how improbable the beginning of our epoch was: one part in ten raised to the power ten to the power one hundred twenty-three (Chapter 7). Penrose's number depends

on a count that tells all the possible beginnings apart. No comparison makes that count, as we found in Chapter 10: the old outside world has emptied, and the new outside world registers none of the old distinctions and finds none of the other possible beginnings. Penrose proposed, as a separate principle, that the geometry of the universe began with no distortion of shape. In the account, Penrose's condition follows from the restoration of context, the re-forming of a large outside world at the turn, because the new outside world re-forms with nothing inherited from the old outside world (Chapter 7).

What the Turn Made, and What It Left

Three things in our epoch come from the turn and the moments just after the turn, and from nothing later: the dark matter, the hum of the settling and the residue of matter.

The first thing that comes from the turn is the dark matter. In Chapter 18 I identified the dark matter as a conserved excitation of the six held dimensions, made at the turn. An excitation is a raised state of vibration. The dark matter is a vibration of context, and not an item of content. A gyration is a vibration of the held space of the six, as a phonon (one step of the vibration of a crystal) is a vibration of the crystal, and a gyration is not an extra thing inside the six. The pieces of the dark matter are the gyrations. Each gyration weighs between about six tenths and one and four tenths Planck masses, and both values are computed (Chapter 18). A Planck mass is about twenty-two millionths of a gram. Gyrations show themselves by their weight alone, and they do not decay. Gyrations are cold: slow, with no motion from heat that could matter. The mass of the gyrations is about 5.36 times the mass of ordinary matter. The exact factor is six, less two divided by pi: six minus the fraction two over pi. An identity of the present measurement fixes the ratio of the two masses, as we found in Chapter 19. An identity of the present measurement is a rule that states what a measurement made now, from inside the epoch, registers.

Each epoch holds only the gyrations that its own turn makes. The law that protects the gyrations is the same in every epoch: the six re-form with the same shape at every turn, so every epoch has the same odd levels and the same swap of opposite points (Chapter 18). The odd levels are the vibrations of the six that flip their sign when each point of the sphere is swapped with the point opposite it.

The second thing that comes from the turn is the hum of the settling. The new epoch itself made the hum as the floor caught the six. At the catch, the energy of the motion of the six went into breathing vibrations, and the breathing vibrations turned at once into gravitational waves of the four large dimensions (Chapter 16). The count of light species is the radiation other than light, counted in units of the energy one kind of neutrino carries (Chapter 16). The measured count of light species limits the waves of the settling to less than about three percent of the radiation of the content, at the time when about a hundred particle states shared the heat. *The Compactification* computes the pitch of the hum from three premises (Chapter 16): the energy of each wave at its birth, the time of its birth, and the stretching of space since. The computation shows that the hum sounds today at a pitch above about five times ten to the eleven hertz, which is more than five hundred billion rises and falls each second. The account predicts a ceiling on the energy the hum carries, and no particular energy below the ceiling.

The third thing that comes from the turn is a residue: the small excess of matter over antimatter, carried as a value of B minus L, the count of baryons less the count of leptons, each count taken as particles less antiparticles. Baryons are the protons, the neutrons and their relatives. Leptons are the electron, its heavier copies and the neutrinos. The excess of matter is a residue from the earliest moments that our history goes back to: the turn leaves the residue, and the residue stays the same through every change after the turn. No process among the kinds of particle makes the residue. The kinds of particle fix only the share of the residue held as baryons. The share is about one third, set when the sphaleron passages froze (came to a stop). The sphaleron passages are passing states of the

field of the weak force. Each passage changes the baryons and the leptons together and keeps B minus L the same. The third of the residue held as baryons is the measured 6.13 baryons for every ten billion particles of light (Chapter 20). How the turn leaves the residue is a question for the account of the turn.

The amount of gyrations sets conditions on the turn, because the gyrations are made only at the turn. *The Weight of Possibility* is the book of this collection that identifies the dark matter with a lasting vibration of the six. *The Weight of Possibility* states three of the conditions (Chapter 4). The first condition is on the temperature of the hot gas of the new epoch. If the hot gas formed at once, the gas was never hotter than about one thirtieth of a gyration's mass. A stage that behaves as matter is a stage in which energy is held in heavy, slow things that later decay. If the energy reached the gas first through a stage that behaved as matter, then at the end of that stage the gas was cooler than about one hundredth of a gyration's mass. The second condition is that the waves of the settling stayed below about three percent of the radiation of the content. The third condition is that the six re-formed gently, smooth over about ten swings of the lightest odd level, so the six made about a billion billion billion times fewer gyrations than a sudden re-forming would make. The three conditions are conditions on the turn, and not results of the account: the account requires the three conditions, given the amounts we measure, and the account does not derive that the turn satisfies the three conditions.

What the Six Left Fixed

The six re-form at every turn with the same shape, the complex projective plane times the sphere. The floor catches the six at the same held size every time. So the six leave one vacuum, the same at every point of the large world and the same in every epoch (Chapter 5). The hold is the state in which the six are held small, and the case for the hold is the argument of Chapter 4. The argument of Chapter 4 starts from the modal boundary.

The modal boundary is the Planck scale, where each of the six has the Planck curvature. String theory threads a field called a flux through the loops of its six small dimensions. In the account, gravity is built from the geometry alone, so the six carry no flux (Chapter 5). Because the six carry no flux, the account has no landscape of vacua: no range of alternatives among which the world had to be chosen. The held size is the size of the six taken as the setting of the large world. The kinds of particle are read on the six taken another way, as a space of states. The space of states has no size (Part I).

The one vacuum fixes the energy of empty space once, through the present measurement. A measurement made inside the vacuum takes in a share of the energy of the vacuum: three parts out of the count of the particle horizon, the edge of everything whose light has reached us since the expansion began. Each of the three parts is an equal share of the total energy, once the part that the curvature of the six draws away has been trimmed from the total (the trim of Chapter 8). The share gives a value of the energy of empty space, and the value follows from the structure once the identifications of Chapter 8 are taken, with the measured size of our particle horizon. The value is 2.56 times ten to the minus forty-seven GeV to the fourth power, an energy in each unit of volume written in the particle physicists' unit (Chapter 8), against the observed 2.52 times ten to the minus forty-seven. Friedmann's equation ties the rate of expansion to what space holds (Chapter 8). When the value of the energy of empty space is joined with Friedmann's equation, the value sets the matter share at 31.1 percent, against a measured 31.5 percent. The matter share is the part of the universe's energy held as matter. The cosmological constant is the energy of empty space, which drives the accelerating expansion of the universe. The cosmological constant has one value at every era of the reconstructed history, so the energy of empty space stays exactly constant.

You may ask whether the energy of empty space stops the cycle. The cosmological constant drives the accelerating expansion of the outside world. Could the cosmological constant keep the black holes from ending the epoch? The cosmological constant does not stop the cycle. The *outside*

description is the description of the large world outside the black holes. The cosmological constant belongs to the outside description. The cosmological constant is a structural property of the relation between an observer and that observer's horizon. Picture a speed limit, which belongs to a road: past the end of the road, the speed limit does not hold you back, and the speed limit is nothing at all, because there is no road for the speed limit to belong to. In the picture, the road stands for the outside world, and the speed limit stands for the cosmological constant. The *common interior* is the shared end in which the interiors of an epoch's black holes are one. Inside the common interior, the outside world has dissolved. Inside the common interior, the cosmological constant is nothing at all, because the conditions for defining the cosmological constant are no longer met, so the cosmological constant does not act there as a force resisting the collapse. The faster expansion of the outside world stops none of four things: black holes still form at every scale, bound black holes still merge, the ends of all interiors are still one, and the conditions that hold the six still fail, since those conditions depend on the outside world and not on the cosmological constant.

What the First Light Records

The first ripples are the faint departures from uniformity with which our epoch began and from which the galaxies of our epoch grew. The first ripples are the form of the content carried through the turn: the distinctions of the old epoch, averaged and not erased (Chapter 11). *At the Edge of Reality* states plainly that the faint pattern in the microwave sky is the averaged form of what the black holes of the epoch before took in.

Four marks of the pattern follow from two premises (Chapter 12): the first premise is my identification, that the faint pattern is the averaged form of what the old black holes took in. The second premise is that the new context re-forms with no prior identity. First, the pattern is correlated across the whole sky. Second, every ingredient of the universe ripples to-

gether. Third, the ripples are ripples of density, and waves of space arise from the ripples only at second order, from ripples acting on ripples, at far below one millionth of the power of the ripples. Fourth, space as a whole is flat. Measurement agrees with every mark so far.

Two numbers of the ripples come from measurement, and the account sets a question beside each number. The strength of the ripples is a power of about two parts in a billion. The question is what count of states sets the strength. The tilt of the ripples is an index of 0.9649, from large sizes to small, and the question is what length sets the tilt, a length lying between the length of the turn and the de Sitter radius. The de Sitter radius is the edge that a universe holding nothing but the energy of empty space would have. Inflation is the proposal that an early phase of ever faster expansion stretched a small region to far beyond everything we can now see. No inflation is needed to make the ripples, and a long inflation is excluded (Chapter 13).

What the Coming Measurements Will Find

The account makes definite predictions for the measurements of the coming decades, and Chapters 14 and 17 set out each prediction. Below, the predictions are gathered in one list, each with what the account says will be found.

The matter share will be found at 31.1 percent. The rate of expansion follows from the matter share and the amount of matter that measurements of the first light show, and the rate will be found at sixty-seven and eight tenths kilometers per second for every megaparsec. A megaparsec is a unit of distance of about three and a quarter million light years. The matter share and the rate of expansion are one test, and the two numbers are the predictions that set the account apart from the standard picture of cosmology, which leaves both numbers free. The energy of empty space will be found exactly constant, with a ratio of pressure to energy of exactly

minus one. Space will be found flat, curved only by its ripples, by about one part in a hundred thousand. Every ingredient will be found rippling together, with no separate pattern of the dark matter's own.

The Simons Observatory is a set of telescopes in the high desert of northern Chile, and LiteBIRD is a satellite. The first waves are gravitational waves present from the beginning of an epoch. The Simons Observatory and LiteBIRD will measure the ratio of the power of the first waves to the power of the first ripples, down to a ratio of about a thousandth. The account says the ratio is far below one millionth, so the two instruments will find no first waves. The Simons Observatory will also measure the count of light species to within about five to seven hundredths, and the account requires the hum of the settling to stay within the measured bound. LISA is a planned detector in space that will listen for slow gravitational waves by measuring, with laser light, the changing distances between spacecraft far apart. LISA will register backgrounds of waves from the sky, from pairs of white dwarfs and from pairs of black holes, and no background from a change of state of the early universe.

The dark matter will be found by its weight alone, with no decay and no cut-off in its structure on small scales. No short loop will be found anywhere in space: no path that goes all the way around the universe and back to its start and is shorter than the width of the horizon (Chapter 19). Two small masses will become entangled through gravity alone when each mass is held in two places at once. For the design of the physicist Sougato Bose and colleagues, the entangling phase will be 0.314 radians (a radian is a measure of angle, and a full turn is a little over six and a quarter radians). The entanglement comes from the step we took in Chapter 9: each alternative of the matter carries its own geometry. The entanglement also agrees with the description of small ripples by quanta. The quanta of that description belong to the description and never to the arena. So finding the entanglement would not show gravitation quantized, as we found in Chapter 15 for the single click of a detector. Gravitation is not quantized.

In Chapter 17 we estimated when the measurements above will arrive, from the instruments now being built or proposed. Within about twenty years, the Simons Observatory and LiteBIRD measure the ratio of first waves to first ripples down to about a thousandth. In the same twenty years, the Simons Observatory measures the count of light species to within five to seven hundredths. LISA registers the waves of its band. Appendix B sets out each of the measurements above, the instruments now coming, and the rule that states when a result confirms the account.

A Cycle with No First Turn

Suppose the collapse of black holes and the origin of the cosmos are one transition, approached from its two sides. Then the history of the cosmos is a cycle, and not a single arc from a Big Bang to a cold, empty end. *The Gravitational Passage* argues that the collapse and the origin are one transition, and so that the history of the cosmos is a cycle. Each epoch makes structure, the structure makes black holes, and the interiors of the black holes share one end. At the shared end the six are released from their hold. A new epoch re-forms: three directions open while the six close again. When observers in an epoch reconstruct the history of their epoch, the history runs back to a beginning, the Big Bang of the epoch. The Big Bang of each epoch is the turn approached by the collapse of the interiors of the epoch before.

The turns of the epochs are distinct. Every comparison made inside an epoch shows the turn from which the epoch emerged, as the beginning that the reconstructed history of the epoch runs back to. No comparison made inside an epoch shows anything of the turn at which the interiors of the epoch end, since that turn lies in the future of the epoch. No act of comparison spans the turn, so no finding made in one epoch is real in the other epoch, and no one ever sets a record of one epoch beside a record of the other. Two things join the epochs: the sameness of their structure, and the form of the content. The form of the content crosses the turn as struc-

ture, with no act crossing. So the epochs follow one another in a single sequence. Picture the cars of one train, each car coupled to the car before it, and not the branches of a tree: the cars stand for the epochs, one after another, while the branches of a tree would stand for many new universes growing from one epoch. All the interiors of an epoch are one at their end, because a difference that no comparison can register is no difference (Chapter 1). So in the account all the interiors of an epoch together give one new epoch. Earlier proposals gave one new universe for each black hole, and the account does not.

Two tendencies take turns in the cycle. The first tendency is gravity, which gathers the content toward collapse: the smooth character of the arena tends toward concentration, curvature and collapse. The second tendency is the restoration of context, the re-forming of a large outside world when the six are released from their hold (Chapter 1). When a large outside world re-forms, the content is spread out again and has a world, and the restoration of context occurs when the conditions that hold the six small dissolve. Picture day and night on the one spinning Earth. Day and night each follow the other, neither is more basic, and both come from the one spinning of the Earth. Like day and night, neither tendency is more basic than the other. In the picture, the spinning of the Earth stands for one constraint, and both tendencies follow from that constraint. The constraint is the one that the Incommensuration Theorem places on the relation between the two characters of the world, the smooth and the stepwise, which trade places at the Planck scale as the one that rules (Chapter 1).

The cycle has no first iteration, and nothing piles up from one epoch to the next. In 1934 the physicist Richard Tolman objected to histories made of cycles: the entropy, the count of states of Chapter 6, would grow from cycle to cycle, so the cycles would need a first cycle. In the account, each epoch begins its own count, so no count piles up and no first cycle is needed. Gyrations do not pile up either, since no gyration passes the turn. Newton's constant, the measured strength of gravity, sets only the unit of length. If Newton's constant changed and every quantity measured in

Planck units stayed the same, the change would be a change of unit. No comparison registers a change of unit. No comparison across the turn is ever made, and setting the value of Newton's constant in one epoch beside its value in the next epoch would be a comparison across the turn, so the account assigns Newton's constant no value in another epoch. A further question is whether the measured proportions of matter repeat from one epoch to the next. The question is about the make-up of the content, which lies outside the scope of *The Gravitational Passage*. The question is a topic for another day. In the terms of the play, the script repeats. Every epoch has the same numbers of dimensions, the same shape of the six, the same held size and the same vacuum.

Where You Stand

You are inside an epoch whose beginning is the common end of the interiors of the black holes of the epoch before. The faint pattern in the microwave sky is the averaged form of what the black holes of the epoch before took in. Take the black hole at the center of our Galaxy, or any other black hole of our epoch: anything that falls into the black hole approaches, on its own clock, the one end from which the next epoch begins.

At the Edge of Reality named three edges of reality, and in this book I have studied each of the three edges. The first edge is the cosmological event horizon, beyond which nothing can ever be compared with you. The second edge is the Planck scale, where the smooth description of space and time reaches the edge of the range in which that description applies. The six are held at the Planck scale, and the dark matter rings at the Planck scale as a vibration of the six. The third edge is the turn, where the comparisons of one epoch end and the comparisons of the next epoch begin. No comparison registers the turn. Three things come from the turn: the ripples, the gyrations and the residue of matter. In this book, everything real is everything given in an actual act of comparison made

from inside the world. Everything real lies within all three edges. Your reading of this sentence is an actual act of comparison made from inside the world.

We now know what the beginning left behind, which answers the question this book began with. Each thing the beginning left can be looked for. The form of the content can be looked for in the ripples of the oldest light, the one vacuum in the measured energy of empty space, the dark matter by its weight, and the matter we are made of in its measured amount. The settling hum can be looked for through the ceiling that the measured count of light species places on the hum. All five things show the same story: one form carried across, one law returned, and a new world that begins its own count from its own start.

> **Key idea.** An epoch runs in seven scenes: the approach, the turn, the re-forming, the capture and settling, the start of the reconstructed history, the history, and the emptying. No count, no length, no geometry and no field passes the turn, while the form of the content passes, and the first light records the form of the content as the ripples. The turn makes the gyrations of the dark matter and leaves the residue of matter, the settling of the six leaves a faint hum, and the six leave one vacuum, the same in every epoch. The cycle has no first turn, and what returns is the law.

Summary

In this book I took up the story where *At the Edge of Reality* left the story, at the turn, and I asked what the beginning left behind. In Part I we found the six held again: three directions open while six close along one arc, and the floor catches the six at radii of the square root of six Planck lengths and one Planck length. The six have one vacuum, with no landscape of vacua: no range of alternatives among which the world had to be chosen. In Part II we found why the beginning was orderly. An entropy is a count of states, and every count of states belongs to the context in which the

count is made. No count passes the turn. The counts of an epoch run from a high count of the content and a small count of the geometry to the largest count, at the particle horizon. The particle horizon fixes the energy of empty space once. In Part III we found what the first light records. The beginning was even, but never perfectly even, because the Incommensuration Theorem requires small departures from evenness, and the small departures carry the form of the content, averaged and not erased. Four marks of the pattern agree with measurement so far. Two numbers are taken from measurement, each with its question, and no inflation is needed. In Part IV we found the waves that the beginning left. The settling of the six left a hum, whose whole energy the measured count of light species holds below a ceiling, and ripples acting on ripples made a faint hum of waves. The beginning left no waves of space present from the start, in the largest part of the effect, and no hum from a change of state. In Part V we found what the turn made and what the turn left. The turn made the gyrations of the dark matter, six, less two divided by pi, times the mass of the ordinary matter. The turn left the residue of matter, and the kinds of particle fix the share of the residue held as baryons. The epochs follow one another in a single sequence, each epoch with its own count. The cycle has no first turn, and what returns from epoch to epoch is the law. One test sets the account apart: the matter share will be found at 31.1 percent, and with the matter share the rate of expansion at sixty-seven and eight tenths kilometers per second for every megaparsec. And you stand inside all three edges of reality: the cosmological event horizon, the Planck scale and the turn. Every actual act of comparison is inside all three edges, and your reading of this book is one of those acts.

The beginning left behind one form, one vacuum, the dark matter, a faint hum and the matter we are made of. The beginning carried no count, no length and no geometry across. So our world begins its own count of states from its own start. And the beginning of our world is one beginning in a cycle with no first turn.

Appendix A

For the Physicist

This appendix gives the formulas and technical details that the chapters of this book say in words, for a reader who wants the formulas. Every chapter states its results in full without this appendix, so no step of the chapters depends on reading the formulas. The items are arranged by chapter: each item is placed under the chapter of this book to which the item belongs. Each item gives three things. First comes the formula or technical statement, exactly as printed by its authority, the source book from which the item is taken. Then comes the place where the authority prints the statement, with the standing of the statement. Last comes the formula or statement read aloud in plain words. The standing is the one that the authority itself gives: a theorem; established physics; an identification, which is a step from the structure of comparison to physics that I make in the first person; argued; computed; or measured. The symbols are the ones the authorities use. Throughout this appendix, M_{Pl} is the full Planck mass, 1.221×10^{19} GeV, and never the reduced Planck mass. The symbol l_{Pl} is the Planck length formed from the measured value of Newton's constant G_N. The Planck length is $l_{Pl} = \hbar/(M_{Pl}c)$, about 1.616×10^{-35} m. The equations use natural units, $\hbar = c = k_B = 1$, in which the reduced Planck constant, the speed of light and Boltzmann's constant are each counted as one, unless a unit is written (*The Compactification*, Preface, Section IV). When an item belongs to one chapter and is used again in a later chapter, the item is given once, under the earlier chapter, and the entry for the later chapter names the item.

Chapter 1, Where the Last Book Left Us

The action of the metric alone

$$S_{10} = \int d^{10}x \, \sqrt{(-g_{10})} \, R_{10}$$

Source: *The Compactification*, Chapter 1, Section IV, equation (1.2), after *The Gravitational Passage*, Chapter 11, Section I, equation (11.1). The authority adds: "Lovelock's theorem in ten dimensions allows further terms of higher order in the curvature, whose coefficients *The Gravitational Passage* does not fix (Chapter 11, Section I); the Einstein–Hilbert term is the leading one, the only one linear in the curvature". Standing: argued; the coefficients of the higher-order terms are not fixed.

The formula above is the action of the ten dimensions. The action is the one quantity from which the rule that governs the geometry of all ten dimensions follows. The action is built from the geometry and nothing else. The action adds up the curvature at every point of the ten dimensions of space and time, and at each point the action weights the curvature by the volume of a small piece of space and time at that point. In the formula, g_{10} is the determinant of the ten-dimensional metric, the measuring rule of the geometry. The determinant turns coordinates into true volume. R_{10} is the scalar curvature: the single number that sums up how curved the geometry is at a point. A theorem of David Lovelock allows further terms with higher powers of the curvature. The account does not fix how large the further terms are. The term written in the formula above is the leading term, and the only term in which the curvature appears to the first power.

The two instances of the internal space

"On the content instance, $\mathbb{C}P^2$ is the space of pure states of the summand $M_3(\mathbb{C})$ and S^2 is the ray space of the summand $\mathbb{H}$ (*The Quantum Passage*, Chapter 5, Section VIII); the kinds and the values of the particles are read there, and nothing on that instance has a length".

Source: *The Compactification*, Chapter 1, Section II. Standing: definition of the two instances.

The six small dimensions have one shape: the complex projective plane (written $\mathbb{C}P^2$, four of the six dimensions) times the sphere (written S^2, the other two dimensions). The shape of the six is taken in two ways, which this book calls the two instances of the internal space. On the content instance, the complex projective plane is the set of all the pure states of a system with three alternatives, the system that is described by three-by-three complex matrices. On the content instance, the sphere is the set of the states of a system with two alternatives: the system of the quaternions, written $\mathbb{H}$. The kinds of particle are read on the content instance, and nothing on the content instance has a size. On the context instance, the shape of the six has radii and a curvature, and on the context instance the shape holds the content of an epoch, which is one era of the world.

How the ten differ from the ten of superstring theory

"its dimensions are of three kinds; its six are not spatial and are never traversed; the signature of one against three holds within the four of time and space; and nothing in it curls the six or gives them a size".

Source: *The Compactification*, Chapter 1, Section II, quoting *Measurement; The Steps from Comparison to Physics*, Chapter 11, Section XII. Standing: a statement of scope (what the count of ten carries); the shape and the size of the six are given under Chapter 3.

The ten dimensions of this account differ from the ten dimensions of superstring theory. Superstring theory is the theory in which the smallest pieces of matter are tiny vibrating strings. In superstring theory the extra six dimensions are directions of space, curled up small. In this account the ten dimensions come in three kinds: one of time, three of space and six of possibility. The six are not directions of space, and nothing ever travels along the six. The four large dimensions hold the split of one direction of time against three directions of space. And the count of ten neither curls the six nor gives the six a size. The difference about curling concerns the count alone: the count of ten gives the six no shape and no size. *The Gravitational Passage* derives the shape and the size of the six, on the context instance. The shape and the size are given under Chapter 3.

No field threaded through the six

"With a field of its own, the action would no longer be built from the metric alone; the field's classical energy would enter the external value of Chapter 13, Section II, beside the bare curvature term, and so change the predicted matter fraction; and its coupling would be a second constant beside G_N. This book takes no such field".

Source: *The Gravitational Passage*, Chapter 12, Questions, as *The Compactification*, Chapter 2, Section VI, quotes it. Standing: argued (the action of the metric alone; a further field refused, with its reason).

The account threads no extra field through the six, and *The Gravitational Passage* gives three reasons for taking no extra field. First, with an extra field the law of the ten-dimensional geometry would no longer be built from the geometry alone. Second, the energy of the extra field would change the predicted share of matter in the universe. Third, the extra field would couple to matter with a strength of its own. The strength of the extra field would be a second constant beside Newton's constant. The whole numbers that the six do carry are given under Chapter 5, with the phase of the hypercharge force.

What is taken as given, and what is obtained

"The book takes a metric geometry as given, as general relativity does; what it obtains from comparison is the number of dimensions, the signature and the product form of that geometry, and the form of the field equations then follows from Lovelock's theorem (Chapter 12, Section VII)".

Source: *The Gravitational Passage*, Preface. Standing: a statement of scope.

The account uses, as given, a geometry of space and time with lengths and angles, as Einstein's theory does. The account does not derive the geometry with lengths and angles. In the account, three things follow from comparison: how many dimensions the geometry has, which of the dimensions is time, and that the geometry splits into a large part and a small part. Given the three things that follow from comparison, Lovelock's theorem fixes the form of the equations of the geometry. The account also uses the measured value of Newton's constant. And the account uses one rule as given: the count of states of a horizon grows with the area of the horizon (*The Gravitational Companion*, What the Passage Obtains That Others Assume).

The standing of each link

The Gravitational Passage opens with a table of the chain of its argument. "The table gives each link, the chapter that gives it, the links it rests on, and its standing: a theorem; a standard technique; an identification (a reading from comparison to physics, not a theorem); argued (from Part I and the stated identifications, each cited); computed; or measured".

Link	Chapter	Rests on links	Standing
1. Comparison as the ground of science; six intrinsics; four compound concepts	1, 2, 5	none	the base: what any comparison involves, and any denial is itself a comparison
2. The Incommensuration Theorem; the three axioms and the modalities	3, 4	1	theorem (enumeration); the axioms found in the structure of comparison; the readings in physics (Chapters 3 and 4) identifications
3. Content and context: gravity as context, the quantized fields as content	6, App. A	2	identification; the markers argued
4. The six metrics and the grid	7	2, 3	identification
5. The product $M^4 \times K$	8	2, 3, 4	identification; the separation of the factors argued from the theorem

Link	Chapter	Rests on links	Standing
6. The internal space $\mathbb{C}P^2 \times S^2$	9	4, 5, and the identifications below the table	classification theorem, given the identifications
7. Four external dimensions; Lorentzian signature; a finite signal speed	10, App. C	2, 4, 18	argued, with identifications
8. The radii $R_O = \sqrt{6}\, l_{Pl}$, $R_T = l_{Pl}$	9, 17	6, 18	ratio and scale by theorem, given one identification: the Ricci curvature of every internal direction is the Planck curvature (each direction at the modal boundary), which makes the internal metric an Einstein metric
9. The ten-dimensional action and its reduction	11	5, 6	standard technique
10. The internal space held; the masses of the moduli	11, App. D	6, 8, 18	the hold argued (each direction at the modal boundary, the same throughout the exterior); the masses computed at first order for one mechanism
11. The Einstein equations in four dimensions	12	1, 4, 9, 10	Lovelock's theorem, with covariance identified with the omniscient modality; the source of the curvature identified as the count of findings, which energy and momentum measure
12. One geometry for each alternative of the matter	12	3	identification

Link	Chapter	Rests on links	Standing
13. A finding made in the first person; no common surface of simultaneity	12	12	argued, with the theorem that local results do not depend on the surface
14. Λ from the information deficit	13	6, 8, 9, 18, 22	identification; computed
15. $I^2\Omega_\Lambda = 8 - 3/\pi$; $\Omega_m = 0.311$	13	14, 22	computed; compared with measurement
16. $w = -1$	13	14	argued
17. Equivalence, geodesics, the Newtonian limit	14 to 16	11	standard results; the equivalence extended to the inertial forces, with a test that tells the three apart
18. G_N and the modal boundary at the Planck scale	17	2, 3	G_N measured; the boundary argued
19. The interior passage and the cycle	18, 19, App. B	6, 8, 14, 18	argued; the items this book does not fix are listed in the Questions of Chapters 18 and 19
20. The turn as a limit; the Big Bang as its projection	19	19	identification
21. Scalar, adiabatic primordial departures; a spatially flat epoch	18	19	argued; amplitude and tilt not fixed in this book

Link	Chapter	Rests on links	Standing
22. The flatness used by the identity	18	21	argued

Source: *The Gravitational Passage*, The Chain at a Glance. Here K is the six-dimensional internal factor, $\mathbb{C}P^2 \times S^2$.

The table gives the standing of every step of the account of gravity that this book uses. The base of the chain is comparison itself: the act of finding whether two things are the same or different, in some respect. The Incommensuration Theorem is a theorem, proved by going through every case. The Incommensuration Theorem is a structural fact about the nature of knowing. The Incommensuration Theorem says that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. The steps from comparison to physics are identifications. I make each identification in the first person, at the place where the identification is made. Once the identifications are made, the shape of the six small dimensions follows by a theorem of classification. The sizes of the six follow by a theorem from one more identification. The further identification is that every direction of the six is curved as strongly as the Planck curvature, which is the bending of the surface of a ball whose radius is one Planck length. The energy of empty space, the cosmological constant, comes from an identification and a computation. The matter share is computed and compared with measurement. The energy of empty space stays the same over time ($w = -1$ in the table), and the constancy is argued. Newton's constant is the one measured constant on the side of gravity.

The three constants and their roles

$$M_{Pl} = \sqrt{(\hbar c / G_N)}$$

"The three constants have different roles. Two of them are limits of the domain of comparison, and the third is a coupling within it. The speed of light, c, is the macroscopic limit of the domain. [...] Planck's constant, ħ, is the microscopic limit of the domain. [...] The gravitational constant, G_N, is the coupling of the context phenomenon: it measures how strongly the content curves the context". And: "This book makes no claim on the value of any such ratio. It takes G_N from measurement, as its single measured constant, and expresses every other dimensionful quantity in the unit G_N fixes".

Source: *The Gravitational Passage*, Chapter 17, Section I, equation (17.1). Standing: identification (the roles of c and ħ); measured (the magnitude of G_N).

The Planck mass is the square root of a ratio: the reduced Planck constant times the speed of light, divided by Newton's constant. The three constants in the formula have different roles. The speed of light is the limit of the domain of comparison on the large side. Planck's constant is the limit of the domain of comparison on the small side. Newton's constant has a different role: Newton's constant measures how strongly matter curves space and time, and the value of Newton's constant is a fact about our world, taken from measurement. A measurement finds ratios, such as the mass of a particle divided by the Planck mass, but the account makes no claim on the values of those ratios and does not fix those ratios.

The full Planck mass and the Planck units

$$l_{Pl} = \hbar/(M_{Pl}\, c), \quad t_{Pl} = l_{Pl}/c$$

"The boundary is a scale formed from those three constants alone, so it is marked by the full Planck mass, not by the reduced mass $M_{Pl}/\sqrt{(8\pi)}$: the 8π belongs to the coupling of the field equations, $8\pi G_N$ (Chapter 12, Section I)".

Source: *The Gravitational Passage*, Chapter 17, Section III; *The Compactification*, Chapter 1, Section III, which gives the value 1.221×10^{19} GeV. Standing: argued (the role, and the full Planck mass); measured (the location, through G_N).

The Planck length is the reduced Planck constant divided by the product of the Planck mass and the speed of light, and the Planck time is the Planck length divided by the speed of light. The Planck mass used throughout is the full Planck mass, about 1.221 times ten to the nineteen billion electron volts. Physicists often use a smaller "reduced" Planck mass, the full Planck mass divided by the square root of eight pi. The account does not use the reduced Planck mass, because the eight pi belongs to the strength with which matter enters Einstein's equations, and not to the scale at which the description of gravity and the description of the quantum fields each reach the edge of their range.

What happens at the boundary

"The boundary is not a wall that matter cannot cross, and it is not a regime of a third kind of physics". "The corrections to the Einstein action are of relative size the curvature times l_{Pl}^2, so they are negligible far below the Planck curvature and of order one at it (Chapter 12, Section VII)".

Source: *The Gravitational Passage*, Chapter 17, Sections II and IV. Standing: argued.

At the Planck scale, the description of gravity is as strong as the description of the quantum fields. So neither description dominates at the Planck scale, and no single description includes both. The modal boundary at the Planck scale is not a wall that matter cannot cross, and no third kind of physics takes over there. Compared with the leading term, the corrections to Einstein's equations grow as the curvature times the square of the Planck length. So the corrections are tiny where the curvature is small, and as large as the leading term where the curvature equals the Planck curvature, one over the square of the Planck length, because there the curvature times the square of the Planck length is one.

Rotating and charged holes

"For rotating and charged holes, the inner horizon of the exact solutions is unstable to the infall that forms real holes, and the generic end is expected to be of the same kind: a limit approached by the oscillating directions that Appendix B describes".
Source: *The Gravitational Passage*, Chapter 18, Section I. Standing: established physics; argued.

For spinning and charged black holes, the exact solutions of Einstein's equations have a second horizon inside the first, called the inner horizon. Matter falling into a real black hole makes the inner horizon unstable. So real spinning and charged holes are expected to end in the same way as holes that neither spin nor carry charge: at a limit approached by directions that stretch and squeeze in turn.

The principle of identity, stated exactly

"Stated exactly, the principle is this. Two things are one when, for every possible comparison and for every thing whatever (the two themselves included), the comparison of the first with that thing gives the same verdict as the comparison of the second with that thing; a comparison that cannot take one of them as content gives, for that one, the verdict that it cannot. So stated, the principle defines an equivalence". Its scope: "It counts every

possible comparison: every comparison that any comparer could make, in any domain, at any place and at any time. [...] I take a comparison to register nothing that it only predicts, since a prediction is a possibility and not a finding". The weaker reading is excluded: "The weaker reading, on which two things are one when no comparison of the one with the other finds them different, defines no equivalence". The principle "counts comparisons as structure. Every comparison that could be made exists, as a possibility, and the principle does not need any of them to be real".
Source: *The Gravitational Passage*, Chapter 18, Section II. Standing: principle of comparison, stated with its scope.

The principle of identity says that two things are one thing when, for every possible comparison and every thing whatever, the comparison of the first thing with that thing has the same result as the comparison of the second thing with that thing. The possible comparisons are all the comparisons that any comparer could make, at any place and at any time, and the principle counts those comparisons as possibilities, none of which needs to be real. So each thing has a pattern of results, and two things are one when their patterns match. Matching of patterns is an equivalence, a relation like equality, so a chain of matches joins its two ends: if a first thing matches a second, and the second matches a third, then the first matches the third. A weaker rule would call two things one whenever a comparer, comparing the two things directly, finds no difference. *The Gravitational Passage* excludes the weaker rule, because the weaker rule defines no equivalence. Suppose a comparer finds no difference between a first thing and a second thing, and none between the first thing and a third thing, but finds the second and the third different. Under the weaker rule the first thing would count as equal to the second and to the third, although the second and the third differ. And under the weaker rule a graded series could run, in steps too small to see, from one end to a different end.

Why the ends of the interiors are one

"The identity of the ends does not rest on a comparison made at the end, of which there is none; it rests on there being no comparison, before the end or after the turn, that registers a difference between them". "The two turns receive different verdicts, and the principle does not identify them". Source: *The Gravitational Passage*, Chapter 18, Section II. Standing: argued.

No comparison is made at the end of the interior of a black hole. The ends of the interiors are one end because no comparison registers any difference between the ends, either in the old epoch before the end or in the new epoch after the turn. The principle of identity also shows that the turns of different epochs are distinct. Comparisons made in an epoch register the turn that the epoch came from as the beginning of the epoch. Comparisons made in an epoch do not register the later turn at the end of the epoch's own interiors. The reason is that the later turn lies in the epoch's future. So the two turns receive different verdicts, and the principle does not make the two turns one. The principle is applied to the comparisons made before the end and after the turn, and not at the end itself. If the principle were applied at the end, where no comparison is made, the end would count as the same as every other thing at which no comparison is made, the beginning of the epoch among those things. So applying the principle at the end would identify too many things with one another (*At the Edge of Reality*, Chapter 11).

The time left in a fall, and the tides near the end

$$\tau_{horizon} = 4GM/(3c^3); \quad \textit{radial stretching } 4/(9\tau^2), \textit{ transverse compression } 2/(9\tau^2); \textit{ mass-dependent part} \propto (c^3\tau/GM)^{2/3}$$

"A comparer inside an interior registers that interior and does not register another, and it registers the mass of its hole by the proper time that remains when it crosses the horizon: $4GM/(3c^3)$ for a fall from rest at infinity into a non-rotating hole". "An observer falling from rest at infinity into

a non-rotating hole, with proper time τ left before the end, measures a radial tidal stretching of $4/(9\tau^2)$ and a transverse compression of $2/(9\tau^2)$, whatever the mass of the hole. On any other radial path the part of these values that depends on the mass falls in proportion to $(c^3\tau/GM)^{2/3}$ as the end is approached".

Source: *The Gravitational Passage*, Chapter 18, Section II. The subscript on the first τ is added here to name the time left at the horizon. Standing: computed (established general relativity).

Someone falls from rest, starting far away, into a black hole that does not spin. Call the person who falls the faller. When the faller crosses the horizon of the hole, the faller has a time left, on the faller's own clock, of four times Newton's constant times the mass of the hole, divided by three times the cube of the speed of light. Near the end, the tides of the hole stretch the faller along the line of the fall. The tides also squeeze the faller across the line of the fall. When a time τ is left, the stretching along the fall is four ninths divided by the square of the time left, and the squeezing across the fall is two ninths divided by the square of the time left. Both values are the same whatever the mass of the hole. On other falling paths, the tides have a part that depends on the mass of the hole. That part shrinks as the end is approached, in proportion to the two-thirds power of a ratio: the cube of the speed of light times the time left, divided by Newton's constant times the mass. So the tides differ less and less from one hole to another as the faller comes closer to the end.

The one-way order inside each act

"The one-way order inside each act of comparison (Chapter 5, Section IV) is not reversed; at the end of an interior there is no comparable content for it to order".

Source: *The Gravitational Passage*, Chapter 18, Section IV. Standing: argued.

At the end of an interior, time on the large scale loses its direction. Inside each single act of comparison, the order of before and after is not reversed at the end. At the end of an interior no comparable content is left for the order of before and after to arrange.

The cosmological constant inside a hole

"Inside the hole its curvature, Λ, is about 10^{-45} of the tidal curvature at the horizon of a hole of one solar mass, $2GM/(c^2 r_s^3)$, and the ratio falls further on the way in". What reaches a comparer during its time inside, "about 0.28 r_s/c of the exterior's advanced time for a fall from rest far away into a hole without rotation, and the delay of that light by the hole's own field, change that count by parts in 10^{23} to 10^{22} for a hole of one solar mass".

Source: *The Gravitational Passage*, Chapter 18, Section V, and Appendix B, Section VI. Standing: argued; computed (the ratio 10^{-45}).

A comparer is anyone or anything that makes a comparison, such as an observer with an instrument. A comparer who falls into a black hole keeps the count of the comparer's particle horizon. In the account the cosmological constant depends on the particle horizon through the identity of Chapter 8, which ties the share of the energy of empty space to the radius of the particle horizon. So the cosmological constant is the same inside the hole as outside. Take a hole of one solar mass, and the tidal curvature at its horizon. Inside the hole, the curvature due to the cosmological constant is about one part in ten to the forty-five of the tidal curvature at the horizon. The ratio falls further as the comparer falls inward. While the comparer is inside the hole, some light still reaches the comparer from outside, and that light was sent out during a stretch of outside time of about 0.28 of the time light takes to cross the radius of the hole. For a hole of one solar mass, the change in the comparer's count is only parts in ten to the twenty-three to ten to the twenty-two. The change includes the effect of the light that reaches the comparer and of the delay of that light by the hole's own field.

Spinors across the turn

"The ten-dimensional geometry admits no spin structure (Chapter 12, Questions), so no description of half-integer spin as a field on the undivided ten directions is available, and the account offers none. In the new epoch the product re-forms with the same internal topology (Questions), and its four-dimensional factor is the product of a line of time with an orientable three-manifold, spatially flat (Section IX). Every such four-manifold is parallelizable, so it admits a spin structure whatever metric it carries (Geroch 1970)".

Source: *The Gravitational Passage*, Chapter 18, Section VII. Standing: argued; theorem (Geroch 1970).

Particles of half-integer spin, such as electrons, are described by fields called spinors. Spinors need a property of the geometry called a spin structure. The undivided ten-dimensional geometry has no spin structure, so no spinor field can be defined on all ten directions at once. At the turn no geometry is built, so no spinor field is defined at the turn, and the spin of the content passes the turn as one of the labels of the content, not as a field. In the new epoch the four large dimensions are a line of time times a flat three-dimensional space. Geroch proved in 1970 that every space and time built as a line of time times a flat three-dimensional space admits a spin structure, whatever its metric, so the spinors of the new epoch are defined in the same way as the spinors of the old epoch were.

The singularity theorems

$$R_{\mu\nu\alpha\beta}\, R^{\mu\nu\alpha\beta} \to \infty$$

"A singularity in general relativity is a point at which a curvature invariant diverges: $R_{\mu\nu\alpha\beta}\, R^{\mu\nu\alpha\beta} \to \infty$. The singularity theorems of Penrose and Hawking establish that under generic conditions (positive energy and an

appropriate causal structure) geodesics end in finite proper time (Penrose 1965; Hawking 1966)". "In this book the singularity is a breakdown of the context-modal description".
Source: *The Gravitational Passage*, Chapter 19, Section I. Standing: theorem (the singularity theorems); argued (the reading).

In general relativity, a singularity is a place where a measure of curvature grows without bound. The measure in the formula is the sum of the squares of all the parts of the curvature. Two physicists proved the singularity theorems in 1965 and 1966: Roger Penrose and Stephen Hawking. The singularity theorems show that, under general conditions, paths through space and time end after a finite time. The general conditions are that energy is positive and that the order of cause and effect in space and time has a suitable form. In this account the singularity is the place where the smooth description of space and time, which is the description of context, is carried past the curvature at which that description holds.

Beyond the boundary

"Beyond the Planck scale no description takes over (Chapter 17, Section IV): the full axiom structure holds there without the macroscopic partition into separate content and context geometries, and the book constructs no description of that region. Loop quantum gravity, which constructs a model of that region, also reaches a finite curvature at the Planck scale (Ashtekar, Pawlowski and Singh 2006; Ashtekar and Bojowald 2006)".
Source: *The Gravitational Passage*, Chapter 19, Section IV. Standing: argued.

The account contains no description of the region past the Planck scale. In the region past the Planck scale, the full structure of comparison still applies, with all three axioms. In that region the structure of comparison applies without the split into a separate geometry of content and a separate geometry of context. Loop quantum gravity is a research program whose physicists do build a model of the region past the Planck scale. In loop quantum gravity, too, the curvature stops growing at about the Planck curvature. In loop quantum gravity the curvature stays finite

because the geometry is quantized, that is, treated as made of quantum units. In the account, the smooth description of space and time ends at the Planck scale because of the modal boundary, where each description reaches the edge of its range, and nothing is quantized at the modal boundary. Nothing is quantized because a quantized arena would need a pairing that the Incommensuration Theorem excludes (*The Gravitational Companion*, Comparisons at Single Steps, Section IV). A quantized arena would be both stepwise and uneven, and so would need discontinuity and asymmetry together. The Incommensuration Theorem says that discontinuity and asymmetry cannot both be applied, simultaneously and fundamentally, to any event or comparison. Gravitation is not quantized.

Chapter 2, Three Open While Six Close

The Kasner form and its two conditions

$$ds^2 = -d\tau^2 + \Sigma_i \, \tau^{2p_i} \, dx_i^2 \quad (1.3)$$

$$\Sigma_i \, p_i = 1, \quad \Sigma_i \, p_i^2 = 1 \quad (1.4)$$

Source: *The Compactification*, Chapter 1, Section VI, equations (1.3) and (1.4), after Kasner (1921): "where τ is the proper time counted from the limit (the time remaining, read toward a limit in the future; the time elapsed, read away from a limit in the past), x_i is a coordinate along the i-th direction of space, and p_i is the exponent of that direction". In ten dimensions the sums run over the nine directions of space. Standing: established physics (the vacuum Einstein equations).

A limit is a moment such as the end of the interior of a black hole or the start of an epoch. Near a limit, each direction of space grows or shrinks as a power of the time counted from the limit. Each direction has its own power, called the exponent of that direction. In ten dimensions there are nine directions of space, so there are nine exponents. Einstein's

equations for empty space require two things of the nine exponents: the nine exponents add up to one, and the squares of the nine exponents also add up to one.

Three directions growing, in earlier physics

"Cosmologies in which three dimensions expand while the others contract are known. Chodos and Detweiler (1980) found a vacuum solution of Kasner type in five dimensions, and Freund (1982) studied such cosmologies in higher dimensions, including ten- and eleven-dimensional supergravity, in which three space dimensions are preferred in the expansion. What this appendix adds is the curve for the grouping of three, four and two directions, and the reading of its upper arc in both directions of time". "In pure higher-dimensional Einstein theory no group of three directions is preferred in the expansion (Freund 1982); here also the grouping comes from the product geometry, not from the constraints".
Source: *The Gravitational Passage*, Appendix B, Section IV and Questions. Standing: established physics, cited; the grouping comes from the product geometry (given under Chapter 3).

The physicist Peter Freund showed in 1982 that Einstein's equations alone, in more dimensions than four, single out no group of three directions to grow. In this account the grouping of three directions comes from the product form of the geometry (given under Chapter 3), and not from Einstein's equations.

Earlier physics already contains worlds in which three directions expand while the other directions contract. In 1980 the physicists Alan Chodos and Steven Detweiler found a world of the Kasner kind with three expanding directions, empty of matter, in five dimensions. In 1982 Freund studied expanding and contracting worlds in larger theories, with ten and eleven dimensions. In Freund's larger theories, three directions of space are preferred in the expansion. *The Gravitational Passage* adds two things to the earlier work of Chodos, Detweiler and Freund. The first addition is the curve of exponents for the grouping of three, four and two directions. The

second addition is a reading of the upper stretch of the curve of exponents in both directions of time. Read toward a limit in the future, the upper stretch describes the end of an interior. Read away from a limit in the past, the upper stretch describes the start of an epoch.

The curve for three, four and two directions

$$3p + 4q + 2s = 1, \quad 3p^2 + 4q^2 + 2s^2 = 1 \quad (1.5)$$

$$24q^2 + (24p - 8)q + (15p^2 - 6p - 1) = 0 \quad (1.6)$$

$$(24p - 8)^2 - 96(15p^2 - 6p - 1) = -32(3p + 1)(9p - 5) \quad (1.7)$$

$$q = 1/6 - p/2 \pm (1/12)\sqrt{(10 + 12p - 54p^2)}, \quad s = 1/6 - p/2 \mp (1/6)\sqrt{(10 + 12p - 54p^2)} \quad (1.8)$$

Source: *The Compactification*, Chapter 1, Section VI, equations (1.5) to (1.8), which are the equations of *The Gravitational Passage*, Appendix B, Section III, and of *At the Edge of Reality*, Appendix A. The letter s is *The Compactification*'s; *The Gravitational Passage* writes r for the sphere's exponent. The first condition gives s = (1 − 3p − 4q)/2. "The discriminant is zero at p = −1/3 and at p = 5/9 and positive between them, so real exponents exist exactly for −1/3 ≤ p ≤ 5/9". Standing: theorem (algebra), given the Kasner form.

Let the three large directions share one exponent, p. Let the four directions of the complex projective plane share a second exponent, q. Let the two directions of the sphere share a third exponent, s. The two Kasner conditions are that the nine exponents add up to one, and that the squares of the nine exponents add up to one. With the three shared exponents, the two Kasner conditions become two equations. The first equation says that three times p, plus four times q, plus two times s, equals one. The second equation says that three times the square of p, plus four times the square of q, plus two times the square of s, also equals one. Solving the two equations together gives one closed curve of allowed exponents, which this

book calls the Kasner curve. Real solutions exist exactly when p, the exponent of the large directions, lies between minus one third and five ninths, both ends included.

The two ends, and the arc

At the lower end, $p = -1/3$ and $q = s = 1/3$; at the upper end, $p = 5/9$ and $q = s = -1/9$. "On the branch on which s rises through zero, $s = 0$ where $7p^2 - 2p - 1 = 0$, at $p = (1 + 2\sqrt{2})/7 \approx 0.547$; on the other branch, $q = 0$ where $15p^2 - 6p - 1 = 0$, at $p = (3 + 2\sqrt{6})/15 \approx 0.527$. Along the curve from $p \approx 0.547$ through the upper end at $p = 5/9$ to $p \approx 0.527$, all six internal exponents are negative. That stretch is the arc".
Source: *The Compactification*, Chapter 1, Section VI. Standing: theorem (algebra); the placement of the end of every interior and the beginning of an epoch at the upper end is a choice of *The Gravitational Passage*, Appendix B, Section IV, "made where the internal space moves as a whole; the shared arc follows from the constraints".

At each end of the Kasner curve, all six small directions move together as one block, because q and s are equal there. At the lower end, the three large directions have the exponent minus one third, and each of the six has the exponent one third. At the upper end, the three large directions have the exponent five ninths, and each of the six has the exponent minus one ninth. The arc is a short stretch of the curve near the upper end, on which all six small exponents are negative, and in p the arc runs from about 0.547, through five ninths at the upper end, to about 0.527. The curve and the arc follow by algebra from the two conditions that Einstein's equations set on the exponents. *The Gravitational Passage* places the end of every interior and the beginning of an epoch at the upper end, where the six move as a whole, and that placement is a choice.

Volumes, and the slow growth of the six

three large directions: $\tau^{5/3}$*; six internal:* $\tau^{-2/3}$*; all nine:* τ*; growth of each internal direction from* τ_1 *to* τ_2*:* $(\tau_1/\tau_2)^{1/9}$

Source: *The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Section IV: "Between two depths τ_1 and τ_2, with τ_2 nearer the singularity, each internal direction grows by the factor $(\tau_1/\tau_2)^{1/9}$: the ninth root of the ratio of the two times". *The Compactification* adds: "a billionfold change of τ changes each of the six tenfold and each of the three a hundred-thousandfold". Standing: theorem (algebra).

At the upper end, the volume of the three large directions goes as the time to the limit raised to the power five thirds. The volume of the six goes as the time to the limit raised to the power minus two thirds, and the volume of all nine directions goes as the time to the limit itself. The powers add because the volume of a set of directions multiplies the sizes of those directions: three times five ninths is five thirds, and six times minus one ninth is minus two thirds. Take two moments, and the two times to the limit at those moments. Between the two moments, each of the six grows toward the limit by the ninth root of the larger time divided by the smaller. So when the time to the limit changes a billionfold, each of the six changes only tenfold: the ninth root of a billion is ten. Each of the three large directions changes a hundred-thousandfold over the billionfold change, because a billion raised to the power five ninths is a hundred thousand.

The spread of the rates

$$\Sigma_i (H_i - \bar{H})^2/\theta^2 = \Sigma p_i^{\,2} - (\Sigma p_i)^2/9 = 1 - 1/9 = 8/9 \quad (1.9)$$

Source: *The Compactification*, Chapter 1, Section VI, equation (1.9), with $H_i = p_i/\tau$, $\theta = \Sigma H_i = 1/\tau$ and $\bar{H} = \theta/9$. "This is an identity of the Kasner conditions in nine directions of space: the ratio 8/9 of the squared spread of the rates to the squared rate of the volume holds at every point of the curve, and at every Kasner point off it". Standing: computed.

The nine directions of space change at nine rates, one rate for each direction, and the nine rates have a mean. Each direction changes at a rate equal to the exponent of the direction divided by the time. The whole volume changes at a rate of one divided by the time, because the exponents add up to one. Divide the rate of a direction by the rate of the whole volume, and what is left is the exponent of the direction. The mean of the nine exponents is one ninth, because the nine exponents add up to one. So the mean rate is one ninth of the rate of the whole volume. Take the squared spread of the nine rates about their mean. Divide the squared spread by the square of the rate of the whole volume. The quotient is always eight ninths. The two Kasner conditions are that the exponents add up to one and that the squares of the exponents add up to one. The two Kasner conditions fix the value eight ninths at once, for every Kasner form in nine directions. Measured in units of the rate of the volume, each departure from the mean is an exponent less one ninth. Squaring each departure and adding the nine squares gives three pieces. The first piece is the sum of the squares of the exponents, which is one, by the second condition. The second piece takes away twice the mean times the sum of the exponents: twice one ninth times one, which is two ninths. The third piece adds back nine times the square of the mean: nine times one eighty-first, which is one ninth. So the spread is one, less two ninths, plus one ninth: eight ninths. So every Kasner form in nine directions is strongly uneven,

with directions that change at widely different rates, and the re-forming of the six after the turn is uneven because every Kasner form in nine directions is uneven.

What the Kasner algebra does not settle

"The ordinary interior of a black hole that neither rotates nor carries charge has, near its end, the exponents $(-1/3, 2/3, 2/3)$ for its three directions of space, and in ten dimensions the six internal directions keep their size, with exponent zero. Those exponents form a valid Kasner point, and the point does not lie on the curve (1.5), whose three large exponents are equal. What carries an interior from that form onto the arc, and at what depth, is not derived (*The Gravitational Passage*, Chapter 19, Section II). Nor is a single Kasner form the generic behavior: empty-space gravity in ten spacetime dimensions passes near a generic spacelike singularity through an endless sequence of Kasner forms (Demaret, Henneaux and Spindel 1985), and how the product geometry selects one form is not derived. A form field threading the internal space would also drive that oscillation (Damour and Henneaux 2000); the action (1.2) has none. The algebra of this section depends on none of the points this note names".
Source: *The Compactification*, Chapter 1, Section VI, "Method-note: what the Kasner algebra does not settle". Standing: not fixed (not derived); the last sentence states the scope of the algebra of the curve.

Take the interior of an ordinary black hole, one that neither spins nor carries charge. Near the end of the interior, the interior takes a Kasner form of its own. One direction of space has the exponent minus one third, and the other two large directions have the exponent two thirds, while in ten dimensions the six small directions keep their size, with the exponent zero. The interior's exponents obey both Kasner conditions, so the interior's exponents form a valid Kasner form. But the exponents of the interior do not lie on the Kasner curve of Chapter 2, because on the curve the three large directions share one exponent, and in the interior the three large directions have two different exponents. The account does not derive

what carries an interior from the Kasner form of the interior onto the arc. Nor does the account derive at what depth the interior reaches the arc. A single Kasner form is also not the general behavior. Near a general end of an interior, empty-space gravity in ten dimensions passes through an endless sequence of Kasner forms, as the physicists Demaret, Henneaux and Spindel showed in their paper of 1985. The account does not derive how the product of the large dimensions and the six selects one Kasner form. A field threaded through the six would also drive an endless sequence of Kasner forms, as Damour and Henneaux showed in 2000, but the action of the metric alone has no field threaded through the six. The Kasner algebra depends on none of the open questions just named.

The depth at which each density is reached

"The equivalent density rises inward as $1/r^3$". "As a fraction of the Schwarzschild radius, that depth falls as $M^{-2/3}$". "The internal space begins to change when the external curvature approaches the curvature of the internal space, of order $1/R^2$ for an internal radius R", and "That depth scales as $R^{2/3}$".

Source: *The Gravitational Passage*, Chapter 19, Section II; Appendix B, Section V. Standing: established general relativity; the densities of standard physics are used "only as markers of depth".

Each depth inside a black hole has an equivalent density. The equivalent density is the density at which the mass of the hole would fill a ball whose radius is that depth. The equivalent density rises inward as one over the cube of the radius. A given density is reached at some radius inside the hole. The radius at which the density is reached can be written as a share of the radius of the hole. The share falls as the mass of the hole raised to the power minus two thirds, because the radius at which a given density is reached grows only as the cube root of the mass. The radius of the hole, by contrast, grows in proportion to the mass, so in heavier holes a given density lies relatively deeper. The curvature of the six is about one over the square of the radius of the six. The six begin to change at the

depth where the curvature of the large dimensions approaches the curvature of the six, and that depth goes as the radius of the six raised to the power two thirds.

Chapter 3, The Shape and Size of the Six

The classification that selects the shape

"For a compact homogeneous space G/H, χ is positive exactly when H has the same rank as G (Hopf and Samelson 1941). The only homogeneous surface with $\chi = 2$ is the sphere S^2. Among the connected compact homogeneous four-manifolds with positive Euler characteristic (Ishihara 1955), the only one with $\chi = 3$ is $\mathbb{C}P^2$. The simply connected ones are S^4 ($\chi = 2$), $\mathbb{C}P^2$ ($\chi = 3$) and $S^2 \times S^2$ ($\chi = 4$). A quotient by a free action of a finite group of order k has Euler characteristic χ/k, so the non-trivial quotients of S^4 and $S^2 \times S^2$ have $\chi = 1$ or 2, and $\mathbb{C}P^2$ has no non-trivial quotient, since every self-map of it has a fixed point".

Source: *The Compactification*, Chapter 1, Section III, after *The Gravitational Passage*, Chapter 9, Section VI, which adds that "by the Lefschetz theorem every self-map of $\mathbb{C}P^2$ has a fixed point". Standing: theorems of topology; "the conditions are identifications, and the uniqueness holds at their standing".

The Euler characteristic, written χ, is the number that this book calls the Euler count. When every independent class of a shape lies in an even dimension, χ counts the independent classes of the shape. Every comparison has three roles: what is possible, what is happening, and what is known. The conditions of Chapter 3 require a count of three for the factor of the six that carries the three roles. The grid of six measures has two rows, content and context. The conditions of Chapter 3 also require a count of two for the factor that carries the two rows. The sphere is the only surface with a count of two among the shapes that look the same at every point. The complex projective plane is the only four-dimensional shape

with a count of three among the shapes that look the same at every point. A new shape can also be made by folding a shape onto itself, so that several points of the original shape become one point. Folding divides the count by the number of points that become one, so no folded shape has a count of three. And the complex projective plane itself cannot be folded, because every map of the complex projective plane to itself leaves some point in place.

Two shapes that the conditions exclude

"The six-dimensional flag manifold, $SU(3)/(U(1) \times U(1))$, also has Euler characteristic six; it is a single homogeneous space and not a product (its cohomology ring has no factorization into the rings of two smaller spaces), so the product condition excludes it. The triple product $S^2 \times S^2 \times S^2$ has Euler characteristic eight".

Source: *The Gravitational Passage*, Chapter 9, Section VII. Standing: theorem.

One other six-dimensional shape, the flag manifold, also has a count of six. But the flag manifold is a single shape and not a product of two shapes. The six must be a product of a factor for the roles and a factor for the rows. So the condition of being a product excludes the flag manifold. The six need a count of six: three for the roles, times two for the rows. Three spheres multiplied together have a count of eight, not six, so the product of three spheres is excluded too.

The same pair in earlier physics

"The product $\mathbb{C}P^2 \times S^2$ is the base of the seven-dimensional spaces M^{pqr} that Witten (1981) studied".

Source: *The Gravitational Passage*, Chapter 9, Section III. Standing: established physics, cited.

Physicists studied the product of the complex projective plane and the sphere in earlier work. In 1981 the physicist Edward Witten studied a family of seven-dimensional spaces, in his search for a realistic theory with

extra dimensions. Each of Witten's seven-dimensional spaces is built from the product by attaching a small circle at every point. The product is called the base of each space. Witten chose the seven-dimensional spaces of his family for their symmetries. *The Gravitational Passage* derives the product from the grid of comparison, and not from a group of symmetries. The internal space of *The Gravitational Passage* is the base without the circle. So the symmetries of the internal space include no turning around the circle of Witten's spaces. And *The Gravitational Passage* makes no claim on the spectrum of the content, which is the list of the particles and of their values (*The Gravitational Companion*, Comparisons at Single Steps, Section II).

Curvature and discreteness on one connection

"By the Ambrose–Singer theorem (Ambrose and Singer 1953), the curvature of a connection generates the Lie algebra of its holonomy group. A connection whose holonomy group is discrete therefore has zero curvature everywhere; a connection whose smooth curvature is non-zero anywhere has a holonomy group of positive dimension, which is continuous". "The gauge connection has a compact structure group, whose irreducible unitary representations form a discrete set; that set, and not the holonomy, is the source of the discrete spectra of the quantized gauge field".

Source: *The Gravitational Passage*, Chapter 9, Section V. Standing: theorem (Ambrose and Singer); argued (the reading as the geometric counterpart of the Incommensuration Theorem).

A connection is the rule that says how to carry a direction from one point to another. Carrying a direction around a closed loop can turn the direction. Warren Ambrose and Isadore Singer proved in 1953 that the curvature of a connection produces all of the turning that closed loops can give. So where the turnings of a connection come only in separate steps, the connection has no curvature anywhere, and where a connection has a smooth curvature that is not zero at some point, the turnings of the con-

nection vary continuously. So no connection is both curved and stepwise. *The Gravitational Passage* argues that the absence of any connection both curved and stepwise is the counterpart in geometry of the Incommensuration Theorem. The quantum fields do come in separate steps. The separate steps of the quantum fields come from the representations of the symmetry group of each field, and not from the turning. A representation is one of the ways in which the symmetry group can act. For a symmetry group of the compact kind, the basic representations form a separate, countable set.

Not a Calabi–Yau space

"A Calabi–Yau space has vanishing first Chern class and admits a Ricci-flat metric. The first Chern class of $\mathbb{C}P^2 \times S^2$ does not vanish, so the internal space is not a Calabi–Yau space and admits no Ricci-flat Kähler metric. This is a fixed structural fact".

Source: *The Gravitational Passage*, Chapter 9, Section VIII. Standing: theorem (topology).

The six small dimensions of string theory are usually taken to be Calabi-Yau spaces. Mathematicians call a certain count of a shape its first Chern class, and in a Calabi-Yau space the first Chern class is zero. A Calabi-Yau space can also carry a geometry in which the total bending of every direction is zero. For the complex projective plane times the sphere, the first Chern class is not zero, so the internal space is not a Calabi-Yau space, and the internal space carries no geometry of the Calabi-Yau kind with zero bending. The difference from a Calabi-Yau space is a fixed fact of the structure of the internal space, a theorem of topology.

Why the Ricci curvature, and no other

"Independence alone fixes no curvature: directions that cannot be composed from one another remain so when either factor is enlarged or reduced, so independence holds at every ratio of the radii". "A plane with one direction in each factor has sectional curvature zero, while a plane

within $\mathbb{C}P^2$ has sectional curvature between $1/R_O^2$ and $4/R_O^2$, so the sectional curvatures cannot all agree. The holomorphic sectional curvature of a direction X [...] takes different values on different directions at every ratio of the radii, and so does the largest sectional curvature among the planes through X. The least sectional curvature among the planes through X is zero for every direction at every ratio, and the scalar curvature is the same at every point at every ratio, so neither fixes the ratio. The Ricci curvature can be the same in every direction, and it is so at exactly one ratio of the radii".

Source: *The Gravitational Passage*, Chapter 9, Section IX. Standing: theorem of Riemannian geometry.

The independence of directions, by itself, sets no amount of bending. Directions are independent when no one of the directions is a combination of the others: the directions cannot be composed from one another. Independent directions stay independent under any enlarging or shrinking of either factor of the six. So independence holds at every ratio of the two radii. A shape has several measures of curvature. Of the measures named in the rest of this item, only the Ricci curvature can fix the ratio of the radii. The sectional curvature is the curvature of a flat slice through two directions. In the six the sectional curvature differs from slice to slice. A slice with one direction in each factor has curvature zero, while a slice inside the complex projective plane is curved. So the sectional curvature cannot be the same for every slice. Two other measures differ from direction to direction at every ratio of the radii. The two measures are the holomorphic sectional curvature and the largest sectional curvature through a direction. The least sectional curvature through a direction is the same at every ratio of the two radii, and so is the scalar curvature. So the ratio is fixed by neither the least sectional curvature nor the scalar curvature. The Ricci curvature of a direction is the curvature summed over all the slices through that direction. The Ricci curvature can be the same in every direction. The Ricci curvature is the same in every direction at exactly one ratio of the two radii.

A metric the account does not take

"Suppose the distinguishability of two pairs of states is measured by one rule for both factors, with the squares of the two distances added. Then the states that superpose two given alternatives form a round sphere of the same radius whichever factor carries them, $R_O/R_T = 2$, and the Ricci constants stand in the ratio 3 to 2, so that metric is not an Einstein metric. I do not take that metric as the internal metric".

Source: *The Gravitational Passage*, Chapter 9, Section IX, with Wootters (1981) for distinguishability as a metric on states. Standing: computed; the choice of the Einstein metric is the identification of that section.

Take each factor of the six as a space of states. Then the geometry of each factor is fixed, up to its size, by how easily one state can be told from another. Suppose that one rule measured the distance between states on both factors, with the squares of the two distances added. Then the ratio of the radii would be two. The Ricci curvatures of the two factors would be in the ratio three to two, so the two factors would not be curved equally. I do not take that metric as the internal metric. The metric I take counts the square of the distance on each factor once for each alternative of that factor. The complex projective plane has three alternatives and the sphere has two, so the metric I take counts the square three times on the complex projective plane, and twice on the sphere. The weighting of three and two follows from the identification of the internal metric as an Einstein metric, a metric in which every direction has the same Ricci curvature, but the weighting is not the ground of the identification.

The radii, the volume and the slowest variation

$$R_O/R_T = \sqrt{6}, \quad R_T = l_{Pl}, \quad R_O = \sqrt{6}\, l_{Pl} \quad (1.1)$$

$$g_K = R_O{}^2\, \hat{g}_{FS} \oplus R_T{}^2\, \hat{g}_{S^2} \quad (2.1)$$

$$\lambda_{\mathbb{C}P^2} = 6/R_O{}^2, \quad \lambda_{S^2} = 1/R_T{}^2 \quad (2.2)$$

$$6/R_O{}^2 = 1/R_T{}^2 = 1/l_{Pl}{}^2 \quad (2.3)$$

$$\pi R_O{}^2/(4\pi R_T{}^2) = 6/4; \quad \text{volume } (\pi^2/2) R_O{}^4 \times 4\pi R_T{}^2 = 72\pi^3\, l_{Pl}{}^6; \quad \text{first eigenvalue } 12/R_O{}^2 = 2/R_T{}^2$$

Source: *The Compactification*, Chapter 1, Section III, equation (1.1), which is equation (9.2) of *The Gravitational Passage*; *The Compactification*, Chapter 2, Sections III and IV, equations (2.1) to (2.3); *The Gravitational Passage*, Chapter 17, Section V, for the first eigenvalue: "the slowest variation the internal space admits, the first eigenvalue of its Laplacian, $12/R_O{}^2 = 2/R_T{}^2$, is twice the Planck curvature: the internal space begins where the boundary lies". The identification, in the first person: "I take the modal boundary to be one curvature, the Planck curvature $1/l_{Pl}{}^2$, for every internal direction, and I take each internal direction to reach it: the Ricci curvature of every unit internal direction is $1/l_{Pl}{}^2$" (*The Gravitational Passage*, Chapter 9, Section IX). R_O is the radius of the round five-sphere whose quotient by the circle of phases is $\mathbb{C}P^2$, so that "every projective line of $\mathbb{C}P^2$ is a round sphere of radius $R_O/2$"; R_T is the radius of S^2. Standing: theorem (algebra of Einstein products), given the identification.

The metric of the six has two parts, set side by side. One part is the metric of the complex projective plane, scaled by the square of the radius R_O. The other part is the metric of the sphere, scaled by the square of the radius R_T. The Ricci curvature of the complex projective plane is six over the square of R_O. The Ricci curvature of the sphere is one over the square

of R_T. As an identification, I take every internal direction to reach the Planck curvature, one over the square of the Planck length. Setting both curvatures equal to the Planck curvature gives the sphere a radius of one Planck length, and gives the complex projective plane a radius of the square root of six Planck lengths, about two and a half Planck lengths. Each straight line of the complex projective plane is itself a small round sphere, whose radius is half of R_O. Each straight line of the complex projective plane has three halves of the area of the sphere S^2, a ratio that needs no choice of scale. The volume of the six is seventy-two times pi cubed, times the sixth power of the Planck length. The first eigenvalue of a shape is the sharpness of the gentlest ripple the shape allows. The first eigenvalue measures the slowest variation the shape admits. For the six, the first eigenvalue is twice the Planck curvature: two over the square of the Planck length. So the internal space begins where the modal boundary lies.

The volume, and the unit in which it is counted

$$Vol(\mathbb{C}P^2 \times S^2) = 2\pi^3 R_O{}^4 R_T{}^2 = 2(R_O/R_T)^4\, \pi^3 R_T{}^6 = 72\, \pi^3 R_T{}^6 = 72\, \pi^3 l_{Pl}{}^6$$

$$\pi l_{Pl}{}^2 = \pi\hbar = h/2; \quad 72(h/2)^3 = 9h^3; \quad \textit{six-ball}\ \pi^3 R_T{}^6/6, \quad 3! = 6$$

“In the unit $\pi^3 R_T{}^6$ the volume is 72, which is 2×6^2, twice the fourth power of the ratio of the radii”. “One such disc in each of the three complex directions, taken as a product, is the Planck polydisc, whose volume is $(\pi R_T{}^2)^3 = \pi^3 R_T{}^6$”. “Since $l_{Pl}{}^2 = \hbar G_N/c^3$, in units in which G_N and c are one the area of a disc of Planck radius is $\pi l_{Pl}{}^2 = \pi\hbar = h/2$, half of Planck’s constant $h = 2\pi\hbar$. The Planck polydisc is then half a cell of action in each complex direction, $(h/2)^3$, and the volume of the internal space is $72(h/2)^3 = 9h^3$”. “The six-dimensional ball of Planck radius has volume $\pi^3 R_T{}^6/6$, six times smaller than the polydisc”, and “the factor between the two units is $3! = 6$, the number of orders of three. The volume of the internal space is therefore counted in polydiscs, and the reason it must be is Ax-

iom III". "The identification sets a coefficient of order one: the factor 1 that multiplies $1/l_{Pl}^2$, whose value this book takes from the identification and does not fix otherwise (Chapter 13, Questions)". The scalar curvature, "the sum of the Ricci curvatures of six orthonormal directions", is fixed by the identification "at $6/l_{Pl}^2$", and "The ratio of the radii meets no observation of this book: an internal metric with another ratio and the same scalar curvature gives the same bare term, and so the same predictions in Chapter 13".

Source: *The Gravitational Passage*, Chapter 9, Section IX. Standing: computed (the volume, given the radii); argued, from Axiom III (the polydisc as the unit of count).

The complex projective plane has a volume of pi squared over two, times the fourth power of R_O, and the sphere S^2 has an area of four pi times the square of R_T. With the radii that the identification sets, the product of the volume and the area is seventy-two times pi cubed times the sixth power of the Planck length. The volume of the six is counted in a unit called the Planck polydisc. A Planck disc is a flat disc whose radius is one Planck length. The six directions of the internal space make up three complex directions, and each complex direction is a pair of real directions. The Planck polydisc is three Planck discs, one in each of the three complex directions, held together as a product. The volume of the Planck polydisc is pi cubed times the sixth power of the Planck length. Counted in Planck polydiscs, the volume of the six is seventy-two. Seventy-two is twice the fourth power of the ratio of the radii: two times six squared. In units in which Newton's constant and the speed of light are one, the area of one Planck disc is half of Planck's constant h. A cell of action is a patch of the size of Planck's constant. So each Planck polydisc is half a cell of action in each of its three complex directions. So the volume of the six is seventy-two times the cube of half of h, which is nine times the cube of h. A single six-dimensional ball of Planck radius has one sixth of the volume of the Planck polydisc, and that factor of six is the number of ways of putting three things in order. The third axiom says that the three roles of compar-

ison are distinct, and none can do the work of another. The volume is counted in polydiscs, and not in balls, because under the third axiom the three complex directions stay three, distinct from one another. The identification also sets a coefficient: the factor one that multiplies the Planck curvature in the statement that every internal direction reaches the Planck curvature. The value of the factor one comes from the identification, and *The Gravitational Passage* does not fix the factor in any other way. The factor enters the cosmological constant through the energy that the bending of the six leaves in our four dimensions, and that energy is set by the total curvature of the six alone. The total curvature is the sum of the Ricci curvatures of six perpendicular directions. The identification fixes the total curvature at six times the Planck curvature: six over the square of the Planck length. If the ratio of the radii were different but the total curvature the same, the energy would be the same, and so would the predictions. So no observation that *The Gravitational Passage* discusses tests the ratio of the radii.

The Planck mass and the second length

$$M_{Pl}{}^2 = M_{10}{}^8 \times Vol(K) \quad (11.2)$$

"The ten-dimensional action defines a second length, from its coupling M_{10}; by Chapter 11, Section III, $M_{10}{}^8 = M_{Pl}{}^2/\mathrm{Vol(K)}$, so that length takes the internal volume as an input besides G_N. No located comparison supplies that volume: the internal directions are not traversed, and they present to a located comparer as compact (Appendix C, Section VIII). The volume is what the boundary fixes, so the boundary is the length a comparison supplies, the Planck length formed from G_N".

Source: *The Gravitational Passage*, Chapter 17, Section V, and Chapter 11, Section III, equation (11.2). Standing: standard technique (the reduction).

The square of the Planck mass equals the eighth power of the ten-dimensional mass scale M_{10}, times the volume of the six. The ten-dimensional mass scale is the strength of the coupling in the action of the ten di-

mensions. A second length can be formed from the ten-dimensional mass scale. But forming the second length needs the volume of the six as well as Newton's constant. So the second length is not the Planck length that the account uses. Nothing travels along the six. So a comparer in the large world has no comparison that supplies the volume of the six. To a comparer at a place in the large world, the six appear only as compact: finite and without edge. The volume of the six is what the modal boundary fixes. So the length that a comparison supplies is the modal boundary itself: the Planck length formed from Newton's constant.

Why two rows give a dimension equal to the count of cells

"For a grid of a columns and b rows carried by projective factors $\mathbb{C}P^{a-1} \times \mathbb{C}P^{b-1}$, the count of cells is ab, since $\chi(\mathbb{C}P^n) = n + 1$, and the real dimension is $2(a - 1) + 2(b - 1)$. The two are equal exactly when $(a - 2)(b - 2) = 0$".

Source: *The Gravitational Passage*, Chapter 9, Questions. Standing: theorem (arithmetic).

Take a grid of a columns and b rows, carried by two complex projective spaces, one space for the columns and one for the rows. The grid has a times b cells. The two spaces together have twice (a less one), plus twice (b less one), real dimensions. The two counts agree exactly when there are two columns or two rows. The reason is that the count of cells less the count of dimensions is (a less two) times (b less two). With two rows, the count of dimensions equals the count of cells for any number of columns. For three columns and two rows, both counts are six.

The spin^c structure

"$\mathbb{C}P^2$ admits no spin structure, only a spin^c structure, because the reduction modulo two of its first Chern class, three times the class of a projective line, is not zero. The spin^c structure the account uses is the one the complex structure induces, with the connection the metric induces (*The Gravitational Passage*, Chapter 12, Questions). That structure is a property

of the space and of its metric. No field is added to the action to carry it; the flux of degree two in *The Story of the Particles*, note 7, is the degree of a line bundle over the sphere, a topological count, and not a field". And: "The count of generations, the Kähler property and the spin^c structure are not conditions on this route".

Source: *The Compactification*, Chapter 1, Section III, and its sidebar. Standing: theorem (the topology); the use of the induced structure is argued.

The complex projective plane has no spin structure, the property of a geometry that spinors need. The reason is the first Chern class of the complex projective plane: the class is three times a basic class, and three is odd. The complex projective plane does carry a variant of the spin structure, called a spin^c structure. A spin^c structure is a phase at each point of a shape, a setting like the angle of a clock hand. Some of the waves on a shape need the phase of the spin^c structure in order to be defined. The account uses the spin^c structure that comes from the complex structure of the shape and from the metric of the shape. No field is added to the action to carry the spin^c structure. *The Story of the Particles* names a "flux of degree two" in its note 7. The flux of degree two is the number of times a family of phases winds over the sphere. The flux of degree two is a count fixed by the shape, and not a field. The count of generations of particles is not among the conditions that select the shape, and neither is the spin^c structure.

Homogeneity in its exact sense

"Homogeneous has here the sense of Section II: a quotient G/H of a compact Lie group G by a closed subgroup H, with G acting transitively by isometries. Every connected manifold is homogeneous as a topological space, so the classification uses the stronger sense throughout".

Source: *The Gravitational Passage*, Chapter 9, Section VI. Standing: definition; the homogeneity of each factor is an identification (Chapter 9, Section II).

A rigid motion of a shape is a motion that keeps every distance unchanged. The classification uses a strict sense of the word homogeneous. In the strict sense, a shape is homogeneous when a group of rigid motions can carry any point of the shape to any other. In the loose sense of topology, every connected shape looks alike everywhere, so the classification uses the strict sense throughout.

The scope of the uniqueness

"They are read from the structure of comparison, and none refers to the particles to be reproduced or to the symmetry group the internal space should have. From them the classification of compact homogeneous spaces selects $\mathbb{C}P^2 \times S^2$ by theorem (Section VI). A referee who rejects one of the identifications rejects the uniqueness at that step". Of the route through algebra (given under Chapter 5): "The two routes use different premises and meet on one space. This book uses the space; it does not add a third route".

Source: *The Gravitational Passage*, Chapter 9, Questions; *The Compactification*, Chapter 1, Section III. Standing: a statement of scope.

I read the conditions that select the shape from the structure of comparison alone. No condition refers to the particles that the account must reproduce, or to the symmetries that the six ought to have, and the number of families of particles is not among the conditions. Neither are two finer properties of the shape that the books on the particles use: the Kähler property and the spin^c structure. From the conditions, the classification of compact homogeneous shapes selects the complex projective plane times the sphere, by theorem. The conditions are identifications, so the uniqueness of the shape holds at the standing of those identifications: a reader who rejects one of the identifications rejects the uniqueness at the step of the rejected identification. The complex projective plane times the sphere also follows from different premises by a second route, through algebra, which is given under Chapter 5. I use the shape and add no third route.

Chapter 4, Caught by the Floor

Chapter 4 describes two things separately, as *The Compactification* does in its Chapter 2, Section V. The first is the hold, which rests on an argument. The second thing that Chapter 4 describes is one mechanism of the stiffness with which the six are held. That mechanism is computed, and the hold does not rest on the computation. The items below state the argument for the hold first, then the computation. The items also contain the authority's own statement of what rests on the computation and what does not. The action of the metric alone, equation (1.2), is given under Chapter 1 above.

The ground of the hold

Axiom III: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable" (*An Immanent Metaphysics*, aphorism 1.3-1). "A size held at zero with no spread is excluded by the uncertainty relation. The internal space therefore has a floor: a least extent, with a least motion that cannot be removed".
Source: *The Compactification*, Chapter 2, Section V, quoting *The Gravitational Passage*, Chapter 11, Section II. Standing: argued (Axiom III and the uncertainty relation).

Axiom III says that the three modes of every comparison are distinct, cannot be separated, and cannot stand in for one another. The six small dimensions are the context of possibility. Because the modes cannot be separated, content cannot be without its context, so the six cannot shrink to nothing. The uncertainty relation of quantum physics excludes a size held exactly at zero, with no spread at all. So the six have a floor: a least size, with a least motion that cannot be taken away.

The energy of the floor, at first order

$$\rho_{floor} = {R_O}^{-4} [\ a + b\ ln(\mu R_O) + C\ ] \quad (2.4)$$

$$b = -\zeta(-2)/(32\pi^2), \quad a = -(\zeta'(-2) + 3\zeta(-2)/2)/(64\pi^2), \quad \zeta(s) = \Sigma\ m^{-2s}$$

$$G(h,k) = \int \sqrt{g}\ (h{\cdot}k - \tfrac{1}{2}\ tr\ h\ tr\ k)$$

"where a and b are fixed by the zeta function of the masses of the metric's levels on the internal factor, μ is the scale at which the logarithm is referred, and C is the least-extent term". The computation uses this method: "The zero-point energy is computed in the form that does not depend on the gauge, the unique effective action (Vilkovisky 1984; DeWitt 1987); in the Landau–DeWitt gauge only the Christoffel symbol of the metric on the space of fields enters (Fradkin and Tseytlin 1984)". The metric on the space of fields is the one that the Einstein-Hilbert action supplies. "The background is flat M^4 times $\mathbb{C}P^2$, with the Fubini–Study metric of Ricci constant $6/R_O^2$, times the round S^2 of radius R_T". The stabilization of Candelas and Weinberg (1984) holds an internal space by the zero-point energy of its fields. In the account, as *The Compactification* says, "here the field is the metric itself".

Source: *The Compactification*, Chapter 2, Section V, equation (2.4); *The Gravitational Passage*, Appendix D, Sections I and II. Standing: computed at first order, for one mechanism.

Equation (2.4) gives the energy of the floor in each unit of four-dimensional volume. That energy is one over the fourth power of the radius of the complex projective plane, times a sum of three terms. The first term is a constant. The second term is a second constant times the logarithm of the radius, measured against a reference scale. The third term is the least-extent term. The two constants come from the zeta function of the masses of the levels of the metric's small waves on the six. The zeta function is a standard way of adding up the zero-point energies of infinitely many levels. A gauge is one of the many equivalent ways of describing the same

geometry in numbers. The computation uses the form of the zero-point energy that gives the same answer in every gauge. The computation measures the distance between two metrics by the rule that Einstein's action itself supplies. In one known method, an internal space is held by the zero-point energy of its fields. The physicists Philip Candelas and Steven Weinberg used the method in 1984. In the account, the geometry itself is the field whose zero-point energy holds the six.

The constants of the least extent, and what the computation finds

"At the ratio $\sqrt{6}$, $\zeta(-2) R_O^4 = -6596269937/3326400$ (exact), with a closed form in the shape that is negative at every shape; for a scalar field it is 71237/51975 (minimal coupling) and 317/10395 (conformal coupling), of the opposite sign. The finite part is $\zeta'(-2) R_O^4 = 3662.009$ (a small-time series to t^{12}). In the four-dimensional Einstein frame, at any stationary point in size, every minimum lies above $\sqrt{(\pi b/9)}\ l_{Pl} = 1.48\ l_{Pl}$". "Its two constants, fixed by the balance at the derived point, are $\kappa = 8.98 \times 10^{-5}$ for the term κ tr Ric^5 and $\mu = 0.651\ M_{Pl}$. The derived point is then a stable minimum, the only one in shape between $\sqrt{3}$ and $\sqrt{12}$, with its barrier at $R_O = 1.79\ l_{Pl}$, past the modal boundary: at that radius the Ricci curvature of $\mathbb{C}P^2$, $6/R_O^2 = 1.87/l_{Pl}^2$, exceeds the Planck curvature". "The minimum stays stable for ratios up to 2.50 at the derived radius and for radii above $1.87\ l_{Pl}$ at the derived ratio, so its location is not a test of the computation; its stability and the sign of the least-extent term are tests, and the derived point passes both. The moduli have masses of $2.25\ M_{Pl}$ (mostly size) and $3.09\ M_{Pl}$ (mostly shape). At the derived point the floor's own energy is about $+0.071\ M_{Pl}^4$, counted within M_{Pl}^4 (Chapter 13, Section II); with the curvature energy, whose bare part $-3/(8\pi)\ M_{Pl}^4$ stands beside M_{Pl}^4, the internal space contributes $-0.049\ M_{Pl}^4$ in all".

Source: *The Gravitational Passage*, Appendix D, Section III; *The Compactification*, Chapter 2, Section V. Standing: the values of $\zeta(-2)$ are exact; κ and μ are set by the balance at the derived point (not a test); the stability of the

derived point and the sign of the least-extent term are computed at first order (tests, passed); the masses of the moduli are computed at first order.

At the derived ratio of the two radii, the square root of six, the sum over the levels is an exact fraction. The sum is negative at every shape, while a single scalar field gives a sum of the opposite sign. Every energy minimum lies above about one and a half Planck lengths. The least-extent term is the fifth power of the curvature in each of the six directions, summed over the six directions. The least-extent term has two constants: about nine parts in a hundred thousand, and about 0.65 of the Planck mass. The two constants are set so that the minimum falls at the derived point. So the place of the minimum is not a test of the computation. The minimum stays stable wherever the two constants place it within two ranges. The first range is ratios of the two radii up to about two and a half (2.50), at the derived radius. The second range is radii of the larger factor above about one and nine tenths Planck lengths (1.87), at the derived ratio. Two results are tests: the minimum is stable, and the least-extent term has the sign that the balance needs. The derived point passes both tests. The minimum's barrier lies at a radius of about 1.79 Planck lengths. At that radius the complex projective plane is curved more strongly than the Planck curvature, so the barrier lies past the modal boundary. The six can change in two ways, in size and in shape, and the two ways are the two moduli of the six. The two moduli have masses of about two and a quarter Planck masses and about three Planck masses. The floor's own energy is about seven hundredths of the Planck density. With the curvature energy added, the six contribute about minus five hundredths of the Planck density in all.

Checks of the method

"The method reproduces the published value for six-dimensional gravity on $M^4 \times S^2$, $-2249/840$ (Cho and Kantowski 2000, equation 124), and, for the present geometry, the count of 35 polarizations of ten-dimensional gravity in the leading coefficient, and the local value of its next coefficient".

Source: *The Gravitational Passage*, Appendix D, Section II. Standing: computed.

The physicists Cho and Kantowski published a value for gravity in six dimensions on a sphere: minus 2249 over 840. Applied to gravity in six dimensions on a sphere, the method of the floor computation produces the same value. Applied to the present shape of the six, the method produces the right count in its leading term. The count is thirty-five, the number of independent ways in which a wave of ten-dimensional gravity can stretch space.

One disputed level of the sum

"The singlet spin-two level, constant on the internal space, is the arena's own wave. On the flat background its eigenvalue is $R/2 = 18/R_O^2$ [...] and it adds $+1620$ to $\zeta(-2)R_O^4$. On the arena's own background, which solves the Einstein equations with the observed Λ [...], it is massless and adds nothing. Without it, $\zeta(-2)R_O^4 = -11985037937/3326400$, every minimum lies above $1.996\, l_{Pl}$, the net is $-0.040\, M_{Pl}^4$, and the derived point is again a stable minimum". "Does the result depend on the disputed level, the arena's own wave? Its numbers do; its conclusion does not. [...] In both cases the derived point, $\sqrt{6}$ Planck lengths, is a stable minimum, and the moduli have masses of the order of the Planck mass".

Source: *The Gravitational Passage*, Appendix D, Section IV and Questions. Standing: computed; the floor on a background that solves the four-dimensional equations is not computed.

One level of the sum is the wave of the large dimensions themselves. That level is counted in two ways, according to the background used. On the flat background, the level adds to the sum. On a background that solves Einstein's equations with the observed cosmological constant, the level has no mass and adds nothing. With the level counted, every minimum lies above about one and a half Planck lengths. With the level left out, every minimum lies above about two Planck lengths. In both counts the derived point is a stable minimum. In both counts, too, the size and the shape of the six have masses of the order of the Planck mass.

What rests on the computation, and what does not

"It is a first-order description at the boundary: at every balance of this kind the zero-point energy is between one half and three quarters of the curvature energy, 0.59 at the derived point, and the balance itself fixes that ratio. At the modal boundary neither description dominates (Chapter 17, Section II), and these numbers are that statement in energies; no parameter of the theory makes the next order small, since the field that holds the space is the metric itself. The radius and the ratio are derived in Chapter 9, Section IX, and do not rest on this computation. The depth of the minimum and the masses of the moduli are fixed here at first order only, and the order of those masses, far above what the inverse-square tests ask, is what the argument of the book takes from it". Appendix D of *The Gravitational Passage* opens with this line: "Used by: no later step; the chain takes the hold of the internal space as a requirement".
Source: *The Gravitational Passage*, Appendix D, opening and Section VI; *The Compactification*, Chapter 2, Section V. Standing: a statement of standing, at the scope of the mechanism.

The computation describes the floor at first order only. The floor is at the Planck scale, where the smooth description and the stepwise description of nature meet and neither one dominates. At every balance between the zero-point energy and the curvature energy, the zero-point energy is between one half and three quarters of the curvature energy. So neither

energy is small beside the other, and nothing in the theory makes the next order small. The account uses one result from the computation. The size and the shape of the six have masses of the order of the Planck mass, far above any mass that the tests of gravity at short range can detect. The hold rests on the argument from Axiom III and the uncertainty relation. The radii and their ratio rest on the identification of Chapter 3, that every direction of the six is curved at the Planck curvature. No later step uses the computation. *The Gravitational Passage* leaves three things about the mechanism of stiffness unfixed. The first thing that *The Gravitational Passage* leaves unfixed about the mechanism of stiffness is the energy of the mechanism beyond first order. The second thing that *The Gravitational Passage* leaves unfixed about the mechanism of stiffness is the floor on a background that solves the four-dimensional equations. *The Gravitational Passage* does not compute that floor. The third thing that *The Gravitational Passage* leaves unfixed about the mechanism of stiffness is the measure of distance between geometries. The measure of distance meant here is the measure taken once the coefficients of the higher Lovelock terms are fixed (Appendix D, Section IV and Questions). The higher Lovelock terms are the further terms, with higher powers of the curvature, that Lovelock's theorem allows. The hold rests on none of the three.

The external value of the vacuum's energy

$$f\,{M_{Pl}}^4 = {M_{Pl}}^4 - (3/(8\pi))\,{M_{Pl}}^4, \;\; f = 1 - 3/(8\pi) = 0.8806 \quad (2.5)$$

Source: *The Compactification*, Chapter 2, Section V, equation (2.5); *The Gravitational Passage*, Chapter 13, Section II, equation (13.1), which prints 0.881. Standing: structural value (the trim); the ground-state part is taken as ${M_{Pl}}^4$ by identification.

The external value is the vacuum's total energy at the Planck scale. The external value is the Planck density, less the bare curvature term of the six. The bare curvature term is three over eight pi of the Planck density. So the trim is one minus three over eight pi: about 0.88, or eighty-eight hundredths. The trim is the factor that multiplies the Planck density.

The Lovelock terms, beside the mechanism of stiffness

"The Lovelock terms (Lovelock 1971), of which the term of Gauss and Bonnet is the first, can by themselves hold a curved internal space under flat open dimensions, without a cosmological constant, once terms cubic in the curvature are present (Müller-Hoissen 1985); the same class of theories gives four dimensions with a small cosmological constant (Canfora, Giacomini, Troncoso and Willison 2009). So their coefficients bear on the hold as well as on the decay of the breathing excitations".
Source: *The Weight of Possibility*, Chapter 4, Section III. Standing: established physics, cited. The quotation names a further possible mechanism of stiffness, beside the computed mechanism. The hold itself rests on the argument from Axiom III and the uncertainty relation, given above under the ground of the hold.

The action of the metric allows further terms with higher powers of the curvature, the Lovelock terms. In other theories, the Lovelock terms can by themselves hold a curved set of small dimensions under flat large dimensions. So the sizes of the Lovelock terms affect how stiffly the six are held. The sizes also affect how fast the breathing of the six decays. The account does not fix the sizes of the Lovelock terms.

The breathing excitations

"The higher Lovelock terms leave the form of four-dimensional gravity unchanged" (*The Gravitational Passage*, Chapter 11, Section I). *The Weight of Possibility* says that the coupling of the breathing to the open waves "vanishes at the order the Passage computes, but the term of Gauss and Bonnet, which the ten-dimensional action admits with a coefficient the Pas-

sage does not fix (the Passage, Chapter 11, Section I), couples the breathing of the internal volume to the waves of the open geometry, and no symmetry forbids that coupling". "For a coefficient of natural size, the lowest breathing excitation decays within about a Planck time. The estimate from perturbation theory, about 10^{-46} seconds, is shorter than the Planck time of 5.4×10^{-44} seconds, so it shows only that nothing suppresses the decay. A lifetime above 10^{11} years would need the coefficient below about 8.8×10^{-33} Planck areas, and the floor's own energy at one loop is expected to produce a larger effective coefficient even where the coefficient of the action is zero". "Other even levels, which are not invariant under the isometries of the internal space, cannot pass their energy to the waves of the open geometry in this way, since those waves are invariant under the isometries; their other channels of decay are open".
Source: *The Weight of Possibility*, Chapter 4, Section III; *The Gravitational Passage*, Chapter 11, Section I, for the form of four-dimensional gravity. Standing: computed for a Gauss-Bonnet coefficient of natural size; argued (no symmetry forbids the coupling).

The Lovelock terms change the values of three constants and leave the form of gravity in the four large dimensions unchanged. The three constants are the Planck mass of the large dimensions, the bare curvature term, and the strength with which the size and the shape of the six couple to other things. All three constants are named again under Chapter 5. The breathing of the six is a swelling and shrinking of their size. The term of Gauss and Bonnet couples the breathing of the volume of the six to the waves of the geometry of the large world. No symmetry forbids that coupling. For a coefficient of natural size, the breathing decays within about a Planck time. The rough estimate of the lifetime is about ten to the minus forty-six seconds, which is shorter than the Planck time itself, so the rough estimate shows only that nothing holds the decay back. If the breathing were to last a hundred billion years, the coefficient would have to be smal-

ler than about nine parts in ten to the thirty-three of a Planck area. The energy of the floor is expected to produce a larger effective coefficient even where the action itself has no term of Gauss and Bonnet.

The smoothness the re-forming needs

$$10^{-27} \approx e^{-62};\ \ yield \propto e^{-\omega\tau},\ \omega\tau \approx 62;\ \ e^{2\pi} \approx 535,\ \ 535^{10} \approx 10^{27}$$

"A suppression of 10^{-27} is about e^{-62}. Where the yield falls as $e^{-\omega\tau}$, with ω the frequency of the level and τ the time over which the change is smooth, that needs $\omega\tau$ of about 62, a change smooth over about ten periods of the level".

Source: *The Weight of Possibility*, Chapter 4, Section VI; *The Compactification*, Chapter 1, Section VII. The last two figures of the displayed line are the arithmetic behind a sentence of this book's Chapter 4: "each swing of the gyration over which the change is spread cuts the yield about five hundredfold". One period of the level is $\omega\tau = 2\pi$, and $e^{2\pi}$ is about 535. Standing: a condition on the turn, not a result; the numbers are arithmetic.

The re-forming of the six must directly make about ten to the twenty-seven times fewer gyrations than a sudden re-forming would make. Take the yield to fall as the exponential of minus a product. The product is the frequency of the level times the time over which the change is smooth. Then a suppression of ten to the twenty-seven needs a product of about sixty-two. One period of the level is a product of two pi. Each period cuts the yield by about five hundred and thirty-five times, and ten periods cut the yield by about ten to the twenty-seven. Put another way, ten to the minus twenty-seven is about e to the minus sixty-two: one divided by e, sixty-two times over. The number e, about two and seven tenths, is the number on which natural growth and decay are built. *The Weight of Possibility* uses an estimate for a change that is smooth over a stretch of time. In that estimate, the yield falls by a factor of e for each part in two pi of a

period that the stretch lasts. One part in two pi of a period is about a sixth of a period. Sixty-two steps of a sixth of a period each make about ten periods. So the change must be smooth over about ten periods of the level.

The ceiling on the first heat

"The divisor of the mass runs from about twenty-eight to about thirty-four according to the strength of the coupling; for a gyration of $\sqrt{2}$ Planck masses the limit is about 5×10^{17} GeV, and for 0.6 Planck masses about 2×10^{17} GeV. This is the production studied for dark matter that interacts with the plasma through gravity alone, whose yield falls exponentially once the mass exceeds the temperature (Garny, Sandora and Sloth 2016; Garny, Palessandro, Sandora and Sloth 2018). The divisor is computed here from their method and their cross sections; they do not state it. The limit concerns the highest temperature the epoch reaches, which can exceed any later temperature of reheating (Chung, Kolb and Riotto 1999)". "The limit on the temperature assumes couplings suppressed by the Planck mass; exchange of a massive particle of spin two coupled more strongly than gravitation, as posited in the account of a portal of spin two (Bernal and others 2018), would produce more gyrations from the plasma and so tighten it. The fields of the rotations of the sphere, which couple to the gyrations at a strength of about 6 (Section V), would tighten the limit only to about one thirty-fourth of the mass if they were present and free in the plasma, and would leave it unchanged if they are confined". "Stable excitations of a compact space would be about as numerous as the photons if the epoch began near the Planck temperature (Kolb and Slansky 1984)".

Source: *The Weight of Possibility*, Chapter 4, Section VI. Standing: a requirement on the turn, from the measured amount of dark matter; the divisor is computed.

A hot gas makes pairs of anything with weight, through gravity alone. The yield of the pairs falls steeply once the mass of the particle is larger than the temperature of the gas. The gas of the new epoch can never have

been hotter than about one thirtieth of the mass of a gyration, because a hotter gas would have made more gyrations than the measured amount of dark matter allows. The exact divisor lies between about twenty-eight and about thirty-four, according to the strength of the coupling. Garny, Sandora and Sloth and their colleagues are the physicists who studied production through gravity. *The Weight of Possibility* computes the divisor from the method and the rates of Garny and his colleagues. Garny and his colleagues do not state the divisor themselves. For a gyration of the square root of two Planck masses, the limit is about five times ten to the seventeen billion electron volts. For a gyration of six tenths of a Planck mass, the limit is about two times ten to the seventeen billion electron volts. A stronger channel of production would lower the limit. If the fields of the rotations of the sphere were free in the gas, those fields would lower the limit only to about one thirty-fourth of the mass, but the fields would not lower the limit at all if the fields are confined.

Chapter 5, One Vacuum

The items under Chapter 3 contain the radii and the volume, equations (1.1) and (2.1) to (2.3). The items under Chapter 4 contain the floor and the external value, equations (2.4) and (2.5).

Why two moduli

"Let G/H be a homogeneous space, and let H act on the tangent space at one point by its isotropy representation. A G-invariant Riemannian metric on G/H is the same thing as an inner product on that tangent space which the isotropy representation preserves. If the isotropy representation is irreducible over the real numbers, Schur's lemma shows that any two such inner products differ by a positive constant factor, so the G-invariant metrics form a single ray: one metric up to scale (Wolf 1968; Besse 1987, Chapter 7)". "$\mathbb{C}P^2$ is the quotient $SU(3)/S(U(2) \times U(1))$, and the isotropy group acts on the four-dimensional tangent space as $U(2)$ acts on $\mathbb{C}^2$,

which is irreducible over the real numbers. S^2 is the quotient SO(3)/SO(2), and the isotropy group acts on the two-dimensional tangent space by rotation, which is irreducible over the real numbers". On the product of the two factors, the two isotropy representations are inequivalent. So every invariant metric has the form (2.1), with two parameters and no others. Of the two parameters, *The Compactification* says: "Their logarithms, taken in the combinations that change the volume and the ratio R_O/R_T, are the volume modulus and the shape modulus".

Source: *The Compactification*, Chapter 2, Section III. Standing: theorem (Schur's lemma on the isotropy representation), given the identification that each factor is homogeneous.

On a shape that looks the same at every point, take a metric that respects all the symmetries of the shape. The value of that metric at one point fixes the metric everywhere. At that one point, the metric must respect the symmetries that keep the point in place. For the complex projective plane and for the sphere, the symmetries that keep a point in place mix all the directions at the point together. So each factor has only one metric that respects all its symmetries, up to the size of the metric: a theorem of the mathematician Issai Schur, known as Schur's lemma. The symmetries never mix the directions of one factor with the directions of the other factor, so the six have exactly two numbers that can change: the overall size, and the ratio of the two radii.

The Einstein point

"A Riemannian product of two Einstein spaces is itself an Einstein space exactly when the two Ricci constants are equal, because the Ricci tensor of a product is the sum of the Ricci tensors of its factors (Besse 1987, Chapter 1)".

Source: *The Compactification*, Chapter 2, Section IV. Standing: theorem of Riemannian geometry; the common value is fixed by the identification given under Chapter 3.

Each factor is curved equally in every direction of its own. The product of the two factors is curved equally in every direction exactly when the two factors are curved by the same amount. The reason is that the curvature of a product is the sum of the curvatures of its factors. Equal curvature in the two factors fixes the ratio of the two radii at the square root of six.

The higher terms of the action

The Compactification describes the higher Lovelock terms as terms "whose coefficients *The Gravitational Passage* does not fix, adding only to the four-dimensional Planck mass, to the bare curvature term and to the couplings of the moduli".
Source: *The Compactification*, Chapter 2, Section VI, with equation (2.6), which repeats the action (1.2). Standing: argued.

The terms with higher powers of the curvature are of the same sort as the action of the metric. The higher terms, too, are built from the geometry alone. The higher terms change only three things: the Planck mass of the large dimensions, the bare curvature term, and how strongly the size and the shape of the six couple to other things. None of the higher terms brings in a field.

No flux, and the integers the space does carry

"The integers that the internal space does carry are topological counts of its own, not flux quanta: its Euler characteristic, $3 \times 2 = 6$, and the count $c_1 = \chi(\mathbb{C}P^2) = 3$, which enters the cosmological constant as a count of channels and in no other way: 'No property of the first Chern class beyond this equality is used here' (*The Gravitational Passage*, Chapter 13, Section II)". In a sidebar of its Chapter 2, Section VI, *The Compactification* speaks of "a field the action refuses, and a field it already has". On the content side, the phase of the hypercharge boson comes from the spin^c structure of $\mathbb{C}P^2$ as a space of states, on which "no field lives" (*The Kinds of Particle*, Chapter 3, Section IV). On the content side, too, "Neither the labels nor

the count involves a field, a flux or a size". On the context side, the excitations of the internal factor "couple to content only through gravity" (*The Branches of the Kinds*, Chapter 2, Section III). The forms of the internal factor carry the spin^c structure "with the connection that the metric induces" (*The Gravitational Passage*, Chapter 12, Questions).

Source: *The Compactification*, Chapter 2, Section VI, and its sidebar. Standing: argued (the action of the metric alone; a further field refused, with its reason).

The many vacua of string theory come from fields that thread the holes of the small dimensions, and each threading is counted by a whole number that could be chosen. This account has no field that threads the six, and so no whole number can be chosen. The whole numbers that the six do carry are counts of their own shape: six cells in all, and three for the complex projective plane. The count of three enters the energy of empty space as the count of channels, and in no other way. The account contains no further field on the six, and the absence of a further field takes away no force of matter. One of the forces of matter carries a phase: hypercharge, a part of the joint electromagnetic and weak force. The phase of hypercharge comes from the spin^c structure of the complex projective plane, read on the space of states, on which no field lives, while on the six of the context instance the phase is the one that the geometry supplies. Neither phase is carried by a field with an energy of its own, and neither phase carries a whole number that could be chosen.

No second vacuum to tunnel to

"In a theory with several vacua, a region held in one of them can pass to another of lower energy by the nucleation of a bubble, at a rate set by the action of a Euclidean bounce (Coleman and De Luccia 1980)". "At the order computed, and within the shapes from $\sqrt{3}$ to $\sqrt{12}$ that the computation covers, there is one minimum and no second vacuum to which a bubble could lead".

Source: *The Compactification*, Chapter 2, Section VII; the numbers 1.48 l_{Pl} and 1.79 l_{Pl} are given under Chapter 4. Standing: computed at first order; the hold does not rest on the computation.

A vacuum can decay into a lower vacuum by forming a bubble. The physicists Sidney Coleman and Frank De Luccia showed how to compute the rate of that decay. Decay by a bubble needs a second, lower vacuum to lead to. The computation covers the shapes whose ratio of the two radii lies between the square root of three and the square root of twelve, and within that range of shapes there is only one minimum, at first order. Toward larger size, the energy rises to zero with no barrier.

The vacuum of the content: the Higgs field

$$\lambda_{tree} = a^2/2 = (1/2)^2/2 = 1/8 \quad (2.7)$$

$$m_H = v\,\sqrt{(2(1/8 + 1/(24\pi^2)))} = 125.17\ GeV \quad (2.8)$$

"Fluctuations along the finite directions are the commutators $[D_F, a]$; since D_F acts along the edge I–O (Proposition 8.4), they form a field that is a doublet under $\mathbb{H}$: the Higgs field" (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section II). The form of the potential comes from the heat-kernel expansion of the spectral action (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section III). In (2.7), a = 1/2 is "the doublet's half (the weak isospin of each side)". In (2.8), v = 246.22 GeV, and "The value m_H = 125.17 GeV, from $\lambda = 1/8 + 1/(24\pi 2) = 0.129222$ and the measured v, is one measured agreement". "The loop term is positive and carries no logarithm", and the loop term "is not produced by the running of a coupling". The loop term differs in sign and form from the top quark's Coleman-Weinberg contribution in the Standard Model. "Whether the couplings of the Standard Model, taken at their measured values and run by the renormalization group to scales far above the weak scale, leave the electroweak vacuum stable or metastable is a question about the content that this book does not take up. In the

Standard Model as usually computed, with the measured masses of the Higgs boson and the top quark, the quartic coupling runs to small or negative values at high scales and the vacuum is found to be metastable with a lifetime far longer than the age of the universe (Buttazzo and others 2013)"; "This chapter makes no claim about the stability of the content's vacuum under such running. Its claim concerns the context instance: the geometry of the internal space has one held configuration".
Source: *The Compactification*, Chapter 2, Section VIII, after *The Measures of the Kinds*, Chapter 13, Sections II, IV and VI. Standing: $\lambda = a^2/2$ is an identification, with $a = 1/2$ by three routes; $m_H = 125.17$ GeV is a measured agreement (measured: 125.13 ± 0.11 GeV); the stability of the content's vacuum under running to high scales is a statement of scope: not taken up.

The Higgs field is a ripple of the operator that measures distance in the small, finite part of the geometry. The ripple lies across the edge between two pieces of that finite part. The Higgs field comes as a pair, a doublet. The Higgs field is not a change of the six. At tree level (the simplest level of the calculation), the self-coupling of the Higgs field is one half squared, divided by two: one eighth. The one half is the weak charge of each member of the pair. *The Measures of the Kinds* derives that one half by three separate routes and sets the three routes side by side. The self-coupling then takes a small positive correction of one over twenty-four pi squared. With the correction, the Higgs mass comes out at 125.17 billion electron volts. The calculation uses the measured strength of the Higgs field, 246.22 billion electron volts. The measured Higgs mass is 125.13 billion electron volts, give or take 0.11. The correction brings the self-coupling from one eighth to about thirteen hundredths, and the correction is positive. The correction does not come from a strength that changes with the energy at which the strength is measured. In sign and in form, the correction differs from the correction that the top quark makes in the Standard Model. The Standard Model is physics' established account of the particles and of every force but gravity. In this book I do not take up one question about

the content. Physicists can follow the strengths of the Standard Model up to energies far above the scale of the weak force. The question is whether the Higgs vacuum stays stable at energies that high. Physicists usually compute the Standard Model with the measured masses of the Higgs boson and the top quark. In that computation, the self-coupling of the Higgs field falls to small or negative values at very high energies. The Higgs vacuum then comes out metastable, with a lifetime far longer than the age of the universe (Buttazzo and others, 2013). That computation is established physics. I make no claim about the stability of the Higgs vacuum at very high energies.

One algebra, one space

Non-Commutative Geometry and the Immanent Metaphysics, Theorem 12.1, "states that a comparison triple satisfying its postulates P-III, P-I and P-Rep has the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, KO-dimension 6, and the structural constants $(c_1, N, H) = (3, 4, 5)$. The theorem holds at the standing of its postulates, each of which is an identification. The pure states of $M_3(\mathbb{C})$ form $\mathbb{C}P^2$, and the ray space of $\mathbb{H}$, the space of complex lines in its module $\mathbb{C}^2$, is the sphere S^2 (*The Quantum Passage*, Chapter 5, Section VIII); their product is the internal space". "*The Kinds of Particle*, Chapter 3, Section II, derives the same uniqueness for the space read as a space of states: among all products of compact homogeneous spaces that satisfy its seven conditions, $\mathbb{C}P^2 \times S^2$ is the only solution".
Source: *The Compactification*, Chapter 2, Section IX, with *The Kinds of Particle*, Chapter 3, Section II, as it cites it. Standing: theorem under postulates; theorems of topology under identifications.

The shape of the six follows from algebra too, by a second route. The algebra that comparison requires is the complex numbers beside the quaternions beside the three-by-three complex matrices. The three-by-three matrices have pure states that form the complex projective plane, and the quaternions have states that form the sphere. So the algebra produces the same product as the route through geometry. *The Kinds of Particle* derives

the same product for the six read as a space of states. *The Kinds of Particle* sets seven conditions. Among all products of compact homogeneous spaces that satisfy the seven conditions, the complex projective plane times the sphere is the only one.

The standing of the steps of the one vacuum

Step	Kind	Where
The content instance has no size, no fields and no moduli	definition of the content instance	*The Kinds of Particle*, Ch. 1, §XIII
Each factor of the context instance is homogeneous	identification	*The Gravitational Passage*, Ch. 9, §II
A homogeneous $\mathbb{C}P^2 \times S^2$ has exactly two metric parameters	theorem (Schur's lemma on the isotropy representation)	Section III; Wolf 1968; Besse 1987
A product of Einstein spaces is Einstein iff the Ricci constants are equal	theorem of Riemannian geometry	Section IV; Besse 1987
Every internal direction has Ricci curvature $1/l_{Pl}^2$	identification	*The Gravitational Passage*, Ch. 9, §IX
$R_O = \sqrt{6}\, l_{Pl}$, $R_T = l_{Pl}$	theorem (algebra of Einstein products), given the identification	(2.3)
The internal space has a floor and is held while its conditions hold	argued (Axiom III and the uncertainty relation)	*The Gravitational Passage*, Ch. 11, §II
$\kappa = 8.98 \times 10^{-5}$, $\mu = 0.651\, M_{Pl}$	set by the balance at the derived point (not a test)	*The Gravitational Passage*, App. D, §III
Stability of the derived point; sign of the least-extent term	computed at first order (tests, passed)	*The Gravitational Passage*, App. D, §III

Step	Kind	Where
Moduli masses 2.25 and 3.09 M_{Pl}	computed at first order	*The Gravitational Passage*, App. D, §III
No form field and no flux integer	argued (the action of the metric alone; a further field refused, with its reason)	*The Gravitational Passage*, Ch. 11, §I; Ch. 12, Questions
No second minimum within the range computed	computed at first order	Section VII
The hold ends only where its conditions fail	argued	*The Gravitational Passage*, Ch. 18, §VI
$\lambda_{tree} = 1/8$	identification ($\lambda = a^2/2$), with $a = 1/2$ by three routes	*The Measures of the Kinds*, Ch. 13, §IV
$m_H = 125.17$ GeV	measured agreement	*The Measures of the Kinds*, Ch. 13, §VI
One algebra; one internal space by two routes	theorem under postulates; theorems of topology under identifications	NCG Theorem 12.1; *The Gravitational Passage*, Ch. 9, §VII

Source: *The Compactification*, Chapter 2, Section X; the sections named in the third column are that chapter's.

The table sets each step of the argument for one vacuum beside the standing of the step. The statement that the six look alike at every point is an identification. Two further statements are theorems once the identification of the Planck curvature is made. The first of the two further statements is that, once the identification of the Planck curvature is made, the six have exactly two numbers that can change. The second of the two further statements is that both of the two numbers that can change are fixed at the derived point. The floor is argued. The constants of the least-extent term are set by the balance at the derived point. Two results are computed at first order: the stability of the derived point, and the masses of the size and the shape of the six. The statement that no field threads the six is ar-

gued. The hold ends only where the conditions of the hold end, at the turn, and that statement is argued too. The self-coupling of the Higgs field is an identification, and the Higgs mass is one measured agreement.

Chapter 7, A Clean Slate, Counted

The items under Chapter 9 contain the count of a black hole, one quarter of the area of its horizon. The items under Chapter 9 also contain the reading of the count as a count relative to the outside.

The ground of the reset

"The standing of the step is argued: it is drawn from the definition of an entropy as a count relative to a context, and from the account of the turn (*The Gravitational Passage*, Chapter 18, Sections I to VII), in which the old exterior has no moment at the end of the interiors. The reset does not depend on the dynamics by which the internal space returns to its compact form in the new epoch, or on the temperature at which the reconstructed history begins (Chapter 1, Sections VII and VIII)". *The Gravitational Passage* states the limit of its own claim: "The reset is given here by principle; a count of states across the turn, in numbers, is not given in this book".
Source: *The Compactification*, Chapter 3, Section V, quoting *The Gravitational Passage*, Chapter 18, Section VIII. Standing: argued; a count across the turn in numbers is outside the scope of both books.

The reset is drawn from two things: the definition of an entropy as a count relative to a context, and the description of the turn in *The Gravitational Passage*. The reset does not depend on the way the six return to their small held size in the new epoch, or on the temperature at which the reconstructed history begins. Neither *The Gravitational Passage* nor *The Compactification* gives a count of states across the turn in numbers. The reset is given as a principle, and the principle is the whole scope of the reset. So Chapter 10 makes its argument from the reset in words. The physicist Roger Penrose gives a figure for the improbability of the beginning. Chapter

10 argues that Penrose's figure needs a context that tells the possible beginnings apart, and that at the turn no such context exists. Penrose's figure rests on a comparison of the actual beginning with the possible beginnings. A numerical count of states across the turn would give that comparison a numerical form. Neither book gives such a count.

The even beginning

"The account does not say that the whole beginning was improbable and then explain the improbability away. It says that the part of the beginning that the statistical arrow needs to be low, the gravitational part, is low because the geometry of a new context has nothing to be lumpy with".

Source: *The Compactification*, Chapter 3, Section VI, after *The Gravitational Passage*, Chapter 18, Section VIII. Standing: argued.

The growth of entropy needs one part of the beginning to be low, the gravitational part, and the account says that the gravitational part is low because the geometry of a new context has nothing to be lumpy with. The account does not call the whole beginning improbable and then explain the improbability away.

How far the even beginning reaches

"The cosmological constant is fixed once, by the present measurement (Chapter 13, Sections V and VI), and de Sitter space is the geometry of an idealized content (Chapter 10, Section III); the geometry found carries content, its comparers among it. The step requires the departures, and it fixes neither the spectral index nor w".

Source: *The Gravitational Passage*, Chapter 18, Section VIII, quoted in part in *The Compactification*, Chapter 3, Section VI. Standing: a statement of scope.

The Gravitational Passage adds two statements of scope, which say what the even beginning does and does not fix. The cosmological constant is the energy of empty space that speeds up the expansion, and the present measurement fixes the cosmological constant once. The perfectly even de

Sitter space is the geometry of an idealized content. But the geometry actually found carries content, with the comparers among that content. The step of the even beginning requires the departures from uniformity. The step fixes neither the spectral index nor w. The spectral index says how the strength of the departures changes from small sizes to large. The number w is the ratio of the pressure of the energy of empty space to its amount in each unit of volume.

Information that falls into a hole formed from a star

"For a hole formed from a star, *The Unquantized Arena* requires only 'that the information is not destroyed and is not accessible from outside while the hole stands', and it asserts neither the return of that information to the outside world nor its loss to it".
Source: *The Compactification*, Chapter 3, Section IX, after *The Unquantized Arena*, Chapter 17, its opening summary and Section VII. Standing: a statement of scope.

For a black hole formed from a star, *The Unquantized Arena* requires only two things: the information is not destroyed, and the information is not accessible from outside while the hole stands. *The Unquantized Arena* asserts neither that the information returns to the outside world nor that the information is lost to the outside world. Chapter 9 of this book has the same scope in its main text.

Chapter 8, The One Edge That Sets the Vacuum

The formula for the cosmological constant

$$\rho_\Lambda = c_1 f {M_{Pl}}^4/S, \quad \Lambda = 8\pi G_N \rho_\Lambda, \quad S = \pi R^2/{l_{Pl}}^2 \quad (4.4)$$

"where R is the radius of the observer's particle horizon and S is counted in nats", with $c_1 = 3$ and $f = 1 - 3/(8\pi) = 0.8806$ (*The Gravitational Passage* prints 0.881), and M_{Pl} the full Planck mass, 1.221×10^{19} GeV, so that ${M_{Pl}}^4$

$= 2.222 \times 10^{76}$ GeV^4. "The particle horizon is not a thermodynamic horizon in the sense of Gibbons and Hawking. I assign it the Bekenstein–Hawking count by an identification: the bound of what has been compared is the bound of the information available to the comparison".
Source: *The Compactification*, Chapter 4, Section III, equation (4.4); *The Gravitational Passage*, Chapter 13, Section II, equations (13.1) to (13.3). Standing: identification (the accessible share c_1/S, and the count of the particle horizon); computed.

The energy of empty space equals three, times the trim, times the fourth power of the Planck mass, divided by the count of the particle horizon, and the cosmological constant is eight pi times Newton's constant times that energy. The count of the particle horizon is pi times the square of the radius of the horizon, divided by the square of the Planck length, and the count is made in nats. A nat is the natural unit of a count of information. The account sets the ground-state part of the external value equal to the Planck density, but *The Gravitational Passage* does not fix the number in front of the Planck density (its Chapter 13, Questions). By a further identification, the ground-state part includes the energy of the floor that holds the six.

The identity, in three lines

$$I(\Omega_m) = \int_0^\infty dz/E(z), \quad E(z) = \sqrt{(\Omega_\Lambda + \Omega_m(1+z)^3 + \Omega_r(1+z)^4)} \quad (13.4)$$

$$\rho_\Lambda = c_1 f M_{Pl}{}^2 H_0{}^2/(\pi I^2) \quad (13.5)$$

$$\rho_\Lambda = \Omega_\Lambda \times 3H_0{}^2 M_{Pl}{}^2/(8\pi) \quad (13.6)$$

$$I^2 \times \Omega_\Lambda = 8c_1 f/3 = 8f = 8 - 3/\pi \quad (13.7)$$

"The integral is the comoving distance light has traveled over the whole expansion, and the particle horizon is $R = I/H_0$, in units with $c = 1$", with $\Omega_r \approx 9 \times 10^{-5}$. "Equating the two, the Hubble constant and the Planck mass cancel". "The 8 has a simple reading. The 8π of the Einstein equa-

tions, divided by the π of the horizon entropy, gives 8; the three channels, $c_1 = 3$, cancel the 3 of the critical density". *The View from Inside* adds: "Notice that the book now has three different threes, and they should not be confused: the three channels, from the roles of comparison (Chapters 9 and 12); the 3 in the critical density, from the three dimensions of space; and the 3 in the trim, from the calculation of the compact internal space (Chapter 9). Here the first cancels the second, and the third stays inside f". For a spatially flat universe, $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$, and the solution is $\Omega_m = 0.311$, $\Omega_\Lambda = 0.689$ (13.8). *The View from Inside* gives the unrounded values $\Omega_m = 0.3108$, $\Omega_\Lambda = 0.6891$.

Source: *The Gravitational Passage*, Chapter 13, Section III; *The View from Inside*, Chapter 13, "Setting the Two Equal", and Appendix C. Standing: computed, from the identification; the flatness used is argued (*The Gravitational Passage*, Chapter 18, Section IX).

The number written I is the size of the particle horizon, measured in Hubble radii. The number I is the distance that light has traveled over the whole expansion, and so the number I depends only on the shares of matter, radiation and empty space. The formula of the account states the energy of empty space in terms of the Hubble rate and I, while the astronomers' pie of Chapter 14 states the same energy as the vacuum's slice of the critical density, the whole pie. When the two expressions for the energy of empty space are set equal, the Hubble rate and the Planck mass cancel. What remains is the identity: the square of I, times the vacuum's share, equals eight less three over pi, about 7.05. In the identity, the eight is the eight pi of Einstein's equations divided by the pi of the count of the horizon. The three channels cancel the three of Friedmann's equation, which is the three of the critical density. The three over pi comes from the trim, because eight times the trim is eight less three over pi. In a flat universe the shares fill the whole pie, so the identity has one unknown, the matter share, and the solution of the identity is a matter share of 0.311, thirty-one point one percent.

The identity in other units

$$R = 2\sqrt{2}\ \sqrt{f}\, r_\Lambda = 2.65\, r_\Lambda,\quad \Lambda = 3/r_\Lambda{}^2;\quad t_0 H_\Lambda = 0.792,\quad H_\Lambda = \sqrt{(\Lambda/3)}$$

"With $\Lambda = 3/r_\Lambda{}^2$, r_Λ the de Sitter radius, it says that the present particle horizon is $2\sqrt{2}\ \sqrt{f} = 2.65$ times the de Sitter radius. With the measured parameters the ratio is 46.1/17.5 = 2.63 (both in billions of light years). Equivalently, the identity fixes the age of the universe in units of the de Sitter time, $1/H_\Lambda$ with $H_\Lambda = \sqrt{(\Lambda/3)}$: it gives $t_0 H_\Lambda = 0.792$, against 0.787 $\pm$ 0.009 from the measured parameters". On any horizon of radius R, the value of the formula is "$8f/(R\ H_\Lambda)^2$ times the constant". On the Hubble radius and on the event horizon, the value "exceeds 8f = 7.05 times the constant at every epoch".

Source: *The Gravitational Passage*, Chapter 13, Sections III and VI. Standing: computed; the same test as Ω_m.

Stated without Planck units, the identity says that the particle horizon is about two and sixty-five hundredths times the de Sitter radius. The de Sitter radius is the radius of the horizon of a universe that holds only the energy of empty space. Stated in time, the identity says that the age of the universe is about 0.792 of a de Sitter time. The measurements give 0.787, give or take 0.009. The de Sitter time is the de Sitter radius divided by the speed of light. The agreement in time is the same test as the agreement in the matter share, stated in time. On the Hubble radius and on the event horizon, the formula always gives more than seven times the constant, because both edges are always smaller than the de Sitter radius. So no measurement could find the constant on either edge.

The numbers, with one set of parameters

$$\rho_\Lambda = 3 \times 0.881 \times (2.222 \times 10^{76}\ GeV^4)/(2.29 \times 10^{123}) = 2.56 \times 10^{-47}\ GeV^4 \quad (13.9)$$

"The parameters are those of Planck 2018, used throughout as one set: h = 0.674, $\Omega_m = 0.315$, $\Omega_r = 9 \times 10^{-5}$, and a flat universe. With them the particle horizon is 46.1 billion light years, and its entropy is $S = 2.29 \times 10^{123}$". From the parameters of Planck 2018, the observed value is $\rho_{\Lambda,obs} = 2.52 \times 10^{-47}$ GeV^4 (13.10). "With one set of parameters, the ratio of the predicted density to the observed one is exactly $8f/(I^2\Omega_\Lambda)$, with I evaluated at the measured Ω_m". *The View from Inside*, Appendix C, gives the ratio as 1.0168. "With the ground-state part alone, the predicted density is 2.91×10^{-47} GeV^4, about fifteen percent above the observed one". With f = 1 the identity is $I^2\Omega_\Lambda = 8$, and the solution of that identity is $\Omega_m = 0.280$. "Counting the floor outside the ground-state part would give $\Omega_m = 0.292$". *The Gravitational Passage* names three coefficients of order one that would change the numbers above, and the comparison with measurement sets none of the three. The first of the three coefficients of order one is the coefficient of the ground-state part. The ground-state part is taken as exactly one Planck density, and *The Gravitational Passage* does not fix the coefficient of the ground-state part. The second of the three coefficients of order one is the internal curvature. The internal curvature is taken as exactly the Planck curvature, by the identification of Chapter 3. The third of the three coefficients of order one is the classical energy of the higher Lovelock terms, and *The Gravitational Passage* does not fix that classical energy. That classical energy would shift the bare curvature term, and with it the trim (Chapter 13, Questions).

Source: *The Gravitational Passage*, Chapter 13, Sections III and IV, with Figure 3, which draws $I^2\Omega_\Lambda$ against the matter share with the levels $8 - 3/\pi$ and 8; *The Compactification*, Chapter 4, Section III; *The View from Inside*, Appendix C. Standing: computed; measured (the parameters of Planck 2018).

With the measured parameters, the particle horizon is about 46.1 billion light years. The count of the particle horizon is then about 2.29 times ten to the one hundred twenty-three. The value of the formula is 2.56 times ten to the minus forty-seven, in billions of electron volts to the fourth power: three times 0.881 times the fourth power of the Planck mass, divided by the count of the particle horizon. The observed value is 2.52 times ten to the minus forty-seven, in the same unit. With the ground-state part alone, without the trim, the formula gives 2.91, about fifteen percent too much. The matter share would then be twenty-eight percent. With the energy of the floor counted outside the ground-state part, the matter share would be 29.2 percent.

Why nats, and what the count gives each degree of freedom

"I count S in nats, the measure in which S is stated and in which $dE = T\,dS$ holds with Boltzmann's constant equal to one. In that measure Section III finds that the observed density gives each degree of freedom at the de Sitter radius the energy $\hbar c/(2\pi r_\Lambda)$, the value at which Bekenstein's bound is saturated. The choice of measure decides the number: in bits the same horizon counts $S/\ln 2$, and one bit for each channel would give $\Omega_m = 0.407$ in place of 0.311". "With the observed density, at the particle horizon, it is $I^2\Omega_\Lambda$ times the corresponding value there, $\hbar c/(2\pi R)$. The formula requires 8f times that value. Equating the two is the identity".
Source: *The Gravitational Passage*, Chapter 13, Sections II and III. Standing: argued (the measure); computed.

The count is made in nats. In nats, the law that ties energy, temperature and count holds in its plainest form: a change of energy equals the temperature times the change of count. Counted in nats, the observed en-

ergy of empty space gives each degree of freedom at the de Sitter radius a definite energy. That energy is Planck's constant times the speed of light, divided by two pi times the de Sitter radius. That energy is the most that the bound of the physicist Jacob Bekenstein allows. Counted in bits instead, with one bit for each channel, the matter share would come out at 40.7 percent.

The ground of the count of three

"The ground of the count is not that content and context are inseparable. The three modal roles are inseparable too (Axiom III), so inseparability treats the rows and the columns alike: counting inseparable terms once would give one channel, and counting distinct terms separately would give six. The ground is the difference between the two members of the row pair: content is what a measurement registers, and context is where the measurement registers it". "The classes that share one class of S^2 number three, whichever class of S^2 it is: $\chi(\mathbb{C}P^2) \times 1 = 3$". "The first Chern class of the product is the sum of the first Chern classes of its factors: three times the class of a line, from $\mathbb{C}P^2$, and twice the class of a point, from S^2. It is a pair of integers, $(3, 2)$, and not one count". $\mathbb{C}P^n$ "has $n + 1$ cells, one in each even dimension from 0 to $2n$", and the first Chern class of $\mathbb{C}P^n$ is $n + 1$ times the class of a hyperplane (Milnor and Stasheff 1974). "I write the count c_1, the coefficient of the first Chern class of $\mathbb{C}P^2$, because on every complex projective space the two integers are equal, and for the same reason: each counts the alternatives of the system whose pure states the space is"; "No property of the first Chern class beyond this equality is used here".

Source: *The Gravitational Passage*, Chapter 13, Section II. Standing: identification (the channels as the contents a measurement registers); theorem (the counts of the shape).

The count of three channels rests on the difference between the two rows of the grid of Chapter 8, content and context. Content is what a measurement registers, and context is where the measurement registers

the content. The count of three does not rest on content and context being inseparable, because the three roles of comparison are inseparable too. In the shape of the six, the classes of the complex projective plane that share any one class of the sphere number three: three times one. *The Gravitational Passage* writes the count of three as "c one". The name "c one" is borrowed from a second count of the shape of the plane, its first Chern class, which has the same number, three. The two counts agree on every complex projective space. A complex projective space is the space of the pure states of a system with some number of alternatives. On every complex projective space, the coefficient of the first Chern class equals the number of alternatives. The number of alternatives is also the number of cells of the space. The account uses no property of the first Chern class beyond the equality of the two counts, and the count is three.

The other counts, and what they would give

"Six counts each context as a channel. Two counts the rows. Five, the sum of the two parts of the first Chern class of the product, adds a count of rows to a count of roles. One, the largest integer that divides that class, counts nothing in the grid. The identity of Section III gives $\Omega_m = 0.163$ with six channels, 0.197 with five, 0.418 with two and 0.311 with three, against the measured 0.315 ± 0.007". "The share is of a count, not a fluctuation in one. A share of $1/\sqrt{S}$ would be the relative size of a statistical fluctuation in a count of S, and log S grows with S, so it cannot give a suppression at all".

Source: *The Gravitational Passage*, Chapter 13, Questions. Standing: computed; "the reason for three is the one above, which does not refer to them".

The other whole numbers that the shape offers have no reading as the count of channels. With six channels the matter share would be 16.3 percent; with five channels, 19.7 percent; and with two channels, 41.8 percent. With three channels the matter share is 31.1 percent. If a share were one over the square root of the count, the share would be the size of a chance

fluctuation in the count, and the size of a chance fluctuation is not a share of the count. If a share were built on the logarithm of the count, the share could not make the energy small either, because the logarithm of the count grows with the count.

The five choices, varied together

Edge (number of channels across)	1	2	3	4	5	6
Hubble radius	3.9	7.8	11.7	15.6	19.5	23.4
Event horizon	2.9	5.9	8.8	11.8	14.7	17.7
de Sitter radius	2.7	5.3	8.0	10.7	13.3	16.0
Particle horizon	0.38	0.77	1.15	1.54	1.92	2.31

The entries are the formula's value as a multiple of the observed energy of empty space, with the ground-state part of the external value alone. "The entry the reasons select, the particle horizon with 3, is 1.15, and the classical part of the external value brings it to 1.017. The nearest other entries are 0.77 and 1.54. The reduced Planck mass in place of the full one divides every entry by $(8\pi)^2 \approx 632$. The ratio of exactly 8 at the de Sitter radius with 3 is an identity of de Sitter thermodynamics, $\rho_\Lambda = (3/8)\ M_{Pl}{}^4/S_{dS}$". *The View from Inside* varies all five choices at once: four edges; six whole numbers; the trim or not; the holding energy inside or outside the ground-state part; the full or the reduced Planck mass. The five choices give "$4 \times 6 \times 2 \times 2 \times 2 = 192$ combinations. Exactly one of them gives an energy of empty space within the measured uncertainty, about 2.8 percent either way: the combination the reasons select, at 1.017. The next nearest is the same combination with the holding energy added outside, at 1.10". *The Gravitational Passage* leaves out the integer five and counts 160 combinations, of which again exactly one falls within the measured uncertainty. "With the reduced Planck mass in place of the full one, the formula would be divided by $(8\pi)^2$, and every horizon would have an epoch of that kind;

the full Planck mass is fixed by the modal boundary (Chapter 17, Section III), not by this condition" (*The Gravitational Passage*, Chapter 13, Section VI).

Source: *The View from Inside*, Appendix A, Section IV; *The Gravitational Passage*, Chapter 13, Section VI and Questions; *The Compactification*, Chapter 6, Section X. Standing: computed; each choice is fixed by a reason that does not refer to the comparison with measurement.

The table shows what the formula would give on each of four edges, with each whole number of channels from one to six. Each entry is a multiple of the observed energy of empty space, before the trim. The particle horizon with three channels gives 1.15, and the trim brings the value to 1.017, within two percent of the observed energy. *The Compactification* records the search made over the choices of edge and of count. *The View from Inside* varies all five choices at once. The edge may be any of the four edges of the observer. The count may be any whole number from one to six in place of three. The trim may be used or left out. The floor may be counted inside the Planck density part or added to it. The Planck mass may be the full one or the reduced one. The five choices give four times six times two times two times two combinations, which is 192 combinations. (*The Gravitational Passage*, which leaves out the whole number five, counts 160.) Exactly one of the 192 combinations gives an energy of empty space within the measured uncertainty. That combination is the one that the account's reasons select. The next nearest combination gives a value ten percent too large. Some physicists use a reduced Planck mass, smaller than the full Planck mass by the square root of eight pi. With the reduced Planck mass, the share would be divided by eight pi, squared, about 632. Every edge would then give the constant at some era. The modal boundary fixes the full Planck mass, and the arithmetic of the edges plays no part in that choice.

The constant is one number

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

"In $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ the divergence of the Einstein tensor vanishes by the contracted Bianchi identity, the divergence of the metric vanishes, and the stress-energy of the content is conserved (Chapter 12, Section VII). If Λ varied from place to place or from epoch to epoch, the divergence of the left side would be the gradient of Λ, and the equations would set that gradient to zero. So within one geometry Λ does not vary. This is a theorem of the geometry, and it uses no measurement and no value of w". "A vacuum energy that is the same for every comparer, in whatever uniform motion, has a stress-energy proportional to the metric, and so it enters the field equations as such a constant term. A density that followed a horizon from epoch to epoch would be a property of the content at each epoch, with a frame of rest of its own, and it would not be the energy of the vacuum".
Source: *The Gravitational Passage*, Chapter 13, Section VI. Standing: theorem of the geometry; argued (that a density following a horizon would belong to the content).

In Einstein's equations, the curvature side and the matter side each have a total change of zero: the curvature side by an identity of geometry, and the matter side by the conservation of energy. If the cosmological constant changed from place to place or from era to era, the change would appear on one side of the equations with nothing to balance it, so the equations would force the change to zero. So within one geometry the cosmological constant is one number. The energy of empty space is the same for every comparer in steady motion, and so the energy of empty space enters Einstein's equations as the one constant term of the equations. If a density followed a horizon from era to era, the density would belong to the matter instead, with a frame of rest of its own, and so the density would not be the energy of empty space. A density that followed a horizon would not

speed up the expansion either. Einstein's equations make the expansion speed up only when the ratio of pressure to energy lies below minus one third. An energy that followed the particle horizon would have a ratio of pressure to energy of about minus 0.12 today. *The Gravitational Companion* gives minus 0.125 (Comparisons at Single Steps, Section VII), and *The View from Inside* gives minus 0.124 (Appendix A, Section V). The share of the account has the form of holographic dark energy. Holographic dark energy was proposed by the physicists Cohen, Kaplan and Nelson (1999), Hsu (2004) and Li (2004). Holographic dark energy is a density in proportion to the square of the Planck mass divided by the square of the length of a horizon. In the holographic program, the density follows the horizon as the universe expands. *The Gravitational Passage* fixes the density once (Chapter 13, Section VI), and the argument of *The Gravitational Passage* uses nothing from the holographic program (Preface).

A black hole's count, and the factor eight

$$c_1 {M_{Pl}}^4 / S_{BH} = 3{M_{Pl}}^4 {l_{Pl}}^2 / (\pi R^2) = 3{M_{Pl}}^2 / (\pi R^2) = 3/(\pi G_N R^2) \quad (4.6)$$

$$\bar{\rho} = (R/2G_N)/(4\pi R^3/3) = 3/(8\pi G_N R^2) \quad (4.7)$$

"So the ratio (4.6) is exactly eight times the mean density (4.7). This is arithmetic". "Restored to ordinary units, the ratio is $3c^4/(\pi G_N R^2)$, which carries no Planck constant". For a hole of one solar mass, eight times the mean density is "about 1.5×10^{20} kilograms in a cubic meter, about 0.64 GeV^4". That density is "about 2.5×10^{46}" times the measured density of the vacuum. "The total entropy of all the black holes in the observable universe, about 10^{104}, is a small part of the 2.29×10^{123} of the particle horizon (Egan and Lineweaver 2010). Those states enter the observer's count once, there". *The Compactification* gives the estimate of Egan and Lineweaver as "about 3×10^{104}" (Chapter 3, Section VIII).

Source: *The Compactification*, Chapter 4, Section IV, after *The Gravitational Passage*, Appendix B, Section VI. Standing: arithmetic; computed from the measured constants.

One can put the count of a black hole into the formula for the energy of empty space, in place of the count of the particle horizon. The value of the formula is then exactly eight times the mean density of the hole's mass within its horizon. The result contains no Planck constant at all. For a hole of one solar mass, the result would be about ten to the forty-six times the density of empty space. No density like that is seen near black holes. So the count of a black hole enters no energy of empty space. Together, the black holes in the observable universe carry a count of about ten to the power one hundred four. The astronomers Egan and Lineweaver estimated the count in 2010 at about three times ten to the power one hundred four. The count of all the black holes is a small part of the count of the particle horizon. The states of the black holes enter the observer's count once, as part of the count of the particle horizon.

Chapter 9, Black Holes Within an Epoch

The count of a horizon

$$S_{BH} = A/(4\, {l_{Pl}}^2) \quad (4.1); \quad S_{BH} = \pi R^2/{l_{Pl}}^2 \quad (4.2)$$

"For a hole of one solar mass, R is about 2.95 kilometers and S_{BH} is about 1.05×10^{77} (computed from the measured constants)". The reading: "I read it as the number of states of the hole that no comparison from outside tells apart, a count relative to the outside as context" (*The Unquantized Arena*, Chapter 17, Proposition 17.3), and the identification "makes no claim on the coefficient one quarter".

Source: *The Compactification*, Chapter 4, Section II, with Bekenstein (1973) and Hawking (1975). Standing: established physics; the reading is an identification of *The Unquantized Arena*.

The count of a black hole is a quarter of the area of the hole's horizon, measured in Planck areas. A Planck area is the square of the Planck length. For a hole that neither spins nor carries charge, the count is pi times the square of the horizon's radius, divided by the square of the Planck length. For a hole of one solar mass, the radius is about three kilometers, and the count is about ten to the seventy-seven. The factor of one quarter is Hawking's, from his calculation of 1975. I read the count as a count of the hole's states: the states that no comparison made from outside the hole can tell apart. So the count is relative to the outside. My reading makes no claim about the factor of one quarter. The factor of one quarter stays at the value that established physics gives.

The area theorem and the horizons of general relativity

"Hawking (1971) proved that, in a spacetime satisfying Einstein's equations with matter obeying the null energy condition, and under the assumption of cosmic censorship, the area of the event horizon of a black hole never decreases". "A closed two-dimensional spacelike surface is marginally trapped if the light rays leaving it outward, orthogonally, neither diverge nor converge: the expansion of the outgoing null normals vanishes on the surface, and the expansion of the ingoing null normals is negative"; "a surface on which only the first condition holds is a marginally outer-trapped surface. A three-dimensional surface foliated by marginally outer-trapped surfaces is a trapping horizon; where it is spacelike it is a dynamical horizon, and where it is null and unchanging it is an isolated horizon". For a dynamical horizon the increase of area "is balanced by the flux of energy across it, which is non-negative when matter obeys the dominant energy condition (Ashtekar and Krishnan 2004)".

Source: *The Compactification*, Chapter 4, Section V. Standing: theorems of general relativity, under their stated conditions.

In 1971 Hawking proved that the area of a black hole's horizon never shrinks. Hawking's proof assumes three things: Einstein's equations hold, the energy of matter is never negative along rays of light, and no singular-

ity lies in the open, outside every horizon. Whether a surface is the event horizon of a hole depends on the whole future of space and time. So general relativity also defines horizons from the geometry near a surface at one time. The building block is a marginally trapped surface. Outgoing light from a marginally trapped surface neither spreads apart nor gathers together, while light sent inward gathers together. A horizon built of marginally trapped surfaces grows while energy flows in across the horizon.

A merger of two equal holes

$$A_i = 8\pi {G_N}^2 M^2/c^4 \quad (4.8)$$

$$A_f = 8\pi {G_N}^2 M_f^2 (1 + \sqrt{(1 - \chi_f^2)})/c^4 \quad (4.9)$$

$$A_f/A_i = (0.95162)^2 \times (1 + \sqrt{(1 - 0.68646^2)}) \approx 0.9056 \times 1.7272 \approx 1.564 \quad (4.10)$$

The remnant is a Kerr black hole of mass M_f and dimensionless spin χ_f; numerical relativity gives $M_f = 0.95162$ M and $\chi_f = 0.68646$ for two equal holes without spin (Scheel and others 2009), and "about 4.8 percent of the initial mass-energy, $1 - 0.95162$, leaves as gravitational waves".
Source: *The Compactification*, Chapter 4, Section VI. Standing: established physics (numerical relativity); the ratio is arithmetic.

Take two equal black holes without spin, of total mass M. The two horizons together have an area of eight pi times the square of Newton's constant, times the square of M, divided by the fourth power of the speed of light. The two holes merge into one spinning hole. The final hole has about 95 percent of the mass M, and a spin of about 0.69 of the most spin a hole can have. The horizon of the final hole has about 1.56 times the area of the two first horizons together. About 4.8 percent of the mass M leaves as gravitational waves. The numbers of the merger are of three kinds. First, the final mass and the final spin are results of numerical relativity, cited from Scheel and others (2009). Second, the ratio 1.564 and the 4.8 percent are arithmetic on the final mass and the final spin. Third, this

book reads the growth of the area as a growth of the count. Reading area as count is the one identification by which this book reads the entropy of every black hole. None of the account's quantities is adjusted to fit the merger.

The glow of a black hole and the Page curve

$$T_H = \hbar c^3/(8\pi G_N M k_B) \quad (4.11)$$

$$S_{rad}(t) = min[S_{emit}(t), S_{BH}(t)] \quad (4.12)$$

$$S_{BH}(\tau) = S_0(1-\tau)^{2/3}; \quad \tau_P = 1 - [r/(1+r)]^{3/2} \quad (4.13)$$

For one solar mass T_H is "about 6.2×10^{-8} kelvin"; the evaporation "takes about 2.1×10^{67} years for one solar mass, emitting photons only, as a black body without greybody factors". Equation (4.12) holds "to within half a nat", under the two conditions (C) and (H) of *The Unquantized Arena*; τ is the fraction of the evaporation time that has passed, and r the ratio of the entropy emitted to the Bekenstein-Hawking entropy lost. "By the generalized second law, $r \geq 1$, and so the turning point comes before the hole has lost half its entropy, and no later than 0.646 of the evaporation time (Proposition 17.5)". "The small waves of the geometry enter this count in the description's own terms, as quanta of their description, which the account admits; a quantum of that description is not a quantum of the arena (Section IX). The curve is conditional, and the book states it so: it holds under (C) and (H), for a hole that evaporates while an outside world remains to receive the radiation. Its last stage, as the mass approaches the Planck mass, lies where the two descriptions meet, and there it rests on condition (C) alone".

Case	r	S_{BH}/S_0 at the turning point	τ_P
Bound from the generalized second law	1	0.500	0.646
Black-body emission into empty space	4/3	0.571	0.568
Photons and the small waves of the geometry, with greybody factors (Page 2013)	1.4847	0.598	0.538

Source: *The Compactification*, Chapter 4, Section VII, and Table 4.1, after *The Unquantized Arena*, Chapter 17, Propositions 17.4 and 17.5 and Table 17.1. The "greybody factors" are the part of the glow that the hole's own surroundings turn back. Standing: established physics (Hawking's result); the Page curve is a conditional result, holding under (C) and (H).

A black hole glows, at a temperature inversely proportional to the hole's mass. For a hole of one solar mass, the temperature is about six hundredths of a millionth of a kelvin, far colder than the oldest light. So a hole of one solar mass takes in more energy from the oldest light than the hole gives out, for a hundred billion years or more. A hole of one solar mass evaporates in about 2.1×10^{67} years. The figure of 2.1×10^{67} years counts the glow as light alone, given off as by a perfect warm body. *The Unquantized Arena* computes the figure from established physics. For a hole that evaporates while an outside world remains to receive the glow, and under two stated conditions, the entanglement entropy of the glow first rises and then falls. The first condition is that the matter inside the horizon and the radiation outside, taken together, began pure and stay pure, as described from one outside world. The second condition is that the entanglement entropy of the matter inside the horizon cannot exceed the hole's count. The entanglement entropy of the glow rises with the entropy the glow has carried away, and then falls with the count the hole still holds. The rise turns into the fall no later than about 65 percent of the way through the evaporation.

The time of the turning point depends on one ratio: the entropy the radiation gains, divided by the entropy the hole loses. The table above lists three worked cases, from Table 17.1 of *The Unquantized Arena*. The first case is the bound that the generalized second law sets, where the radiation gains exactly as much entropy as the hole loses. At the bound, the turning point comes when half the hole's count remains, at 0.646 of the evaporation time. In the second case the hole gives off its glow as a perfect warm body into empty space. The radiation then gains four thirds as much entropy as the hole loses. In the second case the turning point comes when 0.571 of the count remains, at 0.568 of the evaporation time. The third case is Page's. In 2013 Page computed the glow of a hole that gives off light and the small waves of the hole's geometry, allowing for the part of the glow that the hole's own surroundings turn back. In Page's case the radiation gains about one and a half times as much entropy as the hole loses (1.4847 times). In Page's case the turning point comes when 0.598 of the count remains, at 0.538 of the evaporation time. By then the hole has lost about two fifths of its count. Put Page's ratio into the rule for the turning point, equation (4.13) above. The rule then agrees with Page's own result: the radiation's entanglement peaks a little more than halfway through the hole's life. All three cases hold under the two conditions of the Page curve. Page counted the small waves of the geometry in the terms of the description of the small waves, by quanta. The account admits the description by quanta. Each quantum of the description is a quantum of the description of a small wave, and never a quantum of space and time.

The Page curve is a conditional result. The account states the Page curve together with its conditions. For a hole that evaporates while an outside world remains to receive the hole's glow, the Page curve holds under its two conditions. The last stage of the Page curve comes as the hole's mass approaches the Planck mass. The last stage lies where two descriptions meet: the smooth description of general relativity, and quantum theory. In the last stage, the Page curve rests on the first condition alone.

Where Hawking's result does not depend on the shortest lengths

"The actual derivation is a Bogoliubov transformation: the vacuum state defined before the collapse differs from the vacuum state defined after, and the mismatch produces a thermal spectrum at the Hawking temperature". "The modes that eventually become Hawking radiation were, at the moment of collapse, trans-Planckian". "Unruh (1995) and Corley and Jacobson (1996) changed the dispersion of the field at short wavelengths, and Corley and Jacobson (1998) put the field on a lattice. The thermal spectrum survived whenever the surface gravity lies far below the cutoff"; "An analog black hole in a Bose–Einstein condensate, whose short-wavelength physics is atomic, shows a thermal spectrum at the predicted temperature (Muñoz de Nova and colleagues 2019)".

Source: *The Gravitational Passage*, Chapter 18, Section X. Standing: established physics, cited.

Hawking's glow comes from the mismatch between the vacuum before the star collapsed and the vacuum after the collapse. If the waves that make the glow are followed back in time, the waves were once shorter than the Planck length, where the smooth description no longer holds. Physicists have tested whether the glow depends on waves shorter than the Planck length. In 1995 Unruh changed how the field behaves at very short waves. In 1996 Corley and Jacobson also changed how the field behaves at very short waves. In 1998 Corley and Jacobson set the field on a lattice, a grid with a smallest spacing. In every one of the three models, the warm glow survived whenever the hole's pull at the horizon was far gentler than the scale at which the model changed the field. The temperature of the glow still depended only on the hole's pull at the horizon. Physicists have also made a model black hole in a laboratory, in a cold gas of atoms (Muñoz de Nova and colleagues, 2019). The model black hole glows at the predicted temperature, even though in the cold gas the physics at short lengths is the physics of atoms. So the glow does not rest on what happens beyond the Planck scale.

The island formula, read as a calculation

The island formula (Penington 2020; Almheiri, Engelhardt, Marolf and Maxfield 2019; Almheiri, Mahajan, Maldacena and Zhao 2020) gives the same shape as (4.12). "Each saddle of the gravitational path integral is a classical solution; a replica wormhole is a saddle of a calculation, and 'It belongs to the calculation, and it is not the geometry that any one alternative of the matter carries'". "On the same ground the account keeps the smooth horizon and needs no firewall: past the turning point of the curve the partner modes behind the horizon are encoded in the early radiation"; the firewall argument is that of Almheiri, Marolf, Polchinski and Sully (2013).

Source: *The Compactification*, Chapter 4, Section VIII, after *The Unquantized Arena*, Chapter 17, Sections VI and VIII. Standing: argued.

Physicists have recently found the rise and fall of the Page curve by another method. In 2019 and 2020, physicists working separately computed the radiation's entanglement with a sum over geometries: Geoff Penington, and Ahmed Almheiri with several colleagues. Past the turning point, Penington and Almheiri's group add to the radiation a region behind the horizon, called the *island*, and count the island with the radiation. The result of the method is called the island formula. The island formula has the same shape as the Page curve: a rise, a turning point, and a fall with the count of the hole. In the account, the agreement is agreement in the shape of the curve. The answer of the sum is settled by geometries that are smooth classical solutions of Einstein's equations. One of the geometries that settle the answer is called a replica wormhole. The replica wormhole belongs to the calculation, and the replica wormhole is not the geometry that any one alternative of the matter carries. The account declines one reading of the sum over geometries. On the declined reading, the sum over geometries is space and time held in a quantum blend of shapes. The declined reading would put the arena into the quantum state. So the declined reading would repeat the premise of one quantum state: the

premise that the geometry and the matter together form one quantum state. The account has a smooth horizon and no wall of fire, on the same ground: the arena has no quantum state. Each late piece of radiation has a partner behind the horizon, and past the turning point the partner is encoded in the early radiation.

The coupling of matter to geometry

$$G_{\mu\nu}[g^{(i)}] = (8\pi G_N/c^4)\, T^{(i)}{}_{\mu\nu} \text{ for each } i \quad (4.14)$$

"Each alternative of the quantum matter carries its own classical geometry, shaped through Einstein's equations by the stress-energy of that alternative alone, and the relative phases of the alternatives are kept" (*The Unquantized Arena*, Proposition 7.1), for a state of the matter $|\Psi\rangle = \Sigma_i\, c_i |\psi_i\rangle$. The alternative, one geometry sourced by the expectation value, $G_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu}\rangle / c^4$, "is not the coupling of this account": by Proposition 7.3 of the same book, without reduction of the state the experiment of Page and Geilker (1981) contradicts it, since "the measured field followed the outcome, not the average over outcomes", and with reduction of the state it signals faster than light.

Source: *The Compactification*, Chapter 4, Section IX. Standing: identification (one geometry for each alternative; *The Gravitational Passage*, The Chain at a Glance, link 12).

When the matter is in several alternatives at once, each alternative has its own geometry, shaped by the matter of that one alternative alone. The phases between the alternatives are kept. The rival way of joining matter to geometry gives all the alternatives one geometry, shaped by the average of the alternatives. The rival way is not the account's way, for a reason found by experiment. In 1981 Page and Geilker set the masses of a balance by a random quantum event. The measured pull followed the outcome, not the average. In this book, a graviton is at most a quantum of the de-

scription of the small waves of a classical geometry. A graviton is never a quantum of space and time, never a particle, never a carrier of gravitation, and never one of the kinds of particle.

Chapter 10, The Counts of an Epoch

The count of the present horizon, and the room above the matter

$$S = \pi R^2 / {l_{Pl}}^2 \approx 2.29 \times 10^{123} \quad (3.2)$$

$$(8\pi^2/135)({g_*}^{1/2}/1.66)(T/M_{Pl}) \approx 3.64\, T/M_{Pl} \quad (3.3)$$

For the region within the Hubble radius R = 1/H, "In a flat epoch the Friedmann equation gives the density $3H^2/(8\pi G_N)$, so the mass-energy within the Hubble radius is $R/(2G_N)$, and a black hole of that mass has a horizon of radius R itself", so the largest count grows as t^2; the count of the matter, of density $(2\pi^2/45)\ g_*\ T^3$, with T falling as $t^{-1/2}$ and the volume growing as t^3, grows as $t^{3/2}$ (Frautschi 1982, after Layzer 1975). Equation (3.3) is the count of the matter within the Hubble radius divided by the largest count of that region, with $H = 1.66\ {g_*}^{1/2}\ T^2/M_{Pl}$ and $g_* = 106.75$; at 2 to 5×10^{17} GeV it is 0.060 to 0.149. "Applied, on paper, to the particle horizon of other eras of the one reconstructed history, the formula gives these multiples of the measured constant: 1.224 a billion years ago, 7.049 at redshift 1, and 0.852 a billion years from now"; "none of these is a finding; each is a number computed from the structure of the history" (*The View from Inside*, Appendix A, Section V).

Source: *The Compactification*, Chapter 3, Section VIII; Chapter 5, Section VII, equation (5.4); *The View from Inside*, Appendix A, Section V. Standing: computed; established physics for the counts (Frautschi 1982, after Layzer 1975, as *The Compactification* computes them).

The count of the present particle horizon is about 2.29 times ten to the one hundred twenty-three. Take the region within the Hubble radius at any moment of the radiation era. The largest count the region could have is the count of one black hole made of all the mass in the region. The largest count grows as the square of the time, while the count of the matter grows only as the time to the power three halves. The matter held at most about fifteen percent of the largest count of the region at the start of the history. The room above the matter is the gap between the count of the matter and the largest count. The room above the matter widened from then on.

Chapter 8 states the account's formula for the cosmological constant, and the formula divides by the count of the observer's particle horizon. On paper, the formula can be applied to the horizon that an observer would have had at another era. For an observer a billion years ago, the formula gives 1.224 times the measured constant. For an observer when distances were half of today's, the formula gives 7.049 times the measured constant. For an observer a billion years from now, the formula gives 0.852 times the measured constant. *The View from Inside* says of the three numbers, "None of these is a finding; each is a number computed from the structure of the history." The cosmological constant is fixed once, and the reconstructed history carries the one fixed value at every era.

The settling waves counted as light species

$$\Delta N_{eff} = (8/7)(11/4)^{4/3}(106.75/2)(3.909/106.75)^{4/3}\,\varepsilon \approx 2.86\,\varepsilon, \quad 3.909 = 2 + (7/8)(6)(4/11) \quad (5.1)$$

Here ε is the share of the energy of the radiation that the waves carried when the count of species was 106.75, and "The bound of three percent is therefore an excess of light species below about 0.09".

Source: *The Compactification*, Chapter 5, Section II, equation (5.1), computed there. Standing: computed.

The settling waves are the gravitational waves made as the six settle into their held size. Suppose the settling waves carried some fraction of the energy of the radiation when every kind of particle was present. The settling waves then count today as an excess in the number of light species of about 2.86 times the fraction. The bound of three percent on the fraction sets the excess at less than about nine hundredths of a species.

The count of a hot gas, and the photons today

$$s = (2\pi^2/45)\, g_s\, T^3 \quad (5.2)$$

Each fermionic state is weighted 7/8, and "a species at a temperature other than T is weighted by the cube of its temperature over T". "The microwave background is at $T_0 = 2.72548 \pm 0.00057$ K (Fixsen 2009), which is 11.902 inverse centimeters in the units of equation (5.2)"; with $g_s = 2$, "the density of their count is about 1,479 per cubic centimeter, 3.602 times their number density of 411 per cubic centimeter", and the region within the particle horizon has "a volume of about 3.48×10^{86} cubic centimeters".

Source: *The Compactification*, Chapter 5, Sections III and V. Standing: established physics; computed.

In each cubic centimeter, a hot gas has a count of two pi squared over forty-five, times the number of the gas's kinds of state, times the cube of the gas's temperature. The oldest light is at 2.72548 kelvin. The oldest light carries a count of about 1,479 in each cubic centimeter. The count is about 3.6 times the number of the light's photons in each cubic centimeter, 411.

The count of the black hole at the center of the Galaxy

$$S_{BH} = A/(4\,l_{Pl}{}^2) = \pi R_s{}^2/l_{Pl}{}^2, \quad R_s = 2GM/c^2 \quad (5.3); \quad 1.05 \times 10^{77} \times (4 \times 10^6)^2 \approx 1.7 \times 10^{90}$$

Source: *The Compactification*, Chapter 5, Section VI, computed there. Standing: established physics; computed.

The count of a black hole grows as the square of the hole's mass. The hole at the center of our Galaxy has about four million solar masses. So the hole at the center of our Galaxy holds a count of about 1.05 times ten to the seventy-seven (the count of a hole of one solar mass), times sixteen million million (the square of four million): about 1.7 times ten to the ninety. The Galaxy's central hole holds more count than all the photons and neutrinos of the early universe in the whole region we can see.

The count of a sphere of the de Sitter radius

The formula for the cosmological constant is given under Chapter 8. "The count of a sphere of the de Sitter radius, $\pi\, r_\Lambda{}^2/l_{Pl}{}^2$, is about 3.31×10^{122}, computed here. Put into equation (5.5) in place of S, it gives exactly 8f, or 7.05, times the observed constant (*The View from Inside*, Chapter 12), and *The Gravitational Passage* (Chapter 13, Questions) gives the ratio as 8.0 without the trim", with $r_\Lambda = \sqrt{(3/\Lambda)} = 17.53$ billion light years.

Source: *The Compactification*, Chapter 5, Section VII. Standing: computed.

The de Sitter radius is about 17.5 billion light years. A sphere of the de Sitter radius has a count of about 3.31 times ten to the one hundred twenty-two. Put the count of the sphere in place of the count of the particle horizon. The account's formula for the cosmological constant then gives about seven times the observed constant (7.05). Without the trim, the formula gives 8.0 times the observed constant on the sphere of the de Sitter radius. *The Gravitational Passage* gives the ratio without the trim as 8.0.

The moments of equal shares and of the release of the light

"that redshift is $z_{eq} = 3402 \pm 26$ (Aghanim and others 2020, Table 2; *The Coming of the Kinds*, Chapter 13, Section I), when the photons were at $2.72548 \times 3403 = 9{,}275 \pm 71$ kelvin, about 0.80 eV". "The light of the microwave background last scattered at the measured redshift 1089.92 ± 0.25, when it was at about 2,973 kelvin".

Source: *The Compactification*, Chapter 5, Section V. Standing: measured.

Matter came to hold as much energy as radiation when distances were 3,403 times smaller than today, give or take 26. At the moment of equal shares, the light was at about 9,275 kelvin. The oldest light went free when distances were about 1,091 times smaller than today. The light was then at about 2,973 kelvin.

Temperatures as energies

The Coming of the Kinds, Box 2.1: $k = 8.617333262 \times 10^{-5}$ eV K^{-1}, exact; "1 eV/k = $1/(8.617333262 \times 10^{-5})$ K = 11,604.5 K"; "$kT_0 = 2.72548 \times 8.617333 \times 10^{-5}$ eV $= 2.349 \times 10^{-4}$ eV"; the ceiling of 2 to 5×10^{17} GeV "is 0.016 to 0.041 of M_{Pl}. Its ratio to m_t is $2 \times 10^{17}/172.60 = 1.2 \times 10^{15}$ to $5 \times 10^{17}/172.60 = 2.9 \times 10^{15}$", with $M_{Pl} = {G_N}^{-1/2} = 1.220890 \times 10^{19}$ GeV.

Source: *The Coming of the Kinds*, Chapter 2, Box 2.1. Standing: measured constants; arithmetic.

One electron volt of energy is a temperature of about 11,600 kelvin. The oldest light today is at about 2.35 ten-thousandths of an electron volt. The highest temperature of our epoch lies below a ceiling of two to five times ten to the seventeen billion electron volts. The ceiling is between about one and a half and four percent of the Planck mass. The ceiling is also more than a thousand million million times the mass of the heaviest kind of particle, the top quark.

The count of states, and the race with the expansion

$$g_* = 28 + (7/8) \times 90 = 28 + 78.75 = 106.75$$

$$H = (8\pi G_N \rho/3)^{1/2} \simeq 1.66\, N(T)^{1/2}\, T^2/M_P$$

Box 3.2: "Bosons: 8 gluons × 2 = 16; 3 weak bosons × 2 = 6; 1 hypercharge boson × 2 = 2; Higgs doublet 4; total 28. Fermions, per generation: [...] total 30. Three generations: 90", and "The Particle Data Group's table of N(T) gives 4N(T) = 427 for $m_t < T$, which is N = 106.75"; the right-handed neutrino is left out. Box 3.3: "At T = 1000 GeV: H = [...] 1.40 × 10^{-12} GeV; the expansion time is ħ/H = [...] 4.7 × 10^{-13} s. Electromagnetic reaction at 1000 GeV: $\Gamma \approx \alpha^2 T$ [...], so $\Gamma/H \approx 4 \times 10^{10}$"; for the right-handed neutrino, with $y_\nu = 2.87 \times 10^{-13}$, Γ/H is 3.7 × 10^{-10} at 159.5 GeV, "and at every higher temperature it is smaller". Box 3.1: a kind much heavier than the temperature holds, per state, against the photon, the share $(x/2\pi)^{3/2} e^{-x}/0.1218$, with x = m/T: 7.5 × 10^{-4} at x = 10, 9.6 × 10^{-8} at x = 20, and 5.6 × 10^{-16} at x = 40.

Source: *The Coming of the Kinds*, Chapter 3, Boxes 3.1, 3.2 and 3.3. Standing: established physics; computed.

When every kind of particle is present, and the mass of each kind is small beside the temperature, the gas has twenty-eight states of the force-carrying kinds and ninety states of the matter kinds. Each state of a matter kind is weighted seven eighths, so the count of species is 106.75 in all. At a temperature of a thousand billion electron volts, the distances of the universe grew by an appreciable share of themselves in about half a million-millionth of a second. At a thousand billion electron volts, electric reactions ran some forty billion times faster than the expansion, so the charged kinds stayed in balance. The right-handed neutrino's reactions run at less than four ten-billionths of the expansion rate, so the right-handed neutrino never comes into balance. When the mass of a kind is ten times the temperature, the kind has about seven ten-thousandths as many

particles per state as light does. When the mass is twenty times the temperature, the kind has about one ten-millionth as many particles per state as light does.

The measured temperatures of the changes of state

The electroweak crossover is centered at 159.5 $\pm$ 1.5 GeV (D'Onofrio and Rummukainen 2016); the strong crossover at 156.5 $\pm$ 1.5 MeV (HotQCD Collaboration 2019; Aoki and others 2006); a first-order electroweak transition "occurs only for a Higgs mass below the endpoint 72.4 $\pm$ 1.7 GeV" (Csikor, Fodor and Heitger 1999); the sphalerons leave balance at $T_* = 131.7 \pm 2.3$ GeV (D'Onofrio, Rummukainen and Tranberg 2014).
Source: *The Coming of the Kinds*, Chapters 4 and 8, and Box 4.3; *The Compactification*, Chapter 7, Section VIII. Standing: computed on the lattice (established physics), with the measured Higgs mass 125.13 $\pm$ 0.11 GeV.

For the measured Higgs mass, 125.13 GeV (give or take 0.11), the electroweak change is a crossover. A crossover is a smooth change with no sudden jump. The electroweak crossover is centered at 159.5 GeV (give or take 1.5), and its steepest part is about 5 GeV wide. A sudden change would happen only for a Higgs mass below an end point of 72.4 GeV (give or take 1.7), far below the measured mass. Quarks and gluons bind into protons, neutrons and the other particles made of quarks in a crossover at 156.5 MeV (give or take 1.5). The values of this section come from computations on a lattice. The lattice of the computations is a fine grid of points in space and time.

Chapter 11, The Oldest Light

The curvature perturbation

"The curvature perturbation $\zeta(\mathbf{x})$ is defined through the spatial metric $a^2 e^{2\zeta}\delta_{ij}$ of the surfaces of constant total density: it is the local departure of the logarithm of the scale factor on those surfaces, whose spatial

curvature is $-(4/a^2)\nabla^2\zeta$, at a comoving position **x**; on lengths larger than the horizon of the time considered it equals the comoving curvature perturbation, and there it is constant in time when the departures are adiabatic (defined below). Its Fourier components ζ_k are labeled by a comoving wavenumber k, and their variance per logarithmic interval of k is the power spectrum $P_\zeta(k)$".

Source: *The Compactification*, Chapter 6, Section I. Standing: "These are definitions of the standard description; none of them is an identification of this book".

Take the surfaces of space on which the total density is the same everywhere. On each surface of equal density, the local stretch of space differs slightly from place to place. The curvature perturbation is the local difference in the logarithm of the stretch. On lengths larger than the horizon at the time considered, the curvature perturbation stays fixed when every ingredient of matter and radiation ripples together. Split the curvature perturbation into waves of every length. The power spectrum is the strength of the waves in each range of lengths.

The form of the spectrum, and the first ripples' strength

$$P_\zeta(k) = A_s(k/k_*)^{n_s - 1} \quad (6.1)$$

$$|\Delta T/T| = \zeta/5 \quad (\textit{Sachs–Wolfe limit})$$

"where $k_* = 0.05$ Mpc^{-1} is a reference wavenumber chosen by convention, A_s is the amplitude of the power at k_*, and n_s is the scalar spectral index". "The square root of A_s is the strength of the departures, about 5×10^{-5} in ζ, and the temperature differences of the sky on the largest angles are smaller again by a factor of about five". Measured: $\ln(10^{10}A_s) = 3.044 \pm 0.014$, "which is $A_s = 2.10 \times 10^{-9}$", from the final results of the Planck satellite, "for temperature, polarization and lensing together" (Aghanim and others 2020); the first measurement of the differences is that of Smoot and others (1992).

Source: *The Compactification*, Chapter 6, Section I, equation (6.1). Standing: definitions of the standard description; A_s and n_s measured, and not fixed by the account.

At each length, the power of the first ripples is a fixed amount, the power at a reference length, times the ratio of the two lengths raised to a small power. The small power is the spectral index less one. The fixed amount is about two parts in a billion, so the ripples themselves are about five parts in a hundred thousand, the square root of the fixed amount. On the largest angles, the temperature of the sky differs from place to place by a fifth of five parts in a hundred thousand: about one part in a hundred thousand.

The ratio of first waves to first ripples

$$r = P_t(k_*)/P_\zeta(k_*) \quad (6.2)$$

"Waves of the geometry itself, transverse and traceless departures of the spatial metric, have their own spectrum $P_t(k)$". "In this book the letter r without a subscript is used only in this sense. Chapter 1 writes the Kasner exponent of the sphere as s, not r, for that reason".

Source: *The Compactification*, Chapter 6, Section I, equation (6.2). Standing: definition of the standard description.

The ratio written r is the power of the waves of the geometry itself at the reference length, divided by the power of the ripples of density at the same reference length.

When the electrons are bound

$$n_e n_p / n_H = (m_e T / 2\pi)^{3/2} \, e^{-B/T}$$

Box 13.1: "B = 13.605693 × 938.27209/938.78309 = 13.598 eV"; "Solved for T (numerically): X = 0.9 at T = 0.345 eV; X = 0.5 at T = 0.322 eV; X = 0.1 at T = 0.294 eV. By (6), these are 4,010 K, 3,730 K and 3,420 K. At X = 0.5, B/T = 13.598/0.322 = 42", with X the share of the hydrogen's electrons that

is free and 2.16×10^9 photons for each hydrogen nucleus (Saha 1920). Box 13.2: "$T_* = T_{\gamma 0}(1 + z_*) = 2.72548 \times 1090.92 = 2973.3$ K", with uncertainty 0.9 K; "$kT_* = 2973.3 \times 8.617333 \times 10^{-5}$ eV = 0.2562 eV. By (4): $13.598/0.2562 = 53$".

Source: *The Coming of the Kinds*, Chapter 13, Boxes 13.1 and 13.2; for the name, the Particle Data Group (2026), review "Big-Bang Cosmology", which "gives 'last scattering' as the more accurate name for the era commonly called recombination", as *The Coming of the Kinds*, Chapter 13, reports it. Standing: established physics (Saha's equation); measured (the redshift of last scattering, 1089.92 ± 0.25, and the present temperature).

In balance, one rule is true: the free electrons times the free protons, divided by the hydrogen atoms, equals a factor set by the electron's mass and the temperature, times the exponential of minus the binding energy over the temperature. The rule is Saha's equation, from 1920. Hydrogen binds its electron by 13.6 electron volts. But half the electrons are bound only when the gas has cooled to about 3,700 kelvin, because there are about two billion photons for each hydrogen nucleus. At 3,700 kelvin the energy of the heat is about a forty-second of the binding energy. The light went free at 2,973 kelvin, give or take one, when the energy of the heat was about a fifty-third of the binding energy. The Particle Data Group is the body of physicists that publishes the standard reviews of particle physics. The Particle Data Group's review of cosmology says that last scattering is a more accurate name than recombination for the era when the light went free.

The neutrinos' lower temperature

$$(T_\nu/T_\gamma)^3 = 2/(2 + (7/8) \times 4) = 4/11$$

"Before the annihilation, with $T \gg m_e$: $N_s = 2 + (7/8) \times 4 = 2 + 7/2 = 11/2$ [...]. After it, with $T_\gamma \ll m_e$: $N_s = 2$". "The neutrinos today are at $T_{\nu 0} = 1.945$ K, beside the light's 2.7255 K; each kind of neutrino, with its antineutrino, numbers 112 in each cubic centimeter, 336 for the three kinds".

Source: *The Coming of the Kinds*, Chapter 10, Box 10.1. Standing: established physics; computed.

Before the electrons and positrons annihilated, the light and the electron and positron pairs shared eleven halves of states. After the annihilation the light alone held two states, and the light took the pairs' share of the count. The neutrinos had already gone free before the annihilation, so the neutrinos missed the heat that the pairs passed to the light. So the cube of the neutrinos' temperature is four elevenths of the cube of the light's temperature: today about 1.95 kelvin against 2.73.

What is identified, and what follows

"Two steps must be kept apart. The identification of the departures with the form of the content is a first-person step, made with its observation, as above. The four properties of Sections III to VI are argued from that identification together with one premise: that the new context has no prior identity [...]. Each property follows from these in the form *must*: given the identification and the premise, the property holds in the reconstructed history of the epoch. None of the four fixes the amplitude or the tilt, which Section VII treats as not fixed".

Source: *The Compactification*, Chapter 6, Section II. Standing: a statement of the kinds of the steps (an identification; then properties argued from it).

I identify the first ripples with the form of the content. The identification is one step, made in the first person, together with the observation that bears on the identification. What follows from the identification is a second step, and the two steps are kept apart. Four properties of the first ripples follow, and each holds in the form "must". The first property is that the ripples are linked across the whole sky and nearly the same in strength at every length. The second property is that the ripples are one form for every component of the content. The third property is that first waves arise only from ripples acting on ripples. The fourth property is that the large-scale model of the epoch is flat in space. The premise beside the identification is that the new context has no prior identity: the new

context inherits no record, no length and no mark that sets one of its parts apart from another. If the identification and the premise are true, each of the four properties holds in the reconstructed history of the epoch. None of the four properties fixes the strength or the tilt of the first ripples.

Chapter 12, Four Marks of the Pattern

The spectrum (6.1), the ratio (6.2) and the relation $|\Delta T/T| = \zeta/5$ are given under Chapter 11.

A ripple frozen beyond the horizon

"A departure of the metric on a length λ does not change while λ lies beyond the horizon, and it can depend on a reference length ℓ only through the ratio λ/ℓ".
Source: *The Compactification*, Chapter 6, Section VII. Standing: established physics; the tilt and its sign are not fixed by the account.

While the length of a ripple of the geometry lies beyond the horizon, the ripple stays the same. A ripple beyond the horizon can depend on a reference length only through one ratio: the length of the ripple divided by the reference length.

The tilt and the running under a rule not adopted

"If the departures of the metric were largest on the length to which they refer, a reference to the length of the turn would give an index above one, and a reference to the de Sitter radius an index below one; but *The Gravitational Passage* identifies neither the length referred to nor the way the departures depend on it (Chapter 18, Section IX)". "If the reference weakens as the logarithm of the larger length over the smaller, and the departure varies as a fixed power of that logarithm, the size of the running is the departure of the index from one divided by the logarithm: about 6×10^{-4} for a reference to the length of the turn, and about 4×10^{-4} for a reference

to the de Sitter radius, negative in the first case and positive in the second when the index is below one". "These are consequences of a stated rule, which that book states as a condition and does not adopt; this book does not adopt it either. The measured running is consistent with zero, with an uncertainty of about 6×10^{-3}, more than ten times either estimate".
Source: *The Compactification*, Chapter 6, Section VII, and Table 6.1, after *The Gravitational Passage*, Chapter 18, Section IX. Standing: computed under a rule that neither book adopts; the account takes the running from measurement.

If the ripples were strongest on the length to which the ripples refer, a reference to the length of the turn would give an index above one, and a reference to the de Sitter radius would give an index below one. *The Gravitational Passage* names neither the length to which the ripples refer nor the way the ripples depend on that length. So the two indices are a case worked out, and not a result. *The Gravitational Passage* works out one more case, under a stated rule. Under the stated rule, the reference weakens as the logarithm of the larger length over the smaller. The ripple varies as a fixed power of the logarithm. Under the stated rule, the running is the tilt divided by the logarithm. The running is then about six ten-thousandths for a reference to the length of the turn, and about four ten-thousandths for a reference to the de Sitter radius. The measured index is a little below one. For an index below one, the running is below zero for a reference to the length of the turn, and above zero for a reference to the de Sitter radius. *The Gravitational Passage* states the rule as a condition and does not adopt the rule. Neither *The Compactification* nor this book adopts the rule. The measured running is consistent with zero, with an uncertainty of about six thousandths, more than ten times either computed running.

White noise

"Independent pieces, averaged, would leave white noise, with a spectral index of 4 in the curvature or 0 in the density, and the observed index is close to 1".

Source: *The Gravitational Passage*, Chapter 18, Section IX; *The Compactification*, Chapter 6, Section III. Standing: established physics (the arithmetic of averaging independent pieces); the observed index measured.

White noise is the pattern that independent pieces leave when the pieces are averaged. If the curvature of space is the quantity left as white noise, the spectral index is four. If the density of matter is the quantity left as white noise, the spectral index is zero. Both numbers come from the arithmetic of averaging independent pieces. Both numbers lie far from one, and the observed index is close to one.

The waves made at second order

"The waves of the metric do not bear on the sign. Their absence (Section XIII) rests on the scalar character of the primordial departures of the metric, which concerns the kind of departure that the content sources and not the way the departures depend on length. The waves that arise at second order are sourced by the scalar departures, and the logarithm of their power changes with length twice as fast as that of the scalar power. They carry the same reference, at a power far too small to measure it".
Source: *The Gravitational Passage*, Chapter 18, Section IX. Standing: established physics (the waves of second order); argued (that the absence of first waves rests on the kind of ripple, not on the tilt).

The tilt and the first waves are separate questions. The absence of the first waves depends on the kind of ripple the content makes: a ripple of density, with no stress pulling differently in different directions. The kind of ripple is a separate fact from the way the ripples depend on length. When ripples of density act on one another, the ripples make faint waves of the geometry. The power of the faint waves goes as the square of the power of the ripples. So the logarithm of the power of the faint waves changes with length twice as fast as the logarithm of the power of the ripples themselves. The faint waves therefore carry the same reference to a length as the ripples. But the power of the faint waves is far too small for the reference to be measured in the faint waves.

The measure of curvature, and the curvature the account gives

"The spatial curvature of the homogeneous model is reported as Ω_K, the share of the critical density that the curvature term carries in the Friedmann equation; $\Omega_K = 0$ is a flat space". "A radius equal to the de Sitter radius leaves a curvature today of about 10^{-64} as a share of the critical density"; *At the Edge of Reality*, Appendix C, gives 2.5×10^{-64}.
Source: *The Compactification*, Chapter 6, Sections I and VI; *At the Edge of Reality*, Appendix C, Part IV, after *The Gravitational Passage*, Chapter 18, Section IX. Standing: definition; computed (the curvature for a radius equal to the de Sitter radius).

Astronomers report the curvature of space as a share of the critical density: the share that the curvature carries in Friedmann's equation of the expansion. A share of zero is a flat space. If space were curved with a radius as large as the de Sitter radius, the share today would be about two and a half parts in ten to the sixty-four. The share of the curvature would be far too small to measure.

Chapter 13, No Inflation Needed

Why neither modulus can drive slow-roll inflation

$$|\eta| = M_{Pl,red}{}^2 |V''/V| << 1, \quad M_{Pl,red} = M_{Pl}/\sqrt{(8\pi)}; \quad V \approx 3H^2 M_{Pl,red}{}^2 \Rightarrow |\eta| = m^2/(3H^2) << 1$$

"At first order the masses of the two moduli are 2.25 and 3.09 Planck masses (Chapter 11, Section II; Appendix D, Section III). For a potential dominated by its constant part, $V \approx 3H^2 M_{Pl,red}{}^2$, the condition reads $|\eta| = m^2/(3H^2) << 1$ for a field of mass m and an expansion rate H; for masses of about the Planck mass it would need an expansion rate far above the Planck mass, beyond the modal boundary at which the description of the

exterior ends (Chapter 17). The step uses only the order of the masses, as the Passage's bound on the range of the force the moduli carry does (Chapter 11, Section II). Neither modulus can drive slow-roll inflation".
Source: *The Gravitational Companion*, Comparisons at Single Steps, Section IX. The reduced Planck mass appears here only because the standard condition is written with it. Standing: established physics (the slow-roll condition); argued.

Slow-roll inflation needs a field whose energy changes very little as the field changes. One condition of slow roll says that the square of the field's mass must be much less than three times the square of the expansion rate. The six have two moduli, their size and their shape. The masses of the two moduli are two to three Planck masses. So the two moduli could drive inflation only with an expansion rate far above the Planck mass, where the description of the large dimensions has already ended. The step that rules out the two moduli as drivers of inflation uses only the rough size of the masses, about a Planck mass. *The Gravitational Passage* sets a bound on the range of the force the moduli carry. The bound also uses only the rough size of the masses.

The fading of shear

$$H^2 = (\rho + \sigma^2)/3 \quad (8\pi G_N = c = 1); \quad \sigma \propto a^{-3}, \sigma^2 \propto a^{-6}, \rho_{radiation} \propto a^{-4}$$

"In a homogeneous expansion whose slices of space at one time are flat, with no stress that pulls differently in different directions, the shear falls as the inverse cube of the scale of distances (Ellis and MacCallum 1969; Tsagas, Challinor and Maartens 2008, Section 1.5.2)". "From then on the rate of expansion falls as the inverse square of the scale while radiation carries most of the energy, and as the inverse three-halves power once matter does; so the shear's share of the expansion falls as the inverse of the scale while radiation carries most of the energy, and as the inverse

three-halves power once matter does". The route needs flat slices: Collins and Hawking (1973) "concluded that the models expanding at just the escape velocity, which are the flat ones, would in general approach it".
Source: *The Gravitational Companion*, Comparisons at Single Steps, Section IX. Standing: established physics.

Shear is the part of an expansion that changes the shape of a small region without changing the region's volume. In an even expansion with flat slices of space and no stress that pulls differently in different directions, the square of the shear falls as the inverse sixth power of the scale of distances. The energy of radiation falls only as the inverse fourth power of the scale, so the square of the shear falls faster than the energy of radiation. The shear itself falls as the inverse cube of the scale. While radiation carries most of the energy, the rate of expansion falls as the inverse square of the scale. So the shear's share of the expansion falls as one over the scale while radiation carries most of the energy, and falls faster once matter carries most of the energy.

The fall, worked for one stretch of the history

$$3/159{,}500 = 1.9 \times 10^{-5};\ \ \textit{growth of scale} \geq \textit{about } 53{,}000;\ \ \textit{share} < \sqrt{(3/2)} \times 1.9 \times 10^{-5} = 2.3 \times 10^{-5}$$

"From the electroweak crossover at 159.5 GeV (Section VII) to 3 MeV, the temperature falls by the factor $3/159{,}500 = 1.9 \times 10^{-5}$, so the scale grows at least by the inverse of that factor, about 53,000". "By 3 MeV the share is therefore less than $\sqrt{(3/2)} \times 1.9 \times 10^{-5} = 2.3 \times 10^{-5}$: less than three parts in a hundred thousand of the rate of expansion then".
Source: *The Gravitational Companion*, Comparisons at Single Steps, Section IX. Standing: computed.

Take the stretch of the history from the electroweak crossover, at 159.5 billion electron volts, to the release of the neutrinos, at about three million electron volts. Over the stretch, the temperature falls to about two parts in a hundred thousand of its starting value, so distances grow at least about

53,000 times. Suppose the shear was as large as the whole expansion at the start of the stretch. By the release of the neutrinos, the shear would be less than about two and a third parts in a hundred thousand of the rate of expansion.

The collapse grows uneven

"Near the center of a black hole the geometry takes the Kasner form, in which each direction of space scales as a power of the time to the singularity, with exponents $-1/3$, $2/3$ and $2/3$, stretched along one direction and squeezed along the other two".
Source: *The Gravitational Companion*, Comparisons at Single Steps, Section IX, citing *The Gravitational Passage*, Appendix B, Section II. Standing: established general relativity.

Shear grows in a collapse by the law by which shear fades in an expansion, read the other way in time. Near the center of a black hole, one direction of space stretches in proportion to the time left to the end raised to the power minus one third. The other two directions squeeze in proportion to the time left to the end raised to the power two thirds.

Strings, walls and monopoles

"Stable strings form when a symmetry is broken such that the vacuum manifold has non-contractible loops, that is, when the first homotopy group of the vacuum manifold is not trivial (Kibble 1976). In the Standard Model the Higgs doublet breaks $SU(2) \times U(1)$ to the $U(1)$ of electromagnetism, and the vacuum manifold is the three-sphere, whose first homotopy group is trivial". "The three-sphere is also all one piece, and every sphere drawn on it shrinks to a point: its zeroth and second homotopy groups are trivial too. Domain walls form only when the vacuum manifold falls into separate pieces, and monopoles only when its second homotopy group is not trivial (Kibble 1976), so the breaking makes no domain walls and no monopoles either". "Strings of the electroweak field that are

not topologically protected exist as solutions, and they are unstable at the measured value of the weak angle (Vachaspati 1992; James, Perivolaropoulos and Vachaspati 1992). These are results of established physics".
Source: *The Compactification*, Chapter 7, Section IX. Standing: established physics.

When a field settles into its lowest state, the field can trap energy in strings, in walls or at points, according to the shape of the set of lowest states. Physicists call a point at which a settling field traps energy a monopole. For the Higgs field, the set of lowest states is a three-dimensional sphere. The three-dimensional sphere is all one piece, and every loop and every sphere drawn on the three-dimensional sphere can be shrunk to a point. So the Higgs field makes no stable strings, no walls and no monopoles. Some strings of the electroweak field are not protected by the shape of the set of lowest states. The unprotected strings do exist as solutions of the equations. But the unprotected strings are unstable at the measured value of the weak mixing angle (Vachaspati, 1992; James, Perivolaropoulos and Vachaspati, 1992). The weak mixing angle is a measured number that fixes how the weak force and electromagnetism are blended. The results of this section are results of established physics.

The gauge group, unbroken by unification

"the gauge group of the finite algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ is the product of its factors at every scale, and there is no X or Y boson and no force carrier joining quark to lepton (The Branches of the Kinds, Chapter 2; The Grand Synthesis, Chapter 14, Section V). SU(3) and SU(2) act as the isometry groups of $\mathbb{C}P^2$ and S^2 taken as spaces of states, and the U(1) of hypercharge as the phase of a line bundle over $\mathbb{C}P^2$ (The Kinds of Particle, Chapter 4, Section IV); the group is not the remnant of a larger group".
Source: *The Gravitational Companion*, Comparisons at Single Steps, Section IX. Standing: argued, from the particle books of the series.

At every energy, the symmetry of the forces is a product of three pieces. The symmetry of the forces is not what is left of one larger symmetry. So no larger symmetry breaks. Monopoles from the breaking of a larger symmetry therefore never form.

The ceiling on the first heat, and the vacuum's power

The masses of a gyration, $\sqrt{2}\ M_{Pl}$ at the classical level and about $0.6\ M_{Pl}$ at first order, and the ceiling of about one thirtieth of that mass, are given under Chapter 4. "Traced back to near the turn in the reconstructed history, a length observed today lay some 10^{27} times beyond the horizon of that time, and the vacuum of a wave so far beyond the horizon carries a power below 10^{-51}, against the observed 2×10^{-9}".
Source: *The Compactification*, Chapter 6, Section II; the values are computed in *The Gravitational Passage*, Chapter 18, Section IX, and *At the Edge of Reality*, Appendix C, Part IV, records them. Standing: computed.

Follow a length we see today back through the reconstructed history to near the turn. Near the turn, the length was some ten to the twenty-seven times larger than the horizon at that time. The restless trembling of the vacuum gives a power below ten to the minus fifty-one on a wave ten to the twenty-seven times larger than the horizon. The power measured in the sky is two parts in a billion. *The Gravitational Passage* computes the values of this section, and *At the Edge of Reality* records the values. Chapter 11 states the values first.

The standing of the exclusion of a long inflation

"On its identification the account also excludes one kind of inflation. A phase long enough to stretch the lengths observed today from within the early horizon to beyond it would make the departures on those lengths the departures of the inflationary vacuum. The account holds that they are the form of the content carried through the turn, to which the vacuum of the new epoch adds almost nothing (Section II). So, on that identification, a long phase is excluded. A shorter phase, which leaves the observed

lengths outside the early horizon throughout, 'is neither required nor excluded by the account' (*The Gravitational Passage*, Chapter 18, Questions). The kind of this step is *must*, relative to the identification: if the identification of Section II holds, a long phase cannot have occurred".
Source: *The Compactification*, Chapter 6, Section VIII, after *The Gravitational Passage*, Chapter 18, Questions. Standing: argued, in the form *must*, relative to the identification.

A long phase of inflation is a phase long enough to stretch the lengths we observe today from within the early horizon to beyond the early horizon. A long phase would make the ripples on the observed lengths the ripples of the vacuum of inflation. In the account, the ripples on the observed lengths are the form of the content carried through the turn. The vacuum of the new epoch adds almost nothing to the ripples. A long phase is excluded by a step of the kind "must", relative to the identification: if the first ripples are the carried form of the content, a long phase of inflation cannot have occurred. A shorter phase of inflation leaves the observed lengths outside the early horizon throughout. A shorter phase is neither required nor excluded by the account.

Two books, one conclusion about inflation

"The two books are therefore consistent: this account holds that the observed departures were not made by inflation, and *The Weight of Possibility* requires it, since otherwise gyrations of this mass could not be made in the measured amount; the gyrations are made at the turn, not after an inflation".
Source: *The Compactification*, Chapter 6, Section VIII, after *The Weight of Possibility*, Chapter 4, Section VI. Standing: argued.

The Compactification and *The Weight of Possibility* agree about inflation. *The Compactification* holds that the observed ripples were not made by inflation. *The Weight of Possibility* requires that the observed ripples were not made by inflation, because otherwise gyrations of about the Planck mass

could not be made in the measured amount. The gyrations are made at the turn, not after an inflation. Chapter 19 states the agreement between the two books.

Chapter 14, The Ledger of the Sky

The identity (13.4) to (13.7), the numbers (13.9), and the five choices are given under Chapter 8.

The six base parameters

"the density of ordinary matter $\Omega_b h^2$, the density of cold dark matter $\Omega_c h^2$, the angular size of the sound horizon at the release of the background, in the approximation that is the base parameter of the fit, $100\theta_{MC}$, the optical depth to reionization τ, the amplitude $\ln(10^{10} A_s)$, and the index n_s. Here h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}, and Ω_b and Ω_c are shares of the critical density. The model assumes flat space and an equation of state $w = -1$ for the energy of empty space".

Source: *The Compactification*, Chapter 6, Section X. Standing: definitions of the standard model of cosmology.

The standard model of cosmology describes the sky with six numbers. The six numbers are the amount of ordinary matter in each volume, the amount of dark matter in each volume, the angle that the largest sound wave of the early universe makes on the sky, how much the oldest light was scattered after the first stars formed, and the strength and the tilt of the first ripples.

The horizon ratio and the Hubble constant

$$\textit{particle horizon / de Sitter radius} = I\sqrt{\Omega_\Lambda} = \sqrt{(8f)} = 2.654$$

$$h = \sqrt{(\Omega_m h^2/\Omega_m)} = \sqrt{(0.1430/0.3108)} = 0.678;\ \ \Omega_m = 0.1430/0.730^2 = 0.268$$

"The particle horizon over the de Sitter radius is $I\sqrt{\Omega_\Lambda}$, and the identity sets $I^2\Omega_\Lambda = 8f$, so the ratio is $\sqrt{(8f)}$" (*The View from Inside*, Appendix C). "So h = $\sqrt{(0.1430/0.3108)} = 0.678$, with an uncertainty of ± 0.003 from $\Omega_m h^2$: a Hubble constant of 67.8 ± 0.3 km s^{-1} Mpc^{-1}. The radiation fraction itself depends on h ($\Omega_r h^2 = 4.18 \times 10^{-5}$ for the photons and three light neutrinos); solved together, $\Omega_r = 9.1 \times 10^{-5}$, and the result is unchanged at this precision" (*The Gravitational Companion*, Comparisons at Single Steps, Section VIII). A Hubble constant of 0.730 "with the same $\Omega_m h^2$ [...] would require $\Omega_m = 0.268$, which the identity excludes" (*The Gravitational Passage*, Chapter 13, Section IV).

Source: as named in each quotation. Standing: computed; the same test as Ω_m, through one more measurement ($\Omega_m h^2 = 0.1430 \pm 0.0011$, Planck 2018).

The particle horizon is the square root of eight times the trim, about 2.654 times the de Sitter radius. The letter h stands for the Hubble constant, measured in units of a hundred kilometers per second for every megaparsec. In the oldest light, astronomers measure the amount of matter in each volume as the matter share times h squared: 0.1430. To find h, divide 0.1430 by the account's matter share and take the square root of the result: h is 0.678. So in the account the expansion rate is 67.8 kilometers per second for every megaparsec, give or take three tenths. A rate of 73 would need a matter share of 26.8 percent, and the identity excludes a matter share of 26.8 percent.

The density of dark matter from the ratio

$$\Omega_c h^2 = (6 - 2/\pi) \times \Omega_b h^2 = 5.3634 \times 0.02237 = 0.1200 \pm 0.0008$$

Source: *The Compactification*, Chapter 6, Section X, Table 6.2 ("computed from the two rows above"), with $\Omega_b h^2 = 0.02237 \pm 0.00015$ measured; the first light measures 0.1200 ± 0.0012. Standing: computed; it restates the registered ratio at the measured density of ordinary matter.

The ratio of dark to ordinary matter is six less two over pi. The density of dark matter is the ratio times the measured density of ordinary matter, 0.02237: a density of 0.1200, give or take eight ten-thousandths. Astronomers measure the density of dark matter in the first light as 0.1200, give or take twelve ten-thousandths.

The measure of clumping

$$S_8 = \sigma_8 (\Omega_m / 0.3)^{1/2} = 0.823 \pm 0.006$$

"With the Passage's matter fraction, 0.311, the ratio of Chapter 3, and the primordial amplitude and tilt of Planck 2018, the amplitude of clustering today is $S_8 = \sigma_8 (\Omega_m / 0.3)^{1/2} = 0.823 \pm 0.006$. That value is computed at the matter density that keeps the angular size of the acoustic scale the microwave background measures; with the Passage's Hubble constant taken at its central value, $h = 0.678$, it is 0.827 ± 0.007".
Source: *The Weight of Possibility*, Chapter 5, Section VI. Standing: computed; it rests on the two books together.

Sigma eight is the typical difference from the average in the amount of matter in spheres whose radius is eight in the astronomers' units of distance. S eight is sigma eight times the square root of the matter share divided by three tenths. S eight is 0.823, give or take six thousandths, for the account's matter share and the account's ratio of dark to ordinary matter.

The family of expressions near the ratio

Expression	What it takes	Value
$6 - 2/\pi$	the abiding of $\mathbb{C}P^2$ (the rule)	5.3634
6	no abiding	6.0000
$6 - 1/\pi$	the abiding of S^2	5.6817
$6 - 3/\pi$	the abiding of the whole internal space	5.0451
$6 - 6/\pi$	the internal space as one factor of six dimensions, by its Einstein trace	4.0901
$6(1 - 1/(4\pi))$	six shares, each less the bare term of $\mathbb{C}P^2$	5.5225
$6f = 6(1 - 3/(8\pi))$	six shares of the external value	5.2838
$(6 - 3/\pi)/((1 + f)/2)$	the whole abiding, with the ordinary residue at the mean of one and f	5.3653
$(6 - 2/\pi)/(1 - 1/(3\pi))$	the rule, with the ordinary residue also reduced by an abiding; identically 6	6.0000
16/3	a simple fraction	5.3333
27/5	a simple fraction	5.4000
38/7	a simple fraction	5.4286

"The rule was selected from a small family of expressions of one form, and twelve expressions were examined in all". As a function of the shape, with x the ratio of the radius of $\mathbb{C}P^2$ to the radius of S^2 and the whole internal curvature held at 6, the rule reads $\Omega_c/\Omega_b = 6 - 36/(\pi(12 + x^2))$, "which equals $6 - 2/\pi$ exactly at the Passage's $x = \sqrt{6}$", and "At $x = 2$ the rule equals $6 - 9/(4\pi)$, which is 6f exactly".

Source: *The Weight of Possibility*, Chapter 5, Sections IV and VIII; the distances of each expression from the measurements are given in Appendix B. Standing: the counting principle and the rule are the account's principles; the rule was selected by the measurement from this family.

The table lists the twelve expressions examined. The twelve expressions are six less the abiding share of each part of the six, the simple fractions near 5.36, and forms built from the trim. The first row of the table is the rule for the ratio of dark to ordinary matter: six less two over pi, which is 5.3634. The six have two radii: the radius of the complex projective plane and the radius of the sphere. The rule can be written in terms of one ratio: the radius of the complex projective plane divided by the radius of the sphere. The rule is then six less a fraction, with thirty-six above the line and, below the line, pi times the sum of twelve and the square of the ratio. The rule equals six less two over pi exactly when the ratio of the two radii is the square root of six.

Chapter 15, Ripples of the Arena

The description by quanta, and the size of its corrections

"The description is the effective field theory of gravity (Donoghue 1994a; 1994b), whose corrections to the Newtonian potential are of relative size $(l_{Pl}/r)^2$, of order 10^{-70} at a separation of one meter". "The description by quanta ends at the modal boundary (Chapter 17). The claim that quantizing the arena is a category error holds at every scale, and the Passage parts from the description by quanta in what it predicts only at the Planck scale and beyond".

Source: *The Gravitational Passage*, Chapter 12, Questions, as *The Gravitational Companion*, Comparisons at Single Steps, Section V, quotes it; the second quotation is the Companion's own, in the same section. Standing: established physics (the effective field theory of gravity and the size of its corrections); argued (the scope of the claim that the arena has no quanta).

Physicists call the description of the small waves of the geometry by quanta the effective field theory of gravity. The effective field theory contains small corrections to Newton's law. For two masses a meter apart, the corrections are about one part in ten to the power seventy. The size of the corrections is the square of the Planck length divided by the square of the distance between the masses. The quanta of the effective field theory are quanta of the description, and not quanta of the arena. The description by quanta ends at the Planck scale. The claim that the arena has no quantum of its own holds at every scale.

Chapter 16, The Hum of the Settling

The computations of this section are *The Compactification*'s own: "The computation is made here, from stated premises; it is not a number of the authority books, and each premise is named so that a reader can change it" (Chapter 7, Section IV). The excess of light species (5.1) is given under Chapter 10; the decay of the breathing excitations under Chapter 4; and strings, walls and monopoles under Chapter 13.

Why the arc itself makes no wave

"The strong anisotropy of a Kasner geometry in nine directions of space is an anisotropy between the open directions and the internal ones, and in the reduction to four dimensions the motion of the internal radii appears as motion of the two moduli of the internal space, its volume and its shape (*The Gravitational Passage*, Chapter 11, Section II), which are scalar fields of the four-dimensional description, and not as a wave of the four-dimensional metric. This is argued here from the form of the solution and of the reduction: a homogeneous and isotropic open space carries no tensor wave, and the anisotropy of the arc is carried entirely by the moduli". Of the settling background: "It is not a tower of Kaluza–Klein modes

on the content instance: the content instance is a space of states with no size, on which no field lives and no mode is formed (*The Kinds of Particle*, Chapter 1, Section XIII)".

Source: *The Compactification*, Chapter 7, Section III. Standing: argued.

On the arc, the three large directions share one exponent, so the large space grows alike in every direction. The unevenness of the arc lies between the large directions and the small directions. In the four-dimensional description, the unevenness of the arc shows as a change of the size and the shape of the six. The unevenness of the arc does not show as a wave of the large geometry. The settling hum is not a ladder of heavy copies of particles on the content instance of the six. The content instance is the six taken as a space of states, which has no size and on which no field lives.

The frequency of the settling waves today

$$E_e = m_b/2 \approx 1.125\, M_{Pl} \approx 1.37 \times 10^{19}\, GeV \quad (7.1)$$

$$a_e/a_0 = (g_{s,0}/g_{s,e})^{1/3}\, T_0/T_b \quad (7.2)$$

$$f_0 = (E_e/h)(a_e/a_0) \approx 2.6 \times 10^{29}\, Hz \times (1\, GeV/T_b) \quad (7.3)$$

$$f_H = (a_e/a_0)\, H/(2\pi) \approx 2.7 \times 10^{-8}\, Hz \times (T_b/1\, GeV)(g_*/106.75)^{1/6} \quad (7.4)$$

Here m_b is the mass of the breathing excitation, about 2.25 M_{Pl}, taken to decay into two quanta of the wave description; $g_{s,e} = 106.75$ and $g_{s,0} = 3.909$, so $(g_{s,0}/g_{s,e})^{1/3} = 0.332$; $T_0 = 2.72548$ K $= 2.349 \times 10^{-13}$ GeV, and (7.2) gives $a_e/a_0 = 7.80 \times 10^{-14}$ GeV/T_b. "At the upper bound $T_b = 5 \times 10^{17}$ GeV, $f_0 \approx 5.2 \times 10^{11}$ Hz. At $T_b = 2 \times 10^{17}$ GeV, $f_0 \approx 1.3 \times 10^{12}$ Hz". In (7.4), $H = 1.66\, g_*{}^{1/2} T^2/M_{Pl}$ and $k = aH$; "At $T_b = 5 \times 10^{17}$ GeV this is $f_H \approx 1.3 \times 10^{10}$ Hz. Any wave made at the start of the history lies above this frequency today". "The breadth of the spectrum is not computed here".

Source: *The Compactification*, Chapter 7, Section IV. Standing: computed under premises 1 to 3; T_b is not fixed by the account, and it is bounded above.

In the computation, each breathing excitation is taken to decay into two quanta of the description of the settling waves. So each quantum is born with an energy of about half the mass of a breathing excitation, a little more than one Planck mass. When the waves were made, the scale of distances was about eight parts in a hundred million million of the scale now, divided by the starting temperature in billions of electron volts. So the frequency of the waves today is about 2.6 times ten to the twenty-nine hertz, divided by the starting temperature in billions of electron volts. The starting temperature is at most the highest temperature the account allows, so the frequency today is at least about five hundred billion vibrations a second. At the highest starting temperature, about thirteen billion vibrations a second is a lower limit today for every wave made within the horizon of its time.

The strength of the settling waves today

$$\rho_{GW}/\rho_{\gamma} = \varepsilon(g_{*,e}/2)(g_{s,0}/g_{s,e})^{4/3} \approx 0.65\,\varepsilon \quad (7.5)$$

$$\Delta N_{eff} = (8/7)(11/4)^{4/3}(\rho_{GW}/\rho_{\gamma}) \approx 2.86\,\varepsilon \quad (7.6)$$

$$\Omega_{GW}h^2 = (7/8)(4/11)^{4/3}\,\Omega_{\gamma}h^2\,\Delta N_{eff} \approx 5.6\times 10^{-6}\,\Delta N_{eff} \lesssim 5\times 10^{-7} \quad (7.7)$$

$$h_c \approx 1.26\times 10^{-18}(1\ Hz/f)(\Omega_{GW}h^2)^{1/2}$$

ε is the share of the content's radiation carried by the waves when the content had $g_{*,e} = 106.75$ species, "so that the bound reads $\varepsilon \leq 0.03$"; $\Omega_{\gamma}h^2 = 2.47\times 10^{-5}$. "At $\varepsilon = 0.03$ this is $\Delta N_{eff} \approx 0.09$". "In the unit that detectors use, the characteristic strain, a background of strength $\Omega_{GW}h^2$ at frequency f has $h_c \approx 1.26\times 10^{-18}$ (1 Hz/f) $(\Omega_{GW}h^2)^{1/2}$ (Maggiore 2000). At

the bound (7.7) and the frequency 5.2×10^{11} Hz, $h_c \approx 1.7 \times 10^{-33}$; at 1.3×10^{12} Hz, $h_c \approx 7 \times 10^{-34}$". "The bound is an upper bound", and "The account predicts no particular strength below the bound".

Source: *The Compactification*, Chapter 7, Section V, equations (7.5) to (7.7), and Table 7.1. Standing: computed here from the three percent bound, which is a requirement on the turn from the measured N_{eff}.

After the settling waves are made, the waves thin out as light does. But the light is warmed each time a kind of particle falls away, so today the waves carry about 0.65 times their starting share of the light's energy. Counted as light species, the waves today are an excess of about 2.86 times the starting share. At the ceiling of three percent on the starting share, the excess is about nine hundredths of a species. As a share of the critical density, the ceiling on the waves today is about five parts in ten million. At the ceiling and at about five hundred billion vibrations a second, the waves stretch and squeeze a length by about two parts in ten to the thirty-three.

The faint background of second order

$$\Omega_{GW} h^2 \approx \kappa\, \Omega_r h^2 \times 0.82\, P_\zeta^{\,2} \approx 6 \times 10^{-23} \text{ to } 1.5 \times 10^{-22} \quad (7.8)$$

"For a scalar power that is nearly the same at every length, P_ζ, the induced waves during the domination of radiation carry a share $\Omega_{GW} \approx 0.82\, P_\zeta^{\,2}$ of the radiation's energy per logarithmic interval of frequency (Kohri and Terada 2018)", with $P_\zeta \approx 2.1 \times 10^{-9}$; $\Omega_r h^2 \approx 4.2 \times 10^{-5}$; and $\kappa = (g_{*,e}/g_{*,0})(g_{s,0}/g_{s,e})^{4/3}$, "$\kappa \approx 0.39$ for waves that entered while the content had about a hundred species, and $\kappa \approx 1$ for waves that entered after the electrons and positrons had annihilated". "(7.8) assumes the measured power continues to short lengths unchanged".

Source: *The Compactification*, Chapter 7, Section VII. Standing: established physics (Ananda, Clarkson and Wands 2007; Baumann, Steinhardt, Takahashi and Ichiki 2007); computed.

When ripples of density act on one another, the ripples make waves of the geometry. The waves made by the ripples carry a share of the radiation's energy of about 0.82 times the square of the ripples' power. Today the waves made by the ripples carry between about six parts in ten to the twenty-three and one and a half parts in ten to the twenty-two of the critical density. The share today is about nine powers of ten below the smallest share that the most sensitive planned detectors can register.

Chapter 17, Listening for the Beginning

The frequency of the settling waves today is equation (8.2) of *The Compactification*. Equation (8.2) is the same as equation (7.3) of *The Compactification*. This appendix gives equation (7.3) under Chapter 16, together with the ratio of scale factors, about $8 \times 10^{-14}\ \mathrm{GeV}/\mathrm{T_b}$.

The measures of a background

$$h_c(f) \approx 1.26 \times 10^{-18} (1\ Hz/f)[h^2 \Omega_{GW}(f)]^{1/2} \quad (8.1); \quad h_c \approx (f\, S_h)^{1/2}$$

"The energy density of a background of gravitational waves is stated as $\Omega_{\mathrm{GW}}(\mathrm{f})$, the energy density per unit logarithmic interval of frequency divided by the critical density of the universe today; it is quoted multiplied by h^2, where h is the present Hubble constant in units of 100 km s^{-1} Mpc^{-1}, so that the product does not depend on the measured value of h. The characteristic strain $\mathrm{h_c}(\mathrm{f})$ is the dimensionless amplitude of the background per logarithmic interval of frequency; the subscript c distinguishes it from the Hubble parameter h". "Detectors that measure a strain spectral density, in units of strain per square root of a hertz, are compared with the background after their spectral density $\mathrm{S_h}^{1/2}$ is turned into a characteristic strain, $\mathrm{h_c} \approx (\mathrm{f}\, \mathrm{S_h})^{1/2}$".

Source: *The Compactification*, Chapter 8, Section I, equation (8.1), after Maggiore (2000). Standing: established physics (definitions and the standard relation).

Physicists state the energy of a background of gravitational waves as a share of the critical density. The critical density is the energy in each volume of space that a flat universe must hold to expand at the present rate. The share is counted for each equal step in the logarithm of the frequency: for each band across which the frequency grows by one fixed factor, as an octave doubles the frequency. Physicists then multiply the share by h squared, where h is the present Hubble constant in units of 100 kilometers per second per megaparsec. Physicists quote the product because the product does not depend on the measured rate of expansion today. So the quoted number stays the same as astronomers refine their measurement of the rate of expansion.

The characteristic strain is the fraction by which the passing waves of a background stretch and squeeze a length, and the characteristic strain is about 1.26 parts in ten to the eighteen, divided by the frequency in hertz, times the square root of the quoted energy share. Many detectors report their noise as a strain per square root of a hertz. Physicists compare a detector that reports its noise per square root of a hertz with a background by turning the detector's noise into a characteristic strain at the frequency the detector listens at. The characteristic strain of the detector is the square root of that frequency times the reported noise.

The count of light species weighs any early background

$$h^2\Omega_{GW} \approx 5.6 \times 10^{-6}\,\Delta N_{eff} \quad (8.3)$$

"where ΔN_{eff} is the excess of N_{eff} over the value 3.044 that the three kinds of neutrino give, and $h^2\Omega_{GW}$ is the energy density of the waves today integrated over frequency. The relation holds for any background present before the light elements formed, whatever its frequency".

Source: *The Compactification*, Chapter 8, Section X, equation (8.3), after Caprini and Figueroa (2018). Standing: established physics.

If a background of gravitational waves was present before the light elements formed, the background adds to the count of light species. The count of light species measures the energy of all the radiation other than light, in units of the energy that one kind of neutrino carries, and the three kinds of neutrino give a count of 3.044. Today the early background carries an energy share, summed over all frequencies, of about 5.6 millionths of the excess of the count above 3.044, whatever the frequency of the waves.

The entangling phase

$$\Delta\varphi = (G_N m^2 t/\hbar)(1/d_{nn} + 1/d_{ff} - 1/d_{nf} - 1/d_{fn}) \quad (8.4)$$

"where m is the mass of each body, t the time the bodies are held, and d_{ij} the separation of their centers when the first is in place i and the second in place j, with n the place nearer the other body and f the place farther from it (*The Unquantized Arena*, equation 19.1). For the parameters of Bose and colleagues, masses of 10^{-14} kg, each held in two places 250 μm apart, with centers 450 μm apart and a closest approach of 200 μm, for 2.5 s, the separations are 200, 700 and 450 μm". "The arithmetic of (8.4) with these values, checked in this chapter, gives $G_N m^2 t/\hbar \approx 1.58 \times 10^{-4}$ m and a bracket of about 1984 m^{-1}, so $\Delta\varphi \approx 0.314$".

Source: *The Compactification*, Chapter 8, Section XIII; *The Gravitational Passage*, Chapter 12, Questions. Standing: set by the structure once the identification of one geometry for each alternative is taken, with the measured G_N; the arithmetic is computed.

The entangling phase is the angle that marks the entanglement of two bodies through gravity. In the experiment that Bose and colleagues proposed in 2017, each of two bodies of equal mass is held in two places at once: a place nearer the other body and a place farther from the other body. The phase is a first factor times a bracket, and the first factor is New-

ton's constant times the square of the mass times the holding time, divided by the reduced Planck constant. The bracket adds one over the near-and-near distance and one over the far-and-far distance. The near-and-near distance is the distance between the centers of the two bodies, both in their near places. The far-and-far distance is the distance between the centers of the two bodies, both in their far places. The bracket then subtracts one over each of the two mixed distances, the distances between the centers with one body near and the other far. For the 2017 design the first factor is about 1.58 ten-thousandths of a meter. The bracket is about 1,984 per meter. The phase is the product of the first factor and the bracket: about 0.314 radians, one twentieth of a full turn.

What the high-frequency detectors reach

The bulk acoustic resonator MAGE "reaches a peak spectral strain sensitivity of about 6.6×10^{-21} $Hz^{-1/2}$ in several narrow bands between 5 and 16 MHz, and a projected 5 to 10×10^{-22} $Hz^{-1/2}$ at an operating temperature of 20 mK". Magnon detectors "reach spectral densities of 5.4×10^{-22} $Hz^{-1/2}$ at 14 GHz and 8.6×10^{-21} $Hz^{-1/2}$ at 8.2 GHz", on a reinterpretation of data from experiments built to search for axions. For the levitated sensor, "The projected strain sensitivity is 10^{-20} to 10^{-22} $Hz^{-1/2}$, which corresponds to a reach in energy density of about $h^2\Omega_{GW} \sim 10^6$ to 10^{12}". The Holometer "reached a strain sensitivity better than 10^{-21} $Hz^{-1/2}$ between 1 and 13 MHz, and it set the first direct upper limit on a stochastic background at megahertz frequencies: $\Omega_{GW} < 5.6 \times 10^{12}$ at 1 MHz, rising to 8.4×10^{15} at 13 MHz".

Source: *The Compactification*, Chapter 8, Sections III, V and VII. Standing: measured or projected, as each instrument's team reports it.

The best of the high-frequency detectors register stretches of space of a few parts in ten to the twenty-two, per square root of a hertz. The high-frequency detectors listen between a million and twenty billion vibrations a second. The sensitivity of each detector can be turned into an energy share. The weakest background a detector can register then carries a mil-

lion or more times the critical density. The account sets a ceiling on the settling waves of about five parts in ten million of the critical density (Chapter 16). The ceiling lies far below the weakest background that any of the detectors can register. The settling waves lie at frequencies above about five hundred billion vibrations a second (Chapter 16). So the detectors also listen below the band of the settling waves.

Chapter 18, Matter Made of Possibility

The same pair of spaces in earlier physics

"The product $\mathbb{C}P^2 \times S^2$ has served in theories of the Kaluza and Klein kind since the first searches for a realistic one: as the base of seven-dimensional internal spaces in eleven-dimensional supergravity (Witten 1981; Castellani, D'Auria and Fré 1984), and as the internal space itself in ten dimensions (Watamura 1983; Shaya 2026). In the earlier of those uses it is held by fluxes; the most recent leaves its holding to a companion paper (Shaya 2026). The radii, and an internal space held by the metric alone, are the Passage's own". "This book takes nothing about content from the Passage. In particular, it does not assume that the six internal dimensions are the arena of particle physics, and it does not assume that the dark component belongs to the content that particle physics describes". "The book makes no claim about which particles do exist, since that is a claim about content" (Chapter 1, Questions).

Source: *The Weight of Possibility*, Chapter 1, Section III and Questions. Standing: established physics, cited; a statement of scope (what the book does not assume).

Physicists have used the product of the complex projective plane and the sphere before, in theories with extra dimensions. In the earlier uses, fluxes hold the product in place: fields threaded through the extra dimensions. In the most recent use, Shaya (2026) leaves the question of what holds the product to a companion paper. Two things are *The Gravitational*

Passage's own: the radii of the six, and a set of six held by the geometry alone. *The Weight of Possibility* takes nothing about content from *The Gravitational Passage*. In particular, *The Weight of Possibility* does not assume that the six are the arena of particle physics, or that the dark matter belongs to the content that particle physics describes. *The Weight of Possibility* makes no claim about which particles exist, because a claim about which particles exist is a claim about content.

The antipodal parity

Forms on the sphere	Laplacian eigenvalue × R_T^2	Sign under the map
functions of degree l	$l(l+1)$	$(-1)^l$
exact one-forms, degree $l \geq 1$	$l(l+1)$	$(-1)^l$
co-exact one-forms, degree $l \geq 1$ (degree 1: the Killing forms)	$l(l+1)$	$(-1)^{l+1}$

"The action of the geometry contains the metric alone. It contains no orientation of the internal space, and so it is unchanged when the sphere S^2 is mapped to its antipodes". A term of the Chern and Simons kind "is odd under the reflection and would threaten the stability of the lightest odd state (Agashe, Falkowski, Low and Servant 2008)". The orientation-sensitive quantum effects in gravity "come from chiral fields, namely Weyl fermions of spin one half or three halves and self-dual antisymmetric tensor fields (Alvarez-Gaumé and Witten 1984); a theory of the metric alone in ten dimensions contains none of these fields". The symmetry is a parity of the Kaluza and Klein kind (Cheng, Matchev and Schmaltz 2002; Servant and Tait 2003), and in this account the parity is "the antipodal map of the sphere itself, which leaves no point fixed". So no fixed-point terms appear: the terms that break a Kaluza and Klein parity in orbifolds "do not arise (Datta, Raychaudhuri and Shaw 2014)". On the real projective plane, "fields identified without a change of sign keep only their harmonics of

even degree (Dohi and Oda 2010)". On the sign of each harmonic of the sphere: "The sign of a harmonic of degree l under the antipodal map is $(-1)^l$ [...]; it plays the part of the sign $(-1)^n$ of the n-th level on an interval (Hooper and Profumo 2007)". Without a symmetry of the Kaluza and Klein kind, dark matter that acts only through gravity decays through loops of gravitation (Tang and Wu 2016), and excitations of the graviton decay down their tower (Gonzalo, Montero, Obied and Vafa 2023). "Where this book reports those papers, the word 'graviton' names at most a quantum of a description, the description of small waves of a classical geometry, and never a quantum of the arena: gravitation is not quantized".

Source: *The Weight of Possibility*, Chapter 4, Section III. Standing: argued, with established physics cited.

The laws of motion of the geometry follow from one quantity, its action, and the action contains the metric alone. The metric carries no orientation, so the action has no built-in sense of which way round the sphere is. The swap of opposite points sends each point of the sphere to the opposite point. Physicists call the swap of opposite points the antipodal map, and the action stays the same under the swap. Under the swap, every vibration of the six either keeps its sign or flips its sign. A vibration that keeps its sign is even, and a vibration that flips its sign is odd. A vibration of the sphere has a degree, a whole number l, and the vibration changes sign when l is odd. The large dimensions carry only even waves, so the lightest odd vibration has nothing lighter and odd to decay into, and lasts.

Theories with extra dimensions already use a parity, a swap under which the laws stay the same, to protect a particle from decay. In this account the protecting symmetry is the swap of opposite points, and the swap leaves no point in place. If each point of the sphere were glued to its opposite point, the result would be a different space, the real projective plane. The real projective plane holds only even vibrations. In this account the sphere stays whole, so its odd vibrations exist, and the lightest of them is the lasting gyration.

The parity is gauged

"Couplings of the geometry to content are outside this book; a content coupling that distinguished the orientation of the sphere could break the symmetry, and the account requires that none does". In the mainstream account a theory of gravity has no global symmetries (Banks and Seiberg 2011; Harlow and Ooguri 2021). "A gauged discrete symmetry is exact: nothing can violate a discrete gauge symmetry (Krauss and Wilczek 1989)". Harlow and Numasawa (2026): "In quantum gravity there are no global symmetries, so any spacetime inversion symmetries must be gauge symmetries". "That the antipodal map of the internal sphere is such an inversion, and is gauged, is this book's own step". *The Weight of Possibility* takes the step on the ground of the principle of identity, applied to whole configurations of the geometry.

Source: *The Weight of Possibility*, Chapter 4, Section III. Standing: argued (this book's own step); established physics cited.

A gyration has a mass of about the Planck mass, and only an exact symmetry protects a gyration. A symmetry of the ordinary kind, called a global symmetry, relates two arrangements that are different and behave alike. The usual expectation in physics is that a quantum theory of gravity breaks every global symmetry. On that expectation the breaking comes from effects that grow as a thing's mass approaches the Planck mass. For a gyration, the mass divided by the Planck mass is about one. So on the usual expectation a global symmetry would protect no gyration.

On this account gravitation is not quantized. Suppose two whole geometries differ only by the swap of opposite points. No one can tell the two geometries apart by any comparison, so the two geometries are one. Because the two geometries are one, the swap is a symmetry that nothing can break. Physicists call a symmetry gauged when it holds because the arrangements it relates are one and the same. The swap is gauged, so the swap is exact both on this account and on the usual account. On both accounts the swap protects the gyrations. The account requires that no

coupling of content to the geometry tell which way round the sphere is, because a coupling that told the sphere's orientation could break the symmetry.

Conjugation on the complex projective plane

"$\mathbb{C}P^2$ has a discrete isometry of its own that lies outside $SU(3)$: complex conjugation. It keeps the orientation of $\mathbb{C}P^2$, and it changes the sign of the class of degree two of $\mathbb{C}P^2$. The principle of identity applies to it as to the antipodal map, so it too is a discrete gauge symmetry. Conjugation acts on the gauge fields of the isometries of $\mathbb{C}P^2$ as charge conjugation acts on the gauge field of the group $SU(3)$: of the eight fields, three keep their sign and five change it".

Source: *The Weight of Possibility*, Chapter 4, Section III. Standing: argued.

The complex projective plane has a second swap of its own, called complex conjugation. The principle of identity applies to complex conjugation as to the swap of opposite points, so complex conjugation too is a symmetry that nothing can break. The turnings of the complex projective plane give eight force fields in the large dimensions, and conjugation flips the sign of five of them. If the eight force fields are free, a state that flips under conjugation alone can decay into lighter states and three of the eight fields. So no further state lasts because of conjugation. A confined force field does not reach across the large space as a long-range force. If the force fields of the complex projective plane are confined, one state cannot decay: the lightest state of the confined spectrum that flips under conjugation. Unless that state is the lightest odd gyration itself, that state lasts beside the lightest odd gyration. The lasting state would be a vibration of the same six, and it adds nothing to the count of classes that Chapter 19 uses.

No exit by annihilation or collision

"Two odd gyrations can annihilate together into even states, which the symmetry allows; at the density of the dark matter near the Sun, with a cross section of the order of the Planck area, that happens about once in 10^{78} seconds for each gyration, which is no exit at all". "At the speeds of matter in galaxies and in clusters, from a few hundred to a few thousand kilometers per second, the cross section per unit mass for the transfer of momentum is between about 10^{-45} and 10^{-50} square centimeters per gram for a gyration of about the Planck mass".
Source: *The Weight of Possibility*, Chapter 4, Section III. Standing: computed.

The symmetry allows two odd gyrations to meet and annihilate into even states. Near the Sun, each gyration annihilates with another about once in ten to the seventy-eight seconds. So annihilation is no exit at all: it removes practically no gyrations. Gyrations also scatter off one another, and clusters of galaxies that have collided set a limit on the scattering of dark matter. The scattering of gyrations is at least forty-five powers of ten below the limit from colliding clusters, so the dark matter passes through itself.

The lightest odd level

Spin	Components of the metric	Factor on $\mathbb{C}P^2$	Factor on S^2	$SU(3) \times SO(3)$	Sign	Mass squared
2	open with open	constant	function of degree one	$(1, 3)$	odd	2
1	open with $\mathbb{C}P^2$	an isometry (octet)	function of degree one	$(8, 3)$	odd	$4 - 2 = 2$

Spin	Components of the metric	Factor on $\mathbb{C}P^2$	Factor on S^2	SU(3) × SO(3)	Sign	Mass squared
1	open with internal	first harmonic (octet), with its gradient	function of degree one, with its gradient	(8, 3)	odd	$4 - 2 = 2$
0	$\mathbb{C}P^2$ with S^2	gradient of the first harmonic (octet)	gradient of the function of degree one	(8, 3)	odd	$4 - 2 = 2$

"The lowest level of the internal space that holds odd states is the first harmonic of the sphere: a function of degree one on S^2, constant over $\mathbb{C}P^2$. Its Laplacian eigenvalue is $2/R_T^2 = 2/l_{Pl}^2$". "At the classical level four odd multiplets share the lowest mass, a squared mass of 2 in Planck units, which is $\sqrt{2}$ Planck masses, about 1.7×10^{19} GeV". "For spin one and spin zero the squared mass is the eigenvalue of the operator on one-forms or on symmetric tensors less 2, which is twice the Ricci curvature of each direction". The spin-zero member of the first harmonic is the size of $\mathbb{C}P^2$ varying across the sphere. The spin-zero member "would be unstable without the floor; the floor's response, estimated from its response to a uniform change of size, lifts it to about 3.2 Planck masses", and it stays above the lightest odd state "if the floor resists an uneven change of the size of $\mathbb{C}P^2$ with more than about a fifth to a third of the stiffness it shows for a uniform one". "At first order it takes the squared mass from 2 to about 0.4, and so the mass from $\sqrt{2}$ to about 0.6 Planck masses"; the least extent $\kappa \operatorname{tr} \mathrm{Ric}^5$ "multiplies the squared mass of every spin-two level of the internal space by one factor, about 0.19 at first order, and it leaves the gauge fields of the isometries massless". The masses of the table hold "under two conditions: that the higher Lovelock terms do not alter the operators, and that the floor acts as an ordinary source".

Source: *The Weight of Possibility*, Chapter 4, Section V, with Hinterbichler, Levin and Zukowski (2014) for the spin-two masses. Standing: computed (classical, and at first order for one mechanism).

The lowest odd vibration is the simplest vibration of the sphere, and it is the same at every point of the complex projective plane. The simplest wave of the sphere varies as slowly as the six allow (Chapter 3), and its squared mass is twice the Planck curvature. At the classical level, four families of states share the lowest mass. The four families are one family of spin two, two families of spin one, and one family of spin zero. Each family has a mass of the square root of two Planck masses, about 1.7 times ten to the nineteen billion electron volts. The spin-two family has three members, and each of the other three families has eight times three. The floor lowers all four families alike, at first order to about six tenths of a Planck mass, but which family is lightest, the spin-two family or a spin-one family, depends on the parts of the floor not computed and on the term of Gauss and Bonnet.

The sphere's simplest wave holds one more state: the size of the complex projective plane changing from one side of the sphere to the other. The floor lifts that state to about three and two tenths Planck masses. The estimate comes from how the floor resists an even change of size. The state stays above the lightest odd level if the floor's stiffness against an uneven change of size is more than about a fifth to a third of the floor's stiffness against an even change. The fraction is a fifth for the first-order mass of the lightest odd level, and a third for its simplest, classical mass.

What the rotations do and do not protect

"The odd states form triplets under the rotations of the sphere. The rotations do not by themselves keep them from decaying. The fields that the reduction gives for the rotations are also a triplet, so a triplet could pass its charge to one of those fields; only the antipodal map forbids it, since those fields are even". Kolb and Slansky (1984) first drew the distinction between a conserved charge and a class of representations as the source of

stability. "Weinberg's prescription gives each coupling as the ratio of $2\pi(16\pi G)\text{^}1/2$ to a root-mean-square circumference of the internal space (Weinberg 1983). Computed here by that prescription, with each generator normalized so that the trace of its square is one half and so that its flow has a period of 2π, the coupling at the Passage's radii is about 6 for the rotations of S^2 and about 16/3 for the isometries of $\mathbb{C}P^2$, in the units in which electromagnetism has 1/137. These two values are this book's computation; Weinberg gives the prescription, not these values". "These couplings are about 300 times the weak and strong couplings at the Planck scale (Buttazzo and others 2013)". The fields are non-abelian and strongly coupled, so they "may confine there and not reach the open spacetime as a long-range force" (Gross and Wilczek 1973; Politzer 1973). "Were the triplet to carry a charge of the content, it would be tested directly: a charged relic of the Planck mass would be visible to detectors on Earth (Lehmann, Johnson, Profumo and Schwemberger 2019), and slow, heavy, ionizing particles of the Planck mass have been proposed as dark matter and as a target for search (Meissner and Nicolai 2019)".
Source: *The Weight of Possibility*, Chapter 4, Section V; its Note 2 gives the units: lengths in Planck lengths and masses in Planck masses, with c, ħ and G_N equal to one; the full Planck mass, 1.22×10^{19} GeV; scalar curvature 4 on $\mathbb{C}P^2$, 2 on S^2 and 6 on the product. Standing: computed (this book's computation by Weinberg's prescription); established physics cited.

The odd levels come in threes under the turnings of the sphere. The turnings alone do not protect the odd levels from decay. Each turning of the sphere gives a force field in the large dimensions, and those force fields also come in threes. So an odd level could pass its charge under the turnings to one of the force fields of the turnings. Only the swap of opposite points forbids that decay, because the force fields are even.

The physicist Weinberg gave a rule in 1983 for the strength of the force fields that the turnings of an internal space give. By Weinberg's rule, the sphere's force fields have a strength of about six at the account's radii, in

units in which electromagnetism has one over 137, and that strength is some three hundred times the strength of the weak and strong forces at the Planck scale. By Weinberg's rule, the force fields of the complex projective plane have a strength of about five and a third. Fields with strengths of about six may be confined at the Planck scale, so the fields of the six are not the forces of particle physics.

The account requires that the force fields of the six not reach across the large space as a long-range force on charges that do not cancel. Two measurements show the reason. Kesden and Kamionkowski (2006) studied the tidal streams of stars torn from the Sagittarius dwarf galaxy. The streams disfavor a difference of more than ten percent between the ways dark and ordinary matter fall. A second study limits a force acting on the dark matter alone to below about five percent of gravitation (Bean, Flanagan, Laszlo and Trodden 2008). At a strength of about six, the force fields may be confined at the Planck scale. If the force fields are confined, each gyration is dressed into a neutral state, which is still odd, stays dark and lasts.

How many gyrations there are

"With the ratio of Chapter 3, there is about one gyration for every 1.5 to 3.4×10^{18} baryons (for a gyration of 0.6 to $\sqrt{2}$ Planck masses, the first-order and the classical values of Section V), and about 2 to 4×10^{-28} gyrations for every photon of the microwave background. This is the opposite of the asymmetric accounts, in which dark and ordinary particles are about equal in number (Barr, Chivukula and Farhi 1990; Kaplan, Luty and Zurek 2009)". The count is also different from the count in the account of Kaplan (1992). "In the neighborhood of the Sun, at the measured local density of dark matter, there is about one gyration in every few tens of thousands of cubic kilometers. An estimate of the same kind, about one white hole of the Planck scale in every 10,000 cubic kilometers, has been made for remnants of the Planck mass (Rovelli and Vidotto 2018a)".
Source: *The Weight of Possibility*, Chapter 4, Section VI. Standing: computed.

A gyration weighs about as much as ten to the nineteen protons. The dark matter carries about 5.36 times the mass of the ordinary matter (Chapter 19). So there is about one gyration for every one and a half to three and four tenths billion billion protons and neutrons. The range comes from the range of the gyration's mass. The mass is 0.6 Planck masses at first order and the square root of two Planck masses at the classical level. Near the Sun there is about one gyration in every few tens of thousands of cubic kilometers.

How this dark matter differs from others

Against a parity of the Kaluza and Klein kind, "The account differs in four ways. The reflection is the antipodal map of a sphere that is not identified with its antipodes, and it has no fixed points. The action contains the metric alone. The scale is the Planck scale. The lightest odd state is the spin-two triplet of the first harmonic of the sphere, or one of the two spin-one octet-triplets that share its mass". Planckian interacting dark matter is a second proposal (Garny, Sandora and Sloth 2016; Garny, Palessandro, Sandora and Sloth 2018). In that proposal "the amount is made by scattering in the hot plasma after inflation, and it is set by the temperature of reheating", and "the stability is not exact". In this account the parity is gauged, an identity of the present measurement fixes the amount, and the gyrations are made at the turn. Against remnants of evaporated black holes, "The bound from gravitational waves on remnants formed at a peak of a Gaussian primordial spectrum (Trivedi and Loeb 2025) does not apply here, since nothing here needs such a peak". "A gyration of 0.6 to $\sqrt{2}$ Planck masses has a gravitational radius of about one to three Planck lengths, comparable to its reduced wavelength and to the size of the internal space it vibrates, and stable objects of the Planck mass have been described as elementary black holes (Markov 1993) and as the remnants of evaporated black holes (Chen and Adler 2003). A gyration is a level of

vibration of the internal geometry, with a definite energy and a definite sign under the antipodal map; it is not a collapsed region of the open geometry".
Source: *The Weight of Possibility*, Chapter 4, Questions. Standing: argued; established physics cited.

The gyrations resemble other proposals for heavy dark matter that acts only through gravity. The gyrations differ from each of the other proposals in a definite way: in the symmetry that protects the gyrations, in where and how the gyrations are made, and in what the gyrations are. A gyration is a vibration of the six, and not a collapsed region of the large dimensions, as a black hole is.

The three distortions of context, and the three edges

"Each column of the grid is such a domain: pattern is compared in space, force in time, probability in possibility. So the context of each column is continuously asymmetric, and none can be perfectly uniform; a perfectly uniform context would perfect continuity and symmetry together, which the theorem forbids." "In every epoch each context is distorted, by a distortion of its own that is not zero: space by gravitation, around content; time by the cosmological constant, uniformly; possibility by the dark matter, held at the Planck scale and gathered in halos. On this account the dark matter is not an addition to the world that might have been absent. A world with a continuous context of possibility must carry it, as it must carry gravitation and a cosmological constant." "The columns are the three modalities: pattern and space are omniscient, force and time immanent, probability and possibility transcendent." "Distinct (Axiom III). The cosmological constant is uniform and never clumps. Gravitation is local and follows content. The dark matter clumps, and never dissipates or collides." "Inseparable (Axiom III). The three appear together in one equation of the expansion: in a flat epoch the shares of the cosmological constant, of the matter and of the radiation sum to one, and gravitation is the equation itself. [...] The dark matter, acting only through gravitation, can-

not be registered apart from it." "Non-interchangeable (Axiom III). No amount of the cosmological constant makes a halo; no amount of dark matter accelerates the expansion; gravitation is the response of the arena to content and is not a component of the budget at all." "The transcendent precedes the immanent: the internal space is released from the Planck scale, and the turn follows. The immanent precedes the omniscient: the cosmological constant, fixed by the time since the turn, fixes the radius of the horizon of the emptied epoch. The omniscient precedes the transcendent: within the horizon, gravitation clumps matter into black holes, whose interiors reach the Planck scale again. The reverse order matches no step of the account." "The distortion is local: it is where content is, and in proportion to it." "So the distortion of space is all local: gravitation is structure, and has no share of its own in the budget. The distortion of time is all global: the cosmological constant is uniform. The distortion of possibility is both: global in its amount, which Chapter 3 fixes, and local in its halos." "The binding is by what each distortion does, in the slicing that the flat epoch itself picks out". "A triple is three terms that stand to one another as the three modalities stand: the immanent, the omniscient and the transcendent." "A list of three that binds unambiguously, one to one, to a triple already known thereby becomes a known triple." Of the two horizons: "The two are the past face and the future face of one boundary of comparison, and the record determines the bound." "The binding does not rest on P being a length. It rests on the roles that the Passage gives P, and every one of those roles is a role of the transcendent column." "T is fixed by no length at all, since no comparison registers it." "No epoch has one of the three edges without the other two." "So T is the clear immanent, and its place fixes how the other two stand." "H is the limit of perfected symmetry." "P is the limit of perfected continuity." "The cosmological constant is the clear immanent." "Gravitation distorts the context in which pattern is laid out, and what it does is itself laid out: the bending of light maps it. The dark matter distorts the context of what could occur, and what it does is laid out nowhere except through the first: it registers only as a distortion of space, by gravitating. The two meet in exactly that one

way, which is the one way the dark matter is observed." "the three distortions of context are required, they are three, and they stand as a triple, with the cosmological constant as its clear immanent and gravitation and the dark matter as its conjugate pair." "gravitation is the arena's response to content, the cosmological constant is the arena's own constant term, and the dark matter is a distortion of another arena, the internal one, which the open arena feels as mass."
Source: *The Weight of Possibility*, Chapter 2, Sections I to VIII and Questions; Appendix A, Sections I to X; Note 6 (the notion of a triple, from *An Immanent Metaphysics*). Standing: the Incommensuration Theorem, a structural fact about the nature of knowing, and the three axioms; the reading by column is that book's proposal ("This book proposes the reading by column").

Every setting in which something is found must be bent: no setting can be perfectly uniform. The settings are the three contexts in which anything is ever found: pattern is compared in space, force in time, and chance in possibility. The Incommensuration Theorem is a structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity. Symmetry is sameness through a change, and continuity is smoothness. The theorem applies wherever a comparison is made, so the theorem applies in each of the three settings. The theorem forbids a perfectly uniform setting, because a perfectly uniform setting would make sameness and smoothness both perfect at once. The three axioms of the account require each setting to have a bending of its own that is not zero. Gravitation bends space around content. The cosmological constant bends time evenly, the same at every place. The dark matter bends possibility: the dark matter is held at the Planck scale and gathered in halos.

The third axiom requires three things of the three bendings: they go together, none can do another's work, and they are three. First, the bendings go together. In a flat epoch, a single equation of the expansion requires the shares of the cosmological constant, of the matter and of the ra-

diation to add to one, and gravitation is that equation itself. The dark matter acts only through gravitation, so no one can measure the dark matter apart from gravitation. Second, no bending can do another's work. No amount of the cosmological constant makes a halo, and no amount of dark matter speeds the expansion. Gravitation is the response of the arena to content, and not an entry in the budget of the universe's energy. Third, the bendings are three distinct things. The cosmological constant is even everywhere and never clumps. Gravitation is local and follows content. The dark matter clumps, but it never loses its energy and never collides.

The dark matter is required, and not merely allowed, because possibility must be bent and the dark matter is the bending of possibility. A world whose context of possibility is continuous must carry dark matter, as it must carry gravitation and a cosmological constant. The bending of space is all local: it lies where content is, in proportion to content. The bending of time is all overall, the same everywhere. The bending of possibility is both: overall in its amount, and local in its halos.

In this account reality has three edges: the horizon, the turn and the Planck scale. The three edges pair one to one with the three settings: the horizon with space, the turn with time, and the Planck scale with possibility. The pairing goes by what each edge does, and not by the unit in which the edge is measured. The horizon has two faces. The particle horizon bounds what has reached us, and the event horizon bounds what can ever reach us. The two faces are one boundary, because what has reached us fixes the cosmological constant, and the cosmological constant fixes the event horizon.

The Planck scale is a length, but every role the Planck scale plays belongs to possibility. The six dimensions of possibility are held at the Planck scale. The Planck scale bounds the standpoint from which any comparison is made. The Planck scale is where the smooth description and the stepwise description exchange roles. So the Planck scale is the limit of perfect smoothness. No length fixes the turn, because no comparison registers the turn as an event. No epoch has one edge without the other two.

Every comparison has three roles: the immanent, the act itself as it occurs; the omniscient, what is laid out as structure; and the transcendent, the range of what could be found. The second axiom orders the three roles in a one-way cycle: the transcendent, the immanent, the omniscient, and the transcendent again. The three edges follow the same cycle. The six are released at the Planck scale, and the turn follows. The time since the turn fixes the cosmological constant, and the cosmological constant fixes the horizon of the emptied epoch. Within the horizon, gravitation gathers matter into black holes, whose interiors reach the Planck scale again. The reverse order agrees with no step of the account.

A triple is three terms that stand to one another as the three roles of a comparison stand. When three terms pair one to one with a known triple, the three terms are themselves a triple. The three settings are a known triple: space is omniscient, time is immanent, and possibility is transcendent. The three bendings pair one to one with the three settings, so the three bendings are a triple. The act of comparison made from inside fixes the cosmological constant. So in the triple the cosmological constant takes the role of the act itself: the clear immanent. Gravitation and the dark matter are the conjugate pair: the other two terms of the triple, paired as counterparts. The horizon is the limit of perfect sameness. At the horizon an emptied epoch reaches the most symmetric geometry of four dimensions, with almost no gravitation. The Planck scale is the limit of perfect smoothness, at the other end of the range of lengths.

Gravitation bends space, the setting in which patterns are laid out, and astronomers map that bending by the bending of light. The dark matter bends possibility, the setting of what could occur. The dark matter shows itself only through gravitation. Gravitation and the dark matter meet in that one way, and that way is the only way anyone observes the dark matter. In Einstein's field equations, the cosmological constant and the dark matter both curve the large space. The triple fits the field equations. Gravitation is the arena's response to content, and the cosmological constant is the arena's own constant term. The dark matter is a bending of the other space, the internal one, and the large space feels that bending as mass.

Chapter 19, How Much the Turn Made

The rule, and the coefficient it shares with the identity

$$\Omega_c/\Omega_b = 6 - 2/\pi = 5.3634; \quad \textit{with the coefficient k: } \Omega_c/\Omega_b = 6 - 2k/\pi, \quad I^2\Omega_\Lambda = 8 - 3k/\pi$$

"The Passage places each internal direction at a curvature of $1/l_{Pl}^2$, and names that coefficient as one of three of order one that its identity takes as fixed (the Passage, Chapter 13, Questions). Write it k. Both the identity and the rule then depend on it in the same way". "The book's claim is that both find the value the Passage identifies, k = 1".

Source: *The Weight of Possibility*, Chapter 3, Sections V and VI. Standing: registered ratio; the counting principle and the rule are the account's principles.

The dark matter is 5.3634 times the ordinary matter: six less two over pi. *The Gravitational Passage* places each of the six at the Planck curvature, and it takes the coefficient, one, as fixed. Write k for the coefficient: the curvature of each of the six, in units of the Planck curvature. If the curvature were k times the Planck curvature, the rule for the ratio would be six less two k over pi. The identity of Chapter 8 sets the square of the radius of the particle horizon, in Hubble radii, times the share of the vacuum, equal to eight less three over pi. With the coefficient k, the identity would be eight less three k over pi. So a measurement of the ratio measures k, and a measurement of the identity measures k too. *The Weight of Possibility* claims that both measurements find k equal to one, the value *The Gravitational Passage* identifies.

Different identities fix the cosmological constant and the amount of dark matter, but both identities rest on one measure: the present act of comparison. The present act of comparison fixes the cosmological constant through the time since the turn. The dark matter was made at the

turn and is held at the Planck scale. The present act of comparison fixes the dark matter through an identity that the reconstructed history carries back to the turn. Because both identities rest on the present act of comparison, the cosmological constant and the dark matter share one coefficient, and a measurement of either quantity measures the coefficient of both.

The share of a curvature

$$\textit{bare term } R/(16\pi);\quad 1/L^2 = 8\pi\rho/3;\quad S_L = \pi L^2;\quad \textit{share } c_1/S_L = 8\rho,\quad c_1 = 3$$

$$\textit{whole internal space } (R = 6)\textit{: } S_L = \pi\textit{, share } 3/\pi;\quad \mathbb{C}P^2\ (R = 4)\textit{: } S_L = 3\pi/2,$$
$$\textit{share } 2/\pi;\quad S^2\ (R = 2)\textit{: } S_L = 3\pi\textit{, share } 1/\pi$$

"The curvature of the internal space enters the four-dimensional equations as a bare term (the Passage, Chapter 11, Section III), an energy density of magnitude $R/(16\pi)$ in Planck units, where R is the scalar curvature: 6 for the whole internal space, 4 for $\mathbb{C}P^2$, 2 for S^2. A density ρ has, through the Friedmann equation, a curvature radius L with $1/L^2 = 8\pi\rho/3$, and a horizon of that radius has the count $S_L = \pi L^2$, in the measure the Passage uses for the particle horizon. That is the count A/4 of Bekenstein and Hawking for a sphere of radius L". "The bare term has the sign of anti-de Sitter space, which has no horizon, so S_L is a formal count; no observer has a horizon of that radius". A second reading: "The density of the curvature term that one degree of freedom induces, through its quantum fluctuations at the Planck scale, in units of the coefficient of the Einstein and Hilbert action, is $R/(6\pi)$ in four dimensions; so $2/\pi$ is the curvature term that three degrees of freedom, one for each channel, induce from the curvature of $\mathbb{C}P^2$".

Source: *The Weight of Possibility*, Chapter 3, Section III. Standing: the account's measure of the reduction, read from the Passage's identity; computed.

Each piece of the six has a curvature. Written as an energy, the curvature of a piece sets a radius through Friedmann's equation. A horizon of that radius has a count: its area in Planck units, divided by four. The horizon is formal, because the curvature term has the sign of anti-de Sitter space, and anti-de Sitter space has no horizon. No observer has a horizon of that radius. The share of a piece's curvature is three channels, one for each role of a comparison, divided by the count of the piece's formal horizon. For the whole six the share is three over pi, the same three over pi that appears in the identity for the energy of empty space, while for the complex projective plane the share is two over pi and for the sphere the share is one over pi.

Which curvature abides

$$\chi = (1/4\pi)\int R\, dA; \quad \textit{Einstein trace } R(1 - n/2)\textit{: } -4 \textit{ for } \mathbb{C}P^2\textit{, 0 for } S^2; \quad \chi(S^2)/S_L = 2/\pi$$

"By the theorem of Gauss and Bonnet, the Euler number of a surface is its whole linear curvature, $(1/4\pi)\int R\, dA$, at any radius"; "$\mathbb{C}P^2$'s Euler number, three, comes wholly from an integral of the squared curvature, half from its Weyl part and half from its scalar part; it includes none of $\mathbb{C}P^2$'s ordinary curvature, which grows with its size". "Exempting S^2 in the Passage's identity too would give the matter fraction 0.300 in place of 0.311. One statement gives both numbers. The principle is a principle of bookkeeping, and it is the account's: it states what a count already contains, and it is not a statement about the dynamics of the turn. Near the turn no internal curvature acts [...], and the six internal directions move as one block". "One principle accounts for it, and four further considerations agree with it": "The factors are used apart"; "A surface exerts no stress on what it holds"; "The rows divide, and their curvature does not abide"; "The abiding falls on distortions of context". "In n dimensions the trace of the Einstein tensor is $R(1 - n/2)$: for $\mathbb{C}P^2$, of four dimensions, it is -4; for S^2, of two, it is zero". "Read from the accelerations, the abiding in the dark

residue would be $3/\pi$, and the ratio would be $6 - 3/\pi = 5.045$, which the data exclude. Taken together as one space of six dimensions, the abiding in the dark residue would be $6/\pi$, and the ratio would be 4.09, which the measurement excludes". "The two forms agree because the ratio of the curvatures of $\mathbb{C}P^2$ and of the whole, $4/6$, equals the ratio of the Euler numbers of S^2 and of $\mathbb{C}P^2$, $2/3$; for a grid of a columns carried by $\mathbb{C}P^{a-1} \times S^2$, that equality holds only when a is three".

Source: *The Weight of Possibility*, Chapter 3, Section IV. Standing: the principle is the account's principle of bookkeeping, supported by the theorem of Gauss and Bonnet.

In this account a curvature abides, or holds fast, in a measurement when the curvature stays as a term of its own. An abiding curvature is subtracted from a count of classes. The theorem of Gauss and Bonnet says that the Euler number of a surface equals the surface's whole curvature, at any radius. So, for a surface, the whole curvature is already held in any count that includes the surface's Euler number. The surface's curvature does not reduce that count a second time. The count of the dark matter includes the two classes of the sphere, and the sphere is a surface. So only the curvature of the complex projective plane reduces the count of the dark matter, and that curvature lowers the ratio from six by two over pi, because the complex projective plane is not a surface: its Euler number, three, includes none of its ordinary curvature.

The two over pi of the rule can be read a second way: the sphere's two rows, content and context, over the formal count pi of the whole six, and the two readings agree only for a grid of three columns. The grid of the account has three columns, the three settings, and two rows, content and context. If the sphere were exempted in the identity of *The Gravitational Passage* too, the share of matter in the universe would be three tenths, in place of the thirty-one percent that the identity gives.

The principle of bookkeeping states that a count that already includes a surface's Euler number is not reduced again by the surface's curvature. Four further considerations agree with the principle of bookkeeping. First,

the two factors of the six are used apart where a question concerns each factor on its own. *The Gravitational Passage* already uses the two factors apart when it counts its channels. Second, a surface puts no stress on what it holds, however curved the surface is. The four-dimensional complex projective plane does put stress on what it holds. Third, the sphere is the factor that carries the two rows, content and context. The split between ordinary matter and dark matter is a split between content and context. So the sphere's curvature abides on neither side of the split, because the sphere is the factor that does the dividing. Fourth, the abiding share applies only to the distortions of context, the cosmological constant and the dark matter. The abiding share never applies to the ordinary matter, which is content. The principle of bookkeeping states what a count already contains, and it is not a statement about how the six move at the turn. Near the turn no internal curvature acts, and the six move as one block.

The count of classes, and the short loop

What is counted	$\mathbb{C}P^2$	S^2	The ball within the horizon	Count
the channels of the registering act (the Passage)	ranged over: 3	fixed to one class: 1	not a factor	3
the dark residue	ranged over: 3	ranged over: 2	one class: 1	6
the ordinary residue	no extent	no extent	one class: 1	1

"A registration counts the independent classes of what it counts: every class of a factor over which the counted thing ranges, and one class of a factor at one class of which the counted thing is fixed". "On $\mathbb{C}P^2$, on S^2 and on their product, the usual ways of counting classes agree: the number of independent classes over any field, the Euler number, the number of cells, and the number of points that a generic rotation fixes all give

three, two and six. On a ball, the number of independent classes and the Euler number both give one". "No count of holders of energy gives six to one. A wave of the ten-dimensional metric has 35 independent polarizations. Counted by direction, the internal and the open stand 6 to 3; counted by polarization, 33 to 2 or 21 to 2; counted by the massless modes, 12 to 1 or 11 to 1". "If the region within the horizon held N independent classes, the ordinary residue would count N, [...] and the dark residue 6N", the abiding is taken once, "and the ratio would be $6 - 2/(N\pi)$"; "For two classes the ratio is $6 - 1/\pi = 5.682$". Of the three contents, "only pattern remains as a residue: force leaves the turn as radiation, which is the bulk of its energy and not a residue, and probability carries the form of the departures from uniformity, not a density of energy"; and "the ordinary residue is one kind of mass on either reading of mass: the single measure, or the baryonic one of the three". Bai and Chen (2026) divide six against one on other grounds, "domain walls of a vacuum with seven states", and the rule shares their form $6(1 - 1/(3\pi))$. "There the reduction depends on the strong interaction; here it is a pure number".
Source: *The Weight of Possibility*, Chapter 3, Sections II and V. Standing: the account's principle (the rule of counting); theorem (the agreement of the ways of counting).

A measurement counts kinds, not holders of energy. A measurement counts the independent classes of the shape over which the counted thing ranges. The dark matter ranges over both factors of the six. So the dark matter counts six classes: three of the complex projective plane times two of the sphere. The ordinary matter is registered in the large space, inside the horizon. The region inside the horizon is a ball, and the ordinary matter counts one class. So the two counts are in the proportion six to one.

On the complex projective plane, on the sphere and on their product, every usual way of counting classes gives the same answers: three, two and six. On a ball, every usual way gives one. So the way of counting is not a free choice. Counts of holders of energy give other proportions, and none of them gives six to one. Counted by direction, for example, the counts are six small directions to three large ones.

The theorem of equal parts says that degrees of freedom that nothing distinguishes hold equal parts of the energy. Near the turn the large and the small directions scale differently, so something does distinguish them. So the theorem of equal parts fixes no ratio between the large and the small directions, and the rule for the ratio does not use the theorem.

The count does not depend on how finely the measures are resolved. Mass is one measure where all its forms turn into energy. Mass is three measures where the photon, the electron and the proton each carry their own kind. The six classes are three kinds times two rows of possibility, and the ordinary matter is one kind of mass on either reading. Suppose the ordinary matter were counted as one of the classes of the six, and not as the one class of the large space. Then the ordinary matter would take that class's part of the abiding share, and the ratio would be exactly six. The measurement excludes a ratio of exactly six.

A short loop is a path that goes all the way around the universe and cannot be pulled tight to a point. A short loop is also shorter than the diameter of the horizon. If space wrapped around within the horizon in a short loop, the region within the horizon would count N classes, more than one. The ratio would then be six less two over N pi; for two classes, about 5.68.

The shares of the two residues

$$1/(7 - 2/\pi) = 0.15715;\ \ \Omega_m = 0.3108, \Omega_\nu = 0.0014, \text{cold matter } 0.3094;\ \ \Omega_b = 0.0486, \Omega_c = 0.2608$$

"The share of the ordinary matter in the cold matter is $1/(7 - 2/\pi) = 0.15715$. The Passage's identity gives the matter fraction 0.3108, which the Passage rounds to 0.311. With that fraction, and the neutrinos of the smallest masses oscillation experiments allow ($\Omega_\nu = 0.0014$), the cold matter is 0.3094". "The two densities are then $\Omega_b = 0.0486$ and $\Omega_c = 0.2608$, each rounded from a pair whose ratio is exactly $6 - 2/\pi$".

Source: *The Weight of Possibility*, Chapter 3, Section V. Standing: computed, from the two books together.

The cold matter is the ordinary matter and the dark matter together. The dark matter is six less two over pi times the ordinary matter. So the cold matter is seven less two over pi times the ordinary matter. So the ordinary matter is one part in seven less two over pi of the cold matter, about 15.7 percent. The identity of *The Gravitational Passage* gives a matter share of 31.08 percent. The cold matter is that matter share less the share of the lightest neutrinos that oscillation experiments allow. So the ordinary matter is about 4.86 percent of the whole budget of the universe's energy, and the dark matter about 26.08 percent.

The shape the ratio reaches

$$\Omega_c/\Omega_b = 6 - R_{\mathbb{C}P^2}/(2\pi)$$

"A product of two Einstein spaces is an Einstein space exactly when the two factors have equal Ricci constants (Besse 1987); for $\mathbb{C}P^2$ and S^2 this happens when the ratio of their radii is $\sqrt{6}$. A flux of the kind used by Freund and Rubin (1980), filling the open dimensions, would also select it: with such a flux alone, the field equations make the internal space an Einstein space. The rule depends on the curvature of $\mathbb{C}P^2$ alone, $6 - R_{\mathbb{C}P^2}/(2\pi)$, so the measured ratio fixes how the internal curvature divides between the factors". This appendix gives the rule for the ratio as a function of the ratio of the radii, $6 - 36/(\pi(12 + x^2))$, under Chapter 14.
Source: *The Weight of Possibility*, Chapter 3, Section VI, and Chapter 5, Section IV. Standing: theorem (Besse); the rule as stated.

The ratio depends on the curvature of the complex projective plane alone: the ratio is six less that curvature over two pi. So a measured ratio shows how the curvature of the six divides between the complex projective plane and the sphere, and *The Gravitational Passage* says that the division is the one fact about the shape that none of its own observations measures.

The ground of the identity

"The ground of that step is the Passage's Axiom I, which 'makes the immanent act prior to the structure reconstructed from it' (the same section)". This appendix gives two related items under Chapter 4. The items are the making of the gyrations, with the yield $e^{-\omega\tau}$ and $\omega\tau \approx 62$, and the ceiling on the first heat. "On the Passage's arc each internal direction contracts as the inverse ninth root of the time (the Passage, Appendix B, Section IV), which is slow compared with that period once the time exceeds about the time light takes to cross the internal radius". "Mechanisms that fix the ratio from the side of the content act on baryons already formed, and so at or after the transition in which the proton acquires most of its mass (Farrar 2018; Brzeminski and Hook 2024; Bai and Chen 2026; Banerjee, Brzeminski and Hook 2026); the rule does not need them". "The Passage does not fix whether the ratios of the content repeat from epoch to epoch", and "The rule does not ask the turn to set a dark mass per baryon".

Source: *The Weight of Possibility*, Chapter 3, Section I, and Chapter 4, Section VI. Standing: argued, from Axiom I.

The ratio is fixed at the moment of measuring, as the cosmological constant is. The reason is the first axiom of *The Gravitational Passage*: the present act of comparison comes before the history reconstructed from it. On the arc, each of the six contracts only as the inverse ninth root of the time, so the contraction is slow beside the period of the lightest odd vibration once the time is longer than light takes to cross the six. Other accounts fix the ratio from the side of the ordinary matter. The mechanisms of the other accounts act on protons and neutrons already formed, at or after the change in which the proton gains most of its mass, but the rule for the ratio needs none of those mechanisms. *The Gravitational Passage* does not fix whether the proportions of the content repeat from one epoch to the next. The rule does not depend on whether the proportions repeat, because the rule concerns what the present measurement registers.

Chapter 20, The Residue of Matter

The count of B, and the measured amount of matter

$$\eta = 274 \times 10^{-10}\, \Omega_b h^2; \quad n_\gamma = (2\zeta(3)/\pi^2)\, T^3$$

"Each quark counts one third and each antiquark minus one third, so a proton or a neutron, made of three quarks, counts one, and every lepton counts zero" (Chapter 4, Section I). Box 1.1: with $\Omega_b h^2 = 0.02237 \pm 0.00015$ (Planck 2018), "$\eta = 274 \times 10^{-10} \times 0.02237 = 6.13 \times 10^{-10}$, with uncertainty $274 \times 10^{-10} \times 0.00015 = 0.04 \times 10^{-10}$"; $n_\gamma = 0.2436 \times 11.902^3$ cm^{-3} = 411 cm^{-3}, "so about 2.5×10^{-7} baryons in each cubic centimeter on average". Note 2 of Chapter 12: the review's own value from the microwave background is "$(\eta_{10})_{\text{CMB}} = 6.12 \pm 0.04$", and "the two differ by a quarter of either uncertainty".

Source: *The Coming of the Kinds*, Chapter 1, Box 1.1; Chapter 4, Section I; Chapter 12, note 2. Standing: measured; the number is a number of history, not of the structure of the kinds.

Physicists write the baryon number as B. The baryon number counts each quark as one third and each antiquark as minus one third. So a proton or a neutron counts one, and every lepton counts zero. The number of baryons for each photon is 274 ten-billionths times the measured density of ordinary matter. So there are 6.13 baryons for every ten billion photons, give or take four hundredths. Space holds about 411 photons of the oldest light in each cubic centimeter. So space holds about one baryon in every four million cubic centimeters, on average.

The sphaleron

$$\Delta B = 9 \times 1/3 = 3, \;\; \Delta L = 3 \times 1 = 3, \;\; \Delta(B - L) = 0, \;\; \Delta(B + L) = 6$$

$$T = 18\alpha_W{}^5 M_{Pl}/(1.66\, g_*{}^{1/2}) = 5.6 \times 10^{11}\ GeV; \;\; \textit{zero-temperature rate} \propto$$
$$exp(-4\pi/\alpha_W) \approx exp(-372) \approx 10^{-161}$$

"One passage changes nine quark doublets (three colors in each of three generations) by one unit each, and three lepton doublets by one unit each". "The factors this estimate does not carry [...] move the crossing by a factor of a few, so it lies at about 10^{12} GeV, within a factor of ten". "Sphaleron transitions run in balance from about 10^{12} GeV down to about 132 GeV, and their rate today carries a factor of about 10^{-161}".
Source: *The Coming of the Kinds*, Chapter 4, Box 4.1, after 't Hooft (1976) and D'Onofrio, Rummukainen and Tranberg (2014). Standing: established physics; computed.

A sphaleron passage is a passing state of the field of the weak force, and it is the one process among the kinds of particle that changes the number of baryons. One sphaleron passage changes nine quark doublets, three colors in each of three generations, by one unit each. Each quark counts one third, so the passage changes the baryon number by three. The passage also changes three lepton doublets by one unit each, so the lepton number, L, changes by three in the same direction as B. So each passage keeps the difference of the two numbers, B minus L, and changes their sum by six.

From a temperature of about a trillion billion electron volts down to about 132 billion electron volts, sphaleron passages run faster than the universe expands, and so they stay in balance. Today the rate of the passages carries a factor of about one in ten to the one hundred sixty-one.

The quark phase is far too small

$$\delta_{CP} = J \prod(m_i^2 - m_j^2)_u \prod(m_i^2 - m_j^2)_d / T^{12} = 4.4 \times 10^{-22} \text{ at 159.5 GeV;} \quad \eta / \delta_{CP} = 1.4 \times 10^{12}$$

"At the temperature of the electroweak crossover the strength of the quark phase is about 4.4×10^{-22}, more than a trillion (10^{12}) times smaller than the measured excess of baryons per photon", with $J = 3.16 \times 10^{-5}$, following Cline (2006) and Jarlskog (1985). The strong angle: "that angle is zero, set by the structure (*The Branches of the Kinds*, Chapter 3, Section V, Box 3.6). The angle of the strong force that measurement reads is that angle together with a phase from the masses of the quarks. Measurement places that angle below 1.8×10^{-10}, so the phase from the quark masses is zero within that bound", the bound coming from the neutron's electric dipole moment. "So the kinds break CP through two phases, one in the mixing of the quarks and one in the mixing of the leptons, and through nothing else" (Section III). "The account of Chapter 5 holds whatever their sizes" (Key Concepts).

Source: *The Coming of the Kinds*, Chapter 4, Box 4.2, Section III and Key Concepts. Standing: computed (δ_{CP}); set by the structure (the angle of the strong force); measured (the bound).

The quarks tell matter from antimatter through one phase, an angle in the mixing of the quarks. At the temperature of the electroweak change, that phase has a strength of about 4.4 parts in ten to the twenty-two, written as a pure number, which is more than a million million times too small to make the measured excess of baryons. The strong force has an angle of its own, and the structure sets that angle at zero. The angle that measurement reads is the strong force's own angle together with a phase from the masses of the quarks. Measurements of the neutron's electric dipole moment place the measured angle below 1.8 parts in ten billion.

CP is the double swap: particle for antiparticle, together with left for right. Under the double swap, the kinds of particle tell matter from antimatter through two angles of mixing and through nothing else. One angle lies among the quarks, and one lies among the neutrinos. Chapter 20 argues that B minus L never changes, and the next sections of this appendix give the argument. The argument uses neither angle, so its conclusion does not depend on the sizes of the two angles.

Why the sphalerons stay in balance through the crossover

$$T_c - T_* = 159.5 - 131.7 = 27.8\ GeV$$

"A first-order transition occurs only for a Higgs mass below the endpoint 72.4 $\pm$ 1.7 GeV (Csikor, Fodor and Heitger, 1999; Kajantie, Laine, Rummukainen and Shaposhnikov, 1996, bound it between 70 and 95 GeV)". "$T_c - T_* = 159.5 - 131.7 = 27.8$ GeV, which is more than ten times the combined uncertainty of the two temperatures (2.7 GeV) and more than five times the width of the steepest part". "By (7) the gathering holds no kind that the computations of (1) and (3) leave out, apart from a state that it never makes in numbers and that carries no weak charge; so the crossover of (1) and the temperature T_* of (3) are those of the kinds". "No process among the kinds departs from balance while changing baryon number at the transition, so the third condition is not satisfied there, and electroweak baryogenesis is excluded in the Standard Model with the measured Higgs mass. The crossover and the sphalerons' leaving balance that the lattice computes for the Standard Model are those of the kinds, so the exclusion holds among the kinds" (Section IV).
Source: *The Coming of the Kinds*, Chapter 4, Box 4.3 and Section IV. Standing: computed on the lattice (established physics); argued (that the lattice of the minimal Standard Model computes the transition of the kinds).

The sphaleron passages fall behind the expansion at a temperature of 131.7 billion electron volts. That temperature is 27.8 billion electron volts below the center of the smooth electroweak change. The gap is more than

ten times the combined uncertainty of the two temperatures. So through the steepest part of the change the passages stay in balance, and no excess of baryons can be made there, because any process that makes an excess must depart from balance.

Physicists find the two temperatures for the Standard Model by lattice computations. A lattice computation is a calculation made by computer on a fine grid of points in space and time. The lattice computations leave out only the right-handed neutrino, which never comes into balance and carries no weak charge. So the lattice computations hold for the kinds of particle of this account. Therefore, in the Standard Model with the measured Higgs mass, no excess of baryons can be made at the electroweak transition, and none can be made there among the kinds either.

Why B − L is kept

$$\partial_\mu {j_B}^\mu = \partial_\mu {j_L}^\mu = (N_f/32\pi^2)(g^2 W\tilde{W} - g'^2 Y\tilde{Y}); \quad \partial_\mu({j_B}^\mu - {j_L}^\mu) = 0$$

$$\textit{left: } 6 \times (1/3) + 2 \times (-1) = 0; \quad \textit{right: } 3 \times (1/3) + 3 \times (1/3) + 1 \times (-1) + 1 \times (-1) = 0$$

"The color field's windings leave B and L unchanged, because SU(3) acts on both chiralities alike and the vector currents of B and L have no anomaly under it". "Without the right-handed neutrino the right-handed sum would be 1". The absent Majorana mass: "No Majorana mass exists, for either chirality". The ground of the absent Majorana mass: "No state of content is its own antiparticle; only the act of comparing (the photon) is". The Z boson and the Higgs boson are also their own antiparticles, and they are "columns of context", "not states of content" (*The Branches of the Kinds*, Chapter 1, as *The Coming of the Kinds* quotes it). Box 5.2: with the sphalerons in balance and the total hypercharge zero, B = c (B − L), "with c a fixed number set by the kinds and their charges".

Source: *The Coming of the Kinds*, Chapter 5, Boxes 5.1 and 5.2. Standing: established physics (the anomaly); argued (the closed list of kinds and the absent Majorana mass).

The baryon number and the lepton number change only through one quantum effect, the anomaly. The anomaly changes both numbers alike, so their difference, B minus L, never changes. In a curved and twisted geometry, B minus L is kept too. The reason is that in each generation the left-handed sum of B minus L is zero, and the right-handed sum is zero as well. Without the right-handed neutrino the right-handed sum would be one. No particle of content is its own antiparticle. So no Majorana mass exists, and no mass term changes the lepton number by two. While sphalerons run in balance, the baryon excess is a fixed share of B minus L, so an excess with no B minus L is erased.

The share at which the baryon number froze

$$B = (8N_f + 4N_H)/(22N_f + 13N_H)\ (B - L) = (28/79)(B - L) = 0.3544\ (B - L);\quad L = -(51/79)(B - L)$$

$$f = 4(77 + 27r)/(869 + 333r),\quad r = \varphi^2/T^2 : f(0) = 28/79,\quad f \to 12/37 = 0.3243$$

$$B - L = \eta/C_{sph} = 6.13 \times 10^{-10}/0.3328 = (1.84 \pm 0.01) \times 10^{-9}\ \text{per photon};\quad 1/0.3328 = 3.005;\quad L = -2.005\ B$$

Box 7.1: for three generations and one Higgs doublet, with the right-handed neutrino out of balance, the chemical potentials in balance give the share 28/79. Box 7.2, from Laine and Shaposhnikov (2000): "Deep in the broken phase, where the field's value is large against the temperature, B = (12/37)(B − L) = 0.3243 (B − L)", and "At 131.7 GeV the two further terms of Eq. (7) together are below two parts in a hundred thousand of the leading term". Box 7.3: "The baryon number froze at 0.3328 ± 0.0005 of B − L, between 12/37 and 28/79" (Benso and others 2026).
Source: *The Coming of the Kinds*, Chapter 7, Boxes 7.1, 7.2 and 7.3. Standing: computed (established physics); measured (η).

When the sphalerons and all the ordinary reactions run in balance, the baryon number is a fixed share of B minus L. The share is twenty-eight seventy-ninths where the Higgs field is zero, and twelve thirty-sevenths where the Higgs field is large against the temperature. Between a zero Higgs field and a large one, the share falls steadily from twenty-eight seventy-ninths to twelve thirty-sevenths. The baryon number froze, and stopped changing, when the sphaleron passages fell behind the expansion at 131.7 billion electron volts. At that moment the share was 0.3328, a value computed across the crossover. The measured amount is 6.13 baryons for every ten billion photons. Dividing the measured amount by the share 0.3328 gives the residue the turn left: 1.84 units of B minus L for every billion photons. The residue is about three times the baryon number. The lepton number is the baryon number less B minus L, so the lepton number is about minus two times the baryon number.

Chapter 21, What the Beginning Left Behind

Where the description of the new epoch begins

"Two facts of the account are not fixed, and the book does not fix them: where on the arc the description of the new epoch begins, and the dynamics by which the internal space returns to its compact form in the new epoch". In the sequence of an epoch: "Read away from the turn, on the same arc, three directions grow as $\tau^{5/9}$ while each of the six contracts as $\tau^{-1/9}$. Where on the arc the description begins is not fixed".

Source: *The Compactification*, Chapter 1, Sections VII and IX. Standing: a statement of scope.

Near the turn, each direction of space changes in size as the time raised to its own exponent. Einstein's equations set two rules for the exponents: the exponents add to one, and their squares add to one. The two rules fix the arc on which the new epoch opens, and on that arc three directions grow as the time to the power five ninths, and each of the six

shrinks as the time to the power minus one ninth. The account leaves two facts open. The first open fact is where on the arc the description of the new epoch begins. The second open fact is the detailed motion by which the six return to their held size in the new epoch. Chapter 4 states both as facts the account does not fix.

Summary

This appendix gathered, chapter by chapter, the formulas and technical details that the chapters of this book state in words. For Chapters 1 to 5 the appendix gave the action of the metric alone, and how the ten dimensions differ from the ten of superstring theory. The appendix gave the reason no further field is threaded through the six, the standing of every link of the chain, and the Planck units. The appendix gave the Kasner curve and the arc, with the earlier worlds in which three directions grow. The appendix gave the classification that selects the complex projective plane times the sphere, and the reason the product is not a Calabi-Yau space. The appendix gave the radii and the volume of the six. The appendix gave the floor and its first-order computation, with the source book's statement of what rests on the floor. The appendix gave the external value of the vacuum's energy, and the one vacuum with the table of its standing.

For Chapter 7 the appendix gave the ground and scope of the reset, of the even beginning, and of the information that falls into a black hole. For Chapters 8 to 14 the appendix gave the formula for the cosmological constant, and the identity in three lines and in other units. The appendix gave the five choices varied together, and the counts of horizons and of an epoch. The appendix gave the definitions of the measures of the first ripples, and the binding of the electrons. The appendix gave the two steps of identification and consequence, and white noise. The appendix gave the standing of the exclusion of a long inflation, and the formulas of the ledger. For Chapters 16 to 20 the appendix gave the frequency and

strength of the settling waves, and the faint background of second order. The appendix gave the measures of a background, the entangling phase, the antipodal parity and the lightest odd level. The appendix gave the rule of the ratio with the count of classes, and the counts of the residue of matter. For Chapter 21 the appendix gave the scope of where the description of the new epoch begins.

Each item has the strength and scope that its source book states, and the standing of each item is named: theorem, established physics, identification, argued, computed or measured. Chapters 6 and 15 state their results without formulas, and this appendix holds no item for those two chapters.

How the Coming Measurements Will Test the Account

The account makes definite statements about the sky. The instruments of the coming decades will measure each statement of the account more closely than any earlier instrument has. This appendix gathers the statements and sorts them by the quantity that is measured. For each quantity this appendix gives four things. First, this appendix gives what the account says. Each value that the account gives comes with its standing, the ground on which the value stands. The Preface names five sorts of standing: set by the structure; set by the structure once a stated step is taken; measured; measured, with its reason still a question; and bounded, with its value not given. Second, this appendix gives what the present measurements find, with their uncertainties as the authorities report them (the authorities are the books of the collection from which this book takes its claims). Third, this appendix names the instruments now coming. Fourth, this appendix gives the authorities' own statement of which measurement separates the account from which other account. The chapters state each measurement in the main text positive first, as what the measurement measures and what it found, while the comparisons in standard deviations, the rules of decision and the forecasts of the instruments are collected in this appendix.

How a Result Is Counted

Every measurement in cosmology comes with an uncertainty, written $\pm$ and read "give or take". *The View from Inside* is the book of the collection for the general reader on why empty space weighs so little. *The View from Inside* explains the sign: "0.315 ± 0.007 means the measured value is most

likely within 0.007 of 0.315. That $\pm$ is one standard deviation, the usual measure of how far a measurement might wander by chance". Physicists state the distance between a prediction and a measurement in standard deviations. A distance of five standard deviations is, in the words of *The View from Inside*, "a margin so wide that chance would produce it less than once in a million tries".

The account fixes in advance its rule for when a result counts as confirmed. The rule is stated in *The Gravitational Passage*, the book of the collection that sets the whole account out for physicists. "Here, as throughout the book, confirmed means established at five standard deviations by at least two independent kinds of measurement". The second kind of measurement guards against an error in any one method. In this appendix the word "confirmed" has the meaning of that rule and no other. *At the Edge of Reality* is the book for the general reader whose story this book takes up. *At the Edge of Reality* says how the account treats a difference smaller than five standard deviations: "A smaller difference is a measurement of interest, and it is reported as such". This appendix reports each difference below in that way, with the authority's own reading of the difference.

The account makes some statements about the sky that an instrument can reach, and some that no instrument can reach. *The Compactification* is the main source of this book. *The Compactification* states the use of each kind of statement. "A statement about the sky that no instrument can reach still has a use. It can constrain the account internally, bound a neighboring computation, or fix what a further step must deliver. A statement that an instrument can reach has a use of a different kind: a definite result, at a definite sensitivity, decides it" (Chapter 8, Section I). In plain words, each kind of statement has its own use. A statement that no instrument can reach still holds the account together, limits a neighboring computation, or fixes what a further step must deliver. A statement that an instrument can reach is decided by a definite result at a definite sensitivity.

The predictions of the source books are fixed at their date. *The View from Inside* names the record for *The Gravitational Passage*. "Every prediction of *The Gravitational Passage*, with the measurements at its date, is recorded in *The Registered Predictions of The Gravitational Passage*" (*The View from Inside*, Appendix B). *The Weight of Possibility* is the book that identifies the dark matter with a lasting vibration of the six small dimensions. *The Weight of Possibility* keeps a dated record of its own. In that record the predictions of *The Weight of Possibility* "are fixed at their date, with their standing against the measurements of that date" (Notes, note 3).

The Predictions in One List

The Gravitational Passage lists the account's principal predictions. "The account makes six principal predictions: a dark-energy equation of state of exactly $w = -1$; a matter fraction of 0.311; two masses, each held in two places and related only through gravitation, that become entangled; no primordial gravitational waves; no primordial isocurvature mode; and a spatial curvature of order 10^{-5} (Chapter 20)". In that book, "no primordial gravitational waves" means none at first order, in the largest and simplest part of the effect. The first ripples of the epoch are ripples of density. In the words of *The Gravitational Passage*, "waves of the metric arise from them only at second order, with a ratio of tensor to scalar power far below 10^{-6}" (Chapter 18, Section XIII). The ratio of tensor to scalar power is the power of the first waves divided by the power of the first ripples. *At the Edge of Reality* lists the predictions again for the general reader. "The coming decade of measurement will look for what the story says is there: no primordial gravitational waves, a flat space, every component of the early universe departing from evenness together, and a cosmological constant that stays constant (Appendix B)".

The Weight of Possibility adds predictions of its own. The first added prediction is the ratio of dark to ordinary matter, $6 - 2/\pi$. The other added predictions are six predictions of kind, which say what sort of thing

the dark matter is. The six predictions of kind read: "the dark matter acts only through gravity; it does not decay; it is perfectly cold, with no cut-off in its structure on small scales; it is made of gyrations of about the Planck mass; no short loop exists anywhere in space (Chapter 3, Section II); and the dark component carries no separate pattern of primordial departures (Chapter 3, Section I)".

What Separates the Account from the Standard Picture

Astronomers report the measurements in terms of the standard picture of cosmology. The standard picture takes the energy of empty space as a constant. The standard picture fits two numbers to the sky: the matter share (the part of the energy of a flat universe held as matter, ordinary and dark together) and the expansion rate. Chapter 8 calls the matter share the matter fraction. *The Gravitational Passage* puts the question of which of the account's predictions separate the account from the standard picture. The answer of *The Gravitational Passage* is: "Two do: the matter fraction, and the Hubble constant it implies, which are one test". The table of *The Gravitational Passage* sets out each prediction, the step on which the prediction rests, and whether the standard picture expects the prediction. The last column of the table below gives the answer of *The Gravitational Passage* in another form: what each measurement separates.

Prediction	**Step it rests on**	**Expected in the standard picture?**	**What the measurement separates**
$w = -1$ exactly	Λ fixed once (Chapter 13, Sections V and VI)	Yes: the default of standard cosmology	The standard picture expects this result too, so this measurement does not separate the two

Prediction	Step it rests on	Expected in the standard picture?	What the measurement separates
$\Omega_m = 0.311$	the accessible share c_1/S (Chapter 13, Section II)	No: a free parameter of the standard fit	Separates the account from the standard picture
$h = 0.678$, given $\Omega_m h^2$ from the microwave background	the same step; the same test as Ω_m (Chapter 13, Section IV)	No: free	Separates the account from the standard picture, as one test with the row above
Entanglement of two masses through gravitation alone	one geometry for each alternative (Chapter 12, Section VI)	Yes, in linearized quantum gravity	Separates the account from accounts with one geometry for all alternatives
Ratio of tensor to scalar power far below 10^{-6}	the scalar character of the primordial departures (Chapter 18, Section IX)	No: many inflationary models give a ratio within reach of coming measurements	A detection of first waves separates the two decisively; a finding of none is what the account says, and inflationary models with a smaller ratio allow it too
Spatial curvature of order 10^{-5}	the fixing of the curvature by the restoration (Chapter 18, Section IX)	Yes, in inflationary cosmology	The standard picture expects this result too, so this measurement does not separate the two
No primordial isocurvature mode (adiabatic departures)	one form for every component of the content (Chapter 18, Section IX)	Yes, in single-field inflation	Separates the account from accounts with several independent fields

Source: *The Gravitational Passage*, Chapter 20, Questions. The first three columns are the columns of *The Gravitational Passage*. The chapters named in the first three columns are chapters of *The Gravitational Passage*. The fifth row of the table concerns the ratio of tensor to scalar power. The fifth row follows what *The Compactification* says of the inflationary models in Chapter 6, Section IX. "The account parts from those models at one entry, the tensor ratio, decisively only if primordial gravitational waves are found".

In plain words, the matter share and the expansion rate set the account apart from the standard picture. The reason is that the standard picture leaves both free, and the account fixes both. The expansion rate follows from the matter share, so the matter share and the expansion rate make one test. The standard picture also expects the energy of empty space to stay constant and space to be flat, so a measurement that finds either one agrees with both the account and the standard picture. If every component of the first ripples is found to have one form, the finding separates the account from accounts with several independent fields at the beginning. The search for first waves separates the account from the many inflationary pictures that give first waves strong enough for the coming instruments to find. Inflation is a proposed burst of extremely fast expansion at the very start, which Chapter 13 describes. One more test is whether two masses become entangled through gravitation alone. The entanglement test separates the account from every account in which one geometry serves all the alternatives of the matter (the several definite possibilities in which the matter can be held at once).

The Matter Share and the Expansion Rate

What the account says

The identity of Chapter 8 is one statement in pure numbers. The identity is what is left when the two ways of finding the energy of empty space, from its share of the whole and from Friedmann's equation for the expansion, are set equal. The identity gives a matter share of 0.311, give or take 0.001: thirty-one point one percent of the energy of a flat universe. The standing of that value is "identification; computed". The small spread of the value comes only from how precisely the radiation is known. "The measured radiation density, from 7×10^{-5} to 10^{-4}, moves it from 0.3119 to 0.3103, so this input gives $\Omega_m = 0.311 \pm 0.001$" (*The Gravitational Passage*, Chapter 13, Section IV). The oldest light measures an amount of matter, $\Omega_m h^2$ in the table above, of 0.1430, give or take 0.0011. When joined with that amount, the identity gives h = 0.678, give or take 0.003. The letter h stands for the Hubble constant divided by a hundred. So h = 0.678 is an expansion rate of sixty-seven and eight tenths kilometers per second for every megaparsec, give or take three tenths. *The Gravitational Passage* says of that value of h: "This is the same test as Ω_m, not an independent one".

The test of the matter share can be read in five ways. *The Compactification*, Chapter 6, Section X, sets the five ways side by side, as the table below shows, and the ledger of the sky's numbers in Chapter 14 counts the five ways once, as one test.

Quantity	The account	Measured
Matter share Ω_m	0.311 ± 0.001	0.315 ± 0.007 (Planck); 0.2975 ± 0.0086 (DESI alone)
Vacuum share Ω_Λ	0.689	0.685 ± 0.007
Energy of empty space ρ_Λ (GeV^4)	2.56×10^{-47}	2.52×10^{-47}
Particle horizon over de Sitter radius	$\sqrt{(8f)} = 2.654$	2.632
Age in de Sitter times	0.792	0.787 ± 0.009
h	0.678 ± 0.003	0.674 ± 0.005 (Planck); 0.730 ± 0.010 (Cepheids)

What the measurements find

The Planck satellite measures a matter share of 0.315, give or take 0.007. In words, the Planck share is thirty-one and a half percent, give or take seven tenths of a percent. The Dark Energy Spectroscopic Instrument, DESI, maps how galaxies are spread through space. DESI measures distances with the baryon acoustic oscillations, a ruler of fixed size that sound in the early universe left in that spread. Taken alone, the baryon acoustic oscillations of DESI give a matter share of 0.2975, give or take 0.0086: about twenty-nine and three quarters percent. DESI also measures the Lyman alpha forest, the forest of hydrogen absorption. In the standard model, the forest gives 0.325, give or take 0.018 (DESI Collaboration 2026). The value 0.325 lies 0.8 standard deviations from the account's 0.311. *The View from Inside* reports how far the prediction lies from the Planck value and from

the DESI value. “The prediction for the matter fraction lies 0.6 standard deviations from the Planck value, and 1.6 from the DESI value, on the other side”. So the account’s value of 0.311 lies between the Planck value and the DESI value. *The Gravitational Passage* reads the comparison of 0.311 with the measured matter share as a statement about the present universe. The measured matter share places the present universe at the account’s value: in the words of *The Gravitational Passage*, “the measured matter fraction places it there to within 0.6 standard deviations”.

The Gravitational Passage also states the comparison for the energy of empty space in each volume. “The predicted density lies about 1.7 percent above the observed one. The uncertainty of the measured Ω_m, ± 0.007, moves the ratio by about ± 2.8 percent, so the prediction lies within it” (*The Gravitational Passage*, Chapter 13, Section IV). The comparison can also be read the other way, from the measurement back to the account. Read that way, “the comparison with 0.315 ± 0.007 measures the coefficient of that part as 0.99 ± 0.02”. The part in question is the ground-state part, the energy of the lowest, quietest quantum state. The account identifies the ground-state energy with the Planck density. The measurement puts the ground-state energy at ninety-nine hundredths of the Planck density, give or take two hundredths.

The prediction of 0.311 depends closely on its inputs. *The Gravitational Passage* states how far a change in each input would move the prediction. One input is the trim, written f. The trim is the share of the Planck density that is left once the curvature of the six small dimensions has drawn its part away. “A change of one percent in f moves Ω_m by ∓ 0.0025, so the measured ± 0.007 corresponds to about ± 3 percent in f, and the coefficients of order one named in the Questions must be known to a few percent before the test is decisive”. The six small dimensions are held at a small size. The floor is what that holding rests on: the internal space cannot shrink to nothing. The account counts the energy of the floor within the ground-state part, and that count is an identification. *The Gravitational Passage* shows the size of the difference that the count decides: “if the first-

order energy of the floor, +0.071 M_{Pl}^4 (Appendix D), were added to the external value instead, f would be 0.951 and Ω_m 0.292, about three standard deviations below Planck 2018". The external value is the energy of empty space as it would be found from outside, if there were an outside. The external value is taken at the Planck scale.

The expansion rate

Astronomers measure the expansion rate in two ways today: from the oldest light, and from distance ladders. Astronomers fit the standard model to the oldest light and get 67.4, give or take 0.5, in kilometers per second for every megaparsec. *The Gravitational Companion*, another book of the collection, puts that value 0.8 standard deviations from the account's 67.8. A distance ladder is a method of measuring the expansion. The ladder climbs, rung by rung, from nearby stars of known brightness to stars farther out. One distance ladder is built on the brightest red giant stars. The red-giant ladder gives 70.39, give or take 1.22 from chance and 1.33 from method (Freedman and colleagues 2025). The red-giant value lies 1.4 standard deviations from 67.8. A second distance ladder is built on Cepheids and exploding stars. Cepheids are nearby pulsating stars whose rhythm of pulsing tells how bright they are. The Cepheid ladder gives 73.04, give or take 1.04 (Riess and colleagues 2022). The Cepheid value lies 4.9 standard deviations away from 67.8, with the Cepheid uncertainty and the account's uncertainty added in quadrature (each uncertainty squared, the squares added, and the square root of the sum taken). *The Gravitational Passage* gives the Cepheid value as $h = 0.730 \pm 0.010$, "about five standard deviations away".

The Gravitational Passage states plainly where the predicted $h = 0.678$ places the account. "It places the account with the value of the Hubble constant inferred from the early universe: the local distance ladder gives $h = 0.730 \pm 0.010$ (Riess and colleagues 2022), about five standard deviations away, and with the same $\Omega_m h^2$ that value would require $\Omega_m = 0.268$, which the identity excludes" (*The Gravitational Passage*, Chapter 13,

Section IV). *The Gravitational Companion* gives the range of the Hubble constant that the identity allows. "With the microwave background's $\Omega_m h^2$, a Hubble constant outside 65.6 to 70.2 would put the matter fraction more than three standard deviations of the measured matter fraction (0.007) from the identity's 0.311. The identity therefore excludes 73 and does not exclude 70". *The Gravitational Companion* also states what would test the account if the measured rate settled near 73: "the test is then a matter fraction confirmed away from 0.311, the value the Passage predicts". *The Compactification*, Chapter 6, Section XII, adds a remark on the two largest differences between the account and the measurements. "The largest differences, the Cepheid Hubble constant at about five standard deviations and the ratio with the DESI distances at 4.7, each rest on one kind of measurement". The rule for a confirmed result requires two kinds. The second of the two largest differences concerns the ratio of dark to ordinary matter. A later section of this appendix takes up that ratio with the studies of DESI.

The coming instruments

The View from Inside names the coming instruments for the matter share, and says where the account expects their measurements to arrive. "The Dark Energy Spectroscopic Instrument, the Euclid space telescope, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope will narrow both, toward an uncertainty of about 0.001. The account says they will converge on 0.311" (*The View from Inside*, Appendix B). For the expansion rate, Appendix B of *The View from Inside* adds: "it says the present dispute over the Hubble constant will settle near that value". The value meant is sixty-seven and eight tenths kilometers per second for every megaparsec.

The clumping of matter, S_8

S_8, read "S eight", is the number that says how clumped the matter is today. *The Weight of Possibility* computes S_8 from the account's matter share and the account's ratio of dark to ordinary matter. The computed value of S_8 is 0.823, give or take 0.006. With the account's Hubble constant as well, *The Weight of Possibility* gets 0.827, give or take 0.007, while the fit of the standard model to the oldest light gives 0.832, give or take 0.013. *The Weight of Possibility* sets the measurements beside the account's value. "The lensing of the microwave background agrees with the account within one standard deviation (Qu and others 2026; Omori and others 2026), as do the complete Kilo-Degree Survey (Wright and others 2025) and the growth rate measured from the motions of galaxies (Alam and others 2021). The final galaxy clustering and weak lensing of the Dark Energy Survey lies 2.5 standard deviations below the account (Abbott and others 2026). The clusters counted by eROSITA lie 3.2 standard deviations above it (Ghirardini and others 2024). Those two measurements differ from each other by 4.5 standard deviations".

The Weight of Possibility gives the reason the account's value differs from the standard model's value. "The account's value is lower, and its uncertainty half as large, because the account fixes the matter fraction where the standard model fits it" (*The Weight of Possibility*, Chapter 5, Section VI). In plain words, the account's value of S eight is lower than the standard model's value, and its spread is half as large. The reason is that the account sets the matter share at one value, where the standard model fits the matter share to the sky. S_8 is computed from the matter share, so S_8 is not a separate prediction. *The Weight of Possibility* says what a value of S_8 confirmed away from the account's value would bear on. "The amplitude of clustering is not independent of the matter fraction: a value confirmed away from 0.823 bears first on the identity for the cosmological constant, unless the matter fraction is confirmed at 0.311 by a separate measurement" (*The Weight of Possibility*, Chapter 5, Section VI).

The Energy of Empty Space and w

The account says the energy of empty space is exactly constant at every era of the rebuilt history (the one history that the present requires, pieced together from what arrives now). In the language of physics, the energy of empty space has an equation of state, written w, of exactly minus one, where w is the ratio of the pressure of a form of energy to the amount of that energy in each unit of volume. The standing of the constant energy is argued. On the account, the cosmological constant is fixed once, from the horizon of the measurement made now, as the constant term of one geometry. If a change of the constant between the eras of our history were confirmed, the change would bear on one step of the account: the step that fixes the constant once, as the constant of the reconstructed history (*The Gravitational Companion*, Refutation Conditions Named in This Book, Section I). The Planck satellite has been combined with the ruler of galaxy distances and with exploding stars. The combination puts w at minus one and three hundredths, give or take three hundredths.

Recent combinations of measurements report a departure from a constant. *The Gravitational Passage* reports the departure exactly: "the final analysis of the Dark Energy Survey, with its own supernovae, galaxy clustering and weak lensing, combined with DESI and the cosmic microwave background, gives $w_0 = -0.82 \pm 0.05$ and $w_a = -0.63$ $(+0.21/-0.18)$, about three standard deviations from a cosmological constant (Abbott and colleagues 2026). The departure does not rest on the supernovae: without them the same analysis still finds 2.6 standard deviations, and leaving out each probe in turn gives between 2.3 and 3.2". DESI itself reports a preference for a changing w at 2.7 and 3.2 standard deviations (DESI Collaboration 2026). *At the Edge of Reality* notes that "these analyses share much of their data". *The View from Inside* sets the reported departures against the rule for a confirmed result. In the words of *The View from Inside*, "the rule above asks for five, by two kinds of measurement".

The account states what the coming surveys will find. *The Gravitational Passage* says: "The account predicts that the reported departure will not be confirmed" (*The Gravitational Passage*, Chapter 13, Section V). *At the Edge of Reality* names three coming surveys and what the account says they will find. "The Euclid space telescope, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope will settle the question, and the account says they will find the energy of empty space constant" (*At the Edge of Reality*, Appendix B). A cosmological constant is the default of standard cosmology, so the standard picture expects a constant too, and so a constant found by the coming surveys does not separate the account from the standard picture.

The account has a formula for the vacuum share, the share of the energy of the universe held by empty space. *The View from Inside* gives how much the value of that formula at the present changes over a span of measuring. "Over thirty years, the length of a career in astronomy, the formula's value at the present changes by about 5 parts in a billion" (*The View from Inside*, Appendix A, Section V). Over the thirty years in which the energy of empty space has been measured, the vacuum share at the present has changed in value by about five parts in a billion.

The Flatness of Space

The account says the smooth model of the new epoch is flat, because no length of the old epoch passes the turn, which is the passage at the end of the interiors of the black holes in which a large outside world is restored. On the account, space is curved only by its ripples, by about one part in a hundred thousand. The standing of the flatness is argued. Suppose space had a curvature with a radius as large as the de Sitter radius. The curvature would then leave a share today of about 10^{-64}, two and a half parts in ten to the sixty-four. A share that small is far too small to measure. The de Sitter radius is the edge that a universe holding nothing but the energy of empty space would have.

Astronomers report the curvature of space as a share of the total density, and a share of zero is flat. One measurement combines the oldest light, its lensing and the baryon acoustic oscillations. The combination gives a curvature share of 0.0007, give or take 0.0019 (Aghanim and others 2020). The curvature share of 0.0007 is seven ten-thousandths, give or take nineteen ten-thousandths, which is zero within the measurement. DESI with the oldest light gives 0.0023, give or take 0.0011 (Abdul-Karim and others 2025), and *At the Edge of Reality* reports the DESI value as 2.1 standard deviations from flat. *At the Edge of Reality* also says what the account expects. "The coming surveys will sharpen both, and the account says they will close in on zero" (*At the Edge of Reality*, Appendix B). The inflationary picture expects a flat space too, so the measurement of the curvature does not separate the account from the inflationary picture.

The Pattern of the First Ripples

The first ripples are the faint differences of density with which our epoch began, and from which its galaxies grew. The account says the first ripples are correlated across the whole sky, with nearly the same strength at every length. Independent pieces would leave a different pattern, with a spectral index of four, and the account says the first ripples do not have that pattern. The spectral index is the number that says how the power of the ripples changes from band to band, so that an index of one means the same power in every band. The correlated pattern rests on one step of the account, the oneness of the ends of the black-hole interiors, so a finding of first ripples with the pattern of independent pieces on large lengths would bear on that step. *The Gravitational Passage* states the step of the oneness in Chapter 18, Section II. *The Gravitational Companion* names the step of the oneness among the conditions that would refute the account (Refutation Conditions Named in This Book, Section I). The standing of

the correlated pattern is argued. The account does not fix the strength and the tilt of the first ripples. The account uses measured values for both. The tilt is how far the spectral index lies from one.

The measured index is 0.9649, give or take 0.0042 (Aghanim and others 2020; Akrami and others 2020). The measured change of the index with length is minus 0.0045, give or take 0.0067, which is consistent with zero. Combined measurements that add the Atacama Cosmology Telescope give an index of 0.974, give or take 0.003 (Louis and others 2025). With the Atacama Cosmology Telescope, the change of the index with length is plus 0.0062, give or take 0.0052 (Calabrese and others 2025). A change of plus 0.0062 is also consistent with zero. *At the Edge of Reality* states what the account expects. "The observed pattern is nearly the same at every length, with a spectral index of about 0.97, and the account says every finer map of the sky will keep it so" (*At the Edge of Reality*, Appendix B). The inflationary models now favored also give an index below one, so the measurement of the index does not separate the account from those models.

The First Waves

What the account says, and what is measured

The first waves are the waves of the geometry present from the beginning of the epoch. The account says the first waves have a power far below one millionth of the power of the first ripples. The reason is that the first ripples are ripples of density. Waves arise from ripples of density only at second order, from ripples acting on ripples. The standing of the statement is argued. Astronomers measure the ratio of the power of the first waves to the power of the first ripples, and write the ratio r. The BICEP and Keck telescopes place the ratio below 0.036 when combined with

Planck and WMAP, at 95 percent confidence (Ade and others 2021). A joint analysis with the Planck polarization data places the ratio below 0.032 (Tristram and others 2022): about three parts in a hundred.

The coming instruments

The Compactification names the two coming instruments and says what the account expects of them. "The Simons Observatory forecasts an uncertainty in r of about 1.2×10^{-3} (Simons Observatory Collaboration 2025), and the LiteBIRD satellite targets an uncertainty below 0.002 (LiteBIRD Collaboration 2026); the account says that both will find nothing" (*The Compactification*, Chapter 6, Section V). The Simons Observatory is an array of telescopes in the Atacama Desert of Chile. LiteBIRD is a Japanese satellite. The Simons Observatory is forecast to measure the ratio to about one part in a thousand. LiteBIRD aims at better than two parts in a thousand. The account's ratio lies "three decades or more below the forecast reach" (*The Compactification*, Chapter 8, Section XI). A decade is a factor of ten, so three decades or more is a thousand times or more. The faint waves of second order carry an energy share of order ten to the minus twenty-two. The energy share of a background of waves is the share of the critical density that the waves carry. *The Compactification* places the waves of second order beyond every instrument. In the words of *The Compactification*, the waves lie "below every instrument considered in this chapter, at every frequency, by nine decades or more". No planned measurement aims at the waves of second order.

Which measurement separates which account

The Compactification names what a finding of no first waves tests. "The primordial tensor null: the Simons Observatory and LiteBIRD reach r of about 10^{-3}, which tests the many models of inflation that predict a ratio in that range" (*The Compactification*, Chapter 8, Section XIV.A). Many models of inflation predict a ratio within reach of the Simons Observatory and LiteBIRD, so a detection of first waves at that level would separate

those models from this account decisively. A finding of no first waves is what this account says. The inflationary models that predict a smaller ratio also allow a finding of none, so a finding of none does not by itself separate the account from those models.

The search for first waves also bears on a second account of dark matter. In the second account, the dark matter is made of particles of about the Planck mass. The second account is called Planckian interacting dark matter. *The Weight of Possibility* states that "its most natural masses, above about a hundredth of the Planck mass, are already excluded by the absence of primordial gravitational waves at the level they need". *The Weight of Possibility* also states that "A detection of primordial gravitational waves at the level that proposal needs would support its production, which this account does not need". So first waves found at that level would be what Planckian interacting dark matter needs, while this account needs no first waves.

A third question is often tied to the first waves. Would finding the first waves show that space and time are made of quanta? Quanta are the smallest units of energy that a wave can carry, as light comes in photons. The arena is space and time together, the stage on which everything happens. In 2014 the physicists Lawrence Krauss and Frank Wilczek argued that an imprint of first waves would show the arena quantized. A detection of first waves would not show the arena quantized. *The Unquantized Arena*, another book of the collection, gives the answer. *The Compactification*, Chapter 7, Section VI, quotes the answer. If a background of first waves exists, "its amplitude is computed from the vacuum fluctuations of the small waves of the geometry in their description by quanta. The account allows that description, as a description: its quanta are quanta of the description, never quanta of the arena". *The Unquantized Arena* adds: "The answer holds whether or not such a background is found, and the account rests on neither outcome". In the usual story of inflation, the first waves are small waves of the geometry on a smooth background, described by quanta. In that story, the power of the first waves is set by the restless trembling of the quanta in the vacuum. The arena of space and time has

no smallest step and no quantum of its own at any scale. The small waves of the arena can be described by quanta, and those quanta belong to the description of the waves, not to the arena. So a background of first waves, if one existed, would not show the arena quantized. Chapter 12 predicts that the turn makes no first waves at first order. The prediction of Chapter 12 does not rest on the question of quanta at all. Gravitation is not quantized.

The account makes two statements about first waves. The two statements concern two different steps, and both statements hold. *The Gravitational Passage* speaks of the beginning of an epoch, and says what the turn makes. The turn makes no first waves at first order, because the ripples that the turn passes on are ripples of density, and because no burst of early expansion stretched the waves of empty space out to the lengths we observe. *The Unquantized Arena* says what a detected background would show about the arena. A detected background would not show the arena quantized, because the strength of a detected background would be set by the lowest, unremovable motion of small waves counted in steps. The steps belong to the description of small waves, which the account allows. So the statement of *The Gravitational Passage* says what the turn makes, while the statement of *The Unquantized Arena* says what a finding would mean.

The Separate Pattern of One Ingredient

The account says every ingredient of the young epoch departed from evenness together. In the language of physics, the first ripples are adiabatic: every component of matter and radiation departs together. The account also says that no isocurvature ripple will be found. An isocurvature ripple is a ripple of one ingredient against another, with the total density unchanged at first. The standing of the statement is argued, and the statement follows by two routes. The first route starts from the one form that passes the turn (*The Compactification*, Chapter 6, Section IV). The second

route starts from the identity that fixes the ratio of dark to ordinary matter wherever a measurement is made (*The Weight of Possibility*, Chapter 3, Section I). The identity of the ratio "predicts that no separate pattern of departures from uniformity will be found in the dark component".

The measurements find no isocurvature ripple. In models that mix the adiabatic and the isocurvature ripples and correlate the two, an isocurvature ripple of the cold dark matter carries less than 1.3 percent of the variance of the temperature of the oldest light, at 95 percent confidence (Akrami and others 2020). Three kinds of isocurvature ripple are commonly tested. The bounds for the three kinds are 1.3, 1.7 and 1.7 percent (*At the Edge of Reality*, Appendix C). *The Coming of the Kinds* is the book of the collection that tells how the kinds of particle came to be after the Big Bang. *The Coming of the Kinds* gives the bound for a ripple that is uncorrelated with the main ripples. "Departures of one component against another, of the kind uncorrelated with the main departure, can account for no more than 1.55 percent of the variation of its temperature (Planck 2018 results X, Table 14, for the cold dark matter)".

Single-field inflation is inflation driven by a single field. Single-field inflation also expects no isocurvature ripple, so the search for an isocurvature ripple separates the account only from accounts with several independent fields, as *The Gravitational Passage* states. The authorities name no particular coming instrument for the search for an isocurvature ripple. Each finer map of the oldest light narrows the bound.

The Count of Light Species and the Settling Waves

What the account says

At the turn the six small dimensions settled into their held size. The settling released waves of the large dimensions, the settling waves. The count of light species, written N_{eff}, is the energy of all the radiation other than light, counted in units of the energy that one kind of neutrino carries. The

account requires the settling waves to stay within the bound that the measured count of light species sets. The account predicts no particular value below that bound. "The account predicts no particular excess in the effective number of light species; it requires only that the waves stay within the measured bound" (*The Weight of Possibility*, Chapter 4, Section III). The three kinds of neutrino give a count of 3.044 (Bennett and others 2021). The settling waves can add less than about 0.09, nine hundredths of a species. Nine hundredths of a species is less than about three percent of the radiation of the content at the time when the content had about a hundred species.

What the measurements find

The Planck team combined the oldest light with the baryon acoustic oscillations. The combination gives a count of 2.99, give or take 0.17 (Aghanim and others 2020). In words, the count is two point nine nine, give or take seventeen hundredths. The Atacama Cosmology Telescope gives 2.86, give or take 0.13, when combined with Planck and further surveys (Calabrese and others 2025). *The Compactification* names the measurements behind the bound. "Later analyses that add the Atacama Cosmology Telescope and the abundances of the light elements narrow the bound further (Calabrese and others 2025; Goldstein and Hill 2026); these are the measurements on which *The Weight of Possibility* rests its bound of three percent" (*The Compactification*, Chapter 8, Section X).

The coming instrument, and the form of the test

For the coming instrument the forecast reads as follows. "The Simons Observatory forecasts an uncertainty on N_{eff} of about 0.05 to 0.07 (Ade and others 2019)". An uncertainty of 0.05 to 0.07 is five to seven hundredths of a species. *The Compactification* states the form of the test. "Since no further light kinds exist (*The Kinds of Particle*, Chapter 6, Section IV), a confirmed excess ΔN_{eff} well above 0.1 would measure the share of the radiation carried by the settling waves, and the condition on the turn would be re-

stated with that share in place of three percent". *The Compactification* adds: "A measurement that finds N_{eff} consistent with 3.044 at an uncertainty of 0.05 narrows the room for the settling waves without deciding whether they are present".

The condition on the turn is one of three requirements that the account places on the turn. For the present bound of three percent, the condition on the turn reads: "either the internal space arrives at its held size displaced by no more than about one part in a thousand of its radius, or a stage that behaves as matter carries at least as much energy as the waves and decays into the content". The internal space is the one small space that the six form together. In a stage that behaves as matter, energy is held in heavy, slow things. The heavy things later decay into the hot gas of particles. The source adds: "That condition is a condition on the turn, not a result, and the measurement of N_{eff} sets it".

The count of light species is the strong test of the settling waves, for the reason the authority gives. "A direct detector registers the part of a background in its own band. The count of light species registers the energy of the whole background, in every band at once, because it measures the expansion rate the energy produces". The settling waves are "tested, for the present, only through N_{eff}, which counts it whatever its frequency" (*The Compactification*, Chapter 7, Section V).

The Ratio of Dark to Ordinary Matter

What the account says

Take any region of space. The ratio of dark to ordinary matter is the mass of dark matter in the region divided by the mass of ordinary matter in the region. The account registers the ratio at six less two over pi, 5.3634. The ratio has a standing of its own, the standing of a registered ratio. The counting principle is the account's own, and so is the rule that turns the principle into a number. The rule was selected by comparison with the

measurement from a small family of expressions of one form, of which twelve were examined in all. Together with the matter share of Chapter 8, the ratio gives the share of ordinary matter in all the matter at 0.15644, with the lightest neutrinos: a little more than fifteen and a half percent.

What the oldest light finds

The Weight of Possibility reports the measurements of the oldest light. "Planck alone gives 5.364 ± 0.065 (Aghanim and others 2020)". "The combination of the South Pole Telescope, Planck and the Atacama Cosmology Telescope gives 5.370 ± 0.048 (Camphuis and others 2026), and every analysis of the microwave background alone lies within 1.3 standard deviations of the prediction. These two ratios are computed here from the two published densities, treated as uncorrelated. A correlation of -0.4 between the densities would enlarge each uncertainty by about 15 to 18 percent. The neighbors of the rule within its form, $6 - 1/\pi$ and $6 - 3/\pi$, would then still lie more than five standard deviations from the combined measurement" (*The Weight of Possibility*, Chapter 5, Section I).

The Studies with DESI

Some studies add the baryon acoustic oscillations of DESI, and those studies find a lower ratio. The second data release of DESI gives densities of the two kinds of matter, and those densities yield a ratio of 5.199, give or take 0.035 (Louis and others 2025, Table 5). *The Compactification* reports how far 5.199 lies from the account's ratio. The distance is "4.7 standard deviations from the ratio, or about 4.0 with a correlation between the two fitted densities".

The Weight of Possibility reads the DESI measurement, and begins with what the oscillations measure. "The oscillations do not measure the dark density: they measure the matter fraction, and they lower the fitted dark density where they prefer a lower matter fraction. The same analyses press on the matter fraction of the Passage. Read through the coefficient of Section III, the two pressures pull in opposite directions: in the most pre-

cise of these analyses the ratio asks for a coefficient of 1.26 ± 0.06 and the matter fraction for 0.68 ± 0.11, 4.7 standard deviations apart, while the ratio itself lies 4.7 standard deviations from the prediction, or about 4.0 with that correlation. Both figures rest on one kind of measurement, and a single kind can carry an error of its own that its stated uncertainty does not include. Within the standard model itself, the oscillations of DESI and the microwave background differ by 2.3 standard deviations (DESI Collaboration 2025), so a combination of the two measures that difference along with the ratio. The account holds the matter fraction at 0.311 and the cosmological constant fixed at one value (the Passage, Chapter 13, Sections III and VI), so it expects the pull of the oscillations toward a lower matter fraction to recede as their data grow".

In plain words, the oscillations measure how much matter there is in all. Where the oscillations prefer less matter, the studies lower the dark matter to match. The ratio and the identity of Chapter 8 have one number in common, a coefficient. In the DESI studies the ratio requires a larger coefficient and the matter share a smaller one. The two required values lie 4.7 standard deviations apart. The oscillations and the oldest light also differ from each other by 2.3 standard deviations within the standard model. The account fixes the matter share at 31.1 percent. So the account expects the pull of the oscillations toward less matter to recede as the data of DESI grow.

A second kind of measurement

The Weight of Possibility gives a second kind of measurement, one that does not use the oldest light. "Routes that do not use the microwave background measure the share of the ordinary matter in all the matter, for which the prediction, with the lightest neutrinos, is 0.15644. The baryon density from primordial nucleosynthesis (Schöneberg 2024) with the oscillations of the Dark Energy Spectroscopic Instrument (DESI Collaboration 2025) gives 0.1587 ± 0.0066; the same baryon density with the full-shape analysis of that instrument (Forero-Sánchez and others 2026) gives

0.1542 ± 0.0063; the gas of galaxy clusters gives 0.156 ± 0.034 (Mantz and others 2022). Each agrees with the prediction; the most precise of them is about five times less precise than the microwave background, and the cluster gas about thirty times less" (*The Weight of Possibility*, Chapter 5, Section II).

The shared coefficient and the shape of the six

The ratio and the matter share have one coefficient in common. The coefficient is the curvature of the six in Planck units, which *The Gravitational Passage* identifies as one. *The Weight of Possibility* reports how closely the ratio measures the coefficient. "The ratio measures it at 0.99 ± 0.08, two to three times more closely than the matter fraction does, and the two agree with the microwave background alone" (*The Weight of Possibility*, Chapter 5, Section III).

The Weight of Possibility also computes the shape of the six from the measured ratio. In the quotation that follows, $R_{\mathbb{C}P^2}$ is the curvature of the complex projective plane, the larger of the two factors of the internal space (the other factor is the sphere), and x is the ratio of the radii of the two factors. "If the rule holds, the combined measurement of Section I gives $R_{\mathbb{C}P^2} = 3.96 \pm 0.30$, against the 4 of the Passage's Einstein condition, and x = 2.49 ± 0.28, against $\sqrt{6} = 2.45$. Planck alone gives x = 2.45 ± 0.38, and the analysis with the oscillations that gives 5.199 (Section I) gives x = 1.52 ± 0.21. From its published forecast (Ade and others 2019), the Simons Observatory with Planck would measure x to about 8 percent, which is the ratio to about 0.6 percent". *The Weight of Possibility* warns that "The shape test and the choice of the rule are one measurement, not two". *The Weight of Possibility* adds that "a measured ratio can confirm the rule at the Passage's shape, or fit the rule at another shape, but it cannot do both. The dated record fixes the shape at the Passage's $x = \sqrt{6}$" (*The Weight of Possibility*, Chapter 5, Section IV).

The forms near the rule, and what would separate them

The Weight of Possibility examined twelve expressions. *The Weight of Possibility* gives the distance of each expression, in standard deviations, from the two measurements of the oldest light. One measurement is from Planck alone, 5.364 ± 0.065, and the other is from the combination, 5.370 ± 0.048.

Expression	What it takes	Value	Planck	Combined
$6 - 2/\pi$	the abiding of $\mathbb{C}\mathrm{P}^2$ (the rule)	5.3634	−0.01	−0.14
6	no abiding	6.0000	+9.8	+13.2
$6 - 1/\pi$	the abiding of S^2	5.6817	+4.9	+6.5
$6 - 3/\pi$	the abiding of the whole internal space	5.0451	−4.9	−6.8
$6 - 6/\pi$	the internal space as one factor of six dimensions, by its Einstein trace	4.0901	−19.7	−26.8
$6(1 - 1/(4\pi))$	six shares, each less the bare term of $\mathbb{C}\mathrm{P}^2$	5.5225	+2.4	+3.2
$6\mathrm{f} = 6(1 - 3/(8\pi))$	six shares of the external value	5.2838	−1.2	−1.8
$(6 - 3/\pi)/((1 + \mathrm{f})/2)$	the whole abiding, with the ordinary residue at the mean of one and f	5.3653	+0.01	−0.10
$(6 - 2/\pi)/(1 - 1/(3\pi))$	the rule, with the ordinary residue also reduced by an abiding; identically 6	6.0000	+9.8	+13.2
$16/3$	a simple fraction	5.3333	−0.5	−0.8
$27/5$	a simple fraction	5.4000	+0.6	+0.6
$38/7$	a simple fraction	5.4286	+1.0	+1.2

Source: *The Weight of Possibility*, Chapter 5, Section VIII: "The distances are computed from the unrounded ratios of the two measurements, 5.3643 ± 0.0646 and 5.3701 ± 0.0478".

The Weight of Possibility states which future measurement separates the rule from each near form. "Two other forms built from the Passage's quantities lie within two standard deviations of the measurement: six shares of the external value, $6f = 5.284$; and $(6 - 3/\pi)/((1 + f)/2) = 5.365$ [...]. The second differs from the rule by 0.036 percent, which no foreseeable measurement separates; the rule needs one step fewer. The first would be separated from the rule at about three standard deviations by a ratio measured to half a percent. The same measurement would separate the rule from $2\pi - 1 = 5.283$ (Agyemang 2026), which lies within 0.001 of the first form" (Chapter 5, Section I). "One simple fraction near the rule, 16/3, is the value that a relaxation of the masses of the axion and the proton gives for a gauge group of eight colors (Banerjee, Brzeminski and Hook 2026). It lies 0.56 percent below the rule, and a ratio measured to about 0.2 percent would separate the two at three standard deviations" (Section VIII). Sarici (2025) proposed a ratio built from counts of dimensions, $(26 - 4)/4 = 5.5$. *The Weight of Possibility* states that the proposal of Sarici "lies outside the present measurement" (Chapter 3, Questions). *The Weight of Possibility* sums up the grounds of the rule. "The weight of the rule rests on its form, on the exclusion of its neighbors within that form, and on the coefficient it shares with the Passage's identity".

In plain words, the Simons Observatory with Planck is forecast to measure the ratio to about six tenths of a percent. A measurement to half a percent separates the rule from six times the trim and from two pi less one. A measurement to two tenths of a percent separates the rule from sixteen thirds. No foreseeable measurement separates the rule from the form that lets the whole six abide, and that form needs one step more than the rule. In this book, to abide is to hold fast in a measurement, as a term of its own.

Short Loops

The account says that no short loop exists anywhere in space. A short loop would be a path that goes all the way around the universe and returns, that cannot be pulled tight to a point, and that is shorter than the diameter of the horizon, wherever the path lies. The prediction of no short loop follows from an identity. The identity fixes the ratio of dark to ordinary matter for every measurement, wherever the measurement is made. *The Weight of Possibility* counts the absence of a short loop among the predictions that belong to the account alone. "The ratio, the coefficient it shares with the Passage's identity (Section III), and the absence of a short loop are the account's alone" (Chapter 5, Section IX).

The Weight of Possibility reports the searches made so far. "Searches of the microwave background find that every closed path through our location that goes around the universe is longer than 98.5 percent of the diameter of the last scattering surface, the width of the visible microwave sky (Petersen and others 2023). So the region within the horizon is shown to be a ball out to 96.5 percent of the horizon's radius" (*The Weight of Possibility*, Chapter 3, Section II). A short loop would change the count of classes, and so would change the ratio, which depends on the count. "For two classes the ratio is $6 - 1/\pi = 5.682$, the value that the abiding of S^2 alone would give, which the combined measurement excludes (Chapter 5, Section I); more classes give larger ratios, which it excludes further". The authorities name no coming instrument for the search for short loops.

The Dark Matter's Other Ways of Acting

The Weight of Possibility names which of its predictions of kind other accounts share. "The first four it shares with other accounts of a stable, cold dark matter that acts only through gravity, and the sixth with every account in which the dark matter and the ordinary matter vary together

from place to place" (Chapter 5, Section IX). The first four predictions of kind say that the dark matter acts only through gravity, does not decay, is perfectly cold, and is made of gyrations of about the Planck mass. The fifth says that no short loop exists anywhere in space, and the sixth says that the dark component carries no separate pattern of departures. A gyration is a lasting vibration of the geometry of the six small dimensions. The account shares the first four predictions with other accounts of a lasting, cold dark matter that acts only through gravity, and the account shares the sixth with every account in which dark and ordinary matter vary together from place to place.

Searches by any interaction but gravity

The Weight of Possibility states the prediction. "The dark matter is registered through its weight alone: on the identification of Chapter 1, Section IV, every search for it through any interaction but gravity finds nothing, at any sensitivity. That is a prediction of the identification" (*The Weight of Possibility*, Chapter 5, Section VII). *The Weight of Possibility* also states what a lasting null result of the searches by any interaction but gravity separates. "Other accounts of dark matter that interacts only through gravity make the same prediction for these searches (Feng, Rajaraman and Takayama 2003), and so do relics of the Planck mass if they are neutral; if they carry electric charge, as has been argued, detectors on Earth would see them (Lehmann, Johnson, Profumo and Schwemberger 2019). So a lasting null result separates this account from particles of weak interaction, not from those accounts".

Registering a gyration by its weight

The Weight of Possibility reports how far detectors are from registering a single gyration through its weight. "No detector yet built registers a single one through gravity alone; arrays of sensors proposed for the purpose would need reductions of quantum noise far beyond present reach. Such arrays were proposed by Carney, Ghosh, Krnjaic and Taylor (2020). A

realistic study finds that they would need about 80 decibels of reduction of quantum noise (Qin and others 2025)" (*The Weight of Possibility*, Chapter 4, Section VI). The gyrations are spread thinly: near the Sun there is about one gyration in every few tens of thousands of cubic kilometers.

Decay

The account says the dark matter does not decay. *The Weight of Possibility* gives the measured bound on a decay. "A component that decayed into anything unseen would have changed the expansion since then, and the measurements bound such a decay to lifetimes beyond about 160 billion years (Poulin, Serpico and Lesgourgues 2016). A decay into anything seen would be bounded far more tightly" (*The Weight of Possibility*, Chapter 1, Section I). Conformal cyclic cosmology is the account proposed by the physicist Roger Penrose. In that account the dark matter is made of particles that Penrose calls erebons. The erebons of Penrose also have about the Planck mass. Yet the erebons "must decay within each epoch, with a lifetime of the order of a hundred billion years". *The Weight of Possibility* states which measurement separates this account from conformal cyclic cosmology. "The bound on the lifetime of the dark component in Chapter 1, Section I, already presses on that decay" (Chapter 4, Questions). Conformal cyclic cosmology needs a decay of the dark component, seen in the expansion. This account says the dark component does not decay. So the measured lifetime of the dark component separates the two accounts.

Coldness and small scales

The account says the dark matter is perfectly cold, with no cut-off in its structure on small scales. Cold dark matter is dark matter that moves slowly. *The Weight of Possibility* states: "A perfectly cold, collisionless component meets every present bound that separates cold dark matter from its warmer or wavier alternatives" (*The Weight of Possibility*, Chapter 5, Section V). A combined analysis of the faintest satellite galaxies and of

strong lensing "places the half-mode mass [...] below about ten million solar masses at 95 percent confidence (Nadler and others 2021)". The forest of hydrogen absorption "places the mass of a thermal relic of warm dark matter above 5.3 keV at 95 percent confidence (Palanque-Delabrouille and others 2020)". The bounds on the half-mode mass and on warm dark matter agree with the account and with every other account of cold dark matter, so neither bound separates the account from the other accounts of cold dark matter.

The Weight of Possibility also states, in Chapter 5, Section V, the scope of the account on the motions of galaxies. The radial acceleration relation is a rule found in disc galaxies. The relation ties the acceleration of the stars and gas to the acceleration that the visible matter alone would give the stars and gas. Astronomers have measured the relation at 2,693 points in 153 disc galaxies, with a scatter of about 0.13 dex. A dex is one power of ten. The scale of the relation lies near 1.2×10^{-10} meters per second per second. *The Weight of Possibility* says: "The account offers no explanation of that tightness beyond what galaxy formation under general relativity provides". The physicist Mordehai Milgrom proposed a modified dynamics, which changes the law of gravity at small accelerations. Chapter 5, Section V, also states one measurement that separates the account from the modified dynamics of Milgrom. Because the field equations hold exactly below the Planck curvature, "a galaxy falling freely in the field of a larger one feels that field only through its tide, as general relativity requires, and not through the external field effect that the modified dynamics predicts".

The scale of the relation lies close to the speed of light times the Hubble constant divided by two pi. With the account's Hubble constant of 67.8 kilometers per second per megaparsec, the speed of light times the Hubble constant divided by two pi is 1.05×10^{-10} meters per second per second. The field equations hold Newton's constant and the cosmological constant, and no constant of acceleration. The cosmological constant draws two free bodies apart with an acceleration of its own. The size of

the acceleration follows from the account's Hubble constant and from the account's share of the cosmological constant, 0.689. Across ten kiloparsecs, the size of a galaxy, the acceleration is about 1.0×10^{-15} meters per second per second, which is less than a hundred-thousandth of the scale of the relation. So on this account the scale of the relation comes from how the galaxies formed under general relativity, as the small scatter of the relation does. The field equations give the scale no value at any place in a galaxy (*The Weight of Possibility*, Chapter 5, Section V). The modified dynamics changes the law of gravity instead. Yet the modified dynamics still leaves clusters of galaxies short of mass (Sanders 2003; Famaey, Pizzuti and Saltas 2025). Without dark matter, the modified dynamics predicts a third peak of the oldest light lower than the one measured (McGaugh 2004; Aghanim and others 2020). The third peak is the third crest in the chart of how strongly each size of pattern shows in the oldest light. The account has the other open questions of cold dark matter on small scales as well, as *The Weight of Possibility* says. Computer simulations of cold dark matter alone give sharp peaks of density, called cusps, at the centers of dwarf galaxies. Astronomers find flat cores at the centers of dwarf galaxies instead. The simulations also give more small halos, the dark clouds around galaxies, than there are faint satellite galaxies. And the simulations give dense, massive satellites that are not seen, the problem known as too big to fail. Within cold dark matter, three explanations are the most studied: the feedback of stars on the gas of small galaxies, the suppression of star formation in the smallest halos after reionization, and the faint satellites that deep surveys continue to find (Bullock and Boylan-Kolchin 2017).

Collisions

1E 0657-56 is a colliding cluster of galaxies. The collision bounds the scattering of the dark component on itself below about one square centimeter per gram (Markevitch and others 2004). The cross section per unit mass measures how readily the dark matter scatters on itself. For gyrations of

about the Planck mass, at the speeds of matter in galaxies and clusters, the cross section per unit mass is between about 10^{-45} and 10^{-50} square centimeters per gram. The cross section of the gyrations is "at least forty-five orders of magnitude below the bound" (*The Weight of Possibility*, Chapter 4, Section III).

The neutrinos are not the dark matter

The Weight of Possibility gives two grounds on which the neutrinos are not the dark matter. One ground is the identification, and the other is a measurement of galaxies. "The neutrinos are content, a residue of neither kind (Section V), and on the identification of Chapter 1, Section IV, the dark matter is not content. A measurement of galaxies agrees, from a premise of its own: the exclusion principle, by which no two neutrinos of one kind can share a state. In a galaxy of a given size whose stars move at given speeds, the states slow enough to stay in the galaxy number in proportion to the cube of a particle's mass, and each state holds one particle, so the galaxy can hold of one kind of neutrino at most a mass that grows as the fourth power of that neutrino's mass (Tremaine and Gunn 1979). The dwarf spheroidal galaxies, small and faint galaxies that hold much dark matter in a small region and whose stars move slowly, would need a particle that obeys the exclusion principle to have a mass of at least 130 electronvolts to hold their dark matter (Alvey and others 2021, at 95 percent confidence). The sum of the three neutrino masses lies below 0.064 electronvolts in the standard model of cosmology (DESI Collaboration 2025), and the mass of the electron's neutrino, measured directly, lies below 0.45 electronvolts (Aker and others 2025, at 90 percent confidence), so the neutrinos fall short of that need by a factor of at least about three hundred in mass" (*The Weight of Possibility*, Chapter 3, Questions).

In plain words, no two neutrinos of one kind can be in the same state. A galaxy of a given size, whose stars move at given speeds, has only so many states slow enough to stay in the galaxy, so the galaxy can hold only so much of one kind of neutrino. The dwarf galaxies are small and faint,

they hold much dark matter in a small space, and their stars move slowly. The dwarf galaxies would need a particle like the neutrino to weigh at least 130 electron volts to hold their dark matter. The three neutrino masses together lie below 0.064 electron volts. A direct measurement puts the mass of the electron's neutrino below 0.45 electron volts, so the neutrinos fall short of 130 electron volts by a factor of at least about three hundred in mass. The measurement of the dwarf galaxies agrees with the identification of the dark matter, from different ground, that the neutrinos are not the dark matter.

Backgrounds of Gravitational Waves

The settling background, heard directly

The settling background is the background of gravitational waves that the settling of the six left. Today the settling background lies above about five hundred billion vibrations a second, 5×10^{11} hertz. The energy share of the settling background is below about five parts in ten million, and the stretch of space that the background makes is about two parts in ten to the thirty-three. On the account, the settling background has a ceiling, an upper limit on its energy, and no lower limit. *The Compactification*, Chapter 8, sets every class of detector at high frequency against the settling background. "Eight of the nine classes lie below the settling band in frequency, and every class with a stated reach lies at least twelve decades above its ceiling in energy density. The one class whose band contains the target has no demonstrated sensitivity". The Holometer is a pair of laser interferometers at Fermilab, side by side, which has run and finished. "The best direct limit in the megahertz band, from the Holometer, lies nineteen decades above it".

The estimates of time in *The Compactification* are not results of the account: "They extrapolate from the instruments under construction or proposed". Suppose that detectors at high frequency improved within about

fifty years as the detectors at the pitches of sound did. Then the reach of the detectors at high frequency "might come to about $h^2\Omega_{GW} \sim 10^8$ [...]. That remains about fourteen decades above the ceiling of the settling background, and still below its band in frequency". In plain words, the detectors at the pitches of sound gained about eight powers of ten in strain over fifty years. An equal gain at high frequency is a generous guess, because no source in the sky drives the work at high frequency. The energy share goes as the square of the strain, so eight powers of ten in strain are sixteen in energy share. So the reach at gigahertz frequencies might come to an energy share of about ten to the power eight. A direct registration of the settling background would need an instrument at half a terahertz or above with a strain sensitivity near 10^{-33}. Or the instrument would need a reach to an energy share below one part in a million at that frequency. *The Compactification* says: "No roadmap to such an instrument exists. The background is not lost for that reason: its energy is registered by the count of light species, and the account requires of it only that it stay within that count's bound" (Section XIV.C).

LISA

The Compactification states the account's prediction for LISA. "LISA, the space interferometer adopted by the European Space Agency (Amaro-Seoane and others 2017; Colpi and others 2024), will register gravitational waves between about 0.1 and 100 mHz, with a sensitivity to a stochastic background of order $h^2\Omega_{GW} \sim 10^{-13}$ near 3 mHz. A transition of first order at the electroweak scale would put its background in that band. The electroweak transition of the content is a crossover, so the account says that LISA will register no stochastic background from a cosmological phase transition. The backgrounds LISA will register are those of astrophysical sources, among them the foreground of binaries of white dwarfs in the Galaxy" (*The Compactification*, Chapter 8, Section XII.A). The statement about LISA is, in the words of its source, "a prediction, stated as following from the cited results and not quoted from them". The elec-

troweak crossover is the smooth change in which the Higgs field takes its value. The crossover rests on established physics. A sudden change would need a Higgs mass below 72.4, give or take 1.7, billion electron volts, and the measured Higgs mass is 125.13, give or take 0.11, billion electron volts.

The pulsar-timing arrays

A pulsar-timing array is a set of pulsars timed over years. When a slow gravitational wave passes between the pulsars and us, the wave shifts the arrival times of the pulses. *The Compactification* reports what the arrays found. "The North American Nanohertz Observatory for Gravitational Waves reported evidence for a background in its fifteen-year data, with a characteristic strain of about 2.4×10^{-15} at a frequency of one per year (Agazie and others 2023), and the European array reported evidence of the same kind (Antoniadis and others 2023). By (8.1) that amplitude corresponds to $h^2\Omega_{GW} \approx 4 \times 10^{-9}$ at one per year, a value computed here". The strong force changed near 156.5 million electron volts, and the change was a crossover. So the change put no background in the band of the pulsar-timing arrays. The settling background lies some twenty decades higher in frequency than that band. The second-order background lies about fourteen decades lower in energy than the observed signal. "So the account has no cosmological source for the observed signal, and it reads the signal as astrophysical: the summed waves of binaries of supermassive black holes, the source that the pulsar-timing collaborations take as their leading interpretation. That reading follows from the account's closed list of kinds and the crossovers of the content; it is argued, not measured" (*The Compactification*, Chapter 8, Section XII.A). If the first ripples on short lengths are near their measured strength, the astrophysical reading holds. A far larger strength on short lengths is one that the account "neither provides nor excludes" (Chapter 7, Section X). *The Compactification* itself states that condition, with the word "if". The conclusion is argued, not measured, from three findings of Chapter 16 of this book. The three findings are that there was no transition with bubbles, that there are

no strings, and that the induced hum is faint. *The Compactification* states how the astrophysical reading is tested: "it is tested as the arrays measure the shape of the spectrum and the anisotropy that a finite population of binaries produces" (Chapter 7, Section X).

Strings

A cosmic string is a thin line of trapped field energy. A network of cosmic strings would make a background of gravitational waves over a wide band of frequencies. The Higgs field makes no stable strings, walls or monopoles. *The Compactification* concludes that the content makes no network of strings. "With the Standard Model as the whole of the content, and with no unifying group and no further scalar field [...], the content's history makes no stable network of strings, and no background from one" (*The Compactification*, Chapter 7, Section IX).

The speed and the polarizations

The Unquantized Arena states the prediction: "Gravitational waves travel at the speed of light with the two polarizations of general relativity" (Prediction 19.4). A polarization of a gravitational wave is one of the patterns in which the wave stretches and squeezes space across its path. In the table of *The Compactification*, the standing of the prediction is "prediction; argued from the uniqueness of the Einstein form". "The coincidence of the gravitational waves of GW170817 with the gamma rays of GRB 170817A bounds the difference of the two speeds: $(v_{gw} - c)/c$ lies between -3×10^{-15} and $+7 \times 10^{-16}$ (Abbott and others 2017a)". In plain words, the speed of the waves differs from the speed of light by at most three parts in a thousand million million on the slow side, and by at most seven parts in ten million billion on the fast side. "The three-detector event GW170814 favors purely tensor polarization over purely vector and purely scalar polarization (Abbott and colleagues 2017b)" (*The Gravitational Passage*, Chapter 16, Section V). "A network of ground-based detectors with differing orientations, and LISA with its own geometry, tests the

number of polarizations with every signal". The account shares the prediction of speed and polarizations with general relativity. So the measurements of speed and polarization do not separate the account from general relativity. The waves also arrive with no spreading of their frequencies on the way. At ninety percent credibility, the absence of spreading bounds the mass that a quantum of the description of the waves could carry, at no more than 2.42×10^{-23} electron volts (Abbott and others 2025). Abbott and others state the bound as a mass of the graviton. In this book the word "graviton" names at most a quantum of the description. *The Gravitational Companion* names the step on which the prediction rests (Refutation Conditions Named in This Book, Section I). If a further polarization were found, or a speed other than the speed of light, the finding would bear on the uniqueness of the Einstein form of the equations in four dimensions once the six are held.

If a detector registered single quanta of a gravitational wave, the detector would not show the arena quantized. *The Unquantized Arena* gives the reason in Proposition 9.2: "a click of such a detector is produced equally by a classical wave acting on a quantum detector (Carney, Domcke and Rodd 2024)".

The area of horizons

The area theorem says that, in general relativity, the area of a horizon never shrinks: when two holes merge, the area of the final horizon is at least the areas of the two starting horizons added together. "For the first detected event, GW150914, Isi and others (2021) measured the total area of the two horizons before the merger from the inspiral part of the signal, and the area of the remnant from its ringdown, and found the area increased, as the theorem requires, at a probability of about 97 percent" (*The Compactification*, Chapter 4, Section VI). The area theorem is a theorem of general relativity, and on the account general relativity is valid below the Planck curvature.

The Instruments at High Frequency, One by One

Chapter 17 tells how each sort of detector at high frequency listens, and in which band of frequencies each sort listens. Below are the exact figures for each sort. The settling background is the background of gravitational waves made as the six small dimensions of the account settle into their held size. The settling background lies today in the settling band, above about five hundred billion hertz. Every figure in this section is set beside the settling background. The account sets a ceiling on the settling background. An energy share is the share of the critical density that gravitational waves carry in one band of frequency. The ceiling is an energy share of about five parts in ten million. The energy share of the settling background lies below the ceiling. Every comparison in this section is made in energy share, because every instrument can be set beside the ceiling in energy share.

Two gaps state how far a detector falls short of the settling background. The frequency gap is the number of powers of ten by which the top of the detector's band falls below five hundred billion hertz, the lowest frequency of the settling band. The amplitude gap is the number of powers of ten by which the weakest energy share the detector can register lies above the ceiling. The bands and sensitivities that follow are taken from the published work on each instrument, as a record of the field, and *The Compactification* computes the two gaps from those published figures. *The Compactification* surveys nine sorts of detector, each listening for gravitational waves at high frequency in its own way. *The Compactification* calls the nine sorts detector classes. Each sort below is described in the same order: what the instrument is, where in frequency it listens, its reach (the weakest energy share it can register), and how it compares with the settling background. The readiness of a sort is how far its method has come

from idea to working instrument. The description of each instrument repeats the words of Chapter 17, so that a reader can read this appendix on its own.

Quartz resonators. The most developed of the nine sorts is a crystal of very pure quartz mounted between two electrodes. Quartz makes a small electric voltage when it is squeezed, so the electrodes turn the crystal's vibrations into an electric signal. The crystal is then a ringing antenna: a passing gravitational wave squeezes the crystal, and the crystal rings. The leading instrument among quartz resonators is MAGE, the Multi-mode Acoustic Gravitational wave Experiment at the University of Western Australia. MAGE reads two nearly identical quartz crystals, deeply cooled, with very sensitive magnetic amplifiers. MAGE listens in several narrow bands between five and sixteen million vibrations a second (megahertz). MAGE is projected to grow several times more sensitive when it is run at twenty thousandths of a degree above absolute zero. Researchers have proposed an array of quartz crystals like those of MAGE. The array would cover a tenth of a megahertz to twenty megahertz at about the sensitivity of MAGE. A quality factor is the measure of how slowly a ringing dies away. The crystals of quartz resonators ring for a very long time before the ringing fades, and their quality factor is about a billion, approaching ten billion. The crystals could in principle listen up to nearly a gigahertz, through their higher notes. A top note of nearly a gigahertz lies about five hundred times below the settling band. So the frequency gap of quartz resonators is two and seven tenths powers of ten. The working band of the crystals lies four to five powers of ten below the settling band. The reach of quartz resonators in energy share is about ten to the power thirteen to ten to the power seventeen, which lies nineteen to twenty-three powers of ten above the ceiling. The readiness of quartz resonators is high. Quartz resonators cannot register the settling background.

Superconducting cavities. The second sort is a hollow metal cavity, cooled until its walls carry current with no resistance. The cavity holds two nearly identical patterns of microwaves. One pattern is filled, and the other pattern is kept quiet. A passing gravitational wave moves a little en-

ergy from the filled pattern into the quiet pattern. The wave moves the energy either by flexing the walls or by acting on the microwaves directly. Precise electronics watch the quiet pattern. Physicists first proposed the superconducting cavity as a detector in 1971 and 1978. The main program for superconducting cavities today is MAGO 2.0, a joint effort of the DESY laboratory in Hamburg and Fermilab near Chicago. The prototype cavity of MAGO 2.0 was built in Genoa. The prototype has been refurbished and tested at four degrees and at one and eight tenths degrees above absolute zero, and the tests used the equipment built for the European X-ray laser. Superconducting cavities work from about ten thousand vibrations a second (ten kilohertz) to about a hundred megahertz. No sensitivity target for superconducting cavities has been published in a form that can be set beside a background from the beginning. The top of the band of the cavities lies about five thousand times below the settling band. So the frequency gap of the cavities is three and seven tenths powers of ten. Heat and quantum noise limit superconducting cavities. No projection for the cavities comes near an energy share of one part in ten million at any frequency. The readiness of superconducting cavities is moderate. Superconducting cavities cannot register the settling background.

Magnon detectors. In a magnetic crystal every atom is a tiny magnet, and the tiny magnets can wobble together in a shared wave. A magnon is one step of the shared wobble. A gravitational wave can set off magnons by resonance when the pitch of the wave matches the pitch of the wobble, and because an applied magnetic field tunes the wobble, magnon detectors work naturally at billions of vibrations a second. Some experiments with magnons were built to search for axions, a particle that other theories propose, and their data have been read again as records of magnons set off by gravitational waves, at fourteen gigahertz and at eight and two tenths gigahertz. A related instrument, the haloscope, works in the band of magnon detectors. A haloscope is a strong magnet around a microwave cavity, built to turn axions into microwaves. A haloscope can turn a gravitational wave into microwaves, as it turns axions into microwaves. The CAPP-12T haloscope is at the Center for Axion and Precision Physics in

Korea. The data of the CAPP-12T haloscope have been read again. A strain is the fraction by which a passing wave stretches and squeezes a length. The Korean reading bounds single waves of one pure pitch to a strain below about three and nine tenths parts in ten to the power twenty-one, over a span of two megahertz centered at five point three one one gigahertz. The Korean bound is the strain of one pure wave, which is a different measure from the strain of a background. Magnon detectors work between about one and twenty gigahertz, the highest band of any laboratory method now running. Twenty gigahertz, the top of the magnon band, lies about twenty-five times below the settling band, so the frequency gap of magnon detectors is one and four tenths powers of ten. The reach of magnon detectors in energy share is about ten to the power twenty-four to ten to the power twenty-five, which lies thirty to thirty-two powers of ten above the ceiling. When the Korean bound is read as a strain, the bound lies about twelve powers of ten above the strain the ceiling allows at five hundred billion hertz: two parts in ten to the power thirty-three. The readiness of magnon detectors is low to moderate. Magnon detectors cannot register the settling background.

Conversion into light. In 1962 the physicist Mikhail Gertsenshtein described how, inside a magnetic field, gravitational waves and light can turn into one another. Because of the conversion Gertsenshtein described, a magnetic field can serve to detect gravitational waves at high frequency, or to bound them: to set an upper limit on how strong the waves are. The conversion is used in two ways, and the two ways count as two of the nine sorts. The first way uses magnetic fields in space. Clusters of galaxies carry magnetic fields of about a millionth of a gauss (the field at the Earth's surface is about half a gauss). The field in each cluster holds its direction over about ten kiloparsecs, some thirty thousand light-years. The thin gas between galaxies carries a field of about a tenth of a billionth of a gauss. Both kinds of field would turn a background of gravitational waves into a little extra radio and microwave light. Measurements already made with radio and microwave instruments have been read as limits on the extra light. The instruments are the Atacama Cosmology Telescope,

EDGES, LOFAR, the Murchison Widefield Array and ARCADE 2. Read with the fields of clusters, the measurements bound the energy share below about ten to the power sixteen near a hundred gigahertz. Read with the field between galaxies, the same measurements bound the energy share below about ten to the power eleven to ten to the power thirteen, between a hundred megahertz and ten gigahertz. The Square Kilometre Array is a very large radio telescope now being built. The Square Kilometre Array could tighten the bounds between a hundred megahertz and ten gigahertz by about ten powers of ten, to an energy share between about ten and about a million. The second way uses magnets in the laboratory: strong superconducting magnets with microwave cavities or guides that collect the light made inside the magnets. A study of 2021 estimated that superconducting magnets with microwave cavities or guides could in principle register strains of about one part in ten to the power thirty, enough to register pairs of planet-mass black holes made in the early universe. The laboratory detectors would need large magnets of very high field, and the counting of single particles of light, to reach strains of one part in ten to the power thirty, and both requirements are far beyond present practice at frequencies of megahertz to gigahertz.

Of all the measurements in this section, the bound from clusters near a hundred gigahertz is the closest in frequency to the settling band. A hundred gigahertz lies about five times below the settling band, a frequency gap of seven tenths of a power of ten. In energy share, the bound from clusters lies about twenty-two powers of ten above the ceiling. The reach of conversion in space lies seventeen to twenty-two powers of ten above the ceiling, from a hundred megahertz to a hundred gigahertz. The projected bounds of the Square Kilometre Array lie seven to twelve powers of ten above the ceiling. The projected bounds lie at frequencies two to four powers of ten below the settling band. The laboratory method works in its own band, from megahertz to gigahertz. At the strain the study of 2021 estimated, the laboratory method would come near the ceiling in energy share at its own frequency. As a strain, the laboratory estimate lies about three powers of ten above the strain the ceiling allows at five hundred bil-

lion hertz. The band of the laboratory method lies at least two and seven tenths powers of ten below the settling band. The readiness of the method in space is moderate. The laboratory method is a concept so far, and its readiness is low. Neither the method in space nor the laboratory method can register the settling background.

Levitated sensors. In the proposal of levitated sensors, laser light holds up and cools tiny beads or discs of glass inside a cavity between two mirrors. The beads or discs serve as a tunable ringing sensor for waves between about fifty thousand and three hundred thousand vibrations a second. In the band of the levitated sensor, the sensor grows more sensitive as the pitch rises, just where a large laser detector such as LIGO loses sensitivity to the graininess of its own light. So a compact instrument could do better than LIGO in the band of the levitated sensor, at a length of ten to a hundred meters. A prototype one meter long is being built at Northwestern University. The projected reach of the levitated sensor is an energy share of about ten to the power six to ten to the power twelve. The band of the levitated sensor lies about a million times too low in frequency. The frequency gap of the levitated sensor is six and two tenths powers of ten. The projected reach lies twelve to nineteen powers of ten above the ceiling in energy share. The readiness of levitated sensors is low. Levitated sensors cannot register the settling background.

The Li-Baker concept. The Li-Baker concept is a proposed detector for waves between about three and ten gigahertz. The concept would use a microwave beam of smooth, bell-shaped profile, a steady magnetic field, and patterned reflectors, and the detector would read the waves through the small change that the waves make in the flow of light in the beam. The band of the Li-Baker concept lies about two powers of ten below the settling band. The frequency gap of the concept is one and seven tenths powers of ten. The proponents claim a sensitivity to strains of one part in ten to the power thirty-two to one part in ten to the power thirty-seven. The claimed sensitivity is not established, and *The Compactification* does not use the claimed sensitivity, for reasons that *The Compactification* states. First, no independent analysis or experiment supports the claimed sensit-

ivity. Second, the JASON advisory group reviewed the concept. The JASON advisory group is a panel of scientists that advises the United States government. The review found that the design reasoning of the concept confuses the conversion effect of Gertsenshtein with the effect the concept relies on. Third, light scattered by the reflectors cannot be told apart from the light of the signal. Fourth, no prototype has been built or tested. The Li-Baker concept is listed in the survey for completeness, and its readiness is very low.

The Holometer. The Holometer has run and finished. An interferometer is a detector that measures tiny changes of distance with light. The Holometer was a pair of laser interferometers at Fermilab, side by side, each thirty-nine meters long, which listened between one and thirteen megahertz. The Holometer set the first direct limit on a background at megahertz frequencies from a hundred and thirty hours of data: an energy share below five and six tenths times ten to the power twelve at one megahertz, rising to eight and four tenths times ten to the power fifteen at thirteen megahertz. The Holometer's limits are measured. The Holometer's limits lie nineteen to twenty-two powers of ten above the ceiling. The band of the Holometer lies more than four powers of ten below the settling band. The frequency gap of the Holometer is four and six tenths powers of ten.

Phonon detectors in crystals. A phonon is one step of energy in one way the lattice of a crystal can vibrate. A proposal of 2023 showed that detectors able to register single phonons in crystal targets could search for gravitational waves between about a trillion and a hundred trillion vibrations a second. Phonons in the band of the proposal have energies from about four thousandths to about one tenth of an electron volt. The proposal of 2023 exists as a calculation. No experiment has been mounted, and the readiness of phonon detectors is very low. Phonon detection in crystals is the one concept whose band contains the settling band.

The settling waves follow a frequency rule: the frequency at which the background of the settling waves peaks today is set by the starting temperature, the temperature at which the reconstructed history of our epoch

begins. A cooler start gives a higher frequency. By the frequency rule, the background peaks at a trillion hertz when the starting temperature is about two point six times ten to the power seventeen GeV (billion electron volts). The background peaks at a hundred trillion hertz when the starting temperature is about two point six times ten to the power fifteen GeV. Any starting temperature between the two values puts the peak inside the band of the proposal. At the ceiling, a phonon detector would have to register a strain of about nine parts in ten to the power thirty-four at a trillion hertz, and a strain of about nine parts in ten to the power thirty-six at a hundred trillion hertz. *The Compactification* takes no sensitivity figure from the proposal, because no sensitivity has been shown for the proposal. *The Compactification* makes one comparison: the band of the proposal is the right band, and the strain the proposal would have to register lies far below any strain a laboratory instrument has registered at any frequency.

Besides the nine sorts there are two further efforts. The first of the two further efforts is GravNet, a proposed worldwide network of synchronized detectors, working from megahertz to gigahertz. GravNet would compare the records of instruments far apart, so that a signal common to all the instruments stands out. GravNet is at the stage of design, and it has no projected sensitivity specific enough to set beside a background from the beginning. The second effort is a new reading of the data of several axion searches (ADMX, CAPP and HAYSTAC). The data were searched again for narrow-band waves near a few gigahertz. The Korean CAPP-12T result is the most sensitive of the new readings. All the new readings lie below the settling band in frequency, and many powers of ten above the ceiling.

The table sets the nine sorts side by side.

Sort of detector	Band	Frequency gap (powers of ten)	Amplitude gap (powers of ten)	Readiness
Quartz resonators (MAGE)	5 MHz to 1 GHz	2.7	19 to 23	High

Sort of detector	Band	Frequency gap (powers of ten)	Amplitude gap (powers of ten)	Readiness
Superconducting cavities (MAGO 2.0)	10 kHz to 100 MHz	3.7	Not stated	Moderate
Magnon detectors	1 to 20 GHz	1.4	30 to 32	Low to moderate
Conversion into light, in space	100 MHz to 100 GHz	0.7	17 to 22	Moderate
Conversion into light, in the laboratory	MHz to GHz	at least 2.7	Not stated	Low
Levitated sensors	50 to 300 kHz	6.2	12 to 19	Low
Li–Baker concept	3 to 10 GHz	1.7	Not used	Very low
Holometer	1 to 13 MHz	4.6	19 to 22	Completed
Phonon detectors in crystals	10^{12} to 10^{14} Hz (a trillion to a hundred trillion hertz)	None: the band contains the target	Not stated	Very low

Eight of the nine sorts work below the settling band in frequency, while every sort with a stated reach lies at least twelve powers of ten above the ceiling in energy share. Phonon detection in crystals is the one sort whose band contains the settling band, and it has no demonstrated sensitivity. Each power of ten in strain is two powers of ten in energy share, so a gap of twelve or more powers of ten in energy share is a gap of six or more in strain.

The Entanglement of Two Masses

Two things are entangled when they are joined so that neither can be described without the other. The account says that two masses become entangled when each mass is held in two places at once and the two masses are related only through gravity. The reason is that each alternative of the matter (each definite possibility in which the matter is held at once) carries its own geometry, its own shape of space and time. Bose and colleagues designed an experiment to test whether two masses become entangled through gravity: masses of ten to the minus fourteen kilograms, each held in two places a quarter of a millimeter apart, centers 0.45 millimeters apart, held for two and a half seconds. The entangling phase is the angle that marks the entanglement of the two masses through gravity. For the design of Bose and colleagues, the account states the phase: "For these numbers the entangling phase is 0.314 radians, and the account predicts that the entanglement appears with that phase" (*The Gravitational Passage*, Chapter 12, Questions). A phase of 0.314 radians is about a third of a radian, eighteen degrees. The phase has the standing set by the structure once a stated step is taken: the step is the identification of one geometry for each alternative, and the phase also uses the measured strength of gravity.

The experiments are under way: "Experiments to test this are being built; the test masses on which the pull of gravity has been measured have reached below a milligram" (*The Compactification*, Chapter 8, Section XIII). The cited books name the accounts that the result of the experiment will separate from the account. One rule covers every account with one shared geometry: "A single shared geometry predicts no entanglement at all" (*Two Theories, One World*, Chapter 13). *The Gravitational Companion* names the accounts one by one. The first two accounts are "sourcing by a continuous measurement record (Tilloy and Diósi 2016), which also predicts a specific gravitational decoherence, a second channel of test; and postquantum classical gravity (Oppenheim 2023)", and each of the two

predicts no entanglement. The third account is collapse driven by gravity (Penrose 1996), "in which, for masses of 10^{-14} kilograms of the density of diamond, about two micrometers across, each superposition decays within about a hundredth of a second, before entanglement can build". The fourth is the semi-classical theory of Møller (1962) and Rosenfeld (1963), which "keeps one geometry for all alternatives and predicts no entanglement". The test has one limit: "The test separates the account from accounts in which one geometry serves all the alternatives of the matter; it does not separate the account from linearized quantum gravity, which predicts the same phase below the Planck scale" (*The Compactification*, Chapter 8, Section XIII). Linearized quantum gravity describes the small waves of the geometry by quanta of the description, never by quanta of the arena. Quanta of the description are steps that belong to the way the small waves are described, and not to the arena that ripples.

Measured Agreements Beneath the Account

Several measurements already made support steps of the account. The measurements run from the Cassini measurement of the bending of radio waves to the counts of the kinds of neutrino. I name those measurements in the chapters of this book, and here I collect the comparisons of those measurements.

The six small dimensions of the account have two moduli, their size and their shape: numbers that describe the six and could differ from place to place in the large world. The size and the shape of the six have masses of about two to three Planck masses, so the force that the size and the shape carry has a range shorter than the Planck length. The range is "some thirty orders of magnitude shorter than the shortest range that any of these tests reaches: the Cassini value $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ (Bertotti, Iess and Tortora 2003), and the inverse-square tests to about a twentieth of a millimeter (Kapner and colleagues 2007)" (*The Gravitational Passage*, Chapter 11, Section II). The Cassini spacecraft measured the bending

of radio waves by the Sun, and the Cassini measurement excludes a light modulus (a modulus of small mass, whose force would reach far) at the strength of force that a light modulus would carry.

The account's Higgs mass is 125.17 billion electron volts, from the self-coupling of one eighth with its loop term. The self-coupling is how strongly the Higgs field acts on itself, and the loop term is a small correction to the self-coupling. The measured Higgs mass is 125.13 billion electron volts, give or take 0.11. The two values agree, and the agreement is recorded as "one measured agreement" (*The Compactification*, Chapter 2, Section VIII, after *The Measures of the Kinds*, Chapter 13).

The amount of ordinary matter is measured twice, by two routes. Physicists state the amount as baryons per photon: the number of baryons (protons, neutrons and their relatives) for every particle of the oldest light. "The deuterium left from the first minutes, about 2.5 for every hundred thousand hydrogen nuclei, gives $(6.04 \pm 0.12) \times 10^{-10}$ baryons per photon; the microwave background gives $(6.13 \pm 0.04) \times 10^{-10}$. They differ by 0.7 of their combined uncertainty: the two measures agree" (*The Coming of the Kinds*, Box 12.3).

The excess of matter over antimatter has one sign everywhere: in every region, matter outnumbers antimatter. The single sign is measured by two routes. The first route is the pattern of the oldest light, which bounds a separate pattern of one component at 1.55 percent. A separate pattern of one component is a departure from uniformity in which one ingredient departs against the others, while the total density is unchanged at first. The second route is the gamma rays across the sky, from which "Andrew Cohen, Alvaro De Rújula and Sheldon Glashow conclude that 'a matter-antimatter symmetric universe is empirically excluded' (1998)" (*The Coming of the Kinds*, Chapter 5, Section VI).

The sphaleron passages, or sphalerons, are passing states of the field of the weak force. The sphalerons are the one process among the kinds of particle that changes the number of baryons. The sphalerons fall behind the expansion at a temperature of 131.7, give or take 2.3, billion electron

volts. The smooth electroweak change is centered at 159.5, give or take 1.5, and it is the cooling in which the Higgs field settles at its value. The two temperatures are far apart: "$T_c - T_* = 159.5 - 131.7 = 27.8$ GeV, which is more than ten times the combined uncertainty of the two temperatures (2.7 GeV) and more than five times the width of the steepest part" (*The Coming of the Kinds*, Box 4.3). So the sphalerons stay in balance (running as often forward as backward) through the steepest part of the change, because they fall behind the expansion only after the steepest part is over.

The number of kinds of neutrino is counted twice, by independent routes. In a collider, physicists count the decays of the Z boson, a carrier of the weak force, and find 2.9963, give or take 0.0074, kinds of light neutrino (Janot and Jadach 2020). The count of light species is the energy of all the radiation of the early universe other than light, counted in units of the energy that one kind of neutrino carries. Measured in the oldest light, the count of light species is 2.99, give or take 0.17, against 3.044 for three kinds of neutrino. *The Coming of the Kinds* concludes: "The two are measured by independent routes, one in a collider and one in the microwave background, and both find three" (Chapter 9, note 3).

Summary

This appendix gathered what the coming measurements will measure, what the account says each will find, and which other accounts each measurement separates from this one. The account's matter share is 31.1 percent: the part of the energy of the universe held as matter, ordinary and dark together. The account's expansion rate follows from the matter share, together with the amount of matter that the oldest light measures, and it is sixty-seven and eight tenths. The matter share and the expansion rate are one test, and they are the measurements that set the account apart from the standard picture of cosmology, which leaves both values free. The Planck satellite measures a matter share of 31.5 percent, while DESI alone measures 29.75 percent, so the value of the account lies between the

two, at 31.1 percent. The measurements of both values will be narrowed toward a thousandth by DESI, the Euclid space telescope, the Rubin Observatory and the Roman Space Telescope.

The standard picture expects the same as the account on the constancy of the energy of empty space, on the flatness of space and on the one pattern of the first ripples. The coming surveys will measure the constancy, the flatness and the pattern more closely. Physicists write w for the pressure of a form of energy divided by the amount of that energy in each unit of volume. The account says the surveys will find w exactly minus one and space flat. Astronomers can tell the account apart from accounts with several fields at the beginning by searching for a separate pattern of one ingredient. The Simons Observatory and LiteBIRD will search for first waves, the gravitational waves present from the beginning of an epoch, down to a ratio of first waves to first ripples of about a thousandth. The search for first waves separates the account from the many inflationary pictures that give first waves within the reach of the search, and the account says the two instruments will find none.

The count of light species weighs the settling waves, because the energy of the settling waves adds to the radiation that the count measures, and the Simons Observatory will measure the count of light species to five to seven hundredths. In the account, the ratio of dark to ordinary matter is 5.3634. The account's ratio is set beside measurements of the oldest light and measurements of other kinds. A near form is another formula built from the account's quantities that gives a value close to 5.3634, and the cited books state the precision at which each near form would be separated from the account's value. The account also says that no short loop exists anywhere in space. The account says that the dark matter does not decay, has no cut-off in its structure on small scales, and acts only through gravity. The coming measurements can look for a departure from every one of these statements about short loops and about the dark matter.

Eight of the nine sorts of detector at high frequency work below the settling band in frequency. Every sort with a stated reach lies at least twelve powers of ten above the ceiling of the settling background in en-

ergy share. Only phonon detection in crystals has a band that contains the settling band, and it has no demonstrated sensitivity. So the settling background is weighed by the count of light species, and not by a detector. LISA, a planned detector in space, will register backgrounds of gravitational waves from astrophysical sources in the sky, and none from a change of state of the matter. The pulsar-timing arrays, sets of pulsars timed over years, will map the shape of the background signal they report. Two masses held in two places at once will become entangled through gravity alone. Entanglement through gravity alone separates the account from every account with one geometry for all the alternatives of the matter. Gravitation is not quantized.

Appendix C

The Numbers and Their Standing

This appendix lists every number that carries physics in the main line of this book. A number that carries physics states something about the physical world, such as a mass, a temperature or a share of the universe's energy. The main line of the book is the Preface and the chapters. The numbers are listed Part by Part, in one table for each Part of the book. For each number, the number's row gives the value as the chapters print the value, the number's standing, where the number comes from, and the chapters of the book that print the number. Each number is listed once, in the table of the Part where the number first appears. The last column of each table, headed Chapters, names every chapter that prints the number. Where a chapter contains a number only in part, and Appendix A or Appendix B contains the number in full, the last column says which chapters contain the number in part and which appendix contains the number in full. Where only an appendix contains a number, the column names the appendix.

The words "the account" mean my account of the world, which other books of the same collection as this book set out in full. A number's standing says whether the number is derived within the account, is a measured value that the account uses, or has only a ceiling that the account sets. A number's standing is the ground the number stands on. The account's own numbers have five standings, which the opening section of this book, Before We Begin, sets out. The first standing is set by the structure: the number follows from the structure, which is the shape of the world as derived within the account. The second standing is set by the structure once a stated step is taken: the number follows only after a named step from the structure to the world, and I name the step where the step is made. The third standing is measured: the value comes from an instrument. The

fourth standing is measured, with its reason still a question: the account uses the measured value and says what would set the number. One number with the fourth standing is the strength of the faint ripples in the oldest light in the sky, the subject of Part III. The fifth standing is bounded, with its value not given: the account sets only a ceiling on the number. The temperature at which our history begins has the fifth standing.

Two further terms mark values that are not the account's own. The term *established physics* marks a result of standard physics, taken from the source of that result. The term *computed* marks a value found by arithmetic on stated values, and I name the values that the arithmetic uses. Each measurement is named by its authors and its year (for example, Aghanim and others 2020, a measurement named in many rows below), as the book named in the Source column cites the measurement. The tables leave out numbers that only count steps or items. The tables also leave out amounts inside a picture, an everyday scene that a chapter describes to make an idea plain, such as the cups of a recipe or the tiles of a floor laid by many workers. Wherever the tables of this appendix name the Planck mass, the tables mean the full Planck mass, about 1.221×10^{19} GeV. One GeV is about the energy held at rest in one proton.

Part I: The Six Held Again

Quantity	As printed	Standing	Source	Chapters
The counts of the dimensions	one of time, three of space, six of possibility; ten in all	Set by the structure (argued from the structure of comparison)	*The Gravitational Passage*, Chapter 10, Section I, and Appendix C; *The Compactification*, Chapter 1, Section II	Preface, 1, 2, 3
The Planck mass	about twenty-two millionths of a gram, about the mass of a grain of sand a quarter of a millimeter across; about 1.22×10^{19} GeV (1.221)	Measured (Newton's constant fixes it, with the speed of light and Planck's constant); the grain of sand computed	*The Compactification*, Preface, Section IV; *At the Edge of Reality*, Appendix C (2.18×10^{-8} kilograms)	1, 3, 4, 5, 8, 10, 13, 18

Quantity	As printed	Standing	Source	Chapters
The Planck length beside a proton	about a hundred billion billion times smaller than a proton	Measured (through Newton's constant); the comparison computed	*At the Edge of Reality*, Appendix C: 1.616×10^{-35} meters against a proton of about 1.68×10^{-15} meters, a ratio of 1.04×10^{20}	Preface, 1, 3, 5, 12
The Planck time	about 5×10^{-44} seconds (Chapters 4 and 16); about five and four tenths times ten to the minus forty-four of a second (Chapter 18)	Computed from the Planck length and the speed of light	*The Weight of Possibility*, Chapter 4, Section III (5.4×10^{-44} seconds); *At the Edge of Reality*, Appendix C (5.39×10^{-44} seconds)	4, 16, 18
Longest time from the horizon to the end, a hole of one solar mass	at most about fifteen millionths of a second	Established physics	*The Gravitational Passage*, Chapter 18, Section I (1.55×10^{-5} seconds)	1

Quantity	As printed	Standing	Source	Chapters
The emptying of the outside world	over about a hundred billion to a trillion years	Established physics, computed	*The Gravitational Passage*, Chapter 18, Sections III and X; *The Compactification*, Chapter 4, Section VII	1, 9, 10
The evaporation of a hole of one solar mass	about ten to the power sixty-seven years (Chapter 1); about 2.1×10^{67} years (Chapters 9 and 10); Appendix A gives it as the glow counted as light alone	Established physics	*The Compactification*, Chapter 4, Section VII, from *The Unquantized Arena*; *The Gravitational Passage*, Chapter 18, Section X	1, 9, 10
The exponents at the upper end of the Kasner curve	five ninths for each of the three large directions; minus one ninth for each of the six	Set by the structure, given the Kasner form (algebra on the two rules of Einstein's equations)	*The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Section III	2, 4, 21

Quantity	As printed	Standing	Source	Chapters
The exponents at the lower end, and the span of the curve	minus one third for each of the three and one third for each of the six; the exponent of the three runs from minus one third to five ninths	Set by the structure, given the Kasner form	*The Compactification*, Chapter 1, Section VI	2
The two ends of the arc	about 0.547, exactly one plus twice the square root of two, all divided by seven; about 0.527, exactly three plus twice the square root of six, all divided by fifteen	Set by the structure, given the Kasner form	*The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Section IV	2
The volumes at the upper end	the three grow as the time to the power five thirds; the six shrink as the time to the power minus two thirds; all nine change in direct proportion to the time	Set by the structure (sums of the exponents)	*The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Section IV	2
A billionfold change of the time, at the upper end	each of the six changes tenfold; each of the three a hundred thousand times; the volume of the three a million billion times and of the six a millionfold	Computed from the exponents	Arithmetic on *The Gravitational Passage*, Appendix B, Section IV	2, 4

Quantity	As printed	Standing	Source	Chapters
The spread of the rates	eight ninths, at every Kasner form in nine directions	Set by the structure (from the two rules alone)	*The Compactification*, Chapter 1, Section VI, equation (1.9)	2
The depths inside a black hole	about a billion billion kilograms in a cubic meter, passed at the horizon; ten to the thirty to ten to the thirty-one kilograms in a cubic meter at about two ten-thousandths of the radius; for four million solar masses, about a ten-thousandth and a hundred-millionth of the radius; the Planck density near one part in a hundred million billion billion of the radius, for one solar mass	Established physics (the densities); the depths computed	*The Gravitational Passage*, Chapter 19, Section II, and Appendix B, Section V	2
The mass of the hole at the center of the Galaxy	about four million solar masses	Measured (Event Horizon Telescope 2022)	*At the Edge of Reality*, Appendix C	2, 10

Quantity	As printed	Standing	Source	Chapters
The counts of basic pieces (Euler counts)	three for the complex projective plane, two for the sphere, three times two, six, for the six	Set by the structure (theorems of topology)	*The Gravitational Passage*, Chapter 9; *The Compactification*, Chapter 1, Section III	3, 5, 19
The radii of the six	the sphere one Planck length; the complex projective plane the square root of six Planck lengths, about two and a half (2.45); a few Planck lengths across	Set by the structure once a stated step is taken: the identification that every direction of the six reaches the Planck curvature, then a theorem of Einstein products	*The Gravitational Passage*, Chapter 9, Section IX; *The Compactification*, Chapter 2, Section IV	Preface, 1, 3, 4, 5, 16, 18, 21
The shape of the six, said without a convention	the ratio of the radii is the square root of six; each simplest sphere inside the plane has one and a half times the area of the sphere of the rows	Set by the structure once the same identification is taken	*The Gravitational Passage*, Chapter 9, Section IX	3, 5

Quantity	As printed	Standing	Source	Chapters
The curvature of the six	every direction at exactly the Planck curvature; the total, over six directions, six times the Planck curvature	Set by the structure once the same identification is taken	*The Gravitational Passage*, Chapter 9, Section IX, and Chapter 17, Section V	3, 5, 21
The volume of the six	seventy-two times pi cubed times the sixth power of the Planck length, about two thousand two hundred thirty (2,230); seventy-two Planck polydiscs	Computed from the radii	*The Gravitational Passage*, Chapter 9, Section IX; *The Compactification*, Chapter 2, Section IV	3, 5
The polydisc and the ball	a disc one Planck length in radius has the area of half of Planck's constant; the volume is nine whole cells of action, seventy-two being nine times eight; the ball has one sixth the volume of the polydisc	Computed from the radii	*The Gravitational Passage*, Chapter 9, Section IX	3

Quantity	As printed	Standing	Source	Chapters
The slowest variation of the six	twice the Planck curvature; its levels lie at about the Planck mass	Computed from the radii	*The Gravitational Passage*, Chapter 17, Section V; *The Compactification*, Chapter 2, Section III	3, 5, 18
The count of channels	three	Set by the structure (equal to the Euler count of the complex projective plane)	*The Gravitational Passage*, Chapter 13, Section II; *The Compactification*, Chapter 6, Section X	3, 5, 8, 10, 14, 19
The reach of the force of the two settings	shorter than a Planck length, some thirty powers of ten shorter than the shortest range any test of gravity reaches, about a twentieth of a millimeter	Computed at first order; the range of the tests measured (Kapner and others 2007)	*The Compactification*, Chapter 2, Section V; *The Gravitational Passage*, Chapter 11, Section II	3, 4, 5

Quantity	As printed	Standing	Source	Chapters
The two constants of the least-extent term	about nine parts in a hundred thousand (0.0000898); about two thirds of a Planck mass (0.651)	Set by the balance at the derived point	*The Gravitational Passage*, Appendix D, Section III; *The Compactification*, Chapter 2, Section V	4
The shapes over which the computed minimum is the only one	ratios of the radii between the square root of three and the square root of twelve: about 1.7 and about 3.5 (Chapter 4); about 1.73 and 3.46 (Chapter 5)	Computed at first order	*The Gravitational Passage*, Appendix D, Section III	4, 5
The barrier	at a plane radius of 1.79 Planck lengths, past the modal boundary, where the plane curves by 1.87 times the Planck curvature	Computed at first order	*The Gravitational Passage*, Appendix D, Section III	4, 5
Where the minimum stays stable	for ratios up to 2.50 at the derived radius, and for radii above 1.87 Planck lengths at the derived ratio	Computed at first order	*The Gravitational Passage*, Appendix D, Section III	Appendix A

Quantity	As printed	Standing	Source	Chapters
The least size of any minimum	above a plane radius of about one and a half Planck lengths (1.48)	Computed at first order	*The Gravitational Passage*, Appendix D, Section III; *The Compactification*, Chapter 2, Section VII	5
The masses of the two settings (moduli)	2.25 Planck masses, mostly size, and 3.09, mostly shape; "about two and a quarter" and "about three (3.09)" (Chapter 5); "about two and a quarter" and "about three and a tenth" (Chapters 4, 16 and 21)	Computed at first order, in one mechanism of the hold	*The Gravitational Passage*, Appendix D, Section III	4, 5, 13, 16, 21
The floor's own energy	about plus seven hundredths of the Planck density (0.071)	Computed at first order	*The Gravitational Passage*, Appendix D, Section III	4
The bare energy of the curvature of the six	minus three divided by eight pi of the Planck density, about minus twelve hundredths (−0.119)	Set by the structure	*The Gravitational Passage*, Appendix D, Section III; *The Compactification*, Chapter 2, Section V	4, 5, 8

Quantity	As printed	Standing	Source	Chapters
The depth of the well	about minus five hundredths of the Planck density (−0.049); a well about one twentieth of the Planck density deep	Computed at first order	*The Gravitational Passage*, Appendix D, Section III; *The Weight of Possibility*, Chapter 4, Section III	4, 5, 16
The trim: the vacuum energy at the boundary	one less three divided by eight pi, about eighty-eight hundredths of the Planck density (0.8806); "about 0.881" (Chapter 10)	Set by the structure	*The Compactification*, Chapter 2, Section V, equation (2.5); *The Gravitational Passage*, Chapter 13, Section II	4, 5, 8, 10, 14
The decay of the breathing excitations	within about a Planck time; the step-by-step estimate about ten to the minus forty-six seconds	Computed (an estimate by perturbation theory, for a term of natural size)	*The Weight of Possibility*, Chapter 4, Section III	4, 10, 16, 21
The energy the settling releases	between a tenth of the depth of the well and most of it	Computed	*The Weight of Possibility*, Chapter 4, Section III	4, 16

Quantity	As printed	Standing	Source	Chapters
The hottest the new epoch's plasma can have been	about one thirtieth of the mass of a gyration, the divisor running from about twenty-eight to about thirty-four with the strength of the coupling	Bounded, value not given: a condition on the turn, set by the measured amount of dark matter	*The Weight of Possibility*, Chapter 4, Section VI, and Chapter 6, Section I	4, 10, 13, 16, 19, 21
The end of a stage that behaved as matter, if there was one	below about one hundredth of the mass of a gyration	Bounded, value not given: a condition on the turn	*The Weight of Possibility*, Chapter 4, Section VI, and Chapter 6, Section I	4, 10, 13, 16, 19, 21
The waves of the settling	below about three percent of the radiation of the content, at the time when the content had about a hundred kinds of particle (one hundred six and three quarters species)	Bounded, value not given: a condition on the turn, from the measured count of light species (Aghanim and others 2020; Calabrese and others 2025; Goldstein and Hill 2026)	*The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I	4, 10, 15, 16, 17, 19, 21

Quantity	As printed	Standing	Source	Chapters
The release against a cool enough plasma	about forty to three hundred times the largest energy that such a plasma can hold	Computed	*The Weight of Possibility*, Chapter 4, Section III	4, 16
How far the six may arrive from their held size	no more than about one part in a thousand of their radius, unless a stage that behaved as matter carried at least as much energy as the waves	Bounded, value not given: a condition on the turn	*The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I	4, 16, 17
The gyrations made directly as the six reform	about a billion billion billion (ten to the twenty-seven) times fewer than an abrupt re-forming would make; ten to the minus twenty-seven is about e to the minus sixty-two (Appendix A)	Bounded, value not given: a condition on the turn, set by the measured amount of dark matter	*The Weight of Possibility*, Chapter 4, Section VI	4, 19, 21 (in part); Appendix A (in full)

Quantity	As printed	Standing	Source	Chapters
How smooth the re-forming must be	smooth over about ten swings (periods) of the lightest gyration; each swing cuts the yield about five hundredfold (Chapter 4); the yield falls by a factor of e for each part in two pi of one period, about a sixth of a period, and sixty-two such steps make about ten periods (Appendix A); about ten periods (Chapters 19 and 21)	Bounded, value not given: a condition on the turn, set by the measured amount of dark matter; the cut per swing computed (e to the power two pi, about 535)	*The Weight of Possibility*, Chapter 4, Section VI	4, 19, 21 (in part); Appendix A (in full)
The valleys of string theory	ten to the five hundred and beyond; some hundreds of independent loops in the spaces string theory uses	Established physics (estimates of string theory: Douglas 2003; Douglas and Kachru 2007)	*The Compactification*, Chapter 2, Sections I and VI	5

Quantity	As printed	Standing	Source	Chapters
The energy of the fields' lowest motion, added up at the Planck scale, against the measured energy of empty space	a factor of about a one followed by 123 zeros; pictured as a hundred numbers, each about a hundred and twenty-three digits long, adding up to a number with one digit	Established physics (the standard estimate, against a measured value)	*The View from Inside*, Chapter 1, "The Number That Shouldn't Exist", and Chapter 2	5, 8
The number of settings (moduli) of the six	two: the volume and the shape	Set by the structure once a stated step is taken: homogeneity, an identification; then a theorem	*The Compactification*, Chapter 2, Section III	4, 5, 13, 15, 16, 21
The self-coupling of the Higgs field at tree level	one eighth, from a weak charge of one half (half of one quarter)	Set by the structure once a stated step is taken: an identification	*The Compactification*, Chapter 2, Section VIII, from *The Measures of the Kinds*, Chapter 13, Section VI	5

Quantity	As printed	Standing	Source	Chapters
The loop term and the full self-coupling	one over twenty-four times pi squared, about four thousandths; about thirteen hundredths in all (0.1292)	Computed	*The Compactification*, Chapter 2, Section VIII	5
The strength of the settled Higgs field	about two hundred forty-six billion electron volts (246.22)	Measured	*The Compactification*, Chapter 2, Section VIII	5
The mass of the Higgs boson	one hundred twenty-five and seventeen hundredths GeV (125.17) from the account; measured one hundred twenty-five and thirteen hundredths GeV (125.13), give or take eleven hundredths	Computed from the identification and the measured strength (one measured agreement); measured	*The Compactification*, Chapter 2, Section VIII	5, 10, 16, 20

Part II: Why the Beginning Was Orderly

Quantity	As printed	Standing	Source	Chapters
Penrose's figure for how improbable the beginning was	one part in ten to the power ten to the power one hundred twenty-three	Established physics (Penrose 1979; Penrose 1989), a ratio of two volumes in a space of states	*The Compactification*, Chapter 3, Sections I and VIII	7, 10

Quantity	As printed	Standing	Source	Chapters
The Weyl curvature at the start of the new epoch	near zero, and different from zero only to the size of the departures from uniformity, about one part in a hundred thousand	Set by the structure (argued from the new geometry's lack of a prior identity); the size of the departures measured	*The Gravitational Passage*, Chapter 18, Sections VIII and IX; *The Compactification*, Chapter 3, Section VI	7
The count of a black hole	one quarter of the area of its horizon, measured in Planck areas; one tile for every four Planck squares	Established physics (Bekenstein 1973; Hawking 1975); the reading of it as a count relative to the outside is an identification that makes no claim on the factor of one quarter	*The Compactification*, Chapter 4, Section II; *The View from Inside*, Chapter 12	7, 8, 9, 10
The particle horizon	about forty-six billion light years (46.1; 46.14)	Measured (worked out from the measured history of the expansion)	*The Gravitational Passage*, Chapter 13, Section IV; *The View from Inside*, Appendix C	8, 10, 14
The age of the universe	about thirteen point eight billion years	Measured	*The View from Inside*, Chapter 12	8, 14

Quantity	As printed	Standing	Source	Chapters
The cosmological event horizon	about sixteen point seven billion light years today (16.68)	Measured	*The View from Inside*, Appendix C; *The Compactification*, Chapter 5, Section VII	8, 10
The Hubble radius	about fourteen and a half billion light years	Measured	*The View from Inside*, Appendix C (14.51)	8
The de Sitter radius	about seventeen and a half billion light years (17.53); the de Sitter time about 17.5 billion years	Measured	*The View from Inside*, Appendix C; *The Compactification*, Chapter 5, Section VII	8, 10, 12, 14
A black hole of one solar mass: radius and count	about three kilometers (2.95); about 1.05×10^{77}, a one followed by seventy-seven zeros	Established physics, computed from the measured constants	*The Compactification*, Chapter 4, Section II	8, 9, 10

Quantity	As printed	Standing	Source	Chapters
The count of the particle horizon	about 2.29×10^{123}; the horizon spans about ten to the power sixty-one Planck lengths	Set by the structure once a stated step is taken (the identification of the horizon's count as the one that enters a vacuum energy); computed from the measured horizon	*The Gravitational Passage*, Chapter 13, Sections IV and VI; *The Compactification*, Chapter 4, Section III	8, 10
The share of the vacuum's energy that a measurement takes in	three parts in the count of the particle horizon, of 0.8806 of the Planck density; three tiles hold about one part in a one followed by 123 zeros of the external value	Set by the structure once a stated step is taken: my identification of the share, made in three steps	*The Gravitational Passage*, Chapter 13, Sections II and IV	8, 10, 14, 21

Quantity	As printed	Standing	Source	Chapters
The energy of empty space	two and fifty-six hundredths times ten to the minus forty-seven GeV to the fourth (2.56) from the share; measured two and fifty-two hundredths (2.52); about six times ten to the minus twenty-seven kilograms in each cubic meter (5.8), the mass of three or four hydrogen atoms	Set by the structure once the identification is taken, computed with the measured horizon; measured	*The Gravitational Passage*, Chapter 13, Section IV; *The Compactification*, Chapter 4, Sections III and IV	8, 10, 14, 21
The share on other edges	on the de Sitter radius about seven times the measured constant (eight times the trim, 7.05); 8.0 without the trim (Appendix A); more on any smaller edge	Computed	*The Compactification*, Chapter 4, Section III, and Chapter 5, Section VII; *The Gravitational Passage*, Chapter 13, Section VI	8, 10 (in part); Appendix A (in full)

Quantity	As printed	Standing	Source	Chapters
The edge on which the share equals the constant	about 2.65 times the de Sitter radius (2.654, the square root of eight times the trim); the measured particle horizon stands at about 2.63 times it (2.632, from 46.1 and 17.5)	Set by the structure once the identification is taken; measured	*The Gravitational Passage*, Chapter 13, Section III; *The Compactification*, Chapter 6, Section X	8, 14
The reduced Planck mass in place of the full one	divides the share by eight pi squared, about 632	Computed	*The Gravitational Passage*, Chapter 13, Section VII; *The View from Inside*, Appendix A, Section IV	Appendix A
The five choices varied together	four times six times two times two times two, which is 192 combinations, of which exactly one gives an energy of empty space within the measured uncertainty; *The Gravitational Passage*, leaving out the whole number five, counts 160 (Appendix A); Chapter 14 says that exactly one combination lands, without the count	Computed	*The View from Inside*, Appendix A, Section IV; *The Compactification*, Chapter 6, Section X; *The Gravitational Passage*, Chapter 13, Section VII	14 (in part); Appendix A (in full)

Quantity	As printed	Standing	Source	Chapters
The radiation's share of the universe's energy	about nine parts in a hundred thousand	Measured	*The Gravitational Passage*, Chapter 13, Section III	8, 14
The identity of the two ways	the square of the particle horizon in Hubble radii, times the vacuum's share, equals eight less three over pi, about 7.05	Set by the structure once the identification is taken	*The Gravitational Passage*, Chapter 13, Section III	8, 14
The matter share of the universe's energy	0.311, thirty-one and one tenth percent, give or take one tenth of a percent; "thirty-one percent" (Chapter 19); measured 0.315, give or take 0.007 (Aghanim and others 2020)	Set by the structure once the identification is taken, computed with the measured radiation share; measured	*The Gravitational Passage*, Chapter 13, Section III; *The Compactification*, Chapter 6, Section X	8, 10, 14, 19, 21
The vacuum's share	0.689, 68.9 percent; measured 68.5 percent, give or take 0.7	The same result as the matter share, seen another way; measured	*The Gravitational Passage*, Chapter 13, Section III; *The Compactification*, Chapter 6, Section X	8, 14

Quantity	As printed	Standing	Source	Chapters
The age in de Sitter times	0.792 from the identity; measured 0.787, give or take 0.009	The same result as the matter share, seen another way; measured	*The Gravitational Passage*, Chapter 13, Section III	14
The share worked out for other eras on paper	for an astronomer at half the present size, a matter fraction of 0.79 and 7.049 times the measured constant; 1.224 times a billion years ago; 0.852 times a billion years from now	Computed	*The Gravitational Passage*, Chapter 13, Section VI; *The Compactification*, Chapter 4, Section III, and Chapter 5, Section VII	8
The change of the share's present value over thirty years	about five parts in a billion	Computed	*The View from Inside*, Appendix A, Section V	Appendix B
The era of the present act	about ten to the power sixty-one Planck times after the beginning of the reconstructed history	Measured (taken from the measured particle horizon)	*The Gravitational Passage*, Chapter 13, Section VII	8

Quantity	As printed	Standing	Source	Chapters
The count of all black holes in the observable universe	about ten to the power one hundred four (Chapter 8); about ten to the one hundred four, about three times that in the estimate of Egan and Lineweaver (Chapter 10)	Established physics, cited	*The Compactification*, Chapter 4, Section IV, Chapter 5, Section VI, and Chapter 3, Section VIII (3×10^{104})	8, 10
A hole's own density against empty space	for one solar mass, an average of about 1.8×10^{19} kilograms in a cubic meter, eight times it about 1.5×10^{20}; against the density of empty space, a ratio of about 2.5×10^{46}, "about forty-six orders of magnitude"	Computed from the measured constants	*The Compactification*, Chapter 4, Section IV; *The Gravitational Passage*, Appendix B, Section VI	8, 9
Light reaching a hole while inside	changes its count by parts in ten to the twenty-two or twenty-three	Computed	*The Gravitational Passage*, Appendix B, Section VI; *The Compactification*, Chapter 4, Section IV	Appendix A
A merger of two equal holes: final mass and spin	0.95162 of the starting mass; 0.68646 of the fastest spin	Established physics (numerical relativity: Scheel and others 2009)	*The Compactification*, Chapter 4, Section VI	9

Quantity	As printed	Standing	Source	Chapters
A merger of two equal holes: growth of the count and energy radiated	about 1.56 times (1.564) the starting areas added together, about half again; about 4.8 percent of the mass and energy leaves as waves	Computed from the final mass and spin	*The Compactification*, Chapter 4, Section VI	9
The two patterns of a gravitational wave	two polarizations, traveling at the speed of light	Set by the structure (Einstein's equations, once the six are held); measured (GW170814; GW170817)	*The Compactification*, Chapter 7, Section II	9, 15, 17
The Hawking temperature of a hole of one solar mass	about 6.2×10^{-8} kelvin, about sixty-two billionths of a degree	Established physics	*The Compactification*, Chapter 4, Section VII; *The Gravitational Passage*, Chapter 18, Section X	9, 10

Quantity	As printed	Standing	Source	Chapters
The temperature of the microwave background today	2.72548 kelvin; 2.7255 kelvin; about 2.7 degrees above absolute zero; under three; about 2.35 ten-thousandths of an electron volt	Measured (Fixsen 2009)	*The Coming of the Kinds*, Chapter 13, Section III, and Chapter 2, Section IV; *The Compactification*, Chapter 5, Section V	9, 10, 11, 16, 20
The hole against the background	colder by a factor of tens of millions, nearly eight powers of ten	Computed	*The Compactification*, Chapter 4, Section VII	9
The cooling of the background	to a little more than a third of its value about every seventeen billion years	Computed	*The Gravitational Passage*, Chapter 18, Section X; *The Compactification*, Chapter 4, Section VII	9
A hole made in the early universe that has evaporated by now	less than about 5×10^{11} kilograms, five hundred billion kilograms	Established physics	*The Gravitational Passage*, Chapter 18, Section I; *The Compactification*, Chapter 4, Section VII	9

Quantity	As printed	Standing	Source	Chapters
The Page curve	holds to within half a nat	A theorem under two stated conditions (*The Unquantized Arena*)	*The Compactification*, Chapter 4, Section VII	9
The turning point of the Page curve	no later than 0.646 of the evaporation time, half the count left; a perfect warm body: four thirds as much, 0.571 of the count left, at 0.568 of the time; Page's case: 1.4847 times as much, 0.598 left, about two fifths lost, at 0.538 of the time	The bound a theorem; the cases computed (Page 2013 for the last)	*The Compactification*, Chapter 4, Section VII, Table 4.1	9 (in part); Appendix A (in full)
The entangling phase for the design of Bose and colleagues	0.314 radians: about a third of a radian, about eighteen degrees, a twentieth of a full turn	Computed from the identification of one geometry for each alternative	*The Gravitational Passage*, Chapter 12, Sections VI and VIII; *The Compactification*, Chapter 8, Section XIII	9, 17, 21

Quantity	As printed	Standing	Source	Chapters
The masses of a gyration	the square root of two Planck masses (about 1.41) at the classical level; about six tenths of a Planck mass at first order in the floor; between about six tenths and one and four tenths Planck masses	Computed	*The Weight of Possibility*, Chapter 4, Section V	10, 13, 18, 19, 21
The temperature at which our history begins	never hotter than about 5 and about 2 × 10^{17} GeV, for the two masses of a gyration; about 2.3 to 5.8 × 10^{30} kelvin; between about 1.6 and 4.1 hundredths of the Planck mass; between about 1.2 and 2.9 thousand million million times the mass of the top quark	Bounded, value not given: the measured amount of dark matter bounds it from above	*The Weight of Possibility*, Chapter 4, Section VI; *The Coming of the Kinds*, Chapter 2, Section IV	10, 13, 16, 17, 19, 21
A temperature given as an energy	one GeV, about the energy held at rest in one proton, is about eleven and a half million million kelvin; an MeV is about eleven and a half billion kelvin	Established physics	*The Coming of the Kinds*, Chapter 2, Section IV (1.16 × 10^{13} kelvin)	10, 16, 19, 20

Quantity	As printed	Standing	Source	Chapters
The waves of the settling counted as light species	an excess of 2.86 times their starting share, in units of one kind of neutrino; under the three percent bound, less than about 0.09 of one kind	Computed from the three percent bound	*The Compactification*, Chapter 5, Section II, and Chapter 7, Section V	10, 16, 17
The count of species sharing the heat	106.75 above the top quark (twenty-eight states of the bosons plus seven eighths of ninety of the fermions); 86.25; 61.75 just above the binding of the quarks; 17.25 just below; 10.75 by a few MeV; 3.909 today (2 for the light and 1.909 for the neutrinos), "about three point nine" (Chapter 16)	Established physics	*The Compactification*, Chapter 5, Section III; *The Coming of the Kinds*, Chapters 3 and 8	10, 16
The right-handed neutrino's reactions	no more than 3.7 ten-billionths of the expansion rate	Computed	*The Compactification*, Chapter 5, Section III	10
When the pions and the muon leave the crowd	near 14 MeV and near 10 MeV	Established physics	*The Compactification*, Chapter 5, Section III	10

Quantity	As printed	Standing	Source	Chapters
When the neutrinos go free	near 3 MeV for the muon and tau neutrinos and near 2 MeV for the electron's (Chapter 10); between about 3 and 1 MeV, in the first second, the computation taking 3 (Chapters 13 and 20)	Established physics	*The Compactification*, Chapter 5, Section III; *The Coming of the Kinds*, Chapter 9; *The Gravitational Companion*, Chapter 3, Section IX	10, 13, 20
The light against the neutrinos	warmer by the cube root of eleven fourths, about 1.40; the neutrinos at 1.945 kelvin today	Established physics	*The Coming of the Kinds*, Chapter 10, Section II	10, 11, 20
The effective number of neutrino kinds	3.044 from the standard computation (Bennett and others 2021); measured 2.99, give or take 0.17 (Aghanim and others 2020), and 2.86, give or take 0.13 (Calabrese and others 2025)	Established physics; measured	*The Compactification*, Chapter 5, Section III, and Chapter 7, Section V	10, 16, 17

Quantity	As printed	Standing	Source	Chapters
The settling of the Higgs field	a crossover centered at 159.5 GeV, give or take 1.5, steepest over about 5 GeV; "near a hundred and sixty GeV" (Chapter 17); "near 160 GeV" (Chapter 20); a sudden change only below an end point of 72.4 GeV, give or take 1.7, "about 72 GeV" (Chapter 20)	Established physics (lattice computations: D'Onofrio and Rummukainen 2016; Csikor, Fodor and Heitger 1999)	*The Compactification*, Chapter 5, Section IV, and Chapter 7, Section VIII; *The Coming of the Kinds*, Chapter 4, Section IV	10, 13, 16, 17, 20
The binding of the quarks	a crossover at 156.5 MeV, give or take 1.5	Established physics (lattice computations; Aoki and others 2006)	*The Compactification*, Chapter 5, Sections III and IV	10, 16, 17, 20
The sphaleron passages	in balance from about ten to the twelve GeV (a million million GeV) down to 131.7 GeV, give or take 2.3	Established physics (lattice computation: D'Onofrio, Rummukainen and Tranberg 2014)	*The Compactification*, Chapter 5, Section IV; *The Coming of the Kinds*, Chapter 4, Section II	10, 20

Quantity	As printed	Standing	Source	Chapters
Neutrons against protons	near 0.8 MeV the ratio is left at about one in six; by the first nuclei about one in seven	Established physics	*The Coming of the Kinds*, Chapters 11 and 12; *The Compactification*, Chapter 5, Section V	10, 20
The last scattering of the light	at about 2,973 kelvin, give or take about one; distances then about 1,091 times smaller (Chapter 10), 1,090.92 times (Chapter 11), about one thousand and ninety times (Chapter 18); at a measured redshift of 1089.92, give or take 0.25	Measured (Aghanim and others 2020; Fixsen 2009); the temperature computed from the two	*The Coming of the Kinds*, Chapter 13, Section III	10, 11, 16, 18, 20
Baryons for every photon	6.13 for every ten billion particles of light, give or take 0.04; "about six in ten billion"	Measured (Aghanim and others 2020)	*The Coming of the Kinds*, Chapter 1, Section VI; *The Compactification*, Chapter 5, Section V	10, 11, 19, 20, 21

Quantity	As printed	Standing	Source	Chapters
When matter came to outweigh radiation	when distances were about 3,403 times smaller than today, give or take 26, the light at about 9,275 kelvin, about 0.80 electron volts (Chapter 10); near 9,300 kelvin (Chapter 11)	Measured (Aghanim and others 2020); the temperature computed	*The Compactification*, Chapter 5, Section V	10, 11
The light in a cubic centimeter today	about 411 photons; a count of about 1,479, which is 3.602 times their number	Computed from the measured temperature	*The Compactification*, Chapter 5, Section V	10, 11, 20
The volume within the particle horizon	about 3.48×10^{86} cubic centimeters	Computed from the measured horizon	*The Compactification*, Chapter 5, Section V	10
The count of the content today	the light about 5.2×10^{89}; the neutrinos about 4.9×10^{89}; together about ten to the ninety	Computed from measured values	*The Compactification*, Chapter 5, Section V	10
The count of the hole at the center of the Galaxy	about 1.7×10^{90}	Computed from measured values	*The Compactification*, Chapter 5, Section VI	10

Quantity	As printed	Standing	Source	Chapters
The counts set side by side	the black holes exceed the content by about ten to the fourteen; the particle horizon exceeds the black holes by about ten to the nineteen	Computed	*The Compactification*, Chapter 5, Sections VI and VII	10
The count of a sphere of the de Sitter radius	about 3.31×10^{122}	Computed from measured values	*The Compactification*, Chapter 5, Section VII	10
The ceiling of a region and its content's count	the ceiling grows as the square of the time, a hundred times the time giving ten thousand times the ceiling; the content's count grows as the time to the power one and a half, a hundred times the time giving a thousand times the count	Established physics (Frautschi 1982, after Layzer 1975)	*The Compactification*, Chapter 3, Section VIII	10
The content's count against the ceiling at the start	about 3.64 times the temperature divided by the Planck mass, with 106.75 species; 0.060 to 0.149 at the bounding temperature; at most about fifteen percent	Computed	*The Compactification*, Chapter 3, Section VIII	10

Part III: What the First Light Records

Quantity	As printed	Standing	Source	Chapters
Electrons over positrons, before they annihilated	about two extra electrons for every billion positrons	Computed (established physics, from the measured baryons per photon)	*The Coming of the Kinds*, Chapter 10, Section IV	11
Photons for each nucleus	about 2.2 billion for every hydrogen nucleus; 1.6 billion for every proton and neutron; about three protons and neutrons in four are hydrogen nuclei	Computed from measured values	*The Coming of the Kinds*, Chapter 13, Section II, and Chapter 12, Section II	11
When half the electrons of hydrogen would be bound, in balance	near 3,700 kelvin, about one forty-second of the energy that binds hydrogen's electron	Established physics (Saha's rule)	*The Coming of the Kinds*, Chapter 13, Section II	11
The light at last scattering	about one electron in ten still free; some fifty-three times below the energy that binds hydrogen's electron; about 370,000 years into the reconstructed history	Established physics (Particle Data Group 2026); the ratio computed	*The Coming of the Kinds*, Chapter 13, Section III	11, 15, 20

Quantity	As printed	Standing	Source	Chapters
The first atoms	about twelve hydrogen atoms for every atom of helium; helium about a quarter of the mass of ordinary matter	Measured (helium mass share 0.245, give or take 0.003: Particle Data Group 2026)	*The Coming of the Kinds*, Chapter 12, Section III	11, 20
The temperature differences of the sky	about one part in a hundred thousand, first measured in 1992; on a sky at 2.7255 kelvin about twenty-seven millionths of a degree	Measured (Smoot and others 1992); the degrees computed	*The Compactification*, Chapter 6, Section I; *The Weight of Possibility*, Chapter 1, Section I	7, 11, 12, 13, 14, 18, 21
The strength of the first ripples	about five parts in a hundred thousand in the curvature of space; the temperature differences smaller again by a factor of about five	Measured, with its reason still a question: what count of states sets it	*The Compactification*, Chapter 6, Sections I and VII	11, 12, 14
The power of the first ripples (the amplitude)	about two and one tenth billionths (2.10×10^{-9}); about two billionths; in the coded form of the standard fit, 3.044, give or take 0.014	Measured, with its reason still a question: what count of states sets it (Aghanim and others 2020)	*The Compactification*, Chapter 6, Sections I, VII and X	11, 12, 13, 14, 16, 21

Quantity	As printed	Standing	Source	Chapters
The spectral index	0.9649, give or take 0.0042, a little below one; a tilt of about three and a half hundredths; "about 0.97" (Chapters 11 and 13)	Measured, with its reason still a question: what length, between the turn's length and the de Sitter radius, sets it (Aghanim and others 2020)	*The Compactification*, Chapter 6, Sections I and VII	11, 12, 13, 14, 21
The running of the index	minus 0.0045, give or take 0.0067, consistent with zero	Measured (Aghanim and others 2020)	*The Compactification*, Chapter 6, Section I	11, 12

Quantity	As printed	Standing	Source	Chapters
First waves against first ripples	the account: far below one millionth, from second order only; measured below 0.036 at ninety-five percent confidence (Ade and others 2021), below 0.032 in a reanalysis (Tristram and others 2022; Appendix B); said in later chapters as "below about three hundredths" (Chapter 12), "below about three parts in a hundred" (Chapter 17), "below about three and a half hundredths" (Chapter 14), "below about three and a half in a hundred" (Chapter 16)	Set by the structure once a stated step is taken: my identification of the first ripples with the form of the content carried through the turn, with the premise that the new context has no prior identity; the bound measured	*The Compactification*, Chapter 6, Sections I and V; *The Gravitational Passage*, Chapter 18, Section XIII	11, 12, 14, 15, 16, 17, 21

Quantity	As printed	Standing	Source	Chapters
A separate pattern of one ingredient (isocurvature)	the account: none; measured below 1.3 percent of the power of the temperature differences, at ninety-five percent confidence; for the three kinds commonly tested, "about one and a third to one and seven tenths percent" (Chapter 12); "down to about 1.3 percent" (Chapter 19)	Set by the structure once the same step is taken; the bound measured (Akrami and others 2020)	*The Compactification*, Chapter 6, Sections I and IV (1.3 to 1.7 percent); *The Weight of Possibility*, Chapter 1, Section I, and Chapter 3, Section I	11, 12, 14, 18, 19
The curvature share of space	the account: flat in the smoothed-out model, curved only by its ripples, about one part in a hundred thousand; measured seven ten-thousandths, give or take nineteen ten-thousandths (0.0007 ± 0.0019), consistent with zero to about two thousandths	Set by the structure once the same step is taken; measured (Aghanim and others 2020)	*The Compactification*, Chapter 6, Sections I and VI	11, 12, 14, 21

Quantity	As printed	Standing	Source	Chapters
A curvature radius equal to the de Sitter radius	a curvature share today of about ten to the minus sixty-four	Computed	*The Compactification*, Chapter 6, Section VI; *At the Edge of Reality*, Appendix C (2.5×10^{-64})	12
The vacuum of the new epoch on the lengths we observe	such a length lay some ten to the twenty-seven times beyond the horizon of that time; the vacuum's power below ten to the minus fifty-one; short of the observed power by more than ten to the forty-two	Computed	*The Compactification*, Chapter 6, Section II; *The Gravitational Passage*, Chapter 18, Section IX	11, 12, 13
An observed length near the turn	some ten to the twenty-seven Planck lengths, and some ten to the minus thirty-four of the de Sitter radius	Computed within the reconstructed history	*The Compactification*, Chapter 6, Section VII; *The Gravitational Passage*, Chapter 18, Section IX	12

Quantity	As printed	Standing	Source	Chapters
Directions never in reach of each other in the standard history	more than about a degree apart, about twice the width of the full Moon	Established physics (Baumann and Peiris 2009)	*At the Edge of Reality*, Appendix C	12
The index of white noise	four if the curvature of space is left as white noise; zero if the density of matter is	Established physics	*The Gravitational Passage*, Chapter 18, Section IX; *The Compactification*, Chapter 6, Section III	Appendices A and B
The waves that arise at second order	counted in powers of ten, their power changes from one length to the next twice as fast as the ripples' power does	Established physics	*The Gravitational Passage*, Chapter 18, Section IX	12

Quantity	As printed	Standing	Source	Chapters
The coming measurements of the first waves	the Simons Observatory and LiteBIRD to within about one or two thousandths (Chapter 12); the Simons Observatory to within about one and two tenths thousandths and LiteBIRD better than two thousandths (Appendix B); "about a thousandth" (Chapters 17 and 21)	Forecast (Simons Observatory Collaboration 2025; LiteBIRD Collaboration 2026)	*The Compactification*, Chapter 8, Section XI	12, 17, 21
The fading of shear	each time the universe doubles in size, the shear's term falls sixty-four-fold and the light's sixteen-fold	Established physics (Ellis and MacCallum 1969)	*The Gravitational Companion*, Chapter 3, Section IX	13
From the settling of the Higgs field to the release of the neutrinos	the temperature fell to about 1.9 parts in a hundred thousand of what it was; the universe grew at least about fifty-three thousand-fold; the shear's share then less than 2.3 parts in a hundred thousand, less than three parts in a hundred thousand	Computed	*The Gravitational Companion*, Chapter 3, Section IX	13

Quantity	As printed	Standing	Source	Chapters
The exponents near the center of an ordinary black hole	one direction with the power minus one third, two with the power two thirds; the squares one ninth, four ninths, four ninths	Established physics (the Kasner form in three directions)	*The Gravitational Passage*, Appendix B, Section II	13
Dark matter against ordinary matter	the account: six less two divided by pi, 5.3634, about five and thirty-six hundredths (Chapters 19 and 21; "about 5.36", Chapter 20); "a little more than five and a third" (Chapter 14); measured "about five and four tenths" (Chapters 13 and 18)	A registered ratio, an identity of the present measurement: it follows from the counting principle and the rule of the abiding share, which are the account's principles, and the rule was selected by the measurement from a small family of expressions of one form, twelve examined in all	*The Weight of Possibility*, Chapter 3, Section V, Chapter 5, Section VIII, and Chapter 1, Section I ("about 5.4")	13, 14, 18, 19, 20, 21

Quantity	As printed	Standing	Source	Chapters
The Hubble constant and the megaparsec	about sixty-seven to sixty-eight kilometers per second for every megaparsec; a megaparsec about 3.26 million light years; h of 0.678 means sixty-seven and eight tenths	Measured; the unit established	*The Compactification*, Chapter 6, Section X	14
The density of ordinary matter	0.02237, give or take 0.00015, a little more than two hundredths; about five percent of the pie	Measured (an input of the account: Aghanim and others 2020)	*The Compactification*, Chapter 6, Section X	14
The density of dark matter	the account: 5.3634 times 0.02237, which is 0.1200, give or take eight ten-thousandths; measured 0.1200, give or take twelve ten-thousandths; about twenty-six percent of the pie	Computed from the ratio and the measured density of ordinary matter (a restatement of the ratio); measured	*The Compactification*, Chapter 6, Section X	14
The angle of the sound horizon	a little more than half a degree, a little wider than the full Moon; in the fit's form 1.04092, give or take 0.00031; a radian a little over fifty-seven degrees	Measured (an input of the account)	*The Compactification*, Chapter 6, Section X	14

Quantity	As printed	Standing	Source	Chapters
The optical depth	0.0544, give or take 0.0073; about one photon of the first light in twenty scattered on the way	Measured (an input of the account)	*The Compactification*, Chapter 6, Section X	14
The measured ratio of dark to ordinary matter	5.364, give or take 0.065 (Planck); 5.370, give or take 0.048 (South Pole Telescope, Planck and the Atacama Cosmology Telescope); 5.199, give or take 0.035 (with DESI)	Measured (Aghanim and others 2020; Camphuis and others 2026; Louis and others 2025), the ratios computed from the published densities	*The Weight of Possibility*, Chapter 5, Section I; *The Compactification*, Chapter 6, Section X	14, 19
The coefficient the ratio shares with the identity	the curvature of the six in Planck units, one by identification; read from the measured ratio, ninety-nine hundredths, give or take eight hundredths; the plane's curvature three and ninety-six hundredths, give or take three tenths, where the Passage's shape gives four	Computed from the measured ratio through the rule	*The Weight of Possibility*, Chapter 5, Sections III and IV, and Chapter 3, Section VI	19

Quantity	As printed	Standing	Source	Chapters
The share of ordinary matter in all the matter	the account: 0.15644, a little more than fifteen and a half percent, with the neutrinos at their least masses; measured 0.1587, 0.1542 and 0.156	Computed from the ratio and the matter share; measured (light elements with DESI; the full DESI map; hot gas in clusters: Mantz and others 2022)	*The Weight of Possibility*, Chapter 5, Section II	14
The matter share from DESI alone	29.75 percent, give or take 0.86	Measured (Abdul-Karim and others 2025)	*The Compactification*, Chapter 6, Section X	14
The Hubble constant from the account	the measured matter in each volume, 0.1430, divided by 0.311: h of 0.678, sixty-seven and eight tenths, give or take three tenths; measured 67.4, give or take 0.5, from the first light; 73.0, give or take 1.0, from Cepheids (Riess and others 2022); a rate of 73 would need a matter share of 26.8 percent	The same result as the matter share, through one more measurement; measured	*The Compactification*, Chapter 6, Section X; *The Gravitational Passage*, Chapter 13, Section IV	14, 21

Quantity	As printed	Standing	Source	Chapters
The equation of state of the energy of empty space	the account: minus one exactly; measured minus one and three hundredths, give or take three hundredths	Set by the structure (argued: the constant is fixed once); measured (Aghanim and others 2020, with galaxy distances and exploding stars)	*The Gravitational Passage*, Chapter 13, Section V; *The Compactification*, Chapter 6, Section X	14
S eight, the clumping of matter today	0.823, give or take 0.006, at the measured angle of the sound horizon; 0.827, give or take 0.007, at h of 0.678; the standard model's value from the same first light 0.832, give or take 0.013	Computed from the matter share, the ratio and the measured amplitude and index	*The Weight of Possibility*, Chapter 5, Section VI; *The Compactification*, Chapter 6, Section X	14
How far a gyration travels in the whole history	less than a meter, even leaving the turn at the Planck temperature with a hundred times its mass in momentum; about two centimeters from the hottest gas the account allows	Computed	*The Weight of Possibility*, Chapter 5, Section VI	14, 18

Part IV: The Waves the Beginning Left

Quantity	As printed	Standing	Source	Chapters
The first gravitational wave caught	it changed each four-kilometer arm of LIGO by far less than the width of a proton; it swept from about thirty-five to about two hundred fifty hertz in a fraction of a second	Measured (Abbott and others 2016)	The published measurement (Abbott and others 2016)	15, 17
The speed of gravitational waves	the waves and the light of GW170817 arrived within two seconds of each other; equal to the speed of light within three parts in a million billion on the slow side (minus 3×10^{-15}) and seven parts in ten million billion on the fast side (plus 7×10^{-16})	Measured (Abbott and others 2017)	*The Compactification*, Chapter 7, Section II, and Chapter 8, Section XII	15, 17
The pitches of light and of LIGO	visible light rises and falls several hundred million million times a second; LIGO hears tens to hundreds of hertz	Established physics; measured	Standard values	15

Quantity	As printed	Standing	Source	Chapters
The band of the planned space detector (LISA)	near a thousandth of a hertz, one rise and fall in about seventeen minutes (Chapters 15 and 17); from one vibration every ten thousand seconds (about three hours) to one every ten seconds, with a reach of about ten to the minus thirteen near one vibration every five or six minutes (Appendix B)	Projected (Amaro-Seoane and others 2017; Colpi and others 2024)	*The Compactification*, Chapter 8, Section XII	15, 17; Appendix B
The hum found by pulsar timing	near a billionth of a hertz, one rise and fall in about thirty years (Chapters 15 and 16); in fifteen years of data, a strain of about two and four tenths parts in ten to the fifteen at one vibration a year, matching an energy share of about four parts in a billion (Appendix B)	Measured (Agazie and others 2023); the energy share computed	*The Compactification*, Chapter 7, Section X, and Chapter 8, Section XII	15, 16, 17; Appendix B
The energy of each quantum of the settling waves	a breathing vibration of about two and a quarter Planck masses turns into two quanta, each about one and an eighth Planck masses, about 1.4 times ten to the nineteen GeV	Computed, with the vibration identified as the one mostly of size	*The Compactification*, Chapter 7, Section IV, equation (7.1)	16

Quantity	As printed	Standing	Source	Chapters
The stretching since the start	every length has grown by about 1.3 times ten to the thirteen for each GeV of the starting temperature (Chapters 16 and 17); by the starting temperature divided by about eight hundredths of a trillionth of a GeV (Appendix A)	Computed from the counts of species and the measured temperature of the light	*The Compactification*, Chapter 7, Section IV, and Chapter 8, Section IX	16, 17; Appendix A
The pitch of the settling hum today	about 2.6 times ten to the twenty-nine hertz divided by the starting temperature in GeV; above about five times ten to the eleven hertz (5.2×10^{11}), five hundred twenty billion, half a terahertz; about 1.3 times ten to the twelve hertz at two times ten to the seventeen GeV; about 2.6 times ten to the thirteen at ten to the sixteen GeV	Computed under three stated premises; bounded from below by the bounded temperature	*The Compactification*, Chapter 7, Section IV, equation (7.3), and Chapter 8, Section II	16, 17, 21
The floor of frequency (the size of the horizon)	about 1.3 times ten to the ten hertz, thirteen billion, at the upper bound; the settling waves about forty times higher	Computed	*The Compactification*, Chapter 7, Section IV	16

Quantity	As printed	Standing	Source	Chapters
The strength of the settling hum today	about sixty-five hundredths of the starting share times the energy of the light, about two percent of the light's energy at the ceiling; the light's share about 2.47 times ten to the minus five; the hum's share below about five ten-millionths	Computed from the three percent bound; the light's share computed from the measured temperature	*The Compactification*, Chapter 7, Section V	16, 17
The share against the excess of light species	about five point six millionths of the excess above 3.044; an excess of nine hundredths matches about five parts in ten million	Computed	*The Compactification*, Chapter 7, Section V, and Chapter 8, Section X	17
The strain of the settling hum	about 1.7 times ten to the minus thirty-three at five hundred twenty billion hertz; about seven times ten to the minus thirty-four at 1.3 times ten to the twelve hertz; below about two times ten to the minus thirty-three at the peak	Computed	*The Compactification*, Chapter 7, Section V	16, 17

Quantity	As printed	Standing	Source	Chapters
The waves induced by the first ripples	about eighty-two hundredths of the square of the ripples' power while radiation fills the universe; a share today of about six times ten to the minus twenty-three to one and a half times ten to the minus twenty-two, about ten to the minus twenty-two; about nine powers of ten below the most sensitive planned interferometers; some fourteen powers of ten too weak for the pulsar hum	Established physics; computed from the measured power	*The Compactification*, Chapter 7, Section VII, and Chapter 8, Sections XI and XII	16, 17
A sudden change at the weak scale	near a hundred GeV it would peak today near a millihertz, about seventeen minutes	Established physics	*The Compactification*, Chapter 7, Section VIII	16
The settling hum against the pulsar hum	about twenty powers of ten higher in frequency	Computed	*The Compactification*, Chapter 8, Section XII	17
The pitches of everyday objects	at or below about a billion vibrations a second; the settling hum five hundred times higher	Established physics	*The Compactification*, Chapter 8, Section IX	17

Quantity	As printed	Standing	Source	Chapters
Quartz resonators	narrow bands between five and sixteen million vibrations a second; twenty thousandths of a degree above absolute zero; an array proposed for a tenth of a megahertz to twenty megahertz; ringing quality about a billion, approaching ten billion; top note about five hundred times below the settling band, a gap of two and seven tenths powers of ten; working band four to five powers of ten below; reach ten to the thirteen to ten to the seventeen, nineteen to twenty-three powers of ten above the ceiling	Measured or projected, as the published record reports them; the gaps computed	*The Compactification*, Chapter 8, Sections III and VIII	17 (in part); Appendix B (in full)
Superconducting cavities	tested at four and at one and eight tenths degrees above absolute zero; about ten kilohertz to about a hundred megahertz; top of the band about five thousand times below, a gap of three and seven tenths powers of ten; no projection near one part in ten million	Measured or projected; the gap computed	*The Compactification*, Chapter 8, Sections IV and VIII	17 (in part); Appendix B (in full)

Quantity	As printed	Standing	Source	Chapters
Magnon detectors	about one to twenty gigahertz; data read again at fourteen and at eight and two tenths gigahertz; the Korean bound a strain below about three and nine tenths parts in ten to the twenty-one over two megahertz centered at 5.311 gigahertz; top about twenty-five times below, a gap of one and four tenths; reach ten to the twenty-four to ten to the twenty-five, thirty to thirty-two powers above the ceiling; the Korean bound about twelve powers of ten above the allowed strain	Measured; the gaps computed	*The Compactification*, Chapter 8, Sections V and VIII	17 (in part); Appendix B (in full)

Quantity	As printed	Standing	Source	Chapters
Conversion into light in space	cluster fields about a millionth of a gauss over about ten kiloparsecs (some thirty thousand light years), against half a gauss at the Earth's surface; fields between galaxies about a tenth of a billionth of a gauss; bounds below about ten to the sixteen near a hundred gigahertz and ten to the eleven to ten to the thirteen between a hundred megahertz and ten gigahertz; the Square Kilometre Array about ten powers of ten tighter, between about ten and about a million; closest in frequency, about five times below, a gap of seven tenths; twenty-two powers above the ceiling, seventeen to twenty-two across the band; the array's projections seven to twelve above, two to four too low in frequency	Measured or projected; the gaps computed	*The Compactification*, Chapter 8, Sections VI and VIII	17 (in part); Appendix B (in full)
Conversion into light in the laboratory	strains of about one part in ten to the thirty in principle; about three powers of ten above the allowed strain; the band at least two and seven tenths powers of ten below	Estimated (a study of 2021); the gaps computed	*The Compactification*, Chapter 8, Section VI	Appendix B

Quantity	As printed	Standing	Source	Chapters
Levitated sensors	about fifty thousand to three hundred thousand vibrations a second; an instrument ten to a hundred meters long; reach ten to the six to ten to the twelve; about a million times too low, a gap of six and two tenths; twelve to nineteen powers above the ceiling	Projected; the gaps computed	*The Compactification*, Chapter 8, Section VII	17 (in part); Appendix B (in full)
The Li–Baker concept	about three to ten gigahertz; a gap of one and seven tenths powers of ten; strains of one part in ten to the thirty-two to ten to the thirty-seven, as its proponents claim	Proposed; the gap computed	*The Compactification*, Chapter 8, Section VII	17 (in part); Appendix B (in full)
The Holometer	two interferometers each thirty-nine meters long; a hundred and thirty hours of data between one and thirteen megahertz; a share below five and six tenths times ten to the twelve at one megahertz, rising to eight and four tenths times ten to the fifteen at thirteen; nineteen to twenty-two powers above the ceiling; a gap of four and six tenths	Measured; the gaps computed	*The Compactification*, Chapter 8, Section VII	17 (in part); Appendix B (in full)

Quantity	As printed	Standing	Source	Chapters
Phonon detectors in crystals	about a trillion to a hundred trillion vibrations a second, phonons of about four thousandths to one tenth of an electron volt; a peak at a trillion hertz goes with a starting temperature of about 2.6 times ten to the seventeen GeV, at a hundred trillion with about 2.6 times ten to the fifteen; strains at the ceiling of about nine parts in ten to the thirty-four and nine in ten to the thirty-six	Proposed; the matching temperatures and strains computed	*The Compactification*, Chapter 8, Section VII	17 (in part); Appendix B (in full)
The nine classes together	eight of nine below the settling band, from seven tenths to six and two tenths powers of ten; every class with a stated reach at least twelve powers of ten above the ceiling, and as many as thirty-two	Computed	*The Compactification*, Chapter 8, Section VIII	17 (in part); Appendix B (in full)
The pace of past improvement	the first resonant bars about eight powers of ten less sensitive in strain than Advanced LIGO; closing that gap took about fifty years; eight powers in strain are sixteen in energy share, which might bring gigahertz reach to about ten to the eight, still about fourteen above the ceiling	Measured (the history); the projection computed, a generous guess	*The Compactification*, Chapter 8, Sections IX and XIV	17 (in part); Appendix B (in full)

Quantity	As printed	Standing	Source	Chapters
What hearing the settling hum directly would take	half a terahertz or above, a strain near one part in ten to the thirty-three, or a share below one part in a million at that frequency; single quanta of a few thousandths of an electron volt	Computed	*The Compactification*, Chapter 8, Sections IX and XIV	17 (in part); Appendix B (in full)
The coming measurement of light species	the Simons Observatory to within about five to seven hundredths; a confirmed excess well above a tenth of a species would measure the settling waves' share	Forecast (Ade and others 2019)	*The Compactification*, Chapter 8, Section X	17, 21
The design of Bose and colleagues	each mass a hundred-billionth of a gram, about ten to the minus fourteen kilograms; two places a quarter of a millimeter apart; centers forty-five hundredths of a millimeter apart, closest approach two tenths; the far-and-far distance seven tenths; held two and a half seconds; a full turn is a little over six and a quarter radians	The stated inputs of a proposed experiment (Bose and others 2017)	*The Gravitational Passage*, Chapter 12, Section VIII; *The Compactification*, Chapter 8, Section XIII	17 (in part); Appendix B (in full)

Part V: What the Turn Made

Quantity	As printed	Standing	Source	Chapters
The weight of a galaxy's magnetic field	about two parts in a billion of the dark matter's weight nearby	Computed	*The Weight of Possibility*, Chapter 1, Section I	18
How long any decay of the dark matter must take	longer than about a hundred and sixty billion years	Measured (Poulin, Serpico and Lesgourgues 2016)	*The Weight of Possibility*, Chapter 1, Section I	18
The decay Penrose's cyclic account requires	within each epoch, over about a hundred billion years	The requirement of another account (Penrose 2018)	*The Weight of Possibility*, Chapter 4, Questions	18
How hard the dark matter bounces off itself	a measured limit of about one square centimeter per gram, from colliding clusters	Measured (Markevitch and others 2004)	*The Weight of Possibility*, Chapter 1, Section I	18

Quantity	As printed	Standing	Source	Chapters
Why ordinary matter alone could not make the galaxies	matter departed about three times as much as the temperature, about three parts in a hundred thousand; distances have grown about one thousand and ninety times; so about three parts in a hundred; a galaxy forms where the excess reaches about one, about twice the average density	Established physics; computed from measured values	*The Weight of Possibility*, Chapter 1, Section I	18
Two gyrations meeting near the Sun	about once in ten to the seventy-eight seconds for a given gyration	Computed	*The Weight of Possibility*, Chapter 4, Section III	18
The next scale at which evaporation could act	some thirty powers of ten away	Computed	*The Weight of Possibility*, Chapter 4, Section IV	18
A gyration against a proton	about ten billion billion times the mass of a proton	Computed	*The Weight of Possibility*, Chapter 4, Section VI	18, 19

Quantity	As printed	Standing	Source	Chapters
The lightest odd levels	four families share the lowest mass at the simplest calculation; the spin-two family has three members, each of the other three eight times three; spin values two, one and zero	Computed	*The Weight of Possibility*, Chapter 4, Section V	18 (in part); Appendix A (in full)
One further state of the sphere's simplest wave	lifted by the floor to about three and two tenths Planck masses; it stays above the lightest odd level if the floor resists an uneven change of size with more than about a fifth to a third of its stiffness against an even one	Computed (an estimate from the floor's response to an even change of size)	*The Weight of Possibility*, Chapter 4, Section V	Appendix A
A gyration's gravitational size	about one to three Planck lengths	Computed	*The Weight of Possibility*, Chapter 4, Questions	18

Quantity	As printed	Standing	Source	Chapters
The forces of the six	a strength of about six for the sphere and about five and a third for the larger part, on the scale on which electromagnetism has one part in a hundred and thirty-seven; about three hundred times the weak and strong forces at the Planck scale	Computed	*The Weight of Possibility*, Chapter 4, Section V	18 (in part); Appendix A (in full)
Gyrations bouncing off one another	at a few hundred to a few thousand kilometers a second, between about ten to the minus forty-five and ten to the minus fifty square centimeters for each gram; at least forty-five powers of ten below the measured limit	Computed	*The Weight of Possibility*, Chapter 4, Section III	18

Quantity	As printed	Standing	Source	Chapters
How many gyrations there are	about one for every one and a half to three and four tenths billion billion protons and neutrons; about two to four for every ten billion billion billion particles of the oldest light; near the Sun about one in every few tens of thousands of cubic kilometers	Computed from the ratio and measured amounts	*The Weight of Possibility*, Chapter 4, Section VI, and Chapter 3, Section I	18, 19
The departure the gyrations leave by their number	about five to eight billion-billion-billionths across a region twenty megaparsecs wide, about sixty-five million light years	Computed	*The Weight of Possibility*, Chapter 4, Section VI	18
The binding of helium	lowered the mass of the ordinary matter by about eighteen hundredths of one percent	Established physics	*The Weight of Possibility*, Chapter 3, Section I	19

Quantity	As printed	Standing	Source	Chapters
The share of a curvature	for the whole six, count pi, share three divided by pi; for the plane, count one and a half times pi, share two divided by pi, about sixty-four hundredths; for the sphere, count three times pi, share one divided by pi, about thirty-two hundredths	Set by the structure once a stated step is taken: the counting principle	*The Weight of Possibility*, Chapter 3, Section III	19
Exempting the sphere in the Passage's rule too	would put the matter share at three tenths, in place of thirty-one percent	Computed	*The Weight of Possibility*, Chapter 3, Section IV	Appendix A
The neighbors of the rule	six; six less one divided by pi, about five and sixty-eight hundredths; six less three divided by pi, about five and five hundredths; six lies more than six tenths above the measured value, the others about three tenths above and below	Computed; excluded by the measurement	*The Weight of Possibility*, Chapter 3, Section VII, and Chapter 5, Section VIII	19

Quantity	As printed	Standing	Source	Chapters
The budget of the universe	neutrinos about fourteen ten-thousandths of the whole; ordinary matter one part in seven less two divided by pi of the cold matter, about sixteen percent of it; about four and nine tenths percent ordinary and about twenty-six percent dark; about seven tenths the cosmological constant, a quarter dark matter, a twentieth ordinary matter	Computed from the matter share and the ratio	*The Weight of Possibility*, Chapter 3, Section V	19
The shape of the region within our horizon	every loop around the universe through our place is longer than ninety-eight and a half percent of the width of the visible sky of the oldest light; a ball out to ninety-six and a half percent of the horizon's radius	Measured; the radius computed	*The Weight of Possibility*, Chapter 3, Section II	19

Quantity	As printed	Standing	Source	Chapters
Accounts that tie dark to ordinary matter at the turn	a tie across nineteen powers of ten; a dark particle of about five protons	Established physics (Kaplan, Luty and Zurek 2009; Zurek 2014)	*The Weight of Possibility*, Chapter 3, Section I, and Questions	19
An even annihilation, with no excess	about one baryon for every billion billion particles of light, with as many antibaryons; the measured amount hundreds of millions of times more; about one baryon in every four cubic meters	Established physics (Kolb and Turner 1990); the four cubic meters computed from 6.13 baryons for every ten billion photons and 411 photons in a cubic centimeter	*The Coming of the Kinds*, Chapter 5, Section VII, and Chapter 1, Section VI	20
The sphaleron passage	each makes or removes nine quarks and three leptons together, changing the baryons and the leptons by three each; today suppressed by about one in ten to the one hundred sixty-one	Established physics	*The Coming of the Kinds*, Chapter 4, Section II	20

Quantity	As printed	Standing	Source	Chapters
The angle in the masses of the quarks	measured at about three parts in a hundred thousand; computed near 160 GeV at about 4.4 parts in ten thousand billion billion, more than a trillion times smaller than the excess	Measured (Particle Data Group 2026); computed	*The Coming of the Kinds*, Chapter 4, Section III	20
A Majorana mass	would change the lepton count by two	Established physics	*The Coming of the Kinds*, Chapter 5	20
The share of the residue held as baryons	twenty-eight seventy-ninths (about 0.354) while the passages run; twelve thirty-sevenths (about 0.324) deep in the settled Higgs state; 0.3328, give or take 0.0005, where the passages froze, about one third; from three families and one Higgs field	Set by the structure of the kinds; the last computed through the crossover (Benso and six colleagues 2026)	*The Coming of the Kinds*, Chapter 7, Sections III and V	20, 21
The size of the residue	about 1.84 units of B minus L for every billion photons, about three times the baryons; the antileptons over the leptons about twice the baryons	Computed from the measured baryons per photon and the share	*The Coming of the Kinds*, Chapter 7, Section VI	20

Quantity	As printed	Standing	Source	Chapters
The ages of the cooling	159.5 GeV about nine millionths of a millionth of a second; 131.7 GeV about thirteen; 156.5 MeV between about twelve and twenty-four millionths of a second; 30 MeV between about six and eight ten-thousandths; 3 to 1 MeV in the first second; 0.8 MeV about one second; annihilation of electrons and positrons between about three seconds and six minutes; 0.1 MeV about two minutes; light nuclei fixed by about three minutes	Established physics	*The Coming of the Kinds*, The Story in Brief, "The Map of the Cooling"	20
Quarks over antiquarks before the binding	about four in a billion	Computed from the measured baryons per photon	*The Coming of the Kinds*, Chapter 8, Section III	20
The antibaryons by 30 MeV	a few in a million of the baryons	Computed	*The Coming of the Kinds*, Chapter 8, Section V	20

Quantity	As printed	Standing	Source	Chapters
The neutrinos in a cubic centimeter today	about 336 neutrinos and antineutrinos	Computed from the measured temperature of the light	*The Coming of the Kinds*, Chapter 10, Section II	20
Baryons per photon from the heavy hydrogen	about 6.04 per ten billion, give or take 0.12	Measured (Particle Data Group 2026)	*The Coming of the Kinds*, Chapter 12, Section IV	20
The kinds of light neutrino, from the decays of the Z boson	2.9963, give or take 0.0074	Measured (Janot and Jadach 2020)	*The Coming of the Kinds*, Chapter 9, Section IV	Appendix B

Summary

A short list of the book's numbers is set by the structure. Each number set by the structure follows from the shape of the world that this book derives. The list holds the counts of the dimensions, the exponents of the arc, the count of three channels, the trim, and the two limiting shares of the residue held as baryons. The counts of the dimensions are one of time, three of space and six of possibility. The arc is the short stretch of the Kasner curve on which all six small directions have negative exponents: as a new epoch opens along the arc, the three large directions grow and the six small directions shrink. A channel is one of the ways in which a measurement takes in what the measurement finds. There are three channels, one for each of the three roles that every comparison has. The trim is the share

of the Planck density that remains after the curvature of the six small dimensions draws part of the density away. The share held as baryons is the fraction of the residue of matter that ends as protons and neutrons. The share held as baryons has two limiting values: one while the sphaleron passages run, and one deep in the settled Higgs state.

Once a stated step is taken, the structure also sets two more results: the shape and size of the six small dimensions, and the share of the vacuum's energy that fixes the cosmological constant. A stated step is a step from the structure to the world. I name each stated step at the place where the step is made. For the shape and size of the six, the step is the identification that every direction of the six reaches the Planck curvature. For the share of the vacuum's energy, the step is my identification of the share, which I make in three steps. The ratio of dark to ordinary matter is a registered ratio of the present measurement, which means a ratio fixed by what the present measurement registers, and not by a mechanism at the turn. The registered ratio is the one number in this book with a standing of its own.

The other numbers are measured, computed, or taken from established physics. Established physics means results of standard physics, each taken from its source. Among the other numbers, a computed number is worked out either from measured values or from the account's own mechanism at first order, which means for the largest and simplest part of an effect, with the smaller corrections left aside. Two numbers are measured with their reasons still questions. The two are the strength and the tilt of the first ripples, the departures from uniformity with which our epoch began. Both values come from measurement. I name what sets each of the two values: a count of states, and a length lying between the turn's length and the de Sitter radius. Two numbers are bounded, with their values not given. Measurement sets an upper bound on each of the two: the temperature at which our history begins, and the energy that the settling waves carry. The settling waves are the gravitational waves made at the turn as the six small dimensions settle into their held size.

Notes

These notes give, chapter by chapter, the place in the source books where each claim of this book is made, in the order in which each chapter makes its claims. For a claim of the account, a note names the book of the collection that makes the claim, with the chapter and the section of that book. For a result of established physics, or for a measured value, a note names the original work by author and year, as the books of the collection cite the original work, and the full entry for the work is in the Works Cited. Where one book of the collection quotes or cites another book of the collection, the note names both books. Each note states a claim of the chapter, or names a topic or a picture of the chapter, and after the colon gives the places where the claim is made or the topic is treated.

Before We Begin

The world has ten dimensions, one of time, three of space and six of possibility, and nothing ever travels through the six dimensions of possibility: *At the Edge of Reality*, Chapters 5, 6 and 7; *The Compactification*, Preface, Section II. A dimension is an independent way in which things can differ: *At the Edge of Reality*, Chapter 5. The six are held a few Planck lengths across, and the Planck length is about a hundred billion billion times smaller than a proton: *At the Edge of Reality*, Chapter 8. The end of the interiors of the black holes, the emptying of the outside world, the failure of the conditions that hold the six, and the restoration of a large outside world: *At the Edge of Reality*, Chapters 9 to 16; *The Gravitational Passage*, Chapter 18, Sections I to VII. I identify the restored outside world with the Big Bang of the next epoch, and no comparison registers the turn as an event: *The Gravitational Passage*, Chapter 18, opening, and Chapter 19, Sec-

tion III; *At the Edge of Reality*, Chapters 16 and 17. The five kinds of standing that a number can have, and the kinds of step (theorem, established physics, identification, argued, computed): *The Compactification*, Preface, Section III. The count of three in the cosmological constant: *The Gravitational Passage*, Chapter 13, Section II. Newton's constant is the one measured constant on the side of context: *The Compactification*, Preface, Section III. The strength of the first ripples is taken from measurement, with the reason for its value still a question: *The Compactification*, Chapter 6, Section VII. The temperature at which our history begins is bounded from above: *The Compactification*, Chapter 1, Section VIII. Every measurement is a comparison: *At the Edge of Reality*, Chapter 4. The Incommensuration Theorem, in the words this book uses: *The Gravitational Passage*, Chapter 3; *At the Edge of Reality*, Chapter 4. The source of the Incommensuration Theorem is *An Immanent Metaphysics*, Technical Appendix, aphorisms 1R1 to 1R20, and the proof of the Incommensuration Theorem is set out in *Incommensuration; The Theorem and the Three Axioms*, Chapter 6. The beginning was even, but never perfectly even: *The Compactification*, Chapter 3, Section VI.

Part I

Chapter 1. The picture of the film of the world run backward, and the three questions that a reader of *At the Edge of Reality* brings to this book: *The Compactification*, Chapter 1, Section I. What a comparison is, what content and context are, and the picture of the thermometer: *At the Edge of Reality*, Chapter 4; *The Gravitational Passage*, Preface. The Incommensuration Theorem, and the two characters of the world (smooth and uneven, stepwise and symmetric): *The Gravitational Passage*, Preface; *At the Edge of Reality*, Chapter 4. The ten dimensions are of three kinds and are never traveled through, the ten dimensions differ from the dimensions of superstring theory, and the count of the dimensions gives no shape and no size: *The Compactification*, Preface, Section II, and Chapter 1, Section II, quoting *Measurement; The Steps from Comparison to Physics*, Chapter 11, Section XII;

At the Edge of Reality, Chapters 5, 6 and 7. The internal space is made of the states of a three-way choice and the states of a two-way choice: *At the Edge of Reality*, Chapter 7. The Planck scale, the Planck mass and the Planck length, the modal boundary, Newton's constant as the one measured constant, and an internal space a few Planck lengths across: *The Gravitational Passage*, Preface and Chapter 17, Sections I to V; *At the Edge of Reality*, Chapter 8. What an epoch is: *The Compactification*, Preface, Section I. The two instances of the six (the content instance and the context instance), the context instance as the instance this book uses, and the facts about the particles that I take from the books of the collection about the particles: *The Compactification*, Preface, Section II, and Chapter 1, Section II. What compactification means in this book: *The Compactification*, Preface, Section I, and Chapter 1, Section II; *The Gravitational Passage*, Chapter 18, Section VI. The compactification conditions, the hold of the six, the floor, the action of the metric alone, and the absence of any field threaded through the six: *The Compactification*, Chapter 1, Section IV, quoting *The Gravitational Passage*, Chapter 11, Sections I and II, Chapter 12, Questions, and Chapter 18, Section VI. The pictures of the room and the gyroscope, and inertia as a comparison: *At the Edge of Reality*, Chapter 8. The eight steps by which an epoch ends: *The Compactification*, Chapter 1, Section V; *The Gravitational Passage*, Chapter 18, Sections I to VII. For a black hole of the Sun's mass, the time from the horizon to the end, at most about fifteen millionths of a second, and the time to evaporate, about ten to the sixty-seven years: *The Gravitational Passage*, Chapter 18, Section I. The radiation of black holes: Hawking (1975). The emptying of the outside world, over a hundred billion to a trillion years: *The Gravitational Passage*, Chapter 18, Sections I and III. The question "which black hole am I inside?": *The Gravitational Passage*, Chapter 18, Section II. Each act of comparison has a one-way order inside it, and that order is not reversed at the end of an interior: *The Gravitational Passage*, Chapter 18, Section IV. The form of ordinary matter depends on the six being held: *The Gravitational Passage*, Chapter 18, Section V. The standing of each of the eight steps by which an epoch ends: *The Gravitational Passage*, Chapter 18, opening; *The Compactification*, Chapter 1, Section

X. The pictures of the friend who watches the fall from far away, of the funnels, and of the edge of a map: *At the Edge of Reality*, Chapters 1, 9, 11 and 15. I identify the restoration of context with the Big Bang of the new epoch: *The Gravitational Passage*, Chapter 18, opening, and Chapter 19, Section III; *At the Edge of Reality*, Chapter 17. At the turn no field is defined, the form of the content passes the turn, the six re-form with the same internal shape as before the turn, and the new space and time have no prior identity: *The Compactification*, Chapter 1, Section V; *The Gravitational Passage*, Chapter 18, Sections VII and VIII, and Chapter 19, Sections III and IV; *At the Edge of Reality*, Chapters 16, 17 and 18. The three consequences of the turn (there is no initial state, the turn is a passage, and the cycle of epochs has no first time round), and each turn kept distinct from the next turn: *The Compactification*, Chapter 1, Section V; *The Gravitational Passage*, Chapter 18, Sections II, VII and XII; *At the Edge of Reality*, Chapters 16 and 20. Our history is reconstructed from what arrives now, and each count "at" a past era is a number within the reconstruction: *The Compactification*, Chapter 5, Section I; *At the Edge of Reality*, Chapter 17; *The Gravitational Passage*, Chapter 19, Section III. The plan of this book, the starting temperature of our history bounded from above, and the finding that no inflationary phase is needed and a long inflationary phase is excluded: *The Compactification*, Preface, Section I, and Chapter 1, Abstract and Section VIII.

Chapter 2. Three directions grow as the time to the power five ninths while each of the six shrinks as the time to the power minus one ninth, with the time counted away from the turn: *The Compactification*, Chapter 1, Section VI ("Volumes, and the two directions of time"); *The Gravitational Passage*, Appendix B, Section IV. The six are held at a tiny size: *The Gravitational Passage*, Chapter 11. The Kasner form is a solution of Einstein's equations for empty space: Kasner (1921); *The Compactification*, Chapter 1, Section VI ("The Kasner form"); *The Gravitational Passage*, Appendix B, Section I. The worked example of what an exponent is: *At the Edge of Reality*, Chapter 14. The time remaining before the turn and the time elapsed since the turn, and duration, which applies to the approach to the turn only: *The*

Compactification, Chapter 1, Sections V and VI. The two rules on the exponents (the exponents add to one, and their squares add to one), and the three facts that follow from the two rules (sign, volume, direction of time): *The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Section I. The nine directions fall into three groups, and the grouping comes from the shape of the six: *The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Questions. Einstein's equations alone single out no group of three directions: Freund (1982), as *The Gravitational Passage*, Appendix B, Questions, cites it. Earlier proposals of worlds in which three directions expand while the others contract: Chodos and Detweiler (1980); Freund (1982); as *The Gravitational Passage*, Appendix B, Section IV, and Chapter 18, Section VII, cite them. One Kasner form is consistent within the product geometry, with each group of directions moving together: *The Gravitational Passage*, Appendix B, Section VII, quoted in *The Compactification*, Chapter 1, Section VI. The closed curve of the allowed exponents, its span, its two solutions for each value, and its ends: *The Compactification*, Chapter 1, Section VI ("The curve"); *The Gravitational Passage*, Appendix B, Section III; *At the Edge of Reality*, Appendix A. The checks of the two ends of the curve against the two rules: *The Compactification*, Chapter 1, Section VI ("The two ends, checked"); *The Gravitational Passage*, Appendix B, Section III; *At the Edge of Reality*, Appendix A. The arc, its ends and the exact values at its ends: *The Compactification*, Chapter 1, Section VI ("The arc"); *The Gravitational Passage*, Appendix B, Section IV; the decimal values are arithmetic on the exact forms. The volumes, and the example of a billionfold change of the time: *The Compactification*, Chapter 1, Section VI; *At the Edge of Reality*, Chapter 14 and Appendix A; the volume figures of the example are arithmetic on the exponents. The end of an interior and the opening of an epoch lie on one arc, in the two directions of time: *The Compactification*, Chapter 1, Section VI; *The Gravitational Passage*, Appendix B, Section IV, and Chapter 19, Section II. The picture of the room: *At the Edge of Reality*, Chapter 14. The slow growth of the six, which is completed at the modal boundary: *The Gravitational Passage*, Appendix B, Section IV. The lower end of the arc: *The Gravitational Passage*,

Appendix B, Section IV; *The Compactification*, Chapter 1, Section VI ("The placement"). *The Gravitational Passage* places both readings (the end of an interior and the opening of an epoch) at the upper end of the curve, collapse and origin are one transition, and the scope of the shared exponents is an interior that has come to the arc: *The Gravitational Passage*, Appendix B, Section IV and Questions; *The Compactification*, Chapter 1, Section VI ("The placement") and Section IX. The three accounts of how the six come to be released at the end of an interior (the whole picture, the account by depth, and the Kasner geometry): *The Gravitational Passage*, Appendix B, Section VII. The account by depth: *The Gravitational Passage*, Chapter 19, Section II, and Appendix B, Section V. The picture of halving the radius: *At the Edge of Reality*, Chapter 10. Many black holes end in one origin: *The Gravitational Passage*, Appendix B, Section VI. The anisotropy is eight ninths at every point of the Kasner form, where the anisotropy is the spread of the rates of the directions: *The Compactification*, Chapter 1, Section VI ("The anisotropy is generic"); the arithmetic of the spread over the three groups of directions, and the checks of that arithmetic at both ends of the curve, are computed on the value of eight ninths. The picture of the drum, and the contraction of the six, which ends where the six are held: *At the Edge of Reality*, Chapter 8; *The Compactification*, Chapter 1, Section VII.

Chapter 3. The pictures of the lamp, the globe, the ways light vibrates, the tin can, the cube, the pyramid and the drawstring bag, and the counting of pieces: *The View from Inside*, Chapter 9 ("Two Shapes Side by Side", "Counting Pieces", "Why Four and Two", "The Shape That Fits"). The picture of the globe with one alternative at each pole: *The Weight of the Unseen*, question 2.3. The two instances of the six, the content instance and the context instance: *The Compactification*, Chapter 1, Section II. The grid of the six measures, the six separate ways in which the content can vary, the three conditions that translate the grid into geometry, and the standing of the three conditions: *The Gravitational Passage*, Chapter 9, Section I; *The Compactification*, Chapter 1, Section III ("How the space is reached"). The six measures in plain words: *At the Edge of Reality*, Chapter 4. Every point

of each shape is alike, and the reason for that sameness: *The Gravitational Passage*, Chapter 9, Section II; *The Compactification*, Chapter 2, Section III. The product of the complex projective plane and the sphere, realized: *The Gravitational Passage*, Chapter 9, Section III. The count of the pieces of the complex projective plane and of the sphere: *The Gravitational Passage*, Chapter 9, Sections IV and VII; Hopf and Samelson (1941), as *The Gravitational Passage* cites them. Smooth bending and separate steps, and the theorem of Ambrose and Singer, which makes the link between smooth bending and separate steps exact: Ambrose and Singer (1953); *The Gravitational Passage*, Chapter 9, Section V, with one geometry for each alternative of the content. The mathematicians' classification of shapes, which leaves one shape: *The Gravitational Passage*, Chapter 9, Section VI, citing Ishihara (1955); *The Compactification*, Chapter 1, Section III, citing Hopf and Samelson (1941) and Ishihara (1955). The uniqueness of the shape, the counts of six and six, and three times two as four plus two: *The Gravitational Passage*, Chapter 9, Section VII and Questions. The sentence that states the standing of the shape, and the finding that no condition refers to particles: *The Gravitational Passage*, Chapter 9, opening and Questions; *The Compactification*, Chapter 1, Section III. A second route to the internal space, through the algebra of comparison: *The Compactification*, Chapter 1, Section III, citing *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 12, and *The Quantum Passage*, Chapter 5, Section VIII. The measures of bending, and the Ricci curvature as the one measure of bending that can be the same in every direction: *The Gravitational Passage*, Chapter 9, Sections VIII and IX. The constants of the two pieces of the internal space, the complex projective plane and the sphere, which are equal exactly when the product is an Einstein space: *The Gravitational Passage*, Chapter 9, Section VIII; *The Compactification*, Chapter 2, Section IV. The internal space is not a Calabi-Yau space: *The Gravitational Passage*, Chapter 9, Section VIII. The identification that sets the size of the internal space, and the independence of that identification: *The Gravitational Passage*, Chapter 9, Section IX; *The Compactification*, Chapter 1, Section III, and Chapter 2, Section IV. The radii, the convention for the radius of the complex project-

ive plane, the area ratio of three halves, the factor of one, and the ratio of the radii, on which no observation bears: *The Gravitational Passage*, Chapter 9, Section IX; *The Compactification*, Chapter 1, Section III, and Chapter 2, Section IV. The weighting of three and two: *The Gravitational Passage*, Chapter 9, Section IX. The Planck length, built from Newton's constant, and the full Planck mass: *The Gravitational Passage*, Chapter 9, Section IX, and Chapter 17, Sections I, III and V; *The Compactification*, Chapter 1, Section III. The Planck length and the Planck mass in everyday sizes: *At the Edge of Reality*, Chapter 8. The internal space has the same size and shape everywhere: *The Compactification*, Chapter 2, Section IV, quoting *The Gravitational Passage*, Chapter 11, Section II. The Planck curvature on the six alone: *The Gravitational Passage*, Chapter 9, Questions. Where the internal space begins, a few Planck lengths across, and the modal boundary, which separates the two parts of the ten-dimensional product, the four large dimensions and the six: *The Gravitational Passage*, Chapter 17, Section V; *The Compactification*, Chapter 1, Section III. The volume of the internal space, its unit, the seventy-two discs, pi cubed, the modal reading of the polydisc, the cells of action, the ball and the Borromean rings: *The Gravitational Passage*, Chapter 9, Section IX; *The Compactification*, Chapter 2, Section IV. The integer three and the count of channels: *The Gravitational Passage*, Chapter 9, Section VII, and Chapter 13, Section II; *The Compactification*, Chapter 1, Section III ("The integer three, and the spinc structure"); *The View from Inside*, Chapter 9 ("The Number Three"). What an observation can test of the shape and the size of the six: *The Gravitational Passage*, Chapter 9, Questions. The table of what is taken and what follows: *The Compactification*, Chapter 1, Section X; *The Gravitational Passage*, Chapter 9.

Chapter 4. The short answer to what holds the six, the two questions (where the six sit, and how stiffly the six are held), and the answer to the second question, given in two separate steps: *The Compactification*, Chapter 1, Sections IV and VII, and Chapter 2, Section V. The floor of the six (the third axiom, the least extent and the least motion, and the lowest state, which cannot radiate): *The Gravitational Passage*, Chapter 11, Section II; *The Compactification*, Chapter 1, Section IV, and Chapter 2, Section V. The third

axiom in plain words: *At the Edge of Reality*, Chapter 4. The picture of the restless trembling of the fields at their lowest: *The View from Inside*, Chapter 1 ("What Is 'Empty' Space?"). The compactification conditions: *The Gravitational Passage*, Chapter 18, Section VI, as *The Compactification*, Chapter 1, Section IV, quotes it. The picture of the room: *At the Edge of Reality*, Chapter 8. The six are held with the same size and shape everywhere, and the moduli are fixed: *The Gravitational Passage*, Chapter 11, Section II; *The Compactification*, Chapter 1, Section IV. The moduli are the volume and the shape of the six, and the volume sets the four-dimensional Planck mass: *The Gravitational Passage*, Chapter 11, Sections II and III. The one mechanism for how stiffly the six are held, computed at first order, in a form independent of the bookkeeping choices, with its term for the least extent: *The Gravitational Passage*, Chapter 11, Section II, and Appendix D, Sections I to III; *The Compactification*, Chapter 2, Section V. The independence of the form from the bookkeeping choices: Vilkovisky (1984), as *The Gravitational Passage* and *The Compactification* both cite it. An internal space held by the zero-point energy of its fields: Candelas and Weinberg (1984). The two constants of the term for the least extent, the stable minimum, the only stable minimum in shape, the barrier, the range of stability, the tests, and the masses of the two settings of the six (the volume and the shape): *The Gravitational Passage*, Appendix D, Section III; *The Compactification*, Chapter 2, Section V. The bending of radio waves by the Sun, measured by the Cassini spacecraft: Bertotti, Iess and Tortora (2003). The tests of the inverse-square law of gravity: Kapner and others (2007). The range of the force that the two settings of the six carry, and gravity in the four large dimensions as general relativity: *The Gravitational Passage*, Chapter 11, Section II; *The Compactification*, Chapter 2, Section V. The energy of the floor is counted within the Planck density, and the energy of the floor bears on the moduli and not on the cosmological constant: *The Gravitational Passage*, Chapter 11, Sections II and III, and Appendix D, Section III; *The Compactification*, Chapter 2, Section V. The bare curvature term, its sign, and the energy of the held six at about minus five hundredths of the Planck density: *The Gravitational Passage*, Chapter 11,

Section III, and Appendix D, Section III. The six sit in a well about one twentieth of the Planck density deep, and the well rises to zero with no barrier: *The Weight of Possibility*, Chapter 6, Section I; *The Compactification*, Chapter 1, Section VII. The total energy of the vacuum is one value and not a range, and the observed cosmological constant is the share of that energy that a measurement takes in: *The Compactification*, Chapter 2, Section V; *The Gravitational Passage*, Chapter 11, Section III. The six contract slowly along the arc, and the contraction ends at the floor: *The Compactification*, Chapter 1, Section VII. Where the description of the new epoch begins: *The Weight of Possibility*, Chapter 6, Section I, on *The Gravitational Passage*, Chapter 18, Section VII, and Chapter 19, Section I; *The Compactification*, Chapter 1, Section VII. What the account does not fix (where on the arc the description begins, the course of the return of the six, and the starting temperature): *The Compactification*, Chapter 1, Section VII; *The Gravitational Passage*, Chapter 18, Section XI, and Chapter 19, Section I; *The Weight of Possibility*, Chapter 6, Section I. The picture of the crystal: *The Weight of the Unseen*, question 3.1. The further terms of the ten-dimensional geometry: Lovelock (1971). The form of four-dimensional gravity: *The Gravitational Passage*, Chapter 11, Section I. The breathing vibrations of the six, the term of Gauss and Bonnet and its natural size, the decay of the breathing vibrations within about a Planck time, the energy of the settling leaving as gravitational waves, and the release of a tenth to most of the depth of the well, registered as extra radiation: *The Weight of Possibility*, Chapter 4, Section III; *The Compactification*, Chapter 1, Section VII. The Planck time: *At the Edge of Reality*, Appendix C. All three conditions on the turn: *The Weight of Possibility*, Chapter 6, Section I, and Chapter 4, Sections III and VI; *The Compactification*, Chapter 1, Section VII. The making of particles in pairs by a hot plasma through gravity alone: Garny, Sandora and Sloth (2016); Garny, Palessandro, Sandora and Sloth (2018); as *The Weight of Possibility*, Chapter 4, Section VI, uses them. The measured number of light kinds of particle: Aghanim and others (2020); Calabrese and others (2025). The gyration, a lasting vibration of the six: *The Weight of*

Possibility, Chapter 1, Section IV, as *The Compactification*, Chapter 1, Section VII, quotes it. The gyrations made at the turn remain: *The Compactification*, Chapter 1, Section IX.

Chapter 5. What a vacuum is, and the difference between history and law: *The Compactification*, Chapter 2, Section I. The landscape of string theory, with its fluxes and its ten to the five hundred valleys, each valley with its own cosmological constant: Bousso and Polchinski (2000); Kachru, Kallosh, Linde and Trivedi (2003); Douglas (2003); Douglas and Kachru (2007); as *The Compactification*, Chapter 2, Section I, cites them. Why physicists turn to the landscape: *The View from Inside*, Chapter 3 ("The Shape of the Problem", "Strategy Two: Accept the Landscape"). String theory, as the reader met string theory in *At the Edge of Reality*: *At the Edge of Reality*, Chapter 3. The two instances of the six, and the two senses of the word vacuum: *The Compactification*, Chapter 2, Section I. What a modulus is, and on which of the two instances of the six a modulus can exist: *The Compactification*, Chapter 2, Section II. The content instance has no size: *The Kinds of Particle*, Chapter 1, Section XIII, as *The Compactification*, Chapter 2, Section II, quotes it. The content is described by an algebra: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 10, Section V, as *The Compactification*, Chapter 2, Section II, cites it. The volume and shape moduli, named: *The Gravitational Passage*, Chapter 11, Section II. Each piece of the internal space is homogeneous, alike at every point, by an identification, and the ground of that identification: *The Gravitational Passage*, Chapter 9, Section II; *The Compactification*, Chapter 2, Section III. The theorem that leaves exactly two moduli, applied to both pieces and to the product, and the two consequences of the theorem: *The Compactification*, Chapter 2, Section III, citing Wolf (1968) and Besse (1987). The Einstein condition, the identification of the Planck curvature, the radii, the area ratio of three halves, the volume, the sameness at every point, the factor of order one, and the ratio of the radii, on which no observation bears: *The Compactification*, Chapter 2, Section IV; *The Gravitational Passage*, Chapter 9, Sections VIII and IX, Chapter 11, Section II, and Chapter 17. The floor, the masses of the moduli, the range of the force of the moduli, and the external value

as one value and not a range: *The Compactification*, Chapter 2, Section V; *The Gravitational Passage*, Chapter 11, Sections II and III, Chapter 13, and Appendix D, Section III. The absence of any flux and of any flux integer, the action of the metric alone, the further terms of the geometry, and the three reasons against a further field (the law of gravity would no longer be built from the geometry alone, the energy of the field would change the predicted share of matter, and the strength of the field would be a second constant beside the strength of gravity): *The Compactification*, Chapter 2, Section VI; *The Gravitational Passage*, Chapter 11, Section I, Chapter 12, Questions, and Chapter 13, Section II; *The Kinds of Particle*, Chapter 3, Section IV; *The Branches of the Kinds*, Chapter 2, Section III. A vacuum decays by tunneling, at a rate physicists know how to compute: Coleman and De Luccia (1980). The six have one valley in the range computed, with the bound of 1.48, no barrier toward large size, and the barrier at 1.79: *The Compactification*, Chapter 2, Section VII; *The Weight of Possibility*, Chapter 6, Section I. The hold ends where its conditions fail, the turn is not a decay of a false vacuum, the one held configuration is captured at the same point at each turn (with many valleys, the physics of the epochs would differ), the topology and the radii are the same at each turn, and no value of Newton's constant is assigned to another epoch: *The Compactification*, Chapter 2, Section VII; *The Gravitational Passage*, Chapter 18, Sections VI and VII, and Questions. What carries over from epoch to epoch, and every ratio measured within one epoch: *At the Edge of Reality*, Chapter 20 ("What Carries Over"). The account does not fix whether the measured ratios of matter repeat from epoch to epoch: *The Gravitational Passage*, Chapter 18, Questions; *The Weight of Possibility*, Chapter 3. The pictures of the gyroscope, the drum and the room with an outside: *At the Edge of Reality*, Chapter 8. The Higgs field is a fluctuation of the finite Dirac operator, and the Higgs condensate is a value of the Higgs field and not a change of the six: *The Compactification*, Chapter 2, Section VIII, citing *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section II, and *The Kinds of Particle*, Chapter 3, Section VI. The form of the Higgs potential: *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Sec-

tion III, as *The Compactification*, Chapter 2, Section VIII, cites it. The self-coupling of one eighth by identification, the one half reached by three routes, the mass of 125.17 GeV with its loop term, and the standing of that mass as one measured agreement: *The Measures of the Kinds*, Chapter 13, Sections II, IV and VI, as *The Compactification*, Chapter 2, Section VIII, quotes it. The measured mass of the Higgs boson, 125.13 GeV: Particle Data Group (2026), as *The Coming of the Kinds*, Chapter 4, Section IV, collects it. The self-coupling has the value one eighth at the scale of the weak force, and the account makes no claim on the stability of the Higgs vacuum at far higher energies: *The Compactification*, Chapter 2, Section VIII, where the question of that stability is set beside Buttazzo and others (2013). Two routes lead to one space, no vacuum is selected among many vacua, and the cosmological constant is fixed by the identity: *The Compactification*, Chapter 2, Section IX, citing *Non-Commutative Geometry and the Immanent Metaphysics*, Theorem 12.1, *The Quantum Passage*, Chapter 5, Section VIII, *The Gravitational Passage*, Chapter 9, Sections I and VII, and *The Kinds of Particle*, Chapter 3, Section II. What the turn gives back: *The Compactification*, Chapter 2, Sections VII and XI.

Part II

Chapter 6. The laws of physics are reversible for nearly every process, and the world runs one way (heat, records, the cup): *The Compactification*, Chapter 3, Section I. The statistical account of the one-way running of things: Boltzmann (1877), as *The Compactification*, Chapter 3, Section I, cites it; the standing of the statistical account: Section X. The gap in the counting, and the Past Hypothesis, proposed as a law: Albert (2000), as *The Compactification*, Chapter 3, Section I, cites it. Two arrows of time are two different things that point the same way: *The Compactification*, Chapter 3, Section II, citing *Measurement; The Steps from Comparison to Physics*, Chapter 12, Section VII. Lawful and faithful reversal, symmetry in the strict sense, a state specified without gap as continuity, and the exclusion

of the pair of perfect symmetry and perfect continuity: *The Compactification*, Chapter 3, Section II ("The structural arrow"), from *The Basis of Physics*, Essay 11, "The Structural Arrow of Time". "Perfect time-reversal is therefore precluded": *The Basis of Physics*, Essay 11, Abstract, as *The Compactification*, Chapter 3, Section II, quotes it; the standing of that conclusion: Sections II and X. The Incommensuration Theorem, in the words that no finding of any comparison can be both perfectly symmetric and perfectly continuous: *The Compactification*, Chapter 3, Sections III and VI. Symmetry and continuity in plain words: *At the Edge of Reality*, Chapter 4; *The View from Inside*, Chapter 7. The picture of the photographer, the three roles of a comparison (the transcendent, the immanent and the omniscient), the reading of the second axiom, and the words "no print comes before its click": *The View from Inside*, Chapter 7 ("A Photographer at Work", "Three Roles", "The Third Axiom", "The Second Axiom", "The First Axiom"). The second axiom, quoted: *An Immanent Metaphysics*, aphorism 1.3-1, as *The Compactification*, Chapter 3, Section II, quotes it. The cycle of comparison has an orientation, the orientation is one of precedence and not of clock time, the cycle can be entered at any point, and the cycle is irreversible: *The Compactification*, Chapter 3, Section II, citing *Incommensuration; The Theorem and the Three Axioms*, Chapter 1, Section IV. "In every comparison there is first a stance": *The Compactification*, Chapter 3, Section II. The record shapes the next stance: *At the Edge of Reality*, Chapter 4; *The View from Inside*, Chapter 7. Class, instance and stance in plain words: *At the Edge of Reality*, Chapters 4 and 5; *The View from Inside*, Chapter 7. The step from the order of comparison to time: *Incommensuration; The Theorem and the Three Axioms*, Chapter 8, Section X, as *The Compactification*, Chapter 3, Section II, quotes it. I identify the structural arrow with the direction of time, and the observation that bears on that identification: *Measurement; The Steps from Comparison to Physics*, Chapter 12, Section III; *Comparison; The Necessary Basis*, Chapter 7, Section II; as *The Compactification*, Chapter 3, Section II, quotes them; the standing of the identification: Section X. The order present in one comparison, made once: *Physics Is Not Math*, Chapter 8, Section II, as *The Compactification*, Chapter 3,

Section II, quotes it. The entropic arrow is statistical, belongs to many parts, and needs a less mixed beginning: *Physics Is Not Math*, Chapter 8, Section II, as *The Compactification*, Chapter 3, Section II, quotes it; the standing of the entropic arrow: Section X. The record joins the structural arrow and the entropic arrow (the darkened grain and the registered count): *The Compactification*, Chapter 3, Section II, quoting *Measurement; The Steps from Comparison to Physics*, Chapter 12, Section VII; the standing of that step as argued: Section X. The Past Hypothesis is a condition on the entropic arrow, and not the source of the structural arrow: *The Basis of Physics*, Essay 11, Section IV, as *The Compactification*, Chapter 3, Section II, quotes it. What lies outside the scope of the structural arrow, and the question passed on to the account of the beginning of an epoch: *The Compactification*, Chapter 3, Section II, citing *The Basis of Physics*, Essay 11, Abstract. The structural arrow and the entropic arrow, defined: *The Compactification*, Chapter 3, Key Concepts. The limit set by the Incommensuration Theorem is one of principle, and the limit holds for every instrument because every instrument makes comparisons: *The Compactification*, Chapter 3, Section III, quoting *Incommensuration; The Theorem and the Three Axioms*, Chapter 6, Section XIII; *At the Edge of Reality*, Chapter 4; *The View from Inside*, Chapter 4. Every comparison, in any epoch, runs from stance to act to record: *The View from Inside*, Chapter 7.

Chapter 7. Penrose's two facts about gravity and order: Penrose (1979); Penrose (1989); as *The Compactification*, Chapter 3, Section I, cites them. The Weyl curvature, and Penrose's proposal that the Weyl curvature was near zero at the beginning: Penrose (1979). The question that Penrose presses against every account in which a collapse becomes a new expansion: Penrose (1979); Penrose (2010); *The Gravitational Passage*, Chapter 18, Section VIII. The picture of the clean slate, recalled from *At the Edge of Reality*: *At the Edge of Reality*, Chapter 18. What the axioms do not do, the first axiom quoted, the absence of an initial state, the exclusion that the Incommensuration Theorem makes, and the model set apart from the finding: *The Compactification*, Chapter 3, Section III, quoting *An Immanent Metaphysics*, aphorism 1.3-1, *Incommensuration; The Theorem and the*

Three Axioms, Chapter 6, Section XIII, and Chapter 9, Section I, *Comparison; The Necessary Basis*, Chapter 5, Section II, and *Measurement; The Steps from Comparison to Physics*, Chapter 6, Section VI. The pictures of the photographer and the map: *The View from Inside*, Chapter 7. Physics divides into content and context, gravity is on the side of context, there is no quantum state of the arena, and quanta are quanta of the description, not of the arena: *The Compactification*, Chapter 3, Section IV, quoting *Measurement; The Steps from Comparison to Physics*, Chapter 10, Sections I and VII, and *The Unquantized Arena*, Propositions 7.1, 9.1 and 17.1. The category error: *The Unquantized Arena*, Chapter 6, as *The Compactification*, Chapter 3, Section IV, cites it. The pictures of the crystal, the phonon and the held structure: *Two Theories, One World*, Chapter 14. The entropy of a black hole is one quarter of its horizon area in Planck areas: Bekenstein (1973); Hawking (1975). I read the entropy of a black hole as a count relative to the outside, and that reading is an identification: *The Unquantized Arena*, Chapter 17, as *The Compactification*, Chapter 3, Section V, quotes it. Every count has a context, the count is reset at the turn, the standing and the scope of the reset, and the census: *The Compactification*, Chapter 3, Section V; *The Gravitational Passage*, Chapter 18, Section VIII; *At the Edge of Reality*, Chapter 18. One process, seen from its two sides: *The Gravitational Passage*, Chapter 18, Section VIII. The even beginning, the geometry that has nothing to be lumpy with, the departures from evenness of about one part in a hundred thousand, and the Weyl condition reached as a consequence: *The Compactification*, Chapter 3, Section VI; *The Gravitational Passage*, Chapter 18, Section VIII. The scope of the even beginning: *The Compactification*, Chapter 3, Section VI; *The Gravitational Passage*, Chapter 18, Section VIII ("The scope of the step"). The objection to histories made of cycles: Tolman (1934). The answer to Tolman's objection: *The Compactification*, Chapter 3, Section VII; *The Gravitational Passage*, Chapter 18, Section VIII; *At the Edge of Reality*, Chapter 18. Information in two senses: information is not destroyed, is not accessible from outside while a black hole formed from a star still exists, and neither its return nor its loss is asserted: *The Compactification*,

Chapter 3, Section IX, citing *The Unquantized Arena*, Chapter 17, Proposition 17.3 and Section VII, and *The Gravitational Passage*, Chapter 18, Section IX; *At the Edge of Reality*, Chapter 18.

Chapter 8. Horizons are edges that signals cross one way, and the scope of horizons in four dimensions: *The Compactification*, Chapter 4, Section I. The horizon of a black hole is a feature of the shape of the geometry, with no material lying there: *Two Theories, One World*, Chapter 25 ("What a Black Hole Is"). The distances of 46.1, 16.7 and 17.5 billion light years: *The Compactification*, Chapter 4, Section III, and Chapter 5, Section VII. The Hubble radius of 14.5 billion light years and the age of 13.8 billion years: *The View from Inside*, Chapter 12. The ratio of 2.65 against the measured 2.63: *The Compactification*, Chapter 4, Section III; *The Gravitational Passage*, Chapter 13, Sections III and VI. The count of a black hole, its values for a hole of the Sun's mass, my reading of the count as a count relative to the outside, the two senses of the word information (the form of the matter, and a count of states against a context), area against volume, and the first axiom: *The Compactification*, Chapter 4, Section II, quoting *The Unquantized Arena*, Chapter 17. The factor of one quarter: Hawking (1975). The entropy of a horizon: Bekenstein (1973). Being inside, the deficit, and my identification of the deficit as an instance of the Incommensuration Theorem at the scale of the cosmos: *The Gravitational Passage*, Chapter 13, Section I. The picture of the fish: *The View from Inside*, Chapter 11. My identification of the cosmological constant, the external value and its two parts, the floor counted within the external value, the three steps of the share, the three channels (one for each role of a comparison), the integer three, the count in nats, and one over the count: *The Gravitational Passage*, Chapter 13, Section II and Questions; *The Compactification*, Chapter 4, Section III. The trim is a structural value: *The Compactification*, Chapter 6, Section X. The particle horizon is the edge that sets the cosmological constant, for two reasons (the particle horizon bounds what a present measurement contains, and only on the particle horizon can the share equal one constant): *The Compactification*, Chapter 4, Section III; *The Gravitational Passage*, Chapter 13, Sections II and VI. Holographic dark energy: Cohen,

Kaplan and Nelson (1999); Hsu (2004); Li (2004), as *The Gravitational Passage*, Preface, cites them. The five choices (the particle horizon as the edge, three channels, the trim included, the energy of the floor counted inside the Planck density part, and the full Planck mass), each with its reason, and the 192 combinations: *The Compactification*, Chapter 6, Section X; *The View from Inside*, Appendix A, Section IV. The count of 160, which leaves out the whole number five: *The Gravitational Passage*, Chapter 13, Questions. The equation of the expanding universe: Friedmann (1922), as *The View from Inside* cites it. The value of 2.56 against the observed 2.52 times ten to the minus forty-seven GeV to the fourth power, the identity, the matter share of 0.311 and the vacuum share of 0.689, one test seen two ways, and nothing chosen to fit: *The Compactification*, Chapter 4, Sections III and X; *The Gravitational Passage*, Chapter 13, Sections III, IV and VII. The measured matter share of 0.315, give or take 0.007: Aghanim and others (2020). The energy of empty space in kilograms per cubic meter: *The Compactification*, Chapter 4, Section IV; the count of hydrogen atoms is arithmetic on that energy. The cosmological constant fixed once, the identity of Bianchi, what exists and what is real, records, values on paper, the observer at redshift one, the question of why now, ten to the sixty-one Planck times, and a ratio of pressure to energy of exactly minus one: *The Gravitational Passage*, Chapter 13, Sections V and VI, and Questions; *The Compactification*, Chapter 4, Section III, quoting *The View from Inside*, Appendix A, Section V. The count of all the black holes in the observable universe: Egan and Lineweaver (2010). Black holes and the vacuum, the black holes entering the count once, and the ratio of about forty-six powers of ten: *The Compactification*, Chapter 4, Section IV; *The Gravitational Passage*, Appendix B, Section VI. One ratio is computed in *The Compactification* from the measured constants: eight times the average density of a hole of the Sun's mass, divided by the measured density of empty space, which comes to about two and a half times ten to the power forty-six. *The Gravitational Passage* gives the ratio as "about forty-six orders of magnitude".

Chapter 9. Horizons change, and three results govern the change of a horizon (the area theorem, the account of dynamical horizons, and the generalized second law): *The Compactification*, Chapter 4, Section V. The area theorem: Hawking (1971). Dynamical horizons: Ashtekar and Krishnan (2004). The generalized second law: Bekenstein (1974); *The Unquantized Arena*, Chapter 17, Section V, as *The Compactification*, Chapter 4, Section V, cites it. The count of a horizon and my reading of the count, and the count of a hole of the Sun's mass: *The Compactification*, Chapter 4, Section II; *The Unquantized Arena*, Chapter 17, Sections III and IV. The final mass and spin of a merger, computed by numerical relativity: Scheel and others (2009). The area ratio, the energy radiated, the three kinds of number, and the cosmological constant left unchanged by a merger: *The Compactification*, Chapter 4, Section VI. The first merger detected, on 14 September 2015: Abbott and others (2016). The area theorem, checked in the first merger detected: Isi and others (2021). The waves of general relativity travel at the speed of light, in two patterns: Abbott and others (2017a); Abbott and others (2017b); *The Unquantized Arena*, Prediction 19.4; *The Compactification*, Chapter 4, Section VI. What the waves of a merger carry, and the information hidden while the black hole still exists: *The Compactification*, Chapter 4, Section VI; *The Unquantized Arena*, Chapter 17, Section VII. The glow of a black hole: Hawking (1974); Hawking (1975). Hawking radiation is real, how Hawking radiation is derived, the misleading picture of pairs, and the step of Hawking's calculation that uses the smooth description beyond the Planck scale: *The Gravitational Passage*, Chapter 18, Section X; *The Compactification*, Chapter 4, Section VII; *Two Theories, One World*, Chapter 25 ("The Radiation Hawking Found"). The models that change the field at very short waves: Unruh (1995); Corley and Jacobson (1996); Corley and Jacobson (1998). The glow of a model black hole in a chilled cloud of atoms: Muñoz de Nova and others (2019). The temperature, the wavelength, the cooling of the background, and the time of evaporation of a black hole: *The Compactification*, Chapter 4, Section VII; *The Gravitational Passage*, Chapter 18, Section X; *The Unquantized Arena*, Chapter 17, Section II. The radius of a hole of the Sun's mass: *The*

Compactification, Chapter 4, Section II. The temperature of the microwave background: Fixsen (2009). The emptying of the outside world comes first: *The Compactification*, Chapter 4, Section VII; *The Gravitational Passage*, Chapter 18, Section I; *At the Edge of Reality*, Chapter 12 and Appendix C. Of the holes made in the early universe, those of less than about five hundred billion kilograms have evaporated by now: Carr, Kohri, Sendouda and Yokoyama (2010), as *The Gravitational Passage*, Chapter 18, Section I, cites them. The Page curve, its two conditions, its two further steps, its turning point and the table of its three cases: *The Compactification*, Chapter 4, Section VII and Table 4.1; *The Unquantized Arena*, Chapter 17, Section V, Propositions 17.4 and 17.5, and Table 17.1; *Two Theories, One World*, Chapter 25 ("Small Holes and the Page Curve"). The shape of the Page curve: Page (1993b). The entanglement of a part of a pure whole: Page (1993a). The reading of "nearly every" as a symmetry: *Two Theories, One World*, Chapter 25. The picture of the coded message and its key: *Two Theories, One World*, Chapter 25. The glow of light and of the small waves of the geometry, with the ratio of 1.4847: Page (2013). The information paradox: Hawking (1976). The two premises of the information paradox (the premise of the whole history and the premise of one quantum state) set aside, and information not destroyed: *The Compactification*, Chapter 4, Section VIII; *The Unquantized Arena*, Chapter 17, Sections I and VII, Propositions 17.1, 17.2 and 17.3; *Two Theories, One World*, Chapter 25 ("The Puzzle", "The Premise Underneath", "Information Is Not Destroyed", "Holes Formed From Stars"). The form of what fell in is carried to the turn as one form: *The Compactification*, Chapter 4, Section VIII, and Chapter 3, Section IX; *At the Edge of Reality*, Chapter 18. The island formula: Penington (2020); Almheiri, Engelhardt, Marolf and Maxfield (2019); Almheiri, Mahajan, Maldacena and Zhao (2020). The reading of the island formula that the account declines: *The Compactification*, Chapter 4, Section VIII; *The Unquantized Arena*, Chapter 17, Section VI. The firewall: Almheiri, Marolf, Polchinski and Sully (2013). The horizon kept smooth: *The Unquantized Arena*, Chapter 17, Section VIII. One geometry for each alternative, the rival way of joining gravity to quantum theory with one

geometry shaped by the average of the alternatives, horizons as classical, and the graviton as a quantum of the description: *The Compactification*, Chapter 4, Section IX; *The Unquantized Arena*, Propositions 6.3, 7.1, 7.3, 9.1 and 17.1; *Two Theories, One World*, Chapter 14. The experiment in which the pull of gravity followed the outcome: Page and Geilker (1981). The entangling phase of 0.314 radians: *The Compactification*, Chapter 4, Section IX; *The Gravitational Passage*, Chapter 12, Questions. The design of the experiment: Bose and others (2017). The experiment in plain words: *Two Theories, One World*, Chapter 13. Eight times the mean density, forty-six powers of ten, and one count entered once: *The Compactification*, Chapter 4, Sections IV and XI. The standing of the steps: *The Compactification*, Chapter 4, Section X.

Chapter 10. Entropy is a count relative to a context, in nats, and the count has no reference once the context ends: *The Compactification*, Chapter 5, Section I, citing *At the Edge of Reality*, Chapter 18, and *The Gravitational Passage*, Chapter 18, Section VIII. Counts at past eras are counts within the reconstruction: *The Compactification*, Chapter 5, Section I; *At the Edge of Reality*, Chapter 17; *The Coming of the Kinds*, Chapter 2, Section I. The comoving region and its radius of 46.14 billion light years: *The Compactification*, Chapter 5, Section I; *The View from Inside*, Appendix C. The measurements of the expansion: Aghanim and others (2020). The content is hot and close to balance, and the geometry is even, both argued: *The Compactification*, Chapter 5, Sections II and X; *The Gravitational Passage*, Chapter 18, Section VIII; *The Coming of the Kinds*, Chapter 2, Section II. The gyration is a conserved vibration of the six held small, made at the turn: *The Weight of Possibility*, Chapter 1, Section IV, and Chapter 3, Section I; *The Coming of the Kinds*, Chapter 2, Section III. The starting temperature is not fixed, and its ceiling is about one thirtieth of a gyration's mass: *The Compactification*, Chapter 5, Section II; *The Weight of Possibility*, Chapter 4, Section VI; *The Gravitational Passage*, Chapter 18, Section XI; *The Coming of the Kinds*, Chapter 2, Section IV. The stage that behaved as matter, ending below one hundredth of a gyration's mass: *The Compactification*, Chapter 5, Section II; *The Weight of Possibility*, Chapter 6, Section I. The settling waves

within three percent of the radiation, and the bound stated in kinds of neutrino: *The Compactification*, Chapter 5, Sections II and X; *The Weight of Possibility*, Chapter 4, Section III. The radiation era and the conserved count: *The Compactification*, Chapter 5, Section III; Kolb and Turner (1990), Chapter 3. The count of 106.75, and the right-handed neutrino out of balance: *The Compactification*, Chapter 5, Section III; *The Coming of the Kinds*, Chapter 3, Section IV; *The Kinds of Particle*, Chapter 6, Section II. The picture of the soup and the bowls: *The Coming of the Kinds*, Chapter 3, Section IV. The steps of the cooling, the neutrinos going free, the cube root of eleven fourths, and the present count of 3.909: *The Compactification*, Chapter 5, Section III; *The Coming of the Kinds*, Chapters 6, 8 and 9; Dolgov (2002). The count of light species of 3.044 in the standard description: Bennett and others (2021). The count measured at 2.99 by the Planck satellite: Aghanim and others (2020). The count measured at 2.86 by the Atacama Cosmology Telescope: Calabrese and others (2025). Smooth changes with no bubbles, the endpoint at 72.4 GeV, and the electroweak crossover at 159.5 GeV, about 5 GeV wide: *The Compactification*, Chapter 5, Sections IV and X; Csikor, Fodor and Heitger (1999); D'Onofrio and Rummukainen (2016); *The Coming of the Kinds*, Chapter 4, Section IV. The Higgs field is the one field of its type: *The Compactification*, Chapter 5, Section IV, citing *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 13, Section II, and *The Kinds of Particle*, Chapter 6, Section IV. The binding of the quarks near 156.5 MeV is a crossover: Aoki and others (2006); Bazavov and others (2019); *The Coming of the Kinds*, Chapter 8. The crossovers leave no background of waves: *The Compactification*, Chapter 5, Section IV, and Chapter 7. B minus L is a residue that the turn leaves: *The Coming of the Kinds*, Chapters 5 and 7, and "The Story in Brief". The sphaleron passages from about ten to the twelve GeV down to 131.7 GeV: D'Onofrio, Rummukainen and Tranberg (2014). The freezing of the neutrons, the first nuclei, the release of the light at 2,973 kelvin and redshift 1089.92, and 6.13 baryons for every ten billion photons: *The Compactification*, Chapter 5, Section V; *The Coming of the Kinds*, Chapters 1, 11, 12 and 13; Aghanim and others (2020). Matter and radiation are equal at a redshift of about 3,402:

Aghanim and others (2020), Table 2, as *The Compactification*, Chapter 5, Section V, cites it. The counts of photons and neutrinos today: *The Compactification*, Chapter 5, Section V; Fixsen (2009); *The Coming of the Kinds*, Box 1.1; *The View from Inside*, Appendix C. The count of the content is about ten to the ninety: *The Compactification*, Chapter 5, Section V and Section XI. Clumping raises the count, and the picture of the collapsing cloud: *The Compactification*, Chapter 5, Section VI. The counts of black holes, and the generalized second law: *The Compactification*, Chapter 5, Section VI, and Chapter 3, Section VIII; Bekenstein (1973); Bekenstein (1974); Hawking (1975); *The Unquantized Arena*, Chapter 17; *The Gravitational Passage*, Appendix B, Section VI. The count of all black holes: Egan and Lineweaver (2010). The count of the particle horizon, by identification, and the black holes within the particle horizon: *The Compactification*, Chapter 5, Section VII, and Chapter 3, Section VIII; *The Gravitational Passage*, Chapter 13, Sections II and IV, and Appendix B, Section VI. The energy of empty space, fixed once, its ratio of pressure to energy of minus one, and the values on paper: *The Compactification*, Chapter 5, Section VII; *The Gravitational Passage*, Chapter 13, Sections III, IV and VI; *The View from Inside*, Chapters 11 to 13 and Appendix A, Section V. The event horizon, the de Sitter radius and the count of the de Sitter horizon, and the trim: *The Compactification*, Chapter 5, Section VII; *The View from Inside*, Chapter 12 and Appendix C; *The Gravitational Passage*, Chapter 13, Questions. The room for the count to grow: *The Compactification*, Chapter 3, Section VIII; Frautschi (1982); Layzer (1975). Penrose's figure for how improbable the beginning was, and the argument from the reset: *The Compactification*, Chapter 3, Section VIII ("The improbability estimate"), citing *Incommensuration; The Theorem and the Three Axioms*, Chapter 6, Section XIII, and *The Gravitational Passage*, Chapter 18, Section VIII; Penrose (1989). The standing of the Past Hypothesis: *The Compactification*, Chapter 3, Section VIII ("Where the Past Hypothesis stands"). The emptying, the evaporation, the conditions failing, the interiors becoming one, no count passing the turn, and Tolman's objection: *The Compactification*, Chapter 5, Section VIII; *At the Edge of Reality*, Chapters 12, 16 and 20, and Appendix C; *The Gravitational Passage*,

Chapter 18, Sections II, VIII and X; *The View from Inside*, Chapter 16; Tolman (1934). The entropic and structural arrows across the epoch, joined by the record: *The Compactification*, Chapter 5, Section IX; *Measurement; The Steps from Comparison to Physics*, Chapter 12, Sections III and VII; *Physics Is Not Math*, Chapter 8, Section II. The table of the counts: *The Compactification*, Chapter 5, Section IX, Table 5.1; the standing of each step: Section X.

Part III

Chapter 11. The picture of the fog that clears, the first atoms, and last scattering: *The Coming of the Kinds*, Chapter 13, Overview and Sections I to V, Boxes 13.1 and 13.2. The rule for how many electrons are bound in balance: Saha (1920). The theory of how the electrons are bound and how many stay free: Peebles (1968). Last scattering is the more accurate name for what is often called recombination: Particle Data Group (2026), review "Big-Bang Cosmology", as *The Coming of the Kinds* cites it. The redshift of last scattering, 1089.92: Aghanim and others (2020). The present temperature of the light, 2.72548 kelvin: Fixsen (2009). The pictures of the cloakroom and the balcony, the neutrinos at 1.945 kelvin, and the contents of the universe fixed in kind: *The Coming of the Kinds*, Chapter 10, Overview and Sections II, IV and V, Boxes 10.1 and 10.3. Matter and radiation were equal near 9,300 kelvin: *The Coming of the Kinds*, Chapter 13, Section I and Summary. The oldest light differs in temperature from place to place by one part in a hundred thousand: Smoot and others (1992). The departures from uniformity, the definitions of the standard description, and the measured values: *The Compactification*, Chapter 6, Sections I and XI. The strength and the index of the first ripples: Aghanim and others (2020); Akrami and others (2020). The ratio of first waves to first ripples is below 0.036: Ade and others (2021). The ratio is below 0.032: Tristram and others (2022). The three uses of the pattern in the oldest light (to show that the ripples are adiabatic, to weigh the radiation other than light, and to give the number of protons and neutrons for each photon): *The Coming of the*

Kinds, Chapter 13. The pattern as read in *At the Edge of Reality*: *At the Edge of Reality*, Chapters 2 and 19. What passes the turn, my identification made in the first person, and the observation that bears on the identification: *The Compactification*, Chapter 6, Section II; *The Gravitational Passage*, Chapter 18, Sections IX and XIII; *At the Edge of Reality*, Chapter 19. The first ripples are the averaged form of what the black holes of the epoch before took in: *At the Edge of Reality*, Chapter 19 ("What This Means for You"). The picture of the restless trembling of the fields at their lowest: *The View from Inside*, Chapter 1 ("What Is 'Empty' Space?"). The vacuum of the new epoch adds almost nothing to the first ripples: *The Compactification*, Chapter 6, Section II; *The Gravitational Passage*, Chapter 18, Section IX; *At the Edge of Reality*, Appendix C. The two steps of the argument kept apart, the premise that the new context has no prior identity, and the words "from a geometry in which it had no separate identity": *The Compactification*, Chapter 6, Section II; *The Gravitational Passage*, Chapter 18, Sections VII and VIII. The four properties that I argue the first ripples must have (the ripples are linked across the whole sky and nearly the same in strength at every length, the ripples are one form for every component of the content, the ripples begin as departures of density with a ratio of first waves to first ripples far below one millionth, and the large-scale model of the new epoch is flat): *The Compactification*, Chapter 6, Abstract and Section XI. I take the strength and the tilt of the first ripples from measurement, each with the question of what sets it: *The Compactification*, Chapter 6, Abstract and Section VII. The tilt is a reference to a length, and an index below one means power growing slowly toward longer lengths: *The Gravitational Passage*, Chapter 18, Section IX; *The Compactification*, Chapter 6, Section VII; *At the Edge of Reality*, Chapter 19.

Chapter 12. "From a geometry in which it had no separate identity": *The Gravitational Passage*, Chapter 18, Sections VII and VIII, as *The Compactification*, Chapter 6, Section II, quotes it. The horizon problem, the answer from one domain, and the near sameness of the ripples at every length: *The Compactification*, Chapter 6, Section III; *The Gravitational Passage*, Chapter 18, Sections II and IX, and Questions. The picture of the two

towns: *At the Edge of Reality*, Chapter 19. The Incommensuration Theorem, and the statistics of the ripples set against the one sky: *The Compactification*, Chapter 6, Section III, quoting *At the Edge of Reality*, Chapter 19. The identification from Chapter 11, recalled, and the form of the content: *The Compactification*, Chapter 6, Section II; *The Gravitational Passage*, Chapter 18, Sections IX and XIII. Each mark of the pattern follows from the identification and the premise of no prior identity: *The Compactification*, Chapter 6, Section II and Section IX. Every ingredient rises together, which physicists call adiabatic, with no separate pattern: *The Compactification*, Chapter 6, Section IV; *The Gravitational Passage*, Chapter 18, Sections IX and XIII. The measured bound on a separate pattern: Akrami and others (2020). The dark matter, by two routes: *The Compactification*, Chapter 6, Section IV, quoting *The Weight of Possibility*, Chapter 3, Section I. The first ripples are ripples of density at first order, and waves come only at second order, so that the ratio of first waves to first ripples is far below a millionth: *The Compactification*, Chapter 6, Section V; *The Gravitational Passage*, Chapter 18, Sections IX and XIII. The present bound on first waves: Ade and others (2021). The forecasts of the Simons Observatory and LiteBIRD: Simons Observatory Collaboration (2025); LiteBIRD Collaboration (2026). The arena is not quantized, and quanta are quanta of the description: *The Unquantized Arena*, Chapter 18, Section VI, as *The Compactification*, Chapter 6, Section V, quotes it. Flat space, the one length that the state at the turn holds, the de Sitter radius as belonging to the law, the Planck-length radius excluded, and the smoothed model flat: *The Compactification*, Chapter 6, Section VI; *The Gravitational Passage*, Chapter 18, Sections VIII, IX and XII. The curvature as a number set by the starting state: Misner, Thorne and Wheeler (1973), Chapter 27. The finest precision that any measurement of the curvature can achieve: Waterhouse and Zibin (2008). The flatness problem in plain words: *At the Edge of Reality*, Chapter 2. The measured curvature: Aghanim and others (2020). The size and the slope of the ripples, taken from measurement, with the question beside each, and what the Incommensuration Theorem cannot compute: *The Compactification*, Chapter 6, Section VII; *The Gravitational Passage*, Chapter 18, Section

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Chapter 13. No inflation is needed, a long inflation is excluded, and a short inflation is neither required nor excluded: *The Gravitational Passage*, Chapter 18, Questions; *The Compactification*, Chapter 1, Section VIII, and Chapter 6, Section VIII. Inflation, as first proposed: Guth (1981). The many forms of inflation, and the standing of inflation among cosmologists: *At the Edge of Reality*, Chapter 2. The inflaton, the e-fold and reheating, named to state the scope of the account: *The Compactification*, Chapter 1, Section VIII, and Chapter 6, Section VIII. The problems that inflation was introduced to solve: *The Gravitational Companion*, chapter "Comparisons at Single Steps", Section IX. The horizon and flatness problems in plain words: *At the Edge of Reality*, Chapter 2; *The Gravitational Passage*, Chapter 18, Questions. The sameness of the sky explained from one domain, the form of the content passing the turn as one form, and the index near one: *The Gravitational Passage*, Chapter 18, Questions; *At the Edge of Reality*, Chapter 19. Flatness follows from the turn: *At the Edge of Reality*,

Chapter 19; *The Compactification*, Chapter 6, Section VI. The sameness of the expansion in every direction, its two routes (the restoration of context at the turn, and the fading of shear in a flat expansion), and the dying away of shear: *The Gravitational Companion*, chapter "Comparisons at Single Steps", Section IX; Ellis and MacCallum (1969); Collins and Hawking (1973). The even beginning: *The Gravitational Passage*, Chapter 18, Section VIII. The absence of a monopole problem, the absence of grand unification, the symmetries as turnings, and the charge rule: *The Gravitational Companion*, chapter "Comparisons at Single Steps", Section IX, citing *The Branches of the Kinds*, Chapter 2, *The Kinds of Particle*, Chapter 4, Section IV, and *The Story of the Particles*. Strings, walls and monopoles from the settling of the Higgs field: Kibble (1976). Unstable electroweak strings: Vachaspati (1992); James, Perivolaropoulos and Vachaspati (1992). The absence of a network of strings: *The Compactification*, Chapter 7, Section IX. Method, form and function, and the moduli, which cannot drive a slow roll: *The Gravitational Companion*, chapter "Comparisons at Single Steps", Section IX. The masses of the moduli: *The Gravitational Passage*, Chapter 11, Section II, and Appendix D, Section III. My identification of the departures, and the vacuum of the new epoch adding almost nothing to the first ripples: *The Compactification*, Chapter 6, Section II, citing *The Gravitational Passage*, Chapter 18, Section IX, and *At the Edge of Reality*, Appendix C. A long phase of inflation excluded, relative to the identification, a shorter phase not fixed, and no quantities of the inflationary description: *The Compactification*, Chapter 6, Section VIII; *The Gravitational Passage*, Chapter 18, Questions. The amplitude and the tilt are not fixed: *The Gravitational Passage*, Chapter 18, Section XI and Questions. The starting temperature is not fixed: *The Compactification*, Chapter 1, Section VIII, citing *The Gravitational Passage*, Chapter 18, Section XI. The masses of a gyration: *The Weight of Possibility*, Chapter 1, Section IV, and Chapter 4, Section V; *The Compactification*, Chapter 1, Section VIII. Pair production through gravity, the ceiling of about one thirtieth of a gyration's mass, the stage that behaved as matter, and the dilution: *The Weight of Possibility*, Chapter 4, Section VI; *The Compactification*, Chapter 1, Section VIII; Garny, Sandora and Sloth (2016).

Lasting vibrations of a compact space are about as numerous as the particles of light for a start near the Planck temperature: Kolb and Slansky (1984). Reheating, in the conditional sense only, and no inflaton: *The Compactification*, Chapter 1, Section VIII. The three conditions on the turn are requirements and not results: *The Weight of Possibility*, Chapter 6, Section I; *The Compactification*, Chapter 1, Sections VII and X. The measured bound on first waves: Ade and others (2021). The dark matter as a second ground for excluding a long inflation, and *The Weight of Possibility* and *The Compactification* in agreement: *The Weight of Possibility*, Chapter 4, Section VI, citing Garny, Sandora and Sloth (2016) and Kolb and Long (2024); *The Compactification*, Chapter 1, Section VIII, and Chapter 6, Section VIII. The dark matter is about five and four tenths times the ordinary matter: *The Weight of Possibility*, Chapter 1, Section I, and Chapter 3.

Chapter 14. The ledger of the sky, its margin notes and every row of its table: *The Compactification*, Chapter 6, Section X, Table 6.2, and "The counts behind the table". The three sources of the numbers in the ledger (set by the structure, taken from measurement, and predicted): *The Compactification*, Chapter 6, Section X. The standard model of cosmology and its six base parameters, with flat space and a ratio of pressure to energy of minus one assumed: *The Compactification*, Chapter 6, Section X. The critical density, the picture of the pie, and flatness: *The View from Inside*, Chapter 13 ("The Second Way: The Astronomers' Way"). The Hubble constant and the megaparsec: *The View from Inside*, Chapter 13 ("The Same Number, Seen from Other Sides"). The five entries taken from measurement: Aghanim and others (2020). The amplitude and the index are fixed in kind and measured in size and slope: *The Compactification*, Chapter 6, Sections XI and XII. The registered ratio and its standing: *The Compactification*, Chapter 6, Section X ("Registered ratios"); *The Weight of Possibility*, Chapter 3, Section VII, and Chapter 5, Section VIII. What registration is: *The Weight of Possibility*, Glossary; *The Gravitational Passage*, Chapter 13, Section II. How the first light measures the densities of ordinary matter and of dark matter: *The Weight of Possibility*, Chapter 5, Section I. The ratio measured by the Planck satellite: Aghanim and others (2020); by three

telescopes together: Camphuis and others (2026); with the distances of DESI: Louis and others (2025), as *The Weight of Possibility*, Chapter 5, Section I, collects them. The DESI survey itself: Abdul-Karim and others (2025). What supports the rule of the registered ratio: the form of the rule, the exclusion by measurement of the nearest neighbors of the rule, and the coefficient that the rule shares with the identity of Chapter 8: *The Weight of Possibility*, Chapter 5, Sections III and VIII. The second kind of measurement, which finds the amounts of ordinary matter and dark matter without the first light: *The Weight of Possibility*, Chapter 5, Section II. The light elements with the distances of DESI: Schöneberg (2024); Abdul-Karim and others (2025); with the fuller analysis: Forero-Sánchez and others (2026). The hot gas of clusters: Mantz and others (2022). The picture of the two surveyors: *The View from Inside*, Chapter 13 ("Two Surveyors"). The identity, one equation in one unknown, and the matter share of 0.311: *The Gravitational Passage*, Chapter 13, Section III; *The View from Inside*, Chapter 13 ("One Equation, One Unknown"). The matter share measured at 0.315 by the Planck satellite: Aghanim and others (2020); at 0.2975 by DESI alone: Abdul-Karim and others (2025). The spread of 0.001 that comes from the radiation: *The Gravitational Passage*, Chapter 13, Section IV; *The View from Inside*, Appendix B. The identity fixes the split and the total matter, not the make-up of the matter: *The Gravitational Passage*, Chapter 13, Sections III and VII. The energy of empty space, the horizon ratio and the age: *The Gravitational Passage*, Chapter 13, Sections III and IV; *The View from Inside*, Appendix C. The Hubble constant through one more measurement, 67.8, and the excluded share: *The Gravitational Passage*, Chapter 13, Section IV; *The View from Inside*, Chapter 13 and Appendix C. The Hubble constant measured at 67.4 by the Planck satellite: Aghanim and others (2020); at 73.0 by the distance ladder: Riess and others (2022). One result seen five ways: *The Compactification*, Chapter 6, Section X ("Independent tests"); *The View from Inside*, Appendix C. A ratio of pressure to energy of exactly minus one, argued: *The Gravitational Passage*, Chapter 13, Sections V and VI. The measured value of the ratio: Aghanim and others (2020). Flat space, one form for every ingredient, and a ratio of

first waves to first ripples far below a millionth: *The Compactification*, Chapter 6, Sections IV to VI and XII; *The Gravitational Passage*, Chapter 18, Section IX, and Chapter 20, Questions. The measured bounds: Ade and others (2021); Akrami and others (2020). What the standard picture expects of each prediction: *The Gravitational Passage*, Chapter 20, Questions. S eight, the reason S eight lies lower, and the coldness of the dark matter: *The Weight of Possibility*, Chapter 5, Section VI. The measured inputs, the structural values, the identifications, and the five choices of the share with their 192 combinations: *The Compactification*, Chapter 6, Section X ("The counts behind the table"); *The View from Inside*, Appendix A, Section IV. The channels are the count of the basic pieces of the complex projective plane, one for each role of a comparison: *The Gravitational Passage*, Chapter 13, Section II; *The View from Inside*, Chapter 9 ("The Number Three"). The trim: *The Gravitational Passage*, Chapter 13, Section II; *The Weight of Possibility*, Chapter 5, Section I. What the account fixes: *The Compactification*, Chapter 6, Sections X and XII.

Part IV

Chapter 15. The arena as the context of every comparison and not a force, and a gravitational wave as a ripple of the arena: *The Compactification*, Chapter 7, Section II, citing *Measurement; The Steps from Comparison to Physics*, Chapter 3, Section VII, and *The Gravitational Passage*, Chapter 16, Section V; the arena in plain words: *The View from Inside*, Key Terms. The stretched sheet, and ripples crossing regions where nothing is found: *The View from Inside*, Chapter 8 ("In Proportion to What Can Be Found"). The first gravitational wave caught, on 14 September 2015: Abbott and others (2016). No least step, no least length, area or duration, and no quantum of the arena's own: *The Compactification*, Chapter 7, Section II, citing *The Unquantized Arena*, Chapter 6, Proposition 6.3; the standing of the claim that the arena has no least step and no quantum of its own: Chapter 7, Section XI. Why the bending of the arena is smooth: *Two Theories, One World*,

Chapter 8 (“The Context Carries the Rest”). The Planck length not a least length: *The Weight of the Unseen*, “Words Used in This Book”; *The View from Inside*, Chapter 2 (“The Symbols”). The arena with no quantum state, and a horizon as a feature of a classical geometry, one for each alternative of the matter: *The Compactification*, Chapter 4, Section IX, citing *The Unquantized Arena*, Proposition 17.1. What a phonon is, the crystal and the held structure: *Two Theories, One World*, Chapter 14 (“What a Phonon Is”, “The Small Waves of Geometry”). The description of small waves by quanta, admitted below the Planck curvature, and the quanta of that description as quanta of the description and not of the arena: *The Compactification*, Chapter 7, Section II, citing *The Unquantized Arena*, Chapter 9, Proposition 9.1; *Two Theories, One World*, Chapter 14 (“Where the Description of Small Waves Ends”). The graviton, not a particle, carrier or kind, and the list of kinds with no graviton on it: *The Compactification*, Chapter 7, Section II, and Chapter 4, Section IX; *The Kinds of Particle*, Chapter 6, Section IV, as *The Compactification* cites it. Light and gravitational waves, the two questions kept apart, a wave in LIGO counted as so many gravitons, and the change in the length of LIGO’s arms: *Two Theories, One World*, Chapter 14. A detector of single jumps of energy proposed: Tobar, Manikandan, Beitel and Pikovski (2024). What a single click in a detector of single jumps of energy would show: Carney, Domcke and Rodd (2024); the photoelectric effect set beside the single click, and the ruling on detection: *Two Theories, One World*, Chapter 14 (“Has Anyone Detected a Graviton?”, “The Ruling on Detection”); *The Compactification*, Chapter 7, Section II, citing *The Unquantized Arena*, Proposition 9.2. Two patterns and one speed for gravitational waves, argued from the four-dimensional Einstein equations with the moduli held: *The Compactification*, Chapter 7, Section II, citing *The Gravitational Passage*, Chapter 16, Section V, and Chapter 12, Section VII, and *The Unquantized Arena*, Prediction 19.4. The merger of two neutron stars on 17 August 2017, and the speed of the waves: Abbott and others (2017a); “within two seconds” after many millions of years: *Two Theories, One World*, Chapter 8. The two patterns on a ring of beads: the plus and cross patterns of general relativity; *The View from Inside*, Ap-

pendix B. The hum with no single source, the counterpart of the microwave background, the record the hum keeps, and the five places to listen: *The Compactification*, Chapter 7, Abstract and Section I. The first wave sweeping from about 35 to about 250 hertz: Abbott and others (2016). A space detector near a thousandth of a hertz: *The Compactification*, Chapter 7, Section VIII; Amaro-Seoane and others (2017). Evidence from the timing of pulsars near a billionth of a hertz, and the two candidate origins of the evidence: *The Compactification*, Chapter 7, Section X; Agazie and others (2023). The first maker of a hum, the settling of the six: *The Compactification*, Chapter 7, Section III and Section XI, citing *The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I. The second maker of a hum, the first ripples as scalar departures from uniformity, no first-order waves, and waves at second order: *The Compactification*, Chapter 7, Abstract and Sections VI and VII, citing *The Gravitational Passage*, Chapter 18, Section XIII, and *At the Edge of Reality*, Chapter 19. The third maker of a hum, a change of state of the matter, and the electroweak and strong changes of state both crossovers: *The Compactification*, Chapter 7, Section VIII, citing *The Coming of the Kinds*, Chapter 4, Section IV, and Chapter 8, and *The Kinds of Particle*, Chapter 6, Section IV. The fourth maker of a hum, cosmic strings, and the Higgs breaking with no loop that cannot shrink: *The Compactification*, Chapter 7, Section IX. The fifth maker of a hum, pairs of very heavy black holes: *The Compactification*, Chapter 7, Sections I and X.

Chapter 16. The arc making no wave, the settling into breathing, the decay of the breathing, the masses of the moduli, the bounds on the release, the condition on the turn, the ceiling on the settling hum, and what the settling background is not: *The Compactification*, Chapter 7, Section III, citing *The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I, *The Gravitational Passage*, Chapter 11, Section II, and Appendix D, Section III, *The Kinds of Particle*, Chapter 1, Section XIII, and *Measurement; The Steps from Comparison to Physics*, Chapter 11, Section XII. The radii: *The Compactification*, Chapter 7, Section III; *The Gravitational Passage*, Chapter 9, Section IX. The lowest motion that cannot radiate, the lifetime of the

breathing, and the dark matter not among the breathing: *The Weight of Possibility*, Chapter 4, Section III. The starting temperature not fixed, the ceiling on the starting temperature, and the stage that behaved as matter: *The Compactification*, Chapter 5, Section II, and Chapter 7, Section IV; *The Gravitational Passage*, Chapter 18, Section XI; *The Weight of Possibility*, Chapter 4, Section VI, and Chapter 6, Section I; the conversion of GeV to kelvin: *The Coming of the Kinds*, Chapter 2, Section IV. The three premises, the frequency of the settling hum today, the floor under that frequency, the breadth of the hum not computed, and the release of entropy: *The Compactification*, Chapter 7, Section IV; the present temperature of the light: Fixsen (2009). The count of light species: *The Coming of the Kinds*, Chapter 9, Section V; the standard value of the count, 3.044: Bennett and others (2021); measured at 2.99: Aghanim and others (2020); and at 2.86: Calabrese and others (2025). The strength, the excess of light species, the energy share, the strain and the table: *The Compactification*, Chapter 7, Section V, Table 7.1, and Chapter 5, Section II; the strain of a background: Maggiore (2000). No first waves at the start, and the two statements: *The Compactification*, Chapter 7, Section VI, citing *The Gravitational Passage*, Chapter 18, Section XIII and Questions, and *At the Edge of Reality*, Chapter 19. The argument that an imprint of first waves would show the arena quantized: Krauss and Wilczek (2014); the answer to Krauss and Wilczek: *The Unquantized Arena*, Chapter 18, Section VI, as *The Compactification*, Chapter 7, Section VI, quotes it. The faint hum every epoch makes, at second order: *The Compactification*, Chapter 7, Section VII; Ananda, Clarkson and Wands (2007); Baumann, Steinhardt, Takahashi and Ichiki (2007); Kohri and Terada (2018). No hum from boiling: *The Compactification*, Chapter 7, Section VIII, citing *The Coming of the Kinds*, Chapter 4, Section IV, and Chapter 8, *The Kinds of Particle*, Chapter 6, Section IV, and *Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 19.10; Kajantie, Laine, Rummukainen and Shaposhnikov (1996); Csikor, Fodor and Heitger (1999); D'Onofrio and Rummukainen (2016); Aoki and others (2006); Bazavov and others (2019); the kitchen: *The Coming of the Kinds*, Chapter 4, Section IV. The waves a change of state with bubbles

would make: Caprini and Figueroa (2018); LISA: Amaro-Seoane and others (2017). Strings, walls and monopoles: *The Compactification*, Chapter 7, Section IX; Kibble (1976). What the pulsars hear: *The Compactification*, Chapter 7, Section X; Agazie and others (2023). The standing of the steps: *The Compactification*, Chapter 7, Section XI; what the beginning left in waves: Section XII.

Chapter 17. What we are listening for: the settling background, its frequency and ceiling, and the two nulls: *The Compactification*, Chapter 8, Abstract and Sections I and II. The words of the condition on the turn: *The Weight of Possibility*, Chapter 4, Section III, and Chapter 6, Section I, as *The Compactification*, Chapter 8, quotes them. The second-order background, at about one part in ten to the twenty-two: *The Compactification*, Chapter 8, Section II.B. The radio and its dial, the frequency gap and the amplitude gap: *The Compactification*, Chapter 8, Section II.C and Section IX. The nine classes of detector: *The Compactification*, Chapter 8, Sections III to VII. Quartz crystals and MAGE: *The Compactification*, Chapter 8, Section III; Goryachev and Tobar (2014); Campbell, Goryachev and Tobar (2023); the proposed array: arXiv:2506.17479. Superconducting cavities: *The Compactification*, Chapter 8, Section IV; Braginsky and Menskii (1971); Pegoraro, Radicati, Bernard and Picasso (1978); MAGO 2.0: arXiv:2601.18719. Magnon detectors and haloscopes: *The Compactification*, Chapter 8, Section V; Ito, Ikeda, Miuchi and Soda (2020); Ito and Soda (2023); the CAPP-12T bound: arXiv:2511.17817. Waves turned into light in a magnetic field: Gertsenshtein (1962); in space: *The Compactification*, Chapter 8, Section VI.A; arXiv:2312.17636; in the laboratory, the study of 2021: Herman, Füzfa, Lehoucq and Clesse (2021). The levitated sensor: *The Compactification*, Chapter 8, Section VII.A; Arvanitaki and Geraci (2013). The Li-Baker concept and the reasons the sensitivity claimed for the Li-Baker concept is not used: *The Compactification*, Chapter 8, Section VII.B; Li, Baker, Fang, Stephenson and Chen (2008); Eardley and others (2008). The Fermilab Holometer and the limits the Holometer measured: *The Compactification*, Chapter 8, Section VII.C; Chou and others (2017). Phonon detectors in crystal targets: *The Compactification*, Chapter 8, Section VII.D; Kahn,

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Chapter 19, Section II, as *The Compactification*, Chapter 8, Section XIII, cites it. How long the listening takes: *The Compactification*, Chapter 8, Sections XIV.A to XIV.C; the standing of the steps: Section XV.

Part V

Chapter 18. The dark matter seen by its weight in three places: *The Weight of Possibility*, Chapter 1, Section I. First inferred from the Coma cluster: Zwicky (1933). The rotation of the Andromeda galaxy: Rubin and Ford (1970). The weighing of the Bullet Cluster by the bending of light: Clowe and others (2006). The limit on how strongly the dark matter bounces off itself: Markevitch and others (2004). No decay into anything unseen within about a hundred and sixty billion years: Poulin, Serpico and Lesgourgues (2016). The ratio of about five and four tenths: Aghanim and others (2020); no separate pattern, within about one and three tenths percent: Akrami and others (2020). The growth of the galaxies, the scaffolding, and the halos: *The Weight of Possibility*, Chapter 1, Section I; the slow growth while light ruled the expansion: Mészáros (1974); growth from the measured ripples: Peebles (1982); Smoot and others (1992); halos and the gas that sinks: Rees and Ostriker (1977); White and Rees (1978). Where exchange can happen, and the turn as the one place: *The Weight of Possibility*, Chapter 1, Section II, and Chapter 4, Questions. The space of possibility, the two comparisons, and nothing taken about the content: *The Weight of Possibility*, Chapter 1, Section III, quoting *The Gravitational Passage*, Appendix C, Section VIII; no assumption that the six are the arena of particle physics: *The Weight of Possibility*, Chapter 1, Section III. The identification, the crystal, the hold resting on the modal boundary, the floor as one mechanism, the arena not held, and the three ways out: *The Weight of Possibility*, Chapter 1, Section IV; *The Gravitational Passage*, Chapter 11, Section II, and Appendix D. The three distortions and the three edges: *The Weight of Possibility*, Chapter 1, Section V, and Appendix A, Section IV. Every setting bent, the three distortions as a triple, and the

three edges as one triple (Appendix A): *The Weight of Possibility*, Chapter 2, Sections I to VIII and Questions, and Appendix A, Sections I to X; the notion of a triple: *An Immanent Metaphysics* (Landry 2001), as *The Weight of Possibility*, Note 6, gives it; the approach of an emptied epoch to the de Sitter geometry: Wald (1983); the ten symmetries of the de Sitter geometry: Obukhov and Puetzfeld (2011). The gyration as a Kaluza-Klein state, the claim of role, and the prediction: *The Weight of Possibility*, Chapter 1, Questions. Persistence, and the three ways out closed: *The Weight of Possibility*, Chapter 4, Sections I to IV. Conformal cyclic cosmology, the erebons of conformal cyclic cosmology, and the decay of erebons: Penrose (2010; 2018); the measured lifetime that separates conformal cyclic cosmology from this account: *The Weight of Possibility*, Chapter 4, Questions, as Appendix B quotes it; the bound on a decay: Poulin, Serpico and Lesgourgues (2016). The swap of opposite points, gauged and so exact, the consequences of the swap, and the requirement on the content: *The Weight of Possibility*, Chapter 4, Section III. The masses of the gyrations, the conditions on those masses, the candidates, and the forces: *The Weight of Possibility*, Chapter 4, Section V; the requirement on the forces: Chapter 6, Section II. The numbers of the gyrations, the graininess of the dark matter the gyrations make up, and the making of the gyrations only at the turn: *The Weight of Possibility*, Chapter 4, Section VI. The gyrations cold, whatever speed the turn gave the gyrations: *The Weight of Possibility*, Chapter 5, Section VI. No gyration passes the turn, and what returns is the law: *The Weight of Possibility*, Chapter 6, Section III, quoting *The Gravitational Passage*, Chapter 18, Sections VII and VIII. A gyration not a black hole: *The Weight of Possibility*, Chapter 4, Questions. The Planck mass, the Planck length and the modal boundary in plain words: *At the Edge of Reality*, Chapter 8.

Chapter 19. The recipe card: two residues, the helium share of 0.18 percent, the identity, and the ratio constant since helium formed: *The Weight of Possibility*, Chapter 3, Section I; the gyrations made at the turn: Chapter 1, Section IV. The ratio fixed as the cosmological constant is fixed, by the present measurement: *The Gravitational Passage*, Chapter 13, Section

VI, as *The Weight of Possibility*, Chapter 3, Section I, quotes it. Counting the kinds, six against one: *The Weight of Possibility*, Chapter 3, Section II; *The Gravitational Passage*, Chapter 13, Section II, and Chapter 9, Sections I and IV, as quoted there; the drawstring bag, and adding against multiplying: *The Weight of the Unseen*, questions 5.2 and 5.3. No other account of very heavy dark matter fixing the amount of the dark matter relative to the ordinary matter: *The Weight of Possibility*, Chapter 3, Questions, citing Chung, Kolb and Riotto (1998), Garny, Sandora and Sloth (2016) and MacGibbon (1987). The share that holds fast, the share of a curvature, and which curvature abides: *The Weight of Possibility*, Chapter 3, Sections III and IV; *The Weight of the Unseen*, questions 5.4 and 5.5; *The Gravitational Passage*, Chapter 18, Section XIII, Chapter 11, Section III, Chapter 13, Sections II and III, and Appendix B, as quoted there. The theorem of Gauss and Bonnet, and the result of six less two over pi: *The Weight of Possibility*, Chapter 3, Sections V and VI, and Questions; Chapter 5, Sections III and IV; *The Weight of the Unseen*, questions 5.1, 5.8, 5.9 and 5.10. The number of the gyrations: *The Weight of Possibility*, Chapter 4, Section VI. The ratio measured by the Planck satellite: Aghanim and others (2020); by three telescopes together: Camphuis and others (2026); both ratios computed from the published densities: *The Weight of Possibility*, Chapter 5, Section I. What the oldest light finds, the weight of the match, and the form of the rule: *The Weight of Possibility*, Chapter 5, Sections I and VIII, and Chapter 3, Section VII and Questions; the drum: *The Weight of the Unseen*, question 1.5. Two predictions anyone can look for, no short loop and no separate pattern: *The Weight of Possibility*, Chapter 3, Sections I and II, and Chapter 5, Sections V and IX; *The Compactification*, Chapter 6, Section IV. The turn need not know the proton, and the repetition of the proportions left unfixed: *The Weight of Possibility*, Chapter 3, Section I and Questions; *The Weight of the Unseen*, question 5.7; *The Gravitational Passage*, Chapter 18, Questions. What the amount asks of the beginning: *The Weight of Possibility*, Chapter 3, Questions, Chapter 4, Section VI, and Chapter 6, Section I; *The Weight of the Unseen*, questions 6.6 and 6.7. The method and the rates for making particles through gravity alone: Garny, Sandora and Sloth

(2016); Garny, Palessandro, Sandora and Sloth (2018). The gentle re-forming, and the slow shrinking along the arc: *The Weight of Possibility*, Chapter 3, Questions; *The Gravitational Passage*, Appendix B, Section IV. The starting temperature not fixed: *The Gravitational Passage*, Chapter 18, Section XI, and Chapter 19, Section I. The second ground against a long inflation: *The Weight of Possibility*, Chapter 4, Section VI; the measured limit on first waves: Ade and others (2021); *The Gravitational Passage*, Chapter 18, Questions; *The Compactification*, Chapter 6, Section VIII.

Chapter 20. The residue, the baryon and the antiparticle, and the answer in three parts: *The Coming of the Kinds*, Preface and "The Story in Brief". The balance, the hot crowd, and the even annihilation that would leave about one part in ten to the eighteen: *The Coming of the Kinds*, "The Story in Brief", Chapter 2, Section II, and Chapter 5, Section VII; Kolb and Turner (1990). The 411 photons in every cubic centimeter, from the measured temperature: Fixsen (2009); the 6.13 baryons for every ten billion photons: Aghanim and others (2020), with the Particle Data Group (2026); *The Coming of the Kinds*, Chapter 1, Section VI, and Box 1.1. The ratchet, and the three demands for making matter: Sakharov (1967); *The Coming of the Kinds*, Chapter 4, Section I. The sphaleron passage, nine quarks and three leptons together: 't Hooft (1976); the passages running from about ten to the twelve GeV down to 131.7 GeV: D'Onofrio, Rummukainen and Tranberg (2014); *The Coming of the Kinds*, Chapter 4, Section II, and Box 4.1. The angle in the masses of the quarks: Kobayashi and Maskawa (1973); the strength of the angle: Particle Data Group (2026), review "CKM Quark-Mixing Matrix"; the strength computed against the excess: Cline (2006); *The Coming of the Kinds*, Chapter 4, Section III, and Box 4.2. The electroweak change a smooth crossover, bubbles only below about 72 GeV: Csikor, Fodor and Heitger (1999); the measured Higgs mass: Particle Data Group (2026); the crossover at 159.5 GeV: D'Onofrio and Rummukainen (2016); *The Coming of the Kinds*, Chapter 4, Section IV, and Box 4.3. The teeth of the ratchet: *The Coming of the Kinds*, Chapter 4, Section V. The tally counters, what every process keeps, the closed list, and B minus L: *The Coming of the Kinds*, Chapter 5, Sections I and II, and Box 5.1.

The marble, and the erasure: *The Coming of the Kinds*, Chapter 5, Section III, and Box 5.2. The chain followed backward, the main claim in the words of *The Coming of the Kinds*, and the residue the turn leaves: *The Coming of the Kinds*, Chapter 5, Section IV; *The Compactification*, Chapter 5, Section IV. The three routes with no starting point: Yoshimura (1978); Fukugita and Yanagida (1986); *The Coming of the Kinds*, Chapter 5, Section V; *The Compactification*, Chapter 5, Section IV. The share the kinds fix, the estate and the will: *The Coming of the Kinds*, Chapter 7, Sections I to VI, Boxes 7.1 to 7.3, and Summary; the share of 28/79: Harvey and Turner (1990); of 12/37: Laine and Shaposhnikov (2000); the computation across the crossover, 0.3328, give or take 0.0005: Benso, Bödeker, Kamada, Mukaida, Sagunski, Schicho and Schmitz (2026). The 1.84 units of B minus L for every billion photons: *The Coming of the Kinds*, Box 7.3. "The residue is history; the share is structure": *The Coming of the Kinds*, Chapter 7, Summary. One sign everywhere: *The Coming of the Kinds*, Chapter 5, Section VI. The map of the cooling: *The Coming of the Kinds*, "The Story in Brief" and Chapter 14, Table 14.1; the binding of the quarks near 156.5 MeV: Bazavov and others (2019); the neutrinos going free: Chapter 9; the annihilation of electrons and positrons: Chapter 10; the freezing of the neutrons: Chapter 11; the first nuclei: Chapter 12; the release of the light: Chapter 13. The numbers of the cooling and the residue: *The Coming of the Kinds*, Chapter 14, Section IV. Two leftovers of the turn, and the start of our history in three sentences: *The Weight of Possibility*, Chapter 3, Section I, as *The Coming of the Kinds*, Chapter 2, Section III, and Chapter 5, Section IV, quotes it.

Chapter 21. What returns from epoch to epoch, the same counts of the dimensions, shape of the six and held size: *At the Edge of Reality*, Chapter 20 ("What Carries Over"), and Chapter 21. The seven scenes, in the order of the cycle, with no time measured across the turn: *The Compactification*, Chapter 1, Section IX. The end of every interior placed at the upper end of the arc: *At the Edge of Reality*, Chapter 21 ("The Fall"); *The Compactification*, Chapter 1, Section VI ("The placement"). The turn, the re-forming, the capture and the settling, the start of the reconstructed history, the history of the epoch, and the emptying: *The Compactification*, Chapter 1, Section

IX, and Section XI. The identification of the restoration of context with the Big Bang: *The Compactification*, Chapter 1, Section XI; *At the Edge of Reality*, Chapter 21 ("The Turn"). No count carried across the turn, "for the matter or for the geometry": *The Gravitational Passage*, Chapter 18, Section VIII. No geometry built at the turn, and no field defined there, "of any spin": *The Gravitational Passage*, Chapter 18, Section VII. The old distinctions "averaged and not erased": *The Gravitational Passage*, Chapter 18, Section VIII. Information not destroyed, and neither its return nor its loss asserted: *The Compactification*, Chapter 3, Section IX. The three features of the beginning, Penrose's figure, and his condition reached as a consequence: *The Compactification*, Chapter 3, Sections VI and VIII; *The Gravitational Passage*, Chapter 18, Section VIII; Penrose (1979); Penrose (1989). What the turn made, in brief: the dark matter as a conserved excitation of the six, the ratio of the dark matter to the ordinary matter, and each epoch holding only the gyrations its own turn makes: *The Weight of Possibility*, Chapter 6, Sections III and IV. The hum of the settling and the ceiling on the hum: *The Compactification*, Chapter 7, Sections III to V. The residue of matter and the share the kinds fix: *The Coming of the Kinds*, Chapter 5, Section IV, and Chapter 7. The three conditions on the turn: *The Weight of Possibility*, Chapter 6, Section I. The one vacuum, the hold, the floor, and no landscape: *The Compactification*, Chapter 2; *The Gravitational Passage*, Chapter 11, Section II, and Appendix D, Section III. The energy of empty space fixed once, and the matter share: *The Gravitational Passage*, Chapter 13, Sections II to VI; the measured matter share: Aghanim and others (2020). The cosmological constant not stopping the cycle: *The Gravitational Passage*, Chapter 18, Section XII; *At the Edge of Reality*, Chapter 20. The first light recording the averaged form of what the black holes of the epoch before took in: *At the Edge of Reality*, Chapter 21 ("What This Means for You"); *The Compactification*, Chapter 6, Section II. The four marks, and the two numbers taken from measurement, each with its question: *The Compactification*, Chapter 6, Sections IV to VII and XI; the index of 0.9649: Aghanim and others (2020). The coming measurements, and the two predictions that set the account apart from the standard picture: *The Gravitational Passage*, Chapter 20,

Questions; *The Compactification*, Chapter 6, Section X, and Chapter 8, Sections X to XIII; *The Weight of Possibility*, Chapter 6, Section IV. The cycle with no first turn, and the epochs in a single sequence: *The Gravitational Passage*, Chapter 18, Section XII; *At the Edge of Reality*, Chapter 20. The earlier proposals of a universe born inside a black hole: Pathria (1972); Frolov, Markov and Mukhanov (1989); Frolov, Markov and Mukhanov (1990); Smolin (1992); Popławski (2010); a succession of epochs, each joined to the next: Penrose (2010); as *The Gravitational Passage*, Chapter 18, Section XII, cites them. Smolin proposed that each black hole gives rise to a new universe whose constants differ slightly from the constants of the parent universe, so that the population of universes is selected for making black holes. This account, by contrast, has a single sequence of epochs, and in every epoch the axioms fix the structure. Brandenberger and Vafa (1989) let three of the nine small directions of string theory grow large, because strings wound around the small directions annihilate in at most three directions. In this account, three is the number of directions of space of the large world, and the structure of comparison sets that number. *At the Edge of Reality* sets out the proposal of Smolin and the proposal of Brandenberger and Vafa in its Notes to Chapter 20. The two tendencies that take turns: *The Gravitational Passage*, Chapter 18, Section XII. Tolman's objection answered, and no gyration piling up: Tolman (1934); *The Compactification*, Chapter 3, Section VII; *The Weight of Possibility*, Chapter 6, Section III. Whether the measured proportions of matter repeat, left as a topic for another day: *The Gravitational Passage*, Chapter 18, Questions. Where you stand, and the three edges of reality: *At the Edge of Reality*, Chapter 21 ("What This Means for You").

Terms

Each term below is given as this book uses it, with the chapter where it is first explained; the Preface is the opening section, Before We Begin. Where a later chapter explains a term more fully, or uses it in a second sense, that chapter is named too.

Abiding share. The share of the curvature of one factor of the internal space (the complex projective plane or the sphere, the two shapes whose product is the internal space) that holds fast in a measurement as a term of its own, and that is subtracted from a count of classes. To abide is to hold fast (Chapter 19).

Action. Energy multiplied by time; Planck's constant is the size of the least step of action in quantum physics (Chapter 3). Action is also the one quantity from which the laws of motion follow (Chapter 5).

Adiabatic. Of an expansion: with no heat flowing in or out, so that no heat enters or leaves a comoving region and the entropy count of the content in that region stays the same (Chapter 10). Of a departure from uniformity: every component of matter and radiation departs together, so that the number of particles of any one kind, divided by the number of any other kind, is the same everywhere (Chapter 11).

Algebra. The working-out of unknown numbers from the rules those numbers must obey (Chapter 2); a set of rules by which quantities combine, as numbers add and multiply (Chapter 3); in the mathematician's sense, a stock of quantities together with rules for adding and multiplying the quantities (Chapter 5).

Alternative. One of the several definite possibilities in which a quantum thing can be held at once, as a tiny magnet can point up, point down, or be held in both together (Chapter 9).

Amplitude. The power of the ripples at one reference length, a length that astronomers have chosen by convention (Chapter 11).

Amplitude gap. For a detector of gravitational waves, the number of powers of ten by which the weakest energy share that the detector can register lies above the ceiling that the account sets on the settling hum (Chapter 17).

Anisotropic stress. A stress that pulls differently in different directions. A wave of the geometry needs an anisotropic stress (Chapter 12).

Anisotropy. Moving at different rates in different directions (Chapter 2).

Annihilate. Of a particle and its antiparticle: to meet and both disappear, their energy leaving as light and other particles (Chapter 10).

Anti-de Sitter. The name physicists give to a space that draws itself together. The energy of the curvature of the six enters the description of the large world with the sign that makes a space draw itself together, the anti-de Sitter sign (Chapter 4).

Antimatter. Matter made of the twins of the ordinary particles, each twin with the same mass as its ordinary particle and every charge reversed (Preface): that is, matter made of antiparticles (Chapter 20).

Antiparticle. The twin of a particle, with the same mass and every charge reversed. Every particle that carries a charge has an antiparticle (Chapter 10).

Arc. The short stretch of the Kasner curve through its upper end on which all six small directions have negative exponents (Chapter 2).

Area theorem. The rule that, in general relativity, the area of the horizon of a black hole never shrinks: when two black holes merge, the area of the final horizon is at least the areas of the two starting horizons added together (Chapter 9).

Arena. Space and time together, the stage on which everything happens, smooth and uneven, whose shape is gravity (Chapter 1): the context in which everything is measured (Chapter 7). In this account, gravity is the arena itself, and not a force acting within the arena (Chapter 15).

Argued. Of a step: following from stated premises by a stated argument (Preface).

Arrow of time. The one-way running of time; this book describes two arrows of time, the structural arrow and the entropic arrow, two different things that point the same way (Chapter 6).

Axiom. One of the three starting points of the account, found in the structure of comparison: a statement that holds of every comparison whatever. Every argument of the account is reasoned from the three axioms (Chapter 1).

Axiom II. The second of the account's three axioms. Axiom II states the order of the three roles of a comparison: a one-way cycle, the transcendent, the immanent, the omniscient, and the transcendent again (Chapter 6).

Axion. A particle that theories other than the account of this book propose (Chapter 17).

B minus L. The number of baryons less the number of leptons, each number counting particles less antiparticles (Chapter 10). Every process among the kinds of particle keeps B minus L unchanged (Chapter 20).

B-mode pattern. A pattern with a curl in it, in the polarization of the oldest light. First waves would add a B-mode pattern over large patches of the sky (Chapter 17).

Balance. Of a hot gas of particles: the settled state in which every reaction runs as often forward as backward (Chapter 20).

Barrier. A ridge that would have to be crossed to reach other ground, in the picture of the energy of the six as the height of the ground along a path (Chapter 4).

Baryon. One of the protons, neutrons and their relatives (Chapter 10). Baryons carry nearly all the mass of every atom, so physicists count matter in baryons (Chapter 20).

Baryons per photon. The number of baryons for every photon of the oldest light, the measure in which the amount of ordinary matter is stated (Chapter 20).

Base parameters. The six numbers of the standard model of cosmology (Lambda CDM) from which every other number of the model is worked out (Chapter 14).

Bekenstein-Hawking entropy. The entropy count of a black hole: one quarter of the area of the horizon of the hole, measured in units of the Planck area (Chapter 7). By my identification, the Bekenstein-Hawking entropy counts the states of the hole that no comparison made from outside the hole tells apart: a count relative to the outside as context (Chapter 7).

Big Bang. The beginning that someone finds when tracing a history of the universe backward. In this account, by my identification, the Big Bang is the turn as the turn appears in the history reconstructed from inside the epoch (Chapter 1).

Bit. The answer to one yes-or-no question. One nat is about one and forty-four hundredths bits (Chapter 8).

Black hole. A region where so much matter is squeezed into so small a space that nothing can climb back out, not even light (Chapter 1).

Borromean rings. Three rings, each at right angles to the other two, linked so that no two hold together without the third. The Borromean rings picture the rule by which the volume of the six is counted in Planck polydiscs (Chapter 3).

Boson. One of the particles that are force carriers or belong to the Higgs field; in the count of species, each state of a boson counts one (Chapter 10).

Boundary condition. A condition placed at one end of a history, as the Past Hypothesis is placed at the start of the history of our universe (Chapter 6).

Bounded, with its value not given. The standing of a number on which the account sets a ceiling without giving the number's value, as the account does for the temperature at which our history begins (Preface).

Breathing excitations. The even swellings and shrinkings of the size of the six, above their lowest motion (Chapter 4).

Bubble. A small region of a lower vacuum that forms within a world held in a higher vacuum, and that grows (Chapter 5).

C one. The name that *The Gravitational Passage* gives to the count of three in the cosmological constant. The name is borrowed from a second count of the shape of the complex projective plane, the plane's first Chern class, which has the same number, three. The account uses no property of the first Chern class beyond the fact that the first Chern class of the complex projective plane has the same number, three, as the count in the cosmological constant (Appendix A).

Calabi-Yau space. One of the shapes usually taken for the six small dimensions of string theory (Chapter 5). In a Calabi-Yau space, a certain count of the shape is zero: the count that mathematicians call the first Chern class of the shape. A Calabi-Yau space can also carry a geometry in which the total bending of every direction is zero (Appendix A).

Capture. The end of the re-forming: the floor receives the re-forming six in the floor's well, and the six come to rest at their held radii (Chapter 4).

Category error. A mistake of category (Chapter 7): applying a concept where the concept does not belong (Chapter 9).

Cell of action. A patch of the size of Planck's constant. In quantum counting, one state takes up one cell of action (Chapter 3).

Cepheids. Nearby pulsating stars whose rhythm of pulsing tells how bright each Cepheid is. Cepheids are the first rung of a distance ladder (Chapter 14).

Channel. One of the ways a measurement takes in what the measurement finds, one way for each role of a comparison (Chapter 3): that is, a content that a measurement registers (Chapter 8).

Characteristic strain. The fraction by which the passing waves of a background stretch and squeeze a length. Characteristic strain is the unit that detectors use (Chapter 16).

Chirality. The handedness of a particle, left-handed or right-handed: the two ways the particle can turn about its direction of motion (Chapter 20).

Class. A type of thing, taken as a whole and standing ready (Chapter 6). In counting the pieces of a shape, each independent basic piece of the shape (Chapter 19).

Classical. Of equations: those of general relativity, which take no account of the quantum (Chapter 2); of a geometry: smooth, with no steps in the geometry (Chapter 9). A classical geometry is a smooth shape of the arena, described without quanta (Chapter 15).

Cold. Of the dark matter: moving slowly (Chapter 14); of the gyrations: slow, with no motion from heat that could matter (Chapter 18).

Color. The label the strong force acts on (Chapter 20).

Comoving region. A region that grows with the expansion and always holds the same matter (Chapter 10).

Compact. Of a space: finite and without edge, as the surface of a ball has a finite area and no edge (Chapter 1).

Compactification. In this book, the six being held compact: held small, as the content of the large world of four dimensions, which is the context of the six (Chapter 1). In this book the word never means that six directions of space were once spread out and then curled up. The six were never directions of space (Chapter 1).

Compactification conditions. The conditions under which the six are held compact: the large world is present as a context for the six, as a domain of the smooth description, with a reference for inertia fixed in the large world (Chapter 1).

Comparer. Anyone or anything that makes a comparison, such as an observer with an instrument (Chapter 5).

Comparison. The act of finding whether two things are the same or different, in some respect (Preface). Every observation, every measurement and every test is a comparison (Chapter 1).

Complex projective plane. The space of all the states of a three-way choice (Chapter 1): a shape of four dimensions, the larger of the two factors of the internal space (Chapter 3).

Computed. Of a value: arithmetic on stated inputs (Preface).

Condensate. The settled value of the Higgs field, the same everywhere (Chapter 5).

Conditions on the turn. Three requirements that the account sets for the turn: the plasma of the new epoch is never too hot, the settling waves stay within the measured bound, and the re-forming is smooth over about ten swings of the lightest gyration. The three conditions on the turn are conditions, not results: the account contains no derivation showing that the turn meets the three conditions (Chapter 4).

Conformally flat. Of a geometry: with the Weyl curvature of the geometry zero, so that circles may swell or shrink but stay circles (Chapter 7).

Conjugate. Paired as counterparts (Chapter 7).

Conserved. Kept: a conserved thing, once made, does not go away; lasting (Chapter 4; in these words, Chapter 18).

Constant of integration. A number that the law of a history does not fix, set once by the state the history starts from and carried unchanged afterward. The curvature of space as a whole is a constant of integration (Chapter 12).

Content. What the person or instrument making a comparison finds, as a thermometer comparing morning and evening finds the temperature (Chapter 1).

Content instance. The internal space taken as a space of states, with no size, no radius and no fields. Nothing on the content instance has a length, and the kinds of particle and their values are read on the content instance (Chapter 1).

Context. The respect in which the content of a comparison is found, as the two times of day are the context for a thermometer comparing morning and evening. Place, time and possibility are the three contexts in which anything is ever found (Chapter 1).

Context instance. The internal space taken as the space of possibility in which the content of an epoch is held, with sizes and a curvature. Gravitation rests on the context instance, and so do the cosmological constant and the dark matter (Chapter 1).

Continuity. Smoothness, with nothing sudden (Preface): a sameness without a break (Chapter 6).

Conversion into light. Turning a gravitational wave into light inside a magnetic field, as the physicist Mikhail Gertsenshtein described in 1962. With conversion into light, an instrument built around a magnetic field can set a bound on gravitational waves at high frequency, or detect those waves (Chapter 17).

Cosmic string. A thin line of trapped field energy (Chapter 13).

Cosmological constant. The energy of empty space, which drives the accelerating expansion of the universe and appears as one constant term in Einstein's equations (Preface). The constant term is the same at every place and time (Chapter 8). I identify the cosmological constant with the share of the energy of the vacuum that a measurement inside the vacuum engages, and not with the total (Chapter 8).

Cosmological event horizon. The edge of everything whose light will ever reach the observer, in all the future (Chapter 8).

Count of species. The number of particle states in balance that share the heat, each fermion state counted as seven eighths (Chapter 10).

Critical density. The total energy in each volume of space that a flat universe, expanding at the present rate, must hold (Chapter 14).

Crossover. A smooth change, in which the state follows the temperature continuously, with no persisting old state, no bubbles and no latent heat (Chapter 10).

Curvature. Bending, like the bending of the surface of a ball (Chapter 1). For a sphere, the curvature is one over the square of the sphere's radius (Chapter 3).

Curvature of space. Whether space on the largest scale is flat like a sheet of paper or curved like a ball or a saddle. Astronomers report the curvature of space as a share of the total density, and a share of zero is flat (Chapter 11).

Curvature perturbation. The quantity that says, place by place, by what fraction space has stretched more or less than on average, measured on surfaces where the total density is the same (Chapter 11).

Cycle, the. The sequence in which each epoch makes stars, the stars make black holes, the ends of the black holes are one end, and the turn restores a large world: the next epoch (Chapter 1).

Dark matter. The unseen matter whose pull is measured in every galaxy (Preface). I identify the dark matter as a conserved excitation of the six held internal dimensions, a vibration of context, not an item of content, made at the turn. The dark matter is made of gyrations (Chapter 18).

De Sitter radius. The edge that a universe holding nothing but the energy of empty space would have (Chapter 8).

De Sitter space. A model of an expanding universe with exactly the same shape at every place and in every direction. The exact symmetry of de Sitter space belongs to the model on paper, and not to anything that someone making a comparison finds (Chapter 7).

De Sitter time. The de Sitter radius divided by the speed of light (Chapter 14).

Degree of freedom. One independent way in which what lies inside a horizon can hold energy or vary, as each string of a piano can ring on its own (Chapter 8).

Departures from uniformity. The faint differences of density with which an epoch begins (Chapter 9). At the beginning of an epoch the departures from uniformity are also called the primordial departures, and in this book I call them the first ripples (Chapter 11).

DESI. The Dark Energy Spectroscopic Instrument, which maps how galaxies are spread through space and measures distances with a ruler of fixed size that sound in the early universe left in that spread (Chapter 14).

Detector classes. The nine kinds of instrument that *The Compactification* surveys, each kind listening for gravitational waves at high frequency in its own way (Chapter 17).

Dimension. An independent way in which things can differ (Preface).

Dirac mass. The usual kind of mass, which joins the left-handed part of a particle to the right-handed part of the same particle (Chapter 20).

Distance ladder. A method of measuring the expansion step by step, from nearby pulsating stars called Cepheids out to exploding stars farther away (Chapter 14).

Domain wall. A sheet of trapped energy between regions that settled on different choices (Chapter 13).

Doublet. A pair of fields that the weak force turns into one another, as the Higgs field is (Chapter 5).

Dynamical horizon. A horizon found from the geometry near a black hole at each moment, rather than from the whole future: the name physicists give a trapping horizon while the trapping horizon grows (Chapter 9).

E-fold. A step in counting how much an inflation stretched space: one e-fold is a growth by a factor of about two and seventy-two hundredths, the number mathematicians call e. This account counts no e-folds (Chapter 13).

Effective number of light species. The energy of all the radiation other than light, counted in units of the energy that one kind of neutrino carries; written N eff (Chapter 16).

Eigenvalue. The first eigenvalue of a shape is the sharpness of the gentlest ripple the shape allows. The first eigenvalue of a shape measures the slowest variation the shape admits (Chapter 3).

Einstein condition. The condition met by a geometry in which every direction has the same Ricci curvature: every direction bends by the same total amount (Chapter 3).

Einstein's equations. The equations of gravitation of general relativity, which I derive from the structure of comparison in *The Gravitational Passage* (Chapter 1).

Electron volt. The energy an electron gains in crossing one volt (Chapter 5). A billion electron volts, written GeV, is the unit in which particle physicists measure mass and energy (Chapter 5). A million electron volts, written MeV, is a thousandth of a GeV (Chapter 10).

Electroweak transition. The change in which the Higgs field takes its value (Chapter 16): the cooling through the scale of the weak force, in which the Higgs field settles at its value and the masses switch on (Chapter 20).

Energy share per band of pitch. The share of the critical density that gravitational waves carry within one band of frequency, where a band is a stretch of frequencies across which the pitch grows by one fixed factor, as an octave doubles the pitch (Chapter 16).

Entangled. Of things: joined so that none of the things can be described without the others (Chapter 3); joined so that part of what is true of each thing is held only in the relation between the things (Chapter 8).

Entanglement entropy. The count that measures how far the radiation of a black hole is entangled with the hole: how much the radiation and the hole share in quantum correlation (Chapter 9).

Entangling phase. The angle that marks the entanglement of two specks of matter through gravity: the part of the drift of the clock hands of the two specks that no adjustment of either speck on its own could produce or undo (Chapter 9).

Entropic arrow. The growth of entropy in a thing left to itself. The entropic arrow is statistical, and the entropic arrow is the arrow of time that needs an orderly beginning (Chapter 6).

Entropy. A count of the states that a context cannot tell apart, reckoned relative to that context: many ways for the parts to be, all looking alike from outside (Preface; in full, Chapter 6).

Epoch. One era of the world, pieced together from inside the era, in which stars, galaxies and black holes form (Chapter 1).

Equality of matter and radiation. The moment at which the matter comes to hold as much energy as the radiation. After the equality of matter and radiation, the matter holds more of the energy than the radiation does, and so the matter sets how the expansion proceeds (Chapter 10).

Equilibrium. The settled state in which nothing further changes on average (Chapter 7).

Equivalent density. At a depth inside a black hole, the density at which the mass of the hole would fill a sphere whose radius is the radius of that depth (Chapter 2).

Established physics. A result of physics, taken from the source where the result was established (Preface).

Euler count. Corners minus edges plus faces, and more generally the number of the basic pieces of a shape counted with signs (Chapter 3).

Evaporation. The slow loss of mass that the Hawking radiation of a black hole carries away (Chapter 9).

Even. Of the beginning of an epoch: without the curvature that distorts shapes, up to the small departures the Incommensuration Theorem requires (Chapter 7). The sense of the word "even" in this entry is different from the sense of the word in the even and odd levels of vibration.

Even and odd. Of a level of vibration under a parity: the level is even if the level keeps its sign under the parity, and odd if the level flips its sign (Chapter 18).

Event horizon. Of a black hole: the boundary of the region from which no signal reaches the outside (Chapter 8).

Excitation. A raised state of vibration, a ringing above the quietest state (Chapter 18).

Exist and real. What exists stands as structure, laid out the same for everyone who looks (Chapter 7), timeless, like a law. What is real is given in an actual act, here and now (Chapter 8).

Expansion rate. How fast every distance grows, as a share of itself, in each unit of time (Chapter 10).

Exponent. The power to which the time counted from the limit is raised, in the Kasner form. Each direction of space has its own exponent (Chapter 2).

External value. The energy of empty space as an observer outside would find that energy, if there were an outside, with the energy taken at the Planck scale (Chapter 8).

Faithful reversal. A reversed process that retraces exactly the states the original process passed through (Chapter 6).

Fermion. A quark or a lepton. In the count of species, each state of a fermion counts seven eighths (Chapter 10).

Field. Something spread through space with a value at every point, as the temperature of a room has a value at every point in it (Chapter 1).

Finite Dirac operator. A table of numbers that links the pieces of the algebra of the content and carries the masses of the matter particles and the ways the matter particles mix with one another (Chapter 5).

Firewall. A wall of energy at the horizon that would burn whatever falls in (Chapter 9).

First atoms. Nuclei holding their electrons. Once the first atoms form, nothing turns the light aside, and the light travels free (Chapter 20).

First nuclei. The cores of the lightest atoms, formed in the first minutes of our history (Chapter 20).

First order. In the largest and simplest part of an effect, leaving the smaller corrections aside (Preface).

First-order transition. A change in which the old state persists below the temperature at which the new state would be favored, the new state forms in bubbles, and the bubbles grow, collide and release a latent heat (Chapter 10).

First ripples. The departures from uniformity with which our epoch began, and from which the galaxies of our epoch grew (Chapter 11). See Departures from uniformity.

First waves. Gravitational waves present from the beginning of an epoch (Chapter 11). See Primordial gravitational waves.

Flat universe. A universe in which, on the largest scales, space has no overall bend, so that lines that start out parallel stay parallel (Chapter 8).

Flatness problem. The question why the beginning was so flat, since in the standard history any small curvature at the beginning would have grown enormously (Chapter 12).

Floor. What the hold rests on: the internal space cannot shrink to nothing (Chapter 1). The floor is a least extent, with a least motion that cannot be removed (Chapter 4).

Flux. In string theory, a field threaded through the loops of the six small dimensions (Chapter 5).

Flux integer. The whole number that counts how many times a flux threads one loop (Chapter 5).

Form of the content. Which alternatives the content has taken, that is, what has been selected. The form of the content is what passes the turn (Chapter 1).

Frequency. The number of times a wave rises and falls each second (Chapter 15). For a sound, the frequency is the pitch (Chapter 16).

Frequency gap. For a detector of gravitational waves, the number of powers of ten by which the top of the detector's band falls below five hundred billion hertz, the lowest frequency of the settling band (Chapter 17).

Friedmann's equation. The equation that the physicist Alexander Friedmann found from Einstein's theory in 1922. In a flat universe, Friedmann's equation says that the rate of the expansion fixes how much energy the universe must hold in all (Chapter 8).

Gauged. Of a symmetry: holding because the arrangements the symmetry relates are one and the same. A gauged symmetry is exact, and nothing can violate a gauged symmetry (Chapter 18).

General relativity. Einstein's smooth description of the arena (Chapter 1).

Generalized second law. The old law that the entropy of a closed system never falls, widened to take in black holes: the entropy of the black holes plus the entropy of everything outside the black holes never decreases (Chapter 9).

Gentle re-forming. A re-forming of the six, together with the capture by the floor at the end of the re-forming, that is slow and gentle compared with one swing of the lightest odd level of vibration (Chapter 19).

Global symmetry. A symmetry of the ordinary kind, which relates two arrangements that are different and behave alike (Chapter 18).

Gluon. A carrier of the strong force (Chapter 10).

Grand unification. The route to an excess of matter through theories in which the strong, weak and electromagnetic forces join into one force at very high energy, inside one larger pattern of symmetry called a unifying group (Chapter 20).

Grand unified theory. A theory in which the strong force, the weak force and electromagnetism are, at very high energy, one force with one larger symmetry, and the one force breaks apart into the three forces as the universe cools (Chapter 13).

Gravitational waves. Ripples in the shape of space and time themselves (Preface). As gravitational waves pass, they stretch distances one way and squeeze distances the other way (Chapter 4).

Graviton. At most a quantum of the description of a small wave of the arena: one step in the energy of one way a small wave of the geometry can ripple. A graviton is never a quantum of the arena itself, never a particle, never a carrier of gravitation, and never one of the kinds of particle (Chapter 7; in full, Chapter 15).

Ground state. The lowest and quietest quantum state (Chapter 7).

Gyration. A conserved excitation of the six held internal dimensions, a lasting vibration of their geometry, of about the Planck mass (Chapter 4). A gyration is a vibration of the held space, as a phonon is a vibration of a crystal, and not an extra thing inside the held space. The dark matter is made of gyrations, made at the turn (Chapter 18).

h. The Hubble constant divided by a hundred (Chapter 14).

Halo. The larger dark cloud around the bright core of stars and gas of each galaxy (Chapter 18).

Haloscope. A strong magnet around a microwave cavity, built to turn axions into microwaves. A haloscope can also turn a gravitational wave into microwaves, as the haloscope turns axions into microwaves (Chapter 17).

Hawking radiation. The faint glow of a black hole, the way any warm body glows; the smaller the hole, the warmer its glow. Hawking radiation is a result for quantum fields on a classical geometry (Chapter 9).

Hawking temperature. The temperature of the Hawking radiation of a black hole (Chapter 9).

Held compact. Of the six: held small, as the content of the large world of four dimensions, which is the context of the six (Chapter 1).

Held structure. A structure whose parts are held in place by bonds, as the bonds of a crystal hold its atoms in the lattice (Chapter 7).

Hertz. The unit of frequency: one hertz is one rise and fall each second (Chapter 15).

Higgs boson. The particle that is a small ripple of the Higgs field (Chapter 5).

Higgs field. A field that fills all of space and, by settling at a value that is not zero, gives many of the elementary particles their masses (Chapter 5).

Hold, the. The state of the six while the compactification conditions are met: the six are held as content, and each of the six directions lies at the modal boundary, where the curvature that the direction meets is the Planck curvature (Chapter 1).

Holometer. A pair of laser interferometers at Fermilab, side by side, each thirty-nine meters long. The Holometer has run and finished (Chapter 17).

Homogeneous. Of a shape: every point of the shape is like every other point, in the strict sense that a symmetry of the shape, one that keeps every distance unchanged, can carry any point to any other (Chapter 3).

Horizon. Of a black hole: the surface from inside which no signal can reach the outside (Chapter 1). In general, a boundary across which signals pass in one direction only (Chapter 8).

Horizon of a time. For a given time, the greatest distance a signal could have crossed by that time (Chapter 11).

Horizon problem. The question how regions that never exchanged a signal could agree so closely (Chapter 12).

Hubble constant. The present rate of the expansion: the speed at which a distant galaxy moves away from us for each unit of the galaxy's distance (Chapter 14).

Hubble radius. The distance at which the expansion carries things away from us at the speed of light (Chapter 8).

Hypercharge. A part of the joint electromagnetic and weak force, tied to electric charge (Chapter 13).

Identification. A step from the structure of comparison to something in the world, which I make in my own voice, with the words "I identify", and which the book states together with any observation that bears on the identification (Preface).

Identity, the. The one statement in pure numbers left when the two ways of finding the energy of empty space, the share and Friedmann's equation, are set equal: the square of the radius of the particle horizon, measured in Hubble radii, times the share of the vacuum in the whole, equals eight, less three divided by pi (Chapter 8).

Identity of the present measurement. A rule that states what a measurement made now, from inside the epoch, registers. *The Weight of Possibility* fixes the ratio of dark to ordinary matter by an identity of the present measurement (Chapter 19).

Immanent. The role of what is happening in a comparison: the act itself, as it occurs, like the click of a camera (Chapter 6).

Incommensuration Theorem. A structural fact about the nature of knowing: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and neither can asymmetry and discontinuity (Preface).

Indirect test. The count of light species, as a test of the settling hum: the count of light species registers the waves through the effect of the waves on the expansion, not through their effect on a detector (Chapter 17).

Induced background. A background of gravitational waves that the first ripples make as the ripples act on themselves, rather than a background present at the start beside the ripples. An induced background is also called second-order (Chapter 16).

Inertia. The tendency of a body to keep its state of motion (Chapter 1).

Inflation. The proposal, made by the physicist Alan Guth in 1981, that an early phase of ever faster expansion stretched a small region, whose parts had once been in contact with one another, to far beyond everything we can now see (Chapter 12; in full, Chapter 13).

Inflaton. A field spread through space whose energy pushes space apart while the field drifts slowly down a gentle slope of its own energy. In the usual form of inflation, an inflaton drives the expansion. This account has no inflaton (Chapter 13).

Information. A word with two senses, which I keep apart in *The Unquantized Arena*: the form of the content, carried with the matter wherever the matter goes; and a count of states relative to a context (Chapter 7).

Information paradox. The question how information that falls into a black hole can be recovered in the outside world, since nothing comes back out, and the hole may in time evaporate (Chapter 7).

Infrared. Light of lower frequency than red, which the eye cannot see (Chapter 15).

Initial conditions. The conditions at the start of a history, as the factory order is the initial condition of a deck of cards (Chapter 6).

Initial state. The state from which a description of a system is run forward (Chapter 1).

Instance. One of the two ways in which the internal space, one pattern, is taken: the content instance and the context instance of the internal space (Chapter 1). In general, one particular case of a type, here and now (Chapter 6).

Interferometer. A detector that measures tiny changes of distance with light (Chapter 16): an interferometer splits a laser beam, sends the halves down two arms, and brings the halves back together, so that a tiny change in the length of one arm against the other shows as a change in the light (Chapter 17).

Interior. Of a black hole: everything inside the horizon (Chapter 1).

Internal space. The one small space that the six form together: the range of what content could be (Chapter 1).

Internal value. What a measurement finds of the energy of empty space: a share of the external value (Chapter 8).

Island. A region behind the horizon of a black hole that physicists, in one method of computing the entanglement of the hole's radiation, add to the radiation and count with the radiation, past the turning point of the Page curve (Chapter 9).

Isocurvature. Of a departure from uniformity: one component departs against another, with the total density unchanged at first; from words meaning "the same curvature" (Chapter 11).

Isotropy. Sameness in every direction (Chapter 7).

Kasner curve. The loop of exponents allowed when three, four and two directions each move together (Chapter 2).

Kasner form. The simple form space takes near a limit such as the end of the interior of a black hole or the start of an epoch: each direction of space has a size that changes as the time counted from the limit, raised to a fixed number (Chapter 2).

Lambda CDM. The standard model of cosmology; Lambda stands for the cosmological constant, and CDM for cold dark matter, dark matter that moves slowly (Chapter 14).

Landscape of vacua. The whole range of held arrangements a theory allows, each arrangement a valley of its own (Chapter 5).

Last scattering. The last turning aside of the light by the free electrons of the early universe. Last scattering is often called recombination (Chapter 11).

Latent heat. Heat that the old state held hidden, released in a first-order transition (Chapter 10).

Lattice computation. A calculation made by computer on a fine grid of points in space and time (Chapter 10).

Lawful reversal. A reversed process that obeys the laws (Chapter 6).

Leptogenesis. The route to an excess of matter in which physicists add heavy neutrinos with Majorana masses. The decays of the heavy neutrinos out of balance leave an excess of leptons, and the sphaleron passages carry part of that excess over into baryons (Chapter 20).

Lepton. One of the electron, its heavier copies and the neutrinos (Chapter 10); in full, the electron, its two heavier copies the muon and the tau, or one of the three neutrinos (Chapter 20).

Levitated sensor. Tiny beads or discs of glass, held up and cooled by laser light inside a cavity between two mirrors, that serve as a tunable ringing sensor for gravitational waves (Chapter 17).

Li-Baker concept. A proposed detector that would register gravitational waves with a microwave beam of smooth, bell-shaped profile, a steady magnetic field and patterned reflectors, reading the waves through the small change the waves make in the flow of light in the beam (Chapter 17).

Light cone. All the places, and the later moments, that a signal sent from a place and a moment can reach (Chapter 9).

Light year. The distance light travels in a year (Chapter 8).

LIGO. The pair of detectors that registered the first merger of black holes ever detected, on 14 September 2015 (Chapter 9).

LISA. A planned detector in space that will listen for slow waves by measuring, with laser light, the changing distances between spacecraft far apart (Chapter 16). The full name of LISA is the Laser Interferometer Space Antenna, and the European Space Agency has adopted LISA (Chapter 17).

LiteBIRD. A satellite, forecast to measure the ratio of first waves to first ripples (Chapter 12).

Logarithm. The power to which a fixed base must be raised to give a number. In this book the base is the number e, and a count stated by its logarithm to the base e is a count in nats (Chapter 10).

Lower end. The end of the Kasner curve at which each of the three large directions has the exponent minus one third, and each of the six has the exponent one third (Chapter 2).

Magnetic monopole. A particle that carries a single magnetic pole, a north with no south (Chapter 13).

Magnon. One step of the shared wobble of the tiny magnets of a magnetic crystal (Chapter 17).

Majorana mass. A mass that joins a particle to its own antiparticle (Chapter 20).

Marginally trapped. Of a surface: a surface on which the outgoing light neither spreads apart nor draws together, but holds level, while light sent inward draws together (Chapter 9).

Matter fraction. The part of the energy of a flat universe that is matter, ordinary and dark together (Chapter 8).

Matter share. The part of the universe's energy held as matter, ordinary and dark together. Chapter 8 calls the matter share the matter fraction (Chapter 14). See Matter fraction.

Measured. The standing of a number that comes from an instrument and that the account uses as measured, as the account uses Newton's constant of gravity (Preface).

Measured input. A value copied from measurement: the account uses the measured value and offers no value of its own (Chapter 14).

Measured, with its reason still a question. The standing of a number whose measured value the account uses while saying what would set the number, as the account does for the strength of the first ripples (Preface).

Megaparsec. A unit of distance of about three and a quarter million light years (Chapter 14).

Merger. The whole event in which two black holes become one, from the last circuits of the two holes to the settled hole (Chapter 9).

Metastable. Of a vacuum: held for a long time, and able to pass by tunneling to a lower valley, a vacuum of lower energy (Chapter 5).

Metric. The measuring rule of a geometry: a rule that says, at every point, how long each small step in each direction is (Chapter 5).

Microwave background. The faint afterglow of the early universe that fills all of space (Chapter 9): light left from the early universe, stretched by the expansion, the oldest light there is (Chapter 11).

Modal boundary. The Planck scale, where the two characters of the Incommensuration Theorem, smooth and stepwise, trade places as the character that rules. The range of each description ends at the modal boundary, and beyond the boundary no third description takes the place of the two (Chapter 1).

Modulus. One of the numbers, called moduli, that describe the size and shape of an internal space, settings like the dials of an instrument. The six have two moduli, the volume of the six and the shape of the six (Chapter 4). In the precise sense of Chapter 5, a number that describes the size or the shape of the six and could differ from place to place in the large world (Chapter 5).

Monopole problem. The difficulty that grand unified theories predict so many magnetic monopoles that the universe would hold more mass than its measured expansion allows (Chapter 13).

Muon. The heavier copy of the electron (Chapter 10).

Nat. The natural unit of information: one nat is about one and forty-four hundredths bits (Chapter 8).

Neutrinos. The lightest particles of matter, which barely touch anything else. There are three kinds of neutrino (Chapter 4).

Neutrinos go free. The neutrinos stop colliding with the rest of the hot crowd of particles and travel on their own, within the first second (Chapter 13).

Neutron star. The collapsed core of a large star, packed as densely as the nucleus of an atom (Chapter 9).

Numerical relativity. General relativity solved by computer, where no formula on paper will do (Chapter 9).

Omniscient. The role of what is laid out as structure in a comparison, which can be held in view as a whole, like a map, a record or the print of a photograph (Chapter 6).

Optical depth. The number that says how thick the fog of electrons freed by reionization is. The fog of freed electrons scatters a little of the first light on the light's way to us (Chapter 14).

Page curve. The rise and fall of the entanglement entropy of the radiation of a black hole, under two conditions: first, that the content inside the horizon and the radiation outside, taken together, began pure and stay pure, as described from one outside world; and second, that the entanglement entropy of the content inside the horizon cannot exceed the Bekenstein-Hawking entropy of the hole. In *The Unquantized Arena* I derive the Page curve as a result under the two conditions (Chapter 9).

Page time. The turning point of the Page curve (Chapter 9).

Parity. A swap under which the laws stay the same (Chapter 18).

Particle horizon. The edge of everything whose light has reached us since the expansion began (Chapter 5).

Past Hypothesis. The condition that the universe began in a state of extraordinarily low entropy, named by the philosopher of physics David Albert in 2000 (Chapter 6).

Phase. How far the tiny clock hand on each alternative has turned compared with the clock hands on the other alternatives (Chapter 3).

Phonon. One step of energy in one way the lattice of a crystal can vibrate: a quantum of the description of how a held structure vibrates, and not a piece of the crystal (Chapter 7).

Phonon detectors in crystals. Detectors able to register single phonons in crystal targets. A proposal of 2023 showed that phonon detectors in crystals could search for gravitational waves (Chapter 17).

Photon. Any particle of light (Chapter 11).

Placement. The choice of the one point on the arc at which both histories, the end of the interior of a black hole and the opening of an epoch, are set. *The Gravitational Passage* sets both histories at the upper end (Chapter 2).

Planck area. The square of the Planck length (Chapter 7).

Planck curvature. The bending of the surface of a ball whose radius is one Planck length (Chapter 1).

Planck density. The density at which the smooth description of space and time reaches the Planck scale, one Planck mass in every cube one Planck length on a side (Chapter 2).

Planck length. The length at the edge of the domain of the smooth description of space and time: below the Planck length, the smooth description no longer holds. The Planck length is about a hundred billion billion times smaller than a proton (Preface).

Planck mass. The one mass for which the gravitational size of a thing (the radius to which the thing would have to be squeezed to become a black hole) and the quantum size of the thing (the wavelength of the thing's quantum wave) are equal: about twenty-two millionths of a gram (Chapter 1).

Planck polydisc. Three discs, each one Planck length in radius, one in each of the three pairs of directions of the six, held together as a product. The volume of the Planck polydisc is the unit in which the volume of the six is counted (Chapter 3).

Planck scale. The scale at which gravity and the quantum act on equal terms (Chapter 1).

Planck temperature. The temperature at which a typical particle carries the energy of a Planck mass (Chapter 4).

Planck time. The time light takes to cross a Planck length (Chapter 4); the briefest interval built from the constants of gravity, quantum theory and light, which is not a least step of time (Chapter 15).

Planck's constant. The size of the least step of action in quantum physics (Chapter 3).

Plasma. The gas of particles so hot that nothing in the gas holds together (Chapter 4).

Polarization. Of a gravitational wave: one of the two patterns in which the wave stretches and squeezes space across the wave's path (Chapter 15). Light is polarized when the light vibrates more in one direction than in another (Chapter 17).

Possibility. The range of what someone making a comparison could find. With place and time, possibility is one of the three contexts in which anything is ever found (Chapter 1).

Power. The strength of the ripples multiplied by itself, the quantity the measurements compare (Chapter 11).

Power of ten. One factor of ten. Ten to the power five is five tens multiplied together, a hundred thousand (Chapter 2).

Power spectrum. The power of the ripples in each band of length, each band spanning the same factor of length: how strong the ripples are at each length (Chapter 11).

Precedence. An order that says what must come before what, as a question must come before its answer. Precedence is not an order of clock time (Chapter 6).

Price of being inside. The part of the external value of the energy of empty space that the internal value, the share a measurement finds, leaves out (Chapter 8).

Primordial gravitational waves. Gravitational waves present from the beginning of an epoch. In this book I call primordial gravitational waves the first waves (Chapter 11).

Product. A shape in which every point is a pair, one point taken from each of two parts (Chapter 1). Within a product, each of the two parts keeps its own character (Chapter 3).

Production through gravity. A hot gas making pairs of anything that has weight, through gravity alone, when two particles of the gas meet (Chapter 19).

Pulsar. A spinning dead star that sweeps a beam of radio waves past the Earth as regularly as a clock (Chapter 15).

Pulsar-timing array. A set of pulsars timed over years: a slow gravitational wave passing between the pulsars and us shifts the arrival times of the pulses, in a pattern shared across the sky (Chapter 16).

Pure. Of a quantum state: as fully specified as quantum theory allows (Chapter 9).

Quality factor. The measure of how slowly a ringing dies away (Chapter 17).

Quanta of the description. Steps that belong to the way small waves of the arena are described, and not to the arena that ripples (Chapter 7; in full, Chapter 15).

Quantum. One least step of something that comes only in equal steps, as light comes in photons (Chapter 7; in full, Chapter 15).

Quantum state. Quantum theory's description of matter: a quantum state holds the alternatives of the matter, the different ways the matter could be found (Chapter 7).

Quantum theory. The stepwise description of what fills the arena (Chapter 1).

Quark. One of the particles of which protons and neutrons are made, three to each (Chapter 2).

Quartz resonator. A crystal of very pure quartz mounted between two electrodes, which turn the vibrations of the crystal into an electric signal, since quartz makes a small voltage when quartz is squeezed. A passing gravitational wave squeezes the crystal, and the crystal rings (Chapter 17).

Radian. A unit of angle; a full turn is a little over six and a quarter radians (Chapter 9). A radian is a little over fifty-seven degrees (Chapter 14).

Radiation. Light and other fast particles (Chapter 8).

Radiation era. The stretch of the history in which nearly all the energy of the content is radiation: light, together with every particle moving so fast that it behaves as light does (Chapter 10).

Radius. The distance from the center of a sphere to its surface (Chapter 3).

Ratio of dark to ordinary matter. The mass of dark matter in a region divided by the mass of ordinary matter in the same region (Chapter 12). The standing of the ratio is the registered ratio (Preface; Chapter 14).

Ratio of first waves to first ripples. The power of the first waves divided by the power of the first ripples, both taken at the reference length. Physicists call the ratio of first waves to first ripples the tensor-to-scalar ratio (Chapter 11).

Re-forming. The return of the six, released at the turn, to their compact form in the new epoch. The re-forming ends when the floor catches the six, at the capture (Chapter 4).

Reconstructed history. The one history that the present requires, pieced together from impressions arriving now. The reconstructed history exists as structure, as a map or a record exists (Chapter 1).

Record. Anything that holds a finding after the act that found it (Chapter 6).

Redshift. The stretching of the waves of light by the expansion: light arriving now from a moment in the past has waves longer than when the light set out, by the factor by which every distance has grown since; astronomers quote a redshift as that factor less one (Chapter 11).

Reduced Planck mass. A Planck mass smaller than the full Planck mass by the square root of eight pi. Some physicists use the reduced Planck mass (Chapter 8).

Reference for inertia. The standard of not accelerating and not rotating, fixed by matter within reach, against which motion is compared; also called the Mach reference (Chapter 1).

Registered ratio. A ratio fixed by what the present measurement registers, and not by a mechanism at the turn; the one number in this book that keeps a standing of its own (Preface; explained in Chapter 14).

Registration. The act by which a measurement made inside the world fixes what the measurement registers (Chapter 14).

Reheating. In this book, only the end of a stage of the content that behaved as matter, if there was such a stage: the decay that handed the energy of that stage to the plasma (Chapter 10). In the usual sense of the word, the handing on of the energy of an inflaton to ordinary hot matter and light. This account has no inflaton whose decay the word "reheating" names (Chapter 13).

Reionization. The second charging of the gas between the galaxies, when the light of the first stars broke the gas back into charged pieces (Chapter 14).

Release of the six. The six coming to be no longer held compact, at the end of the interiors of the black holes, at the turn (Chapter 2).

Reset, the. The fact that at the turn no count reckoned against the old outside world is carried across, while the content, and the form of the content, are carried across (Chapter 7).

Residue. What remains when nearly everything else has gone one way (Chapter 20). The amount of ordinary matter is a residue from the earliest moments, and no process among the kinds of particle makes that residue. The kinds of particle fix only the share held as baryons (Chapter 20).

Resonance. The way almost every detector of gravitational waves at high frequency works: a part of the instrument has a natural pitch at which the part swings most readily, and a gravitational wave arriving at that pitch builds up the swing, as pushes timed to a child's swing build up the motion of the swing (Chapter 17).

Resonant bars. Large metal cylinders built to ring when a gravitational wave passed (Chapter 17).

Restoration of context. The re-forming of a large outside world at the turn, with an outer edge of its own and, as the matter of that world spreads, a reference for inertia of its own (Chapter 1).

Ricci curvature. How much a direction curves, taking every slice through the direction into account (Chapter 3).

Ringdown. The ringing of a newly merged black hole as the hole settles into its final shape (Chapter 9).

Room to grow. The gap between the entropy count of a region and the region's ceiling, the count of one black hole made of all the mass and energy in the region (Chapter 10).

Running. The change of the spectral index itself from one end to the other of the row of columns in a display of the power, band by band (Chapter 11).

S eight. The number that says how clumped the matter is today (Chapter 14).

Scalar. Of a ripple of the geometry: a change from place to place in the amount of curvature, the kind a ripple of density produces (Chapter 12).

Scale invariant. Of a pattern of ripples: with the same power in every band, so that no length stands out from any other (Chapter 11).

Second law. The law that the entropy of a thing left to itself grows (Chapter 6).

Second order. From ripples acting on ripples, a far smaller effect (Chapter 12).

Self-coupling. How strongly the Higgs field acts on itself, which sets how steeply the walls of the energy valley of the Higgs field rise (Chapter 5).

Set by the structure. The standing of a number that follows from the shape of the world as the account derives that shape (Preface).

Set by the structure once a stated step is taken. The standing of a number that follows only after a named step from the structure of comparison to the world, a step that the book names at the place where the step is made (Preface).

Settling hum. The hum of gravitational waves made as the six settle into their held size: vibrations of the volume of the six turn into waves of the four large dimensions (Chapters 4 and 16).

Share held as baryons. The fraction of the residue of matter that ends as protons and neutrons. The list of the kinds of particle, with their charges, fixes the share held as baryons (Chapter 20).

Shear. The part of an expansion that changes the shape of a small region without changing the region's size, as when shear stretches a round region into an egg while keeping the region's volume (Chapter 13).

Short loop. A path that goes all the way around a universe, that cannot be pulled tight to a single point, and that is shorter than the diameter of the horizon, wherever the path lies (Chapter 19).

Simons Observatory. A set of telescopes in the high desert of northern Chile, forecast to measure the ratio of first waves to first ripples (Chapter 12).

Singularity. Where Einstein's equations give curvature without bound: inside a black hole, a moment and not a place, the future end of every path inside. In this account, the singularity is what the smooth equations produce when the equations are carried past the edge of their range (Chapter 1).

Six, the. The six dimensions of possibility, held very small, a few Planck lengths across. The six are never traveled through, and are met only as what could be found (Preface).

Slow-roll inflation. The usual kind of inflation, driven by a field that drifts slowly down a long, almost level slope of its own energy (Chapter 13).

Sound horizon. The distance each sound wave in the hot glowing gas of the early universe had traveled by the time the light went free (Chapter 14).

Species. One distinct state that a kind of particle can be in, apart from where the particle is and how the particle moves (Chapter 4). A light species is the unit in which the radiation of the early universe other than light is counted: the radiation that one kind of neutrino carries (Chapter 4).

Spectral index. The number that says how the strength of the ripples changes from small sizes to large: how the power of the ripples changes from band to band, an index of one meaning the same power in every band (Chapter 11).

Sphaleron passages. Passing states of the field of the weak force, the one process among the kinds of particle that changes the number of baryons. Each passage changes the baryons and the leptons together and keeps B minus L (Chapter 10).

Sphere. The space of all the states of a two-way choice (Chapter 1): the surface of a ball, with two dimensions (Chapter 3).

Spin. A number by which physicists label each family of vibrations of the six. For the gyrations, the spin says only how the vibration is laid out against the directions of the large space (Chapter 18).

Spin-c structure. A phase, a setting like the angle of a clock hand, at each point of a shape. Some waves on the shape need a spin-c structure in order to be defined (Chapter 5).

Square Kilometre Array. A very large radio telescope now being built (Appendix B).

Stable minimum. The bottom of a hollow: step off the bottom in any direction and the ground rises, so anything nudged off the bottom is pushed back (Chapter 4).

Stage that behaves as matter. A stage of the content in which energy is held in heavy, slow things that later decay into the plasma (Chapter 4).

Stance. The position from which a comparison is made: the comparing, together with what is compared (Chapter 6).

Standard Model. The established account in physics of the particles and of every force but gravity (Chapter 5).

Statistical. Describing what happens almost always, among a great many possible arrangements, and not what happens in each single case (Chapter 6).

Stochastic background. A random sum of gravitational waves arriving from every direction, with no single source that can be picked out. The word "stochastic" means random (Chapter 15). In this book, a stochastic background is called a hum (Chapter 10).

Strength. Of the first ripples: the typical size of the curvature perturbation (Chapter 11).

String theory. The theory in which the smallest pieces of matter are tiny vibrating strings. In string theory, the extra six dimensions are directions of space, curled up small (Chapter 1).

Strong force. The force that binds the quarks. The carriers of the strong force are the gluons (Chapter 10).

Structural arrow. The arrow of time that comes from the structure of comparison itself. The structural arrow needs no special beginning of the world (Chapter 6).

Structural value. A value set by the structure of the account alone, before any measurement is consulted (Chapter 14).

Superconducting cavity. A hollow metal cavity, cooled until its walls carry current with no resistance, holding two nearly identical patterns of microwaves, one filled and one kept quiet. A passing gravitational wave moves a little energy from the filled pattern into the quiet one (Chapter 17).

Supermassive black holes. Black holes each of millions to billions of times the mass of the Sun. Pairs of supermassive black holes circle one another at the centers of galaxies (Chapter 17).

Symmetry. Sameness through a change (Preface); in the sense the Incommensuration Theorem uses, a sameness of content across a difference of context (Chapter 6).

Term of Gauss and Bonnet. The first of the further terms of the law of gravity, built from the curvature squared, that a theorem of the physicist David Lovelock allows in ten dimensions. *The Gravitational Passage* does not fix the size of the term of Gauss and Bonnet (Chapter 4).

Theorem. A result of mathematics, proved (Preface).

Three conditions. The conditions that the physicist Andrei Sakharov set out in 1967: the three things that any process needs, all together, to make an excess of matter in a universe that starts with none (Chapter 20).

Three-sphere. A sphere one dimension up from the surface of a ball (Chapter 13).

Tidal stretching. The difference in the pull of gravity from one part of a body to another, which stretches the body along one direction and squeezes the body along the others (Chapter 2).

Tilt. How far the spectral index lies from one: the lean of the row of columns in a display of the power, band by band (Chapter 11).

Tolman's objection. The argument of the physicist Richard Tolman, made in 1934, that entropy carried from cycle to cycle gives a history of cycles a first cycle (Chapter 7).

Topology. The mathematics of shape (Chapter 3).

Total curvature. The total of the Ricci curvature over all the directions at a point (Chapter 3).

Transcendent. The role of what is possible in a comparison: the stance from which an act proceeds, and the range of what could be found, like the position of a photographer with all the pictures she might take from that position (Chapter 6).

Trapping horizon. A horizon made by stacking marginally trapped surfaces one after another through time. While a trapping horizon grows, physicists call it a dynamical horizon, and while a trapping horizon stays the same, physicists call it an isolated horizon (Chapter 9).

Tree level. The simplest level of a calculation, before the smaller corrections (Chapter 5).

Trim. The share of the Planck density that remains after the part due to the curvature of the six is subtracted. The trim is written f, and the trim is set by the structure (Chapter 8).

Tunneling. Passing through a ridge rather than over it: a ball held in a valley that is not the lowest can pass through the ridge into a lower valley without ever climbing over the ridge (Chapter 5).

Turn, the. The passage at the end of the interiors of the black holes in which the ten-dimensional geometry that remains restores a large outside world: three directions open up while the six close in. No comparison registers the turn itself as an event (Preface).

Uncertainty relation. The rule that nothing can have both a perfectly sharp place and a perfectly sharp motion: the more exactly the place of a thing is pinned down, the more widely its motion spreads (Chapter 4).

Upper end. The end of the Kasner curve at which each of the three large directions has the exponent five ninths, and each of the six has the exponent minus one ninth (Chapter 2).

Vacuum. One lowest, settled state of empty space (Preface); the state a system settles into when the system has the least energy that the system can have nearby; for the six, a held size and shape, an arrangement of the geometry of the six that the laws of motion hold (Chapter 5).

Vacuum energy. The energy that empty space holds in itself: the restless trembling of all the fields at their lowest (Chapter 8).

Vacuum manifold. The set of lowest-energy states a field can settle into (Chapter 13).

w. The ratio of the pressure of a form of energy to the amount of that energy in each unit of volume. For the energy of empty space, w is exactly minus one (Chapter 8).

W and Z bosons. The carriers of the weak force (Chapter 10).

Weak force. The force at work in some sorts of radioactive decay (Chapter 2).

Weak mixing angle. A measured number that fixes how the weak force and electromagnetism are blended (Appendix A).

Weyl curvature. The part of the curvature of space and time that distorts shapes, as distinct from the part that changes volumes (Chapter 7).

Weyl curvature hypothesis. The proposal that the physicist Roger Penrose made in 1979, as a separate principle, that the Weyl curvature was near zero at the beginning (Chapter 7).

What every change keeps. B minus L, which every process among the kinds of particle keeps, at every temperature, from the earliest moment our history reaches to today (Chapter 20).

White dwarfs. The dense, burnt-out cores of stars like the Sun (Chapter 16).

White noise. The pattern that independent pieces leave when averaged, in which the ripples die away on large lengths (Chapter 12).

Zero-point energy. The energy a thing keeps in its lowest state, the energy of the lowest motion that cannot be removed (Chapter 4).

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Six Books of the Collection

Every claim in this book comes from other books in this book's collection, and those books set out in full the account of the world that this book teaches. I name six books of the collection most often: *At the Edge of Reality*, *The Compactification*, *The Gravitational Passage*, *The Weight of Possibility*, *The Coming of the Kinds* and *The View from Inside*. For each of the six books, the description below agrees with the description of that book in the opening section of this book, Before We Begin.

At the Edge of Reality. Here I continue the story of an earlier book, *At the Edge of Reality*. The book *At the Edge of Reality* followed a black hole to the black hole's end, and here I take up the story at the turn. The turn is the passage at the end of the interiors of the black holes, in which three directions open up while the six small dimensions of possibility close in.

The Compactification. *The Compactification* is the main source of this book. *The Compactification* sets out for physicists how an era of the world begins and what the beginning of an era leaves in the sky.

The Gravitational Passage. *The Gravitational Passage* derives Albert Einstein's equations of gravity, the equations of general relativity, from the structure of comparison, where a comparison is the act of finding whether two things are the same or different, in some respect. *The Gravitational Passage* also sets out the whole account for physicists. *The Gravitational Passage* is the source of what this book says about the turn, the six small dimensions of possibility and the cosmological constant, which is the energy of empty space.

The Weight of Possibility. *The Weight of Possibility* identifies the dark matter, the unseen matter whose pull is measured in every galaxy, with a lasting vibration of the six small dimensions of possibility. *The Weight of Possibility* is the source of what this book says about the dark matter.

The Coming of the Kinds. *The Coming of the Kinds* tells how the kinds of particle came to be after the Big Bang. *The Coming of the Kinds* is the source of what this book says about the ordinary matter that remains, which is the small excess of matter over antimatter that the turn leaves.

The View from Inside. *The View from Inside* is a book for the general reader on the cosmological constant and on why empty space weighs so little. *The View from Inside* tells parts of the account that this book teaches, in plain words.

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