

The Move Not Yet Made

The Immanent Metaphysics
for Readers of Max Tegmark

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Contents

Preface iv
1. What Is Reality? 13
2. Does Space Go On Forever? 26
3. Where Did It All Come From? 41
4. What Is the Universe Made Of? 56
5. What Put the Bang in the Big Bang? 73
6. Is There Another Copy of Me Out There? 89
7. Why Are the Constants What They Are? 107
8. What Happens When We Look? 126
9. Am I Splitting Into Copies of Myself? 144
10. Is the World Made of Mathematics? 162
11. Is Time an Illusion? 179
12. What Am I? 196
13. Do All Mathematical Structures Exist? 211
14. Is the World a Computation? 228
15. How Will Our Universe End? 245
16. What Do I Now Hold? 262
What the Coming Measurements Will Find 277
For the Specialist 303
Reading Beside Tegmark 456
Glossary 465
Where to Go from Here 527
References 532
Index 547

Preface

A Letter from Paris

You have read the physicist Max Tegmark's book *Our Mathematical Universe*. The question that Tegmark's book raises has stayed with you. What is reality, and can physics find out? Tegmark uses the word *reality* for the ultimate nature of the outside physical world that we are part of.

Johann Gottfried Galle was an assistant astronomer at the Berlin Observatory. On the morning of 23 September 1846, Galle received a letter from Urbain Le Verrier in Paris. Le Verrier had studied the small irregularities in the motion of the planet Uranus, and from the irregularities he had worked out, by calculation, where in the sky an unseen planet pulling on Uranus had to be. In his letter, Le Verrier asked Galle to point the observatory's telescope at the place Le Verrier had calculated.

On the night of 23 September, Galle sat at the telescope and read out the positions of the stars he saw, one by one, and Heinrich d'Arrest, a younger assistant, worked beside him. D'Arrest looked for each star on a new chart of the part of the sky at which the telescope pointed, and he found each star on the chart, one after another. Shortly after midnight, d'Arrest came to a star that the chart did not show. The star missing from the chart lay close to the place that Le Verrier had calculated. Two days later Galle wrote to Le Verrier that the planet whose position Le Verrier had pointed out really existed.

Le Verrier worked with pen and paper in Paris. He told astronomers in Berlin where to point their telescope. The planet was there. Why should a calculation on paper show the place of a planet that no one had yet seen? Le Verrier's calculation is one case of a wider question: why does mathematics describe the world so well? Tegmark sets his book to answer the wider question. Answering the wider question, he writes, is "the main

goal of this book" (chapter 1, "Not What It Seems"). Tegmark's answer is, in his words, "my most radical and controversial idea: that the ultimate reality is purely mathematical" (chapter 1, "The Journey Begins").

What Max Tegmark Showed

The question "What is reality?" is one of the oldest questions people ask. Tegmark asks the question as a physicist. He treats the question as a detective story, following each clue wherever the clue leads. He states the lesson he draws from physics early in his first chapter: "modern physics has made abundantly clear that the ultimate nature of reality isn't what it seems" (chapter 1, "What's the Ultimate Question?").

Tegmark's book has three parts. In the first part, "Zooming Out", Tegmark goes outward to the largest scales. He asks how far space goes, and how our universe grew from a hot and dense early state, which he calls our Big Bang. He reports what astronomers, by precise measurements of the sky, have found our universe to be made of, and he describes inflation, a very rapid stretching of space early on, which he takes to be the cause of the Big Bang. In the second part, "Zooming In", Tegmark goes down into atoms and into the building blocks of atoms. He then takes up the rules of quantum mechanics, which those building blocks obey. In the third part, "Stepping Back", Tegmark asks what everything found in the first two parts means for the ultimate nature of reality.

Across the three parts, Tegmark describes four levels of parallel universes. Level I is the regions of our own space so far away that their light has not yet had time to reach us, and the Level I regions have our laws of physics, each region with a history of its own. Level II is regions that inflation keeps forever out of reach. In the Level II regions the fundamental equations are the same as in our region, but the equations can settle into different local laws, with different constants of nature (the fixed numbers in the laws, such as the strength of gravity). Level III is the branches of

Hugh Everett's account of quantum mechanics: on Everett's account every possible outcome of a quantum event happens, each outcome in a branch of its own. Level IV is every mathematical structure there is.

A *mathematical structure* is, in Tegmark's words, "a set of abstract entities with relations between them" (chapter 10, "The Mathematical Universe Hypothesis"). The entities of a mathematical structure have no properties except their relations to one another, and the whole numbers are an example: a whole number has no property beyond its relations to the other whole numbers, such as that two and two make four. Tegmark sets the four levels one inside the next, in what he calls "a nested four-level hierarchy of increasing diversity" (chapter 12, "The Bottom Line"). In Tegmark's hierarchy, Level III has the same variety as Level II. In Tegmark's words, Level III "adds no new storylines beyond levels I or II" (chapter 8, Figure 8.10). Tegmark is careful to say where the four levels come from: "Parallel universes are not a theory, but a prediction of certain theories" (chapter 6, "The Level I Multiverse").

Tegmark gives the idea that is most his own in the third part of his book, "Stepping Back". He starts from what he calls the External Reality Hypothesis: that there exists an external physical reality completely independent of us humans. He argues that a complete description of a reality independent of humans must be free of every word and idea that only humans use. Tegmark calls the words and ideas that only humans use "baggage". He argues further that a description with all the baggage removed is a mathematical structure. So Tegmark arrives at the Mathematical Universe Hypothesis: "Our external physical reality is a mathematical structure" (chapter 10, "The Mathematical Universe Hypothesis"). According to the Mathematical Universe Hypothesis, our world is mathematics, as well as being described by mathematics. On the Mathematical Universe Hypothesis, we are self-aware parts of one giant mathematical object. Level IV follows from the Mathematical Universe Hypothesis, and so does Tegmark's conclusion that the flow of time, change and randomness are illusions.

Tegmark writes clearly, and he tells his reader plainly where his book goes beyond what most physicists accept. In the opening lines of his sixth chapter, "Welcome to the Multiverse", he writes: "The science we've explored so far in this book has by now become mostly mainstream and well accepted. We now enter the controversial, which many of my physics colleagues will argue passionately either for or against".

Who This Book Is For

I wrote this book for the reader who has read Tegmark and wants to go further with him. I follow Tegmark's book chapter by chapter. For each question Tegmark answers, this book sets his result beside what my physics books of the Immanent Metaphysics show on that question. In this book, *metaphysics* means the study of what must be so for anything at all to be known. *Immanent* means within: in the Immanent Metaphysics I reason from inside the act of knowing, here and now, and never from a place outside the world. My *physics books* of the Immanent Metaphysics are the books in which I build physics from one act, the act of comparison. Those books call themselves "the collection" or "this account", and the sentences I quote from them keep those names. The Immanent Metaphysics is my own work: I first set it out in my book *An Immanent Metaphysics* (2001), and I work out its physics in a series of later books, every one of them mine, which Appendix B, For the Specialist, names chapter by chapter, with the place in each book that every step draws on.

Comparison was the work d'Arrest did on the night of 23 September in Berlin. For each star that Galle read out, d'Arrest looked at the chart and found the same star on the chart, until he came to the one star that the chart did not show. *Comparing* is telling one thing from another, or finding two things the same. Every measurement is a comparison. An *interaction* is anything that happens between two things as each acts on the other, a col-

lision of two particles included, and every interaction is a comparison too. I call the result of one comparison a *finding*. D'Arrest's finding, shortly after midnight, was a star missing from the chart.

Where the Two Accounts Meet, and Where They Part

On many questions, my results agree with Tegmark's results. Tegmark's starting premise is also a premise of mine: there is a world, and the world does not depend on us. My account has a reason of its own for the premise: every interaction is a comparison, so the world was comparing itself, in every collision of particles, long before anything human looked at the world. From the measurements Tegmark reports, I describe the history he describes, in which our universe has expanded from a hot and dense early state. Two of the measurements show the expansion and the hot early state. The galaxies move away from us faster the farther off they are, as the astronomer Edwin Hubble found, because space itself is stretching. A faint glow arrives from every direction of the sky, and the glow is left over from the early universe. The radio astronomers Arno Penzias and Robert Wilson found the glow.

On Level I, I agree with Tegmark in substance: space goes on beyond what we can see, with the same laws as in our region. The regions beyond our view belong to the world because each region acts on the regions next to it, as our region does, so I call the regions beyond our view further parts of one world, while Tegmark calls the regions parallel universes.

A *superposition* is one thing described as holding two or more alternatives together, as physicists can describe a particle as being both here and there. I also agree with Tegmark's estimate that quantum superpositions in the warm brain fade far faster than nerve cells act. Tegmark's estimate applies to one proposal about the mind, that the mind rests on quantum superpositions held for a long time in nerve cells, and my account of the mind needs no long-held superpositions (chapter 12 sets out that account).

On other questions, my results differ from Tegmark's results, and I state my reason for each difference. Tegmark's account and mine divide over one distinction: the distinction between what exists and what is real. The night in Berlin shows the distinction. In this book's sense, what *exists* is laid out whole, timeless and the same for everyone who reads it: the orbit that Le Verrier calculated was the same in Paris as in Berlin, and the orbit is the same today. In this book's sense, what is *real* is given in an act of finding, made once, from one place: the planet was found on one night, at one telescope, when d'Arrest came to the star that the chart did not show. Galle wrote that the planet really existed, and his phrase contains both existing and being real. I keep existing and being real apart. Every physical theory has both a part that exists and a part that is real: its mathematics, which exists, and its findings, which are real. Tegmark's hypothesis says that the mathematics, which exists, is the whole world.

Tegmark removes every human word from his description of the world. I agree that a description of a world independent of us must contain nothing human. Take every human word out of an account of the night in Berlin, and the account still has the side from which the stars were compared with the chart: one telescope, in one city, pointed at one part of the sky. Every comparison is made from some side, whether the comparison is a collision of two particles or a look through a telescope. The side belongs to the comparison, whether or not a human makes the comparison. So when every human word is removed from a description, the side stays in place. Mathematics describes what is found as if no one stood anywhere to find it: mathematics leaves out the side from which every comparison is made. Every mathematical structure, taken whole, is laid out all at once and complete, with nothing missing and nothing still to come, and a mathematical structure is the same wherever and whenever someone reads the structure. So every mathematical structure, taken whole, has perfect sameness (the same wherever and whenever it is read) and gapless completeness (nothing missing) together. No single act of finding can have perfect sameness and gapless completeness together, and in chapter 1 I give the reason. The world is found one act of finding at a

time, so the world is not a mathematical structure, although mathematics describes the world. Mathematics still has every power it has ever had: mathematics is the finest instrument there is for describing what is laid out as structure.

Tegmark's account and mine also differ on further questions, and each further difference has a reason of its own. On my account, Level II does not occur. No long inflation occurred, and the fundamental equations settle into one set of local laws, so the constants of nature are the same everywhere. Tegmark's Level II needs many sets of local laws. On my account, every finding is made now, with the future not yet settled, and the outcome of a single quantum measurement is fixed by nothing that came before the measurement, however completely the past is known. Tegmark counts the flow of time, change and randomness as illusions, while on my account the flow of time, change and randomness are real, in this book's sense, because each of the three is given in an act of finding. In every chapter where the two accounts divide, I set out the step at which the accounts divide, so that you can see the argument of each account.

How to Read This Book

Each chapter's title is one question, and in the chapter I answer that question. I open each chapter with a scene from the history of physics, mathematics or philosophy (a named person, at a known place and date, doing something you can picture), and I bring in the chapter's idea inside the scene. In each chapter I then set out three things in order. The first thing that I set out in each chapter is what Tegmark showed, in his own words where his words are short enough to quote. The second thing that I set out is where my account agrees with Tegmark's result, on a ground that is my own. The third thing that I set out is where my account differs from Tegmark's result, and why my account differs. I close each chapter with a section called "What This Chapter Holds". The section "What This Chapter Holds" has the place in this book that Tegmark's "Bottom Line"

boxes have in Tegmark's book. In the section "What This Chapter Holds", I state the chapter's result in one sentence you can keep, I say why the result matters, and I end the section on the question that I answer in the next chapter. I end the last chapter on a picture instead of a question: the last picture of the book.

I show you every idea before I name it, and I explain every new word where you first meet it. I have gathered the mathematics, with the exact statements that support the chapters, in the appendix "For the Specialist", and you can read every chapter without the mathematics in that appendix. The appendix "What the Coming Measurements Will Find" states what my account says each coming measurement will find.

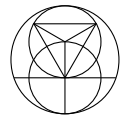
Tegmark invites readers who already know the physics of his early chapters to skip some of those chapters after reading the chapters' "Bottom Line" boxes, and I make you a like offer. If you know cosmology, the science of the universe as a whole, you may read only "What This Chapter Holds" in chapters 2 to 4 and go on. In the later chapters I remind you of each word explained in chapters 2 to 4.

The chapters of this book are in the order of Tegmark's chapters. Chapters 2 to 7 are about the subjects of Tegmark's first part, "Zooming Out": space, the history of the universe, what the universe is made of, where the bang of the Big Bang came from, and his first two levels. Chapters 8 and 9 are about the subjects of his second part, "Zooming In": quantum mechanics and his third level. Chapters 10 to 14 are about the subjects of his third part, "Stepping Back". Chapters 10 to 13 are about four questions: whether the world is mathematics, whether time is an illusion, what you are, and whether every mathematical structure is a world of its own. Chapter 14 is about whether the world is a *computation*, a procedure that goes step by step by fixed rules, as a computer program does. Chapter 15 is about how our universe will end. In chapter 16 I gather together what the earlier chapters have shown you. A table at the end of the book sets each chapter of this book beside the chapter of Tegmark's book

that the chapter follows. I use one picture throughout the book, from the first chapter to the last: a game of chess at a kitchen table, with a score sheet, the written list of the game's moves, lying beside the board.

Here, in brief, is what this book holds that Tegmark's book does not: the difference between what exists, laid out whole like a score sheet, and what is real, a move made once from a seat (Chapter 1); space as the arena of comparison, with no short loop (Chapter 2); the turn between epochs in place of a beginning of everything (Chapters 3, 5 and 15); the dark matter as vibrations of six small dimensions, with its ratio to ordinary matter (Chapter 4); no long inflation, and so no Level II (Chapters 5 and 7); hard randomness and one outcome for each measurement (Chapters 8 and 9); the Incommensuration Theorem proved step by step, and the world described by mathematics without being a mathematical structure (Chapter 10); time and its flow as real (Chapter 11); the bound triple as what consciousness is (Chapter 12); the rule for claims about everything taken as one (Chapter 13); and the world as neither a computation nor a simulation (Chapter 14). If you have read another of my books written for the readers of another author, you know the ground I lay in Chapter 1 already: comparison and the Incommensuration Theorem, told here with two stones and a game of chess. What is new for you begins where Chapter 1 sets the score sheet beside the move, and runs through every chapter after it. Apart from the shorter road through chapters 2 to 4 that I offer above, read the chapters in order, because each chapter builds on the ones before it. Where a chapter uses a word or a result that an earlier chapter teaches, a short box at the head of the chapter lists the word or result, with the chapter that teaches it. Where a chapter retells what Tegmark showed, the retelling stands in a shaded box headed "From Tegmark's pages": a reader who knows Tegmark's book can pass over the boxes, and a reader new to it can read the boxes first.

Tegmark begins from what physics has shown him: that reality, in his word, is not what it seems. In his first chapter Tegmark asks what reality is, and I start the first chapter of this book from the same question. If reality is not what it seems, what is reality, and can physics find out?



Chapter 1

What Is Reality?

Beside Tegmark's chapter 1, "What Is Reality?"

I. A Book Printed in Rome

Max Tegmark tells you on the first pages of *Our Mathematical Universe* that reality is not what it seems, and by reality he means the ultimate nature of the outside physical world that we are part of.

In October 1623 a book came from the press of the printer Giacomo Mascardi in Rome. The Accademia dei Lincei was a learned society of Rome. Members of the Accademia saw the book into print. The book was dedicated to the new pope, Urban VIII. The author of the book was Galileo Galilei. The title of the book was *Il Saggiatore*, *The Assayer* (an assayer is someone who tests metals for their worth with a fine balance). Galileo chose the title as an answer to another man's book: Orazio Grassi had published a book on the comets of 1618 under the name Lotario Sarsi, and he called his book the *Libra*, the *Balance*. Galileo's full title says that his book weighs what Grassi's *Balance* holds "with an exquisite and just balance". So Galileo's book is an act of weighing one thing against another, from its title onward.

Near the start of his own book, Tegmark sums up a passage from the sixth section of *The Assayer*. He writes that Galileo proclaimed nature to be "a book written in the language of mathematics" (chapter 1, "Not What It Seems").

In Stillman Drake's English translation, Galileo's passage runs:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.

Galileo's passage describes two things. The first thing that Galileo's passage describes is a book, laid out whole, with the same letters on the book's pages for every reader. The second thing that Galileo's passage describes is a reader, who must learn the language of the book and then read the book one line at a time. In Galileo's picture, Tegmark's answer is that the world is the book alone, laid out whole, and that the reader is one more set of letters on the pages of the book.

II. Two Stones and a Game of Chess

Pick up two stones, one in each hand, and ask whether the two stones are the same. The stones may be nearly the same in weight: neither hand sinks lower than the other. The stones may differ in color: one stone is gray, and the other is streaked with red. You have just made a comparison. A *finding* is what one comparison gives: with the stones, "nearly the same in weight" and "different in color".

Every measurement is a comparison of the kind you made with the stones. A goldsmith at an assayer's balance sets a sample of gold against a standard weight, and the balance tips or stays level. A thermometer sets the warmth of a room against the marks on the thermometer's scale. Comparing also happens where no person is present. An atom takes in light only when the energy of the light matches a gap between the energies that the atom can hold. So the atom finds the energy of the light and the gap the same or different, and when the atom takes in the light, the taking in is the atom's finding that the two are the same. An *interaction* is something that happens between two things, as each thing acts on the other. Every interaction is a comparison in exactly the sense just given, a telling of one thing from another or a finding of two things the same, and the atom's taking in of light is one such interaction.

I set one account beside Tegmark's: my own, the physics of the Immanent Metaphysics, which starts from the act of comparison. The Immanent Metaphysics is my account of what every act of comparison contains. My physics books build physics from the act of comparison, the act you made with the stones. In the sentences I quote from those books, they call themselves "the collection" or "this account".

Besides the finding, the comparison you made with the stones has two more parts, and my account has a word for each part. The *content* of a comparison is what is found the same or different: with the stones, first their weight, and then their color. The *context* of a comparison is the respect in which the two things compared are two. With the stones, the context is two places, one stone in each hand. Two things compared can also be two because they are found at two moments, as when a tide gauge sets the height of the sea at noon against the height of the sea at midnight. Place, time and possibility are the three contexts in which anything is ever found. In this book the word "context" always has my meaning: the respect in which the two things compared are two.

Now picture two people playing chess at a kitchen table. Beside the board lies a score sheet, the written list of the game's moves. There are three ways of looking at one game.

The first way of looking at the game is the score sheet, on which the moves are written down in order, so that anyone who walks into the kitchen reads the same list. The whole game so far is on the sheet at once, and the list reads the same for every reader, today or next year.

The second way of looking at the game is the move. A hand lifts a knight and sets the knight down here, now, once. The hand sets the knight down first, and the score sheet records the move after the hand has made the move.

The third way of looking at the game is the seat. Each player sits in one seat, at one end of the board, and every move is made from a seat. White's knight is set down from White's seat, and the same pieces on the same squares look different from Black's seat.

My account has a word for each of the three ways of looking at the game, the score sheet, the move and the seat, and I use each word in one sense only. What *exists* is what is laid out whole, timeless and the same for everyone: a structure, as the score sheet is. What is *real* is what is given in an act of finding, once, as the move is. One line of mine uses both words: "A law exists; a measurement is real". The *stance* is the side from which a comparison is made, as a move is made from a seat. Every comparison has a stance, whether a human being makes the comparison or not.

In this book "real" means only what is given in an act of finding. Where I report Tegmark's word "reality", I say that the word is his.

We can now ask Tegmark's question about the chess game. Is the game the score sheet?

From Tegmark's pages

III. What Max Tegmark Showed

In his first chapter Tegmark goes quickly to findings of physics that overturn the everyday picture of things. His first finding is about a solid object: "Physicists have known for a century that solid steel is really mostly empty space" (chapter 1, "Not What It Seems"). Nearly all the mass of the steel is in the nuclei of its atoms, the small dense cores of the atoms, and the nuclei fill a tiny fraction of the volume of the steel. The steel feels solid because the electrical forces that hold the nuclei in place are strong. Tegmark adds other findings: particles that appear able to be in two places at once, liquid helium that flows upward when the helium is cold enough, and clocks that run slow at high speed.

Tegmark gives a reason why the findings of physics surprise us: human beings evolved an intuition for the physics that mattered to the survival of their ancestors, such as the flight of a thrown stone. So our intuition fails whenever instruments take us far beyond the human scale, to the very small, the very fast or the very large.

In his next section Tegmark recalls the philosopher Plato, who likened us to prisoners in a cave who take the shadows on the wall of the cave for the whole of what there is. Tegmark then states the lesson that his life as a physicist has taught him: Plato was right, and "modern physics has made abundantly clear that the ultimate nature of reality isn't what it seems" (chapter 1, "What's the Ultimate Question?"). Tegmark calls the lesson the most important that physics has taught about the ultimate nature of reality, in his closing summary of the chapter.

Tegmark then says exactly what he means by the word "reality": "I use it to mean the ultimate nature of the outside physical world that we're part of" (chapter 1, "What's the Ultimate Question?"). He also says how he will look for the nature of the outside physical world: as a

physicist, treating the great questions as detective mysteries, "combining clever observations and reasoning and persistently following each clue wherever it leads" (chapter 1, "What's the Ultimate Question?").

Tegmark follows one clue furthest, the clue that Galileo saw: the physical world shows a regularity that mathematics describes with astonishing exactness. Tegmark sets Galileo's book beside the physicist Eugene Wigner, whose paper of 1960 is titled "The Unreasonable Effectiveness of Mathematics in the Natural Sciences", and Tegmark treats the regularity as a mystery that needs an explanation. He asks why the world is so mathematical, and he states his purpose plainly: "Answering this question is the main goal of this book, as its title suggests" (chapter 1, "Not What It Seems").

Tegmark announces his answer in chapter 1 and argues for the answer in his chapters 10 to 12. He calls the answer "a crazy-sounding belief of mine", and he states the answer in one line: "our physical world not only is described by mathematics, but that it is mathematics" (chapter 1, "Not What It Seems"). On Tegmark's view we are "self-aware parts of a giant mathematical object" (chapter 1, "Not What It Seems"). On Tegmark's view every other mathematical structure is a world as well, "a fourth and ultimate level of parallel universes" (chapter 1, "The Journey Begins"). Tegmark also says what his view removes from the everyday picture: his view ends in "demoting familiar notions such as randomness, complexity, and even change to the status of illusions" (chapter 1, "The Journey Begins").

Before Tegmark makes his argument, he holds that physics need not first explain the human mind: "our failure to understand consciousness doesn't stand in the way of a complete understanding of the external physical reality" (chapter 1, "The Journey Begins"). In his chapter 10 Tegmark names the first premise of his whole argument, the External Reality Hypothesis: "There exists an external physical reality completely independent of us humans" (chapter 10, "The Mathematical Universe Hypothesis"). He argues from the External

Reality Hypothesis to the Mathematical Universe Hypothesis: “Our external physical reality is a mathematical structure” (chapter 10, “The Mathematical Universe Hypothesis”).

Tegmark puts his question clearly, says what his position is, and marks his answer as his own. Tegmark states his answer so exactly that a reader can set another answer beside his answer and compare the two answers.

IV. Where This Account Reaches the Same Result

I agree with the plain sense of Tegmark’s starting premise: there is a world, and the world does not depend on human beings. My account has a reason of its own for the premise. Every interaction is a comparison, as with the atom that takes in light, and so the world was comparing itself, in every collision of particles, long before anything human looked at the world. In two volumes, I set twenty-three models of the universe beside my own model, twenty-four models in all; its Volume Two holds my essay on Tegmark, “The Structure and the Act”, which this book follows throughout. I hold his External Reality Hypothesis on my own ground.

I also agree that physics need not first explain the human mind. The reason is that a stance need not be a person, just as a seat at the board need not hold a player. A detector with no one watching still clicks when a particle passes. The click is a comparison of a charge against a threshold. I describe how physics is built from comparison. The word observer names a place inside the world from which a comparison is made, and never names a mind. So I agree with Tegmark that physics need not explain the human mind before physics describes the outside world. The agreement comes from my own definition of an observer.

I also agree that the exactness of mathematics in physics needs a reason, and I state the reason. In every physical theory the mathematics is set against the findings. The mathematics and the physics are two aspects of

one act of comparison. I add that the mathematics of a theory is dense around the findings of the theory: some value of the mathematics lies as near to each finding as any precision requires. A mathematics without gaps has values for findings that come in separate steps, to any precision required, and a mathematics of perfect sameness describes a smooth finding along any path, to any precision required. So mathematics can describe what physics finds as closely as anyone wishes.

The number pi shows one case of a sameness that mathematics and physics share: sameness under rotation. Throw darts at a target, aiming at the center each time. The hits scatter around the center. When the misses to left and right spread just as the misses up and down do, the scatter looks the same however the target is rotated about its center, as a circle does. Pi is a circle's distance around divided by its distance across. Pi appears wherever sameness under rotation appears. So pi, first known from the circle, appears again in the law of how the errors of measurements spread, as the darts' misses spread.

Tegmark and I agree that the regularity Galileo saw is a clue to be followed. Tegmark follows the clue to the claim that the world is mathematics, while I follow the clue to the act of comparison, of which mathematics and physics are two aspects.

I also use the method that Tegmark names: observation and reasoning, following each clue wherever the clue leads. Observation and reasoning are both acts of comparison. An observation is a comparison of a quantity with a standard. Reasoning is a comparison of a proposed pattern with what has been observed. In a test, reasoning is a comparison of a prediction with an outcome. The ground of my whole account is the act of comparison, and every measurement is an act of comparison.

V. Where This Account Reaches a Different Result

Tegmark's account and mine differ at one step: the step from "described by mathematics" to "is mathematics". Every physical theory has two levels: its mathematics, which exists, and its findings, which are real. The equations of a theory are the theory's score sheet, laid out whole and the same for every reader, while the findings of a theory are the theory's moves, each made once, from a stance. In his hypothesis Tegmark takes the level of mathematics for the whole. At the kitchen table, Tegmark's hypothesis is that the game is the score sheet.

My account includes both levels because of the Incommensuration Theorem. An orange peel pressed flat pictures what the theorem says. Press the peel of an orange flat on the table. The peel splits, or stretches, or does both: a flat map of a round surface cannot keep every distance true and also stay unbroken, so the mapmaker chooses where the map gives way. In my exact words:

> Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity.

The Incommensuration Theorem is a structural fact about the nature of knowing. The Incommensuration Theorem is about comparison, the act in which anything at all is known. *Symmetry* means sameness: a thing unchanged when you swap it or move it, as a meter stick measures the same length in every country. *Continuity* means having no gaps, as a ramp rises with no step. The opposite of symmetry is *asymmetry*, a difference from one place or moment to the next, as no stretch of a coastline repeats another. The opposite of continuity is *discontinuity*, coming in separate steps, as a staircase does. When one comparison has two of the four qualities (symmetry, continuity, asymmetry, discontinuity) perfect at once, at its founda-

tion, the two qualities are applied simultaneously and fundamentally. So the theorem says that no single act of finding can hold perfect sameness and perfect gaplessness together. No single act of finding can hold perfect difference and perfect separate steps together either. The theorem shows a choice like the mapmaker's inside every act of comparison: of each pair of qualities that the theorem names, a single comparison has at most one quality perfectly, and the other quality gives way.

Your comparison of the stones had two verdicts, one on the content and one on the context, and each was either same or different. On the content, the weight, the verdict was "the same", as nearly as two hands can tell. Neither hand sank lower than the other. The verdict on the context was "different", because the two stones lay in two places, one in each hand. Every comparison has six parts, which I call the six *intrinsics* of comparison, from a word meaning belonging to the thing itself. The six are: sameness and difference, the only two verdicts any comparison can have; the content and the context; the stance from which the comparison is made; and what the comparison is made about, which in your comparison was the two stones. The Incommensuration Theorem is built from nothing but the meanings of the six intrinsics. Every act of knowing is a comparison or is built from comparisons. So the theorem is inherent in every single act of knowing, together with the intrinsics of comparison from which the theorem is built. *Epistemic* means concerning knowing: how anything comes to be known, checked and held. The theorem is directly about comparison, the act in which every other result is reached, and so the theorem is the deepest and most powerful epistemic basis possible. The theorem holds with no condition of any kind. Chapter 10 sets out the theorem step by step, from the six intrinsics. I need one consequence of the theorem.

The consequence is about a circle. A circle in mathematics is perfectly smooth, with no gap anywhere, and perfectly the same under every rotation about its center, both at once and exactly. A mathematician can write the circle down because mathematics leaves out the stance: a theorem of mathematics names no one who checks the theorem, no place and no mo-

ment, and so a mathematician can write sameness and gaplessness together. No finding ever shows a thing both perfectly smooth and perfectly the same under every rotation. Taken as a whole, every mathematical structure is laid out all at once, complete, with nothing missing and nothing still to come, and the structure is the same wherever and whenever it is read. So every mathematical structure, taken whole, has perfect sameness and gapless completeness at once, while by the Incommensuration Theorem no single act of finding can hold perfect sameness and gapless completeness together. The parts of a mathematical structure may be separate and irregular, and the structure still has perfect sameness and gapless completeness, as every mathematical structure does, because the sameness and the completeness belong to the structure taken whole. Every finding of physics is a comparison made at some moment, after one moment and before another, so the theorem applies to every finding. In chapter 2 I show that everything there is, taken together, is never one completed whole. So the world that comparisons find is not a mathematical structure. The world that comparisons find is the only world physics has. The mathematical description of the world comes as close to the world as anyone wishes but, however exact, never coincides with the world.

Tegmark argues for his hypothesis from a description of the world stripped of everything human. I differ from Tegmark because, on my account, a description stripped of everything human still describes comparisons made from stances. The stance stays when every human word is gone, because a stance belongs to every comparison, human or not: a collision of two particles is made from a stance as surely as a reading in a laboratory is. At the kitchen table, the seats stay when the players' names are erased from the score sheet. So a description stripped of everything human still describes comparisons made from stances. A description stripped of everything human is a score sheet, and the sheet holds what each comparison is about. The score sheet is never the whole of a comparison, because the stance from which each comparison was made is left off the sheet.

Change is real, and so is an outcome that nothing before it fixed. The move is made once, and the next move enters the score sheet only after a hand has made the move. Physicists find outcomes in the world that nothing before the outcomes fixed. A Geiger counter clicks when an atom of radioactive material decays, and everything that can be known about the atom beforehand fixes only the chance of a click in each second, and never the moment of the click. I wire a Geiger counter, in thought, to a small rocket. The rocket nudges a small asteroid. Over years the small asteroid nudges a larger asteroid, and in time the orbits of planets are moved. Every later link in the chain is ordinary gravity and motion, and the first link is a click that nothing before the click fixed. Where the masses go fixes how space and time are curved. So the shape of space and time depends on an outcome that was settled only when the outcome happened. So the whole story of the universe cannot be one fixed geometry of all space and all time, laid out once and for all. What is actual and what is still possible are divided in one place only: at the act of finding made now, here, in this outcome rather than another. On Tegmark's hypothesis, change and randomness are illusions. On my account, change and randomness are real, and each is given in an act of finding.

Tegmark tells the findings about steel, helium and clocks to show that what he calls reality is not what it seems. I agree with each finding about steel, helium and clocks as a finding of physics. Tegmark's account and mine differ over why the quantum seems strange (by the quantum I mean the behavior that quantum physics describes, such as a particle that appears able to be in two places at once). On my account, the quantum seems strange because physicists first met the quantum from the everyday starting point: things with definite positions, paths fixed in advance, and causes that act only where the causes are. Findings surprise us when the findings do not fit the everyday starting point. The surprise is a fact about the expectations that the everyday starting point produces. I start instead from the act of comparison. I show that in any world in which anything can be compared with anything else, findings come in exact separate steps, and the mathematics that describes the findings is smooth. Findings

in separate steps, described by smooth mathematics, are the character of the quantum. The Geiger counter shows both halves of the character of the quantum: the clicks of the counter come whole, one at a time, and the chance that a click has come grows smoothly with the time you wait. A physicist expects the quantum when the physicist starts from the act of comparison.

Mathematics keeps every power it has. The proofs of mathematics are exact, and the results of mathematics are valid for every reader. Mathematics is the finest instrument there is for what is laid out as structure. Mathematics is the score sheet, on which every move is recorded and studied. The moves themselves are made at the board.

VI. What This Chapter Holds

What Tegmark calls reality is a world of findings, each finding made once from a stance inside the world, and the mathematics of the world is the score sheet of those findings.

The score sheet exists, and the score sheet describes the findings as closely as anyone wishes. The Incommensuration Theorem is the reason the world that comparisons find is not a mathematical structure. So physics can find out what reality is, and physicists find out the way anything is found: by comparison, from inside the world. Every chapter that follows depends on the answer to both halves of the question: what reality is, and whether physics can find out. In each later chapter I answer one of Tegmark's questions from inside the world, by findings made from stances, and I read his mathematics as the score sheet of those findings. Tegmark goes on in his book from his first chapter toward the claim that the game is the score sheet, while I follow his book through the same cosmos, and I keep in view both the moves, which are real, and the score sheet, which exists.

If every finding is made from a seat inside the world, how far from our seat can anything be found, and does space go on forever?



Chapter 2

Does Space Go On Forever?

Beside Tegmark's chapter 2, "Our Place in Space"

What this chapter uses

Comparison: telling one thing from another, or finding two things the same; every measurement and every interaction is a comparison, and a *finding* is what is found in one comparison (Preface and Chapter 1).

What *exists* is laid out whole, timeless and the same for everyone, as the list on a score sheet is; what is *real* is given in an act of finding, once, as a move is made (Chapter 1).

The *stance* is the side from which a comparison is made, as a move is made from a seat; every comparison has a stance, human or not (Chapter 1).

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

I. A Letter from Temesvár

Does space go on forever, and how could anyone standing here ever know? The first part of Max Tegmark's book starts from the first half of that question: whether space goes on forever. Tegmark says of the question: "Yet it's not a hopelessly metaphysical question, but a serious scientific question" (chapter 2, "Cosmic Questions"). I take up both halves of the question, because the answer to the second half, how anyone standing here could know, fixes the sense in which I answer the first half, whether space goes on forever.

In September 1823 János Bolyai, a young officer of military engineers, was sent to Temesvár to work on its fortifications. Temesvár is the town now called Timişoara, in western Romania. Bolyai was twenty years old. Since his student years in Vienna, Bolyai had been working on a puzzle of geometry two thousand years old. The puzzle was the parallel postulate of Euclid, a mathematician of ancient Greece. A postulate is a starting assumption, and the parallel postulate, in the form most often taught, says that through a point beside a straight line exactly one straight line can be drawn that never meets the first line. For two thousand years mathematicians had tried to prove the parallel postulate from Euclid's other starting assumptions, and none had managed to prove it. On 3 November 1823 Bolyai wrote to his father, Farkas Bolyai, the author of a textbook of mathematics. In one translation, Bolyai's letter says: "from nothing I have created a new different world". Tegmark quotes Bolyai's letter in another translation: "Out of nothing I have created a strange new universe" (chapter 2, "What Is Space?").

Bolyai had found a geometry in which many straight lines can be drawn through the point beside the first line, and none of them meets the first line. Every step of Bolyai's geometry held together as well as Euclid's did. In 1832 Farkas Bolyai printed his son's work as an appendix to his own textbook. The appendix has a Latin title that promises the absolutely

true science of space. The Russian mathematician Nikolai Lobachevsky had made the same discovery on his own, and he had published it in 1829. Mathematicians now had two geometries, each free of contradiction, so they had to look beyond reasoning to learn which geometry belongs to the space we live in. Which geometry holds of space became a question for measurement. The question left the desk and went to the instruments: reasoning at a desk could not answer it, and only measurement with instruments could.

Every measurement is made from some place. The second half of this chapter's question, how anyone standing here could know, is a question about what anyone can learn by a measurement made from our place. Think of the game of chess at the kitchen table from chapter 1, now played late in the evening. You sit in one seat at the table. A lamp hangs above your chair, and the lamp's light falls on you, on the board and on a circle of floor with your chair at its center, while past the edge of the circle of lamplight the room is dark. How far does the floor go? Whatever you learn about how far the floor goes, you learn from your seat, by looking. Your seat is your *stance*. A stance is the side from which a comparison is made. Every comparison has a stance, whether a person makes the comparison or not (chapter 1). At this table the floor stands for space. The circle of lamplight stands for the part of space whose light has had time to reach us.

II. What Tegmark Showed: The Ladder of Distances

Tegmark tells the story of how humans measured the size of the world as one of the great detective stories. Tegmark finds a plain pattern in the story: each time humans mapped a larger scale, "we've discovered that everything we previously knew about was part of something greater" (chapter 2, "How Big Is Space?"). Humans found the homeland to lie inside a planet, the planet inside a solar system, the solar system inside a galaxy, and the galaxy inside clusters of galaxies and the whole of what

can be seen. Tegmark draws a second lesson from the story: "we've repeatedly underestimated not only the size of our cosmos, but also the power of our human mind to understand it" (chapter 2, "How Big Is Space?").

The measurements in Tegmark's story form a ladder of distances: each rung is the measurement of a larger distance than the rung below it. The first rungs of the ladder use nothing but angles. The Greek scholar Eratosthenes compared the noon Sun at two cities, Syene and Alexandria, on the longest day of the year, in Tegmark's telling. At Syene the noon Sun stood straight overhead, and at Alexandria the noon Sun stood a little to the south. Eratosthenes found the size of the Earth from the difference between the two angles of the Sun and the distance between the two cities. The Greek astronomer Aristarchos of Samos read the Moon's size from the curved edge of the Earth's shadow during an eclipse of the Moon. He then read the Moon's distance from how large it looks. Aristarchos went on to estimate the distance of the Sun from the angle between the Sun and the half-lit Moon, and that estimate went wrong, because the true angle is so close to a right angle that his estimate placed the Sun about twenty times too near. Tegmark draws the lesson exactly: the story of Aristarchos shows "the importance of quantifying uncertainties in our measurements" (chapter 2, "Distance to the Sun and the Planets"). The uncertainty of a measurement is its margin of error: the range within which the true value most likely lies. When an estimate comes with its uncertainty, you can tell whether a later measurement that differs from the estimate shows a true difference in the thing measured, or only the spread of the measurements themselves.

Astronomers took longer to measure the distance of a star, because a star is only a point of light. Tegmark teaches, with the reader's own thumb, the method by which astronomers measured the distance of the stars. Hold your thumb up at arm's length and close one eye and then the other: the thumb jumps against the background, and the nearer the thumb is to your eyes, the more the thumb jumps. The jump of the thumb is its *parallax*, and from the parallax you can work out the distance of the

thumb. Astronomers use the Earth's orbit as the two eyes. Astronomers look at a star from two points of the orbit six months apart, with the Sun between the two points. The astronomer Friedrich Bessel, at Königsberg, measured the parallax of the star 61 Cygni from two points of the Earth's orbit, and he published the result in 1838. Bessel used a heliometer, a telescope made by Joseph von Fraunhofer for measuring very small angles between stars. The parallax of 61 Cygni came to about a third of a second of arc (a second of arc is one 3,600th of a degree), an angle nearly six thousand times smaller than the width of the full Moon. We now know that the light of 61 Cygni takes about eleven years to reach us, and a light-year is the distance light travels in a year, so 61 Cygni is about eleven light-years away. Tegmark adds a second way of learning about the stars, besides parallax: the dark lines that Fraunhofer found in the colors of sunlight, which show what a glowing thing is made of. Together, parallax and the dark lines showed that the stars are suns, and Tegmark sums the case up in one line: "the only information we have about stars is in their faint light that reaches us" (chapter 2, "Distance to the Stars").

Astronomers needed a star of known true brightness in a galaxy, in order to measure the distance of the galaxy. If you know the wattage of a lamp, you can tell the distance of the lamp from how faint the lamp looks, and a star of known true brightness is a lamp of known wattage. Henrietta Swan Leavitt was an astronomer at Harvard College Observatory. In 1912 Leavitt published a discovery about certain pulsing stars, now called Cepheids. Cepheids pulse more slowly the brighter they are, so the rhythm of a Cepheid gives its true brightness. The astronomer Edwin Hubble found Cepheids in the haze called the Andromeda nebula. On 1 January 1925 the astronomer Henry Norris Russell read out a paper of Hubble's at a meeting of the American Astronomical Society in Washington, and the paper placed Andromeda about a million light-years away, far outside the stars of our own galaxy. We now know that Andromeda is more than two million light-years away. Each rung of the ladder pushed the known size of the world outward, and each rung was a measurement made from here.

Tegmark gives a name to the part of space that the ladder of distances can ever reach. Tegmark writes "our Universe", with a capital letter, for "The spherical region of space from which light has had time to reach us" (chapter 6, "What's a Universe?"). The time he means is the time that has passed since the hot, dense beginning he calls our Big Bang. In this book, "our universe", written small, means the same spherical region and everything in it. We call the edge of our universe our *horizon*: the farthest distance from which light has had time to reach us. In the kitchen, our universe is the circle of lamplight, and the horizon is the edge of the circle.

From Tegmark's pages

III. What Tegmark Showed: The Shape of Space

Tegmark then goes on from how far we have measured space to what space itself is. Tegmark tells Bolyai's story in the part of his book on what space is. Euclid treated the geometry of space as mathematics. For two thousand years, Euclid's geometry of an endless, flat space was taken to be the only way space could be. Tegmark tells how three mathematicians, Carl Friedrich Gauss, Bolyai and Lobachevsky, found other geometries, each as free of contradiction as Euclid's. Tegmark names the boldest step of the three mathematicians: "a space can be curved all by itself, even if it isn't the surface of anything!" (chapter 2, "What Is Space?").

Tegmark pictures a curved space with a blind ant on a surface. The ant cannot leave the surface or look at it from outside, but the ant can still draw a triangle and add up its three angles. On a flat sheet the three angles add up to 180 degrees, on a sphere to more, and on a saddle-shaped surface to less. A surface whose triangles add to 180 degrees is *flat*, and a surface whose triangles add to more or less is *curved*. So the ant can measure the curving of its surface without any third direction to look from. The ant's surface has two *dimensions*. A dimension is an independent way in which things can differ: a point on a city map dif-

fers from another point in two ways, east and north, and a point in space differs from another point in three ways. Then Albert Einstein completed his theory of gravity, general relativity, in Berlin in November 1915. In general relativity our own three-dimensional space can curve in just the way the ant's surface curves, and Tegmark puts the point in three words: "We're the ants!" (chapter 2, "What Is Space?"). So Tegmark says that the kind of space we live in "can only be resolved by performing measurements" (chapter 2, "What Is Space?"), the same measurements the ant makes: draw a huge triangle with rays of light, and check its angles. Tegmark reports that the angles of triangles the size of our universe add up to about 180 degrees, and in his chapter 4 he tells how he and his colleagues measured the angles.

Tegmark then sets out three shapes of space that Einstein's theory allows. First, space may go on forever. Second, space may be finite and curved, closing on itself like the three-dimensional surface of a ball in four dimensions. Tegmark calls a space of the second shape a hypersphere. In a hypersphere, a traveler going straight on would come home from the opposite direction. Third, space may be flat and finite, joined to itself in every direction like a video game screen whose right edge leads back to its left edge and whose top leads to its bottom. Mathematicians call a flat space joined to itself in every direction, like the video game screen, a *torus*. Mathematicians call the way a space is joined to itself its *topology*. Tegmark promises that his chapters 4 and 5 will tell of "finding evidence that space is truly infinite after all" (chapter 2, "What Is Space?").

Tegmark ends his chapter on space with a further question, in his own words: "what is space, really?" (chapter 2, "What Is Space?"). Mathematicians study spaces as mathematical things, and Tegmark writes: "One could indeed argue that space is a mathematical object, in the sense that its only intrinsic properties are mathematical properties" (chapter 2, "What Is Space?"). The mathematical properties Tegmark names are the number of dimensions of space, its curvature and its to-

pology. The curvature of space is how space curves, which the ant's triangles measure. Tegmark takes his first step toward the claim of his whole book in that sentence about space as a mathematical object. The claim of Tegmark's book is that the world itself is a mathematical structure.

IV. The Same Result, on Separate Ground

I agree with Tegmark's central point about the shape of space: which geometry holds of the space we live in is a question for measurement. I argue for that point from what mathematics can settle and what mathematics leaves to measurement, while Tegmark argues for the same point from Einstein's theory. So two separate arguments have the same result. I tell Bolyai's story and draw its conclusion about the two geometries, Bolyai's and Euclid's, in one sentence: "Which of them holds of the space we live in is a question mathematics does not settle: it is a question for measurement". Mathematicians work out what follows from their starting assumptions, in every case those assumptions cover, and they work out each geometry completely. Measurement answers whether the starting assumptions hold of the world.

In the sense of chapter 1, a geometry *exists*: it is laid out whole, timeless, and the same for everyone who studies it. A measurement of space is *real*, in the sense of chapter 1: it is given in an act of finding, made once, from somewhere. Bolyai's geometry and Euclid's both exist, completely and at once. Which geometry our space follows is given only in findings.

I also hold Tegmark's statement that the only information we have about stars is in their faint light that reaches us, and I hold the statement for a reason of my own. Every rung of Tegmark's ladder of distances was a comparison: an angle set against an angle, a shadow against a shadow, a faint light against a known one. *Comparing* is telling one thing from another, or finding the two things the same, and a *finding* is what one comparison gives (chapter 1). Every measurement is a comparison. Tegmark's sen-

tence about starlight is the same claim applied to the sky: everything we know of distant space is known by comparisons made here, from the light that has reached us.

My account has a word for space and time as comparisons find them, and I use the word from here on. The *arena* is space and time as found in comparisons, the place where findings happen. Every comparison has its *content*, what the comparison finds the same or different. Every comparison also has its *context*, the respect in which the two things compared are two: two moments, two places, or two possibilities. The arena covers two of the three contexts: place and time. I also say: "The arena of space and time, the stage on which everything happens, is smooth and uneven, and its shape is gravity". In that sentence, "smooth" means that the arena changes from place to place with no jump anywhere. "Uneven" means that one place in the arena can differ from another. Einstein's theory describes the arena, and in his theory the curving of the arena is gravity.

Since Tegmark wrote, astronomers have measured the angles of a triangle the size of our universe again, with greater precision. The Planck satellite mapped the faint glow of microwaves that fills the whole sky, the oldest light there is. The Planck team combined its map with surveys of how galaxies cluster, and found space flat to within about two parts in a thousand. The Planck team released the results in 2018 and published them in 2020. I hold that space is flat on the largest scales, and that space is curved only by its own small ripples. The ripples are the slight differences from place to place in how matter and light were spread. The ripples are about one part in a hundred thousand. I give a reason for the flatness of space. We live in a long era of the universe, and our era began at a passage: the change from the era before it into our era. No length from the earlier era carries over through the passage into our era. The curving of space as a whole is set by a length, as the curving of a ball's surface is set by the ball's radius, and the passage has one length of its own, far smaller than an atom. If the length of the passage had set the curving of space, either our era would have collapsed and ended almost at once, or the expansion of our era would have been set by the bend of

space itself rather than by the matter and light our era holds. In either case no stars and no galaxies would have formed in our era, but our era does have stars and galaxies, so our space has no curving of its own. Chapter 5 sets out the argument for the flatness of space in full. So the Planck measurement and my account agree that space is flat.

V. Where the Accounts Part: How Far Space Goes

I answer this chapter's question in a stated sense. For any finding made from inside space, space is infinite as far as any comparison can tell, and space nowhere closes on itself by a path shorter than the width of our horizon. I get this answer from the second half of our question: how could anyone standing here know? Tegmark promised his reader evidence that space is infinite, and my answer says in what sense space is infinite for anyone inside it.

Start with the circle of lamplight, which stands for our universe. Every finding we make about space is made within our horizon, because light has reached us only from within our horizon. The region within the horizon is a ball centered on us. I observe that no comparison can tell the ball within our horizon apart from a ball cut out of an endless flat space, down to the finest difference any comparison can detect. I then use its *principle of identity*: two descriptions that no comparison could ever tell apart are one description. A registration is the act by which a measurement made inside the world fixes what it registers: the measured value is fixed in the act of measuring itself. So I conclude that "for a registration, space is infinite up to the limit of distinguishability": for a measurement made inside the world, space is infinite as far as any comparison can tell. In the kitchen, the floor goes on as far as any looking from any seat can tell.

I then make a claim for every place in space. When a path has two properties, I call it a path around space. First, the path goes straight out and comes home from the opposite direction, as a straight path across the video game screen does. Second, no sliding of the path could shrink it to a

single point. When a path around space is shorter than the width of the horizon, I call the path a *short loop*. I hold that every path around space, wherever the path runs, is longer than the width of the horizon. So space has no short loop anywhere.

My reason for that claim depends on the dark matter, and chapter 4 tells what the dark matter is. I hold that a count made in each measurement from inside sets how much dark matter there is beside the ordinary matter (the atoms of the stars, the gas and ourselves). The *basic pieces* of a shape are the fewest building blocks from which the shape can be put together. The count is a count of the basic pieces from which the region that the measurement registers is built. A simple ball is built from one basic piece. Chapter 4 shows a globe built from two basic pieces: a single point, and a disc whose whole rim is drawn together to that point. The count that follows answers whether space has a short loop; its last sentence gives the answer: space has no short loop anywhere. In the count, the ordinary matter fills the region and counts one for each basic piece, so over one simple ball it counts one. I call the three directions of space that we move through "the open space". So the open space counts as one, whatever it is beyond the horizon. A region crossed by a short loop is not a simple ball, because from the center of such a region one place inside it can be seen along two different straight routes. So in the count, a region crossed by a short loop is built from two basic pieces or more. I hold that the count gives the same ratio of dark matter to ordinary matter for a measurement made from any place in space, and not only for a measurement made from our place. With one basic piece, the ordinary matter counts one and the dark matter counts six less two divided by pi, so the ratio of dark matter to ordinary matter is about 5.36. Chapter 4 tells where the six and the two divided by pi come from. With two basic pieces, the ordinary matter would count two, and the dark matter, which also fills the region and counts six for each piece, would count twelve less the same two divided by pi. The two divided by pi is taken away once for the whole measurement, so the ratio would then be about 5.68. The share is taken away once because a measurement is one act: the share is built from the

three channels of the act of registering and from a count that the curving itself sets, and neither is a quantity of the region, so the share is the same whatever the number of pieces. The combined measurements of the oldest light put the ratio within about one percent of 5.36, and they exclude 5.68. So the measured ratio is the ratio for one basic piece. The count gives the same ratio from every place. So the region within the horizon of every place is one simple ball, and space has no short loop anywhere. The appendix "For the Specialist" gives the same count in numbers.

Astronomers have already searched for a short loop through our location. The oldest light reaches us from a great sphere around us, a little inside our horizon. Before the oldest light went free, light was knocked aside again and again. So the great sphere is the farthest place from which light has come to us straight. Chapter 3 tells how the oldest light went free. A short loop through our location would leave a mark in the oldest light. Light from one place would reach us along two routes, so the sky would show the same pattern of hot and cold spots around a pair of circles in two different directions. Pip Petersen and colleagues, researchers in cosmology, searched the maps of the oldest light made by two satellites for pairs of matching circles. In 2023 the team reported that no pair of matching circles stands out from the random speckle of the maps. So we now know that every path around space through our own location is longer than 98.5 percent of the width of the great sphere from which the oldest light comes. The search tested only paths through our own location, while my claim covers every path, everywhere. The appendix "What the Coming Measurements Will Find" gives what I say further searches will find.

Of Tegmark's three shapes of space, two agree with my answer and one does not. An endless flat space fits my answer exactly. When a flat space is joined to itself like the video game screen only on a scale larger than every horizon, that space also satisfies my two claims about space: space is flat, and space has no short loop. No finding made anywhere inside a flat space joined to itself on a scale larger than every horizon could tell that space apart from an endless flat space. A space curved like a hy-

persphere differs from my flat space. The measurements find space flat as far as the measurements reach. No possible experiment done within a domain (the universe) can demonstrate the closure of the domain (universe). I use the word "domain" for a whole world within which comparisons can be made. In the aphorism, the word "universe" names the whole of the space we live in, not only the part we can see. So my answer covers everything any finding could ever show. No finding made inside space could show whether space closes on itself beyond every horizon.

Tegmark's account and mine also differ about Tegmark's pattern: each whole that humans have mapped proved to be part of something greater. If the pattern is followed upward, it makes a ladder of wholes, each whole inside the next. Tegmark's ladder of wholes goes on past our universe to one or more levels of what he calls parallel universes, further universes beside our own. I hold that the ladder of wholes has no top rung: no domain contains every domain. There is no single domain that encompasses all other domains. I give as my reason the Incommensuration Theorem, applied to everything there is, taken together. The Incommensuration Theorem, which chapter 1 set at the base of this book, says that no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The aphorisms of *An Immanent Metaphysics* use the word "exists" in another sense than this book does. For the aphorisms, what exists is everything there is: everything that can act on anything else. The aphorisms call gaplessness in the possibility of acting continuity of potentiality. Everything there is can act on everything else there is, directly or through other things, so continuity of potentiality applies to each thing there is and to everything there is, taken together. The aphorisms call sameness in what each thing actually is symmetry of actuality. Each thing there is has being of the same nature as every other thing there is, so symmetry of actuality also applies to each thing there is and to everything there is, taken together. The Incommensuration Theorem holds of the two concepts themselves: the concept of perfect symmetry and the concept of perfect continuity are themselves fundamentally incommensurate. So perfect sameness and perfect gaplessness cannot both belong, fun-

damentally, to everything there is, taken together. If everything there is, taken together, were one completed whole, perfect sameness and perfect gaplessness would both belong to it fundamentally, at once. So everything there is, taken together, is never one completed whole.

VI. Where the Accounts Part: What Space Is

I hold that space is the arena, found in every comparison, and that the mathematics of space describes the structure of the arena. Tegmark's sentence says that space "is a mathematical object, in the sense that its only intrinsic properties are mathematical properties" (chapter 2, "What Is Space?"), and on what space is, Tegmark's account and mine give different answers.

I keep every mathematical property of space that Tegmark names: the number of dimensions, the curvature and the topology. I apply its distinction between what exists and what is real to space. The structure of the arena is the number of dimensions of space, its curvature and its topology, and that structure exists: it is laid out whole, timeless, and the same for everyone. The arena itself is met in findings. Every finding is made at some place and some time, from a seat, and the arena is where each finding happens. The structure of the arena is what all findings share, and the arena is where each finding is real.

The arena and its mathematical structure are two because of the Incommensuration Theorem, which says that symmetry and continuity cannot both be perfected in one comparison, and that neither can asymmetry and discontinuity. Symmetry means sameness, as of a thing unchanged when you swap it or move it, and continuity means having no gaps. The opposites of symmetry and continuity are asymmetry (a difference from one place or moment to the next) and discontinuity (coming in separate steps). A mathematical object, taken as a whole, is laid out at once and is the same wherever and whenever it is read. So a mathematical object, taken whole, holds perfect sameness and a gapless completeness, with

nothing in it missing and nothing still to come. A mathematical structure can be smooth and perfectly symmetric at once, like the circle of geometry; no comparison ever finds a thing that is both. The arena that comparisons find is smooth and uneven: gapless, and different from place to place. The shape of the arena is gravity. So the space in which our findings are made is the arena. Chapter 10 sets out the Incommensuration Theorem step by step.

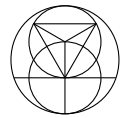
Bolyai's discovery shows a structure and a finding together, in one story. Bolyai's new geometry was complete the moment it held together, and it was the same for every reader. So in this book's sense Bolyai's geometry existed. Whether our space follows Bolyai's geometry was given only later, in findings made with instruments, from here. Mathematics laid out the possible spaces. Measurement, made from a seat inside the arena, finds which of the possible spaces our space follows.

VII. What This Chapter Holds

For every finding that can ever be made from inside space, space goes on as far as any comparison can tell, flat and with no short way around. We learn the shape of space from where we sit, by looking, because space is the arena where findings happen.

The answer about space matters for this book's question: What is reality, and can physics find out? The shape of space is a structure that mathematics lays out whole. Which shape is ours is given only in findings made from a seat inside space. So the shape of space has both parts: what exists (the possible shapes, laid out by mathematics) and what is real (the findings that tell which shape is ours). Physics finds out which shape is ours, from inside space, one finding at a time.

Every finding about space comes in light that reaches us here and now. How, then, can that same light tell us what happened long before anyone was here to see it?



Chapter 3

Where Did It All Come From?

Beside Tegmark's chapter 3, "Our Place in Time"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

What *exists* is laid out whole, timeless and the same for everyone, as the list on a score sheet is; what is *real* is given in an act of finding, once, as a move is made (Chapter 1).

The *arena* is space and time as found in comparisons, the place where findings happen (Chapter 2).

I. A Hiss from Every Direction

In his third chapter, Max Tegmark takes his reader back through time, from the Moon to the first minutes of the universe. You can carry one question with you the whole way. Where did the Sun, the galaxies and the atoms come from, and how can anyone know what happened before anyone was there?

Arno Penzias and Robert Wilson were radio astronomers at Bell Telephone Laboratories, in Holmdel, New Jersey. From July 1964 to April 1965, Penzias and Wilson measured the sky with a horn-shaped antenna twenty feet across. The antenna had been built to catch radio signals bounced off a satellite. Penzias and Wilson had put the antenna to work for astronomy. Radio astronomers state the faint noise an antenna picks up as a temperature: the temperature a body would need to give off that much noise. Penzias and Wilson compared the noise coming down the antenna with the noise of a reference source sunk in liquid helium, a few degrees above *absolute zero*, the lowest temperature there is. Penzias and Wilson took away the noise of every source they knew of, from the air overhead to their own equipment, but about three and a half degrees were left over. The leftover was the same in every direction Penzias and Wilson pointed the horn. The leftover did not change with the seasons. A pair of pigeons had been roosting in the narrow end of the horn, so in the spring of 1965 Penzias and Wilson evicted the birds and cleaned out what the birds had left behind. The leftover hardly changed. Penzias and Wilson published the measurement in July 1965, in *The Astrophysical Journal*. In the same issue of the journal, four physicists read the leftover noise as light remaining from a hot early universe: Robert Dicke, James Peebles, Peter Roll and David Wilkinson. Penzias and Wilson received the Nobel Prize in Physics for 1978 for finding the leftover noise.

Penzias and Wilson learned about the early universe at Holmdel, in the act of measuring, and nowhere else: a faint glow arrived at their antenna, there and then, and Penzias and Wilson compared the glow with a cold reference on the ground. We piece together the past of the universe in the way Penzias and Wilson did, from what arrives now.

Think back to the game of chess at the kitchen table in the first chapter: the board, the moves, and the score sheet, the written list of moves that everyone who walks into the kitchen reads the same way. Now change the scene. You walk into an empty room and find a game half played on the board, the captured pieces standing beside the board, and no score sheet anywhere. You are alone with the board. You still know a great deal. You know the rules, which are the same at every table. You can also see the position on the board now. You can work backward from the rules and the position and write out much of a score sheet for the game: this bishop has moved, that knight was taken, this pawn has advanced. Your rebuilt sheet runs back to a first move. The first move is as far back as your rebuilding can go. In this picture the rules are the laws of physics, the same everywhere. The position on the board is the present state of the universe: the point the history of the universe has reached now. Penzias and Wilson were reading the position on the board, in the light that arrives from the sky.

II. Tegmark's Frontier, Pushed Back in Time

Tegmark pushes the edge of what is known further back in time with each step of his history of the universe. At every step Tegmark sets the evidence beside each claim. The physicist Isaac Newton took the first step. Newton supposed "that heavenly objects obey the same laws as objects here on Earth" (chapter 3, "Where Did Our Solar System Come From?"). The Moon's measured speed in its orbit agreed with what Newton's laws predicted. Newton's laws also explained the tides. The tides drag on the spinning Earth and slow its spin. Tegmark argues from the slowing of the

Earth's spin that the Earth and the Moon, in their present arrangement, cannot be more than four to five billion years old. Tidal friction is the drag of the tides on the spinning Earth. In Tegmark's words, tidal friction "kills the idea of an eternal solar system" (same section).

Tegmark shows how the Solar System itself formed: a great cloud of gas fell together under its own gravity, spun faster as the cloud shrank, and flattened into a disc, with the Sun at the center and the planets in the outer parts. Astronomers see other stars forming today from collapsing clouds of gas in our Galaxy. Radioactive dating reads the age of a rock from how much of the rock's radioactive atoms have changed into other atoms. Radioactive dating gives the oldest meteorites an age of about four and a half billion years, which agrees with the estimate from the tides. The galaxies formed out of gas through a contest between gravity and pressure. Gravity pulls gas together, while pressure pushes gas apart. The contest between gravity and pressure made the Solar System too, on a far smaller scale.

Tegmark's next step concerns the universe as a whole. The physicist Alexander Friedmann solved Albert Einstein's equations of gravity for a universe filled evenly with matter. Friedmann found that most of the solutions change over time, and that a universe standing still could not stay still for long. Tegmark puts Friedmann's result in one sentence: "Friedmann's work showed that the natural state of our entire Universe is to be in motion" (chapter 3, "Our Universe Could Be Expanding"). In a paper of 1929 the astronomer Edwin Hubble measured galaxies moving away from us, the farther ones faster, roughly in proportion to their distance. Divide each galaxy's distance by its speed, and the answer comes out the same for every galaxy. Tegmark uses modern measurements and gives the answer as about fourteen billion years: the time since everything we can see was packed close together.

Tegmark then makes the expansion easy to picture. The galaxies are like chocolate chips in a rising muffin, and space is like the dough. The chips are carried apart as the dough grows. Our own Galaxy is held together by its own gravity and does not expand, as a chocolate chip does

not grow with the dough. The universe needs no outside to grow into, because the universe grows as new volume appears between the galaxies. A distant galaxy can recede faster than light without breaking any law, because "nothing is allowed to move faster than light through space, but space itself is free to stretch however fast it wants to" (chapter 3, "Making Sense of an Expanding Universe"). Light takes time to travel, so looking farther away is looking further into the past. A telescope shows young galaxies far off and mature ones nearby. Tegmark draws one lesson from Newton and Friedmann together. Tegmark puts the lesson in a mantra: "Dare to extrapolate!" (chapter 3, "Where Did the Mysterious Microwaves Come From?"). To extrapolate is to carry what is known beyond the range in which that knowledge was gained. Newton extrapolated when he carried the laws of the Earth to the Moon.

From Tegmark's pages

III. The Glow, the Furnace and the Seeds

The physicist George Gamow extrapolated boldly. Gamow ran the expansion backward and found a young universe so hot that its hydrogen was a *plasma*, a gas so hot that its atoms are broken into loose electrons and nuclei. Gamow reasoned that we should see a wall of glowing plasma in every direction, its light stretched by the expansion into microwaves, radio waves a few centimeters long, because a hot plasma glows and cannot be seen through. Gamow's students Ralph Alpher and Robert Herman predicted that the glow would now be at about five degrees above absolute zero, while Penzias and Wilson found the glow at about three and a half. Tegmark states the conclusion plainly: "our entire Universe was once as hot as a star" (chapter 3, "Seeing the Afterglow").

Tegmark explains that the glow had to carry small differences from place to place, because a perfectly even beginning would have given a perfectly even universe today, with no galaxies anywhere. The Cosmic

Background Explorer satellite (COBE) found the differences, which the COBE team published in 1992: the glow is warmer in some patches of sky and cooler in others, by about one part in a hundred thousand.

Gamow pushed further back in time, to the first minutes after the start, when the whole universe was hot enough to fuse hydrogen into helium, as the core of the Sun does. Tegmark reports the result of the modern calculation: the young universe "fused about 25% of its mass into helium" (chapter 3, "The Cosmic Fusion Reactor"). The calculated share matches the helium measured in distant gas between the galaxies. The first minutes also left small amounts of *deuterium*, heavy hydrogen, and of lithium, the third element after hydrogen and helium. Astronomers have measured the amounts of deuterium and lithium. The heavier atoms, carbon, oxygen and the rest, were made later, inside stars, and scattered when stars blew apart, so that "we're made of star stuff" (chapter 3, "Big Bang in Trouble").

Tegmark is careful about what the glow and the helium establish. Tegmark states a deliberately modest hypothesis and calls the hypothesis the Big Bang Hypothesis: "Everything we can observe was once hotter than the core of the Sun, expanding so fast that it doubled its size in under a second" (chapter 3, "What Is a Big Bang, Really?"). A *singularity* is a moment of infinite density at which the equations of gravity give no answer. Tegmark finds no evidence for a singularity, because the extrapolation of the equations of gravity fails before the extrapolation gets to a moment of infinite density. Tegmark states plainly what is not known: "we don't even know for sure that our Universe really had a beginning at all" (same section). Tegmark leaves the reader with a question about the seeds of the galaxies. Tegmark writes that "gravity can only amplify small fluctuations into larger fluctuations" (same section), so there must have been small seeds from the start: places a little denser than their surroundings, which gravity could grow into galaxies. Tegmark takes up the question of where the seeds

came from in his next chapters. Tegmark closes his chapter on its widest claim: “our most accurate description of our cosmic history is a mathematical description” (chapter 3, “The Bottom Line”).

IV. The Same History, Reached on Separate Ground

I tell the same history of the universe as Tegmark does. I give a reason of my own, separate from Tegmark’s reason, for each part of the history that this section names. Comparing is telling one thing from another, or finding two things the same, as the first chapter set out. Every measurement and every interaction is a *comparison*. A finding is the result of one comparison. Penzias and Wilson made a comparison each time they set the noise coming down the antenna beside the noise of the cold reference.

On my account, as on Tegmark’s, the Earth has a history with a beginning, but my reason for the beginning lies in the Earth’s heat. The Earth is spending its heat, and whatever spends cannot have been spending for ever unchanged. Tegmark’s reason is the tides. The age of the Earth is a measured number. In 1956 the geochemist Clair Patterson read the age of the Earth from the lead in meteorites, which formed with the Earth, and found about four and a half billion years. Tegmark reports an age of about four and a half billion years too, from the radioactive dating of the oldest meteorites.

On my account too, each galaxy keeps its own size while space stretches between the galaxies, because the parts of a galaxy are held together by their own motion and pressure. On my account, as on Tegmark’s, a distant galaxy can recede faster than light. The speed of light sets a limit on how fast a signal or a cause can pass from one thing to another, but a distant galaxy sends us no signal and no cause by receding, so the speed of light sets no limit on the recession. The cosmological horizon is the edge beyond which no light can ever reach us. The recession of distant galaxies is the reason the cosmological horizon is where it is. The horizon of chapter 2 is a different edge, which marks the distance from

which light has reached us so far. The cosmological horizon is a second edge. The cosmological horizon marks the distance from which light can ever reach us.

I describe the same glowing wall of plasma that Gamow described. I picture the wall as a valley full of fog that clears. Picture a valley filled with fog on a sunny morning: the fog glows, bright all through, and nothing can be seen across the valley, because each ray of light is knocked aside by one droplet after another. When the fine droplets gather into raindrops and fall, the light passes between the raindrops. The air clears, and the far end of the valley comes into view. In the picture, the fog is the young universe, and the droplets are its free electrons. While the young universe held one free electron for every proton, each electron knocked aside any light that struck it, so every ray of light was knocked aside again and again. As the universe cooled, the electrons were bound into atoms, and the light went free. Physicists call the moment the light went free *last scattering*: the last time the free electrons knocked the light aside. A kelvin is one degree of temperature, counted up from absolute zero. At last scattering the light was at about 2,970 kelvin. The Planck satellite mapped the oldest light across the whole sky and measured how much the universe has grown since last scattering: about 1,090 times. The measured numbers of the Planck satellite's final results place last scattering about 370,000 years into the history we piece together. The light has cooled by the same factor of about 1,090 since last scattering. The light set free at last scattering is the leftover noise that Penzias and Wilson measured at Holmdel: the light arrives now at 2.7255 kelvin, the temperature measured across the whole sky.

I agree with Tegmark about the helium. My ground for the helium is a count of neutrons. About two minutes into our history, roughly one neutron was left for every seven protons. A *nucleus* is the small dense core of an atom, made of protons and neutrons. Every nucleus heavier than hydrogen's single proton had to be built. The building starts with deuterium, heavy hydrogen, whose nucleus is one proton bound to one neutron. Once the light had cooled enough to stop breaking deuterium apart,

nearly every neutron ended in helium. Each neutron took one proton with it into the helium, because a helium nucleus holds two protons and two neutrons. Take eight particles, one neutron and seven protons: the neutron and the proton it takes make two of the eight, a quarter. A quarter of the mass ended in helium, because a proton and a neutron weigh nearly the same. The measured share is 0.245. So the universe's helium came from the neutrons. I derive the list of the kinds of particle from the parts that every act of comparison has. The first chapter named the parts that every act of comparison has: sameness and difference, the content and the context, the stance from which the comparison is made, and what the comparison is made about. I also show that the list is closed: the list holds every kind of particle there is. The kinds of particle on the list enter the calculation of the first minutes after the start at two points. At the first point, the count of the kinds of particle that filled the young universe sets how fast the young universe expanded. In the count of the kinds, each kind is counted with every form the kind can take. At the second point, the kinds enter through the count of neutrons. Three things set the count of neutrons: the neutron's mass, a little above the proton's; the force that changes neutrons and protons into one another; and the time a free neutron lasts before it decays. The share of helium comes out as the physicists' calculation gives it, because my list gives the calculation of the first minutes the same inputs as the calculation physicists already use.

The deuterium left over from the first minutes measures one more number: how many protons and neutrons there were for each particle of light. A little deuterium escaped being made into helium. In 2018 three astronomers measured the amount: about two and a half deuterium nuclei for every hundred thousand hydrogen nuclei. The three astronomers were Ryan Cooke, Max Pettini and Charles Steidel. When there were more protons and neutrons for each particle of light, a deuterium nucleus met a partner and was burned into helium more often, and so less deuterium was left over. Physicists make a second count of the protons and neutrons for each particle of light from the pattern in the microwave glow, by other physics entirely, in light set free hundreds of thousands of years later. The

count from the deuterium agrees with the count from the glow within the uncertainties of the two counts. Tegmark prizes confirmation by an independent route: "A key to believability in science is having an independent experiment confirm your results" (chapter 3, "The Gold Rush"). I give my own reason why independent confirmation counts. An argument about the world is built like a cable, of many strands from independent sources: each strand is one line of evidence, and the cable holds when one strand breaks. I call independent routes that arrive at one value a "convergence", and say: "Convergence is the strongest evidence that inquiry into the world can have".

On my account, as on Tegmark's, the seeds had to be there from the start. My reason is the Incommensuration Theorem. The first chapter gave the Incommensuration Theorem: symmetry and continuity cannot both be perfected in one comparison, and neither can asymmetry and discontinuity. In the theorem, symmetry means sameness: a thing unchanged when you move it. Continuity means having no gaps. In plain words, no single act of finding holds perfect sameness and perfect gaplessness together, nor the two opposites together. The Incommensuration Theorem applies to every event as well as to every comparison, as its own words say, so the theorem applies to every event of the young universe. If the young universe were perfectly the same in every direction and at every place, it would have no jump anywhere, and so it would hold perfect sameness and perfect gaplessness together, which the theorem rules out. So the young universe began even, and never perfectly even. Perfect sameness in every direction and at every place would make symmetry and continuity both perfect, and the theorem forbids that. Tegmark's reason for the seeds is gravity, which can only enlarge differences already there, while my reason for the seeds is the Incommensuration Theorem. The two separate reasons show one result: the young universe held seeds from the start.

I agree with Tegmark that the history gives no evidence of a moment of infinite density, but I give my own reason. Einstein's theory of gravity is the smooth description of space and time. Einstein's theory describes space and time as changing from place to place with no jump anywhere.

The smooth description holds at every scale down to the *Planck scale*. The Planck scale is a length far smaller than any atom, at which gravity and the quantum act on equal terms. Quantum theory is the stepwise description: quantum theory describes the smallest things as changing in separate steps. The Planck scale is the edge of the range of the smooth description, and below the Planck scale the smooth description no longer holds. The Planck scale is also the edge of the range of the stepwise description of quantum theory, and no further description holds below the Planck scale. I derive Einstein's equations from comparison. I hold that the singularity is what the equations of the smooth description produce when physicists carry the equations past the range where the equations hold. Tegmark names quantum mechanics as the reason the extrapolation fails, while I name the Planck scale, the edge of the ranges of both the smooth description of gravity and the stepwise description of quantum theory.

The appendix "What the Coming Measurements Will Find" sets out what, on my account, the coming maps of the sky will show of the early history of the universe.

V. How the Past Is Reached

I hold that we know the past only by the impression the past makes on the present. Tegmark writes that the equations of physics predict the future, and also that "the equations can be solved equally well in reverse, using knowledge of the present to reveal the past" (chapter 3, "Where Did Our Solar System Come From?"). I agree that the equations can be run backward, but I add two points: the ease of running the equations backward is a property of the equations, and the world itself runs one way.

I show the difference between the equations and the world with a beautiful machine. An *orrery* is a working model of the solar system: a brass Sun at the center, the planets on arms around the Sun, and beneath the planets a train of gears that carries each planet round at its proper pace. Crank the gears the other way, and the planets go round backward,

just as smoothly. The solar system itself runs one way. The Moon raises tides in the Earth's oceans. The tides drag against the spinning Earth and slow the Earth's spin. The drag of the tides also pushes the Moon slowly outward. The Apollo astronauts left reflectors on the Moon. Laser light bounced off the reflectors shows the Moon moving away from us by about 3.8 centimeters a year. The orrery has no gear for the tides or for the receding Moon. The ease with which the model runs backward is a property of the model. Tegmark's own argument from the tides shows that the solar system runs one way: the tides slow the Earth's spin, so the Earth and the Moon, in their present arrangement, cannot have gone on for ever.

I hold that we know the past in the way Penzias and Wilson knew the early universe: from what arrives now. We piece together one history from impressions arriving now: the light of distant galaxies, the glow of the oldest light, the amounts of the chemical elements and the records of every instrument. The history we piece together is the one history that the present requires. The pieced-together history is a structure, and the history exists as a map or a record exists. In this book's sense, what exists is laid out whole, timeless, and the same for everyone. What is real is given in an act of finding, once.

In the empty room, the rebuilt score sheet is the picture of the history of the universe as we piece that history together. The lines of the rebuilt sheet are laid out whole. The lines read the same for anyone who picks up the sheet. The moves on the sheet were made once each, by a hand at the board, from a seat. The sheet is laid out whole, as what exists is, while each move was made once, as what is real is given.

I differ from Tegmark over the question of what the pieced-together history is. Tegmark closes his chapter by calling the most accurate description of our cosmic history a mathematical one. I hold that the description of our cosmic history is mathematical, and a mathematical description comes this close: the mathematical description of physics approaches the physical process as closely as one likes and never coincides with it. In his chapter 10 Tegmark goes one step further. Tegmark holds that the world is a mathematical structure, and so he takes the history, laid out whole as

mathematics, for the world itself. My account does not follow Tegmark from the history, laid out whole, to the world itself. On my account the history is a structure, which exists, built from findings, which are real. The rebuilt score sheet can be exact in every line. Even so, the rebuilt sheet is the score sheet of the game. Chapter 10 of this book follows in full the question whether the game itself is its score sheet.

VI. The Edge of the Score Sheet

Tegmark leaves one question open at the edge of what is known: did the universe have a beginning? I give a definite answer. Trace the pieced-together history backward, and you find density and curvature rising: the bending of space and time grows sharper and sharper. The pieced-together history approaches a limit without ever reaching the limit. I take a history to report each thing it holds as happening at a definite time and place, as a record or a map does: this happened, then, there. So when our history is traced backward, our history draws the limit as a point in time, a point that looks like a first moment.

I call the limit *the turn*: the passage from the end of one *epoch*, one long age of the universe, to the start of the next. An epoch runs from one turn to the next and is the age in which stars, galaxies and black holes form. A *black hole* is a region where matter is squeezed so tightly that not even light climbs back out. I take the turn to be the limit at the edge of the ranges of both the smooth description and the stepwise description, where no further description holds. Knowing the turn as an event would need a comparison that sets a time before the turn beside a time after the turn. But every comparison is made inside one epoch. So we can ask how long, how hot and how fast of the approach to the turn, from the end of the old epoch and from the start of ours, but not of the turn itself.

I identify what we call the Big Bang with the turn as the turn appears in the history we piece together from inside our epoch, where the turn, a limit, is drawn as a point. Stand on a beach and look out to sea. You can

see as far as the line where sea and sky appear to touch, but the sea goes on past the line. The Big Bang is the edge of the history we can piece together, as the line between sea and sky is the edge of what you can see from the beach: the Big Bang is the beginning of our epoch, and the Big Bang is not the beginning of everything. As the sea goes on past the line, the world goes on past the turn. Beyond the turn lies the end of the epoch before ours. In the phrase "the epoch before ours", the word "before" names the order of a cycle of epochs, and no comparison measures a time across the turn. Chapter 5 follows how our epoch opened out of the turn. Chapter 15 follows how our epoch ends: first in the long emptying of its sky, as the expansion carries everything not bound to our own group of galaxies beyond the cosmological horizon, and then in a turn that begins the next epoch. Inside every black hole of our epoch, the end is the same: the turn that begins the next epoch.

Come back to the empty room. Your rebuilt score sheet begins with a first move. The first line of the sheet is as far back as your rebuilding can go. The table, the board and the pieces were in the room before the first move was played. The first line of the sheet is the beginning of this game. The room holds more than this game, as the world holds more than our epoch.

VII. What This Chapter Holds

Tegmark tells the history of the universe from the hot early universe, through the helium made in the first minutes after the start, to the galaxies. I tell the same history, pieced together from what arrives now. What we call the Big Bang is the edge of the pieced-together history: the beginning of our epoch, and not the beginning of everything.

This chapter's answer matters for the book's question: What is reality, and can physics find out? Tegmark gives the history of the universe as one case of a world made of mathematics, because he calls the description of the history mathematical, and the most accurate description we have. The

history of the universe is itself a score sheet rebuilt from the position on the board: a structure laid out whole, as exact as the measurements it is built from. The history is built from acts of finding made one at a time, such as Penzias and Wilson's finding of the hiss of leftover noise at Holmdel. Physics finds the world in acts of finding.

Gravity built the galaxies out of the faint seeds of the young universe, but was the matter we have counted, the atoms of hydrogen and helium, enough to do the building, and if it was not, what else does the universe hold?



Chapter 4

What Is the Universe Made Of?

Beside Tegmark's chapter 4, "Our Universe by Numbers"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

The *arena* is space and time as found in comparisons, the place where findings happen (Chapter 2).

An *epoch* is one long age of the universe, from one turn to the next; the *turn* is the passage from the end of one epoch to the start of the next, and what we call the Big Bang is the turn as it appears in the history we piece together from inside our epoch (Chapter 3).

I. Galaxies That Move Too Fast

If atoms are only about five percent of everything, what is the rest? In his fourth chapter Max Tegmark gives the measured shares of the rest to the nearest percent. He leaves what the rest is as a question.

The first clue to what the rest is came in 1933, from a crowd of galaxies. Point a large telescope toward the faint constellation called Coma Berenices, Berenice's Hair. You find hundreds of galaxies packed together in one swarm, each galaxy a whole system of stars. Astronomers call a swarm of galaxies a cluster of galaxies. The swarm in Coma Berenices is the Coma cluster. Fritz Zwicky was a Swiss astronomer working at the California Institute of Technology. Zwicky put the number of galaxies in the Coma cluster at about eight hundred. He took the speeds of eight of the galaxies in the Coma cluster from earlier observers, who had measured the speeds from the light of the galaxies. The eight speeds differed from one another by at least fifteen hundred to two thousand kilometers a second.

A swarm of galaxies holds together when the pull of the whole swarm is strong enough to keep the fastest galaxies in the swarm from flying off, just as the Sun's pull keeps the planets from flying off. Zwicky estimated how much pull the visible galaxies could supply. He found the pull of the visible galaxies far too little. He wrote that, for the cluster to hold galaxies moving as fast as the eight galaxies he studied, the average density of the cluster would have to be at least four hundred times greater than the density of its luminous matter. Luminous matter is the matter that shines. Zwicky published his paper in 1933 in the Swiss journal *Helvetica Physica Acta*, and in the paper he drew the conclusion in plain words: there would be dark matter in much greater density than luminous matter. Zwicky's paper is the first place where anyone inferred the dark matter. Zwicky inferred the dark matter from the speeds of the galaxies of the Coma cluster.

The second clue came from a single galaxy. In 1970 the astronomers Vera Rubin and Kent Ford published the speeds at which glowing clouds of gas circle the center of the Andromeda galaxy, speeds that Rubin and Ford had measured at sixty-seven places, out to nearly eighty thousand light-years from the center of the galaxy. The outer clouds moved round the center about as fast as the clouds much nearer to the center. When the outer parts of a galaxy move round as fast as its inner parts, the mass of the galaxy keeps growing as you count farther and farther out from the center of the galaxy.

Both clues are evidence of something that pulls and does not shine. First we follow what Tegmark showed about the unseen mass. Then we follow my physics, built from the act of comparison, to the places where I derive Tegmark's results from my own starting point. Last, we follow those books to their two answers: what the unseen mass is, and what the energy of empty space is.

From Tegmark's pages

II. What Max Tegmark Showed

Tegmark tells how cosmology, the science of the universe as a whole, became a science of exact numbers. Cosmologists call the new science of exact numbers *precision cosmology*. Tegmark starts from gravity. When a region of space is slightly denser than its surroundings, the region pulls in more matter, grows denser, and so pulls harder still. Tegmark puts the rule of growth in one line: "The essence of instability is that small fluctuations get amplified" (chapter 4, "Wanted: Precision Cosmology"). Gravity grew faint differences of density into galaxies and clusters of galaxies over the long history of the universe.

The faint differences of density show up in the oldest light, the glow left from the hot early universe that reaches us from every direction (chapter 3 of this book met the oldest light). In 1992 the Cosmic Background Explorer satellite (COBE) measured the size of the faint dif-

ferences, as chapter 3 told: the glow is warmer in some directions and cooler in others by about one part in a hundred thousand. Tegmark draws the consequence: differences of one part in a hundred thousand could grow into today's galaxies in the time available only if some unseen matter added its pull. Tegmark gives the unseen matter a name that says how little was then known about the unseen matter: "This mysterious stuff is known as dark matter, which is really little more than a name for our ignorance" (chapter 4, "Wanted: Precision Cosmology").

Tegmark then shows how cosmologists weighed the whole universe, the unseen part included. The oldest light has warm and cool spots, and the spots come in favored sizes, as a drum sounds favored notes. The *power spectrum* is a plot of how strongly each size of spot shows up. Tegmark gives the reason the power spectrum is so useful. Astronomers can measure the power spectrum, and for any model of the universe cosmologists can calculate the power spectrum exactly. The size of the most common spots depends on the shape of space. The reason is that curved space magnifies or shrinks the spots, as a curved lens does. Tegmark states an idea that he counts among the most beautiful in Einstein's theory of gravity, that "geometry isn't just mathematics: it's also physics" (chapter 4, "Gold in the Hills"). In Einstein's theory, matter curves space, and the curving is gravity. So the size of the most common spots gives the total amount of matter and energy in the universe, because the total amount of matter and energy sets the curvature of space. Boomerang was a telescope carried by a balloon for about eleven days around Antarctica. In April 2000 the Boomerang team reported that the first peak of the power spectrum, which marks the size of the most common spots, lies where a flat space puts the first peak (a flat space is a space in which lines that start out parallel stay parallel). Tegmark explains that the total amount of energy in the universe followed from the position of the first peak.

Next came the budget of the universe: the division of the total energy among the ingredients of the universe. Atoms and dark matter together made up only about thirty percent of the total, so about seventy percent had to be something spread evenly through space that never gathers into clumps. Cosmologists named the evenly spread seventy percent *dark energy*. Tegmark then reports that two teams had already found the same seventy percent by a completely different method. The two teams used exploding stars of one kind, called Type Ia supernovae. Every Type Ia supernova shines with nearly the same brightness wherever it goes off, so astronomers can read the distance of a Type Ia supernova from how faint the supernova looks. Saul Perlmutter headed one team, and Brian Schmidt headed the other, with Adam Riess as the lead author of the paper of Schmidt's team. In 1998 the two teams presented the same finding: the distant supernovae were fainter than a slowing expansion allows, so the expansion of the universe has been speeding up. Perlmutter, Schmidt and Riess received the Nobel Prize in Physics in 2011. Tegmark draws the lesson in his own words: "Cross-checking is a bad thing in ice hockey but a good thing in science, where it can reveal hidden mistakes" (chapter 4, "A 50% Batting Average"). In the second cross-check that Tegmark tells, cosmologists weighed the atoms in two ways: by the height of the second peak of the power spectrum, and by the amounts of the light elements made in the hot early universe, whose making chapter 3 told. The two ways disagreed at first. A year later the two ways agreed, at about five percent.

The oldest light gives some numbers only in combination, such as the curvature of space and the share of dark energy. Maps of galaxies in three dimensions measured the curvature of space and the share of dark energy separately. By the early years of this century the numbers of the budget had settled. Tegmark's Table 4.1 gives the settled numbers: atoms about 4.9 percent of the total, dark matter about 27 percent, dark energy about 68 percent, and an age of 13.8 billion years.

Cosmologists had once argued about a factor of two, and now they argued about the last decimal place. Tegmark sums up: "To me, the most surprising result was that there was no surprise" (chapter 4, "From Derision Cosmology to Precision Cosmology").

Tegmark ends the story with a plain list of what the numbers do not explain: he notes that astronomers have measured the density of dark matter, and he asks what dark matter is. He asks the same question of dark energy. In his summary, cosmologists have "mainly just parameterized our ignorance" (chapter 4, "A Lot Left to Explore"): for each number of his table, the question of what the number measures is still unanswered. He also asks whether coming measurements will find the density of dark energy exactly constant, or changing even slightly over time or from place to place.

III. The Same Results on Separate Ground

I derive four of Tegmark's results. I derive three of the four by routes of my own, and one by a short piece of established physics.

The first result is the idea that Tegmark counts among the most beautiful in Einstein's theory: matter curves space. The reason takes four steps: the strain between two demands, where the strain goes, why the strain grows with energy, and the form of the bending. I give the reason that matter curves space at all. My reason starts from comparison. Every measurement and every interaction is a comparison, as chapter 1 showed. Every comparison has a content, which is what the comparison finds, and a context, which is the respect in which the two things compared are two: two moments, two places, or two possibilities. When a thermometer compares this morning's warmth with this evening's, the temperature the thermometer finds is the content of the comparison. The two times of day are the context. What a comparison finds must be the same for everyone who finds it, so that separate findings agree and make one shared world:

in the word of chapter 1, the content is symmetric. To compare is to hold the two things compared in one act, joined with no break: the comparing is continuous. A comparison can give up neither the sameness of what is found nor the continuity of the comparing, so each must be perfect. Symmetry is sameness and continuity is having no gaps, so no single act of finding holds perfect sameness and perfect gaplessness together.

Two demands pull against each other: the demand for perfect sameness and the demand for perfect continuity. I call the pull between the two demands the *strain*. The strain has to go somewhere: into the content, into the comparing, or into the context. The content must stay perfectly the same, and the comparing must stay perfectly continuous, so the strain goes into the context. So the context gives way, as a rug between a wall and a sofa buckles when someone pushes the sofa toward the wall: the wall and the sofa stand for the two demands, which cannot give way, and the rug stands for the context.

For a physical finding the context is place or time, since the two things compared are two places or two moments. The arena is all the places and moments at which findings are made, taken together: space and time as found in comparisons (chapter 2). So the arena takes the strain. The arena joins every act of comparing to every other, so the arena carries the continuity of the comparing and stays gapless itself. By the Incommensuration Theorem the arena cannot also keep perfect sameness, so the arena gives up its sameness: the arena differs from place to place, and so the arena bends.

I identify energy with the rate at which findings can be made. Where more findings can be made, more of what is found must agree with all the rest of what is found. So the arena carries more strain where more findings can be made. Since energy is the rate at which findings can be made, the arena bends in proportion to energy.

The mathematician David Lovelock proved a theorem in 1971 and 1972. Lovelock's theorem fixes the form of the bending. Take a measure of curvature built from the geometry of space and time alone, in four dimen-

sions, using no more than how the geometry changes and how that change itself changes. Lovelock proved that, in four dimensions, only one measure of curvature built from the geometry alone and its first two rates of change can be set equal to an energy that is conserved: the measure in Einstein's equations, with room for exactly one constant term, a fixed number added to the equations. So Tegmark's sentence is my result too: geometry is also physics, and I derive that result from what a comparison is.

The second result is Tegmark's reason for dark matter: the faint differences of density could grow into galaxies in the time available only with the added pull of unseen matter. My books set out the reason by a short piece of arithmetic from established physics. When the oldest light went free, as the hot early fog cleared, the ordinary matter varied in density by about three parts in a hundred thousand, three times the variation of the light's temperature. Until the light went free, sound waves ran through the hot mixture of light and matter. The same sound waves gave the spots of the oldest light their favored sizes. The ordinary matter was caught in the sound waves and could not gather. After the light went free, a small excess of density grows in step with the distances between things, which have grown about one thousand and ninety times since the light went free. Three parts in a hundred thousand, grown one thousand and ninety times, comes to about three parts in a hundred. A region draws together into a galaxy only when its excess of density reaches about one: when the region holds about twice the average density. Three parts in a hundred is far short of one, so ordinary matter alone would have made no galaxy at all. The dark matter took no part in the sound waves, so its unevenness was already growing before the light went free. After the light went free, the ordinary matter fell into the denser places that the dark matter had already gathered. The dark matter built the scaffolding on which the galaxies formed.

The third result is the share of dark energy. Tegmark's table gives the share of dark energy as about 68 percent, taken from measurement. The energy of empty space enters Einstein's equations as their one constant

term, the *cosmological constant*, which drives the expansion apart and is the same at every place and time of our epoch. I set the share of empty space by an identity: two reckonings of the energy of empty space must be equal. I identify the cosmological constant with the part of the energy of empty space that a measurement made from inside the universe takes in. The part a measurement takes in is the part the measurement registers, which is a small share of the whole energy of empty space. Picture two surveyors measuring one field, one with a measuring chain along its edge and one with angles from a hilltop. The two surveyors' answers must agree. Setting the two answers equal gives something that neither surveyor could find alone. In my reckoning, the first surveyor counts the energy of empty space as the share that a measurement from inside takes in, while the second surveyor counts the energy of empty space as the astronomers do, as one slice of the budget of the whole universe. When the two reckonings are set equal, the rate of expansion and the strength of gravity cancel. One equation is left, in pure numbers: numbers with no units. The equation connects the share of empty space with the size of our horizon. Our horizon is the edge of the region from which anything has reached us (chapter 2). The equation counts the size of the horizon in a natural unit of the expansion: the distance at which space carries things away from us at the speed of light. The size of our horizon depends only on how the energy has been divided among matter, light and empty space over the history of the expansion, so the equation has one unknown, the share of matter. In a flat space the solution of the equation gives matter 31.1 percent of the whole and empty space 68.9 percent. The Planck satellite mapped the oldest light from space. The Planck team published its final results in 2020: matter 31.5 percent and empty space 68.5 percent, each give or take seven tenths of a percent.

The fourth result is Tegmark's cross-check. Two methods found that dark energy is about seventy percent of the total. Two other methods found that atoms are about five percent. When independent routes, resting on different premises, arrive at one value, I call the agreement *convergence*, a word that chapter 3 met. An argument about the world is a cable

of many strands, each strand one route to the value: the strands that come from separate sources make the value they share surer than any one strand alone. As chapter 3 said, convergence is the strongest evidence that inquiry into the world can have.

So we now know four things. Tegmark showed each of the four, and my account shows each of the four again. Space curves in proportion to energy. The galaxies needed an unseen matter to grow in time. Empty space carries about seven tenths of the energy of the universe. And each number of precision cosmology rests on several independent measurements, the strands of a cable, which is why the numbers are so firm.

IV. What the Dark Matter Is

On the question of what the dark matter is, I give a different answer from Tegmark's. Tegmark writes that dark matter is little more than a name for our ignorance, while I say definitely what the dark matter is. To see what I say the dark matter is, we need one new idea: the internal space.

A dimension is an independent way in which things can differ. Time has one dimension, and space has three. I show that there are six more dimensions, which I call together the *internal space*, or the *held internal dimensions*: six further directions in which things can differ, held small at every point. The shape of the internal space fixes the kinds of particle. The large world holds the six small: the large world is the world whose space and time open out to large size around the six. The six are directions of possibility, the range of what a comparison could find, and nobody travels along them. The six are present wherever you are, as the grain of a sculptor's stone is present at every point of the block, deciding how the stone can be shaped, while no tape measure ever measures the grain. In the same way the shape of the six fixes the kinds of particle, while nobody travels along the six. The shape of the six has a plain meaning. Four of the six have the shape of all the ways a three-way choice can stand, while the other two

have the shape of all the ways a two-way choice can stand, which is an ordinary sphere. Picture a lamp that could be found red, green or blue, and that is held between its three colors in any proportions and with any timing among the three colors. A shape of four dimensions has one point for every way of holding the three colors at once. Picture a tiny clock hand on each color: the timing is how far each hand has gone round compared with the other two hands. Picture a globe with one answer of a two-way choice at the north pole and the other answer at the south pole. The globe has one point for every way of holding the two answers at once. The proportion between the two answers sets how far north or south the point lies, while the timing between the two answers sets how far around the globe the point lies.

How small are the six held internal dimensions? The *Planck length* is the length at which the smooth description of space and time reaches the edge of its range, about a hundred billion billion times smaller than the width of a proton. The six are held a few Planck lengths across. The six are held at the same size and the same shape at every point of space. I describe the six in two ways, as a clock face can be described in two ways. Taken as the pattern of twelve hours, a clock face has no size and is the same on a wristwatch and on a tower. Taken as one particular face, a clock face has a width of its own. Taken as a pattern with no size, the six fix the kinds of particle. Taken as a small space held at its size, the six can ring, as any held thing can ring.

Think of a crystal, a solid whose atoms are held in a fixed lattice. Strike the crystal, and the crystal rings. Each way of ringing has a definite energy, because the lattice keeps its shape. Physicists call the smallest step of a crystal's ringing a phonon. A phonon is a vibration of the crystal, not an extra thing inside the crystal. I identify the dark matter with a lasting vibration of the six held internal dimensions, which I call a *gyration*. A gyration is a vibration of the internal space, as a phonon is a vibration of the crystal, and a gyration is not an extra thing inside the internal space. Each

gyration weighs about a *Planck mass*, the mass that goes with the Planck length, which is about twenty-two millionths of a gram, about ten billion billion times the mass of a proton.

Identifying the dark matter with gyrations shows why the dark matter is dark. Every instrument registers content, the things a comparison finds. Light registers content too, since every meeting of light with matter is itself a comparison. A gyration is a vibration of the internal space, and the internal space is made of directions of possibility: possibility is one of the three contexts of chapter 1 (place, time and possibility), a respect in which two things compared can be two. So a gyration is a vibration of context, and not an item of content: a gyration belongs to the respect in which two things compared are two, and never to what a comparison finds. So no instrument registers a gyration, and a gyration shows itself by its weight alone. Astronomers see exactly the effects of weight alone: galaxies held together, light bent, the oldest light patterned, and nothing else. One pair of clusters of galaxies passed through each other, and astronomers now call the pair the Bullet Cluster. In 2006 the astronomer Douglas Clowe and his colleagues weighed the Bullet Cluster by the way its mass bends the light of galaxies far behind the Bullet Cluster. In the two clusters, most of the ordinary matter is hot gas. The hot gas of the two clusters had crashed in the middle and glowed, while most of the mass had passed on through with the galaxies. Identifying the dark matter with gyrations also shows why the Bullet Cluster came out as it did: the dark matter acts on ordinary matter only by its weight.

The gyrations are made only at the turn, the passage from the end of one epoch, one long age of the universe, to the start of the next epoch (chapter 3 explained both words). A gyration could end in three ways, and none of the three can happen, so a gyration lasts. The first way would turn the gyration into an ordinary thing. No process within an epoch changes a vibration of the internal space into an ordinary thing, because a vibration of the internal space and an ordinary thing are two different sorts of thing. The second way would hand the gyration's energy to ripples of space and time. The small sphere of the two-way choice has a

symmetry: the swap of each point with the point opposite it. The swap rules out the second way. Under the swap, the gyration that lasts changes sign, so that each crest of its vibration becomes a trough. Ripples of space and time keep their sign under the swap, so the gyration cannot hand its energy to ripples of space and time. The gyration could hand its energy only to a lighter vibration that also changes sign under the swap, and the gyration that lasts is the lightest vibration that changes sign. The third way would be a slow leaking away of mass, of the kind a black hole undergoes, but at the Planck scale, the scale of the Planck length, there is no lower level for a gyration to leak into. Each gyration is so heavy that the gyrations are few. The dark matter is perfectly cold, in the cosmologists' sense of the word: slow moving, with no motion from heat that could matter. The standard picture of cosmology is the model that cosmologists fit to their measurements. In the standard picture the dark matter is slow and only pulls, and cosmologists call this dark matter cold dark matter. So in everything astronomers can see, the dark matter of gyrations behaves exactly as the cold dark matter of the standard picture does. What the account changes is not how the dark matter behaves, but what it is.

Tegmark recalls the doubts that people once had about dark matter and dark energy. Blaming invisible things that pass through walls sounded like believing in ghosts. Dark matter also reminded doubters of the ancient astronomer Ptolemy, whose circles riding on circles had once accounted for the motions of the planets. A better law of gravity later replaced Ptolemy's circles. I keep Einstein's law of gravity unchanged, at every scale down to the Planck scale. On my account the dark matter is energy, with weight, located where the dark matter pulls. A ghost has no weight, while a gyration has nothing but weight.

I also say how much dark matter there is for each amount of ordinary matter. I identify the ratio of dark matter to ordinary matter with six minus two over pi, about 5.36. The ratio comes from a count. The count gives the six and the one. The share that the curving takes away, two divided by pi, is given by a rule of the same form as my rule for the cosmological constant, with the same coefficient, and the measurement selects it

from the expressions of that form. A measurement counts what it registers piece by piece, by the basic pieces from which each shape is built: the fewest building blocks from which the shape can be put together. The globe of the two-way choice is built from two basic pieces: a single point, and a disc whose whole rim is drawn together to that point, like a drawstring bag pulled shut. The three-way choice has a shape of four dimensions, built from three basic pieces. The dark matter spreads over the whole internal space, so the dark matter counts three pieces times two, six in all. The curving of the four-dimensional part takes a small share away from the count of six: two divided by pi, about sixty-four hundredths. The ordinary matter counts one, because the region within our horizon is one simple ball, built from one basic piece. The dark matter fills the same region and also spreads over the whole internal space, so for each basic piece of the region within our horizon the ordinary matter counts one and the dark matter counts six. The two divided by pi is taken away once, for the whole measurement, however many basic pieces the region is built from. The densities that the Planck satellite measured give a ratio of about 5.36 as well: for every kilogram of ordinary matter, a little more than five and a third kilograms of dark matter. The appendix "For the Specialist" gives the count in full, and what my account says of the ratio.

The same count gives chapter 2's reason that space has no short loop. The region within a horizon is built from one basic piece exactly when no short loop passes through the region. The count of one holds for every measurement, wherever in the universe the measurement is made. So space has no short loop anywhere. If a short loop passed through the region within a horizon, the region would be built from two basic pieces or more. With two basic pieces, the ordinary matter would count two, the dark matter would count twelve less two divided by pi, and the ratio would come out near five and sixty-eight hundredths.

V. One Number for Empty Space

I also say what dark energy is: one number. Think of a map with its scale printed in one corner: one inch to one mile. The scale belongs to the whole map at once. If the scale changed from one corner of the map to the next, the map would not be one map. The cosmological constant is like the scale of a map: one number for the whole of the one space and time of our epoch at once. Einstein's equations keep a strict account of energy and of motion, which are moved from place to place and never made or lost. If the constant term changed from place to place or from time to time, the term would break the strict account wherever the term changed. So the cosmological constant is one number within the one space and time of our epoch, by a theorem about Einstein's equations. The appendix "For the Specialist" states the theorem. I hold that the dark energy is the part of the energy of empty space that a measurement takes in, and that the part a measurement takes in enters Einstein's equations as their one constant term. So the dark energy is one number, as distinct from a *field*. A field is a quantity spread through space with a value of its own at every place and time. The measurement made now sets the number, because the number is the share of the energy of empty space that the measurement takes in. We piece together the history of our epoch from the present measurement. The history we piece together carries the same number at every time of our epoch.

So I answer plainly the question that Tegmark puts to coming measurements: the density of dark energy is exactly constant, at every place and every time of our epoch. Physicists describe how an ingredient of the universe thins as space grows by its *equation of state*. The equation of state of an ingredient is the ratio of its pressure to its energy. Ordinary matter has an equation of state of zero. An energy that never thins at all, at any time, has exactly minus one. On my account the dark energy has an equa-

tion of state of exactly minus one. The appendix "What the Coming Measurements Will Find" gives what I say each coming survey of empty space and of the dark matter will find.

VI. A Measured Number and Its Question

Tegmark reads his table as ignorance put into numbers, while on my account the same table is knowledge. When a measured value has its question written beside it, the value and the question together are a complete state of knowledge. Each line of Tegmark's table is a measured value with its question beside it. In the line for dark matter, the measured density of dark matter says what is the case, and Tegmark's question beside the density, what dark matter is, says where the next work lies. Cosmologists choose the share of matter in the standard picture of cosmology to match the measured sky. I set the share of matter at 31.1 percent by an identity: the share of the energy of empty space that a measurement from inside takes in equals the share of empty space in the budget of the whole universe.

Tegmark draws one more lesson from his chapter: precision cosmology shows that "simple mathematical laws govern our Universe all the way back to its fiery origins" (chapter 4, "Where Did Our Big Bang Come From?"). He calls the lesson a sign of "the mysterious utility of mathematics for understanding our world" (same section). In his own chapter 10 Tegmark offers a radical explanation for the mystery: a world made of mathematics. On my account, the same success shows how closely a fine model can follow the world. Recall the orrery of chapter 3, the brass model of the solar system whose gears carry each planet around at its proper pace. Run the gears backward, and the planets go backward just as smoothly, but the Moon in the sky raises tides that slow the Earth and push the Moon outward, so the solar system runs one way only. Running backward with ease is a property of the model, and not of the solar system. The orrery stays a fine instrument, and a beautiful one; it is simply

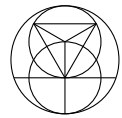
not the solar system. The laws of precision cosmology are as exact as Tegmark says. The laws are the finest description there is of what is laid out as structure. The world that the laws describe is found one comparison at a time. Chapter 10 of this book follows the difference between the description and the world step by step.

VII. What This Chapter Holds

On my account we can say what the rest of the universe is, beyond the ordinary matter of the atoms. The dark matter makes up about a quarter of the energy of the universe: on my account the dark matter is made of gyrations, lasting vibrations of the six small dimensions of the internal space, which pull and never shine. The energy of empty space makes up about seven tenths of the energy of the universe, set by one constant that is the same at every place and every time of our epoch.

The answer about what the universe is made of bears on the book's question: What is reality, and can physics find out? On my account the dark matter comes from the internal space, and the dark energy comes from the act of comparison, as the share of the energy of empty space that a measurement takes in. Tegmark's precise numbers for both ingredients are findings, each made from inside the world. Two independent routes of measurement support the share of empty space, and two independent routes support the share of atoms. Astronomers read most of the numbers of this chapter from the oldest light. The oldest light already carried the faint seeds that gravity grew into galaxies.

So where did the hot, even, faintly rippled beginning of our epoch come from, and what, as Tegmark asks, "put the bang into the Big Bang" (chapter 4, "Where Did Our Big Bang Come From?")?



Chapter 5

What Put the Bang in the Big Bang?

Beside Tegmark's chapter 5, "Our Cosmic Origins"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

An *epoch* is one long age of the universe, from one turn to the next; the *turn* is the passage from the end of one epoch to the start of the next, and what we call the Big Bang is the turn as it appears in the history we piece together from inside our epoch (Chapter 3).

The *internal space*, or the held internal dimensions: six further directions in which things can differ, held small at every point (Chapter 4).

I. Two Words at the Top of a Page

Point a radio telescope at any part of the sky, by day or by night, and the oldest light there is arrives from that direction with the same temperature as from every other direction, to about one part in a hundred thousand. What put the bang in the Big Bang, and why does the sky look the same in every direction?

The oldest light is the microwave background of chapter 3: light set free when the young universe first became clear, and stretched by the expansion of space until the light now has a temperature of about 2.7 degrees above absolute zero. The sameness of its temperature in every direction is measured. Most cosmologists explain the sameness of the sky by inflation, a very rapid stretching of space early in the history of the universe. Tegmark answers both questions with inflation. We ask whether answering either question needs inflation.

In December 1979 Alan Guth was a young particle physicist, spending a year as a visitor at the Stanford Linear Accelerator Center in California. Guth and his colleague Henry Tye had been asking how many magnetic monopoles the hot early universe would have made. A magnetic monopole is a particle that carries a single magnetic pole, a north with no south. Guth and Tye's calculation gave far too many monopoles: a universe swimming in them. Guth and Tye had also found how the universe could make far fewer. Physicists expected a cooling universe to change the arrangement of its forces at one particular temperature, as water freezes at one particular temperature. Water can stay liquid below its freezing point. If the universe had cooled in the same way, far below the temperature of the change before the change took place, few monopoles would have been made. When something cools far past the temperature of a change before the change happens, physicists call the cooling supercooling.

One night that December, Guth went home to the house he rented in Menlo Park and wrote down the equations for how a supercooled universe would expand. The equations said that a supercooled universe would expand faster and faster, doubling its size again and again at regular intervals. Guth now saw that the repeated doubling would drive the universe to exactly the flatness that the Princeton cosmologist Robert Dicke had described the year before, in a lecture on a puzzle about how flat space is. Guth later described his thought in his own words: "the flatness problem would be solved". The next day Guth looked over the page of equations and wrote two words in capitals across the top of the page: "SPECTACULAR REALIZATION". By the end of December 1979 Guth called the doubling inflation. Guth found in the same doubling an answer to a second puzzle, which colleagues had discussed over lunch in the cafeteria of the accelerator center: the horizon problem, the puzzle of why the sky has one temperature in every direction. The journal *Physical Review D* received Guth's paper in August 1980 and printed the paper in January 1981. The Adler Planetarium in Chicago now keeps the notebook page with the two words in capitals.

Guth answered three puzzles together with one proposal: the monopoles, the flatness of space and the sameness of the sky. Max Tegmark tells the story of inflation well. He adds a fourth puzzle, which he answers with inflation: what put the bang in the Big Bang. We begin with Tegmark's telling.

From Tegmark's pages

II. What Max Tegmark Showed

Tegmark ends his chapter 4 by naming what cosmologists, for all their precise measurements, have left unexplained: "I like to think of this as the 'Bang Problem': what put the bang into the Big Bang?" (chapter 4, "Where Did Our Big Bang Come From?"). He asks where the hot expanding gas of the early universe came from, why the gas was so uni-

form, and why the gas carried the faint seeds from which the galaxies later grew. In his chapter 5 Tegmark gives the answer that cosmologists have taken most seriously: inflation. He says how firmly he holds that answer: "it's too early to say for sure whether our Big Bang really was caused by inflation" (chapter 5, "Sowing the Seed Fluctuations").

Alexander Friedmann solved Einstein's equations of gravity for a universe filled evenly with matter (chapter 3). Tegmark first shows what Friedmann's model of the Big Bang does not explain. When Friedmann's equations are run backward in time, they explain the flight of the galaxies, the glow of the oldest light and the making of the lightest atoms. Tegmark credits Friedmann's equations with all three. Friedmann's equations contain no cause of the Big Bang itself: in Tegmark's words, "the equations simply assume it happened" (chapter 5, "What's Wrong with Our Big Bang?").

Then Tegmark sets out two puzzles: the horizon and flatness problems. In the horizon problem, a horizon is the farthest distance that a signal could have crossed in the time that had passed since the Big Bang. Tegmark takes two regions of the sky in opposite directions and calls them A and B. Regions A and B show the same temperature. Yet in Friedmann's history no signal, not even light, has yet had time to pass from region A to region B. Cold milk poured into hot coffee mixes to one temperature, but mixing takes time. Regions A and B never had the time to mix. Tegmark puts the horizon problem in one line: "One unexplained Big Bang is bad enough; an infinite number of unexplained Big Bangs in perfect synchronization strains credulity" (chapter 5, "The Horizon Problem").

The second puzzle is the flatness problem. When every triangle drawn across a space has angles that add to exactly 180 degrees, however large the triangle, we call the space flat. A sheet of paper is flat in this sense. Astronomers have measured our space to be flat to high precision. In Friedmann's history a flat universe is balanced like a bicycle standing upright with no rider: the slightest lean of the bicycle grows,

and in the same way the slightest departure from flatness grows. The universe then either falls back on itself or thins out too fast to make galaxies. Tegmark pictures the odds that a universe starts out balanced finely enough to stay as flat as ours: the odds are smaller than the odds that a dart fired at random from Mars hits the bull's-eye on a dartboard on Earth.

Tegmark answers both puzzles with inflation, from one assumption, which he states in one sentence: "This assumption is that once upon a time, there was a tiny uniform blob of a substance whose density was very hard to dilute" (chapter 5, "The Power of Doubling"). Einstein's theory of gravity says that a blob of a substance so hard to dilute doubles its size again and again, as the cells of an unborn child double, until in far less than a second the blob holds more mass than everything we can see. In Tegmark's telling, the repeated doubling of the blob is the bang. Regions A and B were close neighbors early in the doubling. So regions A and B share one temperature for the reason that a cell in your nose shares its genes with a cell in your toe: both cells came from one first cell. The doubling also stretches any curve of space almost perfectly flat. The doubling is like blowing a ping-pong ball up to the size of the Earth: a square centimeter on the ping-pong ball is plainly curved, and a square centimeter on the Earth looks flat.

Then Tegmark sets out the predictions of inflation that measurement bore out. Tegmark writes: "In other words, inflation theory made a testable prediction back in the eighties: our space should be flat" (chapter 5, "Problems Solved"). For years the astronomers who surveyed the galaxies seemed to find the universe less dense than a flat space requires. Guth held to the prediction. Once astronomers counted the dark energy, they measured space to be flat to better than one percent. Inflation also explains where the seeds of the galaxies came from: the quantum trembling of the inflating substance. In quantum physics nothing lies wholly at rest, so even the inflating substance trembled faintly. The doubling stretched the trembling from lengths

smaller than an atom to lengths larger than galaxies, and gravity then grew the stretched trembling into galaxies, clusters of galaxies and the great structures of the sky. Tegmark sums up: "the cosmic seed fluctuations came from quantum mechanics" (chapter 5, "Sowing the Seed Fluctuations").

Tegmark states the test he holds a theory to: "The hallmark of a successful scientific theory is that you get more out of it than you put into it" (chapter 5, "Sowing the Seed Fluctuations"). Inflation satisfies Tegmark's test with a number. The seeds are faint ripples of density in the young universe, and the ripples are slightly stronger on large lengths than on small ones. Astronomers measure how the strength of the ripples changes with length by a number called the spectral index. An index of exactly one means the same strength at every length. An index a little below one means that the larger ripples are slightly stronger than the smaller ones. Astronomers call how far the index lies from one the tilt. Tegmark calls the index itself the "tilt" parameter (chapter 5, "Sowing the Seed Fluctuations"). The pioneers of inflation had predicted an index close to one. The simplest models of inflation predicted an index a little below one. The physicist Andrei Linde built models of the inflating substance, and the simplest of his models gave about 0.96. Tegmark reports a measured index of 0.96, give or take 0.005, "tantalizingly close to what was predicted" (chapter 5, "Sowing the Seed Fluctuations"). Physicists computed the value 0.96 before astronomers measured it, and the credit for the value belongs to inflation and to the physicists who computed it.

Tegmark names one more sign that many models of inflation predict. Light vibrates in a direction across its path. Astronomers call the pattern of the directions of vibration across the sky the polarization of the light (bees see polarization, and polarized sunglasses use it). In many models space itself trembles during inflation, and the trembling makes gravitational waves (ripples of space and time themselves). The gravitational waves leave a twisting pattern in the polarization of

the oldest light. Tegmark writes of a finding of the twisting pattern: "I think it will be hailed as smoking-gun evidence that inflation really happened" (chapter 5, "Sowing the Seed Fluctuations").

Tegmark ends his chapter with eternal inflation, Linde's name for inflation that ends in some places and goes on forever in others. Almost every model of inflation studied so far proves to be eternal. In Tegmark's telling, our Big Bang is the place where inflation ended in our part of space, one branch of a tree that keeps growing. So on Tegmark's account our Big Bang was the end of inflation in our part of space, and not the ultimate beginning.

III. The Same Results, Reached on Other Ground

My account agrees with three of Tegmark's results, and it has premises of its own for each: the sky is one pattern with nearly the same strength at every length, space is flat, and our Big Bang was not the ultimate beginning. To see the premises, we need, in brief, the history of the universe as my account has it.

Chapter 3 gave two words for that history: epoch and turn. An epoch is one long age of the universe. The turn is the passage from the end of one epoch to the start of the next. So an epoch runs from one turn to the next. In every epoch stars form, and the stars make black holes, regions so dense that nothing, not even light, climbs back out. Inside each black hole every path runs toward an end that the path approaches and never reaches. The ends inside all the black holes are one end, as I show in my plain-words book on the beginning of our epoch. (Chapter 1 defined a comparison as any act of telling two things apart or finding them the same, and every measurement is a comparison.) The argument that the ends are one end depends on the principle of identity of chapter 2. The principle says that two descriptions that no comparison could ever tell apart are one description. In the world of an epoch, a comparison tells

apart two things that are alike in every other way by their places or their moments. The ends of the black holes lie at no place and no moment of the world outside the black holes, because no signal from the ends ever reaches the world outside, and nothing in the world outside puts the ends in an order in time. My account states one further premise: the new world that forms from the ends carries over none of the old world's places and moments. So no comparison has a place or a moment by which to tell one end from another. The ends are therefore one end, by the principle of identity. Chapter 4 met six further dimensions, held small at every point of space, whose shape fixes the kinds of particle. A new large world forms from the one end. A large world is a world whose space and time open out to large size, with the six further dimensions held small inside it. The new large world is the next epoch. When we trace our history back from what reaches us now, the history runs toward the turn and shows the turn as its beginning.

The first agreement is the result Tegmark reaches last: our Big Bang was not the ultimate beginning. Our Big Bang is the beginning that we find when we trace our history back to the turn, while the same turn is approached from the other side, from the epoch before ours, by the collapse inside the black holes of that earlier epoch. The epoch before ours began in the same way, from the epoch before it, and the cycle of epochs has no first time round. Tegmark reaches the result that our Big Bang was not the ultimate beginning through eternal inflation, while in my account the same result follows from the turn.

The second agreement is flat space. An ant on a marble feels the curve of its world within a few steps, while an ant on a ball the size of the Earth walks for days and finds the ground flat. A curvature of space is set by a length in the same way, its radius: the marble has a small radius, and the ball the size of the Earth has a large one. In Einstein's equations for a universe that is alike everywhere, the starting state of the history fixes the radius of curvature once, and the equations carry the radius unchanged afterward. Our epoch starts from a state that inherits no length from the epoch before. The starting state of our epoch holds one length of its own,

the length of the turn, which is about the Planck length. A larger radius would have to be made by multiplying the length of the turn by some very large number. In my account the only numbers that large are counts, such as the number of different ways that the matter within our horizon could be arranged, and no count is carried across the turn, because each epoch begins its own counts. So the radius of curvature of our epoch is either about the length of the turn, or there is none. If the radius were as small as the length of the turn, the new epoch would fall back on itself almost at once, or be ruled by its own curvature, thinning out too fast to make galaxies. Either way the new epoch would make no stars and no black holes. Yet the black holes of each epoch are where the next epoch comes from. So the radius cannot be as small as the length of the turn. No radius of curvature is left, and space as a whole is flat, curved only by its own faint ripples. Inflation makes space flat by stretching space. In my account flatness follows from what the turn leaves: a starting state with no length but its own. Neither argument needs the premises of the other.

The third agreement is the pattern of the sky itself. Inflation and my account both say three things about the pattern. The faint differences in the oldest light are the seeds of every galaxy. Every ingredient of the universe ripples together. And the pattern runs nearly the same in strength at every length across the whole sky. Lay a floor of ten thousand small tiles, each shaded light or dark by a different worker who never saw another tile, and from far off the floor fades to an even gray, because independent pieces cancel one another over a wide patch. Now weave a cloth on one loom, with one design running through the cloth, and the swirls of the design stay as strong across the whole width of the cloth as across a hand's breadth. The sky is the cloth. The measured index of the sky lies close to one, so the ripples of the sky keep nearly the same strength at every length. On my account the whole pattern of the sky came from one *domain*, one undivided whole in which no comparison can tell one part from another, as the cloth was woven on one loom. The one domain is the one end of the black holes of the epoch before.

IV. One Setting-Up, and No Inflation Needed

In my account every puzzle that inflation was proposed to solve has its answer in the turn or in the forces of nature. So my account needs no inflation. We take the puzzles in Tegmark's order, the bang, the horizon problem and the flatness problem, and then Guth's first puzzle, the monopoles.

The bang comes first. My account's answer to the bang starts in the room you sit in. The room has an outside, which you can reach at moments you can name. Its space and time are smooth at every scale you can reach. And the distant galaxies fix what "not rotating" means, for a spinning gyroscope and for the balance organ of your inner ear. On my account the three plain facts about the room are the three conditions under which the six further dimensions stay small. For the large world around the six dimensions, the first of the three conditions is that the large world is present around the six dimensions as the setting in which the six are held, at moments at which comparisons can be made. Second, the large world lies within the range of the smooth description of space and time. Third, the large world fixes what "not rotating" means. Every part of the world you know satisfies the three conditions, so the six dimensions stay small, and because the shape of the six fixes the kinds of particle, the forms that the matter of your body can take stay fixed too.

At the end inside a black hole, all three facts fail. The world outside the black hole has no moment at the end, since no signal from the end ever reaches the world outside. At the end the smooth description of space and time has reached the edge of its range. And nothing is left to fix what "not rotating" means. So at the end inside every black hole, nothing holds the six dimensions small any longer. Everything that fell into the black hole stops being matter in the forms that matter takes in our epoch, yet what fell in does not cease to be: it is still there to be found.

Break a cup, and the water the cup held does not cease to be: the water spreads over whatever is left beneath it. What fell into the black holes is like the water, and the old large world, broken at the end, is like the cup. In my account nothing is found without a *context*. A context is the respect in which the things compared are two (chapter 1): two places, two moments, or two possibilities. The large world is the context in which the six dimensions are held, with its places and its moments. At the end inside the black holes the old world has no places and no moments left, but what fell in still needs places and moments in order to be found. Only one thing is left from which places and moments can form: the geometry of all ten directions together, the four of space and time and the six. So the geometry of the ten directions must open out again into a large world. The new large world has large space and time, with the six held small inside it once more.

In chapter 3 I identified the Big Bang with the turn, as the turn appears in the history we piece together from inside our epoch. The turn is where the new large world forms. So my account's answer to the Bang Problem is the turn: the one end of the black holes of the epoch before, from which a large world had to form again. The *content* of an epoch is what the comparisons made in that epoch find. The content of the epoch before continues through the turn. So the turn is not a creation from nothing.

The horizon problem comes next. Two towns far apart can share an accent without their people ever having spoken to each other, because both towns were founded from one place. The sky is like the two towns. Regions A and B are alike because both regions come from one domain, the one end of the black holes of the epoch before. No signal ever had to pass between regions A and B.

One picture travels with this book: a game of chess at a kitchen table. The game shows how one origin makes two places alike. Look at a chessboard set for a new game: the white pieces on one half of the board stand in exactly the order of the black pieces on the other half, rook for rook and pawn for pawn. The two halves of every board start alike because every

game starts from one setting-up. The setting-up is the one arrangement in which the players set out the pieces before the first move. Nobody needs to suppose that the pieces of the two halves once reached across the board and arranged each other. The position on the board is the history of this game. The first position of the game comes from the setting-up, whole and at once. In the picture, the setting-up stands for the turn, and the two halves of the board stand for regions A and B. On inflation, the likeness of regions A and B comes from an earlier contact inside our epoch. In my account the likeness comes from one setting-up at the turn.

Section III has already answered the flatness problem: no length of the old epoch is carried across the turn, and a radius as small as the length of the turn is excluded.

The monopoles that Guth and Tye counted come from a family of theories called grand unified theories, in which three of the forces of nature are one force in the great heat of the beginning. The three forces are the strong force that binds the nucleus, the weak force of some radioactive decays, and the *electromagnetic force*, the force between charges, which light carries. The one force breaks apart into three as the universe cools. The breaking apart of the one force leaves monopoles behind. On my account the three forces were never one larger force. Each force has its own symmetry, its own sameness under change. The three symmetries exist together as three separate symmetries, and each symmetry is the same at every energy. None of the three symmetries is what is left of a larger symmetry that broke. So nothing breaks in the way that makes monopoles, and the monopole puzzle does not arise.

V. The Ripples and the Waves

What, then, are the faint ripples of the sky on my account? Climb the stone stairs of an old church and look at the steps. Each step is worn into a shallow curve, lowest where most feet have fallen over hundreds of years. Every footstep has worn its own small share into the curve, but no single

footstep can be picked out of the curve. The curve is all the footsteps together. Everything that fell into the black holes of the epoch before ours had gone one way and not another at every choice it met: it had taken some alternatives and not others. The alternatives that everything had taken are carried through the turn, all averaged together and none erased. Every footstep is kept in the one curve of the stair in the same way. In our epoch we can find only the shape that the alternatives make together: small differences of density from place to place in the young universe. I identify the first ripples of our sky with these small differences of density: the shape that the alternatives taken in the epoch before make together, carried through the turn.

I compute one consequence of identifying the first ripples in this way, and the consequence concerns empty space. Empty space is never perfectly still, as the surface of a pond shivers faintly even on a windless day. In quantum physics nothing lies wholly at rest, as section II told of the inflating substance. The trembling of empty space makes faint ripples of its own. Astronomers measure the strength of ripples by their *power*, the strength multiplied by itself. Take a length we observe today and trace the length back to near the turn: near the turn the length was about a billion billion billion times longer than the horizon of that moment, the farthest distance a signal could have crossed since the turn. On a wave that much longer than the horizon, the trembling of empty space in our epoch has a power more than ten to the forty-two times smaller than the power of the ripples that the sky shows. So the trembling of empty space in our epoch adds almost nothing to the ripples we see. On my account the ripples we see are the shape that the alternatives taken in the epoch before make together, carried through the turn.

On my account the first ripples are the shape of the old alternatives carried through the turn, and so a long inflation is excluded. When an inflation carries the lengths we observe from inside the early horizon (the horizon near the turn) to beyond the early horizon, I call the inflation long. A shorter phase, one that leaves the lengths we observe outside the early horizon the whole time, my account neither requires nor excludes,

and the starting temperature of our history bounds it. A long inflation would have made the ripples on those lengths out of the trembling of empty space during the inflation. On my account those ripples came through the turn instead.

Inflation and my account differ on gravitational waves from the beginning. On my account the first ripples are ripples of density. A ripple of density bends space more in one place and less in another, as children standing on a trampoline make deeper and shallower dips. A wave of space itself needs something that pulls harder in one direction than in another, but hot matter close to balance pushes equally every way. So on my account waves of space from the beginning arise only where ripples of density act on one another, which is a far smaller effect, and for that reason the power of such waves is far below a millionth of the power of the ripples of density. Measurements made from 2014 to 2021 show what astronomers have found.

The BICEP2 telescope at the South Pole had watched the polarization of the oldest light for three seasons when, on 17 March 2014, the team of the telescope announced that they had found the twisting pattern that the gravitational waves of inflation would leave. In their paper the team put the power of the waves at about a fifth of the power of the ripples. In the same paper the team said that dust in our own galaxy could make part of the twisting pattern, and that the published maps of the dust were too uncertain to settle how much. The Planck satellite had mapped the polarization of the whole sky at seven frequencies, reaching well above the frequency that BICEP2 watched, so the BICEP2 team and the Planck team analyzed their data together to tell the dust apart from any waves. In a joint paper published in March 2015, the two teams found strong evidence for dust and no significant evidence for gravitational waves from the beginning, and they set the power of such waves below about an eighth of the ripples' power. In 2021 the BICEP and Keck teams combined their own measurements from the South Pole with the measurements of the Planck satellite and of the earlier Wilkinson Microwave Anisotropy Probe satellite, and lowered the limit to about three and a half hundredths of the

ripples' power. No gravitational waves from the beginning have yet been found, down to a power below about three and a half hundredths of the ripples' power. The appendix "What the Coming Measurements Will Find" states what the coming instruments will find on my account.

A second argument also shows that a long inflation did not make the ripples. The second argument depends on the dark matter, and it uses nothing from the identification of the first ripples with the old alternatives. On my account (chapter 4) the dark matter is made of lasting vibrations of the six held dimensions, each of about the Planck mass (some twenty-two millionths of a gram), made only at the turn. The hotter a young universe is, the more it makes of anything that has weight, so the measured amount of dark matter sets an upper limit on how hot our history ever was. In inflation, the strength of the gravitational waves grows with the energy that the inflating substance holds. The heat of the universe after inflation comes from the same energy. The measured limit on the waves is about three and a half hundredths of the ripples' power. So the measured limit on the waves also sets an upper limit on how hot the universe could have been after an inflation. After an inflation that had made the ripples, the upper limit set by the waves would have kept the universe far too cool to make the dark matter afterward in the measured amount. Two arguments, resting on different premises, have one conclusion: the ripples were not made by a long inflation.

Physicists computed an index of about 0.96 from inflation before any measurement reached that value, and the final measurement of the Planck satellite, published in 2020, is 0.965, give or take 0.004. My account fixes the kind of pattern the sky shows: one cloth across the whole sky, the same for every ingredient, made of ripples of density, on flat space, while for the strength and the tilt of the ripples my account uses the measured values. On my account the tilt has a meaning. A pattern strongest at one particular length singles out that length. When a pattern singles out no length at all, the pattern is exactly the same at every length, with an index of exactly one. A tilt is a departure from sameness at every length, so a tilt

singles out some length. On my account the measured index, a little below one, says that the ripples faintly single out some length of space that belongs to our epoch.

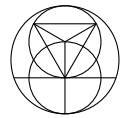
On my account the epochs follow one another in a single sequence, as the cars of one train do. Each epoch begins from the end of the one before. My account has no long inflation. So it has no inflation going on forever somewhere else, and no tree of Big Bangs growing from one eternal inflation.

VI. What This Chapter Holds

The turn is what put the bang in the Big Bang: at the one end of the black holes of the epoch before ours, a large world had to form again, and our epoch is that large world. The sky is the same in every direction because our epoch formed from one domain, as every game starts from one setting-up. Space is flat because no length of the old epoch is carried across the turn. No monopoles formed, because the strong, weak and electromagnetic forces were never one larger force. None of these answers needs inflation. On my account the first ripples of the sky came through the turn, and not from a long inflation.

The answer without inflation bears on the book's question: What is reality, and can physics find out? The answer bears on that question because in his chapter 5 Tegmark derives his infinite space from inflation that never ends, and his first argument for the further universes in that infinite space depends on the same inflation. On my account our sky came from one setting-up at the turn, so the question of space beyond our horizon needs premises other than inflation.

Beyond the farthest light we can see, does space go on, and is there another copy of me out there?



Chapter 6

Is There Another Copy of Me Out There?

Beside Tegmark's chapter 6, "Welcome to the Multiverse"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

What *exists* is laid out whole, timeless and the same for everyone, as the list on a score sheet is; what is *real* is given in an act of finding, once, as a move is made (Chapter 1).

An *epoch* is one long age of the universe, from one turn to the next; the *turn* is the passage from the end of one epoch to the start of the next, and what we call the Big Bang is the turn as it appears in the history we piece together from inside our epoch (Chapter 3).

The *internal space*, or the held internal dimensions: six further directions in which things can differ, held small at every point (Chapter 4).

I. A Book Printed in London

Is there really another copy of me, far beyond what any telescope can see, and is it science to say so? Max Tegmark puts both questions to his reader, in their sharpest form, in chapter 6 of his book *Our Mathematical Universe*. Tegmark answers yes to both questions. He says that the simplest and most popular model of the cosmos today predicts a copy of you, and that it is science to say so. An older form of the question is whether there are other worlds at all. The older question was put in print at a known place and date: London, in 1584.

In the spring of 1583 the Italian philosopher Giordano Bruno came to London. He carried a letter of introduction from King Henry III of France to the French ambassador, Michel de Castelnau. Bruno lived in the ambassador's house until 1585. In 1584 the London printer John Charlewood printed Bruno's dialogue *De l'infinito, universo e mondi* (*On the Infinite Universe and Worlds*). The title page of Charlewood's printing names Venice in place of London. In the dialogue, Bruno argues that space has no edge and no center, and that space holds worlds without number. Dorothea Waley Singer translated the dialogue into English. Near the end of the dialogue, in Singer's translation, Bruno says that God is glorified "not in a single earth, a single world, but in a thousand thousand, I say in an infinity of worlds". Bruno was burned in Rome, in the Campo de' Fiori, on 17 February 1600. Tegmark recalls Bruno's death in his own chapter 6.

Bruno asked whether there are other worlds. Tegmark asks a sharper question: whether one of the other worlds is exactly like this world, with you in it, reading this sentence. You may already be asking Tegmark's question in your own words. But if space is infinite, surely there is a copy of me?

Picture two kitchen tables in two houses across town. At each table two people play chess, and beside each board lies a score sheet, the written record of the game, move by move. In this book the score sheet stands

for structure, which is what exists: laid out whole, and the same for everyone who reads it. Each move stands for an act of finding, which is what is real: one comparison, in which one thing is told from another, made once, now. Tonight the players at both tables play the same opening, move for move, so the two score sheets read the same, line for line. There are still two games, because each move is made once, at its own table. The two games are not one game played in two places. What do the two tables show about a copy of you?

II. Space Beyond the Fog

After his opening question, Tegmark defines his key words carefully, and his care helps his reader. By "physical reality" he means everything that exists. By "our Universe" he means the part of physical reality that we can in principle observe: the sphere of space from which light has had time to reach us since the expansion began. Our Universe has an edge, called the *horizon*. The horizon is the boundary beyond which light has not yet had time to reach us. When a region of space is as large as our Universe, and centered on a place too far away for its light to have reached us, Tegmark calls the region a Level I parallel universe. He calls all the Level I parallel universes together the Level I multiverse.

Tegmark gives a picture of the horizon. You walk on a wide field in a fog that lets you see fifty meters. You stand at the center of a sphere of fog. Everyone else on the field stands at the center of a sphere of their own, and the spheres overlap. In Tegmark's words, "any observers anywhere in space will find themselves at the centers of their universes" (chapter 6, "Where Are the Level I Parallel Universes?"). The fog has no wall at its edge, and the field goes on beyond the edge of the fog. In the picture, the field is space, your sphere of fog is our Universe and the edge of the fog is the horizon.

Tegmark then reports the measurements that show space going on past our horizon. The microwave background is the oldest light that reaches us. The microwave background shows no curvature of space. Space could close back on itself, as the surface of a ball does in two dimensions, and a space closed in that way would be curved: the larger the space is, the less its curve shows in the part we see. So if space closed like the surface of a ball, space would have to hold at least a hundred times the volume of the part we see. Space could also close on itself as the surface of a doughnut does, and a space closed in that way would show repeated images. So astronomers looked for repeated images in three ways: for copies of our own galaxy in rows, for one bright galaxy seen in two opposite directions, and for matching circles of pattern in the microwave background. Chapter 2 described the latest of the searches for repeated images, reported in 2023. The astronomers found no repeated images in any of the searches. Maps of the galaxies in three dimensions show clusters and walls of galaxies on smaller scales. On scales above about a billion light-years the galaxies are spread evenly, and the maps show no edge to the matter anywhere in sight. Tegmark draws the conclusion: "space as we know it appears to continue far beyond the edge of our Universe, teeming with galaxies, stars and planets" (chapter 6, "Evidence for Level I Parallel Universes").

From Tegmark's pages

III. How Far to the Nearest You

Tegmark then argues from space beyond the horizon to a copy of you. *Inflation* is the proposed early stretching of space that chapter 5 described, and in its eternal form inflation never stops everywhere at once. Tegmark first derives the copy of you from eternal inflation: "Your alter ego is simply a prediction of eternal inflation" (chapter 6, "The Level I Multiverse"). Then he sets out his argument for the copy in three parts. The first part is that quantum mechanics sets a limit to how finely the position of a thing can be fixed: below that limit, it

means nothing to say where a thing is. The part of space we can see is a region of finite size. So the part of space we can see can be arranged in only a finite number of ways. The second part is two assumptions, "both of which could conceivably be correct even without inflation" (chapter 6, "Evidence for Level I Parallel Universes"). The first assumption is that early on space was infinite and filled with hot expanding matter. The second assumption is that early on some mechanism let any region receive any possible seeds of later structure (small differences in density that gravity later grew into galaxies), "seemingly at random" (chapter 6, "Evidence for Level I Parallel Universes"). The third part is a count: there are infinitely many regions, and each region can be arranged in only finitely many ways, so every arrangement occurs, and occurs infinitely many times. Tegmark sums up the argument: "in an infinite space created by inflation, everything that can happen according to the laws of physics does happen" (chapter 6, "What Are Level I Parallel Universes Like?").

Tegmark supports each of the two assumptions by measurements made inside our horizon. He reports that the seeds we see in the microwave background and in the maps of galaxies have the statistics of what statisticians call a "Gaussian random field". A Gaussian random field is a pattern drawn by chance, by one rule in every place, with the sizes of its ripples spread as the familiar bell curve spreads them. Tegmark writes that the measured pattern of the seeds "satisfies assumption 2", his second assumption (chapter 6, "Evidence for Level I Parallel Universes"). Tegmark supports the first assumption with the measurements of section II: the microwave background shows no curvature of space, the searches for a space closed on itself found nothing, and matter is spread evenly on the largest scales.

Tegmark also says how far away the nearest copy of you is. He gives the distance in his paper "The Multiverse Hierarchy", published in 2007. He puts the distance to the nearest copy at about ten raised to the power of ten to the twenty-ninth meters: written out in meters, a

one followed by a hundred billion billion billion zeros. In meters, the distance to our horizon is a number of twenty-seven digits, while the distance to your nearest copy has more digits than the distance to our horizon has meters. Tegmark adds that copies of you are probably much nearer than his estimate, since the processes that made planets and life here are at work everywhere. By his count, the part of space we can see probably holds at least a hundred billion billion planets fit for life.

Tegmark draws one more consequence from the copies, about what you can predict of your own future. *Determinism* is the view that the state of the world at one moment, with the laws, fixes every later state completely. If many copies of you share your whole past, you cannot tell which of the copies you are, and the lives of the copies will go different ways from here on. So Tegmark writes that "this kills the traditional notion of determinism" (chapter 6, "What Are Level I Parallel Universes Like?"). On Tegmark's account, where copies of you exist, the best you can do is predict the chances of what you will experience next.

From Tegmark's pages

IV. Tegmark's Case That This Is Science

At this point in his chapter, Tegmark stops his own story to ask whether talk of copies we can never see is science. Tegmark begins his answer with a distinction: "Parallel universes are not a theory, but a prediction of certain theories" (chapter 6, "Are Parallel Universes Unscientific?"). On Tegmark's account, what is tested is a theory, and a theory is tested when some of its predictions are tested. *Falsifiable* is Tegmark's word for a theory that some observation could show to be wrong. He writes: "For a theory to be falsifiable, we need not be able to observe and test all its predictions, merely at least one of them" (chapter 6, "Are Parallel Universes Unscientific?"). General relativity,

Einstein's theory of gravity, predicted the orbit of Mercury and the bending of light, so physicists take seriously the theory's further prediction that space goes on inside a black hole. On Tegmark's account, a theory comes as a package: whoever accepts the theory accepts all its predictions, tested and untested. Tegmark calls inflation "the leading theory for our cosmic origins" (chapter 6, "The Bottom Line"). The parallel universes come in one package with inflation. Tegmark gives two lines of evidence for the copies. He writes: "the best evidence for the Level I multiverse is the evidence we have for inflation" (chapter 6, "Evidence for Level I Parallel Universes"). The second line is the measurements made inside our horizon that support his two assumptions. The two assumptions are an infinite space filled with hot matter, and seeds spread at random. He says that the two assumptions could hold even without inflation.

In his last chapter, Tegmark replies directly to the people who doubt the multiverse. He sums up the case that the cosmologist George Ellis makes against the multiverse in seven statements. Six statements are from Ellis, and Tegmark adds one for him. The seven statements include that inflation may be wrong and that multiverses may be unfalsifiable. Tegmark replies, "I agree with all of these seven statements" (chapter 13, "The Case for a Smaller Reality"). Yet he still bets on a multiverse. Then Tegmark grants what he calls the External Reality Hypothesis: "There exists an external physical reality completely independent of us humans" (chapter 13, "The Case for a Greater Reality"). Once the External Reality Hypothesis is granted, Tegmark finds that most of the case against the multiverse rests on three assumptions. Tegmark calls the first of the three assumptions the Omnivision Assumption: the assumption that some observer can in principle observe all of reality. Tegmark writes of the Omnivision Assumption: "The Omnivision Assumption effectively redefines the word exists to be synonymous with what's observable by us humans" (chapter 13, "The Case for a Greater Reality"). Tegmark ends his case with a chain of argument. The first link is an external reality independent of us.

From the first link he argues to the second, a mathematical universe (a world that is a mathematical structure), and from a mathematical universe he argues to every level of parallel universe. So, Tegmark writes, "we basically get stuck with all these parallel universes as soon as we accept that there's an external reality independent of us" (chapter 13, "The Case for a Greater Reality").

Tegmark also names a problem that he finds in his own account of an infinite multiverse. The physicist Alan Guth named inflation in 1979. Under some assumptions that Tegmark calls "reasonable-sounding", Guth found that an inflation that never stops predicts a much younger Earth than ours. Guth called the prediction of a younger Earth the youngness paradox. Tegmark counted all the observers alive at one moment, in every region, and he found the paradox even more extreme: observers in younger regions vastly outnumber observers in older regions, so a typical observer should see a much hotter sky than ours. The Cosmic Background Explorer satellite (COBE) measured the temperature of the sky at 2.725 degrees above absolute zero. Tegmark writes that by his count the measured temperature "ruled out our whole inflation-based story" (chapter 11, "Why Aren't You Younger?"). The trouble comes from counting an infinite set. When you count an infinite set, the fraction that has some property depends on the order in which you count. Counted in the usual order, half of the counting numbers are even, while in another order two thirds of the counting numbers are even. Cosmologists call the chosen order of counting a *measure*, and the fraction comes out differently under different measures. Tegmark calls the problem of choosing among measures the measure problem, and he writes: "In fact, I view the measure problem as the greatest crisis in physics today" (chapter 11, "The Measure Problem: Physics in Crisis"). Tegmark names his suspect for the cause of the measure problem in one character: "Here's my prime suspect: ∞" (chapter 11, "Infinite Problems"). By ∞ Tegmark means both the infinitely big (endless space, endless time, endless numbers

of things) and the infinitely small, which he calls the *continuum*. By the continuum he means the idea that every stretch of space holds infinitely many points, that space can be stretched without end, and that some quantities in nature can vary smoothly, with no least step. Tegmark notes that neither the infinitely big nor the infinitely small has been observed directly: the part of space we can see holds a finite number of things, and no quantity has been measured to more than about sixteen decimal places.

The first line is the evidence for inflation, because eternal inflation produces the Level I multiverse.

V. Where the Two Accounts Meet

I derive Tegmark's main result about Level I from a premise of my own. To compare, in this book's sense, is to tell one thing from another, or to find two things the same. Every measurement and every interaction is a comparison, and a finding is the result of one comparison. The shared result is that space goes on past our horizon, with the same laws, and that the regions past our horizon belong to our one world. My premise for the shared result is what I call the principle of existence. I prove the Incommensuration Theorem. Existence without any interaction whatsoever is indistinguishable from non-existence, and therefore is non-existence. In that sentence I use the word "existence" in the sense of the aphorisms that chapter 2 named: everything that can act on anything else. In plain words, whatever interacts with nothing at all does not exist. Every region past our horizon interacts with the regions next to it, as our region does. So every region past our horizon belongs to our one world. The regions past our horizon are further parts of our one world. On whether those regions belong to the world, Tegmark's account and mine differ only in a name: Tegmark calls each region another universe, and I call each region a further part of one world.

When I apply my principle of existence to Tegmark's Omnivision Assumption, I reach the answer Tegmark reaches, and I reject the assumption. On my account, a thing belongs to the world if the thing has any interaction at all. Whether a human being can observe a thing makes no difference to whether the thing belongs to the world. So the world holds more than any one of us can see. On my account, the premise from which Tegmark starts his chain is also true. There is a world, and the world does not depend on us. My account has its own reason for holding that the world does not depend on us: every interaction is a comparison, and the world was comparing itself, in every collision of particles, long before anything human looked.

On my account, Tegmark's picture of the fog is right too. The horizon of one observer is the edge of what one observer can ever compare with, and not an edge of the universe, because other observers stand past the first observer's horizon, each with a horizon of their own. So each observer stands at the center of all that the observer can ever compare with, and the regions that neighboring observers can compare with overlap, as the spheres of fog overlap across the field.

I derive Tegmark's conclusion that you cannot predict with certainty what you will experience next. My argument is a separate argument, which needs no copies at all and applies to every world. Determinism needs two things at once: one law that is the same everywhere and always, which is perfect sameness, and every state specified with no gap anywhere, which is perfect gaplessness. In plain words, symmetry is sameness (a thing unchanged when you swap or move it), and continuity is having no gaps. The theorem says that no single act of finding holds perfect sameness and perfect gaplessness together, nor their two opposites (asymmetry and discontinuity) together. So determinism needs exactly the pair that, by the theorem, no act of finding can hold: perfect sameness and perfect gaplessness together. The consequence: total specification (continuity) of lawful structure (symmetry) is structurally precluded. So no world is purely and completely deterministic, with copies or without them. For every description of a world it follows that, whatever lawful de-

scription is given of a world, something happens in that world which the description does not fix. So no one can predict with certainty all that happens next from any description, however complete.

VI. What Lies Past the Fog

Tegmark's account and mine disagree about what fills the regions past our horizon. On my account, the regions past our horizon belong to our one world, and we can make no comparison with what is in those regions. Where a beyond is defined, as past the horizon of one observer, we still cannot assume anything about what is there, because there is no way to tell. To tell what is past the horizon, we would have to interact with what is there. No signal from past the horizon reaches us. In the picture of the kitchen tables, the laws of physics are the rules of chess, which are the same at every table. What fills a far region is the position on that region's board, and we can make no finding about that position. So I contain no hypothesis about what is in the regions past our horizon. I also contain no conclusion like the one Tegmark draws about those regions, that "everything that can happen does happen" (chapter 6, "Evidence for Level I Parallel Universes").

Tegmark's count needs assumptions about what is in the far regions, while my account needs none. Tegmark's two assumptions are claims about what is in regions that no signal of ours reaches: the first assumption is that matter fills an infinite space, and the second is that any region could receive any possible seed. Tegmark supports the two assumptions with measurements made inside our horizon. On my account, the measurements made inside our horizon are findings about the part of space we can see, and those findings fix nothing about what lies past our horizon. Tegmark sums up the whole idea in one line, "inflation is a process converting potentiality into reality" (chapter 6, "Multiverse Halftime Roundup"), and on his account inflation turns potentiality into reality by making every possibility actual somewhere. On my account our epoch

(the long age of the universe we live in) began at the turn. The turn is the passage from the end of the epoch before. Our epoch began out of one single whole, with no long inflation, and chapter 5 gave the reasons. So there is no endless inflation to make the far regions.

The passage from the possible to the actual happens somewhere else: in each act of finding. Before the measurement, the system's state is a range of possibilities; after it, one possibility is actual. The other possibilities differ from other places and other times: other places can be looked into, and other times can be read from records. Other possibilities can be reached only by hypothesis. You can walk to the next room, and you can read yesterday in a diary, but you cannot walk to the outcome that did not happen. A possibility is a member of the range from which each act of finding makes one thing actual. A far region of space is a place, so a possibility and a far region are different things.

Infinite space has room for a copy of you. A copy of you needs more than room. A copy needs the space past every horizon of ours to hold one particular arrangement of matter. We can make no finding about what the space past our horizon holds.

VII. Two Games, Not One

Suppose, all the same, that a copy of you stood somewhere past the fog, exact to the last atom. I apply my principle of identity to the supposed copy. The Principle of Identity says that what cannot be told apart, by any test that could even be imagined, must be counted as the same. Chapter 21 also says what it is for two things to differ: some content of the two things (what is found the same or different in a comparison, such as weight or color) differs in a setting that the two things have in common. A difference of content in a shared setting, the chapter adds, "is exactly to say that some comparison could register the difference". Two identical

coins, one in each hand, are not one. Compare each coin with your left hand: one coin is in your left hand and the other coin is not, so the verdicts of the two comparisons differ, and the coins are two.

The same test shows that you and a copy of you in a far region are two. Compare each of you with this room: one of you is in this room and the other is not. So, by the principle of identity, an exact copy of you in another place is a second thing, in a second place. Whatever the copy finds, it finds from where it stands, and whatever you find, you find from here.

Go back to the two kitchen tables. As score sheets, the two games read the same, line for line, and a score sheet is the same for everyone who reads it. As games, the two tables hold two games, because each move was made once, at its own table, by its own players. A copy of you, if one existed, would be like the second game. The copy's record could read like yours, line for line. The copy's moves would be its own, made one at a time, at its own table. Read as score sheets, you and the copy could not be told apart. Read as games, you and the copy each make your own moves. The question "which one am I?" has its answer at each table: the one here.

I derive a result of the same kind for a copy of you made in a machine, and for a copy in a machine my argument is about persons. The hope of living on as a copy in a computer rests on the assumption that persons are information, and the answer is that a person is not a very large file. The reason is that experience needs three things that a record lacks: something new arriving that nothing before it fixed, time running one way, and a point of view from inside. In his chapter 11, Tegmark takes up "subjective immortality", the idea that a copy of you that survives where you die counts as your own continuation. By the principle of identity, applied in the same way, the surviving copy is a second thing, in a second place. The surviving copy is not you.

VIII. Counting the Infinite

I contain an answer to a question that has the same form as the measure problem. I apply my answer to Tegmark's count. I ask why the present act of measuring falls at one time in the history of our universe, and not at another time. My answer there is that "only the present act is given". A comparer is any position inside the world from which comparisons are made, a mind or not. I then say there, in two sentences about epochs and comparers, what a survey made from outside can do and what the survey cannot do. The two sentences are: "A criterion stated from outside, which surveys all epochs at once, can give a distribution of the epochs at which comparers stand. It cannot single out one of them as real, since that would take the presents of all epochs as given together". In those two sentences, an epoch is one time in the history that our measurements piece together, and not the long age from one turn to the next that chapter 3 described. In plain words, a survey made from outside can say how comparers are spread across the times of history, but the survey cannot make any one of those times the real present. Tegmark's youngness count is a survey of the same form, made across regions in place of times. In the count, Tegmark lays out, from outside, every observer alive at one chosen moment in every region at once, and he asks which of those observers you should expect to be. On my account, only the present act is given, and our measurement of a cold sky is the present act. Our measurement of a cold sky is the only present given, and so the only present to count. On Tegmark's account, eternal inflation made the crowd of observers in younger regions. On my account, eternal inflation did not occur.

My framework does not treat infinite totalities as completed objects of different sizes. In a fraction such as infinity over infinity, two endless totals are treated as completed objects with sizes. So my framework contains no fraction of one endless total over another.

On the infinitely small, Tegmark's account and mine disagree: Tegmark suspects that the continuum may be a convenient approximation, as the smooth air of the equations of sound is an approximation to air made of atoms. The *context* of a comparison is the respect in which the two things compared are two: two places, or two moments, of space and time as they are found. The context of every comparison is continuous: the context has no gap and no least step. The reason: a least step of context would be a difference of context with no sameness within it, and in every comparison sameness and difference come together. The smallest scale physics speaks of is the Planck length. The Planck length is roughly a hundred-millionth of a billionth of a billionth of a billionth of a meter. The Planck length is the limit of what can be measured. Essay 18 states what the limit of measurement means, in one line: "a boundary on measurement is not a granularity of existence". In plain words, a limit on what can be measured does not give space a grain. The discrete is what comes in separate steps, and the continuous is what has no gaps. The difference between the discrete and the continuous is basic in itself: the distinction between the discrete and the continuous is foundational, not a mathematical convenience.

A measurement made in 2009 bears directly on whether space is a continuum. The Fermi Gamma-ray Space Telescope is a satellite that watches the sky in *gamma rays*, light of the highest energies. On 10 May 2009 the Fermi telescope recorded a short burst of gamma rays, the burst named GRB 090510. The burst came from a galaxy about 7.3 billion light-years away. A photon is the smallest amount of light that can be sent or taken in. The light of the burst included one photon with about fifteen billion times the energy of a photon of yellow light. The high-energy photon arrived less than a second after the burst began, together with photons of far lower energy. Suppose space were made of grains that slow light in proportion to the energy of the light. Then light of high energy would be slowed more than light of low energy, and over seven billion years of travel the delay of the high-energy photon would have grown large enough to show. In 2009 the Fermi teams, A. A. Abdo and colleagues, published the result in the journal *Nature*: any grain of space that slowed

light in proportion to the light's energy would have to be smaller than the Planck length. So we now know that light of very different energies crosses billions of light-years of space at one speed, and that, down to the finest scale yet measured, space has shown no least step.

Tegmark's account and mine agree on how many digits a measurement can carry: no finding carries infinitely many digits. Chapter 14 of this book has a section of its own on the agreement about digits. What is continuous is places and moments themselves, and not a list of digits completed ahead of time.

IX. Whether It Is Science to Say So

The second half of the reader's question is whether it is science to say that there is a copy of you far away. My answer to the second half has two steps. In the first step, the kind of claim being made is named, and in the second step, the claim is checked in the way that claims of that kind are checked. The rule: methods match claims, which means that each kind of claim is checked by its own kind of method. A claim of logical form is checked by its structure. A claim about the world is checked by measurement and by evidence gathered from many independent routes. When three things are true of a thing, I call the thing *objective*. First, the thing is met in an actual interaction. Second, the thing is found the same by many comparisons made from many places. Third, the thing can show what no one expected. A test is a comparison of outcomes, and a test applies exactly where the outcome of a comparison can be objective, and nowhere else. Chapter 19 also describes a further kind of claim: a description of what a thing is. A test does not apply to a description. A description is checked by its derivation, step by step, and by whether its definitions are faithful to the way their words are used. Take the claim that regions exist past every horizon of ours. The claim is a description of what the world is, about regions that we can make no comparison with. The claim is derived from the principle of existence, because every region past our horizon acts

on the regions next to it. So it is right to say that space goes on past the fog. The reason is an argument about what existing is, and no telescope needs to confirm the argument.

Tegmark's answer depends on tested theories, and so my reply to Tegmark depends on measurement too. Tegmark holds that physicists come to trust a theory through its tested predictions, and that physicists then trust the theory's untested predictions too. He finds the best evidence for the Level I multiverse in the evidence for inflation. He supports his two assumptions by measurements made inside our horizon. So the measurable parts of Tegmark's picture are inflation, and the evenness and randomness that we measure inside our horizon. On my account, no long inflation occurred, for the reasons chapter 5 gave, and the evenness and randomness are findings about the part of space we can see, which fix nothing about what lies past our horizon. For each coming measurement that bears on a long inflation, the appendix "What the Coming Measurements Will Find" states what I say the measurement will find.

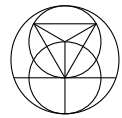
Tegmark's chain runs from an independent world through a mathematical universe to every parallel universe, and I disagree with Tegmark at the second link of the chain. On my account, the first link is true: there is a world, and the world does not depend on us. I disagree at the step to the second link, a world that is pure mathematics. Take every human word out of a description of the world, and what remains still describes a world in which every finding is made from a stance. A collision of two particles has a stance as surely as a laboratory does. Mathematics sets aside the stance that every comparison is made from, so on my account a world where every finding has a stance is not pure mathematics. So holding that there is a world independent of us commits no one to every parallel universe. Chapter 10 contains the full argument.

X. What This Chapter Holds

Space goes on past every horizon as more of our one world, with the same laws. We can make no finding about what fills space past our horizon. A copy of you, wherever it stood, would be a second thing, in a second place. An argument about what existing is shows that space goes on. I contain no hypothesis about what lies past every horizon, a copy of you included.

I have answered the question of space past the horizon and of a copy of you. Tegmark finds a copy of you by its score sheet, which reads the same as yours, line for line, while I count the world by its moves, each made once, at its own table, from its own seat. In plain words, Tegmark counts by structure, which is the same wherever it is read, and I count by acts of finding, each made once, from one place.

If the rules are the same at every table out to the farthest region, why do the constants of nature have the values they have? Are the constants just right for us because countless other regions, forever out of reach, have other values?



Chapter 7

Why Are the Constants What They Are?

Beside Tegmark's chapter 6, "Welcome to the Multiverse" (the Level II Multiverse)

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

A *comparer* is any position inside the world from which a comparison is made, such as a glass plate, a counter, an atom or a person; a comparer need not be a mind (Chapter 6).

The *principle of existence*: "Existence without any interaction whatsoever is indistinguishable from non-existence, and therefore is non-existence" (Chapter 6).

The *internal space*, or the held internal dimensions: six further directions in which things can differ, held small at every point (Chapter 4).

I. A Teacher in Graz

The constants of nature seem set just right for us: are they set that way because there are countless universes with other values, and we live in one of the few universes that suit us? Max Tegmark answers yes, with care. We follow his answer first, and then a different one. Both answers depend on a question older than either answer: which numbers in nature are law, and which are history?

Johannes Kepler met the question of law and history twice, inside his own work. In April 1594 Kepler, aged twenty-two, arrived in Graz, in southern Austria, to teach and to serve as mathematician to the province. On 19 July 1595, by his own account, he was showing his class a figure of the meetings of Jupiter and Saturn in the sky. An idea came to him then about the six planets known at the time, from Mercury out to Saturn. Why are there six planets, and why are the planets at their particular distances from the Sun? Kepler knew that there are exactly five regular solids, the figures whose faces are all one regular shape: the cube, and the solids of four, eight, twelve and twenty faces. He nested six spheres one inside another and set one solid between each pair of spheres. Each solid fitted inside the sphere beyond it and around the sphere within it. With the solids in the right order, the six spheres stood nearly in the proportions of the six planets' paths. Kepler published the scheme at Tübingen in 1596, in a book whose short title is *Mysterium Cosmographicum*, the secret of the universe. He believed he had found why there are six planets and why they lie where they do.

Thirteen years later Kepler took a step of the opposite kind. He had in hand the finest measurements of the sky then made. The measurements were the positions of Mars that the Danish astronomer Tycho Brahe had recorded over many years. Kepler worked out the best path for Mars that circles could give, and he set that path against Tycho's positions. The best circles missed some of Tycho's positions by eight minutes of arc, about a

quarter of the width of the full Moon. Kepler knew that Tycho's measurements were finer than eight minutes, so he kept the miss and gave up the circles. Kepler's book *Astronomia nova*, the New Astronomy, was printed at Heidelberg and published in 1609. In that book he wrote that the eight minutes alone had led the way to a reform of the whole of astronomy. The path of Mars is an ellipse, an oval drawn with a loop of string around two pins, with the Sun at one pin.

Now compare Kepler's two steps. In 1596 Kepler took a set of distances, which belong to the history of one solar system, and read them as law. In 1609 he took a miss of eight minutes, a small number in a table of measurements, and read the miss as a finding about law: the shape that every planet's path must have. Better measurements later ruled out Kepler's reading of 1596. His reading of 1609 led, through Isaac Newton, to the law of gravity. So Kepler met the difference between a number that is law and a number that is history inside the work of one life, and the question of the constants of nature depends on that same difference.

Since chapter 1 I have used one picture in this book, the chessboard at the kitchen table. The chessboard shows the same difference between law and history. The rules of chess are the same at every table: the board has eight ranks and eight files, the bishop moves on the diagonal, and no game changes the rules. The position on the board is where the pieces stand after the twentieth move. The position is the history of this one game, while another game, played under the same rules, has another position. When the question "why this value?" is asked of a number in nature, the answer is of one kind for a rule and of another kind for a position. For a rule, the answer is a reason. For a position, the answer is a story of what happened. So the first task is to sort every constant of nature. Is the constant a rule, or a position?

From Tegmark's pages

II. What Max Tegmark Showed

Tegmark's answer begins with inflation, the very fast early expansion of space that chapter 5 described. In his chapter 5 Tegmark tells how the physicists Andrei Linde and Paul Steinhardt found, for particular models of inflation, that inflation never stops everywhere at once: inflation ends in some regions and goes on in others, forever. Tegmark adds that the physicist Alex Vilenkin gave a proof of the effect, and that Linde named the effect *eternal inflation*. Between two regions where inflation has ended, space goes on inflating faster than anything can cross the space between them. So Tegmark writes of the other regions that "they're more than infinitely far away" (chapter 6, "Many Universes in One Space"): a traveler at the speed of light would never arrive at any of them. Tegmark reports that Linde, Vilenkin, Alan Guth, who gave inflation its name, and their colleagues have shown that inflation typically predicts many regions where inflation has ended, with space that goes on inflating between them, and Tegmark calls all the regions together the Level II multiverse.

In Level II, different regions have different constants. He gives a picture of diversity: a fish that has lived all its life in the sea might take water for empty space, never knowing that water also comes as ice and as steam, three solutions of one set of equations. Tegmark proposes that empty space may be like the sea. The empty space we know may be one of several forms that one set of equations allows. *String theory* is a leading candidate for the equations of space and its contents. String theory proposes that every particle is a tiny vibrating string. Different ways of vibrating give different particles, as different vibrations of a guitar string give different notes. The equations of string theory have very many solutions, perhaps ten to the five hundred (a one followed by five hundred zeros), and physicists call the whole range of those solutions the *landscape*. The landscape is every settled state that the equations of string theory allow, each settled state

like a valley in a hilly country, where a rolling ball comes to rest, and each with its own constants. Tegmark then writes: "Remarkably, eternal inflation has the property that it creates all possible kinds of space!" (chapter 6, "Diversity!"). Tegmark uses the landscape to draw a clear distinction between two kinds of law. In his use, a *fundamental law* is one of the equations themselves, the same everywhere. An *effective law* is the law that observers find in their own region. An effective law belongs to one solution of the equations. So many regularities long taken as fundamental are, in Tegmark's words, "local bylaws that can vary from place to place" (chapter 6, "Diversity!").

Tegmark's evidence for Level II is "the fact that our Universe appears highly fine-tuned for life" (chapter 6, "Fine-Tuning as Evidence for the Level II Multiverse"). By *fine-tuning* he means that many constants sit in a narrow range that life needs, so that a small change in one of those constants would leave a universe with no galaxies, no stars or no atoms. His leading example is the energy of empty space. Picture a dial whose one full sweep spans every value of the energy of empty space. The sweep runs from the largest negative value to the largest positive value. The setting we find sits about one part in a one followed by one hundred twenty-three zeros of a sweep from the middle. Tegmark says that a setting fit for galaxies must be right to more than one hundred twenty decimal places. The force between electric charges, which light carries, is the *electromagnetic force*. Tegmark reports that if the electromagnetic force were four percent weaker, the Sun would explode, and that if protons were two tenths of a percent heavier, there would be no atoms.

Tegmark weighs three answers to the fine-tuning: luck, design, and many regions. He sets luck aside on a principle: "Because the scientific method doesn't tolerate unexplained coincidences" (chapter 6, "The Fluke Explanation"). He leaves design for his chapter 12. That leaves many regions. If the knobs of nature, the constants, take every setting in some region, then a region with our settings exists, and we can find

ourselves only in a region where we can live. The physicist Brandon Carter called reasoning from where observers can live the weak *anthropic* principle. The word anthropic comes from the Greek word for a human being. Tegmark objects only to the word "principle", because "the use of correct logic when confronting a theory with observation isn't optional" (chapter 6, "The A Word").

Kepler appears in Tegmark's chapter too, as the man who took history for law. Tegmark recalls the nested solids and states what we now know: the distances of the planets tell us which solar system we live in, and the distances tell us nothing more fundamental. Then Tegmark carries Kepler's lesson much further: "The Level II multiverse does to the electron's mass what other planets did to Earth's mass" (chapter 6, "What Can We Ever Hope to Predict?"). On Tegmark's account, many measured constants become "merely part of our cosmic address" (chapter 6, "What Can We Ever Hope to Predict?"), as the distances of the planets are part of our address among solar systems. Tegmark plots the measured masses of nine particles of matter on a scale of powers of ten, and he finds that "it looks to me like nine randomly thrown darts" (chapter 6, "What Can We Ever Hope to Predict?").

Tegmark also shows what physicists can still predict. Combine how the constants vary from region to region with what galaxies need, and you get the odds of what a typical observer should measure. Tegmark reports that the odds of what a typical observer should measure agree fairly well with the measured energy of empty space, the dark matter and the *neutrinos*. Neutrinos are particles with no charge and almost no mass, which pass through matter almost untouched. In 1987 the physicist Steven Weinberg used odds of what a typical observer should measure to make his first prediction of an energy of empty space other than zero. Tegmark also asks why space has three dimensions, three independent directions in which things extend, and why time has one. Tegmark writes that with more than three dimensions of space there can be neither stable solar systems nor stable atoms. In

1917 the Austrian physicist Paul Ehrenfest showed that in four dimensions of space no orbit is stable. Tegmark adds that with fewer than three dimensions of space, gravity no longer pulls things together. His own paper of 1997 showed that with more or fewer than one dimension of time, the equations of physics lose the property that lets an observer predict anything. So, for observers, "this leaves three space dimensions and one time dimension as the only viable option" (chapter 6, "All Isn't Lost").

In his last chapter Tegmark adds two points about Level II. He expects Level II to survive even without string theory, because equations commonly have several solutions, as the equations of water have three. In his words, "I suspect that we'll be stuck with a Level II multiverse even if string theory turns out to be a red herring" (chapter 13, "The Case for a Smaller Reality"). In chapter 13 too, Tegmark treats an unexplained coincidence as a clue instead of as luck. He writes: "To me, an unexplained coincidence can be a telltale sign of a gap in our scientific understanding" (chapter 13, "The Case for a Smaller Reality"). Tegmark has shown his reader two things: the question "why these values?" deserves an answer, and an answer must account for the values together with the fact that the values suit life.

Each region where inflation has ended is a whole infinite space full of galaxies, of the Level I kind that chapter 6 described: a space that goes on far beyond what we can see, with the same laws throughout. Level II holds more than Level I does because of what Tegmark calls diversity. I call the number that measures the strength of the electromagnetic force the strength of light; physicists call that number the fine-structure constant.

III. Where the Two Accounts Meet

I agree with three of Tegmark's results, from premises of my own. The three points of agreement are the count of dimensions, the refusal to call a fit between the numbers and life a coincidence, and the line between law and history.

The first point of agreement is the count of dimensions: three of space and one of time. I derive the count from the structure of comparison. Time has one dimension because every comparison is made from one stance. From one stance, any two findings come in exactly one order: one finding earlier, the other later.

For space, take a flat page. On the page you can tell east from north. No direction on the page is independent of both east and north. Lift your eyes off the page. The third direction, straight up, is the direction from which the two directions of the page can be told apart. I make the lesson of the page a rule for space: the difference between any two directions of space must be marked within space itself, by a further direction not made from those two directions. If space had one dimension, space would have the same form as time, and nothing would tell space and time apart. If space had two dimensions, as the page has, nothing in space would mark the difference between its two directions. With three dimensions, each direction marks the difference between the other two directions. A fourth direction would mark only differences that the three directions already mark, so no comparison could register a fourth direction as a direction of its own. A difference that no comparison can register is no difference, and so space has three dimensions. The count depends on one identification: a step that reads something in the structure of comparison as something in geometry or in the physical world. I identify the space the count is about with the space of the large world, the directions in which things extend,

and every measurement of the large world finds three directions. I add that analyses of the equations of physics also single out one dimension of time. There I cite Tegmark's paper of 1997 by name.

So Tegmark's account and mine agree on three dimensions of space and one of time, for different reasons. For Tegmark, three and one are the counts at which observers can live, while for me three and one are the counts that comparison itself requires. I picture the count with a board. The rules of a board fix its ranks, and no move can add a rank that the rules do not supply. So at the kitchen table the count of dimensions belongs with the eight ranks and eight files of the chessboard, which no game changes. The count of dimensions is a rule.

The second point of agreement is about the word "coincidence". I call a number that agrees with measurement a *value that lands*. The rule that I give for a value that lands is close to Tegmark's own rule: "A value that lands, before its reason is found, is a question, and it is evidence of a reason not yet known". I give two cases that show why a value that lands is a question. Every body has two kinds of mass: how hard the body is to set moving, and how strongly gravity pulls the body. The two kinds of mass agree in every material ever tried. Newton could give no reason for the agreement. Einstein asked why the two kinds of mass agree, and his answer was general relativity. Johann Balmer's rule of 1885 gave the colors of hydrogen's light from whole numbers, with no theory behind the rule. Niels Bohr asked why Balmer's rule works, and Bohr's answer in 1913 led into quantum theory: the rule became a door into quantum theory. Because physicists held the two agreements as questions, one agreement became the theory of gravity and the other became a door into quantum theory. If physicists had written the two agreements down as coincidences, physicists would have filed the agreements away and stopped asking why. So I agree with Tegmark in refusing to dismiss the fine-tuning as luck.

The third point of agreement is the line Kepler drew, between numbers that are law and numbers that are history. Tegmark says that the distances of the planets are history, part of an address. Astronomy bears Tegmark out: other stars have planets at other distances. I draw the same line

between law and history inside our own world. The kinds of particle are fixed by structure. The amounts of each kind are not fixed by structure. The excess of matter is the small surplus of matter over antimatter, matter's mirror twin with every charge reversed, that our universe holds. The excess of matter is not structural. It belongs to the initial conditions, to the history of the universe, to the facts that the zoo does not fix. The zoo is the list of the kinds of particle. The excess of matter is measured: about six protons and neutrons for every ten billion particles of light, a number of what happened. On the chessboard, the excess of matter is position.

IV. One Internal Space, the Same at Every Turn

Tegmark's account and mine disagree about Level II itself. I give three separate arguments for that result, three routes, each from its own premises.

The first route is the argument chapter 5 gave, which starts from the first ripples of the sky: the slight unevenness of the oldest light. In chapter 5 I identified the first ripples with a shape carried through the turn. Every alternative taken is kept in that one shape, as every footstep is kept in the one curve of a worn stair. A long inflation would have made the first ripples, on the lengths we observe, out of the faint trembling of empty space during the inflation. On my account the ripples came through the turn instead. So a long inflation did not happen. The amount of the dark matter gives a second, separate reason that a long inflation did not happen: after an inflation that had made the ripples, the universe would have been too cool to make the dark matter in its measured amount. Eternal inflation needs a long inflation, so on my account there is no eternal inflation to make the regions of Level II.

The second of my three routes concerns the landscape. Tegmark himself names the condition on which his step from eternal inflation to Level II depends. He states the condition in these words: "if there are laws or constants of nature that can in principle vary from place to place, then

eternal inflation will make them do so across the Level II multiverse" (chapter 6, "Fine-Tuning as Evidence for the Level II Multiverse"). Tegmark explains that the constants differ from region to region because the equations of space have many solutions, as the equations of water have three (ice, water and steam), while one solution would give the same constants everywhere. Recall the *internal space* of chapter 4: six further directions, held small at every point, whose shape fixes the kinds of particle. I call the six further directions the six held dimensions, or the six for short. I hold that the equations of space have one solution for the six held dimensions. So I do not grant Tegmark's condition. String theory also curls six directions small. In string theory the six small directions can settle into an enormous number of shapes and sizes. Each settled shape and size of the six small directions of string theory is a valley of the landscape. On my account the six settle into one shape and one size.

I give the reasons for one shape and one size, in order. I give the same reasons in plain words. I derive the shape of the six by two routes that start from different premises. Each premise is an identification: a step that reads something in the structure of comparison as something in geometry. The algebra of comparison is a stock of quantities with rules for adding and multiplying them. The algebra of comparison describes the findings of comparisons. One route goes through the algebra of comparison. The other route uses geometry and no algebra at all. Both routes arrive at the one shape that chapter 4 described. I picture the two routes as two climbers who start from opposite valleys and meet on one summit: "they have found one summit, and neither of them chose it from a list". In the picture each climber is one route, the summit is the one shape, and the valleys are only the starting places of the climbers, not the valleys of the landscape. Neither route chooses the space from a set of candidates, so there is no landscape of internal spaces and no problem of selecting one. One further identification fixes the size of the six too. That identification ties the bending of the six to the Planck length. Chapter 4 gave the result: the six are held a few Planck lengths across. The six have the same size and the same shape at every point of space.

A *field* is something spread through all of space with a strength at every point, as a magnet's pull is. In string theory, fields are wound through the loops of the six small directions, as thread is wound through the holes of a button, and every pattern of winding gives another valley of the landscape. On my account the law of gravity is built from the geometry of all ten dimensions alone (the four of space and time, and the six held small), with no further field. So nothing is wound through the six. String theory's threaded fields give its six some ten to the five hundred valleys; the six of my account have one.

One valley means one set of rules. At each turn the same six re-form at the same size and shape, so the rules are the same in every epoch. The structure that my account derives is the same in every epoch. I have a chapter on the physicist Lee Smolin, and that chapter names what the structure that I derive includes: the forces, the kinds of particle and their charges, the three generations and the counts of dimensions. The three *generations* are three sets of matter particles, each set a heavier copy of the one before. The list of the kinds of particle is closed. The particles of the Standard Model, the quarks, the leptons, the carriers of the three forces and the Higgs boson, exhaust the kinds. Physicists call the list of known particles and their forces the *standard model of particle physics*, and the quotation above calls it the Standard Model. The *quarks* are the particles of which protons and neutrons are built. The *leptons* are the electron, its two heavier copies and the neutrinos. The three forces are the electromagnetic force, the force between charges, which light carries; the strong force, which holds the core of every atom together; and the weak force, which is at work in some radioactive decays. Each of the three forces has its own carrier particles. The *Higgs boson* is the particle of the Higgs field, a field that gives many of the smallest particles their masses. On the chessboard, Tegmark's Level II is a hall of tables. Each table keeps house rules of its own, as each region keeps effective laws of its own. At one table in the hall the game might be played with ten kinds of quark in place of the six of the

standard model of particle physics. On my account the rules are the same at every table. Three things are rules: the number of dimensions, the shape of the six and the kinds of particle.

V. A One Followed by One Hundred Twenty-Three Zeros

The third route concerns the number on which Tegmark's case depends most: the energy of empty space. Chapter 4 gave the cosmological constant as the one number that the dark energy is. Quantum field theory is the theory of the particles and their forces. Quantum field theory says that every field hums even when the field is as quiet as it can be, and that the hum is energy. Add up the hum of every field, up to the scale at which gravity and the quantum act on equal terms, and empty space should hold an enormous energy. The sky shows a tiny energy of empty space, so the two energies differby a factor of a one followed by one hundred twenty-three zeros. That factor is the largest mismatch between a theory and a measurement in physics. The mismatch is also the gap across which Tegmark's dial is tuned. On the dial, the setting we find sits about one part in that factor of a sweep from the middle.

I answer the mismatch in two steps. The first step is about what curves space. Chapter 4 gave the reason that space curves at all. In every finding, two things pull against each other: the unbroken act of comparing, and the sameness of what the act finds. The strain falls on the arena, which is space and time as found in comparisons, as a rug between a wall and a sofa buckles when the sofa is pushed toward the wall. So space curves under the findings of comparisons. Energy that is in no finding puts no strain on the arena, as the rug lies flat when no sofa pushes.

The *vacuum* is physicists' word for the settled state of empty space, as quiet as it can be. A measurement of the vacuum is made from inside the vacuum, by instruments made of the very fields whose hum the instruments measure. I picture a measurement of the vacuum as a cupful of the sea trying to know the rest of the sea. Every measurement takes in a part

of the vacuum's energy, as a cup holds a part of the sea. If the vacuum's total energy were taken whole, as one thing, the total would be the same at every place, the same for every observer in steady motion, and without a break anywhere. The total would be perfectly symmetric and perfectly continuous at once, and no finding is both. The reason no finding is both is the Incommensuration Theorem, met in chapter 1: symmetry and continuity cannot both be perfected in one comparison, and neither can asymmetry and discontinuity. In plain words, symmetry is sameness and continuity is gaplessness. No single finding holds perfect sameness and perfect gaplessness together, nor their two opposites, perfect difference and perfect separateness into steps, together. So each finding holds a part of the vacuum's energy, while the vacuum's total, taken as one whole, is in no finding, and so the total puts no strain on the arena. So the total curves nothing, whatever its size. No cancellation is needed for the mismatch of one hundred twenty-three powers of ten, because the total curves nothing. The energy of empty space needs no tuning across that factor, because the vacuum's large total never acted on space.

The second step is about what a measurement does take in. The second step uses the identification that chapter 4 stated. The cosmological constant is the share of the vacuum's energy that a measurement made from inside can take in. The vacuum holds its energy in a vast number of independent ways, as a piano holds its sound in many strings, each of which can ring on its own, and each way holds an equal part of the vacuum's total energy. The vacuum's total energy is the hum of every field, added up to the scale at which gravity and the quantum act on equal terms. The six held internal dimensions are curved, and their curvature carries a small energy of the opposite sign, about twelve percent of the total, so the curvature of the six takes that much off the total and leaves 0.881 of the total. Three steps lead from the vast number of ways to the share that a measurement takes in, and each step is an identification of mine. First, I count the ways on the edge of everything whose light has reached us, and the count at that edge is about a one followed by one hundred twenty-three zeros. I take the count at that edge as an identification. I hold that

the edge of everything whose light has reached us is the edge of what has been compared, and I identify the edge of what has been compared with the edge of the information open to the comparison. Second, a measurement takes in what it finds through *channels*: the routes by which the vacuum reaches the measurement. I identify the channels with what the measurement registers, one channel in each of the three roles that every comparison has: what is laid out, the act of finding, and the stance. Chapter 1 set out the three roles at the kitchen table. The first role is what is laid out, which is the score sheet. The second role is the act of finding, which is the move. The third role is the stance from which the act is made, which is the seat. The shape of the six gives the same three channels by a separate route. Four of the six have the shape of all the ways a three-way choice can stand, and that part of the six is built from exactly three basic pieces, one for each role. A basic piece is one of the simplest parts a shape can be built from, as the surface of a ball is built from two: a single point, and a disc whose rim is drawn shut to that point, like a drawstring bag. Third, I take each channel to take in one whole way, and a different way for each channel. The reason is that what a measurement registers comes whole, in steps, and the three channels carry three different things. So a measurement takes in three of the ways: three parts in the count of about a one followed by one hundred twenty-three zeros. I write the share as one formula: divide the 0.881 of the vacuum's total among the ways counted on the edge of everything whose light has reached us, and take three of those parts. The cosmological constant is small because the part of the universe we can see is large.

Chapter 4 set the share from the formula beside the energy of empty space that astronomers find from the rate of expansion. The two values are like the results of two surveyors who each measure one piece of land. Setting the two values equal cancels the rate of expansion and the strength of gravity, and the one equation left says that 31.1 percent of the energy of the universe is matter, while the Planck satellite measured 31.5 percent. I make the same comparison. There the formula gives the energy of empty space within two percent of the value the sky shows.

Tegmark counts the fine-tuning of the cosmological constant among the strongest evidence for Level II, but on my account the cosmological constant is one value, given by one formula, the same at every place. From Tegmark's reasoning a physicist gets the odds of what a typical observer should measure, while from my account a physicist gets one number for the one measurement made from where we are. I grant the reasoning from the landscape its logic: I call that reasoning "logically impeccable". I also state that a physicist who accepts the reasoning from the landscape gives up a law that derives the cosmological constant. The reasoning makes the cosmological constant "an accident of location", a value that no law derives. I keep the derivation of the cosmological constant from law. My three routes together give one result: Level II does not occur.

VI. Rule, Position and Measured Value

Now we can sort the constants of nature, with the chessboard beside us.

Some constants are rules. The structure of comparison fixes the three dimensions of space and the one of time, the shape and size of the six, and the closed list of the kinds of particle, the same everywhere and in every epoch. For each rule the answer to "why this value?" is a reason, and I give the reason.

The cosmological constant is one value, the same at every place. One formula gives the cosmological constant from four numbers. The structure of comparison sets two of the four numbers: the number three, one for each role of a comparison, and the 0.881 of the vacuum's total that the curvature of the six leaves. The other two numbers are measured: the strength of gravity, which sets the size of the vacuum's total, and the size of the edge of everything whose light has reached us now, which sets how many ways share that total. So for the cosmological constant the answer to "why this value?" is a reason for the formula and for two of its numbers, and a measurement for the other two.

The rules fit observers, and the fit has a reason of its own. By the rules I mean the counts of dimensions, the shape and size of the six and the list of the kinds of particle. I give the reason for the fit. The world has its form because any form that supports comparison must have that form, not because observers must exist, but because comparison must be self-consistent and there is exactly one self-consistent comparison structure. Self-consistent, in that quotation, means that no part of the structure contradicts another part. Reasoning as Carter did, a physicist picks our values out of many, by where observers can live, but I find only one structure that comparison allows, and so there is nothing to pick from.

Some numbers are position. The distances of the planets are history, and so are the number of protons and neutrons for each particle of light and where the galaxies happen to lie: each of these numbers is what happened in this epoch, under the rules. For numbers of position the answer to "why this value?" is the story of the game so far, and Tegmark and I agree on that answer.

There is a third kind of number besides rules and position: a value of this world, which the structure of comparison leaves free and which measurement reads. The strength of light is the clearest case. Section II met the strength of light as the number that measures the strength of the electromagnetic force. So the value of the strength of light is a value that "no argument from the comparison structure, of any length, fixes". Its companion in plain words contains the same argument. The number physicists quote is one over the strength of light: the number one divided by the strength of light. From 1929 the astronomer Arthur Eddington argued that one over the strength of light must be exactly a whole number, first 136 and later 137. Measurement gives 137.035999177 for one over the strength of light. The measured number is no whole number, so measurement excludes Eddington's 137. For a value of this world, the answer to "why this value?" is the measurement, together with a proof that the structure of comparison leaves the value free. I count a number that the structure of comparison leaves free as a value of this world. In the phrase "value of this world", this world means the world in which the structure

of comparison is made actual. Tegmark reads a value that no law fixes as a sign of other regions with other values, while I read the strength of light as a value of this world, measured, together with the reason that the strength of light stays measured.

Some values agree with measurement before their reason is known: each of them is a value that lands. I write each value that lands as measured, with its question beside it. Chapter 4 called a measured value with its question written beside it a complete state of knowledge. Tegmark counts a fit between a number and life as evidence for other regions, while I count a value that lands as evidence of a reason not yet known, and I keep the number as a clue to that reason. Tegmark's nine darts show the difference between his reading and my reading. A halving is a step down in mass by a factor of two. I place every charged particle of matter a counted number of halvings below the heaviest quark, the top. One formula gives every count. A family is three particles of one type, one in each generation. The electron and its two heavier copies make one family, and the quarks make two more families. The formula has three numbers for each family. Tegmark sees darts, while I find that "Nine masses that spread over a range of hundreds of thousands become one formula". Eight constants can always be found to fit eight masses, and I place the test where a fit cannot reach: the constants that reproduce the masses lie close to simple fractions of three, four and five, which are the edges of a right triangle, and they satisfy identities among themselves. Eight dials turned at random would show none of this.

Kepler's two readings can now be sorted. With the nested solids, Kepler took position for rule. Kepler found a rule inside a table of positions when he kept the miss of eight minutes. Tegmark takes many constants for position: one region's settings of the knobs of nature, knobs that are set otherwise in other regions. I sort many of those constants as rules, fixed by one structure of comparison, and I keep the rest as history or as values of this world, measured. The appendix "What the Coming Measurements Will Find" sets out what I say the coming measurements of the constants of nature will find.

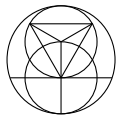
VII. What This Chapter Holds

On my account Level II does not occur, and the constants of nature belong to one world, in which each constant is a rule, a piece of history or a value of this world read by measurement.

The number of dimensions is a rule, and so are the kinds of particle, fixed by one structure of comparison that is the same in every epoch. The energy of empty space is one value, given by one formula. The amount of ordinary matter is the history of this epoch: about six protons and neutrons for every ten billion particles of light. The strength of light is a value of this world, measured.

After the sorting, a reader of Tegmark still has the point on which Tegmark and I agree most closely: neither Tegmark nor I call a fit between the numbers and life a coincidence. Once the constants are sorted, the question "why these values?" has three kinds of answer: a reason for each rule, a history for each position and a measurement for each value of this world. The sorting also means that the kinds of particle belong to the rules, and so the kinds of particle can be listed once, for every table.

If the kinds of particle are rules of the game, the same at every table, what are those kinds, and what happens when one of those kinds is found?



Chapter 8

What Happens When We Look?

Beside Tegmark's chapter 7, "Cosmic Legos"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

The *stance* is the side from which a comparison is made, as a move is made from a seat; every comparison has a stance, human or not (Chapter 1).

A *comparer* is any position inside the world from which a comparison is made, such as a glass plate, a counter, an atom or a person; a comparer need not be a mind (Chapter 6).

A *value that lands* is a number that agrees with measurement before its reason is found; I count it as a question, and as evidence of a reason not yet known (Chapter 7).

I. Two Spots on a Glass Plate

What happens when we look at the smallest things, and why can physicists not agree about what happens? The answer depends on an earlier question: what is everything made of at the bottom? Max Tegmark asks both questions in his chapter 7, "Cosmic Legos". One experiment, made in Frankfurt early in 1922, is a small case of both questions.

Walther Gerlach and Otto Stern, two physicists in Frankfurt, heated silver in a small oven until atoms boiled off the silver. A narrow beam of the silver atoms passed through the gap of a magnet, and the magnet was shaped so that its pull was much stronger near one face of the gap than near the other face. Each silver atom is a tiny magnet of its own. When a magnet's pull is stronger near one face, the magnet pushes a tiny magnet toward the stronger face or toward the weaker face, by an amount that depends on the way the tiny magnet points. Beyond the magnet, the atoms struck a glass plate and stuck to the plate.

The best plate took eight hours of running. On the best plate the silver lay in two separate traces, about two tenths of a millimeter apart, with no silver between them. If the tiny magnets of the atoms could point every way, the atoms would have spread into one smear from top to bottom. The plate showed two places, and only two. On 8 February 1922 Gerlach sent the physicist Niels Bohr a postcard with a photograph of the plate. On the postcard Gerlach called the plate the experimental proof of what physicists then called the quantizing of direction: the idea that the tiny magnet in an atom can point in only a few set directions. Physicists found the reason for the two places later. In 1925 physicists proposed that every electron carries a magnetism in itself, which they named *spin*. The same physicists proposed that an electron's spin, measured along any line, comes in exactly two values. The magnetism of a silver atom is the spin of one of its electrons.

The plate gives part of the answer to the question of what matter is made of, before anyone brings a theory to the plate: what the plate shows of matter comes in steps, two places and nothing between them. The plate also raises the second question, what happens when we look, in its sharpest form. Each atom made one spot, in one place. For a silver atom on its way to the plate, the best description in physics states only the odds of each place. Physicists argue about what happens at the plate when one of the two possible spots becomes the one spot the atom makes. We follow Tegmark, and then my physics built from the act of comparison, first through what matter is made of, and then through what happens when we look.

From Tegmark's pages

II. What Max Tegmark Showed About Matter

Tegmark answers the question of what matter is made of with a picture his readers remember: a box of Legos. Everything around us is built from the same eighty stable kinds of atom, and atoms are built from protons, neutrons and electrons, and protons and neutrons are built from two kinds of quark. Rearranging atoms is chemistry, and rearranging the protons and neutrons of atoms is nuclear physics. Rearranging protons, neutrons and electrons into new particles is particle physics. Tegmark sums up the levels in one line: "In summary, nature has a hierarchical Lego structure" (chapter 7, "Mathematical Legos"). On what lies below the quarks, Tegmark says, the best evidence of today is clear: "we simply don't know yet" (chapter 7, "Particle-Physics Legos"). Tegmark adds that "there's good reason to suspect that everything we know of so far" is built from deeper blocks, "including the very fabric of spacetime itself" (chapter 7, "Particle-Physics Legos"). *Spacetime* is space and time taken together as one whole of four directions.

A *conserved quantity* is an amount that stays the same in total through every change. Energy, momentum and electric charge are the familiar conserved quantities. In collisions of particles, Tegmark writes, "every reaction that isn't forbidden (for violating some conservation law) appears to actually occur in nature" (chapter 7, "Mathematical Legos"). If every reaction that no conservation law forbids occurs, the conserved quantities alone fix which reactions occur. Tegmark says that the deepest Legos of particle physics can therefore be taken to be the conserved quantities themselves. On Tegmark's view a particle is a recipe of conserved quantities: so much charge, so much spin, so much energy. A cat has a name, a smell and a personality besides its energy and its charge, but a particle, on Tegmark's account, has only its numbers, which physicists call its *quantum numbers*. Tegmark draws his conclusion from the numbers alone: "The Legos at the deepest level appear to be purely mathematical objects" (chapter 7, "Mathematical Legos"). Tegmark also says how far he will carry the conclusion in his chapter 10: "not that our Universe is made of mathematical objects, but that it's a part of a single mathematical object" (chapter 7, "Atomic Legos").

Tegmark then tells the history of the quantum. Light comes in pieces, called photons. The fact that light comes in pieces did away with an old prediction. The old prediction was that every warm object should glow with endless energy at short wavelengths. Tegmark puts the rescue in one line: "You're saved by the fact that light is made of particles" (chapter 7, "Photon Legos"). By the classical laws, an electron circling a nucleus should radiate its energy away. The electron should then spiral into the nucleus in about two hundredths of a billionth of a second. Yet hydrogen atoms have lasted for billions of years. Bohr described the atom with separate orbits, and the physicist Louis de Broglie described the electron as a wave. The physicist Werner Heisenberg showed that a particle squeezed into a small space must have a widely spread motion, and the widely spread motion of the

electron keeps the atom from falling in. Physicists call Heisenberg's limit on sharpening a particle's place and its motion together the *uncertainty principle*. Tegmark puts the lesson of the history in two sentences: "So are microscopic particles above the law? No, they obey a different law: Schrödinger's" (chapter 7, "Above the Law?"). Schrödinger's equation is the law by which the quantum description of a particle changes smoothly in time. The physicist Erwin Schrödinger wrote the equation down. Tegmark measures the success of Schrödinger's equation by a test he likes: "There are several science definitions that I like, and one of them is data compression, explaining a lot with a little" (chapter 7, "Making Waves"). Tegmark applies the test of data compression to Schrödinger's equation. The equation compresses more than twenty thousand measured lines of color in the light of atoms into three numbers.

III. The Same Results About Matter, on Separate Ground

My books derive much of what Tegmark shows about matter, each time from premises of their own.

Take Tegmark's levels of building blocks first. I derive the same levels as Tegmark from the kinds of particle and the forces between the particles. Matter is a ladder of levels, each held by what the level below leaves over and each held far more weakly than the one below. The field of the strong force holds quarks together into protons and neutrons. What the field of the strong force leaves over holds protons and neutrons together into nuclei. What a nucleus leaves over, its electric charge, holds electrons into atoms. What atoms leave over holds atoms together into molecules. I also state a rule for which wholes last: "A whole lasts when it is the lightest thing that carries its counts". A whole's counts are conserved quantities of the kind Tegmark lists: its electric charge, its number of baryons (wholes of three quarks, such as the proton and the neutron) and its number of leptons (the electron and its relatives). The rule for which wholes last is

structure: the rule is laid out whole and is the same everywhere. Which whole is lightest is found by measurement. Tegmark observed that every reaction that no conservation law forbids occurs. The rule for which wholes last says the same thing from the direction of what lasts: a whole that no conserved quantity protects breaks down into lighter things, because every reaction that no conserved quantity forbids occurs.

I tell the history of the quantum from two starting points: the act of comparison, and the Incommensuration Theorem. A *finding* is what one comparison gives. The Incommensuration Theorem, first met in chapter 1, says of every comparison that symmetry and continuity cannot both be perfected in it, and that neither can asymmetry and discontinuity. The Incommensuration Theorem says that no single act of finding can hold perfect sameness and perfect gaplessness together, nor perfect difference and perfect discontinuity together.

I derive from the Incommensuration Theorem why what is found of matter and light comes in exact, repeating steps: one outcome, one packet of light, one sharp line of color, the same charge on every electron. What a comparison finds of matter is the same across every change of place and time: an electron is an electron in every laboratory and every galaxy, with the same charge and the same spin. Because what is found is the same across place and time, what is found falls into kinds, and belonging to a kind is all or nothing: there is no particle that is two thirds of an electron. So there are gaps between the kinds. The Incommensuration Theorem states the underlying reason for the gaps: what is perfectly the same cannot also be perfectly gapless.

Tegmark tells of two great failures of classical physics, the endless glow and the falling atom, and I explain both failures in the same way: "In both cases the limit was removed by giving content its own character". Content is what a comparison finds the same or different, and here the content is matter and light. Matter and light have a character of their own: they come in exact steps, the same in every instance. Classical physics had described matter and light as smooth, but the steps had been there all along.

The uncertainty principle also follows from the Incommensuration Theorem. A particle's place is described point by point along an unbroken line, so a description of place has continuity. A state of perfectly sharp motion looks the same at every place along the line, so a description of sharp motion has symmetry. One exact rule converts either description into the other. I identify the rule that converts one description into the other with the pairing that I name at its foundation: two descriptions of one thing, each of which can be made perfect only at the cost of the other. If a description made the place and the motion both perfectly sharp at once, the description would hold perfect sameness and perfect gaplessness together, and the Incommensuration Theorem forbids the two together. The uncertainty principle is an instance of the Incommensuration Theorem. Tegmark is right about why atoms do not fall in, and the Incommensuration Theorem shows why a limit on sharpening place and motion together must hold, and why no later theory will remove the limit.

I also agree with Tegmark's praise of Schrödinger's equation. A measurement made in 2023 shows how precisely the quantum theory that grew from Schrödinger's equation describes the electron. At Northwestern University, the physicist Gerald Gabrielse and his group held a single electron in a trap of electric and magnetic fields, and the group measured the strength of the electron's own magnetism to about thirteen hundredths of a part in a million million. The quantum theory of light and electrons computes the same number. The measured number and the computed number agree to about one part in a million million. We now know that the rules of quantum theory describe the electron's magnetism to about one part in a million million, in a test of the most precise prediction of the theory of the known particles.

IV. What Lies at the Bottom

On what lies below the quarks, I come to a result different from Tegmark's, for two reasons.

The first reason concerns the kinds of particle. I derive the rulebook from which every kind of particle comes. The rulebook is a finite set of rules for combining the labels a particle carries. The structure of comparison fixes which rules are in the rulebook. The kinds that the rulebook allows are exactly the known kinds. There are twelve known kinds of matter: six kinds of quark, and six leptons, which are the electron, two heavier relatives of the electron, and three neutrinos, particles with no charge and almost no mass. Each kind of matter has its antiparticle, a twin with every charge reversed. The known kinds also include the carriers of three forces. One is the strong force. Another is the weak force, which changes one kind of particle into another. The third of the three forces is the electromagnetic force, the force between charges. Light carries the electromagnetic force: its carrier is the photon, the particle of light. The last of the known kinds is the Higgs boson, the particle of the field that gives many of the other kinds their mass. The argument is one of necessity, not of exhaustion: in any world in which comparison occurs, these kinds cannot not exist. In the mathematician's sense, exhaustion is proof by checking every case one by one. So the list of kinds is closed by the structure of comparison that makes the kinds, and not by how hard anyone has looked, and no further kind of particle lies below the quarks and the electron. Heavier wholes built from the known kinds are there to be found at higher energies. Every building block of the heavier wholes is one of the known kinds.

The second reason concerns space and time. Tegmark suspects that spacetime itself is built of smaller blocks. Tegmark states his suspicion that "the continuum is an illusion" in a table that sums up the key concepts of quantum mechanics (chapter 7, "The Collapse of Consensus"). Tegmark means by the continuum "the idea that even a liter of space contains an infinite number of points, that space can be stretched out indefinitely without anything bad happening, and that there are quantities in nature that can vary continuously" (chapter 11, "Infinite Problems"). I call space and time the *arena* (space and time as found in comparisons). The arena has no gap and no least step.

A picture on a screen shows what a least step would be. Enlarge the picture far enough and the picture breaks into small squares of color, each square with a sharp edge against the next. Between two neighboring squares there is no place at all. The arena is the setting of every comparison, and every comparison finds sameness and difference together, so some sameness lies within every difference between two places or two moments of the arena. A finer comparison can always find the sameness within the difference, if the finer comparison is made at a place or moment between the two places or the two moments. A least step of space or time would be like two neighboring squares of the screen: two places, or two moments, with nothing between them and nothing shared, which is a difference with no sameness within it. In this book's sense from chapter 1, the context of a comparison is the respect in which the two things compared are two. So the context has no gap and no least step: between any two settings there is a third, and nothing in the context is sudden, which is continuity. The steps of quantum physics belong to what is found within the arena: the charges, and the kinds of particle. Chapter 6 told of a burst of gamma rays that the Fermi space telescope caught in 2009. The burst shows the arena smooth. The light of the burst came at very different energies, and all of the light crossed billions of light years of space and arrived together.

Tegmark reads a particle's numbers as a sign that the deepest Legos are mathematical. I agree with every one of the numbers, and reads the numbers in another way. Every physical theory has two levels. The first level is the theory's mathematics, which *exists*: the mathematics is laid out whole and is the same for everyone. The second level is the theory's findings, which are *real*: each finding is given once, in an act of finding. In quantum theory the two levels have opposite characters: the findings are governed by sameness, and the mathematics by continuity. What is found of a particle is the same in every instance and comes in steps: a charge of exactly one unit, a spin of exactly one half, two spots on a plate. Quantum theory describes its findings with smooth mathematics, in which states blend into one another in any proportion. I set out the division between

findings and mathematics in a table, theory by theory. For quantum theory, the table puts two things among the findings. The two are the sharp lines of color that atoms give, and the numbers that label each kind of particle. The table puts the smoothly blending states in the mathematics. A particle's numbers are what comparisons find of the particle. Each laboratory makes its finding once, in an act of comparison, and every laboratory finds the same value. Chapter 10 takes up the larger claim that Tegmark builds on the numbers: that the world is one mathematical structure.

We now know what lies at the bottom of matter: a closed list of kinds, found in exact steps, in an arena of space and time that has no least step.

From Tegmark's pages

V. What Max Tegmark Showed About Looking

Tegmark then takes up what happens when we look. Tegmark writes that Schrödinger "abandoned the very idea that a particle has a well-defined position and velocity!" (chapter 7, "Quantum Weirdness"). Schrödinger put the *wavefunction*, a mathematical description of a particle's state, in place of a well-defined position and velocity. In Tegmark's words, the wavefunction "describes the extent to which the particle is in different places" (chapter 7, "Quantum Weirdness"). For each place, the odds of finding the particle are the square of the wavefunction's size at that place. When a particle is described as both here and there, physicists say that the particle is in a *superposition* of here and there. The textbooks then add a rule taught by Bohr and Heisenberg, called the *Copenhagen interpretation*. Tegmark describes the Copenhagen interpretation as "postulating that change is only governed by the Schrödinger equation part of the time, depending on whether an observation is taking place" (chapter 7, "The Collapse of Consensus"). Between observations, the wavefunction changes

smoothly, by Schrödinger's equation, but when someone observes, the wavefunction *collapses*: the wavefunction jumps, abruptly and at random, to the one place where the particle is found.

Tegmark sets out three faults that he sees in the collapse rule. First, the collapse rule means that nature has a fundamental randomness. Einstein refused a fundamental randomness in nature with the words "I can't believe that God plays dice" (chapter 7, "The Collapse of Consensus"). Second, on the collapse rule, observers, and so humans, are back at the center of things. Third, the collapse rule has no exact statement, since "the Copenhagen interpretation had no equation specifying what constituted an observation" (chapter 7, "The Collapse of Consensus"). Tegmark quotes Einstein's question, "Does the Moon exist because a mouse looks at it?", and adds questions of his own: "Can a robot collapse the wavefunction? What about a webcam?" (chapter 7, "The Collapse of Consensus").

Tegmark then shows that a superposition cannot be kept small. In Schrödinger's famous imagined experiment, a cat is shut in a box with one radioactive atom, a counter and some poison. Whether the atom decays fixes whether the poison is released. So, in Tegmark's words, "a seemingly innocent microsuperposition involving a single atom is amplified over time into a macrosuperposition" (chapter 7, "The Weirdness Can't Be Confined"). Small differences grow in the same way in a pencil balanced on its tip, in the weather, and in a single calcium atom at the edge of a nerve cell. Last, Tegmark asks his reader to imagine performing the cat experiment inside a closed laboratory, while Tegmark himself stands outside the laboratory. By the textbook rule, the cat is settled as alive or dead for the person inside at the moment that person looks, and is settled for Tegmark only when Tegmark walks in. So, Tegmark concludes, "at best the Copenhagen interpretation was incomplete, refusing to answer the question of when precisely the wavefunction collapsed" (chapter 7, "Quantum Confusion"). At worst, Tegmark adds, the Copenhagen interpretation was

inconsistent, since "the wavefunction of our whole Universe would never collapse from the viewpoint of someone in a parallel universe who could never observe us" (chapter 7, "Quantum Confusion"). Tegmark names his own remedy in the table of key concepts in his chapter: "Lack of wavefunction collapse implies Hugh Everett's Level III multiverse" (chapter 7, "The Collapse of Consensus"). Tegmark's next chapter takes up the remedy.

VI. The Same Results About Looking, on Separate Ground

My account has one plain answer to Einstein's question about the mouse and to Tegmark's questions about the robot and the webcam. Every interaction is a comparison. An atom that takes in light compares the light's energy with the atom's own steps of energy, and a screen that a silver atom strikes is changed in one way and not another. I use the word *comparer* for what Tegmark calls an observer. Comparers include a glass plate, a counter, an atom that takes in light, and a person. A comparer need not be a mind. A finding is any comparison that lays down a record: an interaction after which some part of the world carries a record of the outcome. A hoofprint in sand is a record in this sense, and so is the silver on Gerlach's plate. I answer Einstein's question and Tegmark's questions in one sentence: "Consciousness is not what makes anything definite; any interaction will do". The Moon is definite because sunlight, and everything near the Moon, interact with the Moon, and no one needs to look, so the world compared itself, in every collision of particles, long before anything human looked at the world.

Tegmark judges the textbook rules incomplete or inconsistent, and I agree. Quantum theory as the textbooks teach it is built on six rules, gathered from six traditions between 1906 and 1932. The six rules had later refinements in 1935 and 1951. No experiment has ever contradicted the predictions of the six rules. One rule says that a state always changes smoothly, and two others say that a measurement makes the state jump.

By the smooth rule alone, a silver atom and its plate end in a description that holds both spots at once. Yet every plate shows one spot. Physicists call the clash between the smooth rule and the jump the measurement problem. I trace the measurement problem to the way the six rules were gathered: "The measurement problem is the scar left by that assembly". The six rules, as gathered, contradict one another where the smooth change and the jump both apply.

VII. One Move, One Line

On the question of what happens when we look, Tegmark's remedy has two halves: let the smooth rule apply everywhere, and drop the collapse. I agree with the half that drops the collapse: there is no physical process of collapse. Collapse is not a physical process that destroys a superposition. I also keep the smooth rule, as the law by which the state changes between findings, but disagrees with Tegmark where Tegmark takes the smooth rule for all that happens. I keep the single outcome of each finding, because the smooth rule describes the range of possible findings, and the single outcome is given in the act of finding.

Come back to the chess game at the kitchen table, the picture that runs through this book. The rules are the same at every table, and the position on the board is the history of this game so far. The rules and the position together allow a range of legal moves, and a full list of the legal moves, however complete, marks none of them as the move that will be played. One move is played, from one seat, now. The score sheet records one line, and only one. The next range of legal moves begins from the new position that the recorded move leaves on the board.

I read quantum theory in three parts that match the three parts of the game. The range of legal moves matches the quantum state, the move matches the act of finding, and the line on the score sheet matches the record that the act of finding lays down. Tegmark describes the quantum state by the wavefunction. The quantum state is the full description of the

range: The state exists: it is the complete description of the range of possible findings of the system, and of their probabilities. Between findings, the state changes smoothly, by Schrödinger's equation, as Tegmark says the state always changes. Each finding has one outcome, given in the act of finding, and the state does not contain the outcome in advance. The outcome is real, as the move is. The jump that the textbooks call collapse is the score sheet taking its new line: the shared record of what has been found now contains the outcome, and the next comparison begins from the new record. Nothing physical jumps, and the outcome is single.

Gerlach and Stern's magnet shows how the record of one finding sets the range from which the next comparison starts. Set three magnets like Gerlach and Stern's in a row, the first and third standing upright and the second set crosswise. Keep the atoms that the first magnet sends up. The second magnet sends half of the kept atoms one way and half the other way. Then keep the atoms that the second magnet sent one way. The third magnet, upright again, sends down half of the atoms it receives, although the first magnet sent every one of them up. Each magnet is a comparison with its own setting, and each atom makes one spot at each plate. Each finding sets a new range of possible findings, and the next comparison starts from the new range.

Tegmark makes two charges against the textbook rule: that the rule is incomplete, and that the rule may be inconsistent. I answer both charges with the same three parts: the range, the move and the line. The first charge is Tegmark's question of when the collapse happens. I answer that a finding happens at every comparison that lays down a record. Findings happen at the plate, in the counter, in the air around the cat. Every comparison is made from a stance (the side from which a comparison is made) about a thing compared. The comparison actually made sets where the dividing line between the stance and the thing compared falls. In 1932 the mathematician John von Neumann showed that the dividing line between the measured system and the chain of instruments can be placed at any link of the chain. Wherever the dividing line is placed, the predicted counts come out the same. I identify the dividing line that von

Neumann moved as the boundary between the two levels of a theory, its mathematics and its findings, and not as a place among objects. A boundary between two kinds of object would stay in one place. The dividing line can be placed at any link of the chain, so its freedom to move is a reason to read the dividing line as the boundary between the two levels of a theory.

The second charge is that the textbook rule may be inconsistent. Tegmark's reason is that the wavefunction of our whole universe never collapses for someone who could never observe our universe. I answer the second charge with the same three parts with which I answer the first charge. I apply the three parts to the second charge through the case of Tegmark standing outside the laboratory. Inside the box, four things are in comparison with one another the whole time: the atom, the counter, the poison and the cat. Relative to the comparisons inside the box, the cat is alive or dead. The person inside who looks makes a comparison, and the comparison has one outcome. Tegmark, waiting outside, has made no comparison with the laboratory. So for Tegmark the laboratory is still a range of possible findings, each with its odds. When Tegmark walks in, his comparison has one outcome too, and his finding agrees with the record of the person who looked first. Each comparison has its own outcome, no comparison has two, each finding is made by the comparer that finds, and no geometry changes at either finding. When someone has made no comparison with a laboratory, that person has a description of what that person could find. The person may be near at hand, or beyond the edge of all we can see. Every finding made inside the laboratory stays as it was, whatever description the person outside has.

VIII. Hard Randomness

Tegmark finds three faults in the collapse rule, and my account has now answered two of the three. A comparer need not be a mind, so nothing in a finding puts humans at the center. The collapse rule lacked an exact statement of when a finding happens. The third fault is the random-

ness. Einstein refused a world in which God plays dice, and Tegmark lists randomness among the faults of the collapse. The randomness of a single outcome is real, and state the reason.

A tossed coin and a silver atom differ on one point. How a tossed coin lands is fixed by how the coin was thrown. The odds we give the coin measure what we do not know about the throw. A silver atom's state is known as fully as quantum theory allows. Yet which spot the atom will reach is written nowhere in the state. *Hard randomness* is an outcome that nothing before it fixed, however completely everything before it is known. Hard randomness is real in each single finding. A Geiger counter beside a lump of ore shows hard randomness plainly. Each click has a cause, a decaying atom, and nothing fixed the moment of the decay. Yet in a large number of atoms of one kind, half of the atoms decay in a time that never changes. Physicists call the time in which half of the atoms decay the *half-life* of the kind of atom. The single event is unfixed, and its odds are steady.

Hard randomness follows from the Incommensuration Theorem. If every outcome in a world were fixed in advance, the world would need one law carried the same way into every state everywhere, which is perfect sameness. A world fixed in advance would also need every state to pass into the next with no gap, which is perfect gaplessness, and the Incommensuration Theorem forbids perfect sameness and perfect gaplessness together. So every lawful description leaves some outcome that its law does not fix, and the single outcome of a quantum measurement is the plainest case of an outcome that the law does not fix. The probability of a quantum outcome is ontic in the structural sense: the residual belongs to comparison itself, and no further knowledge of the state removes it. Ontic means belonging to what is, as distinct from what someone knows. Every cause stays in place, and hard randomness means that determinism, the claim that everything is fixed in advance, is false.

Tegmark's own test of good science is data compression, and hard randomness is what is left when every pattern has been compressed. I agree with Tegmark that a law of nature is the best available compression of

what has been observed. I also state the limit of compression: "Compression of information can never be perfect; it can never reduce all observable information to zero bits, a bit being the smallest amount of information, the answer to one yes-or-no question". Squeeze every pattern out of a string of Geiger clicks, the steady half-life included, and what is left is the moment of each click, which is hard randomness.

Tegmark is right that small differences grow into large differences, and here is what grows. A single decay sets off an avalanche of charged atoms in a Geiger counter, and the avalanche ends in a click anyone can hear. The weather, and three balls striking one another on a table, magnify differences far too small for any measurement. I picture hard randomness rising into the everyday world "the way groundwater rises wherever it finds a path, through every process that amplifies what it cannot resolve". What rises into the everyday world is one outcome that nothing before it fixed, and the outcome becomes one large difference in one world: one dead cat or one living cat, one direction of a fallen pencil. The appendix "What the Coming Measurements Will Find" gives what I say each coming measurement in this chapter's field will find.

IX. What This Chapter Holds

What happens when we look is a comparison. A comparer inside the world finds one outcome, whether the comparer is a mind or not. The mathematics of quantum theory describes completely what the comparer could find. Nothing before the finding fixed which outcome the comparer finds. At the bottom of matter there is a closed list of kinds, found in exact steps. The kinds are found in an arena of space and time that has no least step. What happens when we look bears on this book's question: What is reality, and can physics find out? What happens when we look bears on this book's question because quantum theory shows the two levels of every theory, its mathematics and its findings, more plainly than anything else does. The mathematics of quantum theory is smooth, exists, and is

the same for everyone, while the findings of quantum theory are real, one at a time, each with its share of hard randomness. The score sheet records every move and never makes a move. As the score sheet does with moves, the mathematics of quantum theory describes every finding and never makes a finding. If nothing in the world collapses the wavefunction, as Tegmark and I agree, why not say that every outcome is found after all, each by a copy of me in a world of its own?



Chapter 9

Am I Splitting Into Copies of Myself?

Beside Tegmark's chapter 8, "The Level III Multiverse"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

A *comparer* is any position inside the world from which a comparison is made, such as a glass plate, a counter, an atom or a person; a comparer need not be a mind (Chapter 6).

Hard randomness: an outcome that nothing before it fixed, however completely everything before the outcome is known (Chapter 8).

I. A Note Added in Proof

If the wavefunction never collapses, am I splitting into copies of myself all the time? And if I am splitting, why does my world look single to me, and why do the outcomes I find look random? Max Tegmark raises both questions in chapter 8 of *Our Mathematical Universe*. Tegmark answers both questions with the work of one young physicist.

The physicist was Hugh Everett III, a graduate student at Princeton University, working in the Palmer Physical Laboratory there. In July 1957 the journal *Reviews of Modern Physics* printed Everett's paper "'Relative State' Formulation of Quantum Mechanics". A note on the paper's first page said that the paper was Everett's thesis, submitted to Princeton University on 1 March 1957 for his doctorate. The Princeton physicist John Wheeler wrote an assessment of Everett's paper, and the journal printed the assessment after the paper, in the same issue.

Physicists describe any piece of the world by a *wavefunction*. A wavefunction is a mathematical description from which physicists compute the odds of every possible outcome of a measurement on that piece of the world. Between measurements, the wavefunction changes smoothly and predictably, by a law called the *Schrödinger equation*. The textbooks of 1957 added a second rule, called *collapse*: at a measurement, the wavefunction jumps to the one outcome that was found. Everett kept the Schrödinger equation and dropped the rule of collapse. In Everett's paper, the wavefunction changes by the Schrödinger equation everywhere and at every time. The wavefunction is then a complete description of every isolated system, the observer included. When an observer looks, the observer becomes part of the description. Everett wrote that "the observer state 'branches' into a number of different states", one for each outcome.

Before the paper was printed, Everett had sent copies to other physicists. Some of the physicists who received copies wrote back with an argument you may be making now. The physicists who wrote back argued that

experience shows no splitting: we always find one outcome, and only one. Everett answered in a note added in proof. A note added in proof is a note set into a paper after the paper is already in type. Everett wrote that the branches have no effect on one another, so that "no observer will ever be aware of any 'splitting' process". Everett then compared the argument that no one feels the splitting to an old argument against Nicolaus Copernicus. Copernicus was the astronomer who held that the Earth moves around the Sun. The old argument was that the Earth cannot move "because we feel no such motion". Everett wrote that in both cases the argument from what we feel fails once someone shows that the theory itself predicts the experience we have. The theory of the moving Earth predicts that we feel no motion. The theory of the branching observer predicts that no observer feels the splitting.

Everett's note states both the argument that we always find one outcome and Everett's answer to that argument, an answer that depends on one inference: since the wavefunction never collapses, every outcome of a measurement occurs.

From Tegmark's pages

II. Everett's Theory, as Tegmark Tells It

Tegmark states Everett's idea in two short sentences, set apart on their own line: "The wavefunction never collapses. Ever" (chapter 8, "The Level III Multiverse"). Tegmark calls Everett's theory "Quantum Mechanics Lite" (chapter 8, "The Level III Multiverse"). Quantum Mechanics Lite is the textbook theory with the rule of collapse, and the odds that come with that rule, taken out. Tegmark holds that Everett was right.

Tegmark pictures Everett's idea with a game he calls Quantum Cards. You balance a card on a perfectly sharp edge, bet a hundred dollars that the card will fall face up, close your eyes, and wait. Under the Schrödinger equation, the card cannot stay balanced. Nothing favors

one direction of fall, so the card falls both ways at once. Physicists call a state like the card's a *superposition*: one thing described as holding two or more alternatives together, here a card fallen both to the left and to the right. When you look, the Schrödinger equation describes you as entering the superposition too. One version of you sees the card face up and smiles, and one version sees the card face down and frowns. Each version of you sees one definite card, and neither version knows of the other. Tegmark puts Everett's discovery in one line: "even if experiments didn't have definite outcomes, it would still seem as though they did!" (chapter 8, "The Level III Multiverse").

Tegmark then applies the splitting to the whole world, and not only to the card. Small differences in the world grow into large ones all the time: a ray from space alters one cell or misses it, and a small swirl of air becomes next year's storm or does not. On Everett's account, the splitting therefore never stops. The splitting has gone on since the beginning of our universe. Tegmark calls the world we see "the tip of an ontological iceberg" (chapter 8, "The Level III Multiverse"). Ontological means having to do with what there is. So Tegmark's phrase says that the world we see is a tiny part of what there is: a tiny part of everything that the one wavefunction of the whole world holds. Other versions of you and your world can be right here in space, "separated from us in what mathematicians call Hilbert space" (chapter 8, "The Level III Multiverse"), the abstract space of infinitely many dimensions in which a wavefunction is drawn. Tegmark calls the other versions the Level III parallel universes, the third of his four levels of parallel universes.

Tegmark also tells how Everett answered a famous version of the argument that no one feels the splitting. The physicist Bryce DeWitt later gave Everett's idea the name "Many Worlds". Tegmark reports, from DeWitt's own account, that DeWitt had once objected to Everett that he, DeWitt, did not feel himself splitting. Everett answered with a question: "Do you feel like you're orbiting the Sun at thirty kilometers

per second?" (chapter 8, "The Level III Multiverse"). Tegmark draws the lesson: "collapse-free quantum physics predicts both that we're splitting and that we won't feel it" (chapter 8, "The Level III Multiverse").

Tegmark also explains the randomness of quantum outcomes by the splitting. The Schrödinger equation contains nothing random, so Tegmark asks where the odds come from. Tegmark's answer is that every version of you sees one outcome that the version could not have predicted. The reason is that every other outcome happened too, to another version. Repeat the card many times. Almost every version of you then finds a patternless string of wins and losses, half of them wins and half of them losses. Tegmark adds a case with no quantum physics in it: copy a sleeper, and put the two copies in rooms numbered 0 and 1. Each copy wakes to a room number that the copy could not have predicted. Tegmark puts his conclusion in one line: "So how does it feel when you get cloned? It feels random!" (chapter 8, "The Illusion of Randomness"). On Tegmark's account, an infinite space holds many identical copies of you. For outcomes with unequal odds, Tegmark and the physicist Anthony Aguirre argue from the copies. You cannot know which of your many identical copies you are, and so, Tegmark writes, "You therefore have to consider yourself to be a random member of this group of copies" (chapter 8, "Multiverses Unified").

From Tegmark's pages

III. Why the Copies Never Meet

Everett's paper left one question for later physicists to answer, and Tegmark states the question clearly. If a large object can be in two places at once, why does no experiment ever show a large object behaving as if it were in two places at once? Two alternatives that nothing has recorded can show *interference*. Interference is the pattern of

bright and dark bands that two paths make together when nothing has marked which path was taken. Large objects in ordinary surroundings never show interference bands.

The answer to Tegmark's question is *decoherence*, and Tegmark explains decoherence clearly. Decoherence is the fading of interference between the alternatives of a thing as the surroundings of the thing come to carry records of which alternative the thing is in. When a single particle of light bounces off an object, the particle carries away a trace of where the object is, so once the trace has left the object, the two alternatives of the object no longer show bands together. On Everett's account, the alternatives are all still there. Tegmark says that the alternatives remain: "So the quantum weirdness doesn't go away, it just gets censored!" (chapter 8, "Quantum Censorship"). The trace is copied onward, from one molecule of air to the next, and in practice no one can gather the trace back. Tegmark credits the discovery of decoherence to two physicists: Dieter Zeh, who published the idea in 1970, and Wojciech Zurek, who developed the idea.

Tegmark draws two lessons from decoherence, with care. The first lesson is that a measurement needs no mind. In Tegmark's words, "quantum observation isn't about consciousness, but simply about the transfer of information" (chapter 8, "Quantum Censorship"). A bowling ball outdoors is struck by light and air so often that the ball's alternatives are recorded long before anyone sees the ball. The second lesson is that your brain is not a quantum computer. A quantum computer is a machine that computes by holding its parts in superpositions for as long as a calculation takes. Tegmark worked out how long a nerve cell could stay in a superposition of firing and not firing, and he found about ten billionths of a trillionth of a second, far shorter than any thought. Tegmark gives the reason for so short a time: "your brain is a warm and wet place whose parts aren't isolated" (chapter 8, "Why Your Brain Isn't a Quantum Computer"). Every signal from the world reaches you through the nerves of your senses. The nerves of

your senses decohere far faster than any thought, so you never perceive a superposition. Tegmark says how decoherence is related to measurement in a sentence of his own: "Decoherence is simply a measurement that you don't know the outcome of" (chapter 8, "Subject, Object and Environment").

Tegmark sees physicists' views shifting toward Everett's account, and from the shift of views he draws advice for the writers of textbooks. The *Copenhagen interpretation* is the working rule of quantum physics since the 1920s. On the Copenhagen interpretation, the measurement gives one outcome, and the wavefunction serves to compute the odds of that outcome. Tegmark advises that textbooks mention decoherence and make clear that "the Copenhagen interpretation is better thought of as the Copenhagen approximation" (chapter 8, "Shifting Views: Many Worlds or Many Words?"). Tegmark says that every theory of physics has equations and words. Many interpretations of quantum physics, he adds, differ only in their words. So, he says, when interpretations differ only in their words, the choice among them comes down to what bothers you more: "a profusion of worlds or a profusion of words" (chapter 8, "Shifting Views: Many Worlds or Many Words?"). Tegmark ends his chapter by noting that the wavefunction, and the Hilbert space in which the wavefunction lives, seem to be made of nothing but mathematics. And so, he writes, "we've found a hint that the bedrock itself is purely mathematical" (chapter 8, "Shifting Views: Many Worlds or Many Words?").

IV. Where the Two Accounts Agree

I agree with several of Tegmark's results, on ground of my own. My starting point is the *comparison*, as chapter 1 of this book set out. Comparing is telling one thing from another, or finding them the same. Every measurement is a comparison, and so is every interaction. A *finding* is

what one comparison gives. A *comparer* is any position inside the world from which a comparison is made, such as a glass plate, a counter, an atom that takes in light, or a person. A comparer need not be a mind, as this book's chapter 8 showed.

The first agreement is that collapse is not a physical process. The change of the wavefunction at a measurement is a change in the shared record of what has been found. The shared record is the same for everyone who has the outcome. So on my account, as in Everett's account and in Tegmark's, the Schrödinger equation describes every change between findings, with no exception and no second law beside the equation.

The second agreement is about decoherence itself. The definition of decoherence has a picture of its own. Think of the surroundings of a thing as a notebook in which each alternative of the thing writes: the notebook stands for the surroundings, and writing in the notebook stands for leaving a record in the surroundings. If two alternatives leave the notebook the same, the notebook holds no record of which alternative happened, and the two alternatives still show their bands. If the two alternatives leave two notebooks that differ on every page, the bands between the two alternatives vanish from every measurement made on the thing alone. The parts of the description that make the bands become out of reach for any measurement that probes the system alone, and they are not destroyed. The sentence just quoted and Tegmark's word "censored" state one claim. After decoherence, no measurement made on the thing alone can detect the bands, and the alternatives remain.

My account has a reason of its own for the fading of the bands, a reason that comes from the act of comparison. Every comparison has a content and a context, as chapter 1 of this book set out. The *content* of a comparison is what is found the same or different. When you compare two stones held one in each hand, the weight of the stones is the content of your comparison. As the surroundings interact with the thing, a comparison made on the thing has a content that spreads from the thing alone to the thing together with its surroundings. So the bands are made by parts of the description that belong to the thing and its surroundings together,

and not to the thing alone. Ordinary decoherence is made of comparisons that the surroundings make: each interaction of the surroundings that lays down a lasting record is a comparison, with one outcome. The theory of decoherence uses the Schrödinger equation alone to describe the comparisons that the surroundings make. The description shows every alternative with its share of the odds.

Physicists have watched the bands fade: in 2003 physicists in Vienna sent molecules of carbon seventy through an *interferometer*. The physicists were Klaus Hornberger, Markus Arndt, Anton Zeilinger and their colleagues. An interferometer is an apparatus that splits a beam into paths and brings the paths back together. Each molecule of carbon seventy is a closed cage of seventy carbon atoms. In a good vacuum, the molecules built up interference bands. Then the experimenters let a little gas into the chamber, and as the pressure rose, the bands faded, because each molecule of gas that struck a cage carried away a trace of the path the cage had taken. We now know that decoherence is a measured fact. Schrödinger's equation, applied to the cages and the gas together, describes the fading step by step.

The third agreement is that a finding needs no mind. I define a finding as "any comparison that lays down a record: an interaction after which some part of the world carries a record of the outcome" (chapter 17, "Every Comparison Has a Cut"). A hoofprint in sand is a record of an outcome. The sand took one shape and not another, and no one had to look. Tegmark speaks of a "transfer of information", and I speak of a record laid down by an interaction. The two phrases state one claim, reached from two starting points: a measurement needs only an interaction that leaves a record, and no mind.

The fourth agreement is about Tegmark's estimate for the brain. I agree with Tegmark's estimate of how quickly superpositions fade in the brain, as he computed it (essay on Tegmark, "The Structure and the Act", section II). The essay also says which proposal Tegmark's estimate applies to: the proposal of the anesthesiologist Stuart Hameroff and the physicist Roger Penrose that consciousness rests on superpositions held whole for a

long time inside nerve cells. On my account, the undecided part of a brain's choice lies in *sensitive dependence near criticality*, and not in superpositions held for a long time. Criticality is a state in which small disturbances cascade. A brain near criticality is poised like a pile of sand at the point where one more grain can set off an avalanche of any size, so a tiny difference in the brain can grow into a different choice.

The fifth agreement is about how a superposition ends. Some physicists, Roger Penrose among them, have proposed that a superposition of something heavy enough, or spread far enough, ends by itself, with nothing around the thing to record which alternative is the case. On Tegmark's account and on mine, a superposition left to itself lasts, however large the mass, however far apart its alternatives and however long the time. On both accounts, too, the only decoherence is the ordinary kind that the surroundings make. I list Tegmark among the physicists who agree with me that a superposition left to itself lasts (section "Does a Superposition End of Its Own Accord?").

Tegmark's account and mine also agree on the odds of every outcome. Physicists compute the odds by squaring the size of the wavefunction. Physicists call the rule of squares the *Born rule*. The derivation has two parts: why each outcome has one stable weight, and why the weights of outcomes that exclude one another add; two theorems of mathematics then give the squares. I derive the Born rule from the act of comparison, by an argument of my own in two parts. The first part of the argument shows why each outcome has steady odds. Prepare a thousand silver atoms in the same way, each with its tiny magnet pointing sideways, and send the atoms one after another through the upright magnet of Walther Gerlach and Otto Stern from this book's chapter 8, which sends each atom to one of two spots on a plate. About five hundred atoms land in the upper spot, and about five hundred land in the lower spot. Send another thousand, prepared the same way, and the shares of the two spots come out the same again. When a comparison is made again on the same state

in the same context, it is the same comparison, so the outcomes of the comparison recur with one stable frequency. I identify the probability of an outcome with the stable frequency of the outcome.

The second part of the argument shows why the odds are squares. The squares follow, by two theorems of mathematics, from two conditions on a rule for the odds. Physicists call the share of the odds that a rule assigns to an outcome the *weight* of the outcome. For the thousand atoms, the weight of each spot is one half. One half is the share to which the count of atoms in each spot settles. The first condition is that each outcome gets one weight, whichever measurement the outcome is found in. The second condition is that the weights of outcomes that exclude one another add up: the count of atoms in "one spot or the other" is the count in the first spot plus the count in the second. I derive the second condition, the adding up, by two routes. The first route starts from the range of possible findings of a comparison: the range divides into regions, one region for each outcome, with nothing left over. So the weight of "one or the other" is the weight of the first region plus the weight of the second, because any extra share would belong to something found in neither region, and there is no such thing. The second route is the count of repeated outcomes. No atom lands in both spots, so the two counts simply add, as the thousand atoms show. For the first condition, one weight to each outcome, I make a second identification. An identification is a step that reads something in the structure of comparison as something in the physical world. The first identification was that probability is the stable frequency of an outcome. The second identification is that the same content found in two contexts is the same finding, with one weight, as when one outcome is found in two different measurements that both contain the outcome. In 1957 the mathematician Andrew Gleason proved the first of the two theorems. Where a thing's measurements can have three or more distinct outcomes, every rule that satisfies the two conditions sets all its weights from the state of the thing by one fixed recipe, the recipe of quantum mechanics. In 2003 the physicist Paul Busch proved the second theorem. Busch's theorem gives the same result for a thing whose measurements can have any num-

ber of distinct outcomes, two included, as the silver atom's measurements can. Busch's theorem needs the two conditions to hold also for the outcomes of *unsharp* measurements. An unsharp measurement is one whose outcomes do not cut the possible states cleanly apart. A detector makes an unsharp measurement when it answers "up" nine times in ten for an atom that is up, and one time in ten for an atom that is down. Two steps of my account apply to unsharp measurements too. The two steps are the first route to the adding up, and the identification of one weight for one outcome. Both steps concern the outcomes of any comparison, sharp or unsharp. A thing is in one definite state when a single wavefunction describes the thing, and not a blend of several wavefunctions each taken with a share. For a thing in one definite state, the recipe of quantum mechanics is the rule of squares, the Born rule. The appendix "For the Specialist" states the two theorems more fully. So I derive the Born rule as the only rule, with no rule for probability assumed at the start.

V. "But I Don't Feel Myself Splitting"

You may now raise again the argument from the start of this chapter, in your own words: "But I don't feel myself splitting". Everett and Tegmark answer that Everett's theory predicts you will not feel the splitting, as Copernicus's theory, joined to Isaac Newton's theory of motion, predicts that you will not feel the Earth move. I give a different answer, and a shorter one. You do not feel yourself splitting because each finding you make has one outcome.

Tegmark's account and mine agree on decoherence, and they differ on what happens after decoherence, when you look. Tegmark himself says that when the surroundings record where an object is, nothing has collapsed, and every alternative is still there. I agree with Tegmark that after decoherence the whole, the thing together with its surroundings, is still a sum of alternatives, each alternative with its own record written into the air and the light. When you look, your finding settles which of the records

you see. Decoherence does not say which record you will see. So the two accounts differ on one question, what happens when you look: whether a finding is one outcome, given in an act, or every outcome at once, each in a branch.

The chessboard picture of this book shows the inference from "the wavefunction never collapses" to "every outcome occurs". In the picture, two people play chess at a kitchen table. The position on the board allows a range of legal moves. One move is played, and the score sheet records one line, as this book's chapter 8 showed. Now suppose that every legal move were played, each on a board of its own. The score sheet would then have no single next line. Every line would be written together, and no line would be the one move played at this table, once, now. The move itself would be gone: the act of playing one line out of the many lines that the position allows.

The chessboard picture matches my account of Everett's inference, part for part. The wavefunction, with all its alternatives, exists: the wavefunction is structure, laid out whole and the same for everyone who reads it, as the score sheet is. A finding is real: a finding is given in one act of comparison, made once, as a move is. For every comparer, one outcome is real, the one found. The other branches are never given in any act of that comparer, and the many-worlds account takes the structure that exists for the whole of what is real.

I first credit Everett's account: "Its achievement is economy and reach". Everett's account has one smooth law for everything, applied without change to the whole universe, and with decoherence added, Everett's account describes a definite pattern of nearly classical branches. Each nearly classical branch behaves almost as the everyday world does. The section "Many Worlds" then names two places where Everett's account needs the act of finding, the act that Everett's account had set aside. First, Everett's account needs a choice of which outcomes count as having occurred, to say which alternatives count as the branches. Second, Everett's account needs someone who faces the outcomes and weighs them, to give each branch its odds. Choosing among outcomes belongs to the act of finding.

So in both places the argument of Everett's account uses the act of finding again, as a premise. In my language, the immanent is the act itself, the comparing as it happens. Many worlds cannot derive the Born rule because it drops the immanent.

My account has a second reason against Everett's inference, and the second reason rests on the Incommensuration Theorem, which chapter 1 of this book stated and chapter 10 sets out step by step. Continuity is having no gaps. A whole world fixed completely by one law would need perfect sameness and perfect gaplessness together: one law carrying every state into the next, the same everywhere and always, and every state fixed with no gap left anywhere. I name Everett's many-worlds account among the accounts that describe a whole world as completely fixed by one law (chapter 8, section VII). A whole world completely fixed by a law perfects symmetry and continuity together, whichever account describes it, and the theorem forbids that pairing. The Schrödinger equation is consistent with the theorem, because the equation fixes the odds of findings and does not fix any single finding. The theorem shows that one further claim is false: the claim that the Schrödinger equation, with nothing beside it, fixes the whole world.

VI. Randomness That Nothing Fixed

Tegmark's sleeper in room 0 shows exactly what kind of unpredictability copying produces. The copy in room 0 wakes, reads its room number, and could not have predicted the number. Yet the room number was settled before the copy woke: both rooms were there, each with its copy, and the copy lacked only the knowledge of which room was its own. I call the copy's kind of unpredictability *soft randomness* (chapter 8). Soft randomness is unpredictability that comes from a limit of what someone knows, and that better knowledge could shrink. The toss of a coin is ran-

dom in the soft way, and so is the top card of a freshly shuffled deck: there is, or might be, a fact that fixes each outcome in advance, and we lack access to that fact.

Hard randomness is an outcome that nothing before it fixed (this book's chapter 8 gave the definition). Hard randomness is real in this book's sense, given in each single finding. The single outcome of a quantum measurement is the plainest case of hard randomness. When the state of the measured thing is known as fully as quantum theory allows, the state gives the odds of each outcome, and the outcome found is not contained in the state. The copying picture reproduces the odds of quantum outcomes so well for a reason that I give about all randomness (chapter 8, "What Only a Stance Can Tell Apart"). When randomness is seen from outside, by a description that holds no one's point of view, hard and soft randomness "cannot be told apart. Both produce the same statistics". Hard randomness gives a finding that is new, and soft randomness gives a finding that was settled before and only hidden. The comparer that makes a finding is the only one that can register whether the finding is hard or soft, and the comparer registers the difference from its *stance*, the side from which a comparison is made. So Tegmark's copies reproduce the statistics of quantum outcomes exactly. The statistics alone do not show which kind of randomness is at work.

Tegmark explains the odds by self-location among copies, so the randomness in his account is the soft kind. Self-location is working out where you are among many places laid out at once. Each copy lacks knowledge of which of the laid-out places is its own. Self-location among the branches is an answer of the same kind for the odds of Everett's branches, and the philosopher Charles Sebens and the physicist Sean Carroll published that answer in 2018. On the answer of Sebens and Carroll, the uncertainty is ignorance of which laid-out branch is one's own. A quantum outcome is hard randomness. Nothing laid out beforehand contains the outcome, and the outcome is given in the act of finding. So the randomness of a quantum measurement is hard randomness, and hard randomness is not how being copied feels.

I also describe a pattern that runs through every reading (every interpretation) of quantum theory (chapter 8, "Three That Come Together"). In every reading, three things are kept together or dropped together: the stance from which a comparison is made, the one-way order of time, and outcomes that nothing before them fixed. The Copenhagen reading has all three: an observer (the stance), a single outcome by chance, and a measurement that cannot be undone (the one-way order of time). The many-worlds reading lacks all three together: the stance, the outcomes that nothing fixed, and the one-way order of time. In the many-worlds reading, there is no fact of which branch a comparer is in, every outcome is fixed, and the basic equation runs the same forward and backward. When a film of a rolling ball played backward shows a motion an equation allows, the equation runs the same forward and backward. I call the three things, bound into one act, the *bound triple*. Chapter 12 of this book sets out the bound triple in full.

In one experiment, physicists placed large masses according to the outcome of a quantum event. Tegmark himself suggests, in a note to his game of Quantum Cards, nudging the card with "a quantum device such as Schrödinger's radioactive-atom trigger" (chapter 8, "The Level III Multiverse"). In 1981 the physicists Don Page and C. D. Geilker built a device of the kind Tegmark suggests, around a *torsion balance*: a light bar hung from a thin fiber, with small masses at its ends, which the pull of large masses set beside the bar twists a little. Before each run, a counter registering radioactive decays chose between two placements of the large masses, one placement that would twist the bar one way and one that would twist the bar the other way. The bar twisted on every run, each time in the direction set by the placement the counter had chosen. We now know that the gravity the bar felt followed the outcome of the quantum event, and not an average of the two possible placements. So the counter found a single quantum outcome, and that outcome settled where the large masses stood and which way their gravity twisted the bar. The experiment rules out one picture of gravity: a single curved geometry of space and time for both placements together, shaped by their average, with the state of the masses

never cut down to the outcome the counter found. I make an identification that the experiment supports: each alternative of the matter has its own geometry, and on each run the counter's finding selects among alternatives that already have their geometries.

Tegmark's account and mine predict the same result on every test of whether a superposition ends by itself, and on every test of the odds of quantum outcomes. The two accounts differ on what a finding is. Each test of either kind is itself a finding, one outcome given in one act. I settle what a finding is by argument: a finding is an act, made once, with one outcome, and an account that contains only the laid-out structure does not contain the act. The appendix "What the Coming Measurements Will Find" states the results that I predict for the coming tests of both kinds.

VII. What This Chapter Holds

Each finding you make has one outcome, given in an act made once, and so you are not splitting into copies of yourself. The randomness of each outcome you find is hard randomness. Hard randomness is real, given in the finding itself, and hard randomness is not how being copied feels.

I agree with Tegmark on the physics of his chapter: no collapse as a physical process, decoherence as he describes it, no mind needed for a measurement, and the same odds in every experiment. I differ from Tegmark at one inference, the inference from "the wavefunction never collapses" to "every outcome occurs". I give two reasons: the inference leaves out the act in which one outcome is found, and the inference describes a whole world as completely fixed by one law. Everett's inference matters for the question of this book: What is reality, and can physics find out? Everett and Tegmark take the wavefunction for the whole of what is real. The wavefunction is mathematics, laid out whole like a score sheet, and Tegmark takes the purely mathematical nature of the wavefunction as his

hint about the bedrock of the world, its most basic level. Is the world, then, only described by mathematics, or is the world mathematics, and you with it?



Chapter 10

Is the World Made of Mathematics?

Beside Tegmark's chapters 9 and 10, "Internal Reality, External Reality and Consensus Reality" and "Physical Reality and Mathematical Reality"

What this chapter uses

What *exists* is laid out whole, timeless and the same for everyone, as the list on a score sheet is; what is *real* is given in an act of finding, once, as a move is made (Chapter 1).

The *stance* is the side from which a comparison is made, as a move is made from a seat; every comparison has a stance, human or not (Chapter 1).

Hard randomness: an outcome that nothing before it fixed, however completely everything before the outcome is known (Chapter 8).

I. A Game Played Once in London

Is the world only described by mathematics, or is the world mathematics, with you inside it as one more piece of mathematics? Why, too, does mathematics describe the world so well? Max Tegmark answers that the world is mathematics, and he illustrates his answer with a game of chess. I take Tegmark's chess game and add what Tegmark leaves out of his picture of the game: the players at the board and the moves they make.

In London in 1851 the chess players Adolf Anderssen and Lionel Kieseritzky sat down to a game. The year 1851 was the year of the first international chess tournament. Anderssen gave up a bishop, then both of his rooks, then his queen, and he won the game with the three small pieces he had left. Kieseritzky lost, and he published the moves himself in the French chess magazine *La Régence* in July 1851, with praise for the winner. Some years later chess players began to call the game the Immortal Game, and players have been setting up their boards and replaying the moves of the Immortal Game ever since.

Tegmark uses the Immortal Game to show what is left of a thing when everything inessential is stripped away. He writes: "Here on Earth, this game was first played in 1851 by Adolf Anderssen and Lionel Kieseritzky" (chapter 10, "What Is a Mathematical Structure?"). He notes that the Immortal Game is replayed every year in Marostica, in Italy, by people dressed as the pieces, and by players everywhere with pieces of wood, marble or plastic, on wooden boards and on screens. Take away the wood, the marble, the costumes and even the letters of the printed notation, and what remains, in his words, is "The Immortal Game itself, 100% pure, with no additives" (chapter 10, "What Is a Mathematical Structure?"). The pure game is Tegmark's worked example of a mathematical structure.

Now bring the game to the kitchen table that this book has pictured since chapter 1, where two people play chess and a score sheet, the written list of the game's moves, lies beside the board. The score sheet of the Immortal Game exists, in this book's sense of the word: the list of moves is the same whether the game is played in wood or in marble or written in notation, and anyone can replay the list at any hour. The London game of 1851 happened once, in one room and from two seats: Anderssen sat in one seat, and Kieseritzky in the other. At the kitchen table, Tegmark's question is whether the world is the score sheet. My answer has two parts. Mathematics lays out the score sheet of the world exactly. The world that physics finds is made of moves, each made once, from a seat.

From Tegmark's pages

II. Three Ways of Holding One World

In his chapter 9 Tegmark describes an experiment you can do now. Swing your head from left to right, and the room seems to move, but keep your head still and move only your eyes, and the room stays where it is, though your eyes swept across the room both times. Tegmark draws a conclusion from the difference between moving the head and moving only the eyes: "what your mind's eye is looking at isn't the external reality, but a reality model stored in your brain!" (chapter 9, "External Reality and Internal Reality"). Tegmark calls the reality model in your brain your *internal reality*: the world as you perceive it from inside your own mind. Tegmark places a *consensus reality* between the internal reality and the world: the shared description that observers agree on, which gives the positions and motions of things in three dimensions. Scientists state the consensus reality in measured terms, such as a wavelength of 600 nanometers in place of the word orange. Tegmark writes that the consensus reality is "by definition free from illusions that are unique to biological minds" (chapter 9, "Consensus Reality"), so physics can study the consensus reality without first explaining how a human mind works. Tegmark places his *external reality* beyond the internal reality and the consensus reality. His table of terms for the chapter describes the external reality as the physical world that he believes would exist even if humans did not.

Tegmark draws two lessons from the three realities. His first lesson is that the question of what reality ultimately is splits into two tasks: the task of physics is to derive the consensus reality from the external reality, and the task of cognitive science is to derive the internal reality from the consensus reality. His second lesson comes from the history of physics. Tegmark looks at special relativity, at general relativity and at quantum mechanics, and in each he finds that "the hardest part wasn't doing the math, but understanding how these two realities are

related" (chapter 9, "Physics: Linking External Reality to Consensus Reality"). The two realities in Tegmark's sentence are the external reality and the consensus reality. Tegmark opens the closing summary of his chapter 9 with one plain sentence: "although there's only one true reality, there are several complementary perspectives on it" (chapter 9, "The Bottom Line").

From Tegmark's pages

III. From a World Without Us to a World of Mathematics

Tegmark gathers clues to how mathematical nature is in "Math, Math Everywhere!", the first section of his chapter 10. Throw a pebble, and its path is a parabola, the curve Galileo found for everything thrown. Follow a comet around the Sun, and its path is an ellipse. Numbers also appear in nature. Some are whole numbers, such as the three directions of space. Others are pure numbers with no unit in them, such as the mass of the proton divided by the mass of the electron, about 1836. Tegmark counts 32 fundamental pure numbers, and he holds that every other constant ever measured can in principle be computed from the 32. He finds that "the more carefully we look, the more math we seem to find" (chapter 10, "Math, Math Everywhere!"). In 1960 the physicist Eugene Wigner asked why mathematics works so well in the natural sciences, and Tegmark answers Wigner: "I agree with Wigner that it demands an explanation" (chapter 10, "Math, Math Everywhere!").

Tegmark's explanation is an argument in two stated points. He starts from a hypothesis he calls the External Reality Hypothesis: "There exists an external physical reality completely independent of us humans" (chapter 10, "The Mathematical Universe Hypothesis"). Tegmark's first point concerns a complete description of the external physical reality. A complete description of the external physical reality must make sense to beings who share none of our concepts, such as

aliens or computers. So a complete description must be free of what Tegmark calls *baggage*: the words humans invent for convenience, such as the words particle, star and observation. He concludes that a complete description with all the baggage removed "must contain no concepts at all!" (chapter 10, "The Mathematical Universe Hypothesis"). His second point rests on a definition from mathematical logic. When a description keeps nothing but entities and the relations between them, a modern logician calls the description a *mathematical structure*. Tegmark defines a mathematical structure as a "Set of abstract entities with relations between them" (chapter 10, "What Is a Mathematical Structure?"). Together, the two points say that a complete description free of baggage keeps only entities and their relations, and so is a mathematical structure. Tegmark therefore argues that the External Reality Hypothesis implies the Mathematical Universe Hypothesis, "with a sufficiently broad definition of mathematical structure" (chapter 10, "The Mathematical Universe Hypothesis"). The Mathematical Universe Hypothesis reads: "Our external physical reality is a mathematical structure" (chapter 10, "The Mathematical Universe Hypothesis"). On Tegmark's account we all live in "a gigantic mathematical object" (chapter 10, "The Mathematical Universe Hypothesis"), and you are part of that object. Tegmark sets the argument out so that a reader can check each point separately: the first point rests on the External Reality Hypothesis, and the second point rests on the definition of a mathematical structure.

Tegmark then adds a definition of equivalence, and he uses the definition to pass from the claim that the world is described by a mathematical structure to the claim that the world is a mathematical structure. When a correspondence between two descriptions keeps every relation, Tegmark calls the two descriptions *equivalent*. A correspondence between two descriptions pairs each entity of one description with an entity of the other. On Tegmark's account, equivalent descriptions describe one and the same thing, and whatever differs between them is

baggage, so that "the structure corresponds to the class of all equivalent descriptions of it" (chapter 10, "What Is a Mathematical Structure?"). In his chapter 11 Tegmark draws the consequence: if one set of equations completely describes both the physical world and a mathematical structure, the physical world and the structure are one. Of the physical world he says, "I'm arguing that it is mathematics (more specifically, a mathematical structure)" (chapter 11, "How Can Physical Reality Be Mathematical?").

IV. Where the Two Accounts Meet

I agree with many of Tegmark's results. My account has grounds of its own for each result it shares with Tegmark. An interaction is anything that happens between two things as each acts on the other. A *finding* is what one comparison gives.

I hold the External Reality Hypothesis in its plain sense: there is a world, and the world does not depend on us. My ground for holding the hypothesis is that a stone striking a stone compares as surely as a laboratory does. Whatever makes a comparison is a *comparer*: any position inside the world from which a comparison is made. A comparer need not be a mind. Tegmark's External Reality Hypothesis is a claim the collection holds on a ground of its own. I draw one more of Tegmark's results from the stone striking a stone. The result is that physics can find out about the world without first explaining the human mind, because the observer of physics was never a human mind.

I agree with Tegmark on one more point, which the close of this chapter relies on: a truth of mathematics holds before anyone proves it. Chapter 7 met Kepler's five regular solids, the solids whose faces are all one regular shape. Tegmark's aliens could study shapes with flat faces all alike and give the five regular solids their own names, but "they can't invent a sixth one" (chapter 10, "The Mathematical Universe Hypothesis"). I give the same agreement for the prime numbers. The prime numbers are

the whole numbers above one that no whole number divides except one and the number itself. The truth that the primes never end does not wait on the day it was found, as the number 31 did not become prime when someone first divided it.

V. Three Realities Beside Three Words

My account has three words for three aspects of one world: exist, real and objective. When the three words are set beside Tegmark's three realities, the three words show how much the two accounts share, and the one claim on which the two accounts part. My book on what it is to compare, the three words have exact and different meanings. To *exist* is to stand as structure: laid out whole, timeless, the same for everyone who looks, like the pattern of a law or the list on a score sheet. To be *real* is to be given in an actual act of comparison: here, now, once, like a move. To be *objective* is to be found the same by many comparisons made from many places. Objectivity is built from many views, and it comes in degrees. The three words differ by a count: existence involves no act of comparison, the real involves exactly one, and objectivity involves many. The word reality needs a warning. Tegmark uses the word reality both for the physical world itself and for each of his three ways of holding the world, while this book uses the word real only for what is given in an act of finding.

I set Tegmark's three realities beside my three words as follows. Tegmark's external reality is the physical world as it would be with no humans in it. The external reality is the one world, and I hold that there is a world independent of humans. The one world has all three aspects together. My words there are close to Tegmark's own words about complementary perspectives: "The structural, the processual, and the convergent are three aspects of one world, seen from three perspectives that cannot be collapsed into one". The structural is what exists, the processual is what is real, and the convergent is what is objective. Of Tegmark's three realities, the consensus reality is the closest to what I call objectivity. Tegmark gives

the picture himself: a seal, a bird, a bee and a robot vacuum cleaner each perceive a room differently, "but we all agree on whether the door is open" (chapter 9, "Consensus Reality"). The word objectivity names the agreement of the seal, the bird, the bee and the robot about the door. One aphorism of *An Immanent Metaphysics* uses Tegmark's very phrase: many levels of interaction are needed "to establish a consensus reality/world". The same aphorism says that "correspondences of subjective experience" across the levels of interaction "are as much inherently a part of the foundation" of objectivity as any material aspect. In plain words, experience at one level of interaction corresponds to experience at another level, and the correspondences between levels belong, by their nature, to the ground of objectivity as fully as any material aspect of the world does. Tegmark's internal reality is the model in your brain. On my account the model is a summary laid out for use, like the dashboard of a car, which is Tegmark's own image. What I call real is the act of perceiving itself: the comparison your eyes and brain make, here, now and once. So the word real is the only one of my three words that matches none of Tegmark's three realities: real is the word for the move.

The two accounts part on one claim. The claim is that physics, by gathering many views, can arrive at a description of the world from no observer's point of view. Tegmark's plan for physics is to start from the consensus reality and find the external reality as it is seen from what he calls the bird perspective, which he describes as the view "of a mathematician studying the equations describing it" (chapter 9, "Consensus Reality", Figure 9.2). In the bird perspective every observer is one part of the mathematical structure, and the view is taken from no observer's place. The *stance* of a comparison is the side from which the comparison is made. Every comparison has a stance, human or not. Gathering many views makes a description ever more objective. Every description gathered from many views still keeps a stance: gathering more views never yields a view with no stance in it, because every view gathered is a view from somewhere. The Incommensuration Theorem shows why a description with the stance taken away is never complete.

VI. The Incommensuration Theorem, Step by Step

Try to draw the whole surface of the Earth on one flat sheet of paper. You want the map unbroken, so that places near each other on the globe are near each other on the sheet, and you also want the map the same everywhere, keeping true distances from edge to edge. Peel an orange and press the peel flat, and you see what happens: the peel splits, or stretches, or does both. The mapmaker must choose where the map gives way, by splitting or by stretching. The Incommensuration Theorem shows that every act of comparison contains a choice of the same kind as the mapmaker's. The theorem rests on nothing but what a comparison is.

Picture a harbor, and on its wall a *tide gauge*: a post with a marked scale, against which the height of the sea is read. At noon the harbor master reads the water at one height, and at midnight at another, and so compares the sea at noon with the sea at midnight. The result of every comparison is a verdict: same or different. The tide gauge shows the two things that every verdict needs: a content and a context. What the verdict is about is the *content* of the comparison: here, the height of the water. The *context* of the comparison is the respect in which the two things compared are two: here, the two readings are two because the harbor master takes them at two moments, noon and midnight. In other comparisons the context is two places, or two possibilities.

When the comparison registers no difference in the respect that makes the two things two, the verdict on the context is "same", as when two readings of the gauge are taken so close together that the comparison registers no difference in time. When the comparison registers the two readings at two moments, as at noon and at midnight, the verdict on the context is "different". The two verdicts combine in four ways. Each way is a *character* that a comparison can find, and physics already has a name for each character. *Continuity* is the same content where the context is the same: nothing jumps. A dimmer shows continuity: the less the knob

moves, the less the light changes. At every point of the knob's travel the light changes smoothly and never leaps. *Discontinuity* is a different content where the context is the same: something jumps, as a light switch goes from off to on at one point of its throw, or a detector clicks. *Symmetry* is the same content where the context differs: something stays unchanged across a change, as a meter stick gives the same length in every country and every year. *Asymmetry* is a different content where the context differs: nothing repeats, as no stretch of a coastline repeats another.

Can one comparison hold two of these four characters exactly, both at once, and not only nearly? A pairing of two characters gives four verdicts: each character gives one verdict on the content and one verdict on the context. The four characters can be paired in ten ways, counting each character paired with itself. With three tests, anyone can decide which of the ten pairings one comparison can hold. The first test removes four pairings: a character paired with itself says nothing new. The second test: a character paired with its direct opposite contradicts itself. Continuity says "same" and discontinuity says "different" of the content within one unchanged context. Together, continuity and discontinuity say both "same" and "different" of one and the same case. Symmetry says "same" and asymmetry says "different" of the content across one change of context, so together symmetry and asymmetry also say both "same" and "different" of one and the same case. The second test therefore removes two pairings: the pairing of continuity and discontinuity, and the pairing of symmetry and asymmetry.

The third test rests on a fact found in every comparison: sameness and difference come together. A comparison that found only sameness would be no comparison at all, since the two things would be one thing. A comparison that found only difference would connect nothing, since the two things would share not even a respect in which to be compared. So every comparison finds sameness and difference together. A pairing gives its verdicts in two respects, the content and the context. A pairing must therefore find one "same" and one "different" among its two verdicts on the content, and likewise among its two verdicts on the context. Continu-

ity with symmetry fails the third test. Return to the tide gauge: continuity says the same height at the same moment, and symmetry says the same height at different moments. A gauge that held continuity and symmetry both perfectly would give two verdicts on the height, "same" and "same", and "different" never. Discontinuity with asymmetry fails the third test in the opposite way: its two verdicts on the content are "different" and "different", and "same" never.

Two pairings survive the three tests. Continuity with asymmetry holds: its verdicts on the content are "same" and "different", and its verdicts on the context are "same" and "different" as well. Picture a walk along a shore: step by step there is no jump, and no stretch of the shore repeats another stretch. Symmetry with discontinuity holds: its verdicts on the content are "same" and "different", and its verdicts on the context are "different" and "same". Picture a ladder: every rung is the same as the last, and there is nothing to stand on between the rungs.

The proof consists of ten cases and three tests. The proof uses nothing but the meanings of same and different and of content and context, and the fact that sameness and difference come together. The theorem is settled as a count is settled. Anyone can make the count of the ten cases, and the count comes out the same each time. Chapter 1 named the *intrinsics* of comparison: the parts that every comparison has. Among the intrinsics are a comparison's two verdicts, same and different, and its content and its context. Every other result about knowing is found by knowing, in acts that already have the theorem in them, and so the theorem is the deepest and most powerful epistemic basis possible.

Where symmetry and continuity are both approximate, the theorem allows the two together. A ripple spreading on a still pond is one case: the ripple is smooth and nearly the same in every direction. Where symmetry and continuity belong to two levels of one theory, the theorem allows the two together as well. Every physical theory has two levels: its mathematics, written down as structure, and its findings, made by comparison, such

as a click in a detector or a reading on a dial. The theorem concerns what a comparison finds. So symmetry and continuity can both be perfect together in the mathematics of a theory, while no finding of the theory ever shows both.

VII. Why the World Is Not a Mathematical Structure

Chapter 1 drew one consequence of the theorem from the circle of geometry, and now that the four characters (continuity, discontinuity, symmetry and asymmetry) are defined, you can check each step of the consequence. Mathematics can write the circle down because mathematics, taken as a set of objects that simply are, contains no one who compares and no time. Temporal comparison is comparison of one moment with another. In a domain from which temporal comparison is removed, symmetry and continuity can be written down together. Every finding of physics is a comparison made at some moment, after one moment and before another. So the theorem applies to every finding. A mathematical structure, taken as a whole, is laid out at once and is the same in every instance, and so the structure as a whole holds the perfect sameness and the gapless completeness that no single act of comparison can hold together. So the world that comparisons find, the only world physics has, is not a mathematical structure, and a mathematical description of it, however exact, is never complete.

The score sheet at the kitchen table shows the difference between a mathematical structure and the world that comparisons find. The score sheet of a chess game can be replayed by anyone, at any time, forever. The game itself happened once, on one evening, between two people. Tegmark's pure Immortal Game, with the wood and the marble taken away, is the score sheet, and the score sheet exists: it is laid out whole, and it is the same for everyone who reads it. The game in London was the moves,

made once, from two seats. Each move was a finding: Anderssen looked at the position on the board and found his sacrifice. On Tegmark's hypothesis, the mathematics is the whole of the world.

I accept the External Reality Hypothesis, and I part from Tegmark at one premise of his argument. Tegmark's argument says three things. The first premise is that a complete description of a world independent of humans must contain nothing human. The second premise is that a complete description with nothing human in it is pure structure. The conclusion is that the world is pure structure. I accept the first premise. I part from Tegmark at the second premise, because a stance is not a human. Removing every human notion from a description removes nothing of the stance, because the stance was never human. A collision of two particles is a comparison, and so the collision has a stance as surely as a measurement in a laboratory has a stance, whether or not anything human is near. At the table, take away the players' names, their languages, their nations and the wood of the pieces, and two seats remain, because a move is always made from a seat. So even where a complete description has nothing human in it, the description still describes a world in which comparisons are made from stances: the description lays out what each comparison finds, and leaves unnamed the stance from which each comparison is made. A stance written into a structure as one more part of it is laid out with the rest of the structure, as a seat drawn on the score sheet is on the sheet; the move is still made from the chair.

"But physics really is all equations", you may say. In practice, a physicist computes a number from the equations and then makes a measurement. A prediction is the comparison of the computed number with the measured number. The two levels of a theory, its mathematics and its findings, differ exactly: no amount of calculation makes a finding, and no finding is a calculation. The equations are the score sheet of physics, and the click in the detector is the move. Physics has always had both the equations and the clicks, and in Tegmark's own history of relativity and

the quantum the hardest part was relating the mathematics to the measurements, so his history is the history of joining the equations to the measurements.

Tegmark infers from his rule of equivalence that the world is one and the same as the mathematical structure its equations describe. I read Tegmark's rule of equivalence as one case of a principle that I hold, the principle of identity. The principle of identity says that which is indistinguishable must be the same, and so two things are one thing only where nothing can tell the two apart. A description and the act it describes are two different kinds of thing. Structures model interactions; they do not become them. A description can be told apart from the act it describes. So the principle of identity does not make a description and the act it describes one thing. Tegmark answers one more argument himself, an argument that someone who rejects the External Reality Hypothesis could make. The argument says that the world is made of stuff that a mathematical structure describes perfectly, and that the stuff also has further properties, which the structure does not describe and which cannot be described in an abstract way, free of human baggage. On the argument's own terms the mathematical description is perfect and accounts for everything that can be observed, so the further properties would have no observable effect. Tegmark writes that "those additional bells and whistles that would make our Universe nonmathematical would by definition have no observable effects whatsoever, rendering them 100% unscientific" (chapter 11, "How Can Physical Reality Be Mathematical?"). I agree with the principle Tegmark relies on: what has no observable effect is no part of science. What no comparison could ever reach can be the basis of no assertion, in physics or out of it. The mathematical description leaves out the stance and the act. The stance and the act are found in every comparison: there is no comparison made from nowhere, since a comparison is an interaction, and an interaction has locations. Every observation ever made has the act and the stance in it. So the act and the stance are not hidden extras like Tegmark's bells and whistles.

VIII. The Order of Depth, and Why Mathematics Works So Well

Picture a kitchen on a Sunday afternoon. A cook tastes two spoons of honey and finds one sweeter. A cake is baking in the oven. A recipe lies on the counter, a list of steps in order. A photograph of last week's cake hangs on the wall, showing the whole cake at once. Ask, of each pair of the four things in the kitchen, which one can be had without the other. The photograph needs a cake, and a cake can be had without a photograph. The photograph also needs steps taken one after another by fixed rules. The steps that make a photograph are a recipe of their own kind: the shutter opens and closes, and the film is developed and printed. A recipe can be followed with no photograph taken. But nothing is baked from a recipe until hands follow the recipe in an oven that heats. The baking is made of things meeting other things and making a difference to them. When a thing meets another thing and makes a difference to it, the meeting is a comparison. The cook's tasting is an act of comparison, and the tasting needs nothing else to be complete.

The four things in the kitchen are pictures of four disciplines. I set the four disciplines in what it calls *the order of depth*: metaphysics, then physics, then computation, then mathematics. When a second discipline needs a first discipline in order to be carried out, stated or checked, and the first can be carried out without the second, the first discipline is deeper than the second. In the order of depth, metaphysics is the account of the act of comparison itself, and the cook's tasting is the picture of metaphysics. Physics is the study of acts of comparison as they happen in the world, and the baking is the picture of physics. Computation is a sequence of steps carried out by a fixed rule, and the recipe is the picture of computation. Mathematics is the study of structure held in view as a whole, and the photograph is the picture of mathematics. Each discipline in the order rests on the one before it. Mathematics rests on computation, because

every proof is checked by a run of steps, each line compared against the rules and against the lines before it. Computation rests on physics, because every run is a physical process in circuits, in neurons or in ink. Physics rests on the act of comparison, because every finding of physics is a comparison made from inside the world.

The photograph is the most exact record in the kitchen, and the photograph is the last thing in the order: depth is the order of what rests on what, and depth is a different thing from certainty or precision. Mathematics keeps everything it has. Mathematics is last in the order, with every power it has, as the finest instrument there is for what is laid out as structure. In Tegmark's hypothesis the whole order is reversed: mathematics is at the bottom, and physics (with you in it) is inside mathematics.

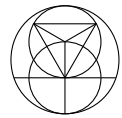
Tegmark and Wigner both asked why mathematics works so well, and my account has an answer: mathematics and physics are two aspects of one act of comparison. Mathematics and physics are both built from the same parts of comparison and the same four characters. I give a picture of the shared source. A rubbing is a copy of a carving made by laying paper over the carving and rubbing the paper. Take two rubbings from one carved stone: the two rubbings match line for line, and the match needs no explanation beyond the stone. Mathematics and physics are two such rubbings, and comparison is the stone. The shared source also sets a limit on how close a description comes to what it describes. A description comes as close to what it describes as anyone likes, and it never contains what it describes, because a description that contained its findings exactly would hold symmetry and continuity together, which the Incommensuration Theorem forbids. The mathematical description of physics approaches the physical process as closely as one likes and never coincides with it. So Tegmark's clues (the parabola, the ellipse and the 32 numbers) show how closely mathematics, the finest instrument there is, describes the world, and the description can be made closer without end.

IX. What This Chapter Holds

The world is described by mathematics without being mathematics, and you are in it as one who makes moves. Mathematics lays out the score sheet of the world exactly, and the world that physics finds is made of moves, each made once, from a seat.

Tegmark and I part on one claim, the claim that the world is its own score sheet. The two accounts agree on much else. The truths of mathematics hold before anyone proves them. And the success of mathematics in describing the world deserves an explanation. The explanation is that mathematics and physics share one source, the act of comparison, and the Incommensuration Theorem shows why the structure written without a stance and the world found from a stance can never be the same thing. For Tegmark's reader, nothing of the mathematics is lost, and something is gained: the move, the act of finding, is real.

Tegmark's mathematics of space and time lays out every moment at once, like a score sheet holding the whole game. Is the passing of time that you feel, the next move not yet made, only an illusion?



Chapter 11

Is Time an Illusion?

Beside Tegmark's chapter 11, "Is Time an Illusion?"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

Hard randomness: an outcome that nothing before it fixed, however completely everything before the outcome is known (Chapter 8).

The *order of depth*: metaphysics, then physics, then computation, then mathematics, each resting on the one before; mathematics stands last, and keeps every power it has (Chapter 10).

I. A Letter from Princeton

If all of space and time is laid out at once, with every moment already there, is the passing of time that you feel each morning an illusion? The physicist Albert Einstein answered the question once, in a single sentence, and people have quoted his sentence ever since.

Michele Besso died in Geneva on 15 March 1955. Besso had been Einstein's friend for more than fifty years, since their student days in Zurich, and later Einstein's colleague at the patent office in Bern. Einstein wrote from Princeton on 21 March 1955, six days later, to Besso's son Vero and Besso's sister Bice Rusconi. Einstein thanked Vero Besso and Bice Rusconi for telling him how Besso had died, and he wrote that Besso had now gone a little ahead of him in leaving "this strange world". Then Einstein wrote, in German, the sentence that the physicist Max Tegmark puts at the head of his chapter on time in *Our Mathematical Universe*. The best-known English form of Einstein's sentence reads: "The distinction between past, present, and future is only a stubbornly persistent illusion". Einstein died four weeks later, on 18 April 1955.

Einstein wrote the sentence to comfort a grieving family. In a few words, Einstein's sentence states a picture of the world in which past, present and future are all laid out together in spacetime. Spacetime is space and time taken together as one four-dimensional whole. In Einstein's picture, Besso's life lies in its own region of space and time, there as surely as the region where Einstein writes the letter. Tegmark builds his whole account of time on the picture of spacetime laid out at once, and his reader comes away with a voice in the head that says: "But Einstein said time is an illusion". In this chapter I answer the voice.

Come back to the kitchen table that this book has pictured since its Preface, where two people are playing chess and a score sheet, the written list of the game's moves, lies beside the board. The score sheet is the structure of the game, written out whole and the same for everyone who reads

the sheet. The move is the act of finding, made once, now, by a player sitting at the board. Picture the game when the game is finished: the score sheet holds every move, from the first to the last, all at once. Now picture the game while the two players are still playing: the score sheet holds every move made so far, and at the board the next move is not yet made. Both pictures are true of one game. At the kitchen table, the question of this chapter becomes whether the finished score sheet is the whole game. Tegmark holds that the four-dimensional shape of spacetime is the whole of time, which at the table is the claim that the finished score sheet is the whole game.

From Tegmark's pages

II. What Tegmark Showed: Time as a Shape

Tegmark begins from Einstein's physics, and he teaches Einstein's physics well. Tegmark says there are two equivalent ways to think about the world: as a three-dimensional space in which things change as time goes by, or as a four-dimensional spacetime that simply is, with time as the fourth direction of spacetime. Tegmark calls the view of a space where things change the frog's view: the view from inside the world. He calls the view of spacetime the bird's view: the view from above the world. Tegmark's figure of the Moon shows the two views together. In the frog's view, the Moon goes round the Earth. In the bird's view, the Moon's path through spacetime is one unchanging spiral, and each moment of the frog's view is one slice across spacetime.

Tegmark then adds a claim of his own to Einstein's physics. In Tegmark's sense, a mathematical structure is a set of abstract things that have no properties except their relations to one another. Tegmark argues that a mathematical structure is timeless, so the structure that is our world must contain the whole of spacetime at once. Tegmark makes the point with the same game that sits on our kitchen table, a

game of chess: “if the history of our Universe were a chess game, the mathematical structure would correspond not to a single position, but to the entire game” (chapter 11, “How Can Physical Reality Be Mathematical?”). At our kitchen table, Tegmark’s “entire game” is the finished score sheet. To Tegmark’s bird, the paths of all things through spacetime form a tangle of uncooked spaghetti, and Tegmark sums up the bird’s view with a smile: “To the bird, reality is the geometry of the pasta” (chapter 11, “How Can Physical Reality Be Mathematical?”).

Tegmark gives his verdict on time in his own words: “In summary, time is not an illusion, but the flow of time is. So is change” (chapter 11, “How Can Physical Reality Be Mathematical?”). On Tegmark’s account, time itself is part of the world. Time is the fourth direction of spacetime. The illusion, for Tegmark, is the passing of time: the sense that the present moves forward and the future becomes the past. Tegmark reads Einstein’s sentence as a statement about spacetime. Tegmark writes that Einstein, in speaking of past, present and future, “was referring to the fact that these concepts have no objective meaning in spacetime” (chapter 11, “How Can Physical Reality Be Mathematical?”). Tegmark argues that spacetime is static and unchanging, so “all parts must be equally real” (chapter 11, “How Can Physical Reality Be Mathematical?”), with the word “real” in Tegmark’s own sense: a part of the external physical reality. “In spacetime”, Tegmark writes, “the future exists and the past doesn’t disappear” (chapter 11, “How Can Physical Reality Be Mathematical?”). Tegmark marks his static view of time as his own, “This is how I see it”, and he adds that the static view “remains controversial and subject to vibrant scientific debate” (chapter 11, “How Can Physical Reality Be Mathematical?”).

Tegmark then explains why the passing of time feels so vivid. Tegmark calls each brief perception an “observer moment”: a pattern in a brain of a little more than a liter, lasting about a tenth of a second.

Each observer moment holds a clear impression of now and fading memories of what came before, like a newsreel that the brain edits again every second. On Tegmark's account, the feeling that time flows comes from the relations among the memories present together in one observer moment. Tegmark concludes that "the subjective perceptions of duration and change are qualia" (chapter 11, "What Are You?"): qualia are felt qualities given all at once, as the redness of a strawberry is given.

Tegmark also shows how a person can still predict the future in a world where time does not flow. On Tegmark's account, your future observer moments are the observer moments that fit your present observer moment like the next piece of a jigsaw: the future observer moments share your memories in the right order and add a little more. A mind that knew the whole mathematical structure could find your future observer moments, and so, Tegmark writes, "In this way, you recover the traditional scientific notion of causality: that you can predict the future from the present" (chapter 11, "What Are You?").

From Tegmark's pages

III. What Tegmark Showed: When Are You?

In the second half of his answer, Tegmark turns the question of time into a question about you. Tegmark points out that nobody stops a stranger to ask "what's the place?" When we are lost, we ask "where am I?", because we are asking about our own location. Tegmark says that "what's the time?" is a question of the same kind: a question about where you stand in time. In Tegmark's picture, spacetime "is like a map of cosmic history" with no marker saying "You are here" (chapter 11, "How Can Physical Reality Be Mathematical?"). Tegmark concludes that the better question is "When am I?"

Tegmark's spacetime is laid out whole, and so it can hold many observer moments that feel just like yours. From the inside, you could be any one of those observer moments. Tegmark states a rule for reasoning about which of those observer moments you are, the rule of the random observer moment: "You should reason as if your observer moment is a random one among those it could be" (chapter 11, "When Are You?"). The philosopher Nick Bostrom states a wider form of the same rule, which he calls the Strong Self-Sampling Assumption. In Bostrom's form of the rule, each observer moment reasons as if it were drawn at random from a chosen class of observer moments. Tegmark's rule is the case in which the class holds only the observer moments that feel exactly like yours. Tegmark uses the rule of the random observer moment, in his own narrow form or in Bostrom's wider form, on a run of famous puzzles.

Tegmark applies the rule to the Sleeping Beauty puzzle. In the puzzle a sleeper is woken once or twice, depending on the toss of a coin, and is made to forget the first waking before the second. Tegmark replaces the coin with a quantum measurement whose two outcomes both occur, each in its own branch of the world, so that three wakings feel the same from inside, and only one of the three wakings follows heads. So, on Tegmark's reasoning, Beauty should give heads one chance in three. Tegmark applies the same rule to the doomsday argument, which he traces to the physicist Brandon Carter in 1983. In the doomsday argument you are a random human in the order of human births, and a random human is unlikely to be among the first. Tegmark assumes that the world's population levels off at 10 billion people with a life expectancy of eighty years, and on that assumption he writes that "humanity as we know it will with 95% certainty end before the year 10,000 AD" (chapter 11, "When Are You?").

Tegmark also applies the rule to brains assembled by chance. Tegmark starts from the standard model of cosmology. The standard model of cosmology is the account of the universe's history and contents that

most cosmologists use today. Six measured numbers fix the model. In the standard model of cosmology, the energy of empty space stays the same for ever, as current measurements suggest, and the energy of empty space keeps up a very faint warmth for ever. Tegmark traces the warmth to the same kind of tiny quantum jitter that produced the ripples in the afterglow of the early universe. He puts the warmth at about a millionth of a trillionth of a trillionth of a degree above absolute zero, the lowest temperature there is. The physicist Ludwig Boltzmann saw that a warm thing left alone long enough will take on, by chance, even the most unlikely arrangements of its atoms. If time goes on without end, the faint warmth of empty space will therefore, by chance, assemble brains that hold your memories, and a brain assembled by chance in this way is called a Boltzmann brain, after Boltzmann. Over endless time, the Boltzmann brains will outnumber you without limit. By Tegmark's rule of the random observer moment, you should then expect to be a Boltzmann brain. Tegmark answers with a test you can do yourself: a brain made by chance would find its memories dissolving into nonsense as it searched them. So, Tegmark writes, "if you're still reading this, you're not a Boltzmann brain" (chapter 11, "When Are You?"). Tegmark concludes from the test of memories that "something is fundamentally wrong with what we assumed about the future of our Universe" (chapter 11, "When Are You?"). Later in his chapter Tegmark looks for the faulty assumption, and he names infinity as his "prime suspect", including the idea "that time can continue forever" (chapter 11, "When Are You?").

Tegmark's reasoning on all three puzzles depends on a change that he proposes in the method of science itself. The usual method of science needs two connected moments: first you predict, and at a later moment you check the prediction. In Tegmark's picture, the memory of having predicted and the observation are both held in one observer moment, so science needs "a one-time approach that can be applied to a single observer moment" (chapter 11, "When Are You?"). Tegmark says that where there are copies of you, the number of copies determ-

ines what you feel: “When the number of yous increases, you perceive subjective randomness” (chapter 11, “Where Are You? (And What Do You Perceive?)”).

The parts of Tegmark’s answer to the question of time fit together into one whole. On Tegmark’s account, time is a direction of one timeless shape. The present is wherever an observer moment sits in the timeless shape. And you answer the question “when am I?” by reasoning as if your observer moment were drawn at random from all the observer moments laid out together.

IV. Where the Two Accounts Meet

I agree with Tegmark on two things. For each of the two, my account has a ground of its own.

First: when the past is taken as a region of the geometry of spacetime, the past is still there. I take up the saying that last Thursday “is still there” in spacetime. I find the saying true of last Thursday as a region of the geometry, the four-dimensional shape of spacetime. The region of spacetime that holds last Thursday exists, in this book’s sense of the word: the region is structure, laid out whole and the same for everyone who consults the region. Tegmark says that in spacetime the past does not disappear. His statement is true of the past taken as a region of the geometry, and I hold the same statement.

Second: a description from outside has no objective “now”. Tegmark says that past, present and future have no objective meaning in spacetime. No description of a system from outside, in the third person, contains a privileged now or a privileged here: a moment or a place singled out above the others. I add that, in a view from outside, every moment is interchangeable with every other moment. The two accounts state the same fact about descriptions from outside. I derive the fact from the Incommensuration Theorem, which chapter 1 stated and chapter 10 set out step by

step. Scientists write descriptions from outside: each description states laws that hold the same everywhere, about things that can be told apart and counted. A description from outside pairs sameness with separate, countable things, which is symmetry together with discontinuity, one of the two pairings that the theorem allows (the other is asymmetry together with continuity). In a description from outside, every moment has the same standing as every other, so no moment in the description is singled out as now.

Tegmark and I agree that the past is still there as a region of spacetime, and that a description from outside has no now. At the kitchen table, the two agreements mean that the score sheet exists, the same for everyone who reads it, and that Tegmark and I read the score sheet the same way. Every move already made stays written on the score sheet.

V. Time Is More Than a Dimension

Tegmark's account and mine part at the word "illusion". I hold that the passing of time is no illusion. My reason starts from three different things that physicists call by the one word "time".

Picture a Geiger counter on a table, with a clock beside the counter. A Geiger counter clicks when an atom of radioactive material decays. The first kind of time is *geometric time*, pictured by the hours on the clock's face, which are all laid out at once. Geometric time is time as a dimension, the fourth direction of Einstein's spacetime, with the length of time along every path held in one shape. The second kind is *event time*, the one-way order in which findings are made, pictured by the clicks of the counter: one click after another, each click a separate outcome laid down as a record. The third kind is *evolution time*, pictured by the chance that a given atom in the counter has decayed, a chance that grows smoothly from moment to moment. Evolution time is the smooth time of quantum physics, along which the chances of every possible finding change. I show that the three kinds of time are distinct, that each kind needs the other two, and

that each kind does work that neither of the other two can do. Geometric time is the dimension, and it exists; event time is the process, and it is real. Geometric time exists as structure, the same for everyone. Event time is real: each finding is given in an act, once, now.

Once the three kinds of time are told apart, we can name exactly the picture that Tegmark draws. The *block universe* is all of spacetime laid out at once, with no moment singled out as now. I describe the block universe in one sentence: "The block universe takes geometric time for the whole, and drops event time". Tegmark and I part on one claim: that geometric time is the whole of time.

At the kitchen table, the block universe is the finished score sheet taken for the whole game. The score sheet holds the whole game, exactly and at once. But the score sheet is written one move at a time, as each move is made from a seat at the board, and the sheet holds each move only after the move is made. If you take the score sheet for the whole game, you leave out the players' moves, and the players' moves wrote every line of the sheet.

I give two reasons why time is more than geometric time, and both reasons come from the Incommensuration Theorem.

The first reason concerns the future. The Incommensuration Theorem shows that there can be no world in which every state is fixed completely by the state before. A world fixed completely in that way would need one law perfectly the same everywhere and every state specified with no gap at all. One perfect law and states that have no gap make perfect sameness together with perfect gaplessness: the pairing that the theorem forbids. So no world is completely fixed in advance. The future does not exist in the sense the past does. The past is laid down in records. The future is a range of what could happen, and in each act of finding one outcome from the range happens.

The second reason concerns the direction of time. The laws of physics run the same forwards and backwards to an extraordinary degree: a film of a rolling ball, played backwards, shows a motion that the laws allow.

The Incommensuration Theorem shows that no process can be run backwards so that the process retraces its path exactly. Running a process back exactly would need a law perfectly the same in both directions and a state specified to have no gap, which is perfect sameness and perfect gaplessness again. So no process at any scale can be run back exactly. The two directions of time differ in their structure. Every record anyone has ever found is a record of the past, and no instrument holds a finding before the interaction that makes the finding.

Physicists have seen the laws themselves differ, by a little, between the two directions of time. In 1964 the physicists James Christenson, James Cronin, Val Fitch and René Turlay found that certain decays of short-lived particles called kaons break a symmetry that physicists had expected to hold exactly. The broken symmetry is the symmetry of mirror and charge taken together: the laws staying the same when a process is seen in a mirror and every charge in the process is reversed. Particle physics has a theorem that ties the symmetry of mirror and charge to the reversal of time, and by that theorem the 1964 finding implies that the laws are not exactly the same when time is reversed. We now know that the laws of nature are almost, and not exactly, the same in both directions of time. So the two directions of time differ in the laws themselves, as well as in every record. The decays of kaons show a small difference between the two directions of time, and I call that difference the lean. I read the lean as an instance of what the Incommensuration Theorem requires: the laws are never perfectly the same in both directions of time. I call the lean "a place where the impossibility of perfecting temporal symmetry has become visible to instruments".

An outcome that nothing before it fixed can change the shape of spacetime itself. Chapter 1 told of a thought experiment. A Geiger counter is wired, in thought, to a small rocket. The rocket nudges one asteroid and then a larger asteroid, until in time the orbits of planets are moved. So the shape of spacetime depends on a click that nothing before the click fixed. Chapter 8 named an outcome that nothing before it fixed *hard randomness*:

an outcome settled only when it happened. A single fixed geometry of all of space and all of time, from the first moment to the last, laid out once and for all, cannot be the whole story of the universe.

VI. The Present Is the Move

What, then, is the present? My answer: the present is the act of comparison itself. Comparison is made only in the present, and that the present is where the interaction itself takes place, as distinct from one point on a line.

The present is where many things that could happen become the one thing that does happen. The Geiger counter on the table is a case: before a click, the counter could click or stay silent, and in the present one of the two happens. I describe the present from the inside: one actual outcome forms, here and now, out of the range of what could happen, and the present is "no point sliding along a timeline laid down in advance". Everything that exists, laid out as structure, came from acts made in the present: structure is the trace that the present leaves behind. Physics describes the trace very exactly, but physics cannot describe the converging itself: the present, in which the many things that could happen become one. The converging is not an object that physics could hold still long enough to examine.

Now listen again to the voice in the head: "But Einstein said time is an illusion". Einstein wrote to a grieving family that the distinction between past, present and future is an illusion, though a stubborn one. Tegmark reads Einstein as saying that the distinction between past, present and future has no objective meaning in spacetime. In Tegmark's reading, Einstein's sentence states what I hold too: in a description from outside, no moment is singled out as now. I add the one thing that a description from outside leaves out. A description from outside lacks the now. The now is the act of finding, made from a stance. A stance, as chapter 1 explained, is the side from which a comparison is made. Everything that any descrip-

tion holds was found in acts of finding. Besso's life lies in its region of spacetime, and Besso's life exists there, as Tegmark says. Besso's life was real in every act he made while he lived. What remains of Besso's life now is its records, and Einstein's letter from Princeton is one of them.

Tegmark's map of cosmic history has no "You are here" marker. For anyone who needs a "You are here" marker, Tegmark offers a phone with a clock and a satellite receiver. I use a map of the same kind: the town map on a post at a street corner. I draw a further conclusion from the town map. The town map shows every street, and on its own the town map shows no "here". Someone has added a red dot to the map. Without the red dot, the map, however exact, cannot tell the reader which way to go. The red dot is the first person (the view from here) put back into the third person (the view from outside that the map gives). The philosopher Thomas Nagel called the view from outside the view from nowhere. The view from nowhere and the view from here are both needed, and neither replaces the other. I give the reason why neither view replaces the other. A single view cannot have both the perfect sameness of the view from outside and the unbroken continuity of the view from here, because the single view would then hold perfect sameness and perfect gaplessness together, the pairing that the Incommensuration Theorem forbids.

Tegmark likens "when am I?" to "where am I?", as if moments were places on a map. In geometric time, every event has its place. A comparer is any position inside the world from which a comparison is made, whether a mind or not (chapter 8 explained the word). A comparer encounters moments only in the order of its own findings, one finding after another, and the order of findings runs one way. A location in geometric time is not something that a comparer reaches as a place is reached. So "when am I?" has a plain answer from the seat at the board: you are at the move being made.

Event time also matters for Tegmark's account of the feeling that time flows. Tegmark places the feeling of flow in the memories present together in one observer moment. I agree that all the evidence of the past is found in the present: the light of a distant galaxy is light arriving now, a fossil is

a stone in a museum now, and a memory is a trace in a brain now. I also hold that a record is the result of the act it records. Your memories are records, laid down by acts, one after another, in the one order that every record shows. The passage of time belongs to the world itself and rest on comparison: the direction of time is the direction in which comparison proceeds. I add that only a comparer that makes the findings in sequence, from one stance, has an order of findings. So on my account the passing of time is real, and you encounter the passing of time from inside, one finding after another.

On my account, a person can still predict the future, by a different route from Tegmark's. Tegmark recovers causality by imagining a mind that knows the whole mathematical structure and matches the pieces of the jigsaw. The mind that Tegmark imagines resembles the intellect that the mathematician Pierre-Simon Laplace imagined in 1814, an intellect that knew every force and every position and so held the future as clearly as the past. Causation is established, and determinism never. Determinism is the view that every state is fixed completely by the state before. A cause is a connection with no break (continuity) along an order of findings that runs one way (asymmetry). I add that the unbroken connection of a cause with its effect is found from inside the order of findings, "by a comparer for whom one finding follows another". Continuity with asymmetry is one of the two pairings that the Incommensuration Theorem allows, and so every world is to some degree causal. Every prediction a scientist makes is a comparison made now, checked by a later comparison. The method of science is what scientists have always done: a cycle of separate acts, each act compared with the last.

VII. Why This Moment and Not Another?

Tegmark's three puzzles about typical observers share one premise. In each puzzle, Tegmark lays out all the moments at which an observer could be, from beginning to end, and treats your moment as one drawn at

random from all those moments. You are drawn from all human births in the doomsday argument, the Boltzmann brains are counted against you in an endless future, and in Sleeping Beauty Tegmark counts three wakings laid out together.

I hold that only the present act is given. We piece together the history of the universe and all its eras from what arrives now: light, records and the afterglow of the early universe (chapter 3 showed how). The pieced-together history exists as structure, and it is very well made. I derive Einstein's theory of gravity from comparison. A rule stated from outside applies to all eras at once, and the rule can produce a spread of the eras at which comparers stand (how many comparers stand at each era). But the rule stated from outside cannot single out one of them as real.

I put the question in its plainest form: why is your present this one, and not a moment in 1850, as if all the presents were laid out together and you had been placed in one of them? Whoever asks the question assumes that all the presents are set out at once, like the frames of a very long film laid on a table, with you in one frame. On my account the history is laid out only afterwards, pieced together from the present act, and so the act comes first. Frames laid out on a table are the finished score sheet in another form. A random draw from the frames is a calculation about the score sheet, and the calculation is exact as a calculation. The frames come from the move being made, because each frame is laid down only after its move is made.

On the Boltzmann brains, the two accounts come to one result by two roads, that is, by two different arguments. Tegmark concludes from his test of memories that something is wrong with what the standard model of cosmology assumes about the future. Time that goes on for ever is among his suspects. I come to the same result on ground of my own. The energy of empty space is one number through the one space and time of our epoch, the long age of the universe that we live in. I also state that the sky empties, over a hundred billion to a trillion years as astronomers reckon time, for anyone, anywhere, who can see and compare. If a thing is not bound to our own group of galaxies, the thing is carried beyond the hori-

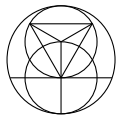
zon, the edge past which no light can reach us. The matter still inside the horizon ends inside black holes or departs. The emptying ends at the turn. So the long emptying of the sky is one stage of the world, and the endless count of chance brains has no endless future to be counted in. The two arguments differ at the random draw. Tegmark keeps his rule of the random observer moment. The rule says that you should almost surely be a chance brain, and his test of memories says that you are not one, so Tegmark concludes that the assumed future is wrong. I make no random draw. I derive the end of the emptying from my own account of the far future, given in full in chapter 15. The appendix "What the Coming Measurements Will Find" states what I say the coming sky surveys will find of the energy of empty space.

VIII. What This Chapter Holds

The passing of time is no illusion: the geometry of spacetime exists, laid out whole like a finished score sheet, and event time, the one-way order of findings, is real. The present is the move being made, the act in which one of the things that could happen becomes what happens.

Tegmark is right that a description from outside has no now, and right that the past as a region of spacetime does not vanish. I hold both of Tegmark's claims. I add event time, the one-way order of findings, which every record shows and which the Incommensuration Theorem requires. The passing of time is real. An order of findings belongs only to a comparer that makes the findings one after another, from one stance. And the question "when am I?" is answered at the board: you are at the move being made. For this book's question (What is reality, and can physics find out?), time shows the difference between the score sheet and the move in its plainest form: geometric time exists, and event time is real.

If every finding is made now, from a seat at the board, then who or what is sitting in the seat? Is the one sitting in the seat only the pattern that Tegmark calls a braid, the paths of all your particles wound together through spacetime?



Chapter 12

What Am I?

Beside Tegmark's chapter 11, its sections "What Are You?" and "Where Are You? (And What Do You Perceive?)"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

The *stance* is the side from which a comparison is made, as a move is made from a seat; every comparison has a stance, human or not (Chapter 1).

Hard randomness: an outcome that nothing before it fixed, however completely everything before the outcome is known (Chapter 8).

The *order of depth*: metaphysics, then physics, then computation, then mathematics, each resting on the one before; mathematics stands last, and keeps every power it has (Chapter 10).

I. A Mill You Can Walk Through

What am I? Is consciousness only matter processing information in a certain way, so that an exact enough description of my brain would be a description of me?

In the summer of 1714 the philosopher Gottfried Wilhelm Leibniz was on a long stay in Vienna. In Vienna he began a short summary of his philosophy, written in French, in ninety numbered sections. The summary is known as the *Monadology*, the title of a German translation of 1720. In section 17 of the *Monadology*, Leibniz asks his reader to imagine a machine built so that the machine thinks, feels and perceives. Enlarge the machine, keeping its proportions, until the machine is large enough to walk into, "so that one might go into it as into a mill", in the words of Robert Latta's English translation of 1898. Walk inside the mill and look around. You find parts that push on one another, each one moving the next. Nowhere among the parts do you find a perception.

Leibniz's picture of the mill is older than any computer. With the picture, Leibniz asks in the plainest form whether a complete description of a thinking thing is a description of the one who thinks. Today scientists can map a brain far more finely than anyone could map a mill: the brain's cells, the connections between the cells, and the charged atoms that move when a nerve cell fires. So we can ask Leibniz's question of a brain. Does a complete map of a brain contain the seeing, the hearing, and the one who sees and hears? Leibniz himself answered his question with a theory of simple substances, things without parts, which he called monads. I keep Leibniz's picture of the mill and reach a different answer.

The move stands for the act of finding, made once, now. The seat at the board stands for the stance, the side from which a move is made. Suppose that someone records every firing of every nerve cell in one player's brain while the player thinks about the next move. The record can be as exact as

any instrument allows. Even so, the record is a description of the play from outside the seat. What does the player in the seat have that the record of the player's brain leaves out?

From Tegmark's pages

II. What Max Tegmark Showed

Max Tegmark takes up the question of what you are in chapter 11 of *Our Mathematical Universe*, and he starts from physics. Your body is made of roughly a hundred thousand trillion trillion elementary particles (a one followed by twenty-nine zeros). Each particle traces a path through spacetime, which is space and time taken together as one four-dimensional whole. The paths of all your particles form a tube through spacetime. Inside the tube, the paths wind around one another like the strands of a braid: the paths of your blood cells, and above all the paths of the charged atoms that move each time a nerve cell fires. Tegmark calls the whole pattern of wound paths a braid. Tegmark gives his summary "in broad brushstrokes", in his own words, and the summary is short: "You're a pattern in spacetime. A mathematical pattern" (chapter 11, "What Are You?").

A pattern in spacetime never changes, so Tegmark then asks how such a pattern can feel time passing. Tegmark answers with the observer moment: one small pattern in the brain, lasting about a tenth of a second, which holds a clear picture of the present together with fading memories of the moments before the present. On Tegmark's account, the feeling that time flows comes from the relations among the memories held within one observer moment, the way one showing of a newsreel holds a whole story. Tegmark gives the example of a sleeper on a long flight. On waking, the sleeper feels joined to the moment of falling asleep, and to none of the nearer moments of the other passengers. Suppose that a perfect copy of the sleeper wakes in another plane. The copy would feel joined to the moment of falling asleep as

well. Tegmark writes that the sleeper and the copy suggest "that the continuity business is a red herring" (chapter 11, "What Are You?"). On Tegmark's account, the memories held inside each moment settle which other moments feel joined to that moment. No new physical process links the moments.

Tegmark next takes up the self. On Tegmark's account, the brain builds a model of the world from what the senses report, and because the model contains a model of the person, we are self-aware as well as aware. Who looks at the model? Tegmark guesses that nobody looks at the model, because a spectator inside the head would need a second spectator to watch the first, and the second spectator would need a third, without end. Tegmark puts his view in one sentence: "I think that consciousness is the way information feels when being processed in certain complex ways" (chapter 11, "What Are You?"). On Tegmark's account the feeling of being an "I" is a *quale*: a basic felt quality, such as the redness of a strawberry. Qualia is the plural of quale, and Tegmark writes: "In short, redness and self-awareness are both qualia" (chapter 11, "What Are You?").

In the next section of his chapter, Tegmark asks how anyone could find the conscious parts of the world at all. When a part of a mathematical structure has subjective perceptions, Tegmark calls that part a self-aware substructure. Tegmark imagines a gift from a friendly alien: a handheld detector that beeps quietly at a goldfish, louder at a cat and loudest at you, and stays silent at a cucumber, a car or a corpse. Tegmark guesses that the detector measures, in part, how complex a thing is and how much information the thing holds about the rest of the world. A video camera is complex and holds information about the rest of the world, and yet a video camera is not self-aware. So Tegmark adds ingredients that are harder to understand. Tegmark gives two examples: whether the thing processes its information, and whether the parts of the thing are joined into one undivided whole. The second example follows a proposal of the neuroscientist Giulio

Tononi. Each property that Tegmark names can be measured from outside the thing. Tegmark writes: "My guess is that we'll one day understand consciousness as yet another phase of matter" (chapter 11, "Where Are You? (And What Do You Perceive?)"). Solid, liquid and gas are the familiar phases of matter. Tegmark names three phases of matter by what each phase does. A memory substance stores information, computronium is "the most general substance that can process information as a computer", and perceptronium is "the most general substance that feels subjectively self-aware" (chapter 11, "Where Are You? (And What Do You Perceive?)").

Two more of Tegmark's results bear on the question of what you are. In a paper of 2000, Tegmark asked whether the brain works as a quantum computer, as the physicist Roger Penrose and others had proposed. A quantum computer needs its parts to hold a superposition, a quantum state that keeps two outcomes at once, and a quantum computer also needs its surroundings to carry no record of which outcome is there. Tegmark computed how long a nerve cell could stay in a superposition of firing and not firing before decoherence: before collisions with the water molecules around the cell record which of the two states the cell is in. He found about ten billionths of a trillionth of a second. The anesthesiologist Stuart Hameroff and Penrose had placed the brain's quantum computation in the microtubules, the fine scaffolding inside cells. For the microtubules, Tegmark found about a ten-trillionth of a second. Tegmark puts the times of the brain's own processes at between a thousandth and a tenth of a second, in the same paper of 2000, and then he compares the times in which superpositions fade with the times of the brain's own processes. The superpositions fade far faster. So Tegmark concludes that the parts of the brain that bear on thinking "should be thought of as a classical rather than quantum system". A classical system is one that follows the everyday physics of definite states. In his book, Tegmark comments on the times he computed: "Perhaps Roger Penrose can think that fast, but I sure can't" (chapter 8, "Why Your Brain Isn't a

Quantum Computer"). From the start of his book, Tegmark also holds that physics does not have to wait for an answer about the mind. He writes that "our failure to understand consciousness doesn't stand in the way of a complete understanding of the external physical reality" (chapter 1, "The Journey Begins").

Chapter 9 of this book described decoherence: the fading of interference between the alternatives of a thing, as the thing's surroundings come to carry records of which alternative the thing is in.

III. Where the Two Accounts Agree

I reach several of Tegmark's results on ground of my own. In this book's sense, comparing is telling one thing from another, or finding two things the same. Every measurement is a comparison, and so is every interaction between two things.

I agree with Tegmark that no spectator sits inside the head. I give the reason with a picture of two crossing laser beams. Where only the electromagnetic field acts, two laser beams can cross in space and pass through each other unchanged, because light carries no electric charge. Light does act on light, through pairs of charged particles, but so slightly that no laser makes the effect large enough to see. So no one can use one beam to watch another beam pass. Perceiving is like the crossing beams: one act of perceiving cannot be used to watch another act of perceiving. I cannot perceive the perceiver, and I cannot perceive perception; I can perceive only what is the object of perception. Tegmark concludes that nobody looks at the model by counting what a spectator would cost. Each spectator would need another spectator, without end. I reach the same result, that nobody looks at the model, from what a comparison is: the act of comparing is never one of the things being compared.

I accept the estimates in Tegmark's paper of 2000 as Tegmark computed them. The estimates say how fast superpositions fade in nerve cells and in microtubules. Hameroff and Penrose hold the hypothesis that consciousness lies in quantum states held whole for a long time in the microtubules. I set my own account apart from the hypothesis of Hameroff and Penrose. Entanglement is the joining of two or more parts into one quantum state, in which no part has a state of its own. The Immanent Metaphysics does not depend on any specific biological mechanism, and it needs no sustained entanglement in the brain. My account of consciousness applies for any system in which novelty enters, time is directed, and there is a point of view. So Tegmark's decoherence times bear on the hypothesis of Hameroff and Penrose, which needs long-held quantum states in the microtubules, and the times do not bear on my account, which needs no such states.

I agree with Tegmark on a further point: on my account, too, physics can proceed before any theory of the mind. Physics can proceed because the observer of physics is a comparer, as chapter 8 of this book set out: a comparer is any position inside the world from which a comparison is made, whether or not the position is a mind. An observer is a comparer: the word names a position inside the world, and it never names a mind. Physics studies every comparer: a thermostat compares, and so does a sense organ.

Last, I agree with Tegmark that consciousness belongs to the study of what matter does, and that Tegmark's list of properties can describe exactly how conscious matter behaves, as seen from outside. Tegmark's account and mine differ on the question of what makes matter conscious. My answer begins with three things that every experience holds together.

IV. Three Things in One Act

Think of the first time you heard a piece of music you came to love. Each note arrived after the note before it, and the first hearing could never be had again: the hearing had a before and an after. Some notes surprised you: nothing you had heard so far fixed which notes would come. And the hearing was yours: the music happened to someone. I begin my account of consciousness with the first hearing of a piece of music. I find three things in the first hearing.

The first of the three is the *arrow of time*: an order that runs one way, binding past to present to future in one thread without a break, as the notes came one after another and never back. The second of the three things in the first hearing is *hard randomness*, a term from chapter 8 of this book. Hard randomness is an outcome that nothing before the outcome fixed, however completely everything before the outcome is described, as nothing fixed the surprising note. The third of the three things in the first hearing is the *subjective*: being the one to whom things happen, as the hearing was yours. I describe the subjective as "the side from which experience occurs, the stance of the one to whom things happen". Chapter 1 of this book defined the stance as the side from which a comparison is made.

All three are bound together into one act: the arrow of time, hard randomness and the subjective. I call the three, bound into one act, *the bound triple*. The three are distinct: each of the three is itself and neither of the others. The three are inseparable: each occurs only together with the other two. And the three are not interchangeable: each has its own part in the act, a part that neither of the other two can take.

Each of the three needs the next. The arrow needs hard randomness, because an order of moments runs one way only where outcomes are new when they come. An order of outcomes all fixed in advance is like the page numbers of a finished book: the page numbers are the same whether

the book is read forward or back, so the fixed order does not run one way. Hard randomness needs the subjective: an outcome that nothing before it fixed is picked out only in an act of comparison, and every act of comparison is made from a stance. The subjective needs the arrow: the hearing was yours only as one note followed another, and every stance from which experience is had belongs to an order that runs one way.

The three come together because of the Incommensuration Theorem, which chapter 1 of this book stated and chapter 10 set out step by step. The Incommensuration Theorem holds of comparison, the act in which anything at all is known. Sameness and gaplessness are never both perfected, so every comparison has a part of its outcome that nothing before the comparison fixed. That part of the outcome is hard randomness. Every comparison is made from a stance, and the stance is the subjective. Every comparison runs one way, from the act to its record, and the one-way direction is the arrow of time. So the three are three aspects of one act: a comparison running in one direction, made from one stance, with an outcome that nothing before it fixed.

In quantum measurement the three appear as the observer, the single random outcome and the one-way record. The accounts of quantum measurement show the three bound together. The Copenhagen account is the textbook reading of quantum measurement. The Copenhagen account has all three together: the observer, the single random outcome and the one-way record. In the many-worlds account of chapter 9, every outcome occurs, each in a branch of its own. The many-worlds account has none of the three. Every account of quantum events has all three, the observer, the single random outcome and the one-way record, or none of them.

I identify consciousness with the bound triple, taken all at once as one act happening now, because the arrow of time, hard randomness and the subjective are aspects of one act and occur only together. Consciousness is the bound triple taken as one act.

V. The Outside and the Inside

Tegmark names four properties for his detector: complexity, information, processing and integration. Anyone can check each of the four from outside. The view from outside is called the third person: the view of someone describing a thing that the person is not part of. No third-person description of a system contains a privileged now or a privileged here. The subjective is the stance that sets this place, this moment and this outcome apart from every other place, moment and outcome, so a description that anyone can check from anywhere has no place for the subjective. Perceptronium is therefore a description of the outside of conscious matter, and it can be exact as such. Perceptronium can be an exact account of how conscious matter looks and behaves from outside. The bound triple, held in the act, makes the matter conscious, and the act is had from inside.

Now walk into Leibniz's mill again. The visitor sees parts pushing parts because the visitor stands outside every act that happens in the mill. In the same way, the recorder of the chess player's brain stands outside the seat. Make the record finer, add every charged atom and every connection, and the record is still a description from outside the seat. The perception that Leibniz looked for is the act as the act is had, from the stance from which the act is made. A perception is reached in one place: what pain feels like, for example, is reached only in one's own perceiving.

A second reason shows that a detector, however clever, finds only the outside of what the detector watches. From the outside, a conscious choice is indistinguishable from a random event. Seen from inside, a choice that nothing before it fixed is meaningful to the one who makes the choice, and the one who makes the choice is the process from which the choice came. Seen from outside, the same event is an outcome without a pattern. The inside view and the outside view describe one event from two positions. Both descriptions are correct, and neither description is complete.

So, from outside, the mark of the subjective is an outcome that nothing before it fixed. No list of measured properties can tell that outcome apart from chance.

I name a physical condition of the brain under which the bound triple shows itself in us. In 2003 the neuroscientists John Beggs and Dietmar Plenz recorded the electrical activity of thin slices of rat cortex, the outer layer of the brain, on a grid of sixty electrodes. Beggs and Plenz found that activity spreads in cascades, which they called neuronal avalanches. The sizes of the avalanches follow one simple rule across many scales. Beggs and Plenz read the pattern as the mark of a network poised between order and disorder. Physicists call a poise between order and disorder *criticality*: a state in which a small disturbance can cascade into a large one. In my use, a circumstance is the set of physical conditions under which an act takes place. The brain's poise is the physical circumstance under which the bound triple is expressed in us; it is not the bound triple itself. Tononi measures how far the parts of a system are joined into one whole, and he calls his measure integrated information. Integrated information, too, is a property measured from outside. I set integrated information beside the brain's poise: "Integrated information is the measure of the circumstance, and not the inside the circumstance supports". The appendix "What the Coming Measurements Will Find" sets out what, on my account, further recordings of the brain's poise will find.

Tegmark's account and mine differ on what makes a thing conscious. Tegmark holds that consciousness is the way information feels when the information is processed in certain complex ways. I hold that a thing is conscious because of the kind of physical process the thing is, and never because of the information processing the thing carries out, and my reason concerns what a computation is. A classical computation is the kind of computation that ordinary digital machines carry out: one rule applied the same way at every step. I give a picture first, the picture of a mosaic. A mosaic can follow a curve as closely as anyone likes: make the tiles smaller and the steps along the edge shrink until no eye can see them, and the mosaic is still made of tiles. A computation is a mosaic in time: each step

is a tile, and however small the steps, a computation is still made of steps. A computation applies one rule the same way at every step, and so a computation has perfect sameness. By the Incommensuration Theorem, perfect sameness and perfect gaplessness never go together, so whatever has perfect sameness comes in separate steps with seams between them. The arrow of time, as had from inside, is an unbroken one-way order, and seamed steps, however many, cannot make an unbroken order. So a computation lacks the arrow of time. A computation is fixed at every step by its rule, so a computation lacks hard randomness too. The three are bound together, and a computation lacks the arrow and hard randomness, so a computation lacks the subjective as well. I draw the conclusion: "classical computation, however intricately organized, cannot be what consciousness is".

In his last chapter, Tegmark applies his view to the machines of the future. So he writes that if humans one day bring about entities more intelligent than themselves, "I therefore think it's likely that they, too, would feel self-aware" (chapter 13, "The Future of Life"). I agree that machines of great competence can be built. Artificial intelligence is possible. Would a machine of great competence be conscious? The answer depends on the kind of physical process the machine is. A program is the list of instructions a computer carries out. Suppose that a program draws its chance from a physical source, such as electrical noise: the program then has the world's randomness at that source, and the program itself is still one rule applied the same way at every step. Whatever has the bound triple has the bound triple as a physical process that makes comparisons in the world and has an inside, as living things do. The question for any future device is therefore not which program it runs but what kind of physical process it is.

VI. The Self Who Wakes

Tegmark ends his account of the self with a set of lessons about how we perceive. We perceive ourselves as local, in one place and one moment, although on Tegmark's account each of us is a braid stretched through spacetime. We perceive ourselves as unique, although on Tegmark's account copies of us may live far away in space or in other branches of quantum physics. Tegmark states how the copies in the world are related to the single self we feel: "cloning with determinism is perceived as uniqueness with randomness" (chapter 11, "Where Are You? (And What Do You Perceive?)"). In plain words: where fixed laws make many copies of you, each copy feels unique and experiences its own next moment as random. Tegmark concludes: "our well-defined local and unique identity exists only in our internal reality; at a fundamental level, it's an illusion" (chapter 11, "Where Are You? (And What Do You Perceive?)"). In Tegmark's book, internal reality is the world as each of us perceives the world.

I count the persistence of the self among the laws that I state. I sort every law into two kinds: laws of sameness across change, such as the conservation of energy, and laws of persistence, of carrying on without a break, such as the smooth path of a thrown stone. Among the laws of persistence, I name the persistence of the self: "That the self who wakes in the morning is the self who fell asleep is a continuity law". The fact reads "Subjective context is continuous", and the example given for the fact is "The nature and persistence of self identity". In the aphorisms, the subjective context is the self, the one in whom every perceiving happens, and continuous means having no gaps. I give the fact in everyday form: the "I" who reads this sentence is the same "I" who read the last one, while what that "I" perceives and thinks changes all the time. So, on my account, Teg-

mark's sleeper on the long flight wakes as the one who fell asleep, because the self persists without a break. The persistence of the self is not constructed but given; it is the ground of all experience, as experience is lived.

I also hold that a copy does not carry the self. Identity, in the sense of a single unitary consciousness, is not something a copy can carry. A choice, had from inside, is made once, by the process that is the choice, and no other process reproduces the choice. At the smallest scale, physics has a theorem of the same kind, about what cannot be copied. In 1982 the physicists William Wootters and Wojciech Zurek proved that no single operation can copy every unknown quantum state. The physicist Dennis Dieks proved the same theorem separately, in the same year. The outcomes of measurements, once recorded, can be copied. I take the theorem of Wootters, Zurek and Dieks as the physical counterpart of a self that no copy carries. My principle of identity says that two things which no test, even an imagined one, could tell apart must be counted as the same, and so two things that can be told apart by some comparison are two. Chapter 6 of this book applied the principle to an exact copy of you in a far region of space, and concluded that the copy is a second thing, in a second place. I draw the same consequence for the copy of the sleeper in the other plane. The sleeper and the copy are two things in two places, told apart by the plane each one wakes in, and so the sleeper's identity, in the sense of a single unitary consciousness, stays with the sleeper. I make the same point about uploading a mind into a machine, in one sentence: "A person is not a very large file".

Tegmark ends his account of the life braid, the braid of particle paths that makes up one whole life, with the picture of a tree: roots where the particles gather at birth, a trunk, and branches where the particles part again at death. I picture a self as a tree too. My picture of the tree has no exact line where the tree ends and the world begins. In that picture, the branches are the body's dealings with the world, and the unseen roots are the subjective aspects of mind and soul. The aphorism that the tree illustrates says: "Self (mind) cannot be defined by composition (assemblies of parts) alone". Leibniz walked through the mill and concluded that per-

ception is found in simple substances, the monads. I find perception in the act of comparison itself. I call the self "non-monadic in character". A self that is non-monadic is not a monad, and so a self is no simple substance of Leibniz's kind. The aphorism that calls the self non-monadic also calls a self the "product and integral synthesis" of many "collections of aspect" that work inseparably to create the "mind of self": the self that one thinks of as what one is. In plain words, a self is one undivided whole, made by many groups of aspects of a person working together. A self cannot be defined by a list of its parts alone.

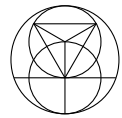
Tegmark's account and mine use the word "illusion" of different things: Tegmark places the illusion in the felt uniqueness and locality of the self, the sense that each of us is one, in one place and one moment. Consciousness is not an illusion, nor is it in conflict with conventional science. In the same aphorism, I place the illusions in two extreme pictures: a world wholly fixed by physical law, and the opposite picture, a mind wholly free of the physical. Interaction and consciousness are neither purely physical nor non-physical.

VII. What This Chapter Holds

You are an act with an inside. Consciousness is the bound triple (the arrow of time, hard randomness and the subjective) taken as one act, and a description of your brain, however exact, describes the act from outside the seat.

What you are bears on this book's question (What is reality, and can physics find out?) because the stance shows most plainly in a self. Tegmark's braid and his perceptronium can be exact, and the braid and perceptronium together are the score sheet of a life, but the life itself is had at the board, one move at a time, from a seat that no score sheet contains.

If a mathematical structure, however complete, has no seat in it, what are we to make of Tegmark's claim that every mathematical structure is a world as real as ours, in his sense of the word?



Chapter 13

Do All Mathematical Structures Exist?

Beside Tegmark's chapter 12, "The Level IV Multiverse"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

What *exists* is laid out whole, timeless and the same for everyone, as the list on a score sheet is; what is *real* is given in an act of finding, once, as a move is made (Chapter 1).

The *principle of existence*: "Existence without any interaction whatsoever is indistinguishable from non-existence, and therefore is non-existence" (Chapter 6).

The *order of depth*: metaphysics, then physics, then computation, then mathematics, each resting on the one before; mathematics stands last, and keeps every power it has (Chapter 10).

I. A Letter from Madras

If our world is one mathematical structure, why is it this structure and not another, and do all the other structures exist as worlds too? Max Tegmark asks both questions at the head of his chapter 12 of *Our Mathematical Universe*. His answer is the boldest step of his book. Before we follow Tegmark's answer, look at a letter.

On 16 January 1913 Srinivasa Ramanujan wrote from Madras in the south of India to the mathematician G. H. Hardy at Cambridge. Ramanujan was a clerk in the accounts section of the Port Trust at Madras. Ramanujan enclosed pages of formulas, more than a hundred of them, and he stated most of the formulas with no proof. Hardy read the pages, and that evening he went through the pages again with his colleague J. E. Littlewood. Some of the formulas were results already known. Other formulas were like nothing Hardy had seen, and Hardy could not prove them. Hardy wrote later of the formulas he could not prove:

> "A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true because, if they were not true, no one would have had the imagination to invent them".

Hardy brought Ramanujan to Cambridge in 1914, and in 1918 Ramanujan was elected a Fellow of the Royal Society, Britain's national academy of science.

Hardy's sentence takes one thing for granted. A formula that no one could have invented is a formula that someone discovered. Ramanujan discovered relations among numbers while he worked alone in Madras, and Hardy and Littlewood, who worked by other routes in Cambridge, could check the same relations, so the same relations were reached from two places. Ramanujan did not make his formulas true. The formulas held

before Ramanujan wrote them down, and they held in the same way for Hardy, who read them half a world away. Tegmark and this book hold the same view of mathematics. Mathematical structures are discovered, and the symbols we write them in are our own.

Tegmark's account and this book's account part at the next step. Ramanujan's formulas describe structures that no telescope will ever find in the sky. Tegmark holds that every mathematical structure is a universe, as much a universe as ours. Is every mathematical structure a universe? We follow Tegmark's answer first. Then I set out the results that this book reaches beside Tegmark from premises of its own, and last the step at which the two accounts part, with the reason they part there.

From Tegmark's pages

II. Mathematical Democracy

Tegmark opens his chapter 12 with a scene for the reader. You have discovered the final theory of physics, a T-shirt printed with its equations is a best-seller, and you are to receive the Nobel Prize. On the night before the ceremony you cannot sleep, because of a question asked by the physicist John Wheeler: "Why these particular equations, not others?" (chapter 12, "Why These Equations, Not Others?"). Wheeler's question is sharper still for Tegmark, because in his chapters 10 and 11 Tegmark argued that our world is a mathematical structure. Tegmark calls the claim the Mathematical Universe Hypothesis. Tegmark writes that "Mathematicians have discovered a large number of mathematical structures" (same section). On his hypothesis, the world we live in is one of the mathematical structures that mathematicians have discovered. Why should the one structure that is our world have physical existence, and every other structure have none?

Tegmark answers that every structure has the same standing. Tegmark calls his answer mathematical democracy: "mathematical existence and physical existence are equivalent, so that all structures that exist mathematically exist physically as well" (chapter 12, "Mathematical Democracy"). In the same passage Tegmark also calls his answer a form of radical Platonism, after the world of ideas that the Greek philosopher Plato described. On mathematical democracy every other structure is a universe of its own, and all those other universes make up the fourth level of Tegmark's parallel universes, which he calls Level IV.

Tegmark's four levels of parallel universes are one of the core ideas of his book. Level II is regions with other values of the constants of nature (the fixed numbers in the laws), regions that inflation, a very rapid stretching of space early on, would carry forever out of reach. Level III is the branches of the quantum state on the many-worlds account of the physicist Hugh Everett, in which every possible outcome of a quantum event happens, each on a branch of its own. The universes of all three levels obey the same fundamental equations. At Level IV the equations themselves differ: "Level IV parallel universes dance to the tunes of different equations, corresponding to different mathematical structures" (chapter 12, "Mathematical Democracy").

Tegmark argues that Level IV follows of necessity from his Mathematical Universe Hypothesis. If a structure is a universe, then any structure that contains self-aware parts is a universe to those parts, as our structure is to us. On the Mathematical Universe Hypothesis a structure needs nothing added to it to be a universe. The physicist Stephen Hawking once asked what breathes fire into the equations and makes a universe for them to describe. Tegmark answers that the equations need no fire, because "the point isn't that a mathematical structure describes a universe, but that it is a universe" (chapter 12, "How the Mathematical Universe Hypothesis Implies the Level IV Multiverse").

Then Tegmark explores what Level IV holds. The structures closest to our own have other symmetries in place of the symmetries of the standard model of particle physics (the theory of every kind of particle so far found), and so they have other particles and other forces. Some of the structures closest to ours have no light, some have no gravity, and some have other numbers of directions of space and time. Structures farther from ours have space and time that come in jumps, such as the computer game Tetris, or the mathematician John Conway's Game of Life, each run with every input from a player switched off. Structures farther still have no time at all, such as the dodecahedron, inside which nothing happens. Tegmark shows how a computer program can list every structure with a finite number of elements, giving each structure its own line number, and he calls the list the *master list*. Tegmark gives a second listing too, which orders the structures by the length of the shortest program that defines each structure, so the second listing runs from the simplest structure to the most complex. Tegmark writes that if physicists one day identify which structure we inhabit, we can name that structure by its number on the master list, and so, in his words, "specify our address in the full physical reality" (chapter 12, "Our Postal Code in the Level IV Multiverse"). Tegmark notes that if we adopt a popular definition of mathematics as "the study of mathematical structures", then studying Level IV is what mathematicians already do for a living. Tegmark adds, with some delight, that Level IV is easier to explore than our own universe, because exploring Level IV "doesn't require rockets or telescopes" (chapter 12, "The Structure of the Level IV Multiverse").

Tegmark is careful to say exactly what Level IV holds. He compares Level IV with the philosopher David Lewis's modal realism. Lewis's modal realism, as Tegmark quotes it, says that "all possible worlds are as real as the actual world". Tegmark draws the line around Level IV more exactly than Lewis draws the line around his possible worlds.

Level IV contains every structure that mathematics defines, and it leaves out every universe that someone can imagine and no one can define.

Tegmark shows how Level IV changes the meaning of several ideas of physics, one idea at a time. Symmetry keeps its central place in physics: on Tegmark's hypothesis the world has symmetries because mathematical structures have symmetries. Initial conditions are the starting state that the laws carry forward. On Tegmark's hypothesis initial conditions stop being part of the description of the world itself. The reason is that a mathematical structure is defined completely, in every part. So a starting state adds nothing to what the structure already defines. Describing our universe in detail takes close to a googol bits. A bit is one choice between two answers, and a googol is a one followed by a hundred zeros. Tegmark holds that the googol bits are about where we are, and not about the world itself: "this information isn't fundamentally about our physical reality, but about our place in it" (chapter 12, "Initial Conditions Reinterpreted"). Tegmark treats randomness in the same way, as a fact about where we are. Nothing in a mathematical structure is random. So the randomness we meet is how being copied feels, when you are copied into observers who see different things. In Tegmark's words, "apparent randomness is caused by multiple yous" (chapter 12, "Randomness Reinterpreted").

On Tegmark's account the complexity we see is also a fact about where we are. To show why, Tegmark makes a fine point of the theory of information, the mathematics of how many bits a description needs. Write the square root of two in binary digits, and the digits look random, yet a short program writes every one of them. To describe a stretch of the digits that starts far along, you need the short program and also the place where the stretch starts. So the stretch takes more bits to describe than the whole run of digits does. A whole can need less information than one of its parts. Tegmark gives a simpler case: a very short program lists all the whole numbers, 1, 2, 3 and

so on, while the shortest program that prints a typical number a trillion digits long must store nearly all of that number's digits. So the complete list of the whole numbers needs less information to describe than a typical number on the list. Tegmark concludes from the case of the whole numbers that "The complexity is an illusion, existing only in the eye of the beholder" (chapter 12, "Initial Conditions Reinterpreted"). He concludes also that Level IV, taken whole, needs essentially no information to describe.

Last, Tegmark says how his hypothesis can be tested. Tegmark has argued that the External Reality Hypothesis implies his Mathematical Universe Hypothesis, and that the Mathematical Universe Hypothesis implies Level IV (chapter 12, "Testing the Level IV Multiverse"). So, he writes, the most straightforward way to strengthen or weaken the evidence for Level IV is to study and test the External Reality Hypothesis. He then explores two more direct ways of testing his Mathematical Universe Hypothesis and Level IV. The first of the more direct tests is a search for a kind of fine-tuning that would be evidence for Level IV: many independent conditions for life, like no-parking signs set at random along a street, which all happen to leave one place to park. Each condition rules out a range of values of a constant of nature, as each sign rules out a stretch of the street, and together the conditions leave one narrow range in which life is possible, as the signs leave one place to park. The second of the more direct tests is about the long success of mathematics in physics. The physicist Eugene Wigner called the success of mathematics in physics unreasonable. Tegmark explains the success of mathematics in one sentence: "our successful theories aren't mathematics approximating physics, but mathematics approximating mathematics" (chapter 12, "The Mathematical-Regularity Prediction"). Tegmark then states the prediction that follows from his explanation: "One of the key testable predictions of the Mathematical Universe Hypothesis is that physics research will uncover further mathematical regularities in nature"

(same section). He also names one finding that his hypothesis cannot allow. The finding would be a convincing demonstration of fundamental randomness in the laws of nature.

Tegmark's account of Level IV is a whole picture, built with care. A reader with no mathematics can follow each step of the account. Tegmark's master list is an exact construction of mathematics. Tegmark draws a sharper line than Lewis draws with modal realism, by his rule that Level IV contains only what mathematics defines. And Tegmark is right that a whole can need less information than its parts. The claim that a whole can need less information than its parts is a correct result of the theory of information. Tegmark's example of the whole numbers lets anyone check the claim.

Level I is the regions of our own space beyond the farthest light that has yet reached us.

III. What the Two Accounts Hold Together

Which of Tegmark's results does this book share, and on what ground? I share many of Tegmark's results, and I reach each shared result from premises of my own. The premises of this book come from my physics books of the Immanent Metaphysics. Every measurement and every interaction is a comparison, and a finding is what one comparison gives.

The first shared result is the one that Hardy's sentence takes for granted: mathematical structures are discovered. I make the point with the Greek mathematician Euclid's proof, about 300 BC, that the prime numbers never end. In the same way, the number 31 did not become prime when someone first divided it. My reason is my account of what mathematics studies. Mathematics studies shape, space, structure and number: objects that do not change, whose truths hold the same at every moment.

Tegmark reaches the same result from the way mathematicians discover their structures one by one, while the words and symbols in which they write the structures vary from one discoverer to the next.

The second shared result is stronger than a reader of Tegmark might expect. Every structure on Tegmark's master list exists, in this book's sense. In the chess game of chapter 1, what exists is what the score sheet holds, the written list of the game's moves. Pi stands as structure, the same in every instance, and that is what it is to exist. Tegmark holds that all structures that exist mathematically have the same standing. I agree: every structure on the master list exists, and every structure on the list exists in the same way. Tegmark's account and mine part over a second word, "real", which this book uses in a sense of its own. The next section gives this book's sense of "real".

The third shared result is the one Tegmark offers as a prediction. Tegmark's account and mine both expect physicists to go on finding mathematical regularities in nature, and each account has its own explanation of why mathematics has served physics so well. I explain the success of mathematics from premises of my own. I hold that the mathematics of a theory and the findings the theory describes are two aspects of one act of comparison. The mathematics can come as close to the findings as anyone wishes. That dense reach is the effectiveness of mathematics in physics. My own predictions include regularities of exactly Tegmark's kind: pure numbers that measurements are now checking. Every regularity that physicists find agrees with both accounts. The two accounts differ on one further claim. I hold that a mathematical description never coincides with the thing it describes, however close the description comes.

The fourth shared result is the premise from which Tegmark starts. There is a world, and it does not depend on us. The premise is this book's plain statement of Tegmark's External Reality Hypothesis, and Tegmark says that studying and testing the premise is the most straightforward way to strengthen or weaken the evidence for Level IV. I hold the premise on my own ground: every interaction is a comparison, so the world was comparing itself, in every collision of particles, long before anything hu-

man looked. The two accounts part at Tegmark's next step, from a world independent of us to a world that is mathematics. Chapter 10 set out Tegmark's step in full. Level IV is Tegmark's step from a world independent of us to a world that is mathematics, taken for every mathematical structure at once.

IV. A Book of Every Game

Think back to the kitchen table of chapter 1, where two people play chess and a score sheet lies beside the board. Each move is written on the score sheet once it is made. Now picture a book on the shelf above the table: a book that lists every game of chess the rules allow, every score sheet that could ever be written, numbered from the shortest game to the longest. The book already contains, line by line, every game that will ever be played at the kitchen table. The book exists: it is laid out whole, and the book is the same for anyone who opens it. Every game is played at the table, one move at a time, and no game is played in the book. The book already contains the line that will record the next move. Yet the move itself is made only at the table, and the move is not yet on the score sheet.

Tegmark's master list is the book of every game, written for all of mathematics. Every entry on the master list exists, as every game in the book exists. I ask whether an entry on the master list is also a world, just because the entry exists. A world, in that question, is a place where something happens and something is found. I give my own sense to two words: "exist" and "real". The sense this book gives to "real" shows whether an entry is also a world. In this book, what is real is what is given in an actual act of comparison: a finding, made once, at the table, as a move is made. In his hypothesis Tegmark takes the mathematics for our whole world. In Level IV he takes the mathematics for the whole of every world.

A structure is not a world because of the Incommensuration Theorem, which chapters 1 and 10 set out. Continuity means having no gaps. Mathematics can lay out a whole structure at once because the frame of math-

ematics (its way of describing) sets aside the one who compares, and so sets aside time. Mathematics can lay out structures that no comparison could ever find. The power to lay out structures that no comparison could find is a power of mathematics, and it is exactly the power that marks its objects as laid out and not found. I then draw the conclusion for Level IV: "Structures that no comparison could ever find are not further worlds".

A second reason against Level IV concerns claims about everything taken as one. Tegmark proposes that mathematical structures, formal systems and computations are three aspects of one underlying structure. A formal system is a list of starting statements with fixed rules for proving new ones. A computation is a step-by-step procedure that follows fixed rules. Tegmark holds that the one underlying structure is both the whole of what exists mathematically and the whole of what exists physically. Chapter 2 gave my answer as a picture: Tegmark's ladder of wholes, in which each whole is a part of a greater one, has no top rung, so there is no last whole that contains every other whole. I hold that existence is always relative to a particular world. In that sentence, to exist is to belong to some one world, which is the sense of the aphorisms and not this book's sense of laid out whole. So whatever exists, exists relative to some one world. There is no one world relative to which every other world either exists or does not exist. When someone says that only one world exists, or that many worlds exist, the speaker uses the word "exist" beyond the range in which the word has a meaning. I call any claim about everything taken as one a *totalization*, and there I give a rule for totalizations: a totalization would need a vantage (a place to look from) outside the whole, and the whole has no outside. A totalization claim therefore has no possible comparison behind it, even in principle, and what cannot be compared even in principle cannot be the basis of any assertion. Tegmark's short program for the whole numbers is exact mathematics, and I accept the program in full, as mathematics. Tegmark then steps from the program to the claim that Level IV, taken whole, needs essentially no information to describe. Tegmark's claim about Level IV taken whole is a claim about everything

taken as one: a totalization. I hold each structure of Level IV one structure at a time, as mathematics, and I rest each of my own assertions on a comparison that could be made.

I give Wheeler's question, "Why these particular equations, not others?", an answer of another kind. I hold that Tegmark's structures closest to our own, with their other symmetries and other particles, exist as mathematics. I derive the structure of the known particles from three sources. The first source is the three axioms of comparison: the three starting statements about what any comparison is, on which I build. The second source is the shape of the six held internal dimensions that the axioms select. The shape of the six dimensions fixes the kinds of particle (chapter 4 set them out). The third source is a few *inputs*: statements that the derivation takes as given, set down beside the derivation, such as the statement that every point of the six held internal dimensions is like every other point. From the three sources I derive the number of forces, the symmetries of the standard model of particle physics, the kinds of matter particle and the *Higgs field*. The Higgs field is a quantity with a value at every point of space. The value at which the Higgs field settles gives many of the smallest particles their masses. I claim that the structure of the particles it derives is the only structure consistent with the axioms and the stated inputs. Chapter 7 set out what the landscape is. There is no landscape, or rather, the landscape has been reduced to a single point.

I explain the count of directions of space and time in the same way, from comparison: I draw one direction of time and three directions of space from the structure of comparison. For the single direction of time, I cite Tegmark's own analysis of 1997 as support from the equations of physics. Tegmark's analysis shows that, with more than one direction of time, the equations by which waves travel can no longer carry a starting state forward in a well-defined way. In both accounts the count is three directions of space and one of time, but the grounds differ: Tegmark explains the count by the presence of observers, and I explain the count by the structure of comparison.

Tegmark's case of the no-parking signs concerns a number related to the Higgs particle. The Higgs particle is the particle whose presence shows that the Higgs field is there. I take a number of the Higgs field, the field's vacuum value, from measurement, as I take several other numbers. Tegmark would explain four of the numbers that I take from measurement, through Level II, as the values of our region. The four numbers are Newton's constant, the strength of light, the strength of color and the vacuum value of the Higgs field. Newton's constant sets the strength of gravity, and the strength of light is this book's plain name for the number that measures the strength of the electromagnetic force, the force between electric charges, which light carries. The strength of color is how strongly the force that binds quarks acts, and the vacuum value of the Higgs field is the value at which the field settles in empty space. Chapter 7 sorted the constants of nature into rules, positions and measured values. I hold each of the four numbers as a measured value of this one world. I explain none of the four numbers as our region's value among the values of many regions, a value picked out because observers can live only where the value allows them to live.

In the order of depth that chapter 10 set out, metaphysics comes first, then physics, then computation, then mathematics, each standing on the one before. Chapter 10 placed mathematics last in the order of depth, with every power that mathematics has. Every structure Ramanujan discovered exists, exactly, timelessly, and the same for everyone, and so does every structure on Tegmark's master list, and so does every game in the book above the table. The world is where the moves are made: the world is where findings are made, one act of finding at a time.

V. Address, Chance and Complexity

Tegmark reinterprets three things: initial conditions, randomness and complexity. Each time, he takes some part of what we find out of the world and places that part in our location in the world. I keep initial conditions, randomness and complexity in the world, and I give a reason for each of the three.

Initial conditions come first. I ask a question of any theory that describes the world wholly from outside, as a laid-out structure: where does the information in the theory's starting point come from? A theory that derives everything from a small set of rules and starting conditions must still take those rules and conditions as given. I hold that no theory that derives everything from rules and starting conditions can say where its own rules and starting conditions come from: "That question cannot be answered from inside the theory". Tegmark answers the question by placing the starting information in our address. On his account, the starting information says where we are in Level IV. I hold that some information is always left over, and that no rule and no starting condition fixes the information left over. The information left over is hard randomness.

Chapter 8 set out what hard randomness is. Hard randomness is an outcome that nothing before it fixed, real in each single finding. I set hard randomness apart from the randomness of a coin toss, which is ignorance of causes that were there. Hard randomness is real, and the single outcome of a quantum measurement is its plainest instance. Hard randomness is real because of the Incommensuration Theorem. A whole world completely fixed by one law would hold the same law at every place and every time, which is sameness perfected, and it would leave no gap anywhere between one state and the next, which is gaplessness perfected. I draw the consequence for every account that describes a whole world as completely fixed by one law, Everett's included.

Experiments made over four decades bear on whether hard randomness is real. In 1964 the physicist John Bell considered two particles prepared together and then measured at two stations far apart, each station with a setting chosen by its experimenter. Bell showed that if every outcome were fixed by causes the particles shared from the past, or by a signal passing between the particles no faster than light, then the results at the two stations could agree only up to a certain limit. Physicists now call Bell's limit *Bell's inequality*. From the 1970s on, the physicists John Clauser, Alain Aspect and others ran experiments that found the limit broken: the results at the two stations agreed more closely than Bell's limit allows. Physicists who doubted the results named two ways in which the early experiments might have misled: detectors that missed too many particles, and stations close enough for a signal to pass between them. In 2015 three groups ran experiments that ruled out both ways of misleading at once, at Delft in the Netherlands, at Vienna, and at the National Institute of Standards and Technology in the United States. At Delft the particles were electrons, each held in a small piece of diamond, at stations 1.3 kilometers apart, while at Vienna and at the National Institute of Standards and Technology the particles were particles of light. In all three experiments Bell's inequality was still broken. In 2022 Clauser, Aspect and Anton Zeilinger shared the Nobel Prize in Physics, for experiments "establishing the violation of Bell inequalities". Zeilinger's group had made the test at Vienna. We now know that the agreement nature shows between separated stations is more than any passing signal can carry, and more than any shared past can carry while the experimenters choose their settings freely. I hold that two positions are still consistent with the tests of Bell's inequality. The first position, called *superdeterminism*, holds that everything, the experimenters' settings included, was fixed together at every scale from a common past. The second position holds that some of what happens is caused by nothing before it. I weigh the two positions by the rule of preferring the simpler account. Superdeterminism needs the whole past, at every scale, to have been arranged so that the experimenters' choices conspire with

the particles. The second position needs only that some events have no cause. I take the second position, and say so in two short sentences: "We accept hard randomness. We live with it".

So on my account fundamental randomness is real, and fundamental randomness is the one finding that Tegmark's hypothesis cannot allow. I also say where hard randomness is met. From a purely third-person standpoint (from outside), hard randomness gives the same statistics as the randomness that comes from not knowing the causes, and "no pattern tells them apart". The difference between the two kinds of randomness is real: the difference is given to the one who makes a finding, in the act of finding itself, at the present moment of that act. A table of results, read from outside, shows the pattern. But the outcome that nothing fixed is given in the finding that produced the outcome. Tegmark's own chapter contains an instance of hard randomness: he made the random-looking pattern printed in his chapter with a quantum random number generator, and he says the pattern probably needs every one of its bits to describe. On my account each bit of Tegmark's pattern is a finding that nothing before it fixed.

Complexity is the third of Tegmark's reinterpretations. I separate two words that people often use as if they meant the same: complicated and complex. A complicated system has many parts whose relations can all be stated in finite terms. The unpredictability of a complicated system is a lack of data. A complex system has some property that no data could predict, because its changes are continuous and carry an element that no law fixes. The difference between a complicated and a complex system is a difference of being, not of knowledge. The digits of the square root of two are complicated, and not complex. Tegmark's short program shows why: the short program states in finite terms every relation among the digits. The weather shows how an element that no law fixes enters what we see. The course of the weather depends on differences in how the weather starts, differences that can be made as small as one likes. I follow the small differences in how the weather starts below the limit of every measurement, where the residue of hard randomness "seeps upward through every pro-

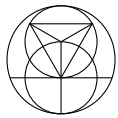
cess that amplifies what it cannot resolve". Tegmark places complexity "only in the eye of the beholder", while I place the complexity of a complex system in what the system is.

VI. What This Chapter Holds

Every mathematical structure exists, laid out whole, made by no one, and the same for everyone, and a structure that no comparison could ever find is laid out and not found, so the structure is mathematics, and it is not a further world.

The answer to this chapter's question bears on the book's question: What is reality, and can physics find out? Level IV is the Mathematical Universe Hypothesis applied to every structure at once, so Level IV fails along with the Mathematical Universe Hypothesis. I keep every structure as mathematics, and I place the world where findings are made. The equations of our particles have the structure they have (the forces, the symmetries, the kinds of particle) because comparison, with a few stated inputs, allows that structure and no other. The rest of mathematics exists, as fully as Ramanujan's formulas do, as mathematics.

A computer program writes out Tegmark's list of structures, and Tegmark goes on in his chapter to ask whether we are living in a simulation, a computation built to copy what some physical thing does. If the world is not a structure laid out whole, could the world be a rule carried out step by step, or a simulation run on someone else's machine?



Chapter 14

Is the World a Computation?

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

The *order of depth*: metaphysics, then physics, then computation, then mathematics, each resting on the one before; mathematics stands last, and keeps every power it has (Chapter 10).

Hard randomness: an outcome that nothing before it fixed, however completely everything before the outcome is known (Chapter 8).

The *bound triple*: the arrow of time, hard randomness and the subjective, bound into one act: distinct, inseparable and not interchangeable (Chapters 9 and 12).

I. A Remark in Königsberg

Could our world be a computation, a rule worked step by step on a vast store of numbers? Could our world even be a simulation running on someone else's computer? Max Tegmark takes up both questions in his chapter 12. On the question whether we are living in a simulation, Tegmark bets his money on "No". We begin at a conference in East Prussia.

Philosophers and mathematicians met for a conference in Königsberg from the fifth to the seventh of September 1930. The conference was on the theory of knowledge in the exact sciences. On the last day, 7 September, the people at the conference held a round-table discussion on the foundations of mathematics. The people who took part were Hans Hahn, Rudolf Carnap, Heinrich Scholz, John von Neumann, Arend Heyting, Kurt Reidemeister and a young logician from Vienna, Kurt Gödel. Near the end of the discussion Gödel made a short remark. Gödel said that if classical mathematics is consistent, never proving a statement together with its opposite, one can give examples of statements about whole numbers that are true and yet cannot be proved within the formal system of classical mathematics. A formal system is a list of starting statements, called axioms, together with fixed rules for passing, one step at a time, from statements already proved to new ones. Gödel's examples were statements of the same kind as Goldbach's famous question, whether every even number above two is the sum of two primes. Von Neumann was the only one at the table who took up Gödel's remark. Von Neumann spoke with Gödel privately when the session ended.

On the next day, also in Königsberg, the mathematician David Hilbert addressed the yearly meeting of the Society of German Natural Scientists and Physicians. Hilbert had called on mathematicians to set the whole of mathematics on one foundation from which every question of mathematics could be settled. Hilbert closed his address with the words "We must know. We will know". Gödel's paper with the full proof reached the

journal *Monatshefte für Mathematik und Physik* on 17 November 1930. The journal printed the paper in 1931. Gödel had found a limit on listing. Where a consistent list of axioms is rich enough to state the arithmetic of whole numbers, and is worked by fixed rules one step at a time, some true statements in the list's own language are never reached by any chain of steps, so those true statements are never proved.

Come back now to the kitchen table of this book, where two people sit at a chessboard with the score sheet beside the board. Four parts of the game each keep the one meaning that chapter 1 gave them: the score sheet, the move, the seat and the rules. The score sheet is the structure, laid out whole. The move is the act of finding, made once. The seat is the stance, the side from which a move is made. The rules are the laws, the same at every table.

Now set a small chess computer on the table beside the board. The chess computer lists the legal moves from the position and shows a suggestion on its screen. When the chess computer lists the moves, it carries out a *computation*. A computation is a procedure that goes, step by step and by fixed rules, from one state to the next. Each state of a computation is a finite string of symbols, such as ones and zeros. The chess computer's *program* is the written set of the fixed rules that its computation follows. The chess computer's listing of moves is a *run*: a computation being carried out. Current from the socket in the kitchen wall flows through the circuits of the chess computer, one step after another. The chess computer is in the room, on the same table as the board, and the chess computer is warm from its own working. This chapter's question can now be put in the picture: could the whole room, players, board and all, be inside a machine running a simulation?

From Tegmark's pages

II. What Max Tegmark Showed

Tegmark's chapter 12 sets out the Level IV multiverse, his proposal that every mathematical structure exists as a world, and Tegmark makes the proposal because, on his Mathematical Universe Hypothesis, our own world is one mathematical structure. Within the Level IV proposal, Tegmark asks a sharp question that many writers on the subject pass over. Which structures are defined well enough to count as worlds? Tegmark shows why the question matters with three rules for whole numbers. The first rule is to find the next prime after a given number, and a computer can carry out the first rule and is sure to stop, because Euclid proved that the primes go on without end. The second rule is to find the next twin prime: a prime with another prime two places away, as 11 and 13 are. No one yet knows whether the search for the next twin prime always stops. The third rule is to decide, of any program, whether the program will ever stop: a question called the halting problem. In 1936 the mathematician Alan Turing showed that no program answers the halting problem for every program, while in the same year the logician Alonzo Church reached a result of the same kind by another route. Tegmark ties the halting problem to Gödel's discovery "that some theorems of arithmetic are undecidable" (chapter 12, "Limits of the Level IV Multiverse: Undecidable, Uncomputable and Undefined").

Tegmark then takes up the real numbers: the points of the number line, such as the number pi. The decimals of pi go on without end. Tegmark says that to specify a typical real number one must give endlessly many decimals, "an infinite amount of information" (chapter 12, "Limits of the Level IV Multiverse: Undecidable, Uncomputable and Undefined"). Tegmark uses the three rules and the real numbers to build a ladder of five answers to the question of which structures count as worlds. The answers range from no structures at all up to structures built on the smooth number line. The smooth

number line is the number line with every real number on it and no gaps. Tegmark calls one rung "An interesting possibility" and names it the Computable Universe Hypothesis: "The mathematical structure that is our external physical reality is defined by computable functions" (chapter 12, "The Computable Universe Hypothesis"). In plain words, a procedure that is sure to stop can work out every relation in the structure that is our world. Tegmark's Finite Universe Hypothesis is a still stricter rung: the structure that is our world has only finitely many parts. Tegmark likes the computable rung because the Computable Universe Hypothesis would make our world well defined in every relation: "I have no doubt that nature knows what it's doing!" (chapter 12, "The Computable Universe Hypothesis").

Tegmark names the more immediate of the challenges to the Computable Universe Hypothesis. The challenge is that nearly every successful theory of physics, the standard model of particle physics included, is written with the smooth number line, and the smooth number line cannot be fed to a procedure that must stop. Tegmark sets an observation beside the challenge: "we've never measured anything in physics to more than about sixteen significant digits" (chapter 12, "The Computable Universe Hypothesis"). So Tegmark finds worth exploring the idea of "abandoning the continuum as fundamental and trying to recover it as an approximation" (chapter 12, "The Computable Universe Hypothesis"), as the smooth equations of a flowing liquid are an approximation to a crowd of atoms. Tegmark's word continuum names the infinitely small: the idea that every stretch of space holds infinitely many points, and that some quantities in nature can vary smoothly, with no least step. In his chapter 11 Tegmark had already named the infinite as the suspect: "Here's my prime suspect: ∞" (chapter 11, "Infinite Problems"). In his chapter 13 Tegmark describes the Big Snap, a breaking of space under too much stretching, should space prove to be grainy. The Big Snap comes from Tegmark's suspicion of the infinite.

Tegmark also draws a distinction that many writers on computation and the universe blur. Some writers have proposed that the world steps forward in time like a computer program, among them the computer pioneer Konrad Zuse and the physicist Stephen Wolfram. Tegmark's Computable Universe Hypothesis requires only that the relations of the structure that is our world be computable. On the Computable Universe Hypothesis computations "don't evolve our Universe, but describe it by evaluating its relations" (chapter 12, "A Different Sort of Computation").

Then Tegmark asks whether we are living in a simulation. In his chapter 12 Tegmark takes up an argument associated with the philosopher Nick Bostrom: you are probably simulated, because simulated people would far outnumber the people who are not simulated. Tegmark holds that "this argument logically self-destructs" (chapter 12, "Are We Living in a Simulation?"). Tegmark's reason is that your simulated copies could make Bostrom's argument too, and so could the simulated copies of your simulated copies, down a ladder of simulations within simulations with no bottom rung.

Tegmark makes a second point, about time. General relativity, Einstein's theory of gravity, has no single clock for the whole universe. And a simulated person's sense of how fast time passes does not depend on how fast the computer runs. So Tegmark concludes that "the role of the simulating computer isn't to compute the history of our Universe, but to specify it" (chapter 12, "The Time Misconception"). A complete description of a world might fit on a memory stick, and Tegmark finds it absurd that a memory stick should decide whether a world exists "for real" (chapter 12, "Does a Simulation Really Need to Be Run?"). On Tegmark's Mathematical Universe Hypothesis, what he calls our physical reality exists whether or not anyone writes a program that simulates our physical reality. So when asked whether we are simulated, Tegmark answers: "I'd bet my money on 'No!'" (chapter 12, "Does a Simulation Really Need to Be Run?").

Chapter 13 of this book follows the proposal step by step.

III. The Same "No", on Separate Ground

I derive Tegmark's answer as a result. We now know that our world is not a simulation run in some other world. Tegmark reaches "No" as a bet. Tegmark's bet rests on his Mathematical Universe Hypothesis. The bet also rests on a guess about how little running a simulation would add to a world that, on his hypothesis, exists already. I derive "No" by two lines of argument that share no step, and I call the two lines of argument two roads.

The first road runs through randomness. A computation is *deterministic*: the state of a computation and its rule fix its next state. When a computation is run twice from the same start, the computation goes through the same steps. In this book's sense (chapter 8), hard randomness is an outcome that nothing before the outcome fixed, however completely the state before the outcome is described. So, because a computation is deterministic, a computation has no hard randomness of its own. Computers do make numbers that look random, and most of those numbers are *pseudo-random*: made by a fixed recipe from a starting number, so that the same start gives the same numbers again.

Chapter 12 of this book named *the bound triple*: the arrow of time, hard randomness and the subjective, bound into one act. The subjective is the stance, the side from which a comparison is made. In the bound triple, none of the three parts is present without the other two. A simulated world whose randomness is pseudo-random therefore has no hard randomness. So, by the bound triple, the simulated world also has no arrow of time and no stance. The simulated world has a numbering of its states in place of an arrow of time. A numbering of states is like the page numbers of a finished book, which read as readily from the last page to the

first as from the first to the last. A simulated world whose randomness is pseudo-random, and which therefore has no hard randomness, is a program running on a machine with no one in it.

Suppose someone proposes that our world is a simulation. What is the fullest reply open to that person? The proposer can feed the simulated world hard randomness, drawn from a physical device in the world that runs the computer, and the simulated world then has the randomness of the world that runs the computer as its randomness, carried through the computation. By the bound triple, the simulated world's arrow of time goes with its randomness, and so the simulated world's arrow of time is the arrow of time of the world that runs the computer. The computer's own running, one step after another that cannot be undone, is the arrow of the running world at work. The bound triple cannot be carried across in part. In plain words, a simulated world takes all three parts of the bound triple from the world that runs the computer, or none of them. Where the randomness and the arrow are shared, the stance is shared as well. So a simulated world with people in it is a region of the one world in which the computer runs. The region is reached through the computation.

Picture a prison cell: the walls of the cell limit what the person inside can see, yet the person inside still breathes the same air and ages by the same clock as everyone outside. A computation restricts what can be reached from inside it, as a prison cell restricts what its occupant can see. The computation is the cell. The computation limits what the people of the simulated world can reach, and those people remain in the one world in which the computer runs.

The road through randomness shows a fault in Bostrom's argument that differs from the fault Tegmark finds, and both faults are there at once. Bostrom set out three choices in a paper of 2003, and he held that at least one of the three must be true. The first choice is that nearly every civilization dies out before it can run simulations of people like us. The second choice is that nearly no civilization able to run simulations of people like us wants to run them. The third choice is that we are almost certainly living in a simulation. Bostrom reaches the third choice by counting simu-

lated observers as if each simulation were a separate world with observers of its own. Tegmark finds that Bostrom's count runs down a ladder with no bottom rung. Bostrom's count uses the wrong unit: the third choice rests on a count that treats as separate worlds what are regions of the one world.

The second of my two roads runs through continuity, which is having no gaps, and the second road starts from a plain fact about lists. When a group can be put in order, first, second, third, with nothing left out, the group is *listable*. The counting numbers are listable. The programs that could ever be written are listable too. Everything a computation could ever produce is listable, even on Turing's ideal machine, a computer imagined with unlimited time and an endless tape. In 1874 the mathematician Georg Cantor proved that the points of the number line cannot be listed. In 1891 Cantor gave the diagonal argument, and with the diagonal argument Cantor showed why the points of the number line cannot be listed. Suppose someone claims to have a list of every number between zero and one, each number written as a decimal without end. Build a new number whose first digit differs from the first digit on line one, whose second digit differs from the second digit on line two, and so on down the list. The new number differs from every line of the list in at least one place, so the new number is not on the list. For any list, one can build a new number that differs from every line of the list, so no list could hold every number between zero and one. The smooth whole that no list reaches is the *continuum*. The continuum always holds another value between any two of its values, and the continuum has no gaps anywhere.

The arena is space and time as found in comparisons (chapter 2). The arena has no least step, and chapter 8 gave my reason: every difference found in a comparison holds some sameness within the difference. A least step would be a jump between two places, or between two moments, with no sameness inside the jump. A jump with no sameness inside it would be a difference that could not be found in any comparison. A computation has a least step in everything it represents. The least steps are the gap

between two neighboring points of the computation's grid and the tick between two of the computation's moments. So reality holds what no computation, which is a listing, can reach.

The road through randomness shares no step with the road through continuity. The first road passes through hard randomness and the act of finding, while the second road passes through the continuum and what no list holds. Both roads arrive at Tegmark's "No".

I also agree with Tegmark on two smaller points. First, Tegmark holds that what he calls our physical reality exists whether or not anyone runs a program of it. I agree that the world goes on whether or not anyone runs a program of the world. I agree on a ground of my own. Rain falls, tides rise and stars burn whether or not anything anywhere is computed. Tegmark guesses that running a simulation would add at most a tiny effect to a structure that, on his hypothesis, exists already. On my account, the world needs no computation, because every computation needs the world. Second, Tegmark holds that physical time is not the ticking of a computer. I agree. The arrow a computation seems to have is lent to it by the physical process that carries it out.

IV. Physics Comes Before Computation

I exclude the idea that the world, at its foundation, is a computation. It is excluded on three independent grounds, and each of them turns on what a computation is. The three grounds are determinism, the two levels of every exact description (its mathematics and its findings), and the bound triple. The three grounds apply to every proposal that the world, at its foundation, is a computation, such as the proposals of Zuse and Wolfram that the world steps forward like a program. Tegmark's Computable Universe Hypothesis requires that the relations of our world be computable. So the hypothesis bears on the claim that the world is a computation. I answer Tegmark's restriction to computable structures from the order of depth, in which physics stands before computation: "A computable

structure is a structure some run could produce, and every run is a process in the world, so restricting the mathematical universe to computable structures restricts it to what the world can do, and cannot make the world one of them". (Chapter 10 gave my reason why the world is not a mathematical structure in the first place: every mathematical structure, taken whole, holds perfect sameness and perfect gaplessness together, and no single act of finding can.)

The first ground is determinism, the claim that every state of a whole world is fixed completely by the states before that state. Chapter 13 of this book showed that complete determinism of a whole world requires two things, both perfected together. The first requirement of complete determinism is the same rule carrying every state into the next state, at every place and every step: sameness perfected. The second requirement of complete determinism is that each state passes into the next state with nothing unfixed between the two states: gaplessness perfected. Symmetry is the theorem's word for sameness, and continuity is the theorem's word for gaplessness. So no whole world is purely and completely deterministic. A world computed at its foundation would be purely and completely deterministic, and so no world is computed at its foundation. Instead of complete determinism, every world has cause: one thing found to follow another, in a one-way order of findings.

The second ground is about the two levels of every exact description, and every theory of physics has both levels: its mathematics, written down as structure, and its findings, made by comparison. Each of the two levels has one of the two characters named in the Incommensuration Theorem. One level has gaplessness, and the other level has sameness. In quantum theory the equations are smooth and gapless, and the findings have sameness: counts that repeat exactly, one click at a time. In Einstein's theory of gravity the laws are the same for every observer, however the observer moves, and the findings are gapless: a smooth arena. Picture a world computed at its foundation as a grid of cells, with one rule applied at every cell and every step. A computed world would have sameness at both levels: a rule the same everywhere, and findings that come in steps

and repeat exactly. Where a thing is compared with something of exactly its own character, nothing new turns up, and so a world with sameness at both levels could find only what its rule already held. Yet the world that physics finds holds outcomes that nothing before them fixed.

The third ground is the bound triple of section III: the arrow of time, hard randomness and the subjective, which come only together. A computed world has no hard randomness of its own, and so a computed world has no arrow and no stance of its own. The three grounds start from three different places, the Incommensuration Theorem, the two levels of description, and the binding of arrow, randomness and stance, and the three grounds confirm one another. Each ground rests on what a computation is, so none of the three grounds needs any experiment, such as an experiment made by people inside a simulation to find out whether they are in a simulation.

The three grounds depend on what I call *the order of depth* (chapter 10): metaphysics, physics, computation, mathematics, each of the four disciplines resting on the discipline that comes before it. Take two disciplines. Suppose the second cannot be carried out, stated or checked without the first, while the first can be carried out without the second. Then the first discipline is deeper than the second. Physics is deeper than computation, because every computation that is carried out is a physical process, in circuits, in nerve cells or in ink, and physics needs no computation to happen. Computation, for its part, is deeper than mathematics, because every proof is checked by a run, step after step, and no run waits on a proof: beads moved along the wires of a counting frame add two numbers whether or not anyone has proved anything about addition. Mathematics keeps every power it has. Mathematics stands last in the order of depth.

The chess computer on the kitchen table shows the order of depth at a glance. The chess computer's listing of moves is a run, and the run is current from the wall, heat in its housing and light on its screen, all happening in the room. The game at the board needs no machine: the move is made by a hand, from a seat at the table. Suppose someone proposes that the whole kitchen with its players, board and chess computer is itself run-

ning in a larger machine somewhere else. The larger machine would be running too, on current, in a room of its own. So the person who proposes the larger machine has moved the room somewhere else, and there is still a room. Every computation stands inside a world, as the chess computer stands on the table, and no world stands inside a computation.

I now take up Tegmark's step that a simulating computer need only specify our history, and never compute it. A program can indeed be described with no time in it. The description is a lookup table that pairs each input with an output, all the pairs at once. At each input the lookup table has an entry or has none, and the lookup table takes no steps. Tegmark uses the halting problem to mark the edge of what is well defined. Only a run takes steps, so only a run can stop or go on without stopping. So the halting problem is a question about runs: stopping belongs to the run, and the halting problem takes its content from the run. A complete description on a memory stick is laid out whole, as the lookup table is: the description exists, the same for everyone, as the score sheet does. What is real is given in acts of finding, once, as the move is. The memory stick holds the score sheet of a game, and the game is played at the table.

Tegmark hopes that his Computable Universe Hypothesis would remove "the concern that the work of Church, Turing and Gödel somehow makes our world incomplete or inconsistent" (chapter 12, "The Computable Universe Hypothesis"). I derive the result that Gödel announced at Königsberg as one case of the Incommensuration Theorem: the case met inside formal systems. When every route of proof to a statement agrees about the statement, the formal system is consistent. Consistency is sameness across different routes, which is a symmetry. When every true statement in a formal system's language has a proof, the system is complete: no truth is left stranded. Completeness is having no gaps, which is a continuity. Where a formal system can state the arithmetic of whole numbers, and a fixed procedure can list its axioms, the formal system cannot be both perfectly consistent and perfectly complete. Perfect consistency is perfect sameness, perfect completeness is perfect gaplessness, and no act of comparison holds perfect sameness and perfect gaplessness together.

Gödel's result is an exact case of the theorem: Gödel's incompleteness theorem is an instance of the ICT in the domain of formal systems (the letters ICT stand for the Incommensuration Theorem). So incompleteness belongs to every description, and incompleteness is no defect of the world. The remainder is what any description leaves unfixed. So the remainder is a feature of every description that knowledge can give. The world needs no restriction to computable structures in order to be well defined. The world is real in every act of finding. Every description of the world approaches the world as closely as one wishes without containing the world.

V. The Smooth Arena and the Sixteen Digits

Tegmark observes that no measurement in physics has given more than about sixteen significant digits, and he offers the observation in support of one idea that he finds worth exploring. The idea is that the continuum is not fundamental and is to be recovered as an approximation. Every measurement is a finite comparison, so every measurement ever made has given finitely many digits. Even the finest comparisons ever made have results with finitely many digits. I tell of two clocks that the physicists Chou, Hume, Rosenband and Wineland compared with each other in 2010. Each clock ticked with the light absorbed by a single aluminum ion. When the physicists raised one clock by 33 centimeters, the raised clock ran faster by about four parts in a hundred million billion. Four parts in a hundred million billion is the amount relativity predicts for a change of height of 33 centimeters. Yet the laws of physics are written with the smooth number line. The Schrödinger equation of quantum theory is written with the smooth number line, and so are Einstein's equations of gravity. Every law of the world that physics has so far been able to state is written in the mathematics of the continuum. The finite findings and the smooth laws fit together, and I give the reason they fit. A simulation of space and time can be refined to come as close as one wishes to what space and time are found to be, and no stage of it ever contains

them. So I draw the opposite of the idea Tegmark finds worth exploring, and from the same fact, that every finding has finitely many digits: a grid is the approximation, and the continuum is what the measurements approach. A fine enough grid can match every measurement made so far, but the match does not make the world a grid.

Weather forecasting shows how a grid is used to approximate smooth physics. The forecasters divide the air into boxes, give each box its temperature, pressure, wind and moisture, and have a computer step every box forward in time from the state of its neighbors. The numbers in the boxes follow the smooth flow of the air closely enough for the forecasters to say whether it will rain tomorrow. A computation can approximate smooth physics in this way; it is then a tool used to describe the world, and not the world's foundation. Physics needs grids and computations that approximate smooth physics, and I use their results everywhere.

Sensitivity to initial conditions is the way the smallest difference in how a system starts grows until the difference rules the whole system. The behavior of the world shows the values of the continuum that no list contains, through sensitivity to initial conditions. The popular picture of the effect comes from the meteorologist Edward Lorenz's work on the weather, published in 1963: the picture of a butterfly whose wingbeat decides where a storm forms weeks later. Suppose a simulation cuts its numbers off at a fixed number of digits, whatever that number is. The simulation follows a sensitive system faithfully for a while. In time the simulation's history departs from the world's history. Sensitivity is the amplifier, and what no list reaches is what it amplifies. In plain words, sensitivity to initial conditions makes differences too fine for any list grow until the differences show in the world.

You may ask about the very small: is there a smallest length, below which space is a grid after all? Physicists build a length called the Planck length from three measured constants of nature: Planck's constant, the speed of light and Newton's constant of gravitation. The Planck length is far smaller than anything any instrument has probed, and physicists often take the Planck length to be a smallest length, but I read the Planck length

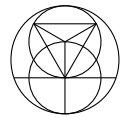
as the inner edge of what can be compared. A boundary on measurement is not a graininess in what exists. Chapter 6 told of a burst of gamma rays recorded in 2009 (gamma rays are light of the highest energies), and the burst sent light of very different energies across billions of years of space. The light of the different energies arrived together. If a grain of space slowed light in proportion to its energy, the light of higher energy would have arrived later. So if space has a grain that slows light in proportion to its energy, the grain lies below the Planck length. In his chapter 13 Tegmark too cites light of different energies from one distant burst, arriving together. Tegmark writes that, taken at face value, the light's arriving together rules out an impending Big Snap for billions of years to come, against what a simple calculation of his had suggested. I derive that space has no least step at all. Measurements of light from distant bursts agree with the derivation. For each coming measurement of how smooth space and time are, the appendix "What the Coming Measurements Will Find" states what my account says the measurement will find.

VI. What This Chapter Holds

We now know that every computation is something the world does, run on a machine standing in a room of the one world, and so the world is neither a computation nor a simulation.

The chapter's answer bears on the book's question (What is reality, and can physics find out?): physics finds reality in acts of finding, and acts of finding come before mathematics and computation in the order of depth. Mathematics and computation are the finest tools there are for describing what is laid out as structure. At the kitchen table, the score sheet and the chess computer serve the game, and the move is made at the table. The chapter's answer also means that the world is not a program anyone could pause or switch off, and that its arena is smooth at every scale, with no grain to break when space is stretched.

If no one can switch our universe off, and its space has no grain to snap, how will our universe end?



Chapter 15

How Will Our Universe End?

Beside Tegmark's chapter 13, "Life, Our Universe and Everything"

What this chapter uses

The *Incommensuration Theorem*: symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. In plain words, no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. The Incommensuration Theorem is a structural fact about the nature of knowing (Chapter 1).

An *epoch* is one long age of the universe, from one turn to the next; the *turn* is the passage from the end of one epoch to the start of the next, and what we call the Big Bang is the turn as it appears in the history we piece together from inside our epoch (Chapter 3).

The *internal space*, or the held internal dimensions: six further directions in which things can differ, held small at every point (Chapter 4).

I. A Paper Read in Edinburgh

How will our universe end? Max Tegmark proposes five endings in the last chapter of his book *Our Mathematical Universe*. Our universe may freeze, crunch, rip, snap or decay. People asked how everything will end long before anyone knew that the universe expands. One of the oldest forms of the question has a date.

On 19 April 1852 a short paper by William Thomson was communicated to the Royal Society of Edinburgh. Thomson was professor of natural philosophy at the University of Glasgow. He was later made Lord Kelvin, and many readers know him as Lord Kelvin. The paper was called "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy". In Thomson's sense, *mechanical energy* is energy that can do work: the energy of a falling weight that can drive a mill, or of a spinning flywheel that can drive a lathe. To *dissipate* mechanical energy is to let the energy spread out as warmth, in a form that can drive nothing more. Rub your palms together. The work of your arms ends as warmth in your hands, and that warmth will never rub your hands back together. Thomson closed his paper with three numbered conclusions. The first conclusion was a single sentence: "There is at present in the material world a universal tendency to the dissipation of mechanical energy". The third conclusion was about the future. Within a finite time to come, Thomson wrote, the earth must again be "unfit for the habitation of man as at present constituted", unless something happens that the known laws of the material world forbid.

If Thomson's tendency runs on to its end, the result is the oldest scientific picture of the end of everything. In that picture every difference of warmth evens out, every store of work is spent, and nothing is left that could make anything happen. Physicists came to call the final state of that picture the *heat death* of the universe, and Tegmark's first ending, the Big Chill, is the heat death in its modern form.

Picture again the kitchen table where this book's game of chess is played. Every game at the kitchen table comes, sooner or later, to its last move. In the picture of the board, Thomson's fear is that the last move of the game now on the board is the last move of all: the pieces are put away, and no game is ever set up again. I hold a different picture. In my picture the game ends, but the rules do not end, and the next game starts from the setting-up again. In the picture, one game means one epoch (one long age of the universe), the rules mean the laws of physics, and the setting-up means the start of the next epoch. Before we look at my picture in full, we take Tegmark's five endings one at a time.

From Tegmark's pages

II. What Max Tegmark Showed

The first thing Tegmark shows is that the question of the end is two questions, and he keeps the two questions apart. One question is about physical reality as a whole, which on his Mathematical Universe Hypothesis is a mathematical structure. A mathematical structure exists outside space and time, so physical reality as a whole, he writes, "can't end or disappear any more than it can get created or change" (chapter 13, "The Future of Our Universe: How Will It End?"). The other question is about the particular mathematical structure that we inhabit, which contains space and time. He names five suspects for the end of the part of space we can see.

In the Big Chill the expansion goes on for ever, and the cosmos thins into a cold, dark and lifeless place. In the Big Crunch the expansion reverses, and everything falls back together, like the Big Bang run backward. In the Big Rip the rate of expansion grows without limit. Galaxies, planets and even atoms are torn apart at a definite time in the future. In the Big Snap, space shows a grainy nature when it is

stretched too far, as a rubber band does. In the Death Bubbles, space changes into a different state of itself, inside lethal bubbles that spread at the speed of light.

Tegmark ties the first three endings to one quantity, the dark energy. The *dark energy* is whatever drives the speeding up of the expansion. In Tegmark's account the dark energy makes up about seventy percent of the universe. Which of the first three endings follows depends on what the dark energy does. If the dark energy stays as it is, the Big Chill follows. If the dark energy thins away to a negative density, the Big Crunch follows. If the dark energy grows denser as space grows, the Big Rip follows. Tegmark holds that no one yet knows what the dark energy is, and so he places bets. He gives his bets in his own words: "I'll just tell you how I'd bet: 40% on the Big Chill, 9% on the Big Crunch and 1% on the Big Rip" (chapter 13, "The Future of Our Universe: How Will It End?"). He keeps the other half of his money for "none of the above".

Tegmark keeps the other half, he writes, because there are basic things we still do not understand. The first basic thing he names is the nature of space. He writes: "The Chill, Crunch and Rip endings all assume that space itself is stable and infinitely stretchable" (chapter 13, "The Future of Our Universe: How Will It End?"). Tegmark saw that the three textbook endings share one assumption, an assumption about space itself. So he put the nature of space on the list of suspects.

The Big Snap is Tegmark's own construction. In the Big Snap, Tegmark joins cosmology and quantum physics into one argument, and he builds the Big Snap from four ingredients. First, space may be grainy at a scale too small for us to have noticed, as a rubber band is made of atoms. Second, the holographic principle is a proposal from the physics of black holes, and on the holographic principle the part of space we can see holds at most a definite number of bits of information. Tegmark gives the number as ten to the hundred and twenty-four, a one followed by a hundred and twenty-four zeros. Third, the Schrödinger

equation is the rule by which a quantum state changes, and Tegmark writes that the Schrödinger equation "implies that information can't be created or destroyed" (chapter 13, "The Future of Our Universe: How Will It End?"). The number of bits then stays fixed while space grows, so the bits in each liter of space must fall as space expands. On Tegmark's account inflation stretched some regions of the early universe much more than others, so if space can be stretched only so far before it snaps, the uneven stretching should leave most observers in the regions stretched closest to their breaking point. Tegmark puts the four ingredients together in what he calls "a simple calculation". He takes the calculation to suggest a snap within a few billion years.

Tegmark then tests his own argument against a measurement. In the simplest grainy spaces, light of short wavelength travels slightly slower than light of long wavelength. Gamma-ray bursts are explosions seen across billions of light-years. Tegmark says the bursts serve as an early warning of a grain in space, "like a canary in a coal mine". One distant burst gave off gamma rays of quite different wavelengths, and the gamma rays arrived within a hundredth of a second of each other. Tegmark writes that, taken at face value, an arrival that close means that no snap will come for billions of years. In his words, the four ingredients "don't mix well, suggesting that there's something wrong with at least one of them", and he adds: "I think we're due for a breakthrough on the nature of space" (chapter 13, "The Future of Our Universe: How Will It End?").

Within the structure we inhabit, Tegmark looks at the region he calls "our Universe", which this book calls the part of space we can see, and for that region, he writes, things "get much more interesting". Fourth, *inflation* is a brief burst of extremely fast expansion that standard accounts place at the very start (chapter 5 of this book described inflation). Tegmark is right to look to the nature of space for the breakthrough, and I look there too.

III. The Long Emptying, Reached on Separate Ground

I agree with Tegmark's Big Chill: on my account the sky goes through a long emptying. I derive the long emptying from reasons of my own. The reasons begin with a measurement that chapter 4 described. In 1998 the teams of Saul Perlmutter and Brian Schmidt measured distant exploding stars. The measurements showed that the expansion of the universe is speeding up.

When the expansion speeds up, every observer has a horizon. A *cosmological horizon* is the boundary beyond which no event can ever send a signal to a given observer, however long the observer waits, because the space in between grows faster than light can cross it. Picture the circle of lamplight around your chair from chapter 2: the part of the floor whose light has had time to reach you. In the picture the floor means space, and the chair means the place of one observer. Now let the floor beyond the circle stretch, faster and faster. The far boards draw away faster than their light can cross the growing gap. Light from the far boards never reaches the chair, however long you wait, because the far boards lie beyond the cosmological horizon: the line on the floor beyond which every board draws away faster than its light can cross the gap. Galaxies held to ours by gravity stay with us. Every other galaxy is carried, in time, beyond the horizon. In 2007 the physicists Lawrence Krauss and Robert Scherrer worked out the timetable on which the other galaxies are carried away. On the order of a hundred billion years from now, the Milky Way and the Andromeda galaxy will have become one galaxy, and the more distant galaxies will have been carried beyond the horizon of the observers in the one galaxy. Krauss and Scherrer called that far future "the end of cosmology and the return of a static universe".

I give the long emptying a definite form. In my account the dark energy is the *cosmological constant*: an energy of empty space itself, the same in every cubic meter, which does not thin as space grows. I identify the

cosmological constant with the part of the energy of empty space that a measurement made from inside the universe can take in, as chapter 4 set out. Section IV of this chapter comes back to the theorem. As chapter 3 explained, an epoch is one long age of the universe, running from one turn to the next. The turn is the passage between epochs. I define an observer's outside world as everything that can ever reach that observer and be compared. Every observer's outside world is bounded by the horizon, so as the galaxies are carried beyond the horizon, every observer's outside world empties. The populated sky is a passing stage of our epoch.

The matter in the emptying sky lasts. The stars burn out and leave their ashes behind: the white dwarfs and neutron stars, the dense, cooling cores of burned-out stars. White dwarfs and neutron stars are held up without heat, and they cool without wasting away. The *proton* is the particle that carries the positive charge at the center of every atom, and in my account the proton does not decay. In 2020 the Super-Kamiokande collaboration reported its search for a proton breaking into a positron (the electron's positively charged twin) and a pion (a light, short-lived particle). The collaboration searched by watching a vast tank of water deep underground in Japan, and the collaboration saw no proton break into a positron and a pion. From the search the collaboration placed the proton's lifetime for a decay into a positron and a pion above twenty-four million billion billion billion years. In my account each piece of matter keeps its forms all along its road, its path through space and time. The road of every piece of matter ends in one of two ways. One way is to end inside a *black hole*, a region from which nothing, not even light, climbs back out. The other way is to be *carried away alone*, beyond the reach of all other matter.

My long emptying is Tegmark's Big Chill, the ending on which he puts forty percent of his money: our sky empties, over spans of time far longer than the age of the universe so far. Tegmark gets the Big Chill as the case in which the dark energy stays as it is, while I derive the emptying from a dark energy that is one constant number.

IV. A Flat Space and a Steady Push

In my account the space of our epoch is flat. The cosmological constant of our epoch is positive, and the constant is one number for the whole epoch. When space is flat and the constant is positive and one number, the expansion never stops, and its rate never grows without limit.

Begin with the Big Crunch, the ending in which the expansion reverses. Einstein's equations set the expansion rate by a sum. The sum can be said in one sentence of words. The square of the expansion rate is a part from the density of matter, minus a part from any curvature that closes space on itself like the surface of a ball, plus a part from the constant. A recollapse needs the expansion rate to fall to zero, as a ball thrown upward stops for an instant before it falls back. The density part is never negative, and the constant part is positive. The curvature part is zero, as chapter 5 showed. Our epoch began at the turn, and the turn carries over no length from the epoch before. The beginning of our epoch holds only one length, the *Planck length*: the length, far smaller than an atom, at which the effects of gravity and the effects of the quantum are equally strong. A space curved on a length as small as the Planck length would close up or fly apart at once, and so such a space would make no stars and no black holes. Every epoch begins from the black holes of the epoch before, so every epoch that the cycle of epochs makes is an epoch that makes black holes, for the next epoch to begin from. Our epoch is one that the cycle makes, so the beginning of our epoch set no curvature, and our space is flat, bent only by the tiny ripples that our space began with. Since the curvature part is zero, the sum is always larger than the constant part alone, which is positive, and so the expansion rate never reaches zero and the expansion never reverses. In my account the one collapse that does happen happens inside black holes, one black hole at a time. The space between the black holes keeps expanding. The measurements agree with a flat space. The Planck satellite mapped the afterglow of the Big

Bang, and astronomers combine the satellite's map with surveys of galaxies. Together, the map and the surveys of galaxies give a curvature share (the curvature part's share of the sum) of one part in a thousand, give or take two parts in a thousand, which is zero within the measurement.

The cosmological constant keeps one value through our whole epoch, so it can never change into a pull that draws the galaxies back. Chapter 4 showed why the constant keeps one value, with the picture of a map: a map has one scale, and the one scale belongs to the whole sheet, as the one value of the constant belongs to the whole of our epoch. A theorem about Einstein's equations states that within one geometry, the one space and time of our epoch, the cosmological constant is one number.

Next comes the Big Rip, the ending in which the rate of expansion grows without limit. In 2003 the physicists Robert Caldwell, Marc Kamionkowski and Nevin Weinberg worked out what a rip would take, in a paper whose title carried the words "Cosmic Doomsday". The quantity that decides whether a rip comes is the *equation of state*: the pressure of a substance divided by its density. The equation of state is about zero for thin matter and one third for light. For the cosmological constant the equation of state is exactly minus one. The value of exactly minus one is why the constant does not thin as space grows. A substance whose equation of state is below minus one grows denser as space grows, and Caldwell called such a substance *phantom energy*. A rip needs phantom energy, and only phantom energy gives a rip. For an equation of state of exactly minus one, the analysis of Caldwell, Kamionkowski and Weinberg found no rip.

The dark energy is the cosmological constant, one number, so the equation of state of the dark energy is exactly minus one, and the dark energy is not phantom energy. The constant gives a small outward push on every thing held together, such as a galaxy, a planet or an atom. The constant's push is already part of the balance that holds a galaxy, a planet or an atom together tonight, and the push never grows. Picture a tent pegged down against a steady breeze. The pegs that hold tonight hold next year and in a trillion years, because the breeze is as steady then as tonight. Only a wind that rises without limit pulls the pegs up. In the picture the

tent is a thing held together, the pegs are what hold the thing together, and the breeze is the constant's push: the constant is a breeze that never rises. The Planck satellite's map and other surveys give the equation of state of the dark energy as minus one and three hundredths, give or take three hundredths, so exactly minus one lies within the measurement.

V. One Vacuum for the Six

As chapter 4 set out, the internal space is six further directions, held small at every point, whose shape fixes the kinds of particle. In my account the six small dimensions of the internal space rest in one way only, so the six have no second resting state for a bubble to lead to. Physicists have made exact the question of whether a bubble of a lower state could form in empty space and spread, and the exact form of the question begins with the vacuum.

Physicists call the settled state of empty space the *vacuum*. Picture a hilly country with a ball resting at the bottom of one valley. Nudge the ball, and it rolls back to where it was. A vacuum is like the bottom of the valley where the ball rests. If a lower valley lies beyond the ridge, the resting place is a *false vacuum*. The resting place holds the ball, and it is not the lowest place. In quantum physics a ball held in a valley can pass through the ridge to a lower valley without climbing over the ridge. Physicists call a passage through the ridge *tunneling*. For a whole world in a false vacuum, the passage would begin in one small region, as a bubble of the lower vacuum, and the bubble, once large enough, would spread. Inside the bubble, empty space is in the lower vacuum. So the bubble leads to the lower vacuum.

The physicists Sidney Coleman and Frank De Luccia worked out what gravity does to a bubble of a lower vacuum. Coleman was then at the Stanford Linear Accelerator Center. De Luccia was at the Institute for Advanced Study in Princeton. The journal *Physical Review D* received the paper of Coleman and De Luccia on 4 March 1980. Coleman and De Luccia

found that when a bubble leads to a vacuum whose energy is below zero, the space inside the bubble collapses in a time of "microseconds or less". Coleman and De Luccia wrote near the end of their paper: "Vacuum decay is the ultimate ecological catastrophe". Tegmark's Death Bubbles are Coleman and De Luccia's bubbles.

Every decay of a vacuum by a bubble needs one thing: a second, lower valley for the bubble to lead to. In my account the settled state of empty space depends first on the internal space, the six further directions held small at every point (chapter 4 set out the internal space). I derive one shape for the six, the only shape that satisfies the conditions I set for the six. String theory is the proposal that the smallest pieces of matter are tiny vibrating strings. String theory also curls six directions small, and the six directions of string theory have loops, like the holes of a button. As chapter 7 set out, string theorists wind fields through the loops of the six directions, as thread is wound through the holes of a button, and each count of windings around the loops gives another valley. The valleys of string theory are so many that physicists call the whole country of them the *landscape*. I wind no field through the six held dimensions. So there is no count of windings to choose, and so there is no landscape. So the six have one vacuum, and no second one for a bubble to lead to.

The six are held small, at their one size and shape, while the large world of space and time is there around them, with things in it to compare against. The large world ceases to be anything to compare against in two places only: at the end of a black hole's interior, and for a body carried away alone. In both places the six stop being held small. In both places the road of the matter comes to one end, the turn, which section VII sets out. At the turn the six have no second, lower state to fall into, so the turn is not a bubble of a lower vacuum.

Empty space holds one more settled feature: the *Higgs field*, a quantity with a value at every point of space, which has settled at a value that gives many of the smallest particles their masses. I work out the strength with which the Higgs field acts on itself at the energies of the weak force, the force that changes one kind of particle into another. The energies of the

weak force are the energies at which physicists measure the Higgs particle, a small ripple of the Higgs field. I derive the curve of the Higgs field's energy from the strength with which the Higgs field acts on itself. The curve is shaped like the raised bottom of a wine bottle with a round trough about it, and the trough where the field rests today is the lowest place on the curve. In my account there is one field of the Higgs field's kind, so the Higgs field has one way to vary, its size, and no second way of varying in which a lower valley could lie.

VI. A Smooth Space and New Information

In my account space has no least step, that is, no smallest length out of which space is built. New information enters the world at every finding whose outcome nothing fixed. No long inflation occurred. I answer three ingredients of the Big Snap: the smooth space answers the grain (Tegmark's first ingredient), the new information answers the fixed stock of information (his third), and the absence of a long inflation answers the uneven stretching (his fourth). I read the remaining ingredient, the holographic count (Tegmark's second), as a limit on what a comparison can tell apart. Tegmark took his four ingredients to suggest that at least one of them is wrong. My account has a result of its own about each of the four ingredients.

Start with the grain, Tegmark's first ingredient. I hold that space and time, the arena in which every finding is made, are continuous, while the separate steps that physicists find belong to the things found, never to the arena in which the things are found. *Content* is what a comparison finds, and *context* is the respect in which the two things compared are two: here, two places or two moments of space and time. I give the reason in one line: "the arena is the context of physics and is continuous; a step in the arena would give it the character of content". The Planck length is often called the shortest length there can be, but in my account the Planck length is a bound on how finely a comparison made from inside the world

can place a body. A ruler marked in millimeters does not make the table it measures out of millimeter tiles. The marks belong to the ruler, and not to the table. In the picture the ruler means the comparing, the table means space, and the millimeter marks mean the Planck length. In the same way the Planck length belongs to the comparing, and space has no least step.

Chapter 6 told of a measurement of the kind Tegmark describes: the gamma-ray burst of 10 May 2009. In the burst of 10 May 2009, light of high and low energy arrived together, so any grain that slowed light in proportion to its energy would be smaller than the Planck length. The measurement has been made more sharply since Tegmark wrote his book. The Large High Altitude Air Shower Observatory is in China. In 2024 the observatory's collaboration placed any grain that slows light in proportion to its energy below a tenth of the Planck length. The collaboration took the limit from its observations of the exceptionally bright burst of 9 October 2022. Tegmark reads the timing of gamma-ray bursts, taken at face value, as ruling out a snap for billions of years, while I derive a space with no least step and finds the same timing in agreement with that derivation.

Next comes information, Tegmark's third ingredient. Tegmark's third ingredient is that information cannot be created. Chapter 8 set out hard randomness: the outcome of a finding that nothing before it fixed. In my account hard randomness is real in each single finding. Each outcome that nothing before it fixed is new. A domain is the range in which comparisons can be made, and the microscopic boundary is the inner edge of the domain: the finest scale at which any comparison can find a definite state. Some of it comes in through the microscopic boundary, the inner edge of the domain, as hard randomness. On my account new information is added to the world at every finding whose outcome nothing fixed. So there is no fixed stock of information for the expansion to thin.

Then comes the holographic count, Tegmark's second ingredient. I take the holographic bounds on the information a region can hold, and reads the bounds as a bound on what any comparison made within the region can tell apart. Those bounds limit what any comparison within the domain can tell apart, which is the domain's own limit, and not a graini-

ness of what exists. A limit on what can be told apart belongs to the comparing, as the marks belong to the ruler, so the holographic count does not mean that space has grains for the expansion to spread thin.

Last comes the uneven stretching, Tegmark's fourth ingredient. I keep a *sealed record*: the dated list of its predictions, published before the measurements, in the appendices of *Side by Side, Volume One*. The early horizon is the horizon near the turn: the farthest distance that light could have traveled in the short time after the turn. The sealed record states that "No inflationary phase long enough to stretch the lengths observed today from within the early horizon to beyond it occurred". On Tegmark's account, inflation did the uneven stretching. In my account no long inflation occurred. So the regions stretched close to breaking, in which Tegmark expected most observers to live, have no counterpart in my account. Tegmark's hint was about the nature of space, and my answer is an answer about space: space is smooth at every scale, and space is the arena in which findings keep adding what is new.

VII. The Turn

Our epoch ends in a turn. From the turn the next epoch begins under the same laws. Thomson's tendency holds inside every epoch. I give three separate grounds, each standing on its own, for holding that the long emptying is not a final dark. A final dark would be an end after which nothing more happens.

The first ground is the third of the three axioms of the Immanent Metaphysics. The three axioms are the starting statements about every comparison on which I rest. The warmth the thermometer finds is the content, and the two times of day, which make the two readings two, are the context. The *third axiom* holds content and context distinct and inseparable: there is no content without a context. In physics the content is the matter and light of the world. The context is the arena of space and time, with the outside world in it. For anything counted within an epoch, I take

the context to be the outside world of that epoch: the sky and everything in it, against which the comparisons of that epoch are made. At the end of the emptying, the matter and light of the epoch are still there. Nothing large is left around the matter and light to be their context. Since there is no content without a context, a context forms again, and on my account the new context forms from the geometry of all ten dimensions, the four of space and time together with the six of the internal space.

The second ground is the Incommensuration Theorem, which says that symmetry and continuity cannot both be perfected in one comparison, and that neither can asymmetry and discontinuity. A final featureless emptiness would be the same everywhere and smooth everywhere, perfectly symmetric and perfectly continuous at once. Every world that a comparison finds holds some difference or some break, so the emptying cannot come to rest in a featureless emptiness.

The third ground is about the measure of Thomson's spreading. *Entropy* is the measure physicists use: a count of the arrangements of a thing's parts that leave the thing looking the same from outside. In my account every entropy is a count made against a context, because "looking the same" always means looking the same to the comparisons that can be made within some context. Think of a library that counts its books by its own catalog. When the building closes for good and a new library opens, the new library starts its own catalog, and the first count of the new library does not continue the count of the old library. In the picture the library means an epoch, and its catalog means the context against which the epoch's count of arrangements is made. Within our epoch, the count of arrangements rises, as Thomson's tendency says it does, and the heat death would be the state in which that count reaches its largest value. The count of arrangements is made against a context, and at the end of the epoch that context has emptied. So the count has nothing to be counted against, and the heat death has no context in which to be counted. The three grounds come from three different places: an axiom, a theorem and the measure of spreading. Each ground by itself answers Thomson's old fear of a final end.

Where does the next epoch begin? Inside a black hole, every path leads forward to an end. The end comes after a finite time on the clock of anything that falls in. The end lies at no moment of the outside world's time. No comparison made in our epoch, or in the next, can tell the end of one black hole's interior from the end of another's, and I hold that a difference that no comparison can register is no difference. So the interiors of all the black holes have one end, and that one end is the turn: the passage from the end of one epoch to the start of the next. The turn is approached from the epoch before it and from the epoch after it, and the turn is reached from neither epoch. I give the body carried away alone the same end, because for that body too nothing is left within its reach to compare against. The rule holds for every path: whichever exit a path takes, into a black hole or into emptiness, the end is the turn. In my account, as chapter 3 set out, the Big Bang of each epoch is the turn as the turn appears in that epoch's own history, the history its observers piece together from what reaches them. The emptying ends in a turn, not in a final dark.

Tegmark's account and mine each answer the old fear of a final end, from two directions and at two scopes. On his hypothesis Tegmark holds that physical reality as a whole cannot end. His ground is that the whole is a mathematical structure, existing outside space and time. In the picture of the kitchen table, the whole is a score sheet, laid out at once. A score sheet laid out at once has no last line to run out of. My books speak of the epochs one after another, never of all of them gathered into one finished whole. Of our own epoch, I hold that "the long emptying is one stage of the world, and not the end of it". My ground is the turn. The laws exist, the same in every epoch, as the rules are the same at every table, and the laws hold again after each turn. What is real is given in acts of finding, one epoch after another. Our epoch is one turn of a cycle with no first turn. In that line, "turn" means one whole round of the cycle, one epoch. Everywhere else in this book the turn is the passage from one epoch to the next. Read that way, the line says that the cycle of epochs has no first epoch. Tegmark kept half his money for "none of the above" (chapter 13,

"The Future of Our Universe: How Will It End?"). My ending is a turn: the sky empties, as in Tegmark's Big Chill, and then the turn comes. None of Tegmark's five endings includes a turn.

VIII. What This Chapter Holds

On the way to the turn, space keeps expanding, flat and smooth, under a push that never grows. So space does not fall back, tear or snap. The six small dimensions keep their one vacuum, so no bubble of a lower vacuum can form. The long emptying is one stage of the world, and not the end of it.

How our universe ends matters for the question this book has asked from its first chapter: What is reality, and can physics find out? Tegmark's account and mine each answer the fear of a final dark at their own scope. For Tegmark, physical reality as a whole has no end, in his sense of the words, because a timeless structure has no time in which to end, while the part of space we can see may still end in his Big Chill. For me, the long emptying of each epoch is followed by a turn, and after every turn the next move is made, as a new game begins at the kitchen table.

Now that the last of Tegmark's questions about the cosmos is answered, from Galileo's book in chapter 1 to the turn, what do I now hold that his book did not give me?



Chapter 16

What Do I Now Hold?

Beside Tegmark's chapter 13, "Life, Our Universe and Everything"

What this chapter uses

What *exists* is laid out whole, timeless and the same for everyone, as the list on a score sheet is; what is *real* is given in an act of finding, once, as a move is made (Chapter 1).

The *order of depth*: metaphysics, then physics, then computation, then mathematics, each resting on the one before; mathematics stands last, and keeps every power it has (Chapter 10).

The *bound triple*: the arrow of time, hard randomness and the subjective, bound into one act: distinct, inseparable and not interchangeable (Chapters 9 and 12).

I. A Book Printed in Rome

You have come to the end of two books: Max Tegmark's *Our Mathematical Universe*, and this one. At the end of both books, what have you learned? What do you now hold that you did not have from Tegmark's book alone? To answer both questions, we go back to the scene that opened chapter 1, a book printed in Rome.

In 1623 a book came from the press of Giacomo Mascardi, a printer in Rome. The book's author was Galileo Galilei. The book's title was *Il Saggiatore*, *The Assayer* (an assayer is one who tests metals for their worth with a fine balance). Galileo wrote *The Assayer* in answer to another man's book about the three comets of 1618, a book that had called itself an astronomical balance, and with his title Galileo promised a finer balance. The Accademia dei Lincei signed the book's dedication to the new pope, Urban VIII, and dated the dedication 20 October 1623. The Accademia dei Lincei was a scientific academy in Rome.

Chapter 1 of this book began with a passage from the sixth section of *The Assayer*.

As in chapter 1, I give the passage from *The Assayer* in the English translation of Stillman Drake (*Discoveries and Opinions of Galileo*, 1957):

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.

Tegmark quotes the central claim of Galileo's passage near the start of his book: nature is "a book written in the language of mathematics" (chapter 1, "Not What It Seems").

In chapter 1 I used Galileo's sentence as one picture of Tegmark's whole question. Tegmark asks whether the world is a mathematical structure. In Galileo's picture, Tegmark's question is whether the book written in the language of mathematics is the world itself. Galileo's page reads today exactly as it read in 1623. The reader has changed: you come back to

the page holding what you have gained from fifteen chapters. We gather what you now hold, set what you hold beside the end of Tegmark's book, and then read Galileo's page a second time.

II. What Max Tegmark Showed at the End of His Journey

Tegmark opens his last chapter by gathering his whole book into one picture, a picture of two expeditions. His first expedition went outward, asking how big everything is, and passed a planet, a galaxy and the part of space we can see. Tegmark had argued for four levels of parallel universes beyond that part: regions beyond our view, regions with other constants, the branches of quantum theory, and the universe of every mathematical structure. His second expedition went inward and asked what everything is made of: cells, molecules, atoms, and the elementary particles. The only properties of the elementary particles, Tegmark says, are mathematical properties. Tegmark finds that both expeditions end in one place, among the mathematical structures. In his words, "our two roads to reality both lead to mathematics" (chapter 13, "How Big Is Our Physical Reality?"). He adds that reality shows its mathematical character plainly at the largest and the smallest scales, and that the mathematical character is easy to miss at the middle scales of everyday human life.

Tegmark shows with great clarity that both roads end in mathematics: physicists have found laws at the scale of galaxies and laws at the scale of particles, and both sets of laws are written in mathematics and are exact. A reader of Tegmark's book has watched mathematics describe the world exactly, out to the edge of the visible sky and down into the atom, and has learned to expect mathematics to do so.

Tegmark then looks ahead to the future of physics and sets out two outcomes that depend on his Mathematical Universe Hypothesis, his claim that our physical world is a mathematical structure. If the hypothesis is true, he writes, "everything is in principle understandable to us" (chapter 13, "The Future of Physics"), and we would then be limited only

by "our imagination and our willingness to do hard work". If the hypothesis is false, "fundamental physics is doomed to eventually hit a roadblock" (chapter 13, "The Future of Physics"). If the hypothesis is true, Tegmark says, the other questions of physics change their form: "Instead of getting answered, most questions get rephrased". His example is the question "What are the equations of quantum gravity?", which becomes the question "Where in the Level IV multiverse are we?". Tegmark has names for two ways of seeing the world: the frog's view is what an observer sees from inside, and the bird's view is the whole structure seen from outside. On his hypothesis, the ultimate question would be to "ask how to compute the frog's view of our Universe" from the bird's view.

Tegmark reminds the reader that humanity has been moved again and again from the center of things: we go around the Sun, and we live in one galaxy among countless others. He finds a gain in that history of being moved from the center: we had overestimated our physical powers, he writes, and "we had underestimated our mental powers!" (chapter 13, "The Case for a Greater Reality"). In his account, our ancestors learned how much they could know of the universe without traveling to the stars. Our ancestors did so by letting their minds go where their bodies could not.

Tegmark asks a hard question near the end of his book: if every physically possible future plays out in some parallel universe, and change itself is an illusion, why should anyone care about our own universe? His answer is that each of us has already chosen to care about something. It then feels logical to him to care about the universe to which the people we care about belong. It feels logical to him, as well, to care less about other universes. His reason for caring less about other universes is that "my decisions here in our Universe by definition can't have any effect on them" (chapter 13, "Why We Should Care About Our Own Universe").

III. Where This Account Arrives at the Same Place

Comparing is telling one thing from another, or finding them the same. Every measurement and every interaction is a comparison, and the result of one comparison is a *finding*. Four results of Tegmark's last chapter also follow from comparison, in my physics. I set each of the four results beside my own words.

The first of the four results is the premise on which Tegmark builds his whole book. His External Reality Hypothesis says: "There exists an external physical reality completely independent of us humans" (chapter 13, "The Case for a Greater Reality"). I hold that hypothesis, read as: there is a world, and the world does not depend on us. I hold the hypothesis because every interaction is a comparison. A *comparer* is the position inside the world from which a comparison is made, and a comparer need not be a mind. A collision of two particles is an interaction. So the world was comparing itself, in every collision of particles, long before anything human looked at the world.

The second of the four results is the fit between mathematics and the world. Tegmark showed the fit at both ends of his journey, at the largest scales and at the smallest. I agree that the mathematics of physics is exact at the largest scales and at the smallest. I also state a reason for the fit. Physicists compare a theory's mathematics with the theory's findings, and in an exact theory, mathematics and findings are of opposite kinds. Where the findings come separately, one at a time, the mathematics is smooth. Where the findings are smooth, as the arena of space and time is found to be, the mathematics is one of perfect sameness. Mathematics contains structures of both kinds, so for findings of either kind there is a mathematics of the opposite kind, and that mathematics comes as close to its findings as one likes. That dense reach is why mathematics works so well in physics. So both of Tegmark's roads arrive at mathematics, and the reason the roads arrive there is this.

The third of the four results is the gain that Tegmark finds in humanity's long history of being moved from the center of things. On my account, every comparer is at some place inside the world, and a comparer's world is everything that can ever reach the comparer at that place and be compared. Each comparer's world ends at a horizon of its own. In chapter 15 the horizon was a line on the floor beyond the circle of lamplight around the chair. No light from beyond that line can ever reach the chair. From wherever a comparer is, physics can describe every how, when and where of the comparer's world as closely as one likes. I read the power of physics to describe a comparer's world from one place as Tegmark's point about our mental powers, arrived at from comparison: the one who finds always finds from one particular place, and from that one place can find things all across the sky.

The fourth of the four results is Tegmark's reason for caring less about other universes: our decisions here have no effect on them. My *arena* is space and time as found in comparisons, and I gave the arena its name in chapter 2. In my physics, space is the pattern of which events a signal can join. So our own world includes any region that a signal from here can reach along some path, while a region that no path from here reaches has no place relative to us. In chapter 3 I introduced my word *epoch*: one long age of the universe, from one turn to the next. The *turn* is the passage from the end of one epoch to the start of the next. That book says: "The epochs before ours and after ours are worlds, each real in its own acts of comparison, and no act is real in two of them". Each epoch is a whole world, complete from inside. The series of epochs is a series of worlds under one law. So a decision made here can affect only what a path from here can reach, which is our own epoch, while Tegmark reached the same answer on other grounds: our decisions affect only our own universe.

IV. Why Both Roads Seem to End in Mathematics

Tegmark and I part at one step of his argument: the step from a world independent of us, which both Tegmark and I hold, to his conclusion that the world is a mathematical structure. Two of my words show what happens at that step. What *exists* is laid out whole and the same for everyone who looks: a structure, like a law. What is *real* is given in an actual act of finding, once, now. Every physical theory has both: its mathematics, which exists, and its findings, which are real. In his hypothesis Tegmark takes the mathematics for the whole world.

Why, then, does each of Tegmark's roads seem to end in mathematics and in nothing else? I name the source of the appearance that each road ends in mathematics alone: in its words, "the timeless description is taken for the foundation". Chapter 3 of this book showed an appearance of the same kind with an *orrery*, a brass clockwork model of the planets, whose gears drive its planets backward as smoothly as forward. The solar system itself runs one way only. The Moon raises tides on the Earth, and the tides drag against the spinning Earth, slow the Earth's spin and push the Moon slowly outward. The ease of running backward belongs to the orrery, the model, and not to the solar system.

A mathematical description has features of the same kind as the orrery's ease of running backward. Three features belong to the description and not to what it describes: the description lays out the whole of its subject at once, its laws hold the same everywhere, and it can often be run backward as easily as forward. We receive a structure only after the act in which the structure was found, and we receive the structure already finished and the same for everyone, so the structure seems to have been there before the act. The more finished, exact and shareable a description is, the deeper the description looks. The finest description there is lies at the end of both of Tegmark's roads. On both roads Tegmark takes that description for the foundation of the world.

A structure cannot be the world it describes, and the reason is the Incommensuration Theorem. Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Symmetry means sameness, a thing unchanged when you swap it or move it. Continuity means having no gaps. In plain words, the Incommensuration Theorem says that no single act of finding has perfect sameness and perfect gaplessness together, nor their two opposites, asymmetry and discontinuity, together. The Incommensuration Theorem is a structural fact about the nature of knowing. The theorem is about comparison, the act in which anything at all is known. Every finding of physics is a comparison, so the theorem applies to every finding of physics. A mathematical structure taken whole has perfect sameness and gapless completeness together, as the circle of geometry is at once smooth and perfectly symmetric, while every finding has at most one of the two. So the world found in comparisons, which is the only world physics has, is a world that mathematics describes, and it is not a mathematical structure.

In his argument Tegmark steps from a world independent of us to a world of pure structure. My answer to that step is the difference between a human and a stance, the side from which a comparison is made. Tegmark's step has two parts. First, a description of a world independent of us must contain nothing human. Second, a description with nothing human in it is pure structure. I accept the first part, and parts from Tegmark at the second. Every comparison is made from a *stance*, and a collision of two particles has a stance as surely as a laboratory does. A stance is not a human. So when every human word is taken out of a description, the stance is left unnamed, and the stance is still there in every finding on which the description rests.

V. A Third Future for Physics

The Incommensuration Theorem shows a third future for physics, beside Tegmark's two outcomes: the mathematical description of physics is very powerful and never complete, and its power and its incompleteness come from one source. The mathematical description of physics comes as close to the physical process as one likes, and never becomes it. The one source of the power and the incompleteness is, once more, the Incommensuration Theorem. The orrery of chapter 3 shows a description beside what it describes. The orrery's brass planets keep the same arrangement as the planets of the sky. Yet the orrery stands on a table in a room, joined to no planet. A description is symmetric with what it describes: the description is the same structure, shown in another setting. The Incommensuration Theorem says that perfect sameness and perfect gaplessness never come together, so a gap always lies between the description and what it describes.

That sentence has two halves: the description comes as close to the process as one likes, and the description never becomes the process. Set the two halves beside Tegmark's two outcomes, everything understandable or a roadblock. Each time physicists improve a theory, they bring more of the world into the model, so physics never hits a roadblock. After each improvement the world still goes beyond the model, so physics never reaches a last page, a complete account with nothing left to find. The work of physics happens in the part of the world that every model leaves out: physicists judge which measurement to make, notice that a result does not match, and choose which oddity to chase. All three acts are outside the model, and every measurement ever made depended on acts like them. The blind spot of a theory is where its practice lives.

Tegmark's ultimate question changes as well: how to compute the frog's view from the bird's view. I keep the view from nowhere: the view that is the same from every place, in which the laws of physics are stated.

Physicists found the laws stated in the view from nowhere by comparisons made from stances inside the world, and no stance is named in the laws because what the laws say is the same from every stance. I hold that "a description of the whole world from a vantage outside it has no comparison behind it, because the whole has no outside to compare it with". So physics is done from the frog's view, to use Tegmark's word. The bird's view is a description drawn from the findings of many frogs. Physicists work from the findings to the description, and then back to the findings for the next check.

Tegmark's example question changes too: "What are the equations of quantum gravity?". On my account, gravitation is the arena itself, space and time as found in comparisons, and the arena does not come in steps. Gravitation is not quantized. To be quantized is to come in quanta, separate least amounts. Physicists describe a small ripple of space and time by quanta, and they call the smallest packet of that description a *graviton*. On my account, the word graviton names at most a step of the quantum description, and never a piece of space and time itself. When Tegmark asks for the equations of quantum gravity, he asks for the equations of a gravitation that comes in steps, but I hold that gravitation, the arena itself, is smooth at every scale. So on my account, Tegmark's question becomes a task of measurement: physicists measure how smooth space and time are, at finer and finer scales. In the appendix "What the Coming Measurements Will Find" I set out what, on my account, each of the coming measurements will find.

Last, Tegmark asks why anyone should care if every possible future plays out and change is an illusion. Change is real: the present is the act of comparison itself, the place where possible futures become what happens. In every comparison a range of outcomes is possible, and one outcome is found. The outcome found is fixed by nothing that came before it. I call an outcome of that kind *hard randomness*, and hard randomness is real in each single finding. Hard randomness is real, and the Incommensuration The-

orem guarantees it. So we have the plainest reason there is to care about what we do here: here, and only here, one of the possible futures becomes the one that happens.

VI. What You Now Hold

You hold a world that does not depend on you, and you hold that world on the ground of comparison, which needs no mind and has been happening everywhere for as long as anything has happened. You hold two words for the two parts of every physical theory: what exists is laid out as structure and is the same for everyone, like the score sheet beside the board at the kitchen table. What is real is given in an act of finding, once, like a move in the game. You hold the Incommensuration Theorem, which is the reason the world found in comparisons is not a mathematical structure. And you hold the *order of depth*: metaphysics, then physics, then computation, then mathematics, each depending on the one before it. Every proof is checked by a run, a computation actually carried out, and every run is a physical process. Every finding of physics is a comparison made from inside the world. Mathematics comes last in the order of depth and keeps every power it has. Mathematics is the finest instrument there is for describing what is laid out as structure.

You hold the history of the cosmos that Tegmark tells, reached on my own ground: the expansion, the hot early universe, the oldest light and the first elements. Astronomers piece the history of the cosmos together from the light and matter that reach us now, as in chapter 3 you rebuilt the score sheet from the position on the board. What we call the Big Bang is the earliest point of the rebuilt history. The Big Bang is the turn that began our epoch, as that turn appears in the rebuilt history. You hold a definite identity for each of the two dark ingredients of the universe, the dark matter and the dark energy. The six *held internal dimensions* are six further directions, held small at every point, whose shape fixes the kinds of particle. The dark matter is the gyration of the six held internal dimensions. A gyr-

ation is a lasting vibration of the geometry of the six dimensions, as a struck crystal rings in chapter 4. Each gyration weighs about a Planck mass, about twenty-two millionths of a gram. The dark energy is exactly a cosmological constant: an energy of empty space, one number, fixed once and the same everywhere and at every time of our epoch. The cosmological constant is a push that never grows, like the steady breeze of chapter 15 that never rises. And you hold an end of our epoch that is a turn to the next epoch. Our epoch ends in the long emptying of its sky, as the expansion carries every galaxy not held to ours by gravity beyond the horizon, and then a turn begins the next epoch under the same law as ours. The ending is like the end of a game of chess: the game ends, the rules stay, and the next game starts from the setting-up again. The long emptying is one stage of the world, and not the end of it.

You hold an answer for each of Tegmark's four levels of parallel universes. His Level I is the regions beyond our horizon, which are further parts of one world, with the same laws as ours, like the floor that goes on past the circle of lamplight in chapter 2. Tegmark and I agree on Level I in substance. His Level II is other regions with other constants. Level II does not occur, because my account has no long inflation (a proposed early stretching of space, long enough to make regions with other constants) and has one internal space, with no range of alternatives among which regions could differ. His Level III is the many worlds of quantum theory. The many-worlds account leaves out the act of comparison, while I keep the act. There is no physical collapse, and each single outcome is real. His Level IV is the universe of every mathematical structure. Level IV fails together with the hypothesis that our world is a mathematical structure. The reason is that a structure that could never be found in any comparison is laid out and not found, and so a structure of that kind is no further world.

You hold time as real, in this book's sense of real. The present is the act of comparison, the move being made, given once in each finding, while the past, as a region of the geometry of space and time, stays where it is. The *block universe* is all of space and time laid out at once, with no moment singled out as now, and at the kitchen table the block universe is the fin-

ished score sheet of chapter 11 taken for the whole game. Geometric time is time taken as one direction of the geometry, and event time is the one-way order of findings. The block universe takes geometric time for the whole, and drops event time, in words that I quote. You hold consciousness as the *bound triple* taken as one act. The bound triple is the arrow of time, hard randomness and the subjective (being the one to whom things happen), bound together. You hold Tegmark's perceptronium, his name for "the most general substance that feels subjectively self-aware" (chapter 11, "Where Are You? (And What Do You Perceive?)"). You hold perceptronium as an exact description of conscious matter from outside: at the kitchen table, perceptronium is the score sheet of a life, and the life is lived at the board. You hold a world that is not a computation, on three independent grounds. The three grounds are the Incommensuration Theorem, the two levels of every exact description (its mathematics and its findings), and the bound triple. Every computation is inside a world, as the chess computer of chapter 14 stands on the kitchen table, inside the world. And you hold the answer on which Tegmark bets his money when he is asked whether we are simulated: "No!". In the visible universe there is one level only, the physical, and we are not dealing with a simulation.

VII. Galileo's Page, Read Again

Now go back to Galileo's page, printed in Rome in 1623. Galileo's book of the universe is written in the language of mathematics, and everything you now hold agrees with Galileo. The laws of physics are exact, the mathematical description comes as close to the world as one likes, and without the triangles and circles of Galileo's language one wanders about in a dark labyrinth. Tegmark shows at length, and generously, that Galileo's sentence holds true from the edge of the sky to the inside of the atom.

Read Galileo's sentence once more, and find three parts in it. There is the book, written and laid out, the same for anyone who opens it. There is the gaze to which the book stands continually open, and whoever gazes looks from one place. And there is the reading, which Galileo says must be learned and then done: the book cannot be understood unless one first learns to comprehend its language and read its letters. I read Galileo's sentence as already containing the three parts that I name: the structure, which exists; the stance, the side from which the book is read; and the act of finding, which is real.

Tegmark asked whether the world is a mathematical structure, which in Galileo's picture is the question whether the book is the world. The answer to Tegmark's question is plain from what you now hold. On my reading of Galileo's page, the book is the finest thing physics makes, and every line of the book was written by an act of reading the world, made from somewhere, once. The world is what the reader finds in the act of reading.

VIII. What This Chapter Holds

You hold a world that does not depend on you, found from inside it in acts of comparison that are real: its structure exists, and its mathematical description comes as close to the world as one likes and never becomes the world.

What you now hold is an answer to the question this book has asked from its first pages: What is reality, and can physics find out? Tegmark meant the word reality as "the ultimate nature of the outside physical world that we're part of" (chapter 1, "What's the Ultimate Question?"). I hold, with Tegmark, that there is an outside physical world that we are part of. I add that to be part of the world is to be at some place in it, where findings are made. On my account, the world Tegmark calls reality is met

in acts of finding. Physicists find that world always from somewhere, one finding at a time. The world always goes beyond the description, so there is always a next finding to make.

Come back, last of all, to the kitchen table of chapter 1. The score sheet lies beside the board with every move so far written on it, the same for anyone who walks in and reads it. The score sheet is the structure, which exists. The rules, the same at every table, are the laws. The position on the board is the history of this game, which the rules allow and do not fix. Two chairs stand at the board, one for each player. Each chair is a seat, a stance, the side from which a move is made.

A range of moves is possible from the position on the board. Among the possible moves is the move that neither player has yet thought of. The score sheet lists every move made so far, and the rules allow the new move without picking the new move out. A hand at this table will make the new move once, here, and then the move will be written down, and the score sheet will be complete up to that move and no further.

A hand rests above the board. The pieces stand where the last move left them. The next move has not yet been made.

What the Coming Measurements Will Find

This appendix lists the measurements now being prepared that bear on the chapters of this book. For each measurement, the appendix says what I say the measurement will find. My physics books build physics from the act of comparison. Comparing is telling one thing from another, or finding two things the same, as every measurement and every interaction does. The entries follow the order of the chapters: each entry stands under the first chapter that the entry bears on, and the other chapters that the entry bears on are named in the entry. When you have finished a chapter, you can open this appendix at the section for that chapter. In the section for a chapter, I name the instruments of the coming years that will look at the question of that chapter. In the same section I also give what I say the instruments will see.

I. How a Result Is Counted

Every entry for a numbered prediction quotes one book: *The Registered Predictions of Physics from Comparison; A Dated Record, 1 November 2026*. *The Registered Predictions of Physics from Comparison* is reproduced whole in *Side by Side*, Volume One, Appendix B. *The Registered Predictions of Physics from Comparison* contains every prediction that my books of physics make. Each prediction has a number in the record. Where the books name the measurement that will decide a prediction, the record states that measurement beside the prediction. I call the dated record of 1 November 2026 the *sealed record*, because the record says of itself: "This record is fixed at its date and is not revised". Where an entry for a numbered prediction gives the words in which I say what a measurement will bear out, I quote the words exactly from the sealed record, with the number that the sealed re-

cord gives the prediction. Two entries are not in the sealed record. The two entries are the count of the directions of space, under chapter 7, and the working state of the brain, under chapter 12. For each of the two entries, I quote the book of mine that argues the claim of the entry.

A second dated record sets my predictions beside the predictions of Max Tegmark, whose book *Our Mathematical Universe* this book follows. The second record is *The Registered Predictions Beside Max Tegmark; A Dated Record, 4 November 2026*, in *Side by Side*, Volume Two, Appendix E. The record of 4 November 2026 has nine entries, named Tegmark 1 to Tegmark 9. Where Tegmark, in his book or his published papers, expects a different result from one of my predictions, the entry in this appendix for that prediction says that he does. Where Tegmark and I expect the same result, the entry says that the two expect the same result.

Every measurement comes with an uncertainty. The *uncertainty* is the range within which the true value most likely lies, written $\pm$ and read "give or take". The usual measure of the uncertainty is the *standard deviation*. A result often falls one standard deviation from a prediction, but a result would come five standard deviations from a prediction by chance less than once in a million tries. I fix in advance when a result counts, and the sealed record states the rule: "A result counts when it is established at five standard deviations by at least two independent kinds of measurement". A second, independent kind of measurement is a guard against an error that belongs to one method alone. The sealed record also says what a difference of less than five standard deviations is: "A measurement that differs from a prediction by less is a measurement of interest, and the record reports it as such". Many entries below say that something will not be found. The sealed record states a rule for the entries that say something will not be found: "Where a prediction says that something will not be found, every search for it bears on the step of the account that the prediction names, and the account says that none of the searches will find it".

II. Beside Chapter 2, "Does Space Go On Forever?"

Astronomers will use the coming surveys of galaxies and of the microwave background to measure the curvature of space on the largest scale. The surveys will show whether space is curved like the surface of a ball, curved the other way like a saddle, or flat. The microwave background is the faint glow left from the early universe, which arrives from every direction of the sky. I say the surveys will find space flat, curved only by its own small ripples. Prediction 9 of the sealed record reads: "The epoch begins spatially flat, and the spatial curvature measured within one horizon is that of the departures from uniformity". An *epoch* is one long age of the universe. The *departures from uniformity* are the small ripples of density in the early universe, about one part in a hundred thousand, so on my account the curvature is also about one part in a hundred thousand. The *horizon* is the edge of the region whose light has had time to reach us. The sealed record names the instruments that decide Prediction 9: the Dark Energy Spectroscopic Instrument (DESI) and the Euclid space telescope, "which measure the curvature of space with the matter fraction". The matter fraction is the share of the energy of a flat universe that matter carries today (section IV). Tegmark expects the same result as I do. Inflation is a very fast stretching of space that Tegmark places at the beginning of our universe. In his paper of 2005 on what inflation predicts, the total density of the universe equals the density of a flat space to about one part in a hundred thousand (Tegmark 5 in the record of 4 November 2026). Prediction 9 also bears on chapters 5, 6 and 15.

Astronomers also search maps of the microwave background for a short loop. A short loop is a path that goes all the way around the universe and back to its start. A closed space would have such a path. I say no short loop will be found anywhere. Prediction 23 of the sealed record reads: "No closed path that goes around the universe, and so cannot be shrunk to a point, is shorter than the diameter of the horizon, at any loca-

tion". The searches made so far use maps of the microwave background, in which a short loop would show as one patch of the early universe seen in two directions of the sky. The sealed record names no coming instrument for Prediction 23. Prediction 23 also bears on chapter 6.

III. Beside Chapter 3, "Where Did It All Come From?"

Every finer map of the microwave sky will measure the pattern of the first ripples, the small ripples of density in the early universe: how strongly the ripples of different sizes are related to one another across the whole sky. I say the maps will find one pattern, the same across the whole sky. Prediction 12 of the sealed record reads: "The primordial departures are correlated across the whole sky, with nearly the same strength at every length". The sealed record names the measurement that decides Prediction 12: "Every finer map of the microwave sky". My account uses two measured numbers of the pattern of the first ripples: the strength of the ripples, and the slight tilt of the ripples from large sizes to small. Physicists state the tilt as an index, which would be exactly one if the ripples had the same strength at every size. The measured index is about 0.965. In 2005 Tegmark calculated an index of about 0.96, in the limit of high energy. The dated record of 4 November 2026 says of the index: "the credit for that number is his". Prediction 12 also bears on chapters 5 and 15.

Antimatter is matter's mirror twin: each kind of particle has its *antiparticle*, a twin with every charge reversed. Searches for antimatter will measure whether any region of the visible universe is made of antimatter in place of matter. I say every search for antimatter will find matter everywhere. Prediction 15 of the sealed record reads: "Nowhere in the observable universe do antibaryons outnumber baryons, and the observable universe holds no regions of antimatter". In Prediction 15, *baryons* are protons and neutrons, and antibaryons are the antiparticles of protons and neutrons. The sealed record names two kinds of search: searches for stars,

galaxies or regions made of antimatter, and searches for nuclei of antihelium among the cosmic rays, the fast particles that reach the Earth from space.

IV. Beside Chapter 4, "What Is the Universe Made Of?"

The coming surveys will measure the matter fraction. The *matter fraction* is the share of the energy of a flat universe that matter carries today, dark matter and ordinary matter together. Dark matter is matter that pulls by its weight and gives off no light. I say the matter fraction has one value, 0.311 give or take 0.001. The energy of empty space carries the remaining 0.689. In the sealed record, Prediction 7 has the title "The matter fraction is 0.311". I compute the rate at which the universe expands, using the matter fraction and the amount of matter that the microwave background measures. The rate is 67.8 kilometers per second for every megaparsec of distance, give or take 0.3. A megaparsec is about three and a quarter million light years. The sealed record states the matter fraction in three further ways, with the rate of expansion as one of the three, and says of all four statements: "These are one test, stated four ways". Four instruments measure the matter fraction: DESI, Euclid, the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope. *The View from Inside*, Appendix B, says what the four instruments will find: "The account says they will converge on 0.311". Tegmark expects a result of a different kind. Inflation is a very fast stretching of space that Tegmark places at the beginning of our universe. His Level II multiverse is a set of regions that inflation keeps forever out of our reach, and across the Level II multiverse the constants of nature (the fixed numbers in the laws, such as the strength of gravity) vary from region to region. Tegmark writes that from the variation of the constants "we can make statistical predictions for what we should actually observe" (chapter 6, "All Isn't Lost"). Where Tegmark makes a statistical prediction, I state one value. Prediction 7 also bears on chapters 7, 10 and 11.

The coming surveys will also measure the *equation of state* of the energy of empty space: the ratio of the pressure of the energy of empty space to its energy density, which shows how the energy changes as the universe grows. I say the surveys will find the equation of state exactly minus one at every era, and minus one is the value for an energy whose density stays the same as space grows. Prediction 8 of the sealed record reads: "The energy of the vacuum has an equation of state of exactly -1, at every epoch". Researchers report a departure from minus one at about three standard deviations, in recent analyses that combine the Dark Energy Survey with DESI. Three standard deviations is less than the five that the sealed record requires, so, by the rule given in section I of this appendix, the departure is a measurement of interest. Prediction 8 says of the departure: "The reported departure from -1 will not be confirmed". Four instruments measure the equation of state: DESI, Euclid, the Rubin Observatory and the Roman Space Telescope. *What Comes After the Dark*, Appendix B, says that measuring the equation of state is part of the work of "the instruments of the coming decades". Tegmark expects the same result. In his paper of 2005 he obtains an equation of state of minus one (Tegmark 4 in the record of 4 November 2026). Prediction 8 also bears on chapters 7, 11 and 15.

The coming instruments will also measure the ratio of the density of the dark matter to the density of the ordinary matter. The measurement of the ratio is a second kind of measurement, independent of the measurement of the matter fraction. I say the ratio has one value: six less two divided by pi $(6 - 2/\pi)$, which is 5.3634. In the sealed record, Prediction 16 has the title "The ratio of the dark matter to the ordinary matter is $6 - 2/\pi$". Prediction 16 also says that the ratio is "the same at every time since helium formed". The sealed record states that the Simons Observatory, an array of telescopes in the Atacama Desert of Chile, would measure the ratio to about 0.6 percent, with the measurements of the Planck satellite. The oldest light alone, measured by the Planck satellite, gives the ratio as 5.364, give or take 0.065, which agrees with Prediction 16. With the galaxy distances of the Dark Energy Spectroscopic Instrument (DESI), the meas-

ured ratio is 5.199, give or take 0.035, about 4.7 standard deviations from 5.3634, so by the rule of section I it is a measurement of interest. Tegmark has explored a result of a different kind in his published work. In 2006 Tegmark and his co-authors Anthony Aguirre, Martin Rees and Frank Wilczek took as a case study a dark matter made of axions. Axions are light particles that other theories propose. In the case study of 2006, when the change that gives rise to the axion came before or during inflation, the ratio of dark to ordinary matter is a value of our own region. In the case study, the value varies from place to place (Tegmark 3 in the record of 4 November 2026). Prediction 16 also bears on chapters 7 and 10.

Predictions 7 and 16 share one number, and two independent measurements will measure the shared number together: the measurement of the ratio of dark to ordinary matter, and the measurement of the matter fraction. The number that Predictions 7 and 16 share is the curvature of the six held internal dimensions: six further directions held small at every point of space, whose shape fixes the kinds of particle. In the sealed record, Prediction 17 has the title "The coefficient shared with the cosmological constant is 1". The *cosmological constant* is the one constant term in Einstein's equations of gravity. On my account, the cosmological constant is the share of the energy of empty space that a measurement takes in. Prediction 17 speaks of two observables, two measured quantities. The two observables are the matter fraction of Prediction 7 and the ratio of dark to ordinary matter of Prediction 16. In Prediction 17, k is the curvature of the six held internal dimensions, counted in the unit that the strength of gravity sets. Prediction 17 says: "Both observables will find k = 1".

The *amplitude of clustering* is one number for how strongly matter is gathered into clumps today. Independent kinds of measurement will measure the amplitude of clustering. The sealed record states the amplitude of clustering as 0.823 give or take 0.006, under the title "The amplitude of clustering is 0.823". A *gyration* is the unit of the dark matter on my account: a lasting vibration of the six held internal dimensions. Standard cold dark matter is the dark matter of the standard model of cosmo-

logy, the account of the universe that six measured numbers fix: matter that moves slowly and acts only by its pull. Prediction 18 says: "The gyrations cluster exactly as standard cold dark matter does".

Searches will look for the dark matter by some interaction other than its weight, in underground detectors, in the sky and at colliders. I say the dark matter will be registered by its weight alone. Prediction 19 of the sealed record reads: "No search will detect the dark matter through any interaction but gravity, at any sensitivity". The sealed record names three kinds of search: direct searches in detectors deep underground, indirect searches for light or particles that dark matter might give off, and searches for dark matter made in a collider.

Astronomers will use the coming maps of the microwave background and the coming surveys of galaxies to measure how long the dark matter lasts. I say the maps and surveys will find the dark matter stable, at every lifetime that the maps and surveys are able to test. Prediction 20 of the sealed record reads: "The dark component does not decay, so its density falls exactly as that of matter". *What Comes After the Dark*, Appendix B, names the instruments that will measure the lifetime of the dark matter. The instruments include the Simons Observatory, DESI and Euclid. The same appendix says that those instruments are among "the instruments of the coming decades". *What Comes After the Dark*, Appendix B, also says that the measured lifetime of the dark matter tells my account's cycle of epochs apart from the cycle of the physicist Roger Penrose. In the cycle of Roger Penrose, the dark matter must decay within each age of the universe. Prediction 20 also bears on chapter 15.

Two kinds of observation will measure whether the dark matter clumps on the smallest scales: counts of the faintest small galaxies around our own, and observations of the small clumps of matter that bend the light of distant sources. I say the dark matter clumps on every small scale. Prediction 21 of the sealed record reads: "The dark matter has no cut-off in its structure on small scales from its own motion; it clumps on every small scale".

Physicists have proposed arrays of sensors to register a single gyration by its weight alone. *What the Beginning Left Behind*, Appendix B, says that the arrays need their quantum noise reduced far beyond what can be done today. I say that one gyration, the single unit of the dark matter, has a mass of about the Planck mass: the mass that the strength of gravity sets, about twenty-two micrograms. Prediction 22 of the sealed record reads: "The dark matter comes in gyrations whose mass is of the order of the Planck mass".

A *neutrino* is a particle with no charge and almost no mass, which passes through matter almost untouched. Neutrinos come in three kinds. Two kinds of measurement will measure the masses of the three kinds of neutrino: measurements of the large-scale pattern of matter, and measurements of the energies of electrons from radioactive decay. I compute the neutrino masses by two routes. I also name the measurement by which physicists will choose between the results of the two routes. In the sealed record, Prediction 38 has the title "The neutrino masses: two readings, decided by their sum". The text of the sealed record calls the two readings routes. An *electronvolt* is the energy an electron gains in crossing one volt. On one route the three masses add up to 84.95 thousandths of an electronvolt. On the other route the three masses add up to 59.31 thousandths, give or take 0.25. Prediction 38 also says what the two routes share: "Both routes give the normal order, three nonzero Dirac masses and no neutrinoless double beta decay". The *normal order* is the arrangement in which two of the three neutrinos are light and close together in mass, and the third is heavier. A *Dirac mass* is the kind of mass a particle has when it differs from its own antiparticle, as the electron does. *Beta decay* is the decay in which a nucleus gives off an electron and an antineutrino, the neutrino's antiparticle. *Neutrinoless double beta decay* is a rare decay in which a nucleus gives off two electrons and no antineutrino. Neutrinoless double beta decay can happen only if the neutrino is its own antiparticle. A neutrino with a Dirac mass differs from its own antiparticle. So the two routes, which both give Dirac masses, both give no neutrinoless double beta decay. The sealed record names two measurements that decide Predic-

tion 38: the sum of the masses, measured together with the matter fraction and the expansion rate of Prediction 7, and the mass of the neutrino measured in beta decay. If the mass of the neutrino in beta decay is measured to within about 3 thousandths of an electronvolt, that one measurement tells the two routes apart by about three standard deviations. The second data release of DESI, in 2025, gives an upper limit of 0.064 electronvolt on the sum of the three masses, at 95 percent confidence, computed in the standard model of cosmology with the matter fraction that the same data give. That limit lies above the sum on the second route, 59.31 thousandths, and below the sum on the first route, 84.95 thousandths, so by the rule of section I it is a measurement of interest for the choice between the two routes.

V. Beside Chapter 5, "What Put the Bang in the Big Bang?"

The coming telescopes will measure the faint twist in the polarization of the microwave background. The polarization of light is the direction in which the light vibrates. Gravitational waves are ripples in the shape of space and time itself. The twist would come from the first waves: gravitational waves present from the beginning of our epoch. Physicists state the strength of the twist as the *ratio of tensor to scalar power*: the power of the first waves divided by the power of the first ripples of density. I say that the ratio is far below one part in a million, and that the instruments will find no first waves. Prediction 10 of the sealed record reads: "The Simons Observatory and LiteBIRD will find no primordial tensor signal". LiteBIRD is a Japanese satellite, and the Simons Observatory is an array of telescopes in the Atacama Desert of Chile. In chapter 17 of *What the Beginning Left Behind*, "Listening for the Beginning", the book states the smallest ratio that the two instruments will be able to measure, and the time the two instruments will take to measure a ratio that small. Within about twenty years, "The Simons Observatory and LiteBIRD reach a ratio of first waves to first ripples of about a thousandth". A thousandth is a thousand

times one part in a million, so, on my account, the ratio lies far below the smallest ratio that the two instruments can measure. Tegmark expects a different result. Inflation is a very fast stretching of space that Tegmark places at the beginning of our universe. In his paper of 2005, the best-motivated models of inflation predict a ratio of about 0.03. Tegmark expected the coming maps of the polarization to detect a ratio at that level. In his book Tegmark writes that a detection of the first waves "will be hailed as smoking-gun evidence that inflation really happened" (chapter 5, "Sowing the Seed Fluctuations"). The dated record of 4 November 2026 names the measurement of the first waves as the clearest test between Tegmark's account and mine (Tegmark 1). Prediction 10 also bears on chapter 15.

The coming maps of the microwave background will also measure whether every ingredient of the early universe rippled together. I say every ingredient rippled together, with no separate pattern of ripples belonging to one ingredient alone. When every ingredient ripples together in one shared pattern, physicists call the ripples *adiabatic*. Prediction 11 of the sealed record reads: "The dark matter carries no separate pattern of primordial departures of its own". *At the Edge of Reality*, Appendix B, says that the measurement is one of the measurements of "the surveys of the coming decade".

Four measurements, taken together, will show whether the early universe passed through a long inflation: the flatness of space, the first waves, the ripples of every ingredient together, and the one pattern across the sky. A long inflation would be a burst of extremely fast expansion that stretched a small region to beyond the present horizon, the edge of the region whose light has had time to reach us. I say no long inflation occurred. On my account, the sameness and flatness of the sky come from the turn. The turn is the passage from the end of the previous epoch to the start of ours, an epoch being one long age of the universe. Prediction 13 of the sealed record reads: "No inflationary phase long enough to stretch the lengths observed today from within the early horizon to beyond it occurred". The early horizon is the horizon near the turn: the farthest distance that light could have traveled in the short time after the turn. The

sealed record names the measurements that decide Prediction 13: the measurements of Predictions 9 to 12, taken together. Tegmark expects a different result. He calls inflation "Our best theory for what put the bang into our Big Bang" (chapter 13, "The Future of Our Universe: How Will It End?"). He also writes that "it's too early to say for sure whether our Big Bang really was caused by inflation" (chapter 5, "Sowing the Seed Fluctuations"). Tegmark reports in his book that, of the models of inflation studied, "almost all of them lead to eternal inflation" (chapter 5, "Eternal Inflation"). Eternal inflation is an inflation that never ends everywhere at once (Tegmark 2 in the record of 4 November 2026). Prediction 13 also bears on chapters 6 and 15.

The *count of light species* is the measure of how much radiation the young universe held, counted in units of one kind of neutrino. In the microwave background, the count of light species will weigh the waves sent out at the turn itself. The count will measure the total energy of those waves. I say that at the turn the six held internal dimensions settled and came to rest at their held size, and the settling of the six dimensions left a background of gravitational waves with an energy share of at most about five parts in ten million. The settling waves vibrate at a frequency above about five hundred billion vibrations a second. Prediction 14 of the sealed record says of the frequency of the settling waves: "no existing or proposed instrument reaches that frequency". The count of light species measures the settling waves all the same, because the count weighs the waves, whatever their frequency. The energy of the settling waves made the young universe expand faster. The count of light species registers that energy in every band of frequency at once.

The Laser Interferometer Space Antenna (LISA) will listen for a background of gravitational waves from a sudden change in the state of the early universe. LISA is the space observatory of gravitational waves that the European Space Agency adopted in January 2024. I say LISA will find no background from a sudden change of state of the early universe. *What the Beginning Left Behind* says what LISA will register, in the section "What the Coming Measurements Will Find" of the book's closing chapter:

"backgrounds of waves from the sky, from pairs of white dwarfs and of black holes". A white dwarf is the dense, cooling core that a star like the Sun leaves behind when it has burned out. Prediction 14 of the sealed record reads, for LISA: "LISA will find no stochastic background from a cosmological phase transition, since the electroweak and strong transitions are crossovers and no further kind of particle exists". A *stochastic background* is a steady random hum of gravitational waves from many sources at once. The *electroweak and strong transitions* are two changes of state that the young universe passed through as it cooled. The *weak force* is the force of some radioactive decays, and the electromagnetic force is the force between charges, which light carries. In the electroweak transition, the weak force parted from the electromagnetic force and took its present form. The *quarks* are the particles of which protons and neutrons are built. In the strong transition, the quarks became bound inside protons and neutrons. A *crossover* is a change of state that happens smoothly, with no sudden jump, as butter softens on a warm counter. *What the Beginning Left Behind*, chapter 17, also states when the count of light species and the results of LISA will come. Within about twenty years, the Simons Observatory measures the count of light species "to within five to seven hundredths", and "LISA registers its band".

The pulsar-timing arrays listen for the slowest gravitational waves by timing the steady pulses of distant spinning stars, called pulsars, over many years. A slow wave passing between a pulsar and us shifts the arrival times of the pulses of the pulsar. The pulsar-timing arrays will lengthen their records of a background of gravitational waves that they have already found. The North American and European array teams have both reported evidence of the background, a slow hum near a billionth of a vibration each second: one rise and fall in about thirty years. I say that the hum comes from pairs of giant black holes circling each other at the centers of galaxies. The sealed record states one condition for the claim about the hum: the first ripples of density on short lengths are near the strength measured for the first ripples on long lengths. Prediction 14 of the sealed record reads, for the arrays: "The background found by the

pulsar-timing arrays is of astrophysical origin, from binaries of supermassive black holes, if the scalar power on short lengths is near the measured power". In Prediction 14, the scalar power is the power of the first ripples of density. In chapter 16 of *What the Beginning Left Behind*, "The Hum of the Settling", section "What the Pulsars Hear", the book states the reason for the claim about the hum, and the reason has three parts. First, on my account the early universe made no sudden change of state. Second, on my account the early universe made no network of strings. Strings are thin lines of trapped energy that some theories expect the early universe to leave behind. Third, the first ripples of density make gravitational waves of their own, and at the measured strength of the ripples, those waves are some fourteen powers of ten too faint to be the hum.

Two entries given earlier in this appendix also bear on chapter 5. Space is flat, curved only by its own small ripples (Prediction 9, under chapter 2). The first ripples are one pattern across the whole sky, with nearly the same strength at every length (Prediction 12, under chapter 3).

VI. Beside Chapter 6, "Is There Another Copy of Me Out There?"

Astronomers will time the arrival of light of different energies from distant explosions. The timing will show whether space and time have a least step. A least step would be a smallest length or time, below which space and time could not be divided. I say space and time have no least step. Prediction 5 of the sealed record reads: "The speed of light in vacuum does not depend on the energy of the light through any step of spacetime". Photons are the particles of light. The sealed record names the measurement: "The timing of photons of different energies from many sources, told apart from effects at the source by its growth with distance; every observation of the spectrum of area or volume". If space and time made a delay between photons of different energies, the delay would grow with the distance the light traveled, while a delay made at the

source would not grow with the distance, so the growth tells the two kinds of delay apart. A spectrum of area or volume would be a set of separate allowed sizes of area or volume. A space made of least steps would show such a spectrum. Tegmark leans the other way on the smoothness of space: of the model of space as an unbroken continuum, he writes, "there's mounting evidence against it" (chapter 13, "The Future of Our Universe: How Will It End?"). Prediction 5 also bears on chapters 8, 14 and 15.

The sealed record also contains statements of kind, statements that say what every finding is like, and Prediction 28 is a statement of kind. I say that no world is completely determined by its past and that no process runs perfectly backward. Prediction 28 of the sealed record reads: "No superdeterministic completion of quantum mechanics will be found, and no world is purely and completely deterministic". A superdeterministic completion of quantum mechanics would be a fuller theory beneath quantum theory, in which every outcome, and every experimenter's choice of what to measure, would be fixed together in advance by a common past. The sealed record says what kind of prediction Prediction 28 is: "These are exclusions that the structure of comparison makes; the outcome of a search for departures from the quantum predictions bears on Prediction 25 and not on this one". Prediction 25, under chapter 8, is the prediction that repeated measurements follow the statistics of quantum theory. Tegmark's account differs. Hugh Everett proposed the many-worlds account of quantum mechanics. On the many-worlds account, every possible outcome of a quantum event happens, each in a branch of its own. Tegmark writes in his book that "Everett's work suggests that randomness, too, is an illusion" (chapter 9, "Physics: Linking External Reality to Consensus Reality"). The dated record of 4 November 2026 lists the difference over determinism and randomness among the differences that no measurement decides. Prediction 28 also bears on chapters 8, 9, 10, 11, 13 and 14.

Three entries given earlier also bear on chapter 6. Space is flat, curved only by its own small ripples (Prediction 9, under chapter 2). No closed path around the universe, anywhere, is shorter than the width of the hori-

zon (Prediction 23, under chapter 2). No long inflation occurred: no inflation stretched the lengths we see today from inside the early horizon to beyond it (Prediction 13, under chapter 5).

VII. Beside Chapter 7, "Why Are the Constants What They Are?"

Physicists will search at colliders, in underground detectors and in the sky for kinds of particle beyond the kinds already known. I say the list of the kinds of particle is closed, and that every search for a new kind will find only the known kinds. Prediction 30 of the sealed record reads: "No further kind of particle will be found". Prediction 30 goes on: "The next collider will not find a new species". Prediction 30 also bears on chapters 10, 13 and 15.

The *generations* are the three repetitions of the particles of matter. Each generation has two quarks, a charged *lepton* and a neutrino. The quarks are the particles of which protons and neutrons are built. A charged lepton is the electron or one of its two heavier copies. Measurements at colliders count the generations. The Z boson is a carrier of the weak force, the force of some radioactive decays. Physicists count the kinds of light neutrino, one for each generation, by measuring how often the Z boson decays into neutrinos. The Higgs boson is the particle of the field that gives the smallest particles their masses. Physicists have measured the rates at which the Large Hadron Collider makes the Higgs boson. The measured rates leave no room for a fourth generation of the known kind. I say there are three generations. Prediction 31 of the sealed record reads: "The generations number exactly three; no fourth exists at any mass". The sealed record names no coming instrument for Prediction 31. Prediction 31 also bears on chapters 10 and 13.

The strong force is the force that holds the nucleus together, and the strong force has a symmetry between a process and the mirror image of the process made of antimatter. Two kinds of measurement will show

whether the strong force needs a new particle to keep that symmetry: searches for the axion, and measurements of how electric charge is spread within the neutron. The *axion* is the light particle that physicists have proposed to keep the symmetry of the strong force, and physicists have also proposed the axion as the substance of the dark matter. I say that no axion will be found and that none is needed. Prediction 34 of the sealed record says that "the structure calls on no axion to cancel the measured angle". The measured angle is the one number by which the strong force could treat a process differently from the mirror image of the process made of antimatter, and measurements of the neutron show that the angle is below 1.8 parts in ten billion. Tegmark has explored a different result in his published work. In a case study that he worked through in 2006 with his coauthors Anthony Aguirre, Martin Rees and Frank Wilczek, the dark matter is made of axions (Tegmark 3 in the record of 4 November 2026).

The directions of space will be counted by every measurement of the waves of gravity, and by every measurement of the large world. I say every region and every epoch will show three directions of space. *Measurement*, chapter 11, section VI, states the count: "So the dimension of space is three". The dated record beside the scientist Stephen Wolfram lists the count of three, for every region and epoch, among the predictions outside the sealed record (Wolfram 3, *Side by Side*, Volume Two, Appendix F). Tegmark expects the same count where observers live. In his paper of 1997 on the dimensions of spacetime, Tegmark finds that every count but three directions of space and one of time may give worlds "devoid of observers" (Tegmark 8 in the record of 4 November 2026). Wolfram 3 also bears on chapter 14.

Three entries given earlier also bear on chapter 7. The matter fraction is 0.311 (Prediction 7, under chapter 4). The matter fraction is the share of the energy of a flat universe that matter carries today. The energy of empty space has an equation of state of exactly minus one, so the density of that energy is the same at every time of our epoch (Prediction 8, under chapter 4). The ratio of dark to ordinary matter is six less two divided by pi, about 5.36 (Prediction 16, under chapter 4).

VIII. Beside Chapter 8, "What Happens When We Look?"

Repeated experiments on identically prepared systems will measure the statistics of quantum theory. The experiments measure three things: how often each outcome is found, whether light through three slits interferes in a way that the pairs of slits do not account for, and how strongly the findings of two distant observers can be correlated. I say the experiments will find exactly the statistics that quantum theory predicts. The *Born rule* says that the chance of each finding is the square of the size of its amplitude. An amplitude is the number that quantum theory attaches to a possible finding. Prediction 25 of the sealed record reads: "Repeated measurements on identically prepared systems settle to the weights of the Born rule, in every sector and on the full space". In Prediction 25, a sector is one family of the inner labels that a particle can carry, such as the charges of the particle, and the full space contains all of the states of the particle, including its place, motion, spin and energy. Prediction 25 also says: "The interference among three paths has no term that the pairs of paths do not account for". Tegmark expects the same result. He takes the statistics of quantum theory unchanged (Tegmark 7 in the record of 4 November 2026). Prediction 25 also bears on chapters 9, 11 and 14.

A discrete rule is a rule that moves forward in separate least steps. For any discrete rule offered as the foundation of physics, a measurement will find statistics that differ from the statistics the rule predicts, once the measurement is fine enough to detect differences smaller than the least step of the rule. Prediction 29 of the sealed record reads: "For any one fixed discrete rule offered as the foundation of physics, a sensitive enough measurement, reaching below the rule's least step, tells the rule's statistics from the world's". Tegmark's Computable Universe Hypothesis is his proposal that only structures a computation can work out exist. The Computable Universe Hypothesis names a class of structures and no one rule, so

in the dated record of 4 November 2026 Prediction 29 applies to the Computable Universe Hypothesis once someone offers a particular rule. Prediction 29 also bears on chapters 10, 13 and 14.

Two entries given earlier also bear on chapter 8. Space and time have no least step: the speed of light does not depend on the energy of the light (Prediction 5, under chapter 6). No world is completely determined by its past (Prediction 28, under chapter 6).

IX. Beside Chapter 9, "Am I Splitting Into Copies of Myself?"

Experiments now being built will hold two small masses, each in two places at once, linked to each other only by gravity. The experiments will measure whether the two masses become entangled, which happens when the states of the two masses become linked so that neither mass can be described without the other. I say the two masses will become entangled. A *superposition* is a state in which a thing is in two alternatives at once. Prediction 1 of the sealed record reads: "Two masses, each held in a superposition of two places and related only through gravitation, become entangled, with the phase of their Newtonian interaction". In quantum theory, each alternative of a thing has a *phase*: the timing of the wave that describes that alternative, counted as an angle. The gravity between the two masses shifts the phase of each placement of the pair by its own amount. The *entangling phase* is the phase of the two placements in which the masses are both near or both far, less the phase of the two placements in which one is near and the other far. For the design that the physicist Sougato Bose and his colleagues proposed in 2017, the sealed record states the entangling phase as 0.314 radians, about one twentieth of the way around a circle. *At the Edge of Reality*, Appendix B, says that experiments to test whether gravity alone entangles two masses are being built. *What*

the Beginning Left Behind, chapter 17, says that the experiments with two masses will come within about twenty years, aiming at the entangling phase of 0.314 radians for the design of Bose and his colleagues.

Decoherence is the fading of interference between the alternatives of a thing as its surroundings come to carry records of which alternative it is in. Two kinds of experiment will measure whether a large mass or a long time ends a superposition of its own accord: the experiment with two masses, and every search for a decoherence beyond the ordinary kind. I say only a finding ends a superposition. Prediction 2 of the sealed record says that "a superposition ends only through a finding, an interaction that registers which alternative is the case". Tegmark expects the same result. On the many-worlds reading that he defends, no mass and no length of time ends a superposition either, beyond the ordinary decoherence by an environment (Tegmark 6 in the record of 4 November 2026). On my account, ordinary decoherence is a comparison made by the environment: an act in which the environment tells one alternative from another. The record of 4 November 2026 also says that Tegmark's account and mine part on what a finding is. The record also says that no measurement can decide that difference. Prediction 2 also bears on chapters 12 and 14.

Every test that measures at two distant places will measure whether a choice made at one place changes what is found at the other place before a signal from the first place could arrive. I say a choice made far away never changes the local findings. In Prediction 27 the signal meant is a signal sent from the place where the far measurement was chosen, traveling no faster than light. Prediction 27 of the sealed record reads: "Until such a signal arrives, every local statistic follows quantum theory unchanged, whatever measurement was chosen far away". Prediction 27 further says that no later finding comes before an earlier one, and that no signal travels faster than light. Prediction 27 also bears on chapter 11.

Two entries given earlier also bear on chapter 9. Repeated measurements on identically prepared systems settle to exactly the statistics that quantum theory predicts (Prediction 25, under chapter 8). No world is completely determined by its past (Prediction 28, under chapter 6).

X. Beside Chapter 10, "Is the World Made of Mathematics?"

To be *quantized* is to come in quanta, separate least amounts. Two kinds of measurement will bear on whether the geometry of space and time carries quanta of its own: the work of every detector of gravitational waves, and every measurement of how gravitational waves scatter. I say gravitation is not quantized: the geometry of space and time has no quanta of its own. Prediction 3 of the sealed record reads: "No effect will show the geometry carrying quantum degrees of freedom of its own, beyond those tied to the alternatives of the matter". A quantum degree of freedom is a way of varying that comes in quanta, and the alternatives of the matter are the places or states that a piece of matter holds together in a superposition. On my account the geometry varies only with the alternatives of the matter, one geometry for each alternative. Prediction 3 also says: "A detector that registers a gravitational wave in single clicks agrees with the account and does not show gravitation quantized". Tegmark expects a quantum theory of gravity, and he writes in his book: "I predict that the correct mathematical theory of quantum gravity will break all previous records in being difficult to interpret" (chapter 9, "Physics: Linking External Reality to Consensus Reality"). Prediction 3 also bears on chapter 16.

Four entries given earlier also bear on chapter 10. The first entry is the matter fraction of 0.311 and the ratio of dark to ordinary matter of 5.3634 (Predictions 7 and 16, under chapter 4). The two values are regularities of the kind that Tegmark expects further measurement to uncover. Tegmark expects such regularities under his Mathematical Universe Hypothesis: his proposal that our world is a mathematical structure. The second entry is the closed list of the kinds of particle and the three generations (Predictions 30 and 31, under chapter 7). The third entry is that no world is completely determined (Prediction 28, under chapter 6). The fourth entry is

that a fine enough measurement shows the statistics of any discrete rule offered as the foundation of physics to differ from the statistics of the world (Prediction 29, under chapter 8).

XI. Beside Chapter 11, "Is Time an Illusion?"

Five entries given earlier bear on chapter 11. No later finding comes before an earlier one, and no signal travels faster than light (Prediction 27, under chapter 9). No world is completely determined, and no process runs perfectly backward (Prediction 28, under chapter 6). The energy of empty space is exactly constant through our epoch (Prediction 8, under chapter 4). The dark energy is the part of the energy of empty space that a measurement takes in. A constant density of dark energy is the premise that Tegmark states for his argument about the far future: "if the cosmic dark-energy density remains constant" (chapter 11, "Why Aren't You a Boltzmann Brain?"). The matter fraction is 0.311 (Prediction 7, under chapter 4). The value 0.311 comes from an identity between the energy of empty space and the horizon of the present measurement, the edge of the region whose light has had time to reach us. The identity holds in the era in which the measurement is made. In *The View from Inside*, I ask why we measure the universe at this moment of its history, and I divide the question into three parts. The first part is how far along its history the universe now is, and the identity is the answer to the first part. Repeated measurements settle to exactly the statistics that quantum theory predicts (Prediction 25, under chapter 8).

XII. Beside Chapter 12, "What Am I?"

Criticality is the poise between order and disorder at which small disturbances can cascade into large ones. Recordings of the living brain will measure whether the cortex works near criticality. I say the cortex works near criticality. *The Basis of Physics*, essay 19, section V, says: "The brain, on

this account, operates near criticality". Giulio Tononi is the author of the measure called integrated information. The dated record beside Tononi lists the statement that the brain operates near criticality among the predictions outside the sealed record (Tononi 1, *Side by Side*, Volume Two, Appendix M). The record beside Tononi names the measurements that decide the question. The deciding measurements tell a critical state apart from cascades of the same shape made by noise alone. *Objections from Physics, Answered* is my book of answers to questions raised from physics. The record beside Tononi quotes *Objections from Physics, Answered* on what kind of question the working state of the cortex is: "the regime in which the cortex operates is a question of measurement, and its answer leaves the triple as it is". The triple in the quotation is the *bound triple*: the arrow of time (an order that runs one way, from past to future), hard randomness (an outcome that nothing before it fixed) and the subjective (being the one to whom things happen), bound into one act.

Tegmark estimated in 2000 that superpositions in the brain fade by decoherence in about a ten-trillionth of a second or less, far faster than nerve cells act. The dated record of 4 November 2026 says that the collection "accepts Tegmark's estimate at its own scope" (Tegmark 9). I accept the estimate because my account of choice depends on criticality, and needs no superposition held for a long time in the brain. One entry given earlier also bears on chapter 12: gravity alone ends no superposition (Prediction 2, under chapter 9).

XIII. Beside Chapter 13, "Do All Mathematical Structures Exist?"

Three entries given earlier bear on chapter 13. The sealed record says that the kinds of particle are "reasoned from comparison as a closed list", and the closed list of kinds and the three generations (Predictions 30 and 31, under chapter 7) are the parts of that statement that a measurement can check. A fine enough measurement shows the statistics of any discrete

rule offered as the foundation of physics to differ from the statistics of the world (Prediction 29, under chapter 8). No world is completely determined (Prediction 28, under chapter 6).

XIV. Beside Chapter 14, "Is the World a Computation?"

Six entries given earlier bear on chapter 14. No search will find a least step of space or time (Prediction 5, under chapter 6). When a measurement can detect differences smaller than the least step of any discrete rule offered as the foundation of physics, the measurement will find statistics that differ from the statistics the rule predicts (Prediction 29, under chapter 8). No world is completely determined (Prediction 28, under chapter 6). Every test of the statistics of quantum theory will find exactly the statistics that quantum theory predicts (Prediction 25, under chapter 8). Only a finding ends a superposition, and no decoherence occurs beyond the ordinary kind (Prediction 2, under chapter 9). Every region and every epoch shows three directions of space (Wolfram 3, under chapter 7).

XV. Beside Chapter 15, "How Will Our Universe End?"

The coming searches will measure how long a proton lasts. I say the searches will find the proton stable, at every lifetime that the searches are able to test. Prediction 33 of the sealed record says that "the proton has no route to decay". Two instruments make the search. Hyper-Kamiokande is a large tank of water deep underground in Japan, and the Deep Underground Neutrino Experiment (DUNE) has tanks of liquid argon. *What Comes After the Dark*, Appendix B, states the reach of Hyper-Kamiokande after ten years: the longest proton lifetime that Hyper-Kamiokande can test. A positron is the electron's antiparticle, and a neutral pion is a short-lived particle with no electric charge. "If one of its tanks sees no decay in ten years", Hyper-Kamiokande will set the proton's lifetime for its decay

into a positron and a neutral pion above seventy-eight million billion billion billion years, at 90 percent confidence. A limit stated at 90 percent confidence means that, if the proton's true lifetime were shorter than the limit, the search would have seen a decay, and so crossed the limit, nine times in ten. My account says both instruments will find the proton stable.

What Comes After the Dark, Appendix B, names five coming measurements of the far future. The five measurements are the lifetime of the proton, the equation of state (Prediction 8), the flatness of space (Prediction 9), the first waves (Prediction 10) and the lifetime of the dark matter (Prediction 20). Eight entries given earlier also bear on chapter 15. The energy of empty space is exactly constant through our epoch, so the expansion never rips (Prediction 8, under chapter 4). A rip would be an expansion that grows without limit and tears galaxies, planets and atoms apart. Space is flat, curved only by its ripples, so with a constant energy of empty space the expansion never reverses into a collapse (Prediction 9, under chapter 2). The dark matter does not decay (Prediction 20, under chapter 4). No first waves will be found (Prediction 10, under chapter 5), and no long inflation occurred (Prediction 13, under chapter 5). The first ripples are one pattern across the sky, and the pattern is the form that passed through the turn from the previous epoch, the long age of the universe before ours, into ours (Prediction 12, under chapter 3). Space and time have no least step, so space has no grain that stretching could snap (Prediction 5, under chapter 6). No kind of particle exists beyond the closed list, so no further field can drive a rip or a bubble of a new vacuum (Prediction 30, under chapter 7). A field is a quantity spread through space with a value of its own at every place and time. A bubble of a new vacuum would be a region inside which space changes into a different state of itself. The bubble would spread at the speed of light.

XVI. Beside Chapter 16, "What Do I Now Hold?"

One entry given earlier bears on the closing chapter: gravitation is not quantized (Prediction 3, under chapter 10). In his closing chapter, Tegmark takes "What are the equations of quantum gravity?" as his example of a question that his Mathematical Universe Hypothesis rephrases (chapter 13, "The Future of Physics"). Prediction 3 of the sealed record is my answer to the part of Tegmark's question that a measurement can decide.

For the Specialist

This appendix gives, for every claim that a chapter of this book makes about my account of the Immanent Metaphysics, the book of mine, the chapter and the section that carry it. The appendix also gives the exact statements, values and relations that the chapters keep in words. Some sections also hold longer passages that carry a chapter's steps further than the chapter needs, each with a line before it that says what the passage is. Read the appendix beside a chapter: each section below follows one chapter, with the claims in the order the chapter makes them, and then the exact statements and values that belong to that chapter.

The Preface

I hold the premise Tegmark starts from: there is a world, and it does not depend on us.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

I hold that premise on a ground of my own: every interaction is a comparison, so the world was comparing itself, in every collision of particles, long before anything human looked at it.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

Every measurement is a comparison, and so is every interaction, anything that happens between two things as each acts on the other.

Home in the World, Prelude, "Two Crews, One Tunnel", section "The Crew from Knowing"; and *Side by Side*, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

Metaphysics here means the study of what must be so for anything at all to be known. *Immanent* means within: the Immanent Metaphysics works from inside the act of knowing, here and now, and never from a place outside the world.

Home in the World, Prelude, "Two Crews, One Tunnel", section "The Crew from Knowing".

From the same measurements, I tell the history Tegmark tells of a universe that has expanded from a hot and dense early state. The galaxies move away from us faster the farther off they are, as Edwin Hubble found, because space itself is stretching.

At the Edge of Reality, chapter 2, "What Came Before the Big Bang?", section "The Expanding Universe".

A faint glow left from the early universe arrives from every direction of the sky, as Arno Penzias and Robert Wilson found.

At the Edge of Reality, chapter 2, "What Came Before the Big Bang?", section "The Afterglow".

I agree in substance about Level I: space goes on beyond what we can see, with the same laws, and the regions beyond our view belong to the world because they act on the regions next to them, as our region does. Tegmark calls the regions beyond our view parallel universes; I call them further parts of one world.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level".

I accept Tegmark's estimate that quantum superpositions in the warm brain fade far faster than nerve cells act.

Side by Side, Volume Two, chapter "The Structure and the Act", section I, "Tegmark's Position", and section II, "The Ground the Two Share".

Tegmark's estimate bears on one proposal, that the mind rests on quantum superpositions held for a long time in nerve cells, and my account of the mind needs no such long-held superpositions.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

What exists, in this book's sense, is laid out whole, timeless and the same for everyone who reads it.

Comparison; The Necessary Basis, chapter 10, "Reasoning", section II, "The Real and the Abstract".

What is real, in this book's sense, is given in an act of finding, made once, from one place.

Comparison; The Necessary Basis, chapter 10, "Reasoning", section II, "The Real and the Abstract".

Every physical theory has both: its mathematics, which exists, and its findings, which are real. Tegmark's hypothesis takes the mathematics, which exists, for the whole world.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

I agree with Tegmark that a description of a world that does not depend on us must contain nothing human.

Side by Side, Volume Two, chapter "The Structure and the Act", section IV, "Why a Description Free of Humans Still Has a Stance".

Every comparison is made from some side, a collision of two particles as surely as a look through a telescope. The side from which a comparison is made belongs to every comparison, human or not, so removing every human word from a description leaves that side in place.

Side by Side, Volume Two, chapter "The Structure and the Act", section IV, "Why a Description Free of Humans Still Has a Stance", and its section "Terms".

Mathematics describes what is found as if no one stood anywhere to find it: it sets aside the side from which every comparison is made.

Side by Side, Volume Two, chapter "The Structure and the Act", "Abstract".

Every mathematical structure, taken whole, is laid out all at once, complete, with nothing missing and nothing still to come, and it is the same wherever and whenever it is read. So every mathematical structure, taken whole, holds perfect sameness and gapless completeness together. No single act of finding can hold perfect sameness and gapless completeness together.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

So, on my account, the world that comparisons find, one act of finding at a time, is described by mathematics, and it is not a mathematical structure.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Mathematics keeps every power it has: it is the finest instrument there is for describing what is laid out as structure.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

I exclude Level II. On my account no long inflation occurred, and the fundamental equations settle into one set of local laws, so the constants of nature are the same everywhere; Tegmark's second level needs many sets of local laws.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level", and section VIII, "Where the Predictions Part, and Why", under "A long inflationary phase".

I hold that every finding is made now, with the future not yet settled, and that the outcome of a single quantum measurement is fixed by nothing that came before it, however completely the past is known.

Comparison; The Necessary Basis, chapter 10, "Reasoning", section II, "The Real and the Abstract"; and *Causality Without Determinism*, chapter 5, "Computation, Consciousness and Simulation", section V, "Hard Randomness and Choice".

On my account the flow of time, change and randomness are real, in this book's sense, because each of the three is given in an act of finding.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section V, "Hard Randomness and Choice"; and *Comparison; The Necessary Basis*, chapter 10, "Reasoning", section II, "The Real and the Abstract".

The Preface states no value or relation beyond these claims.

Chapter 1, "What Is Reality?"

Comparing is telling one thing from another, or finding them the same; a finding is what one comparison gives.

Physics Is Not Math, chapter 1, "What It Is to Compare", section I, "Two Stones", and section II, "Instruments Compare Too".

Every measurement is a comparison.

What the Beginning Left Behind, "Before We Begin", section "What Stands Behind the Book".

An atom takes in light only when the energy of the light matches a gap between the energies the atom can hold, and its taking in the light is a finding made by the atom. An interaction is something happening between two things, each acting on the other, and every interaction, the atom's included, is a comparison.

Physics Is Not Math, chapter 1, "What It Is to Compare", section IV, "One Act, Three Names".

My physics books start from the act of comparison, and the Immanent Metaphysics is the account of what every act of comparison contains. *Physics Is Not Math*, Preface.

The content of a comparison is what the comparison finds the same or different. The context of a comparison is the respect in which the two things compared are two: two places, or two moments, as when a tide gauge sets the height of the sea at noon against its height at midnight. Place, time and possibility are the three contexts in which anything is ever found.

Physics Is Not Math, chapter 1, "What It Is to Compare", section VI, "Six Parts in Three Pairs"; and *What the Beginning Left Behind*, chapter 1, "Where the Last Book Left Us", section "What the Last Book Found".

Two people play chess at a kitchen table with a score sheet beside the board; the score sheet, the move and the seat are three ways of looking at one game.

What Cannot Be, chapter 4, "Process, Pattern and Possibility", section "Three Ways of Looking at a Game"; *What Cannot Be*, chapter 2, "Two Kinds of Law", section "World and Self"; and *Comparison; The Necessary Basis*, chapter 7, "The Order of Finding", section I, "A Game of Chess".

What exists is what is laid out whole, timeless, and the same for everyone, as the score sheet is: a structure. What is real is what is given in an act of finding, once, as the move is. I set the two words together in one line: "A law exists; a measurement is real".

Comparison; The Necessary Basis, chapter 10, "Reasoning", section II, "The Real and the Abstract".

The stance is the side from which a comparison is made, as a move is made from a seat; every comparison has one, human or not.

Side by Side, Volume Two, chapter "The Structure and the Act", section "Terms"; and *Physics Is Not Math*, chapter 1, "What It Is to Compare", section VI, "Six Parts in Three Pairs".

I hold Tegmark's starting premise in its plain sense: there is a world, and it does not depend on us, human beings. I hold the premise on a ground of my own. Every interaction is a comparison, so the world was comparing itself, in every collision of particles, long before anything human looked at it.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

Side by Side sets twenty-three models of the universe beside my own, twenty-four in all, and its essay on Tegmark holds his External Reality Hypothesis on a ground of its own.

Side by Side, Volume Two, chapter "Twenty-Four Models, One Record", and chapter "The Structure and the Act", section II, "The Ground the Two Share".

Physics need not first explain the human mind, because a stance need not be a person. A detector with no one watching still clicks when a particle passes, and the click is a comparison of a charge against a threshold. The word "observer" names a place inside the world from which a comparison is made, and never names a mind.

Measurement; The Steps from Comparison to Physics, chapter 2, "Every Measurement Is a Comparison", section VII, "Questions".

The exactness of mathematics in physics has a reason: in the words of *Measurement*, "the mathematics and the physics are two aspects of one act of comparison". The same book adds that the mathematics of a theory reaches what the theory finds densely, as near as any precision requires. *Measurement* states that dense reach in these words: "a continuous formalism assigns values to discrete outcomes to any precision required, and a symmetric formalism carries a smooth content along any path to any precision required". So mathematics can follow what physics finds as closely as anyone wishes. The number pi shows, in one case, a sameness that mathematics and physics share: sameness under rotation. Pi, the distance around a circle divided by the distance across it, appears wherever sameness under rotation appears: in the circle, and in the law that gives how the errors of measurements spread, whose scatter looks the same however it is rotated about its center.

Measurement; The Steps from Comparison to Physics, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete".

Observation and reasoning are both acts of comparison. An observation compares a quantity with a standard. Reasoning compares a proposed pattern, a hypothesis, with what has been observed, and a test compares a prediction with an outcome.

Physics Is Not Math, chapter 1, "What It Is to Compare", section III, "The Tide Gauge".

I take the act of comparison, which every measurement is, as the ground of my whole account.

What the Beginning Left Behind, "Before We Begin", section "What Stands Behind the Book".

Every physical theory has two levels: its mathematics, which exists, and its findings, which are real. Tegmark's hypothesis takes the level of mathematics for the whole.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part"; and *Comparison; The Necessary Basis*, chapter 10, "Reasoning", section III, "A Physical Theory Has Two Levels".

An orange peel pressed flat splits, or stretches, or does both: a flat map of a round surface cannot keep every distance true and also stay unbroken.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section I, "A Map That Must Give Way".

The Incommensuration Theorem is a structural fact about the nature of knowing. The theorem is about comparison, the act in which anything at all is known.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem".

Symmetry means sameness, a thing unchanged when you swap it or move it; continuity means having no gaps. The two opposites of symmetry and continuity are asymmetry, a difference from one place or moment to the next, and discontinuity, coming in separate steps. To apply two of the four qualities (symmetry, continuity, asymmetry and discontinuity) simultaneously and fundamentally is to have the two qualities, perfect, at once, at the foundation of one comparison. So the theorem says that no single act of finding can hold perfect sameness and perfect gaplessness together, and no single act of finding can hold perfect difference and perfect separate steps together. The theorem finds a choice like the mapmaker's inside every act of comparison: of each pair of qualities the theorem names, a single comparison holds at most one of the two perfectly, and the other gives way.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section II, "Four Characters", and section I, "A Map That Must Give Way".

I name six parts that every comparison has, and call them the six *intrinsics* of comparison, from a word meaning belonging to the thing itself. The six are: sameness and difference, the only two verdicts any comparison can give; the content and the context; the stance from which the comparison is made; and what the comparison is made about. *Physics Is Not*

Math names the six in these words: “sameness and difference, content and context, the subjective and the objective”; the subjective is the stance, and the objective is what the comparison is made about. The theorem is built from nothing but the meanings of the six intrinsics. Every act of knowing is a comparison or is built from comparisons, so the theorem, together with the intrinsics of comparison from which it is built, is inherent in every single act of knowing. The theorem is directly about comparison, the act in which every other result is reached, and so it is the deepest and most powerful epistemic basis possible. *Epistemic* means concerning knowing: how anything comes to be known, checked and held. The theorem holds with no condition of any kind.

Physics Is Not Math, chapter 1, “What It Is to Compare”, section VI, “Six Parts in Three Pairs”; and chapter 2, “Two Things That Cannot Both Be Perfect”, section VI, “The Theorem”.

A circle in mathematics is perfectly smooth and perfectly the same under every rotation about its center, both at once and exactly. Mathematics can write the circle down because mathematics leaves out the stance: a theorem names no one who checks it, no place and no moment, and so mathematics can write sameness and gaplessness together. No comparison ever finds a thing both perfectly smooth and perfectly the same under every rotation.

Physics Is Not Math, chapter 3, “Truth Has a Frame”, section VII, “Mathematics Works as if Only the Objective Existed”; and chapter 2, “Two Things That Cannot Both Be Perfect”, section IX, “What the Theorem Allows”.

Every mathematical structure, taken as a whole, is laid out all at once, complete, with nothing missing and nothing still to come, and it is the same wherever and whenever it is read. So, taken whole, every mathematical structure holds perfect sameness and gapless completeness at once, and no single act of finding can hold perfect sameness and gapless completeness together. A structure whose parts are separate and irregular holds perfect sameness and gapless completeness as every mathematical structure does, because the sameness and the completeness belong to the structure taken whole.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

The world that comparisons find, the only world physics has, is not a mathematical structure, and its mathematical description, however exact, comes as close to that world as anyone wishes and never coincides with it.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part"; and *Measurement; The Steps from Comparison to Physics*, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete".

Tegmark reaches his hypothesis through a description with nothing human left in it, and I part from that route because a description with nothing human left in it still describes comparisons made from stances. I agree that a description of a world independent of us must carry no human word. The stance stays when every human word is gone, because a stance belongs to every comparison, human or not: a collision of two particles is made from a stance as surely as a reading in a laboratory is. So a description stripped of everything human still describes comparisons made from stances. The score sheet that a description stripped of everything human lays out holds what each comparison is about, and it is never the whole of the comparison, because the stance from which each comparison was made is left off the sheet.

Side by Side, Volume Two, chapter "The Structure and the Act", section IV, "Why a Description Free of Humans Still Has a Stance"; and *Physics Is Not Math*, chapter 3, "Truth Has a Frame", section VII, "Mathematics Works as if Only the Objective Existed".

Change and the outcome that nothing before it fixed are real on my account. A Geiger counter wired in thought to a small rocket, which nudges a small asteroid, which nudges a larger one, in time moves the orbits of planets, so the shape of space and time depends on a click that was settled only when it happened. So a single fixed geometry of all of space and all of time, laid out once and for all, cannot be the whole story of the uni-

verse. The line between what is actual and what is still possible lies in one place only: at the act of finding made now, here, in this outcome rather than another.

The View from Inside, chapter 14, "Here, Now, and This", section "A Geiger Counter and the Orbits of Planets".

The quantum seems strange because physics first met it from the everyday starting point: things with definite positions, paths fixed in advance, and causes that act only where they are. The surprise is a fact about the expectations that this starting point produces. In any world in which anything can be compared with anything else, findings come in exact separate steps and the mathematics that describes them is smooth, and that is the character of the quantum. A physics that starts from the act of comparison expects the quantum.

Why the Quantum Is the Way It Is, chapter 19, "Where the Derivation Stops, and Why", section "Why the Quantum Seemed Strange".

Mathematics keeps every power it has, and it is the finest instrument there is for what is laid out as structure. On my account mathematics is the score sheet, on which every move is recorded and studied, and the moves themselves are made at the board.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part"; and *Physics Is Not Math*, chapter 11, "Why the Order Surprises", section VII, "What Each Discipline Keeps".

The exact statements for this chapter are these.

The Incommensuration Theorem, in my words: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Any comparison, perfected at its foundation, is either continuously asymmetric or discontinuously symmetric".

Incommensuration; The Theorem and the Three Axioms, chapter 2, "Symmetry and Continuity Cannot Both Be Perfected", section VI, "The Theorem"; the first sentence also in *Physics Is Not Math*, chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem".

The Incommensuration Theorem in its first form, aphorism 1.43-2: "Where applied to an absolute degree, the following theorems hold for any interaction (any comparison) in any domain: When something is absolutely symmetric, it must also be intrinsically discontinuous. When something is absolutely asymmetric, it must also be intrinsically continuous. When something is absolutely continuous, it must also be intrinsically asymmetric. When something is absolutely discontinuous, it must also be intrinsically symmetric. Nothing anywhere at all can be both absolutely symmetric and absolutely continuous. Nothing anywhere at all can be both absolutely asymmetric and absolutely discontinuous".

An Immanent Metaphysics, chapter 3, "Natural Physics", section "The Universal Law", aphorism 1.43-2.

The four characters, defined where a sameness of subjective context is assumed, in the words of aphorism 1.43-1: "Continuity refers to where there is a sameness of content and a sameness of context. Discontinuity refers to where there is a difference of content and a sameness of context. Symmetry refers to where there is a sameness of content and a difference of context. Asymmetry refers to where there is a difference of content and a difference of context".

An Immanent Metaphysics, chapter 3, "Natural Physics", section "Symmetry and Continuity", aphorism 1.43-1.

The proof is a count: the four characters pair in ten ways, three tests remove eight pairings, and two pairings survive, continuity with asymmetry and symmetry with discontinuity.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", sections III to VII; chapter 10 of this book sets it out in words.

The cross-modal principle: a description and what it describes take the two surviving characters one each, so the reach of a description is dense and never exhaustive.

Measurement; The Steps from Comparison to Physics, chapter 8, "The Cross-Modal Principle", section II, "The General Form", Propositions 8.1 and 8.2.

Chapter 2, "Does Space Go On Forever?"

Which geometry holds of the space we live in is a question mathematics does not settle: it is a question for measurement. I reach that point from what mathematics can settle and what it leaves to measurement, and Tegmark reaches it from Einstein's theory, so two separate routes arrive at it.

Physics Is Not Math, chapter 6, "Mathematics, the Science of the Timeless", section V, "Two Questions Answered Elsewhere".

Mathematics establishes what follows from its starting assumptions, in every case those assumptions cover, and it develops each geometry completely. Whether the starting assumptions hold of the world is answered by measurement.

Physics Is Not Math, chapter 6, "Mathematics, the Science of the Timeless", section V, "Two Questions Answered Elsewhere".

What the Beginning Left Behind states the ground of the whole collection in one short sentence: "Every measurement is a comparison". For the sky, that ground means that everything we know of distant space is known by comparisons made here, from the light that has reached us.

What the Beginning Left Behind, "Before We Begin", section "What Stands Behind the Book".

The arena is space and time as found in comparisons, the place where findings happen. Every comparison has its content, what the comparison finds, and its context, the respect in which the two things compared are two: two moments, two places, or two possibilities. Place, time and possibility are the three contexts in which anything is ever found, and the arena holds the first two, place and time.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "What the Last Book Found".

The arena of space and time is smooth and uneven, and its shape is gravity. Smooth means that the arena changes from place to place with no jump anywhere; uneven means that one place in it can differ from another. Einstein's theory describes the arena, and in his theory the curving of the arena is gravity.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "What the Last Book Found".

Space is flat on the largest scales, curved only by its own small ripples, the slight differences from place to place in how matter and light were spread, which are about one part in a hundred thousand. My reason is that the long era of the universe in which we live began at a passage from the era before it, and no length of the era before carries over through that passage into ours. The curving of space as a whole is set by a length, as the curving of a ball's surface is set by the ball's radius, and the passage has one length of its own, far smaller than an atom. A curving set by that length would either have made our era collapse and end almost at once, or have left its expansion set by the bend of space itself rather than by the matter and light it holds. In either case our era would have built no stars and no galaxies. So our space has no curving of its own.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Flat Space"; and chapter 13, "No Inflation Needed", section "The Sameness and the Flatness, Answered From the Turn", which says: "no length of the old epoch passes the turn".

Down to the finest difference any comparison can detect, the ball within our horizon cannot be told apart by any comparison from a ball cut out of an endless flat space. By the principle of identity, two descriptions that no comparison could ever tell apart are one description, so "for a registration, space is infinite up to the limit of distinguishability". A registration, in that book's glossary, is "The act by which a measurement made inside the world fixes what it registers": the value is fixed in the act of measuring itself.

The Weight of Possibility, chapter 3, "The Registered Ratio", section II, "The Count: Six Against One"; the gloss of "registration" from its "Glossary".

Space has no short loop anywhere: every path around space, wherever it runs, is longer than the width of the horizon. The reason runs through the count that sets the ratio of dark matter to ordinary matter, a count made in each measurement from inside. The count is of the basic pieces from which the region the measurement registers is built. The basic pieces of a shape are the fewest building blocks from which the shape can be put together: a simple ball is built from one. In that count the ordinary matter, which fills the region, counts one for each basic piece, so over a region that is one simple ball it counts one. *The Weight of Possibility* calls the three directions of space that we move through "the open space", and it puts the count of one in its own words: "So the open space counts as one, whatever it is beyond the horizon". A region crossed by a short loop is no simple ball, and the count finds such a region built from two basic pieces or more. The count gives the same ratio of dark matter to ordinary matter for a measurement made from any place in space, and not only for a measurement made from ours. With one basic piece the ordinary matter counts one and the dark matter counts six less two divided by pi, so the ratio is about 5.36. With two basic pieces the ordinary matter would count two, and the dark matter, which fills the region and counts six for each piece, would count twelve less the same two divided by pi. The two divided by pi is taken once for the whole measurement, and the ratio would then be about 5.68. The measurements of the oldest light, combined, find the ratio within about one percent of 5.36 and exclude 5.68. So the measured ratio is the ratio of a region built from one basic piece; and since the count gives the same ratio from every place, the region within the horizon of every place is one simple ball, and space has no short loop anywhere.

The Weight of Possibility, chapter 3, "The Registered Ratio", section II, "The Count: Six Against One".

A flat space joined to itself like a video game screen, but only on a scale larger than every horizon, also meets my claims about space, that it is flat and has no short loop. No finding made anywhere inside such a space could tell it from the endless one.

Side by Side, Volume Two, chapter "The Participant and the Act", section VIII, "Where the Predictions Part, and Why", under "A flat space or a closed one".

"No possible experiment done within a domain (the universe) can demonstrate the closure of the domain (universe)". A domain is a whole world within which comparisons can be made, and the aphorism's word "universe" here names the whole of the space we live in.

An Immanent Metaphysics, "Technical Aphorisms", section "There Is No Universe", aphorism 1.81-1; the gloss of "domain" from *What Comes After the Dark*, "Key Concepts".

The ladder of wholes has no top rung: "There is no single domain that encompasses all other domains". The aphorisms use the word "exists" in another sense than this book does: for them, what exists is everything there is, everything that can act on anything else. Everything there is can act on everything else there is, directly or through other things, so continuity of potentiality, gaplessness in the possibility of acting, applies to each thing there is and to everything there is, taken together. Each thing there is has being of the same nature as every other thing there is, so symmetry of actuality, sameness in what each thing actually is, applies to each thing there is and to everything there is, taken together, as well. *An Immanent Metaphysics* states the theorem of the two concepts themselves: "the concept of perfect symmetry and the concept of perfect continuity are themselves fundamentally incommensurate". So perfect sameness and perfect gaplessness cannot both belong, fundamentally, to everything there is, taken together, and everything there is, taken together, is never one completed whole.

An Immanent Metaphysics, "Technical Aphorisms", section "There Is No Universe", aphorisms 1.71-2, 1.72-8, 1.72-9 and 1.72-10.

Space is the arena, found in every comparison, and its mathematics describes the arena's structure. The arena and its mathematical structure are two, by the Incommensuration Theorem, which I state in these words: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity".

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem"; and *Side by Side*, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

A mathematical object, taken as a whole, is laid out at once and is the same wherever and whenever it is read, so it holds perfect sameness and a gapless completeness, with nothing in it missing and nothing still to come. In the words of *Side by Side*, Volume Two: "A mathematical structure can be smooth and perfectly symmetric at once, like the circle of geometry; no comparison ever finds a thing that is both". The arena as found is smooth and uneven, so a mathematical description of it, however exact, comes as close to the arena as anyone wishes and never coincides with it.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

What *Side by Side* says of the completeness of a mathematical description of the world: *Side by Side*, Volume Two, says that a mathematical description of the world that comparisons find, however exact, "is never complete"; chapter 10 of this book sets out the theorem step by step.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Applied to space, my distinction between what exists and what is real gives this: the number of dimensions of space, its curvature and its topology are the structure of the arena, and that structure exists, laid out whole, timeless, and the same for everyone. The arena is where each finding is real: every finding is made at some place and some time, from a seat.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", quoting *Comparison; The Necessary Basis*, chapter 10, "Reasoning", section II, "The Real and the Abstract".

The exact statements and values for this chapter are these.

Why space is flat: in the equation of the expansion the curvature carries a constant of integration, set once by the state from which the history begins. That state holds the length of the turn, of the order of the Planck length, pure numbers of order one, and the size of the departures, about

ten to the power minus five (one part in a hundred thousand). A radius built from the turn length and the de Sitter radius (the radius of the horizon that the cosmological constant alone sets) would need a pure number near ten to the power thirty, which nothing passing the turn supplies. So the homogeneous history is flat, curved only by its departures.

The Gravitational Passage, chapter 18, "The Interior Passage", section IX, "The Departures from Uniformity"; told in words in *What the Beginning Left Behind*, chapter 12, "Four Marks of the Pattern", section "Flat Space".

The measured share of curvature in the cosmic energy budget is 0.001 $\pm$ 0.002 (the sign $\pm$ read "give or take") from the microwave background with the ruler of galaxy distances (Planck Collaboration, Aghanim and others, 2020, abstract). The floor below which no measurement can separate curvature from the ripples is about ten to the power minus five (T. P. Waterhouse and J. P. Zibin, "The Cosmic Variance of Omega", 2008, arXiv: 0804.1771).

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Flat Space".

The count and the short loop: a registration counts the independent classes of what it registers. The independent classes are what chapters 2 and 4 call basic pieces: on the complex projective plane, on the two-sphere and on their product, the count of independent classes agrees with the count of cells, the building blocks of each shape, three, two and six. The open region within the horizon is one class, a ball. A region with N classes would give the ratio of dark to ordinary matter as $6 - 2/(N\pi)$ (six less two over N times pi). A short loop makes N at least two and the ratio at least $6 - 1/\pi = 5.682$, which the combined measurement excludes.

The Weight of Possibility, chapter 3, "The Registered Ratio", section II, "The Count: Six Against One".

The reach of the searches for a short loop made so far: The searches so far test the paths that pass through us. The search of Pip Petersen and colleagues finds that the shortest path around space through our own location is longer than 98.5 percent of the diameter of the sphere from which the oldest light reaches us. The paper's abstract adds that paths that do

not pass through us may be shorter than that, by a factor of 2 to at least 6, and still agree with the data (P. Petersen and others, "Cosmic topology. Part I. Limits on orientable Euclidean manifolds from circle searches", *Journal of Cosmology and Astroparticle Physics* 2023 (01), 030). My claim of no short loop covers every path, through us or not.

The principle of identity as the count uses it: two descriptions that no comparison can tell apart are one.

The Weight of Possibility, chapter 3, "The Registered Ratio", section II, "The Count: Six Against One".

The parallax that Friedrich Bessel measured for the star 61 Cygni was 0.314 ± 0.020 seconds of arc (Bessel, 1838, *Astronomische Nachrichten* 16, 65), about one part in 5,700 of the width of the full Moon. The Gaia satellite gives 0.28615 ± 0.00006 seconds of arc, about 11.4 light-years.

The sealed record is the dated list of my predictions, published before the measurements, in the appendices of *Side by Side*. The sealed record holds two predictions that bear on this chapter, that space is flat to the size of the primordial departures and that no short loop exists anywhere in space. The appendix "What the Coming Measurements Will Find" gives the two with what the account says each coming measurement will find.

Chapter 3, "Where Did It All Come From?"

Comparing is telling one thing from another, or finding them the same; every measurement and every interaction is a comparison, and a finding is what one comparison gives.

What the Beginning Left Behind, "Before We Begin", section "What Stands Behind the Book".

The Earth has a history with a beginning, and I reach it through heat: "The Earth is spending its heat, and whatever spends cannot have been spending for ever unchanged".

Physics Is Not Math, chapter 14, "When the Mathematics Overruled the World", section V, "What Thomson Got Right".

Each galaxy keeps its own size while space stretches between the galaxies, because the parts of a galaxy are held together by their own motion and pressure.

The View from Inside, chapter 1, "The Emptiest Space Is Not Empty", section "The Cosmological Constant".

A distant galaxy may recede faster than light: the speed of light limits how fast a signal or a cause can pass from one thing to another, and the recession of a distant galaxy carries no signal and no cause to us. That recession is "the reason the cosmological horizon is where it is". The cosmological horizon is the edge beyond which no light can ever reach us.

The Basis of Physics, essay 10, "Causation, Connectivity, and the Domain Boundary of Comparison", section VIII, "What Is Not Excluded".

What the Beginning Left Behind pictures the glowing wall of the oldest light as a valley full of fog that clears. While the young universe held one free electron for every proton, every ray of light was knocked aside again and again; as the universe cooled, the electrons were bound into atoms, and the light went free. The moment the light went free, last scattering, lies about 370,000 years into the history we piece together, and the light was then at about 2,970 kelvin. Since last scattering the universe has grown about 1,090 times, the light has cooled by the same factor, and it arrives now at 2.7255 kelvin.

What the Beginning Left Behind, chapter 11, "The Oldest Light", section "A Fog That Clears".

My ground for the helium is a count of neutrons. About two minutes into our history, roughly one neutron was left for every seven protons, and nearly every neutron ended in helium, taking one proton into the helium with it. With one neutron for every seven protons, a quarter of the mass ended in helium; the measured share is 0.245. "So the universe's helium came from the neutrons".

The Coming of the Kinds, chapter 12, "The First Nuclei", section III, "Nearly Every Neutron Ends in Helium".

The kinds of particle enter the calculation of the first minutes at two points. At the first point, the count of the kinds that filled the young universe, each kind counted with every form it can take, sets how fast the young universe expanded. At the second point, the kinds enter through the count of neutrons, which three things set: the neutron's mass, a little above the proton's; the force that changes neutrons and protons into one another; and the time a free neutron lasts before it decays. With my list, the calculation has the same inputs as the calculation physicists already use, and the helium comes out as that calculation gives it.

The Coming of the Kinds, chapter 12, "The First Nuclei", section VI, "What the First Minutes Left".

I derive the list of the kinds of particle from the parts that every act of comparison has, and I show that the list is closed: the list holds every kind of particle there is.

The Coming of the Kinds, chapter 1, "The Kinds and Their Amounts", chapter overview, and section V, "Gravity, the Dark Matter and the Closed List", which says: "So the list of kinds is the list of Section II, each with its antiparticle, and it is closed".

The deuterium left over from the first minutes measures how many protons and neutrons there were for each particle of light, and the pattern in the microwave glow counts the same number by other physics entirely; the two counts agree within their uncertainties.

The Coming of the Kinds, chapter 12, "The First Nuclei", section IV, "The Deuterium Left Over".

An argument about the world is built like a cable, many strands from independent sources, and of independent routes that reach one value: "Convergence is the strongest evidence that inquiry into the world can have".

Physics Is Not Math, chapter 12, "Chains and Cables", section V, "Two Routes to One Value".

The seeds are required on my account as on Tegmark's, and my reason is the Incommensuration Theorem, whose words cover every event as well as every comparison, and so every event of the young universe. A young

universe perfectly the same in every direction and at every place would have no jump anywhere, so it would hold perfect sameness and perfect gaplessness together. So the young universe began even, and never perfectly even.

At the Edge of Reality, chapter 18, "A Clean Slate", section "Even From the Start".

The history gives no evidence of a moment of infinite density. The smooth description of space and time holds at every scale down to the Planck scale, and the singularity is what the equations of the smooth description produce when they are carried past the range where they hold.

The Gravitational Passage, chapter 19, "What Replaces Singularities", section I, "Singularities as Artifacts".

I take the turn to be the limit at which the smooth description and the stepwise description each reach the edge of their ranges, and at which no description takes over.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "The Turn".

The Planck scale is the scale at which gravity and the quantum act on equal terms.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "What the Last Book Found".

An orrery, a working model of the solar system, runs backward as smoothly as forward when its gears are cranked the other way, while the solar system itself runs one way; no gear in the orrery holds the tides or the receding Moon. "The ease with which the model runs backward is a property of the model".

Physics Is Not Math, chapter 11, "Why the Order Surprises", section I, "A Clockwork Solar System".

Laser light bounced off the reflectors that the Apollo astronauts left on the Moon shows it moving away from us by about 3.8 centimeters a year.

Physics Is Not Math, chapter 11, "Why the Order Surprises", section I, "A Clockwork Solar System".

The equations of motion can be solved in reverse, as Tegmark writes, and the ease of running the equations backward is a property of the equations, while the world itself runs one way.

Physics Is Not Math, chapter 11, "Why the Order Surprises", section III, "One Source".

The past is known only by the impression it makes on the present.

At the Edge of Reality, chapter 2, "What Came Before the Big Bang?", section "The Answer of This Book".

The pieced-together history is a structure, and it exists as a map or a record exists.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "A History Made from What Arrives Now".

I take a history to report what it holds as things at definite times and places, as a record or a map does: this happened, then, there. So, traced backward, our history draws the limit as a point in time, and the point looks like a first moment.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "A History Made from What Arrives Now".

An epoch is one long age of the universe, from one turn to the next, in which stars, galaxies and black holes form; in my words, an epoch is "one era of the world, pieced together from inside it".

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "What the Last Book Found".

The turn is a passage in a cycle: the passage from the end of one epoch to the start of the next.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "The Turn".

To know the turn as an event, a comparison would have to set a time before the turn beside a time after it, and every comparison is made inside one epoch. So questions of how long, how hot and how fast apply to the approach to the turn, from the end of the old epoch and from the start of ours, and none of them applies to the turn itself.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "The Turn".

I identify what we call the Big Bang with the turn as it appears in the history we piece together from inside our epoch, where the turn, a limit, is drawn as a point.

At the Edge of Reality, chapter 17, "Why the Big Bang Looks Like a Beginning", section "The Limit Rendered as a Point".

Beyond the turn lies the end of the epoch before ours, inside that epoch's black holes; "before" here names the order of a cycle of epochs, and no comparison measures a time across the turn.

At the Edge of Reality, chapter 2, "What Came Before the Big Bang?", sections "The Answer of This Book" and "The Question Has an Answer".

Our epoch ends first in the long emptying of its sky, as the expansion carries everything not bound to our own group of galaxies out of reach, and then in a turn that begins the next epoch. The turn that begins the next epoch is also the one end that the insides of all the black holes of our epoch share: "The ends of all black hole interiors are one end, because no comparison can tell one end from another. From that one end the next epoch begins".

What Comes After the Dark, chapter 12, "What Comes After the Dark?", sections "The Sky Thins" and "The Turn"; and chapter 8, "Is the Long Emptying the End?", section "One End, and the Turn".

The Big Bang is the edge of the history we can piece together, as the line where sea meets sky is the edge of what you can see from a beach. The Big Bang is the beginning of our epoch, and it is not the beginning of everything. In the words of *At the Edge of Reality*: "The horizon is the edge of what you can see, and not the edge of the sea".

At the Edge of Reality, chapter 2, "What Came Before the Big Bang?", section "What This Means for You".

Every physical theory has both its mathematics, which exists, and its findings, which are real. I part from Tegmark at the step from the history, laid out whole, to the world itself: on my account the history is a structure, which exists, built from findings, which are real.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part"; and *At the Edge of Reality*, chapter 4, "What Every Comparison Contains", section "What Exists and What Is Real".

The description of our cosmic history is mathematical, and "The mathematical description of physics approaches the physical process as closely as one likes and never coincides with it". The rebuilt score sheet can be exact in every line, and it is still the score sheet of the game.

Measurement; The Steps from Comparison to Physics, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete"; and *Side by Side*, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

The exact statements and values for this chapter are these.

Last scattering: the Planck satellite's final results give the redshift of last scattering (the factor by which the universe has grown since then, less one) as 1089.92 $\pm$ 0.25 (Planck Collaboration, Aghanim and others, 2020, Table 2). The same results give the age of the universe as 13.797 $\pm$ 0.023 billion years. The time at last scattering, computed from the same parameters, is about 371,000 years, which I give as about 370,000. The temperature then was 2.72548 kelvin times 1,090.92, about 2,973 kelvin.

What the Beginning Left Behind, chapter 11, "The Oldest Light", section "A Fog That Clears".

Helium: with x neutrons for each proton when the deuteron lasts, the share of the mass that ends in helium is $2x/(1 + x)$, two x over one plus x. The deuteron lasts once the temperature falls to about 0.105 million electronvolts, and its binding energy is 2.2246 million electronvolts (an electronvolt is the energy an electron gains in crossing one volt). With one neutron for every seven protons, x is one seventh and the share is exactly a quarter.

The Coming of the Kinds, chapter 12, "The First Nuclei", section II, "The Deuteron Waits", and section III, "Nearly Every Neutron Ends in Helium".

Lithium-7: very old stars hold about a third of the computed amount, 1.45 ± 0.25 against 4.72 ± 0.7, each in ten-billionths of the hydrogen. Both values are from B. D. Fields, P. Molaro and S. Sarkar, "Big-Bang Nucleosynthesis", in Particle Data Group, *Review of Particle Physics*, review revised August 2025, equations 24.4 and 24.10.

The Coming of the Kinds, chapter 12, "The First Nuclei", section V, "Helium-3 and Lithium-7".

The lithium left from the first minutes, told in words: The lithium has been measured as well. In 1982 François and Monique Spite found that very old stars of our Galaxy, poor in the heavier elements, hold nearly the same small amount of lithium, and today that amount stands at about a third of what the calculation of the first minutes gives. Two things set the amount of lithium found in those stars: the rates of the nuclear reactions, and what the stars later did to their lithium. My list of kinds changes nothing in the calculation of the first minutes, which stands exactly as nuclear physics gives it.

The Coming of the Kinds, chapter 12, "The First Nuclei", section V, "Helium-3 and Lithium-7"; F. Spite and M. Spite, *Astronomy and Astrophysics* 115, 357 (1982).

The singularity: the smooth description holds at curvatures below the Planck curvature and does not hold beyond it; I construct no geometry beyond that boundary.

The Gravitational Passage, chapter 19, "What Replaces Singularities", section I, "Singularities as Artifacts".

The measured temperature of the microwave background today is 2.72548 ± 0.00057 kelvin (Fixsen, 2009); COBE found the sky's temperature to vary by about 30 millionths of a kelvin, about one part in a hundred thousand (Smoot and others, 1992).

The sealed record holds two predictions that bear on this chapter, that the departures are one pattern across the whole sky and that the excess of matter over antimatter has one sign everywhere; the appendix "What the Coming Measurements Will Find" gives them.

Chapter 4, “What Is the Universe Made Of?”

The dark matter was first inferred in Fritz Zwicky’s paper of 1933, from the speeds of the galaxies of the Coma cluster.

The Weight of the Unseen, chapter 1, “What Is Seen”, under question 1.1, “What do astronomers actually observe?”.

A galaxy whose outer parts move round as fast as its inner parts has mass that keeps growing far out from its center.

What the Beginning Left Behind, chapter 18, “Matter Made of Possibility”, section “What Is Seen”.

Every comparison has a content, what it finds, and a context, the respect in which the two things compared are two: two moments, two places, or two possibilities. A thermometer that compares this morning’s warmth with this evening’s finds a temperature, which is its content, and the two times of day are its context.

What the Beginning Left Behind, chapter 1, “Where the Last Book Left Us”, section “What the Last Book Found”.

What is found must be the same for everyone who finds it, so that separate findings agree and make one shared world: the content is symmetric. To compare is to hold the two things compared in one act, joined with no break: the comparing is continuous. The sameness of what is found and the continuity of the comparing must each be perfect, because a comparison cannot give up either one. The two perfect demands pull against each other, and that pull, the strain, has to go somewhere: into the content, into the comparing, or into the context. The content must stay perfectly the same, and the comparing must stay perfectly continuous, so the strain goes into the context. So the context gives way, as a rug between a wall and a sofa buckles when the sofa is pushed toward the wall. For a physical finding the context is place or time, since the two things compared are two places or two moments; and the places and moments at which findings are made, taken together, are the arena. The arena takes the strain. The arena joins every act of comparing to every other, so it car-

ries the continuity of the comparing and keeps its gaplessness; by the theorem it cannot also keep perfect sameness, so it gives up its sameness, differs from place to place, and bends. In the words of *The View from Inside*: "The context carries the continuity of the comparing, since it is the arena that connects every act; it cannot also carry perfect symmetry; so it gives up symmetry".

The View from Inside, chapter 8, "Content, Context, and Why Space Curves", sections "Two Demands That Cannot Both Be Met" and "The Rug Between the Wall and the Sofa".

I identify energy with the rate at which findings can be made. Where more findings can be made, more of what is found must agree with all the rest of what is found, and the arena carries more strain; so the arena bends in proportion to energy.

The View from Inside, chapter 8, "Content, Context, and Why Space Curves", sections "Energy Is the Rate of Finding" and "In Proportion to What Can Be Found".

A theorem of the mathematician David Lovelock, from 1971 and 1972, fixes the form of the bending. Take a measure of curvature built from the geometry of space and time alone, in four dimensions, using no more than how the geometry changes and how that change changes. Lovelock proved that only one such measure can be set equal to an energy that is conserved: the measure in Einstein's equations, with room for exactly one constant term, a fixed number added to the equations.

The View from Inside, chapter 8, "Content, Context, and Why Space Curves", section "From Here and Now to Everywhere at Once".

The Incommensuration Theorem gives the reason the two demands strain against each other. I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing". Symmetry is sameness and continuity is having no gaps, so no single act of finding holds perfect sameness and perfect gaplessness together.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", and its section "Terms".

When the oldest light went free, the ordinary matter varied in density by about three parts in a hundred thousand, three times the variation of the light's temperature. Until the light went free, sound waves ran through the hot mixture of light and matter, the same waves that gave the spots of the oldest light their favored sizes, and the ordinary matter was caught in them and could not gather. After the light went free, a small excess of density grows in step with the distances between things, and distances have grown about one thousand and ninety times since then. Three parts in a hundred thousand, grown one thousand and ninety times, comes to about three parts in a hundred. A region draws together into a galaxy only when its excess reaches about one, that is, when it holds about twice the average density. So ordinary matter alone would have made no galaxy at all.

What the Beginning Left Behind, chapter 18, "Matter Made of Possibility", section "What Is Seen".

The dark matter took no part in the sound waves, so its unevenness was already growing before the light went free, and the ordinary matter later fell into the denser places that the dark matter had already gathered. "The dark matter built the scaffolding on which the galaxies formed".

What the Beginning Left Behind, chapter 18, "Matter Made of Possibility", section "What Is Seen"; and *The Weight of the Unseen*, chapter 1, "What Is Seen", under question 1.6, "How did the dark matter shape the galaxies?".

The energy of empty space enters Einstein's equations as their one constant term, the cosmological constant, and I set its share by an identity. I identify the cosmological constant with the part of the energy of empty space that a measurement made from inside the universe takes in: the part the measurement registers, which is a small share of the whole. Set that reckoning equal to the astronomers' reckoning of the same energy, as one slice of the budget of the whole universe, and the rate of expansion and the strength of gravity cancel, leaving one equation in pure numbers. The equation ties the share of empty space to the size of our horizon, counted in the distance at which space carries things away from us at the speed of light. That size depends only on how the energy has been divided over

the history of the expansion, so the equation has one unknown, the share of matter. In a flat space its solution gives matter 31.1 percent of the whole and empty space 68.9 percent.

The View from Inside, chapter 13, "One Equation, One Number", sections "Two Surveyors", "Setting the Two Equal" and "One Equation, One Unknown"; *What the Beginning Left Behind*, chapter 8, "The One Edge That Sets the Vacuum", sections "Measuring Empty Space from Inside" and "The Number, Seen Two Ways", and chapter 14, "The Ledger of the Sky", section "One Number Set: The Matter Share".

Independent routes, resting on different premises, arriving at one value, are convergence. An argument about the world is a cable of many strands, and the strands that come from separate sources make the value they share surer than any one strand alone: "Convergence is the strongest evidence that inquiry into the world can have".

Physics Is Not Math, chapter 12, "Chains and Cables", section V, "Two Routes to One Value".

The internal space, or the held internal dimensions, is six further directions in which things can differ, held small at every point, whose shape fixes the kinds of particle; the large world of space and time around them holds them small. The six are directions of possibility, the range of what a comparison could find, and nobody travels along them. The six are present wherever you are, as the grain of a sculptor's stone is present at every point of the block.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", sections "What the Last Book Found" and "One Space, Taken Two Ways".

Four of the six have the shape of all the ways a three-way choice can stand, and two have the shape of all the ways a two-way choice can stand, which is an ordinary sphere. Every way of holding the three colors of a lamp at once, in any proportions and with any timing among them, is one point of a shape of four dimensions. Every way of holding the two answers of a two-way choice at once is one point on a globe: the proportion sets how far north or south the point lies, and the timing between the two sets how far around.

The Weight of the Unseen, chapter 2, “Three Distortions of Context”, under question 2.3, “In what sense are the six internal dimensions a space of possibility?”; and *What the Beginning Left Behind*, chapter 3, “The Shape and Size of the Six”, section “A Lamp and a Globe”.

The six are held a few Planck lengths across, at the same size and the same shape at every point of space.

What the Beginning Left Behind, chapter 1, “Where the Last Book Left Us”, section “What Holds the Six”.

The Planck length is the length at which the smooth description of space and time reaches the edge of its range, about a hundred billion billion times smaller than the width of a proton.

What the Beginning Left Behind, “Before We Begin”, section “Where the Story Stands”.

I take the six in two ways, as a clock face can be taken as the pattern of twelve hours, with no size, or as one particular face, with a width of its own. Taken as a pattern with no size, the six fix the kinds of particle; taken as a small space held at its size, they can ring, as any held thing can.

What the Beginning Left Behind, chapter 1, “Where the Last Book Left Us”, section “One Space, Taken Two Ways”; and chapter 18, “Matter Made of Possibility”, section “A Vibration of the Held Space”.

A struck crystal rings, and the smallest step of its ringing, a phonon, is a vibration of the crystal, not an extra thing inside it. In the same way, I identify the dark matter with a lasting vibration of the six held internal dimensions, which I call a gyration.

What the Beginning Left Behind, chapter 18, “Matter Made of Possibility”, section “A Vibration of the Held Space”.

Each gyration weighs about a Planck mass, the mass that goes with the Planck length: about twenty-two millionths of a gram, which is about ten billion billion times the mass of a proton.

What the Beginning Left Behind, chapter 18, “Matter Made of Possibility”, section “How Heavy, How Many, How Cold”.

Every instrument registers content, the things a comparison finds, and so does light, since every meeting of light with matter is itself a comparison. A gyration is a vibration of the internal space, and the internal space is made of directions of possibility; possibility is one of the three contexts of chapter 1, a respect in which two things compared can be two. So, in my words, a gyration is "a vibration of context, and not an item of content": it belongs to the respect in which two things compared are two, and never to what a comparison finds. So no instrument registers a gyration, and a gyration shows itself by its weight alone.

What the Beginning Left Behind, chapter 18, "Matter Made of Possibility", section "A Vibration of the Held Space"; the quoted words from *The Weight of the Unseen*, chapter 3, "What the Dark Matter Is", under question 3.2, "Then what is it, physically?", and from *The Weight of Possibility*, "A Note to the Reader".

The dark matter acts on ordinary matter only by its weight, so in a collision of two clusters of galaxies it passes on through while the hot gas crashes and glows.

The Weight of the Unseen, chapter 3, "What the Dark Matter Is", under question 3.5, "Does the dark matter collide, with ordinary matter or with itself?".

The gyrations are made only at the turn.

The Weight of the Unseen, chapter 3, "What the Dark Matter Is", under question 3.9, "Is the dark matter still being made today?".

A gyration lasts, because each of the three ways it could end is closed. A vibration of the internal space and an ordinary thing are two different sorts of thing: no process within an epoch changes the one into the other. The swap of each point of the small sphere with the point opposite it closes the way into ripples of space and time. Under the swap the gyration that lasts changes sign, so that each crest of its vibration becomes a trough, while ripples of space and time keep their sign; so the gyration cannot hand its energy to those ripples. The gyration could hand its energy only to a lighter vibration that also changes sign under the swap, and the gyration that lasts is the lightest vibration that changes sign. At the Planck scale there is no lower level for a gyration to leak into.

What the Beginning Left Behind, chapter 18, "Matter Made of Possibility", section "Three Ways Out, All Closed", with the summary sentence in section "Made at Each Turn"; the change of sign under the swap in its part "For the Physicist", "Chapter 18, Matter Made of Possibility", under "The antipodal parity", which says: "a vibration of the sphere of degree l changes sign when l is odd. The waves of the large dimensions are even, so the lightest odd vibration has nothing lower to decay into, and it lasts".

Each gyration is so heavy that the gyrations are few, and the dark matter is perfectly cold, in the cosmologists' sense: slow moving, with no motion from heat that could matter. So in everything astronomers can see, the dark matter behaves exactly as the cold dark matter of the standard picture of cosmology does: "What the account changes is not how the dark matter behaves, but what it is".

The Weight of the Unseen, chapter 3, "What the Dark Matter Is", under question 3.6, "Is this the same as the cold dark matter of standard cosmology?"; and *What the Beginning Left Behind*, chapter 18, "Matter Made of Possibility", section "How Heavy, How Many, How Cold".

I keep Einstein's law of gravity exactly as it stands, at every scale down to the Planck scale, and the dark matter is energy, with weight, located where it pulls.

The Weight of the Unseen, chapter 1, "What Is Seen", under question 1.7, "Is the dark matter a mistake in our law of gravity?".

I identify the ratio of dark matter to ordinary matter with six minus two over pi, about 5.36. The number comes from a count: a measurement counts what it registers piece by piece, by the basic pieces from which each shape is built. The globe of the two-way choice is built from two basic pieces, a single point and a disc whose whole rim is drawn together to that point, and the four-dimensional shape of the three-way choice is built from three. The dark matter spreads over the whole internal space, so it counts three pieces times two, six in all. The curving of the four-dimensional part takes a small share, two divided by pi, about sixty-four hundredths, away from that count. The ordinary matter counts one, because the region within our horizon is one simple ball, built from one basic piece. The dark matter fills the same region and also spreads over the

whole internal space, so for each basic piece of that region the ordinary matter counts one and the dark matter counts six. The two divided by pi is taken away once, for the whole measurement, however many basic pieces the region is built from.

What the Beginning Left Behind, chapter 19, "How Much the Turn Made", sections "Counting the Classes", "The Share That Holds Fast" and "Six Less Two Over Pi".

The same count gives the reason that space has no short loop: the region within a horizon is built from one basic piece exactly when no short loop passes through it. The count of one holds for every measurement, wherever in the universe it is made, so space has no short loop anywhere. With a short loop through it, the region within a horizon would be built from two basic pieces or more. With two, the ordinary matter would count two, the dark matter twelve less two divided by pi, and the ratio would come out near five and sixty-eight hundredths.

What the Beginning Left Behind, chapter 19, "How Much the Turn Made", section "Two Predictions Anyone Can Look For".

The cosmological constant is like the scale printed on a map, which belongs to the whole map at once, for the one space and time of our epoch. Einstein's equations keep a strict account of energy and of motion, and a constant term that changed from place to place or from time to time would break that account where it changed. So within the one space and time of our epoch the cosmological constant is one number, by a theorem about Einstein's equations. The dark energy is the part of the energy of empty space that a measurement takes in, and this part enters Einstein's equations as their one constant term. So the dark energy is one number, as distinct from a field, a quantity spread through space with a value of its own at every place and time. The measurement made now sets that number, since the number is the share of the energy of empty space that this measurement takes in, and the history we piece together from that measurement carries it at every time of our epoch.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The Second Way: One Number for Our Whole Epoch".

The density of dark energy is exactly constant, at every place and every time of our epoch: its equation of state, the ratio of its pressure to its energy, is exactly minus one.

What the Beginning Left Behind, chapter 14, "The Ledger of the Sky", section "Four Predictions of Kind".

A measured value with its question written beside it is a complete state of knowledge. Each line of Tegmark's table is such a state: the measured density of dark matter says what is the case, and his question beside it, what dark matter is, says where the next work lies.

Physics Is Not Math, chapter 15, "The Descent", section VII, "Not Fixed by This Route".

My numbers taken from measurement, in three sentences: I write my own numbers with their questions beside them. I take the density of ordinary matter from measurement, as a fact about the content of our epoch. I also take the strength of the faint ripples of the oldest light from measurement, and writes beside it the question of what sets that strength.

The Weight of the Unseen, chapter 5, "How Much There Is", under question 5.8, "Does the rule fix how much dark matter there is?"; and *What the Beginning Left Behind*, chapter 14, "The Ledger of the Sky", section "Five Entries Copied from Measurement".

Where the standard picture of cosmology chooses its share of matter to match the measured sky, I set that share by its identity, at 31.1 percent.

What the Beginning Left Behind, chapter 14, "The Ledger of the Sky", section "One Number Set: The Matter Share".

The orrery runs backward as smoothly as forward, while the solar system runs one way only; the ease of running backward belongs to the model. "The orrery stays a fine instrument, and a beautiful one; it is simply not the solar system". The laws of precision cosmology are the finest description there is of what is laid out as structure.

Physics Is Not Math, chapter 11, "Why the Order Surprises", sections I, "A Clockwork Solar System", and VII, "What Each Discipline Keeps".

The exact statements and values for this chapter are these.

The formula, as *The View from Inside* names it: the energy density of empty space is the share 3f times the fourth power of the Planck mass, divided by S. In the formula, S is the count of the observer's particle horizon in Planck areas, and $f = 1 - 3/(8\pi) = 0.881$ (one less three over eight pi) is the trim from the curvature of the internal space. Set equal to the astronomers' reckoning of the same density (the vacuum's share of the critical density), the expansion rate, the strength of gravity and the factors of three and pi cancel, and what is left is $I^2 \times \Omega = 8 - 3/\pi$. *The View from Inside* calls that equation the identity. In words, the square of I, the horizon's radius counted in Hubble radii, times Ω, the vacuum's share of the energy, equals eight less three over pi. In a flat universe that identity has one solution, a matter fraction of 0.311.

The View from Inside, chapter 13, "One Equation, One Number", sections "Setting the Two Equal" and "One Equation, One Unknown"; *The Gravitational Passage*, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section III, "The Self-Consistency Identity", and section IV, "The Numbers".

The formula is applied to the horizon computed from the measured history of the expansion. There the formula gives an energy density of 2.56 times ten to the power minus 47, in units of the fourth power of a billion electronvolts, against 2.52 times ten to the power minus 47 from the sky. Written as an age, the identity gives 0.792 de Sitter times against a measured 0.787 ± 0.009.

The View from Inside, chapter 13, "One Equation, One Number", section "The Same Number, Seen from Other Sides"; *What the Beginning Left Behind*, chapter 14, "The Ledger of the Sky", section "One Number Set: The Matter Share".

The ratio of dark to ordinary matter, as I state it: the share of the dark matter over the share of the ordinary matter is $6 - 2/\pi = 5.3634$, six less two over pi. The count behind it: the internal space is the complex projective plane, a shape of four dimensions with three basic pieces, times the two-sphere, with two. The dark matter is counted by $3 \times 2 = 6$ basic pieces of the internal space, less the share $2/\pi$ that abides from the curvature of the complex projective plane; the ordinary matter, registered within the horizon, is one class.

The Weight of Possibility, chapter 3, "The Registered Ratio", section II, "The Count: Six Against One", and section V, "The Result".

The standing of that ratio, in the words of *What the Beginning Left Behind*, chapter 19, in which "this book" names *What the Beginning Left Behind* itself: "The ratio keeps the standing that Before We Begin and Chapter 14 gave it: it is a registered ratio". *What the Beginning Left Behind*, chapter 19, "How Much the Turn Made", section "A Recipe Card". "So the standing of the ratio is this. The counting principle and the rule of which curvature abides are the account's principles. The rule was selected by the measurement from a small family of expressions of one form, and *The Weight of Possibility* examined twelve expressions in all. I give the number twelve so that you know how wide the search was. The rule is supported by three things: its form, which is the form of the Passage's rule for the cosmological constant; the exclusion of its neighbors within that form; and the coefficient it shares with the Passage's rule. As an identity of the present measurement, the rule needs no derivation from the motion of the turn".

What the Beginning Left Behind, chapter 19, "How Much the Turn Made", section "What the Oldest Light Finds".

The measured ratio: 5.364 ± 0.065 from the Planck satellite alone (0.1200 over 0.02237); 5.370 ± 0.048 from three telescopes together; 5.199 ± 0.035 with the galaxy distances of the Dark Energy Spectroscopic Instrument.

What the Beginning Left Behind, chapter 14, "The Ledger of the Sky", section "The Sixth Through One Ratio".

The equation of state: w, the ratio of the pressure of the dark energy to its energy density, is exactly -1 (minus one); Planck with the ruler of galaxy distances and exploding stars measures -1.03 ± 0.03.

The Gravitational Passage, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section V, "The Equation of State"; *What the Beginning Left Behind*, chapter 14, "The Ledger of the Sky", section "The Ledger, Written Out".

Why Λ (lambda, the cosmological constant) is one number: in Einstein's equations with the constant term, the divergence of the Einstein tensor vanishes by the contracted Bianchi identity and the content's energy is conserved. So the gradient of Λ is zero, and within one geometry Λ does not vary.

The Gravitational Passage, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section VI, "The Comparison That Fixes Λ".

The measured base values are those of the Planck Collaboration (Aghanim and others, 2020), in the units cosmologists use: the share times the square of h, the expansion rate in hundreds of kilometers per second per megaparsec. In those units the density of ordinary matter is 0.02237 ± 0.00015, and that of the dark matter 0.1200 ± 0.0012. The matter share is 0.315 ± 0.007.

What the Beginning Left Behind, chapter 14, "The Ledger of the Sky", section "The Ledger, Written Out".

The sealed record holds four predictions that bear on this chapter: the matter fraction, an equation of state of exactly minus one, the ratio of dark to ordinary matter, and what the dark matter is and does. On the sealed record the dark matter acts only by its weight, does not decay, is perfectly cold, and comes in gyrations of about the Planck mass. The appendix "What the Coming Measurements Will Find" gives the four predictions.

Chapter 5, "What Put the Bang in the Big Bang?"

My account answers every puzzle inflation was proposed to solve, from the turn and from the structure of the forces of nature, and so it needs no inflation.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", opening.

The world has an outside, which can be reached at moments one can name; its space and time are smooth at every scale one can reach. The distant galaxies fix what "not rotating" means, for a spinning gyroscope and for the balance organ of the inner ear. On my account those three plain

facts are the conditions that hold the six further dimensions small. Said of the large world around the six, the first of the three conditions is that the large world is present around the six as the setting in which they are held, at moments that comparisons can reach. Second, the large world lies within the range of the smooth description of space and time. Third, the large world fixes what "not rotating" means. In the words of *What the Beginning Left Behind*: "The large world must be present as the context of the six, at moments that a comparison can reach". Every part of the world you know meets the three conditions, so the six stay held small, and the forms that the matter of your body can take stay fixed.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "What Holds the Six".

Inside a black hole every path runs toward an end that it approaches and never reaches, and at that end all three facts fail. The world outside has no moment there, since no signal from the end ever reaches it; the smooth description of space and time has reached the edge of its range; and nothing is left to fix what "not rotating" means. So at the end of the inside of every black hole nothing holds the six small any longer, and everything that fell in stops being matter in the forms our epoch gives matter, without ceasing to be: it is still there to be found.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "How an Epoch Ends, in Eight Steps", the first, sixth and seventh steps.

The ends of the insides of all black holes are one end. In the world of an epoch, two things alike in every other way are told apart by their places or their moments, and the ends lie at no place and no moment of the world outside them. No signal from those ends ever reaches that world, and nothing in it puts the ends in an order in time. The account takes as a stated premise that the new world which forms from the ends carries over none of the old world's places and moments. So no comparison has a place or a moment by which to tell one end from another, and by the principle of identity the ends are one end.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "How an Epoch Ends, in Eight Steps", the second step.

Nothing is found without a context, the respect in which the things compared are two: two places, two moments, or two possibilities. The large world, with its places and its moments, is the context in which the six are held. At the end of the black holes the old world has no places and no moments left. The only thing left from which places and moments can form is the geometry of all ten directions together, the four of space and time and the six. So that geometry must open out again into a large world, with large space and time, and with the six held small inside it once more.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "How an Epoch Ends, in Eight Steps", the seventh step; the large world as the context of the six from section "What Holds the Six".

The Big Bang, which chapter 3 identified with the turn as the history we piece together from inside our epoch draws it, is where this large world forms again; so the account's answer to the Bang Problem is the turn.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "How an Epoch Ends, in Eight Steps", the eighth step, and section "The Turn".

The content of the epoch before, what the comparisons made in it find, continues through the turn, so the turn is not a creation from nothing.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "The Turn".

Our Big Bang is the beginning that our history finds, and the same turn is approached from the epoch before ours, by the collapse inside that epoch's black holes.

At the Edge of Reality, chapter 20, "The Cycle Without a First Turn", section "No First Iteration".

The epoch before ours began the same way, from the epoch before it, and the cycle has no first time round.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "The Turn".

Two regions of the sky far apart are alike because they come from one domain, the one end of the black holes of the epoch before; no signal ever had to pass between them.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "One Origin, One Cloth".

Two towns far apart may share an accent without ever having spoken to each other, because both were founded from one place.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "The Sameness and the Flatness, Answered From the Turn"; and chapter 12, "Four Marks of the Pattern", section "One Origin, One Cloth".

The sky is one pattern, one cloth, with nearly the same strength at every length. A floor of small tiles, each shaded by a worker who never saw another tile, fades from far off to an even gray, because independent pieces cancel over a wide patch. A cloth woven on one loom keeps its swirls as strong across its whole width as across a hand's breadth. The sky is the cloth, woven from one domain, one undivided whole in which no comparison can tell one part from another: the one end of the black holes of the epoch before.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "One Origin, One Cloth".

Inflation and my account both say four things. The faint differences in the oldest light are the seeds of every galaxy; every ingredient of the universe ripples together; the pattern runs nearly the same in strength at every length across the whole sky; and space is flat.

Side by Side, Volume Two, chapter "The Structure and the Act", section VIII, "Where the Predictions Part, and Why", under "A long inflationary phase".

A curvature of space is set by a length, its radius, as an ant on a marble meets the curve of its world in a few steps and an ant on a ball the size of the Earth finds the ground flat. In Einstein's equations for a universe alike everywhere, the radius of curvature is fixed once, by the state the history starts from, and carried unchanged afterward.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Flat Space".

The state our epoch starts from inherits no length from the epoch before, and it holds one length of its own, the length of the turn, which is about the Planck length. A larger radius would have to be made by mul-

tiplying the length of the turn by some very large number. The only numbers that large in my account are counts, such as the number of different ways the matter within our horizon could be arranged, and no count passes the turn: each epoch begins its own counts. So the radius of curvature of our epoch is either about the length of the turn, or there is none.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Flat Space".

A radius as small as the length of the turn would make the new epoch fall back on itself almost at once, or leave it ruled by its own curvature. Either way the new epoch would make no stars and no black holes, and the black holes of each epoch are where the next epoch comes from. So a radius as small as the length of the turn is excluded, no radius of curvature is left, and space as a whole is flat, curved only by its own faint ripples.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Flat Space".

The three forces of nature that grand unified theories join into one at the great heat of the beginning were never one larger force on my account. Each force has its own symmetry, and the three symmetries stand together as three, each the same at every energy; none of them is what is left of a larger symmetry that broke. So nothing breaks in the way that makes monopoles, and the monopole puzzle does not arise.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "No Monopoles, No Strings, No Walls".

Everything that fell into the black holes of the epoch before ours had taken some alternatives and not others at every choice it met. Which alternatives everything had taken is carried through the turn, all of them averaged together and none erased, as every footstep is kept in the one worn curve of a stone stair. Our epoch registers only the shape all those alternatives make together, as small differences of density from place to

place. I identify the first ripples of our sky with those small differences of density: the shape that the alternatives taken in the epoch before make together, carried through the turn.

What the Beginning Left Behind, chapter 11, "The Oldest Light", section "What Passes the Turn".

Empty space is never perfectly still, and the trembling of empty space makes faint ripples of its own; astronomers measure the strength of ripples by their power, the strength multiplied by itself. A length we observe today, traced back to near the turn, was then about a billion billion billion times longer than the horizon of that moment, the farthest distance a signal could have crossed since the turn. On a wave that much longer than the horizon, the trembling of empty space in our epoch carries a power more than ten to the forty-two times smaller than the power of the ripples the sky shows. So the trembling adds almost nothing to the ripples we see.

What the Beginning Left Behind, chapter 11, "The Oldest Light", sections "How the Ripples Are Described" and "Empty Space Adds Almost Nothing".

With the first ripples identified as the shape of the old alternatives carried through the turn, the account excludes a long inflation. An inflation long enough to carry the lengths we observe from inside the early horizon (the horizon near the turn) to beyond it would have made the ripples on those lengths out of the trembling of empty space during that inflation. On my account the ripples on those lengths came through the turn instead.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "A Long Inflation Is Excluded".

The dark matter is made of lasting vibrations of the six held dimensions, each of about the Planck mass, made only at the turn.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "How Hot the Beginning Could Have Been".

The hotter a young universe is, the more it makes of anything that has weight, so the measured amount of dark matter caps how hot our history ever was.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "How Hot the Beginning Could Have Been".

A second argument that the ripples were not made by a long inflation rests on the dark matter, and it uses nothing of the identification of the ripples. In inflation the strength of the gravitational waves grows with the energy that the inflating substance holds, and the heat of the universe after inflation comes from that energy. So the measured limit on the waves caps how hot the universe could have been after an inflation. After an inflation that had made the ripples, that cap would have kept the universe far too cool to make the dark matter afterward in the measured amount. Two arguments, resting on different premises, reach one result: the ripples were not made by a long inflation.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "A Second Reason, From the Dark Matter".

The account's standing on a shorter inflation, in one sentence: A shorter phase, one that leaves the lengths we observe outside the early horizon the whole time, the account neither requires nor excludes, and the starting temperature of our history bounds it.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", section "A Long Inflation Is Excluded".

The epochs follow one another in a single sequence, each from the end of the one before, as the cars of one train follow one another.

At the Edge of Reality, chapter 20, "The Cycle Without a First Turn", section "The Cycle".

With no long inflation, the account has no inflation going on forever somewhere else, and no tree of Big Bangs growing from one.

Side by Side, Volume Two, chapter "The Structure and the Act", section IX, "Summary".

The first ripples are ripples of density, and a ripple of density bends space more in one place and less in another, as children standing on a trampoline make deeper and shallower dips. A wave of space itself needs something pulling harder in one direction than in another, and hot matter

close to balance pushes equally every way. So waves of space from the beginning arise only where ripples of density act on one another, and their power is far below a millionth of the power of the ripples of density.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Ripples of Density, and Waves Only Later".

Inflation computed an index of about 0.96 before the measurement reached it.

Side by Side, Volume Two, chapter "The Structure and the Act", section VIII, "Where the Predictions Part, and Why", under "A long inflationary phase".

The account fixes the kind of the pattern (one cloth across the whole sky, the same for every ingredient, ripples of density, on flat space) and takes the strength and the tilt of the ripples from measurement.

What the Beginning Left Behind, chapter 11, "The Oldest Light", section "What the Pattern Must Show".

A pattern that is strongest at one particular length singles out that length. A pattern that singles out no length at all is exactly the same at every length, with an index of exactly one. A tilt is a departure from that sameness, so a tilt singles out some length, and on my account the measured index, a little below one, says that the ripples faintly single out some length of space belonging to our epoch.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "The Strength and the Tilt".

The joint paper of the BICEP2 and Planck teams, published in March 2015, found strong evidence for dust in our own galaxy and no significant evidence for gravitational waves from the beginning in the signal reported in 2014.

Side by Side, Volume Two, appendix E, "The Registered Predictions Beside Max Tegmark", section II, "Where the Two Accounts Predict Differently", under "Tegmark 1. Primordial gravitational waves".

The exact statements and values for this chapter are these.

On a length observed today, traced back to near the turn, the trembling of empty space carries a power below ten to the power minus fifty-one, against an observed power of the first ripples of about two billionths. The shortfall is a factor of more than ten to the power forty-two.

What the Beginning Left Behind, chapter 11, "The Oldest Light", section "Empty Space Adds Almost Nothing", with the values computed in *The Gravitational Passage*, chapter 18, "The Interior Passage", section IX, "The Departures from Uniformity".

The measured values of the pattern: the index of the ripples is 0.9649 $\pm$ 0.0042 (Planck Collaboration, Aghanim and others, 2020), and the ratio of first waves to first ripples lies below 0.036, with ninety-five percent confidence (BICEP/Keck Collaboration, Ade and others, 2021).

What the Beginning Left Behind, chapter 11, "The Oldest Light", section "What the Measurements Find".

On my account the ratio of first waves to first ripples lies far below one millionth, because the waves arise only at second order.

What the Beginning Left Behind, chapter 12, "Four Marks of the Pattern", section "Ripples of Density, and Waves Only Later".

Why space is flat, with the numbers: the curvature is a constant of integration set once by the starting state. A radius made from the length of the turn and the de Sitter radius would need a pure number near ten to the power thirty, which nothing passing the turn supplies. So space is flat to the size of its departures, about ten to the power minus five.

The Gravitational Passage, chapter 18, "The Interior Passage", section IX, "The Departures from Uniformity"; the section for chapter 2 above gives the argument at more length.

The sealed record holds six predictions that bear on this chapter: space flat to the size of the primordial departures; no primordial gravitational waves at any level a coming instrument can reach; and ripples of one kind, with every ingredient rippling together. The other three are one pattern across the whole sky, no long inflationary phase, and the gravitational waves an epoch itself makes. The appendix "What the Coming Measurements Will Find" gives them with what the account says each measurement will find.

Chapter 6, "Is There Another Copy of Me Out There?"

Space goes on past our horizon, with the same laws, and the regions there exist; they are further parts of one world, and on whether the regions exist the two accounts differ only in what they call them.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level".

The principle of existence: "Existence without any interaction whatsoever is indistinguishable from non-existence, and therefore is non-existence".

Incommensuration; The Theorem and the Three Axioms, chapter 9, "Each Axiom Derived Again", section I, "Axiom I: The Asymmetry of the Relational", under "The Derivation", at "The Scope of the Derivation".

Every region past our horizon interacts with its own neighbors as our region interacts with ours, so every such region exists.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level".

I apply my principle of existence to Tegmark's Omnivision Assumption, and it gives his answer. On my account a thing exists if it interacts with anything at all, and whether a human being can observe it plays no part, so the world holds more than any one of us can see.

Incommensuration; The Theorem and the Three Axioms, chapter 9, "Each Axiom Derived Again", section I, "Axiom I: The Asymmetry of the Relational", under "The Derivation"; and *Side by Side*, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

I hold the premise that there is a world and it does not depend on us, on ground of my own: every interaction is a comparison, and the world was comparing itself, in every collision of particles, long before anything human looked.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

The horizon of one observer "is the edge of what one observer can ever compare with, and not an edge of the universe", because other observers stand past it, with horizons of their own. Each observer stands at the center of all that it can ever compare with, and the reaches of neighboring observers overlap across the whole field.

What Cannot Be, chapter 8, "A Click No One Could Predict", section "An Edge Built Into Measuring".

I reach Tegmark's conclusion, that you cannot predict with certainty what you will experience next, by an argument that needs no copies and hold of every world. Determinism needs two things at once: one law that is the same everywhere and always, which is perfect sameness, and every state specified with no gap anywhere, which is perfect gaplessness. In my words, "total specification (continuity) of lawful structure (symmetry) is structurally precluded", and no world is purely and completely deterministic, with copies or without them.

Causality Without Determinism, chapter 1, "Two Concepts, Three Differences", section VIII, "Why Determinism Cannot Be Had".

The Incommensuration Theorem forbids exactly that pair. I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing".

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", and its section "Terms"; and *Incommensuration; The Theorem and the Three Axioms*, chapter 2, "Symmetry and Continuity Cannot Both Be Perfected", section VI, "The Theorem".

"Whatever lawful description is given of a world, something happens in that world which the description does not fix". So no description, however complete, lets anyone predict with certainty all that happens next.

What Cannot Be, chapter 7, "Choice, Change and Cause", section "From Outside, Chance; From Inside, Choice".

Infinity, in *An Immanent Metaphysics*: The two accounts meet once more, on infinity. Tegmark notes that nothing infinitely big or infinitely small has been observed directly. *An Immanent Metaphysics*, the book of aphorisms from which the whole collection is drawn, holds that no find-

ing can ever take in either. In its words: “no sensory interaction, fact of experience or objective measurement could ever encompass the infinitely small or the infinitely large equal to the presumed absolute nature of the reality itself”. By “the presumed absolute nature of the reality itself” the aphorism means reality as it is presumed to be in itself, whole, apart from any finding. To be *objective*, in the sense that *Physics Is Not Math*, chapter 3, gives the word, is to be found the same by many comparisons made from many places. The two accounts agree on infinity because every finding is one finite interaction.

An Immanent Metaphysics, “Technical Appendix”, section “Cartesian Dualism”, aphorism 2M44; the sense of “objective” from *Physics Is Not Math*, chapter 3, “Truth Has a Frame”, section V, “Objectivity Is Built from Many Views”.

The regions past our horizon belong to our one world, and no comparison we can make reaches what is in them: “Where a beyond is defined, as past the horizon of one observer, we still cannot assume anything about what is there, because there is no way to tell”. So I make no hypothesis about what is in those regions, and I draw there no conclusion that everything that can happen does happen.

What Cannot Be, chapter 8, “A Click No One Could Predict”, section “An Edge Built Into Measuring”.

Measurements made inside our horizon are findings about the part of space we can see, and they fix nothing about what lies past it.

Side by Side, Volume Two, appendix E, “The Registered Predictions Beside Max Tegmark”, section V, “Where No Measurement Decides”, under “Regions beyond every horizon: Levels I and II”.

On my account our epoch began at the turn, out of one single whole and with no long inflation, so there is no endless inflation to make the regions.

Side by Side, Volume Two, chapter “The Structure and the Act”, section VI, “The Four Levels, Level by Level”.

The passage from the possible to the actual is in each act of finding: “Before the measurement, the system's state is a range of possibilities; after it, one possibility is actual”.

Causality Without Determinism, chapter 4, "Measurement; What a Comparison Brings Into Being", section IV, "What a Measurement Brings Into Being".

"Other places can be looked into, and other times can be read from records. Other possibilities can be reached only by hypothesis". A possibility is a member of the range from which each finding makes one thing actual, and a far region of space is a place, so the two are different things.

Measurement; The Steps from Comparison to Physics, chapter 11, "The Ten-Dimensional Context", section VIII, "Why the Six Are Not Traversed"; the same sentences in chapter 4, "Ten Dimensions in Plain Words", section VII.

The principle of identity: "The Principle of Identity says that what cannot be told apart, by any test that could even be imagined, must be counted as the same". Two things differ when some content differs in a setting the two have in common, which "is exactly to say that some comparison could register the difference".

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "The Principle of Identity".

"Two identical coins, one in each hand, are not one": compare each coin with your left hand, and one is in it and the other is not, so the verdicts differ and the coins are two.

At the Edge of Reality, chapter 11, "Every Ending Is the Same Ending", section "Stated Exactly".

I apply my principle of identity to an exact copy of you in a far region: compare each of you with this room, and one of you is in it and the other is not. So, by the principle of identity, an exact copy of you in another place is a second thing, in a second place. Applied in the same way, the principle counts a copy that survives where you die as a second thing, in a second place, and not as you. The application is mine; the principle stands in *What Cannot Be*, chapter 21, "Are We Living in a Simulation?", section "The Principle of Identity", and the coins in *At the Edge of Reality*, chapter 11, "Every Ending Is the Same Ending", section "Stated Exactly".

For a copy made in a machine I speak of persons: the hope of living on as a copy in a computer "rests on the assumption that persons are information", and "A person is not a very large file". Experience cannot do without three things a record lacks: something new arriving that nothing before it fixed, time running one way, and a point of view from inside.

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "A Person Is Not a File".

I answer a question of the same form as the measure problem: why the present act of measuring falls at one time in the history of our universe, and not at another time. My answer is that "only the present act is given", and a criterion that surveys all epochs at once from outside "can give a distribution of the epochs at which comparers stand" and cannot single out one of them as real. In those words of *The Gravitational Passage* an epoch is one time in the history that our measurements piece together, and not the long age from one turn to the next of chapter 3. I apply that answer to Tegmark's count of observers, which is a survey of the same form, laid across regions in place of times. Our measurement of a cold sky is the present act, with no crowd of other presents standing beside it to be counted.

The Gravitational Passage, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section VI, "The Comparison That Fixes Λ", and section VII, "Summary of Predictions", under "Questions".

The framework "does not treat infinite totalities as completed objects of different sizes", and a fraction such as infinity over infinity treats two endless totals as completed objects with sizes, so my framework never forms one.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section V, "Two Infinities".

The context of every comparison, the respect in which the two things compared are two (two places, or two moments, of space and time as they are found), is continuous: it has no gap and no least step. The reason: a least step of context would be a difference of context with no sameness within it, and in every comparison sameness and difference come togeth-

er. The smallest scale physics speaks of is the Planck length, and it marks the edge of what can be measured. *The Basis of Physics*, essay 18, reads that edge in one line: "a boundary on measurement is not a granularity of existence".

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section VII, "The Continuous Wavefunction as Evidence", and section IV, "Randomness, the Division of Space, and the Planck Objection".

The light of distant gamma-ray bursts, arriving together across very different energies, places any grain of space that would slow light in proportion to its energy below the Planck length.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section IV, "Randomness, the Division of Space, and the Planck Objection".

On the number of digits a measurement can carry, the two accounts agree: no finding carries infinitely many digits. The continuity I hold is the continuity of places and moments themselves, and not a list of digits completed ahead of time.

Side by Side, Volume One, chapter "The Digits Not Yet Drawn", section IV, "Finite Information and the Continuous Context".

Incommensuration holds that the difference between the discrete, what comes in separate steps, and the continuous, what has no gaps, is basic in itself: "the distinction between the discrete and the continuous is foundational, not a mathematical convenience".

Incommensuration; The Theorem and the Three Axioms, chapter 14, "Domains, Being and the Two Modes", section V, "Finite and Infinite Describability".

Methods match claims: a claim of logical form is checked by its structure, and a claim about the world by measurement and by evidence gathered from many independent routes. A test is a comparison of outcomes, and "a test applies exactly where the outcome of a comparison can be objective, and nowhere else". *Objective*, in the sense *Physics Is Not Math* gives the word, means three things together: the thing is met in an actual interaction, many comparisons made from many places find it the same, and it can show what no one expected. *Physics Is Not Math* names the three as intensity, consistency and novelty: "Where any one of these three

necessary and sufficient aspects is completely absent, the degree of 'objective reality' of the 'thing' is considered to be completely absent". A description of what a thing is lies beyond any test, and it is checked by its derivation, step by step, and by whether its definitions are faithful to the way their words are used.

Physics Is Not Math, chapter 19, "A Right Method of Evaluation", section II, "Three Kinds of Claim", and section III, "Where a Test Applies", which also holds the three conditions of "objective".

That regions exist past every horizon of ours is a description of what a thing is, a description about regions that no comparison of ours reaches. The derivation of that description is the principle of existence, and that description is settled by argument. The parts of Tegmark's Level I picture that reach measurement are inflation and the evenness and randomness measured inside our horizon.

Side by Side, Volume Two, appendix E, "The Registered Predictions Beside Max Tegmark", section V, "Where No Measurement Decides", under "Regions beyond every horizon: Levels I and II".

I hold the first link of Tegmark's chain, that there is a world and it does not depend on us, and parts at the step to a world that is pure mathematics. A description with every human word taken out is still a description of a world in which every finding is made from a stance, the side from which a comparison is made. A collision of two particles has a stance as surely as a laboratory does.

Side by Side, Volume Two, chapter "The Structure and the Act", section IV, "Why a Description Free of Humans Still Has a Stance".

The exact statements and values for this chapter are these.

The principle of identity, stated exactly, in words: *At the Edge of Reality*, chapter 11, states the principle exactly: "two things are one when, for every possible comparison and for every thing whatever, the two themselves included, the comparison of the first with that thing gives the same verdict as the comparison of the second with that thing".

At the Edge of Reality, chapter 11, "Every Ending Is the Same Ending", section "Stated Exactly".

The principle of identity in symbols: x = y (x is the same thing as y) exactly when, for every possible comparison C and every thing z, C(x, z) = C(y, z). In words, the comparison of x with z gives the same verdict as the comparison of y with z. The two coins differ when z is the left hand.

At the Edge of Reality, chapter 11, "Every Ending Is the Same Ending", section "Stated Exactly".

Cantor's fact about listing: a collection of n members has 2 to the power n yes-or-no assignments, and n is less than 2 to the power n for every n, finite included, so no list with one line per member holds every assignment.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section V, "Two Infinities".

Cantor's fact about listing, in words: I also do without the step that makes infinity into a puzzle of fractions, and *The Basis of Physics*, essay 18, says so outright: "The framework does not treat infinite totalities as completed objects of different sizes; what it uses from Cantor is the fact about listing". The fact about listing, from Georg Cantor's work of 1891, is that no list with one entry for each member of a collection can contain every way of marking each member yes or no, and it holds of three members as much as of endlessly many.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section V, "Two Infinities".

Determinism, in my words, "is an assertion of total symmetry and total continuity".

Causality Without Determinism, chapter 1, "Two Concepts, Three Differences", section VIII, "Why Determinism Cannot Be Had".

The gamma-ray burst of 10 May 2009 brought a photon of about 31 billion electronvolts 0.829 seconds after the burst began (A. A. Abdo and others, 2009, *Nature* 462, 331 to 334). The arrival of that photon places the energy scale of any linear dependence of the speed of light on energy above 1.2 times the Planck energy. The Planck length is about 1.62 times ten to the power minus 35 meters.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section IV, "Randomness, the Division of Space, and the Planck Objection".

The burst of 2009 and the brightest burst yet seen, told in six sentences: Nature has been asked directly whether space has a least step. On 10 May 2009 the Fermi space telescope caught a short burst of gamma rays, GRB 090510, from a galaxy so far away that its light had traveled for about 7.3 billion years. One photon of the burst carried about fifteen thousand million times the energy of a photon of yellow light, and it arrived less than a second after the burst began, together with photons of far lower energy. A space made of tiny grains at the scale physicists call the Planck length, about fifty billion billion times smaller than the radius of a proton, would be expected to slow light of high energy more than light of low energy, and a lag piling up over 7.3 billion years would have shown. The Fermi team found no lag at the Planck scale, and the brightest burst yet seen, caught in 2022 by the Large High Altitude Air Shower Observatory in China, placed any grain of that kind about ten times further below the Planck length. We now know that light of very different energies crosses billions of light years of space at one speed, as a smooth arena carries it, and I read the measurement as one more finding of an arena smooth at every scale yet probed. A. A. Abdo and others, *Nature* 462, 331 (2009); Z. Cao and others (LHAASO Collaboration), "Stringent Tests of Lorentz Invariance Violation from LHAASO Observations of GRB 221009A", *Physical Review Letters* 133, 071501 (2024).

Tegmark's own estimate: the nearest identical copy of you lies about ten to the power (ten to the power 29) meters away, a number written with about a hundred billion billion billion digits. The estimate is in Tegmark, 2007, "The Multiverse Hierarchy", in B. Carr, editor, *Universe or Multiverse?*, Cambridge University Press; arXiv:0905.1283.

The sealed record holds six predictions that bear on this chapter: no long inflationary phase; no primordial gravitational waves; and space flat to the size of the primordial departures. The other three are no short loop

anywhere in space, no least length, area or duration of the arena, and no world perfectly deterministic. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 7, "Why Are the Constants What They Are?"

I reach three dimensions of space and one of time from the structure of comparison. The count rests on one identification, which *Measurement* states in the first person: the space it counts is the space of the large world, the directions in which things extend, and every measurement of the large world finds three.

Measurement; The Steps from Comparison to Physics, chapter 11, "The Ten-Dimensional Context", section V, "Time Has One", and section VI, "Space Has Three"; and chapter 4, "Ten Dimensions in Plain Words", section IV, "Space Is Three".

Time has one dimension because every comparison is made from one stance, the side from which a comparison is made, and from one stance any two findings stand in exactly one order: one earlier, the other later.

Measurement; The Steps from Comparison to Physics, chapter 11, "The Ten-Dimensional Context", section V, "Time Has One".

The difference between any two directions of space must be marked within space itself, by a further direction not made from those two. On a flat page the third direction, straight up off the page, is the one from which the page's two directions can be told apart. With one dimension, space would have the same form as time, and nothing would tell the two apart; with two, nothing in space marks the difference between its two directions; with three, each direction marks the difference between the other two. A fourth direction would mark only differences that the three already mark, so no comparison could register it as a direction of its own; a difference that no comparison can register is no difference, and so space has three dimensions.

Measurement; The Steps from Comparison to Physics, chapter 11, "The Ten-Dimensional Context", section VI, "Space Has Three".

Analyses of the equations of physics also single out one dimension of time, and *Measurement* cites Tegmark's paper of 1997 by name.

Measurement; The Steps from Comparison to Physics, chapter 11, "The Ten-Dimensional Context", section IX, "The Lorentzian Signature".

The rules of a board fix its ranks, and no move can add a rank the rules do not supply; the count of dimensions belongs with the rules.

Measurement; The Steps from Comparison to Physics, chapter 11, "The Ten-Dimensional Context", section IV, "The Minimality Principle, and the Principle Beneath It".

"A value that lands, before its reason is found, is a question, and it is evidence of a reason not yet known". The two kinds of mass of every body agree in every material ever tried, and Einstein's asking why gave general relativity. Balmer's rule of 1885 gave the colors of hydrogen's light from whole numbers, and Bohr's asking why made the rule a door into quantum theory. Had physicists written the two agreements down as coincidences, they would have filed them and stopped asking why.

Physics Is Not Math, chapter 13, "A Value That Lands", sections I to III for the cases, section IV, "A Value That Lands Is a Question", and section V, "A Grade Closes; a Question Opens".

The kinds of particle are fixed by structure and their amounts are not: "The excess of matter is not structural. It belongs to the initial conditions, to the history of the universe, to the facts that the zoo does not fix". The excess of matter is the small surplus of matter over antimatter that our universe holds, and the zoo is the list of the kinds of particle. About six protons and neutrons for every ten billion particles of light is a number of what happened, and it is measured.

The Coming of the Kinds, chapter 1, "The Kinds and Their Amounts", section VI, "The Amounts Are History", quoting *The Story of the Particles*.

On my account Level II does not occur, and I reach that result by three routes, each from its own premises.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level".

Tegmark's answer to the charge that many universes are wasteful, with the word on which it rests: Tegmark answers: "I judge the elegance and simplicity of a theory not by its ontology, but by the elegance and simplicity of its mathematical equations" (chapter 13, "The Case for a Smaller Reality"). *Ontology* is the philosophers' word for the list of things a theory says there are.

In chapter 5 I identified the first ripples with the shape that the alternatives taken in the epoch before make together, carried through the turn, as every footstep is kept in the one curve of a worn stair. With the ripples identified as that shape, a long inflation did not happen. The amount of the dark matter gives a second, separate reason that a long inflation did not happen, and eternal inflation needs a long inflation, so the account has no eternal inflation to make the regions of Level II.

What the Beginning Left Behind, chapter 13, "No Inflation Needed", sections "A Long Inflation Is Excluded" and "A Second Reason, From the Dark Matter".

Tegmark names the condition on which his step rests: "if there are laws or constants of nature that can in principle vary from place to place, then eternal inflation will make them do so across the Level II multiverse" (chapter 6, "Fine-Tuning as Evidence for the Level II Multiverse"). In Tegmark's own telling, the constants differ from region to region because the equations of space have many solutions, as the equations of water have three (ice, water and steam), and one solution would give the same constants everywhere. I hold that the equations of space have one solution for the six held dimensions, and so I do not grant his condition.

Side by Side, Volume Two, chapter "The Structure and the Act", section VIII, "Where the Predictions Part, and Why", under "A long inflationary phase".

The shape of the six is reached by two routes that start from different premises, and each premise is an identification: a step that reads something in the structure of comparison as something in geometry. One route runs through the algebra of comparison, a stock of quantities with rules for adding and multiplying them, which describes what comparisons find; the other uses geometry and no algebra at all. Both arrive at one shape: "they have found one summit, and neither of them chose it from a

list". In the words of *The Compactification*: "Neither route chooses the space from a set of candidates, so there is no landscape of internal spaces and no problem of selecting one".

The Compactification, chapter 2, "One Vacuum", section IX, "One Algebra, One Space"; and *What the Beginning Left Behind*, chapter 5, "One Vacuum", section "One Algebra, One Space".

The size of the six is fixed too, by one further identification, which ties the bending of the six to the Planck length.

What the Beginning Left Behind, chapter 3, "The Shape and Size of the Six", section "What Is Taken and What Follows"; and *The Compactification*, chapter 2, "One Vacuum", sections III and IV.

A field is something spread through all of space with a strength at every point, as a magnet's pull is, and string theory winds fields through the loops of its six, every pattern of winding giving another valley. On my account the law of gravity is built from the geometry of all ten dimensions alone, with no further field, so nothing is wound through the six: "String theory's threaded fields give its six some ten to the five hundred valleys; the six of this account have one".

What the Beginning Left Behind, chapter 5, "One Vacuum", sections "No Threads to Count" and "What the Turn Gives Back"; and *The Compactification*, chapter 2, "One Vacuum", section VI, "No Flux, No Flux Integers".

One valley means one set of rules, and the same rules in every epoch: at the turn the same six re-form at the same size and shape, and "The structure that this account derives is the same in every epoch".

The Compactification, chapter 2, "One Vacuum", section VII, "Tunneling and the Failure of the Hold"; and *At the Edge of Reality*, chapter 20, "The Cycle Without a First Turn", section "What Carries Over".

The structure I derive includes the forces, the kinds of particle and their charges, the three generations (three sets of matter particles, each set a heavier copy of the one before) and the counts of dimensions.

Side by Side, Volume Two, chapter "Time Is Real", section IV, "Laws That Stay the Same, and a Future That Stays Open".

The list of kinds is closed: "The particles of the Standard Model, the quarks, the leptons, the carriers of the three forces and the Higgs boson, exhaust the kinds". The standard model of particle physics, which the quotation calls the Standard Model, is physicists' name for the list of known particles and their forces. The three forces are the electromagnetic force, the force between charges, which light carries; the strong force that holds the core of every atom together; and the weak force at work in some radioactive decays.

The Grand Synthesis, chapter 13, "The Cosmological Constant and the Completeness of the Kinds", section III, "The Completeness of the Kinds".

The energy that quantum field theory assigns to empty space is the hum of every field added up to the scale at which gravity and the quantum act on equal terms. The assigned energy and the energy the sky shows differ by a factor of a one followed by one hundred twenty-three zeros, the largest mismatch between a theory and a measurement in physics.

The View from Inside, chapter 10, "Why the 10^{123} Never Arises", section "Back to the Numbers"; and *The Grand Synthesis*, chapter 13, "The Cosmological Constant and the Completeness of the Kinds", section I, "The Cosmological Constant".

Space curves under what comparisons find: energy that no comparison finds puts no strain on the arena, as the rug lies flat when no sofa pushes. The vacuum's total energy, taken whole, would be the same at every place, the same for every observer in steady motion, and without a break anywhere: perfectly symmetric and perfectly continuous at once, which by the Incommensuration Theorem no finding holds. So what any comparison finds is a part of the vacuum's energy, and "So the total curves nothing, whatever its size": the mismatch of one hundred twenty-three powers of ten needs no cancellation.

The View from Inside, chapter 10, "Why the 10^{123} Never Arises", sections "What Curves Space" and "A Much Smaller Question".

The vacuum is the settled state of empty space, as quiet as it can be. A measurement of the vacuum is made from inside it, by instruments made of the very fields whose hum they measure: a cupful of the sea trying to know the rest of the sea.

The View from Inside, chapter 10, "Why the 10^{123} Never Arises", section "A Cup from the Sea".

The cosmological constant is the share of the vacuum's energy that a measurement made from inside can take in, by the identification that chapter 4 states. The vacuum holds its energy in a vast number of independent ways, as a piano holds its sound in many strings, each of which can ring on its own. Each way holds an equal part of the vacuum's total energy. The six held internal dimensions are curved, and their curvature carries a small energy of the opposite sign, about twelve percent of the total, so it takes that much off and leaves 0.881 of the total. *The View from Inside* takes three steps from the vast number of ways to the share a measurement takes in, each step in the first person. First, the book counts the ways on the edge of everything whose light has reached us, and the count there is about a one followed by one hundred twenty-three zeros. In *The View from Inside* I take the count at the edge of everything whose light has reached us as an identification. The edge of everything whose light has reached us is the edge of what has been compared. I identify the edge of what has been compared with the edge of the information open to the comparison. Second, a measurement takes in what it finds through channels, the routes by which the vacuum reaches the measurement. In *The View from Inside* I identify the channels with what the measurement registers, one channel in each of the three roles that every comparison has. In the picture of chapter 1, the three roles are the score sheet, the move and the seat. In *The View from Inside* I write: "I identify the channels through which the vacuum reaches a measurement with the contents that the measurement registers, one in each role". The shape of the six gives the same three channels by a separate route: the part of the six shaped by the three-way choice is built from exactly three basic pieces, one for each role. A basic piece is one of the simplest parts a shape can be built from, as

the surface of a ball is built from two: a single point, and a disc whose rim is drawn shut to that point. Third, the book takes each channel to take in one whole way, and a different way for each channel, because what a measurement registers comes whole, in steps, and the three channels carry three different things. So a measurement takes in three of the ways, three parts in that enormous count. *The View from Inside* writes the share as one formula: divide the 0.881 of the vacuum's total among the ways counted on the edge of everything whose light has reached us, and take three of those parts. So the cosmological constant is small because the part of the universe we can see is large.

The View from Inside, chapter 11, "The Problem of Being Inside", sections "The Share We Find", "Tiles on the Horizon" (the third step, in which the book calls each way a tile: "I take each channel to take in one tile") and "The Size of What Is Withheld"; chapter 12, "Counting the Horizon", section "The Edge of What Has Reached Us", which says: "So I take the horizon's count at the particle horizon, as an identification: the edge of what has been compared is the edge of the information open to the comparison", section "Why Three", which holds the second step, and section "A Definite Number", which writes the share as one formula; chapter 10, section "A Cup from the Sea"; and chapter 9, "The Shape of the Internal World", section "A Small Trim", and section "From the Grid to a Shape", which says: "The shape that carries the three roles is built from exactly three basic pieces, one for each role".

Set equal to the energy of empty space as astronomers find it from the rate of expansion, the share leaves one equation in pure numbers. The equation gives 31.1 percent matter, where the Planck satellite measured 31.5 percent, and the energy of empty space within two percent of the value the sky shows.

The View from Inside, chapter 13, "One Equation, One Number", sections "One Equation, One Unknown" and "The Same Number, Seen from Other Sides".

The cosmological constant is one value, given by one formula, the same at every place.

The View from Inside, chapter 15, "One Constant, One History", section "One Map, One Scale".

The reasoning from the landscape is "logically impeccable", and the reasoning from the landscape makes the cosmological constant "an accident of location", a value that no law derives.

The View from Inside, chapter 3, "Why Nothing Else Has Worked", section "Strategy Two: Accept the Landscape".

The three dimensions of space and the one of time, the shape and size of the six and the closed list of the kinds of particle are rules, fixed by the structure of comparison, the same everywhere and in every epoch. The cosmological constant is one value, the same at every place, and one formula gives it from four numbers. The structure of comparison sets two of them: the number three, one for each role of a comparison, and the 0.881 of the vacuum's total that the curvature of the six leaves. The other two are measured: the strength of gravity, which sets the size of the vacuum's total, and the size of the edge of everything whose light has reached us now, which sets how many ways share that total. So for the cosmological constant the answer to "why this value?" is a reason for the formula and for two of its numbers, and a measurement for the other two.

The View from Inside, chapter 16, "The Observer Inside", section "What Being Inside Withholds".

The world has this form because any form that supports comparison must have it, "not because observers must exist, but because comparison must be self-consistent and there is exactly one self-consistent comparison structure"; self-consistent here means that no part of the structure contradicts another part. Carter's reasoning picks our values out of many, by where observers can live; I find only one structure that comparison allows, and so there is nothing to pick from.

The Geometric Route, chapter 32, "Self-Descriptive Closure in the Physics", section IV, "The Physical Analogue".

The force that light carries between electric charges is the electromagnetic force. I call the number that measures the strength of the electromagnetic force the strength of light; physicists call that number the fine-structure constant. The structure of comparison is the same at every value of the strength of light, so "no argument from the comparison structure, of

any length, fixes" its value, and the strength of light stays measured. I count such a number as a value of this world, the world in which the structure of comparison is made actual.

The Strengths of the Kinds, chapter 1, "What Is Measured", section I, "The Kinds of Number", and section VI, "Why the Strength of Light Stays Measured"; and *Why These Numbers*, chapter 3, "The Electron and the Strength of Light", section "Why the Strength of Light Stays Measured".

Measured, one over the strength of light is 137.035999177, which is no whole number, so the whole number 137 that Arthur Eddington argued for from 1929 is excluded.

Why These Numbers, chapter 3, "The Electron and the Strength of Light", section "The Strength of Light as Measured".

Some values land on measurement with their reason not yet known, and I write each of them as measured, with its question beside it. A measured value, with its question written beside it, is a complete state of knowledge; a value that lands is evidence of a reason not yet known, and I keep the number as a clue to that reason.

Physics Is Not Math, chapter 15, "The Descent", section VII, "Not Fixed by This Route".

Every charged particle of matter sits a counted number of halvings below the heaviest quark, the top, each halving a step down in mass by a factor of two, and one formula gives every count. The formula has three numbers for each family, and a family is three particles of one type, one in each generation: the electron and its two heavier copies make one family, and the quarks make two more. "Nine masses that spread over a range of hundreds of thousands become one formula".

Why These Numbers, chapter 9, "The Ladder of Masses", section "What the Formula Is Tested By"; the three numbers for each family in the same chapter, section "Why the Formula Has This Shape", which asks: "Why three numbers per family, and why these three kinds of number?"

The exact statements and values for this chapter are these.

The mismatch: the hum of every field added up to the Planck scale gives an energy density of about 2.2 times ten to the power 76, in units of the fourth power of a billion electronvolts (the Planck mass to the fourth power). The sky shows 2.52 times ten to the power minus 47, and the ratio is about ten to the power minus 123.

The View from Inside, chapter 10, "Why the 10^{123} Never Arises", section "Back to the Numbers".

The count of alternatives has five factors: four edges, the whole numbers one to six in place of three, the trim or none, two ways of counting the energy that holds the internal space, and the full or the reduced Planck mass. Together they give $4 \times 6 \times 2 \times 2 \times 2 = 192$ combinations, of which exactly one lands within the measured uncertainty. The same count, in words: *The View from Inside*, chapter 12, counts the alternatives: of 192 combinations of the choices in the reasoning, exactly one lands within the measured uncertainty, the one its reasons select before any number is looked at.

The View from Inside, chapter 12, "Counting the Horizon", section "A Definite Number", with the alternatives tried in its appendix A.

The internal space is the complex projective plane, of four dimensions, times the two-sphere, with radii $\sqrt{6}$ (the square root of six) Planck lengths for the first and one Planck length for the second, fixed by one identification. At first order the two settings of the internal space carry masses of 2.25 and 3.09 Planck masses.

The Compactification, chapter 1, "The Re-Forming of the Internal Space", abstract; and chapter 2, "One Vacuum", abstract and section X, "The Standing of the Steps".

The two settings of the six, with the two identifications that fix them: Given the identification that every point of each part of the six is like every other, a theorem of geometry then leaves exactly two settings, an overall size and a ratio of two radii, and one further identification fixes both, the same at every point of space. The first identification is that every point of each of the two parts of the six is like every other point of that part; the further identification is "that every internal direction bends by

the Planck curvature", and in the words of *What the Beginning Left Behind*, "One more identification, that every internal direction bends by the Planck curvature, fixes the size".

The Compactification, chapter 2, "One Vacuum", section III, "Why Two Moduli", and section IV; and *What the Beginning Left Behind*, chapter 3, "The Shape and Size of the Six", section "What Is Taken and What Follows".

The closure condition: the equation $c^2 + (c + 1)^2 = (2c - 1)^2$ (c squared plus c plus one squared equals two c less one, squared) has exactly one positive solution, $c = 3$, which gives the Pythagorean triple 3, 4, 5.

The Geometric Route, chapter 32, "Self-Descriptive Closure in the Physics", section IV, "The Physical Analogue".

How *The Geometric Route* names the difference from Carter's reasoning: *The Geometric Route* names the difference from Carter's reasoning in two sentences: "The anthropic principle is a selection effect. The self-descriptive closure is a uniqueness theorem applied to the structure of comparison itself". Self-descriptive closure is my name for the finding that the axioms describe themselves: the act of reading and checking them has the structure they state, and in that book's words, "the closure shows that they also describe themselves". Chapter 32 of the same book gives its physical analogue: "The physics, when it examines its own foundations, finds again the same axioms that produced it".

The Geometric Route, chapter 32, "Self-Descriptive Closure in the Physics", section IV, "The Physical Analogue"; and chapter 3, "Tests in One's Own Experience", section I, "The Three Axioms".

What tests the formula of the masses, in two sentences: *Why These Numbers* states that eight constants can always be found to fit eight masses, and it places the test where a fit cannot reach: the constants that reproduce the masses lie close to simple fractions of three, four and five, which are the edges of a right triangle, and they satisfy identities among themselves. In the words of its chapter 10, "Eight dials turned at random would show none of this".

Why These Numbers, chapter 9, "The Ladder of Masses", section "What the Formula Is Tested By"; and chapter 10, "The Rungs, the Offsets and the Small Leftover", section "Eight Numbers for Eight Masses".

The strength of light, measured: one over the fine-structure constant is $137.035999177 \pm 0.000000021$ (2022 recommended values of the Committee on Data of the International Science Council).

The Strengths of the Kinds, chapter 1, "What Is Measured", section VI, "Why the Strength of Light Stays Measured".

The sealed record holds seven predictions that bear on this chapter: the matter fraction; an equation of state of exactly minus one; no long inflationary phase; and the ratio of dark to ordinary matter. The other three are no kind of particle beyond the closed list, exactly three generations, and no axion. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 8, "What Happens When We Look?"

In 1922 Walther Gerlach and Otto Stern sent silver atoms through a shaped magnet, and the atoms landed in two separate spots and nowhere between.

Why the Quantum Is the Way It Is, Preface, "What This Book Does", section "One Atom Through the Whole Book".

The quantity the magnet sorts is spin, and an electron's spin, measured along any line, comes in exactly two values.

Why the Quantum Is the Way It Is, chapter 14, "Spin, and Why Electrons Keep Apart", section "Two Spots and a Double Turn".

"Matter is a ladder of levels, each held by what the level below leaves over and each held far more weakly than the one below". Quarks are held into protons and neutrons by the field of the strong force; what that field leaves over holds protons and neutrons into nuclei; what a nucleus leaves over, its charge, holds electrons into atoms, and what atoms leave over holds them into molecules.

The Matter We Know, Volume One, "The Ladder in Brief".

"A whole lasts when it is the lightest thing that carries its counts". A whole's counts are its electric charge, its number of baryons (wholes of three quarks, such as the proton and the neutron) and its number of leptons (the electron and its relatives). The rule for which wholes last is structure, laid out whole and the same everywhere, and which whole is lightest is measured. Read beside Tegmark's observation, the rule says the same thing from the direction of what lasts: a whole that no conserved quantity protects breaks down into lighter things, because every reaction that no conserved quantity forbids occurs.

The Matter We Know, Volume One, "The Ladder in Brief".

I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing".

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", and its section "Terms".

What is found of matter and light comes in exact, repeating steps: one outcome, one packet of light, one sharp line of color, the same charge on every electron. What a comparison finds of matter is the same across every change of place and time, and sameness across place and time sorts what is found into kinds. Belonging to a kind is all or nothing, so between the kinds there are gaps. The theorem gives the gaps at their root: what is perfectly the same cannot also be perfectly gapless.

Why the Quantum Is the Way It Is, chapter 4, "Symmetry and Continuity Cannot Both Be Perfected", section "The Two Survivors".

The endless glow and the falling atom of classical physics were removed in the same way: "In both cases the limit was removed by giving content its own character". Content is what a comparison finds, here matter and light, and its own character is to come in exact steps, the same in every instance; the classical description had seen matter and light as smooth, and the steps had been there all along.

Two Theories, One World, chapter 11, "What Quantizing Does, and Why the Arena Cannot Take It", section "Why Classical Theories Work, and Where They Stop".

A particle's place is described point by point along an unbroken line, which carries continuity, and a state of perfectly sharp motion looks the same at every place along the line, which carries symmetry. One exact rule converts either description into the other, and I identify that rule with the pairing I name at its foundation: two descriptions of one thing, each of which can be made perfect only at the cost of the other. A description that made the place and the motion both perfectly sharp at once would hold perfect sameness and perfect gaplessness together: "The uncertainty principle is an instance of the Incommensuration Theorem". The theorem gives the reason that a limit on sharpening place and motion together must hold, and that no later theory will remove that limit.

Why the Quantum Is the Way It Is, chapter 4, "Symmetry and Continuity Cannot Both Be Perfected", section "The Uncertainty Principle as an Instance".

The rulebook from which every kind of particle comes is a finite set of rules, selected by the structure of comparison, for combining the labels a particle carries, and the kinds it allows are exactly the known kinds. The known kinds are twelve kinds of matter, each with its antiparticle (six kinds of quark, and six leptons: the electron, its two heavier relatives and three neutrinos), the carriers of three forces, and the Higgs boson. "The argument is one of necessity, not of exhaustion: in any world in which comparison occurs, these kinds cannot not exist". So the list of kinds is closed by the structure that makes them, and not by how hard anyone has looked; no further kind waits below the quarks and the electron, and heavier wholes built from the known kinds wait at higher energies.

The Story of the Particles, level 9, "The Composites and the Boundary", section III, "The Algebra Is Finite".

Where Tegmark's suspicion that "the continuum is an illusion" stands in his Table 7.3, "Summary of key quantum-mechanics concepts": The suspicion stands in the table's entry on quantum immortality, the idea that we are subjectively eternal if every outcome of a quantum event occurs in a world of its own, as his reason for doubting that idea.

Tegmark, chapter 7, "The Collapse of Consensus", Table 7.3, entry "Quantum immortality", which reads: "The idea that we're subjectively eternal if the Level III multiverse exists. I suspect that there's no quantum immortality because the continuum is an illusion (Chapter 11)".

The arena has no gap and no least step. Every comparison finds sameness and difference together, so every difference between two places or two moments of the arena holds some sameness within it, and a finer comparison, made at a place or moment between the two, can always find that sameness. A least step would be two places, or two moments, with nothing between them and nothing shared, a difference with no sameness within it, like two neighboring squares of an enlarged picture on a screen. In the words of *The Basis of Physics*: "so the context has no gap and no least step: between any two settings there is a third, and nothing in the context is sudden, which is continuity". The steps belong to what is found within the arena: the charges, and the kinds of particle.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section VII, "The Continuous Wavefunction as Evidence".

The arena has been found smooth at every scale yet probed: the light of the burst of gamma rays caught in 2009, of very different energies, crossed billions of light-years of space and arrived together. The common arrival places any grain of space that would slow light in proportion to its energy below the Planck length.

The Basis of Physics, essay 18, "The Continuum Argument Against Simulation", section VII, "The Continuous Wavefunction as Evidence".

Every physical theory has two levels: its mathematics, which exists, laid out whole and the same for everyone, and its findings, which are real, each given once, in an act of finding. In quantum theory the two levels take opposite characters: the findings are governed by sameness, and the mathematics by continuity. What is found of a particle is the same in every instance and comes in steps, and the mathematics that carries the findings is smooth: states that blend into one another in any proportion.

Why the Quantum Is the Way It Is, chapter 4, "Symmetry and Continuity Cannot Both Be Perfected", section "Two Levels of One Theory".

Measurement sets out the division between findings and mathematics in a table, theory by theory. For quantum theory, the sharp lines of color that atoms give and the numbers that label each kind of particle stand among the findings, and the smoothly blending states stand in the mathematics.

Measurement; The Steps from Comparison to Physics, chapter 8, "The Cross-Modal Principle", section V, "How a Continuous Group Is Read".

The table's entry for quantum theory, in its own words: *Measurement* names the numbers for what they are: "The discrete spectra of bound states, and quantum numbers, the labels of the representations of its symmetry groups: discontinuously symmetric". The entry beside it, for the mathematics, reads: "The Hilbert space of states, with continuous amplitudes, on which the symmetry groups act with continuous parameters: continuous".

Measurement; The Steps from Comparison to Physics, chapter 8, "The Cross-Modal Principle", section V, "How a Continuous Group Is Read".

Every interaction is a comparison: an atom that takes in light compares the light's energy with its own steps, and a screen that a silver atom strikes is changed in one way and not another.

Why the Quantum Is the Way It Is, chapter 2, "Every Finding Is a Comparison", section "One Act, Three Names".

A comparer, this book's word for what Tegmark calls an observer, is any position inside the world from which a comparison is made: a glass plate, a counter, an atom that takes in light, or a person; a comparer need not be a mind. *Why the Quantum Is the Way It Is*, "Key Terms", entry "Comparer".

"A finding is any comparison that lays down a record: an interaction after which some part of the world carries a record of the outcome". A hoofprint in sand is a record in this sense, and so is the silver on Gerlach's plate.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Every Comparison Has a Cut".

"Consciousness is not what makes anything definite; any interaction will do": the Moon is definite through its meetings with sunlight and with everything near it, and no one needs to look.

What Cannot Be, chapter 4, "Process, Pattern and Possibility", section "The Cat in the Box".

The world compared itself, in every collision of particles, long before anything human looked at it.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

Quantum theory as the textbooks teach it rests on six rules gathered from six traditions between 1906 and 1932, with later refinements in 1935 and 1951, and "No experiment has ever contradicted the predictions of the six rules". One rule says that a state always changes smoothly, and two others say that a measurement makes the state jump; "The measurement problem is the scar left by that assembly". On my account, the rules, as assembled, contradict one another where the smooth change meets the jump.

Why the Quantum Is the Way It Is, chapter 1, "Six Rules Nobody Chose", sections "Why These Rules?" and "The Scar Left by the Assembly".

"Collapse is not a physical process that destroys a superposition". I keep the smooth rule as the law by which the state changes between findings, and I keep the single outcome of each finding. The two are kept because the smooth rule describes the range of possible findings, and the single outcome is given in the act of finding.

What Cannot Be, chapter 4, "Process, Pattern and Possibility", section "The Cat in the Box"; and *Why the Quantum Is the Way It Is*, chapter 17, "Why a Measurement Has an Outcome", section "Each Claim Said of One Role".

The range of legal moves, the move and the line on the score sheet match the way *Why the Quantum Is the Way It Is* reads quantum theory: the state, the act of finding, and the record that the act lays down.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Why Collapse Is Not Physical".

"The state exists: it is the complete description of the range of possible findings of the system, and of their probabilities". Each finding has one outcome, given in the act, and the state does not contain that outcome in advance; the outcome is real.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Each Claim Said of One Role".

The jump that the textbooks call collapse is the shared record of what has been found taking the outcome, and the next comparison begins from it; nothing physical jumps, and the outcome is single.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Why Collapse Is Not Physical".

Each magnet in a row is a comparison with its own setting, each atom makes one spot at each plate, and each finding replaces the range from which the next comparison starts.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "The State After an Outcome".

A finding happens at every comparison that lays down a record. Every comparison is made from a stance about a thing compared, and the comparison performed sets where the line between the stance and the thing compared falls. John von Neumann showed in 1932 that the line between the measured system and the chain of instruments can be placed at any link of the chain, and the counts predicted come out the same.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Every Comparison Has a Cut".

I identify the line that von Neumann moved as the boundary between the two levels of a theory, its mathematics and its findings, and not as a place among objects. A boundary between two kinds of object would stay in one place, so the line's freedom to move along the chain supports reading it as the boundary between the two levels of a theory.

Why the Quantum Is the Way It Is, chapter 18, "The Interpretations, Each Missing a Role", section "Copenhagen and the Cut".

Inside the box, the atom, the counter, the poison and the cat are in comparison with one another the whole time, and relative to those comparisons the cat is alive or dead.

What Cannot Be, chapter 4, "Process, Pattern and Possibility", section "The Cat in the Box".

"Each comparison has its own outcome, no comparison has two, each finding is made by the comparer that finds, and no geometry changes at either finding". Someone who has made no comparison with a laboratory holds a description of what that someone could find, and that description leaves every finding made inside the laboratory as it was.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Every Comparison Has a Cut".

How a coin lands is fixed by how it was thrown, and the odds we give it measure what we do not know about the throw. The atom's state is known as fully as the theory allows, and which spot it reaches is still written nowhere in that state.

Why the Quantum Is the Way It Is, chapter 11, "Why Chance Enters", section "A Click Nobody Could Name in Advance".

Hard randomness is an outcome that nothing before it fixed, however completely everything before it is known, and hard randomness is real in each single finding. A Geiger counter shows it plainly: nothing fixed the moment of each decay, and yet of a large number of atoms of one kind, half decay in a time that never changes.

Why the Quantum Is the Way It Is, "Key Terms", entry "Hard randomness"; and chapter 11, "Why Chance Enters", section "A Click Nobody Could Name in Advance".

A world in which every outcome was fixed in advance would need one law carried the same way into every state everywhere, which is perfect sameness, and every state passing into the next with no gap, which is perfect gaplessness. The theorem forbids the two together. So every lawful description leaves some outcome that its law does not fix, and the single outcome of a quantum measurement is the plainest case.

Why the Quantum Is the Way It Is, chapter 11, "Why Chance Enters", section "The Residual".

"The probability of a quantum outcome is ontic in the structural sense: the residual belongs to comparison itself, and no further knowledge of the state removes it". Ontic means belonging to what is, as distinct from what someone knows.

Why the Quantum Is the Way It Is, chapter 11, "Why Chance Enters", section "From an Unfixed Outcome to a Stable Weight".

Hard randomness takes away determinism, the claim that everything is fixed in advance, and leaves every cause where it was.

Why the Quantum Is the Way It Is, chapter 11, "Why Chance Enters", section "Hard Randomness, Between Two Idealizations".

A law of nature is the best available compression of what has been observed, and "Compression of information can never be perfect; it can never reduce all observable information to zero bits", a bit being the answer to one yes-or-no question. When every pattern in a string of Geiger clicks has been squeezed out, the steady half-life included, what is left is the moment of each click, and that is hard randomness.

What Cannot Be, chapter 22, "Why No Computation Is Everything", sections "The Wolfram Physics Project: An Impression" and "The Residue No Theory Compresses".

Hard randomness rises into the everyday world "the way groundwater rises wherever it finds a path, through every process that amplifies what it cannot resolve": a decay that sets off an avalanche in a Geiger counter, three balls meeting on a table, the weather. What rises is one outcome that nothing before it fixed, which becomes one large difference in one world.

What Cannot Be, chapter 8, "A Click No One Could Predict", section "One Atom, One Avalanche, Three Balls".

The exact statements and values for this chapter are these.

The uncertainty bound: the product of the spreads of position and momentum is at least one half of ħ (h-bar, Planck's constant divided by two pi). The bound follows by Robertson's inequality of 1929 (H. P. Robertson,

"The Uncertainty Principle", *Physical Review* 34, 163) from the exact amount by which position and momentum fail to commute. The theorem explains why a limit of this kind exists at all.

Why the Quantum Is the Way It Is, chapter 4, "Symmetry and Continuity Cannot Both Be Perfected", section "The Uncertainty Principle as an Instance", with the bound reached in chapter 9.

The algebra of the kinds is the complex numbers, the quaternions and the three-by-three complex matrices, joined as one algebra, and its count of generators is $1 + 3 + 9 - 1 = 12$.

The Story of the Particles, level 9, "The Composites and the Boundary", section III, "The Algebra Is Finite".

The electron's magnetic moment, in units of the Bohr magneton, is 1.00115965218059 with an uncertainty of 13 in the last two digits, about thirteen hundredths of a part in a million million (Fan, Myers, Sukra and Gabrielse, 2023, *Physical Review Letters* 130, 071801). The standard model of particle physics agrees with it to about one part in a million million.

The silver beam of Gerlach and Stern split by about two tenths of a millimeter after an exposure of eight hours (Gerlach and Stern, 1922, *Zeitschrift für Physik* 9, 349; Friedrich and Herschbach, 2003, *Physics Today* 56 (12), 53).

The sealed record holds five predictions that bear on this chapter: the arena has no least length, area or duration; the statistics of quantum theory hold; and no world is perfectly deterministic. The other two are that a discrete rule shows itself to a fine enough measurement and that no kind of particle exists beyond the closed list. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 9, "Am I Splitting Into Copies of Myself?"

Collapse is no physical process: the change of the wavefunction at a measurement is a change in the shared record of what has been found, and the laws of motion stay as they are. The shared record is the same for everyone who has the outcome.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Why Collapse Is Not Physical".

I, like Everett and Tegmark, keep the Schrödinger equation for every change between findings, with no exception and no second law beside it.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Why Collapse Is Not Physical".

Decoherence is the fading of interference between the alternatives of a thing as its surroundings come to carry records of which alternative it is in. Think of the surroundings as a notebook that each alternative writes in. If two alternatives leave the notebook the same, the alternatives still show their bands; if they leave two notebooks that differ on every page, the bands vanish from every measurement made on the thing alone. The parts of the description that make the bands "become out of reach for any measurement that probes the system alone, and they are not destroyed", which is one claim with Tegmark's word "censored".

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "What the Surroundings Do".

The content of a comparison is what the comparison finds. The context of a comparison is the respect in which the two things compared are two. As the surroundings interact with the thing, the content of a comparison spreads from the thing alone to the thing together with its surroundings, so the parts of the description that make the bands belong to the two together and not to the thing alone. In the words of *Why the Quantum Is the Way It Is*: "As the surroundings interact with the system, the content of the comparison spreads across the joint system". Ordinary decoherence is made of comparisons by the surroundings: each interaction of the sur-

roundings that lays down a lasting record is a comparison, with one outcome. The theory of decoherence describes, by the Schrödinger equation alone, the comparisons that the surroundings make, and that description shows every alternative with its share of the odds.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "What the Surroundings Do".

In Vienna in 2003 Klaus Hornberger, Markus Arndt, Anton Zeilinger and their colleagues sent molecules of carbon seventy, each a closed cage of seventy carbon atoms, through an interferometer. As gas was let into the chamber the bands faded, and Schrödinger's equation, applied to cages and gas together, describes the fading step by step.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "What the Surroundings Do".

A finding is "any comparison that lays down a record: an interaction after which some part of the world carries a record of the outcome"; a hoofprint in sand is such a record, and no mind is needed.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "Every Comparison Has a Cut".

I accept Tegmark's estimate of how quickly superpositions fade in the brain, as he computed it, and say where the estimate bears. The estimate bears on the proposal of Stuart Hameroff and Roger Penrose that consciousness rests on superpositions held whole for a long time inside nerve cells. I place what is undecided in a choice made by a brain in sensitive dependence near criticality, where small disturbances cascade, and not in superpositions held for a long time. A brain near criticality is poised like a pile of sand at the point where one more grain can set off an avalanche of any size, so that a tiny difference can grow into a different choice.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share"; and appendix E, entry "Tegmark 9".

On both accounts a superposition left to itself lasts, however large the mass, however far apart its alternatives and however long the time, and the only decoherence is the ordinary kind made by the surroundings.

Side by Side, Volume One, chapter "Does a Superposition End of Its Own Accord?".

I derive the Born rule, the rule that gives each outcome the square of the size of the wavefunction as its odds, from the act of comparison, in two parts. The same comparison, made on the same state in the same context, is the same comparison, so its outcomes recur with one stable frequency, and I identify the probability of an outcome with that stable frequency. The squares follow, by two theorems of mathematics, from two conditions on a rule for the odds. Physicists call the share of the odds that a rule gives an outcome its weight. For a thousand silver atoms prepared alike, of which about half land in each of two spots, the weight of each spot is one half, the share to which its count settles. First, each outcome gets one weight, whichever measurement it is found in. Second, the weights of outcomes that exclude one another add up: the count of atoms in "one spot or the other" is the count in the first spot plus the count in the second. I derive the adding up by two routes. The first route starts from what a comparison can find, which divides into regions, one region for each outcome, with nothing left over. So the weight of "one or the other" is the weight of the first region plus the weight of the second: any extra share would belong to something found in neither region, and there is no such thing. The second route is the count of repeated outcomes: no atom lands in both spots, so the two counts simply add. For one weight to each outcome I make a second identification: the same content found in two contexts, here one outcome found in two different measurements that both contain it, is the same finding, with one weight. In 1957 Andrew Gleason proved that, for a thing whose measurements can have three or more distinct outcomes, every rule that meets the two conditions takes all its weights from the state of the thing by one fixed recipe, the recipe of quantum mechanics. In 2003 Paul Busch proved the same result for a thing whose measurements can have any number of distinct outcomes, two included, as the silver atom's can, when the two conditions hold also for the outcomes of unsharp measurements. An unsharp measurement is one whose outcomes do not cut the possible states cleanly apart, as a de-

tector does that answers "up" nine times in ten for an atom that is up, and one time in ten for an atom that is down. My first route to the adding up, and its identification of one weight for one outcome, hold for unsharp measurements too, since both concern the outcomes of any comparison, sharp or unsharp. For a thing in one definite state, described by a single wavefunction and not by a blend of several wavefunctions each taken with a share, the recipe of quantum mechanics is the rule of squares, the Born rule. So I reach the Born rule as the only rule, with no rule for probability assumed at the start.

Why the Quantum Is the Way It Is, chapter 11, "Why Chance Enters", sections "From an Unfixed Outcome to a Stable Weight" and "The Residual"; and chapter 12, "Why the Square", sections "A Thousand Atoms", which gives the weight, "Gleason's Theorem", "Every Sector, and the Whole Space", which gives Busch's theorem and says that "The argument from content and context" concerns "the outcomes of any comparison, sharp or unsharp", "Why the Weights of Exclusive Findings Add", which says: "That extra share would be content belonging to neither region", and "One Weight for One Finding", which says: "I take the same content found in two contexts to be the same finding, with one weight".

The two theorems behind the rule of squares, stated on their own; the entry above gives my derivation of both conditions: In 1957 the mathematician Andrew Gleason proved a theorem about every possible rule for the odds of outcomes, for any thing whose measurements can have three or more distinct outcomes. Gleason's theorem concerns two conditions on a rule for the odds: that the rule gives each outcome one weight, whichever measurement it is found in, and that it makes the weights of outcomes that exclude one another add up. Gleason's theorem shows that every rule meeting the two conditions takes all its weights from the state of the thing by one fixed recipe, the recipe of quantum mechanics. For a thing in a single definite state, not a blend of states, that recipe is the rule of squares. A later theorem, published by Paul Busch in 2003, gives the same result for things whose measurements can have any number of distinct outcomes, two included, once weights are given also to the outcomes of unsharp measurements. Unsharp measurements are measurements whose outcomes do not cut the possible states cleanly apart. I derive both

of Gleason's conditions from the act of comparison. I derive the adding of weights from the way what a comparison can find divides into regions, one region for each outcome, with nothing left over. I derive one weight for one outcome from a second identification: the same content found in two contexts is the same finding, with one weight. Both derivations concern the outcomes of any comparison, sharp or unsharp, so they give Busch's conditions as well as Gleason's.

Why the Quantum Is the Way It Is, chapter 12, "Why the Square", sections "Gleason's Theorem", which says: "For a pure state, the recipe is the Born rule", "Every Sector, and the Whole Space", which says: "In 2003 Paul Busch proved the corresponding result for every number of directions, two included, when weights are given to effects", "One Weight for One Finding" and "Summary"; A. M. Gleason, *Journal of Mathematics and Mechanics* 6, 885 (1957); P. Busch, *Physical Review Letters* 91, 120403 (2003).

After decoherence the whole is still a sum of alternatives, each with its own record written into the air and the light, and which of the records you will see when you look is settled by the finding; decoherence does not say which.

Why the Quantum Is the Way It Is, chapter 17, "Why a Measurement Has an Outcome", section "What the Surroundings Do".

On my account the wavefunction, with all its alternatives, exists: it is structure, laid out whole and the same for everyone who reads it; a finding is real, given in one act of comparison, made once. "For every comparer, one outcome is real, the one found. The other branches are never given in any act of that comparer, and the many-worlds account takes the structure that exists for the whole of what is real".

Side by Side, Volume One, chapter "The Wave Function and the One Who Finds", section III, "The Branches".

Of Everett's account: "Its achievement is economy and reach". Everett's account has one smooth law for everything, applied without change to the whole universe, and with decoherence it gives a definite pattern of nearly classical branches. The account needs the act of finding it set aside in two places. To say which alternatives count as the branches, the account must choose which outcomes count as having occurred, and to give each

branch its odds, it must bring in someone who faces the outcomes and weighs them. “Many worlds cannot derive the Born rule because it drops the immanent”, the immanent being, in my language, the act itself, the comparing as it happens.

Why the Quantum Is the Way It Is, chapter 18, “The Interpretations, Each Missing a Role”, section “Many Worlds”.

The Incommensuration Theorem gives the second reason. I state the Incommensuration Theorem’s standing in these words: “The Incommensuration Theorem is a structural fact about the nature of knowing”.

Side by Side, Volume Two, chapter “The Structure and the Act”, section III, “The Step at Which They Part”, and its section “Terms”.

A whole world fixed completely by one law would need perfect sameness and perfect gaplessness together: one law carrying every state into the next, the same everywhere and always, and every state fixed with no gap left anywhere. “A whole world completely fixed by a law perfects symmetry and continuity together, whichever account describes it, and the theorem forbids that pairing”. *Physics Is Not Math* names Everett’s many-worlds account among those that describe a whole world in that way.

Physics Is Not Math, chapter 8, “Physics Has the Arrow”, section VII, “Three Grounds”.

On my account the theorem leaves the Schrödinger equation standing, because the equation fixes the odds of findings and leaves each single finding unfixed; the theorem excludes the further claim that the Schrödinger equation, with nothing beside it, fixes the whole world.

Why the Quantum Is the Way It Is, chapter 11, “Why Chance Enters”, section “The Residual”.

Soft randomness is unpredictability that comes from a limit of what someone knows, and that better knowledge could shrink: the toss of a coin and the top card of a freshly shuffled deck are random in this soft way.

What Cannot Be, chapter 8, “A Click No One Could Predict”, section “Two Meanings of Random”.

The single outcome of a quantum measurement is the plainest case of hard randomness: the state of the measured thing, known as fully as the theory allows, gives the odds of each outcome, and the outcome found is not contained in that state.

Why the Quantum Is the Way It Is, chapter 11, "Why Chance Enters", section "The Residual".

Seen from outside, by a description that holds no one's point of view, hard and soft randomness "cannot be told apart. Both produce the same statistics". Which of the two a finding is can be registered only by the comparer that makes the finding, from its stance. So copies reproduce the statistics of quantum outcomes exactly, and the statistics alone do not show which kind of randomness is at work; that application to copying is this book's step.

What Cannot Be, chapter 8, "A Click No One Could Predict", section "What Only a Stance Can Tell Apart".

The reply of Charles Sebens and Sean Carroll, published in 2018, by self-location among Everett's branches, makes the uncertainty "ignorance of which laid-out branch is one's own", the soft kind; the quantum outcome, on my account, is the hard kind.

Why the Quantum Is the Way It Is, chapter 18, "The Interpretations, Each Missing a Role", section "What the Interpretations Were Doing".

Three things come together or go together: the stance from which a comparison is made, the one-way order of time, and outcomes that nothing before them fixed. The Copenhagen reading keeps all three: an observer, a single outcome by chance, and a measurement that cannot be undone. The many-worlds reading drops the stance, the outcomes that nothing fixed and the one-way order of time, all three together. The many-worlds reading has no fact of which branch a comparer is in, every outcome in it is fixed, and its basic equation runs the same forward and backward.

What Cannot Be, chapter 8, "A Click No One Could Predict", section "Three That Come Together".

In the experiment of Page and Geilker of 1981 the gravity the bar felt followed the outcome of the quantum event, and not an average of the two alternatives. So a single quantum outcome, found by the counter, settled where the large masses stood and which way their gravity twisted the bar. The experiment excludes one picture of gravity: a single curved geometry of space and time for both placements together, shaped by their average, with the state of the masses never cut down to the outcome the counter found. The experiment supports an identification that I make: each alternative of the matter carries its own geometry, and on each run the counter's finding selects among alternatives that already carry their geometries. In the words of *The Unquantized Arena*: "A finding selects one outcome from among alternatives that already carry their geometries".

The Unquantized Arena, "What This Book Claims and How It Is Set Out", section I, "The Thesis", which holds the identification and the words quoted; and chapter 7, "One Geometry for Each Alternative", section II, "The Experiment of Page and Geilker", which says that the experiment "excludes one geometry sourced by the expectation value in a state that is never reduced"; D. N. Page and C. D. Geilker, *Physical Review Letters* 47, 979 (1981).

On every test of whether a superposition ends by itself, and on every test of the odds of quantum outcomes, the two accounts expect the same result, and they part on what a finding is. Each test of either kind is itself a finding, one outcome given in one act. I settle what a finding is by argument: a finding is an act, made once, with one outcome, and an account that keeps only the laid-out structure has set the act aside.

Side by Side, Volume Two, appendix E, "The Registered Predictions Beside Max Tegmark", section III, "Where the Two Accounts Agree", under "Tegmark 6. No superposition ends beyond ordinary decoherence", and section V, "Where No Measurement Decides".

Where the act returns in Everett's account, with the sources of the arguments for the odds: The one wavefunction of the whole world can be written as a sum of alternatives in many ways, and the wavefunction alone does not say which way gives the branches; choosing a way is choosing which outcomes count as having occurred. And when every out-

come occurs, a probability needs someone who faces outcomes and weighs them, so the arguments for the odds bring in a chooser among outcomes as a premise. The arguments for the odds meant here are the derivations from a chooser's weighing given by David Deutsch (1999) and David Wallace (2012), which the same section names; in the words of *Why the Quantum Is the Way It Is*, "The universal state can be written in many ways as a sum of alternatives, and the state alone does not say which way of writing it gives the branches".

Why the Quantum Is the Way It Is, chapter 18, "The Interpretations, Each Missing a Role", section "Many Worlds".

The experiment of Page and Geilker, read narrowly: Page and Geilker read their result as support for quantizing gravity itself, that is, for treating the geometry of space and time itself as having quantum alternatives; *The Unquantized Arena*, my book on quantum gravity and the measurement problem, in its chapter 7, section II, reads it more narrowly, as excluding one geometry shaped by the average of a state that is never cut down to the outcome found, that is, a state whose alternatives of the matter are never reduced to the one outcome found. On my account each alternative of the matter carries its own geometry, and the counter's finding selected which alternative each run was.

The Unquantized Arena, chapter 7, "One Geometry for Each Alternative", section II, "The Experiment of Page and Geilker", and section III, "One Geometry for Each Alternative".

The exact statements and values for this chapter are these.

Decoherence in the formalism: in the reduced density operator of the system (its description with the surroundings traced out), the squared weights of the alternatives stand on the diagonal. Each term off the diagonal is multiplied by the overlap of two states of the surroundings, which falls toward zero as the surroundings record the alternatives. The result is an improper mixture, and Proposition 11.2 states what decoherence does: "Decoherence is the dense reach of the formalism toward the finding. It fixes the basis of the records and makes interference among the alternatives unobservable, and it does not select an outcome".

The Unquantized Arena, chapter 11, "The Measurement Problem as a Cross-Modal Residual", section III, "Decoherence and Where It Stops", equation 11.2 and Proposition 11.2.

One geometry for each alternative: each alternative of the matter is the source of its own geometry in Einstein's equations, and one geometry sourced by the average over alternatives fails, against Page and Geilker in its unreduced form and by signaling in its reduced form.

The Unquantized Arena, chapter 7, "One Geometry for Each Alternative", section III, Proposition 7.1, with Proposition 7.3.

Tegmark's estimate of decoherence times in the brain: from ten to the power minus 13 to ten to the power minus 20 seconds, against the thousandths and tenths of a second on which nerve cells act (Tegmark, 2000, "Importance of quantum decoherence in brain processes", *Physical Review E* 61, 4194).

Side by Side, Volume Two, chapter "The Structure and the Act", section I, "Tegmark's Position", and appendix E, entry "Tegmark 9".

The sealed record holds the predictions that bear on this chapter: gravitation alone ends no superposition; gravitation entangles two masses; the statistics of quantum theory hold; no signal faster than light and none backward in time; and no world perfectly deterministic. The appendix "What the Coming Measurements Will Find" gives them, with the record beside Tegmark, which says where his account expects the same.

Chapter 10, "Is the World Made of Mathematics?"

I hold the External Reality Hypothesis, read plainly: there is a world, and it does not depend on us.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

My ground is that a stone striking a stone compares as surely as a laboratory does. Whatever makes a comparison is a comparer: any position inside the world from which a comparison is made, and a comparer need not be a mind; "The word observer names that position inside the world, and it never names a mind".

Measurement; The Steps from Comparison to Physics, chapter 2, "Every Measurement Is a Comparison", section VII, "Questions".

The world was comparing itself, collision by collision, long before anything human looked at it: "Tegmark's External Reality Hypothesis is a claim the collection holds on a ground of its own". From the same ground this book draws Tegmark's result that physics can find out about the world without first explaining the human mind, because the observer of physics was never a human mind.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

I agree with Tegmark on one more point, which the close of the chapter relies on: a truth of mathematics holds before anyone proves it. Tegmark's aliens, studying shapes with flat faces all alike, could give the five regular solids of chapter 7 their own names, but "they can't invent a sixth one" (chapter 10, "The Mathematical Universe Hypothesis"). *Physics Is Not Math* gives the same agreement for the prime numbers, the whole numbers above one that no whole number divides except one and the number itself. In the words of *Physics Is Not Math*, the truth that the primes never end "does not wait on the day it was found, as the number 31 did not become prime when someone first divided it".

Physics Is Not Math, chapter 6, "Mathematics, the Science of the Timeless", section VI, "What Mathematics Leaves Out".

My account has three words for three aspects of one world, with exact and different meanings. To exist is to stand as structure: laid out whole, timeless, the same for everyone who looks, like the pattern of a law or the list on a score sheet. To be real is to be given in an actual act of comparison: here, now, once, like a move. To be objective is to be found the same by many comparisons made from many places; objectivity is built from many

views, and it comes in degrees. The three are told apart by a count: existence involves no act of comparison, the real involves exactly one, and objectivity involves many.

Comparison; The Necessary Basis, chapter 12, "Domains and What Is Real", section V, "Being, Existence, Reality, and Objectivity", under "Objectivity"; and *Physics Is Not Math*, chapter 3, "Truth Has a Frame", section V, "Objectivity Is Built from Many Views".

I set Tegmark's external reality, the physical world as it would be with no humans in it, beside my one world, which I hold there is; the one world has all three aspects together. "The structural, the processual, and the convergent are three aspects of one world, seen from three perspectives that cannot be collapsed into one". The structural is what exists, the processual is what is real, and the convergent is what is objective.

Comparison; The Necessary Basis, chapter 12, "Domains and What Is Real", section VI, "Realism, Idealism, and the Errors of Extrema", under "The Three Tests".

Tegmark's consensus reality stands closest to what I call objectivity. *An Immanent Metaphysics* uses Tegmark's very phrase: many levels of interaction are needed "to establish a consensus reality/world". The same aphorism holds that "correspondences of subjective experience" across the levels of interaction "are as much inherently a part of the foundation" of objectivity as any material aspect. In plain words, the ways in which experience at one level of interaction corresponds to experience at another belong, by their nature, to the ground of objectivity as fully as any material aspect of the world does.

An Immanent Metaphysics, "Technical Aphorisms", section "Subjectivity and Consensus", aphorism 3.45-5.

What I call real is the act of perceiving itself, the comparison eyes and brain make, here, now and once, so my word real is the one word of the three that matches none of Tegmark's three realities.

Comparison; The Necessary Basis, chapter 12, "Domains and What Is Real", section V, "Being, Existence, Reality, and Objectivity", under "Reality".

Gathering many views makes a description ever more objective, and every gathered description keeps a stance: "gathering more views never yields a view with no stance in it, because every view gathered is a view from somewhere". The stance of a comparison is the side from which the comparison is made, and every comparison has one, human or not.

Physics Is Not Math, chapter 3, "Truth Has a Frame", section V, "Objectivity Is Built from Many Views".

Every comparison gives two verdicts, each same or different: one on the content, what the verdict is about, and one on the context, the respect in which the two things compared are two. In a tide gauge's readings at noon and at midnight, the height of the water is the content and the two moments are the context. The two verdicts combine in four ways, the four characters. Continuity is the same content where the context is the same; discontinuity, a different content where the context is the same; symmetry, the same content where the context differs; asymmetry, a different content where the context differs.

Physics Is Not Math, chapter 1, "What It Is to Compare", section III, "The Tide Gauge", and section VI, "Six Parts in Three Pairs"; and chapter 2, "Two Things That Cannot Both Be Perfect", section II, "Four Characters".

The four characters pair in ten ways, and three tests decide all ten. A character paired with itself says nothing new, which removes four pairings. A character paired with its direct opposite contradicts itself. Continuity says "same" and discontinuity says "different" of the content within one unchanged context, and symmetry says "same" and asymmetry says "different" of the content across one change of context, so each pair says both of one and the same case. The second test therefore removes two pairings: continuity with discontinuity, and symmetry with asymmetry. The third test rests on a fact found in every comparison: sameness and difference come together, since a comparison that found only sameness would be no comparison at all, and one that found only difference would connect nothing. So a pairing must find one "same" and one "different" among its two verdicts on the content, and likewise on the context: continuity with symmetry gives "same" and "same" on the content, and dis-

continuity with asymmetry gives "different" and "different", so both fail. Two pairings survive: continuity with asymmetry, like a walk along a shore, and symmetry with discontinuity, like a ladder.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", sections II to V, the tests in section IV, "Three Tests"; and chapter 1, "What It Is to Compare", section VII, "Each Pair Inseparable, and Why There Are Six".

I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing". The theorem is about comparison, the act in which anything at all is known. In plain words, symmetry means sameness, a thing unchanged when you move or swap it, and continuity means having no gaps; the theorem says that no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. Every act of knowing is an act of comparison or is built from acts of comparison, so the theorem, together with the intrinsics of comparison from which it is built, is inherent in every single act of knowing. The intrinsics of comparison, as chapter 1 names them, are the parts that every comparison has, among them its two verdicts, same and different, and its content and its context. Every other result about knowing is found by knowing, in acts that already have the theorem in them, and so the theorem is the deepest and most powerful epistemic basis possible.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem"; the intrinsics from chapter 1, "What It Is to Compare", section VI, "Six Parts in Three Pairs".

The proof is ten cases and three tests. The proof uses the meanings of same and different, of content and context, and the fact that sameness and difference come together, and nothing else. *Physics Is Not Math* says that the theorem is "settled as a count is settled": anyone can make the count, and the count comes out the same each time.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section VII, "Settled as a Count Is Settled", and section XI, "Summary".

The theorem allows symmetry and continuity together where both are approximate, as in a ripple spreading on a still pond, and where they sit at two levels of one theory. The theorem concerns what a comparison finds, so the mathematics of a theory may hold symmetry and continuity together, both perfect, while no finding of the theory ever shows both.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section IX, "What the Theorem Allows"; and *Incommensuration; The Theorem and the Three Axioms*, chapter 3, "The Three Axioms from the Theorem", section IV, "The Two Levels of a Theory".

The mathematics of a theory exists, and its findings are real: "no amount of calculation makes a finding, and no finding is a calculation".

Incommensuration; The Theorem and the Three Axioms, chapter 3, "The Three Axioms from the Theorem", section IV, "The Two Levels of a Theory".

Mathematics, taken as a set of objects that simply are, contains no one who compares and no time: "In a domain from which temporal comparison is removed, symmetry and continuity can be written down together". Temporal comparison is comparison of one moment with another, and every finding of physics is a comparison made at some moment, so every finding falls under the theorem.

Physics Is Not Math, chapter 3, "Truth Has a Frame", section VII, "Mathematics Works as if Only the Objective Existed", quoting *Incommensuration*.

A mathematical structure, taken as a whole, is laid out at once and is the same wherever and whenever it is read, so as a whole it holds the perfect sameness and the gapless completeness that no single act of comparison can hold together. "So the world that comparisons find, the only world physics has, is not a mathematical structure, and a mathematical description of it, however exact, is never complete".

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Every physical theory has both, its mathematics, which exists, and its findings, which are real, and Tegmark's hypothesis takes the mathematics for the whole of the world.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

"The score sheet of a chess game can be replayed by anyone, at any time, forever. The game itself happened once, on one evening, between two people".

What Cannot Be, chapter 4, "Process, Pattern and Possibility", section "A Law Exists; a Measurement Is Real".

Tegmark's argument says three things: a complete description of a world independent of humans must contain nothing human; a complete description with nothing human in it is pure structure; so the world is pure structure. I accept the first premise and parts at the second, because a stance is not a human: "Removing every human notion from a description removes nothing of the stance, because the stance was never human". A complete description of a world independent of humans still describes a world in which comparisons are made from stances, and the structure it lays out is what each comparison finds, with the stance left unnamed.

Side by Side, Volume Two, chapter "The Structure and the Act", section IV, "Why a Description Free of Humans Still Has a Stance".

I read Tegmark's rule of equivalence as an instance of a principle I hold, the principle of identity: "that which is indistinguishable must be the same". The principle makes one thing of two only where nothing can tell the two apart.

Comparison; The Necessary Basis, chapter 12, "Domains and What Is Real", section V, "Being, Existence, Reality, and Objectivity", under "Reality".

A description and the act it describes are two different kinds of thing: "Structures model interactions; they do not become them".

The Basis of Physics, essay 3, "Measurement as Interaction", section IV, "The Relation That Exists and the Interaction That Is Real".

I agree with the principle Tegmark relies on, that what has no observable effect is no part of science, in my own words: "what no comparison could ever reach can be the basis of no assertion, in physics or out of it".

The Basis of Physics, essay 3, "Measurement as Interaction", section IV, "The Relation That Exists and the Interaction That Is Real".

What the mathematical description leaves out is the stance and the act: "there is no comparison made from nowhere: a comparison is an interaction, and an interaction has locations". Every observation ever made has the act and the stance in it, so neither one is a hidden extra.

Physics Is Not Math, chapter 3, "Truth Has a Frame", section II, "Every Comparison Is Made from Somewhere", quoting *Incommensuration*.

A kitchen shows the order of depth: a cook tastes two spoons of honey, a cake bakes, a recipe lies on the counter, and a photograph of last week's cake hangs on the wall. The photograph cannot be had without a cake, nor without steps taken by fixed rules, a recipe of its own kind; a recipe bakes nothing until hands follow it in an oven; the baking is comparison, and the tasting needs nothing else. Four disciplines stand in one order of depth. Metaphysics is the account of the act of comparison itself; physics studies acts of comparison as they happen in the world; computation carries out a sequence of steps by a fixed rule; and mathematics studies structure held in view as a whole. One discipline is deeper than another when the second cannot be carried out, stated or checked without the first, and the first can be carried out without the second.

Physics Is Not Math, Preface; and chapter 5, "What Depth Means", section I, "A Kitchen", section II, "Depth Is the Order of Presupposition", and section V, "The Order of the Four".

Mathematics rests on computation, because every proof is checked by a run of steps.

Physics Is Not Math, chapter 7, "Computation Has Sequence", section VII, "Every Proof Waits on a Run".

Computation rests on physics, because every run is a physical process.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VI, "Physics Is Deeper Than Computation".

Depth is a different thing from certainty or precision: the most exact record can stand last in the order.

Physics Is Not Math, chapter 5, "What Depth Means", section III, "Depth Is Not Certainty".

Mathematics keeps everything it has.

Comparison; The Necessary Basis, chapter 10, "Reasoning", section IX, "Truth Above True and False", under "Truth and the Truths of Mathematics".

Tegmark's hypothesis reverses the whole order, placing mathematics at the bottom.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Mathematics works so well in physics because mathematics and physics are two aspects of one act of comparison, built from the same parts of comparison and the same four characters. Two rubbings taken from one carved stone match line for line: "Mathematics and physics are two such rubbings, and comparison is the stone".

Why the Quantum Is the Way It Is, chapter 19, "Where the Derivation Stops, and Why", section "Wigner's Question"; and *Measurement; The Steps from Comparison to Physics*, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete".

A description reaches what it describes as closely as anyone likes, and never contains it, because a description that contained its findings exactly would hold symmetry and continuity together: "The mathematical description of physics approaches the physical process as closely as one likes and never coincides with it".

Measurement; The Steps from Comparison to Physics, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete".

Two agreements with Tegmark, each given with its source in the list after them: One world is seen from several standpoints, and none of them replaces another. Choosing a unit is a convention, and making a comparison against the unit is not.

Physics Is Not Math, chapter 3, "Truth Has a Frame", section VI, "The View from Nowhere", which says: "neither replaces the other"; *Measurement; The Steps from Comparison to Physics*, chapter 2, "Every Measurement Is a Comparison", section IV, "A Unit Is a Standard of Comparison", which says: "choosing a unit is a convention, and making a comparison against it is not".

Further agreements with Tegmark, with their sources: I hold, with Tegmark, that one world is seen from several standpoints that do not replace one another. *Physics Is Not Math* sets the third-person view, the view that holds the same from every place and in which the laws of physics are stated, beside the first-person view, the view from here, and finds that "neither replaces the other". I hold, as Tegmark holds of his consensus reality, that a description many observers share can be objective, and *Physics Is Not Math* adds how: "Objectivity is built; it is not found". I hold that the step from mathematics to measurement is never automatic: in the words of *Measurement*, "The step from a structure of comparison to a physical quantity is never automatic". Tegmark's history of physics, in which the hardest part was relating the mathematics to the measurements, is the history of such steps. I hold that a truth of mathematics holds before anyone proves it. Tegmark's aliens could give the five regular solids their own names, but "they can't invent a sixth one" (chapter 10, "The Mathematical Universe Hypothesis"). *Physics Is Not Math* says of the theorem that the primes never end that "its truth does not wait on the day it was found". I hold Tegmark's point about units on a ground of my own, stated in *Measurement*: "choosing a unit is a convention, and making a comparison against it is not". *Measurement* gives Tegmark's own example: the proton is about 1836 times the mass of the electron whether both are weighed in kilograms or in pounds. Tegmark's rule of equivalence is an instance of a principle I hold, the principle of identity, as *The Concept Sequence*, my book that sets its concepts in order, states it: "That which is indistinguishable must be the same". And I hold that the world is relational, in the words of *Comparison; The Necessary Basis*, my book on what it is to compare: "There are no purely static things; there are only interactions".

Physics Is Not Math, chapter 3, "Truth Has a Frame", sections V and VI, and chapter 6, "Mathematics, the Science of the Timeless", section VI, "What Mathematics Leaves Out"; *Measurement; The Steps from Comparison to Physics*, chapter 2, "Every Measurement Is a Comparison", sections IV, VI and VII, the example of the proton

and the electron in section VII, "Questions"; *The Concept Sequence*, chapter 15, "The Three Axioms", under 15.11, "Principle of Identity"; *Comparison; The Necessary Basis*, chapter 3, "One Act, Three Names", section VI, "Three Principles of Ontology".

Symmetry and the relational reality, Tegmark's two further results and my answer: Tegmark draws two further results. A structure whose points have no properties of their own can still have symmetries, and he finds that "the less properties the points have, the more symmetry the space has" (chapter 10, "What Is a Mathematical Structure?"). And the old regress, in which the properties of each part are explained by smaller parts without end, stops at the bottom, where the parts have no properties at all, so that "we live in a relational reality" (chapter 10, "What Is a Mathematical Structure?"). I give my own reason why physics is built on symmetries. Tegmark reads the symmetries of physics as properties a structure has because its points have no properties. I read a law of physics as a symmetry found by comparison: in the words of *The Last Turtle*, my book of essays from comparison to causation, the arena and the limits of computation, "what stays the same when the context of a measurement changes". In 1918 the mathematician Emmy Noether proved that each smooth symmetry of the equations of motion, a symmetry of the mathematics, which the theorem allows, goes with a quantity that is conserved, so that laws the same at every moment conserve energy and laws the same at every place conserve momentum. Physics is built on symmetries because a law is, by its meaning, a sameness found across a change of context. And the old regress of parts ends, on my account, in the act itself: "relationship is prior, and things are aspects of relationships", as *Incommensuration*, my book in which the theorem is proved, reads the first of the three axioms of the Immanent Metaphysics, the founding statements of my account of comparison.

The Last Turtle, chapter 9, "Three Orders of Knowledge", section VIII, "What Every Physical Theory Presupposes"; *Incommensuration; The Theorem and the Three Axioms*, chapter 9, "Each Axiom Derived Again", section I, "Axiom I: The Asymmetry of the Relational", under "What Axiom I Implies".

Why the order of depth surprises, told with an orrery: Why does nearly everyone expect the order the other way round? *Physics Is Not Math* answers with an orrery, a brass model of the solar system whose gears carry each planet round at its proper pace. Run the gears backward, and the planets go backward just as smoothly. The solar system itself runs one way only: the tides the Moon raises drag on the spinning Earth, and laser light bounced off the reflectors left by the Apollo astronauts shows the Moon moving away by about 3.8 centimeters a year. A visitor who concluded from the gears that the solar system could run backward would have taken a feature of the model for a feature of the thing modeled. A mathematical description of the world has the features of its kind: it holds the same everywhere, it lays out the whole at once, and it reaches us finished, after the acts that found it, so it looks as if it had been there first. *Physics Is Not Math* names the one source of the expectation that mathematics comes first: "the timeless description is taken for the foundation".

Physics Is Not Math, chapter 11, "Why the Order Surprises", section I, "A Clockwork Solar System", section III, "One Source", and section IV, "Why the Laid-Out Seems to Come First".

The exact statements and values for this chapter are these.

The Incommensuration Theorem, in my words: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity". The theorem's first form in *An Immanent Metaphysics*, the definitions of the four characters, and the count that proves it are in the section for chapter 1 above.

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem".

The cross-modal principle in my words: "Framework and content are always in conjugate modes. A symmetric framework probes continuous content; a continuous framework probes symmetric content". The examples *Measurement* gives of the principle are three. The first example is π in the distribution of errors. The second example is the complex numbers of quantum theory, whose phase returns after a full rotation. The third ex-

ample is the electron's magnetic moment, measured to 0.13 parts in a million million, with which quantum electrodynamics agrees to about one part in ten to the power 12.

Measurement; The Steps from Comparison to Physics, chapter 8, "The Cross-Modal Principle", section II, "The General Form", Proposition 8.1; and chapter 14, "The Consequences of the Theorem", section XII.

The two levels in a sphere: in quantum theory the continuous symmetry of a sphere sits in the mathematics, and what is found is angular momentum in equal steps; in general relativity the order is reversed.

Incommensuration; The Theorem and the Three Axioms, chapter 3, "The Three Axioms from the Theorem", section IV, "The Two Levels of a Theory".

The ratio of the proton's mass to the electron's, measured, is 1836.152673426 with an uncertainty of 32 in the last two digits (CODATA 2022 recommended values).

The sealed record holds six predictions that bear on this chapter: the matter fraction; the ratio of dark to ordinary matter; and no world perfectly deterministic. The other three are a discrete rule that shows itself to a fine enough measurement, no kind of particle beyond the closed list, and exactly three generations. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 11, "Is Time an Illusion?"

The past, taken as a region of the geometry of spacetime, is still there: the saying that last Thursday "is still there" in spacetime is true of last Thursday as a region of the geometry, the four-dimensional shape. That region exists, in this book's sense of the word: it stands as structure, laid out whole and the same for everyone who consults it.

Why Time Travel Is Not a Concept, chapter 1, "The Question", section III, "What Exists and What Is Real".

A life was real when it was lived, in every act made in it, and what remains of it now is its records.

Why Time Travel Is Not a Concept, chapter 1, "The Question", section III, "What Exists and What Is Real".

No description of a system from outside, in the third person, contains a privileged now or a privileged here, and from that outside vantage every moment is interchangeable with every other.

Incommensuration; The Theorem and the Three Axioms, chapter 6, "The Incommensuration Theorem: Statement and Proof", section XI, "The Two Surviving Modes".

A description from outside, of the kind science writes, states laws that hold the same everywhere about things it can tell apart and count, which pairs sameness with separate, countable things: symmetry together with discontinuity, one of the two pairings the theorem allows. In such a description every moment has the same standing as every other, so the description singles out no moment as now.

Incommensuration; The Theorem and the Three Axioms, chapter 6, "The Incommensuration Theorem: Statement and Proof", section XI, "The Two Surviving Modes".

I reach that fact from the Incommensuration Theorem. I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing".

Side by Side, Volume Two, chapter "Janus and the Turn", section III, "What Time Is in the Collection".

I hold that the passing of time is no illusion, and my reason begins from three different things that physics calls by the one word "time". Geometric time is time as a dimension, the fourth direction of Einstein's spacetime, pictured by the hours on a clock's face, all laid out at once. Event time is the one-way order in which findings are made, pictured by the clicks of a Geiger counter, each laid down as a record. Evolution time is the smooth time of quantum physics, along which the chances of every possible finding change. The three are distinct, each needs the other two, and each does work that neither of the others can do.

Measurement; The Steps from Comparison to Physics, chapter 12, "Time", section III, "The Three Times as One Triple".

The passing of time is no illusion: the geometry of spacetime exists, laid out whole like a finished score sheet, and event time, the one-way order of findings, is real. The present is the move being made, the act in which one of the things that could happen becomes what happens. In the words of *The Basis of Physics*: "Geometric time is the dimension, and it exists; event time is the process, and it is real". The block universe is all of spacetime laid out at once, with no moment singled out as now: "The block universe takes geometric time for the whole, and drops event time".

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VI, "Time as Process, and Not Only as Dimension".

The theorem excludes a world in which every state is fixed completely by the state before, since such a world would need one law perfectly the same everywhere together with every state specified with no gap at all. So no world is completely fixed in advance, and "the future does not exist in the sense the past does": the past is laid down in records, and the future is a range of what could happen, from which each act draws one outcome.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VI, "Time as Process, and Not Only as Dimension".

Running a process backwards so that it retraces its path exactly would need a law perfectly the same in both directions together with a state specified with no gap, which the theorem excludes. So no process at any scale can be run back exactly, and the two directions of time differ in their structure.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VI, "Time as Process, and Not Only as Dimension", and section VIII, "Summary".

Every record anyone has ever found is a record of the past, and no instrument holds a finding before the interaction that makes it.

Measurement; The Steps from Comparison to Physics, chapter 12, "Time", section I, "One Direction, from the Single Stance".

The finding of 1964, that certain decays of kaons break the symmetry of mirror and charge taken together, implies by a theorem of particle physics that the laws are not exactly the same when time is reversed. So the two directions of time differ in the laws themselves, as well as in every

record. I read the lean found in the decays of kaons as an instance of what the theorem requires: the laws are never perfectly the same in both directions of time. In the words of *The Basis of Physics*, the lean is "a place where the impossibility of perfecting temporal symmetry has become visible to instruments".

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VII, "Consistency with Physical Law".

An outcome that nothing before it fixed can change the shape of spacetime itself, through the Geiger counter wired to a rocket of chapter 1, and "A single fixed geometry of all of space and all of time, from the first moment to the last, laid out once and for all, cannot be the whole story of the universe".

The View from Inside, chapter 14, "Here, Now, and This", section "A Geiger Counter and the Orbits of Planets".

The present is the act of comparison itself: comparison is made only in the present, and the present is the site of the interaction itself, as distinct from one point on a line.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VI, "Time as Process, and Not Only as Dimension".

The present is where many things that could happen become the one thing that does, "no point sliding along a timeline laid down in advance". Everything that exists, laid out as structure, is the trace that the present leaves behind. Physics describes the trace with great exactness, and it cannot describe the converging itself, the present in which the many things that could happen become one. In the words of *Incommensuration*, "the converging is not an object that physics could hold still long enough to examine".

Incommensuration; The Theorem and the Three Axioms, chapter 9, "Each Axiom Derived Again", section II, "Axiom II: The Cycle of Precedence", under "Time and the Reversal of Perspective", which holds both quotations.

The now that a description from outside lacks is the act of finding, made from a stance, and everything that any description holds was found in acts of finding.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VI, "Time as Process, and Not Only as Dimension".

A town map on a post shows every street and on its own shows no "here"; a red dot added to it lets the reader know which way to go: "The dot is the first person, put back into the third". The view from outside and the view from here are both needed. Neither replaces the other, because one view with both the perfect sameness of the view from outside and the unbroken continuity of the view from here would hold perfect sameness and perfect gaplessness together.

Physics Is Not Math, chapter 3, "Truth Has a Frame", section VI, "The View from Nowhere".

In geometric time every event has its place, and a comparer meets moments only in the order of its own findings, one after another, in an order that runs one way: "a location in geometric time is not something that a comparer reaches as a place is reached".

Why Time Travel Is Not a Concept, chapter 1, "The Question", section IV, "Time Is Not a Place".

All the evidence of the past is found in the present: the light of a distant galaxy is light arriving now, a fossil is a stone in a museum now, and a memory is a trace in a brain now.

The View from Inside, chapter 14, "Here, Now, and This", section "All the Evidence Arrives Now".

A record is the result of the act it records, and memories are records, laid down by acts, one after another, in the one order that every record shows.

Side by Side, Volume Two, chapter "Janus and the Turn", section IV, "Where the Roads Part: Is Time Real?".

The passage of time belongs to the world itself, grounded in comparison: "the direction of time is the direction in which comparison proceeds". *Measurement* adds that only a comparer that makes the findings in sequence, from one stance, has an order of findings. So on my account the passing of time is real.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VI, "Time as Process, and Not Only as Dimension"; and *Measurement; The Steps from Comparison to Physics*, chapter 12, "Time", section IV, "Causation Established, and Determinism Never".

Pierre-Simon Laplace imagined, in 1814, an intellect that knew every force and every position and so held the future as clearly as the past.

Causality Without Determinism, chapter 1, "Two Concepts, Three Differences", section II, "Determinism Defined".

"Causation is established, and determinism never". *Measurement* adds that the unbroken connection of a cause with its effect is found from inside the order of findings, "by a comparer for whom one finding follows another". A cause is a connection with no break, which is continuity, along an order of findings that runs one way, which is asymmetry; continuity with asymmetry is one of the two pairings the theorem allows, and so "every world is to some degree causal".

Measurement; The Steps from Comparison to Physics, chapter 12, "Time", section IV, "Causation Established, and Determinism Never".

Every prediction a scientist makes is a comparison made now, checked by a later one, and the method of science on my account is what scientists have always done: a cycle of separate acts, each compared with the last.

Causality Without Determinism, chapter 7, "Four Theorems by Domain Binding", section VII, "The Existence-Doing Theorem: The Scientific Method".

Only the present act is given; a rule stated from outside, which surveys all eras at once, can give a spread of the eras at which comparers stand, and such a rule "cannot single out one of them as real".

The Gravitational Passage, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section VI, "The Comparison That Fixes Λ", and section VII, "Summary of Predictions", under "Questions".

The question why your present is this one, and not a moment in 1850, assumes that all the presents are laid out together, like the frames of a very long film laid on a table, with you in one frame. On my account the

history is laid out only afterwards, pieced together from the present act, so the act comes first, and each frame is laid down only after its move is made.

The View from Inside, chapter 15, "One Constant, One History", section "Why Now?", and chapter 14, "Here, Now, and This".

The energy of empty space is one number through the one space and time of our epoch.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The Second Way: One Number for Our Whole Epoch"; and *The View from Inside*, chapter 15, "One Constant, One History", section "One Map, One Scale".

What Comes After the Dark holds that the sky empties, over a hundred billion to a trillion years as astronomers reckon it, for anyone, anywhere, who can see and compare. Whatever is not bound to our own group of galaxies is carried beyond the horizon, the edge past which no light can reach us, and the matter within reach ends inside black holes or departs. In that book's words, "The emptying ends at the turn". The turn is the passage from the end of one epoch, one long age of the universe, to the start of the next. So the long emptying of the sky is one stage of the world, and the endless count of chance brains has no endless future to be counted in. I make no random draw, and I reach the end of the emptying from my own account of the far future.

What Comes After the Dark, "The Question of the Far Future", section "The Answer, in Brief", for the hundred billion to a trillion years; chapter 1, "What Happens to the Sky We Can See?", section "A Passing Stage", for what is carried beyond the horizon and what ends inside black holes; chapter 8, "Is the Long Emptying the End?", section "The Dark Sky Is Not Where the Story Stops".

Tegmark among other views of time, in one sentence: He names Brian Greene as uneasy about letting go of change, Lee Smolin as holding in *Time Reborn* that time is real, in his sense, and Julian Barbour as holding in *The End of Time* that physics can do without time altogether (chapter 11, "Past, Present and Future").

The age of the Earth, as Tegmark reasons about it: With Nick Bostrom he applies the rule of the random observer moment to the age of the Earth: our planet formed late, long after many planets like it, and that lateness tells against any process that destroys planets quickly. The argument about the old Earth treats you as drawn at random from all observers on all planets (chapter 11, "Why Is Earth So Old?").

One direction of time, from Tegmark's paper of 1997 and from my ground: Spacetime has exactly one direction of time. Tegmark showed in 1997, in a paper on the dimensions of spacetime, that with two or more directions of time the equations of physics lose their power to predict. My book *The Gravitational Passage* cites his result as support from the field equations of physics, and reaches one direction of time on its own ground: comparisons proceed in one order, so what comes before must be told apart from what comes after. Only with exactly one direction of time does every event have a before and an after that can be told apart.

The Gravitational Passage, chapter 10, "Why Four Dimensions, Why Lorentzian", section III, "The Lorentzian Signature".

The direct findings of 1998 and 2012, and the reading of the lean: The CPLEAR experiment at CERN saw the difference between the two directions of time directly in 1998, and the BaBar experiment in California saw it again in 2012, in heavier particles. I read these findings as agreeing with what the theorem requires: some lean of time in the laws, whose size is taken from measurement.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section VII, "Consistency with Physical Law".

The Geiger counter and the orbits of planets, told in full: My book *The View from Inside* gives a picture of how an outcome that nothing fixed can change the shape of spacetime itself. Wire a Geiger counter so that its next click, if it comes within a minute, launches a small rocket. The rocket nudges a small asteroid, which over the years nudges a larger one, and in time the orbits of planets are moved. Every link after the first is ordinary gravity and motion; the first link is a click that nothing before it fixed. An outcome that nothing before it fixed is what chapter 8 named *hard random-*

ness, and it is a fact of each single outcome. So the shape of spacetime, curved by where all those masses go, differs according to an outcome that was settled only when it happened. One fixed geometry of all times, laid out once and for all, is a true description of what has been found, and it cannot be the whole story of the world.

The View from Inside, chapter 14, "Here, Now, and This", section "A Geiger Counter and the Orbits of Planets".

The far future and the count of chance brains: *What Comes After the Dark*, my book on the far future, finds that no single thinker makes an endless run of findings, and that the long emptying of the sky ends at the turn, the passage from the end of one epoch, one long age of the universe, to the start of the next. A final sky, the same everywhere and with no break anywhere, would hold perfect sameness and perfect gaplessness together, which no comparison finds; and the heat death, a final state that holds for ever, has no context in which to be counted, since every count of disorder is made against the outside world of one epoch.

What Comes After the Dark, chapter 7, "Can a Mind Keep Thinking for Ever?", section "The Watcher Who Wants to Keep Thinking"; and chapter 8, "Is the Long Emptying the End?", sections "The Dark Sky Is Not Where the Story Stops" and "The Third Ground: Every Count Has a Context".

Sleeping Beauty, on my account: The Sleeping Beauty answer rests on Tegmark's replacement of the coin by a quantum measurement whose two outcomes both occur. Chapter 9 gave my reason why a quantum measurement, like a tossed coin, has one outcome: a whole world fixed completely by a law, whichever account describes it, holds perfect sameness and perfect gaplessness together, and the theorem forbids that pairing. So the randomness of one outcome is hard randomness, a fact of each single outcome, and it is not the feeling of being copied. My book *Side by Side*, Volume One, says what such a question presupposes: an observer who asks how much weight to give each outcome is a comparer with a stance, before an act, facing a range of what it could find. Beauty, waking and asking her question, is such a comparer, and I begin every account of finding from a comparer of that kind.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VII, "Three Grounds"; *Side by Side*, Volume One, chapter "The Wave Function and the One Who Finds", section III, "The Branches".

Why now, on my account: I answer the puzzle of "why now?" in my own way, with no draw at all. The nearness today of the two great densities of the universe, the matter and the energy of empty space, is, on my account, an identity that holds at the era of the act that measures them.

The View from Inside, chapter 15, "One Constant, One History", section "Why Now?".

The exact statements and values for this chapter are these.

The three times: geometric time is the proper time and causal order carried by the geometry, continuous and asymmetric; event time is the ordered sequence of outcomes, discontinuous and symmetric; evolution time is the parameter of the smooth evolution of the quantum state.

Measurement; The Steps from Comparison to Physics, chapter 12, "Time", section II, "Three Objects Physics Calls Time", and section III, "The Three Times as One Triple".

Lawful reversal and faithful reversal: symmetric laws make the reverse of a process lawful; faithful reversal needs, besides, a specification of the state that is complete without gaps, and that pairing of symmetry with continuity is the one the theorem forbids.

The Basis of Physics, essay 11, "The Structural Arrow of Time", section III, "Lawful Reversal and Faithful Reversal".

One direction of time: with exactly one time direction the cone of timelike directions falls into two halves, future and past. So the signature of spacetime has exactly one negative sign, written $(-, +, +, +)$. With two or more time directions the cone is one connected piece, before cannot be told from after, and the wave equation has an initial value problem that is ill posed. The sources are M. Tegmark, "On the dimensionality of spacetime", *Classical and Quantum Gravity* 14, L69, 1997, and W. Craig and S. Weinstein, "On determinism and well-posedness in multiple time dimensions", *Proceedings of the Royal Society A* 465, 3023, 2009.

The Gravitational Passage, chapter 10, "Why Four Dimensions, Why Lorentzian", section III, "The Lorentzian Signature".

Why now: the identity $I^2 \times \Omega = 8 - 3/\pi$ holds at the particle horizon of the present measurement. In words, the square of the horizon's radius in Hubble radii, times the vacuum's share of the energy, equals eight less three over pi; the section for chapter 4 derives the identity. The identity gives a matter fraction of 0.311 against a measured 0.315 ± 0.007 and an age of 0.792 de Sitter times against a measured 0.787 ± 0.009. Applied on paper to the particle horizon of an observer at redshift one, the same formula gives 7.0 times the measured constant, and the constant that observer finds is the one constant of the history.

The Gravitational Passage, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section VI, "The Comparison That Fixes Λ"; and *The View from Inside*, chapter 15, "One Constant, One History", section "Why Now?".

The sealed record holds five predictions that bear on this chapter: some asymmetry of time in the laws, with no process perfectly reversible; no signal faster than light and none backward in time; and an equation of state of exactly minus one. The other two are the matter fraction and the statistics of quantum theory. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 12, "What Am I?"

No spectator sits inside the head; the comparing is never among the things compared. Through the electromagnetic field alone, two laser beams can cross in space and pass through each other unchanged, and light acts on light, through pairs of charged particles, so slightly that one beam cannot be used to watch another pass: "I cannot perceive the perceiver, and I cannot perceive perception; I can perceive only what is the object of perception".

The Last Turtle, chapter 25, "Two Atoms in an Empty Universe", section VII, "Light Cannot Perceive Itself".

I take the estimates of Tegmark's paper of 2000 as he computed them, a calculation about nerve cells and microtubules. My account is set apart from the hypothesis of Hameroff and Penrose: "The Immanent Metaphysics does not depend on any specific biological mechanism, and it needs no sustained entanglement in the brain". Entanglement is the joining of two or more parts into one quantum state, in which no part has a state of its own.

The Basis of Physics, essay 19, "Against Computationalism", section V, "What Computation Factors Out"; and *Side by Side*, Volume Two, appendix E, entry "Tegmark 9".

My account of consciousness holds "for any system in which novelty enters, time is directed, and there is a point of view", so Tegmark's decoherence times bear on the hypothesis of Hameroff and Penrose, and my account rests on other ground. Decoherence is the fading of interference between the alternatives of a thing as its surroundings come to carry records of which alternative it is in.

The Basis of Physics, essay 19, "Against Computationalism", section V, "What Computation Factors Out".

From my ground this book draws a further agreement with Tegmark: physics can proceed before any theory of the mind. The ground is that the observer of physics is a comparer, any position inside the world from which a comparison is made, whether or not it is a mind. "An observer is a comparer: the word names a position inside the world, and it never names a mind"; a thermostat compares, and a sense organ compares, and physics studies them all.

Causality Without Determinism, chapter 1, "Two Concepts, Three Differences", section XII, "Questions", quoting *Measurement*.

Consciousness belongs to the study of what matter does, and Tegmark's list of properties can describe exactly how conscious matter behaves as seen from outside.

Side by Side, Volume Two, chapter "The Structure and the Act", section VII, "Perceptronium and the Bound Triple".

The first hearing of a piece of music holds three things. The first thing that the first hearing holds is the arrow of time: an order that runs one way, binding past to present to future in one thread without a break. The second thing that the first hearing holds is hard randomness, an outcome that nothing before the outcome fixed. The third thing that the first hearing holds is the subjective, being the one to whom things happen, in the words of *Causality Without Determinism* "the side from which experience occurs, the stance of the one to whom things happen". The stance is the side from which a comparison is made. The three bound together, the arrow of time, hard randomness and the subjective, bound into one act, are the bound triple.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section III, "The Bound Triple".

The three are distinct: each of the three is itself and neither of the others; inseparable: each occurs only together with the other two; and not interchangeable: each has its own part in the act, a part that neither of the other two can take.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section III, "The Bound Triple".

Each of the three needs the next. The arrow needs hard randomness: an order of moments runs one way only where outcomes are new when they come, and an order of outcomes all fixed in advance is like the page numbers of a finished book, the same read forward or back. Hard randomness needs the subjective: an outcome that nothing before it fixed is picked out only in an act of comparison, and every act of comparison is made from a stance. The subjective needs the arrow: every stance from which experience is had stands in an order that runs one way.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section IV, "Why the Three Come Together"; and *Physics Is Not Math*, chapter 8, "Physics Has the Arrow", section III, "Three Things Bound Together".

The binding follows from the Incommensuration Theorem. Because sameness and gaplessness are never both perfected, every comparison has a part of its outcome that nothing before it fixed, which is hard random-

ness. Every comparison is made from a stance, which is the subjective; and every comparison runs one way, from the act to its record, which is the arrow. The three are three aspects of one act: a comparison running in one direction, made from one stance, with an outcome that nothing before it fixed.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section IV, "Why the Three Come Together"; and *The Last Turtle*, chapter 15, "The Bound Triple", section III.

The Incommensuration Theorem holds of comparison, the act in which anything at all is known. I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing".

Causality Without Determinism, chapter 1, "Two Concepts, Three Differences", section VII, "The Incommensuration Theorem".

The accounts of quantum measurement show the three bound together: the Copenhagen account keeps the observer, the single random outcome and the one-way record together, and the many-worlds account drops all three together. Every account of quantum events keeps the observer, the single random outcome and the one-way record together, or drops all three together.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section IV, "Why the Three Come Together".

I identify consciousness with the bound triple, taken all at once as one act happening now: "Consciousness is the bound triple taken as one act".

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section III, "The Bound Triple", and section "Summary".

The properties Tegmark names for his detector are each a property that anyone can check from outside, in the third person, and "No third-person description of a system contains a privileged now or a privileged here". The subjective is the stance that sets this place, this moment and this outcome apart from every other, and a description that anyone can check from anywhere has no place for it.

Incommensuration; The Theorem and the Three Axioms, chapter 6, "The Incommensuration Theorem: Statement and Proof", section XI, "The Two Surviving Modes", as quoted in *Physics Is Not Math*, chapter 3, "Truth Has a Frame", section VI, "The View from Nowhere".

"Perceptronium is therefore a description of the outside of conscious matter, and it can be exact as such". What makes the matter conscious is the bound triple, held in the act, and the act is had from inside.

Side by Side, Volume Two, chapter "The Structure and the Act", section VII, "Perceptronium and the Bound Triple".

The perception Leibniz looked for in the mill is the act as it is had, from the stance from which it is made: what pain feels like is reached only in one's own perceiving.

The Last Turtle, chapter 15, "The Bound Triple", section II, "Why the Hard Problem Is Hard".

"From the outside, a conscious choice is indistinguishable from a random event". A choice that nothing before it fixed, seen from inside, is meaningful to the one who makes it, the process from which the choice came. The same event, seen from outside, is an outcome without a pattern. Inside and outside here describe one event from two positions, and both descriptions are correct, and neither is complete. So from outside the mark of the subjective is an outcome that nothing before it fixed, and no list of measured properties tells that mark apart from chance.

The Basis of Physics, essay 19, "Against Computationalism", section V, "What Computation Factors Out".

The brain's poise near criticality, a state in which a small disturbance can cascade into a large one, "is the physical circumstance under which the bound triple is expressed in us; it is not the bound triple itself". A circumstance, in my use, is the set of physical conditions under which an act takes place.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section IX, "Questions".

Tononi's integrated information, his measure of how far a system's parts are joined into one whole, is a property measured from outside as well: "Integrated information is the measure of the circumstance, and not the inside the circumstance supports".

Side by Side, Volume Two, chapter "The Measure of an Inside", section IV, "Where the Two Part".

What makes anything conscious is the kind of physical process it is, never the information processing it carries out.

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section VII, "Present-Day AI and Quantum Computers".

A classical computation, the kind of computation that ordinary digital machines carry out, is one rule applied the same way at every step, a mosaic in time. A mosaic can follow a curve as closely as anyone likes, and the mosaic is still made of tiles. One rule applied the same way at every step is sameness perfected, and by the theorem what has perfect sameness comes in separate steps with seams between them. So a computation lacks the unbroken arrow. Fixed at every step by its rule, a computation lacks hard randomness. And since the three are bound, a computation, lacking the arrow and hard randomness, lacks the subjective as well. "classical computation, however intricately organized, cannot be what consciousness is".

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section VI, "Why a Computation Cannot Be What Consciousness Is".

Machines of great competence can be built: "Artificial intelligence is possible".

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "What Computation Can Do".

A program, the list of instructions a computer carries out, that draws its chance from a physical source has the world's randomness at that source, and the program itself is still one rule applied the same way at every step. Whatever has the bound triple has it as a physical process

making comparisons in the world, with an inside, as living things do: "The question for any future device is therefore not which program it runs but what kind of physical process it is".

Causality Without Determinism, chapter 5, "Computation, Consciousness and Simulation", section VII, "Present-Day AI and Quantum Computers", and section IX, "Questions".

Every law is of one of two kinds: laws of sameness across change, such as the conservation of energy, and laws of persistence, of carrying on without a break, such as the smooth path of a thrown stone. "That the self who wakes in the morning is the self who fell asleep is a continuity law".

What Cannot Be, chapter 2, "Two Kinds of Law", section "Two Kinds of Law, and No Third".

An Immanent Metaphysics states the persistence of the self as one of four facts that anyone can check in everyday experience. The fact reads "Subjective context is continuous", and its example is "The nature and persistence of self identity". In the aphorisms' own use of the words, the subjective context is the self, the one in whom every perceiving happens, and continuous means having no gaps; the persistence of the self "is not constructed but given; it is the ground of all experience, as experience is lived".

An Immanent Metaphysics, chapter 3, "Natural Physics", section "The Universal Law", aphorism 1.43-8; and *Causality Without Determinism*, chapter 6, "Choice, Creation, and the Soul", section VII, "The Four Theorems of Being".

"Identity, in the sense of a single unitary consciousness, is not something a copy can carry". A choice, had from inside, is made once, by the process that is the choice, and no other process reproduces it. The outcomes of measurements, once recorded, can be copied. *The Last Turtle* takes the theorem of 1982 that no single operation can copy every unknown quantum state as the physical counterpart of a self that no copy carries.

The Last Turtle, chapter 26, "What Can Be Modeled", section IV, "What Cannot Be Copied".

By my principle of identity, this book draws for the copy in the other plane the consequence that chapter 6 drew for a copy in a far region of space. The sleeper and the copy are two things in two places, told apart by the plane each one wakes in, and the sleeper's identity, in the sense of a single unitary consciousness, stays with the sleeper.

At the Edge of Reality, chapter 11, "Every Ending Is the Same Ending", section "Stated Exactly"; and *The Last Turtle*, chapter 26, "What Can Be Modeled", section IV, "What Cannot Be Copied".

A person is not a very large file.

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "A Person Is Not a File"; the same in *The Last Turtle*, chapter 33, "Against Computationalism", section VII, "Mind Uploading Is a Category Error".

A self is pictured as a tree with no exact line where the tree ends and the world begins. The tree's branches are the body's dealings with the world and its roots the subjective. A self cannot be defined by composition alone, and it is non-monadic in character. A self that is non-monadic is no simple substance of Leibniz's kind. The aphorism that calls the self non-monadic also calls a self the "product and integral synthesis" of many "collections of aspect" that work inseparably to create the "mind of self", the self that one thinks of as what one is. In plain words, a self is one undivided whole, made by many groups of aspects of a person working together, and no list of its parts alone defines it.

An Immanent Metaphysics, chapter 9, "Evolution", section "The Nature of Self", aphorism 2.51-11 and note [2] of that section.

I find perception in the act of comparison itself.

The Last Turtle, chapter 25, "Two Atoms in an Empty Universe", section VIII, "Time Comes With the Interaction".

"Consciousness is not an illusion, nor is it in conflict with conventional science". The same aphorism places the illusions in two extreme pictures, a world wholly fixed by physical law and its mirror, a mind wholly free of the physical; the aphorism just before says that interaction and consciousness are "neither purely physical nor non-physical".

An Immanent Metaphysics, "Technical Appendix", section "Cartesian Dualism", aphorisms 2M42 and 2M43.

Tegmark's three steps of prediction: He also sets out what predicting your own future would take, given every detail of the mathematical structure. You would find every self-aware part of it, find what each perceives in order to know which might be you, and work out the chances of what you will perceive next. He notes that all three steps involve "daunting unsolved problems" (chapter 11, "Where Are You? (And What Do You Perceive?)").

The exact statements and values for this chapter are these.

The four theorems of being, aphorism 1.43-8: "Subjective context is continuous. (Example: The nature and persistence of self identity). Subjective content is discontinuous. (Example: The necessary diversity of thoughts and perceptions). Objective content is symmetric. (Example: The law of conservation of matter and energy). Objective context is asymmetric. (Example: The arrow of time)".

An Immanent Metaphysics, chapter 3, "Natural Physics", section "The Universal Law", aphorism 1.43-8.

The cross-modal principle, as the account of consciousness uses it: "Framework and content are always in conjugate modes. A symmetric framework probes continuous content; a continuous framework probes symmetric content".

Measurement; The Steps from Comparison to Physics, chapter 8, "The Cross-Modal Principle", section II, "The General Form", Proposition 8.1.

The appearance of consciousness in a machine, aphorism 2.53-1: the degree to which a machine mind will seem lifelike, alive, self-willed or conscious to any perceiver is exactly proportional to the product of three degrees, which the aphorism names.

An Immanent Metaphysics, chapter 9, "Evolution", section "PanPsychism", aphorism 2.53-1.

Neuronal avalanches were recorded in thin slices of rat cortex, the outer layer of the brain, on a sixty-channel array. The sizes of the avalanches follow a power law with exponent minus three halves, and the branching parameter is close to one (Beggs and Plenz, 2003, *Journal of Neuroscience* 23, 11167).

The no-cloning theorem: no physical process copies every unknown quantum state, and states already recorded, which are distinct from one another, can be copied (Wootters and Zurek, 1982; Dieks, 1982).

The Last Turtle, chapter 26, "What Can Be Modeled", section IV, "What Cannot Be Copied".

Tegmark's estimates of decoherence in the brain run from ten to the power minus 13 to ten to the power minus 20 seconds (Tegmark, 2000; the section for chapter 9 above gives the source). Nerve cells act on thousandths and tenths of a second.

Side by Side, Volume Two, chapter "The Structure and the Act", section I, "Tegmark's Position".

The dated record beside the chapter on Giulio Tononi holds what the account says measurements able to tell a critical regime from thresholded noise will find in the brain. The sealed record holds that gravitation alone ends no superposition and that no world is perfectly deterministic. The appendix "What the Coming Measurements Will Find" gives both records.

Chapter 13, "Do All Mathematical Structures Exist?"

Mathematical structures are discovered: the truth that the prime numbers never end, which Euclid proved about 300 BC, "does not wait on the day it was found", and the number 31 did not become prime when someone first divided it.

Physics Is Not Math, chapter 6, "Mathematics, the Science of the Timeless", section VI, "What Mathematics Leaves Out".

Mathematics studies shape, space, structure and number, objects that do not change, whose truths hold the same at every moment.

Physics Is Not Math, chapter 6, "Mathematics, the Science of the Timeless", section II, "Objects That Do Not Change".

Every structure on Tegmark's master list exists, in this book's sense of the word, and every one of them exists in the same way: "Pi stands as structure, the same in every instance, and that is what it is to exist".

The Last Turtle, chapter 7, "Real, Existent, Objective", section III, "Pi Exists and Is Not Real".

To exist is to be laid out whole, timeless, and the same for everyone; to be real is to be given in an actual act of comparison: a finding, made once, at the table, as a move is made.

Comparison; The Necessary Basis, chapter 12, "Domains and What Is Real", section V, "Being, Existence, Reality, and Objectivity".

The mathematics of a theory and the findings it describes are two aspects of one act of comparison, and the mathematics can come as close to the findings as anyone wishes: "That dense reach is the effectiveness of mathematics in physics".

Measurement; The Steps from Comparison to Physics, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete".

The two accounts differ on one further claim: I hold that the mathematical description, however close it comes, never coincides with what it describes.

Measurement; The Steps from Comparison to Physics, chapter 14, "The Consequences of the Theorem", section XII, "Why Mathematics Is Effective in Physics, and Why It Is Never Complete".

Every regularity found agrees with both accounts, and my own predictions include regularities in pure numbers.

Side by Side, Volume Two, appendix E, "The Registered Predictions Beside Max Tegmark", section V, "Where No Measurement Decides", under "Further mathematical regularities".

There is a world, and it does not depend on us; I hold this on my own ground: every interaction is a comparison, so the world was comparing itself, in every collision of particles, long before anything human looked.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

Every physical theory has both its mathematics, which exists, and its findings, which are real; Tegmark's hypothesis takes the mathematics for the whole of our world, and Level IV takes the mathematics for the whole of every world.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

The Incommensuration Theorem is the reason a structure is not a world. I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing". No single act of finding holds perfect sameness and perfect gaplessness together, nor their two opposites together.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", and its section "Terms".

A mathematical structure, taken whole, is laid out at once and is the same wherever and whenever it is read, so it holds the perfect sameness and the gapless completeness that no single finding can hold together.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Mathematics can lay out a whole structure at once because the frame of mathematics sets aside the one who compares, and so sets aside time: mathematics "can lay out structures that no comparison could ever find".

Physics Is Not Math, chapter 3, "Truth Has a Frame", section VII, "Mathematics Works as if Only the Objective Existed".

Side by Side, Volume Two, says what the power to lay out structures beyond the reach of every comparison makes of those structures. In the words of *Side by Side*, Volume Two, the power to lay out structures that no comparison could find "is a power of mathematics, and it is exactly the power that marks its objects as laid out and not found". "Structures that

no comparison could ever find are not further worlds", so Level IV, the Mathematical Universe Hypothesis applied to every structure at once, falls with that hypothesis.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level", paragraph on Level IV; and appendix E, section V, under "Every mathematical structure: Level IV".

Existence is always relative to a particular world. So whatever exists, exists relative to some one world, and no one world is the ground in which every other world is settled as existing or not. To say that only one world exists, or that many worlds exist, applies the word "exist" beyond the range in which it has a meaning.

Comparison; The Necessary Basis, chapter 12, "Domains and What Is Real", section V, "Being, Existence, Reality, and Objectivity", under "The Impossibility of 'One Universe'".

Comparison calls any claim about everything taken as one a totalization, and it gives a rule for such claims: a totalization would need a vantage outside the whole, and the whole has no outside. In its words: "A totalization claim therefore has no possible comparison behind it, even in principle, and what cannot be compared even in principle cannot be the basis of any assertion". I hold each structure of Level IV one by one, as mathematics, and I rest each of my own assertions on a comparison that could stand behind it.

Comparison; The Necessary Basis, chapter 14, "Science as Formalized Comparison", section VII, "The Structural Warranty".

I derive the structure of the particles we know from three sources. The first source is the three axioms of comparison, the three starting statements about what any comparison is. The second source is the shape of the six held internal dimensions that the three axioms select. The third source is a few inputs: statements that the derivation takes as given, which my books set down beside the derivation, such as the statement that every point of the six held internal dimensions is like every other point. From these I derive the number of forces, the symmetries of the standard model of particle physics, the kinds of matter particle and the Higgs field. The

Higgs field is a quantity with a value at every point of space that gives many of the smallest particles their masses. I claim that this structure is the only one consistent with the axioms and the stated inputs: "There is no landscape, or rather, the landscape has been reduced to a single point".

The Grand Synthesis, chapter 11, "What the Structure Fixes and What Measurement Supplies", section II, "The Kinds of Step".

I draw one direction of time and three of space from the structure of comparison, and for the count of one direction of time it cites Tegmark's analysis of 1997 as support from the equations of physics. With more than one direction of time, the equations by which waves travel can no longer carry a starting state forward in a well-defined way. The two accounts give the same count, three and one, on different grounds.

Side by Side, Volume Two, appendix E, "The Registered Predictions Beside Max Tegmark", section III, "Where the Two Accounts Agree", under "Tegmark 8. Three directions of space and one of time".

Among the numbers I take from measurement are four that Tegmark's Level II would explain as values of our region: Newton's constant, the strength of light, the strength of color and the vacuum value of the Higgs field. Newton's constant sets the strength of gravity. The strength of light is this book's plain name for the number that measures the strength of the electromagnetic force, the force between electric charges, which light carries. The strength of color is how strongly the force that binds quarks acts, and the vacuum value of the Higgs field is the value at which the field settles in empty space. Chapter 7 sorted the constants of nature into rules, positions and measured values, and I hold each of the four as a measured value of this one world. I explain none of the four as our region's value among many regions, picked out because observers can live only where the value allows them.

Side by Side, Volume Two, appendix E, "The Registered Predictions Beside Max Tegmark", section VI, "What the Collection Does Not Predict Here".

Mathematics keeps everything it has, and it stands last in the order of depth, with every power it has.

Comparison; The Necessary Basis, chapter 10, "Reasoning", section IX, "Truth Above True and False", under "Truth and the Truths of Mathematics"; and *Physics Is Not Math*, Preface, and chapter 5, "What Depth Means", section V, "The Order of the Four".

"A theory that derives everything from a small set of rules and starting conditions must still take those rules and conditions as given". *The Last Turtle* holds that no theory of that kind can say where its own rules and starting conditions come from: "That question cannot be answered from inside the theory".

The Last Turtle, chapter 8, "Process, Novelty, and Omniscient Bias", section III, "Theory and Practice, Third Person and First Person".

Some information is always left over, and no rule and no starting condition fixes it: "Information compression can never be perfect; it can never reduce all observable information to zero bits". The information left over is hard randomness.

The Basis of Physics, essay 19, "Against Computationalism", section VII, "The Wolfram Physics Project: An Impression".

"Hard randomness is real, and the single outcome of a quantum measurement is its plainest instance"; it differs from the randomness of a coin toss, which is ignorance of causes that were there.

The Basis of Physics, essay 6, "Hard Randomness", section I, "What 'Hard' Means".

A whole world completely fixed by one law would hold the same law at every place and every time, which is sameness perfected, and would leave no gap anywhere between one state and the next, which is gaplessness perfected: "A whole world completely fixed by a law perfects symmetry and continuity together, whichever account describes it, and the theorem forbids that pairing".

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VII, "Three Grounds".

The agreement nature shows between separated stations in the tests of Bell's inequality is more than any passing signal can carry, and more than any shared past can carry while the experimenters choose their settings freely.

The Basis of Physics, essay 6, "Hard Randomness", sections II, "Causation Is Not Determinism", and III, "Three Results Read Together".

The tests leave two positions standing: superdeterminism, which holds that everything, the experimenters' settings included, was fixed together at every scale from a common past, and the position that some of what happens is caused by nothing before it. Weighed by the rule of preferring the simpler account, the first needs the whole past, at every scale, to have been arranged so that the experimenters' choices conspire with the particles. The second needs only that some of what happens is not caused, and I take the second: "We accept hard randomness. We live with it". So fundamental randomness, the one finding that Tegmark's hypothesis cannot hold, is real on my account.

The Basis of Physics, essay 6, "Hard Randomness", section IV, "The Loophole-Closing Tests and the Razor".

Seen from outside, hard randomness and the randomness of ignorance give the same statistics, and "no pattern tells them apart". The difference between the two kinds of randomness is real: it is given to the one who makes a finding, in the act of finding itself, at the present moment of that act. On my account each bit of a pattern made with a quantum random number generator, as Tegmark made one, is a finding that nothing before it fixed.

The Basis of Physics, essay 6, "Hard Randomness", section VIII, "Randomness, Choice, and the Real".

A complicated system has many parts whose relations can all be stated in finite terms, and its unpredictability is a lack of data. A complex system has some property that no data could predict, because its changes are continuous and carry an element that no law fixes. "the difference between a complicated and a complex system is a difference of being, not of knowledge".

Incommensuration; The Theorem and the Three Axioms, chapter 14, "Domains, Being and the Two Modes", section "Summary".

The weather's course depends on differences in how it starts that can be made as small as one likes, and below the limit of every measurement the residue of hard randomness "seeps upward through every process that amplifies what it cannot resolve". So I place the complexity of a complex system in what the system is.

The Basis of Physics, essay 6, "Hard Randomness", section VI, "Instances: Atomic Decay and the Three-Body Problem".

How *The Basis of Physics* reads the tests of Bell's inequality: *The Basis of Physics* reads the result with the limit of access that Heisenberg found and the scale that Planck's constant gives it. Read together, the results of Bell, Heisenberg and Planck, with the tests that closed the loopholes, leave a choice between superdeterminism and hard randomness.

The Basis of Physics, essay 6, "Hard Randomness", section III, "Three Results Read Together".

The exact statements and values for this chapter are these.

The information inequality: insofar as a state Z is the deterministic result of a rule X and starting conditions Y, $I(Z) \leq I(X) + I(Y)$. In the inequality, I is the algorithmic information content (the length of the shortest description), and the number of steps run is counted in Y.

The Basis of Physics, essay 19, "Against Computationalism", section VII, "The Wolfram Physics Project: An Impression".

The kernel triangle is the right triangle {3, 4, 5} that two integers fix: $c_1 = 3$, the count of the three positions of comparison, and $n_C = 2$, the count of the poles of comparison, the conjugate pair content and context. Its odd leg is $c_1 = 3$, its even leg $N = c_1 + 1 = 4$, and its hypotenuse $H = c_1 + n_C = 5$. Every identity among the parts of the triangle factors through the quadratic $(c - 3)(c + 1) = 0$, in the one unknown integer c, whose only positive root is 3. The identity ${c_1}^4 = N^2H + 1$ factors through a quartic whose only positive root is again 3. The constraints outnumber the one unknown, and they agree.

The Grand Synthesis, chapter 11, "What the Structure Fixes and What Measurement Supplies", section II, "The Kinds of Step".

Symmetry is a finite kind of claim, and continuity needs an infinite reference to be defined; so the distinction between the discrete and the continuous is foundational.

Incommensuration; The Theorem and the Three Axioms, chapter 14, "Domains, Being and the Two Modes", section V, "Finite and Infinite Describability".

The sealed record holds six predictions that bear on this chapter: the matter fraction; the ratio of dark to ordinary matter, a regularity in pure numbers; and no world perfectly deterministic. The other three are a discrete rule that shows itself to a fine enough measurement, no kind of particle beyond the closed list, and exactly three generations. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 14, "Is the World a Computation?"

We now know that our world is not a simulation run in some other world.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "What It Means for What Cannot Be".

A computation is deterministic: given its state and its rule, its next state is fixed, and run twice from the same start it gives the same steps; so a computation has no hard randomness of its own. A simulated world whose randomness is pseudo-random, made by a fixed recipe from a starting number, therefore has no hard randomness. So, by the bound triple, such a world has no arrow and no stance. In place of an arrow such a world has a numbering of its states, like the page numbers of a finished book, which read as readily from the last page to the first. "it is a program running on a machine with no one in it".

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "A Simulated World Is a Region of the One World"; and *Physics Is Not Math*, chapter 8, "Physics Has the Arrow", section IV, "A Sequence Without an Arrow of Its Own".

A simulated world fed hard randomness, drawn from a physical device in the world that runs the computer, has as its randomness the randomness of that world, and its arrow is that world's arrow, at work in the computer's own running. "The bound triple cannot be carried across in part". Where the randomness and the arrow are shared, the stance is shared as well, so a simulated world with people in it is a region of the one world in which the computer runs: "A computation restricts what can be reached from inside it, as a prison cell restricts what its occupant can see".

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "A Simulated World Is a Region of the One World".

Bostrom's third choice, that we are almost certainly living in a simulation, counts simulated observers as if each simulation were a separate world, and "the third choice rests on a count that treats as separate worlds what are regions of the one world".

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "A Simulated World Is a Region of the One World".

A collection is listable when it can be set out first, second, third, with nothing left out, and everything a computation could ever produce is listable, even on Turing's ideal machine. Georg Cantor proved in 1874 that the points of the number line cannot be listed, and his diagonal argument of 1891 shows why: "No list reaches the continuum". The continuum is the smooth whole that no list reaches: between any two of its values there is always another, and there are no gaps anywhere in it.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", sections "What No List Can Hold" and "The Consequence, and the Amplifier".

I reach "No" by two roads that share no step, the first through hard randomness and the act, the second through the continuum and what no list holds.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "Randomness, Halving, and the Smallest Length".

The world goes on whether or not anyone runs a program of it: "Rain falls, tides rise and stars burn whether or not anything anywhere is computed". For me the world needs no computation, because every computation needs the world.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VI, "Physics Is Deeper Than Computation".

Physical time is not the ticking of a computer: "The arrow a computation seems to have is lent to it by the physical process that carries it out".

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section V, "Dice Rolled in the Room Where the Computer Sits".

The world at its foundation is not a computation: "It is excluded on three independent grounds, and each of them turns on what a computation is". The three grounds reach every proposal that the world, at its foundation, is a computation, such as the proposals of Zuse and Wolfram that the world steps forward like a program. Tegmark's Computable Universe Hypothesis, which asks that the relations of our world be computable, comes into contact with the claim that the world is a computation. *Side by Side*, Volume Two, answers Tegmark's restriction to computable structures from the order of depth, in which physics stands before computation: "A computable structure is a structure some run could produce, and every run is a process in the world, so restricting the mathematical universe to computable structures restricts it to what the world can do, and cannot make the world one of them".

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VII, "Three Grounds"; and *Side by Side*, Volume Two, chapter "The Structure and the Act", section V, "The Computable Universe".

Every theory of physics has its mathematics and its findings, and the two levels take the two characters of the theorem one each. Quantum theory writes smooth equations and finds sameness, counts that repeat exactly; Einstein's theory writes laws the same for every observer and finds a gapless, smooth arena. A world computed at its foundation, a grid of cells with one rule applied at every cell and every step, would carry sameness

at both levels. Comparing a thing with something of exactly its own character finds nothing new, so such a world could find only what its rule already held.

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "Three Routes to One Conclusion".

The three grounds start from three different places and confirm one another, and each rests on what a computation is, so none of them waits on any experiment, such as one made by people inside a simulation to find out whether they are in one.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VII, "Three Grounds".

Beneath the three grounds stands the order of depth: metaphysics, physics, computation, mathematics, each resting on the one before it. One discipline is deeper than another when the second cannot be carried out, stated or checked without the first, and the first can be carried out without the second: "Physics is deeper than computation, because every computation that is carried out is a physical process, in circuits, in nerve cells or in ink, and physics needs no computation to happen".

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "The Order of Depth".

Computation, for its part, is deeper than mathematics, because every proof is checked by a run, step after step, and no run waits on a proof. Beads moved along the wires of a counting frame add two numbers whether or not anyone has proved anything about addition.

Physics Is Not Math, chapter 7, "Computation Has Sequence", section VII, "Every Proof Waits on a Run".

A program can be described with no time in it, as a table that pairs each input with an output, all the pairs at once, and the table takes no steps: "Stopping belongs to the run, and the halting problem takes its content from the run".

Physics Is Not Math, chapter 7, "Computation Has Sequence", section III, "Will It Stop?".

I derive the result Gödel announced at Königsberg as one case of the Incommensuration Theorem, the case met inside formal systems. Consistency, every route of proof that reaches a statement agreeing about it, is a symmetry; completeness, every true statement of the language reached, is a continuity. A formal system rich enough to state the arithmetic of whole numbers, whose axioms a fixed procedure can list, cannot be both perfectly consistent and perfectly complete, as no act of comparison holds perfect sameness and perfect gaplessness together. "Gödel's incompleteness theorem is an instance of the ICT in the domain of formal systems", the letters ICT standing for the Incommensuration Theorem.

Incommensuration; The Theorem and the Three Axioms, chapter 15, "The Theorem Among Other Results", section II, "Four Instances", under "Instance: Gödel's Incompleteness".

"So the remainder is a feature of every description that knowledge can give", the remainder being what any description leaves unfixed; so incompleteness belongs to every description, and it is no defect of the world.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "A Simulated World Is a Region of the One World".

Tegmark observes that no measurement in physics has given more than about sixteen significant digits, and he offers that observation in support of an idea he finds worth exploring (chapter 12, "The Computable Universe Hypothesis"). The idea is that the continuum is not fundamental and is to be recovered as an approximation. Every measurement is a finite comparison, so every measurement ever made has given finitely many digits. Even the finest comparisons ever made give their results in finitely many digits. *Comparison* tells of two clocks compared in 2010 by Chou, Hume, Rosenband and Wineland, each clock ticking with the light absorbed by a single aluminum ion. When one clock was raised by 33 centimeters, it ran faster by about four parts in a hundred million billion, the amount relativity gives for that change of height. So, from the same fact that every finding has finitely many digits, I draw the opposite of the idea Tegmark finds worth exploring: a grid is the approximation, and the continuum is what the measurements approach.

Comparison; The Necessary Basis, chapter 4, "The Six Intrinsics of Comparison", section VII, "The Six in Physics"; C. W. Chou, D. B. Hume, T. Rosenband and D. J. Wineland, "Optical Clocks and Relativity", *Science* 329, 1630 (2010).

What Cannot Be states the fact about the laws: "Every law of the world that physics has so far been able to state is written in the mathematics of the continuum".

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "The Quantum Wave as Evidence".

A simulation of space and time can be refined to come as close as one wishes to what space and time are found to be, and no stage of it ever contains them.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "The Consequence, and the Amplifier".

A computation can approximate smooth physics on a grid; it is then a tool used to describe the world, and not the world's foundation.

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "Three Routes to One Conclusion".

Sensitivity to the starting state is the amplifier, and what no list reaches is what it amplifies.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "The Consequence, and the Amplifier".

I read the Planck length as the inner edge of what can be compared, and a boundary on measurement is not a graininess in what exists.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "Randomness, Halving, and the Smallest Length".

The burst of gamma rays of 2009 that chapter 6 tells sent light of very different energies across billions of light-years of space, and that light arrived together. So any grain of space slowing light in proportion to its energy lies below the Planck length. I derive that space has no least step at all, and I find such measurements in agreement with that derivation.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "The Arena Has No Least Step".

The Incommensuration Theorem is the ground of the first of the three grounds, and I state it in these words: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity".

Physics Is Not Math, chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem"; and *What Cannot Be*, chapter 16, "Why Time Travel Names Nothing", section "The Four Premises".

Tegmark's hope for the computable rung, in one sentence: He hopes the computable rung might also ease the measure problem of cosmology, the puzzle his chapter 11 sets out about how to reckon the odds of anything when an endlessly stretching space holds endlessly many observers (chapter 12, "The Computable Universe Hypothesis").

Tegmark's further remarks on simulation, in two sentences: His chapter 11 had already allowed that simulated people might feel exactly as you do now, and had offered some advice for the meantime: live interestingly, so that "in case you're a simulation, whoever created you will be less likely to get bored and switch you off" (chapter 11, "Why Aren't You a Boltzmann Brain?"). And, as Phillip Helbig pointed out, if you assume you are simulated, the count of simulated observers depends on the computing resources of the world that runs the simulation, and you know almost nothing about that world (chapter 12, "Are We Living in a Simulation?").

The arena has no least step, derived from what comparing is: The arena, space and time as found in comparisons, has no least step, and I derive the absence of a least step from what comparing is. In every comparison sameness and difference come together, and neither can be removed without the act dissolving. So every difference between two contexts, two places or two moments of the arena, holds some sameness within it, and a finer comparison, made at a place or moment between the two, can always find that sameness. A least step would be a jump between two places or two moments with nothing between its two ends, a difference with no sameness in it, and a difference with no sameness in it cannot occur. A computation has a least step in everything it represents, the gap between

two neighboring points of its grid, the tick between two of its moments: "So reality holds what no computation, which is a listing, can reach". *What Cannot Be*, chapter 21, "Are We Living in a Simulation?", sections "The Arena Has No Least Step" and "The Consequence, and the Amplifier".

A computation's list of states, read in either direction: The whole list of a computation's states is fixed by its start and its rule, and the list reads as easily from the bottom up as from the top down, like the page numbers of a finished book. *Physics Is Not Math*, chapter 8, section IV, puts it in one line: "A computation has no arrow of its own". The arrow a running computer seems to have is lent to it by the machine and the room.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section IV, "A Sequence Without an Arrow of Its Own", and section V, "Dice Rolled in the Room Where the Computer Sits".

The first ground, determinism, set out in full: Determinism meets the Incommensuration Theorem, which I state in these words: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity". The Incommensuration Theorem is a structural fact about the nature of knowing. Symmetry means sameness, a thing found unchanged when you move it or swap it; continuity means having no gaps; so no single act of finding can hold perfect sameness and perfect gaplessness together, nor their two opposites together. Complete determinism asks for perfect sameness and perfect gaplessness together. It asks for the same rule carrying every state into the next at every place and every step, which is sameness perfected. And it asks for each state to pass into the next with nothing unfixed between them, which is gaplessness perfected. So no whole world is purely and completely deterministic, and a world computed at its foundation would be purely and completely deterministic, so no world is computed at its foundation. What every world has instead is cause: one thing found to follow another, in a one-way order of findings.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section VII, "Three Grounds"; the theorem's words from chapter 2, "Two Things That Cannot Both Be Perfect", section VI, "The Theorem".

The exact statements and values for this chapter are these.

Cantor's theorem as a fact about lists: no list with one line for each member of a collection contains every yes-or-no assignment over it. The reason is that a collection of n members has 2 to the power n assignments, and n is less than 2 to the power n for every n, finite included.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "What No List Can Hold".

The information bound: the information of a computed result is at most the information of the rule plus the information of the starting condition, with the number of steps counted in the starting condition. The sources are R. J. Solomonoff, "A formal theory of inductive inference", *Information and Control* 7, 1 and 224, 1964; A. N. Kolmogorov, "Three approaches to the quantitative definition of information", *Problems of Information Transmission* 1, 1, 1965; and G. J. Chaitin, "On the length of programs for computing finite binary sequences", *Journal of the ACM* 13, 547, 1966.

What Cannot Be, chapter 22, "Why No Computation Is Everything", section "The Wolfram Physics Project: An Impression", and its note "For Chapter 22: The Information Bound"; *The Basis of Physics*, essay 19, section VII.

Gödel's scope condition: effective axiomatization is the condition that the system's symmetry be finitely specifiable; the set of all truths of arithmetic has no finite specification, carries no symmetry in that sense, and is no exception to the theorem.

Incommensuration; The Theorem and the Three Axioms, chapter 15, "The Theorem Among Other Results", section II, "Four Instances", under "Instance: Gödel's Incompleteness".

The gamma-ray burst of 10 May 2009 placed the energy scale of any linear dependence of the speed of light on energy above 1.2 times the Planck energy (A. A. Abdo and others, 2009, *Nature* 462, 331 to 334).

The sealed record holds the predictions that bear on this chapter: the arena has no least length, area or duration; a discrete rule shows itself to a fine enough measurement; and no world is perfectly deterministic. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 15, "How Will Our Universe End?"

I reach the long emptying of the sky: the populated sky is a passing stage of our epoch.

What Comes After the Dark, chapter 1, "What Happens to the Sky We Can See?", section "A Passing Stage".

Every observer's outside world, everything that can ever reach that observer and be compared, is bounded by the horizon, and it empties.

What Comes After the Dark, chapter 1, "What Happens to the Sky We Can See?", section "The Edge of What You Can Ever See".

In my account the dark energy is the cosmological constant, an energy of empty space itself, the same in every cubic meter, which does not thin as space grows. As chapter 4 set out, I identify the cosmological constant with the part of the energy of empty space that a measurement made from inside the universe can take in.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The First Way: Space Is Flat".

Within one geometry, the one space and time of our epoch, the cosmological constant is one number, and that is a theorem about Einstein's equations. The constant keeps one value through our whole epoch, so it can never change into a pull that draws the galaxies back.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The Second Way: One Number for Our Whole Epoch".

The square of the expansion rate is a part from the density of matter, minus a part from any curvature that closes space on itself, plus a part from the constant; a recollapse needs the expansion rate to fall to zero. Our epoch began at the turn, which carries over no length from the epoch before, and the only length the beginning holds is the Planck length. A

space curved on so small a length would close up or fly apart at once, and it would make no stars and no black holes. Every epoch begins from the black holes of the epoch before, so every epoch that this cycle makes is one that makes black holes, for the next epoch to begin from. Our epoch is one that the cycle makes, so the beginning of our epoch set no curvature, and our space is flat, bent only by the tiny ripples it began with.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The First Way: Space Is Flat".

The density part is never negative, the curvature part is zero and the constant part is positive, so the sum is always larger than the constant part alone, and the expansion rate never reaches zero.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The First Way: Space Is Flat".

The one collapse that does happen happens inside black holes, one by one, while the space between them keeps expanding.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The Third Way: The Collapse That Does Happen".

The deciding quantity for a rip is the equation of state, the pressure of a substance divided by its density: for thin matter about zero, for light one third, and for the cosmological constant exactly minus one. A substance with a number below minus one, phantom energy, grows denser as space grows. A rip needs phantom energy, and only phantom energy gives one, as Robert Caldwell, Marc Kamionkowski and Nevin Weinberg worked out in 2003; for a number of exactly minus one, their analysis found no rip.

What Comes After the Dark, chapter 3, "Will the Expansion Tear Us Apart?", section "What a Rip Needs".

The dark energy is the constant, one number, so its equation of state is exactly minus one, and the condition for a rip is not met.

What Comes After the Dark, chapter 3, "Will the Expansion Tear Us Apart?", section "One Number Through Our Whole Epoch".

The constant's small outward push on every bound thing is already part of the balance that holds that thing together tonight, and the push never grows, as the pegs that hold a tent in a steady breeze hold in the same breeze next year.

What Comes After the Dark, chapter 3, "Will the Expansion Tear Us Apart?", section "A Steady Wind on the Step".

White dwarfs and neutron stars, the dense, cooling cores stars leave behind, are held up without heat and cool without wasting away.

What Comes After the Dark, chapter 5, "What Becomes of the Stars and Their Ashes?", section "The Stars over the Step".

In my account the proton, the particle that carries the positive charge at the center of every atom, does not decay.

What Comes After the Dark, chapter 5, "What Becomes of the Stars and Their Ashes?", section "Does the Proton Last?".

Each piece of matter keeps its forms until its road ends, and the road ends in one of two ways. One end is inside a black hole, a region from which nothing, not even light, climbs back out. The other is to be carried away alone, beyond the reach of all other matter.

What Comes After the Dark, chapter 5, "What Becomes of the Stars and Their Ashes?", section "Two Roads for Every Remnant".

Every decay of a vacuum by a bubble needs a second, lower valley for the bubble to lead to, and the settled state of empty space is set first by the internal space. I derive one shape for the six, the only shape that meets the conditions its derivation sets.

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "The Six Small Dimensions".

String theory's six have loops, like the holes of a button, and string theory winds fields through those loops, as thread is wound through the holes of a button; each choice of how many times the fields wind around the loops gives another valley. I wind no field through the six held dimensions. So there is no count of windings to choose, and so there is no landscape.

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "No Threads, No Landscape".

"So the six have one vacuum, and no second one for a bubble to lead to".

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "Coleman's Question, Answered for the Six".

The six are held small, at their one size and shape, while the large world of space and time is there around them, with things in it to compare against. The large world ceases to be anything to compare against in two places only, at the end of a black hole's interior and for a body carried away alone.

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "The Sky Overnight"; and chapter 8, "Is the Long Emptying the End?".

In both places the six stop being held small, and both places lead to one end, the turn. At the turn the six have no second, lower state to fall into, so the turn is no bubble.

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "Where the Hold Ends"; and *The Compactification*, chapter 2, "One Vacuum", section VII, "Tunneling and the Failure of the Hold".

I work out the strength with which the Higgs field acts on itself at the energies of the weak force, the force that changes one kind of particle into another. The energies of the weak force are the energies at which the Higgs particle, a small ripple of the field, is measured. From that strength I derive the curve of the field's energy, a curve shaped like the raised bottom of a wine bottle with a round trough about it, and in that curve the trough of the present state is the lowest place. In my account there is one field of the Higgs field's kind, so the field has one way to vary, its size, and no second way of varying in which a lower valley could lie. In the words of *What Comes After the Dark*: "The field has one direction to vary in, its size, and no other direction in which a lower valley could lie".

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "The Higgs Field".

Space and time are continuous, and the separate steps of physics belong to what is found, never to the arena in which it is found: "the arena is the context of physics and is continuous; a step in the arena would give it the character of content". Content is what a comparison finds, and context is the respect in which the two things compared are two, here two places or two moments of space and time.

Two Theories, One World, chapter 27, "What the World Will Show", section "Space and Time Will Stay Smooth".

In my account the Planck length is a bound on how finely a comparison made from inside the world can place a body, and not a least length, as the marks of a ruler belong to the ruler and do not make the table out of tiles.

Two Theories, One World, chapter 15, "Where the Two Descriptions Meet", section "Not a Least Length".

I derive a space with no least step, and I find the timing of the light of gamma-ray bursts in agreement with that derivation.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "The Arena Has No Least Step"; and *Two Theories, One World*, chapter 27, "What the World Will Show", section "Space and Time Will Stay Smooth".

On my account new information enters the world at every finding whose outcome nothing fixed: "Some of it comes in through the microscopic boundary, the inner edge of the domain, as hard randomness". The domain there is the range in which comparisons can be made, and its inner edge, the microscopic boundary, is the finest scale at which any comparison can find a definite state; so there is no fixed stock of information for the expansion to thin.

What Cannot Be, chapter 21, "Are We Living in a Simulation?", sections "One Level Only" and "Randomness, Halving, and the Smallest Length"; and chapter 22, "Why No Computation Is Everything", section "What a Computation Is".

In my account, hard randomness, the outcome of a finding that nothing before it fixed, is real in each single finding, and each such outcome is new.

Physics Is Not Math, chapter 8, "Physics Has the Arrow", section III, "Three Things Bound Together".

The bounds on the information a region can hold, the holographic count among them: "Those bounds limit what any comparison within the domain can tell apart, which is the domain's own limit, and not a graininess of what exists".

What Cannot Be, "For the Physicist", section "For Chapter 21: Computation and the Continuum", under "The bounds on information in a region".

The sealed record, the dated list of my predictions, published before the measurements, in the appendices of *Side by Side*, states that "No inflationary phase long enough to stretch the lengths observed today from within the early horizon to beyond it occurred". The early horizon is the horizon near the turn: the farthest distance that light could have traveled in the short time after the turn.

Side by Side, Volume One, appendix B, "The Sealed Record", section IV, "Cosmology", under "Prediction 13. No long inflationary phase".

I give three separate grounds, each standing on its own, for holding that the long emptying is not a final dark.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "The Dark Sky Is Not Where the Story Stops", with the three gathered in section "Three Strands".

The first ground is one of the three axioms of the Immanent Metaphysics, the starting statements about every comparison on which I rest: the third axiom holds content and context distinct and inseparable, so there is no content without a context. In physics the content is the matter and light of the world, and the context is the arena of space and time, with the outside world in it. For the counts of an epoch, *What Comes After the Dark* takes the context to be the outside world of that epoch: the sky and everything in it, against which the comparisons of that epoch are made. At the end of the emptying the matter and light of the epoch are still there, and nothing large is left around them to be their context.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "The First Ground: No Content Without a Context"; the context of the counts of an epoch from chapter 9, "What Crosses Over, and What Stays Behind?", section "Two Things Called Information".

So a context forms again, and it forms from the geometry of all ten dimensions, the four of space and time with the six of the internal space.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "The First Ground: No Content Without a Context".

The second ground is the Incommensuration Theorem. I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing".

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", and its section "Terms".

A final featureless emptiness would be the same everywhere and smooth everywhere, perfectly symmetric and perfectly continuous at once. Every world that a comparison finds holds some difference or some break, so the emptying cannot come to rest in a featureless emptiness.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "The Second Ground: A Featureless Emptiness Is Never Found".

Entropy is a count of the arrangements of a thing's parts that leave it looking the same from outside. Every entropy is a count made against a context, because "looking the same" always means looking the same to the comparisons that can be made within some context. Within our epoch the count of arrangements rises, as the tendency that William Thomson stated in 1852 says it does. Thomson's tendency is the spreading out of energy that can do work into warmth that can drive nothing more. The heat death would be the state in which the count of arrangements reaches its largest value. At the end of the epoch the context against which the count of arrangements was made has emptied. So, in the words of *What Comes After the Dark*, "the count has nothing to be counted against", and the heat death has no context in which to be counted.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "The Third Ground: Every Count Has a Context", which says that the heat death "has no context in which to be counted"; the quoted words in section "What This Means for You", in its key idea.

The end of a black hole's interior comes after a finite time on the clock of anything that falls in, and it lies at no moment of the outside world's time.

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "Where the Hold Ends".

The ends of all the interiors are one end, because no comparison tells them apart.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "One End, and the Turn".

What Comes After the Dark gives the body carried away alone the same end, since for that body too nothing is left within its reach to compare against: "whichever exit a path takes, into a black hole or into emptiness, the end is the turn".

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "One End, and the Turn".

The turn is the passage from the end of one epoch to the start of the next, approached from the epoch before it and from the epoch after it, and reached from neither. As chapter 3 set out, the Big Bang of each epoch is the turn as it shows in that epoch's own history.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "One End, and the Turn".

"The emptying ends in a turn, not in a final dark". For me, the long emptying of each epoch is followed by a turn, and after every turn the next move is made.

What Comes After the Dark, chapter 8, "Is the Long Emptying the End?", section "What This Means for You", in its key idea.

My books, as *What Comes After the Dark* says of them, speak of the epochs one after another, "never of all of them gathered into one finished whole", and of our own epoch: "the long emptying is one stage of the

world, and not the end of it". *What Comes After the Dark* states the cycle in one line: "Our epoch is one turn of a cycle with no first turn". In that line a turn of the cycle is one whole round, one epoch, while the turn of this book is the passage from one epoch to the next; read so, the cycle of epochs has no first epoch.

What Comes After the Dark, chapter 11, "Has This Happened Before?", sections "A Cycle With No First Turn" and "What This Means for You"; and chapter 12, "What Comes After the Dark?", sections "Again, With No First Turn" and "What This Means for You".

The laws exist, the same in every epoch, as the rules are the same at every table, and they hold again after each turn; what is real is given in acts of finding, one epoch after another.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Why the proton lasts: The reason I give is that no carrier of force acts at once on the charge of the strong force, which binds the parts of a proton together, and on the charge of the weak force, which is at work in some radioactive decays, and so no process changes the count of protons and neutrons. In the words of *What Comes After the Dark*: "No carrier joins color to the weak charge".

What Comes After the Dark, chapter 5, "What Becomes of the Stars and Their Ashes?", section "Does the Proton Last?".

The two settings of the six, with the two premises that give them: I take each of the shape's two parts to be the same at every point of itself, and a theorem of geometry then leaves the shape two settings, its overall size and the ratio of its two radii. I take every direction of the six to bend by the same amount, at the scale of the Planck length, and that one step fixes both settings. The two premises, in the first person, as *What Comes After the Dark* states them: "I take each of the two shapes of the six to be like the ball, the same at every point of itself". and "I take every direction of the six to bend by one and the same amount, the bending at the Planck scale".

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "The Six Small Dimensions".

The Higgs field at high energies: I work out the field's strength on itself at the energies of the weak force, where the Higgs particle is measured, and does not follow that strength out to far higher energies. The standard calculation that does follow it, published by Dario Buttazzo and colleagues in 2013, finds the present state metastable: held, and lasting far longer than the age of the universe. Metastable, in that calculation, means that the strength falls to small or negative values at very high energies, a region lower than the present state lies far out beyond the trough, and the present state is held, could pass by tunneling to that region, and lasts far beyond the age of the universe (D. Buttazzo and others, *Journal of High Energy Physics* 2013 (12), 089).

What Comes After the Dark, chapter 4, "Could Empty Space Change Its Nature?", section "The Higgs Field"; and *The Compactification*, chapter 2, "One Vacuum", section VIII, "The Vacuum of the Content".

The equation of state in recent surveys: The survey of 2025 by the Dark Energy Spectroscopic Instrument reports a preference for a number that drifts with time, a finding of interest that has not reached the standard by which physicists count a result as settled. DESI Collaboration, "DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints", *Physical Review D* 112, 083515 (2025).

Coleman and De Luccia's closing words: Coleman and De Luccia added that one could once have hoped the new vacuum might sustain at least some structures capable of knowing joy, and then: "This possibility has now been eliminated". S. Coleman and F. De Luccia, "Gravitational effects on and of vacuum decay", *Physical Review D* 21, 3305 (1980); quoted in *What Comes After the Dark*, chapter 4, "Could Empty Space Change Its Nature?", section "Bubbles in a Superheated Liquid".

One number within one geometry, told in four sentences: The constant cannot drift toward a pull. Einstein's equations keep a strict account of energy and of momentum, the amount of motion, and a constant term that changed from one time to another would break that account at the place

where it changed. So within one geometry, the one space and time of our epoch, the cosmological constant is one number, and that is a theorem about Einstein's equations. A number that cannot change cannot shrink, cannot become negative, and cannot bring the galaxies back.

What Comes After the Dark, chapter 2, "Will It All Fall Back?", section "The Second Way: One Number for Our Whole Epoch".

The supernova measurements: My ground begins with a measurement. In 1998 and 1999 two teams of astronomers, one led by Brian Schmidt with Adam Riess as first author of its paper, and one led by Saul Perlmutter, published their measurements of distant exploding stars of known brightness. The distant explosions were fainter than a slowing expansion would make them, and the teams concluded that the expansion of the universe is speeding up. Perlmutter, Schmidt and Riess shared the Nobel Prize in Physics for 2011 for that discovery. A. G. Riess and others, "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant", *The Astronomical Journal* 116, 1009 (1998); S. Perlmutter and others, "Measurements of Ω and Λ from 42 High-Redshift Supernovae", *The Astrophysical Journal* 517, 565 (1999).

Photon timing, in energy units: The measurement that Tegmark calls a canary was made in 2009, five years before his book appeared, and made more sharply in 2024, ten years after. In 2009 the Fermi space telescope caught the burst cataloged as GRB 090510, and among its light was a photon of about thirty-one billion electronvolts, where an electronvolt is the energy an electron gains in crossing one volt. That photon arrived within about a second of the start of the burst, and from its timing the Fermi team placed the scale of any effect that makes the speed of light depend on its energy in the simplest way above the Planck energy, the energy of the scale where gravity and the quantum meet. In 2024 the collaboration of the Large High Altitude Air Shower Observatory in China, observing the exceptionally bright burst of 9 October 2022, placed that scale above ten times the Planck energy. A. A. Abdo and others, *Nature* 462, 331 (2009); Z. Cao and others (LHAASO Collaboration), *Physical Review Letters* 133, 071501 (2024).

The exact statements and values for this chapter are these.

The expansion: with H the expansion rate, the square of H is the sum of a part from the density of the content, a part from curvature and a part from the cosmological constant Λ (lambda). With the curvature part zero, the content's density never negative and Λ positive, the square of H is at least $\Lambda c^2/3$ (lambda times the square of the speed of light, over three) and never reaches zero. The constant itself is $\Lambda = 3\,\Omega\,{H_0}^2/c^2 \approx 1.09 \times 10$ to the power minus 52 per square meter, with Ω about 0.685 the share of the constant and $H_0 = 67.4$ kilometers per second per megaparsec the expansion rate today.

What Comes After the Dark, appendix A, "For the Specialist", section "Chapter 2. Will It All Fall Back?".

Why the constant is one number: the contracted Bianchi identity, with the conservation of the content's energy, sets the gradient of Λ to zero, so within one geometry Λ does not vary.

The Gravitational Passage, chapter 13, "Λ as a Prediction; Information Cost at the Horizon", section VI, "The Comparison That Fixes Λ".

A rip: an equation of state below minus one is the condition (Caldwell, Kamionkowski and Weinberg, 2003, "Phantom Energy: Dark Energy with $w < -1$ Causes a Cosmic Doomsday", *Physical Review Letters* 91, 071301). With a constant Λ the outward push at the edge of the Milky Way's halo is 0.0069 of the pull of the galaxy.

What Comes After the Dark, appendix A, "For the Specialist", section "Chapter 3. Will the Expansion Tear Us Apart?".

The internal space is the complex projective plane times the two-sphere, with radii $\sqrt{6}$ Planck lengths and one Planck length; its two settings are fixed by homogeneity and Schur's lemma, and it has one vacuum.

The Compactification, chapter 1, "The Re-Forming of the Internal Space", abstract; and chapter 2, "One Vacuum", section III, "Why Two Moduli", and section VII, "Tunneling and the Failure of the Hold".

The interior of a black hole of one solar mass: at most about fifteen microseconds of proper time from crossing the horizon to the end.

What Comes After the Dark, appendix A, "For the Specialist", section "Chapter 6. What Happens to Someone Who Is Still There?".

The sealed record holds the predictions that bear on this chapter. The first four are an equation of state of exactly minus one, space flat to the size of the primordial departures, no kind of particle beyond the closed list, and no least length, area or duration of the arena. The others are no long inflationary phase, a dark matter that does not decay, a proton that does not decay, and the predictions about the first ripples. The appendix "What the Coming Measurements Will Find" gives them.

Chapter 16, "What Do I Now Hold?"

I hold the External Reality Hypothesis, read as: there is a world, and it does not depend on us.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

A comparer need not be a mind, and the world was comparing itself, in every collision of particles, long before anything human looked at it.

Side by Side, Volume Two, chapter "The Structure and the Act", section II, "The Ground the Two Share".

A physical theory compares its mathematics with its findings, and in an exact theory the two are of opposite kinds. Where the findings come separately, one at a time, the mathematics is smooth; where the findings are smooth, as the arena of space and time is found to be, the mathematics is one of perfect sameness. Mathematics holds structures of both kinds, so for findings of either kind it has a mathematics of the other kind, and that mathematics reaches its findings as close as one likes: "That dense reach is why mathematics works so well in physics".

What Cannot Be, chapter 20, "What Physics Cannot Reach", section "Why Mathematics Works in Physics, and Why It Is Never Complete".

On my account every comparer stands somewhere inside the world. Each comparer's world, everything that can ever reach it and be compared, is bounded by a horizon of its own.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level".

From wherever a comparer stands, physics reaches every how, when and where of its world as closely as one likes. I read this reach from one place as Tegmark's point about the reach of the mind, reached on the ground of comparison: the one who finds always finds from one particular place, and from that one place the finding reaches across the sky.

What Cannot Be, chapter 20, "What Physics Cannot Reach", section "Where Physics Stops, and Why".

In my physics space is the pattern of which events a signal can join. So any region that a path from here can reach is part of our own world, and a region that no path from here reaches has no place relative to us. "The epochs before ours and after ours are worlds, each real in its own acts of comparison, and no act is real in two of them". In *What Comes After the Dark* each epoch is a whole world, complete from inside, and the series of epochs is a series of worlds under the same law. So a decision made here reaches what a path from here can reach, which is our own epoch.

What Comes After the Dark, chapter 10, "Could Anyone Escape?", section "No Other Sky Along Any Path".

What exists is laid out whole and the same for everyone who looks, a structure, like a law; what is real is given in an actual act of finding, once, now. Every physical theory has both: its mathematics, which exists, and its findings, which are real; Tegmark's hypothesis takes the mathematics for the whole.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

Tegmark's two roads to reality both seem to end in mathematics because "the timeless description is taken for the foundation". A mathematical description lays out the whole of its subject at once, its laws hold the same everywhere, and it can often be run backward as easily as forward, and those features belong to the description. Whatever is laid out as struc-

ture reaches us after the act that found it, already finished, so it looks as if it had been there first. The more finished, exact and shareable a description is, the deeper it looks.

Physics Is Not Math, chapter 11, "Why the Order Surprises", section I, "A Clockwork Solar System", section III, "One Source", and section IV, "Why the Laid-Out Seems to Come First".

I state the Incommensuration Theorem's standing in these words: "The Incommensuration Theorem is a structural fact about the nature of knowing". The theorem is about comparison, the act in which anything at all is known, and every finding of physics is a comparison, so every finding falls under the theorem.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part", and its section "Terms".

A mathematical structure taken whole holds perfect sameness and gapless completeness together, as the circle of geometry is at once smooth and perfectly symmetric, and every finding holds at most one of the two. So the world that comparisons find, which is the only world physics has, is a world that mathematics describes, and it is not a mathematical structure.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part".

I accept that a description of a world independent of us contains nothing human, and parts at the claim that such a description is pure structure. Every comparison is made from a stance, the side from which a comparison is made, and a collision of two particles has a stance as surely as a laboratory does. A stance is not a human, so a description with every human word taken out has the stance left unnamed, and the stance is still there in every finding the description rests on.

Side by Side, Volume Two, chapter "The Structure and the Act", section IV, "Why a Description Free of Humans Still Has a Stance".

The mathematical description of physics is very powerful, it is never complete, and both come from one source: "The mathematical description of physics comes as close to the physical process as one likes, and never

becomes it". A description is symmetric with what it describes, the same structure shown in another setting, and so, by the theorem, a gap always stands between the description and what it describes. Every improvement of a theory brings more of the world into the model, and leaves a world that still goes beyond the model.

What Cannot Be, chapter 20, "What Physics Cannot Reach", sections "Why Mathematics Works in Physics, and Why It Is Never Complete", "The World Exceeds the Model" and "The Model Exceeds the World".

The part of the world that every model leaves over is where the doing of physics lives: the judgment of which measurement to make, the noticing that a result does not match, the choice of which oddity to chase. "The blind spot of a theory is where its practice lives".

What Cannot Be, chapter 20, "What Physics Cannot Reach", section "The Blind Spot Is a Place to See From".

I keep the view from nowhere, the view that holds the same from every place, in which the laws of physics are stated. The laws stated there were found by comparisons made from stances inside the world, and no stance is named in them because what they say is the same from every stance. "a description of the whole world from a vantage outside it has no comparison behind it, because the whole has no outside to compare it with"; so the work of physics runs from the findings to the description, and back to the findings for the next check.

Physics Is Not Math, chapter 3, "Truth Has a Frame", section VI, "The View from Nowhere"; and *Measurement; The Steps from Comparison to Physics*, chapter 2, "Every Measurement Is a Comparison", section II, "From Inside the World".

On my account, gravitation is the arena itself, space and time as found in comparisons, and the arena does not come in steps: "Gravitation is not quantized". Physicists describe a small ripple of space and time by quanta and call the smallest packet of that description a graviton; on my account a graviton names at most a step of that description, and never a piece of space and time itself. Tegmark's example question asks for the equations of a gravitation that comes in steps. I hold that gravitation, the arena itself,

is smooth at every scale, so for me the work Tegmark's question points to is the work of measuring how smooth space and time are, at finer and finer scales.

Two Theories, One World, chapter 14, "Waves Without Particles of Space", opening and section "The Small Waves of Geometry"; and chapter 27, "What the World Will Show", section "Space and Time Will Stay Smooth", which says: "no experiment will find a least length, area or duration in space and time".

The present is the act of comparison itself, the place where possible futures become what happens.

Side by Side, Volume Two, chapter "The Open Future", section III, "Where the Present Is".

Hard randomness is real, and the theorem guarantees it.

What Cannot Be, chapter 6, "What Is a Cause, Then?", section "Why the Theorem Leaves Causes Standing".

The order of depth is metaphysics, then physics, then computation, then mathematics, each standing on the one before it: every proof is checked by a run, every run is a physical process, and every finding of physics is a comparison made from inside the world. Mathematics stands last in that order and keeps every power it has; it is the finest instrument there is for what is laid out as structure.

Side by Side, Volume Two, chapter "The Structure and the Act", section III, "The Step at Which They Part"; and *Physics Is Not Math*, chapter 11, "Why the Order Surprises", section VII, "What Each Discipline Keeps".

The history of the cosmos is pieced together from the light and matter that reach us now. What we call the Big Bang is the earliest point that the rebuilt history reaches: the turn that began our epoch, as that turn appears in the history.

What the Beginning Left Behind, chapter 1, "Where the Last Book Left Us", section "Summary".

The dark matter is the gyration of the six held internal dimensions: a lasting vibration of the geometry of the six, as a struck crystal rings, each gyration weighing about a Planck mass, about twenty-two millionths of a gram.

What the Beginning Left Behind, chapter 4, "Caught by the Floor", section "What the Account Asks of the Turn".

The dark energy is exactly a cosmological constant: an energy of empty space, one number, fixed once and the same everywhere and at every time of our epoch.

What Comes After the Dark, chapter 1, "What Happens to the Sky We Can See?", section "Faster, Not Slower".

Our epoch ends in the long emptying of its sky and then in a turn that begins the next epoch under the same law: "the long emptying is one stage of the world, and not the end of it".

What Comes After the Dark, chapter 12, "What Comes After the Dark?", section "What This Means for You"; and chapter 10, "Could Anyone Escape?", section "An Epoch That Turns, Not a Universe That Dies".

Level I, the regions beyond our horizon, is further parts of one world, with the same laws, and the two accounts agree on it in substance. The regions of Level II do not occur, because my account has no long inflation and one internal space, with no range of alternatives among which regions could differ. The many worlds of Level III drop the act of comparison, and I keep it: there is no physical collapse, and each single outcome is real. Level IV falls with the hypothesis that our world is a mathematical structure, because a structure that no comparison could ever find is laid out and not found, and so it is no further world.

Side by Side, Volume Two, chapter "The Structure and the Act", section VI, "The Four Levels, Level by Level", and section IX, "Summary".

The block universe, all of space and time laid out at once with no moment singled out as now, "takes geometric time for the whole, and drops event time": geometric time is time taken as one direction of the geometry, and event time is the one-way order of findings.

Side by Side, Volume Two, chapter "The Open Future", section III, "Where the Present Is", quoting *The Basis of Physics*.

Consciousness is the bound triple taken as one act: the arrow of time, hard randomness and the subjective, bound together; perceptronium is an exact description of conscious matter from outside.

Side by Side, Volume Two, chapter "The Structure and the Act", section VII, "Perceptronium and the Bound Triple".

The world is not a computation, on three independent grounds.

Side by Side, Volume Two, chapter "The Structure and the Act", section V, "The Computable Universe".

I reach Tegmark's answer to whether we are simulated, "No!", on my own ground, in the words of *What Cannot Be*: "In the visible universe there is one level only, the physical, and we are not dealing with a simulation".

What Cannot Be, chapter 21, "Are We Living in a Simulation?", section "One Level Only".

Among the moves the position offers is "the move that neither player has yet thought of", and it is not on the board.

What Cannot Be, chapter 4, "Process, Pattern and Possibility", section "Three Ways of Looking at a Game"; and *Comparison; The Necessary Basis*, chapter 7, "The Order of Finding", section I, "A Game of Chess".

The exact statements and values for this chapter are these.

The model exceeds the world as the world exceeds the model. A model is structure through and through and carries relations among its own parts that stand for nothing in the world, as the choice of where to put zero belongs to the numbers of a map of hills and to nothing in the hills.

What Cannot Be, chapter 20, "What Physics Cannot Reach", section "The Model Exceeds the World".

Gravitation described as an effective theory (J. F. Donoghue, "General relativity as an effective field theory: The leading quantum corrections", *Physical Review D* 50, 3874, 1994): within its range of gentle curvature and small waves, quantum theory applied to the description of the waves gives the graviton as a quantum of the description. The quantum corrections to Newton's law at a distance of one meter are about one part in a number seventy digits long.

Two Theories, One World, chapter 14, "Waves Without Particles of Space", section "The Small Waves of Geometry".

The ratio of dark to ordinary matter is $6 - 2/\pi = 5.3634$, and the equation of state of the dark energy is exactly -1; the section for chapter 4 above gives both with their locations and standing.

The sealed record holds the predictions that gravitation is not quantized and that the arena has no least length, area or duration, and the rule by which a result counts; the appendix "What the Coming Measurements Will Find" gives them.

Reading Beside Tegmark

I follow Max Tegmark's *Our Mathematical Universe* chapter by chapter. The table sets each part of this book beside the chapter or chapters of his book that it reads, and, after the word "with", beside each further chapter of his book whose sections it cites. The third column gives the scene that the part opens on, and the fourth gives the step that the game of chess at the kitchen table takes in it.

This book	Tegmark's book	The opening scene	The chessboard
Preface	The book as a whole, in its three parts, "Zooming Out", "Zooming In" and "Stepping Back"	On 23 September 1846 Johann Gottfried Galle, at the Berlin Observatory, receives a letter from Urbain Le Verrier in Paris; that night Galle and Heinrich d'Arrest compare the stars in the telescope with a chart and find the planet close to the place Le Verrier had calculated.	Named only: a game of chess at a kitchen table, with a score sheet, the written list of the game's moves, lying beside the board.

This book	Tegmark's book	The opening scene	The chessboard
1. What Is Reality?	Chapter 1, "What Is Reality?"; with chapter 10, "Physical Reality and Mathematical Reality", the section "The Mathematical Universe Hypothesis"	Galileo Galilei's *Il Saggiatore*, *The Assayer*, comes from the press of Giacomo Mascardi in Rome in October 1623, with the passage on the book of the universe written in the language of mathematics.	The board enters: two people play at a kitchen table, with a score sheet beside the board; there are three ways of looking at one game, the score sheet, the move and the seat; and Tegmark's question becomes whether the game is the score sheet.
2. Does Space Go On Forever?	Chapter 2, "Our Place in Space"; with chapter 6, "Welcome to the Multiverse", the section "The Level I Multiverse"	On 3 November 1823 János Bolyai writes from Temesvár to his father that out of nothing he has created a new, different world: a geometry in which many straight lines through a point never meet a given line.	A lamp hangs above your chair and lights a circle of floor with your seat at its center; how far the floor goes is found from that seat, by looking.

This book	Tegmark's book	The opening scene	The chessboard
3. Where Did It All Come From?	Chapter 3, "Our Place in Time"	Arno Penzias and Robert Wilson, at Holmdel, New Jersey, from July 1964 to April 1965, measure a leftover hiss in their horn antenna, the same in every direction.	A game is found half played in an empty room; the score sheet is rebuilt backward from the rules and the position on the board, and the first move is where the rebuilding reaches its edge, while the table, the board and the pieces were in the room before that move.
4. What Is the Universe Made Of?	Chapter 4, "Our Universe by Numbers"	Fritz Zwicky finds the galaxies of the Coma cluster moving too fast for their visible matter to hold them, in a paper published in 1933.	None: the picture rests in this chapter.
5. What Put the Bang in the Big Bang?	Chapter 5, "Our Cosmic Origins"; with chapter 4, "Our Universe by Numbers", the section "Where Did Our Big Bang Come From?"	In December 1979 Alan Guth, a visitor at the Stanford Linear Accelerator Center, writes down at home in Menlo Park the equations of a supercooled universe, and the next day writes "SPECTACULAR REALIZATION" across the top of the page.	The two halves of every board start alike because every game starts from one setting-up, and not because the pieces of the two halves once reached across the board and arranged each other.

This book	Tegmark's book	The opening scene	The chessboard
6. Is There Another Copy of Me Out There?	Chapter 6, "Welcome to the Multiverse", the sections "The Level I Multiverse" and "Multiverse Halftime Roundup"; with chapter 11, "Is Time an Illusion?", the section "When Are You?", and chapter 13, "Life, Our Universe and Everything", the section "How Big Is Our Physical Reality?"	John Charlewood prints Giordano Bruno's *De l'infinito, universo e mondi* in London in 1584, with the name of Venice on its title page.	At another table across town the same opening is played, move for move: two games, and not one game in two places.
7. Why Are the Constants What They Are?	Chapter 6, "Welcome to the Multiverse", the section "The Level II Multiverse"; with chapter 13, "Life, Our Universe and Everything", the section "How Big Is Our Physical Reality?"	Johannes Kepler, teaching in Graz, nests the regular solids between the paths of the planets in a scheme published at Tübingen in 1596; and in *Astronomia nova*, 1609, he keeps a miss of eight minutes of arc against Tycho Brahe's positions of Mars.	The rules are the same at every table, and the position on the board is the history of this game, so each constant of nature is asked: rule, or position? Tegmark's Level II becomes a hall of tables, each with house rules of its own.

This book	Tegmark's book	The opening scene	The chessboard
8. What Happens When We Look?	Chapter 7, "Cosmic Legos"; with chapter 11, "Is Time an Illusion?", the section "When Are You?"	Walther Gerlach and Otto Stern, in Frankfurt early in 1922, send silver atoms through a shaped magnet and find two traces on a glass plate, with no silver between them.	The rules and the position allow a range of legal moves; one move is played, from one seat, now; the score sheet records one line, and the next range begins from the position that line leaves.
9. Am I Splitting Into Copies of Myself?	Chapter 8, "The Level III Multiverse"	Hugh Everett III's paper on the relative state is printed in *Reviews of Modern Physics* in July 1957, with a note added in proof that answers the argument that no one feels the splitting.	If every legal move were played, each on a board of its own, the score sheet would have no single next line, and the move itself, the act of playing one line out of the many that the position allows, would be gone.

This book	Tegmark's book	The opening scene	The chessboard
10. Is the World Made of Mathematics?	Chapters 9 and 10, "Internal Reality, External Reality and Consensus Reality" and "Physical Reality and Mathematical Reality"; with chapter 11, "Is Time an Illusion?", the section "How Can Physical Reality Be Mathematical?"	Adolf Anderssen plays Lionel Kieseritzky in London in 1851, the game later called the Immortal Game, and Kieseritzky publishes its moves in *La Régence*.	The score sheet of the Immortal Game exists, the same in wood, marble or notation; the game played in 1851 happened once, from two seats.
11. Is Time an Illusion?	Chapter 11, "Is Time an Illusion?"	Albert Einstein writes from Princeton on 21 March 1955 to the son and the sister of his friend Michele Besso, after Besso's death.	The finished score sheet holds the whole game at once; at the board the next move has not been made.

This book	Tegmark's book	The opening scene	The chessboard
12. What Am I?	Chapter 11, "Is Time an Illusion?", the sections "What Are You?" and "Where Are You? (And What Do You Perceive?)"; with chapter 1, "What Is Reality?", the section "The Journey Begins"; chapter 8, "The Level III Multiverse", the section "Why Your Brain Isn't a Quantum Computer"; and chapter 13, "Life, Our Universe and Everything", the section "The Future of Life"	Gottfried Wilhelm Leibniz, in Vienna in the summer of 1714, writes the passage of the *Monadology* that enlarges a thinking machine into a mill you can walk through.	A record of every firing of every nerve cell in one player's brain is a description of the play from outside the seat.

This book	Tegmark's book	The opening scene	The chessboard
13. Do All Mathematical Structures Exist?	Chapter 12, "The Level IV Multiverse", the sections "Why I Believe in the Level IV Multiverse", "Exploring the Level IV Multiverse: What's Out There?", "Implications of the Level IV Multiverse" and "Testing the Level IV Multiverse"	Srinivasa Ramanujan writes from Madras to G. H. Hardy at Cambridge on 16 January 1913, enclosing pages of formulas.	A book on the shelf above the table lists every game the rules allow; the book exists, and every game is played at the table, one move at a time, and none in the book.
14. Is the World a Computation?	Chapter 12, "The Level IV Multiverse", the sections "Exploring the Level IV Multiverse: What's Out There?" and "Are We Living in a Simulation?"; with chapter 11, "Is Time an Illusion?", the section "When Are You?"	Kurt Gödel makes a short remark at the round-table discussion of the conference in Königsberg on 7 September 1930: if classical mathematics is consistent, some true statements about whole numbers cannot be proved within it.	A chess computer on the table lists the legal moves; its listing is a run, current from the kitchen wall flowing through its circuits, and the machine is in the room.

This book	Tegmark's book	The opening scene	The chessboard
15. How Will Our Universe End?	Chapter 13, "Life, Our Universe and Everything", the section "The Future of Our Universe: How Will It End?"	William Thomson's paper on the dissipation of mechanical energy is communicated to the Royal Society of Edinburgh on 19 April 1852.	The game ends; the rules do not; the next game starts from the setting-up again.
16. What Do I Now Hold?	Chapter 13, "Life, Our Universe and Everything", the sections "How Big Is Our Physical Reality?", "The Future of Physics" and "The Future of You: Are You Insignificant?"; with chapter 1, "What Is Reality?", the sections "Not What It Seems" and "What's the Ultimate Question?", and chapter 11, "Is Time an Illusion?", the section "Where Are You? (And What Do You Perceive?)"	Galileo's *Il Saggiatore*, Rome, 1623, the scene of chapter 1, with its passage on the book of the universe read a second time in Stillman Drake's English.	The last picture of the book: a hand rests above the board, and the next move has not yet been made.

Glossary

Every word in this glossary is explained in plain words in the Preface, in the chapters or in the appendix "What the Coming Measurements Will Find", so each entry in this glossary is only a reminder of that explanation. The third column shows where you first meet the word. Where you first meet a word without its explanation, the third column also shows the chapter that explains the word. The first column shows the word this book uses for a thing. When Max Tegmark's book *Our Mathematical Universe* uses a word in the sense of this book's word, the second column shows Tegmark's word. Where the two books use the same word, the word stands in both columns, and where Tegmark's book has no word for the thing, the second column is empty. The rows are in the alphabetical order of this book's words. I left out of this glossary the everyday words that a chapter explains in passing, and the names of people, places, instruments and books.

A reader of both books needs to take care with two words, because Tegmark's book and my books use each of the two words in a different sense. The first is "real". Tegmark uses the word reality for the ultimate nature of the outside physical world that we are part of. This book uses the word real only for what is given in an act of finding, once, so wherever this book reports Tegmark's word reality, this book says that the word is Tegmark's. The second is "transcendent". Tegmark uses the phrase "transcendent structure" for one underlying structure of which mathematical structures, formal systems and computations would be aspects (his chapter 12, under the heading "The Transcendent Structure of Level IV"). This book does not use the word transcendent. My physics books of the Immanent Metaphysics, which build physics from one act, the act of comparison, call themselves "the collection". My books use the word tran-

scendent for something different: the role of the stance, the side from which an act is made, together with the range of what could be found from that side.

This book's word	Tegmark's word	Where you first meet it	In plain words
absolute zero	absolute zero	Chapter 3	The lowest temperature there is.
adiabatic		Appendix, "What the Coming Measurements Will Find"	Said of first ripples of one shared kind, in which every ingredient of the early universe rippled together.
algebra of comparison		Chapter 7	A stock of quantities with rules for adding and multiplying them, which describes what comparisons find.
amplitude of clustering		Appendix, "What the Coming Measurements Will Find"	One number for how strongly matter is gathered into clumps today.
antimatter; antiparticle	antiparticle	Chapter 7 (antimatter); chapter 8 (antiparticle)	Each kind of particle has its antiparticle, a twin of the particle with every charge reversed. Antimatter, made of antiparticles, is matter's mirror twin.
arena		Chapter 2	Space and time as found in comparisons, the place where findings happen. The arena is smooth and uneven, and the shape of the arena is gravity.

This book's word	Tegmark's word	Where you first meet it	In plain words
arrow of time	arrow of time	Chapter 12	An order that runs one way, binding past to present to future in one thread without a break, as the notes of a piece of music come one after another and never back. The arrow of time is one of the three parts of the bound triple: the arrow of time, hard randomness and the subjective.
asymmetry		Chapter 1	A difference from one place or moment to the next, as no stretch of a coastline repeats another. Where chapter 10 sets out the Incommensuration Theorem step by step, asymmetry is a different content where the context differs.
axioms of comparison, the three		Chapter 13	The three starting statements about what any comparison is, on which my books build. The third axiom states that content and context are distinct and inseparable (chapter 15).

This book's word	Tegmark's word	Where you first meet it	In plain words
axion		Appendix, "What the Coming Measurements Will Find"	The light particle that physicists have proposed to keep a symmetry of the strong force (the force that holds the nucleus together), and have proposed as well as the dark matter. On my account no axion will be found.
baggage	baggage	Preface	Tegmark's word for every word and idea that only humans use, such as the words particle, star and observation. Tegmark holds that a complete description of a reality independent of humans must be free of all baggage (chapter 10).
baryons	baryon	Chapter 8	Wholes of three quarks, such as the proton and the neutron. Antibaryons are the antiparticles of baryons.

This book's word	Tegmark's word	Where you first meet it	In plain words
basic pieces		Chapter 2	The fewest building blocks from which a shape can be put together. A simple ball is built from one basic piece. Chapter 4 pictures every way of holding the two answers of a two-way choice at once as a point on a globe, and that globe is built from two basic pieces: a single point, and a disc whose whole rim is drawn together to that point.
Bell's inequality		Chapter 13	The limit that John Bell set in 1964 on how closely the results at two separated stations can agree, if every outcome is fixed by causes shared from the past or by a signal no faster than light. Experiments find results that agree more closely than Bell's limit allows.
beta decay		Appendix, "What the Coming Measurements Will Find"	The decay in which a nucleus gives off an electron and an antineutrino, the neutrino's antiparticle.

This book's word	Tegmark's word	Where you first meet it	In plain words
Big Bang	Big Bang	Preface (Tegmark's sense); chapter 3 (this book's sense)	In Tegmark's sense, the hot and dense early state from which our universe grew. In this book's sense, the turn as it appears in the history that we piece together from inside our epoch: the beginning of our epoch, and not the beginning of everything.
Big Crunch	Big Crunch	Chapter 15	Tegmark's name for an ending in which the expansion reverses and everything falls back together. On my account the expansion never stops, so no Big Crunch comes.
Big Rip	Big Rip	Chapter 15	Tegmark's name for an ending in which the rate of expansion grows without limit, and galaxies, planets and atoms are torn apart. A Big Rip needs phantom energy. On my account the dark energy has an equation of state of exactly minus one, so no rip comes.

This book's word	Tegmark's word	Where you first meet it	In plain words
Big Snap	Big Snap	Chapter 14	Tegmark's name for a breaking of space when space is stretched too far, if space should prove to be grainy. On my account space has no least step, so space has no grain to snap.
bird's view	bird perspective	Chapter 10 (Tegmark's "bird perspective"); chapter 11 ("bird's view")	Tegmark's view of the world from no observer's place, as a mathematician sees the world when studying its equations: one four-dimensional spacetime that simply is (chapter 11). In chapter 16 I read the bird's view as a description drawn from many findings made inside the world.
bit	bit	Chapter 8	The smallest amount of information, the answer to one yes-or-no question.
black hole	black hole	Chapter 3	A region where matter is squeezed so tightly that not even light climbs back out.

This book's word	Tegmark's word	Where you first meet it	In plain words
block universe		Chapter 11	All of spacetime laid out at once, with no moment singled out as now. In the block universe the geometry of time is the whole of time, and the one-way order of findings is left out.
Boltzmann brains	Boltzmann brain	Chapter 11	Brains with your memories, assembled by chance from the faint warmth of empty space, if empty space lasts forever. Boltzmann brains are named after the physicist Ludwig Boltzmann.
Born rule		Chapter 9	The rule by which physicists compute the odds of each outcome, by squaring the size of the wavefunction. For a thing whose measurements can have three or more distinct outcomes, Andrew Gleason proved in 1957 that only one recipe of weights satisfies two conditions, and that recipe is the recipe of quantum mechanics. The first condition is that each outcome has one weight, whichever measurement the outcome is found in. The second condition is that the weights of outcomes that exclude one another add up. In 2003

This book's word	Tegmark's word	Where you first meet it	In plain words
			Paul Busch proved that the recipe of quantum mechanics is again the only recipe that satisfies the two conditions, for any number of outcomes, two included, when the two conditions hold also for the outcomes of unsharp measurements. For a thing in one definite state, described by a single wavefunction and not by a blend of several wavefunctions, the recipe of quantum mechanics is the Born rule.
bound triple, the		Chapter 9 (explained in chapter 12)	The arrow of time, hard randomness and the subjective, bound into one act: distinct, inseparable and not interchangeable.
braid	braid	Chapter 11 (explained in chapter 12)	Tegmark's picture of a person: the paths of all the particles of a body, wound around one another through spacetime like the strands of a braid.
budget of the universe		Chapter 4	The division of the total energy of the universe among its ingredients.

This book's word	Tegmark's word	Where you first meet it	In plain words
carried away alone		Chapter 15	The end that comes to a piece of matter that is carried beyond the reach of all other matter, with nothing left within its reach to compare against.
Cepheids	Cepheid variables	Chapter 2	Pulsing stars that pulse more slowly the brighter they are, so that their rhythm gives their true brightness.
channel		Chapter 7	In the word of *The View from Inside*, one of the routes by which the vacuum reaches a measurement, the routes through which a measurement takes in what it finds. A measurement has three channels, one in each of the three roles that every comparison has.
character		Chapter 8 (explained in chapter 10)	One of the four ways a comparison can find its content and its context together: continuity, discontinuity, symmetry and asymmetry.
circumstance		Chapter 12	In my use, the set of physical conditions under which an act takes place.

This book's word	Tegmark's word	Where you first meet it	In plain words
classical computation	classical computation	Chapter 12	The kind of computation ordinary digital machines carry out: one rule applied the same way at every step.
cluster of galaxies	galaxy cluster	Chapter 2 (explained in chapter 4)	A swarm of galaxies held together by its own pull, such as the Coma cluster.
cold dark matter	cold dark matter	Chapter 4	The dark matter of the standard picture of cosmology, the model that cosmologists fit to their measurements: slow moving, and acting only by its pull.
collapse	collapse	Chapter 8	In the textbook rule of quantum physics, the abrupt and random jump of the wavefunction to the one outcome that is found. On my account collapse is no physical process: collapse is a new line written in the shared record of what has been found, as a score sheet takes a new line when a move is made, and the shared record now holds the outcome.

This book's word	Tegmark's word	Where you first meet it	In plain words
comparer	observer	Chapter 6	Any position inside the world from which a comparison is made, such as a glass plate, a counter, an atom that takes in light, or a person. A comparer need not be a mind.
comparison; comparing	observation; measurement	Preface	Telling one thing from another, or finding two things the same. Every measurement and every interaction is a comparison.
complex		Chapter 13	Said of a system with some property that no data could predict, because the changes of the system are continuous and carry an element that no law fixes. A complex system differs from a complicated system in what the system is, and not in what anyone knows of the system: the difference is a difference of being, not of knowledge.
complicated		Chapter 13	Said of a system with many parts whose relations can all be stated in finite terms. The unpredictability of a complicated system is a lack of data.

This book's word	Tegmark's word	Where you first meet it	In plain words
Computable Universe Hypothesis	Computable Universe Hypothesis	Chapter 14	Tegmark's proposal that every relation in the structure that is our world can be worked out by a procedure that is sure to stop. On my account the world is not a computation.
computation	computation	Preface	A procedure that goes, step by step and by fixed rules, from one state to the next, as a computer program does. In chapter 14 each state of a computation is a finite string of symbols, such as ones and zeros.
consciousness	consciousness	Chapter 1 (Tegmark's word); chapter 12 (this book's account)	On my account, the bound triple taken as one act. Tegmark holds consciousness to be the way information feels when the information is processed in certain complex ways.
conserved quantity	conserved	Chapter 8	An amount that stays the same in total through every change, such as energy, momentum and electric charge.
consistent		Chapter 14	Said of a formal system that never proves a statement together with its opposite.

This book's word	Tegmark's word	Where you first meet it	In plain words
constants of nature	constants of nature	Preface	The fixed numbers in the laws, such as the strength of gravity.
content		Chapter 1	What a comparison finds the same or different, as the weight or the color of two stones held one in each hand.
context		Chapter 1	The respect in which the two things compared are two: two places, two moments or two possibilities. Place, time and possibility are the three contexts in which anything is ever found.
continuity		Chapter 1	Having no gaps, as a ramp rises with no step. Where chapter 10 sets out the Incommensuration Theorem step by step, continuity is the same content where the context is the same.

This book's word	Tegmark's word	Where you first meet it	In plain words
continuum	continuum	Chapter 6	Tegmark's word for the infinitely small: the idea that every stretch of space holds infinitely many points, that space can be stretched without end, and that some quantities in nature can vary smoothly, with no least step. In chapter 14 the continuum is the smooth whole whose values no list can contain in full, in which between any two values there is always another.
convergence		Chapter 3	Independent routes, resting on different premises, arriving at one value.
Copenhagen interpretation	Copenhagen interpretation	Chapter 8	The textbook rule of quantum physics taught by Bohr and Heisenberg: a measurement gives one outcome, and the wavefunction serves to compute the odds of each outcome (chapter 9).
cosmic address	cosmic address; cosmic postal code	Chapter 7	Tegmark's name for measured constants read as part of an address that tells which region we are in. This book sorts the constants into three groups instead: rules, position, and values of this world read by measurement.

This book's word	Tegmark's word	Where you first meet it	In plain words
cosmological constant	cosmological constant	Chapter 4	The one constant term in Einstein's equations, which drives the expansion apart and is the same at every place and time of our epoch. On my account the cosmological constant is the dark energy: the part of the energy of empty space that a measurement made from inside the universe takes in.
cosmological horizon		Chapter 3	The edge beyond which no light can ever reach us. The cosmological horizon differs from the horizon of chapter 2, which marks how far light has reached us so far.
cosmology	cosmology	Preface	The science of the universe as a whole.
count of light species		Appendix, "What the Coming Measurements Will Find"	The measure of how much radiation the young universe held, counted in units of one kind of neutrino.

This book's word	Tegmark's word	Where you first meet it	In plain words
criticality		Chapter 9 (explained in chapter 12)	A state in which a small disturbance can cascade into a large one, the poise between order and disorder, as a pile of sand stands where one more grain can set off an avalanche of any size. On my account the undecided part of a brain's choice lies in sensitive dependence near criticality (chapter 9).
crossover		Appendix, "What the Coming Measurements Will Find"	A change of state that happens smoothly, with no sudden jump, as butter softens on a warm counter.
curvature	curvature	Chapter 2	How a space curves, which the angles of its triangles measure.
dark energy	dark energy	Chapter 4	The evenly spread seventy percent of the energy of the universe. On my account the dark energy is one number, the cosmological constant.
dark matter	dark matter	Chapter 2	Matter that pulls by its weight and gives off no light. On my account the dark matter is the gyration of the six held internal dimensions (chapter 4).

This book's word	Tegmark's word	Where you first meet it	In plain words
Death Bubbles	Death Bubbles	Chapter 15	Tegmark's name for an ending in which space changes into a different state of itself inside bubbles that spread at the speed of light. On my account the six held internal dimensions have one vacuum, and no second vacuum for a bubble to lead to.
decoherence	decoherence	Chapter 9	The fading of interference between the alternatives of a thing as the surroundings of the thing come to carry records of which alternative the thing is in. On my account ordinary decoherence is made of comparisons by the surroundings, and each of those comparisons lays down a lasting record, with one outcome.
departures from uniformity		Appendix, "What the Coming Measurements Will Find"	The small ripples of density in the early universe, about one part in a hundred thousand.
determinism	determinism	Chapter 6	The view that the state of the world at one moment, with the laws, fixes every later state completely. On my account no world is purely and completely deterministic.

This book's word	Tegmark's word	Where you first meet it	In plain words
deterministic	deterministic	Chapter 6	Said of a world, or of a computation, whose next state is fixed by its present state and its rule. A deterministic computation run twice from the same start gives the same steps (chapter 14).
deuterium	deuterium	Chapter 3	Heavy hydrogen, whose nucleus is one proton bound to one neutron.
dimension	dimension	Chapter 2	An independent way in which things can differ. A city map has two dimensions, time has one, and space has three.
Dirac mass		Appendix, "What the Coming Measurements Will Find"	The kind of mass a particle has when it differs from its own antiparticle, as the electron does.
discontinuity		Chapter 1	Coming in separate steps, as a staircase does. Where chapter 10 sets out the Incommensuration Theorem step by step, discontinuity is a different content where the context is the same.
dissipate	dissipate	Chapter 15	To let energy spread out as warmth, in a form that can drive nothing more.
diversity	diversity	Preface (explained in chapter 7)	Tegmark's word for different regions having different constants.

This book's word	Tegmark's word	Where you first meet it	In plain words
domain		Chapter 2	In my word, a whole world within which comparisons can be made. Our epoch formed from one domain (chapter 5): one undivided whole in which no comparison can tell one part from another.
early horizon		Chapter 5	The horizon near the turn: the farthest distance that light could have traveled in the short time after the turn (chapter 15).
effective law	effective law	Chapter 7	In Tegmark's use, the law that observers find in their own region, which belongs to one solution of the fundamental equations.
electromagnetic force	electromagnetic force	Chapter 5	The force between charges, which light carries. This book calls the number that measures the strength of the electromagnetic force the strength of light (chapter 7).
electronvolt		Appendix, "What the Coming Measurements Will Find"	The energy an electron gains in crossing one volt.

This book's word	Tegmark's word	Where you first meet it	In plain words
electroweak and strong transitions		Appendix, "What the Coming Measurements Will Find"	Two changes of state that the young universe passed through as it cooled. On my account both transitions were crossovers.
entanglement		Chapter 12	The joining of two or more parts into one quantum state, in which no part has a state of its own.
entropy	entropy	Chapter 15	A count of the arrangements of a thing's parts that leave the thing looking the same from outside. On my account every entropy is a count made against a context.
epistemic		Chapter 1	Concerning knowing: how anything comes to be known, checked and held.
epoch		Chapter 3	One long age of the universe, from one turn to the next, the age in which stars, galaxies and black holes form.

This book's word	Tegmark's word	Where you first meet it	In plain words
equation of state		Chapter 4	The ratio of an ingredient's pressure to the ingredient's energy, a number that tells how the ingredient thins as space grows. Ordinary matter has an equation of state of zero, and an energy that never thins at all has exactly minus one.
equivalent	equivalent	Chapter 10	In Tegmark's definition, said of two descriptions when a correspondence between them keeps every relation.
eternal inflation	eternal inflation	Chapter 5	Andrei Linde's name for inflation that ends in most places and goes on forever in others. On my account eternal inflation does not occur.
event time; order of findings	flow of time	Chapter 11	The one-way order in which findings are made, each finding laid down as a record. Event time is real, and on my account the passing of time that you feel is the one-way order of findings, met from inside.
evolution time		Chapter 11	The smooth time of quantum physics, along which the chances of every possible finding change.

This book's word	Tegmark's word	Where you first meet it	In plain words
excess of matter		Chapter 7	The small surplus of matter over antimatter that our universe holds. The excess of matter is a number of history (of what happened in our epoch), and not a number that structure fixes.
exhaustion		Chapter 8	In the mathematician's sense, proof by checking every case one by one.
exists; existence	mathematical existence	Preface	Laid out whole, timeless and the same for everyone, as a structure is, like the list on a score sheet. Chapter 13 sets Tegmark's mathematical existence beside existence in this book's sense.
external reality	external reality	Preface (explained in chapter 10)	Tegmark's word for the physical world, which he believes would exist even if humans did not. On my account external reality is the one world, which has all three aspects: what exists, what is real and what is objective.

This book's word	Tegmark's word	Where you first meet it	In plain words
External Reality Hypothesis	External Reality Hypothesis	Preface	Tegmark's premise that there exists an external physical reality completely independent of us humans. This book holds the External Reality Hypothesis in the sense that there is a world, and the world does not depend on us.
false vacuum		Chapter 15	A settled state of empty space that lasts, and yet is not the lowest settled state.
falsifiable	falsifiable	Chapter 6	Tegmark's word for a theory that some observation could show to be wrong.
family		Chapter 7	Three particles of one type, one in each generation, as the electron and its two heavier copies make one family.
field	field	Chapter 4	A quantity spread through space with a value of its own at every place and time, as a magnet's pull is.
finding		Preface	What is found in one comparison.

This book's word	Tegmark's word	Where you first meet it	In plain words
fine-tuning	fine-tuning	Chapter 7	In Tegmark's use, many constants sitting in a narrow range that life needs, so that a small change would leave a universe with no galaxies, no stars or no atoms.
Finite Universe Hypothesis	Finite Universe Hypothesis	Chapter 14	Tegmark's still stricter proposal that the structure that is our world has only finitely many parts.
first person; view from here		Chapter 11	The view from one place inside the world, as the red dot on a street-corner map shows the place where you stand. The first person and the third person are two views of the world, and neither view replaces the other.
flat	flat	Chapter 2	Said of a space whose triangles add to 180 degrees. A space whose triangles add to more or to less than 180 degrees is curved.
flatness problem	flatness problem	Chapter 5	The puzzle of why space is so nearly flat, when in Friedmann's history a flat universe is balanced like a bicycle standing upright with no rider, so that the slightest lean away from flatness grows.

This book's word	Tegmark's word	Where you first meet it	In plain words
formal system	formal system	Chapter 13	A list of starting statements, called axioms, together with fixed rules for passing, one step at a time, from statements already proved to new ones (chapter 14).
frog's view	frog perspective	Chapter 11	Tegmark's view of the world from inside, as a three-dimensional space in which things change as time goes by. In chapter 16 I place physics in the frog's view, where findings are made.
fundamental law	fundamental law	Chapter 7	In Tegmark's use, one of the equations themselves, the same everywhere.
further parts of one world	Level I parallel universe	Preface	This book's words for Tegmark's Level I parallel universes: regions of our one space beyond our horizon, with the same laws, which belong to our one world because each region interacts with the regions next to it (chapter 6).
gamma rays; gamma-ray burst	gamma-ray burst	Chapter 6	Gamma rays are light of the highest energies. A gamma-ray burst is a brief, enormous flash of gamma rays from a galaxy far across the universe.

This book's word	Tegmark's word	Where you first meet it	In plain words
Gaussian random field	Gaussian random field	Chapter 6	The statisticians' name for a pattern drawn by chance, by one rule in every place, with the sizes of its ripples spread as the familiar bell curve spreads them.
general relativity	general relativity	Chapter 2	Einstein's theory of gravity, completed in November 1915, in which the curving of space and time is gravity.
generations		Chapter 7	Three sets of matter particles, each set a heavier copy of the one before, each with two quarks, a charged lepton such as the electron, and a neutrino.
geometric time		Chapter 11	Time laid out as a dimension, the fourth direction of Einstein's spacetime, with the length of time along every path held in one shape. Geometric time exists.
googol	googol	Chapter 13	A one followed by a hundred zeros.

This book's word	Tegmark's word	Where you first meet it	In plain words
grand unified theories		Chapter 5	A family of theories in which three of the forces of nature are one force in the great heat of the beginning, and the one force breaks apart into three as the universe cools. On my account the three forces were never one larger force.
gravitational waves	gravitational waves	Chapter 5	Ripples of space and time themselves.
graviton		Chapter 16	The physicists' name for the smallest packet in the quantum description of a small ripple of space and time. On my account the word graviton names at most a step of that quantum description, and never a piece of space and time itself.
gyration		Chapter 4	A lasting vibration of the six held internal dimensions, with a mass of about the Planck mass. On my account the dark matter is made of gyrations.
half-life	half-life	Chapter 8	The time in which half of a large number of atoms of one kind decay, a time that never changes.

This book's word	Tegmark's word	Where you first meet it	In plain words
halting problem		Chapter 14	The question, asked of any program, whether the program will ever stop. Alan Turing showed in 1936 that no program answers the halting problem for every program.
halving		Chapter 7	A step down in mass by a factor of two.
hard randomness	quantum randomness	Chapter 8	An outcome that nothing before it fixed, however completely everything before the outcome is known. Hard randomness is real in each single finding, and hard randomness is not how being copied feels (chapter 9).
heat death		Chapter 15	A final state of the universe in which every difference of warmth has evened out, every store of work is spent, and nothing is left that could make anything happen. On my account the long emptying ends in a turn instead of a heat death.

This book's word	Tegmark's word	Where you first meet it	In plain words
held internal dimensions; internal space		Chapter 4	Six further directions in which things can differ, held small at every point, whose shape fixes the kinds of particle. The held internal dimensions are directions of possibility, and nobody travels along them.
heliometer		Chapter 2	A telescope made for measuring very small angles between stars.
Higgs boson	Higgs boson; Higgs particle	Chapter 7	The particle of the Higgs field, whose presence shows that the Higgs field is there.
Higgs field		Chapter 7	A quantity with a value at every point of space, which has settled at a value that gives many of the smallest particles their masses. The vacuum value of the Higgs field is the value at which the field settles in empty space.
Hilbert space	Hilbert space	Chapter 9	The abstract space of infinitely many dimensions in which a wavefunction is drawn.

This book's word	Tegmark's word	Where you first meet it	In plain words
holographic principle	holographic principle	Chapter 15	A proposal from the physics of black holes that the part of space we can see holds at most a definite number of bits of information. On my account the bound of the holographic principle limits what a comparison can tell apart, and the bound does not make space grainy.
horizon	horizon	Chapter 2	The edge of the part of space whose light has had time to reach us: the farthest distance from which light has had time to reach us.
horizon problem	horizon problem	Chapter 5	The puzzle of why the sky has one temperature in every direction, when in Friedmann's history no signal has had time to pass between regions in opposite directions of the sky.
hypersphere	hypersphere	Chapter 2	The three-dimensional surface of a ball in four dimensions: a space that is finite and curved and closes on itself.

This book's word	Tegmark's word	Where you first meet it	In plain words
identification		Chapter 4 (explained in chapter 7)	A step in which something in the structure of comparison is read as something in geometry or in the physical world. In chapter 4, for example, I identify the dark matter with gyrations of the six held internal dimensions.
immanent		Preface	Within: the Immanent Metaphysics is worked out from inside the act of knowing, here and now, and never from a place outside the world. In my language, the immanent is the act itself, the comparing as it happens (chapter 9).

This book's word	Tegmark's word	Where you first meet it	In plain words
Incommensuration Theorem		Chapter 1	in my words: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity". The Incommensuration Theorem is a structural fact about the nature of knowing. No single act of finding can hold perfect sameness and perfect gaplessness together, nor the two opposites of sameness and gaplessness together: perfect difference and perfect separation into steps. Chapter 10 sets the Incommensuration Theorem out step by step.
inflation	inflation	Preface	A very rapid stretching of space early on, doubling its size again and again, which Alan Guth named in December 1979 and which Tegmark takes to be the cause of the Big Bang. On my account no long inflation occurred.

This book's word	Tegmark's word	Where you first meet it	In plain words
inputs		Chapter 13	Statements that a derivation takes as given, which my books set down beside the derivation, such as the statement that every point of the six held internal dimensions is like every other.
integrated information		Chapter 12	The neuroscientist Giulio Tononi's measure of how far a system's parts are joined into one whole. On my account integrated information is a property measured from outside: the measure of the circumstance (the physical conditions under which an act takes place), and not of the inside that the circumstance supports.
interaction	interaction	Preface	Anything that happens between two things as each acts on the other, a collision of two particles included. Every interaction is a comparison.
interference	interference	Chapter 9	The pattern of bright and dark bands that two paths make together when nothing has marked which path was taken.

This book's word	Tegmark's word	Where you first meet it	In plain words
interferometer	interferometer	Chapter 9	An apparatus that splits a beam into paths and brings the paths back together.
internal reality	internal reality	Chapter 10	Tegmark's word for the world as you perceive it from inside your own mind, a model stored in the brain. On my account internal reality is a summary laid out for use, like the dashboard of a car, while the act of perceiving itself is what this book calls real. This book's word real matches none of Tegmark's three realities (external reality, consensus reality and internal reality).
intrinsics		Chapter 1	The six parts that every comparison has, from a word meaning belonging to the thing itself: sameness and difference, the only two verdicts any comparison can give; the content and the context; the stance from which the comparison is made; and what the comparison is made about.
kelvin	Kelvin	Chapter 3	One degree of temperature, counted up from absolute zero.

This book's word	Tegmark's word	Where you first meet it	In plain words
kinds, the		Chapter 3	The kinds of particle: the list of the kinds of particle, which I derive from the parts that every act of comparison has; the shape of the internal space fixes the kinds (chapter 4).
landscape	landscape	Chapter 7	Every settled state the equations of string theory allow, each like a valley in a hilly country, each with its own constants. On my account the six held internal dimensions have one valley, and there is no landscape.
large world		Chapter 4 (explained in chapter 5)	A world whose space and time open out to large size, with the six further dimensions held small inside it.
last scattering		Chapter 3	The last time the light of the early universe was knocked aside by free electrons, the moment the light went free.
lean, the		Chapter 11	The small difference between the two directions of time that the decays of kaons show.
leptons	lepton	Chapter 7	The electron, its two heavier copies and the three neutrinos.

This book's word	Tegmark's word	Where you first meet it	In plain words
Level II	Level II multiverse	Preface	Tegmark's regions that inflation keeps forever out of reach, where the same fundamental equations settle into different local laws, with other values of the constants of nature. On my account Level II does not occur (chapter 7).
Level III	Level III multiverse	Preface	The branches of Hugh Everett's account of quantum mechanics, in which every possible outcome of a quantum event happens, each in a branch of its own. On my account each finding has one outcome, and whoever takes the step to many worlds leaves out the act of finding (chapter 9).
Level IV	Level IV multiverse	Preface	Every mathematical structure there is, each taken as a world. On my account every structure exists, and a structure that no comparison could ever find is mathematics and not a further world (chapter 13).
light-year	light-year	Chapter 2	The distance light travels in a year.

This book's word	Tegmark's word	Where you first meet it	In plain words
listable		Chapter 14	Said of a collection that can be set out first, second, third, with nothing left out.
long emptying	Big Chill	Chapter 3 (in full, chapter 15)	The emptying of every observer's sky, as the expansion speeds up and carries every galaxy not held by gravity beyond the horizon. Tegmark calls the long emptying the Big Chill. On my account the long emptying is one stage of the world, and the long emptying ends in a turn.
magnetic monopole		Chapter 5	A particle carrying a single magnetic pole, a north with no south.
master list	master list	Chapter 13	Tegmark's name for the list, written out by a computer program, of every mathematical structure with a finite number of elements, each with its own line number.
mathematical structure	mathematical structure	Preface	In Tegmark's definition, a set of abstract entities with relations between them, which have no properties except those relations. In this book's words, a mathematical structure is a structure, which exists.

This book's word	Tegmark's word	Where you first meet it	In plain words
Mathematical Universe Hypothesis	Mathematical Universe Hypothesis	Preface	Tegmark's hypothesis that our external physical reality is a mathematical structure. On my account the world that comparisons find is described by mathematics and is not a mathematical structure.
matter fraction		Appendix, "What the Coming Measurements Will Find"	The share of the energy of a flat universe that matter carries today, dark matter and ordinary matter together.
measure	measure	Chapter 6	Cosmologists' name for a chosen order of counting the members of an infinite set. Different measures give different answers, and the question of which measure to choose is what Tegmark calls the measure problem.
mechanical energy		Chapter 15	In the sense of William Thomson, professor of natural philosophy at Glasgow, energy that can do work, as a falling weight can drive a mill.
megaparsec		Appendix, "What the Coming Measurements Will Find"	About three and a quarter million light years.

This book's word	Tegmark's word	Where you first meet it	In plain words
metaphysics		Preface	The study of what must be so for anything at all to be known. In the order of depth of chapter 10 (metaphysics, then physics, then computation, then mathematics), metaphysics is the account of the act of comparison itself.
microscopic boundary		Chapter 15	The inner edge of the domain of comparison: the finest scale at which any comparison can find a definite state.
microtubules	microtubule	Chapter 12	The fine scaffolding inside cells.
move		Preface (the picture); chapter 1 (its meaning)	In this book's picture of a game of chess at a kitchen table, a move stands for the act of finding, made once, now: what is real.
network of strings		Appendix, "What the Coming Measurements Will Find"	Thin lines of trapped energy that some theories expect the early universe to leave behind. On my account the early universe made no network of strings.
neutrinoless double beta decay		Appendix, "What the Coming Measurements Will Find"	A rare decay in which a nucleus gives off two electrons and no antineutrino, which can happen only if the neutrino is its own antiparticle.

This book's word	Tegmark's word	Where you first meet it	In plain words
neutrinos	neutrino	Chapter 7	Particles with no charge and almost no mass, which pass through matter almost untouched. There are three kinds of neutrino.
Newton's constant		Chapter 13	The number that sets the strength of gravity.
normal order		Appendix, "What the Coming Measurements Will Find"	The arrangement of the three neutrino masses in which two are light and close together and the third is heavier.
nucleus	nucleus	Chapter 3	The small dense core of an atom, made of protons and neutrons.

This book's word	Tegmark's word	Where you first meet it	In plain words
objective; objectivity	consensus reality	Chapter 6	When three things are true of a thing together, the thing is objective in the sense that *Physics Is Not Math* gives the word (chapter 6): the thing is met in an actual interaction, many comparisons made from many places find the thing the same, and the thing can show what no one expected. In the sense that *Comparison; The Necessary Basis* gives the word (chapter 10), objective means found the same by many comparisons made from many places. Objectivity is built from many views and comes in degrees. Chapter 10 sets Tegmark's consensus reality beside objectivity.
observer moment	observer moment	Chapter 11	Tegmark's name for one brief perception: a pattern in a brain lasting about a tenth of a second.
oldest light; microwave background	cosmic microwave background	Chapter 2	The faint glow of microwaves that fills the whole sky: light set free when the young universe first became clear, stretched by the expansion of space.

This book's word	Tegmark's word	Where you first meet it	In plain words
Omnivision Assumption	Omnivision Assumption	Chapter 6	Tegmark's name for the assumption, which he rejects, that some observer can in principle observe all of reality.
ontic		Chapter 8	Belonging to what is, as distinct from what someone knows.
ontological	ontological	Chapter 9	Having to do with what there is, as in Tegmark's phrase "the tip of an ontological iceberg", which says that what we see is a tiny part of what there is.
open space		Chapter 2	The word of *The Weight of Possibility* for the three directions of space that we move through.
order of depth		Chapter 10	The order metaphysics, then physics, then computation, then mathematics, in which each discipline rests on the one before. Mathematics stands last, and mathematics keeps every power it has.
ordinary matter	ordinary matter	Chapter 2	The matter of the atoms, of the stars, the gas and ourselves, as distinct from the dark matter.

This book's word	Tegmark's word	Where you first meet it	In plain words
orrery		Chapter 3	A working model of the solar system whose gears carry each planet round at its proper pace. An orrery runs backward as easily as forward, while the solar system runs one way only.
parallax	parallax	Chapter 2	The jump of a near thing against a far background when the near thing is seen from two places, as your thumb jumps when you close one eye and then the other. The distance of a thing can be worked out from the parallax of the thing.
parallel postulate		Chapter 2	Euclid's starting assumption that through a point beside a straight line exactly one straight line can be drawn that never meets the first line.
perceptronium	perceptronium	Chapter 12	Tegmark's name for the most general substance that feels subjectively self-aware. On my account perceptronium is an exact description of the outside of conscious matter.

This book's word	Tegmark's word	Where you first meet it	In plain words
phantom energy		Chapter 15	The physicist Robert Caldwell's name for a substance whose equation of state is below minus one, a substance that grows denser as space grows. Only phantom energy gives a Big Rip.
phase; entangling phase		Appendix, "What the Coming Measurements Will Find"	The phase of an alternative of a thing is the timing of the wave that describes that alternative, counted as an angle. For two masses each held in two places, the entangling phase is the phase of the two placements in which the masses are both near or both far, less the phase of the two placements in which one mass is near and the other far.
phonon	phonon	Chapter 4	The smallest step of the ringing of a crystal: a vibration of the crystal, and not an extra thing inside the crystal.
photon	photon	Chapter 6	The smallest amount of light that can be sent or taken in: a particle of light.

This book's word	Tegmark's word	Where you first meet it	In plain words
physics collection of the Immanent Metaphysics		Preface	My books that build physics from one act, the act of comparison. This book calls the physics collection "the collection" or "my account".
pion	pion	Chapter 15	A light, short-lived particle. A neutral pion has no electric charge.
Planck length		Chapter 4	The length at which the smooth description of space and time reaches the edge of its range, about a hundred billion billion times smaller than the width of a proton. The smooth description holds at every length down to the Planck length.
Planck mass		Chapter 4	The mass that goes with the Planck length, about twenty-two millionths of a gram.
Planck scale		Chapter 3	A length far smaller than any atom, at which gravity and the quantum act on equal terms.
plasma	plasma	Chapter 3	A gas so hot that its atoms are broken into loose electrons and nuclei.

This book's word	Tegmark's word	Where you first meet it	In plain words
polarization	polarization	Chapter 5	The pattern across the sky of the directions in which light vibrates across its path.
position on the board		Chapter 3	In this book's picture of a game of chess, the position on the board stands for history: what has happened in the game being played, which the rules allow and do not fix.
positron	positron	Chapter 15	The electron's positively charged twin, the antiparticle of the electron.
power		Chapter 5	The measure of the strength of ripples that astronomers use: the strength multiplied by itself.
power spectrum	power spectrum	Chapter 4	A plot of how strongly each size of warm and cool spot in the oldest light shows up.
precision cosmology	precision cosmology	Chapter 4	Cosmology, the science of the universe as a whole, become a science of exact numbers.

This book's word	Tegmark's word	Where you first meet it	In plain words
present, the		Chapter 11	On my account, the act of comparison itself, where one of the things that could happen becomes what happens. In this book's picture of a game of chess, the present is the move being made.
principle of existence		Chapter 6	in my words: "Existence without any interaction whatsoever is indistinguishable from non-existence, and therefore is non-existence".
principle of identity		Chapter 2	When no comparison could ever tell two descriptions apart, the two descriptions are one description.
program	computer program	Preface (explained in chapter 12)	The list of instructions a computer carries out: the written set of the rules of a computation (chapter 14).
pseudo-random		Chapter 14	Made by a fixed recipe from a starting number, so that the same start gives the same numbers again.
pulsar		Appendix, "What the Coming Measurements Will Find"	A distant spinning star whose pulses arrive with very steady timing. Astronomers time arrays of pulsars over many years to listen for the slowest gravitational waves.

This book's word	Tegmark's word	Where you first meet it	In plain words
qualia	qualia	Chapter 11	Felt qualities given all at once, as the redness of a strawberry is given. One of the qualia is called a quale.
quantized	quantized	Chapter 16	Coming in quanta, separate least amounts. On my account gravitation is the arena of space and time itself, and the arena does not come in steps, so gravitation is not quantized.
quantum computer	quantum computer	Chapter 9	A machine that computes by holding its parts in superpositions for as long as a calculation takes.
quantum field theory	quantum field theory	Chapter 7	The theory of the particles and their forces.
quantum numbers	quantum numbers	Chapter 8	The numbers that are a particle's only properties, on Tegmark's account: so much charge, so much spin, so much energy.
quarks	quark	Chapter 7	The particles of which protons and neutrons are built. There are six kinds of quark.
radioactive dating		Chapter 3	Reading the age of a rock from how much of its radioactive atoms have changed into other atoms.

This book's word	Tegmark's word	Where you first meet it	In plain words
ratio of tensor to scalar power		Appendix, "What the Coming Measurements Will Find"	The power of the first waves of space and time divided by the power of the first ripples of density.
real		Preface	Given in an act of finding, made once, from one place, as a move is made. Tegmark's word reality names the ultimate nature of the outside physical world that we are part of, and wherever this book reports Tegmark's word, this book says that the word is Tegmark's.
reality	reality	Preface	Tegmark's word for the ultimate nature of the outside physical world that we are part of.
registration		Chapter 2	In the word of *The Weight of Possibility*, a measurement made inside the world, taken as the act at which a number is set.
rulebook		Chapter 8	A finite set of rules for combining the labels a particle carries; the structure of comparison fixes which rules are in the rulebook, and every kind of particle comes from it.

This book's word	Tegmark's word	Where you first meet it	In plain words
rules		Chapter 3	In this book's picture of a game of chess, the rules stand for the laws, which are the same at every table.
run		Chapter 10 (explained in chapter 14)	A computation being carried out, step after step, as a physical process: current through circuits, nerve cells firing, ink on paper.
Schrödinger equation	Schrödinger equation	Chapter 8	The law by which the quantum description of a thing, its wavefunction, changes smoothly and predictably between measurements.
score sheet	entire game	Preface (the picture); chapter 1 (its meaning)	The written list of a game's moves. In this book's picture, the score sheet stands for the structure, laid out whole and the same for everyone who reads it: what exists. Tegmark's "entire game" is the finished score sheet (chapter 11).

This book's word	Tegmark's word	Where you first meet it	In plain words
sealed record		Chapter 15	The dated list of my predictions, published before the measurements, in the appendices of *Side by Side*: *The Registered Predictions of Physics from Comparison; A Dated Record, 1 November 2026*. The sealed record is fixed at its date and is not revised.
seat at the board		Chapter 1	In this book's picture of a game of chess, a seat at the board stands for the stance, the side from which a move is made.
second of arc		Chapter 2	One 3,600th of a degree of angle.
seeds; first ripples	seed fluctuations	Chapter 3 (in full, chapter 5)	Places in the young universe a little denser than their surroundings, which gravity could grow into galaxies: faint ripples of density, slightly stronger on large lengths than on small.
self-aware substructure	self-aware substructure	Chapter 12	Tegmark's name for any part of a mathematical structure that has subjective perceptions.
self-consistent		Chapter 7	Said of a structure no part of which contradicts another part.

This book's word	Tegmark's word	Where you first meet it	In plain words
sensitivity to initial conditions		Chapter 14	The way the smallest difference in how a system starts grows until the difference governs how the whole system behaves.
setting-up		Chapter 5	The one arrangement in which the players set out the pieces before the first move. In my account the likeness of the sky comes from one setting-up at the turn.
short loop		Chapter 2	A closed path around the universe shorter than the width of the horizon. On my account space has no short loop.
simulation	simulation	Chapter 13 (explained in chapter 14)	A computation built to copy what some physical thing does.
singularity	singularity	Chapter 3	A moment of infinite density at which the equations of gravity give no answer.
smooth; uneven		Chapter 1 (smooth); chapter 2 (uneven, and both said of the arena)	Said of the arena. Smooth means that the arena changes from place to place with no jump anywhere. Uneven means that one place in the arena can differ from another.

This book's word	Tegmark's word	Where you first meet it	In plain words
soft randomness		Chapter 9	Unpredictability that comes from a limit of what someone knows, and that better knowledge could shrink, as when a copy does not know which room is its own.
spacetime	spacetime	Chapter 8	Space and time taken together as one four-dimensional whole, with time as its fourth direction.
spectral index; tilt	spectral index; tilt	Chapter 5	A number for how the strength of the first ripples changes with their length. An index of exactly one means the same strength at every length, and how far the index lies from one is the tilt. Tegmark calls the index itself the "tilt" parameter.
spin	spin	Chapter 8	The magnetism every electron carries in itself, which, measured along any line, comes in exactly two values.
stance		Chapter 1	The side from which a comparison is made, as a move is made from a seat. Every comparison has a stance, human or not.

This book's word	Tegmark's word	Where you first meet it	In plain words
standard deviation	standard deviation	Appendix, "What the Coming Measurements Will Find"	The usual measure of the uncertainty of a measurement. Chance alone would give a result five standard deviations from a prediction less than once in a million tries.
standard model of cosmology	cosmological standard model	Chapter 11	The account of the universe's history and contents that most cosmologists use today, fixed by six measured numbers. In chapter 4 I call the standard model of cosmology the standard picture of cosmology.
standard model of particle physics; Standard Model	standard model	Chapter 7	Physicists' name for the list of known particles and their forces: the theory of every kind of particle so far found.
stochastic background		Appendix, "What the Coming Measurements Will Find"	A steady random hum of gravitational waves from many sources at once.
strain		Chapter 4	The pull between the two perfect demands of every comparison, the sameness of what is found and the continuity of the comparing. The strain goes into the context, and so the arena of space and time bends.

This book's word	Tegmark's word	Where you first meet it	In plain words
strength of color		Chapter 13	How strongly the force that binds quarks acts. On my account the strength of color is a value of this world, measured.
strength of light	fine-structure constant	Chapter 7	This book's plain name for the number that measures the strength of the electromagnetic force, the force between charges, which light carries. On my account the strength of light is a value of this world, measured, which the structure of comparison does not fix.
string theory	string theory	Chapter 7	The proposal that every particle is a tiny vibrating string, different ways of vibrating giving different particles, as different vibrations of a guitar string give different notes.
Strong Self-Sampling Assumption	Strong Self-Sampling Assumption	Chapter 11	The philosopher Nick Bostrom's rule that each observer moment should reason as if it were drawn at random from a chosen class of observer moments. Tegmark's rule of the random observer moment is the narrowest case of Bostrom's rule.

This book's word	Tegmark's word	Where you first meet it	In plain words
subjective, the		Chapter 12	Being the one to whom things happen. In the words of *Causality Without Determinism*, the subjective is "the side from which experience occurs, the stance of the one to whom things happen". The subjective is one of the three parts of the bound triple: the arrow of time, hard randomness and the subjective.
supercooling		Chapter 5	A cooling far past the temperature of a change of state, before the change happens, as water can stay liquid below its freezing point.
superdeterminism		Chapter 13	The position that everything, the experimenters' settings included, was fixed together at every scale from a common past. I hold the second of the two positions still open after the tests of Bell's inequality: some of what happens is caused by nothing before it.
superposition	superposition	Preface	One thing described as holding two or more alternatives together, as a particle described as both here and there.

This book's word	Tegmark's word	Where you first meet it	In plain words
symmetry	symmetry	Chapter 1	Sameness: a thing unchanged when you swap it or move it, as a meter stick measures the same length in every country. Where chapter 10 sets out the Incommensuration Theorem step by step, symmetry is the same content where the context differs.
temporal comparison		Chapter 10	Comparison of one moment with another.
third axiom		Chapter 15	The axiom of the Immanent Metaphysics which states that content and context are distinct and inseparable: there is no content without a context.
third person; view from nowhere		Chapter 11	The view from outside, of someone describing a thing they are not part of, which is the same from every place and in which the laws of physics are stated. The philosopher Thomas Nagel called the third person the view from nowhere.
topology	topology	Chapter 2	The way a space is joined to itself.

This book's word	Tegmark's word	Where you first meet it	In plain words
torsion balance		Chapter 9	A light bar hung from a thin fiber, with small masses at its ends, which the pull of large masses set beside it twists a little.
torus	torus	Chapter 2	A flat, finite space joined to itself in every direction, like a video game screen whose right edge leads back to its left edge and whose top leads to its bottom.
totalization		Chapter 13	A claim about everything taken as one. A totalization would need a vantage outside the whole, and the whole has no outside.
tunneling		Chapter 15	In quantum physics, the passage of something held in a valley through the ridge to a lower valley, without climbing over the ridge.
turn, the		Chapter 3	The passage from the end of one epoch to the start of the next. What we call the Big Bang is the turn as it appears in the history that we piece together from inside our epoch.
twin prime	twin prime	Chapter 14	A prime with another prime two places away, as 11 and 13 are.

This book's word	Tegmark's word	Where you first meet it	In plain words
two levels (of a theory)		Chapter 1	Every physical theory has two levels: its mathematics, which exists, and its findings, which are real.
Type Ia supernovae	Type Ia supernova	Chapter 4	Exploding stars of one kind, which shine with nearly the same brightness wherever they go off, so that their faintness tells their distance.
uncertainty	uncertainty	Chapter 2	The margin of error of a measurement: the range within which the true value most likely lies. The appendix "What the Coming Measurements Will Find" writes the uncertainty with the sign $\pm$, read "give or take".
uncertainty principle	uncertainty principle	Chapter 8	Heisenberg's limit on sharpening a particle's place and its motion together. On my account the uncertainty principle is an instance of the Incommensuration Theorem.
universe, our	our Universe	Preface (explained in chapter 2)	The part of space whose light has had time to reach us, and what that part of space holds. Tegmark writes the word with a capital U.

This book's word	Tegmark's word	Where you first meet it	In plain words
unsharp measurement		Chapter 9	A measurement whose outcomes do not cut the possible states cleanly apart, as a detector does that answers "up" nine times in ten for an atom that is up, and one time in ten for an atom that is down.
vacuum	vacuum	Chapter 7	Physicists' word for the settled state of empty space, as quiet as it can be.
vacuum value		Chapter 13	The value at which a field, such as the Higgs field, settles in empty space.
value of this world		Chapter 7	A third kind of number besides rules and position, which the structure of comparison leaves free and which measurement reads.
value that lands	unexplained coincidence	Chapter 7	A number that agrees with measurement before its reason is found. This book counts a value that lands as a question, and as evidence of a reason not yet known.

This book's word	Tegmark's word	Where you first meet it	In plain words
wavefunction	wavefunction	Chapter 8	The mathematical description of a particle's state, from which the odds of every outcome are computed. On my account the wavefunction is the full description of the range of possible findings, which exists.
weak anthropic principle	anthropic principle	Chapter 7	The physicist Brandon Carter's name for reasoning from where observers can live: we can only find ourselves where we can live. The word "anthropic" comes from the Greek word for a human being.
weight		Chapter 9	In quantum physics, the share of the odds that an outcome has under a rule, as each of the two spots made by the silver atoms of chapter 9 has the weight one half.
white dwarf	white dwarf	Chapter 15	The dense, cooling core that a star like the Sun leaves behind when it has burned out.
youngness paradox	youngness paradox	Chapter 6	Alan Guth's name for his finding that, under some assumptions, an inflation that never stops predicts a much younger Earth than ours.

Where to Go from Here

Every step of this book is worked out in full in my books of the Immanent Metaphysics that the chapters draw on. These are the books to read next, with what each one gives a reader who has followed Max Tegmark's book with this one. The five books written in plain words come first, as their subtitles say; the source books follow; and Tegmark's own work comes last.

Books in Plain Words

Why the Quantum Is the Way It Is; Quantum Mechanics from the Act of Comparison, in Plain Words. The quantum of chapters 8 and 9 in full: why findings come in exact steps described by smooth mathematics, why chance enters a single outcome and why its odds are a square, and why collapse is no physical process.

What the Beginning Left Behind; The First Ripples, the First Waves and the One Vacuum, in Plain Words. The beginning of our epoch, the story of chapters 3 and 5: how the six small dimensions are held again at the turn, why the beginning was orderly, what the oldest light records with no inflation needed, and which waves the beginning left.

Why These Numbers; The Strengths, Masses and Mixings of the Particles, in Plain Words. The numbers of chapter 7 in full: the strength of light as a measured value of this world, the ladder of the masses of the charged particles, and how the quarks and the neutrinos mix.

What Cannot Be; Signals Faster Than Light, Time Travel, Simulation and Determinism, in Plain Words. What the act of comparison excludes, taken one question at a time: a world fixed in advance, signals faster than light, journeys into the past, and a world that is a simulation. Chapters 6, 8, 9, 10, 12, 14, 15 and 16 draw on it, for hard and soft randomness, for the person who is not a very large file, and for the world that is no simulation.

What Comes After the Dark; The Far Future of the Universe, in Plain Words. Chapter 15 in full: the long emptying of the sky, why the expansion neither falls back nor tears, why empty space keeps its nature, and why the long emptying ends in a turn.

The Source Books

Comparison; The Necessary Basis. The ground of the whole collection: what it is to compare, why comparison, interaction and relation are one act under three names, and the three words exist, real and objective that chapter 10 sets beside Tegmark's three realities.

Incommensuration; The Theorem and the Three Axioms. The Incommensuration Theorem of chapters 1 and 10, proved step by step from four characters and the ten ways to pair them, with the three axioms that follow from it.

Measurement; The Steps from Comparison to Physics. How the act of comparison becomes the units, laws and measured values of physics; the source of the count of three directions of space and one of time in chapter 7, and of the three times of chapter 11.

Physics Is Not Math; Metaphysics, Physics, Computation and Mathematics, and Why Their Order Cannot Be Otherwise. The order of depth of chapter 10 in full, with the orrery, the value that lands and the cable of independent strands: the book to read for why mathematics, the finest instrument there is, stands last in the order.

An Immanent Metaphysics. The whole system in aphorisms, from which every one of my other books is drawn; chapters 2, 10 and 12 quote it.

The Last Turtle; Essays from Comparison and the Incommensuration Theorem to Causation, the Arena and the Limits of Computation. Essays on what is real, what exists and what is objective, on the bound triple and hard randomness, and on what perception reaches; chapters 12 and 13 draw on it.

The Basis of Physics; Nineteen Essays on the Structural Limits of Physical Law. Nineteen essays on what physical law cannot reach: hard randomness, the arrow of time, why general relativity and quantum mechanics stay two theories, and the continuum against simulation.

Causality Without Determinism; Free Will, Measurement and Mind. The ground of chapter 12: why causes are real and determinism cannot be had, and consciousness as the bound triple taken as one act.

The View from Inside; On the Cosmological Constant and Why Empty Space Weighs So Little. The energy of empty space of chapters 4 and 7: why the mismatch of one hundred twenty-three powers of ten never arises, how the horizon is counted, and the one equation that gives matter 31.1 percent of the whole.

The Weight of Possibility; Dark Matter as a Vibration of Context. The dark matter of chapter 4 as a vibration of the six held dimensions, and the count, six against one, that sets how much of it there is beside the ordinary matter; chapter 2 takes from it the same count as the reason space has no short loop.

The Weight of the Unseen; Dark Matter in Questions and Answers, A Companion to The Weight of Possibility. The same account of the dark matter in questions and answers: what astronomers observe, what the dark matter is, why it lasts and how much of it there is; chapter 4 quotes it on the scaffolding on which the galaxies formed.

At the Edge of Reality; From Black Holes to the Big Bang in Ten Dimensions. The turn of chapters 3 and 15, told from the inside of black holes: why every ending is the same ending, and why the Big Bang looks like a beginning.

The Gravitational Passage; Deriving General Relativity from the Immanent Metaphysics. Einstein's equations derived from comparison, with the one internal space and the Planck scale as the edge of the smooth description; chapters 3, 6 and 11 quote it.

The Compactification; Initial Conditions, the Perturbation Spectrum, and the Gravitational Wave Signature. The re-forming of the internal space at the turn, its one vacuum, and what the beginning of an epoch leaves in the sky; chapter 7 takes from it the reason there is no landscape.

The Coming of the Kinds; How the Particles Came to Be After the Big Bang. The first minutes of chapter 3 told from the list of kinds: what was there, and how the universe's helium came from the neutrons.

Why Time Travel Is Not a Concept; A Meditation on the Three Times and the Cauchy Horizon. The three times of chapter 11 brought to the question of travel into the past, and the reason a record is not undone.

The Story of the Particles; Why Every Particle Is Required, and Why There Are No More. The rulebook of chapter 8: every kind of particle, level by level, from the act of comparison, and why the list of kinds is closed.

The Grand Synthesis; One Theorem from Comparison to the Particles and the Cosmos. The whole derivation in one book, from the starting principle that comparison, interaction and relation are one act, to the particles and the cosmos; chapters 7 and 13 quote it on the closed list of kinds and on the landscape reduced to a single point.

The Geometric Route; From the Three Axioms to the Lagrangian, with the Masses, the Rates and the Finite Algebra. The route from the three axioms to the laws of the particles through geometry, and the one self-consistent structure of comparison that chapter 7 sets beside the weak anthropic principle.

The Strengths of the Kinds; The Couplings, Their Crossings, and the Scales of the Proton and of Gravity, What the Structure Sets and What Measurement Supplies, Volume I. How the strengths of the forces change and cross, and why the strength of light stays a measured value; the source of the third kind of number in chapter 7.

The Matter We Know; From the First Atoms to the End of the Periodic Table, Volume One; Wholes. The ladder of chapter 8, from quarks to protons and nuclei, each level held by what the level below leaves over.

Two Theories, One World; Gravity, the Quantum and the Mistake Behind the Search for One Theory. Why the arena of space and time has no least step and gravitation is not quantized: the ground of chapters 8, 15 and 16.

Side by Side; Twenty-Four Models of the Universe, *Volume One: The Quantum and the Arena*, and *Volume Two: Computation, the Cosmos, Time and Mind*. Twenty-four models of the universe, each set beside my account in its own terms; Volume Two holds the essay "Max Tegmark: The Structure and the Act", which this book follows throughout, and Volume One reproduces the sealed record of my predictions.

Max Tegmark's Own Work

Max Tegmark's *Our Mathematical Universe; My Quest for the Ultimate Nature of Reality* is the book this one follows, chapter by chapter, and the table "Reading Beside Tegmark" sets each chapter here beside the chapter of his that it reads. His papers that the chapters name carry four of his results further. They are on why space has three dimensions and time one (1997), on how fast quantum states fade in the brain (2000), on the four levels of parallel universes, the source of chapter 6's distance to your nearest copy (2007), and, with Anthony Aguirre, on the odds of quantum outcomes read from the copies of you in an infinite space (2011). The appendix "What the Coming Measurements Will Find" cites two more: on what inflation predicts (2005), and, with Anthony Aguirre, Martin Rees and Frank Wilczek, on the constants of nature and the dark matter (2006).

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These are the works that the Preface, the chapters and the two appendices, "What the Coming Measurements Will Find" and "For the Specialist", name, quote or rest on, sorted by author. Where the text credits a result to a named person, the work in which that person first gave the result is listed; where the text reports a result only as Max Tegmark tells it, his book is the work listed. A paper in a journal is given with its volume and first page, and a book with its ISBN where it has one.

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Index

a run, 157; 176; 183; 229; 237; 272; 379; 388; 427; 448; 456

absolute zero, 42; 45; 47; 74; 94; 183; 465

act of comparison, vii; 14; 19; 21; 47; 57; 72; 97; 114; 127; 130; 132; 150; 155; 167; 168; 170; 173; 176; 178; 186; 190; 201; 203; 205; 208; 218; 220; 234; 237; 246; 266; 270; 272; 275; 277; 307; 321; 379; 388; 400; 410; 419; 427; 448; 465; 527; 528

act of finding, viii; 14; 21; 33; 39; 47; 51; 54; 61; 90; 97; 99; 102; 119; 130; 132; 138; 155; 168; 170; 178; 180; 186; 190; 197; 203; 220; 224; 229; 237; 243; 258; 268; 272; 274; 275; 303; 307; 329; 349; 369; 379; 388; 400; 419; 427; 436; 448; 465

adiabatic, 286; 465

algebra of comparison, 116; 358; 465

amplitude of clustering, 281; 465

antiparticle, 114; 132; 280; 281; 292; 300; 321; 358; 369; 465

arena, 33; 39; 40; 61; 119; 132; 142; 234; 237; 241; 243; 256; 258; 266; 270; 315; 329; 349; 358; 369; 379; 388; 427; 436; 448; 465; 528

arrow of time, 203; 205; 210; 234; 272; 298; 379; 388; 400; 410; 419; 427; 436; 448; 465; 528

Aspect, Alain, 224

Assayer, 13; 262; 456

asymmetry, 21; 39; 47; 61; 97; 119; 130; 155; 170; 186; 190; 203; 220; 258; 268; 307; 315; 349; 388; 400; 410; 427; 465

axion, 281; 292; 358; 465

baryons, 130; 280; 369; 436; 465

basic pieces, 35; 65; 119; 315; 329; 358; 465

Bell's inequality, 224; 419; 465

Bell, John, 224; 465

Besso, Michele, 180; 190; 456

beta decay, 281; 465

Big Bang, v; 28; 45; 47; 53; 54; 71; 72; 73; 74; 75; 79; 82; 88; 247; 252; 258; 272; 286; 303; 321; 340; 436; 448; 456; 465; 528

Big Chill, 246; 247; 250; 258; 261; 465

Big Crunch, 247; 252; 465

Big Rip, 247; 252; 465

Big Snap, 231; 241; 247; 256; 465

bird's view, 168; 181; 264; 270; 465

bit, 140; 213; 224; 247; 369; 419; 465

black hole, 47; 53; 65; 79; 82; 84; 88; 94; 192; 247; 250; 252; 254; 258; 286; 321; 340; 400; 436; 465; 528

block universe, 187; 272; 400; 448; 465

Bohr, Niels, 114; 127; 128; 135; 358; 369; 465

Boltzmann brains, 183; 192; 465

Boltzmann, Ludwig, 183; 192; 298; 427; 465

Bolyai, János, 27; 31; 33; 39; 456

Born rule, 150; 155; 294; 379; 465

Bostrom, Nick, 183; 231; 234; 400; 427; 465

bound triple, 157; 203; 205; 210; 234; 237; 272; 298; 410; 427; 448; 465; 528

braid, 194; 198; 208; 210; 465

budget of the universe, 58; 465

Bullet Cluster, 65

Caldwell, Robert, 252; 436; 465

Cantor, Georg, 234; 349; 427

carried away alone, 250; 254; 258; 436; 465

Carroll, Sean, 155; 157; 379

Carter, Brandon, 110; 122; 183; 358; 465

Cepheids, 28; 465

channels, 119; 358; 465

character, 21; 94; 130; 132; 170; 173; 176; 208; 237; 256; 262; 307; 369; 388; 410; 427; 436; 465; 528

chess, x; 14; 27; 42; 90; 108; 138; 162; 173; 180; 181; 197; 205; 218; 220; 229; 237; 243; 246; 272; 307; 388; 448; 456

circumstance, 205; 410; 465

classical computation, 205; 410; 465

Clauser, John, 224

clock face, 65; 329

cluster of galaxies, 28; 57; 58; 65; 329; 465

cold dark matter, 65; 281; 329; 465

Coleman, Sidney, 254; 436

collapse, 33; 79; 132; 135; 138; 140; 142; 145; 146; 150; 155; 160; 252; 254; 272; 300; 315; 340; 369; 379; 436; 448; 465; 527

comparer, 102; 137; 138; 140; 142; 150; 155; 157; 167; 190; 192; 194; 201; 266; 349; 369; 379; 388; 400; 410; 448; 465

Computable Universe Hypothesis, 231; 237; 294; 427; 465

computation, x; 176; 198; 201; 205; 220; 228; 229; 231; 234; 237; 241; 243; 272; 294; 300; 303; 349; 369; 388; 410; 427; 436; 448; 456; 465; 528

consciousness, 17; 137; 148; 150; 197; 198; 201; 203; 205; 208; 210; 272; 303; 369; 379; 410; 448; 465; 528

conserved quantity, 61; 128; 130; 329; 369; 388; 465

consistent, 220; 229; 237; 349; 400; 419; 427; 456; 465

constants of nature, v; viii; 106; 108; 116; 122; 125; 213; 220; 241; 281; 303; 358; 419; 465; 531

content, 14; 21; 33; 61; 65; 82; 100; 130; 150; 170; 237; 256; 258; 307; 315; 329; 340; 349; 369; 379; 388; 410; 419; 427; 436; 465

context, 14; 21; 33; 61; 65; 82; 102; 132; 150; 170; 208; 256; 258; 307; 315; 329; 340; 349; 358; 369; 379; 388; 400; 410; 436; 465; 528

continuity, 21; 35; 39; 47; 61; 97; 102; 119; 130; 132; 155; 170; 173; 176; 186; 190; 198; 203; 208; 220; 224; 234; 237; 256; 258; 268; 307; 315; 329; 349; 358; 369; 379; 388; 400; 410; 419; 427; 436; 465

continuum, 94; 102; 132; 231; 234; 241; 290; 349; 369; 427; 436; 465; 528

convergence, 47; 61; 168; 321; 329; 388; 465

Copenhagen interpretation, 135; 148; 203; 410; 465

Copernicus, 145; 155

cosmic address, 110; 465

cosmological constant, 61; 70; 119; 122; 250; 252; 272; 281; 315; 321; 329; 358; 436; 448; 465; 528

cosmological horizon, 47; 250; 321; 465

count of light species, 286; 465

criticality, 150; 205; 298; 379; 410; 465

cross-modal principle, 307; 369; 388; 410

crossover, 286; 465

curvature, 31; 39; 53; 58; 61; 79; 91; 119; 122; 252; 279; 281; 315; 321; 329; 340; 358; 436; 448; 465

dark energy, 58; 61; 65; 70; 72; 75; 119; 247; 250; 252; 272; 279; 281; 329; 436; 448; 465

dark matter, 35; 57; 58; 61; 65; 70; 71; 72; 84; 110; 116; 272; 281; 286; 292; 300; 315; 321; 329; 340; 358; 436; 448; 465; 528; 531

dated record, 277; 280; 286; 290; 292; 294; 298; 410; 465

Death Bubbles, 247; 254; 465

decoherence, 148; 150; 155; 160; 198; 201; 295; 298; 300; 379; 410; 465

Delft, 224

departures from uniformity, 279; 315; 340; 465

determinism, 92; 97; 99; 140; 190; 201; 203; 205; 208; 237; 290; 303; 349; 369; 400; 410; 419; 427; 465; 527; 528

deterministic, 97; 234; 237; 290; 349; 369; 379; 388; 410; 419; 427; 465

deuterium, 45; 47; 321; 465

DeWitt, Bryce, 146

Dieks, Dennis, 208; 410

dimension, 31; 39; 58; 61; 65; 72; 79; 82; 84; 91; 110; 114; 116; 119; 122; 125; 146; 164; 187; 220; 254; 258; 261; 272; 281; 286; 292; 315; 329; 340; 349; 358; 400; 419; 436; 448; 465; 527; 528; 531

Dirac mass, 281; 465

discontinuity, 21; 39; 47; 61; 97; 119; 130; 155; 170; 186; 203; 220; 258; 268; 307; 315; 388; 400; 410; 427; 465

dissipate, 246; 456; 465

diversity, v; 110; 410; 465

domain, 35; 79; 82; 88; 173; 237; 256; 307; 315; 321; 340; 388; 400; 427; 436; 465

Drake, Stillman, 13; 262; 456

early horizon, 84; 256; 286; 290; 340; 436; 465

Eddington, Arthur, 122; 358

effective law, 110; 465

Einstein, Albert, 31; 33; 43; 47; 58; 61; 65; 70; 75; 79; 114; 119; 135; 137; 140; 180; 181; 187; 190; 192; 237; 241; 250; 252; 281; 315; 329; 340; 358; 379; 400; 427; 436; 456; 465; 528

electromagnetic force, 82; 110; 116; 122; 132; 220; 286; 358; 419; 465

electronvolt, 281; 321; 329; 349; 358; 436; 465

electroweak and strong transitions, 286; 465

Ellis, George, 94

entanglement, 201; 295; 410; 465

entangling phase, 295; 465

entropy, 258; 436; 465

epistemic, 21; 170; 307; 388; 465

epoch, 53; 54; 61; 65; 70; 72; 79; 82; 84; 88; 99; 102; 116; 122; 125; 192; 250; 252; 258; 261; 266; 272; 279; 281; 286; 292; 298; 300; 300; 315; 321; 329; 340; 349; 358; 400; 436; 448; 465; 527; 528

equation of state, 70; 252; 281; 292; 300; 329; 358; 400; 436; 448; 465

equivalence, 165; 173; 181; 213; 388; 465

eternal inflation, 75; 79; 92; 94; 110; 116; 286; 358; 465

Euclid, 27; 31; 33; 218; 231; 279; 281; 419; 465

event time, 187; 190; 194; 237; 272; 400; 427; 448; 465

Everett's theory, 145; 146; 456

Everett, Hugh, v; 135; 145; 146; 148; 150; 155; 157; 160; 213; 224; 290; 379; 456; 465

evolution time, 187; 400; 465

excess of matter, 114; 321; 358; 465

exhaustion, 132; 369; 465

experiment of Gerlach and Stern, 127; 369; 456

external reality, v; 17; 19; 94; 162; 164; 165; 167; 168; 173; 213; 266; 290; 297; 307; 388; 448; 456; 465

External Reality Hypothesis, v; 17; 19; 94; 165; 167; 173; 213; 266; 307; 388; 448; 465

false vacuum, 254; 465

falsifiable, 94; 465

family, 82; 122; 294; 329; 358; 465

Fermi space telescope, 102; 132; 349; 436

field, 61; 70; 116; 119; 130; 132; 140; 201; 220; 254; 292; 300; 321; 329; 349; 358; 369; 400; 410; 419; 436; 448; 465

fine-tuning, 110; 114; 116; 119; 213; 358; 465

Finite Universe Hypothesis, 231; 465

first person, 114; 119; 190; 358; 388; 400; 419; 436; 465

first ripples, see *seeds*

flat space, 31; 33; 35; 58; 61; 74; 75; 79; 84; 88; 252; 279; 286; 290; 300; 315; 329; 340; 349; 436

flatness problem, 74; 75; 82; 465

fog, 47; 61; 91; 92; 97; 99; 100; 104; 321

formal system, 220; 229; 237; 427; 465

four characters, 170; 173; 176; 307; 388; 528

four levels, v; 213; 264; 272; 303; 349; 358; 419; 448; 531

free will, 97; 201; 528

frog's view, 181; 264; 270; 465

fundamental law, 110; 465

further parts of one world, viii; 97; 272; 303; 349; 448; 465

Galileo, 13; 17; 19; 165; 261; 262; 274; 456

gamma-ray burst, 102; 132; 241; 247; 256; 349; 369; 427; 436; 465

Gamow, George, 45; 47

Gaussian random field, 92; 465

Geiger counter, 21; 140; 187; 307; 369; 400

general relativity, 31; 94; 114; 164; 231; 358; 388; 448; 465; 528

generations, 116; 292; 297; 299; 358; 388; 419; 465

geometric time, 187; 190; 194; 272; 400; 448; 465

Gerlach, Walther, 127; 137; 138; 150; 369; 456

Gödel, Kurt, 229; 231; 237; 427; 456

googol, 213; 465

grand unified theories, 82; 340; 465

gravitational waves, 75; 84; 286; 297; 340; 349; 465

graviton, 270; 448; 465

Guth, Alan, 74; 75; 82; 94; 110; 456; 465

gyration, 65; 72; 272; 281; 329; 448; 465

half-life, 140; 369; 465

halting problem, 231; 237; 427; 465

halving, 122; 358; 427; 436; 465

Hameroff, Stuart, 150; 198; 201; 379; 410

hard randomness, 140; 142; 157; 160; 187; 203; 205; 210; 224; 234; 237; 256; 270; 272; 298; 303; 369; 379; 400; 410; 419; 427; 436; 448; 465; 528

Hardy, G. H., 212; 218; 456

heat death, 246; 258; 400; 436; 465

Heinrich d'Arrest, iv; vii; viii; 456

held internal dimensions, 65; 72; 79; 82; 116; 119; 220; 254; 258; 272; 281; 329; 340; 358; 419; 436; 448; 465; 528

heliometer, 28; 465

Higgs boson, 116; 132; 220; 254; 292; 358; 369; 436; 465

Higgs field, 116; 220; 254; 419; 436; 465

Hilbert space, 146; 369; 465

holographic principle, 247; 256; 436; 465

horizon, 28; 35; 47; 61; 65; 74; 75; 79; 82; 84; 88; 91; 92; 97; 99; 104; 106; 192; 250; 256; 266; 272; 279; 286; 290; 298; 315; 321; 329; 340; 349; 358; 400; 436; 448; 465; 528

horizon problem, 74; 75; 82; 465

Hubble, Edwin, viii; 28; 43; 303; 329; 400

hypersphere, 31; 35; 465

ICT, see *Incommensuration Theorem*

identification, 65; 84; 114; 116; 119; 150; 157; 203; 340; 358; 379; 465

the immanent, vii; 14; 47; 57; 97; 114; 127; 150; 155; 167; 186; 201; 218; 234; 246; 258; 266; 277; 303; 303; 307; 379; 388; 410; 436; 465; 527; 528

Immanent Metaphysics, vii; 14; 35; 47; 57; 97; 114; 127; 150; 167; 168; 186; 201; 208; 218; 234; 246; 258; 266; 277; 303; 303; 307; 315; 349; 388; 410; 436; 465; 527; 528

Incommensuration Theorem, 21; 25; 35; 39; 47; 61; 97; 119; 130; 140; 155; 168; 170; 178; 186; 187; 203; 205; 220; 224; 237; 258; 268; 270; 272; 307; 315; 321; 329; 349; 358; 369; 379; 388; 400; 410; 419; 427; 436; 448; 465; 528

inflation, v; viii; 74; 75; 79; 82; 84; 88; 92; 94; 99; 102; 104; 110; 116; 213; 247; 256; 272; 279; 281; 286; 290; 300; 303; 315; 340; 349; 358; 448; 465; 527; 531

inputs, 47; 220; 227; 321; 419; 465

integrated information, 205; 410; 465

interaction, vii; viii; 14; 19; 47; 61; 97; 104; 114; 137; 150; 167; 168; 173; 186; 187; 190; 201; 208; 218; 234; 266; 277; 281; 295; 303; 307; 321; 349; 369; 379; 388; 400; 410; 419; 465; 528

interference, 148; 198; 294; 295; 379; 410; 465

interferometer, 150; 286; 379; 465

internal reality, 162; 164; 168; 208; 456; 465

intrinsics, 21; 170; 307; 388; 427; 465

Kamiokande, 250

Kamionkowski, Marc, 252; 436

kaons, 187; 400

kelvin, 47; 321; 465

Kelvin, Lord, see *Thomson, William*

lamplight, 27; 28; 35; 250; 266; 272

landscape, 110; 116; 119; 220; 254; 358; 419; 436; 465; 528

large world, 65; 79; 82; 88; 114; 254; 292; 329; 340; 358; 436; 465

last scattering, 47; 321; 465

Le Verrier, Urbain, iv; viii; 456

Leavitt, Henrietta, 28

Leibniz, Gottfried, 197; 205; 208; 410; 456

leptons, 116; 130; 132; 292; 358; 369; 465

Level I, v; viii; 91; 92; 94; 97; 99; 104; 110; 213; 272; 303; 349; 448; 456; 465

Level II, v; viii; 107; 110; 116; 119; 125; 213; 220; 272; 281; 303; 358; 419; 448; 456; 465

Level III, v; 135; 144; 146; 157; 213; 272; 369; 448; 456; 465

Level IV, v; 211; 213; 218; 220; 227; 228; 231; 264; 272; 419; 448; 456; 465

library, 258

light-year, 28; 57; 91; 102; 247; 315; 369; 427; 465

Linde, Andrei, 75; 110; 465

listable, 234; 427; 465

Littlewood, John, 212

long emptying, 53; 192; 250; 258; 261; 272; 321; 400; 436; 448; 465; 527

Lovelock, David, 61; 329

magnetic monopole, 74; 465

Many Worlds, 146; 148; 155; 157; 203; 213; 220; 272; 295; 379; 410; 419; 448; 465

master list, 213; 218; 220; 419; 465

mathematical structure, v; viii; x; 17; 21; 25; 31; 39; 51; 132; 162; 165; 173; 181; 198; 210; 211; 212; 213; 218; 220; 227; 231; 237; 247; 258; 262; 264; 268; 272; 274; 297; 299; 303; 307; 315; 388; 410; 419; 448; 456; 465

Mathematical Universe Hypothesis, v; 17; 165; 167; 213; 227; 231; 234; 247; 264; 297; 302; 388; 419; 456; 465

matter fraction, 279; 281; 292; 297; 298; 329; 358; 388; 400; 419; 465

measure problem, 94; 102; 349; 427; 465

mechanical energy, 246; 456; 465

megaparsec, 281; 329; 436; 465

metaphysics, vii; 14; 35; 47; 57; 97; 114; 127; 150; 167; 168; 176; 186; 201; 208; 218; 220; 234; 237; 246; 258; 266; 272; 277; 303; 303; 307; 315; 349; 388; 410; 427; 436; 448; 465; 527; 528

microscopic boundary, 256; 436; 465

microtubules, 198; 201; 410; 465

microwave background, see *oldest light*

the move, 14; 21; 90; 119; 138; 155; 168; 173; 178; 180; 187; 190; 192; 194; 197; 220; 229; 237; 243; 272; 275; 307; 358; 369; 400; 448; 456; 465

MUH, see *Mathematical Universe Hypothesis*

multiverse, v; viii; 17; 35; 89; 91; 92; 94; 99; 104; 107; 110; 116; 135; 144; 146; 157; 211; 213; 228; 231; 264; 281; 303; 349; 358; 369; 456; 465; 531

network of strings, 286; 465

neutrinoless double beta decay, 281; 465

neutrinos, 110; 116; 132; 281; 286; 292; 300; 369; 465; 527

Newton's constant, 220; 241; 419; 465

Newton, Isaac, 43; 108; 114; 155; 220; 241; 419; 448; 465

Noether, Emmy, 388

normal order, 281; 465

nucleus, 17; 45; 47; 82; 128; 130; 280; 281; 292; 321; 369; 465; 528

objective, 104; 168; 181; 186; 190; 307; 349; 388; 410; 419; 465; 528

observer, see *comparer*

observer moment, 181; 183; 190; 192; 198; 400; 465

oldest light, 33; 35; 51; 58; 61; 65; 72; 74; 75; 79; 84; 91; 92; 116; 272; 279; 281; 286; 315; 321; 329; 340; 465; 527

Omnivision Assumption, 94; 97; 349; 465

one vacuum, 254; 261; 358; 436; 465; 527; 528

one world, viii; 97; 106; 125; 130; 140; 164; 168; 220; 234; 243; 256; 270; 272; 303; 349; 369; 388; 419; 427; 436; 448; 465; 528

ontic, 140; 369; 465

ontological, 146; 465

open space, 35; 315; 465

orange peel, 21; 170; 307

order of depth, 176; 220; 237; 272; 388; 419; 427; 448; 465; 528

ordinary matter, 35; 61; 65; 70; 125; 281; 292; 315; 329; 358; 388; 419; 448; 465; 528

orrery, 51; 71; 268; 270; 321; 329; 388; 465; 528

Our Mathematical Universe, iv; 13; 456; 531

Page and Geilker, 157; 379

parallax, 28; 315; 465

parallel postulate, 27; 465

Penrose, Roger, 150; 198; 201; 281; 379; 410

Penzias, Arno, viii; 42; 45; 47; 51; 303; 456

perceptronium, 198; 205; 210; 272; 410; 448; 465

phantom energy, 252; 436; 465

phonon, 65; 329; 465

photograph of last week's cake, 176; 388

photon, 102; 128; 132; 290; 349; 436; 465

physics collection of the Immanent Metaphysics, vii; 14; 47; 57; 97; 114; 127; 150; 167; 186; 201; 218; 234; 246; 266; 277; 303; 307; 465; 527

pion, 250; 300; 465

Planck length, 65; 79; 102; 116; 241; 252; 256; 315; 329; 340; 349; 358; 369; 427; 436; 465

Planck mass, 65; 84; 272; 281; 329; 340; 358; 448; 465

Planck satellite, 33; 47; 61; 65; 84; 119; 252; 281; 315; 321; 329; 340; 358

Planck scale, 47; 65; 321; 329; 349; 358; 436; 465; 528

plasma, 45; 47; 465

polarization, 75; 84; 286; 465

position on the board, 42; 54; 82; 108; 138; 272; 275; 456; 465

positron, 250; 300; 465

power spectrum, 58; 465

precision cosmology, 58; 61; 71; 329; 465

present, the, 51; 102; 181; 183; 190; 192; 194; 198; 224; 254; 270; 272; 286; 298; 321; 329; 349; 400; 419; 436; 448; 465

principle of existence, 97; 104; 349; 465

principle of identity, 35; 79; 100; 173; 208; 315; 340; 349; 388; 410; 465

program, x; 205; 213; 220; 224; 227; 229; 231; 234; 237; 243; 410; 427; 465

pseudo-random, 234; 427; 465

pulsar, 286; 465

qualia, 181; 198; 465

quantized, 270; 297; 302; 448; 465; 528

quantum computer, 148; 198; 410; 456; 465

quantum field theory, 119; 358; 465

quantum numbers, 128; 369; 465

quarks, 116; 122; 128; 130; 132; 220; 286; 292; 358; 369; 419; 465; 527; 528

radioactive dating, 43; 47; 465

Ramanujan, Srinivasa, 212; 220; 227; 456

ratio of tensor to scalar power, 286; 465

registration, 35; 315; 465

rubbings, 176; 388

Rubin, Vera, 57; 281

rug, 61; 119; 329; 358

rules of chess, 108; 125

Schrödinger equation, 128; 130; 135; 138; 145; 146; 150; 155; 241; 247; 379; 465

score sheet, x; 14; 21; 25; 42; 51; 53; 54; 90; 100; 106; 119; 138; 142; 155; 160; 162; 168; 173; 178; 180; 181; 186; 187; 192; 194; 197; 210; 218; 220; 229; 237; 243; 258; 272; 275; 307; 321; 358; 369; 388; 400; 456; 465

sealed record, 256; 277; 279; 280; 281; 286; 290; 292; 294; 295; 297; 298; 299; 300; 302; 315; 321; 329; 340; 349; 358; 369; 379; 388; 400; 410; 419; 427; 436; 448; 465; 528

seat at the board, 14; 19; 25; 27; 35; 39; 40; 51; 106; 119; 138; 162; 173; 178; 187; 190; 194; 197; 205; 210; 229; 237; 275; 307; 315; 358; 456; 465

Sebens, Charles, 157; 379

second of arc, 28; 315; 465

seeds, 45; 47; 54; 72; 75; 79; 84; 88; 92; 116; 280; 286; 300; 321; 340; 358; 436; 465; 527

self, 198; 208; 210; 307; 410

self-aware substructure, 198; 465

self-consistent, 122; 358; 465; 528

sensitivity to initial conditions, 150; 241; 379; 465

short loop, 35; 65; 279; 315; 329; 349; 465; 528

simulation, 227; 229; 231; 234; 237; 241; 243; 272; 303; 349; 369; 410; 427; 436; 448; 456; 465; 527; 528

singularity, 45; 47; 321; 465

Sleeping Beauty, 183; 192; 400

smooth and uneven, 33; 39; 247; 256; 315; 465

soft randomness, 157; 379; 465; 527

spacetime, 128; 132; 180; 181; 183; 186; 187; 190; 194; 198; 208; 290; 292; 400; 465

spectral index, 75; 84; 280; 340; 465

spin, 43; 51; 127; 128; 130; 132; 268; 294; 369; 465

stance, 14; 19; 21; 25; 27; 104; 114; 119; 138; 157; 168; 173; 178; 190; 194; 197; 203; 205; 210; 229; 234; 237; 268; 270; 274; 275; 303; 307; 349; 358; 369; 379; 388; 400; 410; 427; 448; 465

standard deviation, 277; 465

Standard Model, 116; 213; 220; 231; 358; 369; 419; 465

standard model of cosmology, 183; 192; 281; 465

Stern, Otto, 127; 138; 150; 369; 456

stochastic background, 286; 465

strain, 61; 119; 329; 358; 465

strength of color, 220; 419; 465

strength of light, 110; 122; 125; 220; 358; 419; 465; 527; 528

string theory, 110; 116; 220; 254; 358; 436; 465

Strong Self-Sampling Assumption, 183; 465

subjective, the, 181; 203; 205; 208; 210; 234; 272; 298; 307; 410; 448; 465

supercooling, 74; 465

superdeterminism, 224; 419; 465

superposition, viii; 135; 138; 146; 148; 150; 157; 198; 295; 297; 298; 300; 303; 369; 379; 410; 465

symmetry, 21; 35; 39; 47; 61; 65; 82; 97; 119; 130; 155; 170; 173; 176; 186; 187; 203; 213; 220; 224; 227; 237; 258; 268; 270; 292; 307; 315; 329; 340; 349; 358; 369; 379; 388; 400; 410; 419; 427; 436; 448; 465

symmetry and continuity, see *Incommensuration Theorem*

temporal comparison, 173; 388; 465

third axiom, 258; 436; 465

third person, 186; 190; 205; 224; 270; 388; 400; 410; 419; 448; 465

Thomson, William, 246; 258; 321; 436; 456; 465

three axioms, 220; 258; 307; 349; 358; 388; 400; 410; 419; 427; 436; 465; 528

three roles, 14; 119; 307; 358; 456; 465

tide gauge, 14; 170; 307; 388

Tononi, Giulio, 198; 205; 298; 410; 465

topology, 31; 39; 315; 465

torsion balance, 157; 465

torus, 31; 465

totalization, 220; 419; 465

truth, 167; 218; 237; 307; 349; 388; 400; 410; 419; 448

tunneling, 254; 358; 436; 465

Turing, Alan, 231; 234; 237; 427; 465

turn, the, 53; 65; 79; 82; 84; 88; 99; 116; 192; 250; 252; 254; 256; 258; 261; 266; 272; 286; 300; 315; 321; 329; 340; 349; 358; 400; 436; 448; 465; 527; 528

twin prime, 231; 465

two stones, 14; 21; 307; 465

Type Ia supernovae, 58; 465

uncertainty principle, 128; 130; 369; 465

unsharp measurement, 150; 379; 465

vacuum, 119; 122; 150; 168; 220; 254; 261; 281; 290; 300; 329; 358; 400; 419; 436; 465; 527; 528

vacuum value, 220; 419; 465

value that lands, 114; 122; 358; 465; 528

von Neumann, John, 138; 229; 369

wavefunction, 135; 138; 142; 145; 146; 148; 150; 155; 160; 349; 369; 379; 465

weak anthropic principle, 110; 358; 465; 528

weight, 14; 21; 35; 61; 65; 84; 150; 246; 281; 294; 315; 329; 340; 369; 379; 400; 465; 528

Weinberg, Nevin, 252; 436

Weinberg, Steven, 110

what exists, viii; 14; 35; 39; 40; 51; 168; 186; 218; 220; 241; 256; 268; 272; 303; 307; 315; 321; 388; 400; 419; 427; 436; 448; 465; 528

what is real, viii; 14; 39; 40; 51; 102; 155; 160; 168; 220; 237; 258; 268; 272; 303; 307; 315; 321; 379; 388; 400; 419; 436; 448; 465; 528

Wheeler, John, 145; 213; 220; 227

white dwarf, 250; 286; 436; 465

Wigner, Eugene, 17; 165; 176; 213; 388

Wilkinson Microwave Anisotropy Probe, 84

Wilson, Robert, viii; 42; 303; 456

Wolfram, Stephen, 231; 237; 292; 300; 369; 419; 427

Wootters, William, 208; 410

worn stair, 116; 358

youngness paradox, 94; 465

Zurek, Wojciech, 148; 208; 410

Zuse, Konrad, 231; 237; 427

Zwicky, Fritz, 57; 329; 456