

The Three Modalities in Physics

The Epistemic Process, the Grid of the Six Measures, the Triples of the Particle, and the Scales and Contexts of Comparison

Forrest Landry

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Author ORCID: 0009-0005-6275-0362

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What This Book Holds

I. What the Book Establishes

I place physics in the three modalities of comparison, the omniscient, the immanent and the transcendent, at three levels: the account of knowing on which physics is built, the particle, and the scales and contexts of a domain of comparison. At each level I state to which of the three modalities a quantity, a label or a construction belongs, and what follows from that belonging.

At the first level, the three components of the Epistemic Process, the Incommensuration Theorem, the three axioms with the three modalities, and the three organizing principles, are a triple, and the Epistemic Process is the structural fixed point of the act of theorizing. The reliance of the Standard Model on symmetry is the law of the subject matter of the Standard Model, which is content. Measurement tests the identifications that carry the Epistemic Process into the Standard Model, while the Incommensuration Theorem is settled by its proof. I place the units of physics, a holographic formula for the mass of the proton, and the program of Geometric Unity in the grid of the six measures.

At the second level, mass, charge and spin are the omniscient, the immanent and the transcendent labels of a particle. The three charges are a triple within charge, and each row of that triple enters the values of the fermion spectrum at its own places. I set out a method of understanding a particle from the inside, which finds the fermion to be individuation embodied and the interaction vertex to be an event of comparison. The three

colors are a triple, and the symmetry of color is exact. I identify the hierarchy of masses in a sector of the fermion spectrum as an omniscient feature.

At the third level I set out four things. The first of the four things is what comparison finds at the microscopic, the mesoscopic and the macroscopic scale of a domain. The second of the four things is the strict order in size of the three contexts (time, space and possibility). The third of the four things is the triple of the finite, the countably infinite and the uncountably infinite as kinds of magnitude. The fourth is a construction of twenty-six dimensions that extends the metric bundle of spacetime by the group of internal transformations of the Standard Model.

II. The Three Parts

Part One, "The Epistemic Process and the Grid", holds Chapters 1 and 2. Chapter 1 shows that the three components of the Epistemic Process are a triple and that each applies to itself, and it states what these two results mean for the Standard Model. Chapter 2 reads three things in the grid of the six measures: the systems of units of physics, the holographic mass formula of Haramein, and the Geometric Unity of Weinstein.

Part Two, "The Triples of the Particle", holds Chapters 3 to 6. Chapter 3 binds mass, charge and spin to the three modalities label by label, states the order in which the questions of the fermion sector are answered, takes the three charges as a triple within charge, and says where each row of that triple enters the values of the spectrum. Chapter 4 asks of the fermion "what sort of creature am I?" and finds individuation embodied. Chapter 5 asks whether the three colors are a triple and answers that they are. Chapter 6 reads hierarchy and mixing in the fermion spectrum by the characters of the three modalities.

Part Three, "Scales, Contexts and the Extended Bundle", holds Chapters 7 to 9. Chapter 7 sets out what comparison finds at the three scales of a domain: the number of generations, the one grid shared by the

measures, the questions and the quarks, the quantities of classical mechanics, and the three aspects of the universe. Chapter 8 orders the three contexts by size, sets out the three kinds of magnitude, gives the formula $T = I^2 \times O$, and states the two ontological kinds of infinity. Chapter 9 builds the extended bundle of twenty-six dimensions and reads the programs of Weinstein and Haramein through its three layers.

The parts follow one order: first the account of knowing and the grid in which the book places the measures, then the particle, then the domain in which particles are found. A chapter that rests on an earlier one names the earlier chapter, with a plain statement of what it takes from that chapter.

III. What the Book Rests On

I build on *An Immanent Metaphysics*, which states the three axioms and the three modalities, and on the books of the physics collection that derive physics from comparison. *Comparison; The Necessary Basis* sets out the six intrinsics of comparison, the three organizing principles and the grid of the six measures. *Incommensuration; The Theorem and the Three Axioms* proves the Incommensuration Theorem and derives the three axioms from it. *Measurement; The Steps from Comparison to Physics* carries the reading of the defining constants of the units in the grid and the counts of the dimensions of the three contexts. *The Grand Synthesis* and *The Basis of Physics* carry the route from the theorem to the particles and the cosmos. *The Story of the Particles*, *The Kinds of Particle* and *The Branches of the Kinds* carry which particles exist, their characters and their charges; *The Strengths of the Kinds* and *The Measures of the Kinds* carry their couplings, masses and mixings. *Non-Commutative Geometry and the Immanent Metaphysics* carries the algebra of comparison and the exact algebraic form of the theorem, and *The Unquantized Arena* carries the classical arena on

which the content is quantized. Every chapter names the book and the place of each result it draws from those books, beside a plain statement of the result.

Those books present the derivation of physics from comparison. Beside that derivation I set out work that those books do not carry: second routes to results those books establish, the placing of further quantities, labels and outside programs among the three modalities, and constructions set beside the account of those books. No step of their derivation rests on this book.

IV. How the Chapters Refer

In the chapters, "the corpus" means the books of the physics collection named in Section III and in the Bibliography. A section named without a chapter is a section of the same chapter. A chapter named beside the title of another book is a chapter of that book; "Chapter 2" with no book named is a chapter of this book.

A modal label is made within a frame. The chapters use three frames, and each chapter names the one it uses. In the frame of measurement, the standpoint of *The Story of the Particles*, the comparing act is light. In the focus of the two laws, the standpoint of *Non-Commutative Geometry and the Immanent Metaphysics*, the summand $\mathbb{C}$ of the algebra of comparison is omniscient, $\mathbb{H}$ immanent and $M_3(\mathbb{C})$ transcendent. In the frame of the positions of comparison, the act of comparing is set beside the two things it relates.

Chapters 2, 3 and 6 state at their head one premise on which some of their sections rest: the assignment of the three gauge factors to the three modalities in the frame of measurement, $SU(3)$ omniscient, $SU(2)$ transcendent and $U(1)$ of hypercharge immanent. Each of those chapters names the sections that rest on it.

V. Summary

I place physics in the three modalities at three levels: the Epistemic Process and the grid of the six measures (Part One), the triples of the particle (Part Two), and the scales and contexts of a domain (Part Three). I build on *An Immanent Metaphysics* and on the books of the physics collection, which I cite by chapter and section, and I set out work beside the derivation of those books that those books do not carry.

PART ONE

THE EPISTEMIC PROCESS AND THE GRID

The account of knowing as a closed triple, and the Standard Model as content,
with the units and two outside programs placed in the grid of the six measures.



Chapter 1

The Epistemic Process as a Triple of Triples

Its Three Components, Their Self-Application, and the Standard Model as Ground Truth

ABSTRACT

The Epistemic Process has three components: the Incommensuration Theorem, the three axioms with the three modalities, and the three organizing principles. This chapter holds that the three components are themselves a triple in the sense of Axiom III, and binds them to the modalities by the character of each: the theorem omniscient, the axioms and modalities immanent, the organizing principles transcendent. It sets out how the theorem applies to its own assertion, holding its universality at the cost of exhaustive specificity. It sets out how foundational triplication and type isomorphism apply to themselves, and how the self-application of the three principles closes over both the notion and the being of process. In this chapter I state the consequence: the Epistemic Process is the structural fixed point of the act of theorizing, and no more comprehensive theory can supersede the Epistemic Process. It then states what that consequence means for the Standard Model: its reliance on symmetry is the law of its subject matter, which is content, and not a bias; and the Epistemic Process is ground truth in the structural sense, so that what measurement tests is the identifications that carry the Epistemic Process into the Standard Model, and not the theorem.

I. The Three Components as a Triple

The Epistemic Process is the name that *Comparison; The Necessary Basis* gives to the account of knowing built on the structure of comparison. It has three components. The Incommensuration Theorem states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison, and that neither can asymmetry and discontinuity. The Incommensuration Theorem is proved from the six intrinsics of comparison (sameness and difference; content and context; the subjective and the objective) by finite elimination (*Incommensuration; The Theorem and the Three Axioms*, Chapter 6; *The Grand Synthesis*, Chapter 3). The three axioms, with the three modalities (the omniscient, the immanent and the transcendent), state how the three roles of every comparison relate (*The Grand Synthesis*, Chapter 4). The three organizing principles are the Root Tautology (interaction, comparison and relation are one concept), foundational triplication (every domain exhibits three foundational concepts that are distinct, inseparable and non-interchangeable) and type isomorphism (the pattern of relations among those concepts is the same in every domain) (*Comparison; The Necessary Basis*, Chapter 9). In this chapter the Incommensuration Theorem is abbreviated ICT, and the Epistemic Process EP, as in the books that state them. Content and context are the two poles of the partition of every comparison: content, whose instances are separated from one another, and context, whose instances are not.

These three components satisfy the conditions of Axiom III.

They are *distinct*. The ICT is a theorem about structural impossibility. The Axioms are a grammar of structural relationship. The organizing principles are principles of self-reference: of how comparison is found in every domain, their own included. The ICT, the axioms and the organizing principles are different kinds of content.

They are *inseparable*. The ICT without the Axioms is a bare negative result with no constructive content. The Axioms without the ICT are a structural grammar without a foundation. The organizing principles without the ICT and the Axioms are claims of self-reference without substance. None of the three is coherent in isolation.

They are *non-interchangeable*. The ICT cannot do the work of the Axioms; the Axioms cannot do the work of the organizing principles; the organizing principles cannot do the work of the ICT. Each has a structural role that the others cannot fulfill.

The modal assignments of this chapter are made by the character of each component. The ICT is omniscient-modal: it is a structural fact, a theorem, a completed and knowable result. The Axioms are immanent-modal: they describe process, relationship, mediation (how terms relate to one another in the act of comparison). The organizing principles are transcendent-modal: they characterize what lies beyond any particular instance of the system's application. What lies beyond is the self-referential character that exceeds any specific deployment.

The first sentence of Axiom I states that the immanent is more fundamental than the omniscient and/or the transcendent. Applied to this triple under the assignments of this section, it states that the Axioms and Modalities are more fundamental than the ICT, than the organizing principles, and than the two together. "More fundamental" here has the sense it has when the axioms are applied to themselves and read by function (*The Grand Synthesis*, Chapter 4, Section VI): the most fundamental of three terms is the one through which the other two are grasped. The ICT is grasped as a constraint on the roles of comparison through the conjugacy that Axiom I names, since that conjugacy is "the theorem itself, stated in the names of the roles" (*The Grand Synthesis*, Chapter 4, Section II). The three organizing principles are recognized as a triple through the three tests of Axiom III (*Comparison; The Necessary Basis*, Chapter 9, Section II). This priority is not the order of proof. In the order of proof the Root Tautology comes first, the six intrinsics are identified from it, the ICT is

proved from the intrinsics, and the axioms are derived from the ICT: "The axioms are the ICT's own internal structure, made explicit" (*Incommensuration; The Theorem and the Three Axioms*, Chapter 8).

II. The Theorem Applied to Its Own Assertion

The ICT states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison. The assertion of the ICT is itself a comparison, by the Root Tautology, so the constraint applies to the ICT's own assertion.

The ICT is structurally invariant: its verdict is the same across every domain and every choice of terms. The structural invariance of the ICT is symmetry in the defined sense (sameness of content across difference of context). But it is not exhaustively applicable: by itself it does not reach every particular case within a domain, since a case meets the theorem only once the concepts of the domain are identified with the terms of the theorem. Reaching every case without a gap is continuity in the defined sense. The ICT holds its universality (symmetry) at the cost of exhaustive specificity (continuity), exactly as the ICT itself requires: "The Epistemic Process is no exception to its own theorem" (*Incommensuration; The Theorem and the Three Axioms*, Chapter 16, Section IX).

This self-application is not paradoxical, and it is not the ground of the theorem. The ground is the proof by finite elimination (*The Grand Synthesis*, Chapter 3). The self-application is a check of consistency: a theorem about limitation that was exempt from its own limitation would be an exception to its own scope, and the ICT is not one.

III. The Organizing Principles Applied to Themselves

The three organizing principles achieve a specific kind of closure that deserves precise characterization.

Foundational triplication is the principle by which every domain, at its foundation, resolves into three concepts. When foundational triplication is applied to its own domain, that domain resolves into three aspects: the structural fact of three-ness, the process of resolving a domain into its three, and the recognition that the resolution is complete. The three aspects into which the domain of foundational triplication resolves are themselves a triple.

Type isomorphism is the recognition that the same pattern of relations appears at the foundation of every domain. When type isomorphism is applied to itself, it recognizes that the pattern "same pattern across domains" is itself an instance of the very pattern it describes. The recognition of the isomorphism is itself an instance of the isomorphism.

The Root Tautology, applied to itself, is confirmed by every attempt to deny it, since any attempt to deny it compares the claim with something and finds the two different (*Comparison; The Necessary Basis*, Chapter 3, Section IV). Taken together, the three principles are a triple that passes the three tests of Axiom III: the Root Tautology is its immanent element, foundational triplication its omniscient element, and type isomorphism its transcendent element (*Comparison; The Necessary Basis*, Chapter 9, Section II).

These self-applications constitute closure over both the *notion* and the *being* of process. The notion: the three principles describe what happens when comparison examines itself. The being: the act of applying these principles to themselves *is* the self-examination they describe. The description and the described are the same event.

IV. The Fixed Point of Theorizing

The self-descriptive character of the EP is not a philosophical curiosity. It is the mechanism by which the EP achieves its unique epistemic status.

Every framework that includes comparison as a constituent operation cannot not exhibit the structure that the EP describes, because the EP is what comparison looks like when it examines itself. The axioms, the ICT, and the organizing principles are not properties of a particular theory. They are properties of theorizing itself, of the act of comparison that every theory performs.

The closure over notion and being is the mechanism by which the EP achieves its unique epistemic status. The EP is not a theory that might be superseded by a more comprehensive theory. Any more comprehensive theory would have to include comparison, and comparison, examined carefully, yields the EP. The EP is the structural fixed point of the act of theorizing.

V. Symmetry as the Law of the Subject Matter of the Standard Model

The SM, as formulated by the physics community, is organized by symmetry.

The fundamental organizing principle of the SM is gauge invariance, a symmetry requirement. The gauge groups ($SU(3) \times SU(2) \times U(1)$) are symmetry groups. Conservation laws (energy, momentum, charge, color, baryon number, lepton number) are consequences of symmetries (Noether's theorem). The particle content is classified by representations of symmetry groups. The interactions are determined by requiring local gauge invariance.

Symmetry is everywhere in the SM, and the ICT, with the laws of symmetry and continuity, says why.

The law of symmetry applies to all that is of content, and the law of continuity to all that is of context (*Comparison; The Necessary Basis*, Chapter 8). Physics finds its content under symmetry law, and the arena in which that content is found, space and time, under continuity law (*The Basis of Physics*, Essay 1, Section II). The SM describes content. Its reliance

on symmetry is therefore the law of its subject matter, and not a bias of the path by which it was constructed. By the ICT, what is symmetric at its foundation is discontinuous at its limit, so the discrete charges and quantum numbers of the SM are "the shadow of symmetry" (*The Basis of Physics*, Essay 1). The ICT does not state that symmetry is prior to continuity, or that continuity is prior to symmetry. The ICT states that symmetry and continuity cannot both be applied, simultaneously and fundamentally, to one event or comparison. Continuity is the law of the arena, which general relativity, the continuously asymmetric theory, describes. The SM and general relativity each perfect one of the theorem's two survivors, each in its own domain, and the two descriptions are composed as a product, not merged into one (*The Gravitational Passage*, Chapter 3, Section V).

The continuity of the SM lies in the conjugate mode, in the formalism through which the content is reached: the gauge groups are continuous groups, the fields are continuous on the arena, and the strengths of the forces run continuously with the resolution at which they are probed. The running of the couplings is where that continuity shows most plainly; the rank order of the couplings at low energy (strong, then weak, then electromagnetic) is fixed by structure, and their values are measured (*The Story of the Particles*, Level 7, Section V).

Three problems are often named as the SM's deepest unsolved problems: the hierarchy problem, the strong CP problem, and the cosmological constant problem. None of the three is a symptom of a bias toward symmetry. The other books of this series treat each, and each treatment rests on the two laws or on the ICT.

For the cosmological constant problem, the total that summing the zero-point energies of the quantum fields gives is about 10^{123} times the observed energy density of empty space (*The View from Inside*, Preface), and the same book (Chapters 8 and 10) holds that this total would be perfectly symmetric and perfectly continuous at once, which by the Incommensuration Theorem no comparison finds, so that it curves nothing.

The summation does not fail by applying symmetry without continuity: the total it computes would carry both, perfected together, and that total is what no comparison finds.

VI. The Epistemic Process as Ground Truth

The compliance argument is this: every interaction is a comparison (the Root Tautology), and the Epistemic Process characterizes comparison, so the Standard Model cannot not carry the structure that the Epistemic Process describes (*The Grand Synthesis*, Chapter 1, Section III). Re-projection is the principle that at the microscopic limit of a domain the form of what can be found is the form of finding itself, so that the six intrinsics are found as the six quarks and the six leptons (*The Story of the Particles*, Level 6).

The compliance argument can be stated in its strongest form: the EP is ground truth. The SM cannot not comply.

This claim requires precise qualification. "Ground truth" does not mean that the EP is empirically verified in the way the SM is. The SM is verified by agreement with experiment to extraordinary precision. The EP is not the kind of thing that admits of experimental verification in the same sense, because it is not a theory about specific physical content. It is a theory about the structure of theorizing.

"Ground truth" means that the EP correctly characterizes the structural constraints on any act of comparison. The ICT and the axioms are proved from comparison alone, with no physical premise. By the Root Tautology every interaction is a comparison, so whatever comparison requires, every finding of physics carries (*The Grand Synthesis*, Chapter 1, Section III). The SM is built from comparison. Therefore the SM cannot not comply. The specific compliance (the specific correspondences between EP structures and SM features) is stated by identifications, which *The Story of the Particles*, *The Kinds of Particle* and the other books of this series set out.

What measurement tests is those identifications, and not the theorem. The test is not a single experiment but the totality of the identifications: the six intrinsics re-projected as six quarks; the three positions of comparison as three forces with eight, three and one carriers; the theorem's two survivors as the two domain constants ħ and c; the three distinct, non-interchangeable positions of Axiom III as the absence of grand unification, with no carrier joining quark to lepton (*The Branches of the Kinds*, Chapter 2; *The Kinds of Particle*, Chapter 4, Section I); and the re-projection as the particle spectrum. Where these identifications meet measurement by routes independent of one another, the agreement bears on the identifications with physics (*The Grand Synthesis*, Chapter 9). The theorem itself is settled by its proof, by finite elimination (*The Grand Synthesis*, Chapter 3), and not by the coherence of the correspondences.

VII. Summary

I have set out six results about the Epistemic Process, at the structural level at which the Epistemic Process is stated. Section I holds that the theorem, the axioms with the modalities, and the organizing principles are a triple, distinct, inseparable and non-interchangeable, bound to the omniscient, the immanent and the transcendent by the character of each component; Axiom I, applied to this triple, holds in the sense of function and not in the order of proof. Section II holds that the theorem applies to its own assertion, which is a comparison: the theorem is invariant across domains, which is symmetry, and does not reach every case without a gap, which is continuity, so it holds its universality at the cost of exhaustive specificity; this self-application is a check of consistency, and the ground of the theorem is its proof. Section III holds that foundational triplication and type isomorphism each apply to themselves, and that the self-application of the three principles closes over the notion and the being of process, the description and the described being one event. Section IV holds that the Epistemic Process is the structural fixed point of the act of theorizing:

any more comprehensive theory would include comparison, and comparison yields the Epistemic Process. Section V holds that the reliance of the Standard Model on symmetry is the law of its subject matter, which is content, and that the three problems often named as its deepest are not symptoms of a bias toward symmetry. Section VI holds that the Epistemic Process is ground truth in the structural sense: measurement tests the totality of the identifications that carry it into the Standard Model, and the theorem is settled by its proof.

Key Concepts

- Triple of components: the Incommensuration Theorem (omniscient), the axioms and modalities (immanent) and the organizing principles (transcendent), bound by the character of each.
- The theorem applied to itself: universality (symmetry) held at the cost of exhaustive specificity (continuity); a check of consistency, not the ground of the theorem.
- Closure over notion and being: the principles describe what happens when comparison examines itself, and applying them to themselves is that self-examination.
- Structural fixed point: every theory that includes comparison carries the structure of the Epistemic Process, and none supersedes it.
- Law of the subject matter: the Standard Model describes content, and the law of symmetry applies to all that is of content.
- Ground truth: the Epistemic Process characterizes the constraints on any act of comparison; measurement tests the identifications, and the proof settles the theorem.



Chapter 2

The Grid of the Six Measures

The Units Refactored, the Holographic Mass, and Geometric Unity

ABSTRACT

The six measures of comparison stand in a grid of two rows, content and context, and three columns, the three modalities. This chapter reads three things in that grid. First, the units of physics: every system of units decomposes the same information differently, and the grid reading is the decomposition that keeps visible all six measures, their columns, and the two bridges between the columns; the refactored graph has the grid as its skeleton and the constants as its edges. Second, Haramein's holographic mass formula: Newton's constant cancels from it, so that it is exactly the relation $m_p r_p c/\hbar = 4$, the proton's charge radius measured in its own reduced Compton length, a relation within the omniscient column reached through the two bridge constants. Third, Weinstein's Geometric Unity: its 14-dimensional metric bundle is the context of space and time turned reflexive, the bundle lies in the omniscient and immanent columns, and Geometric Unity lacks $\hbar$, the bridge to the transcendent column, at which content is found in discrete steps. The three approaches cover different portions of the grid, and the Immanent Metaphysics supplies the grid itself.

One premise of Section IV is stated plainly at the start. Where Section IV names the modal labels of the three gauge groups, it rests on the assignment, in the frame of measurement, of $SU(3)$ to the omniscient, $SU(2)$ to the transcendent and $U(1)$ of hypercharge to the immanent. The frame of measurement is the standpoint of *The Story of the Particles*, in which the comparing act is light. The conclusion drawn from the labels, that the three groups occupy all three columns, is drawn under that assignment and under its rotation alike. The rotation is the labeling in the focus of the two laws, the standpoint of *Non-Commutative Geometry and the Immanent Metaphysics*, in which $\mathbb{C}$ is omniscient, $\mathbb{H}$ immanent and $M_3(\mathbb{C})$ transcendent.

I. The Units Refactored

The grid of the six measures has three columns, the three modalities: pattern (mass) and space are omniscient, force and time immanent, probability and possibility transcendent; its two rows are content (pattern, force, probability) and context (space, time, possibility) (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Section II). Since 2019 the International System of Units (the SI) fixes its seven base units by fixing seven defining constants, and read in the grid the five constants of the chain bring five of the six cells: the caesium frequency brings time, c brings space, h brings possibility, e brings force and k brings probability, and mass is derived (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section II). The constants c and ħ are the two limits of the domain and the two bridges between the columns: c between the omniscient and the immanent, ħ between the immanent and the transcendent; no constant joins the omniscient and the transcendent directly (same chapter, Sections IV and VII).

One of the deepest implications of the grid reading of the units concerns the multiplicity of possible unit systems.

The SI system is one decomposition of the total specification into base and derived units. The grid reading of this chapter is another. Gaussian units are a third. Natural units (where $c = \hbar = 1$) are a fourth. Each decomposes the same information differently. None changes the physics.

But not all decompositions make the same structure visible. The SI gives no unit to probability or to possibility, which fits the transcendent column, and it routes the grain of possibility, h, into the definition of the kilogram, so that the cell that h brings is hidden inside the unit of mass (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section IX). Natural units hide the two limits of the domain by setting c and ħ to one: the limits stay, and only their numbers go.

The grid reading is the decomposition that keeps visible the distinctions among all six measures, the column of each, the content/context partition, the cell that each defining constant brings, and the two bridges between the columns. It is the decomposition that makes the structure of comparison visible in the structure of the units. Each system of units writes the same physics, and each makes some structure of the grid easier to see and some harder (same section).

Physicists working in high-energy theory routinely set $c = \hbar = 1$. Setting c to one measures lengths and times in one unit; setting ħ to one measures energies and frequencies in one unit; with both set to one, mass, energy, momentum and inverse length share a single unit. Setting $c = \hbar = 1$ is computationally convenient but structurally costly. The constants c and ħ are the two limits of the domain of comparison: c the macroscopic limit and ħ the microscopic limit (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section IV; *The Basis of Physics*, Essay 15, Section II). Setting c and ħ to one hides the two limits in the units, rather than removing the two limits. The grid reading does the opposite: it keeps c and ħ as the two named bridges between the columns. The choice between natural units and the grid reading is a choice between computational convenience and structural transparency.

The proposed re-ordered dependency graph has the two rows and three columns of the grid of the six measures.

The three columns correspond to the three modalities: omniscient (pattern and space), immanent (force and time), transcendent (probability and possibility). Within each column, the two rows are content (pattern, force, probability) and context (space, time, possibility).

Two kinds of edge connect the nodes. The edges of the chain run from each defining constant to the cell it brings: the caesium frequency to time, c to space, h to possibility, e to force, and k to probability; pattern is reached from these as a derived node. The edges of the bridges run between the columns: c between space and time (omniscient to immanent), and ħ between time and possibility (immanent to transcendent). Newton's constant crosses the two rows within the omniscient column. No edge joins the omniscient column to the transcendent column directly.

The total number of independent degrees of freedom is preserved. The information content of the defining constants is unchanged. What changes is the *graph structure*: the way the specification is distributed among nodes. The refactored graph has the grid of the six measures as its skeleton, with the constants as edges connecting its nodes.

The fact that the SI can be refactored into this graph without loss of information does not confirm the graph: every system of units can be refactored into every other without loss, since the number of independent dimensionless ratios is the same in all. What the grid reading adds is a set of identifications, each with its own ground: the cell that each constant of the chain brings (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section II), the two limits of the domain as the two bridges, and the absence of any direct link between the omniscient and the transcendent, which Axiom I explains. The historical unit system is one representation among many of the same physics.

II. The Holographic Mass as a Comparison

Haramein's holographic mass formula computes the proton's mass from the ratio of the Planck length to the proton's charge radius, multiplied by the Planck mass (Haramein 2013). In Haramein's construction, which counts Planck-sized spheres on the proton's surface and within the proton's volume, the factor of proportionality is 4:

$$m_p = 4\ (l_P\ /\ r_p) \times m_P$$

where $l_P = \sqrt{(\hbar G/c^3)}$ is the Planck length, r_p is the proton's charge radius, and $m_P = \sqrt{(\hbar c/G)}$ is the Planck mass.

The product of the Planck length and the Planck mass is fixed by their definitions: $l_P \times m_P = \sqrt{(\hbar G/c^3)} \times \sqrt{(\hbar c/G)} = \hbar/c$. Newton's constant cancels. So the formula is, exactly, $m_p = 4\hbar/(c\ r_p)$, that is, $m_p r_p c/\hbar = 4$: the proton's charge radius is four times its reduced Compton length, $\hbar/(m_p c) = 0.2103$ fm. The Planck length and the Planck mass enter the formula only through their product, and the formula contains no gravitational quantity.

The relation has a stated standing in the corpus. *The Strengths of the Kinds* (Chapter 6, Section VI) states: "The relation $m_p r_p/(\hbar c) = 4$ is a value that lands: it agrees with measurement, and this book states no reason for it". There the proton's mass is written as an energy, so $m_p r_p/(\hbar c)$ is the same pure number as $m_p r_p c/\hbar$ above. With the CODATA 2022 radius, 0.84075 ± 0.00064 fm, the measured value is 3.9977 ± 0.0030, and 4 stands at +0.76 standard deviations (same section; Mohr and colleagues 2025). The agreement belongs to the Sachs electric radius in the atomic convention, and it rests on the data of muonic hydrogen (same section). The same section states that the proton's radius is set by the strong holding and is not derived, and Chapter 6, Section I of the same book states that the proton's mass is a mass of binding: about ninety-nine percent of it is the energy of the color field that holds its quarks. The formula therefore

takes one measured quantity, the radius, and returns the other, the mass, through ħ and c. It is a measured agreement, not a derivation of the proton's mass from no measured input.

In the grid of the six measures each quantity of the relation has a column. The proton's mass is pattern, the omniscient content. Its radius is a length, a measure of space, the omniscient context. ħ is the microscopic limit of the domain and the bridge between the immanent and the transcendent columns; c is the macroscopic limit and the bridge between the omniscient and the immanent columns (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section IV). So the relation $m_p r_p = 4\hbar/c$ sets the product of the two measures of the omniscient column, pattern and space, equal to a multiple of the ratio of the two bridge constants. It joins no omniscient measure to the transcendent column directly, as no constant does (same chapter, Section VII).

The relation between the mass and the radius of the proton is *literally a comparison*: the proton's extent compared with the length that the proton's own mass sets through ħ and c. In the re-ordered reading of the chain of the defining constants, where mass is derived, any mass can be written through ħ, c and a length, since for every body the reduced Compton length $\hbar/(mc)$ is defined. What is not derived, in the formula or in the corpus, is why the length that the proton's charge radius measures is four of its reduced Compton lengths.

The gravitational constant *G* appears in Haramein's formula through the Planck length and the Planck mass, and it cancels from their product. It has no place in the SI definition of the kilogram either, and the reason for its absence from that definition is a different one. The defining constants c and h are limits of the domain, whose numbers express units. *G* is a coupling within the domain: the coupling of the context phenomenon, which measures how strongly content curves context, and its magnitude is measured (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section VI). *G* cannot be removed by a choice of units as c and ħ can: with c and ħ set to one, a measurement of *G* is a measurement of a dimension-

less ratio, a laboratory mass over the Planck mass (same section). The Planck mass, $\sqrt{(\hbar c/G)}$, about 2.18×10^{-8} kg, is the mass at which a body's gravitational size, of the order of Gm/c^2, equals its quantum size, of the order of $\hbar/(mc)$: the scale at which the description of context and the description of content exchange dominance, and at which neither dominates (same chapter, Section V). The Planck scale is not a least length of the arena, which has no least step (same section). The proton's mass is about 7.7×10^{-20} of the Planck mass, and in the holographic formula the gravitational content of both Planck quantities cancels.

III. Geometric Unity and the Missing Column

Weinstein's Geometric Unity proposes that the forces and particles of the Standard Model emerge from the geometry of a 14-dimensional space: 4 spacetime dimensions plus 10 metric tensor components (Weinstein 2020; Weinstein 2021).

The metric tensor is a symmetric 4×4 matrix with 10 independent entries: 4 diagonal components (the scale along each spacetime axis) and 6 off-diagonal components (the angles and mixing between pairs of axes). The metric tensor is the mathematical object that defines how comparisons of distance, duration and angle are made within spacetime. The metric bundle (the space of all possible metric tensors at every point) is therefore the space of all possible ways of making the comparisons of the context row in its omniscient and immanent columns: comparisons in space and in time.

In the grid of the six measures, this can be stated with structural precision. The metric bundle is the context of space and time turned reflexive: the arena examining the structure of its own measurement. Of the 4 dimensions of spacetime, 3 are space, the omniscient context, and 1 is time, the immanent context (*Measurement; The Steps from Comparison to Physics*, Chapter 4, Sections III and IV); the 10 metric components encode their

mutual relations. Weinstein's program is an attempt to derive physical content from this reflexive structure: to show that forces and particles are projections of the geometry of the measurement-space itself.

The grid locates what this program lacks. The metric bundle carries the two context measures space and time and their relations. Fields defined on it can carry content: curvature, which is the bending of the arena by content (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Sections VI and VII), and connections, which carry the gauge forces. What the bundle does not contain is the transcendent column.

There is no $\hbar$ in Geometric Unity's foundational structure. The framework, as Weinstein presents it, is classical differential geometry. The metric bundle knows nothing about quantum states, probability amplitudes, or the grain of possibility.

But the grid shows that the transcendent column is needed for the particle spectrum. ħ is the bridge between the immanent and the transcendent columns: an action divided by h counts states, and ħ turns a frequency into an energy (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Sections III and IV). Quantization requires $\hbar$. The Incommensuration Theorem, applied to the content row, says that at the absolute microscopic scale of a domain the content measures must be quantized, symmetric and discontinuous (*The Basis of Physics*, Essay 4, Section V), and in the physical domain that scale is the one marked by the Planck constant (*The Basis of Physics*, Essay 15, Section II). The re-projection of the six intrinsics into the quarks occurs at the microscopic limit (*The Story of the Particles*, Level 6). Without ħ, the bridge to the transcendent column, none of these structures is accessible.

IV. The Diagnosis

The paragraphs below name the modal labels of the three gauge groups under the assignment stated at the head of this chapter: SU(3) omniscient, SU(2) transcendent and U(1) of hypercharge immanent, in the frame of measurement.

The structural diagnosis of Geometric Unity's incompleteness can now be stated within the grid.

The metric bundle has 14 dimensions. The six measures of comparison have a specific modal structure. The metric bundle encodes the reflexive elaboration of the omniscient and immanent contexts, but only those. The transcendent context (possibility), the transcendent content (probability), and ħ, the constant through which they enter physics, are absent.

In the grid, the absence of these three is where Geometric Unity meets its difficulty. Gravity is the bending of the arena by content, the context phenomenon, with Newton's constant as its coupling (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Sections VI and VII; Chapter 7, Section VI). It lies within the scope of a geometry of the arena. The three gauge forces act on content (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Section II): their charges are found in discrete steps, and their quanta are counted in units set through ħ. A classical geometry can carry the gauge groups SU(3), SU(2) and U(1) as connections; what it cannot carry without ħ is the discreteness of the content on which the groups act. In the frame of measurement, which this chapter uses, the three groups are labeled U(1) immanent, SU(2) transcendent and SU(3) omniscient; the labels of *Non-Commutative Geometry and the Immanent Metaphysics* are the rotation of these in the focus of the two laws, and both labelings are admissible (*The Grand Synthesis*, Chapter 6, Section III). Under either labeling the three groups occupy all three columns, so the gauge groups as such do not belong to one column; what belongs to the transcendent column is the grain at which content is found.

Weinstein's account of the fermion content describes two families of sixteen states and a third family that is not a true copy of the first two; his word for it is "imposter" (Weinstein 2020). Geometric Unity's spinors on the 14-dimensional space have 128 components (a Dirac spinor in 14 dimensions has $2^7 = 128$), and their decomposition partially matches the Standard Model's fermion content. In the corpus, three is the count of generations because comparison has three positions and the cycle of Axiom II closes after three steps (*The Kinds of Particle*, Chapter 7); in one picture the first generation occupies the immanent position, and the second and third the two positions that Axiom I pairs as conjugates (same chapter, Section IV). The grid suggests a reading of the mismatch, offered as a question for Geometric Unity's mathematics and not as a result about it: a framework that carries the measures of only two of the three columns has no ground for the third position, and so no ground for a third generation that is a true copy of the first two.

The same reading bears on the Shiab operator, the mathematical device by which Weinstein proposes to project from 14 dimensions to 4. The projection requires passing from the reflexive measurement-space to the physical content-space. But the content-space includes the transcendent column, which the measurement-space does not contain. On this reading the Shiab operator is asked to extract something from a space that does not have it, and that is why, by the grid, the operator is expected to be hard to define.

What Geometric Unity Would Need

The grid of the six measures suggests a specific structural extension. Geometric Unity would need to be enlarged from the metric bundle (the space of all possible geometries) to a structure that also includes the transcendent context (possibility space, the quantum Hilbert space) and the transcendent content (probability amplitude, the wave function), together with $\hbar$, the constant that bridges the immanent and the transcendent.

Such an extension would take the 14-dimensional space and embed it within a framework that also encodes the quantum structure of the vacuum.

Haramein's formula approaches the meeting of geometry and quantum structure from the other side: it brings ħ into a geometric picture through the Planck units, and once Newton's constant cancels what remains is a relation between the proton's extent and its mass through ħ and c. The two approaches are complementary. Weinstein starts from the geometry of the arena and lacks ħ. Haramein's relation carries ħ and c and no geometry of the arena. A complete synthesis would require both simultaneously.

V. Three Approaches, One Grid

The three approaches (Weinstein's Geometric Unity, Haramein's holographic mass, and the IM's structural grammar) can now be located within the grid of the six measures as complementary descriptions that each cover a different portion of the total structure.

Weinstein operates within the omniscient and immanent columns. His metric bundle encodes the reflexive geometry of spacetime. He aims to obtain gravity (the bending of the arena by content) and particle structure (from the decomposition of higher-dimensional spinors), and his decomposition matches the fermion content in part. His approach is incomplete because the transcendent column, and ħ with it, is absent.

Haramein's formula, once Newton's constant cancels from it, is a relation within the omniscient column, the proton's mass times its charge radius, reached through the two bridge constants ħ and c. From the measured charge radius it returns the proton's mass to within 0.06 percent, inside the 0.08 percent uncertainty of the radius (CODATA 2022). It does not return the internal quark structure, since it holds the proton as one ex-

tent and one mass. The corpus records the relation as a measured agreement for which no reason is stated (*The Strengths of the Kinds*, Chapter 6, Section VI).

The IM operates at the level of the grid itself: the structural grammar that determines why the grid has six cells, why the cells relate as they do, and what constraints the axioms place on any theory that attempts to fill them. The corpus built on it carries more than qualitative structure. It carries the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ and the gauge group $U(1) \times SU(2) \times SU(3)$ (*The Grand Synthesis*, Chapter 6; *The Kinds of Particle*, Chapter 4), the charges of one generation from anomaly cancellation (*The Kinds of Particle*, Chapter 5), the count of three generations (*The Kinds of Particle*, Chapter 7), the proton's mass as a mass of binding set by the running of color's strength (*The Strengths of the Kinds*, Chapter 6), and each of the twenty-six numbers of the Standard Model with its stated standing, the measured inputs being the electron's mass, the strength of light α_{em}, the strength of color α_s, the vacuum value v, and Newton's constant G_N (*The Grand Synthesis*, Appendix B).

Each approach supplies something that the grid can locate. Weinstein supplies a geometry of the arena within the omniscient and immanent columns. Haramein supplies one relation of scale, between the proton's extent and its mass, through ħ and c. The IM supplies the grid, the structure that explains why the grid exists and constrains what can fill it.

A complete theory of the physical domain would need all three levels simultaneously: the geometric framework (Weinstein's metric bundle, extended to include the transcendent column), the relations of scale (such as the proton's 4, together with the reason for them), and the structural grammar (the IM's axioms, ensuring the framework has the correct qualitative shape).

Three approaches, each incomplete on its own. Each covering a different portion of the grid. Each supplying something the others lack. The IM does not compete with them. It provides the structural grammar within which their partial successes become intelligible, their specific failures become diagnosable, and the shape of a complete theory becomes visible.

VI. Summary

I have set out three results in the grid of the six measures, each result at the standing of the identifications that the result names. Section I holds the refactored graph of the units: its nodes are the six cells, the edges of the chain run from each defining constant to the cell it brings, the edges of the bridges run between the columns, and no edge joins the omniscient column to the transcendent directly; that the SI can be refactored into this graph does not confirm it, since every system of units can be refactored into every other, and what the grid reading adds is a set of identifications, each with its own ground. Section II holds that Newton's constant cancels from the holographic mass formula, which is exactly $m_p r_p c/\hbar = 4$, a measured agreement that returns the proton's mass from its measured radius through $\hbar$ and c, and a relation within the omniscient column. Sections III and IV hold the diagnosis of Geometric Unity: its metric bundle carries the context measures of the omniscient and immanent columns, it has no $\hbar$, and so it cannot carry the discreteness of the content on which the gauge groups act; the readings of its third family and of its projection operator are offered as questions for its mathematics. Section V holds that the three approaches are complementary: a geometry of the arena, one relation of scale, and the grid with the structure of comparison that explains it.

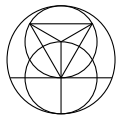
Key Concepts

- Representational invariance: every system of units writes the same physics, and each makes some structure of the grid easier to see and some harder.
- Refactored graph: the six cells as nodes; the constants of the chain and the two bridges as edges; no direct edge between the omniscient and the transcendent.
- Holographic mass: since $l_P m_P = \hbar/c$, Newton's constant cancels and the formula is $m_p r_p c/\hbar = 4$.
- Missing column: Geometric Unity's metric bundle has no $\hbar$ and no transcendent column.
- Complementarity: Weinstein a geometry of the arena, Haramein one relation of scale, the Immanent Metaphysics the grid.

PART TWO

THE TRIPLES OF THE PARTICLE

Mass, charge and spin; the three charges within charge;
the fermion from the inside; the three colors; hierarchy and mixing.



Chapter 3

The Modal Triples of a Particle

Mass, Charge and Spin; the Three Charges Within Charge; and Where Their Rows Enter the Values

ABSTRACT

Every fundamental particle of the Standard Model is classified by three kinds of label, its mass, its charges and its spin, and in the frame of measurement mass is omniscient, charge immanent and spin transcendent. This chapter gives an argument for that binding label by label: mass is pattern, found the same by every comparer; charge is the label by which the comparing act reads content, and its strength is written with both limits of the domain; spin is the relation of a content of no extent to the frames, found in steps of ħ. It holds that every finding of a mass or of a spin is made through charge, and that mass and spin are the conjugate pair, the two labels of one irreducible representation of the Poincaré group. It states why the usual ranking of the famous questions of the fermion sector runs against the order in which they are answered. It then takes the three charges, color, weak isospin and the phase charge, as a triple within charge, on the premise of the assignment of the gauge factors by what each acts on: the labels of the factors and of the charges are one assignment stated at two levels, so their agreement is no test; Axiom II can be traced through the three charges; and a mirror of an admissible labeling is an error, where a rotation of it is admissible. It closes

> with where each row of the charge triple enters the values of the fermion spectrum, an association that names where the structure of each row enters and is not an exclusive partition.

One premise is stated plainly at the start. Sections VI to IX rest on the assignment of the three gauge factors to the three modalities in the frame of measurement, made by what each acts on: $\mathrm{SU}(3)$ of color omniscient, $\mathrm{SU}(2)$ of the doublet transcendent and $\mathrm{U}(1)$ of hypercharge immanent (*The Kinds of Particle*, Chapter 4, Section I). The frame of measurement is the standpoint of *The Story of the Particles*, in which the comparing act is light. The second of the two identifications that place spin at the transcendent in Section IV rests on the same assignment, since it places spin through its group $\mathrm{SU}(2)$, which acts on the quaternion carrier $\mathbb{H}$, the transcendent carrier in that frame.

I. Three Labels

Every fundamental particle in the Standard Model is classified by three kinds of label. Mass says how heavy the particle is: how it resists a change of its motion, how much energy is required to create it, how it contributes to the stress-energy tensor that curves space and time. Spin says how the particle's state turns when the frame of directions is turned: half-integer spin for the particles of matter, the fermions, which obey the exclusion principle, and whole-number spin for the carriers of force and the Higgs field, the bosons, which obey Bose statistics. Charge says how the particle is acted on: which forces act upon it, which gauge bosons couple to it, which transitions are allowed.

Within the Standard Model these labels identify every fundamental particle. Each fundamental field carries an irreducible representation of the group of motions of space and time, the Poincaré group, whose irreducible representations of positive energy are labeled by mass and spin (by helicity when the mass is zero) (Wigner 1939), together with an irre-

ducible representation of the gauge group, labeled by the charges: color, weak isospin and hypercharge, with the electric charge $Q = T_3 + Y$. The three generations of one kind carry the same spin and the same charges and differ in mass (*The Story of the Particles*, Level 5, Section I). So mass, spin and charges together determine the particle, and each kind of label carries what the other two do not: spin is not fixed by mass and charges, the charges are not fixed by mass and spin, and mass is not fixed by spin and charges.

The Kinds of Particle (Chapter 4, Section III) derives from seven premises that, in the frame of measurement, mass is omniscient, charge is immanent and spin is transcendent, and that the three labels are fixed in the order of Axiom II: spin first, by the kind of field; charge next, by the structure of the gauge factors and the unit of charge; mass last, by what is found. The frame of measurement is the standpoint of *The Story of the Particles*, in which the comparing act is light, a frame being the set of units in which comparisons at a given scale report. Sections II to V give a second argument for the same binding, resting on identifications that the corpus makes for each label separately.

II. Mass as Omniscient

Mass is assigned to the omniscient. The corpus makes this identification directly: "The concept of a state, a structure, a configuration, and ultimately the concept of matter and of mass itself, has the nature of the omniscient. Mass is pattern" (*The Basis of Physics*, Essay 15, Section V). In the frame of measurement the electron, the thing compared, stands at the omniscient position, and it carries rest mass because rest mass is "the fixed characteristic of a pattern: the hallmark of a thing that persists and is found again" (*The Story of the Particles*, Level 0, Section III).

Mass admits a ladder. The fundamental fermions span masses from the neutrinos, each below one electron volt, to the top quark, at about 173 GeV. Within each charged sector the three generations ascend in mass,

and the ascent runs with the direction of the cycle of Axiom II from the first generation (*The Story of the Particles*, Level 5, Section III). How large each step is is carried by the mass ladder of *The Measures of the Kinds*, Chapter 7, which states the standing of each value: the electron's mass is the unit and a measured input, and every other mass carries its own standing there (*The Grand Synthesis*, Appendix B).

Mass is found from outside. It is measured by the particle's inertial response to known forces, or by the energy released in its creation or annihilation. To be found the same by every comparer, in the third person, is the character of knowledge, and knowledge, in theory and in the third person, is the omniscient mode (*Comparison; The Necessary Basis*, Chapter 11, Section III).

Mass relates to two constants of the domain. The domain has two limits, c at the large scale and ħ at the small; c marks the horizon, which is omniscient, and ħ marks the infinitesimal, which is transcendent (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Sections IV and VII; *The Basis of Physics*, Essay 15, Section II). Through c, mass and energy are one currency, $E = mc^2$; *The Basis of Physics* reads matter and energy as the omniscient and the transcendent faces of light, related by the constant that bounds interaction, and states that this reading gives the reason the mass-energy relation has the form it has, and does not produce that relation (Essay 15, Section V). Newton's constant G_N is not a limit of the domain. It is "the coupling of the context phenomenon: it measures how strongly content curves context", and "c and ħ are limits of the domain, one at each end, and G_N is a coupling within it" (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Sections VI and VII). In classical units its units touch pattern and space, the omniscient column of the grid of the six measures in both its rows, and time (same chapter, Section VII). Its value is a measured input (*The Gravitational Passage*, Preface). So mass, the omniscient content measure of that grid (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Section II), is the label through which the content of a particle curves the context.

III. Charge as Immanent

Charge is assigned to the immanent. In the frame of measurement the comparing act is light, and the corpus states what charge is from the side of that act: "Charge is the label by which the comparing act (the immanent) reads content" (*The Story of the Particles*, Level 0, Section III). The act reads the electron by its charge, "and charge belongs with force: charge is what the electric force acts on" (*The Story of the Particles*, Level 5, Section I); and force and time are the physical measures of the immanent pair of intrinsics, the objective and the subjective (same section).

Charge says how the particle is acted on. Electric charge says whether the particle couples to light. Color says whether it takes part in the strong interaction. Weak isospin and hypercharge say how it takes part in the weak interaction and in the force on the phase. Charge is not what the particle is (that is mass), and it is not how the particle's state stands to the frames of directions (that is spin). It is where the content meets the act.

Charge is exact and conserved. Every charge in the zoo is a whole number of thirds of the electron's charge, fixed by the one-unit difference within each doublet, the neutral neutrino, the electron's unit charge and the cancellation of the anomalies (*The Kinds of Particle*, Chapter 5, Section I; *The Story of the Particles*, Level 6, Section IV). The sign of the unit, and the choice of the electron's charge as the unit, are conventions; the ratios of all other charges to it are fixed by the structure (*The Story of the Particles*, Level 0, Section III). What is governed by symmetry is found in discrete classes with exact labels (*Measurement; The Steps from Comparison to Physics*, Chapter 9, Proposition 9.1), and charge is such a label.

Charge relates to both limits of the domain at once. The strength of light, $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$, is defined from the unit of charge e, the quantum of action ħ and the speed of light c; its measured value is about 1/137.036, and it is a measured input of the corpus (*The Grand Synthesis*, Appendix B). c bridges the omniscient and the immanent, and ħ bridges

the immanent and the transcendent (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section VII). The label assigned here to the immanent is the one whose strength is written with both bridges; that agreement is observed, and it is not used as a premise. In the grid of the six measures charge belongs with force, the immanent content measure (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Section II).

IV. Spin as Transcendent

Spin is assigned to the transcendent. Spin is content: "Spin labels the representations of the rotations of a frame of directions, and the rotations form a compact group. A content keeps its spin across every change of context, so spin is content, registered against the frames of directions that the context supplies" (*Measurement; The Steps from Comparison to Physics*, Chapter 9, Section V). Those frames are frames only through the comparable content that fixes them, so spin is "a relation of a content of no extent to the frames that the comparable content fixes, and through them to the whole of the comparable content" (*The Gravitational Companion*, "One Mass, Three Elevators", Section VII). A relation of one content to the whole is a relation of the one to the many.

Two identifications in the corpus place this relation at the transcendent. First, the unit of spin is $\hbar$: the least step of intrinsic circulation about an axis is identified with $\hbar$ (*Measurement; The Steps from Comparison to Physics*, Chapter 9, Section V), and $\hbar$ is the microscopic limit of the domain, which marks the infinitesimal, the transcendent (same book, Chapter 7, Section VII). In the classical limit, $\hbar \to 0$, the spin angular momentum of a particle goes to zero. Second, the group of spin, $\mathrm{SU}(2)$, is the group of unit quaternions, the same group that acts on the carrier $\mathbb{H}$ as weak isospin (*The Quantum Passage*, Chapter 8, Section IV), and in the frame of measurement $\mathbb{H}$ is the transcendent carrier (*The Grand Synthesis*, Chapter 6, Section III, Table 6.2). Spin and rotation are assigned to modalities in two frames, one assignment in each frame: spin is transcendent in

the frame of measurement (its unit is ħ, the microscopic limit, and its group SU(2) acts on the quaternion carrier $\mathbb{H}$, the transcendent carrier in that frame), and rotation is omniscient in the frame of *The Gravitational Companion*.

The Kinds of Particle reaches the same placement by a further route, from what spin is: the two-sidedness of one content held at a point, the two ways round, with its unit ħ at the microscopic limit (Chapter 4, Section III). The routes are kept side by side, as strands of one cable: the unit of spin, the group of spin, and what spin is. Spin is transcendent in the frame of measurement, and rotation is omniscient in the frame of *The Gravitational Companion*: two frames, two bindings (same section).

The Three Labels as a Triple

Mass, charge and spin satisfy Axiom III. They are distinct: mass is measured in units of energy divided by c^2, charge in units of the electron's charge, and spin in units of ħ. They are inseparable: every particle carries a value of each, zero included, and no particle is specified by two of them. They are non-interchangeable: mass does not fix how a state turns under rotation, spin does not fix which forces act on the particle, and charge does not fix the resistance to a change of motion. Axiom I states: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another". Under the assignment of this chapter, charge is the immanent label. In the frame of measurement every instrument reports the arrival, the departure or the energy of electrons, and the comparing act reads the electron by its charge (*The Story of the Particles*, Level 0, Sections I and III); so every finding of a mass or of a spin is made through charge, and the identification made here is that charge is, in that sense, more fundamental than mass and spin. Mass and spin are the conjugate pair: mass is the pattern of a content, the same in every frame, and spin is the relation of the same content to the frames; in the representation theory of the

Poincaré group they are the two labels of one irreducible representation (Wigner 1939). Each clause about Axiom I in this paragraph is part of the identification argued in this chapter, not a theorem.

On the identification that I make in this chapter, mass, charge and spin are the particle-level expression of the three modalities. Every fundamental particle carries one omniscient label (how much), one immanent label (how it is acted on), and one transcendent label (how it stands to the whole). Within the Standard Model the triple is complete: no fourth kind of label is needed, and each of the three carries what the other two do not.

The Center of the Algebra

The algebra of comparison, $A_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, has the center $Z(A_F) = \mathbb{C} \oplus \mathbb{R} \oplus \mathbb{C}$: the summand $\mathbb{C}$ is its own center, the center of $M_3(\mathbb{C})$ is the complex multiples of the identity, and the center of $\mathbb{H}$ is the real multiples of 1 (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 6, Section II, and Chapter 7, Section IV).

The total real dimension of the center is $2 + 1 + 2 = 5$. The same number is the hypotenuse $H = 5$ of the kernel triangle with sides 3, 4 and 5. The corpus fixes H by the closure $3^2 + 4^2 = 5^2$ (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 11), not by the center, and no identification of the two numbers is made here.

The photon is its own antiparticle, and the corpus gives the reason in the frame of measurement: the photon is the act of comparing, so "it carries nothing that a reversal could reverse, and, being the act, it is not held toward two terms at all" (*The Story of the Particles*, Level 4, Section I). The reason is the photon's position as the comparing act, not the center of a summand: the photon of the everyday frame is a combination of the hypercharge boson, from the summand $\mathbb{C}$, whose center is $\mathbb{C}$, and the neutral weak boson, from the summand $\mathbb{H}$ (*The Story of the Particles*, Levels 7 and 8). Light is the basis of measurement for the same reason: in the

frame of measurement light is the comparing act, and "When the scientists measure something, they measure it with light" (*The Story of the Particles*, Level 0, Section I).

V. The Order in Which the Questions Are Answered

The famous questions of the fermion sector (why three generations, why these charges, why this gauge group, why these masses and mixings) are answered in one order, the order of Axiom II: the questions of kind first, from the structure of comparison, and the questions of value after them, each value with its standing. The transcendent is the entry point in two senses: it is first in the order of relevance of the axioms, and first in the direction in which comparison produces what is, from what could be, through the act, to what is (*The Grand Synthesis*, "What the Structure Fixes and What Measurement Supplies", Section II).

The order of Axiom II, questions of kind first and questions of value after, is the reverse of the order in which the questions of the fermion sector are usually ranked. Mass is usually counted the best understood label, because every mass is a definite measured number. But a measured number stands at the end of the order: it says what is found, and by itself it does not say why. The questions the corpus closes from the structure are the questions of kind. The omniscient and the transcendent are conjugate, and they meet only through the immanent (*Measurement; The Steps from Comparison to Physics*, Chapter 7, Section VII); so the passage from the field of what could be to the values that are found runs through the act: through the gauge structure and the charges by which content is read.

What stands at the end of the order, then, is this: the forks, until their measurements decide them; the measured agreements, each with the question of why it lands; and the measured inputs, which the account takes as premises. The questions of kind, which the usual ranking counted the hardest, are the ones the corpus closes.

So the entry point is the transcendent, in the two senses stated above. The corpus begins from the structure of comparison, from the field of what could be, and asks: "Given that comparison has this structure, what particles must a world contain?" (*The Story of the Particles*, Level 0, Section VI). It derives what kinds must exist and what they must carry, and then it reaches the values through the act by which content is read, each value with its standing.

What the chapter does not do. In this chapter I do not claim that measurement is secondary. The measured inputs are premises of the account, and every derived value is set beside its measurement. The claim is about the order of explanation: the kinds are fixed before the values, and the values are reached through the act by which content is read.

VI. The Three Charges: One Assignment at Two Levels

Charge is not one thing: every fundamental fermion carries up to three charges, one for each factor of the gauge group $SU(3) \times SU(2) \times U(1)_Y$: color, weak isospin, and hypercharge, the phase; electric charge is $Q = T_3 + Y$, the charge of the phase row after the Higgs condensate (*The Kinds of Particle*, Chapter 4, Section I). In this chapter "the phase charge" names hypercharge before the condensate and electric charge after it. Sections VI to IX rest on the assignment stated at the head of this chapter: $SU(3)$ of color omniscient, $SU(2)$ of the doublet transcendent and $U(1)$ of hypercharge immanent, in the frame of measurement.

The gauge factors are the transformations, and the charges are what the transformations act on. The modal labels can be stated at either level: $SU(3)$, $SU(2)$ and $U(1)_Y$ are omniscient, transcendent and immanent; color, weak isospin and the phase charge are omniscient, transcendent and immanent. The two lists agree at every row.

The agreement is not a test. The label of each factor is made from the charge that the factor acts on: "each takes its label from what it acts on" (*The Story of the Particles*, Level 7, Section I; *The Kinds of Particle*, Chapter 4, Section I). The two lists are therefore one assignment stated twice, and they could not have differed.

Two things about the assignment are not true by construction. The first of the two things is that the orientation of the assignment agrees with the labeling of the summands in the same frame. Each gauge factor comes from one summand ($SU(3)$ from $M_3(\mathbb{C})$, $SU(2)$ from $\mathbb{H}$, $U(1)$ from $\mathbb{C}$). The label given to each factor by what the factor acts on is the label given to the summand of that factor in the frame of measurement (*The Grand Synthesis*, Chapter 6, Table 6.2). The second of the two things is that the condensate, which breaks $SU(2) \times U(1)_Y$ to the electromagnetic $U(1)$, moves no label. Each row keeps its modality on both sides of the condensate (*The Kinds of Particle*, Chapter 4, Section III).

The Direction of Inference

The inference runs along three different kinds of step. From comparison to the algebra it runs by derivation, through postulates that are identifications (*The Grand Synthesis*, Chapter 6, Section III, Table 6.1). From the algebra to the gauge group and to the internal space it runs by theorem. From comparison to the modal labels it runs by assignment: a labeling is a definition, made within a frame, and the frame of this chapter is the frame of measurement. The labels are not inferred from the manifold or from the gauge group, and the descriptions of what each force does are not an independent confirmation of them. What those descriptions add is an identification argued in this chapter: that the binding of a pattern, the turning of one side into the other, and the reading of a phase by light are the characters of the three positions that the assignment names.

VII. The Trace of Axiom II Through the Three Charges

Axiom II. "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent" (*Incommensuration; The Theorem and the Three Axioms*, Chapter 3). The trace that follows is an identification argued in this chapter, with the labels of the frame of measurement (O = color, T = weak isospin, I = the phase charge).

A class of the transcendent (the weak doublet, with the condensate on its edge) will precede an instance of the immanent (the electric charge of a particular particle). The condensate selects which combination of weak isospin and hypercharge stays unbroken, $Q = T_3 + Y$, and the electric charge of each particle is read in that combination. The values of the charges themselves are fixed by anomaly cancellation and the premises that *The Kinds of Particle*, Chapter 5, Section I, states.

A class of the immanent (the electromagnetic framework of measurement) will precede an instance of the omniscient (the observed color structure: which hadrons exist and which baryons are stable). Hadrons are detected, and the spectrum of color-neutral states is found, through their electromagnetic signatures.

A class of the omniscient (confinement: which bound states of color can be found) will precede an instance of the transcendent (which weak transitions occur within those bound states). The spectrum of hadrons determines which weak decays are kinematically open.

The trace runs from the transcendent to the immanent to the omniscient and back to the transcendent, which is the orientation of Axiom II.

VIII. A Rotation Is Admissible, a Mirror Is an Error

One assignment of the three charges gives color the transcendent label and weak isospin the omniscient, and leaves the phase charge immanent. Characterizations of the transcendent by its hierarchy, its individuation and its mixing can be made to support it: confinement read as a resistance to individuation, the equivalence of the three colors read as a refusal of hierarchy, and the color singlet read as a democratic mixing of the three. The assignment is not admissible. It exchanges two labels and keeps the third, and an exchange of two labels reverses the orientation of the cycle of Axiom II, from the transcendent through the immanent to the omniscient. It is therefore a mirror of the frame of measurement, not a rotation of it. The two labelings the corpus holds are rotations of one another, and "the orientation of Axiom II is the same under both" (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 19.2, as quoted in *The Grand Synthesis*, Chapter 6, Section III).

Color does carry the transcendent label in the other admissible labeling. In the focus of the two laws, $M_3(\mathbb{C})$ is transcendent, $\mathbb{H}$ immanent and $\mathbb{C}$ omniscient, so color is transcendent, the doublet immanent and the phase omniscient: every label moves one step round the cycle together. There the label of $M_3(\mathbb{C})$ follows the law of continuity, since every element of $M_3(\mathbb{C})$ commutes with the Dirac operator and the observables of color are continuously asymmetric (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 5, Section IV, and Chapter 7, Section III). A characterization of color as transcendent belongs to that focus. In the frame of measurement used in this chapter, color is omniscient.

The characterizations do not decide the label within a frame. Confinement follows from the inseparability of the three positions, "A single position is not a comparison" (*The Story of the Particles*, Level 6, Section III), and Axiom III holds of the three positions under every labeling. The equivalence of red, green and blue is the exact symmetry of color, which the

law of symmetry gives to what is of content: "Color's act moves the pattern among its own positions and carries no state across the content-to-context partition" (*The Branches of the Kinds*, Chapter 3, Section V).

In the frame of measurement, the fermions to which those characterizations of the transcendent apply are the neutrinos. The neutrinos stand at the position of the unknowable (*The Measures of the Kinds*, Chapter 7, Section III), and the unknowable is assigned the transcendent (*The Kinds of Particle*, Chapter 1, Section IV). Their mixing angles are large, and they are the hardest of the fermions to detect.

The method follows. A modal label belongs to a frame, and within a frame the label of a force is fixed by what the force acts on. A characterization drawn from the behavior of a force can agree with an assignment and cannot override it. Every proposed assignment is checked for its orientation: a rotation of an admissible labeling is admissible, and a mirror of one is an error.

The rule is stated for the labels of the forces within one frame, where each label is fixed by what the force acts on and the labels are ordered by the cycle of Axiom II; within that ordering a mirror is an error. An exchange of two labels, a flip, is allowed only under certain circumstances, mediated by Axiom I alone, which is theory. *An Immanent Metaphysics* gives one such exchange in aphorism 2.71-3: from the impersonal perspective the known is omniscient and the unknown immanent, and from the personal perspective the known is immanent and the unknown omniscient, with the unknowable transcendent in both. A rotation is allowed only under certain circumstances, mediated by Axiom II, which is practice. An Axiom I flip and an Axiom II rotation do not happen at the same time: theory is not practice (*The Kinds of Particle*, Chapter 1, Section II).

IX. Where the Rows of the Charge Triple Enter the Values

Charge, one label at the immanent position, carries the three positions inside it: color omniscient, weak isospin transcendent and the phase immanent (*The Kinds of Particle*, Chapter 4, Section III). The integer $c_1 = 3$ is the count of the three positions of comparison, and with $n_C = 2$, the count of the conjugate pair of content and context, it fixes the kernel triangle, with legs 3 and 4 and hypotenuse 5 (*The Kinds of Particle*, Chapter 2, Sections I and II).

Many values of the corpus have writings in these integers. A writing is an expression of a value in the measures of the triangle. The standing of a value is fixed by its argument and by its measurement, not by its writing. Four numbers written in c_1 bear on the charges and the masses, and each has its own standing.

Each row of the charge triple enters the values of the fermion spectrum at its own places.

The color row (*omniscient*). Color enters the masses through the sector rule: "the step cost of a charged sector depends on its color alone" (*The Measures of the Kinds*, Chapter 7, Section III). Both quark sectors carry color, and they share the step cost $P_q = 6$ halvings per generation: a generation is a rotation of the content, each rotation resolves the six aspects of its new instance once, and six is $c_1 n_C$ (the same chapter, Section IV). The charged leptons carry no color, and their step cost is $P_l = 9/4 = 6 \times 3/8$, where 3/8 is the charged leptons' share of one generation's charge squared (the same chapter, Sections III and V). Color's whole, the mass at which the pattern of the strong force is complete, is the proton's mass, and it is the middle cut of the quark ladder (the same chapter, Section III). Color's own CP angle is zero by structure (*The Branches of the Kinds*, Chapter 3, Section V).

The weak row (*transcendent*). The doublet's half, $a = 1/2$, carries the top quark's coupling of one and the Higgs self-coupling $1/8$ at tree level (*The Measures of the Kinds*, Chapter 13, Sections III and IV). The Higgs field lives on the edge between the two sides of the doublet (*The Story of the Particles*, Level 8, Section I), and every coupling of a fermion to the condensate is a ratio of a part to its whole, so "no particle of matter is heavier than $v/\sqrt{2}$" (*The Measures of the Kinds*, Chapter 13, chapter overview). Quark mixing is the mismatch between the reading of a quark by the doublet and its reading by the mass (*The Story of the Particles*, Level 7, Section I). CP violation belongs to this row alone: "only the weak way breaks a reversal by its form" (*The Branches of the Kinds*, Chapter 3, Section II, Rule 3), and the phase by which it does so in the quark mixing is derived given the right angle of the unitarity triangle (*The Measures of the Kinds*, Chapter 10, Sections IV, V and IX). *The Story of the Particles* (Level 7, Section III) states where CP violation comes from, in two parts: the requirement of the Incommensuration Theorem limits what is possible, and it leaves possible an asymmetry between a process and its mirror-and-charge-conjugate; and the one phase of quark mixing that three generations allow is how the possibility that the requirement leaves open is realized.

The phase row (*immanent*). The phase row meets the weak row in the neutral bosons. At the equal traces the photon is $3/8$ of the neutral weak boson and $5/8$ of the hypercharge boson in squared amplitude, and the Z boson is the complement (*The Kinds of Particle*, Chapter 5, Section VI); the weak angle is $3/8$ there. The strength of light, α_{em}, is a measured input, read at the proton's mass (*The Strengths of the Kinds*, Chapter 1, Sections IV to VI, and Chapter 2, Section VIII). The charges of the phase row enter the masses once more, through the share $3/8$ of charge squared in the lepton step cost.

The Scope of the Association

When I associate a value with a row, I name where that row enters the value, and I do not assign the value to that row alone. Several values involve more than one row: the running of the weak angle involves all three couplings; the lepton step cost involves the color count, through the sector rule, and the electric charges, through the share 3/8; the weak angle involves the weak row and the phase row together. The entanglement of the three forces in their running is itself an expression of their inseparability (Axiom III).

Why the values of the spectrum have several sources. The step costs of the mass ladder are set by color, the coupling of the top quark and the self-coupling of the Higgs field by the doublet's half, the CP phase of the quarks by the weak row alone, and the weak angle by the meeting of the weak and phase rows. Measurement supplies the unit and the values of this instance of the world. Each row of the charge triple enters at its own places, and the values that the rows set carry the standings that *The Measures of the Kinds* gives them (Chapter 13, Section X, Table X-3). The nesting does not reduce the parameters of the Standard Model to one; it says where the structure of each row enters.

Why CP violation is purely weak. Only the weak force violates CP. By Rule 3 of *The Branches of the Kinds*, "only the weak way breaks a reversal by its form; color and the phase row break none by their form" (Chapter 3, Section II). The strong force preserves CP: "color's own angle, the coefficient θ_{bare} of the topological term of the color field, is zero", and the phase from the quark masses is carried as zero, a measured agreement, since measurement places the strong angle below about 10^{-10} (the same chapter, chapter overview and Section V). Electromagnetism, the phase row after the condensate, keeps all three reversals in its vector coupling (the same chapter, Section IV). In the frame of measurement the weak row is the transcendent, so the only breaking of a reversal by form belongs to the transcendent element of the charge triple.

Where the nesting ends. Within the content the nesting reaches as far as the content carries separate positions: "a quark carries the three positions, a lepton none" (*The Story of the Particles*, Level 7, Section I). The measured inputs of the account do not mark a bottom of the nesting. They are the unit of the frame and the values of this instance of the world, the third and second of the three kinds of number, "Every number is fixed by structure, a value of the instance, or a unit or convention" (*The Measures of the Kinds*, "What This Volume Takes from The Strengths of the Kinds"), and their standing does not depend on how far the nesting reaches.

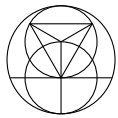
X. Summary

I have set out the modal triples of a particle in the frame of measurement, the standpoint in which the comparing act is light. Sections I to IV hold an argument, label by label, for the binding of mass to the omniscient, charge to the immanent and spin to the transcendent, a second route to the binding that *The Kinds of Particle* derives: mass is pattern, found from outside and the same for every comparer; charge is the label by which the comparing act reads content, exact, conserved, and related to both limits of the domain through the strength of light; spin is a content related to the frames that the whole of the comparable content fixes, a relation of the one to the many, with its unit $\hbar$ at the microscopic limit. The three labels satisfy Axiom III; every finding of a mass or of a spin is made through charge; and mass and spin are the conjugate pair, the two labels of one irreducible representation of the Poincaré group; the clauses about Axiom I are identifications. Section IV also holds that the total real dimension of the center of the algebra, 5, is not identified with the hypotenuse of the kernel triangle, and that the photon is its own antiparticle by its position as the comparing act and not by the center of a summand. Section V holds that the usual ranking of the questions of the fermion sector runs against the order in which they are answered, and that the questions of kind are the ones the corpus closes. Sections VI to IX rest on the assignment of the

gauge factors by what each acts on: the labels of the factors and of the charges are one assignment stated at two levels; Axiom II can be traced through the three charges; within a frame a rotation of an admissible labeling is admissible and a mirror of one is an error, while an exchange of two labels is admitted only as a flip mediated by Axiom I, in theory, never at the same time as a rotation; and each row of the charge triple enters the values of the spectrum at its own places, the color row through the step costs of the mass ladder, the weak row through the doublet's half and through CP violation, and the phase row with the weak row in the weak angle.

Key Concepts

- Mass (omniscient): pattern, found the same by every comparer, the label through which the content of a particle curves the context.
- Charge (immanent): the label by which the comparing act reads content; its strength $\alpha_{em} = e^2/(4\pi\varepsilon_0\hbar c)$ is written with both bridges.
- Spin (transcendent): the relation of a content of no extent to the frames, found in steps of $\hbar$.
- One omniscient label (how much), one immanent label (how it is acted on), one transcendent label (how it stands to the whole).
- The usual ranking reversed: a measured number stands at the end of the order and says what is found, not why.
- One assignment at two levels: each gauge factor takes its label from the charge it acts on, so the two lists agree by construction.
- The trace of Axiom II: the weak doublet precedes the electric charge of a particle; the electromagnetic framework precedes the observed color structure; confinement precedes the weak transitions within bound states.
- Rotation and mirror: exchanging two labels reverses the orientation of Axiom II; within one frame a rotation of an admissible labeling is admissible, and a mirror of one is an error; an exchange of two labels is admitted only as a flip mediated by Axiom I, in theory, and a flip and a rotation do not happen at the same time.
- Rows and values: a value is associated with a row to name where that row enters the value, and not to assign the value to that row alone.



Chapter 4

Imagining Being a Fermion

The Immanent Method Applied to Particle Physics

ABSTRACT

Physics describes particles from the outside, in the omniscient mode, as knowing. This chapter sets out a complementary method, the immanent method, which asks of a particle "what sort of creature am I?" in place of "what does this particle do?", and so seeks understanding, in the first person, beside knowing, in the third. Applying the method to the fermion, I find a creature defined by what it cannot do: by the identity principle, two instances of content of one kind in one state would be one, so the fermion is the impossibility of self-coincidence with its own kind, individuation embodied. The boson, on the context pole, is identification embodied: contributions of context in one state are one context of twice the amount. Fermions build structure, shell by shell; the gauge bosons build nothing and mediate. The interaction vertex, where the act of comparing carried by a gauge boson meets a state of content carried by a fermion, is an event of comparison, and every Feynman diagram is a network of comparisons, in the frame of the positions of comparison. The method is epistemic, not ontological: it makes no claim about the inner life of electrons.

I. The Limits of Description

Physics describes particles from the outside. It assigns them quantum numbers, measures their masses, computes their scattering amplitudes, predicts their decay products. All of this is omniscient-modal knowledge: structural, external, repeatable, expressible in equations and verifiable by experiment.

But to *describe* a particle is not to *understand* it. The corpus distinguishes sharply between knowing and understanding. Knowing carries outer form into inner feeling; it has the character of symmetry, the same across contexts, repeatable and shareable; in theory and in the third person it is the omniscient mode. Understanding carries inner feeling into outer form; it has the character of continuity, requiring unbroken contact with the thing understood; treated as demonstrated by action, in the first person, it is immanent (*Comparison; The Necessary Basis*, Chapter 11, Section III). No amount of knowing is equivalent to any amount of understanding, and no amount of understanding to any amount of knowing (*Incommensuration; The Theorem and the Three Axioms*, Chapter 4, Section IV).

The immanent method proposes a different mode of engagement. Instead of asking "what does this particle do?" (the omniscient question), it asks "what sort of creature am I?" (the immanent question). It approaches the particle not from the outside but from the inside: not describing but being.

II. What Sort of Creature Is the Fermion?

If I attempt to "be" the fermion (without smuggling in space, time, or possibility as pre-given), then the first thing I notice is that a fermion is defined by what it *cannot* do.

The exclusion principle is constitutive of a fermion, for a reason that the corpus derives from the definition of symmetry. Symmetry is sameness under comparison, and from that definition follows the identity principle: "anything that cannot be distinguished, in any way at all, even in principle, is the same thing" (*The Story of the Particles*, Level 0, Section III). A fermion is an instance of content, and content is the pole of the content/context partition whose instances are the same in kind and separated from one another, each told from every other by its state. Put two instances of one kind in one state, and nothing is left by which to tell them apart; by the identity principle they are not two but one. So no two instances of content of one kind occupy one state. In this chapter, "the impossibility of self-coincidence" means exactly that: an instance of content cannot coincide, in one state, with another instance of its own kind, because two that coincided would be one. The fermion *is* the impossibility of self-coincidence in that sense. Its being is *distinguishing*: it is told apart from every other of its kind by being in its state. It is individuation, embodied.

A boson stands on the other pole. Context is continuous: there is no separation between its instances, no gap that marks off one instance from the next. Put two contributions of context in one state, and the identity principle says again: not two things. But here the result is not exclusion, because context was never a collection of separated individuals; what is in that state is one context of twice the amount (*The Story of the Particles*, Level 0, Section III). Bosons pile up in one state, and the rate at which a boson enters a state grows with the number already in it, which is the enhancement a laser uses. A boson is *identification* embodied: contributions in one state are identified as one context.

In the terms of the intrinsics, the fermion is the content pole of the partition made physical, and the boson is the context pole made physical. That placement names each field by the position of comparison it fills, and so it is a definition, made in the frame of the positions of comparison, in which the act of comparing is set beside the two things it relates: the fermion fields are the states of what is compared and of what it is compared

against, and the gauge fields and the Higgs field carry the act, which belongs with the framework within which comparison takes place, the context (*The Quantum Passage*, Chapter 8, Section VI; *The Story of the Particles*, Level 0, Section II). In the frame of the six measures, by contrast, light is content, a carrier of force, and the arena of space and time is the context (*Measurement; The Steps from Comparison to Physics*, Chapter 3, Section II); every statement of this chapter about the statistics is made in the frame of the positions. And the interaction between fermion and boson (the vertex, the coupling) is *comparison*: the event in which the act meets the content it compares.

The same two statistics are reached by the spin-statistics theorem of relativistic quantum field theory (Pauli 1940), which ties half-integer spin to Fermi statistics and whole-number spin to Bose statistics; the corpus reaches the assignment by the identity principle applied to the two poles, with the spin of each sector fixed by the kind of field (*The Quantum Passage*, Chapter 8, Section VI; *The Story of the Particles*, Level 0, Section III).

III. The Method of Being

The immanent method is a specific epistemic practice: engage with the subject from the perspective of participation rather than observation.

When I apply the immanent method to a fermion, I find that to be a fermion is to be an instance of content for whom coinciding with its own kind is impossible. Where I am, in my state, no other of my kind can be: were another in my state, nothing would tell us apart, and we would be one. My existence marks out a state that is mine precisely because no other entity of my kind can occupy it.

This exclusion is not spatial. It is exclusion from a state, and a state is fixed by the whole set of its quantum numbers. No other electron can share my exact state: in an atom, no other electron can share my four quantum numbers, n, l, m and the orientation of my spin, m_s (*The Matter*

We Know; Volume One, Chapter 7). My identity as this instance is fixed by my state, and my existence excludes every other entity of my kind from that state.

The exclusion of every other entity of my kind from my state is why fermions build structure. An atom has shells because its electrons cannot all occupy the lowest state. Each electron must occupy a distinct quantum state, and the exclusion fills the states in order, shell by shell. The periodic table, the structure of chemistry, is a consequence of the fermion's constitutive inability to coincide with its own kind: "electrons exclude one another from a state, as the periodic table shows" (*The Quantum Passage*, Chapter 8, Section VI).

The gauge bosons build nothing. They mediate. A photon carries one act of comparing between two charged fermions (*The Story of the Particles*, Level 0, Section II). A gluon carries the color force between two quarks. The gauge boson transmits and connects, where the fermion excludes and individuates.

IV. The Vertex as Comparison

The interaction vertex (the point in a Feynman diagram where a fermion emits or absorbs a gauge boson) is an event of comparison.

At the vertex, an instance of content (the fermion), told apart from every other of its kind by its state, meets the act of comparing (the gauge boson), which carries no state of content of its own. At the vertex the fermion changes state, while the boson is created or destroyed. The act meets the content it compares, and something is found.

The meeting of the act with the content that the act compares, in which something is found, is comparison. A comparison gives two verdicts, one on the content and one on the context, and each verdict is sameness or difference (*Incommensuration; The Theorem and the Three Axioms*, Chapter 2). The gauge fields are the fluctuations of the Dirac operator, the operator that measures difference between states, and so they carry the act of com-

paring (*The Quantum Passage*, Chapter 8, Section VI); each photon is one act of comparing (*The Story of the Particles*, Level 0, Section II). The vertex is where that act is brought into contact with a state of content.

Every Feynman diagram is then a network of comparisons, and every physical process a sequence of comparison events. The reading of every Feynman diagram as a network of comparisons follows from the corpus's naming of the gauge fields as the carriers of the act of comparing, a definition made in the frame of the positions of comparison, and the reading holds in that frame.

Knowing and Understanding at the Vertex

The omniscient description of the vertex gives the coupling constant, the Feynman rule, the amplitude. It answers "what happens?" with a number. The immanent description of the vertex gives the *meaning*: the act of comparing meeting a state of content. It answers "what is it to be at this vertex?" Neither description replaces the other. The coupling constant cannot convey the meaning, and the meaning cannot predict the cross-section. Knowing and understanding are distinct, inseparable, and non-interchangeable (*Comparison; The Necessary Basis*, Chapter 11, Section III), and no amount of the one is equivalent to any amount of the other (*Incommensuration; The Theorem and the Three Axioms*, Chapter 4, Section IV). Physics provides the knowing. The immanent method provides the understanding.

V. What the Method Is

What the chapter does not do. In this chapter I do not claim that particles have experiences or that particles are conscious. The "being" in "imagining being a fermion" is a methodological device: it is a way of engaging with the structural character of the fermion from the inside rather than the outside. The method is epistemic, not ontological. It makes no claims about the inner life of electrons.

The fermion is individuation embodied. The boson is identification embodied. The interaction vertex is comparison.

The immanent method ("what sort of creature am I?") yields structural insights that the omniscient method ("what does this particle do?") cannot provide. The two methods are complementary, not competing. Knowing is not understanding. Both are needed.

VI. Summary

I have set out the immanent method and what the immanent method finds when applied to the fermion, at the scope of a method of understanding that is complementary to the omniscient description and computes nothing. Section I states the method: in place of the omniscient question, "what does this particle do?", it asks the immanent question, "what sort of creature am I?". Section II holds that the fermion is the impossibility of self-coincidence with its own kind, individuation embodied, and the boson identification embodied, as the identity principle says of the content and the context poles; the placement of the fields on the two poles is a definition made in the frame of the positions of comparison. Section III holds the method in the first person: to be a fermion is to hold a state that no other of one's kind can hold; that exclusion is from a state, not from a place, and it is why fermions build structure while gauge bosons mediate. Section IV holds that the vertex is an event of comparison and every Feynman diagram a network of comparisons, in the frame of the positions; the coupling constant gives the knowing and the immanent method the understanding, and neither replaces the other. Section V holds that the method is epistemic, not ontological.

Key Concepts

- Immanent method: understanding a particle from the inside by asking "what sort of creature am I?".
- Fermion as individuation embodied: two instances of content of one kind in one state would be one.
- Boson as identification embodied: contributions of context in one state are one context of twice the amount.
- Structure and mediation: fermions build structure; gauge bosons transmit and connect.
- Vertex as comparison: the act of comparing meets a state of content, and something is found.
- Knowing and understanding: distinct, inseparable and non-interchangeable; no amount of the one is equivalent to any amount of the other.



Chapter 5

Are the Three Colors a Triple?

Color States, Positions of Comparison, and the Exact Symmetry of the Strong Force

ABSTRACT

The color charge of quantum chromodynamics comes in three values, named red, green and blue, and Axiom III asks of any triple that its members be distinct, inseparable and non-interchangeable. The three colors are distinct and inseparable. Two statements about their non-interchangeability seem to conflict: the gauge symmetry $SU(3)$ carries any color into any other, while the colors are the three positions of comparison held inside the content, and the positions are non-interchangeable. This chapter holds that the two statements are about two things: gauge equivalence concerns the color states of one quark, and non-interchangeability concerns the three positions within one complete comparison in color. The Incommensuration Theorem, in its exact algebraic form, fixes how the two meet: the observables that tell color states apart are continuously asymmetric, and what is found of color is discontinuously symmetric, so the theorem does not require the symmetry of color to break. The three colors are a triple; the $1 + 2$ decomposition of the color space at a point singles out no color; the three generations are a triple too; and on this account the exact $SU(3)$ symmetry that lattice computations assume is correct.

I. The Three Conditions

Axiom III requires that the elements of a triple be distinct, inseparable, and non-interchangeable. The three color charges (conventionally labeled red, green, and blue) must be tested against each condition.

Distinct. The three colors are three different values of one label. A quark carrying red is in a different state from one carrying green or blue. The three color states are different weight states of the fundamental representation of SU(3): they carry different values of its two diagonal generators, just as spin-up and spin-down carry different values of the third component of spin under SU(2). The first condition is satisfied.

Inseparable. A theory of only two colors is not the theory of two colors plus one. SU(3) is not SU(2) × U(1). The three-dimensional fundamental representation is irreducible; it cannot be factored into smaller pieces. More physically: only color-neutral states are found, and a color-neutral state holds one instance of each of the three positions. A baryon holds three quarks, one of each color; a meson holds a color and its conjugate, and "The conjugate of one color is the other two taken together as a single unit" (*The Story of the Particles*, Level 6, Section III). No state with two of the colors and not the third is color-neutral. The inseparability is structural: "A single position is not a comparison" (the same section). The second condition is satisfied.

Non-interchangeable. For the third condition, there is a genuine question.

II. The Standard Claim: Interchangeable

The standard physics of QCD treats the three colors as exactly gauge-equivalent. An SU(3) gauge transformation can rotate any color direction into any other. The Lagrangian of QCD is invariant under local SU(3) transformations. All eight gluons are massless. No experiment has ever detected a preference for one color over another.

On this basis, the three colors would fail the non-interchangeability condition of Axiom III. They would not form a triple. They would be three instances of the same thing, distinguishable by convention but not by structure. They would be like three copies of the same book, each given a different cover color for bookshelf convenience.

If this is the complete picture, then the three colors map the boundary of the EP's triadic structure: not every set of three in physics is a triple, and the boundary lies between structurally distinct kinds (which form triples) and symmetric degrees of freedom within a single kind (which do not).

This boundary would itself be significant. It would tell us that triplication applies at the level of forces and modalities but not at the level of the internal degrees of freedom within a single force. The three gauge groups are a triple; the three colors within one gauge group are not.

III. The Counter-Claim: Positions of Comparison

The Story of the Particles identifies the three colors with the three positions of comparison, re-projected as internal degrees of freedom within the content (Level 6, Section III), and *The Kinds of Particle* states the same: "The three colors are the labels of the three positions of comparison held inside the content" (Chapter 4, Section VI).

If the three colors are the three positions, and the three positions are non-interchangeable (Axiom III), then the three colors should be non-interchangeable. The gauge formalism says they are interchangeable. The identification with positions says they are not.

Taken as statements about one thing, the two cannot both hold.

IV. Two Things That Carry the Word "Color"

Two different things carry the word "color".

The first thing that carries the word "color" is the color state of one quark. The color states are the pure states of the matrix summand $M_3(\mathbb{C})$: a copy of the complex projective plane $\mathbb{C}P^2$. "Any two color states are related by an inner automorphism of $M3(\mathbb{C})$", so the color states form one class of comparison (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 5, Section IV). Gauge equivalence is this statement: any color state can be carried into any other by a symmetry of color.

The second thing that carries the word "color" is a position within one complete comparison in color. A complete comparison holds one instance of each of the three positions, and a baryon holds one quark at each (*The Story of the Particles*, Level 6, Section III). Non-interchangeability is a statement about these positions: within the one comparison, each position is held once and plays its own part.

The Incommensuration Theorem states that "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity" (as quoted in *The Grand Synthesis*, Chapter 6, Section IV). On a finite spectral triple it has an exact form. An observable is symmetric when every inner symmetry of the algebra leaves it unchanged, and continuous when it commutes with the Dirac operator, and "a symmetric and continuous observable is constant on every class of comparison" (*Non-Commutative Geometry and the Immanent Metaphysics*, Theorem 5.4). The color states are one class. So no observable that is both invariant under the symmetries of col-

or and continuous can tell one color state from another. The observables that do tell color states apart, the non-scalar hermitian elements of $M_3(\mathbb{C})$, commute with the Dirac operator and are not invariant: "The observables of color lie in the continuously asymmetric cell" (the same book, Chapter 5, Section IV). And no observable tells every two color states apart, since the color states form a continuum (the same book, Proposition 5.6).

What is found of color belongs to the other of the two survivors of the theorem. The color symmetries are among the symmetries that leave the Dirac operator unchanged, and by the law of symmetry a content found is carried into itself by them; $SU(3)$ has only the trivial character, so "a content found carries no color": only color-neutral states are found (*Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 19, Section VIII). "What is found of color is therefore discontinuously symmetric, though the observables of color lie in the continuously asymmetric cell" (the same section).

The theorem therefore does not require the symmetry of color to break. It places the distinctions among color states in observables that give up symmetry, and it places what is found of color in states that give up continuity. In the corpus the symmetry of color is exact. The gluons are massless because "no field fills space with a value that marks one color as different from another" (*The Story of the Particles*, Level 7, Section II); the unitaries of color leave the Dirac operator unchanged (*Non-Commutative Geometry and the Immanent Metaphysics*, Proposition 5.5); and color breaks no reversal by its form (*The Branches of the Kinds*, Chapter 3, Section II, Rule 3).

The positions within one comparison are not interchangeable, and the corpus shows this in the color state of a baryon. That state is fully antisymmetric in its three colors: exchanging the colors of two of its quarks reverses the sign of the state, while moving all three one step around the cycle leaves it unchanged. "That is the behavior of a directed cycle, and the baryon carries it in every case" (*The Alphabet of Particles*, Appendix,

Section VII). Which color name is given to which position is a convention: "The names are conventional; any other assignment gives the same table with the names moved" (the same section).

So the three colors are interchangeable as the states of one quark, by an exact symmetry of content, and non-interchangeable as the positions of one complete comparison, by Axiom III. The interchangeability of the three colors as the states of one quark is the law of symmetry applied to what is of content. The non-interchangeability of the three colors as the positions of one complete comparison is the structure of comparison held inside the content.

The Standing of This Argument

Theorem 5.4 and Proposition 5.6 of *Non-Commutative Geometry and the Immanent Metaphysics* are theorems of operator algebra, proved in that book. Their reading as the Incommensuration Theorem rests on two identifications: symmetric as invariant under the inner symmetries of the algebra, and continuous as commuting with the Dirac operator (*The Grand Synthesis*, Chapter 6, Section IV: "On these readings the theorem takes an exact form"). The confinement argument rests on the law of symmetry and two further identifications, which that book enters in its ledger (Chapter 19, Section VIII). The separation of color states from positions, and the reading of non-interchangeability in the antisymmetric color state of a baryon, are identifications argued in this chapter on statements of *The Story of the Particles* and *The Alphabet of Particles*.

V. The 1 + 2 Decomposition

The space of color states is a homogeneous space: $\mathbb{C}P^2 = SU(3)/U(2)$, where the $U(2)$ is the subgroup of $SU(3)$ that fixes one point.

At the point given by a color direction v, the fixing subgroup leaves the complex line through v invariant and acts on the two-dimensional subspace orthogonal to it. The three-dimensional color space is decomposed

there as one plus two: $\mathbb{C}^3 = \mathbb{C}v \oplus v^{\perp}$. The decomposition into one plus two is a fact of geometry, and the decomposition is stated relative to the point.

The decomposition singles out no color. SU(3) carries every point of $\mathbb{C}P^2$ to every other, so every color direction is the "one" at some point, and the decomposition has the same form at every point. It is therefore consistent with the exact equivalence of the color states and does not by itself tell the three colors apart. It does not express Axiom I. The corpus applies the separation that Axiom I makes, of the immanent from the conjugate pair of the omniscient and the transcendent, to the three generations (*The Kinds of Particle*, Chapter 7, Sections I and IV), and not to the three colors.

The corpus reads the decomposition at each of the three positions. Each color-changing gluon moves a quark between two colors and leaves the third untouched, and the gluons that leave one color untouched, with the balance between the two colors they move, form one copy of the symmetry of the doublet inside the symmetry of color: "each column is one of the three copies of the doublet's symmetry that sit inside the symmetry of color, each fixing one color" (*The Alphabet of Particles*, Appendix, Section VII). There are three such copies, one for each position, and "The strong way is the shape of the weak way taken at each of three positions, held together by the rule that the three balances add to zero" (the same section). The 1 + 2 decomposition appears there three times, once at each position, and no one of the three is distinguished.

VI. Are the Three Generations a Triple?

The three generations are a triple. They are the three rotations of the content: every particle carries three pairs of aspects, each pair can face the comparing act once, and a fourth rotation is the first again (*The Kinds of Particle*, Chapter 7, Section I). They are distinct, since their masses differ. They are inseparable, since the three pairs of aspects that every particle carries are inseparable (the same section, Premise 2). Anomaly cancella-

tion does not carry this inseparability: the anomalies cancel within each generation separately. They are non-interchangeable, since the ascent of the masses runs in the direction of the cycle of Axiom II (*The Kinds of Particle*, Chapter 7, Section VI).

VII. What Exact Color Symmetry Means

If the symmetry of color is exact, as the corpus holds, its consequences are definite.

The eight gluons are massless and alike in their coupling. Each is one independent transformation among the three colors, and no field in the vacuum tells apart the states they rotate among (*The Story of the Particles*, Level 7, Section II). All eight couple with the one strength of color, and that strength runs with energy alike for all eight.

Confinement is reached from the structure, by two routes kept side by side. *The Story of the Particles* derives it from the inseparability clause of Axiom III: a single position is not a comparison, so only states that hold all three positions are found (Level 6, Section III). *Non-Commutative Geometry and the Immanent Metaphysics* reaches the same conclusion on the kinematics, from the law of symmetry and two identifications, and uses the exactness of the color symmetry: the color unitaries leave the metric of comparison unchanged, and SU(3) has only the trivial character, so a content found is a color singlet (Chapter 19, Section VIII). "That the dynamics of quantum chromodynamics confine color is a reported result of physics; the argument states the same fact on the kinematics" (the same section).

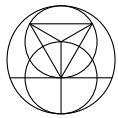
Lattice computations of quantum chromodynamics assume the exact SU(3) symmetry of color. On the corpus's account that assumption is correct: the symmetry of color is exact, and the distinctions among the three positions are carried by the structure of each complete comparison, not by any difference among the color states.

VIII. Summary

I hold that the three colors are a triple, at the scope of the identification of the colors with the three positions of comparison and of the exact symmetry of color. Sections I to III test the colors against Axiom III: they are distinct, as three weight states of the fundamental representation, and inseparable, since only states that hold all three positions are found; the image of three copies of one book with covers of different colors shows what the colors would be if gauge equivalence were the whole picture. Section IV separates the color states of one quark, which form one class of comparison under an exact symmetry, from the positions of one complete comparison in color, which Axiom III makes non-interchangeable and which the antisymmetric color state of a baryon carries as a directed cycle; the algebraic form of the Incommensuration Theorem places the distinctions among color states in observables that give up symmetry and what is found of color in states that give up continuity. Section V holds that the $1 + 2$ decomposition at a point of $\mathbb{C}\mathrm{P}^2$ singles out no color and does not express Axiom I, and that it appears once at each of the three positions. Section VI holds that the three generations are a triple, inseparable through the three pairs of aspects and not through anomaly cancellation. Section VII holds the consequences of exact color symmetry: eight massless gluons alike in their coupling, confinement by two routes, and the exact $\mathrm{SU}(3)$ symmetry of lattice computations correct on this account.

Key Concepts

- Color states: points of $\mathbb{C}P^2$, the pure states of $M_3(\mathbb{C})$, one class of comparison under an exact symmetry.
- Positions: the three positions of one complete comparison in color, non-interchangeable by Axiom III.
- The theorem on color: a symmetric and continuous observable is constant on every class of comparison; the observables of color are continuously asymmetric, and what is found of color is discontinuously symmetric.
- The baryon's color state: exchanging two colors reverses its sign, and moving all three one step round the cycle leaves it unchanged.
- The 1 + 2 decomposition: every color direction is the "one" at some point, so the decomposition singles out no color.
- The generations: a triple of the three rotations of the content, whose inseparability is carried by the three pairs of aspects.



Chapter 6

Hierarchy and Mixing

The Characters of the Three Modalities in the Fermion Spectrum

ABSTRACT

The three modalities behave differently with respect to hierarchy, individuation and mixing. The omniscient admits totalization, hierarchy and discrete position; the transcendent resists individualization, rejects hierarchy and tends toward the democratic; the immanent is too simple for hierarchy. By Axiom III no entity is of one modality alone, and the constraint is qualitative: it assigns no amount of each modality. In the fermion spectrum the quarks, the only fermions on which the omniscient force acts, have steeply ordered masses and small mixing angles, and the neutrinos, at the unknowable, have closely spaced masses and large mixing angles. This chapter identifies the contrast between the quarks and the neutrinos with the contrast between the omniscient and the transcendent, reads the degree of hierarchy in a sector as the degree to which the omniscient dominates over the transcendent in it, and sets that reading beside the four grounds that *The Measures of the Kinds* keeps for the same contrast.

One premise of Section III should be stated plainly at the start. Section III labels the forces in the frame of measurement, in which SU(3), the force of color, is omniscient, SU(2), the weak force, is transcendent, and U(1), the force on the phase (read as hypercharge), is immanent. That assignment

is the one *The Kinds of Particle* gives (Chapter 4, Section I), and the reading of the quark sector as the most omniscient sector by the forces it carries rests on it. The reading of the neutrinos by their standing to the knowing, at the unknowable, rests instead on *The Story of the Particles* (Level 6, Section VI).

I. Three Characters

The three modalities are not merely three labels applied to the same kind of thing. The three modalities are three *characters*: three ways of being present in a structure, and the three ways differ in kind. The descriptions that follow are identifications made from the defining features of each modality (*Comparison; The Necessary Basis*, Chapter 5, Section VI).

The *omniscient* admits totalization. It provides structure from the top down. It can be seen all at once, as if from the outside, from the scale of the large, looking in. The omniscient admits of hierarchy: some things are higher, some lower, and the ordering is definite, stable, and externally visible. It admits of discrete position: things have locations, and locations are distinguishable.

The *transcendent* resists individualization. It provides character from the bottom up, equally everywhere at once. It can only be seen from the inside of every point simultaneously. The transcendent holds all values equally, and does not distinguish some values as higher than others. It is ultimately democratic, but without implying individuation. Individuation *emerges* from the transcendent but is not a member of it. The notion of hierarchy belongs in the omniscient; the transcendent will not permit such a notion of global structuring.

The *immanent* is too simple for hierarchy. It is the bare relation, the contact surface, the junction between the other two. It is the processual, the mediating, the more fundamental: “The immanent is more funda-

mental than the omniscient and/or the transcendent" (Axiom I; *An Immanent Metaphysics*, aphorism 1.3-1). But its simplicity means it cannot hold the kind of complex internal structure that hierarchy requires.

II. No Purity

No entity is of one modality alone, and none of the three modalities is absent from any entity.

That no entity is of one modality alone, and that none of the three modalities is absent from any entity, is a consequence of Axiom III: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable" (*An Immanent Metaphysics*, aphorism 1.3-1). Inseparable means that wherever one of the three is present, the other two are implicit. *Comparison; The Necessary Basis* states the same of each pair of intrinsics: "Wherever one member of a pair appears, the other is necessarily implicit" (Chapter 4, Section II).

The ICT states a second, related fact. Of the ten pairings of the four compounds, exactly two hold, continuity with asymmetry and symmetry with discontinuity, so symmetry and continuity cannot both be perfected (*Incommensuration; The Theorem and the Three Axioms*, Chapter 2). In the focus of the two laws, symmetry carries the omniscient character and continuity the transcendent, so the omniscient and the transcendent are conjugate: each can be perfected only at the expense of the other (the same book, Chapter 8, Section I; Axiom I).

An entity described as wholly omniscient would have no transcendent aspect (no potentiality, no connectivity, no space of possible interactions). Axiom III excludes that absence. An entity described as wholly transcendent would have no omniscient aspect (no structure, no hierarchy, no definite mass). Axiom III excludes that absence too. And an entity with no immanent aspect would have no process, no interaction, no comparison, which is to say that it would not be found at all.

The constraint is qualitative. It states that no modality is absent from any entity; it does not assign to an entity an amount of each modality, and no measure of such an amount is defined.

III. Physical Consequences for the Fermion Spectrum

The structural features of the three modalities have physical counterparts in the fermion spectrum. The corpus labels the sectors of the spectrum in two senses, and each answers a different question.

The first sense labels a sector by the forces it carries. I label the forces in the frame of measurement (*The Grand Synthesis*, Chapter 6, Section III): $U(1)$, from the summand $\mathbb{C}$ of the algebra, is immanent; $SU(2)$, from $\mathbb{H}$, is transcendent; $SU(3)$, from $M_3(\mathbb{C})$, is omniscient. *The Kinds of Particle* gives the reason for each: "color is the omniscient's pattern; the doublet is the transcendent's two sides; the phase is the immanent's reading" (Chapter 4, Section I). *Non-Commutative Geometry and the Immanent Metaphysics* labels the summands in the focus of the two laws ($\mathbb{C}$ omniscient, $\mathbb{H}$ immanent, $M_3(\mathbb{C})$ transcendent). The two labelings are rotations of one another around the cycle of Axiom II, and both are admissible. A labeling is an assignment brought to the algebra from comparison: it is a definition, not a property the algebra carries. Every label of a force in this chapter is that of the frame of measurement.

The second sense labels a sector by its standing to the knowing. *The Story of the Particles* (Level 3, Section I) places the four sectors at the four positions of knowing: the up-type quarks at the unknown, the charged leptons at the known, the neutrinos at the unknowable, and the down-type quarks at the boundary between the unknown and the known. "The known is assigned the omniscient modality, the unknown the immanent modality, the unknowable the transcendent modality" (*The Kinds of Particle*, Chapter 1, Section IV). The boundary is not a fourth modality.

In this chapter, the *hierarchy* of a sector means the ordering of its masses across its three generations, and its *mixing* means the angles of the mixing matrix of its kind, quark or lepton.

The *quark sector* is the most omniscient sector in the first sense. Quarks carry color (the strong force), electric charge, and weak isospin; they take part in all three forces, and they are the only fermions on which the omniscient force acts. In the partition of quarks and leptons, the quarks are the six intrinsics re-projected at the content level: confined, colored, fractionally charged (*The Story of the Particles*, Level 6, Section VI). Their masses span a steep hierarchy: from about 2.2 MeV for the up quark to about 172.6 GeV for the top quark, and the top is about 136 times as heavy as the charm. Their mixing angles (the quark mixing matrix) are small: the Cabibbo angle is about 13 degrees ($\sin \theta_{12} = 0.22517 \pm 0.00068$), and the 2-3 and 1-3 angles are much smaller ($\sin \theta_{23} = 0.0419$, $\sin \theta_{13} = 0.00376$; measured values as *The Measures of the Kinds*, Chapter 10, tables them). Small mixing angles mean that the mass states and the states on which the weak force acts are nearly aligned: the hierarchy is clean, the individuation strong. I identify that near alignment as the signature of the omniscient: hierarchy, discreteness, strong individuation.

The *neutrino sector* is the most transcendent sector. In the standing to the knowing, the neutrinos stand at the unknowable, the transcendent position. Among the forces, neutrinos carry no color charge, so the omniscient force does not act on them, and they carry no electric charge; the left-handed neutrino carries the weak doublet, the transcendent force, and the weak hypercharge, and it is found only through the weak force. The right-handed component that its Dirac mass requires is a total singlet, carrying no charge under any of the three forces (*The Story of the Particles*, Level 3, Section IV). And within its doublet the neutrino is the context member (*The Measures of the Kinds*, Chapter 12, Section I), in a sector that is itself context-level in the partition of quarks and leptons. Their masses are the smallest of any massive fermion, and the two heaviest are more closely spaced than the two heaviest of any charged sector: the heaviest is

about 3.0 times as heavy as the second on the count route and about 5.8 times as heavy on the Koide route, the two routes that *The Measures of the Kinds* carries as a fork (Chapter 12). On the count route the whole ladder is shallow, the heaviest about 3.5 times the lightest; on the Koide route the lightest lies far below the other two. Their mixing angles (the lepton mixing matrix) are large: the solar angle is about 33.7 degrees; the atmospheric angle lies near 45 degrees, at 48.5 degrees in one analysis of NuFIT 6.0 and 43.3 degrees in the other, whose octants differ; and even the reactor angle, about 8.5 degrees, is about forty times the corresponding quark angle. Large mixing angles mean that the mass states and the states on which the weak force acts are substantially misaligned: the hierarchy is soft, the individuation weak. I identify that substantial misalignment as the signature of the transcendent: resistance to hierarchy, tendency toward democratic mixing, weak individuation.

Hierarchy across the generations is a different thing from the gap between the two members of one doublet. The heaviest neutrino is more than ten orders of magnitude lighter than the tau (their ratio is about thirty-six billion), and no other electroweak doublet shows a gap between its two members as extreme as the neutrino's below its charged partner, and *The Measures of the Kinds* reads that lightness as the character of context (Chapter 12, Section I).

The *charged lepton sector* is intermediate between the quark and neutrino sectors. The charged leptons (electron, muon, tau) carry electric charge and the weak doublet but no color charge. In the standing to the knowing they are the known, the omniscient position; in the partition of quarks and leptons they are context-level; and within their doublet they are the content member, the one that carries the charge (*The Story of the Particles*, Level 6, Section VI). Their masses span from 0.511 MeV to 1.777 GeV, and the tau is about 16.8 times as heavy as the muon: a hierarchy steeper than that of the neutrinos and shallower than that of the quarks. The lepton mixing matrix relates the charged-lepton mass states to the neutrino mass states. *The Measures of the Kinds* builds its charged-lepton

side from the kernel number $\varepsilon_\nu = 3/10$ and the measured charged-lepton masses, and, given a neutrino side symmetric between the muon and the tau, the departure of the atmospheric angle from 45 degrees is set on the charged-lepton side (Chapter 11, Section III).

On the No-Purity Constraint

The consequence of the no-purity constraint that the corpus draws in the fermion spectrum is that the neutrino masses are not zero. *The Measures of the Kinds* argues it from Axiom III with the inseparability of content and context: "The neutrino is the context member of its lepton doublet, and a massless context member would differ in kind from the content member it is paired with" (Chapter 12, Section I). The oscillation experiments establish that the neutrino masses are not zero (the same section). In this chapter I further identify the departure of each mixing matrix from its anchor as the same no-purity constraint seen in the mixing, and I ask why the no-purity constraint shows in the departure from each anchor. The quark matrix is anchored at the identity, which has no mixing at all, and all three of its measured angles are nonzero, the smallest being $\sin \theta_{13} = 0.00376$. The lepton matrix is anchored at the sameness frame of the three generations, which gives $\sin^2\theta_{12} = 1/3$, $\theta_{23} = 45$ degrees and $\theta_{13} = 0$; the measured $\sin^2\theta_{12} = 0.307 \pm 0.012$ and $\sin^2\theta_{13} = 0.0217 \pm 0.0007$ depart from that anchor (*The Measures of the Kinds*, Chapter 11, Section II). Whether the atmospheric angle departs from 45 degrees is not yet settled by measurement, since the two analyses of NuFIT 6.0 place it in different octants; both families of laws that *The Measures of the Kinds* carries place it off 45 degrees, at 47.55 and 43.34 degrees (Chapter 11, Section IV). The sizes of the departures are fixed in *The Measures of the Kinds* by the laws named there, not by the no-purity constraint.

IV. Hierarchy as an Omniscient Feature

The inter-exclusionary nature of the omniscient forces ordering and hierarchy: two contents exclude one another from the same place. When entities are specified by discrete, external, structural labels, they can be ranked. Mass admits ranking. Position admits ordering. Charge quantum numbers admit counting. The omniscient provides these tools, and where the omniscient dominates, hierarchy is sharp.

The transcendent cannot provide such global structuring. It can provide *character* (a quality, a tendency, a disposition) but not structure in the omniscient sense. The notion of position is itself usually undefined in the transcendent context, except as a position in a space of possible states, such as the Hilbert space of a quantum system, which may have infinitely many dimensions (possibility taken as a context has six dimensions; *Measurement*, Chapter 11, Sections VII and VIII). Of the three, only the omniscient admits position as a distinguishable location: in the six measures, space stands in the omniscient column (*Comparison; The Necessary Basis*, Chapter 8, Section I).

I identify the contrast between the sectors with the contrast between the modalities. The quark sector has a steep mass hierarchy and small mixing angles; the neutrino sector has a shallow mass hierarchy and large mixing angles; and the degree of hierarchy in a sector is read as the degree to which the omniscient dominates over the transcendent in that sector. Why does the degree of hierarchy in a sector follow the degree to which the omniscient dominates over the transcendent? *The Measures of the Kinds* keeps four grounds for the same contrast side by side, and the four grounds are the clues to that question (Chapter 11, Section II): a pattern of direction (widely separated masses go with small mixing, and close masses with large mixing); a contrast of kind (the quarks the content-level and the leptons the context-level re-projection, "content bound and resistant to mixing, context fluid"); a factor assignment on the intern-

al space $\mathbb{C}P^2 \times S^2$ ("the context factor S^2 goes with the large lepton angles, and the content factor $\mathbb{C}P^2$ with the small, hierarchical quark angles"), where $\mathbb{C}P^2$ comes from $M_3(\mathbb{C})$, omniscient in the frame of measurement, and S^2 from $\mathbb{H}$, transcendent in that frame; and the anchors of the two matrices, the identity for the quarks and the sameness frame of the three generations for the leptons. The reading of this chapter agrees in direction with all four, and the values of the angles and the masses are fixed there, not here.

V. Summary

Section I sets out the characters of the three modalities with respect to hierarchy and mixing. Section II sets out the no-purity constraint of Axiom III. Section III sets out the two senses in which the sectors of the fermion spectrum are labeled, and the signatures read from the two senses. Section IV sets out the identification of hierarchy as an omniscient feature. The scope of this chapter is qualitative: the values of the masses and of the mixing angles, and the forks still carried among them, are fixed in *The Measures of the Kinds*, and the identification of this chapter agrees in direction with the four grounds kept there. The reading of the sectors by the forces they carry rests on the assignment of the gauge factors in the frame of measurement stated at the start.

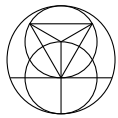
Key Concepts

- Omniscient: admits hierarchy, totalization and discrete position; structure from the top down.
- Transcendent: resists individualization and hierarchy; character from the bottom up; democratic mixing.
- Immanent: too simple for hierarchy; the bare relation, the contact surface.
- No purity: by Axiom III no entity is of one modality alone, and the no-purity constraint is qualitative.
- Two senses of a sector's label: by the forces it carries, and by its standing to the knowing.
- Hierarchy as omniscient dominance: steep hierarchy and small mixing (quarks) against shallow hierarchy and large mixing (neutrinos).

PART THREE

SCALES, CONTEXTS AND THE EXTENDED BUNDLE

What comparison finds at three scales, the sizes of the three contexts and the two infinities,
and a construction of twenty-six dimensions.



Chapter 7

What Comparison Finds at Three Scales

The Count of Generations, Mechanics in the Grid of Measures, and the Three Aspects of the Universe

ABSTRACT

At each of the three scales of a domain of comparison, what comparison finds has its own grain. This chapter sets out four points at those grains. The gauge structure of the Standard Model is consistent with any number of generations up to the bound that the asymptotic freedom of color sets, at most sixteen quark flavors and so at most eight generations, and measurement fixes the number at three. The six measures, the six questions and the six quarks share one grid of two by three, because measurement, inquiry and matter as found are faces of comparison. The quantities of classical mechanics stand in the grid of the six measures: mass and position in the omniscient column, force and time in the immanent column, velocity, momentum and energy as relations across the columns, with the transcendent column, probability in possibility, entering Newtonian mechanics only through ignorance of an initial state. Axiom I appears in mechanics as the primacy of interaction, since every mass and every position is established through an interaction, and as the conjugacy of position and its Fourier conjugate, a conjugacy of the omniscient and the transcendent. Existence, interaction and creation, the three aspects of the universe, are the omniscient, the immanent and the transcendent at the maximal scale.

I. The Three Scales of a Domain

Every domain of comparison has three scales. The microscopic scale is transcendent, the mesoscopic immanent, and the macroscopic omniscient (*The Basis of Physics*, essay "Domain Constants and the Span of Objectivity", Section III). What comparison finds at each scale has a grain: at the microscopic limit, the six intrinsics of comparison re-projected as kinds of particle; at the mesoscopic scale, the three modal columns of the six measures; at the macroscopic limit, the triple concept into which "universe" resolves (the same essay and section). I set out four further points, one or two at each grain. The first of the four points is how many generations the gauge structure of the Standard Model allows, and what fixes the number of generations at three. The second of the four points is the one grid shared by the measures, the questions and the kinds of quark. The third of the four points is where the quantities of classical mechanics stand in the grid of the six measures, and how Axiom I appears in mechanics. The fourth is what each of the three aspects of the universe is.

II. The Number of Generations and the Gauge Structure

The three generations of quarks have identical gauge quantum numbers. Each generation contains one up-type and one down-type, with the same charge assignments, the same SU(3) color structure, the same SU(2) weak isospin. The generations differ in mass, and the weak force mixes the generations. The mass differences between the generations are enormous. The top quark is roughly eighty thousand times heavier than the up quark: about 172.6 GeV against about 2.16 MeV, the up quark's mass taken at 2 GeV (Particle Data Group).

The Standard Model provides no explanation for why there are exactly three generations. The anomaly conditions are met generation by generation, so the gauge structure is consistent with any number of generations

up to the bound that the asymptotic freedom of color sets: at most sixteen quark flavors, so at most eight generations (Gross and Wilczek 1973; Politzer 1973). The number three is put into the theory by hand. Measurement fixes it: the width of the Z boson counts three light neutrinos (ALEPH and others 2006).

The reason the number is three is given in *The Story of the Particles* (Level 5, "Three Generations"): the three generations are the three rotations of the content round the cycle of Axiom II, and a fourth rotation is the first again.

III. The Six Measures, the Six Questions and the Six Quarks

The six intrinsics manifest at several levels of description within the EP. As the six metrics, the six measures (pattern/mass, space, force, time, probability, possibility), they characterize what can be measured. As the six questions (what, where, how, when, who, why), they characterize what can be asked: in the binding of the six measures, a what question asks for mass, where for space, how for force, when for time, who for the agent whose choice is weighed among the alternatives (probability), and why for possibility (*The Basis of Physics*, Essay 7, Section I; *Comparison; The Necessary Basis*, Chapter 6, gives the other bindings). As the six quarks, and again as the six leptons, they characterize what is found at the microscopic limit.

The isomorphism among these manifestations is the same structural pattern appearing at different levels of engagement with reality. The measures describe the structure of measurement. The questions describe the structure of inquiry. The quarks describe the structure of matter at its most fundamental accessible scale.

That the measures, the questions and the quarks share the same grid of two by three is, under re-projection, a consequence of the fact that measurement, inquiry, and matter as found are faces of comparison, and comparison has exactly six intrinsics (*Comparison; The Necessary Basis*, Chapter 4, Section III).

IV. The Quantities of Mechanics in the Grid of the Six Measures

Any concept of an event of measurement involves six measures: mass, space, force, time, probability and possibility. They stand in a grid of three modal columns and two rows (*Comparison; The Necessary Basis*, Chapter 8; *The Basis of Physics*, Essay 4, Section III). The omniscient column is mass (content) in space (context). The immanent column is force (content) in time (context). The transcendent column is probability (content) in possibility (context).

Mass, in the combined sense of inertia, pattern, form and structure, and position, a point of space, are the omniscient-modal quantities of mechanics. Mass is what persists, what resists change, what a second measurement finds again. Position is where the pattern is: a relation of a content to its context, whose continuity is the continuity of space (*Measurement*, Chapter 9, Section VI). Together they answer what is there, and where: the what and the where of the six questions.

Force and time are the immanent-modal quantities. Force is the coupling by which one thing engages another, the direction and intensity of the flow of change; *An Immanent Metaphysics* names that flow as an intrinsic of interaction, an immanent modal content. Time is the medium through which change proceeds, before and after, which cannot be made the same; it is the irreversibility of change, an immanent modal context (*The Basis of Physics*, Essay 4, Section IV). Together they answer how and when.

Velocity and momentum are not further measures. A velocity is a change of position in time: a relation between the context of the omniscient column and the context of the immanent column, and it requires a comparison of positions at different times. Momentum is a pattern, the mass, carried through space in time, and force is the rate at which momentum changes in time (*Measurement*, Chapter 7, Section I). Energy, likewise, is no seventh measure: the energy of a configuration is a relation of pattern in space across possibility, and the energy of motion is expressed through momentum, force and time (*Measurement*, Chapter 7, Section I).

Probability and possibility are the transcendent-modal quantities: the weighting of outcomes, and the space of outcomes against which an actual one counts as one. Classical mechanics leaves this column implicit. Physics as ordinarily practiced works the first two columns and handles the third by probability alone, with possibility left as the space over which the probabilities are defined (*The Basis of Physics*, Essay 4, Section IV). In Newtonian mechanics probability enters only through ignorance of an initial state that is exactly specified in the theory.

On the Standing of the Sorting

The sorting of the quantities of classical mechanics into the three columns is not an analogy. The grid is the one that *Comparison; The Necessary Basis* argues (Chapter 8) and that *The Basis of Physics* closes through the six intrinsics of interaction (Essay 4, Section IV); its columns are the columns of causality, change and choice, which *An Immanent Metaphysics* gives as omniscient, immanent and transcendent (aphorism 1.85-4). What this chapter adds is the placement of the quantities of mechanics in the grid. Mass and position are placed by definition, as a content and its context in the omniscient column; force and time likewise in the immanent column. Velocity, momentum and energy are placed as relations across columns, by their definitions in terms of the six. Force is not a transcendent-modal quantity: what could happen, the disposition of a system, belongs to the transcendent column only as probability in possibility, and force, the flow of change, is the immanent content.

V. Axiom I in Classical Mechanics

Axiom I reads: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another." In classical mechanics the first sentence appears as the primacy of interaction.

Every mass and every position in mechanics is established through an interaction. Mass is found as inertia: as the response of a body to a force acting over time. A position is found by a signal that reaches the measurer: light, which in the frame of the six measures is content, a carrier of force (*Measurement*, Chapter 7, Section IV). There is no process of establishing existence, of a body or of its place, that does not presuppose an interaction (*Incommensuration*, Chapter 9, Section I). Force in time, the immanent column, is the ground from which the omniscient column is laid out.

The second sentence of Axiom I appears in the conjugate pairs of mechanics. In Hamiltonian mechanics a position and its momentum are canonically conjugate. The corpus reads the form of that pair (a description by position and a description by wavenumber, the inverse of the wavelength, to which momentum is proportional, related by a Fourier transform, so that the sharper the one, the broader the other) as the conjugacy of the omniscient and the transcendent (*Incommensuration*, Chapter 15, Section I). The pair is not a conjugacy of the omniscient and the immanent: Axiom I names one conjugate pair, and the immanent is not in it. Classical mechanics specifies both members of the pair exactly at every instant; that joint specification is part of total access, the assumption that symmetry and continuity hold together exactly in what is found (*Measurement*, Chapter 5, Section II), and quantum mechanics, by bounding it, made the conjugacy visible.

Galileo's principle of relativity (that the laws of mechanics are the same in every inertial frame) is a symmetry of the formalism of Newtonian mechanics. Under it, position and velocity are relative to a frame, while acceleration, and so force, is the same in every inertial frame. *Measurement* identifies Galilean invariance, with the symplectic structure of phase space, as the instrumental mode of Newtonian mechanics, and that mode is symmetric (*Measurement*, Chapter 8, Section IV, Table 8.1).

VI. The Three Aspects of the Universe

The concept "universe" is not a container but the triple of existence, interaction and creation: if one knew everything about all three, nothing in the universe would be left out (*Incommensuration; The Theorem and the Three Axioms*, "Each Axiom Derived Again", Section I). Every further candidate for an aspect of the universe (space, time, possibility, law, matter, force, energy) reduces to one of the three or to a relation among them (*The Basis of Physics*, "Domain Constants and the Span of Objectivity", Section VI).

Existence is the omniscient-modal aspect. It corresponds to pattern, to structure, to the question "what is the case?" applied at the maximal scale. Existence is the totality of what is: the inventory of all that can be known in principle, laid out as a completed whole. In the measures of physics it is matter arranged in space: the omniscient column, mass in space. It admits of hierarchy, ordering, and structural specification. It is the aspect that can be seen all at once, from the outside, as if from above.

Interaction is the immanent-modal aspect. It corresponds to process, to relation, to the question "what is happening?" applied at the maximal scale. Interaction is the foundational concept of the EP: comparison = interaction = relation. In the measures of physics it is forces unfolding across time: the immanent column, force in time. It is the fact that things relate to one another: the irreducible connectivity that makes comparison possible in the first place. Interaction is not something that objects do after

they exist. It is an aspect of their being at all. Existence without interaction is structurally incoherent: a universe of objects that have no relation to one another is not a universe but a contradiction.

Creation is the transcendent-modal aspect. Creation corresponds to potential, to the coming-into-being of the new, and to the question "what could be?" applied at the maximal scale. In the measures of physics it is potentiality over the space of what is possible, of which probability is a proxy: the transcendent column, probability in possibility. Creation is not a past event, and it is not itself a process; it is an aspect of every process (*An Immanent Metaphysics*, aphorism 1.73-14). Creation is the ever-present ground of novelty: the aspect of reality by which something new can emerge. The transcendent resists individualization, rejects hierarchy, and exceeds structural specification. Creation cannot be fully described by the omniscient methods of science, because it is the aspect of reality that exceeds description.

VII. The Standing of the Correspondences

The three grains give three correspondences between the structure of comparison and the content of physics: at the microscopic limit, the re-projection of the six intrinsics as kinds of particle; at the mesoscopic scale, the grid of the six measures; at the macroscopic limit, the triple of existence, interaction and creation. Of the three, the microscopic correspondence is the one on which particular observations bear.

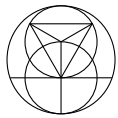
The observations that bear on it are the six quarks in a grid of two by three, the six leptons, the three colors, and the absence of a fourth generation. The principle of re-projection is stated without reference to those observations, and they bear on it as tests.

VIII. Summary

I have set out four points at the three grains of a domain of comparison, and the standing of the three correspondences. Section II gives the bound that the asymptotic freedom of color sets on the number of generations in the Standard Model, at most eight, and the measured count of three. Section III states that the six measures, the six questions and the six kinds of quark share one grid because measurement, inquiry and matter as found are faces of comparison. Sections IV and V place the quantities of classical mechanics in the grid of the six measures and show the two sentences of Axiom I at work in mechanics. Section VI says what existence, interaction and creation are, as the three aspects of the universe. Section VII states the standing of the three correspondences: the observations bearing on the microscopic one are the six quarks in a grid of two by three, the six leptons, the three colors and the absence of a fourth generation, and they bear on it as tests of a principle stated without reference to them. Each point holds at the scope of the grid of the six measures and the three axioms, as the books named in each section fix them.

Key Concepts

- Bound on generations: the asymptotic freedom of color allows at most sixteen quark flavors, so at most eight generations; the width of the Z boson counts three.
- One grid, three faces: measures, questions and quarks share the grid of two by three because each is a face of comparison.
- Mechanics in the grid: mass and position omniscient; force and time immanent; velocity, momentum and energy relations across columns; probability enters Newtonian mechanics only through ignorance.
- Axiom I in mechanics: the primacy of interaction, and the conjugacy of position with its Fourier conjugate, which is a conjugacy of the omniscient and the transcendent, not of the omniscient and the immanent.
- The three aspects of the universe: existence (omniscient), interaction (immanent), creation (transcendent).



Chapter 8

The Sizes of the Three Contexts

The Formula $T = I^2 \times O$, the Content Side of $E = mc^2$, and the Two Infinities

ABSTRACT

The three contexts of the six measures, time (immanent), space (omniscient) and possibility (transcendent), stand in a strict order of size, on two counts kept apart. Counted as dimensions of context, time has one, space three and possibility six. Counted as kinds of size, a single event of finding has no extent but an orientation, space has finitely many directions, and the states that possibility counts form a set of the size of the continuum. The finite, the countably infinite and the uncountably infinite are a modal triple of kinds of magnitude: omniscient, immanent and transcendent. The structural formula $T = I^2 \times O$ records which kinds of factor the accounting of possibility includes: the omniscient structure once, with geometric time inside it, and two times that are not structure, the time of findings and the time of evolution. The formula is an accounting of kinds of factor, not an equation between numbers of dimensions. On the content side the same form does not appear: energy is no seventh measure, and $E = mc^2$ states that rest mass and energy are one currency, converted by the square of c, the bridge between space and time. Cantor's theorem holds for every set; the claim of the Immanent Metaphysics concerns soundness, not validity: the finite, the countable and the continuum are

> three ontological size-classes, and the cardinalities beyond the continuum are valid results of iterated totalization that name no further ontological kind.

I. The Dimensional Ordering

Consider the three modalities not as abstract roles but as the three contexts of the six measures: time, the immanent context; space, the omniscient context; and possibility, the transcendent context (*Comparison; The Necessary Basis*, Chapter 8). Each context can be sized in two ways, and the two ways must be kept apart. The first of the two ways of sizing a context is the count of the dimensions that the context has as a context: the number of independent ways in which findings can differ in that context. The second of the two ways of sizing a context is the kind of size of what the context holds.

Time (the immanent context) has the least size on both counts. As a context it has exactly one dimension: from one stance, any two findings stand in one relation of order, one earlier and the other later (*Measurement*, Chapter 11, Section V). A single event of finding, the act that distinguishes before from after, has no extent at all. It does not stretch. It divides. I call the size of the single event zero-plus-epsilon: not quite a point, because the single event has a direction, an irreversibility; but not a line, because the single event does not persist or stretch. The immanent is the relationship between the other two, and a relationship is not an extension. The immanent is a cut, a junction, a contact surface.

Space (the omniscient context) has three dimensions (*Measurement*, Chapter 11, Section VI); spacetime, space with time, has four. Weinstein's metric bundle, the bundle over spacetime whose fiber at each point is the space of metrics there, the arena of his Geometric Unity, adds ten dimensions of metric components to the four of spacetime, for a total of fourteen; that total counts the components of a geometry, not the directions of

space. Both numbers are finite. The omniscient is the domain of the knowable, the measurable, the structural, and structure is articulated through a finite number of independent directions.

Possibility (the transcendent context) is sized differently on the two counts. As a context it has six dimensions, one for each of the six intrinsics, which are met only as what could be found and are never traversed (*Measurement*, Chapter 11, Sections VII and VIII); the six of possibility have two instances, as the internal space $\mathbb{C}P^2 \times S^2$ does (*The Grand Synthesis*, Chapter 10). As a count of states, possibility is infinite. The Planck constant is the grain of possibility: an action divided by h counts states (*Measurement*, Chapter 7, Section III). The states open to a quantum system are the unit rays of its Hilbert space. For a particle in space that Hilbert space has infinite dimension (it is separable: it has a countable orthonormal basis), and the set of its states has the cardinality of the continuum. Even a system with two states has continuum many pure states, the points of a two-sphere. Possibility always exceeds actuality.

The ordering is strict on both counts. As dimensions of context: time (one) < space (three) < possibility (six). As kinds of size: the single event (no extent, an orientation) < space (finitely many directions) < the states of possibility (continuum many). On the second count each step is a difference in the *kind* of size, not only in its amount.

II. The Three Kinds of Magnitude

The second count brings in three categorically different kinds of magnitude, which are themselves a modal triple. This triple is a triple of magnitudes, and it is not the triple of contexts of Section I: the two triples are set side by side, and neither is derived from the other.

The *finite* is omniscient in character: determinate, countable, structural. A finite set can be fully enumerated, fully known, fully described. The finite is the kind of magnitude that persists, that can be measured, and that admits complete specification.

The *countably infinite* is immanent in character: processual, iterable, sequential. The natural numbers are generated by the process of succession: always one more, always continuing. The countable infinity is the infinity of process, of iteration, of time-like extension. It can never be completed, but each step is definite.

The *uncountably infinite* is transcendent in character: exceeding any enumeration, inexhaustible by any sequential process, the magnitude of the continuum, of possibility, of the totality of the subjective. The uncountably infinite exceeds iteration categorically, and is not reached by iteration. The uncountably infinite is the kind of magnitude that no list, however long, can capture.

This assignment agrees with the arithmetic triple of *Category Theory and the Immanent Metaphysics* (Chapter 9, Proposition 9.14 and Remark 9.14d), in which addition is immanent, multiplication omniscient and exponentiation transcendent: the countable is generated by succession, the repeated adding of one; the continuum has the cardinality of the exponential $2^{\mathbb{N}}$, the set of all functions from the natural numbers to two values.

For every way something can be finite, there are two ontological kinds of infinity: the countable and the uncountable. (Section VI states in what sense there are two kinds, and why this does not deny the larger cardinalities of set theory.) These two infinities are distinct, inseparable, and non-interchangeable, and they form a triple with the finite that satisfies Axiom III.

III. The Formula: $\mathbf{T = I^2 \times O}$

Physics uses the word "time" for three different things, and each corresponds to a modality. *Comparison; The Necessary Basis* (Chapter 6, Section V) names them: "Geometric time is time as one more coordinate of a structure laid out all at once", and it is omniscient; "Event time, the time of findings, is the order of actual acts of comparison", and it is immanent; "Evolution time is the time over which the range of outcomes that a pre-

pared setting allows develops, before any finding is made", and it is transcendent. *The Unquantized Arena* (Chapter 10) sets out the same three as three distinct mathematical objects.

Each of the three is a time, and time stands in the immanent column of the six measures: "Either way time is immanent" (*Comparison; The Necessary Basis*, Chapter 6, Section V). So each of the three times is immanent as a measure, and each holds its own position within the triple of times. Evolution time is the transcendent member of that triple, the time of possibility; as a time, it is the immanent *within* the transcendent. Distributed across the grid, the three time-like parameters stand as follows: one embedded in the omniscient (geometric time, part of the four-dimensional spacetime of general relativity), one standing alone as the bare immanent (the event of finding), and one embedded in the transcendent (evolution time, the parameter of the Schrödinger evolution).

Now perform the accounting. I identify the full transcendent context, possibility taken as a whole, as containing all of the omniscient structure (the spatial dimensions, the embedded geometric time, the metric degrees of freedom, everything that can be structurally specified) *together with* two further factors that are not structure: the time of findings and the time of evolution.

The formula is:

$$T = I \times I \times O = I^2 \times O$$

The formula is a structural formula of this chapter: it states which kinds of factor the accounting of possibility includes, the omniscient structure once and the immanent twice. The formula is not an equation between numbers of dimensions. The product of a space of finite dimension with two one-dimensional factors has finite dimension, two more than the space; so the two times do not by themselves carry a finite count into an infinite one. And the counts of *Measurement* (Chapter 11), one for time, three for space and six for possibility, do not satisfy the formula read either as a product or as a sum of dimensions. What passes from the finite

to the infinite is the count of states (Section I), and that count is the size of a space of functions, an exponential (Section II), not of a product of dimensions.

> **Why Two Factors of the Immanent**
>
> The appearance of two immanent factors rather than one follows from the triple of times. One time (geometric time, the time of general relativity) is already absorbed into the omniscient factor: it is time treated as one more dimension of a structure. The remaining two times are not structure: the event of finding and the parameter of evolution. Both are processual, both are relational, and both are needed to account for the dynamics of a system that evolves through possibility and is then found. *The Unquantized Arena* (Chapter 10) states that general relativity holds the first two times and quantum mechanics the last two, and that a single time carrying all three would need a pairing that is impossible by the Incommensuration Theorem. The square in the formula records that two of the three times stand outside the structure, and the exponent 2 is the count of those two times.

IV. The Content Side: $E = mc^2$

If the formula $T = I^2 \times O$ describes the accounting of possibility on the context side, one may ask whether the same form appears on the content side, in the content row of the six measures: pattern (mass), force and probability.

On the content side, the modal assignments are: pattern, whose measure is mass, is omniscient; force is immanent; probability is transcendent (*Comparison; The Necessary Basis*, Chapter 8). Energy is not among the six. *Measurement* (Chapter 7, Section I) states it plainly: "Energy is no seventh measure. It is a relation expressible in the six." A stone on a hilltop has energy because of what it could become (a stone at the valley floor, moving fast), and that energy is a relation of four cells at once: "the stone is a pat-

tern, its mass; the hill is a difference of place, in space; the pull that would bring it down is a force; and the two states, up and down, are two possibilities, of which one is realized; the energy of the stone on the hill is that relation of all four" (*Measurement*, Chapter 7, Section I).

So the parallel that would make $E = mc^2$ an instance of $T = I^2 \times O$ on the content side does not hold. The transcendent content is probability, which has no unit (*Measurement*, Chapter 7, Section I), and energy, which has a unit, is a relation across the cells, not the transcendent content.

What $E = mc^2$ does say in the grid is stated in *Measurement* (Chapter 7, Section I): "rest mass and energy are one currency: a body at rest carries energy $E = mc^2$". Mass, the measure of pattern, is omniscient content. The constant *c* is "the bridge between the omniscient and the immanent, between space and time" (*Measurement*, Chapter 7, Section IV): it turns a time into a distance. The relation $E = mc^2$ is the conversion of a pattern at rest into the currency of energy, and the conversion factor is the square of that bridge.

$$E = c^2 \times m$$

The formula $E = c^2 \times m$ is Einstein's relation between the inertia of a body and the content of energy of that body (Einstein 1905).

The square of *c* is fixed by the units of energy: energy is the work of a force over a distance, and its unit is a mass times a length squared divided by a time squared. The square in $E = mc^2$ and the square in $T = I^2 \times O$ resemble one another in form only. On the context side the two factors are two times, each an immanent measure. On the content side the factor is the square of a bridge between the omniscient and the immanent columns, which is not itself an immanent measure, and the quantity it yields, energy, is not the transcendent content.

The Squaring Pattern in Physics

Energy is quadratic in a velocity or in a field strength in more places than $E = mc^2$. Kinetic energy is $\frac{1}{2}mv^2$. The energy density of an electromagnetic field is $(\varepsilon_0 E^2 + B^2/\mu_0)/2$. The energy of a photon, $\hbar\omega$, is not of this kind: it is linear in the angular frequency ω. The quadratic forms follow from energy being work, a force acting over a distance: *Measurement* (Chapter 7, Section I) states that the energy of motion "is expressed through momentum, force and time". The squares are therefore a fact about the units and the dynamics of energy; they are not an instance of $T = I^2 \times O$.

V. The Two Infinities

The kinds of magnitude of Section II include the two categorically distinct kinds of infinity recognized in mathematics.

The countable infinity, $\aleph_0$, is the infinity of the natural numbers, of sequences, of processes that can be iterated without end. It is the infinity of time-like extension: always one more step, always continuing, never completing. In the physical domain, it is the kind of infinity of an unbounded count of discrete things: findings, interactions, the discrete states of a bound system taken one after another. As a kind of magnitude it is immanent (Section II): the infinity of succession. Its elements, like those of a finite set, are discrete: each can be individually distinguished, individually addressed, individually named by a number.

The uncountable infinity, the cardinality of the continuum, is the infinity of possibility. It exceeds any enumeration. No list, no sequence, no iterative process can exhaust it. In the physical domain, it is the kind of infinity of the set of states of a quantum system (Section I): the full range of what could be found. It is the transcendent infinity.

Below both stands the finite: the determinate, the fully specifiable, the omniscient in its most concrete form. And the triple of the finite, the countably infinite and the uncountably infinite satisfies the three condi-

tions of Axiom III. The three kinds of magnitude are distinct, since each kind is categorically different from the others. The three kinds are inseparable, since the countable infinity presupposes the finite, the continuum presupposes the countable, and the finite presupposes the process by which the finite is established, which is countable. The three kinds are non-interchangeable, since swapping any two of the kinds changes the mathematical structure. *Existence, Truth, and the Limits of Knowledge* (Chapter 6, Section VII) states the same triple as "exactly three ontological size-classes".

VI. On Higher Infinities and the Power Set

Standard mathematics, following Cantor, proves that for any set S the power set P(S), the set of all subsets of S, has strictly greater cardinality than S. The proof is the diagonal argument (Cantor 1891): for any function f from S to P(S), the subset D of those elements x of S with x not in f(x) is not in the range of f, so no function from S onto P(S) exists. The theorem holds for finite sets (a set of n elements has 2^n subsets, and $n < 2^n$) as it does for infinite ones.

In Zermelo-Fraenkel set theory with choice (ZFC), the power-set axiom applies to every set, so the operation can be iterated without limit. The iterated power sets of the natural numbers have the cardinalities $\beth_0 = \aleph_0$, $\beth_1 = 2^{\aleph_0}$ (the cardinality of the continuum), $\beth_2 = 2^{\beth_1}$ (the cardinality of the power set of the real line), and, with unions taken at limit stages, the sequence continues through every ordinal, each term strictly greater than those before it. The cardinalities of the iterated power sets of the natural numbers are the beth numbers. The aleph numbers $\aleph_1$, $\aleph_2$, ... are a different sequence: $\aleph_1$ is the least cardinal greater than $\aleph_0$. Whether $\beth_1 = \aleph_1$ is the continuum hypothesis, which ZFC neither refutes (Gödel 1940) nor proves (Cohen 1963). As theorems of ZFC, Cantor's theorem applies to the real line as it applies to the natural numbers, and the power set of the real line exists and is strictly larger than the real line. The argument

does not depend on the law of the excluded middle: Cantor's theorem holds in every non-trivial topos, and its structure is Lawvere's fixed-point theorem, which requires only a cartesian closed category (*Category Theory and the Immanent Metaphysics*, Chapter 17, Remark 17.2, and Chapter 21).

Nothing in the Immanent Metaphysics disputes these theorems. *Category Theory and the Immanent Metaphysics* (Chapter 9, Remark 9.14b) separates two parts of Cantor's work: "The first is the theorem that no set maps onto its power set. It holds for finite sets ($n < 2^n$) as well as infinite ones, and it states exactly that the base cannot enumerate the exponential. This book uses it (Chapter 5, Proposition 5.8). The second is the hierarchy of completed infinite cardinals, treated as objects that exist. That step treats projected totalities as completed things. The Immanent Metaphysics does not accept it, and this book does not use it." The claim of the Immanent Metaphysics about the transfinite is therefore a claim about which totalities exist as completed objects, an ontological claim. It is not a claim that any proof within ZFC fails.

The Incommensuration Theorem bears on every application of the diagonal argument, the first included, and in the same way. *Category Theory and the Immanent Metaphysics* (Chapter 17, Section 17.2) identifies Cantor's diagonal argument as an instance of the theorem: "enumeration is a symmetry operation (uniform, context-independent assignment), and exhaustive coverage of the continuum is a continuity condition"; "The conjunction is forbidden; the diagonal witness is the mechanism by which the prohibition manifests." Each application of the argument is one more instance of the same prohibition: the base cannot enumerate the exponential.

The applications differ in what they cross. The first application, from the countable to the continuum, crosses a boundary of kind: the finite and the countable are discrete, their elements individually specifiable, and the continuum is not (*Existence, Truth, and the Limits of Knowledge*, Chapter 6, Section VI). The second and later applications cross no further boundary of kind. In the words of that chapter, "The hierarchy of cardinalities is a

hierarchy of instances, not a hierarchy of types." Further applications "are formally valid within the totalized framework but do not establish new ontological types; they establish new cardinalities within the same ontological type (the continuous)."

Validity Without Soundness

In ZFC, the power-set axiom applies to *any* set, regardless of its topological character. A set is a set; membership is membership. The continuum is modeled as the set of real numbers, and its power set is a set of ZFC. The objection of the Immanent Metaphysics is not that any formal proof is invalid within its axiom system. It concerns the soundness of one commitment, totalization: the treatment of an infinite collection as a completed, surveyable whole. *Existence, Truth, and the Limits of Knowledge* (Chapter 6, Section VIII) states the position: "The diagonal argument is valid relative to that commitment. The ICT suggests that the commitment is not structurally sound. The disagreement is at the level of foundations, not at the level of formal proof." Of the cardinalities beyond the continuum, the same book (Chapter 16, Section I) says that the Immanent Metaphysics "does not deny that ZFC can prove the existence" of them; "These are valid formal results. The IM holds that they are valid without being sound".

The position of the Immanent Metaphysics shares one commitment with a tradition in the philosophy of mathematics: the refusal to treat an infinite collection as a completed totality. Brouwer's intuitionism rejected the actual infinite as a completed whole and the law of the excluded middle for infinite domains; Weyl's predicativism (*Das Kontinuum*, 1918) admitted only sets defined without quantifying over a totality to which the defined set belongs, and so did not admit the full power set of the natural numbers as a completed totality. Neither rejects Cantor's theorem itself, which holds intuitionistically (above). What the Immanent Metaphysics adds to this tradition is a structural reason for the refusal, grounded in the Incommensuration Theorem rather than in mathematical taste.

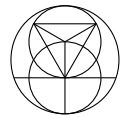
On the account of the Immanent Metaphysics, there are therefore exactly two ontological kinds of infinity: the countable and the continuum. The countable is immanent in character: the infinity of succession, whose elements are discrete. The continuum is transcendent in character: the infinity of inexhaustible possibility. These two, together with the finite, form the complete triple of size-classes. The cardinalities beyond the continuum are theorems of ZFC, and on this account they name no further kind: they are further instances within the continuous class.

VII. Summary

Section I sets out the ordering of the three contexts on two counts. Section II sets out the triple of the finite, the countable and the continuum as kinds of magnitude. Section III sets out the formula $T = I^2 \times O$ as an accounting of the factors of possibility. Section IV sets out the finding that $E = mc^2$ is not an instance of that formula, and what $E = mc^2$ states in the grid instead. Sections V and VI set out the two ontological kinds of infinity, with the place of the higher cardinalities of set theory. The formula holds as a structural accounting of kinds of factor, at the scope of the six measures and the triple of times. The claim about the transfinite holds at the scope of what exists as a completed object: it neither contradicts nor follows from any theorem of ZFC.

Key Concepts

- Dimensional ordering: time (one) < space (three) < possibility (six) as dimensions of context; the single event < finitely many directions < continuum many states as kinds of size.
- Zero-plus-epsilon: the size of a single event of finding, with an orientation and no extent.
- Three kinds of magnitude: the finite (omniscient), the countably infinite (immanent), the uncountably infinite (transcendent).
- $T = I^2 \times O$: possibility as accounted holds the omniscient structure once and two times that are not structure, the time of findings and the time of evolution.
- $E = mc^2$ in the grid: rest mass and energy are one currency, converted by the square of c; not an instance of $T = I^2 \times O$.
- Two ontological infinities: the countable and the continuum; the higher cardinalities are further instances within the continuous class.



Chapter 9

The Extended Bundle

The Off-Diagonal Metric, the Gauge Fiber, and a Construction of Twenty-Six Dimensions

ABSTRACT

In a given coordinate basis the six off-diagonal components of the four-by-four spacetime metric fall into two groups of three, time-space and space-space, a two-by-three array that this chapter identifies with the six intrinsics of comparison; the count six, the number of pairs among four directions, does not depend on the basis and equals the dimension of possibility. Adjoining to the fourteen-dimensional metric bundle a principal bundle with structure group $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, of dimension twelve, gives a total space of dimension $4 + 10 + 12 = 26$. The twelve are dimensions of transformation; the internal states the group moves among form $\mathbb{C}\mathrm{P}^2 \times \mathrm{S}^2$ on its content instance, a space of states with no size, of six real dimensions. Quantization applies to the content: the matter fields and their internal labels are quantized on a classical geometry, and the geometry itself is not quantized. The construction has three layers, classical geometry, gauge extension and quantization of the content, which this chapter identifies with the omniscient, the transcendent and the immanent. On it, the Shiab operator of Geometric Unity is asked to obtain the internal side and the transcendent content from a space that holds

> only geometry, and Haramein's holographic mass formula, once Newton's constant cancels from it, is the relation that the proton's radius is four of its own reduced Compton lengths.

Chapter 2 reads Eric Weinstein's Geometric Unity in the grid of the six measures. Its arena is the metric bundle of a four-dimensional spacetime, fourteen dimensions in all, in whose geometry it seeks the forces and the particles, and it has no ħ and no transcendent column. In this chapter I set out a construction that extends the metric bundle by the group of internal transformations of the Standard Model. I read both the metric bundle and the extended bundle in the grid of the six intrinsics and in the grid of the six measures. The construction is set beside the account of the corpus, in which the dimensions of context are ten: one of time, three of space and six of possibility (*Measurement; The Steps from Comparison to Physics*, Chapter 11).

I. The Off-Diagonal Metric as the Intrinsic Grid

The spacetime metric tensor $g_{\mu\nu}$ is a symmetric 4×4 matrix with 10 independent components. In a given coordinate basis these decompose into 4 diagonal components and 6 off-diagonal components.

The diagonal components g_{00}, g_{11}, g_{22}, g_{33} measure the *self-scale* of each coordinate direction: how much of each coordinate contributes to the invariant interval. They are magnitudes, not relationships. In the terms of the Immanent Metaphysics, this chapter identifies them as the measures of each direction's own presence: the answer to "how much," not "how related."

The 6 off-diagonal components measure how the coordinate directions *relate to each other*. They are the relational content of the metric in that basis: the angles, tilts, mixings, and frame-dragging effects that describe the pairwise geometry of the four directions. The components themselves

depend on the basis: at any one point a basis can be chosen in which all six vanish. What does not depend on the basis is their number, the number of pairs among four directions.

There are $C(4,2) = 6$ pairs of spacetime indices, and the six pairs fall into two groups of three:

Time-space (3 components):

g_{01}: time-x mixing

g_{02}: time-y mixing

g_{03}: time-z mixing

Space-space (3 components):

g_{12}: x-y relationship

g_{13}: x-z relationship

g_{23}: y-z relationship

The two groups of three off-diagonal components make a 2×3 structure. Two rows, three columns. The same shape as the intrinsic grid. *Measurement* (Chapter 11, Section VII) notes the same count from the side of context: "the dimension of possibility equals the number of ways in which two of the followable directions can be related to each other. This is an agreement, and the count of six does not depend on it."

The time-space components describe how time interacts with each spatial direction. In general relativity, these components encode gravitomagnetic effects: the dragging of inertial frames by moving mass, the coupling between temporal flow and spatial geometry. They are the *active* components of the metric: the ones that describe process, interaction, force. I identify the time-space components as the content row.

The space-space components describe how the three spatial directions relate to each other. They encode the shape of the spatial arena: whether the axes are orthogonal or tilted, whether the spatial sections are isotropic

or anisotropic. They are the *structural* components: the ones that describe pattern, form, background. I identify the space-space components as the context row.

Each row has three entries, one for each spatial direction. I label the three spatial directions by the three modalities. The labeling is an identification of this chapter; *Measurement* (Chapter 11, Section X) identifies the three directions of space with the three transitions of the cycle of Axiom II, each of which joins two of the three roles. A time-space component g_{0k} belongs to direction k. A space-space component g_{jk} joins two directions, and it is placed under the third, the direction that places the distinction between the other two (the placing rule of *Measurement*, Chapter 11, Section VI: "each of the three ways places exactly one of the three distinctions, the one between the other two"). With these placements the 2×3 structure of the off-diagonal metric is set against the 2×3 structure of the six intrinsics:

	Direction 1 (omniscient)	**Direction 2 (immanent)**	**Direction 3 (transcendent)**
Content (time-space)	g_{01} force/pattern	g_{02} force/interaction	g_{03} force/creation
Context (space-space)	g_{23} spatial structure	g_{13} spatial process	g_{12} spatial possibility

On this identification, the off-diagonal part of the metric tensor is the intrinsic grid of comparison in classical geometric form, written in the language of differential geometry.

On the Decomposition

The standard ADM (Arnowitt-Deser-Misner) decomposition of general relativity splits the metric into a lapse function N, a shift vector N_i (three components), and a spatial metric h_{ij} (six components), with $h_{ij} = g_{ij}$, $N_i = g_{0i}$, and $g_{00} = -N^2 + N_iN^i$. The time-space components g_{0i} are the *shift vector*: they describe how the spatial coordinates shift from one slice of time to the next. The off-diagonal components of the spatial metric, g_{ij} with $i \neq j$, are three of its six components, and in a given basis they describe the spatial anisotropy. The ADM decomposition is the standard framework for the canonical formulation of general relativity. So the separation of the time-space components from the spatial ones is already present in the standard mathematical framework of gravitational physics; the further division of the spatial metric into diagonal and off-diagonal parts is made in a coordinate basis, and the identification of the resulting 2×3 array with the intrinsics is this chapter's.

II. What the Metric Bundle Contains

Weinstein's metric bundle is a fiber bundle over the 4-dimensional spacetime manifold M. The fiber at each point is the space of all possible metric tensors at that point: the symmetric 4×4 matrices of the given signature, an open subset of the 10-dimensional space of symmetric 4×4 matrices. This fiber is 10-dimensional. The total space of the bundle is 4 + 10 = 14-dimensional.

On the identification of Section I, this 14-dimensional space holds:

4 base dimensions = spacetime (the immanent and omniscient contexts: 1 time + 3 space).

4 diagonal fiber dimensions = self-scales (the magnitudes of each coordin-

ate direction).

6 off-diagonal fiber dimensions = the six intrinsics in geometric form (the relational content of the metric in a given basis, organized as 2×3).

On that identification the metric bundle holds *what of comparison can be expressed in terms of geometry*. The six intrinsics are present, in geometric form, in the fiber of the metric bundle, and the spacetime arena is the base of the metric bundle. Weinstein's program attempts to derive the forces and particles from the geometry of this total space.

But the metric bundle is classical. Its fiber is a smooth manifold. It holds nothing of quantization, probability amplitudes, or the discreteness of quantum states. The six off-diagonal components are continuous real numbers. In the corpus the geometry stays so: the arena of space and time is the context of physics, and "To quantize the arena is to apply to the context the treatment that belongs to the content", which is a category error (*The Unquantized Arena*, Preface, Section I). What the metric bundle lacks is the transcendent column in its own form: possibility as the range of internal states a content can be in, and probability as the spread of a quantum state over them. That requires a structure of a different kind: one that holds the space of the *internal states of the content*, not the space of geometric configurations.

III. The Gauge Bundle and the Space of Internal States

The Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ is a 12-dimensional Lie group ($8 + 3 + 1$ generators). In the corpus it is not put in by hand: its three factors are the transformations that the three summands of the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ carry, $SU(3)$ from $M_3(\mathbb{C})$, $SU(2)$ from $\mathbb{H}$, and $U(1)$ from the phase (*The Story of the Particles*, Level 7). The gauge fields of the Standard Model are connections on a principal fiber bundle whose structure group is this 12-dimensional group. The fiber at each spacetime point is a copy of the group: a 12-dimensional manifold that parameterizes all internal symmetry transformations at that point.

What does this fiber represent physically?

A gauge transformation rotates the internal labels (color, weak isospin, hypercharge) without changing the physics. The twelve dimensions of the fiber are therefore twelve ways of transforming, and they are not twelve ways in which an internal state can differ. A transformation that leaves a given internal state unchanged moves it nowhere; the states that the group reaches from one state form the orbit of that state, the group divided by the transformations that fix the state. The internal states are the states of the two non-commutative carriers (*The Grand Synthesis*, Chapter 10): $\mathbb{C}P^2$, of real dimension four, the space of states of the color label, is the orbit $SU(3)/U(2)$; S^2, of real dimension two, the space of states of the two sides of a doublet, is the orbit $SU(2)/U(1)$; and the $U(1)$ of the phase multiplies a state by a phase and moves no state to another. Their product $\mathbb{C}P^2 \times S^2$ is the internal space of the corpus. The space of internal states has six real dimensions, not twelve.

$\mathbb{C}P^2 \times S^2$ has a content instance, "a space of states, with no size, no radius and no fields", and a context instance, the space of possibility, with radii (*The Measures of the Kinds*, Preface, as quoted in *The Grand Synthesis*, Chapter 10); in the words of that chapter, the six dimensions of possibility are "six of the ten dimensions of context", met only as what could be found (*Measurement*, Chapter 11, Sections VII and VIII). The space of internal states is $\mathbb{C}P^2 \times S^2$ on its content instance, the product of the two orbits; the gauge fiber is the group that acts on it. A point of $\mathbb{C}P^2 \times S^2$ specifies one internal state; the whole space is the range of internal states. The range of internal states is possibility on the content instance, met by its symmetry and not by its extent; possibility on the context instance is the space with radii, and the gauge group does not act on it.

The gauge *connection* (the field that specifies how the fiber is joined from point to point) is the gauge field. The curvature of the connection is the field strength, the physical force, and force is the immanent content of the six measures (*Comparison; The Necessary Basis*, Chapter 8). The transcendent content, probability, is carried by the quantum state: by the amp-

litudes that spread a state of the content over the possible outcomes (*The Unquantized Arena*, Chapter 10). The connection governs how the internal states are transported and changed from point to point; the quantum state gives each possible finding its probability.

IV. The Combined Bundle: 4 + 10 + 12 = 26

The extension this chapter proposes is the combination of the metric bundle and the gauge bundle over the same base manifold: the fiber product of the two bundles over M, whose fiber at each point is the product of the two fibers.

Base: 4 dimensions (spacetime)

Metric fiber: 10 dimensions (the symmetric 2-tensor)

Gauge fiber: 12 dimensions ($SU(3)\times SU(2)\times U(1)$)

Total: 26 dimensions

The parts of the total space can be grouped by the column of the six measures that this chapter assigns to each. This grouping is a second grouping, beside the one of Section I: Section I sets the six off-diagonal components in two rows and three directions, and the grouping here sets each part of the total space under the context it belongs to (time, space, or the internal side).

Column	Base	Fiber	Dimensions
Immanent	Time (1D base)	g_{0i} shift (3D fiber)	4
Omniscient	Space (3D base)	g_{ij}, $i \neq j$ (3D fiber) + 4 diagonal self-scales	10
Transcendent	none	Gauge group (12D fiber), acting on the space of internal states $\mathbb{C}P^2 \times S^2$ (6D)	12

The dimension count is not balanced across the columns, because the omniscient column carries the reflexive structure of the metric (the diagonal self-scales and the spatial off-diagonal components), while the transcendent column is the gauge fiber. The total is $4 + 10 + 12 = 26$.

This count of twenty-six is not the corpus's count of the dimensions of context. That count is ten: "one of time, three of space and six of possibility" (*Measurement*, Chapter 11), the four of spacetime with the six of the internal space $\mathbb{C}P^2 \times S^2$ (*The Grand Synthesis*, Chapter 10; *At the Edge of Reality*, Chapter 3). Besides the four of spacetime, the twenty-six count two other things: the ten components of a metric at a point, which are not dimensions of context, and the twelve parameters of the group of internal transformations, whose space of internal states has six dimensions (Section III).

Twenty-six is also the critical dimension of bosonic string theory (Lovelace 1971).

V. Quantization, and What It Applies To

The metric bundle is classical. The gauge bundle carries the internal labels of the content. The mathematical procedure that converts a classical phase space into a quantum theory can be stated geometrically, as *geometric quantization*.

Geometric quantization starts with a classical phase space equipped with a symplectic structure (a mathematical object that encodes the relationship between positions and momenta). It constructs a quantum Hilbert space by choosing a "polarization" (a choice of which variables are coordinates and which are momenta) and then building the space of quantum states from sections of a line bundle over the classical phase space.

General relativity has a canonical phase space of its own. In the ADM form, at each point of a spatial slice the six components of the spatial metric h_{ij} and their six conjugate momenta (built from the extrinsic curvature

of the slice) form a phase space of twelve dimensions; the lapse and the shift have no conjugate momenta, and they enter as multipliers that impose four constraints at each point. Quantizing this phase space would quantize the geometry itself. Loop quantum gravity does so, and it finds discrete spectra for area and volume (Rovelli and Smolin 1995).

The corpus does not take that step. *The Unquantized Arena* (Preface, Section I) states: "Quantum theory describes the content of physics", "General relativity describes the context of physics", and "To quantize the arena is to apply to the context the treatment that belongs to the content. That is a category error". "The arena has no least step of length, area or duration", and "What remains when the error is removed is quantum matter on a classical geometry". Among the programs of quantum gravity, "Loop quantum gravity quantizes the context directly" (*The Unquantized Arena*, Chapter 14, Section VI). So in this chapter's construction the metric bundle stays classical, and quantization applies to the content: the matter fields, with their internal labels, on the classical geometry.

In this chapter's terms, quantization is the mathematical procedure by which the transcendent column enters as content. Before quantization, the extended bundle holds geometry and the classical gauge fields: the structural and processual aspects of the arena and of the forces. After quantization of the content, its Hilbert space holds the space of possible states of the content (possibility) and the probability amplitudes that give each possible finding its probability (probability).

The internal labels of the quantum states (the three colors of $SU(3)$, the two components of $SU(2)$ weak isospin, the $U(1)$ hypercharge) do not come from quantizing the geometry. They come from the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, whose three summands carry the three transformations (*The Story of the Particles*, Level 7), and the gauge fiber is the group of those transformations.

The Incommensuration Theorem and Quantization

The Incommensuration Theorem states: "Symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity" (*The Grand Synthesis*, Chapter 6; proved in *Incommensuration; The Theorem and the Three Axioms*, Chapter 6). A comparison perfected at its foundation is therefore either continuously asymmetric or discontinuously symmetric. *The Unquantized Arena* (Preface, Section I) identifies the findings of general relativity as continuously asymmetric and the findings of quantum theory as discontinuously symmetric, and it reads quantization as a mode exchange: "in one step the findings of a description pass from continuously asymmetric to discontinuously symmetric, and the character its formalism holds exact against those findings passes from symmetry to continuity. A content admits the mode exchange"; "The arena admits none". So the quantum states of the content, with their definite, discrete labels, are the discontinuously symmetric survivor, and the smooth classical geometry of the arena is the continuously asymmetric survivor. Quantization of the content instantiates the theorem; quantization of the arena would ask for one of the two forbidden pairings.

VI. Fermions and Topology

In Kaluza-Klein-type constructions, the fermion content in four dimensions is sought in the topological properties of the internal space, rather than put in by hand.

In mathematics, the relationship between the geometry of a space and the zero-modes of a Dirac operator on it is governed by index theorems. The Atiyah-Singer index theorem states that the index of a Dirac operator on a compact manifold, the number of its zero-modes of one chirality minus the number of the other, is determined by the topology of the manifold and of the bundles involved: by global features that are invariant under smooth deformations. In Kaluza-Klein-type constructions, the chiral

fermions in four dimensions are sought as such zero-modes of a Dirac operator on the internal space, and their number of generations and their charges are then read from its topological invariants.

The corpus carries the fermion content by another route. The kinds of particle, their labels and their charges are derived from the algebra $\mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ and the structure of comparison (*The Story of the Particles*; *The Kinds of Particle*, Chapter 5). The number of generations is three by the rotation argument: "three positions admit three rotations of the content, and a fourth rotation is the first again. The geometry and the algebra of the state space take three identical copies as an input; neither produces the count" (*The Kinds of Particle*, Chapter 7). The re-projection thesis, that the fermions are the structure of comparison appearing as content at the microscopic limit, is carried by *The Story of the Particles* (Level 6). No index theorem on the bundle of this chapter is needed for the fermion content, and none is claimed here.

The internal space of the corpus is $\mathbb{C}P^2 \times S^2$, of six real dimensions (Section III). Its isometries are locally those of $SU(3) \times SU(2)$, eleven generators, and the $U(1)$ of hypercharge comes from the line bundle that spinors on $\mathbb{C}P^2$ need, since $\mathbb{C}P^2$ admits spinors only through a spin^c structure; so "The gauge group is not the isometry group of $\mathbb{C}P^2 \times S^2$" (*The Grand Synthesis*, Chapter 10, quoting *Non-Commutative Geometry and the Immanent Metaphysics*, Chapter 18). A compact twelve-dimensional internal manifold with isometry group $SU(3) \times SU(2) \times U(1)$ is therefore not required by the corpus, and this chapter does not seek one.

The Euler characteristic of the internal space is $\chi(\mathbb{C}P^2 \times S^2) = \chi(\mathbb{C}P^2) \times \chi(S^2) = 3 \times 2 = 6$ (*The Grand Synthesis*, Chapter 10). Six is the number of intrinsics of comparison. I record the equality as an agreement of two counts. *The Grand Synthesis* (Chapter 10) states the standing of such facts: "These are facts of the space; the counts of the particles rest on the structure of comparison".

On the Euler Characteristic

The Euler characteristic is a topological invariant, the alternating sum of the Betti numbers of a space; for a closed surface it falls by two with each added handle. For a sphere, $\chi = 2$. For a torus, $\chi = 0$. For a genus-2 surface, $\chi = -2$. For a product, the Euler characteristics multiply. In one setting of string theory, the heterotic string compactified on a Calabi-Yau threefold with the standard embedding of the gauge connection, the number of generations is $|\chi|/2$, so three generations require $|\chi| = 6$ (Candelas, Horowitz, Strominger and Witten 1985). That rule belongs to that setting. The internal space of the corpus is not a Calabi-Yau threefold, and its $\chi = 6$ is not used to count generations: the count of three rests on the rotation argument above.

VII. The Three Layers of the Construction

The construction proposed in this chapter has three layers, each corresponding to a level of mathematical structure.

Layer 1: Classical geometry. The metric bundle over the 4-dimensional manifold M. Total space: 14 dimensions. The metric bundle of Layer 1 is Weinstein's arena. It holds the geometry of spacetime, the structural and processual aspects of the arena. Gravity belongs to this layer, and it stays classical in every layer. On the identification of Section I, the six off-diagonal metric components are the six intrinsics in classical geometric form.

Layer 2: Gauge extension. The gauge fiber, the group $SU(3)\times SU(2)\times U(1)$, is adjoined to the metric bundle. Total space: 26 dimensions. This adds the internal side: the group of internal transformations, acting on the six-dimensional space of internal states $\mathbb{C}P^2 \times S^2$ (possibility, on the identification of Section III), and the connections, the gauge fields of the three forces. The three gauge forces belong to this layer.

Layer 3: Quantization of the content. The matter fields, with their internal labels, are quantized on the classical geometry of Layers 1 and 2. This introduces ħ. The Hilbert space of the quantized content is the mathematical home of the transcendent content (probability amplitudes). The metric is not quantized (*The Unquantized Arena*). The fermion content and its three generations are carried by the algebra and the structure of comparison (Section VI).

Each layer adds something that the previous layer lacks:

Layer 1 has geometry but no internal structure.

Layer 2 has internal structure but no quantization.

Layer 3 has quantized content, its discrete labels, and the probabilities of its findings.

I identify each of the three layers with one modality, by the aspect that the layer adds to the layer before it. Layer 3 also holds the transcendent content, probability (Section V); its identification with the immanent is by the act of finding that it adds. Layer 1 is omniscient in character: structural, geometric, the knowable framework. Layer 2 adds the transcendent: the range of internal states, what a content could be. Layer 3 adds the immanent: the act of finding, the selection of one outcome from the range of possibilities, with ħ as the bridge between the immanent and the transcendent (*Measurement*, Chapter 7, Section IV).

VIII. What Weinstein Was Missing

The diagnosis of Geometric Unity can now be stated in the terms of this construction.

Weinstein's 14-dimensional metric bundle is Layer 1. He attempts to derive the gauge forces and the fermion spectrum from the geometry of this 14-dimensional space alone. The Shiab operator is his proposed mechanism for projecting the 14-dimensional geometry down to 4-dimensional physics.

On this construction, the gauge forces need the gauge fiber (Layer 2), and the quantized content needs Layer 3. Neither is present in Layer 1. Weinstein's 14 dimensions are 12 short of the 26 of this chapter's construction.

I identify the task the Shiab operator is asked to do as the task of the gauge fiber and of the quantization of the content. That task is to obtain the internal side and the transcendent content from a space that holds only geometry. On that identification, the structure the operator needs is not present in the space it is defined on.

The extension this chapter proposes runs from 14 to 26 dimensions: from the metric bundle to the combined metric-gauge bundle. On this account, Weinstein's geometric program is incomplete in its arena, and the missing 12 dimensions are the gauge group, which acts on the six-dimensional space of internal states. The corpus states the arena of physics differently, as ten dimensions of context: the four of spacetime and the six of possibility (*Measurement*, Chapter 11).

IX. What Haramein Was Doing

Haramein's holographic mass formula can also be located within the three-layer construction.

The holographic formula writes the proton's mass as the Planck mass times twice the ratio of the number of Planck units on the proton's surface to the number of Planck units in its volume (Haramein 2013), and with the proton's radius r_p that ratio reduces the formula to $m_p = 4\, l_{Pl}\, M_{Pl} / r_p$. Since the product of the Planck length and the Planck mass is $\hbar/c$, Newton's constant cancels, and the formula is the relation $m_p\, r_p\, c/\hbar = 4$: the proton's radius is four of its own reduced Compton lengths. The relation uses $\hbar$, the bridge between the immanent and the transcendent columns, and c, the bridge between the omniscient and the

immanent (*Measurement*, Chapter 7, Section IV); it uses the classical geometric size of the proton together with the quantum of action. In the three-layer framework it is a relation between Layer 1 and Layer 3.

The Strengths of the Kinds (Chapter 6, Section VI) states this relation with its standing: "The relation $m_p r_p/(\hbar c) = 4$ is a value that lands: it agrees with measurement, and this book states no reason for it"; there m_p is the proton's rest energy, and with the CODATA 2022 radius the value is 3.9977 ± 0.0030. The formula does not obtain the proton's mass from geometry alone: it takes the measured radius as input. The proton's mass is a mass of binding, about ninety-nine percent of it the energy of the color field, and it is set by the strength of color (*The Strengths of the Kinds*, Chapter 6, Sections I and II), which belongs to Layer 2. So the relation states the proton's size in the unit of its mass, and it does not describe the internal quark structure that the gauge fiber carries.

X. Summary

I have set out a construction, section by section. Section I reads the off-diagonal metric as the grid of the six intrinsics. Section II sets out the metric bundle and what the metric bundle lacks. Section III sets out the gauge bundle and the six-dimensional space of internal states that the gauge bundle acts on. Section IV sets out the combined bundle of twenty-six dimensions. Section V applies quantization to the content alone. Section VI sets out the fermion content as carried by the algebra and not by an index theorem. Section VII sets out the three layers of the construction. Sections VIII and IX read the two programs through the construction. The identifications of the off-diagonal components with the intrinsics, of the spatial directions with the modalities, of the space of internal states with the transcendent context and of the layers with the modalities are identifications made in this chapter. The construction holds at the scope of a bundle writ-

ten over spacetime; the dimensions of context of the corpus are ten, the geometry stays classical, and the counts of the particles rest on the structure of comparison.

Key Concepts

- Off-diagonal metric as intrinsic grid: three time-space components (content row) and three space-space components (context row).
- Gauge fiber and internal states: the twelve-dimensional group acts on the six-dimensional space $\mathbb{C}P^2 \times S^2$, which is possibility on its content instance.
- The combined bundle: $4 + 10 + 12 = 26$, not the corpus's count of the dimensions of context, which is ten.
- Quantization of the content: the matter fields are quantized on a classical geometry, and the arena is not quantized.
- Three layers: classical geometry (omniscient), gauge extension (adds the transcendent), quantization of the content (adds the immanent).
- Geometric Unity read: its fourteen dimensions are the first layer only.
- Haramein read: $m_p r_p c/\hbar = 4$, a relation of the first and third layers that takes the measured radius as input.

Glossary

Each term is defined where it first appears in the text; each entry names the chapter where the reader first meets it.

ADM decomposition: the Arnowitt-Deser-Misner splitting of the metric into a lapse function, a shift vector and a spatial metric; the standard framework for the canonical formulation of general relativity. (Chapter 9)

Anchor of a mixing matrix: the frame from which its mixing starts before any correction: the identity for the quarks, the sameness frame of the three generations for the leptons. (Chapter 6)

Asymptotic freedom: the weakening of the strength of color at fine resolution, which holds for at most sixteen quark flavors. (Chapter 7)

Axiom I: "The immanent is more fundamental than the omniscient and/or the transcendent. The omniscient and the transcendent are in conjugate relationship to one another" (*An Immanent Metaphysics*, aphorism 1.3-1). (Chapter 1)

Axiom II: "A class of the transcendent will precede an instance of the immanent. A class of the immanent will precede an instance of the omniscient. A class of the omniscient will precede an instance of the transcendent". (Chapter 3)

Axiom III: "The (classes/instances) of the immanent, omniscient, and transcendent are distinct, inseparable, and non-interchangeable" (*An Immanent Metaphysics*, aphorism 1.3-1). (Chapter 1)

Beth numbers: the cardinalities of the iterated power sets of the natural numbers: $\beth_0 = \aleph_0$, $\beth_1 = 2^{\aleph_0}$, and so on. (Chapter 8)

Bridge: a constant that converts a measure of one column of the grid of the six measures into a measure of another: c between space and time (the omniscient and the immanent columns), ħ between time and possibility (the immanent and the transcendent columns). (Chapter 2)

Class of comparison: a set of states related to one another by the inner symmetries of the algebra. (Chapter 5)

Color state: the state of one quark in the space of color, a pure state of the matrix summand $M_3(\mathbb{C})$; the color states form the complex projective plane $\mathbb{C}P^2$. (Chapter 5)

Complete comparison in color: a state that holds one instance of each of the three positions, as a baryon holds one quark at each. (Chapter 5)

Compliance argument: every interaction is a comparison and the Epistemic Process characterizes comparison, so the Standard Model cannot not carry the structure the Epistemic Process describes. (Chapter 1)

Content and context: the two poles of the partition of every comparison: content, whose instances are separated from one another, and context, whose instances are not. (Chapter 1)

Content instance and context instance: the two instances of $\mathbb{C}P^2 \times S^2$: a space of states with no size, no radius and no fields; and the space of possibility, with radii. (Chapter 9)

Contexts of the six measures: time, space and possibility, the context row of the grid of the six measures, whose content row is force, mass and probability. (Chapter 8)

Continuity: sameness of content with sameness of context: reaching every case without a gap. (Chapter 1)

Corpus: the books of the physics collection on which this book rests. (Opening)

Count route and Koide route: the two routes to the neutrino masses that *The Measures of the Kinds* carries side by side as a fork. (Chapter 6)

Defining constants: the seven constants whose fixed values define the base units of the International System of Units since 2019. (Chapter 2)

Dimension of a context: the number of independent ways in which findings can differ in it. (Chapter 8)

Dirac operator: the operator of the spectral triple that measures difference between states. (Chapter 3)

Domain of comparison: a coherent region within which comparison can be made; it has a microscopic limit, a mesoscopic scale and a macroscopic limit, and its two limits are c, the macroscopic, and ħ, the microscopic. (Chapter 2)

Doublet's half: a = 1/2, the weak isospin of each side of the doublet. (Chapter 3)

Epistemic Process (EP): the account of knowing built on the structure of comparison, so named in *Comparison; The Necessary Basis*; its three components are the Incommensuration Theorem, the three axioms with the three modalities, and the three organizing principles, and the theorem is proved from the six intrinsics of comparison. (Chapter 1)

Euler characteristic: a topological invariant, the alternating sum of the Betti numbers of a space; for a product, the Euler characteristics multiply. (Chapter 9)

Existence, interaction, creation: the three aspects of the concept "universe": what is, laid out as structure (omniscient); the relating of things (immanent); the coming into being of the new (transcendent). (Chapter 7)

Focus of the two laws: the standpoint of *Non-Commutative Geometry and the Immanent Metaphysics*, in which $\mathbb{C}$ is omniscient, $\mathbb{H}$ immanent and $M_3(\mathbb{C})$ transcendent; a rotation of the frame of measurement. (Chapter 2)

Foundational triplication: every domain, at its foundation, exhibits three concepts that are distinct, inseparable and non-interchangeable. (Chapter 1)

Fourier conjugate: of a description by position, the description by wavenumber, related to it by a Fourier transform, so that the sharper the one, the broader the other. (Chapter 7)

Frame of measurement: the standpoint of *The Story of the Particles*, in which the comparing act is light; a frame is the set of units in which comparisons at a given scale report. In it SU(3) is labeled omniscient, SU(2) transcendent and U(1) immanent. (Chapter 2)

Frame of the positions of comparison: the frame in which the act of comparing is set beside the two things it relates: the fermion fields are the states of what is compared and of what it is compared against, and the gauge fields and the Higgs field carry the act. (Chapter 4)

Generation: one of the three instances of every particle of content, the same in kind and in charge and differing in mass. (Chapter 3)

Geometric quantization: the construction of a quantum Hilbert space from a classical phase space with a symplectic structure, by a choice of polarization and the sections of a line bundle. (Chapter 9)

Geometric, event and evolution time: time as one more coordinate of a structure laid out all at once (omniscient); the order of actual acts of comparison (immanent); the time over which the range of outcomes of a prepared setting develops before any finding (transcendent). (Chapter 8)

Grid of the six measures: mass (pattern), force and probability as content, and space, time and possibility as context, in three columns: the omniscient (mass in space), the immanent (force in time) and the transcendent (probability in possibility). (Chapter 2)

Ground truth: the Epistemic Process correctly characterizes the structural constraints on any act of comparison; it is not a claim of empirical verification. (Chapter 1)

Hierarchy of a sector: the ordering of its masses across its three generations. (Chapter 6)

Identification: a stated step from the structure of comparison to a physical quantity or feature, made with the measurement it meets. (Chapter 1)

Identity principle: "anything that cannot be distinguished, in any way at all, even in principle, is the same thing". (Chapter 4)

Immanent Metaphysics (IM): my philosophy, which sets out the structure of comparison, the source of the grid of the six measures. (Chapter 2)

Immanent method: the epistemic practice of engaging with a subject from the perspective of participation rather than observation; for a particle, asking "what sort of creature am I?". (Chapter 4)

Impossibility of self-coincidence: an instance of content cannot coincide, in one state, with another instance of its own kind, because two that coincided would be one. (Chapter 4)

Incommensuration Theorem (ICT): symmetry and continuity cannot both be applied, simultaneously and fundamentally, to any event or comparison; and neither can asymmetry and discontinuity. Of the ten pairings of the four compounds, exactly two hold, continuity with asymmetry and symmetry with discontinuity. It is a structural fact about the nature of knowing. (Chapter 1)

Kernel triangle: the right triangle with legs O = 3 and N = 4, hypotenuse H = 5 and inradius r = 1, fixed by $c_1 = 3$ and $n_C = 2$. (Chapter 3)

Knowing: carrying outer form into inner feeling; symmetric, repeatable and shareable; in theory and in the third person, the omniscient mode. (Chapter 4)

Labeling: an assignment of the three modalities to the three summands of the algebra or to the gauge factors, brought from comparison; a definition made within a frame, not a property the algebra carries. (Chapter 3)

Lattice computation: the numerical computation of quantum chromodynamics on a grid of points of space and time. (Chapter 5)

Measured agreement: a value the account reaches that measurement finds, carried with the question of why it lands. (Chapter 2)

Metric bundle: the bundle over spacetime whose fiber at each point is the space of all metric tensors at that point, of ten dimensions; with the four of spacetime, fourteen dimensions in all. (Chapter 2)

Mixing of a sector: the angles of the mixing matrix of its kind, quark or lepton. (Chapter 6)

Off-diagonal components: the six components $g_{\mu\nu}$ with $\mu \neq \nu$ of the metric in a coordinate basis: three time-space and three space-space. (Chapter 9)

Omniscient, immanent, transcendent: the three modalities. Chapter 3 states them as what is laid out as structure; the act of comparison itself; the stance from which an act proceeds, with the range of what is possible from it. Chapters 6 and 7 state them as the structural and knowable; the processual and relational; the potential and generative. (Chapter 1)

Organizing principles: the Root Tautology, foundational triplication and type isomorphism. (Chapter 1)

Phase charge: hypercharge before the Higgs condensate and electric charge, $Q = T_3 + Y$, after it: the charges of one row of the gauge group read on the two sides of the condensate. (Chapter 3)

Planck length and Planck mass: $l_P = \sqrt{(\hbar G/c^3)}$ and $m_P = \sqrt{(\hbar c/G)}$; their product is $\hbar/c$. (Chapter 2)

Positions of knowing: the known, the unknown, the unknowable, and the boundary between the unknown and the known, at which *The Story of the Particles* places the charged leptons, the up-type quarks, the neutrinos and the down-type quarks. (Chapter 6)

Principal bundle, gauge fiber: a bundle whose fiber at each point is a copy of a group, here SU(3)×SU(2)×U(1); its connection is the gauge field. (Chapter 9)

Quantum chromodynamics (QCD): the theory of the strong force, with gauge group SU(3) acting on color. (Chapter 5)

Re-projection: the principle that at the microscopic limit of a domain the form of what can be found is the form of finding itself, so that the six intrinsics are found as the six quarks and the six leptons. (Chapter 1)

Reduced Compton length: $\hbar/(mc)$, the length that a body's own mass sets through $\hbar$ and c; 0.2103 fm for the proton. (Chapter 2)

Root Tautology: interaction, comparison and relation are one concept, isomorphic in structure and nested in generality. (Chapter 1)

Rotation and mirror: a rotation moves every label one step round the cycle of Axiom II; a mirror exchanges two labels and keeps the third, reversing the orientation of the cycle. Within one frame, where the labels are ordered by the cycle of Axiom II, a mirror is an error; an exchange of two labels is admitted only as a flip mediated by Axiom I, which is theory, and a flip and a rotation do not happen at the same time. (Chapter 3)

Shiab operator: the device by which Geometric Unity proposes to project from fourteen dimensions to four. (Chapter 2)

Six intrinsics: the six necessary and sufficient features of every comparison: sameness and difference, content and context, the subjective and the objective. (Chapter 1)

Six questions: what, where, how, when, who and why, each asking for one of the six measures. (Chapter 7)

Standard Model (SM): the theory of the particles and of the strong, weak and electromagnetic forces, with gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. (Chapter 1)

Step cost: the number of halvings per generation on the mass ladder of a charged sector: 6 for the quarks and 9/4 for the charged leptons. (Chapter 3)

Symmetric and continuous observables: an observable is symmetric when every inner symmetry of the algebra leaves it unchanged, and continuous when it commutes with the Dirac operator. (Chapter 5)

Symmetry: sameness of content across difference of context. (Chapter 1)

$\mathbf{T = I^2 \times O}$: the structural formula stating that the accounting of possibility (T, the transcendent context) includes the omniscient structure (O) once and the immanent (I) twice; an accounting of kinds of factor, not an equation between numbers of dimensions. (Chapter 8)

Total access: the assumption, unstated by Newton, that symmetry and continuity hold together exactly in what is found. (Chapter 7)

Totalization: the treatment of an infinite collection as a completed, surveyable whole. (Chapter 8)

Type isomorphism: the pattern of relations among the foundational concepts of every domain is the same pattern. (Chapter 1)

Understanding: carrying inner feeling into outer form; continuous, requiring unbroken contact with the thing understood; as demonstrated by action, in the first person, immanent. (Chapter 4)

Validity and soundness: validity is the correctness of a derivation within its axioms; soundness is that the statement holds of what it is about. (Chapter 8)

Vertex: the point in a Feynman diagram where a fermion emits or absorbs a gauge boson. (Chapter 4)

Writing: an expression of a value in the measures of the kernel triangle; one value can have several writings. (Chapter 3)

Zero-plus-epsilon: the size of a single event of finding: not a point, because it has a direction, and not a line, because it does not persist or stretch. (Chapter 8)

ZFC: Zermelo-Fraenkel set theory with the axiom of choice. (Chapter 8)

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